

Topical questions | 54 questions

8 (a) Find the values of a for which the system of equations

$$ax + 6y - z = 0,$$

$$ay - 2z = 0,$$

does not have a unique solution. [3]

[illegible]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

(b) Use the characteristic equation of $\mathbf{A}$ to find the inverse of $\mathbf{A}^2$. [4]

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9231 Further Mathematics Paper 2 - Matrices

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1. Question part 2/3 | 9231 May-June 2020 P2 V1 Q8

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [7]

[illegible]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]

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does not have a unique solution. [3]

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(b) Use the characteristic equation of $\mathbf{A}$ to find the inverse of $\mathbf{A}^2$. [4]

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2. Question part 2/3 | 9231 May-June 2020 P2 V2 Q8

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [7]

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3 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}.$$

[4]

[illegible]

Topical questions | 54 questions

(b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$.

[4]

[illegible]

Topical questions | 54 questions

3 (a) Show that the system of equations

$$x - 2y + kz = 1,$$

$$-x + 2y + 2z = 1,$$

where k is a constant, does not have a unique solution. [2]

[illegible]

(b) Given that $k=-4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

[illegible]

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4. Question part 2/2 | 9231 Oct-Nov 2020 P2 V1 Q3

- (c) Given instead that $k = -2$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

[illegible]

- (d)** For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

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Topical questions | 54 questions

7 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

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- [illegible]

Topical questions | 54 questions

The 3×3 matrix $\mathbf{A}$ has distinct eigenvalues $b, -1, 1$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

(c) Find $\mathbf{A}$ in terms of b .

[4]

Topical questions | 54 questions

9 It is given that a is a positive constant.

$$3y - z = 3,$$

has a unique solution and interpret this situation geometrically. [3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \mathbf{P}^{-1}. \quad [5]$$
[illegible]

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6. Question part 3/5 | 9231 Oct-Nov 2020 P2 V2 Q9

[illegible]

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6. Question part 4/5 | 9231 Oct-Nov 2020 P2 V2 Q9

(d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$.

[6]

[illegible]

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Topical questions | 54 questions

3 (a) Show that the system of equations

$$x - 2y + kz = 1,$$

$$-x + 2y + 2z = 1,$$

where k is a constant, does not have a unique solution. [2]

[illegible]

(b) Given that $k=-4$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

[illegible]

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7. Question part 2/2 | 9231 Oct-Nov 2020 P2 V3 Q3

- (c) Given instead that $k = -2$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

[illegible]

- (d)** For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [2]

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8. Question part 1/2 | 9231 Oct-Nov 2020 P2 V3 Q7

7 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

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(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

[illegible]

Topical questions | 54 questions

The 3×3 matrix $\mathbf{A}$ has distinct eigenvalues $b, -1, 1$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 \\ -1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

respectively.

(c) Find $\mathbf{A}$ in terms of b .

[4]

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9. Question part 1/5 | 9231 Specimen 2020 P2 Q8

8 (a) (i) Find the set of values of a for which the system of equations

$$\begin{aligned} x - 2y - 2z + 7 &= 0, \\ 2x + (a - 9)y - 10z + 11 &= 0, \\ 3x - 6y + 2az + 29 &= 0, \end{aligned}$$

has a unique solution.

[4]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

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9. Question part 2/5 | 9231 Specimen 2020 P2 Q8

- (ii)** Given that $a = -3$, show that the system of equations in part **(i)** has no solution. Interpret this situation geometrically. [3]

[illegible]

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(b) The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}.$$

[4]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

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9. Question part 4/5 | 9231 Specimen 2020 P2 Q8

(ii) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$.

[4]

[illegible]

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1 (a) Given that a is an integer, show that the system of equations

$$-x + 2y - 5z = 17,$$

has a unique solution and interpret this situation geometrically. [4]

[illegible]

(b) Find the value of a for which $x = 1$, $y = 4$, $z = -2$ is the solution to the system of equations in part **(a)**. [1]

[illegible]

Topical questions | 54 questions

6 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

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11. Question part 2/2 | 9231 May-June 2021 P2 V1 Q6

[illegible]

(b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^3$. [4]

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Topical questions | 54 questions

1 (a) Given that a is an integer, show that the system of equations

$$-x + 2y - 5z = 17,$$

has a unique solution and interpret this situation geometrically. [4]

