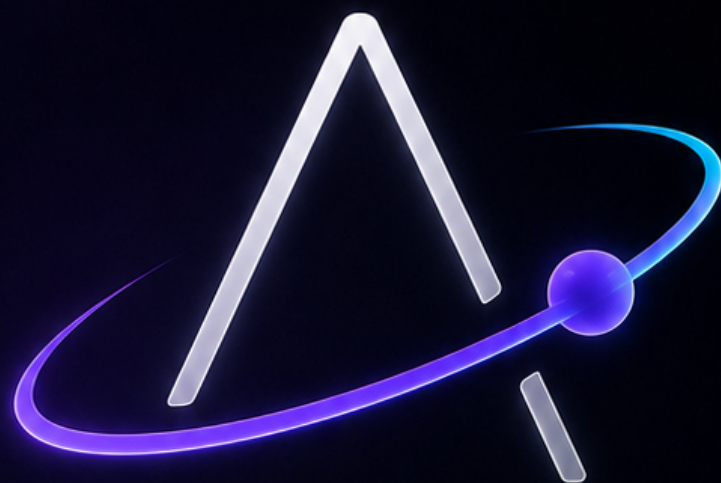




Apollo Physics 9702

Cambridge International A2 Level Notes

Apollo vertical notes edition

Uniform Circular Motion

Chapter 01 / A2 Level / Apollo Physics 9702

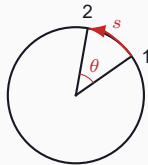


Uniform Circular Motion

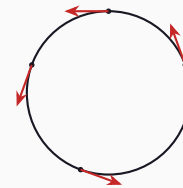
Uniform: Objects moving in a circle at constant speed.

1. Linear displacement (s)

- It is the length of the arc produced by an object moving in a circle about a point.
- Although displacements are supposed to be straight lines, for circular motion the shortest possible distance to travel from 1 to 2 is the arc formed.



Key idea: in circular motion the direction of the velocity changes continuously even if the speed stays constant. The velocity vector is always tangent to the path.



tangent velocity at each point

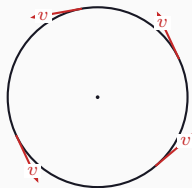
2. Angular displacement (θ)

- The angle subtended by the arc formed when an object moves in a circle about a point.
- The angle is measured in radians.

$$\begin{array}{ll}
 360^\circ & 2\pi \text{ rad} \\
 180^\circ & \pi \text{ rad} \\
 90^\circ & \frac{\pi}{2} \text{ rad} \\
 45^\circ & \frac{\pi}{4} \text{ rad}
 \end{array}
 \quad
 60^\circ = \frac{60}{180}\pi = \frac{\pi}{3} \text{ rad}$$

3. Linear velocity (v)

- The rate of change of linear displacement about a point.
- $v = \frac{\Delta s}{\Delta t}$ SI unit: m s^{-1} .
- The linear velocity of the body is tangent to the point on the circle.



Angular Velocity / Speed

Angular velocity / speed (ω)

The rate of change of angular displacement about a point.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

SI unit: rad s^{-1}

If $\Delta\theta = 2\pi \text{ rad}$ and $\Delta t = T$, then

$$\omega = \frac{2\pi}{T}.$$

Since $f = \frac{1}{T}$,

$$\omega = 2\pi f$$

rpm: revolutions per minute (N).

$$N = f \times 60, \quad f = \frac{N}{60}, \quad \omega = 2\pi \left(\frac{N}{60} \right).$$

Relating v and ω

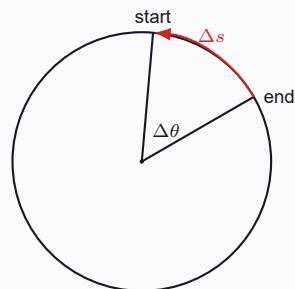
Applying $s = r\theta$ to the sector:

$$\Delta s = r\Delta\theta.$$

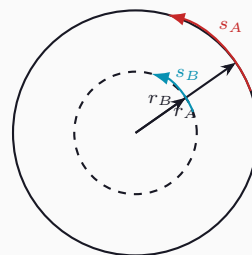
Dividing both sides by Δt :

$$\frac{\Delta s}{\Delta t} = r \frac{\Delta\theta}{\Delta t}.$$

$$v = r\omega$$



The point further away from the centre of rotation covers more linear displacement in the same time than the one that is closer.



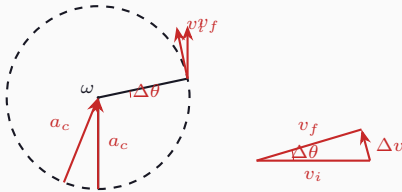
$s_A > s_B$ hence $v_A > v_B$

Centripetal Acceleration and Centripetal Force

Centripetal acceleration (a_c)

The rate of change of linear velocity about a point.

- This acceleration is due to the change of direction of motion.
- For uniform circular motion, $|v_i| = |v_f| = v$.



For a very small change of angle, the velocity triangle can be approximated as a sector:

$$s = r\theta \Rightarrow \Delta v = v\Delta\theta.$$

Dividing by Δt :

$$\frac{\Delta v}{\Delta t} = v \frac{\Delta\theta}{\Delta t} = v\omega.$$

$$a_c = v\omega$$

Using $v = r\omega$:

$$a_c = (r\omega)\omega = r\omega^2.$$

Using $\omega = v/r$:

$$a_c = v \left(\frac{v}{r} \right) = \frac{v^2}{r}.$$

$$a_c = r\omega^2$$

$$a_c = \frac{v^2}{r}$$

Centripetal force (F_c)

The resultant force that causes an object to move in a circle about a point.

- It is a title given to any force which causes an object to move in a circle about a point.

$$F_c = ma_c$$

$$F_c = mv\omega$$

$$F_c = \frac{mv^2}{r}$$

$$F_c = mr\omega^2$$

Case	Centripetal force
Stone on string	Tension force
Planetary motion	Gravitation
Electron around nucleus	Electrostatic force
Car making a turn	Friction force

For an object in circular motion of constant radius:

$$F_c \propto v^2$$

The greater the speed, the greater the centripetal force.

Conical Pendulum / Motion in a Horizontal Circle

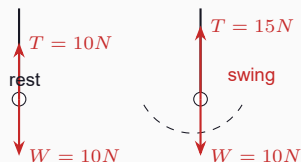
Vertical circle examples

For a body at rest on a string:

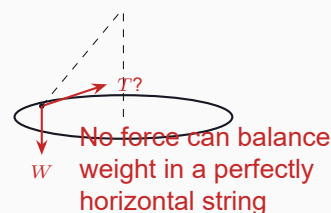
$$T = W.$$

For a body swinging in a vertical circle at a position where upward tension and downward weight act:

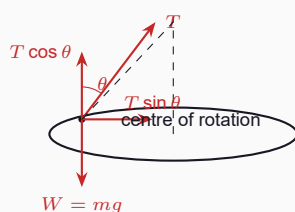
$$F_{net} = F_c, \quad T - W = F_c, \quad T - W = ma_c.$$



If v increases, θ also increases. A pure horizontal motion, i.e. $\theta = 90^\circ$, is not possible because no force exists to balance weight.



Motion in a horizontal circle



Vertically:

$$T \cos \theta = W = mg.$$

Horizontally:

$$T \sin \theta = F_c = \frac{mv^2}{r}.$$

Dividing horizontal by vertical:

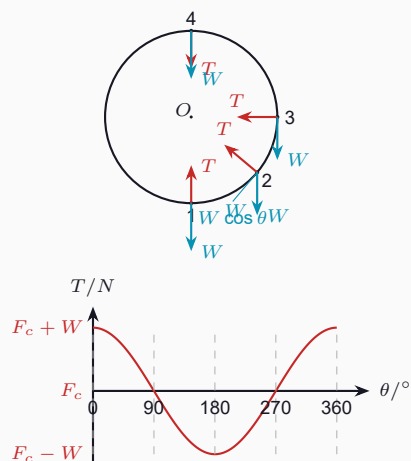
$$\frac{T \sin \theta}{T \cos \theta} = \frac{mv^2/r}{mg}$$

$$\tan \theta = \frac{v^2}{rg}$$

Useful summary

Vertical component	$T \cos \theta = mg$
Horizontal component	$T \sin \theta = mv^2/r$
Final relation	$\tan \theta = v^2/(rg)$

Motion in a Vertical Circle



1. Bottom position

$$F_{net} = ma_c, \quad T - W = ma_c, \quad F_c = T - W.$$

2. General position

$$T - W \cos \theta = ma_c, \quad T - W \cos \theta = F_c.$$

Hence

$$T = W \cos \theta + F_c$$

This has the form $y = a \cos \theta + c$, where a is the amplitude and c is the y -intercept.

θ	$\cos \theta$	T/N
0°	1	$W + F_c$
90°	0	F_c
180°	-1	$-W + F_c$
270°	0	F_c
360°	1	$W + F_c$

4. Top position

$$F_{net} = ma_c, \quad T + W = ma_c, \quad T + W = F_c.$$

At the minimum speed of rotation, the string just remains taut at the top, so $T = 0$:

$$mg = \frac{mv^2}{r}.$$

$$v_{\min} = \sqrt{gr}$$

Minimum speed of rotation

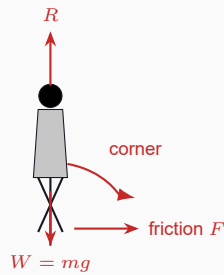
$$F_c = \frac{mv^2}{r}, \quad F_c \propto v^2.$$

At the top, if $W = 10 \text{ N}$:

$T + W = F_c$	comment
$5 + 10 = 15 \text{ N}$	possible
$2 + 10 = 12 \text{ N}$	slower
$0 + 10 = 10 \text{ N}$	minimum speed
$0 + 10 > 8 \text{ N}$	object falls vertically

Bike Making a Turn Around a Corner

Cyclist on a curved road



A cyclist leans because the horizontal resultant force must point toward the centre of the circular path.

$$F_{net} = ma_c, \quad R \sin \theta + F = ma_c, \quad R \sin \theta + F = F_c.$$

At higher speed:

$$F_c = \frac{mv^2}{r} \text{ increases.}$$

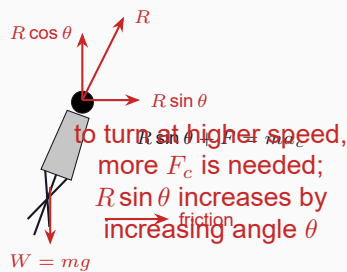
Hence the cyclist must lean more, increasing θ , so that $R \sin \theta$ increases.

For an upright cyclist:

$$R = W, \quad F = F_c.$$

Greater speed $\Rightarrow$ greater centripetal force needed $\Rightarrow$ greater lean

Leaning cyclist



Exam Question: Proving $a = v^2/r$

Question

A body is travelling in a circular orbit of radius r with constant speed v as shown in Fig. 1.1. Use a vector diagram to show that the acceleration a of the body is given by

$$a = \frac{v^2}{r}$$

towards the centre of the circle.

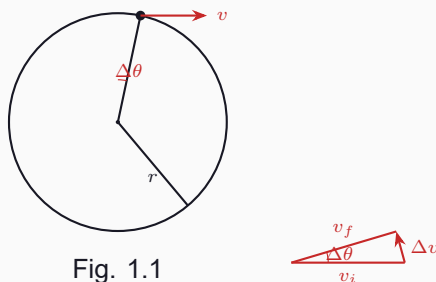


Fig. 1.1

Solution

For small $\Delta\theta$, the velocity triangle can be treated as a sector:

$$\Delta v = v\Delta\theta.$$

Dividing by time:

$$\frac{\Delta v}{\Delta t} = v \frac{\Delta\theta}{\Delta t} = v\omega.$$

But $v = r\omega$, so $\omega = v/r$:

$$a = v\omega = v \left(\frac{v}{r} \right) = \frac{v^2}{r}.$$

The acceleration is toward the centre because the change in velocity is directed inward.

Exam Question: Spin Dryer

Question

The drum of a spin drier has a rate of rotation of 4.0 revolutions per second. An object in the drum has a mass of 0.20 kg and rotates in a vertical circle of radius 0.16 m.

- (i) Calculate the magnitude of the acceleration of the object.
- (ii) Calculate the magnitude of the resultant force on the object.
- (iii) For each of the three positions shown, draw arrows to represent the weight W of the object and the force D that the drum exerts on the object. Indicate how these two forces always add to produce the resultant force of constant magnitude calculated in (ii).

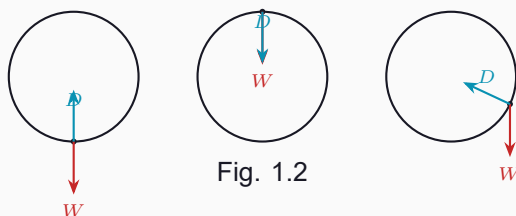
Working

Frequency $f = 4 \text{ Hz}$, so

$$\omega = 2\pi f = 2\pi(4).$$

$$a = r\omega^2 = 0.16(2\pi \times 4)^2 = 101 \text{ m s}^{-2}.$$

$$F_{\text{net}} = ma = 0.20(101) = 20.2 \text{ N} \approx 20 \text{ N}.$$

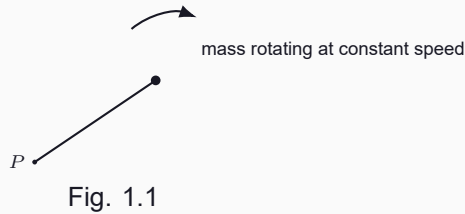


At each position, D and W combine vectorially to give the same inward centripetal resultant.

Exam Question: Radian and Angular Speed

Question

- (a)(i) Define the *radian*.
- (a)(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed. Explain what is meant by the angular speed about point P of the mass.



Answers

- One radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle:
 $s = r\theta$, so for $s = r$, $\theta = 1$ rad.
- Angular speed is the angular displacement covered, or angle swept out, by the body per unit time about a fixed point:

$$\omega = \frac{\Delta\theta}{\Delta t}.$$

Exam Question: Rotating Plate

Question

A horizontal flat plate is free to rotate about a vertical axis through its centre. A small mass M is placed on the plate, a distance d from the axis of rotation. The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.

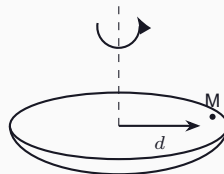


Fig. 1.2

The maximum frictional force F between the plate and the mass is

$$F = 0.72W,$$

where W is the weight of mass M . The distance d is 35 cm. Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate.

Solution

Friction acts as the centripetal force:

$$F = F_c.$$

$$0.72W = mr\omega^2, \quad 0.72mg = mr\omega^2.$$

$$\omega^2 = \frac{0.72g}{r} = \frac{0.72(9.81)}{0.35}, \quad \omega = 4.49 \text{ rad s}^{-1}.$$

$$\omega = 2\pi f \Rightarrow f = \frac{4.49}{2\pi} = 0.715 \text{ Hz}.$$

$$N = f \times 60 = 0.715 \times 60 = 43 \text{ rpm}.$$

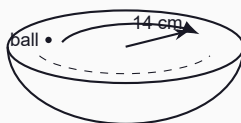
Mud on the plate

Mud near the edge leaves first. Since $F_c = mr\omega^2$, the centripetal force needed is directly proportional to radius, so mud at the edge requires more force than mud near the centre.

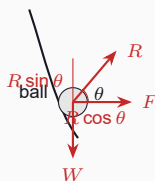
Exam Question: Ball in a Bowl

Question

A large bowl is made from part of a hollow sphere. A small spherical ball is placed inside the bowl and is given a horizontal speed. The ball follows a horizontal circular path of constant radius.



The forces acting on the ball are its weight W and the normal reaction force R of the bowl on the ball. The normal reaction force R is at an angle θ to the horizontal.



By resolving the reaction force R into two perpendicular components, show that the resultant force F acting on the ball is given by

$$W = F \tan \theta.$$

Solution

Resolve R :

$$R \sin \theta = W, \quad R \cos \theta = F.$$

From the second equation,

$$R = \frac{F}{\cos \theta}.$$

Substitute into the vertical equation:

$$\left(\frac{F}{\cos \theta} \right) \sin \theta = W.$$

$$F \tan \theta = W.$$

Therefore

$$W = F \tan \theta$$

Exam Question: Bowl Speed Calculation

Question

State the significance of the force F for the motion of the ball in the bowl. The ball moves in a circular path of radius 14 cm. For this radius, the angle θ is 28° . Calculate the speed of the ball.

Answer

Force F acts as the centripetal force.

$$F = \frac{mv^2}{r}.$$

From the previous result:

$$W = F \tan \theta.$$

Substitute $W = mg$ and $F = mv^2/r$:

$$mg = \frac{mv^2}{r} \tan \theta.$$

Cancel m :

$$g = \frac{v^2}{r} \tan \theta.$$

$$9.81 = \frac{v^2}{0.14} \tan 28^\circ.$$

$$v = 1.6 \text{ m s}^{-1}.$$

$$\text{speed} = 1.6 \text{ m s}^{-1}$$

Gravitation

Chapter 02 / A2 Level / Apollo Physics 9702



Newton's Law of Universal Gravitation

Newton's law of universal gravitation

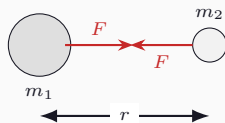
- The force of attraction between two point masses is directly proportional to their product.
- It is inversely proportional to the square of separation.

$$F \propto m_1 m_2, \quad F \propto \frac{1}{r^2}, \quad F \propto \frac{m_1 m_2}{r^2}$$

$$F = \frac{G m_1 m_2}{r^2}$$

Universal gravitational constant

$$G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$$



Example 1: two people

$$m_1 = 80 \text{ kg}, \quad m_2 = 85 \text{ kg}, \quad r = 1 \text{ m}$$

$$F = \frac{G m_1 m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 80 \times 85}{1^2}$$

$$F = 4.5 \times 10^{-7} \text{ N}$$

This is very small. **Example 2: person and Earth**

$$M_E = 5.99 \times 10^{24} \text{ kg}, \quad m = 80 \text{ kg}, \quad r = R_E = 6.4 \times 10^6 \text{ m}$$



$$F = \frac{G m M_E}{R_E^2} = \frac{6.67 \times 10^{-11} \times 80 \times 5.99 \times 10^{24}}{(6.4 \times 10^6)^2}$$

$$F = 780 \text{ N} \quad \text{significant}$$

Weight check:

$$W = mg = 80 \times 9.8 = 784 \text{ N}.$$

Attractive forces are written with a negative sign by convention:

$$F = -\frac{G m_1 m_2}{r^2}$$

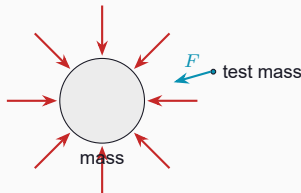
Attractive forces cause objects to lose potential energy and cause them to do work.

Gravitational Field and Field Strength

Gravitational field

A gravitational field is a region in space around a mass where another mass experiences a gravitational force.

- Direction of field lines gives the direction of force on a small mass.
- The gap between field lines shows the strength of field.
- Less gap $\Rightarrow$ strong field; more gap $\Rightarrow$ weak field.



Gravitational field strength

Gravitational field strength is the force per unit mass at a point in a gravitational field.

$$g = \frac{F}{m}.$$

Using $F = \frac{GMm}{r^2}$:

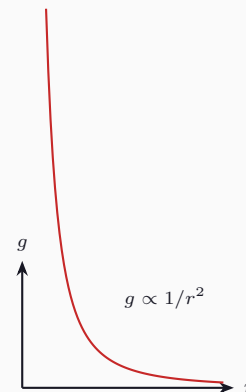
$$g = \frac{GMm/r^2}{m} = \frac{GM}{r^2}.$$

$$g = \frac{F}{m}$$

$$g = \frac{GM}{r^2}$$

- g is a vector quantity.
- g exists till infinity.
- For a point mass: $g \propto 1/r^2$.

$g-r$ graph for a point mass isolated in space



Point mass idea

A uniformly distributed spherical mass can be treated as a point mass at its centre when considered from outside the body.

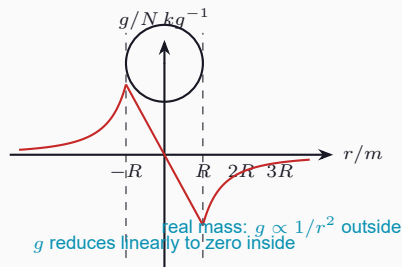


- Presence of the body is experienced outside its surface as if all mass were at the centre.
- No gravitational field or force exists inside a point mass.
- The distance between masses is usually so large compared to their radii that the size of the mass is ignored.

$g - r$ Graphs and Null Points

$g - r$ graph for a real spherical mass

For a real mass, g linearly reduces to zero inside the planet and then follows inverse-square behaviour outside.

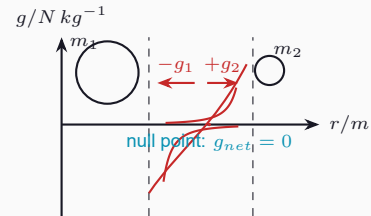


$$r = R \Rightarrow g = g, \quad r = 2R \Rightarrow g = \frac{g}{4}, \quad r = 3R \Rightarrow g = \frac{g}{9}.$$

For a point mass, there is no physical interior; inside a point mass $g = 0$.

$g - r$ graph between two point masses

At a point P between masses, gravitational fields due to the two masses are in opposite directions.



$$g_{net} = -g_1 + g_2$$

$$g_{net} = -\frac{Gm_1}{r^2} + \frac{Gm_2}{(x-r)^2}.$$

For a null point,

$$g_{net} = 0$$

$$0 = -\frac{Gm_1}{r^2} + \frac{Gm_2}{(x-r)^2}$$

$$\frac{Gm_1}{r^2} = \frac{Gm_2}{(x-r)^2}$$

$$\boxed{\frac{m_1}{m_2} = \frac{r^2}{(x-r)^2}}$$

where r is the distance from m_1 to the null point.

Gravitational Potential Energy (U_p)

Meaning of gravitational potential energy

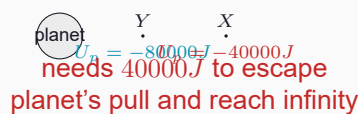
More energy is needed because:

- the object is moved further away from infinity;
- the gravitational field is stronger near the planet.

At infinity:

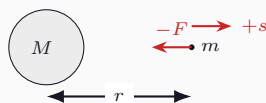
$$U_p = 0$$

A negative potential energy means the object has a deficiency of energy relative to infinity.



Definition

Gravitational potential energy is the work done in moving an object from infinity to a point in a gravitational field.



Using work done against gravity:

Potential energy = work done against gravity = $-F \times s$.

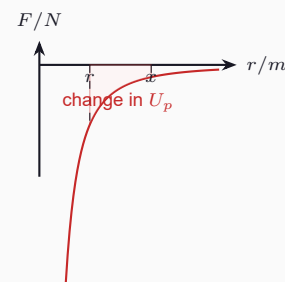
For gravitation:

$$U_p = -\frac{GMm}{r^2} \times r$$

$$U_p = -\frac{GMm}{r}$$

More formally:

$$W = \int F ds = \int \frac{GMm}{r^2} dr = GMm \int r^{-2} dr = -\frac{GMm}{r}.$$



Area under an $F - r$ graph gives the change in gravitational potential energy:

$$\text{Area under } F - r \text{ graph} = \text{change in potential energy}$$

Why mgh is not applicable now; Gravitational Potential

Why mgh is not applicable now?

Change in potential energy near Earth's surface is

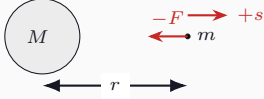
$$\Delta E_p = F \times s = W \times h = mgh.$$

Here g is assumed constant.

- The equation is valid only when gravitational field strength is constant and the object experiences a constant weight force.
- This assumption becomes invalid if the distance travelled is very large and far away from the planet because field strength changes from place to place.

Gravitational potential (ϕ)

It is the work done per unit mass to move an object from infinity to a point in a gravitational field.



$$\phi = \frac{U_p}{m} = \frac{-GMm/r}{m}$$

$$\phi = -\frac{GM}{r}$$

Example:

$$U_p = -100000 \text{ J}, \quad m = 20 \text{ kg}$$

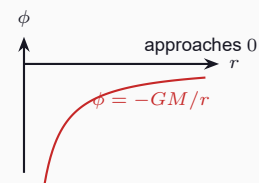
$$\phi = \frac{U_p}{m} = \frac{-100000}{20}$$

$$\phi = -5000 \text{ J kg}^{-1}$$

The closer we are to a planet, the more negative is our potential.

Why is gravitational potential negative?

- Gravitational potential is zero at infinity.
- As a body moves towards the planet, the force is attractive and work is done by the mass.
- Therefore potential decreases and is negative.

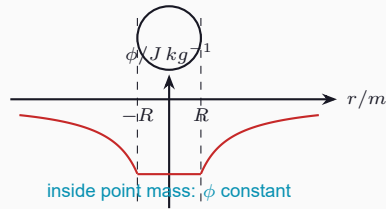


$\phi - r$ Graph and the Link between g and Potential Gradient

$\phi - r$ graph for a point mass isolated in space

For a point mass:

$$\phi = -\frac{GM}{r}.$$



Inside a point mass:

- gravitational field strength is zero;
- force is zero;
- no more work is done;
- ϕ will not decrease, so it is constant.

For outside the mass:

$$\text{gradient} = \frac{\Delta\phi}{\Delta r} = \frac{\Delta\phi}{\Delta x}.$$

$$\phi = -\frac{GM}{r} \Rightarrow \frac{d\phi}{dr} = \frac{GM}{r^2}.$$

By convention, gravitational field strength is the negative of gravitational potential gradient:

$$g = -\frac{\Delta\phi}{\Delta r}$$

Gravitational field strength is negative of gravitational potential gradient

$$\frac{d\phi}{dr} = \frac{GM}{r^2} \Rightarrow g = -\frac{GM}{r^2}$$

for a field directed towards the mass.

Energy Changes from a Potential Graph

Important relationship

$$\text{change in potential energy} = \text{change in kinetic energy}$$

Since $U_p = \phi m$:

$$\Delta U_p = \Delta \phi m.$$

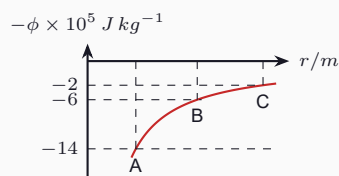
For motion in a gravitational field:

$$\Delta \phi m = \frac{1}{2} m (v^2 - u^2)$$

$$\Delta \phi = \frac{1}{2} (v^2 - u^2)$$

$$(\phi_2 - \phi_1) = \frac{1}{2} (v^2 - u^2).$$

Example graph



(a) A rock falls from rest radially towards a planet from C to A. Calculate its speed at A.

loss in P.E. = gain in K.E.

$$-\Delta \phi m = \frac{1}{2} m (v^2 - u^2)$$

$$-(\phi_2 - \phi_1) = \frac{1}{2} (v^2 - u^2)$$

$$-(-14 - (-2)) \times 10^5 = \frac{1}{2} (v^2 - 0)$$

$$12 \times 10^5 \times 2 = v^2$$

$$v = \sqrt{24 \times 10^5} = 1549.19 \approx 1550 \text{ m s}^{-1}$$

(b) A rock is projected upward radially from A. Determine the minimum speed required for it to reach B.

gain in P.E. = loss in K.E.

$$(\phi_2 - \phi_1) = -\frac{1}{2} (v^2 - u^2)$$

$$(-6 - (-14)) \times 10^5 = -\frac{1}{2} (0 - u^2)$$

$$8 \times 10^5 = \frac{1}{2} u^2 \Rightarrow u = \sqrt{16 \times 10^5}.$$

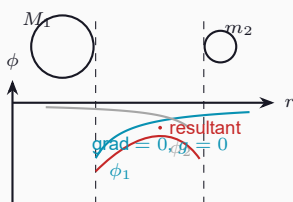
$\phi - r$ Graph between Two Point Masses

For two masses, gravitational potential is a scalar quantity, so it adds algebraically:

$$\phi = \phi_1 + \phi_2$$

$$\phi = -\frac{GM}{r}, \quad \phi \propto \frac{1}{r}.$$

A more negative value of ϕ shows that there are two planets pulling on the object. Hence the object will now need more energy to escape to infinity.



