

Nuclear Physics

Chapter 11 / A2 Level / Apollo Physics 9702



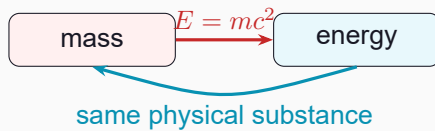
Einstein's mass-energy equation and mass defect

Einstein's mass-energy equation

- Mass and energy are two different manifestations of the same thing.
- During nuclear reactions, mass of the element is converted into energy.
- The energy produced from mass can be calculated using

$$E = mc^2$$

where c is the speed of light.



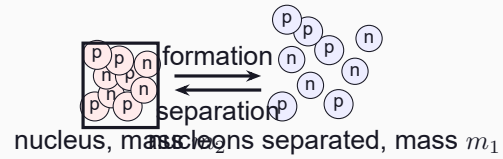
Mass defect

The mass of the nucleus is less than the total mass of the nucleons separated.

$$\Delta m = m_1 - m_2$$

where

- m_1 = mass of nucleons separated,
- m_2 = mass of nucleus.



Mass defect is the difference in mass between the separated nucleons and the nucleons combined as a nucleus.

Mass defect and binding energy

Mass and energy conversion

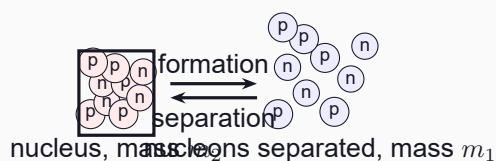
- Mass and energy are two different manifestations of the same thing.
- Therefore they can be converted from one to another.

$$E = mc^2$$

Mass defect and binding energy

When a nucleus is formed from separated nucleons, the released energy is called the **binding energy**.

$$\Delta m = m_1 - m_2, \quad E = \Delta mc^2$$



Definition: Binding energy is the amount of energy needed to separate the nucleons of a nucleus apart to infinity.

Example: tritium nucleus

For ${}^3_1\text{H}$, using masses in u:

$$\Delta m = (m_p + 2m_n) - m_{\text{nucleus}}$$

$$\Delta m = (1.00728 + 2(1.00867) - 3.0155)$$

$$\Delta m = 9.11 \times 10^{-3} \text{ u}$$

Binding energy in joules

$$E = \Delta mc^2$$

$$E = (9.11 \times 10^{-3} \times 1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.36 \times 10^{-12} \text{ J}$$

Binding energy in eV

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$E = \frac{1.36 \times 10^{-12}}{1.6 \times 10^{-19}} = 8.5 \times 10^6 \text{ eV} = \boxed{8.5 \text{ MeV}}$$

Atomic mass unit and binding energy per nucleon

Energy equivalent of 1 u

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

$$E = mc^2 = (1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.494 \times 10^{-10} \text{ J}$$

Converting to eV:

$$1.6 \times 10^{-19} \text{ J} = 1 \text{ eV}$$

$$E = \frac{1.494 \times 10^{-10}}{1.6 \times 10^{-19}} = 9.33 \times 10^8 \text{ eV}$$

$$1 \text{ u} = 933 \text{ MeV} \approx 930 \text{ MeV}$$

Shortcut for binding energy

If Δm is in u,

$$E(\text{MeV}) = \Delta m(\text{u}) \times 933$$

For the tritium example,

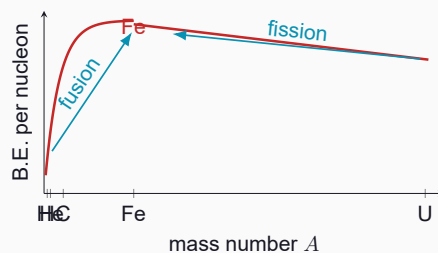
$$E = (9.11 \times 10^{-3})(933) = 8.5 \text{ MeV}.$$

Binding energy per nucleon

$$\text{B.E. per nucleon} = \frac{\text{binding energy}}{\text{number of nucleons}}$$

It tells us about the stability of the nucleus.

- Larger B.E. per nucleon $\Rightarrow$ more stable nucleus.
- Smaller B.E. per nucleon $\Rightarrow$ less stable nucleus.



For a feasible fission/fusion reaction, the product must have greater binding energy per nucleon than the reactant.

