4037 Additional Mathematics Paper 2 - Calculus

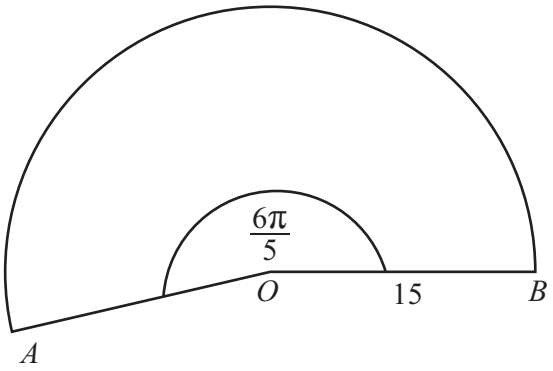
Topical questions | 23 questions

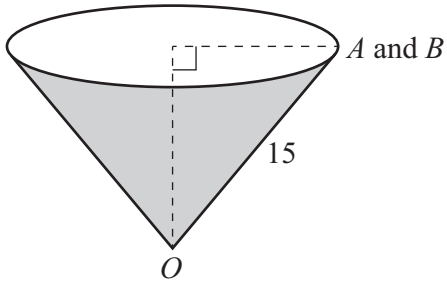
1. Question | 4037 May-June 2025 P2 V1 Q5

5 Given that $y = x^2 \tan\frac{x}{2}$, use calculus to find the approximate change in y as x increases from $\frac{\pi}{3}$ to $\frac{\pi}{3} + h$, where h is small. [4]

2. Question part 1/2 | 4037 May-June 2025 P2 V1 Q9

9 In this question, all lengths are in centimetres and all angles are in radians.





The diagram shows a sector AOB of a circle, centre O, radius 15.
Angle $AOB = \frac{6\pi}{5}$.
The sector is made into a cone with points A and B touching, as shown.

(a) Find the curved surface area of the cone. [2]

The top of the cone is a horizontal circle.

(b) Find the circumference of the circular top. [2]

(c) Hence find the radius of the circular top and the perpendicular height of the cone. [2]

2. Question part 2/2 | 4037 May-June 2025 P2 V1 Q9

(d) Water is poured into the cone.
When the depth of the water in the cone is h, the radius of the circular top of the water is r.

(i) Find an expression for r in terms of h. [1]

(ii) The water is poured into the cone at a constant rate of $27\,\text{cm}^3$ per second.

Find the rate at which the depth of the water is rising when the depth of the water is 4. [5]

3. Question part 1/2 | 4037 May-June 2025 P2 V1 Q10

10 A particle P moves in a straight line.
t seconds after passing a fixed point, O, the acceleration of P, $a\,\text{ms}^{-2}$, is given by

$$\begin{aligned} a &= t^2 - 2 && \text{for } 0 \leqslant t \leqslant 4 \\ a &= 19 - 5\text{e}^{8-2t} && \text{for } 4 \leqslant t \leqslant 10. \end{aligned}$$

When $t = 3$, the velocity of P is $-\frac{1}{3}\text{ms}^{-1}$ and its displacement from O is $-\frac{1}{4}\text{m}$.

(a) (i) Find the velocity of P when $t = 4$. [3]

(ii) Find the displacement of P from O when $t = 4$. [3]

3. Question part 2/2 | 4037 May-June 2025 P2 V1 Q10

(b) Find the displacement of P from O when $t = 10$. [5]

Question 11 is printed on the next page.

4. Question part 1/2 | 4037 May-June 2025 P2 V2 Q2

2 In this question, all lengths are in metres and time is in seconds.

A particle P moves in a straight line such that its displacement s from a fixed point O at time t is given by $s = (t-4)^2(t-1)$ for $t \geqslant 0$.

(a) On the axes, sketch the displacement–time graph of P, stating the intercepts with the axes. [2]



(b) Find an expression for the velocity, v, of P.
Give your answer in a factorised form. [2]

4. Question part 2/2 | 4037 May-June 2025 P2 V2 Q2

(c) On the axes, sketch the velocity–time graph of P, stating the intercepts with the axes. [2]



(d) Find an expression for the acceleration, a, of P. [1]

(e) On the axes, sketch the acceleration–time graph of P, stating the intercepts with the axes. [3]



5. Question | 4037 May-June 2025 P2 V2 Q4

4 **(a)** Given that $y = 4\sin 2x\cos 2x$, find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \frac{\pi}{6}$. [4]

(b) A curve has equation $y = 4\sin 2x\cos 2x$.