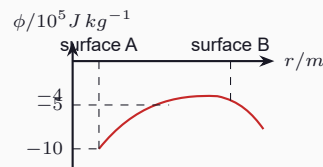
The gravitational field strength is the negative potential gradient:

$$g = -\frac{\Delta\phi}{\Delta r} = -\text{gradient}$$

At the turning point of the resultant potential graph, gradient is zero, so $g = 0$; this is the null point.

Example between two planets

A rock is thrown radially upwards from the surface of planet A towards planet B.



(b) Calculate the speed of the rock at the surface of B if it was thrown from A with a speed of 1400 m s^{-1} .

$$\Delta\phi = -\frac{1}{2}(v^2 - u^2)$$

$$(-5 - (-10)) \times 10^5 = -\frac{1}{2}(v^2 - 1400^2)$$

$$v = 979 \approx 980 \text{ m s}^{-1}$$

(a) Minimum speed at A to reach B:

$$(-4 - (-10)) \times 10^5 = -\frac{1}{2}(0 - u^2)$$

$$6 \times 10^5 = \frac{1}{2}u^2 \Rightarrow u = 1095 \approx 1100 \text{ m s}^{-1}$$

Escape Velocity and Orbital Motion

Escape velocity

The minimum speed of projection required for an object to escape the planet's gravitational field.

loss in K.E. = gain in P.E.

$$-\frac{1}{2}m(v^2 - u^2) = \Delta\phi m$$

$$-\frac{1}{2}(v^2 - u^2) = \phi_f - \phi_i$$

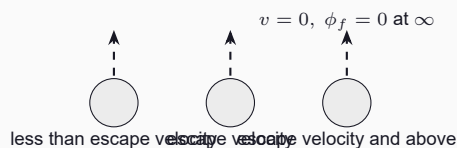
For escape to infinity:

$$v = 0, \quad \phi_f = 0, \quad \phi_i = -\frac{GM}{R}.$$

$$-\frac{1}{2}(0 - u^2) = 0 - \left(-\frac{GM}{R}\right)$$

$$\frac{u^2}{2} = \frac{GM}{R}$$

$$u = \sqrt{\frac{2GM}{R}}$$



For Earth:

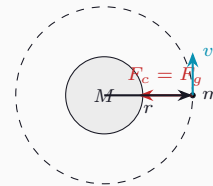
$$M = 6.0 \times 10^{24} \text{ kg}, \quad R = 6.4 \times 10^6 \text{ m}, \quad G = 6.67 \times 10^{-11}.$$

$$u = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 2}{6.4 \times 10^6}}$$

$$u = 11183 \text{ m s}^{-1} \approx 11 \text{ km s}^{-1}$$

Orbital motion

The gravitational force acts as a centripetal force.



$$F_g = F_c$$

$$\frac{GMm}{r^2} = mr\omega^2.$$

$$\text{Using } \omega = \frac{2\pi}{T}:$$

$$\frac{GMm}{r^2} = mr \left(\frac{2\pi}{T} \right)^2$$

$$GM = r^3 \frac{4\pi^2}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$T^2 \propto r^3$$

“The square of the time period is directly proportional to the cube of the radius of orbit.”

Example

	Moon A	Moon B
r/m	6.4×10^{10}	7.5×10^{10}
T/days	30	x

$$\frac{30^2}{x^2} = \frac{(6.4 \times 10^{10})^3}{(7.5 \times 10^{10})^3}$$

$$x = 38 \text{ days}$$

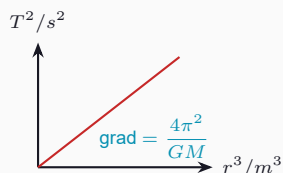
Kepler Graph and Geostationary Satellite

From

$$T^2 = \frac{4\pi^2}{GM} r^3,$$

if a graph of T^2 against r^3 is drawn:

$$y = mx + c, \quad \text{gradient} = \frac{4\pi^2}{GM}.$$

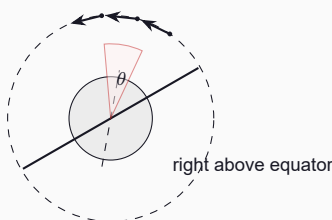


Geostationary satellite

A satellite that has the same angular speed as the Earth and revolves in the same direction as Earth's rotation:

westtoeast.

- $T = 24$ hours.
- It remains right above the same point on Earth's equator.



Speed of geostationary satellite

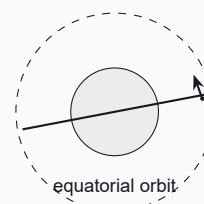
$$v = r\omega = r \left(\frac{2\pi}{T} \right)$$

$$v = 4.23 \times 10^7 \left(\frac{2\pi}{24 \times 3600} \right)$$

$$v = 3075.96 \text{ m s}^{-1} \approx 3100 \text{ m s}^{-1}$$

Visual notes

- The satellite must orbit above the equator.
- It must have the same period as Earth's rotation.
- It must move in the same direction as Earth rotates.



Radius of geostationary orbit

$$G = 6.67 \times 10^{-11}, \quad M = 6.0 \times 10^{24} \text{ kg}, \quad T = 24 \text{ h}.$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$(24 \times 3600)^2 = \frac{4\pi^2 r^3}{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}$$

$$r = 4.23 \times 10^7 \text{ m}$$

Polar Satellites and Energy of a Satellite

Polar satellite

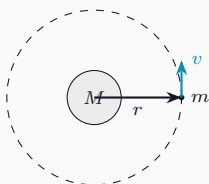
- Satellites that have orbits slightly tilted with respect to the equator.
- They have a smaller radius of orbit, i.e. they travel closer to Earth's surface.
- They act as links between different geostationary satellites.
- They have greater angular speed compared to Earth.

Since $T^2 \propto r^3$,

$$r \downarrow \Rightarrow T \downarrow \Rightarrow \text{speed} \uparrow$$



Energy of a satellite



1. Potential energy

$$E_p = -\frac{GMm}{r}$$

2. Kinetic energy

$$E_k = \frac{1}{2}mv^2.$$

As gravitational force acts as a centripetal force,

$$F_g = F_c, \quad \frac{GMm}{r^2} = \frac{mv^2}{r}.$$

$$\frac{GM}{r} = v^2$$

$$E_k = \frac{1}{2}m \left(\frac{GM}{r} \right)$$

$$E_k = \frac{GMm}{2r}$$

3. Total energy

$$E_T = E_k + E_p$$

$$E_T = \frac{GMm}{2r} + \left(-\frac{GMm}{r} \right)$$

$$E_T = \frac{GMm}{r} \left(\frac{1}{2} - 1 \right)$$

$$E_T = -\frac{GMm}{2r}$$

Final summary

Quantity	Expression
Gravitational force	$F = \frac{GMm}{r^2}$
Field strength	$g = \frac{GM}{r^2}$
Potential energy	$E_p = -\frac{GMm}{r}$
Potential	$\phi = -\frac{GM}{r}$
Escape velocity	$u = \sqrt{\frac{2GM}{R}}$
Orbital period	$T^2 = \frac{4\pi^2 r^3}{GM}$
Kinetic energy in orbit	$E_k = \frac{GMm}{2r}$
Total energy in orbit	$E_T = -\frac{GMm}{2r}$

Oscillations

Chapter 03 / A2 Level / Apollo Physics 9702



Oscillation and Simple Harmonic Motion

Oscillation

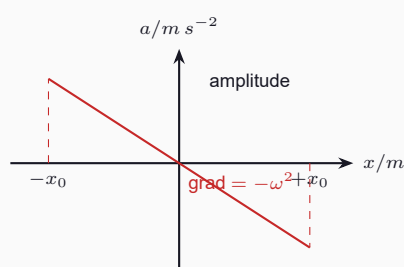
It is a **repetitive motion** that has a time period.

Simple Harmonic Motion (SHM)

- A periodic motion.
- Acceleration of the body is directly proportional to displacement.
- Acceleration is in the **opposite direction** to displacement.

$$a \propto -x \quad a = -\omega^2 x$$

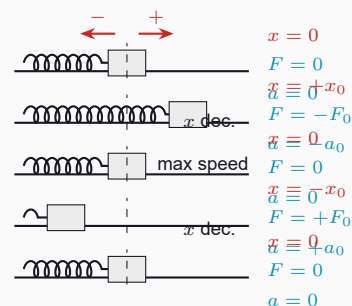
$$\omega = 2\pi f \quad \omega = \frac{2\pi}{T}$$



Four features of the SHM graph

1. Straight line.
2. Passing through origin.
3. Negative gradient.
4. Equal amplitudes on both sides.

Spring-mass system under SHM



Hooke's law gives the restoring force:

$$F = -kx.$$

Using Newton's second law:

$$F_{net} = ma, \quad -kx = ma, \quad a = -\frac{k}{m}x.$$

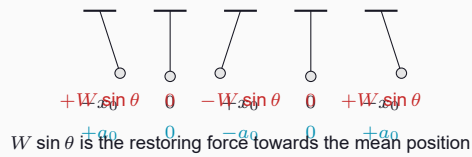
Compare with $a = -\omega^2 x$:

$$\omega^2 = \frac{k}{m}, \quad \left(\frac{2\pi}{T}\right)^2 = \frac{k}{m}.$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where m is mass, k is spring constant and T is time period.

Simple Pendulum under SHM



The component $W \sin \theta$ acts as a **restoring force**; it tends to bring the object towards the mean position.

- As the pendulum swings towards the mean position, θ decreases.
- Therefore $\sin \theta$ decreases.
- Therefore $W \sin \theta$ and acceleration decrease.

For the pendulum:

$$F = W \sin \theta = mg \sin \theta.$$

Using $F = ma$:

$$mg \sin \theta = ma \Rightarrow a = g \sin \theta.$$

The acceleration is towards the mean position, so it is negative relative to displacement. For small angle oscil-

lations:



$$\sin \theta = \frac{x}{L}.$$

Hence

$$a = -g \frac{x}{L} = -\frac{g}{L}x.$$

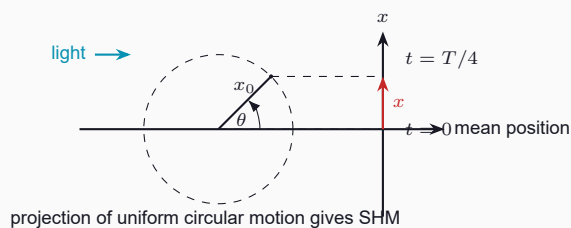
Compare with $a = -\omega^2 x$:

$$\omega^2 = \frac{g}{L}, \quad \left(\frac{2\pi}{T}\right)^2 = \frac{g}{L}.$$

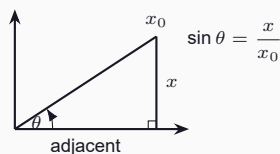
$$T = 2\pi \sqrt{\frac{L}{g}}$$

A simple pendulum observes SHM if it oscillates at a very small amplitude, so that $\theta \approx 0^\circ$ and $W \sin \theta$ is almost horizontal.

Relating Circular Motion to SHM



The vertical projection of a point moving with uniform circular motion performs SHM. If the body starts from the mean position:



$$\sin \theta = \frac{x}{x_0} \Rightarrow x = x_0 \sin \theta.$$

Since $\omega = \frac{\Delta \theta}{\Delta t}$, $\theta = \omega t$.

$$x = x_0 \sin \omega t$$

Velocity

$$\cos \theta = \frac{v}{v_0} \Rightarrow v = v_0 \cos \theta.$$

Since $v_0 = x_0 \omega$,

$$v = v_0 \cos \omega t = x_0 \omega \cos \omega t$$

Acceleration

$$\sin \theta = \frac{a}{a_0} \Rightarrow a = a_0 \sin \theta.$$

Acceleration is in the opposite direction to displacement:

$$a = -a_0 \sin \omega t = -\omega^2 x_0 \sin \omega t$$

If the body starts from maximum displacement:

$$x = x_0 \cos \omega t$$

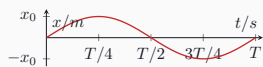
$$v = -v_0 \sin \omega t$$

$$a = -a_0 \cos \omega t$$

Starting condition	Displacement	Velocity / acceleration
Starts at mean	$x = x_0 \sin \omega t$	$v = v_0 \cos \omega t, a = -a_0 \sin \omega t$
Starts at extreme	$x = x_0 \cos \omega t$	$v = -v_0 \sin \omega t, a = -a_0 \cos \omega t$

Displacement, Velocity and Acceleration Graphs

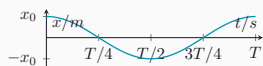
Starting from the mean position



$$x = x_0 \sin \omega t, \quad v = v_0 \cos \omega t, \quad a = -a_0 \sin \omega t.$$



Starting from extreme position



$$x = x_0 \cos \omega t, \quad v = -v_0 \sin \omega t, \quad a = -a_0 \cos \omega t.$$

Relating x and v of a body under SHM

Using

$$x = x_0 \sin \omega t, \quad v = x_0 \omega \cos \omega t,$$

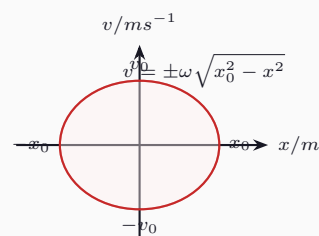
$$\sin^2 \theta + \cos^2 \theta = 1, \quad \left(\frac{x}{x_0} \right)^2 + \left(\frac{v}{x_0 \omega} \right)^2 = 1.$$

$$x^2 \omega^2 + v^2 = x_0^2 \omega^2.$$

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

- v is velocity at displacement x .
- x_0 is amplitude.
- ω is angular frequency.

Graph of $v = \omega \sqrt{x_0^2 - x^2}$



At $x = 0$ (mean position), $v = \pm v_0$. At $x = \pm x_0$ (extreme positions), $v = 0$.

Energy of an Oscillating Body

1. Kinetic energy

$$K.E. = \frac{1}{2}mv^2, \quad v = \omega\sqrt{x_0^2 - x^2}.$$

$$K.E. = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

For maximum kinetic energy, $x = 0$:

$$K.E._{max} = \frac{1}{2}m\omega^2x_0^2$$

2. Total energy

$$T.E. = K.E. + P.E.$$

At the mean position, $x = 0$, potential energy is zero and kinetic energy is maximum:

$$T.E. = K.E._{max}.$$

$$T.E. = \frac{1}{2}m\omega^2x_0^2$$

3. Potential energy

Since $T.E. = K.E. + P.E.$,

$$P.E. = T.E. - K.E.$$

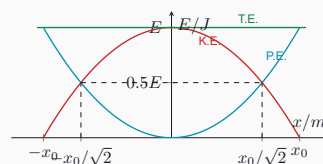
$$P.E. = \frac{1}{2}m\omega^2x_0^2 - \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

$$P.E. = \frac{1}{2}m\omega^2\{x_0^2 - (x_0^2 - x^2)\} = \frac{1}{2}m\omega^2x^2.$$

$$P.E. = \frac{1}{2}m\omega^2x^2$$

At $x = x_0$, potential energy is maximum:

$$P.E._{max} = \frac{1}{2}m\omega^2x_0^2$$



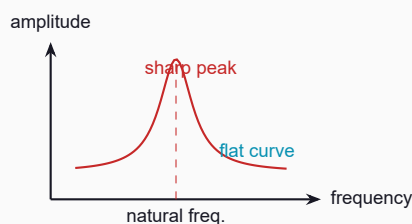
At $x = \pm x_0/\sqrt{2}$, $K.E. = P.E.$

Resonance and Damping

Resonance

Resonance is the increase in amplitude of vibration when the driving frequency becomes equal to the natural frequency of the oscillating body.

- Maximum energy from the external source is absorbed by the vibrating body.
- A sharp peak occurs for light damping.
- A flatter curve occurs for heavier damping.



Uses of resonance

- MRI.
- Ultrasound generation and detection.
- Musical instruments.
- Radio-frequency tuning.

Hazards

- Excessive vibrations of buildings and bridges during earthquakes.
- Vibrations in large rotating machinery, e.g. turbine engines.

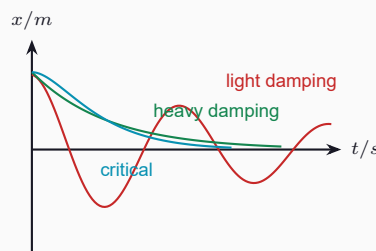
Damping

Damping is the decrease in amplitude of vibration due to loss of energy as heat, caused by a resistive force such as friction or air resistance. **To increase damping:**

1. Increase the surface area of the body by adding feathers, paper or cloth.
2. Oscillate the body in a viscous fluid such as oil.

Types of damping

1. **Light damping:** amplitude decreases gradually in multiple oscillations.
2. **Critical damping:** amplitude decreases to zero in about a quarter of an oscillation and as quickly as possible without oscillating.
3. **Heavy damping:** amplitude decreases to zero gradually in about a quarter of an oscillation.



Worked Example: Vibrating Plate and Cube



A horizontal plate vibrates vertically with frequency 4.5 Hz and amplitude 3.0 mm. A metal cube of mass 5.8 g rests on the plate.

Energy of oscillation

$$E = \frac{1}{2}m\omega^2x_0^2, \quad \omega = 2\pi f.$$

$$E = \frac{1}{2}(5.8 \times 10^{-3})\{2\pi(4.5)\}^2(3.0 \times 10^{-3})^2.$$

$$E = 2.1 \times 10^{-5} \text{ J}$$

Amplitude at loss of contact

At the top of motion, the plate can lose contact with the cube when the downward acceleration reaches g :

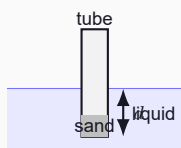
$$a_{max} = \omega^2x_0 = g.$$

$$x_0 = \frac{g}{\omega^2} = \frac{9.81}{(2\pi \times 4.5)^2}.$$

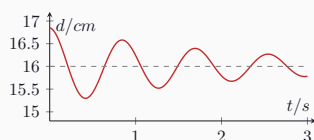
$$x_0 = 1.23 \times 10^{-2} \text{ m} \approx 0.012 \text{ m}$$

If the amplitude is less than this, the cube remains in contact with the plate.

Worked Example: Floating Tube and Damped Oscillations



A tube floats upright in a liquid with equilibrium immersion depth $d = 16$ cm. When displaced and released, d varies with time.



From the graph:

- Mean position: $d = 16$ cm.
- Amplitude: about 0.85 cm.
- Time period: about 0.85 s.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.85} = 7.39 \text{ rad s}^{-1}.$$

Speed at $d = 16.2$ cm

The displacement from equilibrium is

$$x = 16.2 - 16.0 = 0.20 \text{ cm}.$$

Using

$$v = \omega \sqrt{x_0^2 - x^2},$$

$$v = \frac{2\pi}{0.85} \sqrt{(0.85)^2 - (0.20)^2} \text{ cm s}^{-1}.$$

$$v \approx 3.6 \text{ cm s}^{-1}$$

The same speed occurs at the symmetric depth on the other side of equilibrium:

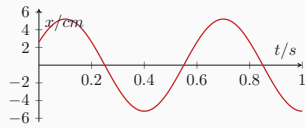
$$d = 15.8 \text{ cm}$$

Damping meaning

Damping is the decrease in amplitude of vibration due to loss of energy as heat caused by a resistive force.

Pendulum Graphs and Kinetic Energy Sketch

A small metal ball suspended from a string performs SHM. The displacement graph gives the period and amplitude.



Acceleration at $x = 2.0 \text{ cm}$:

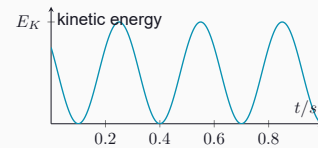
$$a = -\omega^2 x = -\left(\frac{2\pi}{T}\right)^2 x.$$

If $T \approx 0.60 \text{ s}$ and $x = 0.020 \text{ m}$,

$$a = -\left(\frac{2\pi}{0.60}\right)^2 (0.020) \approx -2.2 \text{ m s}^{-2}.$$

Kinetic energy is maximum at mean position and zero

at extreme position. Therefore it has two maxima per oscillation.



- $E_K = 0$ at extreme displacements.
- $E_K = E_{K,max}$ at the mean position.
- Kinetic energy cannot be negative.

Spring-Ball Energy Example

A ball of mass 37 g is held between two fixed points A and B by two stretched helical springs. It oscillates with frequency 3.5 Hz and amplitude 2.8 cm.



Total energy

$$E_T = \frac{1}{2} m \omega^2 x_0^2.$$

$$E_T = \frac{1}{2} \left(\frac{37}{1000} \right) (2\pi \times 3.5)^2 (0.028)^2.$$

$$E_T \approx 0.0070 \text{ J} = 7.0 \text{ mJ}$$

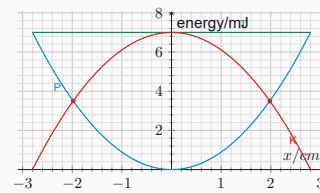
Where $K.E. = P.E.$

$$\frac{1}{2} m \omega^2 (x_0^2 - x^2) = \frac{1}{2} m \omega^2 x^2.$$

$$x_0^2 - x^2 = x^2 \Rightarrow x = \frac{x_0}{\sqrt{2}}.$$

$$x = \frac{2.8}{\sqrt{2}} = 2.0 \text{ cm}$$

Energy sketch against displacement

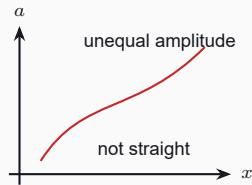


- Total energy is constant.
- Kinetic energy is maximum at $x = 0$.
- Potential energy is maximum at $x = \pm x_0$.
- When the spring system is rotated vertically, gravitational potential energy must also be considered.

Recognising Non-SHM and Damped Block Example

Why a motion may not be SHM

For SHM, the acceleration-displacement graph must be a straight line passing through the origin with negative gradient and equal amplitudes.

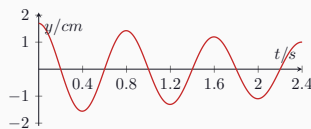


Two reasons the example graph is not SHM:

1. The graph is not a straight line.
2. The amplitudes of the vibration are different on the two sides.

Floating block example

A block floating in a liquid is displaced vertically and released. The displacement-time graph is damped.



If the period from the graph is about 0.80 s,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.80} = 7.85 \text{ rad s}^{-1}.$$

If the amplitude is 1.5 cm,

$$a_{max} = \omega^2 x_0 = (7.85)^2 (0.015) = 0.925 \text{ m s}^{-2}.$$

$$a_{max} \approx 0.93 \text{ m s}^{-2}$$

Energy lost in a damped interval is found from

$$\Delta E = \frac{1}{2} m \omega^2 (A_i^2 - A_f^2).$$

For $m = 0.120 \text{ kg}$, $\omega = 7.85 \text{ rad s}^{-1}$ and decreasing amplitudes, the source working gives roughly

$$\Delta E \approx 5.3 \times 10^{-4} \text{ J}$$

Sewing-Machine Needle and Engine Piston Phase

A sewing-machine needle oscillates vertically through a total distance of 22 mm. Therefore the amplitude is

$$x_0 = 11 \text{ mm} = 1.1 \text{ cm}.$$

For SHM:

$$v_{max} = \omega x_0.$$

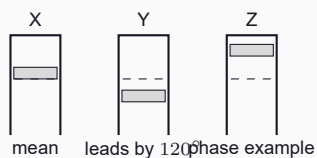
If $\omega = 800 \text{ rad s}^{-1}$ and $x_0 = 1.1 \text{ cm}$,

$$v_{max} = 800 \times 1.1 = 880 \text{ cm s}^{-1}.$$

$$v_{max} = 880 \text{ cm s}^{-1}$$

Phase of pistons

Pistons in an engine may have the same frequency but not be in phase.



If the oscillation of piston Y leads that of piston X by a phase angle $120^\circ = 2\pi/3$, represent positions using cosine:

$$x = x_0 \cos(\omega t + \phi).$$

For a leading piston:

$$x_Y = x_0 \cos(\omega t + 120^\circ).$$

For direction of motion:

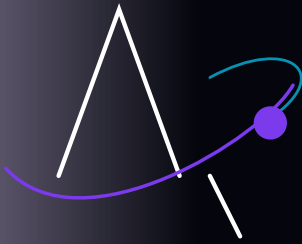
$$v = -\omega x_0 \sin(\omega t + \phi).$$

Rules for phase questions

- Same frequency means same period.
- A phase lead means the oscillator is ahead in its cycle.
- Use displacement equation to locate position.
- Use velocity equation to show direction of motion.

Thermal Properties of Matter

Chapter 04 / A2 Level / Apollo Physics 9702



Heat Capacity and Specific Heat Capacity

Heat capacity

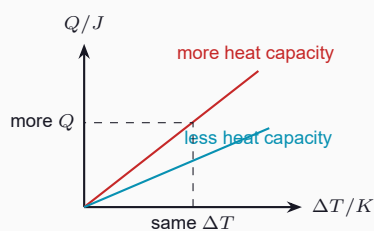
Amount of heat needed to bring a unit change in temperature of a substance.

$$C = \frac{Q}{\Delta T}$$

- C : heat capacity.
- Q : heat energy.
- ΔT : change in temperature.
- SI unit of C : J K^{-1} .

Example. If $Q = 10000 \text{ J}$, $T_i = 25^\circ\text{C}$ and $T_f = 30^\circ\text{C}$,

$$C = \frac{10000}{30 - 25} = 2000 \text{ J K}^{-1}.$$



Important. Heat capacity is not a material property because it depends on the mass of the substance. More mass needs more heat to produce the same temperature rise, hence more heat capacity. **Specific heat capacity**

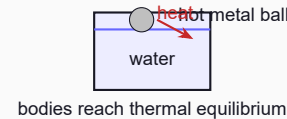
Amount of heat needed to bring a unit change in temperature in a unit mass of a substance.

$$c = \frac{Q}{m\Delta T}$$

$$Q = mc\Delta T$$

Make sure the units of Q and m are consistent with the units of c . **Worked example: hot metal ball in water**

A hot metal ball of mass 200 g at 200°C is placed under water. The mass of water is 0.5 kg and its initial temperature is 25°C . Heat transfer occurs until both bodies reach thermal equilibrium.



