

Ideal Gases

Chapter 05 / A2 Level / Apollo Physics 9702



Assumptions, Boyle's Law and Charles' Law

Assumptions for an ideal gas according to kinetic molecular theory of matter

1. Gas molecules are in continuous random motion.
2. They move in straight lines.
3. Molecules have negligible intermolecular force of attraction.
4. Gas molecules behave as hard, elastic spheres.
5. Collisions between/of molecules are perfectly elastic.
6. Time of collision is negligible compared to time between collisions.
7. Volume of gas molecules is negligible compared with volume of gas.

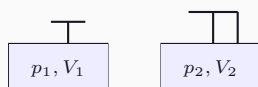
Boyle's law (p - V)

For a fixed mass of gas at constant temperature, volume is inversely proportional to pressure.

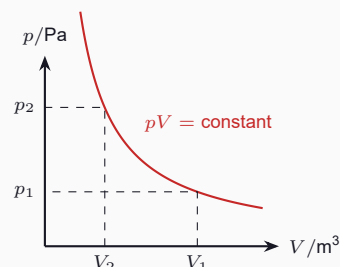
$$V \propto \frac{1}{p}$$

$$pV = \text{constant}$$

$$p_1 V_1 = p_2 V_2$$



When volume decreases, molecules come closer to one another, frequency of collisions with the wall increases, force exerted by molecules increases, hence pressure increases.



At the same volume, if $T_2 > T_1$, then $p_2 > p_1$. **Charles' law** (V - T)

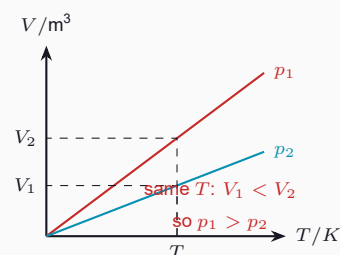
For a fixed mass of gas at constant pressure, volume is directly proportional to absolute temperature.

$$V \propto T$$

$$V = kT$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where T is in Kelvin.



Avogadro's Law and the Ideal Gas Equation

Avogadro's law (V - n)

For a gas at constant temperature and pressure, volume is directly proportional to the number of moles.

$$V \propto n$$

$$V = kn$$

$$\frac{V}{n} = \text{constant}$$

$$pV = nRT \text{ and } n = N/N_A,$$

$$pV = \left(\frac{N}{N_A}\right) RT = NkT, \quad k = \frac{R}{N_A}.$$

$$pV = NkT$$

$$\text{Boltzmann constant : } k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Ideal gas equation

Combining the proportionalities,

$$V \propto \frac{nT}{p} \Rightarrow V = R \frac{nT}{p}$$

$$pV = nRT$$

- p : pressure / Pa.
- V : volume / m^3 .
- n : number of moles / mol.
- T : absolute temperature / K.
- $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$.

$$n = \frac{\text{mass}}{A_r \text{ or } M_r} \quad (\text{mass in grams}), \quad N = nN_A,$$

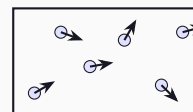
where N is number of particles and $N_A = 6.02 \times 10^{23}$. As

Define an ideal gas

Any gas that obeys $pV = nRT$ or $pV = NkT$ at all temperatures, pressures and volumes is called an ideal gas.

How gases exert pressure

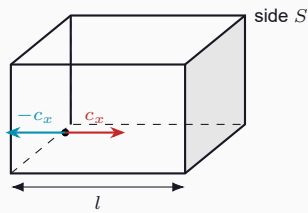
- Gas molecules are in continuous random motion.
- They collide with one another and with the walls of the container.
- Molecules undergo rate of change of momentum, producing force.
- Force per unit area is the pressure of the gas.



continuous random motion

Deriving $p = \frac{1}{3}\rho\langle c^2 \rangle$ – Momentum Argument

Set-up



A molecule of mass m moves inside a cube of side l . The component of velocity along the x -axis is c_x .

$$p_x = mc_x.$$

After collision with the opposite wall, its velocity component becomes $-c_x$.

$$\Delta p = m(v - u) = m(-c_x - c_x) = -2mc_x.$$

The magnitude of the change in momentum is $2mc_x$.