The normal to the curve at the point where $x = \frac{\pi}{6}$ meets the x-axis at the point P.

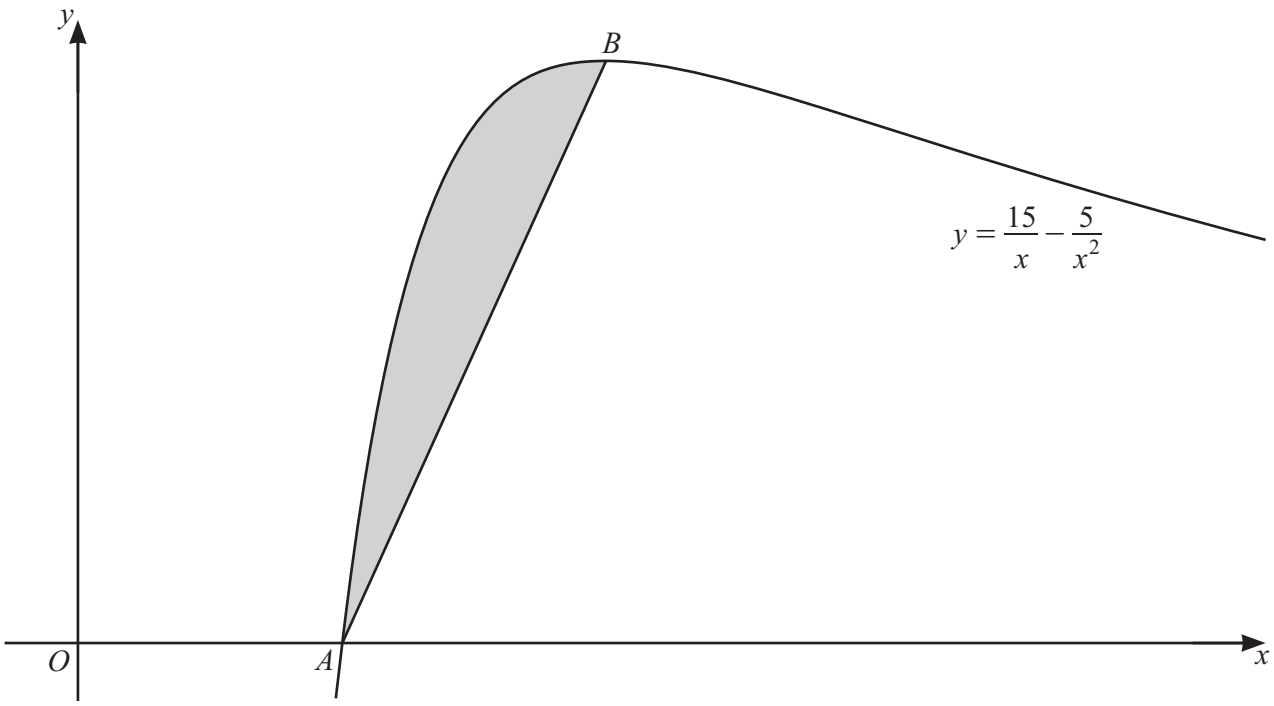
Find the exact coordinates of P. [5]

6. Question | 4037 May-June 2025 P2 V2 Q6

6 The volume, V, of a sphere is increasing at the constant rate of $2\pi\,\text{cm}^3\,\text{s}^{-1}$.
Find the rate of change of the surface area, S, of this sphere when the volume of the sphere is $36\pi\,\text{cm}^3$.
[6]

7. Question part 1/2 | 4037 May-June 2025 P2 V2 Q8

8



The diagram shows part of the curve $y = \frac{15}{x} - \frac{5}{x^2}$.

The curve meets the x-axis at the point A.
The curve has a maximum at the point B.

Find the area of the shaded region enclosed by the line AB and the curve.

Give your answer in exact form. [11]

7. Question part 2/2 | 4037 May-June 2025 P2 V2 Q8

Continuation of working space for Question 8.

