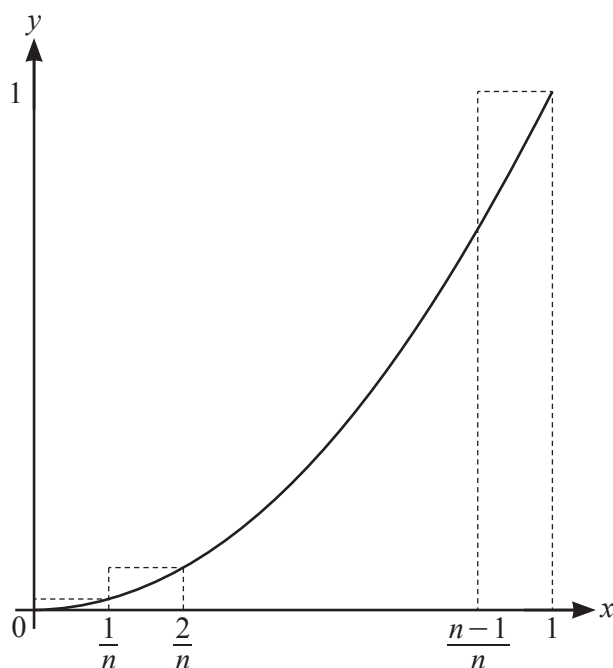


Topical questions | 67 questions

2 It is given that $y = 2^x$.

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[illegible]



(a) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 x^2 dx < \frac{2n^2 + 3n + 1}{6n^2}. \quad [4]$$

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

2. Question part 2/2 | 9231 May-June 2020 P2 V1 Q4

[illegible]

(b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 x^2 dx$. [4]

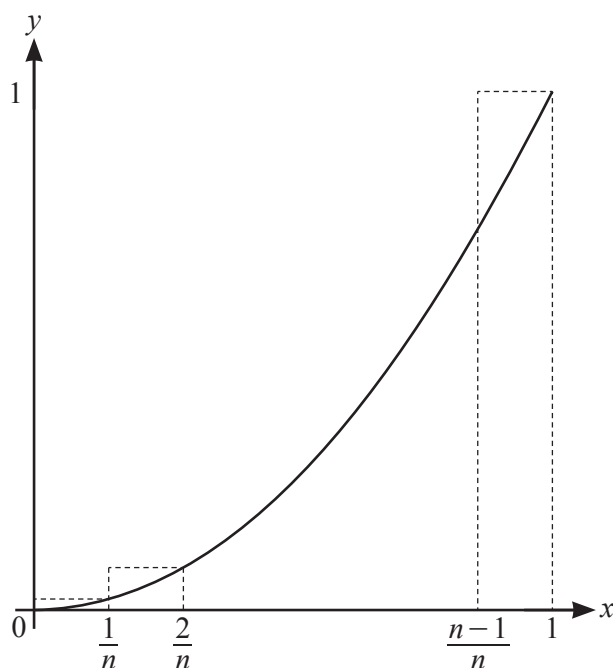
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Topical questions | 67 questions

2 It is given that $y = 2^x$.

[illegible]

[illegible]



(a) By considering the sum of the areas of these rectangles, show that

$$\int_0^1 x^2 dx < \frac{2n^2 + 3n + 1}{6n^2}. \quad [4]$$

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Topical questions | 67 questions

4. Question part 2/2 | 9231 May-June 2020 P2 V2 Q4

[illegible]

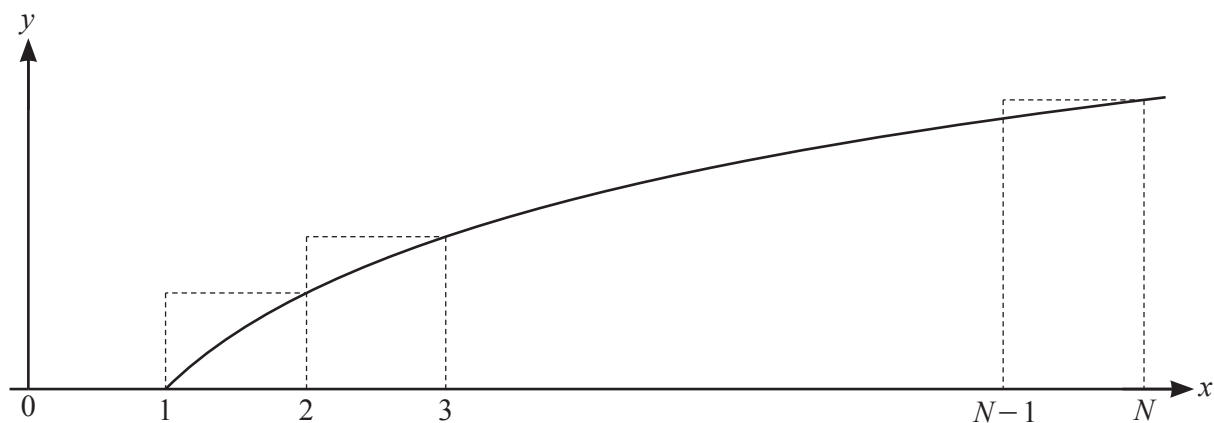
(b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 x^2 dx$. [4]

[illegible]

Topical questions | 67 questions

2 Let $I_n = \int_0^1 (1+3x)^n e^{-3x} dx$, where n is an integer.

[illegible][illegible]



(a) By considering the sum of the areas of these rectangles, show that

$$\ln N! > N \ln N - N + 1. \quad [5]$$

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

6. Question part 2/2 | 9231 May-June 2020 P2 V3 Q4

(b) Use a similar method to find, in terms of N , an upper bound for $\ln N!$.

[3]

This image shows a full page of white paper designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines running from left to right across the entire width of the page. There are no margins, text, or other markings present.

Topical questions | 67 questions

5 The curve C has parametric equations

(a) Find the exact length of C . [5]

[illegible]

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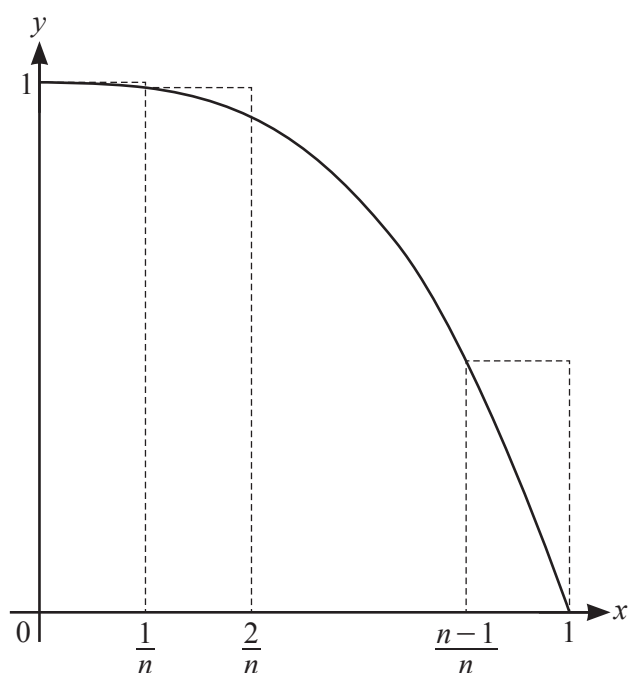
Topical questions | 67 questions

7. Question part 2/2 | 9231 May-June 2020 P2 V3 Q5

(b) Find $\frac{d^2y}{dx^2}$ in terms of t , simplifying your answer.

