

Capacitance

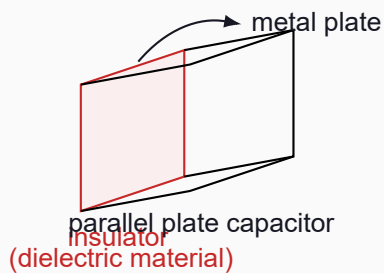
Chapter 07 / A2 Level / Apollo Physics 9702



Capacitor, Charging and Capacitance

Capacitor

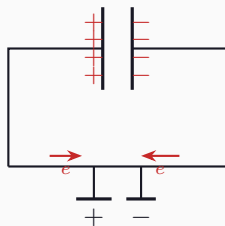
An electronic component that is used to store electrical energy and supply current to the circuit when need be.



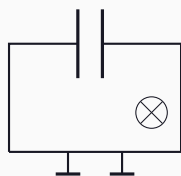
Uses of capacitor (Functions)

1. Stores energy.
2. Time delay circuits.
3. Smoothing in rectifier.
4. Power supplies.
5. Tuning circuits.

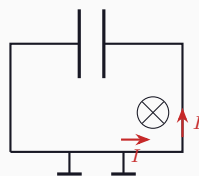
Charging a capacitor



- Positive terminal pulls electrons from one plate and negative terminal deposits them on the other.
- Both plates begin to achieve equal and opposite charge.
- Charging process stops when the p.d. across the capacitor is equal to emf of the battery.



Lamp off after capacitor charges



Lamp still on

Why capacitors are attached in parallel

Hence capacitors are always attached in parallel to a circuit so that the circuit keeps on working even when the capacitor is fully charged. **Explain why capacitor stores energy, not charge**

For a parallel plate capacitor, both plates have equal and opposite charge, hence net charge is zero. Work is done in separating those charges, therefore energy is stored.

Capacitance

It is the charge stored on one plate of a capacitor per unit potential difference.

$$C = \frac{q}{V}$$

$$q = CV$$

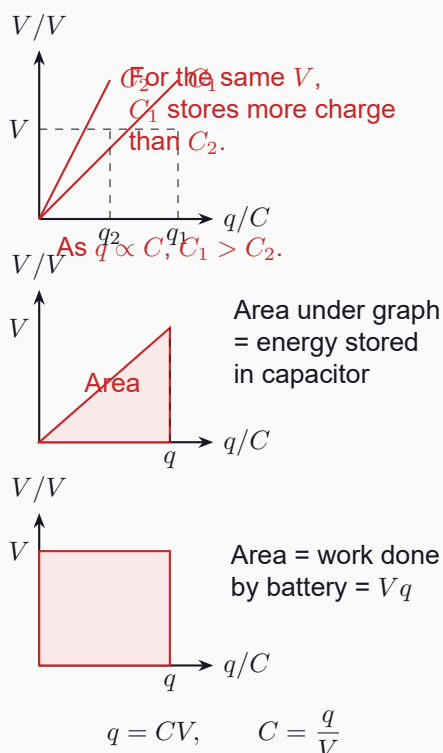
- SI unit: farad (F).
- Define the farad: it is the capacitance of a capacitor when 1 C of charge is stored across a p.d. of 1 V.

Factors affecting capacitance

1. Surface area of plates: bigger surface area of plate → more capacitance.
2. Insulation: better insulators → more capacitance.
3. Distance between plates: less distance → stronger field → more capacitance.