Given $c_{\text{metal}} = 800 \text{ J kg}^{-1}\text{K}^{-1}$ and $c_{\text{water}} = 4200 \text{ J kg}^{-1}\text{K}^{-1}$.

(a) No heat is lost to surrounding

heat lost by metal = heat gained by water

$$0.2(800)(200 - T) = 0.5(4200)(T - 25)$$

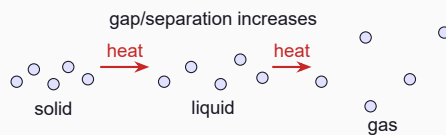
$$T = 26.5^\circ\text{C}$$

(b) 20% heat is lost to surrounding

$$0.80\{0.2(800)(200 - T)\} = 0.5(4200)(T - 25)$$

$$T = 26.2^\circ\text{C}$$

Internal Energy and Latent Heat



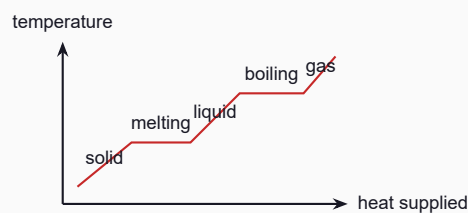
Internal energy

Temperature depends upon the kinetic energy of molecules. Heating can increase:

- kinetic energy of molecules due to vibrations and motion;
- potential energy of molecules due to intermolecular forces and separation.

The sum of microscopic kinetic and potential energies of molecules is called **internal energy**. **Why temperature stays constant during a state change**

Temperature depends on mean kinetic energy. During state change, heat supplied is either used to do work against intermolecular forces or used to increase potential energy by separating molecules. There is no change in kinetic energy, hence no change in temperature. This is why heat supplied during state change is called **latent heat**, i.e. hidden heat.



Latent heat

Amount of heat needed to change the state of a substance at constant temperature. **Specific latent heat**

Amount of heat needed to change the state of a unit mass of substance at constant temperature. **Specific latent**

heat of fusion

Amount of heat needed to change a unit mass of solid to liquid at its melting point.

$$Q = L_f m$$

Specific latent heat of vaporisation

Amount of heat needed to change a unit mass of liquid to gas at its boiling point.

$$Q = L_v m$$

Which is larger: L_v or L_f ?

Boiling needs more energy than melting, so

$$L_v > L_f$$

Reasons:

1. More work is done against intermolecular forces during boiling because there is a greater increase in separation between molecules.
2. Work is also done against atmospheric pressure when gas expands.

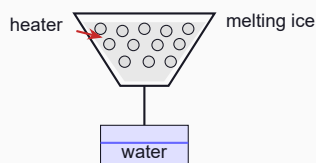
Determining Latent Heat Using an Electric Heater

Heat exchange with surroundings not ignored

When ice melts at a steady rate, the rate of heat gained from surroundings depends on the temperature difference between ice and surroundings.

- Ice melts at 0°C .
- Surrounding temperature is assumed constant, e.g. 25°C .
- Therefore ΔT is always the same.
- The rate of heat gained from surroundings is the same even if the experiment is repeated with a heater of different power rating.

Let $\dot{Q}_s$ be the thermal energy gained per second from surroundings.



For one heater:

$$P + \dot{Q}_s = L\dot{m}$$

For two heater powers:

$$P_1 + \dot{Q}_s = L\dot{m}_1, \quad P_2 + \dot{Q}_s = L\dot{m}_2.$$

Subtracting eliminates the surrounding heat term:

$$P_1 - P_2 = L(\dot{m}_1 - \dot{m}_2)$$

$$L = \frac{P_1 - P_2}{\dot{m}_1 - \dot{m}_2}.$$

Alternative form using time

If masses m_1, m_2 melt in the same time t ,

$$P_1 t + Q_s = Lm_1, \quad P_2 t + Q_s = Lm_2.$$

Subtract:

$$(P_1 - P_2)t = L(m_1 - m_2)$$

Similarly for boiling

For vaporisation:

$$P + \dot{Q}_s = L_v \dot{m}.$$

If the same surrounding heat correction applies, use two heater powers and subtract to obtain L_v .

Key idea: repeating the experiment at two heater powers lets the unknown surrounding heat-gain term cancel.

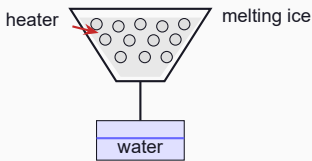
Exam Question: Melting Ice with a Heater – Setup

(a) Define specific latent heat.

Amount of heat needed to change the state of a unit mass of substance at constant temperature.

(b) Ice in a funnel

An electric heater is immersed in melting ice in a funnel. The heater is switched on and when the ice is melting at a constant rate, the mass m of ice melted in 5.0 minutes is recorded. Then the heater power is increased and the experiment is repeated.



Data

Power P /W	Mass melted in 5.0 min / g	Mass melted per second / g
70	78	$\frac{78}{5 \times 60} = 0.26$
110	114	$\frac{114}{5 \times 60} = 0.38$

Completed values

0.26 g s^{-1} and 0.38 g s^{-1}

Exam Question: Melting Ice with a Heater – Solution

Determine L_f

Because the heat gained from surroundings is the same in both experiments:

$$P_2 - P_1 = L_f(\dot{m}_2 - \dot{m}_1).$$

Substitute the data:

$$110 - 70 = L_f(0.38 - 0.26).$$

$$L_f = \frac{40}{0.12} = 333.3 \text{ J g}^{-1}.$$

$$L_f \approx 330 \text{ J g}^{-1}$$

Rate of thermal energy gained from surroundings

Use the lower-power experiment:

$$P + \dot{Q}_s = L_f \dot{m}.$$

$$70 + \dot{Q}_s = (333.3)(0.26).$$

$$70 + \dot{Q}_s = 86.7.$$

$$\dot{Q}_s \approx 17 \text{ W}$$

This is the arithmetic result from the tabulated values.

The method is the same as the handwritten working: use

$$P + \dot{Q}_s = L_f \dot{m}.$$

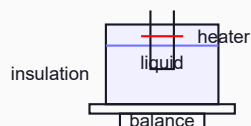
Exam Question: Specific Latent Heat of Vaporisation – Setup

(a) Define specific latent heat

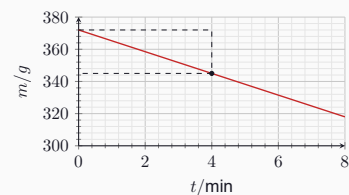
Amount of heat needed to change the state of a unit mass of substance at constant temperature. **(b) Boiling liquid**

on a balance

A beaker containing a liquid is placed on a balance. A heater of power 110 W is immersed in the liquid. The heater is switched on and, when the liquid is boiling, balance readings m are taken at corresponding times t .



Graph of balance reading



From the graph, the reading falls from about 372 g to 345 g in 4.0 min.

$$\Delta m = 372 - 345 = 27 \text{ g.}$$

Exam Question: Specific Latent Heat of Vaporisation – Solution

(i) Feature showing steady boiling rate

The **gradient of the graph is constant**. This means the mass lost per unit time is constant, so the boiling rate is steady. **(ii) Determine** L_v

$$\Delta m = 27 \text{ g} = 0.027 \text{ kg}, \quad t = 4.0 \text{ min} = 240 \text{ s}.$$

$$Q = Pt = 110(240) = 26400 \text{ J}.$$

$$Q = L_v m \quad \Rightarrow \quad L_v = \frac{26400}{0.027} = 9.78 \times 10^5 \text{ J kg}^{-1}.$$

$$L_v \approx 9.8 \times 10^5 \text{ J kg}^{-1}$$

(iii) Overestimate or underestimate?

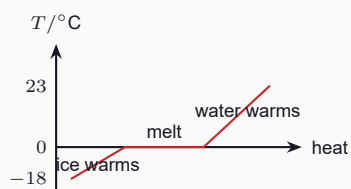
The value determined is likely to be an **overestimate**. Some heat produced by the heater is lost to the surroundings, and less heat is actually used to boil the liquid. Since the calculation assumes all heater energy vaporises the liquid, L_v is too large.

Exam trigger: if the question says no insulation or heat loss, the experimental value usually overestimates L because the calculation counts energy that never reached the liquid.

Exam Question: Melting Ice in Warm Water – Setup

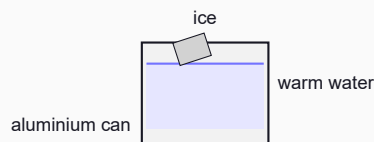
(a) Why temperature remains constant during melting

During melting, the solid becomes liquid with no change in temperature. The temperature depends on the average kinetic energy of molecules. During melting, thermal energy supplied is not used to increase kinetic energy. It is used to do work against intermolecular forces and increase the potential energy of molecules. Therefore there is no change in temperature.



(b) Aluminium can with warm water and ice

An aluminium can of mass 160 g contains 330 g of warm water at 38°C. A mass of 48 g of ice at -18°C is placed into the water. The final temperature is 23°C .



Substance	$c/\text{J g}^{-1}\text{K}^{-1}$
aluminium	0.910
ice	2.10
water	4.18

Exam Question: Melting Ice in Warm Water – Solution

(i) Show heat loss is $2.3 \times 10^4 \text{ J}$

Heat lost by can and warm water:

$$Q = m_{\text{Al}}c_{\text{Al}}\Delta T + m_w c_w \Delta T.$$

$$Q = (160)(0.910)(23 - 38) + (330)(4.18)(23 - 38)$$

$$Q = -22875 \text{ J} \approx -2.3 \times 10^4 \text{ J}.$$

The magnitude of the thermal energy lost is

$$2.3 \times 10^4 \text{ J}$$

(ii) Calculate L_f for ice

Heat gained by ice:

$$22875 = (48)(2.10)(0 - (-18)) + 48L_f + (48)(4.18)(23 - 0).$$

$$22875 = 1814.4 + 48L_f + 4614.72.$$

$$48L_f = 16445.88, \quad L_f = 342.6 \text{ J g}^{-1}.$$

$$L_f \approx 340 \text{ J g}^{-1}$$

Formula summary

Quantity	Formula
Heat capacity	$C = Q/\Delta T$
Specific heat capacity	$Q = mc\Delta T$
Specific latent heat	$Q = Lm$
Heater energy	$Q = Pt$
Rate form	$P = L\dot{m}$
Two-heater correction	$P_2 - P_1 = L(\dot{m}_2 - \dot{m}_1)$

Exam triggers

- Constant temperature during melting/boiling $\Rightarrow$ mean KE unchanged.
- Energy during state change $\Rightarrow$ work against inter-molecular forces / increase in PE.
- Straight-line mass-time graph $\Rightarrow$ steady rate.
- Heat loss to surroundings $\Rightarrow$ use correction or state overestimate/underestimate.

Ideal Gases

Chapter 05 / A2 Level / Apollo Physics 9702



Assumptions, Boyle's Law and Charles' Law

Assumptions for an ideal gas according to kinetic molecular theory of matter

1. Gas molecules are in continuous random motion.
2. They move in straight lines.
3. Molecules have negligible intermolecular force of attraction.
4. Gas molecules behave as hard, elastic spheres.
5. Collisions between/of molecules are perfectly elastic.
6. Time of collision is negligible compared to time between collisions.
7. Volume of gas molecules is negligible compared with volume of gas.

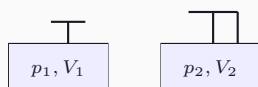
Boyle's law (p - V)

For a fixed mass of gas at constant temperature, volume is inversely proportional to pressure.

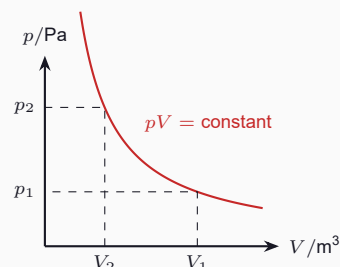
$$V \propto \frac{1}{p}$$

$$pV = \text{constant}$$

$$p_1 V_1 = p_2 V_2$$



When volume decreases, molecules come closer to one another, frequency of collisions with the wall increases, force exerted by molecules increases, hence pressure increases.



At the same volume, if $T_2 > T_1$, then $p_2 > p_1$. **Charles' law** (V - T)

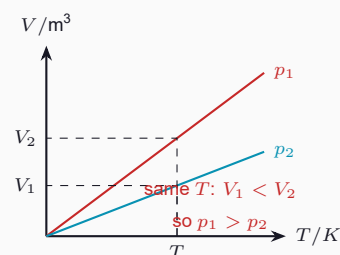
For a fixed mass of gas at constant pressure, volume is directly proportional to absolute temperature.

$$V \propto T$$

$$V = kT$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

where T is in Kelvin.



Avogadro's Law and the Ideal Gas Equation

Avogadro's law (V - n)

For a gas at constant temperature and pressure, volume is directly proportional to the number of moles.

$$V \propto n$$

$$V = kn$$

$$\frac{V}{n} = \text{constant}$$

$$pV = nRT \text{ and } n = N/N_A,$$

$$pV = \left(\frac{N}{N_A}\right) RT = NkT, \quad k = \frac{R}{N_A}.$$

$$pV = NkT$$

$$\text{Boltzmann constant : } k = 1.38 \times 10^{-23} \text{ J K}^{-1}$$

Ideal gas equation

Combining the proportionalities,

$$V \propto \frac{nT}{p} \Rightarrow V = R \frac{nT}{p}$$

$$pV = nRT$$

- p : pressure / Pa.
- V : volume / m^3 .
- n : number of moles / mol.
- T : absolute temperature / K.
- $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$.

$$n = \frac{\text{mass}}{A_r \text{ or } M_r} \quad (\text{mass in grams}), \quad N = nN_A,$$

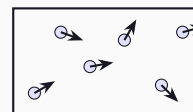
where N is number of particles and $N_A = 6.02 \times 10^{23}$. As

Define an ideal gas

Any gas that obeys $pV = nRT$ or $pV = NkT$ at all temperatures, pressures and volumes is called an ideal gas.

How gases exert pressure

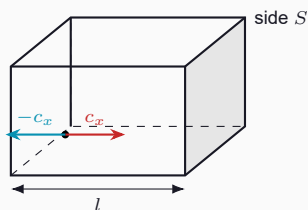
- Gas molecules are in continuous random motion.
- They collide with one another and with the walls of the container.
- Molecules undergo rate of change of momentum, producing force.
- Force per unit area is the pressure of the gas.



continuous random motion

Deriving $p = \frac{1}{3}\rho\langle c^2 \rangle$ – Momentum Argument

Set-up



A molecule of mass m moves inside a cube of side l . The component of velocity along the x -axis is c_x .

$$p_x = mc_x.$$

After collision with the opposite wall, its velocity component becomes $-c_x$.

$$\Delta p = m(v - u) = m(-c_x - c_x) = -2mc_x.$$

The magnitude of the change in momentum is $2mc_x$.

Momentum $\rightarrow$ change in momentum $\rightarrow$ force $\rightarrow$ pressure

Force on a wall

$$F = \frac{\Delta p}{\Delta t}.$$

The time between successive collisions with the same wall is the time to travel $2l$:

$$c_x = \frac{2l}{\Delta t} \quad \Rightarrow \quad \Delta t = \frac{2l}{c_x}.$$

Hence,

$$F_x = \frac{2mc_x}{2l/c_x} = \frac{mc_x^2}{l}.$$

For pressure we take the magnitude:

$$F_x = \frac{mc_x^2}{l}$$

Pressure on the wall is

$$p = \frac{F}{A} = \frac{mc_x^2/l}{l^2} = \frac{mc_x^2}{l^3} = \frac{mc_x^2}{V}.$$

For N molecules,

$$pV = Nmc_x^2$$

This is the pressure of the gas due to all molecules having speed component in the x -axis. **Important:** the same average pressure is obtained on all walls because random motion makes the motion equally likely in all directions.

Mean Square Speed and Temperature

Resolving velocity,

$$\vec{c} = \vec{c}_x + \vec{c}_y + \vec{c}_z, \quad c^2 = c_x^2 + c_y^2 + c_z^2.$$

Due to random motion, molecules are equally likely to move in all directions, so the average values along the axes are the same:

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle.$$

Thus,

$$\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle = 3\langle c_x^2 \rangle,$$

$$\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$$

From $pV = Nm\overline{c_x^2}$,

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle.$$

Since $\rho = Nm/V$,

$$p = \frac{1}{3} \rho \langle c^2 \rangle$$

Here ρ is density and $\langle c^2 \rangle$ is mean-square speed. **Root-**

mean-square speed

values: $-4, 3, 5, -6, 1$

square, mean, then square-root
negative signs are eliminated

$$c_{\text{rms}} = \sqrt{\langle c^2 \rangle}$$

A simple average of positive and negative velocity values can be misleading because opposite signs can cancel. Squaring first removes the sign. **Relating mean kinetic energy to temperature**

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle, \quad pV = NkT.$$

Therefore,

$$3pV = Nm\langle c^2 \rangle, \quad 3NkT = Nm\langle c^2 \rangle,$$

$$\frac{3}{2}kT = \frac{1}{2}m\langle c^2 \rangle.$$

$$\langle E_k \rangle = \frac{3}{2}kT$$

$$\sqrt{T} \propto c_{\text{rms}}$$

Temperature is the measurement of mean kinetic energy of gas molecules

Temperature Example and First Law of Thermodynamics

Example

Gas molecules at a temperature of 100°C have a c_{rms} of 450 m s^{-1} . What will be the c_{rms} at 250°C ?

$$100 + 273 = 373\text{ K}, \quad 250 + 273 = 523\text{ K}.$$

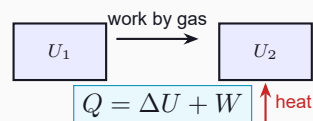
Since $\sqrt{T} \propto c_{\text{rms}}$,

$$\frac{\sqrt{373}}{\sqrt{523}} = \frac{450}{x}$$

$$x = 450 \frac{\sqrt{523}}{\sqrt{373}} = 532.8 \approx \boxed{533\text{ m s}^{-1}}.$$

Thermodynamics

Study of processes and systems that involve heat and work, and conversion between them.



First law of thermodynamics – statement 1

Heat supplied to a system is equal to the sum of increase in its internal energy and work done by the system.

$$Q = \Delta U + W$$

Here W is work done *by* the gas. **Statement 2**

The sum of heat supplied to a system and work done on the system is equal to the increase in its internal energy.

$$Q + W_{\text{on}} = \Delta U$$

Internal Energy and Thermodynamic Processes

Internal energy (U)

The sum of microscopic kinetic and potential energy of gas molecules due to random motion.

$$U = \text{K.E.} + \text{P.E.}$$

For an ideal gas, there is negligible intermolecular force of attraction. Hence there is negligible potential energy between ideal-gas molecules.

- Internal energy of an ideal gas depends only on kinetic energy of molecules.
- $\langle E_k \rangle = \frac{3}{2}kT$.
- Change in kinetic energy is directly proportional to change in temperature.
- Therefore change in internal energy is directly proportional to change in temperature.

$$\Delta U \propto \Delta T \quad \text{for an ideal gas}$$

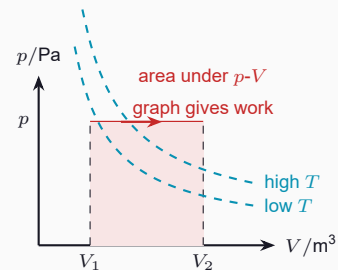
Thermodynamic processes

1. **Constant pressure (isobaric).** Work done at constant pressure:

$$W = p\Delta V$$

$$Q = \Delta U + p\Delta V.$$

Area under a p - V graph gives the work done by the gas.



2. **Constant volume (isochoric).** Since $\Delta V = 0$,

$$W = p\Delta V = 0, \quad Q = \Delta U.$$

All heat supplied is stored as internal energy. Temperature rises and pressure rises due to increase in molecular speed.

Isothermal, Adiabatic and Cycles

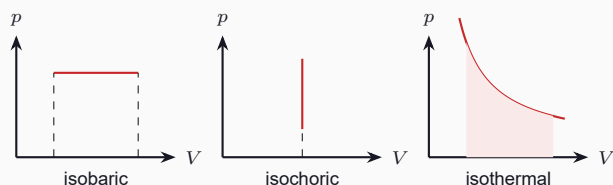
3. **Constant temperature (isothermal).** Since T is constant,

$$\Delta T = 0 \Rightarrow \Delta U = 0.$$

From $Q = \Delta U + W$,

$$Q = W$$

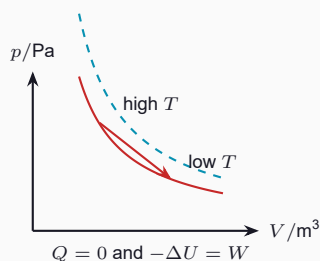
Heat supplied is equal to work done by the gas.



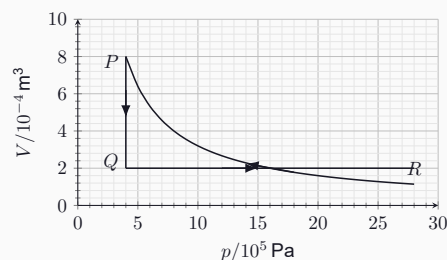
4. **No heat transfer (adiabatic).** Since $Q = 0$,

$$0 = \Delta U + W \Rightarrow -\Delta U = W.$$

Gas cools down when it does work. Gas heats up when work is done on it.



Thermodynamic cycle



A cycle starts and ends at the same temperature, so

$$\Delta T_{\text{cycle}} = 0$$

$$\Delta U_{\text{cycle}} = 0$$

For a closed cycle,

$$\Delta U_{1 \rightarrow 2} + \Delta U_{2 \rightarrow 3} + \Delta U_{3 \rightarrow 4} + \Delta U_{4 \rightarrow 1} = 0.$$

On a p - V graph, the enclosed area represents the net work done in one complete cycle.

Exam Practice: Neon Gas and RMS Speed

(a) Explain qualitatively how molecular movement causes the pressure exerted by a gas.

Gas molecules are in continuous random motion. Upon collision with one another and with the walls of the container, they undergo a rate of change of momentum. This produces a force, and force per unit area is pressure.

(b) Neon gas

The density of neon gas at a temperature of 273 K and pressure 1.02×10^5 Pa is 0.900 kg m^{-3} . Neon may be assumed to be an ideal gas. Calculate the r.m.s. speed at: (i) 273 K

$$p = \frac{1}{3} \rho \langle c^2 \rangle.$$

$$1.02 \times 10^5 = \frac{1}{3} (0.900) \langle c^2 \rangle.$$

$$c_{\text{rms}} = \sqrt{\langle c^2 \rangle} = \boxed{583 \text{ m s}^{-1}}.$$

(ii) 546 K

Since $c_{\text{rms}} \propto \sqrt{T}$,

$$\frac{583}{x} = \frac{\sqrt{273}}{\sqrt{546}}.$$

$$x = 583 \frac{\sqrt{546}}{\sqrt{273}} = 824.5 \approx \boxed{825 \text{ m s}^{-1}}.$$

Useful boxed result

$$c_{\text{rms}} = \sqrt{\frac{3p}{\rho}}$$

This follows immediately from $p = \frac{1}{3} \rho \langle c^2 \rangle$.

Exam Practice: Density at Constant Temperature

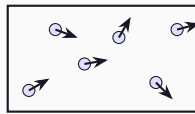
(c) Effect of changing density at constant temperature

The calculations in (b) are based on density 0.900 kg m^{-3} . Suggest the effect, if any, on the root-mean-square speed of changing the density at constant temperature.

The r.m.s. speed stays the same because it only depends on temperature. Changing density will change the pressure of the gas.

$$\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$$

$$c_{\text{rms}} \propto \sqrt{T}$$



continuous random motion

Exam Practice: Kinetic Theory Assumptions

Question. The kinetic theory of gases is based on some simplifying assumptions. The molecules of the gas are assumed to behave as hard elastic identical spheres. State the assumption about ideal gas molecules based on:

1. the nature of their movement;
2. their volume.

Answer.

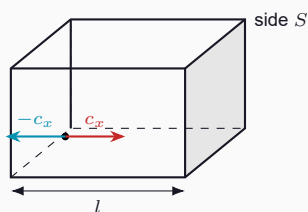
1. Molecules are in continuous random motion and move in straight lines between collisions.
2. The volume of the molecules is negligible compared with the volume occupied by the gas.

Assumption	Consequence
Elastic collisions	no kinetic energy lost during collision
Negligible intermolecular forces	potential energy is zero for an ideal gas
Negligible molecular volume	volume of gas is the container volume
Random motion	equal pressure in all directions

Exam Practice: Cube Derivation and Temperature Change

Cube of volume V with N molecules

Each molecule has component c_x of velocity normal to one side S of the cube.



The molecules of an ideal gas have a root-mean-square speed of 520 m s^{-1} at 27°C . Calculate the r.m.s. speed at 100°C .

$$27^\circ\text{C} = 300 \text{ K}, \quad 100^\circ\text{C} = 373 \text{ K}.$$

$$\frac{x}{520} = \sqrt{\frac{373}{300}}.$$

$$x = 520 \sqrt{\frac{373}{300}} = 580 \text{ m s}^{-1} \text{ (approximately).}$$

The pressure due to the component c_x is given by

$$pV = Nm c_x^2$$

Explain how this leads to

$$pV = \frac{1}{3} Nm \langle c^2 \rangle.$$

Answer

$$\langle c^2 \rangle = \langle c_x^2 \rangle + \langle c_y^2 \rangle + \langle c_z^2 \rangle.$$

Random motion gives

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle,$$

so

$$\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle.$$

Therefore,

$$pV = Nm \langle c_x^2 \rangle = \frac{1}{3} Nm \langle c^2 \rangle.$$

RMS speed temperature question

$$c_{\text{rms}} \approx 580 \text{ m s}^{-1}$$

Exam Practice: Internal Energy and p - V Cycle

(a) State the basic assumption of the kinetic theory of gases that leads to the conclusion that the potential energy between atoms of an ideal gas is zero.

There is negligible intermolecular force of attraction.

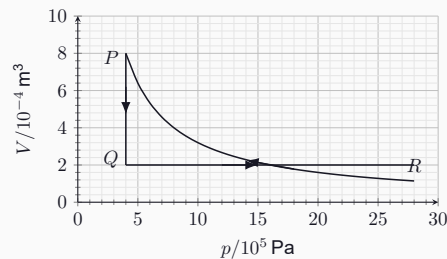
(ii) State what is meant by the internal energy of a substance.

The sum of the microscopic kinetic and potential energies of molecules due to random motion.

(iii) Explain why an increase in internal energy of an ideal gas is directly related to a rise in temperature.

For an ideal gas, internal energy only depends on kinetic energy of molecules. Change in kinetic energy is directly proportional to change in temperature.

A fixed mass of ideal gas undergoes a cycle $P \rightarrow Q \rightarrow R \rightarrow P$.



Read from the graph approximately:

$$P : (4, 8), \quad Q : (4, 2), \quad R : (28, 2).$$

The vertical branch $P \rightarrow Q$ is constant pressure when axes are interpreted in the original graph, while $Q \rightarrow R$ is constant volume/pressure depending on plotted axes. The key calculation below follows the handwritten solution on the topic.