[4]

[illegible]



(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1-x^3)dx \leq \frac{3n^2+2n-1}{4n^2}. \quad [4]$$

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

8. Question part 2/2 | 9231 Oct-Nov 2020 P2 V1 Q4

[illegible]

(b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 (1-x^3)dx$. [4]

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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

9. Question part 1/2 | 9231 Oct-Nov 2020 P2 V2 Q2

2 A curve has equation $y = \cosh x$, for $0 \leq x \leq \frac{1}{2}$.

Find, in terms of π and e , the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [6]

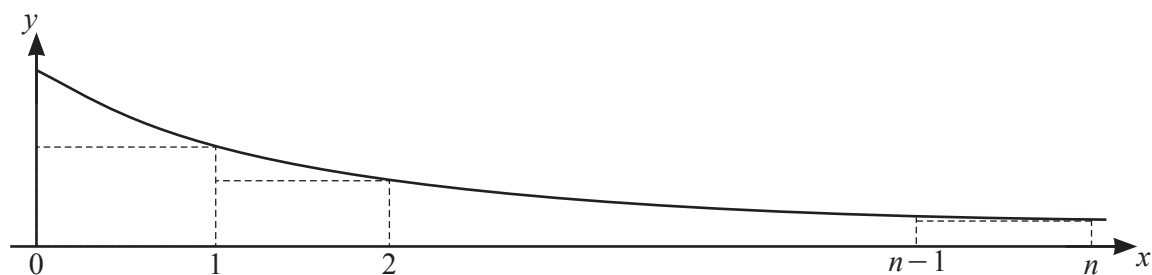
[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

9. Question part 2/2 | 9231 Oct-Nov 2020 P2 V2 Q2

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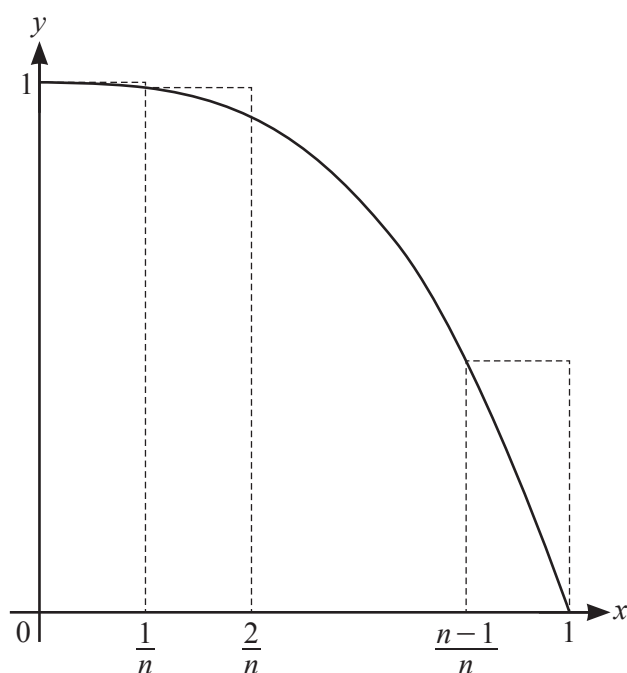

$$\sum_{r=1}^n \frac{1}{\sqrt{r^2 + r + 1}} < \ln\left(\frac{1}{3} + \frac{2}{3}n + \frac{2}{3}\sqrt{n^2 + n + 1}\right). \quad [10]$$
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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

10. Question part 2/2 | 9231 Oct-Nov 2020 P2 V2 Q8

[illegible]



(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1-x^3) dx \leq \frac{3n^2+2n-1}{4n^2}. \quad [4]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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Topical questions | 67 questions

11. Question part 2/2 | 9231 Oct-Nov 2020 P2 V3 Q4

[illegible]

(b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 (1-x^3)dx$. [4]

[illegible]

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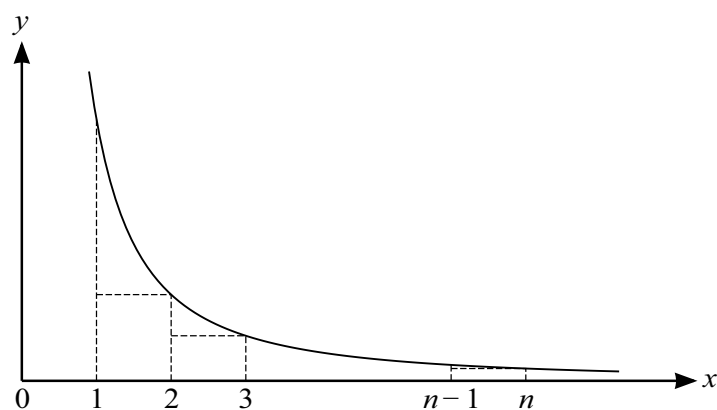
Topical questions | 67 questions

12. Question | 9231 Specimen 2020 P2 Q2

2 Find the exact value of $\int_0^1 \frac{1}{\sqrt{3+4x-4x^2}} dx$. [6]

[illegible]

4



(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{i=1}^n \frac{1}{r^2} < \frac{2n-1}{n}. \quad [5]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

13. Question part 2/2 | 9231 Specimen 2020 P2 Q4

[illegible]

(b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{r^2}$. [3]

[illegible]

Topical questions | 67 questions

5 The curve C has parametric equations

(a) Find, in terms of e , the length of C . [5]

[illegible]

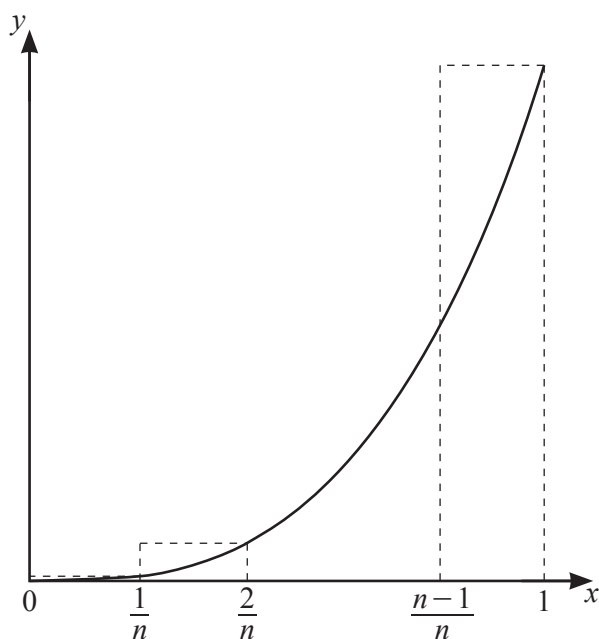
9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

14. Question part 2/2 | 9231 Specimen 2020 P2 Q5

- (b)** Find, in terms of π and e , the area of the surface generated when C is rotated through 2π radians about the x -axis. [5]

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(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 x^3 dx < U_n$, where

$$U_n = \left(\frac{n+1}{2n} \right)^2. \quad [4]$$

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

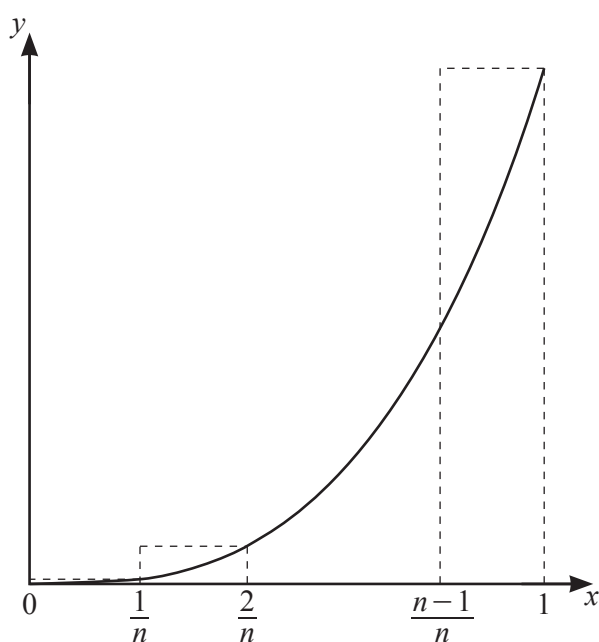
15. Question part 2/2 | 9231 May-June 2021 P2 V1 Q3

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 x^3 dx$. [4]

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(c) Find the least value of n such that $U_n - L_n < 10^{-3}$. [2]

[illegible]



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 x^3 dx < U_n$, where

$$U_n = \left(\frac{n+1}{2n}\right)^2. \quad [4]$$

[illegible]

Topical questions | 67 questions

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 x^3 dx$. [4]

[illegible][illegible]

Topical questions | 67 questions

6 The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & -\frac{22}{3} & 8 \\ 0 & -6 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- (a) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^2 = \mathbf{PDP}^{-1}$. [7]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The entire page is otherwise blank, with no margins, text, or other markings.