Worked example: lanthanum-139 energy release

Given data

particle/nucleus	symbol	mass/u
lanthanum-139	${}_{57}^{139}\text{La}$	138.955
proton	${}_1^1\text{p}$	1.007
neutron	${}_0^1\text{n}$	1.009

Convert to kg and calculate energy

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

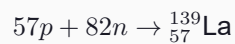
$$\Delta m = 1.182(1.66 \times 10^{-27}) = 1.962 \times 10^{-27} \text{ kg}$$

$$E = mc^2 = (1.962 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.77 \times 10^{-10} \text{ J}$$

Mass of separated nucleons

Lanthanum-139 contains 57 protons and 82 neutrons.



$$m_1 = 57(1.007) + 82(1.009) = 140.137 \text{ u}$$

$$m_2 = 138.955 \text{ u}$$

$$\Delta m = 140.137 - 138.955 = 1.182 \text{ u}$$

For one mole of particles

$$E_{\text{mol}} = (1.77 \times 10^{-10})(6.02 \times 10^{23})$$

$$E_{\text{mol}} = \boxed{1.06 \times 10^{14} \text{ J}}$$

Mass defect in nuclear reactions

Atomic mass unit

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

It is approximately the average mass of a proton/neutron.

Lanthanum-139 calculation continued

$$m_1 = 57(1.007) + 82(1.009) = 140.137 \text{ u}$$

$$\Delta m = 140.137 - 138.955 = 1.182 \text{ u}$$

$$\Delta m = 1.182(1.66 \times 10^{-27}) = 1.96 \times 10^{-27} \text{ kg}$$

For one mole:

$$1 \text{ mol La} \rightarrow 6.02 \times 10^{23} \text{ particles} \rightarrow 139 \text{ g}$$

$$\Delta m_{1 \text{ mol}} = (1.96 \times 10^{-27})(6.02 \times 10^{23})$$

$$\Delta m_{1 \text{ mol}} = 0.00117992 \text{ kg} = \boxed{1.18 \text{ g}}$$

Key notes

- Mass defect happens in nuclear fission, nuclear fusion and radioactive decay because energy is released in them.
- The loss of mass is converted to energy.

$$\Delta E = \Delta mc^2$$

Do not write “mass is conserved” or “energy is conserved” alone in nuclear reactions. Mass is converted to energy, so mass-energy is conserved.

Binding energy and binding energy per nucleon

Binding energy

Binding energy is the amount of energy needed to separate the nucleons of the nucleus apart to infinity.

- Binding energy basically binds the nucleus together.
- To break the nucleus, this much energy must be supplied.

$$\Delta E = \Delta mc^2$$

binding energy	mass defect
ΔE	Δm

Example continued

$$\Delta E = (1.96 \times 10^{-27})(3.0 \times 10^8)^2$$

$$\Delta E = 1.76 \times 10^{-10} \text{ J}$$

Binding energy per nucleon

Energy needed per nucleon to break the nucleus apart and separate nucleons to infinity.

$$\text{B.E. per nucleon} = \frac{\text{binding energy}}{\text{number of nucleons}}$$

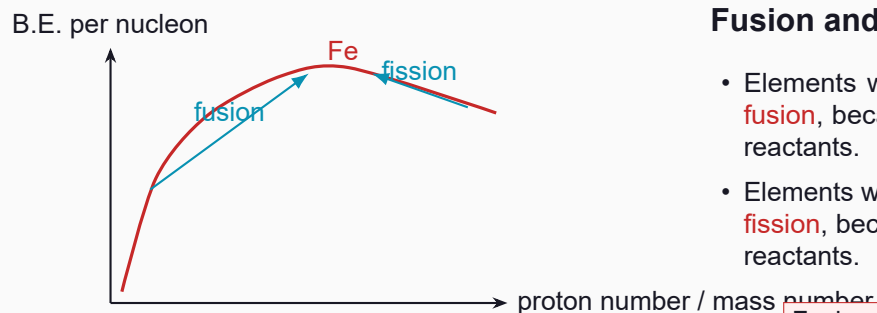
This value gives the idea about stability of the nucleus.

$$\text{B.E. per nucleon} = \frac{1.76 \times 10^{-10}}{139}$$

$$\text{B.E. per nucleon} = 1.27 \times 10^{-12} \text{ J}$$

More binding energy per nucleon means more stable nucleus.

Binding-energy curve and feasible reactions



Fusion and fission

- Elements with proton number less than iron prefer **fusion**, because the product is more stable than the reactants.
- Elements with proton number greater than iron prefer **fission**, because the product is more stable than the reactants.

