

Medical Imaging

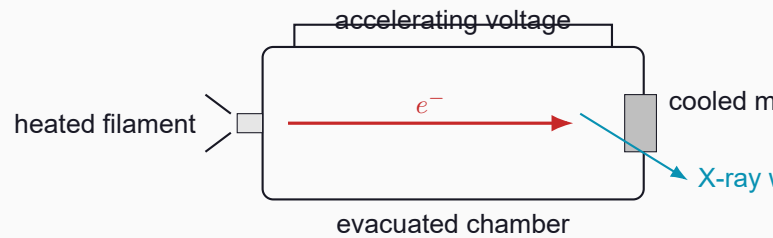
Chapter 13 / A2 Level / Apollo Physics 9702



X-rays: Formation, Image Formation, Sharpness, Contrast and Calculations

Formation of X-rays

- A coil of wire is heated to a very high temperature using a high voltage source.
- The hot coil begins to emit electrons by **thermionic emission**.
- An accelerating voltage speeds up the electrons.
- When the electrons strike the metal target, they lose energy and produce X-ray photons.

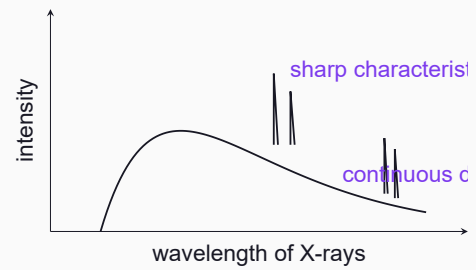


K.E. of electron $\rightarrow$ heat + X-rays

Sharp deceleration of electrons produces X-ray photons.

Characteristic Peaks and Continuous Distribution

- Electrons of the metal anode can de-excite and produce photons of X-rays.
- Some photons are produced in greater quantity because they come from electron transitions in the anode atoms.
- These sharp peaks are characteristic of the metal used.
- The continuous distribution is due to a range of decelerations of the incident electrons, so a range of X-ray photon energies and wavelengths is produced.
- The cut-off wavelength corresponds to a photon produced using the maximum kinetic energy of an electron.



X-ray Intensity, Hardness, Sharpness and Contrast

Intensity

- Intensity depends on the number of incident electrons.
- Therefore it depends on the **heating voltage**.
- Increasing the heating voltage increases the intensity of X-rays.

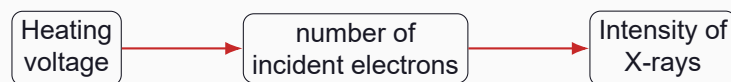
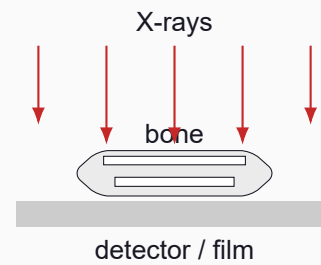


Image Formation



Hardness

- Hardness is the energy of an X-ray photon.
- A hard X-ray has higher photon energy and smaller wavelength.
- A soft X-ray has lower photon energy and longer wavelength, so it is less penetrating.
- Increasing the accelerating voltage increases the kinetic energy of the electrons and hence the hardness of the X-rays.

$$E_{\max} = eV, \quad E = \frac{hc}{\lambda}$$

Sharpness and Contrast

Sharpness of image means a clean boundary of the image. A clear boundary gives a sharp image. It is improved by:

- small anode,
- small aperture,
- lead grid between body and X-ray sheet,
- keeping the body immobile.

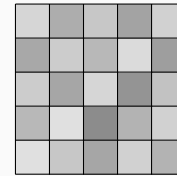
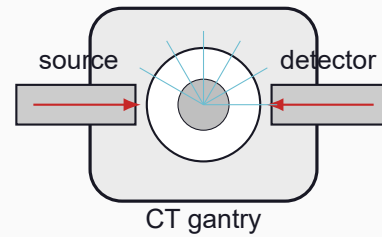
Contrast of image means the difference between black and white regions. It is increased by:

- increasing exposure to X-rays,
- increasing the intensity of X-rays,
- increasing exposure time,
- using a backlit screen.

CT Scans

3D X-ray Imaging

- CT stands for computed tomography.
- It produces a **3D X-ray** image.
- The part of the body to be analysed is divided into smaller parts called **voxels**.
- Each voxel has a particular absorption coefficient for X-rays called a **pixel** in the image slice.
- X-rays are sent from different angles through that part of the body and a 2D X-ray image is produced in the computer.
- The procedure is repeated for different angles and the images formed are joined together to form a 3D image.
- This 3D image can be viewed from different angles and stored.
- The same procedure is followed for all other voxels.



