1. Question | 0607 Feb-March 2025 P6 V2 Q1

Section A

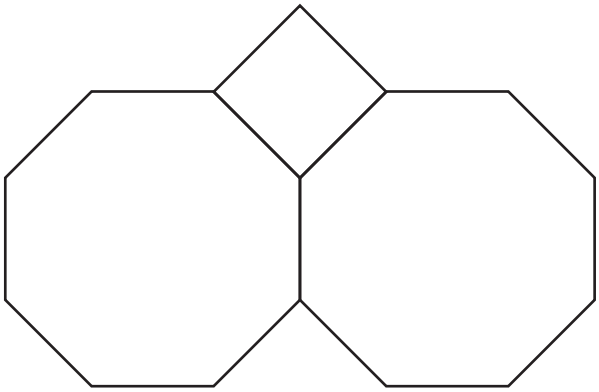
INVESTIGATION REGULAR POLYGONS

You are advised to spend no longer than 45 minutes on this section.

In this task you will investigate fitting 3 regular polygons together at a point.

All the polygons have sides of the same length.
The 3 polygons fit together exactly with one corner from each polygon meeting at a point with no gaps or overlaps.

1 The diagram shows a square and 2 regular octagons fitting together at a point.



This is the formula for the size of each interior angle, $a°$, of a regular polygon with n sides.

$$a = 180 - \frac{360}{n}$$

(a) Show that the size of each interior angle of a regular octagon is 135°.

[1]

(b) Use angles to show that 2 regular octagons and a square fit together at a point.

[2]

2. Question | 0607 Feb-March 2025 P6 V2 Q2

2 Use the formula in **Question 1** to complete the table.

Number of sides, n	5	6	8	9	10
Interior angle, $a°$			135		144

[3]

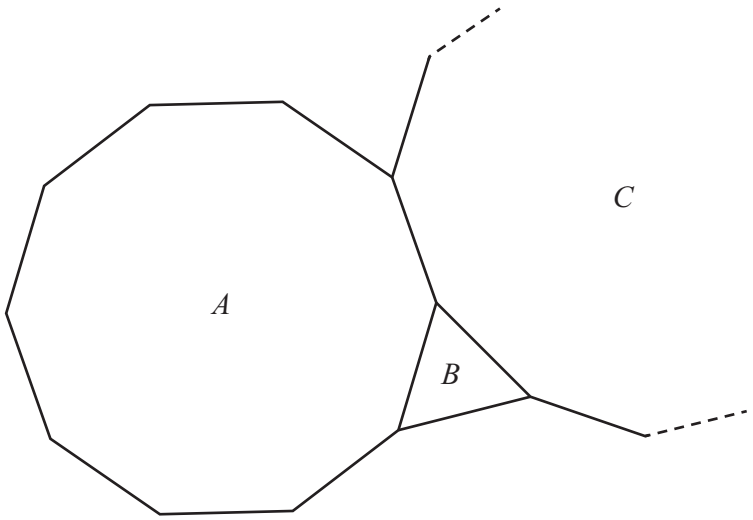
3. Question | 0607 Feb-March 2025 P6 V2 Q3

3 Rearrange the formula $a = 180 - \frac{360}{n}$ to show that $n = \frac{360}{180 - a}$.

[2]

4. Question | 0607 Feb-March 2025 P6 V2 Q4

4 The diagram shows regular polygons *A* and *B* and part of regular polygon *C*.



A is a regular 10-sided polygon.
B is an equilateral triangle.
C is a different regular polygon.

Work out the number of sides of polygon *C*.

..... [3]

5. Question | 0607 Feb-March 2025 P6 V2 Q5

5 Charlie finds other ways to fit 3 regular polygons together at a point.

The first polygon has p sides.
The second polygon has q sides.
The third polygon has r sides.

(a) Consider the angles around a point and use the formula $a = 180 - \frac{360}{n}$ to show that

$$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{1}{2} .$$

[3]

(b) $p \leqslant q \leqslant r$

(i) When $p = 6$, use $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = \frac{1}{2}$ to find the value of q and the value of r.

$q =$ .....

$r =$ .....

[1]

(ii) p cannot be greater than 6.

Give an example to show this.

[1]

6. Question | 0607 Feb-March 2025 P6 V2 Q6

6 In this question, the value of p is 3 and $p \leqslant q \leqslant r$.
Charlie wants to find all the possible ways to fit 3 regular polygons at a point.

(a) Use $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{1}{2}$ to show that $\frac{1}{q}+\frac{1}{r}=\frac{1}{6}$.

[1]

(b) q and r are positive integers.

Give a reason why $q \geqslant 7$.

[1]

(c) Complete the table.

Number of sides of first polygon, p	Number of sides of second polygon, q	Number of sides of third polygon, r
3	7	
3	8	24
3	9	
3	10	
3	12	12

[3]

7. Question | 0607 Feb-March 2025 P6 V2 Q7

7 Find all the possible ways to fit a square and 2 other regular polygons at a point.

..... [4]

8. Question part 1/2 | 0607 May-June 2025 P6 V1 Q1

Section A

INVESTIGATION PRODUCTS OF PAIRS

You are advised to spend no more than 45 minutes on this section.

In this investigation you will look at the differences between the products of pairs of numbers in an increasing linear sequence of positive numbers.

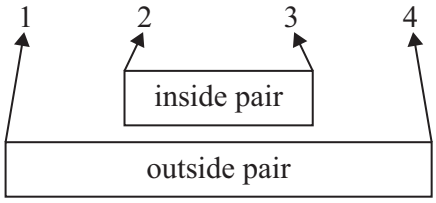
In each sequence:

- there are 3 or more terms
- the difference between each term is the *increase*
- the first term and the last term are the *outside pair*
- the terms next to the first and last terms are the *inside pair*.

To find the *product difference* multiply the numbers in each pair and find the difference.

Example

The sequence is



Product of inside pair $2 \times 3 = 6$
Product of outside pair $1 \times 4 = 4$

Product difference $6 - 4 = 2$

1 The sequences in this question have 4 terms.

(a) Complete the table.

Sequence	Increase	Product of		Product difference
		inside pair	outside pair	
1, 2, 3, 4	1	6	4	2
6, 8, ... , 12	.....	.....	72	.....
7.2, ... , ... , 16.2	.....	.....	116.64	18
..... , ... , 96, 100	.....	.....	.....	.....
..... , ... , ... , ...	5	6000	.....	.....

[5]

8. Question part 2/2 | 0607 May-June 2025 P6 V1 Q1

(b) (i) Complete the table.
Use your answers in **part (a)** to help you.

Sequences with 4 terms	
Increase, n	Product difference
1	2
2	
3	18
4	
5	
6	

[2]

(ii) Find an expression, in terms of n, for the product difference.

..... [2]

(iii) A sequence of 4 terms has an increase of 11.

Use your expression in **part (b)(ii)** to find the product difference for this sequence.

..... [2]

(iv) Use a sequence of numbers to show that your answer to **part (b)(iii)** is correct.

[1]

9. Question part 1/3 | 0607 May-June 2025 P6 V1 Q2

2 **(a)** A sequence has 3 terms.
The multiplication of the inside pair is the middle number squared.

Example
The sequence is 1 2 3

Product of inside pair $2 \times 2 = 2^2 = 4$
Product of outside pair $1 \times 3 = 3$

Product difference $4 - 3 = 1$

(i) Complete the table.
Use your own sequences to help you.

Sequences with 3 terms	
Increase, n	Product difference
1	1
2	
3	
4	

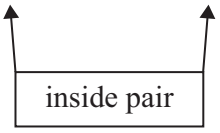
[2]

(ii) Write down an expression, in terms of n, for the product difference.

..... [1]

(b) A sequence has 5 terms.
The inside pair is the two terms that are one away from each end.