Fusion and fission both release energy only when the final nuclei have greater binding energy per nucleon.

Important rule

A reaction is only feasible if the binding energy per nucleon of products is more than the binding energy per nucleon of reactants.

The mass of products will be less than the mass of reactants when a reaction is feasible.

Stability

- The greater the binding energy per nucleon, the more stable the nucleus.
- Iron has the greatest binding energy per nucleon, so it is the most stable.

Practice question: fusion in the Sun

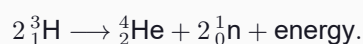
Question style: May/June 2013 Paper 41

(a) State what is meant by a nuclear fusion reaction.

Answer: A nuclear reaction in which small nuclei combine to make a larger, more stable nucleus, releasing a large amount of heat energy.

Reaction

One nuclear reaction in the core of the Sun is represented by



Data:

particle/nucleus	mass/u
proton ${}^1_1\text{p}$	1.00728
deuterium ${}^2_1\text{H}$	2.0140
helium ${}^4_2\text{He}$	4.0015

Energy released

Use step 1: mass defect.

$$\Delta m = m_{\text{products}} - m_{\text{reactants}}$$

The note uses

$$\Delta m = 3.0165 - (1.0072 + 2.0140) = -5.3 \times 10^{-3} \text{ u}$$

Magnitude of mass defect:

$$|\Delta m| = 5.3 \times 10^{-3} \text{ u}$$

Step 2: use $E = mc^2$.

$$E = (5.3 \times 10^{-3})(1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = \boxed{7.9 \times 10^{-13} \text{ J}}$$

Why high temperature is necessary

The temperature in the core of the Sun is approximately $1.6 \times 10^7 \text{ K}$. It is so high because nuclei are both positively charged and repel one another. High temperature gives nuclei high speed and kinetic energy so that the repulsion can be overcome.

Conversion and conservation during nuclear reactions

Conversion of 1 u to MeV

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

$$E = mc^2 = (1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.494 \times 10^{-10} \text{ J}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$E = \frac{1.494 \times 10^{-10}}{1.6 \times 10^{-19}} = 933 \text{ MeV} \approx \boxed{930 \text{ MeV}}$$

Conservation during nuclear reactions

During nuclear reactions the following are conserved:

1. proton number,
2. mass number,
3. mass-energy,
4. momentum.

Do not write just mass conserved or energy conserved, because mass is converted to energy.

Types of nuclear reactions

- Radioactive decay
- Fission
- Fusion

Practice question: polonium nucleus

Question style: Oct/Nov 2014 Paper 41

(a) Explain what is meant by binding energy of a nucleus.

Answer: The energy required to separate all nucleons of a nucleus apart to infinity.

Given masses

particle/nucleus	mass/u
proton	1.00728
neutron	1.00867
lithium/helium data	3.01551
polonium nucleus	208.9822

$$1.0 \text{ u} = 930 \text{ MeV}.$$

Binding energy calculation

$$\Delta m = m_1 - m_2$$

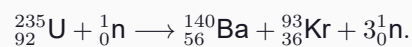
$$\Delta m = 211.0194 - 208.9822 = 2.0372 \text{ u}$$

$$E = 2.0372 \times 930 = 1894.6 \text{ MeV}$$

$$\text{B.E. per nucleon} = \frac{1894.6}{210} = \boxed{9.02 \text{ MeV}}$$

Fission reaction

One fission reaction is represented by



Why is fission energetically possible?

The product nuclei have a greater binding energy per nucleon than the reactant nucleus, so the final nuclei are more stable and energy is released.

Radioactivity and activity

Radioactivity

Radioactivity is the phenomenon of emission of radioactive radiations from the nucleus of an atom.

Types of radiations

- **Alpha particles:** helium nucleus, $2p + 2n$, charge $+2$.
- **Beta particles:** fast moving electrons or positrons, charge -1 or $+1$.
- **Gamma rays:** high-frequency electromagnetic radiation, neutral rays.

Activity, A

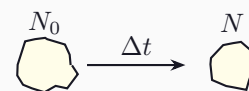
Activity is the rate of radioactive radiations emitted from the nucleus of an atom.

$$A = \frac{\Delta N}{\Delta t}$$

Since $\Delta N = N_{\text{final}} - N_{\text{initial}}$ is negative for decay,

$$A = -\frac{\Delta N}{\Delta t}$$

SI unit: becquerel, Bq.