[illegible]

(b) Find the value of a for which $x = 1$, $y = 4$, $z = -2$ is the solution to the system of equations in part **(a)**. [1]

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13. Question part 1/3 | 9231 May-June 2021 P2 V3 Q8

- $$\begin{aligned} 13x + 18y - 28z &= 0, \\ -4x - ay + 8z &= 0, \\ 2x + 6y - 5z &= 0, \end{aligned}$$

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$$\mathbf{A} = \begin{pmatrix} 13 & 18 & -28 \\ -4 & -1 & 8 \\ 2 & 6 & -5 \end{pmatrix}.$$

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- [illegible]

9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

13. Question part 2/3 | 9231 May-June 2021 P2 V3 Q8

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(d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$ in terms of $\mathbf{A}$.

[2]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]

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14. Question | 9231 Oct-Nov 2021 P2 V1 Q2

2 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined.

[6]

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Topical questions | 54 questions

6 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 6 & 6 \\ 0 & 2 & 6 \\ 0 & 0 & -3 \end{pmatrix}.$$

- (a) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [5]

[illegible]

Topical questions | 54 questions

(b) Find the matrix $\mathbf{A}$ such that

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{pmatrix}. \quad [4]$$

This image shows a full page of white paper with horizontal blue ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

(c) State the eigenvalues and corresponding eigenvectors of $\mathbf{A}^3$. [2]

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16. Question | 9231 Oct-Nov 2021 P2 V3 Q2

2 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}.$$

Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I},$$

where p and q are integers to be determined. [6]

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17. Question part 1/3 | 9231 May-June 2022 P2 V1 Q8

8 (a) Find the value of a for which the system of equations

$$\begin{aligned} 3x + ay &= 0, \\ 5x - y &= 0, \\ x + 3y + 2z &= 0, \end{aligned}$$

does not have a unique solution. [2]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$$

(b) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

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17. Question part 2/3 | 9231 May-June 2022 P2 V1 Q8

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(c) Use the characteristic equation of $\mathbf{A}$ to show that

$$(\mathbf{A} + 6\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where b is an integer to be determined.

[4]

[illegible]

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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8 (a) Find the value of a for which the system of equations

$$5x - y = 0,$$

$$x + 3y + 2z = 0,$$

does not have a unique solution. [2]

[illegible]
$$\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 5 & -1 & 0 \\ 1 & 3 & 2 \end{pmatrix}.$$

(b) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

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[illegible]

$$(\mathbf{A} + 6\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

[4]

[illegible]

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19. Question part 1/2 | 9231 May-June 2022 P2 V3 Q3

3 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -9 & 5 \\ 5 & -8 & 5 \\ 1 & -1 & 2 \end{pmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$.

[4]

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19. Question part 2/2 | 9231 May-June 2022 P2 V3 Q3

- (b) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^{-1} = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are constants to be determined. [3]

[illegible]

2 (a) Show that the system of equations

$$\begin{aligned}x - y + 2z &= 4, \\ x - y - 3z &= a, \\ x - y + 7z &= 13,\end{aligned}$$

where a is a constant, does not have a unique solution. [2]

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(b) Given that $a = -5$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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(c) Given instead that $a \neq -5$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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Topical questions | 54 questions

6 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & -7 \\ 0 & 5 & 7 \\ 0 & 0 & -2 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

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21. Question part 2/2 | 9231 Oct-Nov 2022 P2 V1 Q6

(b) Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = a\mathbf{A}^2 + b\mathbf{I},$$

where a and b are integers to be determined.

[4]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

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22. Question | 9231 Oct-Nov 2022 P2 V2 Q1

- 1 (a)** Find the set of values of k for which the system of equations

$$x + 2y + 3z = 1,$$

$$kx + 4y + 6z = 0,$$

$$7x + 8y + 9z = 3,$$

has a unique solution.