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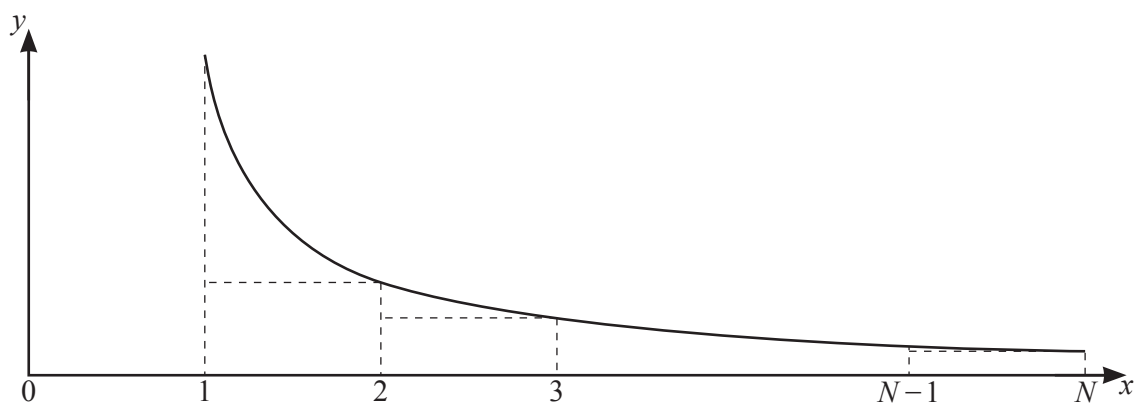
Topical questions | 67 questions

17. Question part 2/2 | 9231 May-June 2021 P2 V2 Q6

[illegible]

(b) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^3$. [4]

[illegible]



(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{r}{2r^2-1} < \frac{1}{4} \ln(2N^2-1) + 1. \quad [7]$$

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical questions | 67 questions

[illegible]

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Topical questions | 67 questions

7 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{3}{2}} (4+x^2)^{-\frac{1}{2}n} dx$.

[illegible]
$$4nI_{n+2} = \frac{3}{2}\left(\frac{2}{5}\right)^n + (n-1)I_n. \quad [5]$$
[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

19. Question part 2/2 | 9231 May-June 2021 P2 V3 Q7

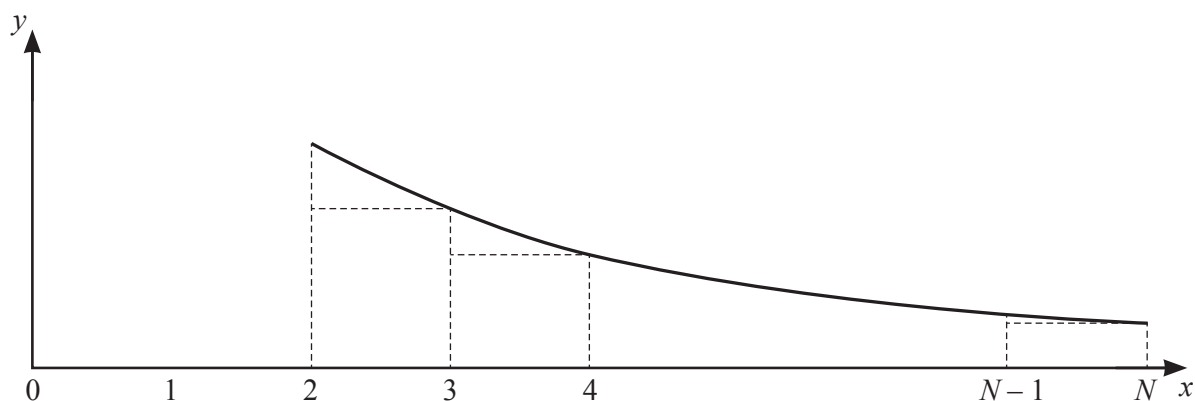
[illegible]

(c) Find the value of I_5 .

[3]

[illegible]

4



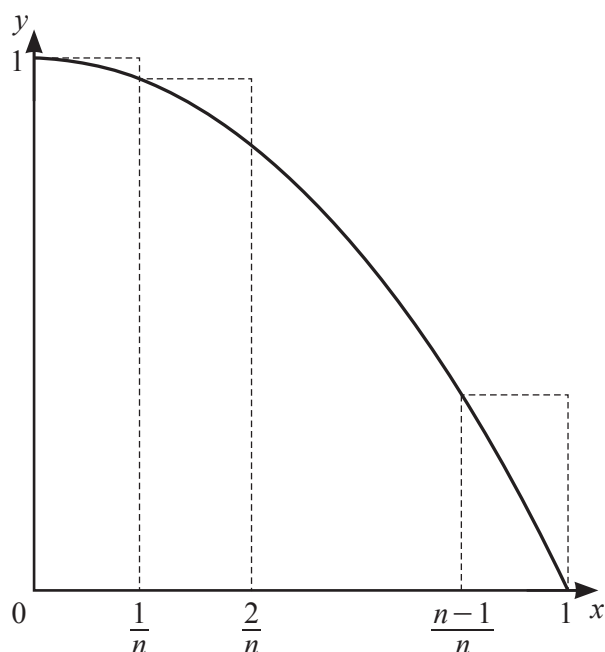
(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{\ln r}{r^2} < \frac{2+3 \ln 2}{4} - \frac{1+\ln N}{N}. \quad [7]$$

[illegible]

Topical questions | 67 questions

(b) Use a similar method to find, in terms of N , a lower bound for $\sum_{r=1}^N \frac{\ln r}{r^2}$. [3]



(a) By considering the sum of the areas of the rectangles, show that

$$\int_0^1 (1-x^2) dx < \frac{4n^2+3n-1}{6n^2}. \quad [4]$$

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Topical questions | 67 questions

21. Question part 2/2 | 9231 Oct-Nov 2021 P2 V2 Q3

[illegible]

(b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 (1-x^2) dx$. [4]

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Topical questions | 67 questions

22. Question part 1/3 | 9231 Oct-Nov 2021 P2 V3 Q8

8 (a) Starting from the definition of cosh in terms of exponentials, prove that

$$2 \cosh^2 A = \cosh 2A + 1. \quad [3]$$

The curve C has parametric equations

$$x = 2 \cosh 2t + 3t, \quad y = \frac{3}{2} \cosh 2t - 4t, \quad \text{for } -\frac{1}{2} \leq t \leq \frac{1}{2}.$$

The area of the surface generated when C is rotated through 2π radians about the y -axis is denoted by A .

(b) (i) Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2 \cosh 2t + 3t) \cosh 2t \, dt$. [4]

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Topical questions | 67 questions

22. Question part 2/3 | 9231 Oct-Nov 2021 P2 V3 Q8

(ii) Hence find A in terms of π and e .

[7]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]

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Topical questions | 67 questions

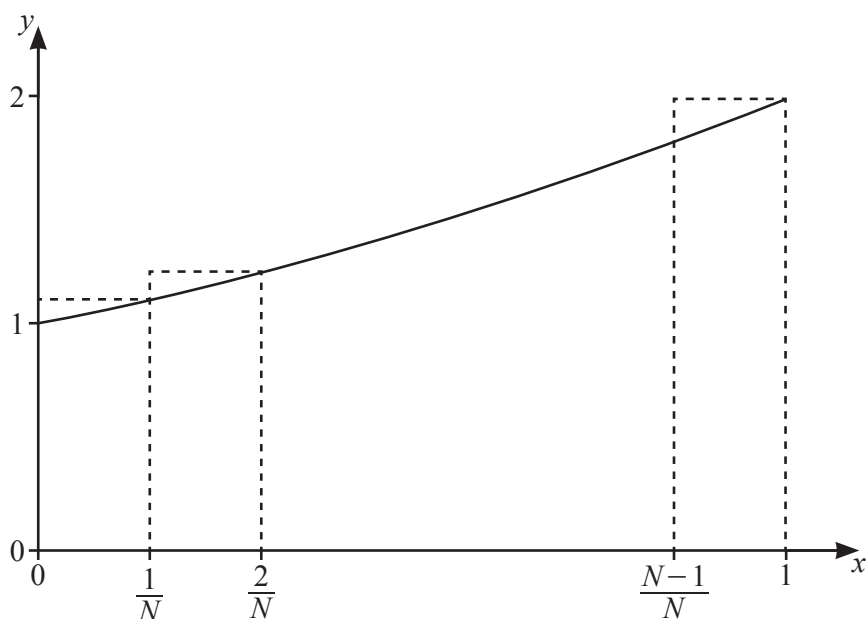
23. Question | 9231 May-June 2022 P2 V1 Q1

- 1 The curve C has polar equation $r = e^{\frac{3}{4}\theta}$ for $0 \leq \theta \leq \alpha$.

Given that the length of C is s , find α in terms of s .

[5]

[illegible]



- $$U_N = \frac{2^{\frac{1}{N}}}{N(2^{\frac{1}{N}} - 1)}. \quad [4]$$

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

25. Question | 9231 May-June 2022 P2 V2 Q1

- 1 The curve C has polar equation $r = e^{\frac{3}{4}\theta}$ for $0 \leq \theta \leq \alpha$.

