

Electromagnetic Induction

Chapter 16 / A2 Level / Apollo Physics 9702

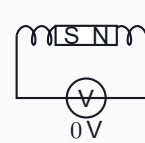
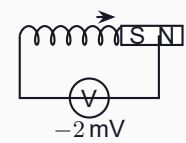


Core idea

In the DC motor, a force was produced using **current and magnetic field**. In electromagnetic induction, the reverse question is asked: can **current/voltage** be produced using **motion/force and magnetic field**?

An induced e.m.f. appears when the magnetic flux linkage through a circuit changes.

Case	Situation	Voltmeter reading
1	Magnet stationary outside the coil	0
2	Magnet moving towards the coil	positive pulse
3	Magnet stationary inside the coil	0
4	Magnet being pulled out of the coil	negative pulse
5	Increasing number of turns of coil	larger induced e.m.f.
6	Increasing strength of magnet	larger induced e.m.f.
7	Increasing speed of motion	larger induced e.m.f.
8	Field lines interacting with coil	magnetic flux linkage

1. Stationary**2. Moving in****3. Stationary inside****4. Moving out****Important pattern**

No induced e.m.f. is produced merely by the presence of a magnet. The e.m.f. is produced only when the magnetic flux linkage changes.

Magnetic flux linkage, Faraday's law and Lenz's law

From cases 1 to 4

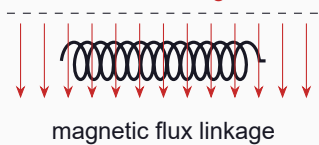
To induce e.m.f. in a conductor, a **change in the interaction of magnetic field lines with the coil** is needed.

$$\mathcal{E} \propto N, \quad \mathcal{E} \propto \Delta\Phi, \quad \mathcal{E} \propto \frac{\Delta\Phi}{\Delta t}$$

Therefore

$$\mathcal{E} \propto \frac{N\Delta\Phi}{\Delta t}$$

field lines interacting with coil



Magnetic flux linkage

Magnetic flux linkage means magnetic field lines interacting with the coil.

$$\Phi = BA \quad (\text{when field is normal to the area}), \quad N\Phi = NBA$$

where N is the number of turns.

Faraday's law of electromagnetic induction

The e.m.f. induced in a conductor is directly proportional to the rate of change of magnetic flux linkage.

$$\mathcal{E} = \frac{\Delta(N\Phi)}{\Delta t}$$

For calculus form:

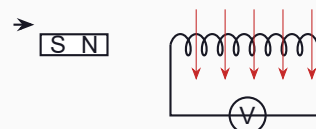
$$\mathcal{E} = \frac{d(N\Phi)}{dt}$$

Lenz's law

The induced e.m.f. is in such a direction that it **opposes the change which produces it**.

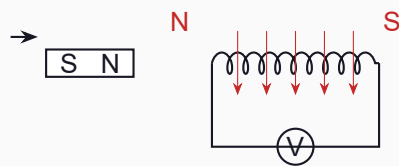
$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

The negative sign gives the opposition predicted by Lenz's law.

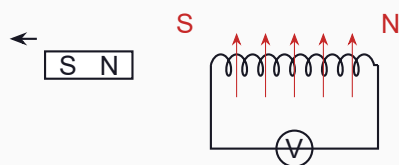


Approaching magnet causes a change in magnetic flux linkage

Lenz's law and conservation of energy



Induced current creates a field that opposes the approach.



When the magnet withdraws, the induced field opposes withdrawal.

Why this agrees with conservation of energy

Lenz's law also relates to the law of conservation of energy:

Energy is neither created nor destroyed; it is converted from one form to another.

For electrical energy to be produced, the magnet must lose kinetic energy. The opposition faced by the magnet achieves this goal. Hence work must be done continuously to keep the coil moving relative to the magnet.

Physical meaning of the minus sign

$$\mathcal{E} = - \frac{d(N\Phi)}{dt}$$

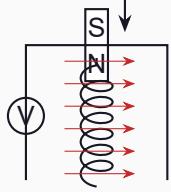
The sign does not make the e.m.f. "negative energy". It tells the direction of the induced e.m.f. and current: the induced current makes a magnetic field that resists the change in flux linkage.

No free energy

If the induced current helped the change, the magnet would accelerate and produce more electrical energy without any external work. Lenz's law prevents this by making the induced effect oppose the cause.