Momentum $\rightarrow$ change in momentum $\rightarrow$ force $\rightarrow$ pressure

Force on a wall

$$F = \frac{\Delta p}{\Delta t}.$$

The time between successive collisions with the same wall is the time to travel $2l$:

$$c_x = \frac{2l}{\Delta t} \quad \Rightarrow \quad \Delta t = \frac{2l}{c_x}.$$

Hence,

$$F_x = \frac{2mc_x}{2l/c_x} = \frac{mc_x^2}{l}.$$

For pressure we take the magnitude:

$$F_x = \frac{mc_x^2}{l}$$

Pressure on the wall is

$$p = \frac{F}{A} = \frac{mc_x^2/l}{l^2} = \frac{mc_x^2}{l^3} = \frac{mc_x^2}{V}.$$

For N molecules,

$$pV = Nmc_x^2$$

This is the pressure of the gas due to all molecules having speed component in the x -axis. **Important:** the same average pressure is obtained on all walls because random motion makes the motion equally likely in all directions.

Mean Square Speed and Temperature

Resolving velocity,

$$\vec{c} = \vec{c}_x + \vec{c}_y + \vec{c}_z, \quad c^2 = c_x^2 + c_y^2 + c_z^2.$$

Due to random motion, molecules are equally likely to move in all directions, so the average values along the axes are the same:

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle.$$

Thus,

$$\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle = 3\langle c_x^2 \rangle,$$

$$\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$$

From $pV = Nm\overline{c_x^2}$,

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle.$$

Since $\rho = Nm/V$,

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

Here ρ is density and $\langle c^2 \rangle$ is mean-square speed. **Root-**

mean-square speed

values: $-4, 3, 5, -6, 1$

square, mean, then square-root
negative signs are eliminated

$$c_{\text{rms}} = \sqrt{\langle c^2 \rangle}$$

A simple average of positive and negative velocity values can be misleading because opposite signs can cancel. Squaring first removes the sign. **Relating mean kinetic energy to temperature**

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle, \quad pV = NkT.$$

Therefore,

$$3pV = Nm\langle c^2 \rangle, \quad 3NkT = Nm\langle c^2 \rangle,$$

$$\frac{3}{2}kT = \frac{1}{2}m\langle c^2 \rangle.$$

$$\langle E_k \rangle = \frac{3}{2}kT$$

$$\sqrt{T} \propto c_{\text{rms}}$$

Temperature is the measurement of mean kinetic energy of gas molecules

Temperature Example and First Law of Thermodynamics

Example

Gas molecules at a temperature of 100°C have a c_{rms} of 450 m s^{-1} . What will be the c_{rms} at 250°C ?

$$100 + 273 = 373\text{ K}, \quad 250 + 273 = 523\text{ K}.$$

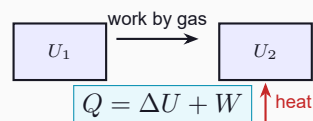
Since $\sqrt{T} \propto c_{\text{rms}}$,

$$\frac{\sqrt{373}}{\sqrt{523}} = \frac{450}{x}$$

$$x = 450 \frac{\sqrt{523}}{\sqrt{373}} = 532.8 \approx \boxed{533\text{ m s}^{-1}}.$$

Thermodynamics

Study of processes and systems that involve heat and work, and conversion between them.



First law of thermodynamics – statement 1

Heat supplied to a system is equal to the sum of increase in its internal energy and work done by the system.

$$Q = \Delta U + W$$

Here W is work done *by* the gas. **Statement 2**

The sum of heat supplied to a system and work done on the system is equal to the increase in its internal energy.

$$Q + W_{\text{on}} = \Delta U$$

Internal Energy and Thermodynamic Processes

Internal energy (U)

The sum of microscopic kinetic and potential energy of gas molecules due to random motion.

$$U = \text{K.E.} + \text{P.E.}$$

For an ideal gas, there is negligible intermolecular force of attraction. Hence there is negligible potential energy between ideal-gas molecules.