[3]

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- (b)** Interpret the situation geometrically in the case where the system of equations does not have a unique solution. [2]

[illegible]

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7 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

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$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 3 \\ 15 & -4 & 3 \\ 3 & 0 & 2 \end{pmatrix}.$$

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(d) Hence find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [2]

[illegible]

- [illegible]

2 (a) Show that the system of equations

$$\begin{aligned}x - y + 2z &= 4, \\ x - y - 3z &= a, \\ x - y + 7z &= 13,\end{aligned}$$

where a is a constant, does not have a unique solution. [2]

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(b) Given that $a = -5$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

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(c) Given instead that $a \neq -5$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

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9231 Further Mathematics Paper 2 - Matrices

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25. Question part 1/2 | 9231 Oct-Nov 2022 P2 V3 Q6

6 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 2 & -3 & -7 \\ 0 & 5 & 7 \\ 0 & 0 & -2 \end{pmatrix}.$$

(a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

25. Question part 2/2 | 9231 Oct-Nov 2022 P2 V3 Q6

(b) Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = a\mathbf{A}^2 + b\mathbf{I},$$

where a and b are integers to be determined.

[4]

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1 (a) Show that the system of equations

$$4x + 5y + 6z = 1,$$

$$7x + 8y + 9z = 1,$$

does not have a unique solution. [2]

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(b) Show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

[illegible]

Topical questions | 54 questions

5 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 18 & 5 & -11 \\ 8 & 6 & -4 \\ 32 & 10 & -20 \end{pmatrix}.$$

- (a) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 4\lambda^2 - 20\lambda + 48 = 0$ and hence find the eigenvalues of $\mathbf{A}$. [4]

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9231 Further Mathematics Paper 2 - Matrices

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27. Question part 2/2 | 9231 May-June 2023 P2 V1 Q5

(b) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{PDP}^{-1}$. [6]

[illegible]

Topical questions | 54 questions

1 (a) Show that the system of equations

$$7x + 8y + 9z = 1,$$

does not have a unique solution. [2]

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(b) Show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

29. Question part 1/2 | 9231 May-June 2023 P2 V2 Q5

5 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 18 & 5 & -11 \\ 8 & 6 & -4 \\ 32 & 10 & -20 \end{pmatrix}.$$

- (a) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 4\lambda^2 - 20\lambda + 48 = 0$ and hence find the eigenvalues of $\mathbf{A}$. [4]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

29. Question part 2/2 | 9231 May-June 2023 P2 V2 Q5

(b) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^5 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [6]

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Topical questions | 54 questions

8 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & -6a & 2a+2 \\ 0 & 1-a & 0 \\ 0 & 2-a & -1 \end{pmatrix}$$

(a) Show that the equation $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ has a unique solution and interpret this situation geometrically.

[3]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

[2]

[illegible]

Topical questions | 54 questions

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^4 = \mathbf{PDP}^{-1}$. [6]

[illegible]

- (d) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^4$ in terms of $\mathbf{A}$ and a . [3]

[illegible]

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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1 Show that the system of equations

$$x + y + kz = 3,$$

$$-21x + 6y - 9z = 14,$$

where k is a constant, does not have a unique solution and interpret this situation geometrically. [4]

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Topical questions | 54 questions

7 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -6 & 2 & 13 \\ 0 & -2 & 5 \\ 0 & 0 & 8 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [7]

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9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

32. Question part 2/2 | 9231 Oct-Nov 2023 P2 V1 Q7

[illegible]

(b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$. [4]

[illegible]

Topical questions | 54 questions

6 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{pmatrix}.$$

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The 3×3 matrix $\mathbf{A}$ has distinct non-zero eigenvalues $a, \frac{1}{2}, 2$ with corresponding eigenvectors

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix},$$

(c) Find $\mathbf{A}^{-1}$ in terms of a .

[5]

[illegible]

Topical questions | 54 questions

1 Show that the system of equations

$$x + y + kz = 3,$$

$$-21x + 6y - 9z = 14,$$

where k is a constant, does not have a unique solution and interpret this situation geometrically. [4]

[illegible]

Topical questions | 54 questions

7 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -6 & 2 & 13 \\ 0 & -2 & 5 \\ 0 & 0 & 8 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [7]

[illegible]

Topical questions | 54 questions

(b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$. [4]

Topical questions | 54 questions

8 (a) Find the set of values of a for which the system of equations

$$x + 5y + 4z = 2$$

has a unique solution. [2]

[illegible]

(b) Show that the system of equations in part **(a)** is consistent for all values of a . [3]

[illegible]

Topical questions | 54 questions

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

36. Question part 3/4 | 9231 May-June 2024 P2 V1 Q8

(d) Use the characteristic equation of $\mathbf{A}$ to show that

$$(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where b is an integer to be determined.