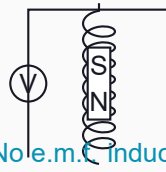
Dropping a small magnet through a long coil

Magnet enters



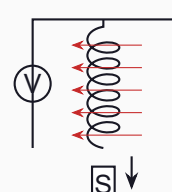
Flux linkage increases;
hence e.m.f. is induced.

Inside coil



No e.m.f. induced.
Equal interaction of field with
both sides gives no change in
magnetic flux linkage.

Magnet exits

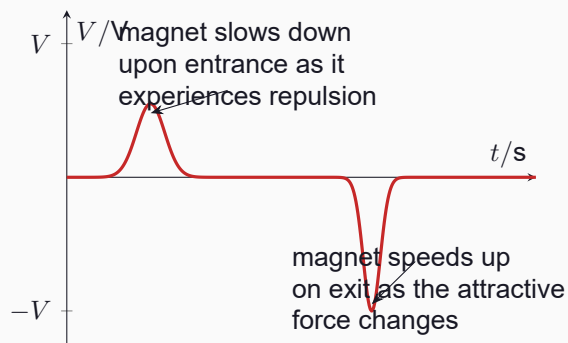


Flux linkage decreases;
e.m.f. is induced again,
with opposite sign.

Lenz force during the drop

As the magnet enters the coil, it experiences repulsion: the magnet slows down upon entrance. Repulsion increases as it approaches closer. As the magnet exits the coil, it experiences attraction in the opposite direction: the magnet may speed up, but the attractive force decreases as its distance from the coil increases.

Induced e.m.f. graph for a falling magnet



Interpreting the graph

- A pulse is produced on entry because flux linkage changes.
- No e.m.f. is produced while flux linkage is not changing appreciably.
- A pulse of the opposite sign is produced on exit because the direction of change of flux linkage reverses.
- The narrower/larger pulse can correspond to faster motion and a greater rate of change of flux linkage.

Rate matters

$$|\mathcal{E}| = \left| \frac{\Delta(N\Phi)}{\Delta t} \right|$$

Greater speed, stronger magnet, or more turns means a larger induced e.m.f. because the rate of change of flux linkage is larger.

Exam-style example: oscillating magnet and coil

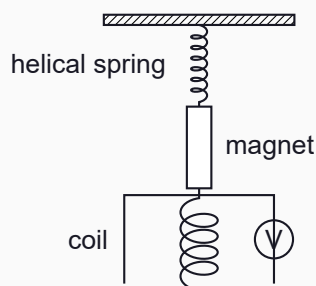


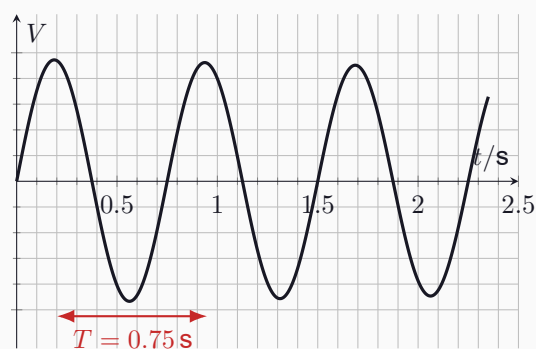
Fig. 5.1

A bar magnet is suspended vertically from the free end of a helical spring. One pole of the magnet is situated in a coil. The coil is connected in series with a high-resistance voltmeter. The magnet is displaced vertically and then released.

(a) Faraday's law

The e.m.f. induced in the conductor is directly proportional to the rate of change of magnetic flux linkage:

$$\mathcal{E} \propto \frac{\Delta(N\Phi)}{\Delta t}.$$



From the graph

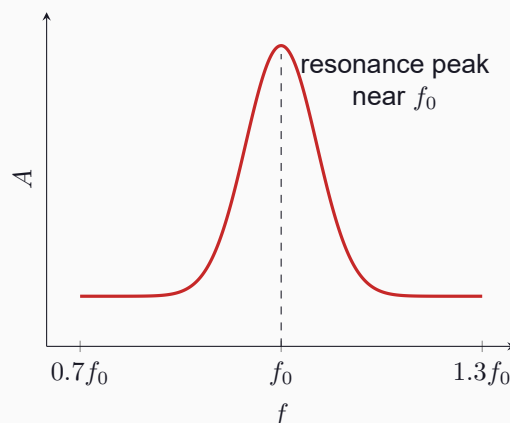
The voltmeter reading alternates between positive and negative values because the direction of motion of the magnet is reversed after every half oscillation.

$$T = 0.75 \text{ s}, \quad f_0 = \frac{1}{T} = \frac{1}{0.75} = 1.33 \text{ Hz}.$$

Resonance of the magnet-coil system

Why the graph has positive and negative values

As the magnet oscillates, the direction of motion changes after every half oscillation. This reverses the direction of change of magnetic flux linkage through the coil. By Lenz's law, the induced e.m.f. also reverses sign.