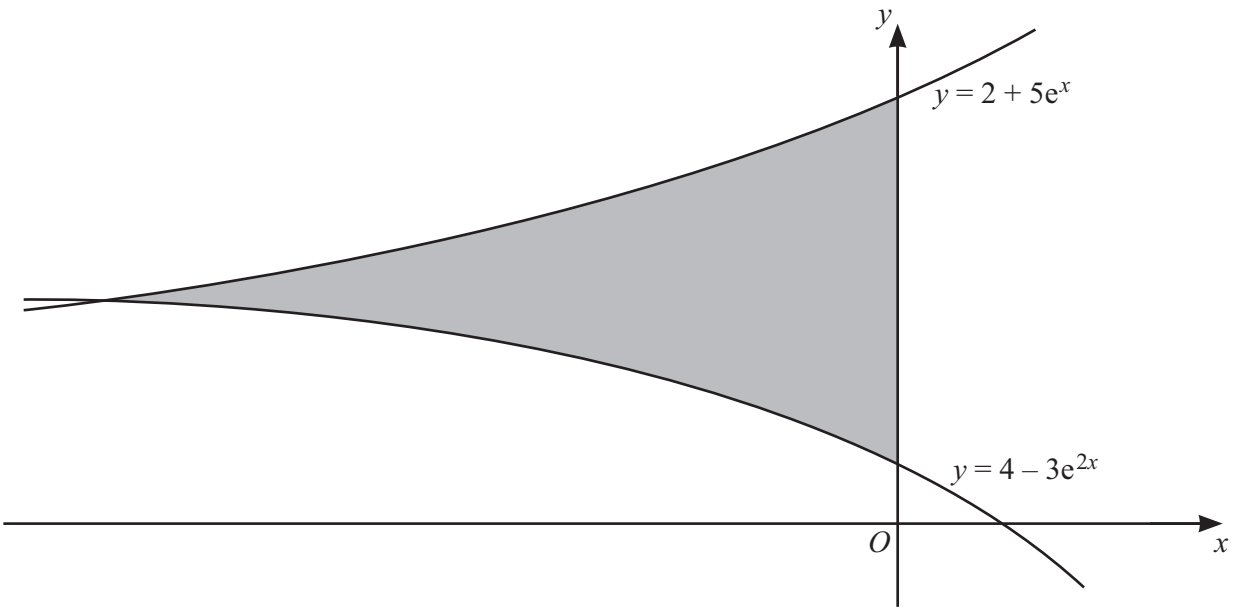
8. Question | 4037 Oct-Nov 2025 P2 V2 Q7

7 **(a)** Given that $y = x\cos 2x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Hence find $\int x\sin 2x \, \mathrm{d}x$. [4]

9. Question part 1/2 | 4037 Oct-Nov 2025 P2 V2 Q10

10



The diagram shows parts of the graphs of $y = 2 + 5e^x$ and $y = 4 - 3e^{2x}$.

Find the area of the shaded region.
Give your answer in the form $a + b \ln 3$, where a and b are exact constants. [10]

9. Question part 2/2 | 4037 Oct-Nov 2025 P2 V2 Q10

Additional working space for Question 10.

10. Question | 4037 Oct-Nov 2025 P2 V3 Q4

4 **(a)** Integrate the following with respect to x.

(i) e^{5x-2} [2]

(ii) $\frac{1}{4-3x}$ where $x < \frac{4}{3}$ [2]

(b) Show that $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \sec^2\left(\frac{1}{2}x\right) dx = 2\left(1 - \frac{\sqrt{3}}{3}\right)$. [3]

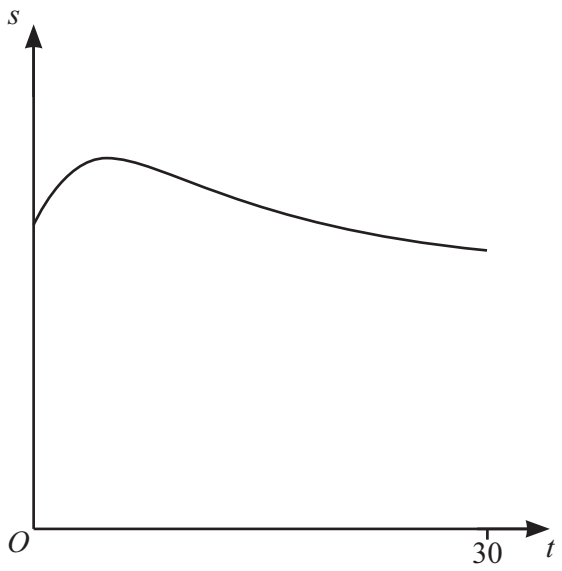
11. Question part 1/2 | 4037 Oct-Nov 2025 P2 V3 Q10

10 In this question the units are metres and seconds.

A particle moves along a straight line through a point A.