- Internal energy of an ideal gas depends only on kinetic energy of molecules.
- $\langle E_k \rangle = \frac{3}{2}kT$.
- Change in kinetic energy is directly proportional to change in temperature.
- Therefore change in internal energy is directly proportional to change in temperature.

$$\Delta U \propto \Delta T \quad \text{for an ideal gas}$$

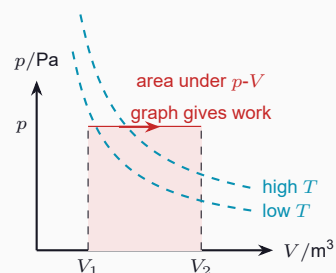
Thermodynamic processes

1. **Constant pressure (isobaric).** Work done at constant pressure:

$$W = p\Delta V$$

$$Q = \Delta U + p\Delta V.$$

Area under a p - V graph gives the work done by the gas.



2. **Constant volume (isochoric).** Since $\Delta V = 0$,

$$W = p\Delta V = 0, \quad Q = \Delta U.$$

All heat supplied is stored as internal energy. Temperature rises and pressure rises due to increase in molecular speed.

Isothermal, Adiabatic and Cycles

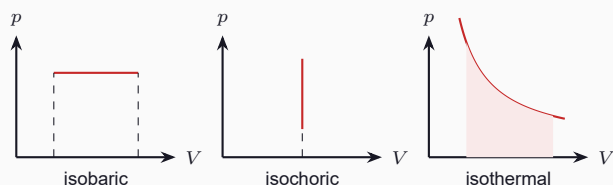
3. **Constant temperature (isothermal).** Since T is constant,

$$\Delta T = 0 \Rightarrow \Delta U = 0.$$

From $Q = \Delta U + W$,

$$Q = W$$

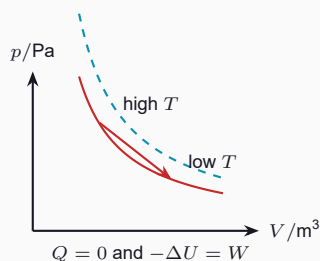
Heat supplied is equal to work done by the gas.



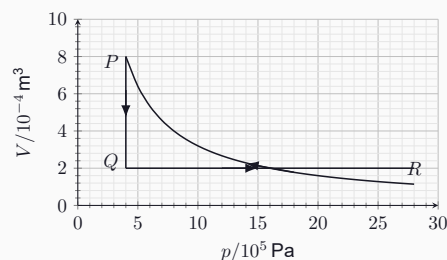
4. **No heat transfer (adiabatic).** Since $Q = 0$,

$$0 = \Delta U + W \Rightarrow -\Delta U = W.$$

Gas cools down when it does work. Gas heats up when work is done on it.



Thermodynamic cycle



A cycle starts and ends at the same temperature, so

$$\Delta T_{\text{cycle}} = 0$$

$$\Delta U_{\text{cycle}} = 0$$

For a closed cycle,

$$\Delta U_{1 \rightarrow 2} + \Delta U_{2 \rightarrow 3} + \Delta U_{3 \rightarrow 4} + \Delta U_{4 \rightarrow 1} = 0.$$

On a p - V graph, the enclosed area represents the net work done in one complete cycle.

Exam Practice: Neon Gas and RMS Speed

(a) Explain qualitatively how molecular movement causes the pressure exerted by a gas.

Gas molecules are in continuous random motion. Upon collision with one another and with the walls of the container, they undergo a rate of change of momentum. This produces a force, and force per unit area is pressure.

Since $c_{\text{rms}} \propto \sqrt{T}$,

$$\frac{583}{x} = \frac{\sqrt{273}}{\sqrt{546}}.$$

$$x = 583 \frac{\sqrt{546}}{\sqrt{273}} = 824.5 \approx \boxed{825 \text{ m s}^{-1}}.$$

Useful boxed result

$$c_{\text{rms}} = \sqrt{\frac{3p}{\rho}}$$

This follows immediately from $p = \frac{1}{3}\rho\langle c^2 \rangle$.