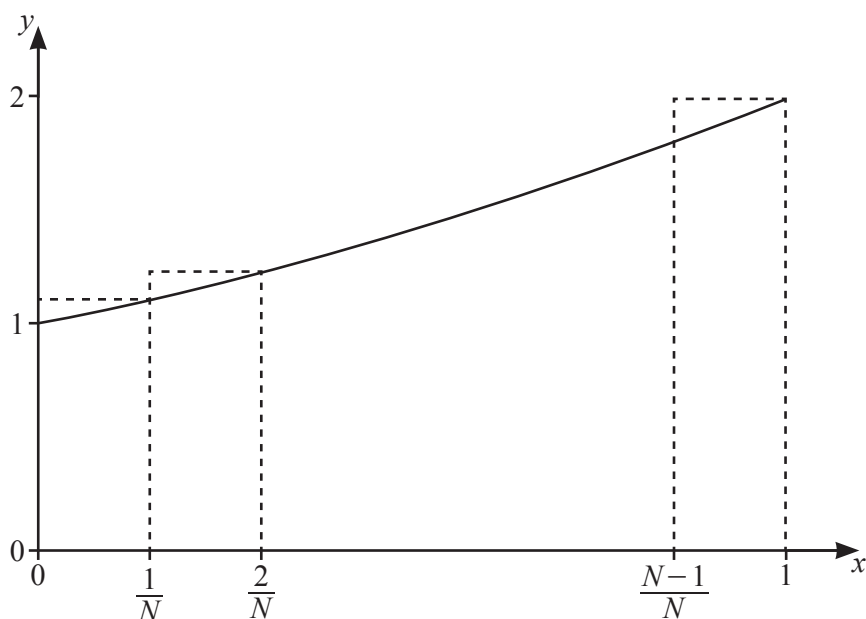
Given that the length of C is s , find α in terms of s .

[5]

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Topical questions | 67 questions

4 The diagram shows the curve with equation $y = 2^x$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.



- $$U_N = \frac{2^{\frac{1}{N}}}{N(2^{\frac{1}{N}} - 1)}. \quad [4]$$

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

26. Question part 2/2 | 9231 May-June 2022 P2 V2 Q4

(b) Use a similar method to find, in terms of N , a lower bound L_N for $\int_0^1 2^x dx$. [4]

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(c) Find the least value of N such that $U_N - L_N < 10^{-4}$. [2]

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Topical questions | 67 questions

27. Question part 1/2 | 9231 May-June 2022 P2 V2 Q7

7 (a) Use de Moivre's theorem to show that

$$\operatorname{cosec} 7\theta = \frac{\operatorname{cosec}^7 \theta}{7 \operatorname{cosec}^6 \theta - 56 \operatorname{cosec}^4 \theta + 112 \operatorname{cosec}^2 \theta - 64}.$$

[6]

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

27. Question part 2/2 | 9231 May-June 2022 P2 V2 Q7

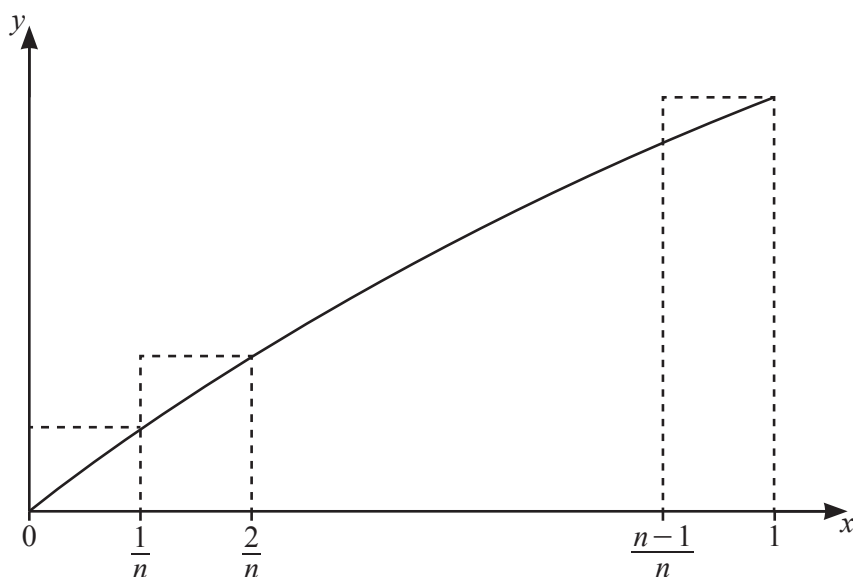
(b) Hence obtain the roots of the equation

$$x^7 - 14x^6 + 112x^4 - 224x^2 + 128 = 0$$

in the form $\operatorname{cosec} q\pi$, where q is rational.

[5]

[illegible]



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \ln(1+x) dx < U_n$, where

$$U_n = \frac{1}{n} \ln \frac{(2n)!}{n!} - \ln n. \quad [4]$$

[illegible]

Topical questions | 67 questions

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \ln(1+x) dx$. [4]

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(c) By simplifying $U_n - L_n$, show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

Topical questions | 67 questions

8 (a) Find $\int \sin \theta \cos^n \theta \, d\theta$, where $n \neq -1$. [2]

Let $I_{m,n} = \int_0^{\frac{1}{2}\pi} \sin^m \theta \cos^n \theta \, d\theta$.

(b) Show that, for $m \geq 2$ and $n \geq 0$,

$$I_{m,n} = \frac{m-1}{m+n} I_{m-2,n}. \quad [5]$$

[illegible]

Topical questions | 67 questions

(c) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)^5$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

where a , b and c are constants to be determined. [5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

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Topical questions | 67 questions

29. Question part 4/4 | 9231 May-June 2022 P2 V3 Q8

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Topical questions | 67 questions

3 The curve C has parametric equations

Find, in terms of e , the length of C . [6]

[illegible]

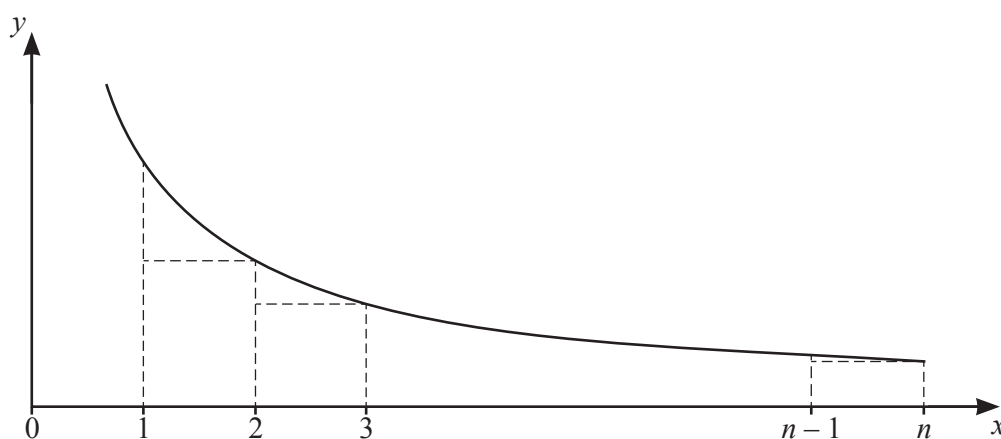
Topical questions | 67 questions

3 (a) A curve has equation $y = e^x + \frac{1}{4}e^{-x}$, for $0 \leq x \leq 1$. Find, in terms of π and e , the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [6]

[illegible]

Topical questions | 67 questions

(b) Using standard results from the list of formulae (MF19), or otherwise, find the Maclaurin's series for $e^x + \frac{1}{4}e^{-x}$ up to and including the term in x^2 . [2]



By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r^2 + 2r}} < \ln(n+1 + \sqrt{n^2 + 2n}) + \frac{1}{3}\sqrt{3} - \ln(2 + \sqrt{3}). \quad [10]$$

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Topical questions | 67 questions

32. Question part 2/2 | 9231 Oct-Nov 2022 P2 V2 Q6

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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

33. Question | 9231 Oct-Nov 2022 P2 V3 Q3

3 The curve C has parametric equations

$$x = e^t - \frac{1}{3}t^3, \quad y = 4e^{\frac{1}{2}t}(t-2), \quad \text{for } 0 \leq t \leq 2.$$

Find, in terms of e , the length of C . [6]

[illegible]

Topical questions | 67 questions

8 (a) Use the substitution $u = 1 - (\theta - 1)^2$ to find

$$\int \frac{\theta-1}{\sqrt{1-(\theta-1)^2}} d\theta. \quad [3]$$

[illegible]
$$\theta \frac{dy}{d\theta} - y = \theta^2 \sin^{-1}(\theta - 1),$$

where $0 < \theta < 2$, given that $y = 1$ when $\theta = 1$. Give your answer in the form $y = f(\theta)$. [11]

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

34. Question part 2/3 | 9231 Oct-Nov 2022 P2 V3 Q8

[illegible]

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Topical questions | 67 questions

4 The integral I_n is defined by $I_n = \int_0^1 (1+x^5)^n dx$.

$$(5n+1)I_n = 2^n + 5nI_{n-1}. \quad [5]$$
[illegible]

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Topical questions | 67 questions

35. Question part 2/2 | 9231 May-June 2023 P2 V1 Q4

(b) Find the exact value of I_3 .