Example
The sequence is 1 2 3 4 5



Product of inside pair $2 \times 4 = 8$
Product of outside pair $1 \times 5 = 5$

Product difference $8 - 5 = 3$

(i) Complete the table.
Use your own sequences to help you.

Sequences with 5 terms	
Increase, n	Product difference
1	3
2	
3	
4	

[3]

(ii) A sequence with 5 terms has a product difference of 432.
An expression for the product difference is $3n^2$.

Find the increase for this sequence.

..... [1]

9. Question part 3/3 | 0607 May-June 2025 P6 V1 Q2

(c) **(i)** Complete the table.
Use your answers to **Question 1(b)(ii) and Question 2(a)(ii)** to help you.

Number of terms in sequence, x	Product difference
3	
4	
5	$3n^2$
6	

[1]

(ii) Find an expression, in terms of x and n, for the product difference.

..... [1]

(d) The number of terms in a sequence is equal to the increase.

Find a sequence with a product difference of 1859.
Write down the first and last terms of your sequence.

First term = ...

Last term = ...

[4]

10. Question | 0607 May-June 2025 P6 V2 Q1

Section A

INVESTIGATION THE CORE AND SHELL OF LINEAR SEQUENCES

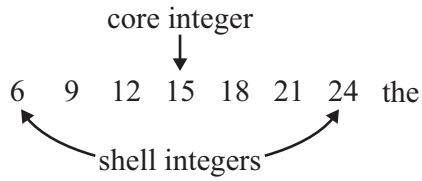
You are advised to spend no more than 45 minutes on this section.

In this investigation you will look at the difference between the values of the shell and the core in linear sequences.
The linear sequences have a constant positive difference between consecutive terms.

Each sequence in this investigation has an odd number of terms.
Each term is a positive integer.

The *core integer* is the integer in the middle of the sequence.
The *shell integers* are the integer at the beginning and the integer at the end of the sequence.

Example



For the sequence 6 9 12 15 18 21 24 the core integer is 15 and the shell integers are 6 and 24.

You calculate the *value of the core* by multiplying the core integer by itself. $15 \times 15 = 225$

You calculate the *value of the shell* by multiplying the shell integers together. $6 \times 24 = 144$

F is the positive difference between the value of the core and the value of the shell. $F = 225 - 144 = 81$

1 **(a)** Write down the core integer and the shell integers for the sequence

5 7 9 11 13 15 17 19 21.

Core integer .....

Shell integers ..... and .....
[1]

(b) Calculate F for the sequence in **part (a)**.

..... [2]

11. Question | 0607 May-June 2025 P6 V2 Q2

2 **(a)** Write down a linear sequence with 5 terms.

..... , ... , ... , ... , ... [1]

(b) For your sequence in **part (a)** find the value of F.
Write F as the square of a number.

..... [3]

12. Question | 0607 May-June 2025 P6 V2 Q3

3 The general linear sequence has 1st term a and common difference d.
So the first 3 terms of this sequence are $a,\ a+d,\ a+2d$.

(a) Write the 7th term of the general linear sequence in terms of a and d.

..... [1]

(b) A general linear sequence has 7 terms.

Find an expression for F.
Write your answer in the form $(kd)^2$ where k is a positive integer.

..... [3]

13. Question part 1/2 | 0607 May-June 2025 P6 V2 Q4

4 **(a)** Complete the table.

Linear sequence with common difference d			Number of terms n	F
1st term	Middle term	Last term		
a	$a+d$	$a+2d$		$(\quad)^2$
a	$a+2d$		5	$(\quad)^2$
a			7	$(\quad)^2$
a	$a+4d$	$a+8d$		$(\quad)^2$
a		$a+10d$		$(5d)^2$

[4]

13. Question part 2/2 | 0607 May-June 2025 P6 V2 Q4

(b) The 1st term of a linear sequence of positive integers is 1.
The value of F is 36.

Use the pattern in the F column in **part (a)** to help you find all the possible sequences.

[5]

14. Question | 0607 May-June 2025 P6 V2 Q5

5 **(a)** A linear sequence has n terms.
The common difference is d.

$F = 100d^2$

Find the number of terms in the sequence.

..... [2]

(b) Another linear sequence has n terms.
The common difference is d.

$F = d^4$.

(i) Find n in terms of d.

..... [2]

(ii) Give an example of a linear sequence with 11 terms where $F = d^4$.

..... [1]

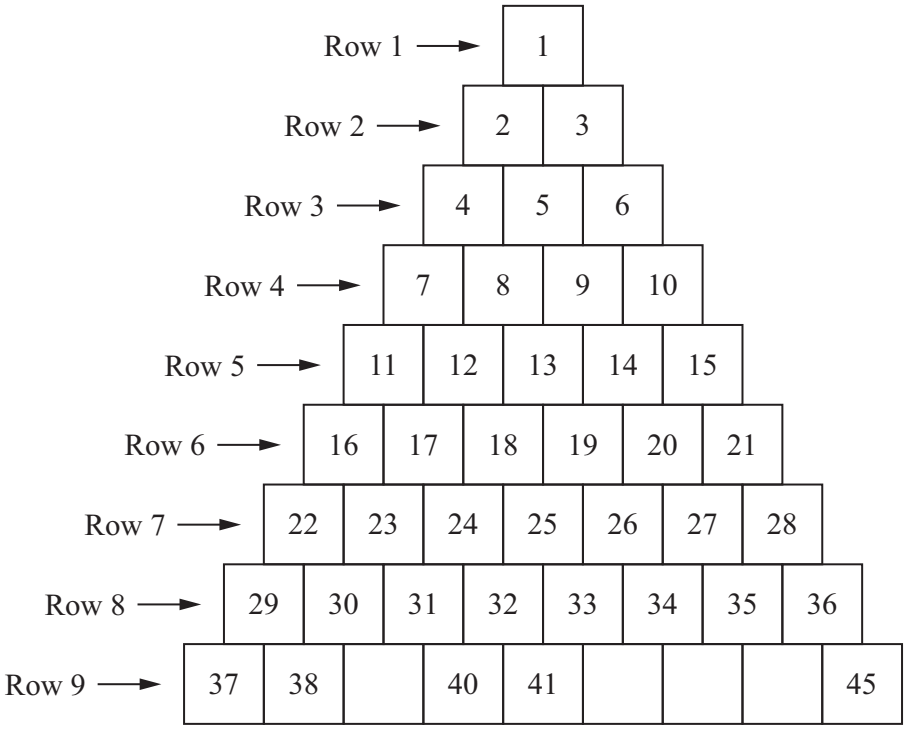
15. Question part 1/2 | 0607 May-June 2025 P6 V3 Q1

Section A

INVESTIGATION NUMBER DIAMONDS

You are advised to spend no more than 45 minutes on this section.

In this investigation you will look at the numbers in a pyramid of numbers.
The numbers are placed in order in rows.
Each row has one more number than the row above.



1 **(a)** Complete row 9 in the pyramid. [1]

(b) In row 4 the 2nd number is 8. We write N(4, 2) = 8.

In row 4 the 3rd number is 9. We write N(4, 3) = 9.

In row 4 the last number is 10. We write N(4, 4) = 10.

Complete these statements.

(i) N(5, ...) = 12 [1]

(ii) N(....., 4) = 25 [1]

15. Question part 2/2 | 0607 May-June 2025 P6 V3 Q1

(c) **(i)** Complete the table.
Use the pyramid of numbers and patterns to help you.

Row number (R)	Calculation	Last number in the row
1	1 × 2 =	1
2	2 × 3 =	3
3	3 × =	6
4	× =	
5	× =	
6	6 × 7 = 42	21

[2]

(ii) Find an expression in terms of R, for the last number in row R.

..... [1]

16. Question part 1/2 | 0607 May-June 2025 P6 V3 Q2

2 To find a number in the pyramid add its position in the row to the last number in the row above.

Example
N(6, 4) is in row 6.
The last number in row 5 is 15.
N(6, 4) is in position 4.
So N(6, 4) = 15 + 4 = 19.

