

Electric Fields

Chapter 06 / A2 Level / Apollo Physics 9702

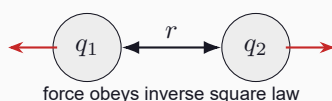


Charge, Coulomb's Law and Electric Field Lines

Charge

- The natural property of matter that it experiences a force in an electric field.
- Two types: positive and negative.
- Proton: $+1.6 \times 10^{-19} \text{ C}$ (+1).
- Electron: $-1.6 \times 10^{-19} \text{ C}$ (-1).
- SI unit: coulomb, C.

Coulomb's law



The force between two point charges is directly proportional to their product and inversely proportional to the square of their separation.

$$F \propto q_1 q_2, \quad F \propto \frac{1}{r^2}, \quad F \propto \frac{q_1 q_2}{r^2}.$$

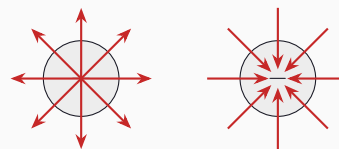
$$F = k \frac{q_1 q_2}{r^2}$$

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9, \quad \epsilon_0 = 8.85 \times 10^{-12}.$$

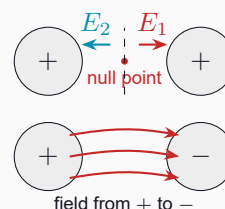
ϵ_0 is the permittivity of free space; it deals with the allowance of a medium to spread the field to the surroundings. **Electric field**

A region in space around a charge where another charge experiences an electric force.



- Direction of field line tells the direction of force on a positive test charge.
- Gap between field lines tells strength of field.
- Less gap $\rightarrow$ strong field; more gap $\rightarrow$ weak field.

Null point and field-line rules



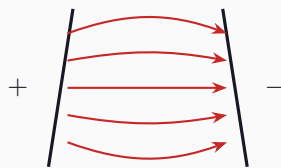
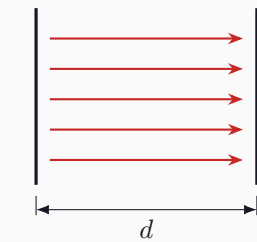
- A null point is a point at which no electric field exists.
- It is in the middle if charges are equal.
- Null point is closer to the weaker charge in case of unequal charges.
- Correct direction is from + to -.
- Gaps between field lines should increase away from charges.
- Radial field lines for point charges.

Comparison with gravitation: $F = Gm_1m_2/r^2$ but $F = kq_1q_2/r^2$; electric forces are much stronger because $k \gg G$.

Parallel Plates, Field Strength and Point-Charge Field

Field lines between parallel metal plates

uniform electric field between plates



increasing gap shows decrease in field strength

Electric field strength (E)

Electric force per unit positive charge at a point in an electric field.

$$E = \frac{F}{q}$$

For parallel metal plates,

$$E = \frac{V}{d}$$

where V is potential difference and d is plate separation.
Since $F = ma$,

$$a = \frac{Vq}{md}$$

Example: proton and alpha particle in a uniform field

$$m_p \approx 1u, \quad m_\alpha \approx 4u, \quad q_p = +1e, \quad q_\alpha = +2e.$$

Both are released from rest in a uniform electric field.

(a) Determine F_p/F_α .

(b) Determine a_p/a_α .

$$F = Eq \Rightarrow \frac{F_p}{F_\alpha} = \frac{Eq_p}{Eq_\alpha} = \frac{1e}{2e} = \boxed{\frac{1}{2}}.$$

$$\frac{F_p}{F_\alpha} = \frac{1}{2}, \quad \frac{m_p a_p}{m_\alpha a_\alpha} = \frac{1}{2},$$

$$\frac{1u a_p}{4u a_\alpha} = \frac{1}{2} \Rightarrow \frac{a_p}{a_\alpha} = \boxed{\frac{2}{1}}.$$