Energy Stored in a Capacitor and Capacitors in Series

Energy stored in a capacitor



For the same voltage, the capacitor with larger capacitance stores more charge.

$$E_{\text{cap}} = \frac{1}{2}qV$$

If $q = CV$,

$$E_{\text{cap}} = \frac{1}{2}(CV)V = \frac{1}{2}CV^2.$$

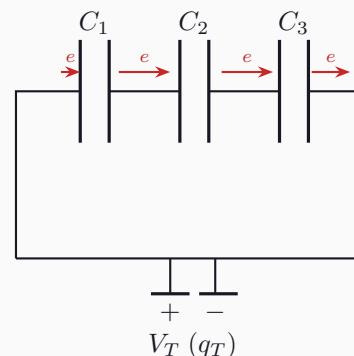
If $V = q/C$,

$$E_{\text{cap}} = \frac{1}{2}\left(\frac{q}{C}\right)q = \frac{1}{2}\frac{q^2}{C}.$$

$$E_{\text{cap}} = \frac{1}{2}CV^2$$

$$E_{\text{cap}} = \frac{1}{2}\frac{q^2}{C}$$

During the charging process, half of the work done by the source is stored in the capacitor and the other half is lost as heat in connecting wires. **Capacitors in series**



Charge: each capacitor gains the same magnitude of charge on one of its plates.

$$q_1 = q_2 = q_3 = q_T$$

Voltage: each capacitor achieves a different p.d. based on its capacitance.

$$q = CV \Rightarrow C \uparrow, V \downarrow.$$

Smaller capacitors achieve more p.d. and vice versa. Sum of all p.d. is equal to emf of source.

$$V_1 + V_2 + V_3 = V_T$$

For capacitance,

$$V_1 + V_2 + V_3 = V_T, \quad V = \frac{q}{C}.$$

$$\frac{q_1}{C_1} + \frac{q_2}{C_2} + \frac{q_3}{C_3} = \frac{q_T}{C_T}.$$

As $q_1 = q_2 = q_3 = q_T$,

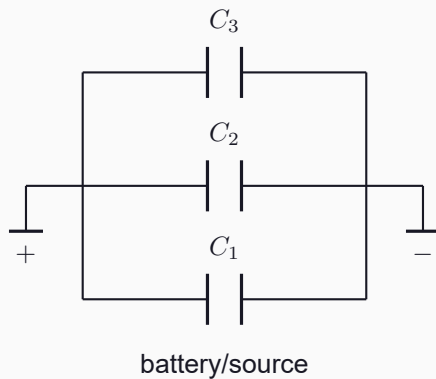
$$q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) = q \left(\frac{1}{C_T} \right).$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For identical capacitors: $C_T = C/n$.

Capacitors in Parallel and Capacitance of a Point Charge

Capacitors in parallel



Voltage: all capacitors charge up to the same p.d. as they are attached across the source directly.

$$V_1 = V_2 = V_3 = V_T$$

Charge: each capacitor holds charge based on its capacitance.

$$q = CV, \quad q \propto C.$$

Bigger capacitors hold more charge.

$$q_1 + q_2 + q_3 = q_T$$

Capacitance:

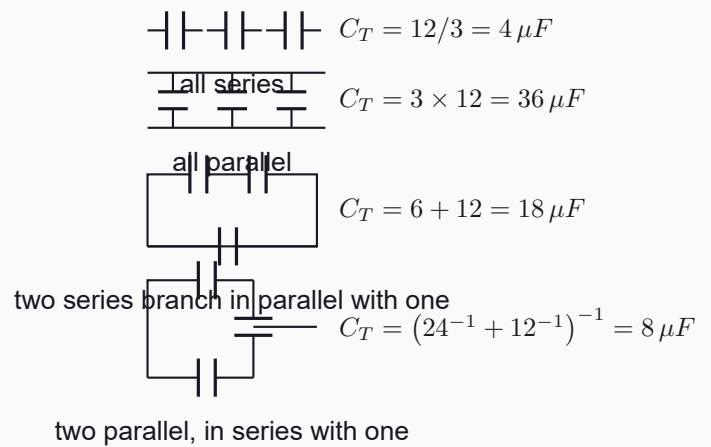
$$q_1 + q_2 + q_3 = q_T, \quad q = CV.$$