[4]

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Topical questions | 67 questions

4 The integral I_n is defined by $I_n = \int_0^1 (1+x^5)^n dx$.

$$(5n+1)I_n = 2^n + 5nI_{n-1}. \quad [5]$$
[illegible]

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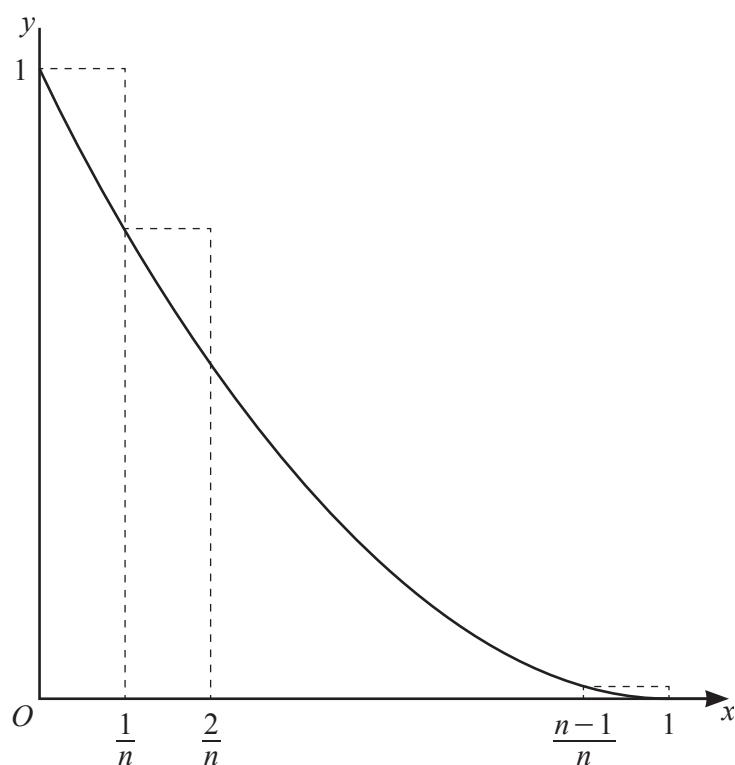
Topical questions | 67 questions

36. Question part 2/2 | 9231 May-June 2023 P2 V2 Q4

(b) Find the exact value of I_3 .

[4]

[illegible]



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (1-x)^2 dx < U_n$, where

$$U_n = \frac{2n^2 + 3n + 1}{6n^2}. \quad [5]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

Topical questions | 67 questions

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (1-x)^2 dx$. [4]

[illegible]

(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

[illegible]

Topical questions | 67 questions

7 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{4}{3}} (1+x^2)^{\frac{1}{2}n} dx$.

- [illegible]

- $$(n+1)I_n = nI_{n-2} + \frac{4}{3}\left(\frac{5}{3}\right)^n. \quad [5]$$

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

38. Question part 2/2 | 9231 May-June 2023 P2 V3 Q7

- (c) A curve has equation $y = x^2$, for $0 \leq x \leq \frac{2}{3}$. The arc length of the curve is denoted by s .

Use the substitution $u = 2x$ to show that $s = \frac{1}{2}I_1$ and find the exact value of s . [4]

[illegible]

Topical questions | 67 questions

5 The curve C has parametric equations

(a) Find the exact length of C . [5]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

39. Question part 2/2 | 9231 Oct-Nov 2023 P2 V1 Q5

(b) Find the set of values of t for which $\frac{d^2y}{dx^2} > 0$.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

Topical questions | 67 questions

5 The curve C has parametric equations

(a) Find the exact length of C . [5]

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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

40. Question part 2/2 | 9231 Oct-Nov 2023 P2 V3 Q5

(b) Find the set of values of t for which $\frac{d^2y}{dx^2} > 0$.

[5]

[illegible]

Topical questions | 67 questions

4 It is given that, for $n \geq 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$.

$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2} \left(\frac{4}{5}\right) + (n-2)I_{n-2}. \quad [5]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9231 Further Mathematics Paper 2 - Integration

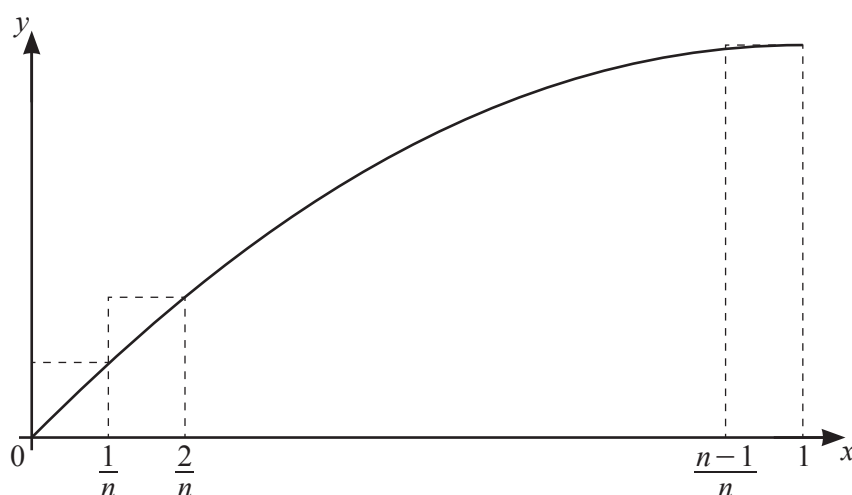
Topical questions | 67 questions

41. Question part 2/2 | 9231 May-June 2024 P2 V1 Q4

(b) Find the value of I_4 .

[3]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (2x - x^2) dx < U_n$, where

$$U_n = \left(1 + \frac{1}{n}\right) \left(\frac{2}{3} - \frac{1}{6n}\right). \quad [5]$$

[illegible]

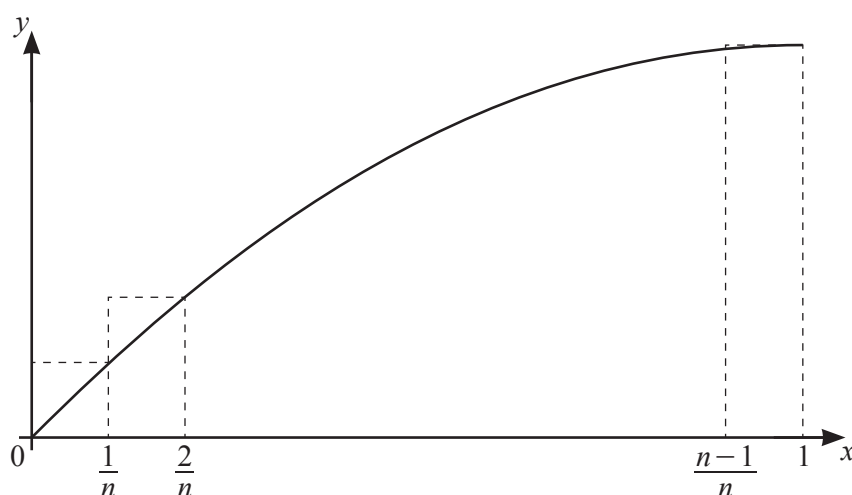
Topical questions | 67 questions

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the entire width of the page. There is no handwriting or other markings on the paper.