Its displacement, s, from A at time t is given by $s = \dfrac{10t + 100}{\sqrt{2t^2 + 100}}$.

The diagram shows the displacement–time graph for the first 30 seconds of the motion.



(a) Find the value of t when s is a maximum. [6]

11. Question part 2/2 | 4037 Oct-Nov 2025 P2 V3 Q10

(b) The particle passes through its starting point again at time $t = T$.

(i) Find the total distance travelled by the particle during the first T seconds of its motion. [2]

(ii) Use algebra to find T. [3]

Question 11 is printed on the next page.

12. Question | 4037 Specimen 2025 P2 Q5

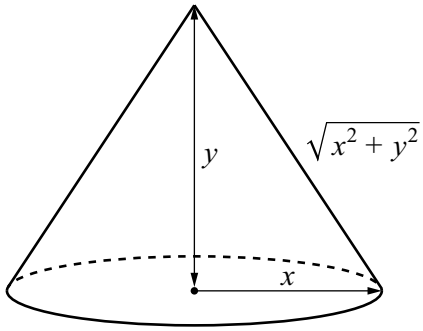
5 Given that $y = \tan x$, use calculus to find the approximate change in y as x increases from $-\frac{\pi}{4}$ to $h - \frac{\pi}{4}$, where h is small. [3]

13. Question | 4037 Specimen 2025 P2 Q6

6 A curve has equation $y = \ln(5 - 3x)$ where $x < \frac{5}{3}$. The normal to the curve at the point where $x = -5$, cuts the x-axis, at the point P.

Find the equation of the normal and the x-coordinate of P. [7]

11 In this question, all lengths are in centimetres.



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The diagram shows a cone of base radius x, height y and sloping edge $\sqrt{x^2 + y^2}$. The volume of the cone is $10\pi\,\text{cm}^3$.

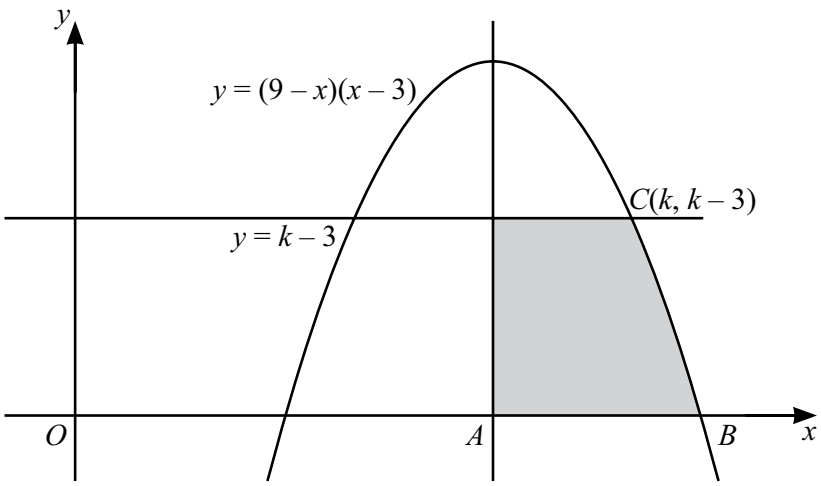
(a) Show that the curved surface area, S, of the cone is given by $S = \dfrac{\pi\sqrt{x^6 + 900}}{x}$. [3]

(b) Given that x can vary and that S has a minimum value, find the value of x for which S is a minimum.

[5]

15. Question part 1/2 | 4037 Specimen 2025 P2 Q12

12



The diagram shows part of the curve $y = (9 - x)(x - 3)$ and the line $y = k - 3$, where $k > 3$.
The line through the maximum point of the curve, parallel to the y-axis, meets the x-axis at A.
The curve meets the x-axis at B, and the line $y = k - 3$ meets the curve at the point $C(k, k - 3)$.

Find the area of the shaded region. [9]

15. Question part 2/2 | 4037 Specimen 2025 P2 Q12

Continuation of working space for Question 12.

