4037 Additional Mathematics Paper 1 - Calculus

Topical questions | 23 questions

1. Question part 1/2 | 4037 May-June 2025 P1 V1 Q6

6 A curve has equation $y = \left(\frac{x^2 - 1}{x^2 + 1}\right)^4$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be written as $\frac{Ax\left(x^2 - 1\right)^3}{\left(x^2 + 1\right)^5}$, where A is a positive integer to be found. [5]

1. Question part 2/2 | 4037 May-June 2025 P1 V1 Q6

(b) (i) Show that the curve has stationary points where $x = -1$, $x = 0$ and $x = 1$. [1]

(ii) Use the first derivative test to determine which two stationary points have the same nature and state whether they are maximum or minimum points. [2]

2. Question part 1/2 | 4037 May-June 2025 P1 V1 Q9

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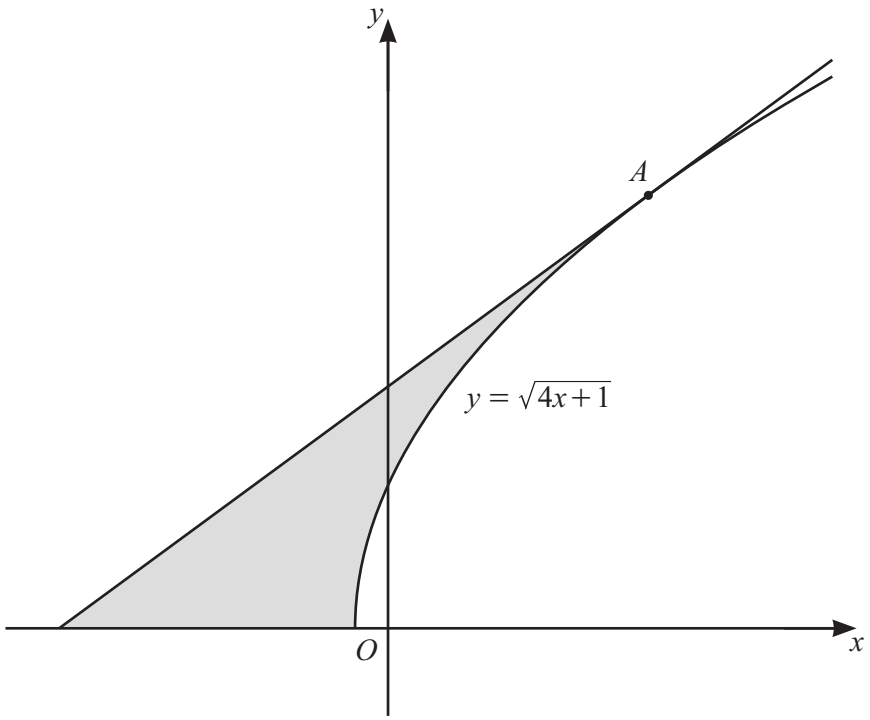

The point A with x-coordinate 2 lies on the curve $y = \sqrt{4x+1}$.
The diagram shows part of this curve and the tangent to the curve at A.

Find the area of the shaded region enclosed by the curve, the tangent and the x-axis. [10]

2. Question part 2/2 | 4037 May-June 2025 P1 V1 Q9

Continuation of working space for Question 9.

3. Question | 4037 May-June 2025 P1 V2 Q8

8 It is given that $y = \dfrac{\ln(3x^2+16)}{x+2}$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 0$.

Give your answer in the form $\ln p$, where p is a constant. [5]

(b) Given that x increases from 0 to h, where h is small, write down the approximate change in y. [1]

4. Question | 4037 May-June 2025 P1 V2 Q12

12 A curve is such that its gradient at the point (x, y) is given by $(5x-2)^{\frac{1}{3}}$.

The curve passes through the point $\left(2, \frac{32}{5}\right)$.

Find the coordinates of the stationary point on the curve. [6]

5. Question part 1/2 | 4037 Oct-Nov 2025 P1 V2 Q9

9 A curve $y = f(x)$ is such that $f''(x) = (2x+5)^{-\frac{3}{2}}$.

The curve has gradient $\frac{2}{3}$ at the point (2, 2).