(a) Complete the statement.

N(..... , ...) = ... + ... = 50 [1]

(b) Use your expression from **Question 1(c)(ii)** to show that

$$\mathrm{N}(R, k) = \frac{R(R-1)}{2} + k.$$

[1]

16. Question part 2/2 | 0607 May-June 2025 P6 V3 Q2

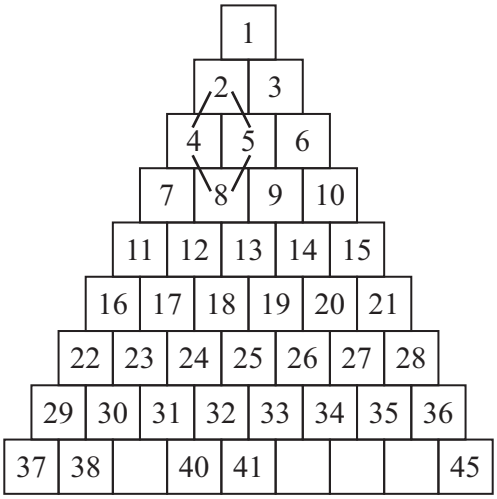
(c) $\mathrm{N}(R, 11) = 362$

Find the value of R.

..... [4]

17. Question part 1/3 | 0607 May-June 2025 P6 V3 Q3

3 Numbers in the pyramid can be joined to make a diamond as shown.
All diamonds in this investigation are similar.

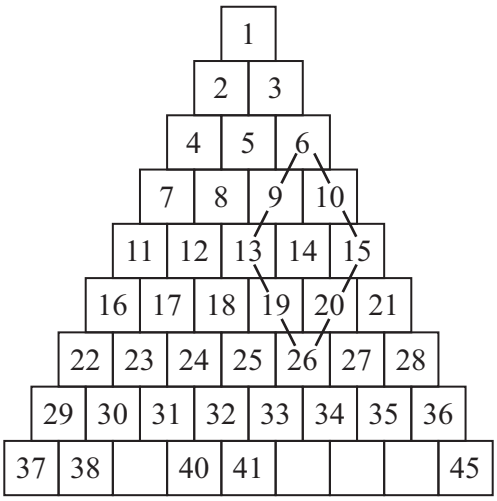


2, 4, 5 and 8 make a diamond.

The four numbers are always written in order from smallest to largest.

This diamond has width 1 because $5 - 4 = 1$.

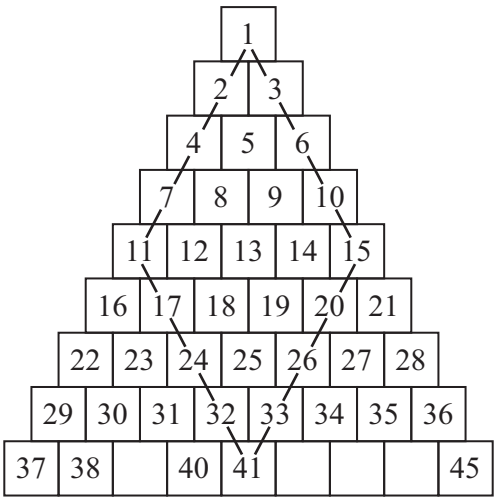
We write D(2, 4, 5, 8) has width 1.



6, 13, 15 and 26 make a diamond.

This diamond has width 2 because $15 - 13 = 2$.

We write D(6, 13, 15, 26) has width 2.

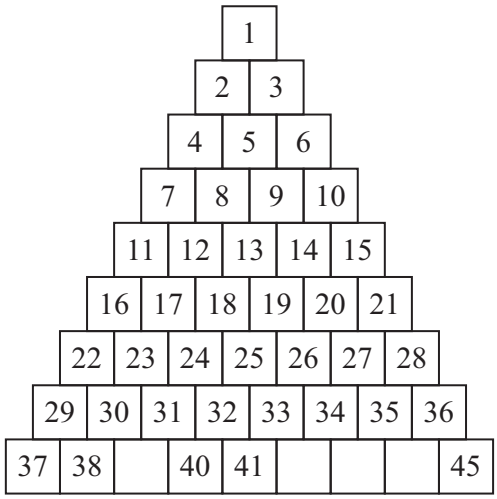


1, 11, 15 and 41 make a diamond.

D(1, 11, 15, 41) has width 4 because $15 - 11 = 4$.

17. Question part 2/3 | 0607 May-June 2025 P6 V3 Q3

(a)



Complete the statements.

D(..... , ... , 18 , 31) has width .....

D(..... , ... , 19 , ...) has width 3.

[3]

(b) A diamond is D(a, b, c, d).
a = N(p , p) and b = N($p+1$, p).

(i) Find c and d in terms of p.

c = N(....., ...)

d = N(....., ...)

[2]

17. Question part 3/3 | 0607 May-June 2025 P6 V3 Q3

(ii) Use the formula from **Question 2(b)** and your answers to **Question 3(b)(i)** to show that

$$b + c = p^2 + 3p + 1.$$

[3]

(iii) $b + c = 701$

Find the values of a, b, c and d.

$a =$ .....

$b =$ .....

$c =$ .....

$d =$ .....

[5]

18. Question part 1/2 | 0607 Oct-Nov 2025 P6 V1 Q1

Section A

INVESTIGATION SQUARE RINGS

You are advised to spend no more than 45 minutes on this section.

In this investigation, you will work with the numbers along the inside edges of two squares drawn on a number grid of width 10.

The diagram shows part of this number grid.
On the grid, a 4 by 4 square surrounds a 2 by 2 square.
The shaded area forms a 4 by 4 *square ring*.
A square ring is always one square thick.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80

The *ring value* is worked out in this way.

Step 1 Add the numbers along the inside edges of the large square to get L.
Step 2 Add the numbers along the inside edges of the small square to get S.
Step 3 Calculate $L - S$. This is the ring value.

Example

Step 1 $L = 3 + 4 + 5 + 6 + 16 + 26 + 36 + 35 + 34 + 33 + 23 + 13$
Step 2 $S = 14 + 15 + 25 + 24$
Step 3 Ring value $= L - S$

1 4 by 4 square rings

The example shows square ring 3.
This is because 3 is the top left number in the ring.

(a) Work out the ring value for the square ring in the example.

..... [2]

(b) Complete this table using **part (a)** and any patterns you notice.

Square ring (n)	L	S	Ring value $L - S$
1			
2	222	74	148
3			
4	246	82	164
5		86	

[2]

(c) Find an expression, in terms of n, for the ring value.

..... [3]

(d) Find the ring value for square ring 731.

..... [2]

19. Question part 1/2 | 0607 Oct-Nov 2025 P6 V1 Q2

2 6 by 6 square rings

On the grid, a 6 by 6 square surrounds a 4 by 4 square.
The square ring has width 6.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90

(a) Complete the working to find the ring value for square ring 1.

$L = 1+2+3+4+5+6+16+26+36+46+56+55+54+53+52+51+41+31+21+11$

$= 570$

$S = 12+13+14+15+25+35+45+44+43+42+32+22$

$= .....$

Ring value $= .....$

[1]

(b) Complete the table.

Square ring (n)	L	S	Ring value $L - S$
1	570		
2	590		236
3	610	366	
4	630	378	
5	650	390	260

[2]

19. Question part 2/2 | 0607 Oct-Nov 2025 P6 V1 Q2

(c) Find an expression, in terms of n, for the ring value.

..... [2]

(d) A square ring has a ring value of 1028.

Find the number in the bottom right square of this square ring.

..... [3]

20. Question | 0607 Oct-Nov 2025 P6 V1 Q3

3 *w* by *w* square rings

The width of a square ring is w.
For the square ring in **Question 1**, $w = 4$.
For the square ring in **Question 2**, $w = 6$.