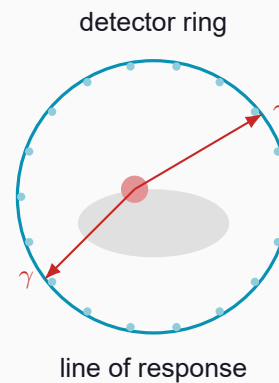
PET: Positron Emission Tomography

Why Use a PET Scan?

- It gives a 3D image.
- It is very precise in locating cancerous cells.
- Brain functioning can be monitored.

What is a PET Scan?

- It is a 3D medical imaging technique.
- An electron and a positron strike one another at high speed.
- They annihilate one another and produce a pair of gamma-ray photons.
- The two photons have equal energies and travel in opposite directions.
- Detectors are used to find the location of tissues to be analysed.



How PET Scan Works

Preparing a Tracer

- Some tissues absorb a specific type of compound or molecule.
- A PET scan uses radiation from those molecules.
- These molecules are made radioactive.
- A cyclotron is used to make tracer molecules.
- It smashes high-speed protons into ordinary atoms, turning loose isotopes into radioactive ones by changing the number of neutrons inside their nucleus.

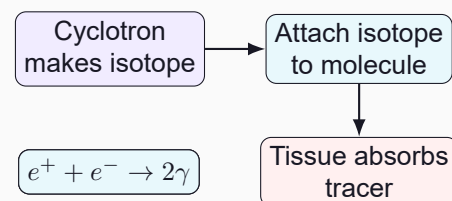
inkpurple

If neutrons < protons: β^+ emission.
If neutrons > protons: β^- emission.

- The isotopes have short half-lives, so the sample is prepared a few hours before the PET scan, e.g. about 2 hours earlier.
- Examples of tracer molecules:
 1. sugar / fluorodeoxyglucose,
 2. protein / RGDfC,
 3. hormone / norepinephrine.
- A radioactive atom is attached to the molecule, making it a tracer.
- When this molecule is absorbed, the location of that part is found.

During a PET Scan

- The tracer is purified from less useful molecules and the amount of radioactivity is checked.
- When the tracer molecule is absorbed by the part of the brain or tissue that is to be analysed, the molecule's radioactive part emits a positron.
- The positron collides with an electron in the surroundings and emits two gamma-ray photons travelling in opposite directions.
- A ring of detectors placed around the patient detects the gamma photons.
- A line of response is mapped by detecting the two photons.
- A small degree of uncertainty exists in detecting the exact location; patients are advised not to move.



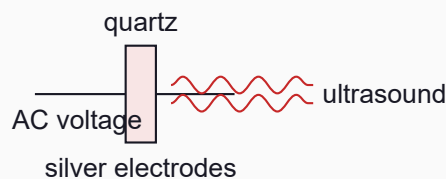
Ultrasound

Definition

Ultrasound: sound waves with $f > 20000 \text{ Hz}$.

Generation

- A piezo-electric quartz crystal is sandwiched between two silver electrodes.
- An alternating voltage is applied across it.
- The crystal is cut into a certain shape and size so that its natural frequency lies within the ultrasonic range.
- Due to the alternating voltage, the crystal resonates and produces an ultrasound wave in air.



Detection

- The same piezo-electric transducer used as the transmitter can act as a receiver.
- The compressions and rarefactions of reflected ultrasound are picked up by the transducer.
- The crystal oscillates and induces opposite charges on the plates.
- The alternating voltage signal is sent to the computer after amplification.
- The time of return and angle of return are used to detect internal structures.

Specific Acoustic Impedance

$$Z = \rho c$$

Z : specific acoustic impedance, ρ : density of medium, c : speed of sound in medium.

At a boundary between media:

$$I = I_R + I_T, \quad \frac{I_R}{I} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2.$$

A large difference in $Z_2 - Z_1$ gives a large reflected intensity.

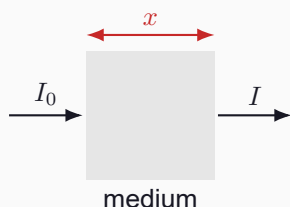
Calculations Regarding X-rays

Exponential Absorption

For X-rays passing through a medium of thickness x :

$$I = I_0 e^{-\mu x}$$

where I_0 is the incident intensity, I is the emerged intensity, μ is the absorption coefficient of the medium, and x is the thickness.