(b) Neon gas

The density of neon gas at a temperature of 273 K and pressure $1.02 \times 10^5 \text{ Pa}$ is 0.900 kg m^{-3} . Neon may be assumed to be an ideal gas. Calculate the r.m.s. speed at: (i) 273 K

$$p = \frac{1}{3}\rho\langle c^2 \rangle.$$

$$1.02 \times 10^5 = \frac{1}{3}(0.900)\langle c^2 \rangle.$$

$$c_{\text{rms}} = \sqrt{\langle c^2 \rangle} = \boxed{583 \text{ m s}^{-1}}.$$

(ii) 546 K

Exam Practice: Density at Constant Temperature

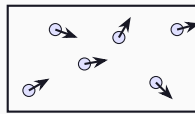
(c) Effect of changing density at constant temperature

The calculations in (b) are based on density 0.900 kg m^{-3} . Suggest the effect, if any, on the root-mean-square speed of changing the density at constant temperature.

The r.m.s. speed stays the same because it only depends on temperature. Changing density will change the pressure of the gas.

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

$$c_{\text{rms}} \propto \sqrt{T}$$



continuous random motion

Exam Practice: Kinetic Theory Assumptions

Question. The kinetic theory of gases is based on some simplifying assumptions. The molecules of the gas are assumed to behave as hard elastic identical spheres. State the assumption about ideal gas molecules based on:

1. the nature of their movement;
2. their volume.

Answer.

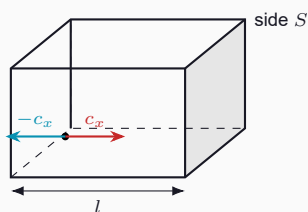
1. Molecules are in continuous random motion and move in straight lines between collisions.
2. The volume of the molecules is negligible compared with the volume occupied by the gas.

Assumption	Consequence
Elastic collisions	no kinetic energy lost during collision
Negligible intermolecular forces	potential energy is zero for an ideal gas
Negligible molecular volume	volume of gas is the container volume
Random motion	equal pressure in all directions

Exam Practice: Cube Derivation and Temperature Change

Cube of volume V with N molecules

Each molecule has component c_x of velocity normal to one side S of the cube.



The molecules of an ideal gas have a root-mean-square speed of 520 m s^{-1} at 27°C . Calculate the r.m.s. speed at 100°C .

$$27^\circ\text{C} = 300 \text{ K}, \quad 100^\circ\text{C} = 373 \text{ K}.$$

$$\frac{x}{520} = \sqrt{\frac{373}{300}}.$$

$$x = 520 \sqrt{\frac{373}{300}} = 580 \text{ m s}^{-1} \text{ (approximately).}$$

The pressure due to the component c_x is given by

$$pV = Nm c_x^2$$

Explain how this leads to

$$pV = \frac{1}{3} Nm \langle c^2 \rangle.$$

Answer

$$\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle.$$

Random motion gives

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle,$$

so

$$\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle.$$

Therefore,

$$pV = Nm \langle c_x^2 \rangle = \frac{1}{3} Nm \langle c^2 \rangle.$$

RMS speed temperature question

$$c_{\text{rms}} \approx 580 \text{ m s}^{-1}$$

Exam Practice: Internal Energy and p - V Cycle

(a) State the basic assumption of the kinetic theory of gases that leads to the conclusion that the potential energy between atoms of an ideal gas is zero.

There is negligible intermolecular force of attraction.

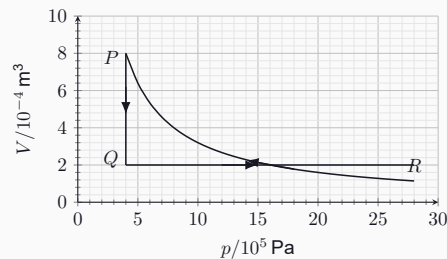
(ii) State what is meant by the internal energy of a substance.

The sum of the microscopic kinetic and potential energies of molecules due to random motion.