For a w by w square ring, an expression for the ring value is $8n + kw - k$.
For an 8 by 8 square ring, an expression for the ring value is $8n + 308$.

Find the value of k.
Write down an expression, in terms of n and w, for the ring value of a w by w square ring.

$k =$ .....

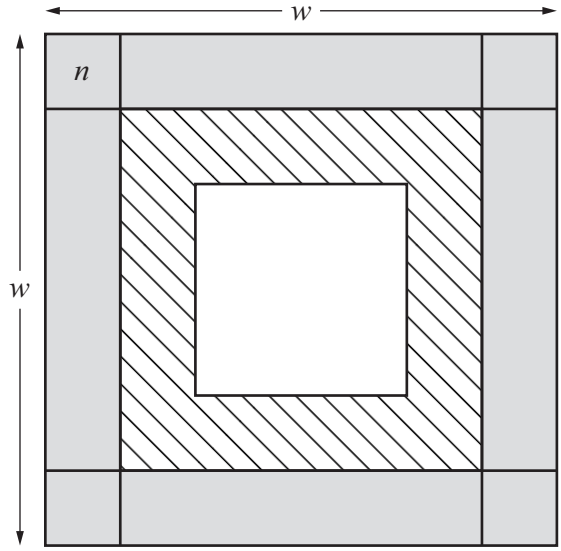
expression for the ring value .....

[3]

21. Question | 0607 Oct-Nov 2025 P6 V1 Q4

4 The diagram shows two square rings, shaded ▢ and ▨ .

The larger square ring has width w.
The number in the top left square is n.



Use algebra to find what is always true about the difference between the two ring values.

..... [5]

22. Question part 1/2 | 0607 Oct-Nov 2025 P6 V2 Q1

Section A

INVESTIGATION SQUARE PATTERNS

You are advised to spend no more than 45 minutes on this section.

In this investigation you will look at the number of lines used to draw square patterns.

| is a vertical line and — is a horizontal line.

1 This is Pattern 1.



There are 2 vertical lines.

There are 2 horizontal lines.

There are 4 lines in total.

This is Pattern 2.



There are 3 vertical lines.

There are 4 horizontal lines.

There are 7 lines in total.

This is Pattern 3.



(a) Complete the table.

Pattern number (n)	Number of vertical lines	Number of horizontal lines	Total number of lines
1	2	2	4
2	3	4	7
3			
4			
5			

[3]

22. Question part 2/2 | 0607 Oct-Nov 2025 P6 V2 Q1

(b) Find an expression, in terms of n, for the total number of lines in Pattern n.

..... [3]

(c) The total number of lines in a pattern is 754.

Work out the number of **horizontal** lines in this pattern.

..... [3]

23. Question part 1/4 | 0607 Oct-Nov 2025 P6 V2 Q2

2 The square patterns in **Question 1** make towers of squares.

This is Tower 1.

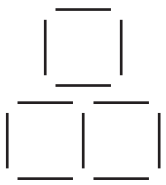


There are 2 vertical lines.

There are 2 horizontal lines.

There are 4 lines in total.

This is Tower 2.

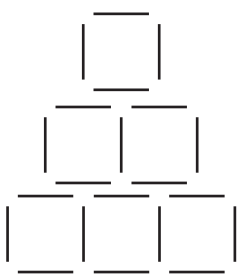


There are 5 vertical lines.

There are 6 horizontal lines.

There are 11 lines in total.

This is Tower 3.



(a) Complete the table.

Tower number (t)	Number of vertical lines	Number of horizontal lines	Total number of lines
1	2	2	4
2	5	6	11
3			
4			
5			

[4]

23. Question part 2/4 | 0607 Oct-Nov 2025 P6 V2 Q2

(b) Find an expression, in terms of t, for the number of **horizontal** lines in Tower t.

..... [2]

(c) This is an expression for the total number of lines in Tower t.

$$0.5t(3t+5)$$

Use this expression and your answer from **part (b)** to find an expression for the number of vertical lines in Tower t.
Give your answer in its simplest form.

..... [3]

(d) A tower has 902 vertical lines.

Find the total number of lines in the bottom row of squares in this tower.

..... [4]

23. Question part 3/4 | 0607 Oct-Nov 2025 P6 V2 Q2

(e) The total number of lines in Tower $(n+1)$ is 175 more than the total number of lines in Tower n.

Work out the value of n.

..... [3]

23. Question part 4/4 | 0607 Oct-Nov 2025 P6 V2 Q2

The modelling task starts on the next page.

Section A

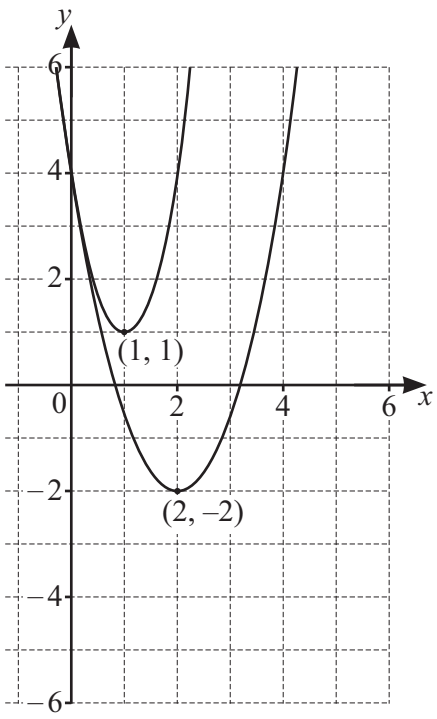
INVESTIGATION THE VERTEX OF THE QUADRATIC FUNCTION

You are advised to spend no more than 45 minutes on this section.

In this task, you will investigate the change in the vertex of the quadratic function $y = ax^2 + bx + c$ as a or b changes.

1 In this question, the value of a changes.
The value of b and the value of c stay the same.

(a)



The diagram shows the graphs of these functions.

$y = 1.5x^2 - 6x + 4$

$y = \quad 3x^2 - 6x + 4$

These functions are quadratic functions of the form $y = ax^2 + bx + c$ where $b = -6$ and $c = 4$.
The value of a changes.

The coordinates of the minimum points of the quadratic functions are shown on the diagram.
These points are the vertices of the quadratic functions.

24. Question part 2/4 | 0607 Oct-Nov 2025 P6 V3 Q1

(i) On the diagram, sketch the graph of $y = x^2 - 6x + 4$. [2]

(ii) The *line of vertices* is the line that joins the vertices of the quadratic graphs.

Find the equation of the line of vertices.

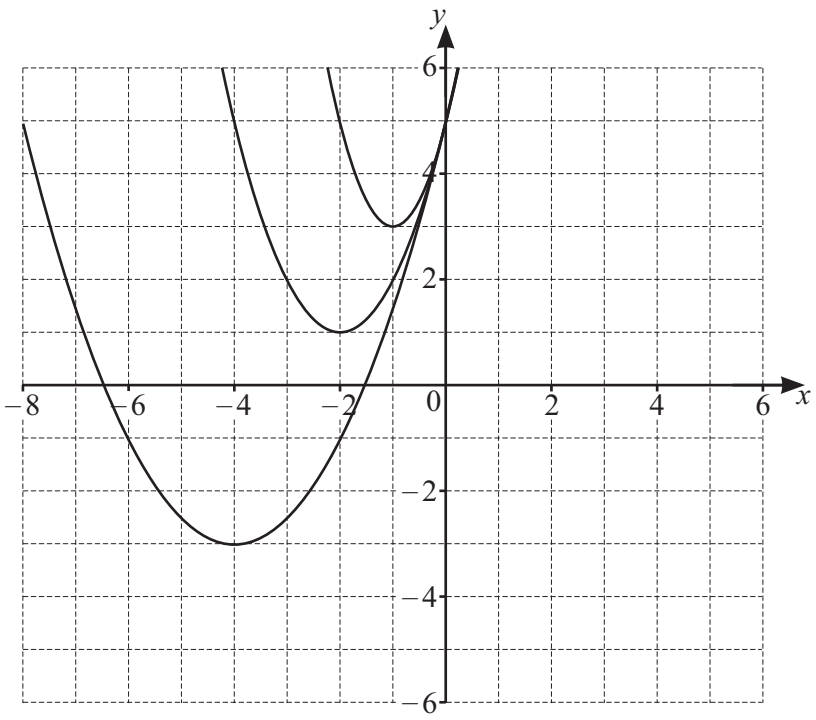
..... [3]

(iii) Does the vertex of $y = 0.5x^2 - 6x + 4$ lie on the line in **part (ii)**?