$$C_1 V_1 + C_2 V_2 + C_3 V_3 = C_T V_T.$$

As V is same,

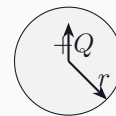
$$C_T = C_1 + C_2 + C_3$$

Three $12 \mu\text{F}$ capacitors: possible combinations



Capacitance of a point charge / isolated sphere

For a point charge at the surface of a sphere,



solid metal sphere

$$V = \frac{kq}{r}, \quad C = \frac{q}{V}.$$

$$C = \frac{q}{kq/r} = \frac{r}{k}.$$

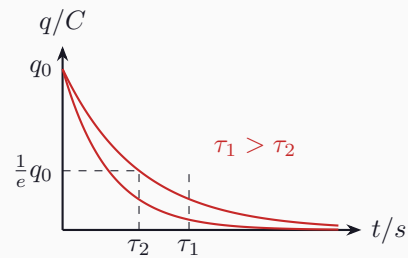
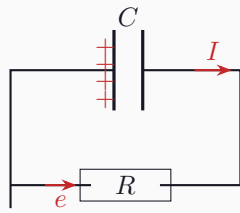
Since $k = \frac{1}{4\pi\epsilon_0}$,

$$C = 4\pi\epsilon_0 r$$

where r is the radius of the sphere.

Discharging a Capacitor and Time Constant

Discharging of a capacitor



$\tau = RC$; larger time constant $\rightarrow$ slower discharge

The rate of discharge of a capacitor is directly proportional to the charge on that capacitor.

$$\frac{\Delta x}{\Delta t} \propto x \quad \Rightarrow \quad x = x_0 e^{-kt}.$$

Such relations follow exponential functions.

$$\frac{\Delta q}{\Delta t} \propto q \quad \Rightarrow \quad q = q_0 e^{-t/RC}.$$

$$\frac{\Delta V}{\Delta t} \propto V \quad \Rightarrow \quad V = V_0 e^{-t/RC}.$$

$$\frac{\Delta I}{\Delta t} \propto I \quad \Rightarrow \quad I = I_0 e^{-t/RC}.$$

- q_0 : initial charge at $t = 0$.
- V_0 : initial voltage at $t = 0$.
- I_0 : initial current at $t = 0$.
- q, V, I : quantities at time t .
- R : resistance; C : capacitance.

$$\tau = RC$$

where τ is the time constant. A larger time constant gives slower discharge. **Definition of time constant**

The time taken for the capacitor to decrease its charge / voltage / current in the circuit to $1/e$ of its initial value.

$$q = q_0 e^{-t/RC}.$$

If $t = RC$, then

$$q = q_0 e^{-RC/RC} = q_0 e^{-1} = \frac{q_0}{e}.$$

$$q = \frac{1}{e} q_0$$

Since $\frac{1}{e} = 0.368$, after one time constant the remaining charge is 36.8% of its initial charge.

Exam Question: Isolated Metal Sphere

A solid metal sphere of radius r is insulated from its surroundings. The sphere has charge $+Q$. The charge is on the surface of the sphere but may be considered to be a point charge at its centre.



solid metal sphere

(a)(i) Define capacitance.

Charge stored per unit potential difference.

(a)(ii)

Show that the capacitance C of the sphere is given by

$$C = 4\pi\epsilon_0 r.$$

$$C = \frac{q}{V}, \quad V = \frac{q}{4\pi\epsilon_0 r}.$$

$$C = q \div \frac{q}{4\pi\epsilon_0 r} = \boxed{4\pi\epsilon_0 r}.$$

(b)(i) The sphere has radius 36 cm. Determine the capacitance.

$$C = 4\pi\epsilon_0 r = 4\pi(8.85 \times 10^{-12})(0.36).$$

$$C = 4.0 \times 10^{-11} \text{ F}.$$

$$\boxed{4.0 \times 10^{-11} \text{ F}}$$

(b)(ii) Charge required to raise the potential from zero to $7.0 \times 10^5 \text{ V}$

$$q = CV.$$

$$\Delta q = (4.0 \times 10^{-11})(7.0 \times 10^5).$$

$$\Delta q = \boxed{2.8 \times 10^{-5} \text{ C}}.$$

(c) Why would the calculation not apply to a plastic sphere?