For point charges

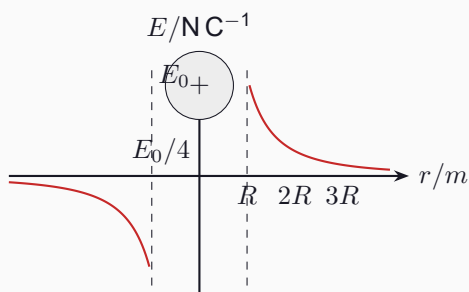
$$E = \frac{F}{q}, \quad F = k \frac{Qq}{r^2} \Rightarrow E = \frac{kQq}{r^2 q}.$$

$$E = \frac{kQ}{r^2}$$

- $E \propto 1/r^2$.
- The further we move away from charge, the weaker the field gets.

E - r Graphs for Point Charges and Resultant Field

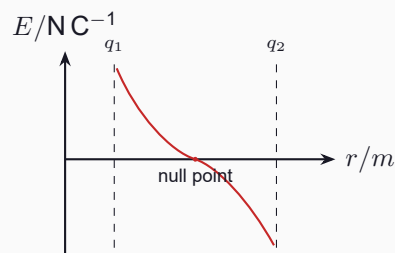
E - r graph for a positive charge



For a positive point charge, field points away from the charge. Magnitude outside the charge follows

$$E \propto \frac{1}{r^2}$$

At R , $E = E_0$. At $2R$, $E = E_0/4$. At $3R$, $E = E_0/9$. Field inside the drawn solid sphere is not treated as a point-charge field in these notes. For a negative charge, directions and signs reverse, but the magnitude still decreases as $1/r^2$. **E - r graph between two point charges**



Like charges:

- Null point is where field strength is zero.
- Null point is closer to weaker charge.
- If both charges are equal, it is in the middle.

At a point P between two like positive charges,

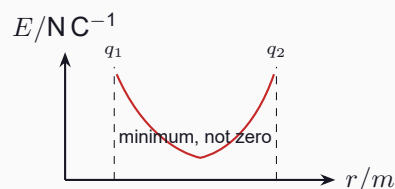
$$E = E_1 - E_2 = \frac{kq_1}{r_1^2} - \frac{kq_2}{r_2^2}.$$

At the null point, $E = 0$:

$$0 = \frac{kq_1}{r_1^2} - \frac{kq_2}{r_2^2} \Rightarrow \frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^2.$$

$$\frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^2$$

Unlike charges:

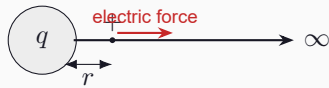


Between opposite charges, the fields reinforce between the charges; the resultant field does not become zero between them.

Electric Potential and Potential of Point Charges

Electric potential (V)

Work done per unit positive charge to move it from infinity to a point in an electric field.



$$V = \frac{kq}{r}$$

- SI unit: volt, V.
- The value can be positive as well as negative.

For a positive source charge, repulsive force means work must be done on a positive test charge to push it towards the charge. Its potential increases from zero and is positive.

$$V = +\frac{kq}{r}$$

For a negative source charge, attractive force means work is done by the field as the positive test charge travels towards the negative charge. Its potential decreases from zero and is negative.

$$V = -\frac{kq}{r}$$

Formula selection

Situation	Formula
Point charges	$V = \frac{kq}{r}$
Parallel plates	$E = \frac{V}{d}$
Point-charge field	$E = \frac{kq}{r^2}$

Potential from multiple charges

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \dots$$

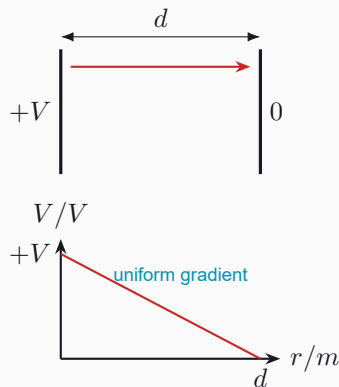
Electric potential is scalar, so potentials are added algebraically. Electric field strength is vector, so fields need direction/sign. **Unlike charges: resultant field**

$$E = E_1 + E_2 = \frac{kq_1}{r_1^2} + \frac{kq_2}{r_2^2}$$

when both field directions are the same at the point under consideration.