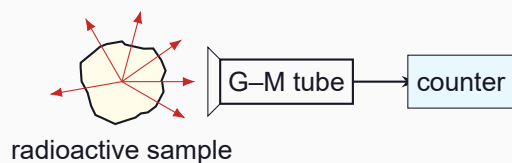
initial nuclei drop to final nuclei in time interval

- N_0 : initial number of radioactive nuclei in sample.
- N : final number of radioactive nuclei in sample.
- Δt : time for N_0 to drop to N .

Count rate and decay constant

Count rate, R

Count rate is the number of radioactive radiations detected per unit time.



Count rate of a sample is different from its activity.

1. Not all emitted radioactive radiations are detected by the G-M tube.
2. The G-M tube also counts background radiation present in the surroundings.

Decay constant, λ

Decay constant is the probability of a nucleus decaying per unit time.

For example,

$$\lambda = 0.4 \text{ s}^{-1}$$

means 40% of the undecayed nuclei decay in one second.

$$\lambda = \frac{\Delta N}{N \Delta t}$$

where ΔN is the number of atoms that decay in time Δt , and N is the number of atoms that could decay.

Activity is proportional to number of undecayed nuclei

Example of decay constant

If $\lambda = 0.4 \text{ s}^{-1}$:

- out of 100, 40 would decay in 1 s,
- out of 50, 20 would decay in 1 s,
- out of 5, 2 would decay in 1 s.

$$\lambda = \frac{\Delta N}{N_0 \Delta t}$$

Link with activity

Since

$$A = \frac{\Delta N}{\Delta t},$$

we get

$$\lambda = \frac{A}{N}$$

$$\boxed{A = \lambda N}$$

where

- A = current activity,
- λ = decay constant,
- N = current number of undecayed nuclei.

Activity is not fixed. It decreases with time as the remaining number of undecayed nuclei decreases with time.

Probability interpretation of the decay constant

Probability

$$\text{probability} = \frac{\text{possible outcomes}}{\text{total outcomes}}$$

Example:

$$N = 500 \text{ atoms}, \quad N_0 = 10000 \text{ atoms}, \quad \Delta t = 1 \text{ s}$$

$$\text{probability} = \frac{500}{10000} = 0.05 \text{ per second}$$

Different sample sizes

N_0	decayed in 1 s	probability
1000	50	0.05 per s
5000	250	0.05 per s
20000	1000	0.05 per s

Decay constant

$$\lambda = \frac{\text{probability}}{\Delta t}$$

For one second intervals, the numerical value of the probability per second is the decay constant.

$$A = \lambda N$$

$$\lambda = \frac{A}{N}$$

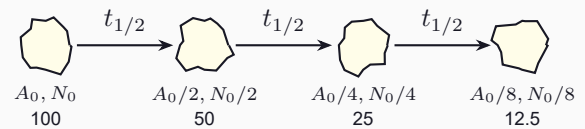
Half-life

Definitions

Half-life is the time taken for a radioactive sample to decrease its activity to half of its initial value.

It is also the time taken for a radioactive sample to decrease its number of radioactive nuclei to half of its initial value.

It is also the time taken for count rate to decrease to half of its initial value.



$$100 \rightarrow 50 \rightarrow 25 \rightarrow 12.5$$

Each arrow represents one half-life.

Equivalent statements

$$A \rightarrow \frac{A_0}{2}, \quad N \rightarrow \frac{N_0}{2}, \quad R \rightarrow \frac{R_0}{2}$$

Exponential decay derivation

In radioactive decay, the rate at which radioactive nuclei emit radiation is proportional to the number of radioactive nuclei themselves.

Any phenomenon behaving in this manner follows an exponential function.

$$\frac{\Delta N}{\Delta t} \propto N$$

In reality, ΔN is negative because

$$\Delta N = N_{\text{final}} - N_{\text{initial}}, \quad N_f < N_i.$$

$$\lambda = -\frac{\Delta N}{N\Delta t}$$

In terms of calculus

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda dt$$

Integrating both sides:

$$\int_{N_0}^N \frac{dN}{N} = \int_0^t -\lambda dt$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

Radioactivity recap

Radioactive decay

The phenomenon of emission of radioactive radiations from the nucleus of an unstable atom.

The process is random and spontaneous.

Random process

Every nucleus has equal probability to decay. It cannot be predicted which nucleus will decay and when.