Frequency calculation

$$T = 0.75 \text{ s}$$
$$f_0 = \frac{1}{T} = \frac{1}{0.75} = 1.33 \text{ Hz.}$$

Sketch requirement

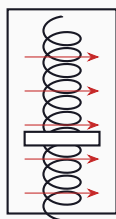
When an alternating supply is connected to the coil, maximum amplitude occurs when the driving frequency equals the natural frequency of the magnet-spring system. Thus the amplitude-frequency curve peaks at $f = f_0$.

Magnetic flux density and flux linkage in a solenoid

Definition of the tesla

One tesla is the magnetic flux density when a wire of length 1 m, carrying a current of 1 A normal to the magnetic field, experiences a force of 1 N.

$$B = \frac{F}{IL}$$



solenoid
area = 28 cm^2
coil C, 160 turns

For a long solenoid,

$$B = \mu_0 n I.$$

Given $n = 1.5 \times 10^3 \text{ m}^{-1}$, $I = 3.5 \text{ A}$, and $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$,

$$B = (4\pi \times 10^{-7})(1.5 \times 10^3)(3.5) = 6.6 \times 10^{-3} \text{ T}.$$

Flux linkage

Area:

$$A = 28 \text{ cm}^2 = 28 \times 10^{-4} = 2.8 \times 10^{-3} \text{ m}^2.$$

$$N\Phi = NBA = 160(6.6 \times 10^{-3})(2.8 \times 10^{-3})$$

$$N\Phi \approx 2.96 \times 10^{-3} \text{ Wb}.$$

Average e.m.f. when current is reversed

Faraday's law

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}.$$

The magnitude of the average induced e.m.f. is

$$|\mathcal{E}| = \left| \frac{(N\Phi)_2 - (N\Phi)_1}{\Delta t} \right|.$$

Worked example

If the flux linkage changes from

$$+2.6 \times 10^{-3} \text{ Wb}$$

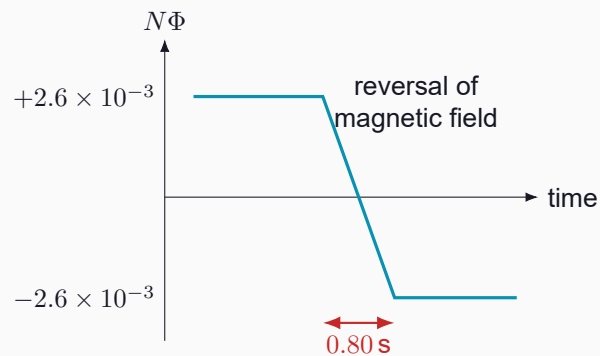
to

$$-2.6 \times 10^{-3} \text{ Wb}$$

in 0.80 s, then

$$\Delta(N\Phi) = -2.6 \times 10^{-3} - (+2.6 \times 10^{-3}) = -5.2 \times 10^{-3} \text{ Wb}.$$

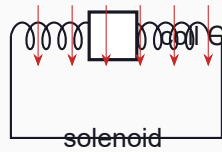
$$|\mathcal{E}| = \frac{5.2 \times 10^{-3}}{0.80} = 6.5 \times 10^{-3} \text{ V}.$$



Sign convention

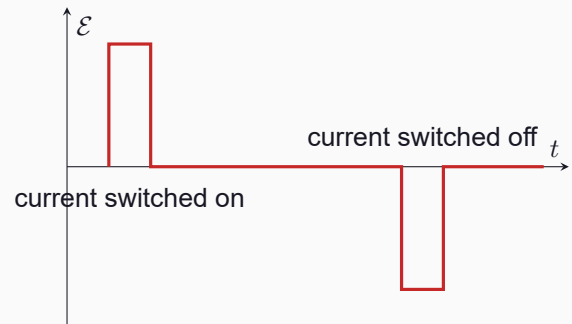
The sign of the induced e.m.f. depends on the chosen direction of the coil and flux. Exam calculations often ask for the magnitude unless a sign is explicitly required.

Switching current on and off: induced e.m.f. pulse



What happens

When current in the solenoid is switched on, magnetic flux linkage through coil C changes from zero to a value, so an e.m.f. pulse is induced. When the current becomes steady, $\Delta(N\Phi)/\Delta t = 0$, so no e.m.f. is induced. When switched off, the flux linkage decreases to zero and an opposite pulse is produced.