Layered Tissue Calculation

For multiple layers:

$$\frac{I}{I_0} = e^{-\mu_1 x_1} e^{-\mu_2 x_2} e^{-\mu_3 x_3} \dots$$

Model Arm Example

The model arm consists of muscle and bone. The linear absorption coefficients are:

$$\mu_{\text{muscle}} = 0.20 \text{ cm}^{-1}, \quad \mu_{\text{bone}} = 12 \text{ cm}^{-1}.$$

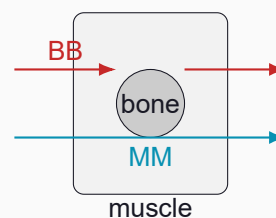
Line MM:

$$\frac{I}{I_0} = e^{-0.20(8)} = 0.20 = 20\%.$$

Line BB:

$$\frac{I}{I_0} = e^{-0.20(2)} e^{-12(4)} e^{-0.20(2)} = e^{-0.8} e^{-48} \approx 6.4 \times 10^{-22}.$$

A large difference between the two ratios gives good contrast; a small difference gives poor contrast.



Calculations for Ultrasound

At the Boundary

If ultrasound enters a layer and is reflected at a boundary:

$$\frac{I_R}{I} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2, \quad \frac{I_T}{I} = 1 - \frac{I_R}{I}.$$

Inside a medium, attenuation is modelled by:

$$I = I_0 e^{-kx}.$$

Fat–Muscle Example

For fat and muscle:

$$Z_{\text{fat}} = 1.3 \times 10^6, \quad Z_{\text{muscle}} = 1.7 \times 10^6.$$

$$\alpha = \frac{I_R}{I} = \left(\frac{1.7 \times 10^6 - 1.3 \times 10^6}{1.7 \times 10^6 + 1.3 \times 10^6} \right)^2 = 0.017.$$

For detected reflected ultrasound $I_S = 0.012I$:

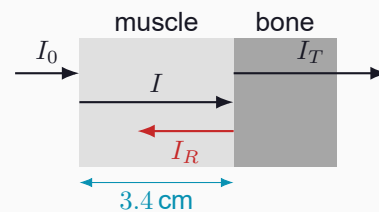
$$0.012 = e^{-48x} \times 0.017 \times e^{-48x} = 0.017e^{-96x}.$$

$$\ln \left(\frac{0.012}{0.017} \right) = -96x, \quad x = 3.6 \times 10^{-3} \text{ m} = 0.36 \text{ cm}.$$

Muscle–Bone Example

Data:

	μ/m^{-1}	$Z/\text{kg m}^{-2} \text{ s}^{-1}$
muscle	23	1.7×10^6
bone	130	6.3×10^6



$$\frac{I}{I_0} = e^{-23(0.034)} = 0.457.$$

$$\frac{I_R}{I} = \left(\frac{6.3 \times 10^6 - 1.7 \times 10^6}{6.3 \times 10^6 + 1.7 \times 10^6} \right)^2 \approx 0.33.$$

$$\frac{I_T}{I} = 1 - 0.33 = 0.67, \quad \frac{I_T}{I_0} \approx 0.457 \times 0.67 \approx 0.31.$$

inkblue

Handwritten notes highlight a final ratio of about 0.21; the algebraic substitution above gives about 0.31 for the single-pass transmitted intensity into bone.

One-page Formula Summary

X-rays

$$E_{\gamma} = hf = \frac{hc}{\lambda}, \quad E_{\max} = eV, \quad I = I_0 e^{-\mu x}.$$

- Heating voltage controls intensity.
- Accelerating voltage controls hardness.
- Hard X-ray: higher energy, shorter wavelength, more penetrating.
- Soft X-ray: lower energy, longer wavelength, less penetrating.

PET

$$e^{+} + e^{-} \rightarrow 2\gamma$$

- A tracer is absorbed by active tissue.
- A positron annihilates with an electron.
- Two gamma photons are detected in opposite directions.

Ultrasound

$$f > 20000 \text{ Hz}, \quad Z = \rho c.$$
$$\frac{I_R}{I} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2, \quad \frac{I_T}{I} = 1 - \frac{I_R}{I}.$$

- Piezo-electric crystal converts electrical oscillations to ultrasound.
- Reflected ultrasound produces voltage in the transducer.
- Return time and angle locate internal structures.