Spontaneous process

It happens on its own. Changes in external conditions such as temperature and pressure have no effect on the rate of emission.

Types

- Alpha: α or ${}^4_2\text{He}$; helium nucleus.
- Beta: β^- or β^+ .
- Gamma: very high-frequency electromagnetic radiation.

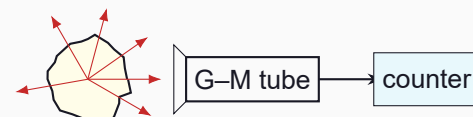
Activity

$$A = -\frac{\Delta N}{\Delta t}$$

SI unit: becquerel (Bq).

Count rate

Count rate is the number of radioactive radiations detected per unit time.



radioactive sample

Count rate = sample activity + background activity, with detector inefficiency taken into account.

Decay equations and half-life

Decay constant

$$\lambda = \frac{\text{probability of decay}}{\text{time interval}}$$

$$\lambda = \frac{A}{N}, \quad A = \lambda N$$

Same form for N , A and R

$$N = N_0 e^{-\lambda t}$$

Half-life

$$A = A_0 e^{-\lambda t}$$

Time taken for the radioactive sample to decrease its activity, count rate or number of radioactive atoms to half of its initial value.

$$R = R_0 e^{-\lambda t}$$

Example:

$$4000 \rightarrow 2000 \rightarrow 1000 \rightarrow 500 \rightarrow 250$$

If each half-life is 2 years, then four half-lives take 8 years.

The units of λ and t must be similar. For example, if λ is in min^{-1} , then t must be in minutes.

Relation between decay constant and half-life

Start with the decay equation

$$N = N_0 e^{-\lambda t}$$

At $t = t_{1/2}$,

$$N = 0.5N_0.$$

Therefore,

$$0.5N_0 = N_0 e^{-\lambda t_{1/2}}$$

$$0.5 = e^{-\lambda t_{1/2}}$$

Take natural log:

$$\ln(0.5) = -\lambda t_{1/2}$$

$$-0.693 = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{0.693}{\lambda}$$

Since $0.693 = \ln 2$,

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

or

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{\ln 2}{t_{1/2}}$$

This relation is only valid for exponential decay, where the decay rate is proportional to the number of undecayed nuclei.

Half-life numerical example

Example by repeated halving

$$A_0 = 4000, \quad t_{1/2} = 2 \text{ years}$$

Given final activity:

$$A = 500$$

Repeated halving:

$$4000 \rightarrow 2000 \rightarrow 1000 \rightarrow 500$$

This is 3 half-lives.

$$3 \times 2 \text{ years} = \boxed{6 \text{ years}}$$

Half-life relation

At $t = t_{1/2}$,

$$N = 0.5N_0, \quad A = 0.5A_0$$

Derivation again

$$N = N_0 e^{-\lambda t}$$

$$0.5N_0 = N_0 e^{-\lambda t_{1/2}}$$

$$0.5 = e^{-\lambda t_{1/2}}$$

$$\ln 0.5 = -\lambda t_{1/2}$$

$$-0.693 = -\lambda t_{1/2}$$

$$\boxed{t_{1/2} = \frac{0.693}{\lambda}},$$

$$\boxed{\lambda = \frac{0.693}{t_{1/2}}}$$

For N , A and R :

$$N = N_0 e^{-\lambda t}, \quad A = A_0 e^{-\lambda t}, \quad R = R_0 e^{-\lambda t}$$

Exponential decay graph

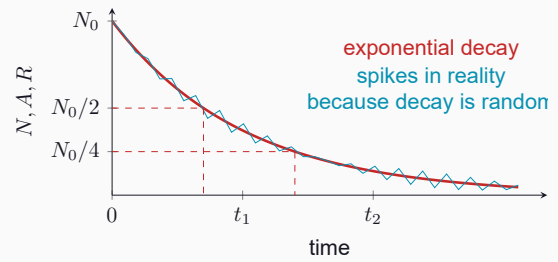
Same exponential form

$$N = N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

$$R = R_0 e^{-\lambda t}$$

Make sure λ and t have similar units. For example, if $\lambda = \text{min}^{-1}$, then t must be in minutes.



Half-life formula

$$N = \frac{N_0}{2}, \quad t = t_{1/2}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$$

- The smooth red line represents the ideal exponential decay.
- In real count-rate graphs, spikes are seen because decay is random.
- Graphs of A and R have the same shape as the graph of N .

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