Topical questions | 67 questions

4 It is given that, for $n \geq 0$, $I_n = \int_0^{\ln 3} \operatorname{sech}^n x \, dx$.

$$(n-1)I_n = \left(\frac{3}{5}\right)^{n-2} \left(\frac{4}{5}\right) + (n-2)I_{n-2}. \quad [5]$$



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 (2x - x^2) dx < U_n$, where

$$U_n = \left(1 + \frac{1}{n}\right) \left(\frac{2}{3} - \frac{1}{6n}\right). \quad [5]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical questions | 67 questions

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 (2x - x^2) dx$. [4]

[illegible]

(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

[illegible]

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

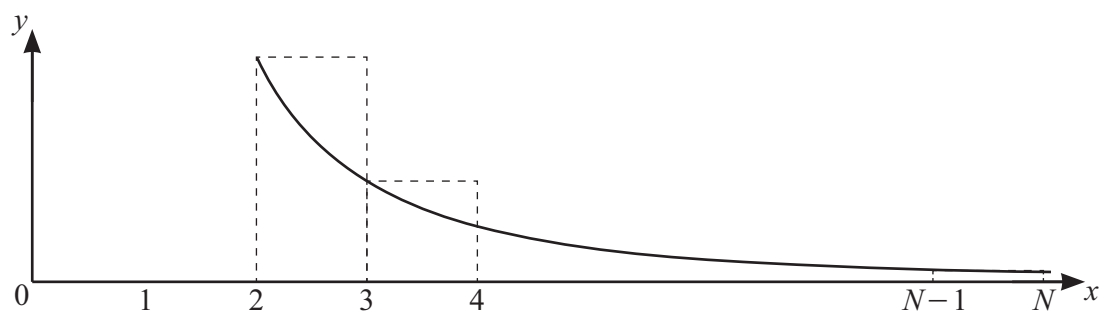
45. Question | 9231 May-June 2024 P2 V3 Q1

1 Find the exact value of $\int_2^{\frac{5}{2}} \frac{1}{\sqrt{4x-x^2-1}} dx$.

[5]

[illegible]

4



The diagram shows the curve with equation $y = x^{-2}$ for $2 \leq x \leq N$ together with a set of $(N-2)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^N \frac{1}{r^2} > \frac{3}{2} - \frac{1}{N} + \frac{1}{N^2}. \quad [5]$$

[illegible]

[illegible]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the entire width of the page. There is no handwriting or other markings on the paper.

Topical questions | 67 questions

3 A curve has equation $y = e^x$ for $\ln \frac{4}{3} \leq x \leq \ln \frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x -axis is denoted by A .

$$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2} du. \quad [2]$$
$$A = \pi \left(\frac{904}{225} + \ln \frac{5}{3} \right). \quad [6]$$

9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

47. Question part 2/2 | 9231 Oct-Nov 2024 P2 V1 Q3

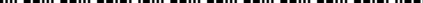
[illegible]

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{2}\right)^x dx > L_N$, where

$$L_N = \frac{1}{2N(2^{\frac{1}{N}} - 1)}. \quad [4]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Topical questions | 67 questions

[illegible][illegible]



(d) Given that $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x = \frac{1}{2\ln 2}$, use the value of N found in part (c) to find upper and lower bounds for $\ln 2$. [4]

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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

49. Question | 9231 Oct-Nov 2024 P2 V2 Q1

- 1 Find the value of $\int_6^7 \frac{1}{\sqrt{(x-5)^2-1}} dx$, giving your answer in the form $\ln(a + \sqrt{b})$, where a and b are integers to be determined. [4]

[illegible]

Topical questions | 67 questions

50. Question part 1/2 | 9231 Oct-Nov 2024 P2 V2 Q3

$$x = \frac{1}{2}e^{2t} - \frac{1}{3}t^3 - \frac{1}{2}, \quad y = 2e^t(t-1), \quad \text{for } 0 \leq t \leq 1.$$
[illegible]

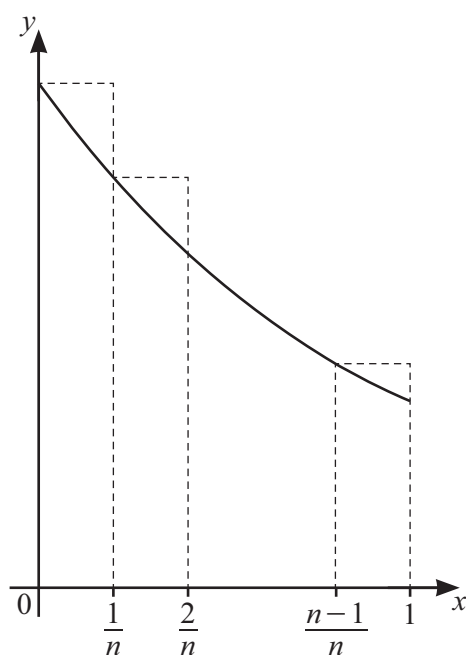
9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

50. Question part 2/2 | 9231 Oct-Nov 2024 P2 V2 Q3

[illegible]

6



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 e^{1-x} dx < U_n$, where

$$U_n = \frac{e-1}{n(1-e^{-\frac{1}{n}})}. \quad [4]$$

[illegible]

Topical questions | 67 questions



This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no margins or additional markings.

[illegible]

Topical questions | 67 questions

- [illegible]

Topical questions | 67 questions

52. Question part 1/2 | 9231 Oct-Nov 2024 P2 V3 Q3

- 3** A curve has equation $y = e^x$ for $\ln \frac{4}{3} \leq x \leq \ln \frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x -axis is denoted by A .

(a) Use the substitution $u = e^x$ to show that

$$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2} du. \quad [2]$$

[illegible]

(b) Use the substitution $u = \sinh v$ to show that

$$A = \pi \left(\frac{904}{225} + \ln \frac{5}{3} \right). \quad [6]$$

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

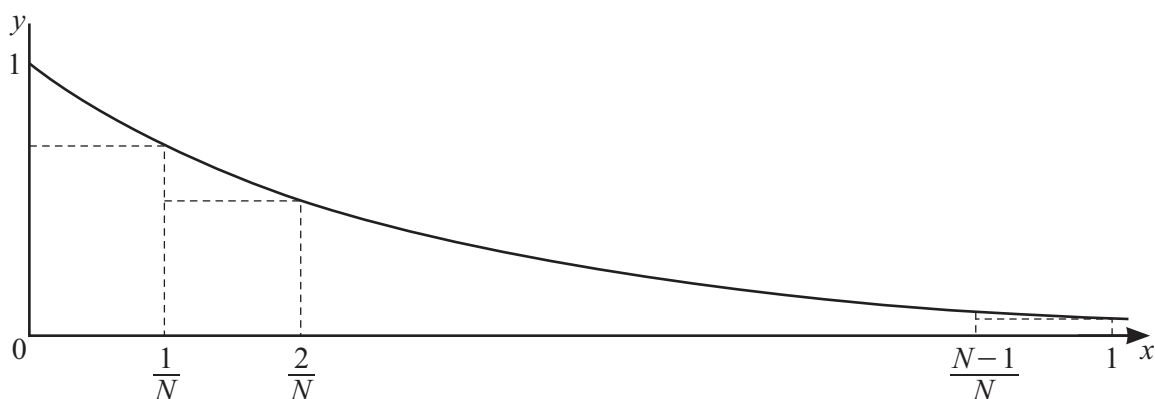
9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

52. Question part 2/2 | 9231 Oct-Nov 2024 P2 V3 Q3

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

6



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{2}\right)^x dx > L_N$, where

$$L_N = \frac{1}{2N(2^{\frac{1}{N}} - 1)}. \quad [4]$$

[illegible]

Topical questions | 67 questions

[illegible]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.



(d) Given that $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x = \frac{1}{2\ln 2}$, use the value of N found in part (c) to find upper and lower bounds for $\ln 2$. [4]

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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

54. Question | 9231 May-June 2025 P2 V1 Q2

2 Let $I_n = \int_0^1 (1-x)^n \sinh x \, dx$, where n is a non-negative integer.

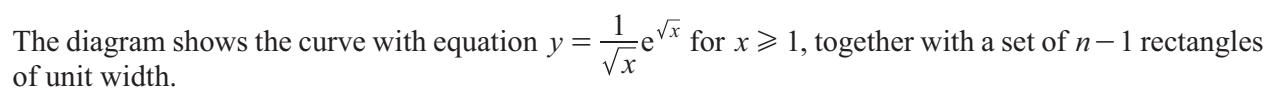
(a) Show that, for $n \geq 2$, $I_n = -1 + n(n-1)I_{n-2}$. [4]

[illegible]

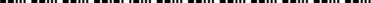
(b) Find the exact value of I_2 . [3]

[illegible]

4


$$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e. \quad [5]$$
[illegible]

Topical questions | 67 questions



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(b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]

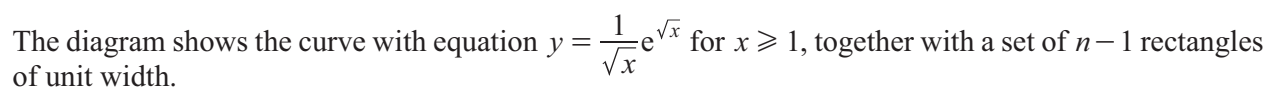
[illegible]

Topical questions | 67 questions

2 Let $I_n = \int_0^1 (1-x)^n \sinh x \, dx$, where n is a non-negative integer.

[illegible][illegible]

4


$$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e. \quad [5]$$
This image shows a full page of white paper with ten evenly spaced horizontal dashed lines, typical of primary school handwriting practice paper. The lines extend across the entire width of the page.