V - r Graphs, Potential Gradient and Energy

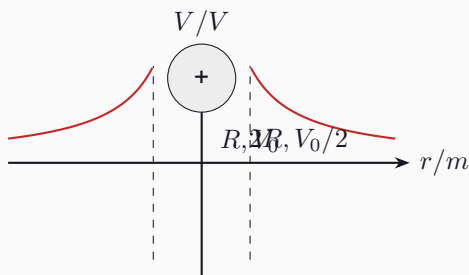
V - r graph between parallel metal plates



Gradient of the V - r graph gives electric field strength.

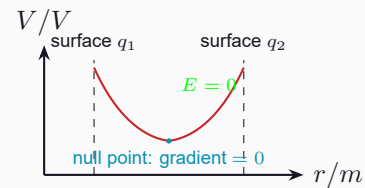
$$E = -\frac{\Delta V}{\Delta r}$$

The negative sign means electric field strength is equal to the negative of the electric potential gradient. For parallel metal plates, graph is straight line because electric field is uniform and gradient is uniform. **V - r graph for an isolated point charge**



- Gradient is zero inside a spherical conductor, so $E = 0$ inside.
- For a point-charge field, $V \propto 1/r$, so gradient decreases as r increases.

V - r graph between like charges



At the null point, the gradient of the V - r graph is zero, hence $E = 0$, $F = 0$, and $a = 0$. **Conservation of energy for a moving charge between two point charges**

$$\Delta V q = \frac{1}{2} m(v^2 - u^2)$$

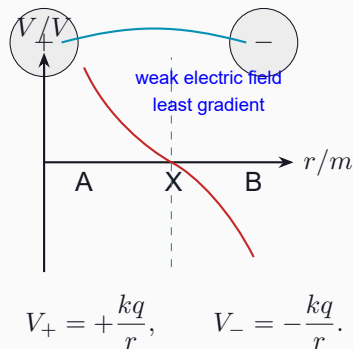
change in electrical potential energy = change in kinetic energy

$$\Delta V q = \frac{1}{2} m(v^2 - u^2), \quad \Delta V = V_{\text{final}} - V_{\text{initial}}$$

This relates change in electrical potential energy to gain/loss of kinetic energy. **Useful statement:** when electric field is strong, the V - r graph has a large gradient. When electric field is weak, the graph has the least gradient.

V - r Graph Between Unlike Charges

Positive and negative charges



From A to X:

- E decreases.
- F decreases.
- a decreases.

From X to B:

- E increases.
- F increases.
- a increases.

A to X to B: speed of the positive charge travelling between A and B keeps on increasing. **Why speed keeps**

increasing

- A positive charge is repelled by the positive sphere A.
- It is attracted towards the negative sphere B.
- Electric force continues to do work on it in the direction of motion.
- Therefore kinetic energy increases and speed increases.

Weak electric field $\Leftrightarrow$ least gradient of potential graph. Strong electric field $\Leftrightarrow$ large potential gradient.

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Exam Graph: Field Strength of a Charged Sphere

Question prompt

The electric field strength at the surface of a charged sphere is E_0 . Sketch a graph to show the variation with distance x of the electric field strength due to the charged sphere, for values of x from $x = 0$ to $x = 4r$.