..... [3]

24. Question part 3/4 | 0607 Oct-Nov 2025 P6 V3 Q1

(b)



The diagram shows the graphs of these functions.

$y = 0.5x^2 + 4x + 5$

$y = \quad x^2 + 4x + 5$

$y = \quad 2x^2 + 4x + 5$

These functions are quadratic functions of the form $y = ax^2 + bx + c$ where $b = 4$ and $c = 5$.
The value of a changes.

Find the equation of the line of vertices.

..... [2]

24. Question part 4/4 | 0607 Oct-Nov 2025 P6 V3 Q1

(c) **(i)** Complete the table.
You may use any patterns you notice.

	Quadratic function	Equation of the line of vertices
part (a)(ii)	$y = ax^2 - 6x + 4$	
part (b)	$y = ax^2 + 4x + 5$	
	$y = ax^2 - 8x + 20$	$y = -4x + 20$
	$y = ax^2 - 12x + 30$	
	$y = ax^2$	$y = 5x - 2$

[3]

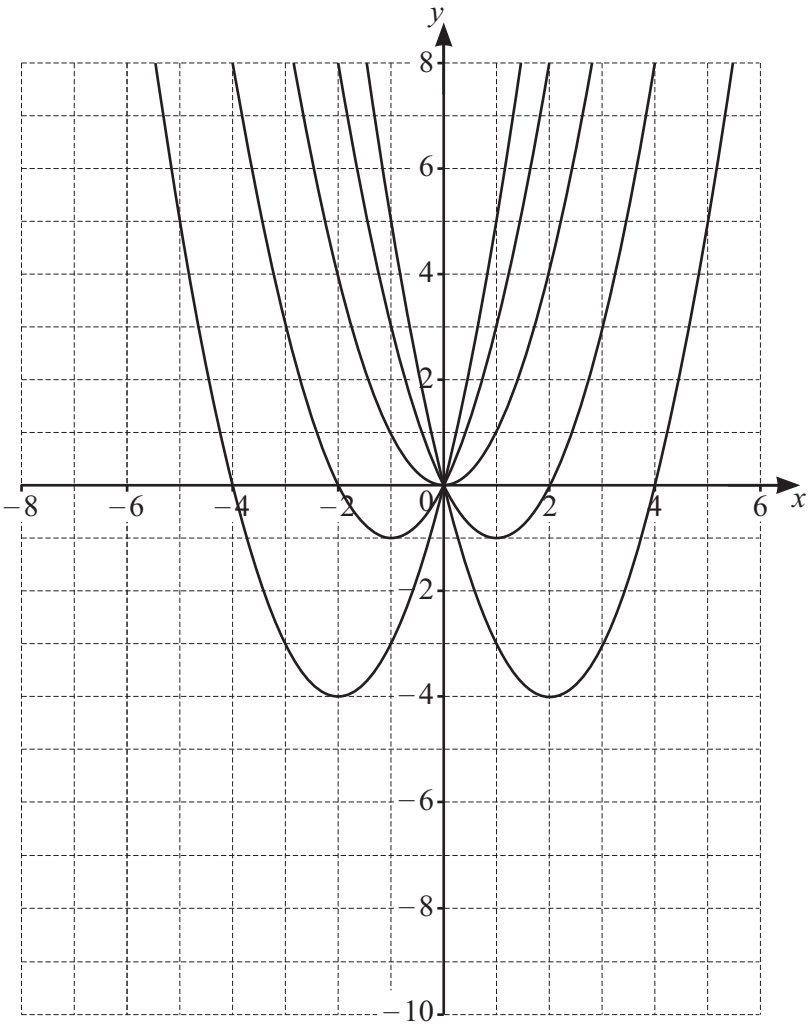
(ii) In quadratic functions of the form $y = ax^2 + bx + c$, the value of b and the value of c stay the same.
The value of a changes.

Complete the equation of the line of vertices.

$y =$ ..... $x +$ ..... [2]

2 In this question, the value of b changes.
The value of a and the value of c stay the same.

(a)



The diagram shows the graphs of these functions.

$y = x^2 + 4x$ $y = x^2 + 2x$ $y = x^2$ $y = x^2 - 2x$ $y = x^2 - 4x$

These functions are quadratic functions of the form $y = ax^2 + bx + c$ where $a = 1$ and $c = 0$.
The value of b changes.

(i) On the diagram, sketch the graphs of $y = x^2 + 6x$ and $y = x^2 - 6x$. [3]

(ii) Draw a smooth curve through the vertices of the 7 graphs. [1]

(iii) Find the equation of the smooth curve through the vertices.

..... [2]

(b) In this part, quadratic functions of the form $y = ax^2 + bx + c$ have $a = 2$ and $c = 0$.
The value of b changes.

Find the equation of the curve through the vertices.
You may use sketches on your calculator to help you.

..... [2]

(c) In this part, quadratic functions of the form $y = ax^2 + bx + c$ have constant non-zero values for a and c.
The value of b changes.

Find the equation of the curve through the vertices.

..... [2]

26. Question | 0607 Specimen 2025 P6 Q1

Section A
INVESTIGATION PICK'S FORMULA

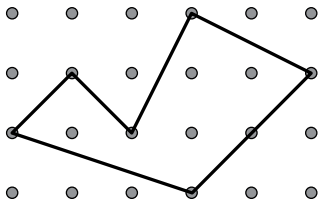
You are advised to spend no more than 45 minutes on this section.

This investigation is about working out the area of a polygon.

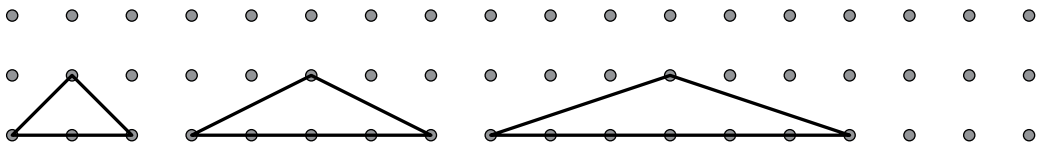
In 1899 the Austrian mathematician Georg Pick found a method to work out the area, A, of any polygon that has its vertices on a square grid.

His method used the number of dots, p, on the perimeter of the polygon and the number of dots, i, inside the polygon.

In the polygon shown, $p = 7$ and $i = 4$.



1 The diagram shows the first three triangles of a sequence with $i = 0$.



The first triangle in the sequence has $p = 4$.
Its area $A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 2 \times 1 = 1$ square.

(a) Complete the table for the first six triangles in this sequence.

	Triangle					
	first	second	third	fourth	fifth	sixth
A	1					
p	4	6				

[3]

(b) Find a formula for p in terms of A.

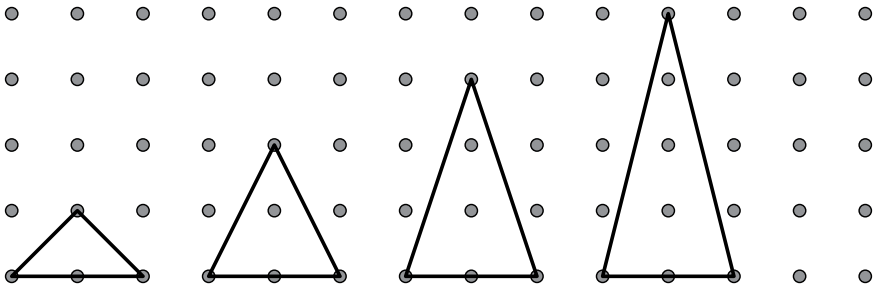
..... [3]

(c) Make A the subject of the formula.