Exam Practice: Cycle Energy Table

(i) Change in internal energy over one complete cycle

$$\Delta U_{\text{cycle}} = 0 \text{ J}$$

(ii) Work done on the gas during $P \rightarrow Q$

$$W = p\Delta V = p(V_2 - V_1) = (4 \times 10^5)(2 - 8) \times 10^{-4} = -240 \text{ J}.$$

For work done *on* the gas,

$$W_{\text{on}} = +240 \text{ J}$$

(iii) Complete the energy table using $Q + W_{\text{on}} = \Delta U$

Change	work done on gas / J	heating supplied to gas / J	increase in internal energy / J
$P \rightarrow Q$	+240	−600	−360
$Q \rightarrow R$	0	+720	+720
$R \rightarrow P$	−840	+480	−360

Checks:

$$-360 + 720 - 360 = 0,$$

so the total change in internal energy over the cycle is zero.

Exam Practice: Kinetic Energy of a Molecule and Heating a Cylinder

(a) Use one of the assumptions of the kinetic theory to explain why the potential energy of the molecules of an ideal gas is zero.

Ideal-gas molecules have negligible intermolecular forces, so there is no intermolecular potential energy.

(b) The average translational kinetic energy E_k of a molecule of an ideal gas is

$$E_k = \frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT.$$

Meanings:

- $\langle c^2 \rangle$: mean-square speed of the molecules.
- T : thermodynamic temperature in Kelvin.

(c) **Cylinder heating question**

A cylinder of constant volume $4.7 \times 10^4 \text{ cm}^3$ contains an ideal gas at pressure $2.6 \times 10^5 \text{ Pa}$ and temperature 173°C . The gas is heated. The thermal energy transferred to the gas is 2900 J.



Since $173^\circ\text{C} = 446 \text{ K}$ and $4.7 \times 10^4 \text{ cm}^3 = 4.7 \times 10^{-2} \text{ m}^3$,

$$N = \frac{pV}{kT} = \frac{(2.6 \times 10^5)(4.7 \times 10^{-2})}{(1.38 \times 10^{-23})(446)}.$$

$$N \approx 1.99 \times 10^{24} \text{ molecules}$$

Exam Practice: Ideal Gas Definition and RMS Speed

(a) State what is meant by an ideal gas.

Any gas that obeys $pV = nRT$ or $pV = NkT$ at all temperatures, pressures and volumes.

$$c_{\text{rms}} = \sqrt{\frac{3pV}{m}}$$

(b) An ideal gas comprised of single atoms is contained in a cylinder and has volume $1.84 \times 10^{-2} \text{ m}^3$ at pressure $2.12 \times 10^7 \text{ Pa}$. The mass of gas in the cylinder is 3.20 kg . Determine the r.m.s. speed of the atoms.

$$c_{\text{rms}} = \sqrt{\frac{3(2.12 \times 10^7)(1.84 \times 10^{-2})}{3.20}}$$

$$= 605 \text{ m s}^{-1}.$$

$$p = \frac{1}{3}\rho\langle c^2 \rangle, \quad \rho = \frac{m}{V} = \frac{3.20}{1.84 \times 10^{-2}}.$$

$$2.12 \times 10^7 = \frac{1}{3} \left(\frac{3.20}{1.84 \times 10^{-2}} \right) \langle c^2 \rangle.$$

$$c_{\text{rms}} = 604.7 \approx \boxed{605 \text{ m s}^{-1}}.$$

Memory hook

Use $p = \frac{1}{3}\rho\langle c^2 \rangle$ when density/mass and volume are given. Use $\sqrt{T} \propto c_{\text{rms}}$ when temperature changes.

Exam Practice: Amount of Gas, Mass of Atom and Nucleon Number

The temperature of the gas in the cylinder is 22°C , hence

$$T = 22 + 273 = 295 \text{ K}.$$

1. Amount of gas

$$pV = nRT$$

$$n = \frac{pV}{RT} = \frac{(2.12 \times 10^7)(1.84 \times 10^{-2})}{(8.314)(295)} = 159.04.$$

$$n = 159 \text{ mol}$$

2. Mass of one atom

$$N = nN_A = 159.04(6.02 \times 10^{23}).$$

$$\text{mass of one atom} = \frac{3.20}{159.04(6.02 \times 10^{23})}.$$

$$m_{\text{atom}} = 3.34 \times 10^{-26} \text{ kg}$$

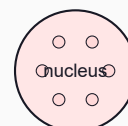
(c) Nucleon number A

Use the mass of one nucleon:

$$1u = 1.66 \times 10^{-27} \text{ kg}.$$

$$A = \frac{3.34 \times 10^{-26}}{1.66 \times 10^{-27}} = 20.1 \approx 20.$$

$$A = 20$$



A = number of nucleons

Exam Practice: Mean Kinetic Energy and Heat Capacity at Constant Volume

(a) The pressure p and volume V of an ideal gas are related to density ρ by

$$p = \frac{1}{3}\rho\langle c^2 \rangle.$$

(i) $\langle c^2 \rangle$ means the *meansquare speed of gas molecules*.

(ii) Show that the mean kinetic energy is

$$E_k = \frac{3}{2}kT.$$

$$p = \frac{1}{3}\rho\langle c^2 \rangle = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle.$$

$$3pV = Nm\langle c^2 \rangle.$$

Since $pV = NkT$,

$$3NkT = Nm\langle c^2 \rangle.$$

Divide by $2N$:

$$\frac{3}{2}kT = \frac{1}{2}m\langle c^2 \rangle.$$

(b)(i) One mole heated at constant volume

At constant volume $\Delta V = 0$, so $W = 0$ and

$$Q = \Delta U.$$

For one molecule,

$$\Delta E_k = \frac{3}{2}k\Delta T.$$

For one mole of molecules, $N = N_A$:

$$Q = N_A \frac{3}{2}k\Delta T = \frac{3}{2}N_A k\Delta T.$$

Since $N_A k = R$,

$$Q = \frac{3}{2}R\Delta T.$$

For $\Delta T = 1.0 \text{ K}$,

$$Q = \frac{3}{2}R$$

Exam Practice: Specific Heat Capacity of Nitrogen

(b)(ii) Nitrogen may be assumed to be an ideal gas. The molar mass of nitrogen gas is 28 g mol^{-1} . Use the answer in (b)(i) to calculate a value for the specific heat capacity, in $\text{J kg}^{-1}\text{K}^{-1}$, at constant volume for nitrogen. For one mole and a temperature rise of 1 K ,

$$Q = \frac{3}{2}R.$$

Mass of one mole of nitrogen:

$$28 \text{ g} = 0.028 \text{ kg}.$$

Specific heat capacity:

$$c = \frac{Q}{m\Delta T} = \frac{\frac{3}{2}R}{0.028 \times 1}.$$

$$c = \frac{3(8.314)}{2(0.028)} = 445.4 \text{ J kg}^{-1}\text{K}^{-1}.$$

$$c \approx 450 \text{ J kg}^{-1}\text{K}^{-1}$$

Final chapter formula set: $pV = nRT$, $pV = NkT$, $p = \frac{1}{3}\rho\langle c^2 \rangle$, $c_{\text{rms}} = \sqrt{\langle c^2 \rangle}$, $\frac{1}{2}m\langle c^2 \rangle = \frac{3}{2}kT$, and $Q = \Delta U + W$.

Electric Fields

Chapter 06 / A2 Level / Apollo Physics 9702

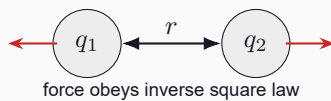


Charge, Coulomb's Law and Electric Field Lines

Charge

- The natural property of matter that it experiences a force in an electric field.
- Two types: positive and negative.
- Proton: $+1.6 \times 10^{-19} \text{ C}$ (+1).
- Electron: $-1.6 \times 10^{-19} \text{ C}$ (-1).
- SI unit: coulomb, C.

Coulomb's law



The force between two point charges is directly proportional to their product and inversely proportional to the square of their separation.

$$F \propto q_1 q_2, \quad F \propto \frac{1}{r^2}, \quad F \propto \frac{q_1 q_2}{r^2}.$$

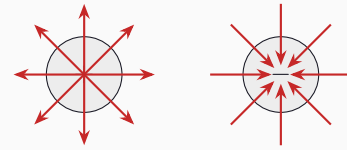
$$F = k \frac{q_1 q_2}{r^2}$$

$$F = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9, \quad \epsilon_0 = 8.85 \times 10^{-12}.$$

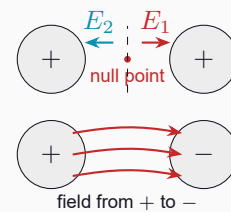
ϵ_0 is the permittivity of free space; it deals with the allowance of a medium to spread the field to the surroundings. **Electric field**

A region in space around a charge where another charge experiences an electric force.



- Direction of field line tells the direction of force on a positive test charge.
- Gap between field lines tells strength of field.
- Less gap $\rightarrow$ strong field; more gap $\rightarrow$ weak field.

Null point and field-line rules



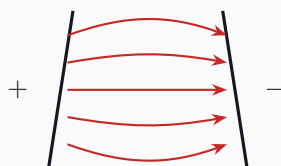
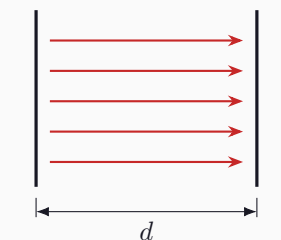
- A null point is a point at which no electric field exists.
- It is in the middle if charges are equal.
- Null point is closer to the weaker charge in case of unequal charges.
- Correct direction is from + to -.
- Gaps between field lines should increase away from charges.
- Radial field lines for point charges.

Comparison with gravitation: $F = Gm_1m_2/r^2$ but $F = kq_1q_2/r^2$; electric forces are much stronger because $k \gg G$.

Parallel Plates, Field Strength and Point-Charge Field

Field lines between parallel metal plates

uniform electric field between plates



increasing gap shows decrease in field strength

Electric field strength (E)

Electric force per unit positive charge at a point in an electric field.

$$E = \frac{F}{q}$$

For parallel metal plates,

$$E = \frac{V}{d}$$

where V is potential difference and d is plate separation.
Since $F = ma$,

$$a = \frac{Vq}{md}$$

Example: proton and alpha particle in a uniform field

$$m_p \approx 1u, \quad m_\alpha \approx 4u, \quad q_p = +1e, \quad q_\alpha = +2e.$$

Both are released from rest in a uniform electric field.

(a) Determine F_p/F_α .

(b) Determine a_p/a_α .

$$F = Eq \Rightarrow \frac{F_p}{F_\alpha} = \frac{Eq_p}{Eq_\alpha} = \frac{1e}{2e} = \boxed{\frac{1}{2}}.$$

$$\frac{F_p}{F_\alpha} = \frac{1}{2}, \quad \frac{m_p a_p}{m_\alpha a_\alpha} = \frac{1}{2},$$

$$\frac{1u a_p}{4u a_\alpha} = \frac{1}{2} \Rightarrow \frac{a_p}{a_\alpha} = \boxed{\frac{2}{1}}.$$

For point charges

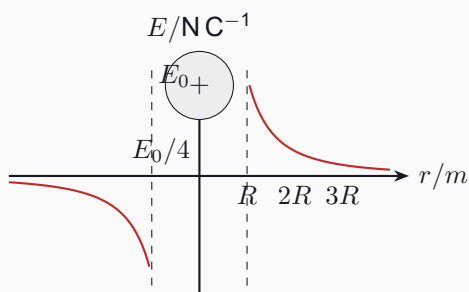
$$E = \frac{F}{q}, \quad F = k \frac{Qq}{r^2} \Rightarrow E = \frac{kQq}{r^2 q}.$$

$$E = \frac{kQ}{r^2}$$

- $E \propto 1/r^2$.
- The further we move away from charge, the weaker the field gets.

E - r Graphs for Point Charges and Resultant Field

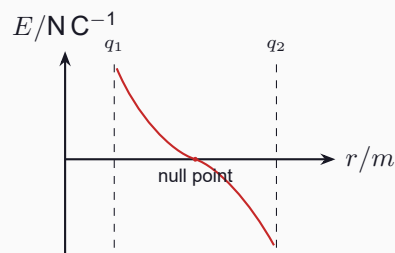
E - r graph for a positive charge



For a positive point charge, field points away from the charge. Magnitude outside the charge follows

$$E \propto \frac{1}{r^2}$$

At R , $E = E_0$. At $2R$, $E = E_0/4$. At $3R$, $E = E_0/9$. Field inside the drawn solid sphere is not treated as a point-charge field in these notes. For a negative charge, directions and signs reverse, but the magnitude still decreases as $1/r^2$. **E - r graph between two point charges**



Like charges:

- Null point is where field strength is zero.
- Null point is closer to weaker charge.
- If both charges are equal, it is in the middle.

At a point P between two like positive charges,

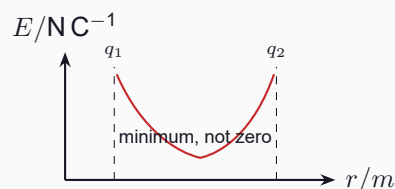
$$E = E_1 - E_2 = \frac{kq_1}{r_1^2} - \frac{kq_2}{r_2^2}.$$

At the null point, $E = 0$:

$$0 = \frac{kq_1}{r_1^2} - \frac{kq_2}{r_2^2} \Rightarrow \frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^2.$$

$$\frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^2$$

Unlike charges:

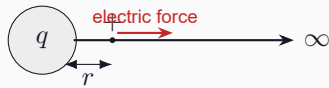


Between opposite charges, the fields reinforce between the charges; the resultant field does not become zero between them.

Electric Potential and Potential of Point Charges

Electric potential (V)

Work done per unit positive charge to move it from infinity to a point in an electric field.



$$V = \frac{kq}{r}$$

- SI unit: volt, V.
- The value can be positive as well as negative.

For a positive source charge, repulsive force means work must be done on a positive test charge to push it towards the charge. Its potential increases from zero and is positive.

$$V = +\frac{kq}{r}$$

For a negative source charge, attractive force means work is done by the field as the positive test charge travels towards the negative charge. Its potential decreases from zero and is negative.

$$V = -\frac{kq}{r}$$

Formula selection

Situation	Formula
Point charges	$V = \frac{kq}{r}$
Parallel plates	$E = \frac{V}{d}$
Point-charge field	$E = \frac{kq}{r^2}$

Potential from multiple charges

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \dots$$

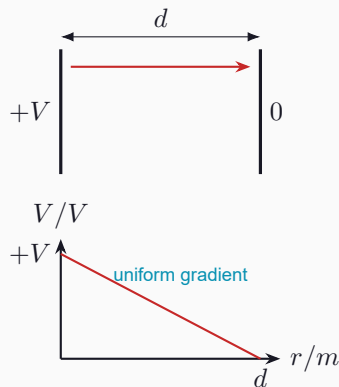
Electric potential is scalar, so potentials are added algebraically. Electric field strength is vector, so fields need direction/sign. **Unlike charges: resultant field**

$$E = E_1 + E_2 = \frac{kq_1}{r_1^2} + \frac{kq_2}{r_2^2}$$

when both field directions are the same at the point under consideration.

V - r Graphs, Potential Gradient and Energy

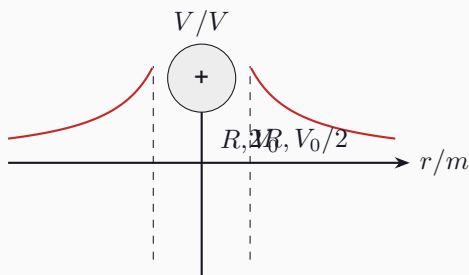
V - r graph between parallel metal plates



Gradient of the V - r graph gives electric field strength.

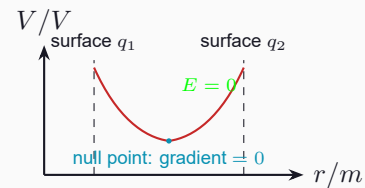
$$E = -\frac{\Delta V}{\Delta r}$$

The negative sign means electric field strength is equal to the negative of the electric potential gradient. For parallel metal plates, graph is straight line because electric field is uniform and gradient is uniform. **V - r graph for an isolated point charge**



- Gradient is zero inside a spherical conductor, so $E = 0$ inside.
- For a point-charge field, $V \propto 1/r$, so gradient decreases as r increases.

V - r graph between like charges



At the null point, the gradient of the V - r graph is zero, hence $E = 0$, $F = 0$, and $a = 0$. **Conservation of energy for a moving charge between two point charges**

$$\Delta V q = \frac{1}{2} m(v^2 - u^2)$$

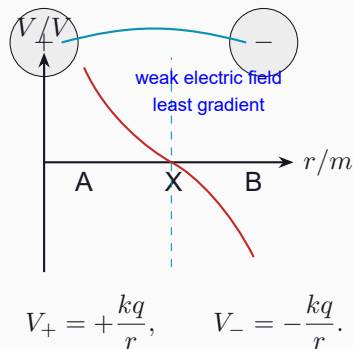
change in electrical potential energy = change in kinetic energy

$$\Delta V q = \frac{1}{2} m(v^2 - u^2), \quad \Delta V = V_{\text{final}} - V_{\text{initial}}$$

This relates change in electrical potential energy to gain/loss of kinetic energy. **Useful statement:** when electric field is strong, the V - r graph has a large gradient. When electric field is weak, the graph has the least gradient.

V - r Graph Between Unlike Charges

Positive and negative charges



From A to X:

- E decreases.
- F decreases.
- a decreases.

From X to B:

- E increases.
- F increases.
- a increases.

A to X to B: speed of the positive charge travelling between A and B keeps on increasing. **Why speed keeps**

increasing

- A positive charge is repelled by the positive sphere A.
- It is attracted towards the negative sphere B.
- Electric force continues to do work on it in the direction of motion.
- Therefore kinetic energy increases and speed increases.

Weak electric field $\Leftrightarrow$ least gradient of potential graph. Strong electric field $\Leftrightarrow$ large potential gradient.

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Exam Graph: Field Strength of a Charged Sphere

Question prompt

The electric field strength at the surface of a charged sphere is E_0 . Sketch a graph to show the variation with distance x of the electric field strength due to the charged sphere, for values of x from $x = 0$ to $x = 4r$.

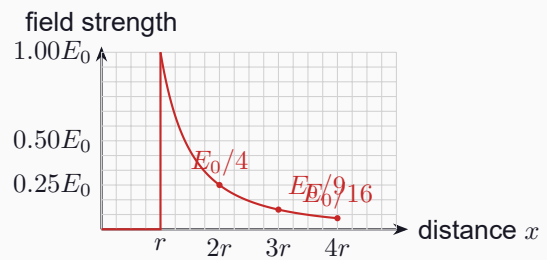
Answer logic

- Inside a conducting sphere: $E = 0$.
- At the surface $x = r$: $E = E_0$.
- Outside, $E \propto 1/x^2$.

$$E = \frac{kq}{r^2}.$$

At distances:

$$2r \Rightarrow 0.25E_0 = \frac{E_0}{4}, \quad 3r \Rightarrow 0.11E_0 = \frac{E_0}{9}, \quad 4r \Rightarrow 0.0625E_0 = \frac{E_0}{16}.$$



Graph features needed for marks

- Horizontal zero field from $x = 0$ to $x = r$.
- Sharp rise to E_0 at the surface.
- Inverse-square decay outside the sphere.
- Correct values around $2r, 3r, 4r$.

Worked Exam: Two Conducting Spheres A and B

Two solid metal spheres A and B are held apart and insulated in vacuum. They carry charges of $+6.0 \mu\text{C}$ and $+12 \mu\text{C}$, respectively. Their centres are separated by 15.0 cm . Point P lies on the line joining the centres and is 2.5 cm from the centre of sphere A.



$$V = V_A + V_B = \frac{kQ_A}{r_A} + \frac{kQ_B}{r_B}.$$

$$V_A = \frac{(8.99 \times 10^9)(6.0 \times 10^{-6})}{0.025} = 2.16 \times 10^6 \text{ V}.$$

$$V_B = \frac{(8.99 \times 10^9)(12 \times 10^{-6})}{0.125} = 8.63 \times 10^5 \text{ V}.$$

Direction of electric field at P

- Field due to A points away from A, to the right.
- Field due to B points away from B, to the left.
- Since P is much closer to A, field due to A is stronger, so resultant field is **to the right**.

$$V_{\text{net}} = \boxed{3.02 \times 10^6 \text{ V}}.$$

Electron accelerated from rest through potential difference If an electron is accelerated through a p.d. ΔV ,

$$q\Delta V = \frac{1}{2}mv^2.$$

Electric field strength at P

$$E = \frac{kQ}{r^2}, \quad E_A = \frac{(8.99 \times 10^9)(6.0 \times 10^{-6})}{(2.5 \times 10^{-2})^2}.$$

$$E_A = 8.63 \times 10^7 \text{ NC}^{-1}.$$

Distance from B is $15.0 - 2.5 = 12.5 \text{ cm} = 0.125 \text{ m}$:

$$E_B = \frac{(8.99 \times 10^9)(12 \times 10^{-6})}{(0.125)^2} = 6.90 \times 10^6 \text{ NC}^{-1}.$$

$$E_{\text{net}} = E_A - E_B = \boxed{7.94 \times 10^7 \text{ NC}^{-1} \text{ to the right}}.$$

Electric potential at P Potential is scalar, so it adds:

For $\Delta V = 2.0 \times 10^4 \text{ V}$,

$$(1.6 \times 10^{-19})(2.0 \times 10^4) = \frac{1}{2}(9.11 \times 10^{-31})v^2.$$

$$v = \sqrt{\frac{2qV}{m}} = \boxed{8.4 \times 10^7 \text{ ms}^{-1}}.$$

Electric Potential at a Point and Moving Charged Particles

Definition Electric potential at a point is the work done per unit positive charge to move it from infinity to that point in an electric field. **Positively charged sphere and alpha particle** A charged particle is accelerated from rest in vacuum through a potential difference V . For an alpha particle of charge $+2e$,

$$qV = \frac{1}{2}mv^2, \quad 2eV = \frac{1}{2}mv^2.$$

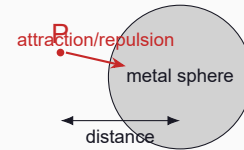
If the particle starts at rest, $u = 0$. Therefore,

$$v = \sqrt{\frac{2qV}{m}}$$

For $q = +2e$,

$$v = \sqrt{\frac{4eV}{m}}.$$

Motion near a metal sphere A particle at P near a charged metal sphere is attracted or repelled depending on the sign of charge. The source notes emphasise using $q\Delta V = \Delta KE$ to calculate the speed.



$$\Delta V q = \frac{1}{2}m(v^2 - u^2)$$

change in electrical potential energy = change in kinetic energy

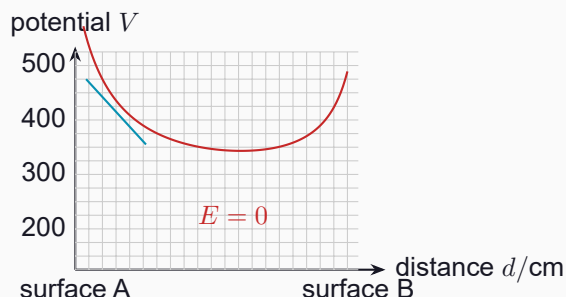
For an alpha particle with charge $+2e$:

$$Vq = \frac{1}{2}mv^2 \quad \Rightarrow \quad V(2e) = \frac{1}{2}mv^2.$$

$$v = \sqrt{\frac{4eV}{m}}$$

Exam Graph: Potential Between Two Spheres

Two positively charged metal spheres A and B are situated in vacuum. A point P lies on the line joining the centres. The graph shows the variation of electric potential V with distance d from the surface of sphere A to the surface of sphere B.



Field direction at the left of the graph

- The field direction is given by the negative gradient of the potential graph.
- If the gradient is negative, E is positive in the direction of increasing distance.

Between A and B

- The field becomes zero where the gradient of the V - d graph is zero.
- At this point, electric force on a charged particle is zero.
- The point is nearer to the weaker/lower charge.

Speed calculation for a proton A proton initially at rest at $d = 0$ is released and moves freely along the line. From the graph, use the change in electric potential to calculate speed.

$$q\Delta V = \frac{1}{2}mv^2.$$

For a proton,

$$q = e = 1.6 \times 10^{-19} \text{ extC}, \quad m_p = 1.67 \times 10^{-27} \text{ extkg}.$$

If $\Delta V \approx 30 \text{ V}$,

$$(1.6 \times 10^{-19})(30) = \frac{1}{2}(1.67 \times 10^{-27})v^2.$$

$$v = \sqrt{\frac{2(1.6 \times 10^{-19})(30)}{1.67 \times 10^{-27}}} = \boxed{7.6 \times 10^4 \text{ m s}^{-1}}.$$

Sketch request from the topic

On a copy of the graph, sketch the variation with distance of electric field strength E . The graph should be based on the gradient of the potential graph: steep gradient gives large $|E|$; zero gradient gives $E = 0$; change in gradient direction changes sign of E .

Formula Sheet and Summary

			Key ideas
Quantity	Formula	Notes	
Coulomb force	$F = k \frac{q_1 q_2}{r^2}$	vector force	• Charge, Coulomb's law, permittivity and field-line rules.
Field strength	$E = \frac{F}{q}$	force per unit positive charge	• Uniform electric field between parallel metal plates.
Point-charge field	$E = k \frac{Q}{r^2}$	inverse square	• Field strength, acceleration in plates and point-charge E -field.
Parallel plates	$E = \frac{V}{d}$	uniform field	• E - r graphs for positive/negative charges and for two charges.
Potential	$V = k \frac{q}{r}$	scalar	• Electric potential, positive/negative potential, and potential graphs.
Potential gradient	$E = -\frac{\Delta V}{\Delta r}$	negative gradient	• Energy conservation for charges moving in electric fields.
Energy	$q\Delta V = \Delta KE$	moving charge	• Exam-style graph sketches and numerical applications.

Capacitance

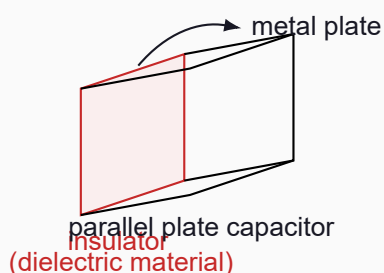
Chapter 07 / A2 Level / Apollo Physics 9702



Capacitor, Charging and Capacitance

Capacitor

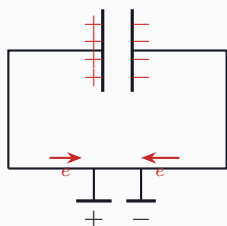
An electronic component that is used to store electrical energy and supply current to the circuit when need be.



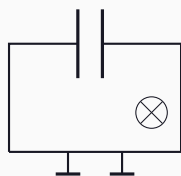
Uses of capacitor (Functions)

1. Stores energy.
2. Time delay circuits.
3. Smoothing in rectifier.
4. Power supplies.
5. Tuning circuits.

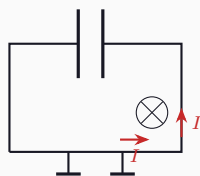
Charging a capacitor



- Positive terminal pulls electrons from one plate and negative terminal deposits them on the other.
- Both plates begin to achieve equal and opposite charge.
- Charging process stops when the p.d. across the capacitor is equal to emf of the battery.