[4]

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Topical questions | 54 questions

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

Topical questions | 54 questions

8 (a) Find the set of values of a for which the system of equations

$$2x - y = 1,$$

$$x + 5y + 4z = 2$$

has a unique solution. [2]

This image shows a full page of white paper with ten horizontal dashed lines, evenly spaced from top to bottom. The lines are thin and black, typical of handwriting practice paper. There are no margins, text, or other markings on the page.

(b) Show that the system of equations in part **(a)** is consistent for all values of a . [3]

[illegible]

Topical questions | 54 questions

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & 0 & 0 \\ 2 & -1 & 0 \\ 1 & 5 & 4 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

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37. Question part 3/4 | 9231 May-June 2024 P2 V2 Q8

(d) Use the characteristic equation of $\mathbf{A}$ to show that

$$(14\mathbf{A} + 24\mathbf{I})^2 = \mathbf{A}^4(\mathbf{A} + b\mathbf{I})^2,$$

where b is an integer to be determined.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

Topical questions | 54 questions

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

Topical questions | 54 questions

8 The planes Π_1 and Π_2 do not intersect and are both perpendicular to $\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The line l intersects Π_1 at the point $(1, 6, 0)$ and intersects Π_2 at the point $(3, -6, 0)$.

[illegible][illegible]

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ \frac{1}{2} & \frac{1}{12} & 0 \end{pmatrix}.$$

- (c) Show that the characteristic equation of $\mathbf{A}$ is $-\lambda^3 + 3\lambda^2 + \frac{7}{4}\lambda = 0$ and hence find the eigenvalues of $\mathbf{A}$. [3]

[illegible]

[illegible]



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Topical questions | 54 questions

- $$\begin{aligned}x + 5y + 6z &= 1, \\ kx + 2y + 2z &= 2, \\ -3x + 4y + 8z &= 3,\end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

[illegible]

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40. Question part 1/2 | 9231 Oct-Nov 2024 P2 V1 Q4

$$\mathbf{A} = \begin{pmatrix} -11 & 1 & 8 \\ 0 & -2 & 0 \\ -16 & 1 & 13 \end{pmatrix}.$$

-
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- This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

40. Question part 2/2 | 9231 Oct-Nov 2024 P2 V1 Q4



(c) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$.

[4]

[illegible]

Topical questions | 54 questions

8 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 7 & 9 \\ 4 & 1 & 7 \end{pmatrix}.$$

- (a) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 12\lambda^2 + 12\lambda + 80 = 0$ and find the eigenvalues of $\mathbf{A}$. [4]

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9231 Further Mathematics Paper 2 - Matrices

Topical questions | 54 questions

41. Question part 2/4 | 9231 Oct-Nov 2024 P2 V2 Q8



(b) Use the characteristic equation of $\mathbf{A}$ to show that

$$\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{A} + r\mathbf{I},$$

where p, q and r are integers to be determined.

[4]

[illegible]

Topical questions | 54 questions



This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings present.

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If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

9231 Further Mathematics Paper 2 - Matrices

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42. Question | 9231 Oct-Nov 2024 P2 V3 Q1

1 Find the set of values of k for which the system of equations

$$x + 5y + 6z = 1,$$

$$kx + 2y + 2z = 2,$$

$$-3x + 4y + 8z = 3,$$

has a unique solution and interpret this situation geometrically. [4]

[illegible]

Topical questions | 54 questions

43. Question part 1/2 | 9231 Oct-Nov 2024 P2 V3 Q4

$$\mathbf{A} = \begin{pmatrix} -11 & 1 & 8 \\ 0 & -2 & 0 \\ -16 & 1 & 13 \end{pmatrix}.$$

- (a) Show that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and state the corresponding eigenvalue. [2]

- (b) Show that the characteristic equation of $\mathbf{A}$ is $\lambda^3 - 19\lambda - 30 = 0$ and hence find the other eigenvalues of $\mathbf{A}$. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical questions | 54 questions



[4]

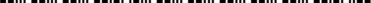
[illegible]

Topical questions | 54 questions

8 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

[illegible]
$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$
[illegible]

Topical questions | 54 questions



- [illegible]

- [illegible]

Topical questions | 54 questions



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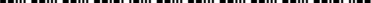
Topical questions | 54 questions

8 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

This image shows a single sheet of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the width of the page. There is no text or other markings on the paper.