..... [2]

27. Question | 0607 Specimen 2025 P6 Q2

2 The diagram shows the first three triangles of a sequence with $p = 4$.
The number of dots, i, inside the triangle, increases by one each time.



(a) The area of the first triangle is 1 square.

Find the area, A, of each of the other three triangles.

....., ..., ... [2]

(b) Explain how the connection between the increase in i and the increase in A changes your answer in **Question 1(c)** to give $A = \frac{1}{2}p + i - 1$.

.....

..... [1]

28. Question | 0607 Specimen 2025 P6 Q3

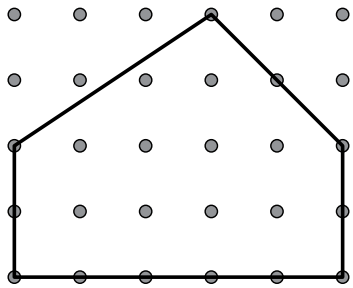
3 $A = \frac{1}{2}p + i - 1$ is *Pick's formula*, which works for any polygon.

Write down the smallest possible value for p.

..... [1]

29. Question | 0607 Specimen 2025 P6 Q4

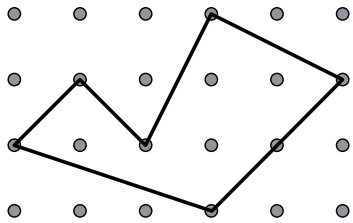
4 Show that Pick's formula gives the correct value for the area of this polygon.



[4]

30. Question | 0607 Specimen 2025 P6 Q5

5 Use Pick's formula to find the area of this polygon.



..... [2]

31. Question | 0607 Specimen 2025 P6 Q6

6 The area of a quadrilateral is 4 squares.

Find how many pairs of values of p and i are possible.

..... [4]

32. Question part 1/2 | 0607 Specimen 2025 P6 Q7

7 A polygon has an area of 104 squares.

It is given that the number of dots inside the polygon equals the square of the number of dots on the perimeter of the polygon.

Find if it is possible to draw this polygon.

[3]

32. Question part 2/2 | 0607 Specimen 2025 P6 Q7

Question 8 is printed on the next page.

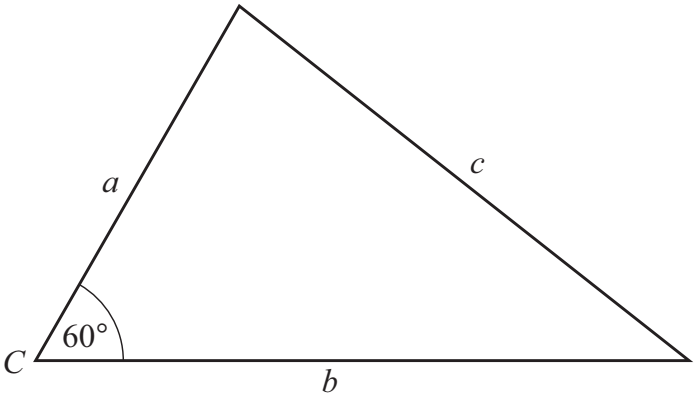
33. Question | 0607 Feb-March 2026 P6 V2 Q1

Section A

Investigation Eisenstein triples

You are advised to spend no more than 45 minutes on this section.

In this investigation you will look at the lengths of the sides of a triangle where one angle is 60°.



In the triangle:

- the length of the shortest side is a centimetres
- the length of the longest side is b centimetres
- the length of the side opposite the 60° angle is c centimetres.

When a, b and c are integers the sides of the triangle make an Eisenstein triple (a, b, c).

1 The cosine rule says that $c^2 = a^2 + b^2 - 2ab\cos C$.
For an Eisenstein triple angle $C = 60°$.

Give a reason why $c^2 = a^2 + b^2 - ab$ for an Eisenstein triple.

..... [1]

34. Question | 0607 Feb-March 2026 P6 V2 Q2

2 Use $c^2 = a^2 + b^2 - ab$ to complete the Eisenstein triple (9, 24, ...).

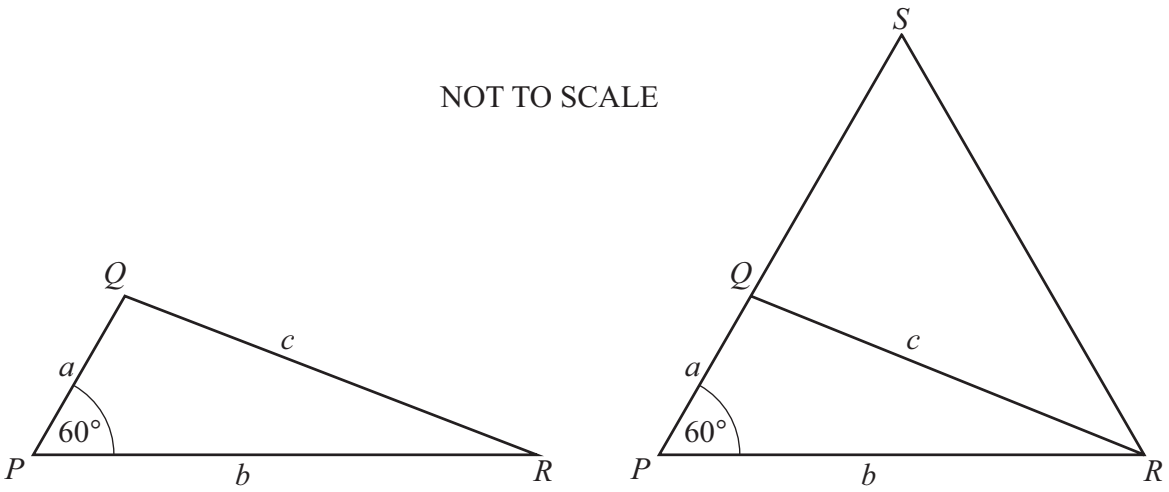
(9, 24, ...) [2]

35. Question | 0607 Feb-March 2026 P6 V2 Q3

3 The diagrams show how you can use one Eisenstein triple to find another related Eisenstein triple.

The sides of triangle PQR make the Eisenstein triple (a, b, c).

NOT TO SCALE



In the diagram on the right, PQ has been extended to S so that $PS = PR$.

(a) Give geometrical reasons why triangle QSR gives the Eisenstein triple $(b - a, \quad b, \quad c)$.

.....

.....

.....

..... [3]

(b) (9, 65, 61) is an Eisenstein triple.

Use the information in **3(a)** to write down the related Eisenstein triple.

(..... , ... , ...) [1]

(c) (32, ... , ...) and (45, ... , ...) are two Eisenstein triples that are related to each other.

Find the missing integers and complete the two Eisenstein triples.

(32, ... , ...) (45, ... , ...) [3]

36. Question part 1/2 | 0607 Feb-March 2026 P6 V2 Q4

4 The table shows the first 7 terms of a sequence of Eisenstein triples (a, b, c).

(a) Complete the table.
n is the term number in the sequence.

n	a	b	c
1	3	3	3
2	5	8	7
3	7	15	
4	9	24	21
5	11	35	
6			43
7			

[5]

36. Question part 2/2 | 0607 Feb-March 2026 P6 V2 Q4

(b) Use patterns in the table to find expressions for each of a, b and c in terms of n.

$a =$ .....

$b =$ .....

$c =$ .....

[5]

(c) Find two Eisenstein triples where the difference between b and c is 35.