Lamp off after capacitor charges



Lamp still on

Why capacitors are attached in parallel

Hence capacitors are always attached in parallel to a circuit so that the circuit keeps on working even when the capacitor is fully charged. **Explain why capacitor stores energy, not charge**

For a parallel plate capacitor, both plates have equal and opposite charge, hence net charge is zero. Work is done in separating those charges, therefore energy is stored.

Capacitance

It is the charge stored on one plate of a capacitor per unit potential difference.

$$C = \frac{q}{V}$$

$$q = CV$$

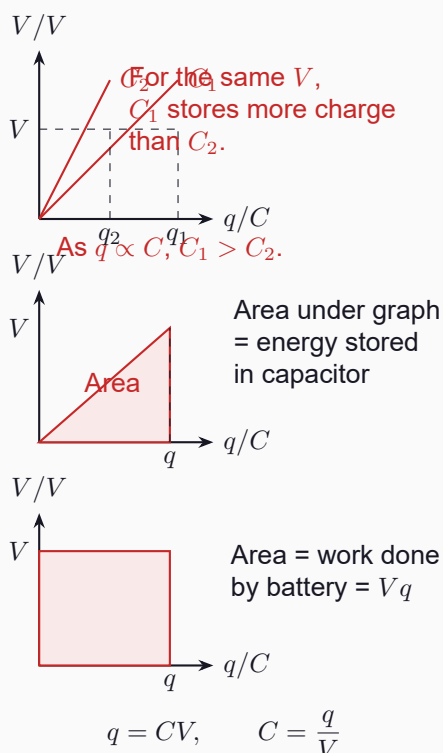
- SI unit: farad (F).
- Define the farad: it is the capacitance of a capacitor when 1 C of charge is stored across a p.d. of 1 V.

Factors affecting capacitance

1. Surface area of plates: bigger surface area of plate → more capacitance.
2. Insulation: better insulators → more capacitance.
3. Distance between plates: less distance → stronger field → more capacitance.

Energy Stored in a Capacitor and Capacitors in Series

Energy stored in a capacitor



For the same voltage, the capacitor with larger capacitance stores more charge.

$$E_{\text{cap}} = \frac{1}{2}qV$$

If $q = CV$,

$$E_{\text{cap}} = \frac{1}{2}(CV)V = \frac{1}{2}CV^2.$$

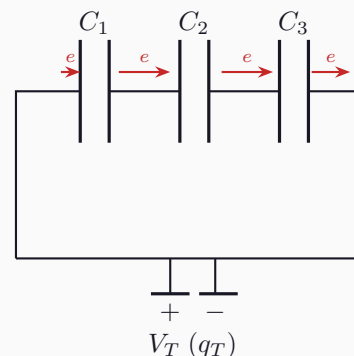
If $V = q/C$,

$$E_{\text{cap}} = \frac{1}{2}\left(\frac{q}{C}\right)q = \frac{1}{2}\frac{q^2}{C}.$$

$$E_{\text{cap}} = \frac{1}{2}CV^2$$

$$E_{\text{cap}} = \frac{1}{2}\frac{q^2}{C}$$

During the charging process, half of the work done by the source is stored in the capacitor and the other half is lost as heat in connecting wires. **Capacitors in series**



Charge: each capacitor gains the same magnitude of charge on one of its plates.

$$q_1 = q_2 = q_3 = q_T$$

Voltage: each capacitor achieves a different p.d. based on its capacitance.

$$q = CV \Rightarrow C \uparrow, V \downarrow.$$

Smaller capacitors achieve more p.d. and vice versa. Sum of all p.d. is equal to emf of source.

$$V_1 + V_2 + V_3 = V_T$$

For capacitance,

$$V_1 + V_2 + V_3 = V_T, \quad V = \frac{q}{C}.$$

$$\frac{q_1}{C_1} + \frac{q_2}{C_2} + \frac{q_3}{C_3} = \frac{q_T}{C_T}.$$

As $q_1 = q_2 = q_3 = q_T$,

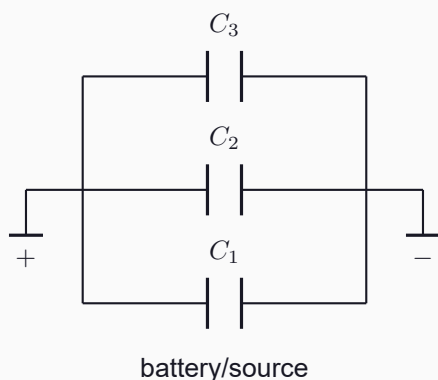
$$q \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) = q \left(\frac{1}{C_T} \right).$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

For identical capacitors: $C_T = C/n$.

Capacitors in Parallel and Capacitance of a Point Charge

Capacitors in parallel



Voltage: all capacitors charge up to the same p.d. as they are attached across the source directly.

$$V_1 = V_2 = V_3 = V_T$$

Charge: each capacitor holds charge based on its capacitance.

$$q = CV, \quad q \propto C.$$

Bigger capacitors hold more charge.

$$q_1 + q_2 + q_3 = q_T$$

Capacitance:

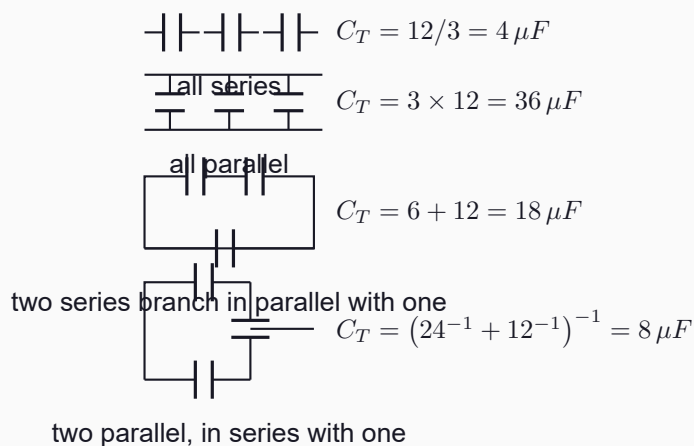
$$q_1 + q_2 + q_3 = q_T, \quad q = CV.$$

$$C_1 V_1 + C_2 V_2 + C_3 V_3 = C_T V_T.$$

As V is same,

$$C_T = C_1 + C_2 + C_3$$

Three $12 \mu\text{F}$ capacitors: possible combinations



Capacitance of a point charge / isolated sphere

For a point charge at the surface of a sphere,



solid metal sphere

$$V = \frac{kq}{r}, \quad C = \frac{q}{V}.$$

$$C = \frac{q}{kq/r} = \frac{r}{k}.$$

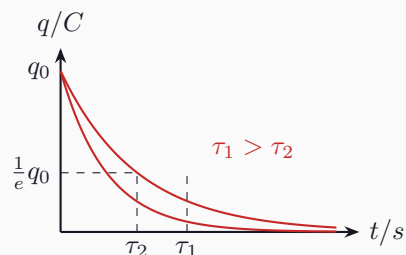
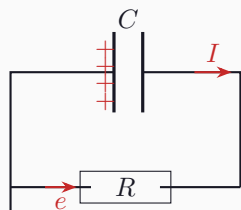
Since $k = \frac{1}{4\pi\epsilon_0}$,

$$C = 4\pi\epsilon_0 r$$

where r is the radius of the sphere.

Discharging a Capacitor and Time Constant

Discharging of a capacitor



$\tau = RC$; larger time constant $\rightarrow$ slower discharge

The rate of discharge of a capacitor is directly proportional to the charge on that capacitor.

$$\frac{\Delta x}{\Delta t} \propto x \quad \Rightarrow \quad x = x_0 e^{-kt}.$$

Such relations follow exponential functions.

$$\frac{\Delta q}{\Delta t} \propto q \quad \Rightarrow \quad q = q_0 e^{-t/RC}.$$

$$\frac{\Delta V}{\Delta t} \propto V \quad \Rightarrow \quad V = V_0 e^{-t/RC}.$$

$$\frac{\Delta I}{\Delta t} \propto I \quad \Rightarrow \quad I = I_0 e^{-t/RC}.$$

- q_0 : initial charge at $t = 0$.
- V_0 : initial voltage at $t = 0$.
- I_0 : initial current at $t = 0$.
- q, V, I : quantities at time t .
- R : resistance; C : capacitance.

$$\tau = RC$$

where τ is the time constant. A larger time constant gives slower discharge. **Definition of time constant**

The time taken for the capacitor to decrease its charge / voltage / current in the circuit to $1/e$ of its initial value.

$$q = q_0 e^{-t/RC}.$$

If $t = RC$, then

$$q = q_0 e^{-RC/RC} = q_0 e^{-1} = \frac{q_0}{e}.$$

$$q = \frac{1}{e} q_0$$

Since $\frac{1}{e} = 0.368$, after one time constant the remaining charge is 36.8% of its initial charge.

Exam Question: Isolated Metal Sphere

A solid metal sphere of radius r is insulated from its surroundings. The sphere has charge $+Q$. The charge is on the surface of the sphere but may be considered to be a point charge at its centre.



solid metal sphere

(a)(i) Define capacitance.

Charge stored per unit potential difference.

(a)(ii)

Show that the capacitance C of the sphere is given by

$$C = 4\pi\epsilon_0 r.$$

$$C = \frac{q}{V}, \quad V = \frac{q}{4\pi\epsilon_0 r}.$$

$$C = q \div \frac{q}{4\pi\epsilon_0 r} = \boxed{4\pi\epsilon_0 r}.$$

(b)(i) The sphere has radius 36 cm. Determine the capacitance.

$$C = 4\pi\epsilon_0 r = 4\pi(8.85 \times 10^{-12})(0.36).$$

$$C = 4.0 \times 10^{-11} \text{ F}.$$

$$\boxed{4.0 \times 10^{-11} \text{ F}}$$

(b)(ii) Charge required to raise the potential from zero to $7.0 \times 10^5 \text{ V}$

$$q = CV.$$

$$\Delta q = (4.0 \times 10^{-11})(7.0 \times 10^5).$$

$$\Delta q = \boxed{2.8 \times 10^{-5} \text{ C}}.$$

(c) Why would the calculation not apply to a plastic sphere?

Metal spheres are conductors and charge in them is uniformly distributed so the charge can be treated as a point charge. Plastic is an insulator; its charge may be at the surface or at the centre / not uniformly distributed, so the same model does not apply. (d) Spark discharge from

$7.0 \times 10^5 \text{ V}$ to $2.5 \times 10^5 \text{ V}$

Energy dissipated equals loss in stored energy:

$$\Delta E = \frac{1}{2}C (V_i^2 - V_f^2).$$

$$\Delta E = \frac{1}{2}(4.0 \times 10^{-11})\{(7.0 \times 10^5)^2 - (2.5 \times 10^5)^2\}.$$

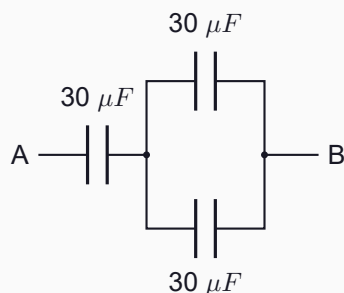
$$\Delta E = 8.55 \text{ J} \approx \boxed{8.6 \text{ J}}.$$

Exam Question: Three $30\ \mu\text{F}$ Capacitors

(a) State two functions of capacitors in electrical circuits.

1. Store energy.
2. Used in tuning circuits / time delay circuits / smoothing circuits.

(b) Three capacitors, each marked ' $30\ \mu\text{F}$, $6\ \text{V max}$ ', are arranged as shown.



The two capacitors on the right are in parallel, so

$$C_p = 30 + 30 = 60\ \mu\text{F}.$$

This $60\ \mu\text{F}$ combination is in series with the left $30\ \mu\text{F}$ capacitor:

$$\frac{1}{C_T} = \frac{1}{30} + \frac{1}{60} = \frac{3}{60}.$$

$$C_T = \boxed{20\ \mu\text{F}}.$$

Maximum p.d. between A and B

For series capacitors, charge is the same and $V = q/C$, so the smaller capacitance gets the larger p.d. The smallest capacitor is $30\ \mu\text{F}$, so assume the maximum safe p.d. across it is $6\ \text{V}$.

$$q_1 = q_2, \quad C_1 V_1 = C_2 V_2.$$

$$30 \times 6 = 60 \times V_2 \Rightarrow V_2 = 3\ \text{V}.$$

$$V_T = 6 + 3 = \boxed{9\ \text{V}}.$$

(c) A $4700\ \mu\text{F}$ capacitor is discharged from $18\ \text{V}$ to $12\ \text{V}$. Calculate energy dissipated.

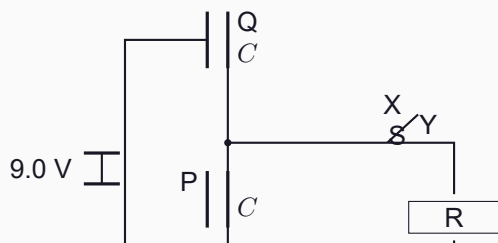
$$\Delta E = \frac{1}{2} C (V_i^2 - V_f^2).$$

$$\Delta E = \frac{1}{2} (4700 \times 10^{-6}) (18^2 - 12^2).$$

$$\Delta E = 0.423\ \text{J} \approx \boxed{0.42\ \text{J}}.$$

Exam Question: Switch X/Y with Three Capacitors

Two capacitors P and Q, each of capacitance C , are connected in series with a battery of e.m.f. 9.0 V . A switch S can connect either a third capacitor T, also of capacitance C , or a resistor R in parallel with capacitor P.



Switch S in position X: combined capacitance

Capacitor T is in parallel with P, so P+T gives $2C$. This is in series with Q:

$$\frac{1}{C_T} = \frac{1}{C} + \frac{1}{2C} = \frac{2+1}{2C} = \frac{3}{2C}.$$

$$C_T = \frac{2C}{3}$$

Potential difference across Q

In series, charge is the same:

$$q_1 = q_2, \quad C_T V_T = C_Q V_Q.$$

$$\left(\frac{2C}{3}\right) 9 = C V_Q.$$

$$V_Q = \boxed{6.0\text{ V}}.$$

Thus the p.d. across the P+T parallel branch is $9 - 6 = 3$

V. Switch moved to position Y

The resistor is connected across capacitor P.

- Capacitor P discharges from 3 V to 0 V because it is connected across the resistor.
- Capacitor Q charges up from 6 V to 9 V because the battery p.d. is greater than 6 V and finally appears across Q when P is discharged.

Capacitor P: discharges from 3 V to 0 V .
Capacitor Q: charges up from 6 V to 9 V .

Concept check

When a branch is placed across a capacitor, it can provide a discharge path. A resistor across a capacitor allows stored energy to be dissipated as heat.

Exam Question: Vibrating Switch and Sensitive Ammeter

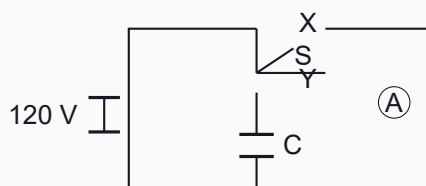
Definition

The capacitance of a parallel plate capacitor is the charge stored on one plate of a capacitor per unit potential difference across the plates:

$$C = \frac{q}{V}$$

where q is charge on one plate and V is the p.d. across them. **Circuit**

A capacitor C is connected to a 120 V battery when switch S is at X . When the switch moves to Y , the capacitor discharges through the sensitive ammeter.



The switch first goes to X , then to Y , then back to X fifty times each second. The current recorded on the ammeter is $4.5 \mu\text{A}$. **Charge passing through the ammeter in 1.0 s**

$$q = It = (4.5 \times 10^{-6})(1) = 4.5 \times 10^{-6} \text{ C}$$

Charge on one plate each time it is charged

There are 50 discharges in 1 second, so the charge per discharge is

$$q = \frac{4.5 \times 10^{-6}}{50} = 9.0 \times 10^{-8} \text{ C}$$

Capacitance of C

$$q = CV \Rightarrow C = \frac{q}{V}$$

$$C = \frac{9.0 \times 10^{-8}}{120} = 7.5 \times 10^{-10} \text{ F}$$

Second capacitor of equal capacitance placed in series

Effective capacitance halves:

$$C_{\text{eff}} = \frac{C}{2}$$

For the same p.d., charge stored on the combination also halves. Since

$$I = \frac{q}{t},$$

the current in the ammeter halves.

Effective capacitance halves, so charge stored each cycle halves; therefore current in the ammeter halves.

Formula Sheet and Summary

Ideas retained from the topics		
Topic	Formula	Key note
Capacitance	$C = q/V$	• Capacitor structure, symbol, charging process and lamp examples.
Stored energy	$E = \frac{1}{2}qV$	charge on one plate
Stored energy	$E = \frac{1}{2}CV^2$	area under $V-q$ graph
Stored energy	$E = \frac{1}{2}q^2/C$	• Why capacitors store energy rather than net charge.
Series capacitors	$1/C_T = 1/C_1 + 1/C_2 + \dots$	use with voltage
Parallel capacitors	$C_T = C_1 + C_2 + \dots$	• Energy from the area under a $V-q$ graph.
Sphere	$C = 4\pi\epsilon_0 r$	• Series and parallel capacitor rules with charge / p.d. explanations.
Discharge	$q = q_0 e^{-t/RC}$	same charge
Discharge	$V = V_0 e^{-t/RC}$	same p.d.
Discharge	$I = I_0 e^{-t/RC}$	• 12 μF combination examples and isolated sphere capacitance.
Time constant	$\tau = RC$	exponential decay
		• RC discharge exponential equations and time constant definition.
		• Exam questions: charged metal sphere, 30 μF network, X/Y switch, and vibrating switch with ammeter.

Magnetism and Electro-magnetism

Chapter 08 / A2 Level / Apollo Physics 9702



Magnetic field and right-hand grip rule

Magnetism

- Magnetism is due to the motion of charged particles.

Magnetic field

A region in space around a magnet where another magnet, magnetic material or current-carrying conductor experiences a magnetic force.

- North pole, South pole.
- Unlike poles attract and like poles repel.



- Direction of field lines tell the direction of force on a North pole.
- Gap between field lines tells strength of field: less gap means stronger field and more gap means weaker field.

Magnetic field around a current-carrying conductor



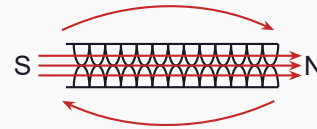
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Right hand grip rule

- Straight wire: thumb = current, curl of fingers = magnetic field.
- Coil of wire: thumb = North pole, curl of fingers = direction of current.



Magnetic field around a coil



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- Field lines around a straight conductor form concentric circles.
- The gap between circles increases away from the conductor: field becomes weaker.
- Arrows must show the correct field direction from the right-hand grip rule.

Current-carrying conductors

Like currents attract; unlike currents repel.



like currents attract unlike currents repel



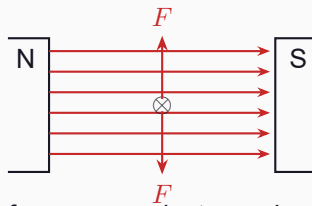
two wires both into page: attraction between the wires



opposite current directions: repulsion between the wires

Force on a current-carrying conductor and Fleming rules

Force on a current-carrying conductor in a magnetic field



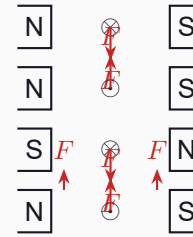
current into page; force on conductor and magnet are opposite

- Due to interaction of magnetic fields of conductor and permanent magnet, a force is produced on the conductor.
- The force acts perpendicular to the direction of current and the magnetic field.
- Force is also experienced by the magnet, equal in magnitude but opposite in direction. **Newton's third law**.

Fleming's left-hand rule

Thumb: Force (F)
First finger: Magnetic field (B)
Second finger: Current (I)

Fleming's left-hand rule is applied when a force is produced using magnetic field and current.



Using Fleming's left-hand rule

Fleming's Left-Hand Rule	Fleming's Right-Hand Rule
Thumb: Force on conductor (F)	Thumb: Applied force / motion of wire (F)
First finger: Magnetic field (B)	First finger: Magnetic field (B)
Second finger: Current (I)	Second finger: Induced current (I)
Memory order: FBI	Memory order: FBI
Known: magnetic field and current	Known: applied force and magnetic field
Unknown: force on conductor	Unknown: induced current
Use: DC motor / moving charge in magnetic field	Use: AC generator

Magnetic flux, magnetic flux density and force equation

Force proportionalities

- $F \propto I$: greater current gives greater force.
- $F \propto l$: greater length of conductor in the field gives greater force.

Combining both proportionalities,

$$F \propto Il \quad \Rightarrow \quad \boxed{F = BIl}$$

where B is the magnetic field strength or **magnetic flux density**.

$$\boxed{B = \frac{F}{Il}}$$

Magnetic flux density, B

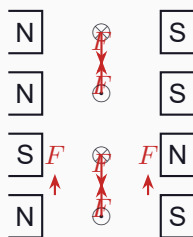
It is the force per unit length on a conductor carrying unit current normal to the magnetic field.

$$\boxed{\text{SI unit of } B = \text{tesla (T)}}$$

Define the tesla, 1 T

It is the magnetic flux density when a wire of 1 m carrying 1 A current experiences 1 N force when placed normal to the magnetic field.

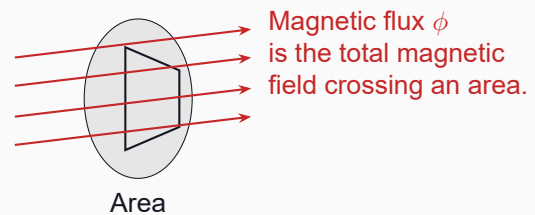
$$1 \text{ T} = 1 \text{ N A}^{-1} \text{ m}^{-1}$$



Using Fleming's left-hand rule

Magnetic flux, ϕ

Magnetic flux is the total magnetic field crossing an area.



Magnetic flux density as flux per unit area

Magnetic flux density is magnetic field in a unit area (1 m^2).

$$\boxed{\phi = BA}$$

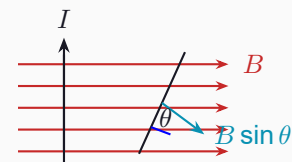
$$\boxed{B = \frac{\phi}{A}}$$

- ϕ : magnetic flux, SI unit weber (Wb).
- B : magnetic flux density, SI unit tesla (T).
- A : surface area through which field passes, SI unit m^2 .

$$\boxed{1 \text{ T} = 1 \text{ Wb m}^{-2}}$$

When the conductor is not normal to the field

Only the component of magnetic field at 90° to the conductor produces force.



only component of B at 90° to conductor produces force

$$\boxed{F = (B \sin \theta)Il = BIl \sin \theta}$$

where θ is the angle between conductor and magnetic field.

- $\theta = 0^\circ$: $\sin 0^\circ = 0 \Rightarrow F = 0$.
- $\theta = 90^\circ$: $\sin 90^\circ = 1 \Rightarrow F_{\text{max}} = BIl$.

Experiment to determine magnetic flux density

Experiment to determine magnetic flux density

Using $F = BIl$,

$$B = \frac{F}{Il}$$

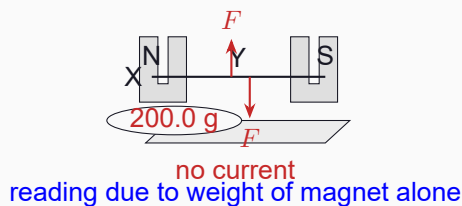
- F : force on conductor (N).
- I : current (A).
- l : length of conductor inside the field measured using a meter rule / tape measure.

Mass balance idea

When no current flows, the reading on the mass balance is due to the weight of the magnet alone:

$$W = m_1 g$$

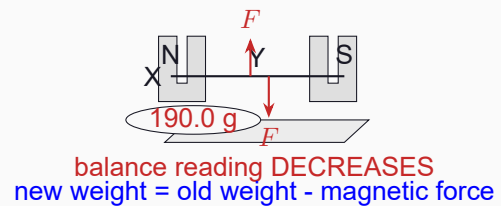
where m_1 is the actual mass of the magnet.



When current flows, a force is exerted on the conductor as well as on the magnet. The forces are equal in magnitude but opposite in direction. **Newton's third law. Case**

1: balance reading decreases

If force on the magnet is upward, apparent weight reduces.



$$W_{new} = W - F$$

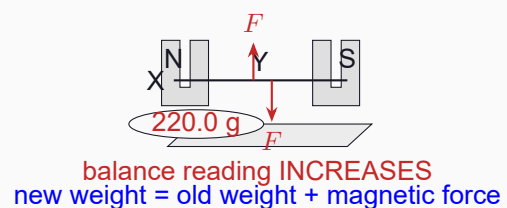
$$m_2 g = m_1 g - F \implies F = (m_1 - m_2)g = \Delta m g$$

Example from page:

$$F = \frac{(200 - 190)}{1000} \times 9.8 = 0.098 \text{ N}$$

Case 2: balance reading increases

If force on the magnet is downward, apparent weight increases.



$$W_{new} = W + F$$

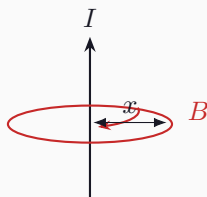
$$m_2 g = m_1 g + F \implies F = (m_2 - m_1)g = \Delta m g$$

Example from page:

$$F = \frac{20}{1000} \times 9.8 = 0.196 \text{ N}$$

Magnetic field around a straight conductor and force between wires

Magnetic field around a straight current-carrying conductor



From the notes:

$$B \propto I \quad B \propto \frac{1}{x}$$

Combining,

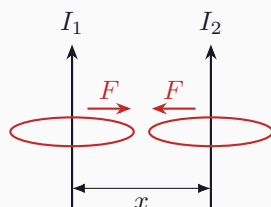
$$B = \frac{\mu_0 I}{2\pi x}$$

where:

- I : current.
- x : distance of a point from the centre of the wire.
- μ_0 : permeability of free space, $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$.

μ_0 tells about the ease for vacuum to allow magnetic field to spread. **Force between two current-carrying conductors**

A current-carrying conductor placed in the field of another current-carrying conductor experiences force.



$$F = BIl$$

For the field produced by wire 1 at wire 2,

$$B = \frac{\mu_0 I_1}{2\pi x}$$

Substituting into $F = BI_2 l$,

$$F = \frac{\mu_0 I_1}{2\pi x} I_2 l$$

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi x}$$

Force per unit length

Dividing both sides by l ,

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi x}$$

- The force between the two wires is mutual.
- $F_{\text{wire 1 on wire 2}} = F_{\text{wire 2 on wire 1}}$
- Even if magnitudes of currents are different, the force on both conductors is the same. This is according to Newton's third law.
- The formula is secondary to this chapter's main results but is useful for force-between-wire calculations.

Useful summaries

Situation	Equation
Straight wire field	$B = \frac{\mu_0 I}{2\pi x}$
Force on conductor	$F = BIl$
Force at angle	$F = BIl \sin \theta$
Magnetic flux	$\phi = BA$
Force per unit length between wires	$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi x}$

Conceptual rule

- Like currents attract.
- Unlike currents repel.
- For both cases the two forces are equal and opposite.