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$
[illegible]

Topical questions | 54 questions



- [illegible]

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- This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the entire width of the page. There is no handwriting or other markings on the paper.

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If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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8 (a) Find the values of a for which the system of equations

$$\begin{aligned}\frac{3}{2}x + 3y + 8z &= 1, \\ ax + 3y + 4z &= 2, \\ ay - z &= 3,\end{aligned}$$

does not have a unique solution. [3]

[illegible]
$$\mathbf{A} = \begin{pmatrix} \frac{3}{2} & 3 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

(b) Given that $\mathbf{B} = \mathbf{A}^{-1}$, use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{B}^2 = p\mathbf{I} + q\mathbf{A}$, where p and q are constants to be determined. [4]

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9231 Further Mathematics Paper 2 - Matrices

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46. Question part 2/3 | 9231 May-June 2025 P2 V3 Q8



(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$.

[7]

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47. Question part 1/2 | 9231 May-June 2025 P2 V4 Q1

$$7x + 2ky + 9z = 3,$$

does not have a unique solution. [3]

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(b) Given that $k = 1$, show that the system of equations in part **(a)** is consistent. Interpret this situation geometrically. [3]

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47. Question part 2/2 | 9231 May-June 2025 P2 V4 Q1



9231 Further Mathematics Paper 2 - Matrices

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48. Question part 1/2 | 9231 May-June 2025 P2 V4 Q7

7 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 7 & 11 \\ 0 & 2 & 5 \\ 0 & 0 & -3 \end{pmatrix}.$$

(a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^6 = \mathbf{PDP}^{-1}$. [7]

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$$\mathbf{A}^6 = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I},$$

where a, b and c are integers to be determined. [4]

[illegible]

9231 Further Mathematics Paper 2 - Matrices

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49. Question | 9231 Oct-Nov 2025 P2 V1 Q1

1 Find the values of k for which the system of equations

$$x - y + z = 0,$$

$$x + ky + 3z = 0,$$

$$x + 2y + kz = 0,$$

does not have a unique solution.

[3]

[illegible]

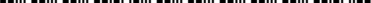
$$\mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & 2a & -1 \\ 0 & 0 & -3a \end{pmatrix}.$$

-

- [illegible]

- [illegible]

Topical questions | 54 questions


$$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2a \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -3a \end{pmatrix},$$

(c) Find $\mathbf{A}$ in terms of a .

[5]

[illegible]

51. Question part 1/3 | 9231 Oct-Nov 2025 P2 V2 Q8

8 (a) Find the values of k for which the system of equations

$$\begin{aligned}x - y + 2kz &= 1, \\ kx + y + 2z &= 2, \\ 2x - y + z &= 3,\end{aligned}$$

does not have a unique solution. [3]

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(b) Given that $k = -\frac{1}{2}$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [3]

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(c) Given instead that $k = -1$, show that the system of equations in part (a) is also inconsistent. Interpret this situation geometrically. [4]

.....

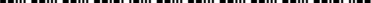
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$$\mathbf{A} = \begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}.$$

- [illegible]

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[illegible]

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52. Question | 9231 Oct-Nov 2025 P2 V3 Q1

1 Find the values of k for which the system of equations

$$x - y + z = 0,$$

$$x + ky + 3z = 0,$$

$$x + 2y + kz = 0,$$

does not have a unique solution.

[3]

[illegible]

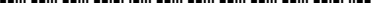
$$\mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & 2a & -1 \\ 0 & 0 & -3a \end{pmatrix}.$$

-

- [illegible]

- [illegible]

Topical questions | 54 questions


$$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2a \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -3a \end{pmatrix},$$

(c) Find $\mathbf{A}$ in terms of a .

[5]

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Topical questions | 54 questions

8 (a) Find the set of values of a for which the system of equations

$$2x - 3y + z = 1,$$

has a unique solution and interpret this situation geometrically. [3]

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(b) Given that $a = -\frac{5}{4}$, show that the system of equations in part (a) is inconsistent and interpret this situation geometrically. [3]

[illegible]

Topical questions | 54 questions


$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(\mathbf{A} + 3\mathbf{I})^2 = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$. [7]

[illegible]



If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

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