16. Question | 4037 Specimen 2025 P2 V2 Q6

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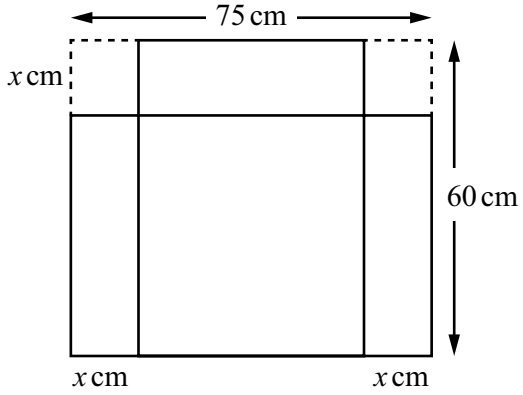
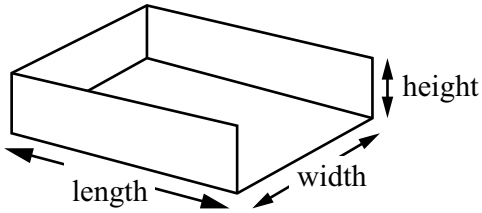




A rectangular sheet of metal measures 75 cm by 60 cm.
A storage tray is made by cutting out squares of side x cm from two corners of the sheet and folding the remainder of the sheet, as shown.
A cuboid has the same length, width and height as the tray.

(a) Show that the volume of the cuboid, V cm^3, is given by $V = ax^3 + bx^2 + cx$, where a, b and c are integers to be found. [3]

(b) Given that x can vary, find the value of x for which V has a stationary value. [3]

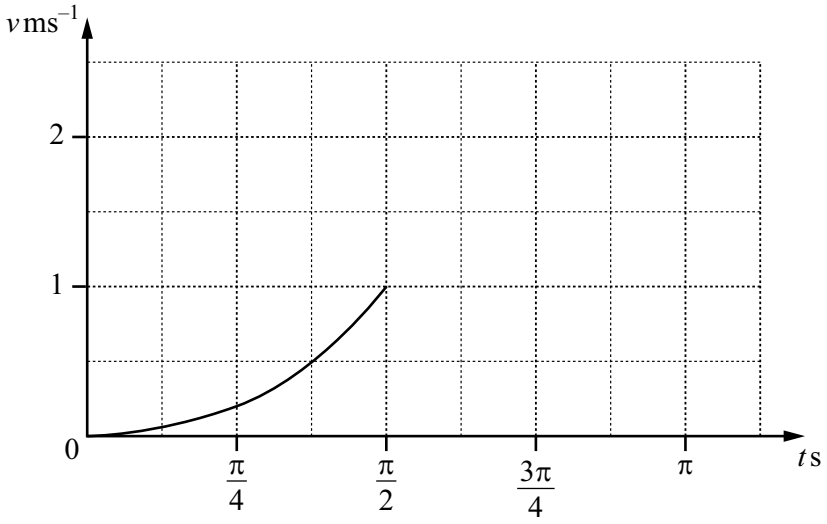
17. Question part 1/2 | 4037 Specimen 2025 P2 V2 Q10

10 **(a)** A particle P travels in a straight line so that t seconds after leaving a fixed point O its velocity, $v\,\text{ms}^{-1}$, is given by

$$v = \sec^2\left(\frac{t}{2}\right) - 1 \quad \text{for } 0 \leqslant t \leqslant \frac{\pi}{2},$$

$$v = \frac{2}{\pi}(\pi - t) \qquad \text{for } t > \frac{\pi}{2}.$$

(i) Complete the velocity–time graph for the first π seconds of the motion of particle P. [1]



(ii) Find the distance, in metres, travelled by P in the first π seconds of its motion.
You must show all your working. [5]

17. Question part 2/2 | 4037 Specimen 2025 P2 V2 Q10

(b) A particle Q travels in a straight line from a fixed point O.

At time t seconds, its displacement from O, s metres, is given by $s = \frac{4 - e^{2t} - 3e^{-2t}}{2}$.

(i) Find the value of t when Q is at instantaneous rest.
Give your answer correct to 4 significant figures. [4]

(ii) Find the distance travelled by Q in the first 0.5 seconds. [3]

18. Question | 4037 May-June 2026 P2 V1 Q5

5 It is given that $y = \tan^3 5x$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\tan 5x$. [4]

(b) Hence solve the equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ for $0 \leqslant x < \frac{4\pi}{5}$. [5]