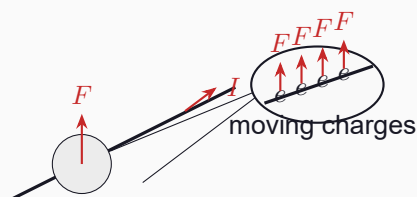
Charged Particles

Chapter 10 / A2 Level / Apollo Physics 9702



Charged Particles in Magnetic Fields

Force on moving charges in a magnetic field



The force on the current-carrying conductor is basically the force on the **moving charges in that conductor**.

Force on a moving charge

For a current-carrying conductor,

$$F = BIl.$$

Since current is the rate of flow of charge,

$$I = \frac{q}{t}, \quad v = \frac{l}{t}.$$

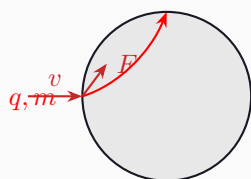
Therefore,

$$F = B \left(\frac{q}{t} \right) l = Bq \left(\frac{l}{t} \right).$$

$$F = Bvq$$

(1)

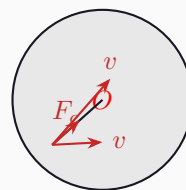
- F : magnetic force / N
- B : magnetic flux density / T
- v : velocity of charged particle / m s^{-1}
- q : magnitude of charge / C



magnetic field into the plane

- The force on the charged particle in a magnetic field acts as a **centripetal force**.
- Therefore the particle moves in a **circular track** in the magnetic field.

Magnetic force acting as centripetal force



magnetic force acts as centripetal force

$$F_m = F_c$$

$$Bvq = \frac{mv^2}{r}$$

Cancel one v :

$$Bq = \frac{mv}{r}$$

$$r = \frac{mv}{Bq}$$

(2)

Since momentum $p = mv$,

$$r = \frac{p}{Bq}$$

$\Rightarrow$

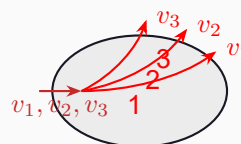
$$p = Bqr$$

If $q = e$ is the elementary charge,

$$p = Ber.$$

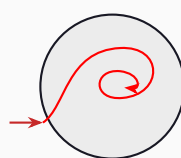
How radius changes

- Since $r \propto v$, the greater the speed, the greater the radius and the lesser the deflection.



$$v_1 > v_2 > v_3 \text{ SO } r_1 > r_2 > r_3$$

- A constant decrease in radius shows that the particle is slowing down.



a constant decrease in radius shows particle is slowing down

Charge-to-mass ratio and particle paths

Effect of magnetic field strength

From

$$r = \frac{mv}{Bq},$$

$$r \propto \frac{1}{B}.$$

Stronger magnetic fields bend charged particles more because the particles experience a larger magnetic force. Hence they have a smaller radius.

Effect of charge-to-mass ratio

From the same equation,

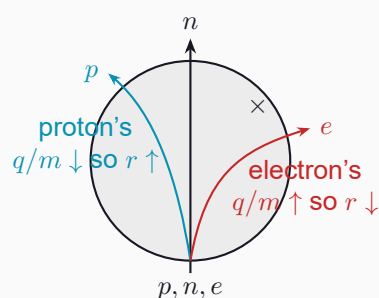
$$r \propto \frac{m}{q} \quad \text{or} \quad r \propto \frac{1}{q/m}.$$

where q/m is the **charge-to-mass ratio**.

Particle	charge q	mass m
proton	$+1.6 \times 10^{-19} \text{ C}$	$1.67 \times 10^{-27} \text{ kg}$
electron	$-1.6 \times 10^{-19} \text{ C}$	$9.11 \times 10^{-31} \text{ kg}$

$$\frac{q}{m} \text{ for proton} \approx 9.6 \times 10^7 \text{ C kg}^{-1}$$

$$\frac{q}{m} \text{ for electron} \approx 1.76 \times 10^{11} \text{ C kg}^{-1}$$

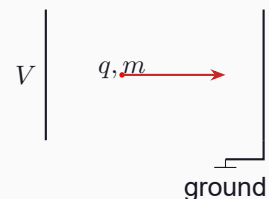


magnetic field into the page: opposite charges bend in opposite directions

Determining charge-to-mass ratio

When a moving charged particle enters a magnetic field, it experiences a magnetic force which acts as a centripetal force.

$$\begin{aligned} F_m &= F_c \\ Bvq &= \frac{mv^2}{r} \\ \frac{q}{m} &= \frac{v}{Br} \end{aligned} \quad (3)$$



To replace v : work done by electric field = gain in kinetic energy

Work done by electric field = gain in kinetic energy.

$$V = \frac{W}{q} \Rightarrow W = Vq$$

$$\Delta K.E. = \frac{1}{2}m(v^2 - u^2)$$

If the particle is initially at rest, $u = 0$:

$$Vq = \frac{1}{2}mv^2$$

$$2Vq = mv^2$$

$$v = \sqrt{\frac{2Vq}{m}}$$

Replacing v in Eq. (3),

$$\frac{q}{m} = \frac{1}{Br} \sqrt{\frac{2Vq}{m}}$$

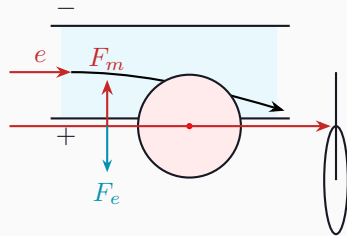
Squaring both sides,

$$\left(\frac{q}{m}\right)^2 = \frac{2V}{B^2 r^2} \cdot \frac{q}{m}$$

$$\frac{q}{m} = \frac{2V}{B^2 r^2} \quad (4)$$

Velocity selector

Velocity selector



The electron will pass straight, as fields tried to deflect it in opposite directions.

A magnetic field and an electric field are placed perpendicular to one another. Their field strengths are adjusted such that the **electric force** is balanced by the **magnetic force**.

Only particles with one particular speed pass straight through. Others are deflected and filtered out.

Condition for straight motion

For a selected charged particle,

$$F_e = F_m.$$

Electric force:

$$F_e = Eq.$$

Magnetic force:

$$F_m = Bvq.$$

Therefore,

$$Eq = Bvq.$$

Cancel q :

$$E = Bv.$$

$$v = \frac{E}{B}$$

Using a specific value of E and B , a particular speed of charged particle is filtered out.

Summary box

Magnetic force	$F = Bvq$
Circular radius	$r = \frac{mv}{Bq} = \frac{p}{Bq}$
Momentum	$p = Bqr$; for charge e , $p = Ber$
Charge-to-mass ratio	$\frac{q}{m} = \frac{v}{Br} = \frac{2V}{B^2r^2}$
Velocity selector	$v = \frac{E}{B}$

Nuclear Physics

Chapter 11 / A2 Level / Apollo Physics 9702



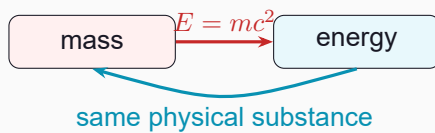
Einstein's mass-energy equation and mass defect

Einstein's mass-energy equation

- Mass and energy are two different manifestations of the same thing.
- During nuclear reactions, mass of the element is converted into energy.
- The energy produced from mass can be calculated using

$$E = mc^2$$

where c is the speed of light.



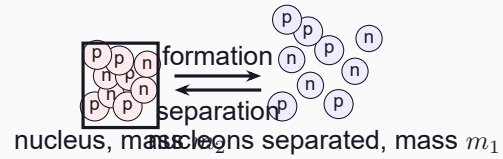
Mass defect

The mass of the nucleus is less than the total mass of the nucleons separated.

$$\Delta m = m_1 - m_2$$

where

- m_1 = mass of nucleons separated,
- m_2 = mass of nucleus.



Mass defect is the difference in mass between the separated nucleons and the nucleons combined as a nucleus.

Mass defect and binding energy

Mass and energy conversion

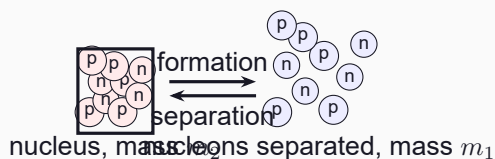
- Mass and energy are two different manifestations of the same thing.
- Therefore they can be converted from one to another.

$$E = mc^2$$

Mass defect and binding energy

When a nucleus is formed from separated nucleons, the released energy is called the **binding energy**.

$$\Delta m = m_1 - m_2, \quad E = \Delta mc^2$$



Definition: Binding energy is the amount of energy needed to separate the nucleons of a nucleus apart to infinity.

Example: tritium nucleus

For ${}^3_1\text{H}$, using masses in u:

$$\Delta m = (m_p + 2m_n) - m_{\text{nucleus}}$$

$$\Delta m = (1.00728 + 2(1.00867) - 3.0155)$$

$$\Delta m = 9.11 \times 10^{-3} \text{ u}$$

Binding energy in joules

$$E = \Delta mc^2$$

$$E = (9.11 \times 10^{-3} \times 1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.36 \times 10^{-12} \text{ J}$$

Binding energy in eV

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$E = \frac{1.36 \times 10^{-12}}{1.6 \times 10^{-19}} = 8.5 \times 10^6 \text{ eV} = \boxed{8.5 \text{ MeV}}$$

Atomic mass unit and binding energy per nucleon

Energy equivalent of 1 u

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

$$E = mc^2 = (1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.494 \times 10^{-10} \text{ J}$$

Converting to eV:

$$1.6 \times 10^{-19} \text{ J} = 1 \text{ eV}$$

$$E = \frac{1.494 \times 10^{-10}}{1.6 \times 10^{-19}} = 9.33 \times 10^8 \text{ eV}$$

$$1 \text{ u} = 933 \text{ MeV} \approx 930 \text{ MeV}$$

Shortcut for binding energy

If Δm is in u,

$$E(\text{MeV}) = \Delta m(\text{u}) \times 933$$

For the tritium example,

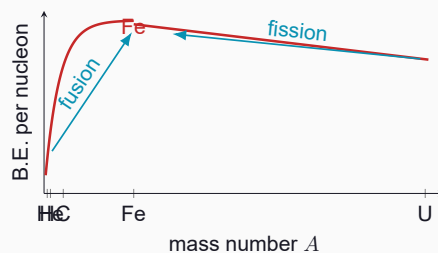
$$E = (9.11 \times 10^{-3})(933) = 8.5 \text{ MeV}.$$

Binding energy per nucleon

$$\text{B.E. per nucleon} = \frac{\text{binding energy}}{\text{number of nucleons}}$$

It tells us about the stability of the nucleus.

- Larger B.E. per nucleon $\Rightarrow$ more stable nucleus.
- Smaller B.E. per nucleon $\Rightarrow$ less stable nucleus.



For a feasible fission/fusion reaction, the product must have greater binding energy per nucleon than the reactant.

Worked example: lanthanum-139 energy release

Given data

particle/nucleus	symbol	mass/u
lanthanum-139	${}_{57}^{139}\text{La}$	138.955
proton	${}_1^1\text{p}$	1.007
neutron	${}_0^1\text{n}$	1.009

Convert to kg and calculate energy

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

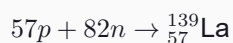
$$\Delta m = 1.182(1.66 \times 10^{-27}) = 1.962 \times 10^{-27} \text{ kg}$$

$$E = mc^2 = (1.962 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.77 \times 10^{-10} \text{ J}$$

Mass of separated nucleons

Lanthanum-139 contains 57 protons and 82 neutrons.



$$m_1 = 57(1.007) + 82(1.009) = 140.137 \text{ u}$$

$$m_2 = 138.955 \text{ u}$$

$$\Delta m = 140.137 - 138.955 = 1.182 \text{ u}$$

For one mole of particles

$$E_{\text{mol}} = (1.77 \times 10^{-10})(6.02 \times 10^{23})$$

$$E_{\text{mol}} = \boxed{1.06 \times 10^{14} \text{ J}}$$

Mass defect in nuclear reactions

Atomic mass unit

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

It is approximately the average mass of a proton/neutron.

Lanthanum-139 calculation continued

$$m_1 = 57(1.007) + 82(1.009) = 140.137 \text{ u}$$

$$\Delta m = 140.137 - 138.955 = 1.182 \text{ u}$$

$$\Delta m = 1.182(1.66 \times 10^{-27}) = 1.96 \times 10^{-27} \text{ kg}$$

For one mole:

$$1 \text{ mol La} \rightarrow 6.02 \times 10^{23} \text{ particles} \rightarrow 139 \text{ g}$$

$$\Delta m_{1 \text{ mol}} = (1.96 \times 10^{-27})(6.02 \times 10^{23})$$

$$\Delta m_{1 \text{ mol}} = 0.00117992 \text{ kg} = \boxed{1.18 \text{ g}}$$

Key notes

- Mass defect happens in nuclear fission, nuclear fusion and radioactive decay because energy is released in them.
- The loss of mass is converted to energy.

$$\Delta E = \Delta mc^2$$

Do not write “mass is conserved” or “energy is conserved” alone in nuclear reactions. Mass is converted to energy, so mass-energy is conserved.

Binding energy and binding energy per nucleon

Binding energy

Binding energy is the amount of energy needed to separate the nucleons of the nucleus apart to infinity.

- Binding energy basically binds the nucleus together.
- To break the nucleus, this much energy must be supplied.

$$\Delta E = \Delta mc^2$$

binding energy	mass defect
ΔE	Δm

Example continued

$$\Delta E = (1.96 \times 10^{-27})(3.0 \times 10^8)^2$$

$$\Delta E = 1.76 \times 10^{-10} \text{ J}$$

Binding energy per nucleon

Energy needed per nucleon to break the nucleus apart and separate nucleons to infinity.

$$\text{B.E. per nucleon} = \frac{\text{binding energy}}{\text{number of nucleons}}$$

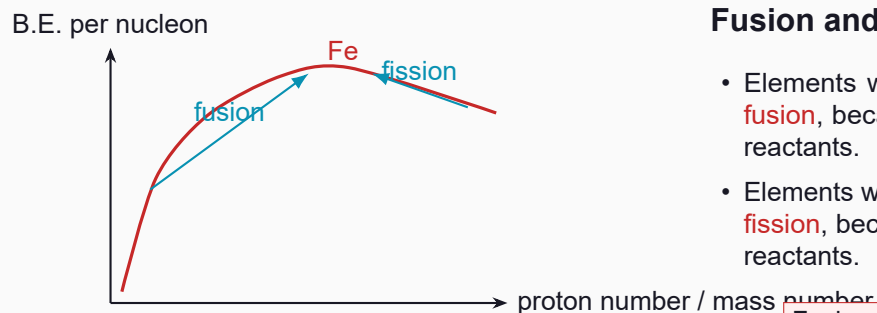
This value gives the idea about stability of the nucleus.

$$\text{B.E. per nucleon} = \frac{1.76 \times 10^{-10}}{139}$$

$$\text{B.E. per nucleon} = 1.27 \times 10^{-12} \text{ J}$$

More binding energy per nucleon means more stable nucleus.

Binding-energy curve and feasible reactions



Fusion and fission

- Elements with proton number less than iron prefer **fusion**, because the product is more stable than the reactants.
- Elements with proton number greater than iron prefer **fission**, because the product is more stable than the reactants.

Fusion and fission both release energy only when the final nuclei have greater binding energy per nucleon.

Important rule

A reaction is only feasible if the binding energy per nucleon of products is more than the binding energy per nucleon of reactants.

The mass of products will be less than the mass of reactants when a reaction is feasible.

Stability

- The greater the binding energy per nucleon, the more stable the nucleus.
- Iron has the greatest binding energy per nucleon, so it is the most stable.

Practice question: fusion in the Sun

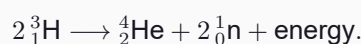
Question style: May/June 2013 Paper 41

(a) State what is meant by a nuclear fusion reaction.

Answer: A nuclear reaction in which small nuclei combine to make a larger, more stable nucleus, releasing a large amount of heat energy.

Reaction

One nuclear reaction in the core of the Sun is represented by



Data:

particle/nucleus	mass/u
proton ${}^1_1\text{p}$	1.00728
deuterium ${}^2_1\text{H}$	2.0140
helium ${}^4_2\text{He}$	4.0015

Energy released

Use step 1: mass defect.

$$\Delta m = m_{\text{products}} - m_{\text{reactants}}$$

The note uses

$$\Delta m = 3.0165 - (1.0072 + 2.0140) = -5.3 \times 10^{-3} \text{ u}$$

Magnitude of mass defect:

$$|\Delta m| = 5.3 \times 10^{-3} \text{ u}$$

Step 2: use $E = mc^2$.

$$E = (5.3 \times 10^{-3})(1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = \boxed{7.9 \times 10^{-13} \text{ J}}$$

Why high temperature is necessary

The temperature in the core of the Sun is approximately $1.6 \times 10^7 \text{ K}$. It is so high because nuclei are both positively charged and repel one another. High temperature gives nuclei high speed and kinetic energy so that the repulsion can be overcome.

Conversion and conservation during nuclear reactions

Conversion of 1 u to MeV

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

$$E = mc^2 = (1.66 \times 10^{-27})(3.0 \times 10^8)^2$$

$$E = 1.494 \times 10^{-10} \text{ J}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$E = \frac{1.494 \times 10^{-10}}{1.6 \times 10^{-19}} = 933 \text{ MeV} \approx \boxed{930 \text{ MeV}}$$

Conservation during nuclear reactions

During nuclear reactions the following are conserved:

1. proton number,
2. mass number,
3. mass-energy,
4. momentum.

Do not write just mass conserved or energy conserved, because mass is converted to energy.

Types of nuclear reactions

- Radioactive decay
- Fission
- Fusion

Practice question: polonium nucleus

Question style: Oct/Nov 2014 Paper 41

(a) Explain what is meant by binding energy of a nucleus.

Answer: The energy required to separate all nucleons of a nucleus apart to infinity.

Given masses

particle/nucleus	mass/u
proton	1.00728
neutron	1.00867
lithium/helium data	3.01551
polonium nucleus	208.9822

$$1.0 \text{ u} = 930 \text{ MeV}.$$

Binding energy calculation

$$\Delta m = m_1 - m_2$$

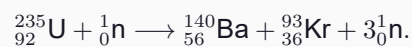
$$\Delta m = 211.0194 - 208.9822 = 2.0372 \text{ u}$$

$$E = 2.0372 \times 930 = 1894.6 \text{ MeV}$$

$$\text{B.E. per nucleon} = \frac{1894.6}{210} = \boxed{9.02 \text{ MeV}}$$

Fission reaction

One fission reaction is represented by



Why is fission energetically possible?

The product nuclei have a greater binding energy per nucleon than the reactant nucleus, so the final nuclei are more stable and energy is released.

Radioactivity and activity

Radioactivity

Radioactivity is the phenomenon of emission of radioactive radiations from the nucleus of an atom.

Types of radiations

- **Alpha particles:** helium nucleus, $2p + 2n$, charge $+2$.
- **Beta particles:** fast moving electrons or positrons, charge -1 or $+1$.
- **Gamma rays:** high-frequency electromagnetic radiation, neutral rays.

Activity, A

Activity is the rate of radioactive radiations emitted from the nucleus of an atom.

$$A = \frac{\Delta N}{\Delta t}$$

Since $\Delta N = N_{\text{final}} - N_{\text{initial}}$ is negative for decay,

$$A = -\frac{\Delta N}{\Delta t}$$

SI unit: becquerel, Bq.



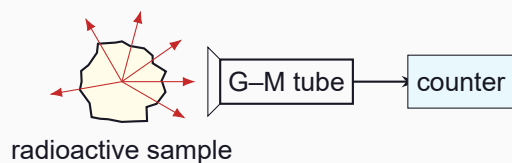
initial nuclei drop to final nuclei in time interval

- N_0 : initial number of radioactive nuclei in sample.
- N : final number of radioactive nuclei in sample.
- Δt : time for N_0 to drop to N .

Count rate and decay constant

Count rate, R

Count rate is the number of radioactive radiations detected per unit time.



Count rate of a sample is different from its activity.

1. Not all emitted radioactive radiations are detected by the G-M tube.
2. The G-M tube also counts background radiation present in the surroundings.

Decay constant, λ

Decay constant is the probability of a nucleus decaying per unit time.

For example,

$$\lambda = 0.4 \text{ s}^{-1}$$

means 40% of the undecayed nuclei decay in one second.

$$\lambda = \frac{\Delta N}{N \Delta t}$$

where ΔN is the number of atoms that decay in time Δt , and N is the number of atoms that could decay.

Activity is proportional to number of undecayed nuclei

Example of decay constant

If $\lambda = 0.4 \text{ s}^{-1}$:

- out of 100, 40 would decay in 1 s,
- out of 50, 20 would decay in 1 s,
- out of 5, 2 would decay in 1 s.

$$\lambda = \frac{\Delta N}{N_0 \Delta t}$$

Link with activity

Since

$$A = \frac{\Delta N}{\Delta t},$$

we get

$$\lambda = \frac{A}{N}$$

$$\boxed{A = \lambda N}$$

where

- A = current activity,
- λ = decay constant,
- N = current number of undecayed nuclei.

Activity is not fixed. It decreases with time as the remaining number of undecayed nuclei decreases with time.

Probability interpretation of the decay constant

Probability

$$\text{probability} = \frac{\text{possible outcomes}}{\text{total outcomes}}$$

Example:

$$N = 500 \text{ atoms}, \quad N_0 = 10000 \text{ atoms}, \quad \Delta t = 1 \text{ s}$$

$$\text{probability} = \frac{500}{10000} = 0.05 \text{ per second}$$

Decay constant

$$\lambda = \frac{\text{probability}}{\Delta t}$$

For one second intervals, the numerical value of the probability per second is the decay constant.

$$A = \lambda N$$

$$\lambda = \frac{A}{N}$$

Different sample sizes

N_0	decayed in 1 s	probability
1000	50	0.05 per s
5000	250	0.05 per s
20000	1000	0.05 per s

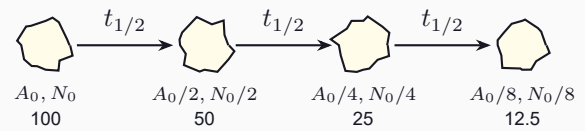
Half-life

Definitions

Half-life is the time taken for a radioactive sample to decrease its activity to half of its initial value.

It is also the time taken for a radioactive sample to decrease its number of radioactive nuclei to half of its initial value.

It is also the time taken for count rate to decrease to half of its initial value.



$$100 \rightarrow 50 \rightarrow 25 \rightarrow 12.5$$

Each arrow represents one half-life.

Equivalent statements

$$A \rightarrow \frac{A_0}{2}, \quad N \rightarrow \frac{N_0}{2}, \quad R \rightarrow \frac{R_0}{2}$$

Exponential decay derivation

In radioactive decay, the rate at which radioactive nuclei emit radiation is proportional to the number of radioactive nuclei themselves.

Any phenomenon behaving in this manner follows an exponential function.

$$\frac{\Delta N}{\Delta t} \propto N$$

In reality, ΔN is negative because

$$\Delta N = N_{\text{final}} - N_{\text{initial}}, \quad N_f < N_i.$$

$$\lambda = -\frac{\Delta N}{N\Delta t}$$

In terms of calculus

$$\frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{N} = -\lambda dt$$

Integrating both sides:

$$\int_{N_0}^N \frac{dN}{N} = \int_0^t -\lambda dt$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 e^{-\lambda t}}$$

Radioactivity recap

Radioactive decay

The phenomenon of emission of radioactive radiations from the nucleus of an unstable atom.

The process is random and spontaneous.

Random process

Every nucleus has equal probability to decay. It cannot be predicted which nucleus will decay and when.

Spontaneous process

It happens on its own. Changes in external conditions such as temperature and pressure have no effect on the rate of emission.

Types

- Alpha: α or ${}^4_2\text{He}$; helium nucleus.
- Beta: β^- or β^+ .
- Gamma: very high-frequency electromagnetic radiation.

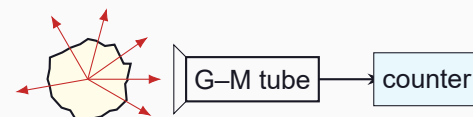
Activity

$$A = -\frac{\Delta N}{\Delta t}$$

SI unit: becquerel (Bq).

Count rate

Count rate is the number of radioactive radiations detected per unit time.



radioactive sample

Count rate = sample activity + background activity, with detector inefficiency taken into account.

Decay equations and half-life

Decay constant

$$\lambda = \frac{\text{probability of decay}}{\text{time interval}}$$

$$\lambda = \frac{A}{N}, \quad A = \lambda N$$

Same form for N , A and R

$$N = N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

$$R = R_0 e^{-\lambda t}$$

Half-life

Time taken for the radioactive sample to decrease its activity, count rate or number of radioactive atoms to half of its initial value.

Example:

$$4000 \rightarrow 2000 \rightarrow 1000 \rightarrow 500 \rightarrow 250$$

If each half-life is 2 years, then four half-lives take 8 years.

The units of λ and t must be similar. For example, if λ is in min^{-1} , then t must be in minutes.

Relation between decay constant and half-life

Start with the decay equation

$$N = N_0 e^{-\lambda t}$$

At $t = t_{1/2}$,

$$N = 0.5N_0.$$

Therefore,

$$0.5N_0 = N_0 e^{-\lambda t_{1/2}}$$

$$0.5 = e^{-\lambda t_{1/2}}$$

Take natural log:

$$\ln(0.5) = -\lambda t_{1/2}$$

$$-0.693 = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{0.693}{\lambda}$$

Since $0.693 = \ln 2$,

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

or

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{\ln 2}{t_{1/2}}$$

This relation is only valid for exponential decay, where the decay rate is proportional to the number of undecayed nuclei.

Half-life numerical example

Example by repeated halving

$$A_0 = 4000, \quad t_{1/2} = 2 \text{ years}$$

Given final activity:

$$A = 500$$

Repeated halving:

$$4000 \rightarrow 2000 \rightarrow 1000 \rightarrow 500$$

This is 3 half-lives.

$$3 \times 2 \text{ years} = \boxed{6 \text{ years}}$$

Half-life relation

At $t = t_{1/2}$,

$$N = 0.5N_0, \quad A = 0.5A_0$$

Derivation again

$$N = N_0 e^{-\lambda t}$$

$$0.5N_0 = N_0 e^{-\lambda t_{1/2}}$$

$$0.5 = e^{-\lambda t_{1/2}}$$

$$\ln 0.5 = -\lambda t_{1/2}$$

$$-0.693 = -\lambda t_{1/2}$$

$$\boxed{t_{1/2} = \frac{0.693}{\lambda}}, \quad \boxed{\lambda = \frac{0.693}{t_{1/2}}}$$

For N , A and R :

$$N = N_0 e^{-\lambda t}, \quad A = A_0 e^{-\lambda t}, \quad R = R_0 e^{-\lambda t}$$

Exponential decay graph

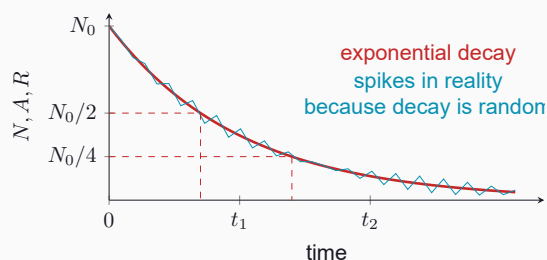
Same exponential form

$$N = N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t}$$

$$R = R_0 e^{-\lambda t}$$

Make sure λ and t have similar units. For example, if $\lambda = \text{min}^{-1}$, then t must be in minutes.