(a) Find the coordinates of the stationary point on the curve. [11]

5. Question part 2/2 | 4037 Oct-Nov 2025 P1 V2 Q9

Continuation of working space for Question 9(a).

(b) Determine the nature of this stationary point. [1]

6. Question | 4037 Oct-Nov 2025 P1 V2 Q12

12 A curve has equation $y = \frac{e^{(x^2)}}{x-2}$ for $x < 2$.

(a) Find the value of $\frac{dy}{dx}$ when $x = 0$. [4]

When $x = 0$, y is increasing at the rate of 0.5 units per second.

(b) Find the corresponding rate of change of x. [2]

Question 13 is printed on the next page.

7. Question | 4037 Oct-Nov 2025 P1 V3 Q2

2 Differentiate $x^2 e^{3x}$ with respect to x. [3]

8. Question | 4037 Oct-Nov 2025 P1 V3 Q3

3 In this question you may use the values in the table below.

θ radians	$\sin\theta$	$\cos\theta$	$\tan\theta$
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Variables x and y are related by the equation $y = \sin 5x$ where $0 \leqslant x \leqslant \frac{\pi}{10}$.

Use calculus to find the approximate change in x when y increases from $\frac{\sqrt{3}}{2}$ by the small amount 0.01. [4]

9. Question | 4037 Oct-Nov 2025 P1 V3 Q4

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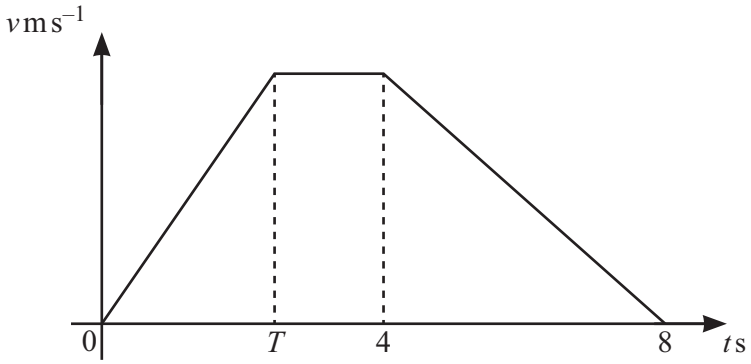

The velocity–time graph represents the motion of a particle moving in a straight line.
The acceleration during the first T seconds of the motion is $2\,\text{m s}^{-2}$.
The total distance travelled is 27 m.

(a) Calculate T. [4]

(b) Calculate the acceleration during the last 4 seconds of the motion. [2]

10. Question | 4037 Oct-Nov 2025 P1 V3 Q5

5 The normal to the curve $y = x^3 + \frac{3}{2}x^2 - 2x + 1$ at the point where $x = 0$ cuts the curve again at two other points.

Find the x-coordinates of these two points. [8]

11. Question | 4037 Oct-Nov 2025 P1 V3 Q8

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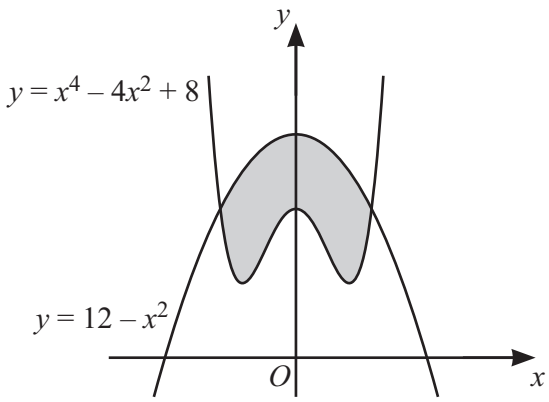


The diagram shows part of each of the curves $y = 12 - x^2$ and $y = x^4 - 4x^2 + 8$.