(iii) Explain why an increase in internal energy of an ideal gas is directly related to a rise in temperature.

For an ideal gas, internal energy only depends on kinetic energy of molecules. Change in kinetic energy is directly proportional to change in temperature.

A fixed mass of ideal gas undergoes a cycle $P \rightarrow Q \rightarrow R \rightarrow P$.



Read from the graph approximately:

$$P : (4, 8), \quad Q : (4, 2), \quad R : (28, 2).$$

The vertical branch $P \rightarrow Q$ is constant pressure when axes are interpreted in the original graph, while $Q \rightarrow R$ is constant volume/pressure depending on plotted axes. The key calculation below follows the handwritten solution on the topic.

Exam Practice: Cycle Energy Table

(i) Change in internal energy over one complete cycle

$$\Delta U_{\text{cycle}} = 0 \text{ J}$$

(ii) Work done on the gas during $P \rightarrow Q$

$$W = p\Delta V = p(V_2 - V_1) = (4 \times 10^5)(2 - 8) \times 10^{-4} = -240 \text{ J}.$$

For work done *on* the gas,

$$W_{\text{on}} = +240 \text{ J}$$

(iii) Complete the energy table using $Q + W_{\text{on}} = \Delta U$

Change	work done on gas / J	heating supplied to gas / J	increase in internal energy / J
$P \rightarrow Q$	+240	−600	−360
$Q \rightarrow R$	0	+720	+720
$R \rightarrow P$	−840	+480	−360

Checks:

$$-360 + 720 - 360 = 0,$$

so the total change in internal energy over the cycle is zero.

Exam Practice: Kinetic Energy of a Molecule and Heating a Cylinder

(a) Use one of the assumptions of the kinetic theory to explain why the potential energy of the molecules of an ideal gas is zero.

Ideal-gas molecules have negligible intermolecular forces, so there is no intermolecular potential energy.

(b) The average translational kinetic energy E_k of a molecule of an ideal gas is

$$E_k = \frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT.$$

Meanings:

- $\langle c^2 \rangle$: mean-square speed of the molecules.
- T : thermodynamic temperature in Kelvin.

(c) **Cylinder heating question**

A cylinder of constant volume $4.7 \times 10^4 \text{ cm}^3$ contains an ideal gas at pressure $2.6 \times 10^5 \text{ Pa}$ and temperature 173°C . The gas is heated. The thermal energy transferred to the gas is 2900 J.



Since $173^\circ\text{C} = 446 \text{ K}$ and $4.7 \times 10^4 \text{ cm}^3 = 4.7 \times 10^{-2} \text{ m}^3$,

$$N = \frac{pV}{kT} = \frac{(2.6 \times 10^5)(4.7 \times 10^{-2})}{(1.38 \times 10^{-23})(446)}.$$

$$N \approx 1.99 \times 10^{24} \text{ molecules}$$

Exam Practice: Ideal Gas Definition and RMS Speed

(a) State what is meant by an ideal gas.

Any gas that obeys $pV = nRT$ or $pV = NkT$ at all temperatures, pressures and volumes.

$$c_{\text{rms}} = \sqrt{\frac{3pV}{m}}$$

(b) An ideal gas comprised of single atoms is contained in a cylinder and has volume $1.84 \times 10^{-2} \text{ m}^3$ at pressure $2.12 \times 10^7 \text{ Pa}$. The mass of gas in the cylinder is 3.20 kg . Determine the r.m.s. speed of the atoms.

$$c_{\text{rms}} = \sqrt{\frac{3(2.12 \times 10^7)(1.84 \times 10^{-2})}{3.20}}$$

$$= 605 \text{ m s}^{-1}.$$

$$p = \frac{1}{3}\rho\langle c^2 \rangle, \quad \rho = \frac{m}{V} = \frac{3.20}{1.84 \times 10^{-2}}.$$

$$2.12 \times 10^7 = \frac{1}{3} \left(\frac{3.20}{1.84 \times 10^{-2}} \right) \langle c^2 \rangle.$$

$$c_{\text{rms}} = 604.7 \approx \boxed{605 \text{ m s}^{-1}}.$$

Memory hook

Use $p = \frac{1}{3}\rho\langle c^2 \rangle$ when density/mass and volume are given. Use $\sqrt{T} \propto c_{\text{rms}}$ when temperature changes.