Metal spheres are conductors and charge in them is uniformly distributed so the charge can be treated as a point charge. Plastic is an insulator; its charge may be at the surface or at the centre / not uniformly distributed, so the same model does not apply. (d) Spark discharge from

$7.0 \times 10^5 \text{ V}$ to $2.5 \times 10^5 \text{ V}$

Energy dissipated equals loss in stored energy:

$$\Delta E = \frac{1}{2}C (V_i^2 - V_f^2).$$

$$\Delta E = \frac{1}{2}(4.0 \times 10^{-11})\{(7.0 \times 10^5)^2 - (2.5 \times 10^5)^2\}.$$

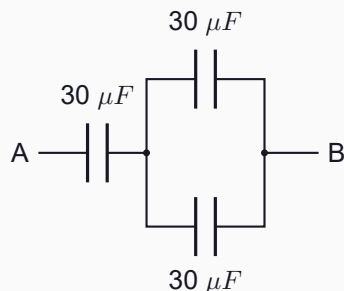
$$\Delta E = 8.55 \text{ J} \approx \boxed{8.6 \text{ J}}.$$

Exam Question: Three $30\ \mu\text{F}$ Capacitors

(a) State two functions of capacitors in electrical circuits.

1. Store energy.
2. Used in tuning circuits / time delay circuits / smoothing circuits.

(b) Three capacitors, each marked ' $30\ \mu\text{F}$, $6\ \text{V max}$ ', are arranged as shown.



The two capacitors on the right are in parallel, so

$$C_p = 30 + 30 = 60\ \mu\text{F}.$$

This $60\ \mu\text{F}$ combination is in series with the left $30\ \mu\text{F}$ capacitor:

$$\frac{1}{C_T} = \frac{1}{30} + \frac{1}{60} = \frac{3}{60}.$$

$$C_T = \boxed{20\ \mu\text{F}}.$$

Maximum p.d. between A and B

For series capacitors, charge is the same and $V = q/C$, so the smaller capacitance gets the larger p.d. The smallest capacitor is $30\ \mu\text{F}$, so assume the maximum safe p.d. across it is $6\ \text{V}$.

$$q_1 = q_2, \quad C_1 V_1 = C_2 V_2.$$

$$30 \times 6 = 60 \times V_2 \Rightarrow V_2 = 3\ \text{V}.$$

$$V_T = 6 + 3 = \boxed{9\ \text{V}}.$$

(c) A $4700\ \mu\text{F}$ capacitor is discharged from $18\ \text{V}$ to $12\ \text{V}$. Calculate energy dissipated.

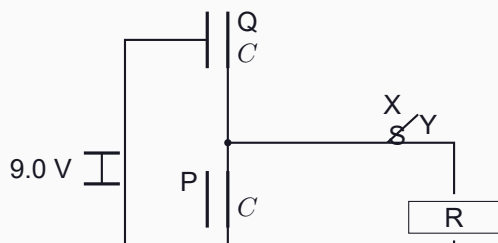
$$\Delta E = \frac{1}{2} C (V_i^2 - V_f^2).$$

$$\Delta E = \frac{1}{2} (4700 \times 10^{-6}) (18^2 - 12^2).$$

$$\Delta E = 0.423\ \text{J} \approx \boxed{0.42\ \text{J}}.$$

Exam Question: Switch X/Y with Three Capacitors

Two capacitors P and Q, each of capacitance C , are connected in series with a battery of e.m.f. 9.0 V . A switch S can connect either a third capacitor T, also of capacitance C , or a resistor R in parallel with capacitor P.