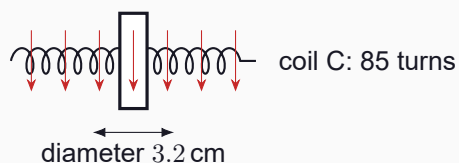


Graph rule

$$\mathcal{E} \propto -\frac{\Delta I}{\Delta t}$$

for a fixed solenoid-coil geometry, because $B \propto I$ and $N\Phi \propto B$.

Solenoid question: flux linkage and mean e.m.f.



Show flux linkage

Diameter $d = 3.2 \text{ cm} = 3.2 \times 10^{-2} \text{ m}$, so

$$A = \pi \left(\frac{d}{2} \right)^2 = \pi (1.6 \times 10^{-2})^2.$$

If

$$B = 3.1 \times 10^{-3} I,$$

then for $I = 2.8 \text{ A}$,

$$B = 8.68 \times 10^{-3} \text{ T}.$$

$$N\Phi = NBA = 85(8.68 \times 10^{-3})\pi(1.6 \times 10^{-2})^2 \approx 6.0 \times 10^{-4} \text{ Wb}.$$

Mean e.m.f. when current is reversed

If current is reversed, flux linkage changes from

$$+6.0 \times 10^{-4} \text{ Wb} \quad \text{to} \quad -6.0 \times 10^{-4} \text{ Wb}.$$

$$\Delta(N\Phi) = 1.2 \times 10^{-3} \text{ Wb}.$$

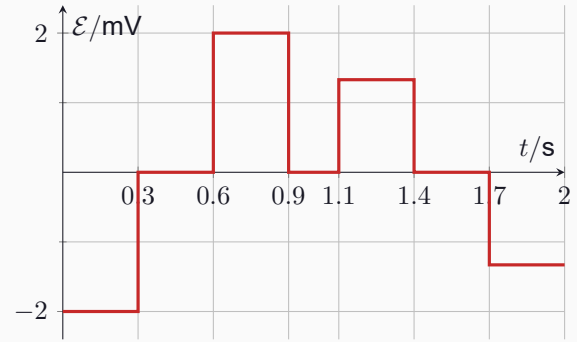
In 0.30 s,

$$|\mathcal{E}| = \frac{1.2 \times 10^{-3}}{0.30} = 4.0 \times 10^{-3} \text{ V} = \boxed{4.0 \text{ mV}}.$$

Reasoning shortcut

Because $N\Phi \propto I$, the sign of the flux linkage reverses when the current is reversed. The change is therefore twice the initial magnitude.

Induced e.m.f. from a changing current waveform



How to sketch $\mathcal{E}$

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

Since $N\Phi \propto I$,

$$\mathcal{E} \propto -\frac{\Delta I}{\Delta t}.$$

Thus the induced e.m.f. is constant wherever the current graph has a constant gradient, zero wherever the current is constant, and changes sign when the gradient changes sign.

Signs

If ΔI is positive, $\mathcal{E}$ is negative. If ΔI is negative, $\mathcal{E}$ is positive. If $\Delta I = 0$, $\mathcal{E} = 0$.

Final practice summary

Common calculation chain

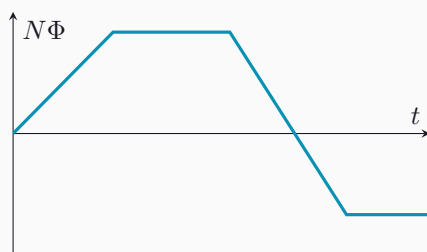
$$B \rightarrow \Phi = BA \rightarrow N\Phi = NBA \rightarrow \mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

For a solenoid where $B \propto I$,

$$N\Phi \propto I, \quad \mathcal{E} \propto -\frac{\Delta I}{\Delta t}.$$

If a graph of $N\Phi$ is given

- Gradient of $N\Phi$ -time graph gives $\Delta(N\Phi)/\Delta t$.
- Induced e.m.f. is the negative of this gradient.
- A horizontal section gives zero e.m.f.
- A steeper section gives larger magnitude e.m.f.



What must be retained in answers

1. State Faraday's law using "rate of change of magnetic flux linkage".
2. State Lenz's law using "opposes the change producing it".
3. Use the correct number of turns N when computing flux linkage.
4. Convert cm^2 to m^2 .
5. When current reverses, flux linkage changes from $+N\Phi$ to $-N\Phi$, so total change is $2N\Phi$.