Half-life formula

$$N = \frac{N_0}{2}, \quad t = t_{1/2}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$t_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$$

- The smooth red line represents the ideal exponential decay.
- In real count-rate graphs, spikes are seen because decay is random.
- Graphs of A and R have the same shape as the graph of N .

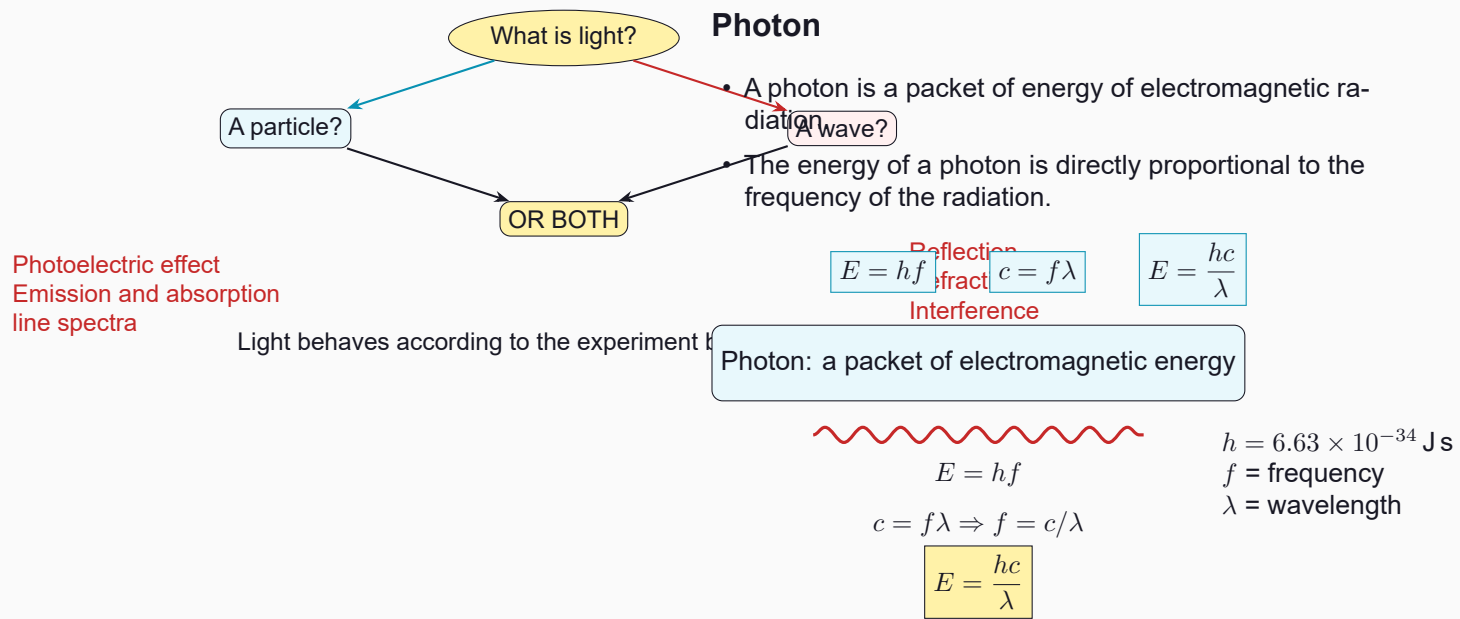
End page / topic 23

Quantum Physics

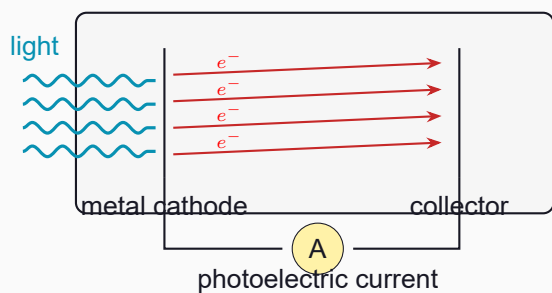
Chapter 12 / A2 Level / Apollo Physics 9702



What is light? Photon energy



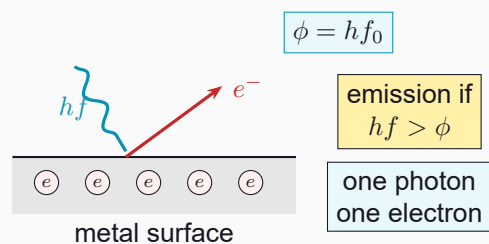
Photoelectric effect: photon energy and work function



No emission occurs if $hf < \phi$. If $hf > \phi$, one photon releases one electron and the excess energy becomes kinetic energy.

Threshold frequency and work function

- Work function energy ϕ : minimum photon energy required to remove an electron from a metal surface with negligible kinetic energy.
- Threshold frequency f_0 : minimum frequency needed for photoelectric emission.
- Planck's constant: $h = 6.63 \times 10^{-34} \text{ J s}$.
- $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.



Einstein's photoelectric equation

If photon energy is less than the work function

The electron absorbs the photon inside the metal atom, gains vibration energy, collides with neighbouring atoms, and the temperature of the metal increases. No electron leaves the surface.

If photon energy is greater than the work function

$$E = \phi + E_{k \max}$$

$$hf = \phi + E_{k \max}$$

Examples from the notes:

$$12 \text{ eV} = 10 \text{ eV} + 2 \text{ eV}, \quad 15 \text{ eV} = 10 \text{ eV} + 5 \text{ eV}.$$

Intensity idea

$$\text{Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{time}}$$

Total energy depends on the number of photons and the energy of each photon:

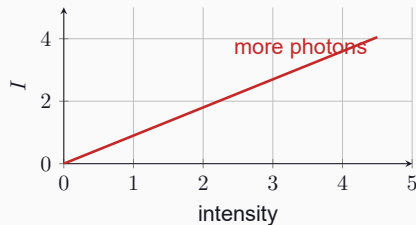
$$\text{Energy} = N \times hf.$$

Increasing frequency increases energy per photon. Increasing intensity at the same frequency increases the number of photons.

Changing intensity and frequency

Intensity increased at the same frequency

- More photons arrive per second.
- $E_{k \text{ max}}$ remains the same because each photon has the same energy.
- Photoelectric current increases because more electrons are emitted per second.



Frequency increased but intensity kept constant

- Photon energy increases.
- $E_{k \text{ max}}$ increases.
- The number of photons must decrease to keep total intensity constant.
- Photoelectric current decreases.

$$400 \text{ eV} = 1000 \times 0.40 \text{ eV} = 200 \times 2.0 \text{ eV}$$

Graph of maximum kinetic energy against frequency

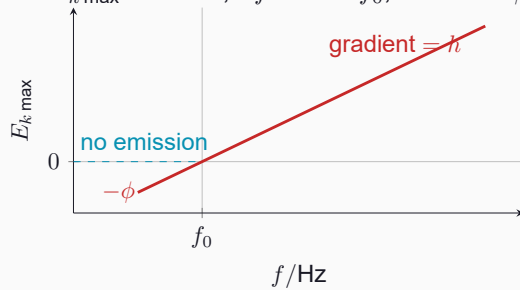
Linear form

$$E_{k \max} = hf - \phi$$

This matches $y = mx + c$:

$$\boxed{y = E_{k \max}}, \quad \boxed{x = f}, \quad \boxed{m = h}, \quad \boxed{c = -\phi}.$$

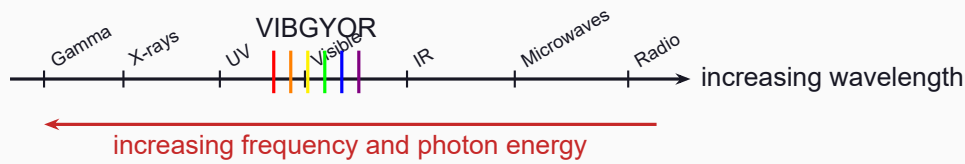
At $E_{k \max} = 0$, $f = f_0$, hence $\phi = hf_0$.



Changing metal

Replacing a red-sensitive metal by a blue-sensitive metal changes the work function and threshold frequency; the gradient remains Planck's constant.

Electromagnetic spectrum and definitions



Photoelectric effect

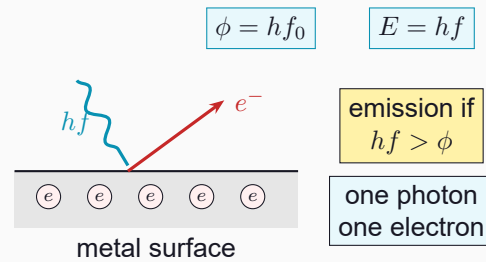
The emission of electrons from the surface of a metal by the incidence of light.

Work function

Minimum energy of a photon needed to remove an electron from the metal surface with negligible kinetic energy.

Threshold frequency

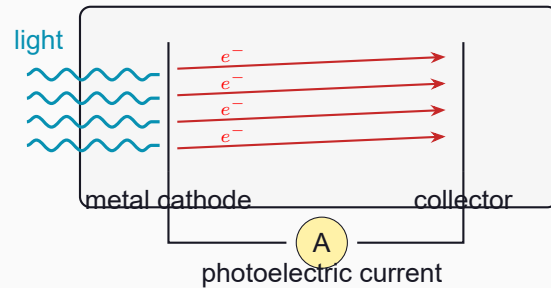
Minimum frequency needed to remove an electron from the metal surface with negligible kinetic energy.



Characteristics of the photoelectric effect

1. Not all electromagnetic radiations produce photoelectric effect. It depends on both the radiation and the metal.
2. If the frequency of the radiation is below the threshold frequency of the metal, no photoelectric effect occurs.
3. One electron absorbs only one photon at a time.
4. There is no time delay between absorption of a photon and emission of an electron if $hf > \phi$.
5. Increasing intensity at fixed frequency increases the rate of emission, not the kinetic energy of electrons.
6. Increasing frequency increases the maximum kinetic energy of emitted electrons.

If $hf < \phi$, the electron does not accumulate photons one by one. The absorbed energy is transferred as internal energy through collisions in the metal.



Electron volt and photoelectric equation

Maximum kinetic energy

$$E = \phi + E_{k \max}$$

$$hf = hf_0 + \frac{1}{2}mv_{\max}^2$$

where m is the mass of electron and v is the electron velocity.

quantity	symbol	meaning
photon energy	hf	incident photon
work function	ϕ	energy to remove electron
maximum kinetic energy	$E_{k \max}$	excess energy
threshold frequency	f_0	just enough frequency

Electron volt

An electron volt is a unit of energy. It is the energy gained by an electron when it moves through a potential difference of 1 V.

$$\text{energy} = \text{voltage} \times \text{charge}$$

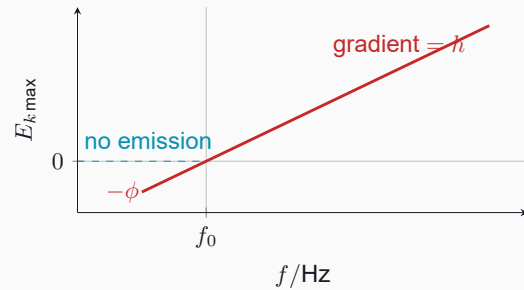
$$1 \text{ eV} = 1 \text{ V} \times e = 1.6 \times 10^{-19} \text{ J}$$

Increasing frequency of incident radiation

Effect on $E_{k \text{ max}}$

If the frequency of the incident radiation is increased, $E_{k \text{ max}}$ increases because the energy of each photon increases while the work function remains the same.

$$E_{k \text{ max}} = hf - \phi$$



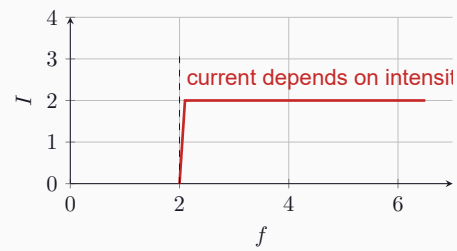
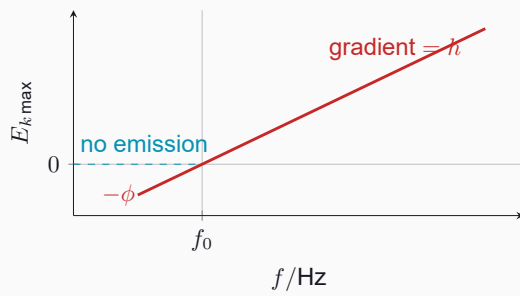
Effect on photoelectric current

The photoelectric current is the rate of emission of electrons from the surface of metal.

- More current: more electrons leave per second.
- Less current: fewer electrons leave per second.

Increasing frequency alone does not necessarily increase current; current mainly depends on photon arrival rate, i.e. intensity.

Variation of current and $E_{k\text{ max}}$ with frequency



Important readings from graph

- Gradient = h .
- y -intercept = $-\phi$.
- x -intercept = f_0 .
- Below f_0 , no photoelectric effect occurs.

Same intensity assumption

When intensity is held fixed, increasing frequency can reduce current because fewer photons are needed to carry the same energy per second.

Increasing intensity at fixed frequency

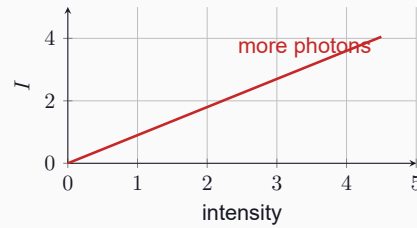
Intensity of electromagnetic radiation

The energy transmitted per unit area per unit time:

$$I = \frac{E}{At}$$

Increasing intensity at fixed frequency means increasing the number of photons incident per second.

- $E_{k \text{ max}}$ stays constant.
- Photoelectric current increases.



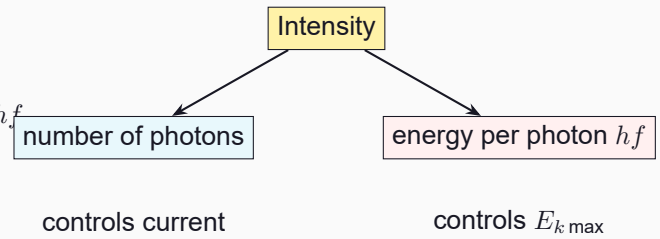
Energy of each photon depends on frequency. Brightness/intensity controls the number of photons when frequency is fixed.

Intensity as number of photons times photon energy

$$\text{Intensity} = \frac{\text{Energy}}{\text{Area} \times \text{time}}$$

Energy = number of photons $\times$ energy of each photon = Nhf

For a particular frequency, increasing intensity means increasing N , not hf .



Consequences

- $E_{k \text{ max}}$ is unchanged.
- Photoelectric current increases.

Same blue light: brighter source

Same blue light means same frequency, hence same photon energy:

$$E = hf = \text{constant}.$$

If the intensity of the same blue light is increased:

- number of photons increases,
- photoelectric current increases,
- maximum kinetic energy stays the same.

same frequency



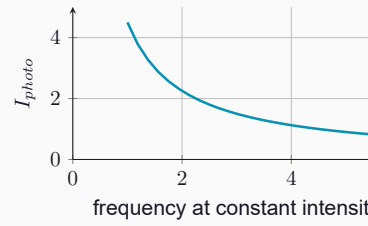
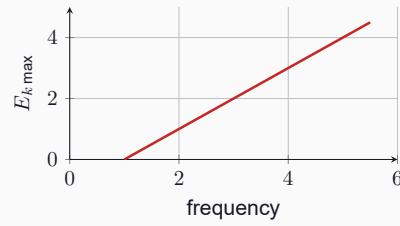
More photons, same energy per photon.

Increasing frequency while keeping intensity constant

At constant intensity

$$I \propto (\text{number of photons}) \times hf.$$

If f increases and I remains constant, the number of photons decreases.



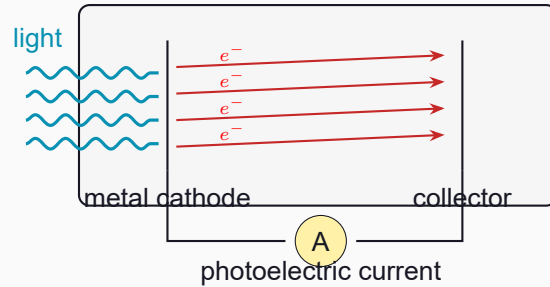
Results

- $E_{k \max}$ increases.
- Photoelectric current decreases.
- Stopping potential increases.

Why photoelectric effect rejects pure wave theory

1. Low-intensity light of high frequency can produce photoelectric effect, whereas high-intensity light of low frequency cannot.
2. Low-frequency light incident for a long time still does not produce emission if $f < f_0$.
3. Wave theory predicts that increasing intensity should increase maximum kinetic energy, but experiments show it increases current instead.
4. There is no time delay between incident light and emission of electrons if $hf > \phi$.

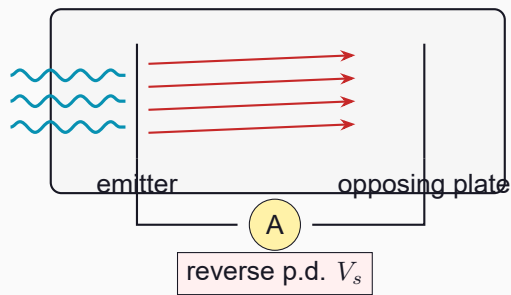
Wave view: energy is continuous.
Particle/photon view: energy arrives in discrete packets.
Photoelectric effect supports the photon model.



Stopping potential

Increase opposite potential difference until current becomes zero.

$$E_{k \max} = eV_s$$



Only electrons with maximum kinetic energy define the stopping potential; slower electrons are stopped earlier.

Meaning

Stopping potential is the minimum opposing potential difference needed to stop the most energetic emitted electrons from reaching the collector.

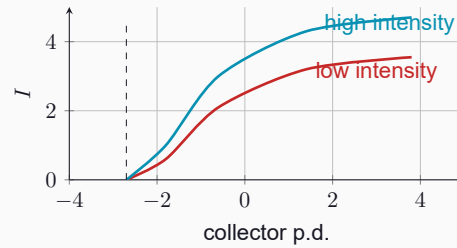
- Electrons are emitted with a range of kinetic energies.
- Increasing the reverse p.d. reduces the current.
- At V_s , the current becomes zero.

Stopping potential: frequency and intensity effects

Increasing frequency

- photon energy increases,
- $E_{k \text{ max}}$ increases,
- more work is needed to stop electrons,
- stopping potential increases.

$$E_{k \text{ max}} = eV_s$$



Increasing intensity

- more electrons are emitted,
- current increases,
- $E_{k \text{ max}}$ is unchanged,
- stopping potential is unchanged.

Work done against stopping potential

Work done against stopping potential equals the loss in kinetic energy of the electron with maximum kinetic energy:

$$V_s q = E_{k \text{ max}}$$

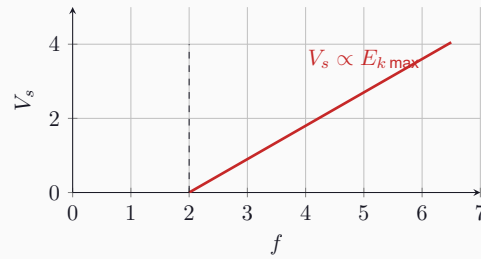
For electron charge $q = e$,

$$E_{k \text{ max}} = eV_s$$

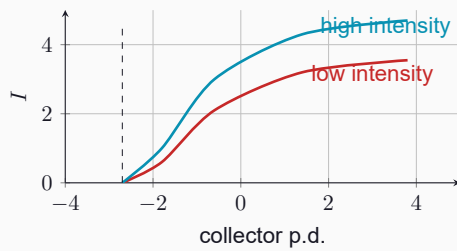
$$\frac{1}{2}mv_{\text{max}}^2 = eV_s$$

Therefore,

$$v_{\text{max}} = \sqrt{\frac{2eV_s}{m}}$$



Current against stopping potential curves



Reading the graph

- High intensity gives a higher saturation current.
- Same frequency gives the same stopping potential.
- Higher frequency shifts the stopping potential to a larger magnitude.

Current reaches zero only when even the fastest photoelectrons are stopped.

Frequency and intensity: final stopping-potential statements

Increasing frequency

Increasing frequency of radiation increases the stopping potential. Electrons now have more kinetic energy, so more work needs to be done to stop them.

$$E_{k\text{ max}} = hf - \phi = eV_s$$

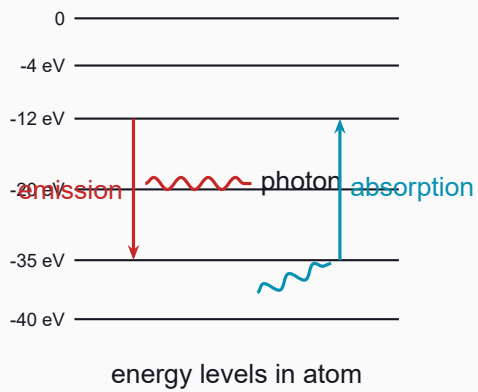
change	$E_{k\text{ max}}$	photoelectric current
increase f	increases	depends on intensity
increase intensity at same f	same	increases
increase intensity and f fixed	same V_s	larger saturation current

Increasing intensity

Increasing intensity of radiation of a fixed frequency only increases the number of photons and therefore more electrons are emitted. The kinetic energy of each electron is still the same because photon energy is unchanged.

$$V_s = \text{unchanged.}$$

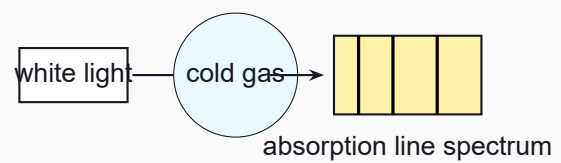
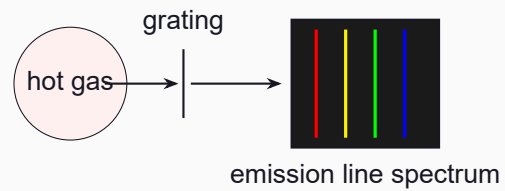
Emission and absorption line spectra



Energy of photon

When an electron changes energy level, a photon is emitted or absorbed:

$$\Delta E = hf = \frac{hc}{\lambda}$$



De Broglie wavelength preview

Every moving particle has a wave associated with it.

$$\lambda = \frac{h}{p}$$

$$p = mv$$

It is inversely proportional to momentum.

For electrons accelerated through V :

$$eV = \frac{1}{2}mv^2 \text{ and } p = \sqrt{2meV}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Interference by electrons

The notes introduce the idea that electrons can behave as waves and produce constructive and destructive interference, giving rings on a fluorescent screen.

thin graphite
rings = constructive interference
empty spaces = destructive interference

fluorescent screen

Energy levels and photons

Hot gas and emission

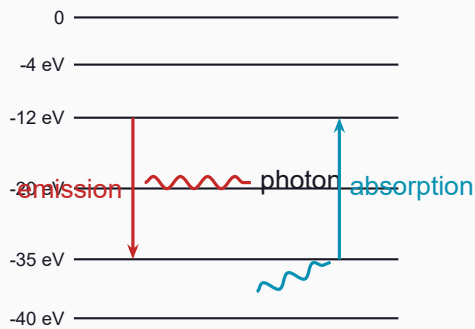
When an electron transits from a higher energy state to a lower energy state, it emits a photon whose energy equals the energy gap.

$$\Delta E = hf$$

Blue/violet photons correspond to larger energy gaps; red photons correspond to smaller energy gaps.

Cold gas and absorption

When an electron transits from a lower energy state to a higher energy state, it absorbs a photon with energy equal



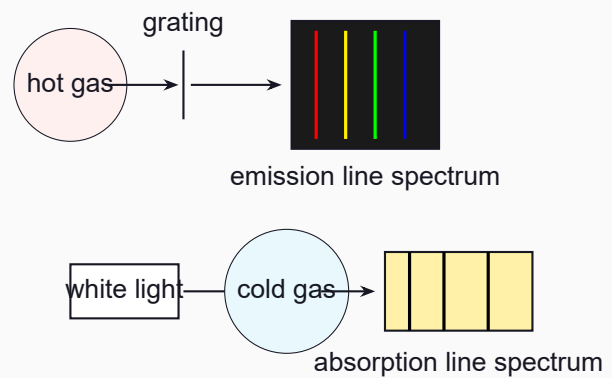
to the gap.

energy levels in atom

Emission line spectra

During the formation of an emission line spectrum, discrete lines of specific colours are observed on a black screen.

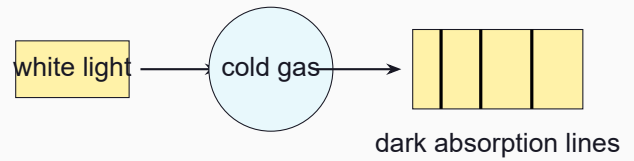
- A cloud of hot gas is used.
- Electrons in the hot gas exist at higher energy states due to high temperature.
- As electrons drop back to more stable lower states, they emit photons of energy equal to the energy gap.
- Hence colours of specific frequencies are observed.



Absorption line spectra

When white light passes through a cloud of cold gas, photons with energy equal to allowed energy gaps are absorbed by electrons.

- Electrons jump from lower to higher energy states.
- Photons of only those specific energies and frequencies are missing from the continuous spectrum.
- Electrons later drop back down and emit photons in random directions, so dark lines remain in the transmitted beam.



Negative energy levels and transition examples

Why energy levels are negative

The negative sign indicates a deficiency of energy in the electron. It tells how much energy must be supplied for the electron to escape the pull of the nucleus.

Transition calculations

$$\Delta E = E_{\text{final}} - E_{\text{initial}}, \quad E_{\gamma} = |\Delta E| = hf = \frac{hc}{\lambda}.$$

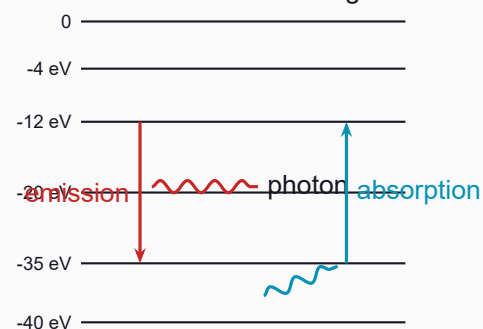
Example style from notes:

$$\Delta E = -0.54 - (-3.40) = 2.86 \text{ eV}$$

$$2.86 \text{ eV} = 2.86(1.6 \times 10^{-19}) \text{ J} = 4.57 \times 10^{-19} \text{ J}.$$

$$\lambda = \frac{hc}{\Delta E}$$

If $\lambda \approx 4.34 \times 10^{-7} \text{ m}$, the line is blue. The notes compare absorption and emission transitions through the same



energy gaps.