(..... , ... , ...) and (..... , ... , ...) [5]

37. Question part 1/2 | 0607 May-June 2026 P6 V1 Q1

Section A

Investigation Frame sums

You are advised to spend no longer than 45 minutes on this section.

In this investigation you will look at calculations in a frame moving on a number grid.

The frame is made of 3 squares.
The frame moves on a number grid.

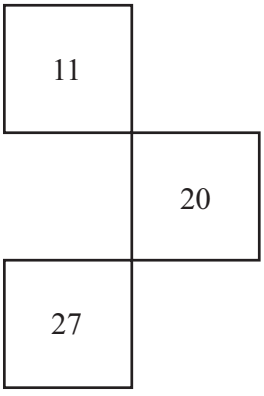
Consecutive whole numbers fill a grid of width 8.
The grid continues downwards.

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40

To find the frame sum for any frame:

- square the number on the right
- multiply the two numbers on the left
- subtract the second answer from the first answer.

Example



$20^2 = 400$

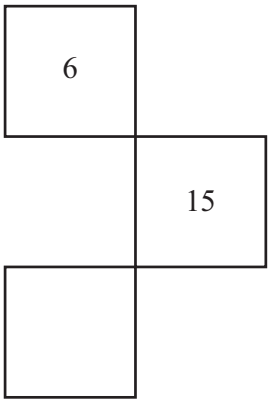
$11 \times 27 = 297$

$400 - 297 = 103$

frame sum $= 103$

37. Question part 2/2 | 0607 May-June 2026 P6 V1 Q1

1 **(a)** Use the grid to complete the frame and find the frame sum.



15^2 = ...

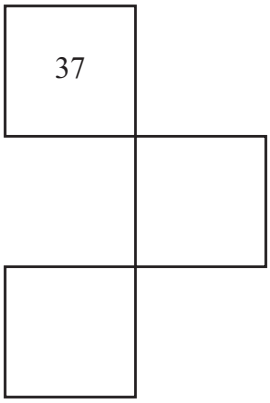
6 × ..... = ...

..... − ..... = ...

frame sum = ...

[2]

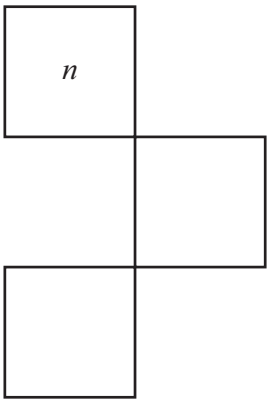
(b) Find the frame sum for this frame.



..... [2]

38. Question part 1/2 | 0607 May-June 2026 P6 V1 Q2

2 The diagram shows the frame on the same grid.
The number in the top square is n.



(a) Complete the other two squares with expressions in terms of n. [2]

(b) Use algebra to show that an expression for the frame sum is $2n + 81$.

[3]

(c) The frame sum is 197.

Find the number in the top square of the frame.

..... [2]

38. Question part 2/2 | 0607 May-June 2026 P6 V1 Q2

(d) The number in the bottom square of the frame is 223.

Find the frame sum.

..... [2]

39. Question part 1/2 | 0607 May-June 2026 P6 V1 Q3

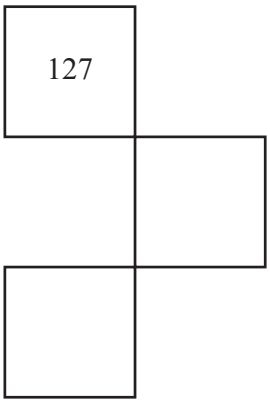
3 The numbers in the grid now increase by a different amount.

In this grid:

- the numbers start at 11
- the numbers increase by 4
- the width of the grid is 7 squares
- the grid continues downwards.

11	15	19	23	27	31	35
39	43	47	51	55	59	63
67	71	75	79	83	87	91
95	99	103	107	111	115	119

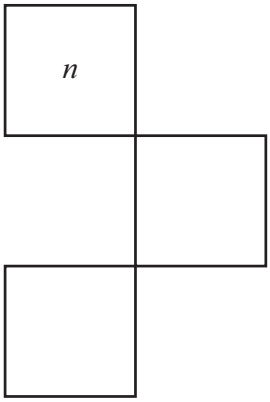
(a) Find the frame sum for this frame.



..... [2]

39. Question part 2/2 | 0607 May-June 2026 P6 V1 Q3

(b) The number in the top square is n.



Find an expression for the frame sum.
Do not simplify your answer.

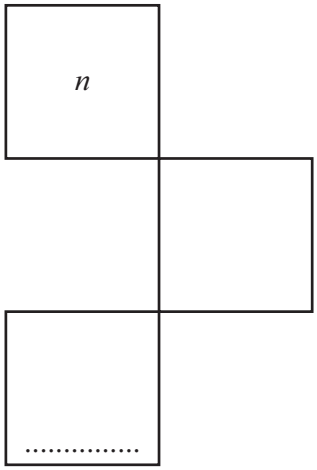
..... [3]

40. Question part 1/3 | 0607 May-June 2026 P6 V1 Q4

4 In another grid:

- the numbers increase by d
- the width of the grid is w squares
- the grid continues downwards.

(a) The number in the top square is n.

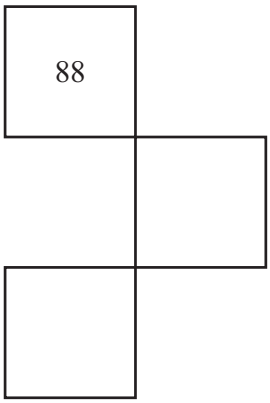


Complete the bottom square with an expression in terms of n, d and w. [1]

40. Question part 2/3 | 0607 May-June 2026 P6 V1 Q4

(b) The formula for the frame sum on this grid is

$$\text{frame sum} = 2dn + d^2(w+1)^2.$$



The frame sum for this frame is 1280.
All the numbers in the grid are positive integers.

(i) Show that $w = \dfrac{\sqrt{1280 - 176d}}{d} - 1$.

[3]

(ii) Complete the table.
Give non-integer values correct to 2 decimal places.

d	***w***
1	32.23
2	
3	
4	
5	
6	1.49
7	

[2]

(iii) Write down all the possible widths of the grid.

..... [1]

41. Question part 1/3 | 0607 May-June 2026 P6 V2 Q1

Section A

Investigation Nim

You are advised to spend no more than 45 minutes on this section.

This investigation looks at the minimum number of moves it takes to win the game of Nim.
Nim is a game for two players using a pile of sticks.

In this game, you and your friend take turns to remove sticks from the pile.
A player wins when they take the last stick.

1 In each move, a player takes 1 or 2 sticks from the pile.

There are 3 sticks in the pile.
Your friend starts the game.

Your friend takes 1 stick from the pile, leaving 2.
You win by taking the remaining 2 sticks.

The table shows these moves.

move	number of sticks taken	number of sticks left in the pile	result
		3	
1	friend takes 1	2	
2	you take 2	0	you win

There is another way for you to win the game when your friend starts.
This time your friend takes 2 sticks from the pile.

move	number of sticks taken	number of sticks left in the pile	result
		3	
1	friend takes 2	1	
2	you take 1	0	you win

41. Question part 2/3 | 0607 May-June 2026 P6 V2 Q1

(a) There are 6 sticks in the pile.
Your friend starts the game.

You should win when there are 3 sticks left after your move.

Complete the tables to show two different ways that you win.

move	number of sticks taken	number of sticks left in the pile	result
		6	
1	friend takes 1	.....	
2	you take	3	
3	friend takes 1	.....	
4	you take	0	you win

move	number of sticks taken	number of sticks left in the pile	result
		6	
1	friend takes 2	.....	
2	you take	3	
3	friend takes 2	.....	
4	you take	0	you win

[2]

(b) Your friend starts the game.
You should win when there are 3, 6 or 9 sticks in the pile.

(i) In a game there are 9 sticks in the pile.
It is your friend's move.

Give a reason why you should win.