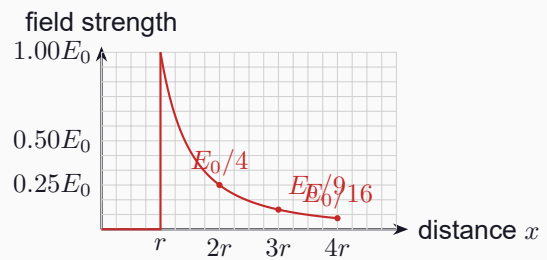
Answer logic

- Inside a conducting sphere: $E = 0$.
- At the surface $x = r$: $E = E_0$.
- Outside, $E \propto 1/x^2$.

$$E = \frac{kq}{r^2}.$$

At distances:

$$2r \Rightarrow 0.25E_0 = \frac{E_0}{4}, \quad 3r \Rightarrow 0.11E_0 = \frac{E_0}{9}, \quad 4r \Rightarrow 0.0625E_0 = \frac{E_0}{16}.$$



Graph features needed for marks

- Horizontal zero field from $x = 0$ to $x = r$.
- Sharp rise to E_0 at the surface.
- Inverse-square decay outside the sphere.
- Correct values around $2r, 3r, 4r$.

Worked Exam: Two Conducting Spheres A and B

Two solid metal spheres A and B are held apart and insulated in vacuum. They carry charges of $+6.0 \mu\text{C}$ and $+12 \mu\text{C}$, respectively. Their centres are separated by 15.0 cm . Point P lies on the line joining the centres and is 2.5 cm from the centre of sphere A.



$$V = V_A + V_B = \frac{kQ_A}{r_A} + \frac{kQ_B}{r_B}.$$

$$V_A = \frac{(8.99 \times 10^9)(6.0 \times 10^{-6})}{0.025} = 2.16 \times 10^6 \text{ V}.$$

$$V_B = \frac{(8.99 \times 10^9)(12 \times 10^{-6})}{0.125} = 8.63 \times 10^5 \text{ V}.$$

Direction of electric field at P

- Field due to A points away from A, to the right.
- Field due to B points away from B, to the left.
- Since P is much closer to A, field due to A is stronger, so resultant field is **to the right**.

$$V_{\text{net}} = \boxed{3.02 \times 10^6 \text{ V}}.$$

Electron accelerated from rest through potential difference If an electron is accelerated through a p.d. ΔV ,

$$q\Delta V = \frac{1}{2}mv^2.$$

For $\Delta V = 2.0 \times 10^4 \text{ V}$,

$$(1.6 \times 10^{-19})(2.0 \times 10^4) = \frac{1}{2}(9.11 \times 10^{-31})v^2.$$

Electric field strength at P

$$E = \frac{kQ}{r^2}, \quad E_A = \frac{(8.99 \times 10^9)(6.0 \times 10^{-6})}{(2.5 \times 10^{-2})^2}.$$

$$E_A = 8.63 \times 10^7 \text{ NC}^{-1}.$$

Distance from B is $15.0 - 2.5 = 12.5 \text{ cm} = 0.125 \text{ m}$:

$$E_B = \frac{(8.99 \times 10^9)(12 \times 10^{-6})}{(0.125)^2} = 6.90 \times 10^6 \text{ NC}^{-1}.$$

$$E_{\text{net}} = E_A - E_B = \boxed{7.94 \times 10^7 \text{ NC}^{-1} \text{ to the right}}.$$

Electric potential at P Potential is scalar, so it adds:

$$v = \sqrt{\frac{2qV}{m}} = \boxed{8.4 \times 10^7 \text{ ms}^{-1}}.$$

Electric Potential at a Point and Moving Charged Particles

Definition Electric potential at a point is the work done per unit positive charge to move it from infinity to that point in an electric field. **Positively charged sphere and alpha particle** A charged particle is accelerated from rest in vacuum through a potential difference V . For an alpha particle of charge $+2e$,

$$qV = \frac{1}{2}mv^2, \quad 2eV = \frac{1}{2}mv^2.$$

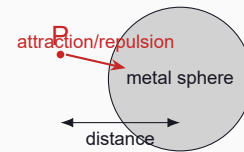
If the particle starts at rest, $u = 0$. Therefore,

$$v = \sqrt{\frac{2qV}{m}}$$

For $q = +2e$,

$$v = \sqrt{\frac{4eV}{m}}.$$

Motion near a metal sphere A particle at P near a charged metal sphere is attracted or repelled depending on the sign of charge. The source notes emphasise using $q\Delta V = \Delta KE$ to calculate the speed.