Find the area of the shaded region enclosed by the two curves. [7]

12. Question part 1/2 | 4037 Specimen 2025 P1 Q8

8 **(a)** Given that $\int_1^a \left(\frac{2}{2x+3} + \frac{3}{3x-1} - \frac{1}{x}\right) \mathrm{d}x = \ln 2.4$, where $a > 1$, find the value of a. [7]

12. Question part 2/2 | 4037 Specimen 2025 P1 Q8

(b) **(i)** Find $\frac{\mathrm{d}}{\mathrm{d}x}(6\sin^3 kx)$, where k is a constant. [2]

(ii) Hence find $\int(\sin^2 2x\cos 2x)\,\mathrm{d}x$. [2]

13. Question part 1/2 | 4037 Specimen 2025 P1 Q9

9 In this question, the units are metres and seconds.

A particle P is travelling in a straight line. Its acceleration, a, away from a fixed point O, at time t, is given by $a = (3t + 2)^{-\frac{1}{3}}$, where $t \geqslant 0$.

When $t = 2$, P is travelling with a velocity of 8 and has a displacement of -4.8 from O.

(a) Find an expression for the velocity of P at time t. [3]

(b) Explain why P is never at rest. [1]

(c) Find the displacement of P from O when $t = \frac{25}{3}$. [4]

Question 10 is printed on the next page.

14. Question | 4037 Specimen 2025 P1 V2 Q8

8

$$\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2} \qquad \cos\frac{\pi}{3}=\frac{1}{2} \qquad \tan\frac{\pi}{3}=\sqrt{3}$$

It is given that $y=\tan x\sin 3x$.

(a) Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x=\frac{\pi}{3}$. [4]

(b) It is given that x is increasing at the rate of 3 units per second.

Find the corresponding rate of change in y when $x=\frac{\pi}{3}$.
Give your answer in its simplest surd form. [2]

(c) Find the approximate change in y as x increases from $\frac{\pi}{3}$ to $\frac{\pi}{3}+h$ where h is small. [1]

15. Question | 4037 Specimen 2025 P1 V2 Q9

9 **(a)** Show that $\frac{1}{2x+1}-\frac{1}{(2x+1)^2}+\frac{4}{4x-1}=\frac{24x^2+14x+4}{(2x+1)^2(4x-1)}$. [2]

(b) Hence find $\int_{\frac{1}{2}}^{1}\frac{24x^2+14x+4}{(2x+1)^2(4x-1)}\mathrm{d}x$.

Give your answer in the form $\frac{1}{2}\ln p+q$ where p and q are rational numbers. [7]

16. Question | 4037 Specimen 2025 P1 V2 Q13

13 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6\mathrm{e}^{3x} + 4x$.

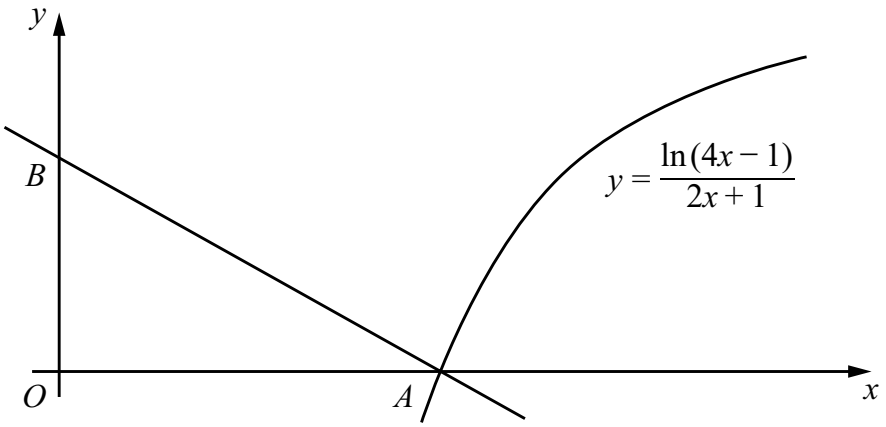
The curve has a gradient of 5 at the point $\left(0, \frac{5}{3}\right)$.