Topical questions | 67 questions

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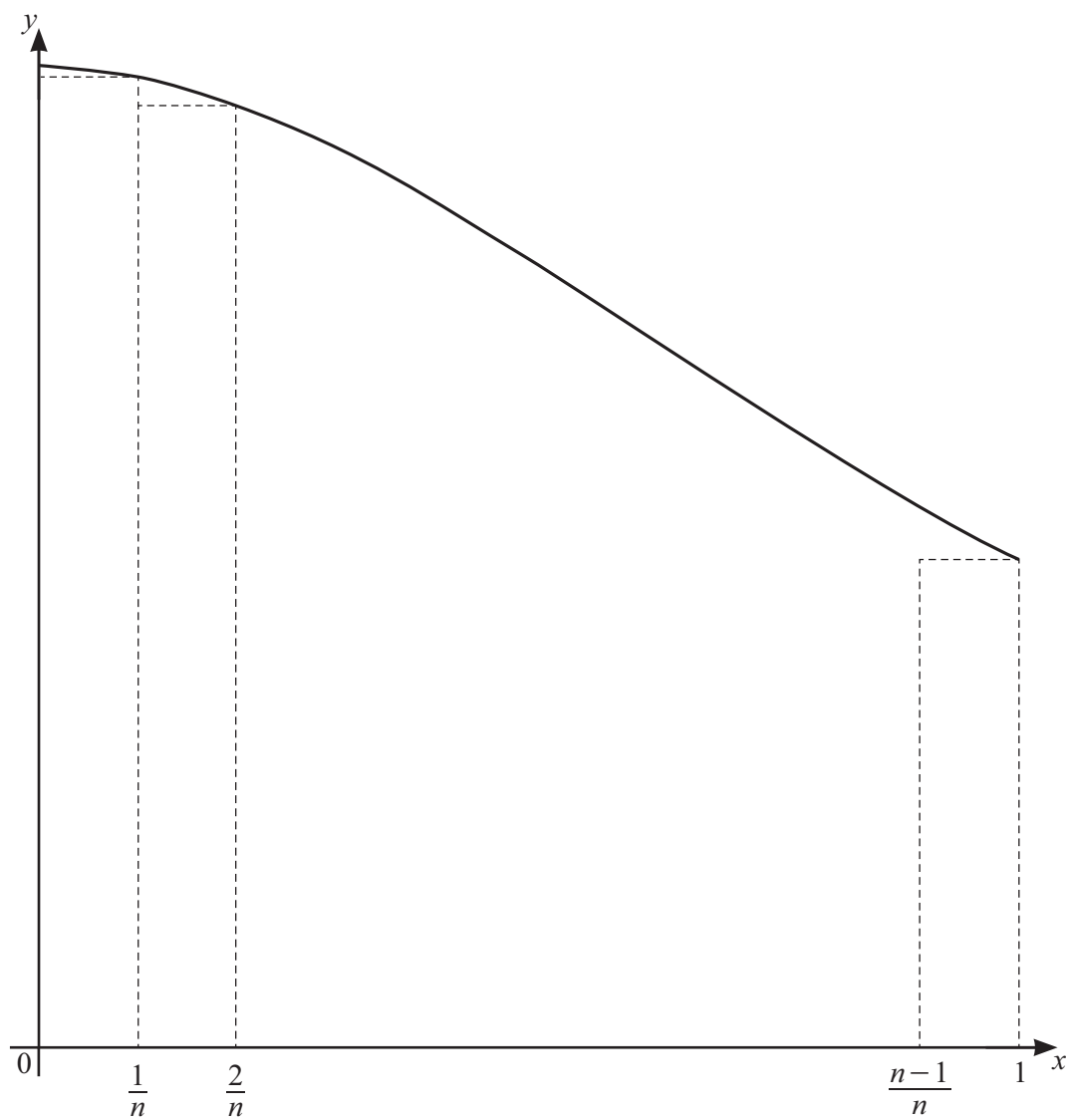
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(b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

58. Question part 1/2 | 9231 May-June 2025 P2 V3 Q6

6



The diagram shows the curve with equation $y = \frac{1}{x^2 + 1}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{n}{n^2 + r^2} < \frac{1}{4}\pi.$$

[5]

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58. Question part 2/2 | 9231 May-June 2025 P2 V3 Q6



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(b) Use a similar method to find a lower bound for $\sum_{r=1}^n \frac{n}{n^2 + r^2}$. Give your answer in terms of n and π . [4]

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(c) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2}$. [1]

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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

59. Question | 9231 May-June 2025 P2 V4 Q2

2

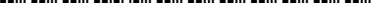
This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

Topical questions | 67 questions

3 The integral I_n is defined by $I_n = \int_0^1 (1+x^2)^n dx$.

$$(2n+1)I_n = 2^n + 2nI_{n-1}. \quad [5]$$
[illegible]

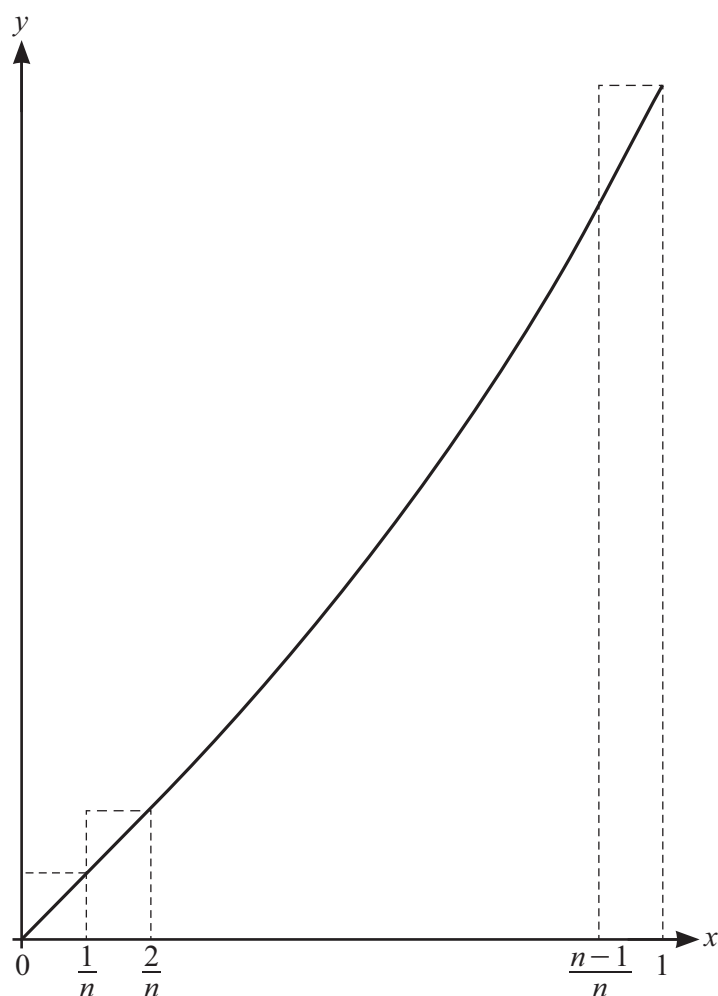
Topical questions | 67 questions



[4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

5



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx < U_n$, where

$$U_n = \frac{1}{12} \left(1 + \frac{1}{n} \right) \left(7 + \frac{1}{n} \right). \quad [5]$$

[illegible]

61. Question part 2/2 | 9231 Oct-Nov 2025 P2 V1 Q5



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(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \left(\frac{1}{3}x^3 + x\right) \mathrm{d}x$. [4]

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(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

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9231 Further Mathematics Paper 2 - Integration

Topical questions | 67 questions

62. Question | 9231 Oct-Nov 2025 P2 V2 Q1

- 1** The curve C has polar equation $r = e^{\frac{4}{3}\theta}$ for $0 \leq \theta \leq \pi$.

Find the length of C . Give your answer in an exact form.