$$\Delta V q = \frac{1}{2}m(v^2 - u^2)$$

change in electrical potential energy = change in kinetic energy

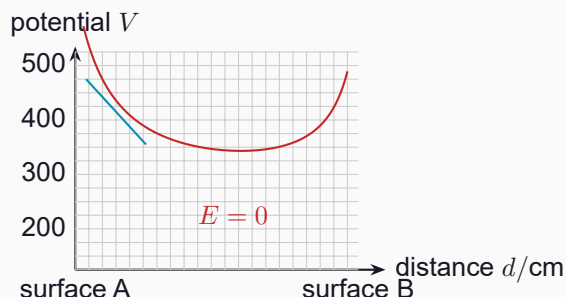
For an alpha particle with charge $+2e$:

$$Vq = \frac{1}{2}mv^2 \quad \Rightarrow \quad V(2e) = \frac{1}{2}mv^2.$$

$$v = \sqrt{\frac{4eV}{m}}$$

Exam Graph: Potential Between Two Spheres

Two positively charged metal spheres A and B are situated in vacuum. A point P lies on the line joining the centres. The graph shows the variation of electric potential V with distance d from the surface of sphere A to the surface of sphere B.



Field direction at the left of the graph

- The field direction is given by the negative gradient of the potential graph.
- If the gradient is negative, E is positive in the direction of increasing distance.

Between A and B

- The field becomes zero where the gradient of the V - d graph is zero.
- At this point, electric force on a charged particle is zero.
- The point is nearer to the weaker/lower charge.

Speed calculation for a proton A proton initially at rest at $d = 0$ is released and moves freely along the line. From the graph, use the change in electric potential to calculate speed.

$$q\Delta V = \frac{1}{2}mv^2.$$

For a proton,

$$q = e = 1.6 \times 10^{-19} \text{ extC}, \quad m_p = 1.67 \times 10^{-27} \text{ extkg}.$$

If $\Delta V \approx 30 \text{ V}$,

$$(1.6 \times 10^{-19})(30) = \frac{1}{2}(1.67 \times 10^{-27})v^2.$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19})(30)}{1.67 \times 10^{-27}}} = \boxed{7.6 \times 10^4 \text{ m s}^{-1}}.$$

Sketch request from the topic

On a copy of the graph, sketch the variation with distance of electric field strength E . The graph should be based on the gradient of the potential graph: steep gradient gives large $|E|$; zero gradient gives $E = 0$; change in gradient direction changes sign of E .

Formula Sheet and Summary

			Key ideas
Quantity	Formula	Notes	
Coulomb force	$F = k \frac{q_1 q_2}{r^2}$	vector force	• Charge, Coulomb's law, permittivity and field-line rules.
Field strength	$E = \frac{F}{q}$	force per unit positive charge	• Uniform electric field between parallel metal plates.
Point-charge field	$E = k \frac{Q}{r^2}$	inverse square	• Field strength, acceleration in plates and point-charge E -field.
Parallel plates	$E = \frac{V}{d}$	uniform field	• E - r graphs for positive/negative charges and for two charges.
Potential	$V = k \frac{q}{r}$	scalar	• Electric potential, positive/negative potential, and potential graphs.
Potential gradient	$E = -\frac{\Delta V}{\Delta r}$	negative gradient	• Energy conservation for charges moving in electric fields.
Energy	$q\Delta V = \Delta KE$	moving charge	• Exam-style graph sketches and numerical applications.