Switch S in position X: combined capacitance

Capacitor T is in parallel with P, so P+T gives $2C$. This is in series with Q:

$$\frac{1}{C_T} = \frac{1}{C} + \frac{1}{2C} = \frac{2+1}{2C} = \frac{3}{2C}.$$

$$C_T = \frac{2C}{3}$$

Potential difference across Q

In series, charge is the same:

$$q_1 = q_2, \quad C_T V_T = C_Q V_Q.$$

$$\left(\frac{2C}{3}\right) 9 = C V_Q.$$

$$V_Q = \boxed{6.0\text{ V}}.$$

Thus the p.d. across the P+T parallel branch is $9 - 6 = 3$

V. Switch moved to position Y

The resistor is connected across capacitor P.

- Capacitor P discharges from 3 V to 0 V because it is connected across the resistor.
- Capacitor Q charges up from 6 V to 9 V because the battery p.d. is greater than 6 V and finally appears across Q when P is discharged.

Capacitor P: discharges from 3 V to 0 V .
Capacitor Q: charges up from 6 V to 9 V .

Concept check

When a branch is placed across a capacitor, it can provide a discharge path. A resistor across a capacitor allows stored energy to be dissipated as heat.

Exam Question: Vibrating Switch and Sensitive Ammeter

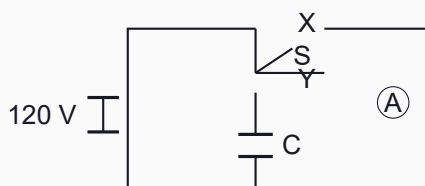
Definition

The capacitance of a parallel plate capacitor is the charge stored on one plate of a capacitor per unit potential difference across the plates:

$$C = \frac{q}{V}$$

where q is charge on one plate and V is the p.d. across them. **Circuit**

A capacitor C is connected to a 120 V battery when switch S is at X . When the switch moves to Y , the capacitor discharges through the sensitive ammeter.



The switch first goes to X , then to Y , then back to X fifty times each second. The current recorded on the ammeter is $4.5 \mu\text{A}$. **Charge passing through the ammeter in 1.0 s**

$$q = It = (4.5 \times 10^{-6})(1) = 4.5 \times 10^{-6} \text{ C}$$

Charge on one plate each time it is charged

There are 50 discharges in 1 second, so the charge per discharge is

$$q = \frac{4.5 \times 10^{-6}}{50} = 9.0 \times 10^{-8} \text{ C}$$

Capacitance of C

$$q = CV \Rightarrow C = \frac{q}{V}$$

$$C = \frac{9.0 \times 10^{-8}}{120} = 7.5 \times 10^{-10} \text{ F}$$

Second capacitor of equal capacitance placed in series

Effective capacitance halves:

$$C_{\text{eff}} = \frac{C}{2}$$

For the same p.d., charge stored on the combination also halves. Since

$$I = \frac{q}{t},$$

the current in the ammeter halves.

Effective capacitance halves, so charge stored each cycle halves; therefore current in the ammeter halves.

Formula Sheet and Summary

Ideas retained from the topics		
Topic	Formula	Key note
Capacitance	$C = q/V$	• Capacitor structure, symbol, charging process and lamp examples.
Stored energy	$E = \frac{1}{2}qV$	charge on one plate
Stored energy	$E = \frac{1}{2}CV^2$	area under $V-q$ graph
Stored energy	$E = \frac{1}{2}q^2/C$	• Why capacitors store energy rather than net charge.
Series capacitors	$1/C_T = 1/C_1 + 1/C_2 + \dots$	use with voltage
Parallel capacitors	$C_T = C_1 + C_2 + \dots$	• Energy from the area under a $V-q$ graph.
Sphere	$C = 4\pi\epsilon_0 r$	• Series and parallel capacitor rules with charge / p.d. explanations.
Discharge	$q = q_0 e^{-t/RC}$	same charge
Discharge	$V = V_0 e^{-t/RC}$	same p.d.
Discharge	$I = I_0 e^{-t/RC}$	• 12 μF combination examples and isolated sphere capacitance.
Time constant	$\tau = RC$	exponential decay
		• RC discharge exponential equations and time constant definition.
		• Exam questions: charged metal sphere, 30 μF network, X/Y switch, and vibrating switch with ammeter.