..... [1]

(ii) In a game there are 20 sticks in the pile.
It is your move.

Write down the number of sticks you should take to make sure you win.

..... [1]

41. Question part 3/3 | 0607 May-June 2026 P6 V2 Q1

(c) There are 18 sticks at the start of the game.
You always take the number of sticks so that you are sure to win.

Complete the table.

move	number of sticks taken	number of sticks left in the pile	result
		18	
1	friend takes 1	17	
2	you take	.....	
3	friend takes 2	.....	
4	you take	12	
5	friend takes 2	.....	
6	you take	9	
7	friend takes 1	.....	
8	you take	.....	
9	friend takes 2	.....	
10	you take	.....	
11	friend takes 1	.....	
12	you take	0	you win

[2]

42. Question | 0607 May-June 2026 P6 V2 Q2

2 In different games of Nim, a player can take different numbers of sticks from the pile.
You win a game of Nim after your friend goes first.

(a) Complete the table.

number of sticks that can be taken in each move	**sequence of possible number of sticks in the pile at the start**				***n*th term of the sequence**
1 or 2	3	6	9	12	
1, 2 or 3	4	8	12	.....	
1, 2, 3 or 4	5	.....	.....	.....	
1, 2, 3, 4, or 5	.....	.....	.....	.....	
1, 2, 3, …. or k					

[3]

(b) The number of sticks in the pile at the start is 30.
In each game, the greatest number of sticks you can take in each move is k.

Find all the different possible values of k.

..... [4]

43. Question | 0607 May-June 2026 P6 V2 Q3

3 Now **you** always start the game.
In each move, a player takes 1 or 2 sticks from the pile.
You always take the number of sticks so that you are sure to win.

Example

There are 5 sticks in the pile at the start.

move	number of sticks taken	number of sticks left in the pile	result
		5	
1	you take 2	3	
2	friend takes 2	1	
3	you take 1	0	you win

There are now 19 sticks in the pile at the start.

Give a reason why you should take 1 stick in your first move.

..... [1]

44. Question part 1/2 | 0607 May-June 2026 P6 V2 Q4

4 You always start the game.
In each move, a player now takes 1, 2 or 3 sticks from the pile.
You always take the number of sticks so that you are sure to win.

In your first move you always take 1 stick so that you are sure to win.
The possible number of sticks in the pile at the start form this sequence.

1 5 9 13 17 ...

(a) Find the nth term for this sequence.

..... [2]

(b) When there is 1 stick in the pile at the start there is 1 move to win.

Complete the total number of moves to win.

number of sticks at the start (p)	1	5	9	13	17
total number of moves to win (m)	1				

You may use the table below to help you.

move	**number of sticks taken**	**number of sticks left in the pile**	**number of sticks left in the pile**
	you take 1		
	friend takes		
	you take		
	friend takes		
	you take		
	friend takes		
	you take		
	friend takes		
	you take		
	friend takes		
	you take		

[3]

(c) Find a formula for the total number of moves, m, in terms of the number of sticks in the pile at the start, p.

..... [2]

45. Question | 0607 May-June 2026 P6 V2 Q5

5 In a game, there are 50 sticks in the pile at the start.
You start the game and you win.

In each move a player can take up to k sticks from the pile, $1 < k < 7$.
You always take the number of sticks so that you are sure to win.

Complete the table.
If you are not sure to win, write X in the cell for the number of sticks you take on the first move.

value of k	**number of sticks you take on the first move**	**total number of moves to win**
2		
3		

[4]

46. Question part 1/2 | 0607 May-June 2026 P6 V3 Q1

Section A

Investigation Factors and multiples

You are advised to spend no more than 45 minutes on this section.

In this investigation you will look at the connection between factors and multiples of numbers.

A factor of a number divides into the number without leaving a remainder.
A multiple of a number is in that number's times table.

Examples

Factors of 10 are 1, 2, 5 and 10

$10 \div 1 = 10$ remainder 0 $10 \div 2 = 5$ remainder 0
$10 \div 5 = 2$ remainder 0 $10 \div 10 = 1$ remainder 0

Any other number leaves a remainder $10 \div 3 = 3$ remainder 1 so 3 is not a factor of 10

Multiples of 7 are 7, 14, 21, 28 and so on

$7 \times 1 = 7$ $7 \times 2 = 14$ $7 \times 3 = 21$ $7 \times 4 = 28$

1 **(a)** Complete this table.

number	factors	first six multiples
1	1	1, 2, 3, 4, 5, 6
2	1, 2	2, 4, 6, 8, 10, 12
3	1, 3	3, 6, 9, 12, 15, 18
4	1, 2, 4	4, 8, 12, 16, 20, 24
5	1, 5	5, 10, 15, 20, 25, 30
6	1, 2, 3, 6	6, 12, 18, 24, 30, 36
7	1, 7	7, 14, 21, 28, 35, 42
8		
9		9, 18, 27, 36, 45, 54
10	1, 2, 5, 10	10, 20, 30, 40, 50, 60

[2]

46. Question part 2/2 | 0607 May-June 2026 P6 V3 Q1

(b) The highest number that is both a factor of 4 and a factor of 10 is called the highest common factor (HCF) of 4 and 10.

The lowest number that is both a multiple of 4 and a multiple of 10 is called the lowest common multiple (LCM) of 4 and 10.

Use the table in **1(a)** to answer these questions.

(i) Write down the HCF and the LCM for 4 and 10.

HCF = ...

LCM = ...
[1]

(ii) Find the HCF and the LCM for 3 and 7.

HCF = ...

LCM = ...
[2]

47. Question | 0607 May-June 2026 P6 V3 Q2

2 When you are looking for the lowest common multiple (LCM) of two numbers you can write as many multiples of each number as you need.

(a) Complete this table.

	numbers (a and b)	**HCF**	**LCM**	$a \times b$
1(b)(i)	4 and 10			40
1(b)(ii)	3 and 7			21
	9 and 12	3		108
	6 and 8	2		48
	5 and 9		45	45
	4 and 6		12	24
	3 and 10	1		30
	8 and 14			112

[3]

(b) Write down a connection between the HCF, the LCM and $a \times b$.

..... [1]

(c) 16 and another number have an HCF of 4 and an LCM of 80.

Find the other number.

..... [2]

48. Question | 0607 May-June 2026 P6 V3 Q3

3 a and b are two numbers.
x and y are different prime numbers.

$a = x \times x \times y \qquad = x^2y$
$b = x \times y \times y \times y = xy^3$

(a) Write down the HCF and the LCM of a and b.
Give each answer in terms of x and y using indices.

HCF = ...

LCM = ...
[2]

(b) Use **3(a)** to show that your connection in **2(b)** is true for a and b.

[2]

49. Question part 1/2 | 0607 May-June 2026 P6 V3 Q4

4 You now use sets of 3 numbers a, b and c.

(a) Complete this table.

numbers a b c	$a \times b \times c$	HCF of a, b, c (P)	LCM of a, b, c (Q)	HCF of a and b (R)	HCF of a and c (S)	HCF of b and c (T)
3 6 9	162	3	18	3	3	3
4 8 10	320	2	40	4	2	2
6 8 12	576	2	24	2		
5 10 4	200	1		5	1	2
12 9 15	1620		180		3	3
20 12 8	1920	4		4	4	
9 12 18	1944			3	9	6

[3]

(b) Complete this relationship using P, Q, R, S and T.

$(a \times b \times c) \times$ ..... = ... × ..... × ..... × ..... [2]

49. Question part 2/2 | 0607 May-June 2026 P6 V3 Q4

(c) In this question there are 3 numbers: 40, d and e.

40, d and e have HCF = 4.
40, d and e have LCM = 360.
$R = 4$, $S = 8$ and $T = 12$.

Find two different pairs of numbers, d and e.

$d =$ ..... and $e =$ .....

$d =$ ..... and $e =$ .....

[5]