Find the equation of the curve. [7]

Question 14 is printed on the next page.

17. Question | 4037 Specimen 2025 P1 V2 Q14

14



The diagram shows part of the curve $y = \frac{\ln(4x-1)}{2x+1}$ and the normal to the curve at the point A.

The curve crosses the x-axis at A.

The normal to the curve at A meets the y-axis at the point B.

Find the equation of this normal and hence the coordinates of B. [9]

18. Question | 4037 May-June 2026 P1 V1 Q4

4 Given that $\int_0^a \frac{1}{3x+2}\mathrm{d}x = \ln 4$, find the value of the constant a. [5]

19. Question part 1/2 | 4037 May-June 2026 P1 V1 Q7

7 A curve has equation $y = \dfrac{4 + e^{2x}}{2 + e^{x}}$.

(a) Show that, when $\dfrac{dy}{dx} = 0$, $e^{2x} + 4e^{x} - 4 = 0$. [5]

19. Question part 2/2 | 4037 May-June 2026 P1 V1 Q7

(b) Hence find the x-coordinate of the stationary point on the curve.
Give your answer in the form $\ln\left(a\sqrt{b}+c\right)$, where a, b and c are integers. [4]

20. Question | 4037 May-June 2026 P1 V1 Q10

10 A curve has equation $y = \mathrm{f}(x)$, where $\mathrm{f}''(x) = (2x+10)^{-\frac{1}{2}} - 2x$.

The curve has a stationary point at $\left(3, \frac{64}{3}\right)$.

Find the equation of the curve. [8]

21. Question | 4037 May-June 2026 P1 V2 Q5

5 Find, in exact form, the equation of the tangent to the curve $y = \ln\left((2x-4)^5\right)$ at the point where $x = 3$. [4]

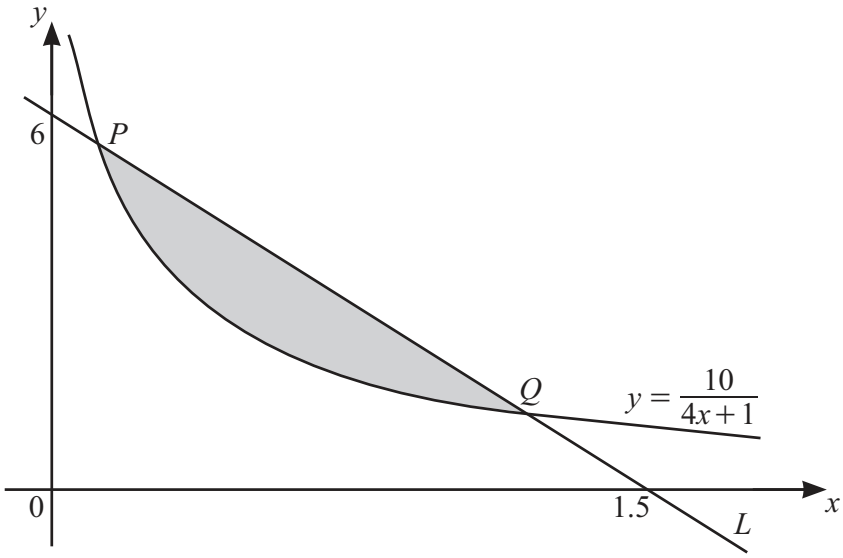
22. Question | 4037 May-June 2026 P1 V2 Q8

8 Variables x and y are related by the equation $y = \frac{5x+1}{x-2}$.

Using calculus, find the approximate change in x when y increases from 4 by the small amount h. [5]

23. Question part 1/2 | 4037 May-June 2026 P1 V2 Q12

12



The diagram shows part of the curve $y = \frac{10}{4x+1}$ and the line L.

L passes through the points (0, 6) and (1.5, 0).
The curve and L intersect at the points P and Q.

Find the exact area of the shaded region. [11]

23. Question part 2/2 | 4037 May-June 2026 P1 V2 Q12

Continuation of working space for Question 12.

Question 13 is on the back page.