Exam Practice: Amount of Gas, Mass of Atom and Nucleon Number

The temperature of the gas in the cylinder is 22°C , hence

$$T = 22 + 273 = 295 \text{ K}.$$

1. Amount of gas

$$pV = nRT$$

$$n = \frac{pV}{RT} = \frac{(2.12 \times 10^7)(1.84 \times 10^{-2})}{(8.314)(295)} = 159.04.$$

$$n = 159 \text{ mol}$$

2. Mass of one atom

$$N = nN_A = 159.04(6.02 \times 10^{23}).$$

$$\text{mass of one atom} = \frac{3.20}{159.04(6.02 \times 10^{23})}.$$

$$m_{\text{atom}} = 3.34 \times 10^{-26} \text{ kg}$$

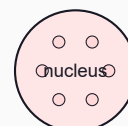
(c) Nucleon number A

Use the mass of one nucleon:

$$1u = 1.66 \times 10^{-27} \text{ kg}.$$

$$A = \frac{3.34 \times 10^{-26}}{1.66 \times 10^{-27}} = 20.1 \approx 20.$$

$$A = 20$$



A = number of nucleons

Exam Practice: Mean Kinetic Energy and Heat Capacity at Constant Volume

(a) The pressure p and volume V of an ideal gas are related to density ρ by

$$p = \frac{1}{3}\rho\langle c^2 \rangle.$$

(i) $\langle c^2 \rangle$ means the *meansquare speed of gas molecules*.

(ii) Show that the mean kinetic energy is

$$E_k = \frac{3}{2}kT.$$

$$p = \frac{1}{3}\rho\langle c^2 \rangle = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle.$$

$$3pV = Nm\langle c^2 \rangle.$$

Since $pV = NkT$,

$$3NkT = Nm\langle c^2 \rangle.$$

Divide by $2N$:

$$\frac{3}{2}kT = \frac{1}{2}m\langle c^2 \rangle.$$

(b)(i) One mole heated at constant volume

At constant volume $\Delta V = 0$, so $W = 0$ and

$$Q = \Delta U.$$

For one molecule,

$$\Delta E_k = \frac{3}{2}k\Delta T.$$

For one mole of molecules, $N = N_A$:

$$Q = N_A \frac{3}{2}k\Delta T = \frac{3}{2}N_A k\Delta T.$$

Since $N_A k = R$,

$$Q = \frac{3}{2}R\Delta T.$$

For $\Delta T = 1.0 \text{ K}$,

$$Q = \frac{3}{2}R$$

Exam Practice: Specific Heat Capacity of Nitrogen

(b)(ii) Nitrogen may be assumed to be an ideal gas. The molar mass of nitrogen gas is 28 g mol^{-1} . Use the answer in (b)(i) to calculate a value for the specific heat capacity, in $\text{J kg}^{-1}\text{K}^{-1}$, at constant volume for nitrogen. For one mole and a temperature rise of 1 K ,

$$Q = \frac{3}{2}R.$$

Mass of one mole of nitrogen:

$$28 \text{ g} = 0.028 \text{ kg}.$$

Specific heat capacity:

$$c = \frac{Q}{m\Delta T} = \frac{\frac{3}{2}R}{0.028 \times 1}.$$
$$c = \frac{3(8.314)}{2(0.028)} = 445.4 \text{ J kg}^{-1}\text{K}^{-1}.$$

$$c \approx 450 \text{ J kg}^{-1}\text{K}^{-1}$$

Final chapter formula set: $pV = nRT$, $pV = NkT$, $p = \frac{1}{3}\rho\langle c^2 \rangle$, $c_{\text{rms}} = \sqrt{\langle c^2 \rangle}$, $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$, and $Q = \Delta U + W$.