19. Question | 4037 May-June 2026 P2 V1 Q10

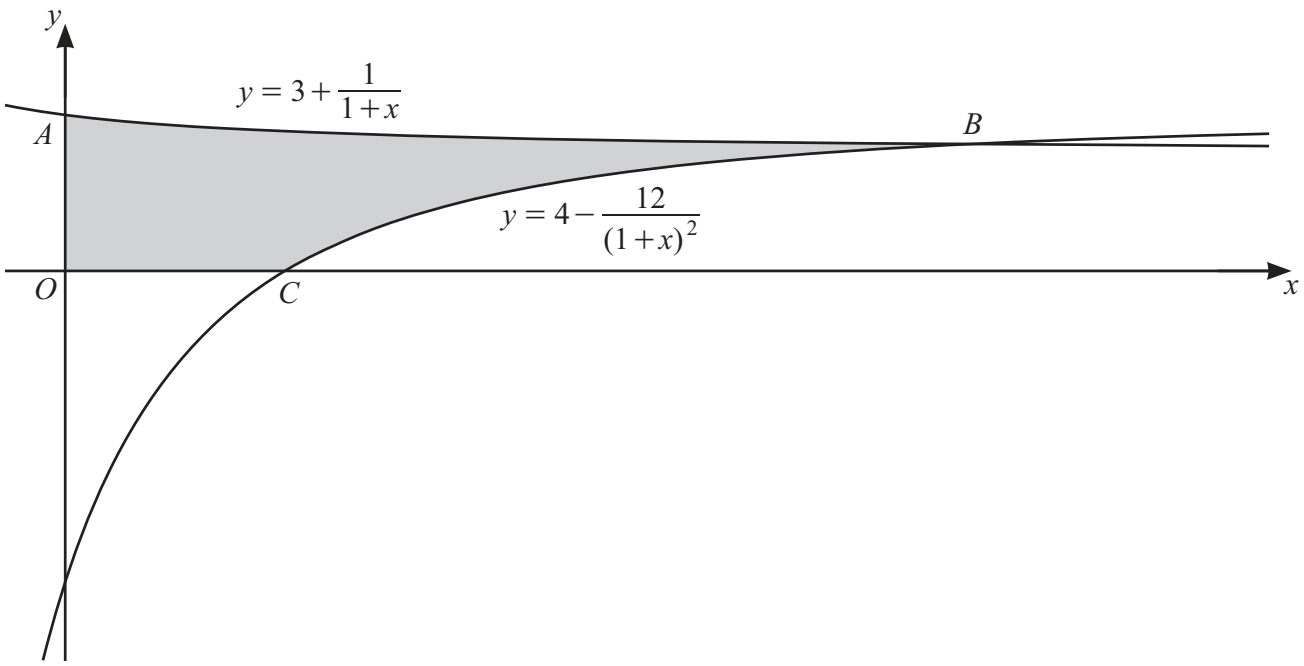
10 A curve has equation $y = x^2 \ln(3x^2 + 5)$.

The normal to the curve at the point where $x = -1$ meets the x-axis at the point P and the y-axis at the point Q.

Find the area of the triangle OPQ, where O is the origin.
Give your answer correct to 3 significant figures. [10]

20. Question | 4037 May-June 2026 P2 V1 Q11

11



The diagram shows part of the curve $y = 3 + \frac{1}{1+x}$ and part of the curve $y = 4 - \frac{12}{(1+x)^2}$.

The curve $y = 3 + \frac{1}{1+x}$ meets the y-axis at the point A.

The curve $y = 4 - \frac{12}{(1+x)^2}$ meets the x-axis at the point C.

The curves intersect at the point B.

Find the area of the shaded region $OABC$.
Give your answer in exact form. [10]

21. Question | 4037 May-June 2026 P2 V2 Q3

3 **(a)** Differentiate $y = \sqrt{x+1} + \cos(5x+3)$ with respect to x. [3]

(b) Find $\int \sec^2 \frac{x}{3} \mathrm{d}x$. [3]

22. Question | 4037 May-June 2026 P2 V2 Q9

9 A particle P moves in a straight line such that t seconds after passing through a fixed point O, its displacement from O, s metres, is given by $s = e^{4t} - 10e^{2t} + 9$.

(a) The velocity of P is $v\,\text{ms}^{-1}$.

Find the value of t for which v has a stationary value. [5]

(b) Find the acceleration of P when $t = \ln 2$. [2]

23. Question | 4037 May-June 2026 P2 V2 Q12

12 A sphere has volume $V\,\text{cm}^3$ and radius r cm.

Given that r varies with time t, find V at the instant when $\frac{\mathrm{d}V}{\mathrm{d}t} = 9\frac{\mathrm{d}r}{\mathrm{d}t}$. [4]