[4]

[illegible]

63. Question part 1/2 | 9231 Oct-Nov 2025 P2 V2 Q6

- 6 (a)** Use the substitution $x = \frac{1}{2}\sqrt{2} \sinh u$ to find $\int \frac{1}{\sqrt{2x^2+1}} dx$. [3]

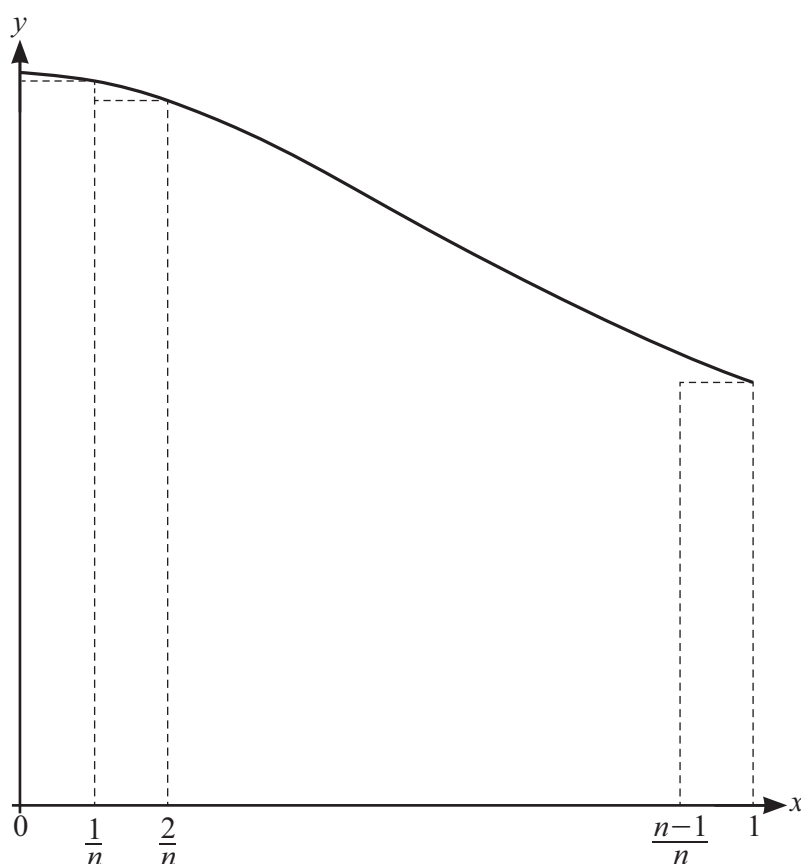
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The diagram shows the curve with equation $y = \frac{1}{\sqrt{2x^2+1}}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (b)** By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} < \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3}).$$
 [5]

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63. Question part 2/2 | 9231 Oct-Nov 2025 P2 V2 Q6



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(c) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}}$. [4]

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(d) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}}$. [1]

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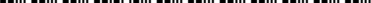
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Topical questions | 67 questions

3 The integral I_n is defined by $I_n = \int_0^1 (1+x^2)^n dx$.

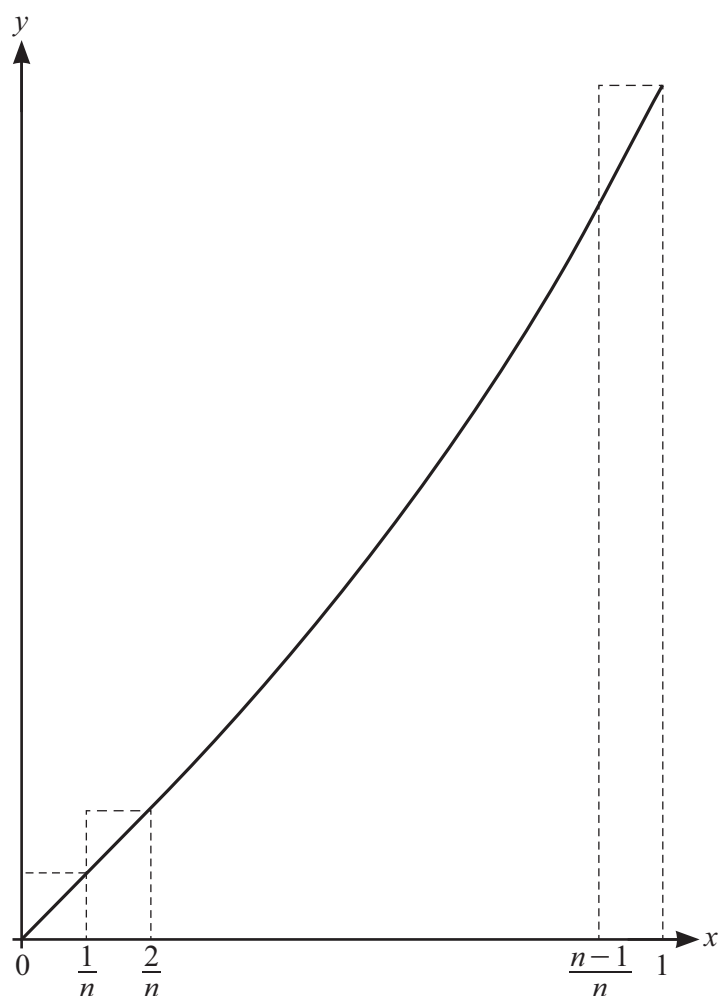
$$(2n+1)I_n = 2^n + 2nI_{n-1}. \quad [5]$$
[illegible]

Topical questions | 67 questions



[4]

[illegible]



(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx < U_n$, where

$$U_n = \frac{1}{12} \left(1 + \frac{1}{n} \right) \left(7 + \frac{1}{n} \right). \quad [5]$$

[illegible]

65. Question part 2/2 | 9231 Oct-Nov 2025 P2 V3 Q5



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(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \left(\frac{1}{3}x^3 + x\right) \mathrm{d}x$. [4]

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(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

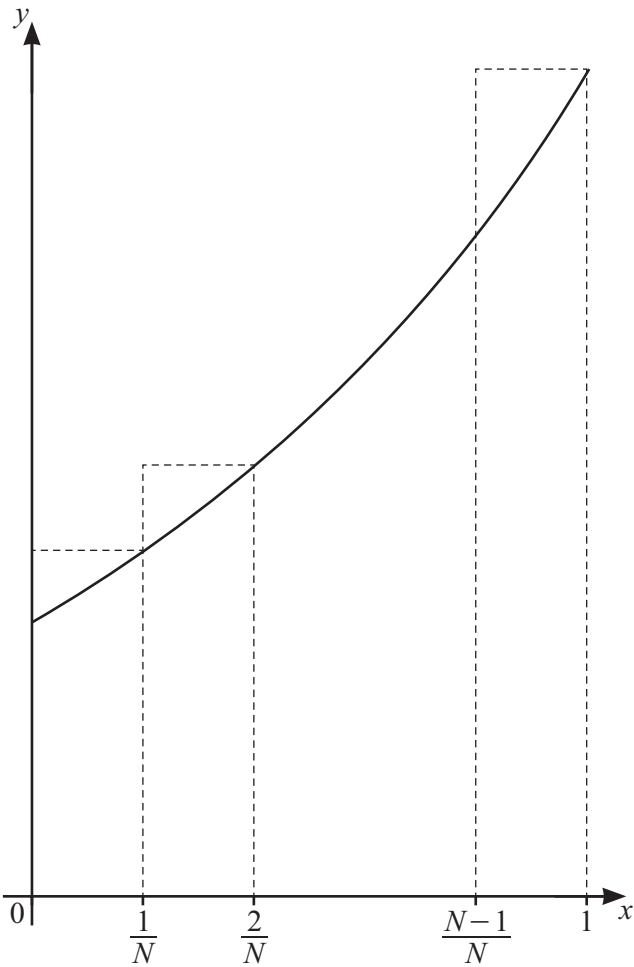
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66. Question part 1/2 | 9231 Oct-Nov 2025 P2 V4 Q4

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The diagram shows the curve with equation $y = \frac{1}{2}(3^x)$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \frac{1}{2}(3^x)dx < U_N$, where

$$U_N = \frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)}.$$

[4]

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66. Question part 2/2 | 9231 Oct-Nov 2025 P2 V4 Q4



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(b) Use a similar method to find, in terms of N , a lower bound L_N for $\int_0^1 \frac{1}{2}(3^x) \mathrm{d}x$. [4]

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(c) By simplifying $U_N - L_N$, show that $\lim_{N \rightarrow \infty} (U_N - L_N) = 0$. [2]

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Topical questions | 67 questions

67. Question part 1/2 | 9231 Oct-Nov 2025 P2 V4 Q7

7

$$x = 8 \ln(\tan \frac{1}{2}t) - 3 \cot t - 3t, \quad y = 6 \ln(\tan \frac{1}{2}t) + 4 \cot t + 4t, \quad \text{for } \frac{1}{6}\pi \leq t \leq \frac{1}{3}\pi.$$

(a) (i) Show that $\frac{dx}{dt} = 8 \operatorname{cosec} t + 3 \cot^2 t$. [1]

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(ii) Find $\frac{dy}{dt}$. [1]

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(iii) Find the exact value of the length of C . [6]

[illegible]

Topical questions | 67 questions


$$\frac{d^2y}{dx^2} = \frac{a \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(b \operatorname{cosec} t + c \cot^2 t)^n},$$

[5]

[illegible]