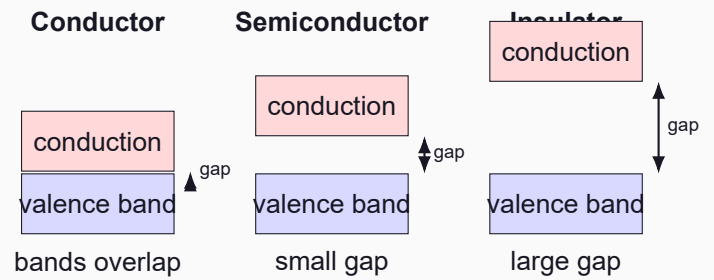
energy levels in atom

Band theory

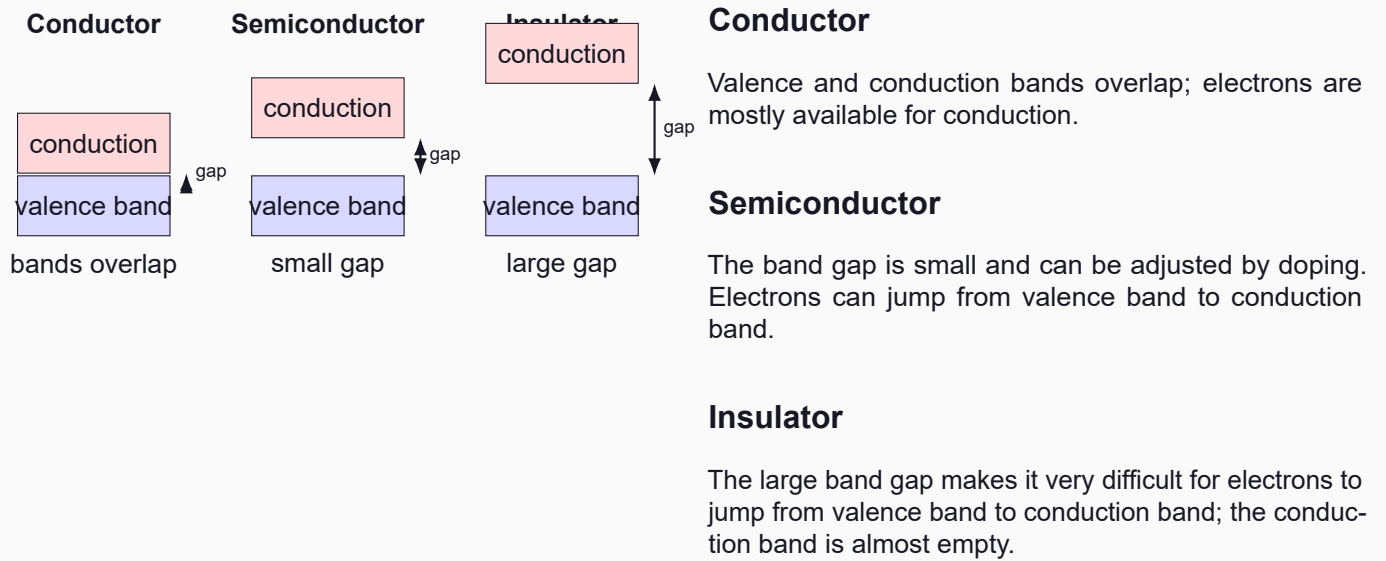
Energy bands in solids and liquids

In gases, atoms are far apart and have distinct energy levels. In solids and liquids, nuclei are close together; neighbouring atoms interfere with each other's energy levels.

- Numerous very close energy levels are produced.
- These levels combine into energy bands.
- Conduction band: electrons here are available for conduction.
- Forbidden band / band gap: region where no electron can exist.
- Valence band: band for valence-shell electrons.



Conductors, semiconductors and insulators



LDR, thermistor and metallic conductors

LDR

- LDR is made from a semiconductor.
- Semiconductors have a small band gap.
- Light provides photons to electrons.
- Electrons gain energy and jump from valence band to conduction band.
- Number density of charge carriers increases, so current increases and resistance decreases.

Thermistor

Increasing temperature gives electrons thermal energy to cross the band gap, increasing number density and decreasing resistance.

Metallic conductor

In metals, valence and conduction bands overlap. The conduction band is already saturated with electrons. When temperature increases, lattice ions vibrate more, causing more collisions with electrons; resistance increases.

Semiconductor: temperature/light increases carriers.

Metal: temperature increases scattering.

De Broglie's wavelength and electron diffraction

Every moving particle has a wave associated with it which is inversely proportional to momentum.

$$\lambda = \frac{h}{p}$$

$$p = mv$$

For electrons accelerated through V :

$$eV = \frac{1}{2}mv^2 \text{ and } p = \sqrt{2meV}$$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

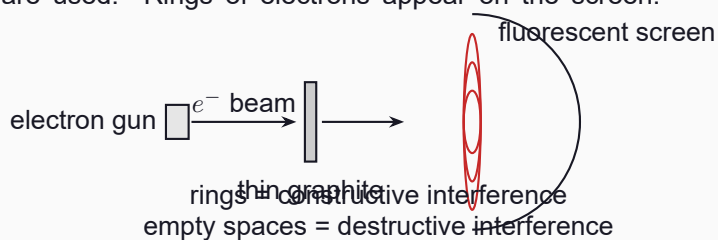
$$\lambda = \frac{h}{p}$$

$$p = mv$$

The energy of a particle can be treated as equivalent to photon energy of the associated wave.

Electron diffraction

A thin sheet of graphite and a fluorescent screen are used. Rings of electrons appear on the screen.



Accelerating electrons and diffraction rings

Electrons are accelerated by an electric field through a potential difference V .

gain in K.E. = work done by electric field

$$\frac{1}{2}mv^2 = eV$$

Since $p = mv$,

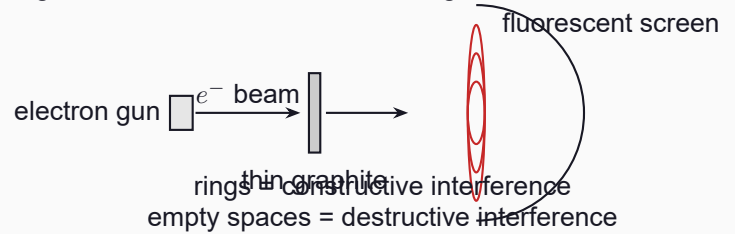
$$E_k = \frac{p^2}{2m} \Rightarrow p = \sqrt{2meV}$$

Therefore,

$$\lambda = \frac{h}{\sqrt{2meV}}$$

Ring spacing

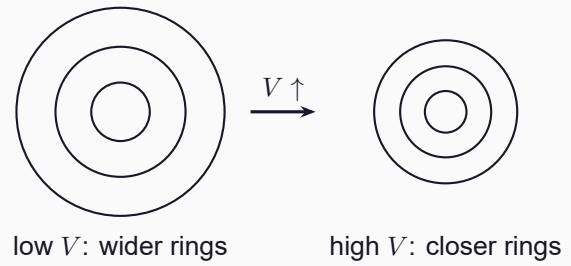
For diffraction, $n\lambda = d \sin \theta$ and $\lambda = h/p$. Larger accelerating voltage gives larger momentum and smaller wavelength, so the diffraction angle becomes smaller and rings come closer.



Increasing accelerating voltage

If the accelerating voltage is increased:

1. Momentum of electrons increases.
2. De Broglie wavelength decreases because $\lambda = h/p$.
3. Wavelength becomes smaller compared with slit/lat-tice spacing, so diffraction is less.
4. The diffraction angle becomes smaller.
5. Ring radius becomes smaller.
6. Rings come closer together and more rings may be observed on the screen.



Blank/checkpoint page

The topic is essentially blank apart from the footer. This rebuilt page keeps the page position and inserts a compact checkpoint summary.

This page is intentionally sparse to match the source sequence and avoid inventing additional notes.

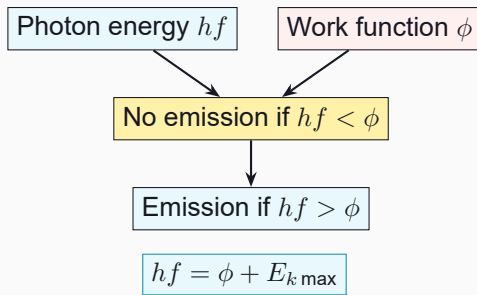
Checkpoint

- Photon model: $E = hf$.
- Photoelectric equation: $hf = \phi + E_{k\text{ max}}$.
- Stopping potential: $E_{k\text{ max}} = eV_s$.
- Energy levels: $\Delta E = hf$.
- De Broglie wavelength: $\lambda = h/p$.
- Accelerated electron: $\lambda = h/\sqrt{2meV}$.

Photoelectric effect summary

Photoelectric chain

photon energy hf compared with work function ϕ

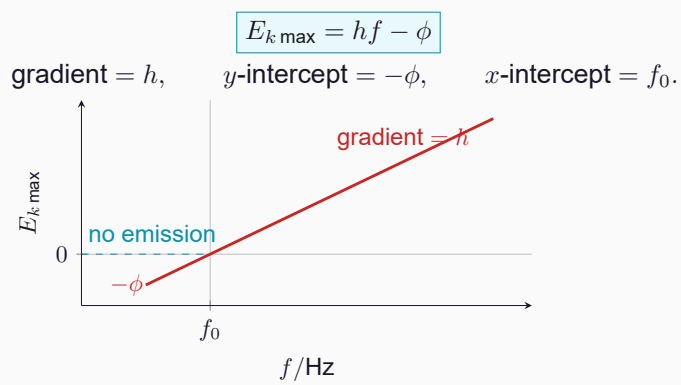


Key effects

- If frequency of radiation increases, $E_{k \max}$ increases.
- If intensity of radiation of the same frequency increases, $E_{k \max}$ stays the same and photoelectric current increases.
- Intensity depends on number of photons and energy of each photon.

$$I = \frac{Nhf}{At}$$

Final $E_{k \text{ max}}$ vs frequency summary



Metal comparison

Different metals have different work functions and threshold frequencies. For all metals, the gradient of an $E_{k \text{ max}}$ vs f graph is Planck's constant.



Medical Imaging

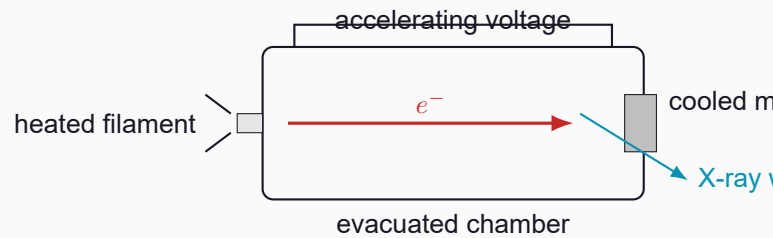
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X-rays: Formation, Image Formation, Sharpness, Contrast and Calculations

Formation of X-rays

- A coil of wire is heated to a very high temperature using a high voltage source.
- The hot coil begins to emit electrons by **thermionic emission**.
- An accelerating voltage speeds up the electrons.
- When the electrons strike the metal target, they lose energy and produce X-ray photons.

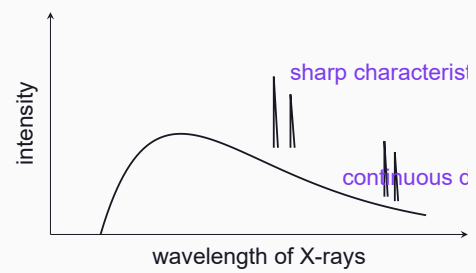


K.E. of electron $\rightarrow$ heat + X-rays

Sharp deceleration of electrons produces X-ray photons.

Characteristic Peaks and Continuous Distribution

- Electrons of the metal anode can de-excite and produce photons of X-rays.
- Some photons are produced in greater quantity because they come from electron transitions in the anode atoms.
- These sharp peaks are characteristic of the metal used.
- The continuous distribution is due to a range of decelerations of the incident electrons, so a range of X-ray photon energies and wavelengths is produced.
- The cut-off wavelength corresponds to a photon produced using the maximum kinetic energy of an electron.



X-ray Intensity, Hardness, Sharpness and Contrast

Intensity

- Intensity depends on the number of incident electrons.
- Therefore it depends on the **heating voltage**.
- Increasing the heating voltage increases the intensity of X-rays.

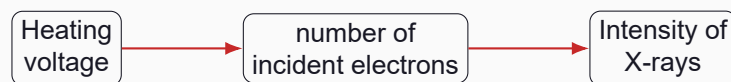
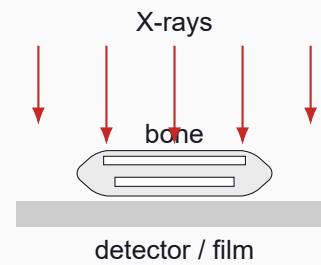


Image Formation



Hardness

- Hardness is the energy of an X-ray photon.
- A hard X-ray has higher photon energy and smaller wavelength.
- A soft X-ray has lower photon energy and longer wavelength, so it is less penetrating.
- Increasing the accelerating voltage increases the kinetic energy of the electrons and hence the hardness of the X-rays.

$$E_{\max} = eV, \quad E = \frac{hc}{\lambda}$$

Sharpness and Contrast

Sharpness of image means a clean boundary of the image. A clear boundary gives a sharp image. It is improved by:

- small anode,
- small aperture,
- lead grid between body and X-ray sheet,
- keeping the body immobile.

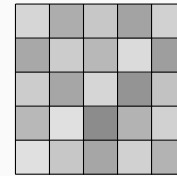
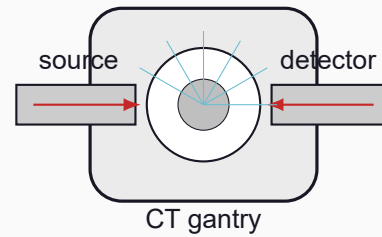
Contrast of image means the difference between black and white regions. It is increased by:

- increasing exposure to X-rays,
- increasing the intensity of X-rays,
- increasing exposure time,
- using a backlit screen.

CT Scans

3D X-ray Imaging

- CT stands for computed tomography.
- It produces a **3D X-ray** image.
- The part of the body to be analysed is divided into smaller parts called **voxels**.
- Each voxel has a particular absorption coefficient for X-rays called a **pixel** in the image slice.
- X-rays are sent from different angles through that part of the body and a 2D X-ray image is produced in the computer.
- The procedure is repeated for different angles and the images formed are joined together to form a 3D image.
- This 3D image can be viewed from different angles and stored.
- The same procedure is followed for all other voxels.



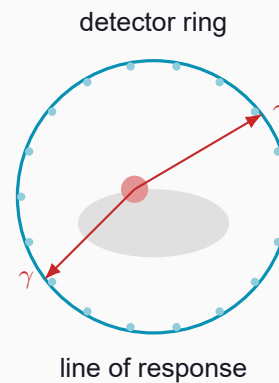
PET: Positron Emission Tomography

Why Use a PET Scan?

- It gives a 3D image.
- It is very precise in locating cancerous cells.
- Brain functioning can be monitored.

What is a PET Scan?

- It is a 3D medical imaging technique.
- An electron and a positron strike one another at high speed.
- They annihilate one another and produce a pair of gamma-ray photons.
- The two photons have equal energies and travel in opposite directions.
- Detectors are used to find the location of tissues to be analysed.



How PET Scan Works

Preparing a Tracer

- Some tissues absorb a specific type of compound or molecule.
- A PET scan uses radiation from those molecules.
- These molecules are made radioactive.
- A cyclotron is used to make tracer molecules.
- It smashes high-speed protons into ordinary atoms, turning loose isotopes into radioactive ones by changing the number of neutrons inside their nucleus.

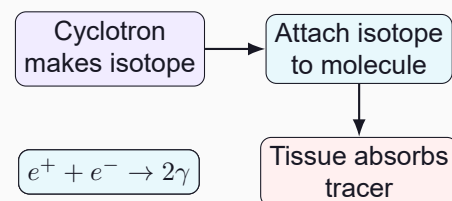
inkpurple

If neutrons < protons: β^+ emission.
If neutrons > protons: β^- emission.

- The isotopes have short half-lives, so the sample is prepared a few hours before the PET scan, e.g. about 2 hours earlier.
- Examples of tracer molecules:
 1. sugar / fluorodeoxyglucose,
 2. protein / RGDfC,
 3. hormone / norepinephrine.
- A radioactive atom is attached to the molecule, making it a tracer.
- When this molecule is absorbed, the location of that part is found.

During a PET Scan

- The tracer is purified from less useful molecules and the amount of radioactivity is checked.
- When the tracer molecule is absorbed by the part of the brain or tissue that is to be analysed, the molecule's radioactive part emits a positron.
- The positron collides with an electron in the surroundings and emits two gamma-ray photons travelling in opposite directions.
- A ring of detectors placed around the patient detects the gamma photons.
- A line of response is mapped by detecting the two photons.
- A small degree of uncertainty exists in detecting the exact location; patients are advised not to move.



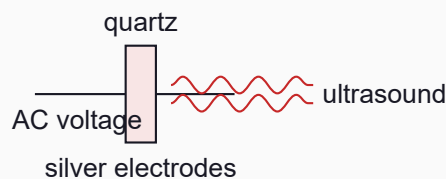
Ultrasound

Definition

Ultrasound: sound waves with $f > 20000 \text{ Hz}$.

Generation

- A piezo-electric quartz crystal is sandwiched between two silver electrodes.
- An alternating voltage is applied across it.
- The crystal is cut into a certain shape and size so that its natural frequency lies within the ultrasonic range.
- Due to the alternating voltage, the crystal resonates and produces an ultrasound wave in air.



Detection

- The same piezo-electric transducer used as the transmitter can act as a receiver.
- The compressions and rarefactions of reflected ultrasound are picked up by the transducer.
- The crystal oscillates and induces opposite charges on the plates.
- The alternating voltage signal is sent to the computer after amplification.
- The time of return and angle of return are used to detect internal structures.

Specific Acoustic Impedance

$$Z = \rho c$$

Z : specific acoustic impedance, ρ : density of medium, c : speed of sound in medium.

At a boundary between media:

$$I = I_R + I_T, \quad \frac{I_R}{I} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2.$$

A large difference in $Z_2 - Z_1$ gives a large reflected intensity.

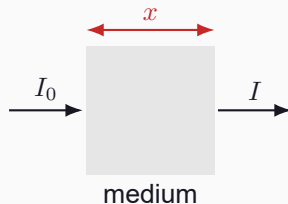
Calculations Regarding X-rays

Exponential Absorption

For X-rays passing through a medium of thickness x :

$$I = I_0 e^{-\mu x}$$

where I_0 is the incident intensity, I is the emerged intensity, μ is the absorption coefficient of the medium, and x is the thickness.



Layered Tissue Calculation

For multiple layers:

$$\frac{I}{I_0} = e^{-\mu_1 x_1} e^{-\mu_2 x_2} e^{-\mu_3 x_3} \dots$$

Model Arm Example

The model arm consists of muscle and bone. The linear absorption coefficients are:

$$\mu_{\text{muscle}} = 0.20 \text{ cm}^{-1}, \quad \mu_{\text{bone}} = 12 \text{ cm}^{-1}.$$

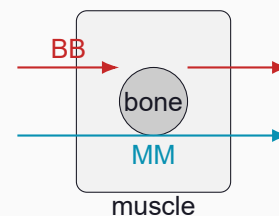
Line MM:

$$\frac{I}{I_0} = e^{-0.20(8)} = 0.20 = 20\%.$$

Line BB:

$$\frac{I}{I_0} = e^{-0.20(2)} e^{-12(4)} e^{-0.20(2)} = e^{-0.8} e^{-48} \approx 6.4 \times 10^{-22}.$$

A large difference between the two ratios gives good contrast; a small difference gives poor contrast.



Calculations for Ultrasound

At the Boundary

If ultrasound enters a layer and is reflected at a boundary:

$$\frac{I_R}{I} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2, \quad \frac{I_T}{I} = 1 - \frac{I_R}{I}.$$

Inside a medium, attenuation is modelled by:

$$I = I_0 e^{-kx}.$$

Fat–Muscle Example

For fat and muscle:

$$Z_{\text{fat}} = 1.3 \times 10^6, \quad Z_{\text{muscle}} = 1.7 \times 10^6.$$

$$\alpha = \frac{I_R}{I} = \left(\frac{1.7 \times 10^6 - 1.3 \times 10^6}{1.7 \times 10^6 + 1.3 \times 10^6} \right)^2 = 0.017.$$

For detected reflected ultrasound $I_S = 0.012I$:

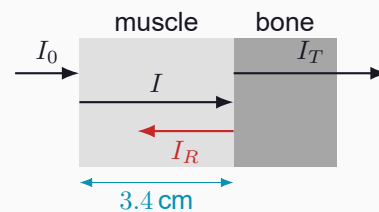
$$0.012 = e^{-48x} \times 0.017 \times e^{-48x} = 0.017e^{-96x}.$$

$$\ln \left(\frac{0.012}{0.017} \right) = -96x, \quad x = 3.6 \times 10^{-3} \text{ m} = 0.36 \text{ cm}.$$

Muscle–Bone Example

Data:

	μ/m^{-1}	$Z/\text{kg m}^{-2} \text{ s}^{-1}$
muscle	23	1.7×10^6
bone	130	6.3×10^6



$$\frac{I}{I_0} = e^{-23(0.034)} = 0.457.$$

$$\frac{I_R}{I} = \left(\frac{6.3 \times 10^6 - 1.7 \times 10^6}{6.3 \times 10^6 + 1.7 \times 10^6} \right)^2 \approx 0.33.$$

$$\frac{I_T}{I} = 1 - 0.33 = 0.67, \quad \frac{I_T}{I_0} \approx 0.457 \times 0.67 \approx 0.31.$$

inkblue

Handwritten notes highlight a final ratio of about 0.21; the algebraic substitution above gives about 0.31 for the single-pass transmitted intensity into bone.

One-page Formula Summary

X-rays

$$E_\gamma = hf = \frac{hc}{\lambda}, \quad E_{\max} = eV, \quad I = I_0 e^{-\mu x}.$$

- Heating voltage controls intensity.
- Accelerating voltage controls hardness.
- Hard X-ray: higher energy, shorter wavelength, more penetrating.
- Soft X-ray: lower energy, longer wavelength, less penetrating.

PET

$$e^+ + e^- \rightarrow 2\gamma$$

- A tracer is absorbed by active tissue.
- A positron annihilates with an electron.
- Two gamma photons are detected in opposite directions.

Ultrasound

$$f > 20000 \text{ Hz}, \quad Z = \rho c.$$

$$\frac{I_R}{I} = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1} \right)^2, \quad \frac{I_T}{I} = 1 - \frac{I_R}{I}.$$

- Piezo-electric crystal converts electrical oscillations to ultrasound.
- Reflected ultrasound produces voltage in the transducer.
- Return time and angle locate internal structures.

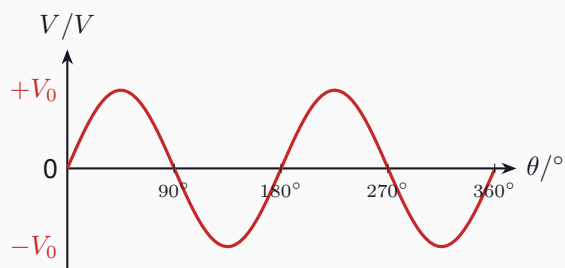
Alternating Currents

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General Equation of Alternating Voltage and Current

General equation



For a sine graph,

$$Y = A \sin(BX) + C$$

A amplitude

B number of cycles in 360°

C y -intercept / mean value

$$Y : V \quad A : V_0 \quad X : \theta$$

$$B : 1 \quad C : 0$$

$$V = V_0 \sin \theta$$

Angular speed:

$$\omega = \frac{\Delta \theta}{\Delta t}$$

hence

$$\theta = \omega t$$

Therefore,

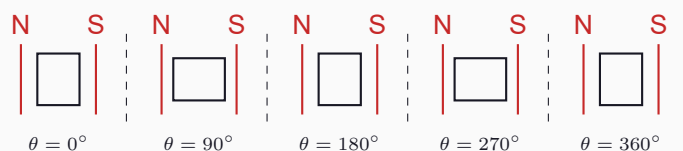
$$V = V_0 \sin \omega t$$

similarly

$$I = I_0 \sin \omega t$$

Generator idea

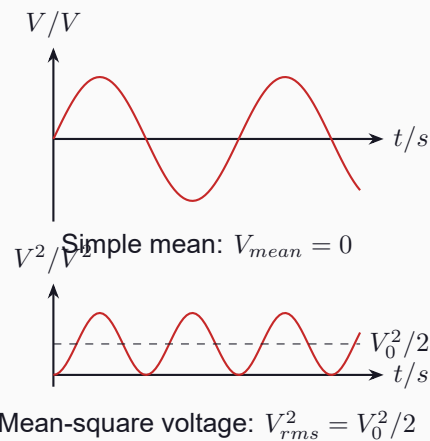
- **Max voltage:** coil crosses horizontal, because rate of change of flux linkage is maximum.
- **Min voltage:** coil crosses vertical, because rate of change of flux linkage is minimum.
- Induced emf is proportional to $N \Delta \Phi / \Delta t$.



The changing orientation of the coil in the magnetic field produces alternating voltage and alternating current.

Root-Mean-Square Voltage and Current

Mean and rms values



For one full cycle of a symmetrical alternating voltage,

$$V_{mean} = 0$$

The square graph is always positive. The mean of the square graph is

$$V_{rms}^2 = \frac{V_0^2}{2}$$

Hence, for a sine wave,

$$V_{rms} = \frac{V_0}{\sqrt{2}}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

Meaning of rms value

- The rms value of alternating voltage/current is the value of direct voltage/current which supplies the same mean power to the same resistor.
- The simple mean value is zero because the positive half cycle cancels the negative half cycle.
- We square first so that both half cycles contribute positive heating effect.

Peak value	rms value
V_0	$V_0/\sqrt{2}$
I_0	$I_0/\sqrt{2}$

Only for a sinusoidal wave form : rms = peak / $\sqrt{2}$.

Power in an A.C. Circuit

What rms tells us

- The rms value of direct voltage/current supplying to an alternating one should give the same mean power output as an alternating voltage/current would.
- 220 V a.c. or 220 V d.c. gives the same mean power output across the same resistor.

$$P = IV$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

For a resistor in an a.c. circuit,

$$P_{\text{mean}} = I_{\text{rms}} V_{\text{rms}} = I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

Since $I = I_0 \sin \omega t$,

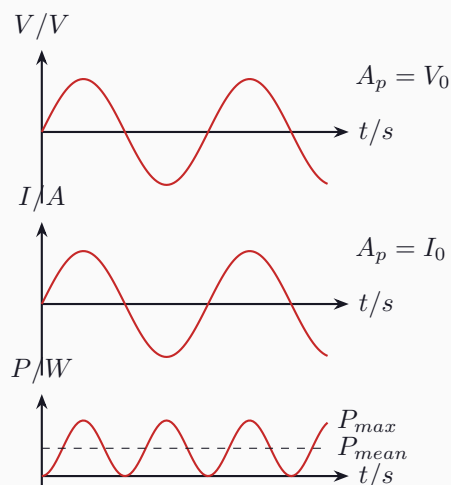
$$P = I^2 R = I_0^2 R \sin^2 \omega t.$$

Thus,

$$P_{\text{max}} = I_0^2 R$$

$$P_{\text{mean}} = \frac{P_{\text{max}}}{2}$$

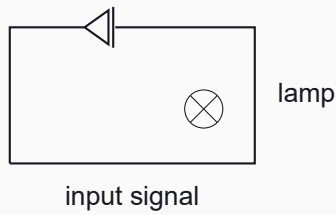
Voltage, current and power graphs



Power is never negative for a resistor because heating depends on I^2 .

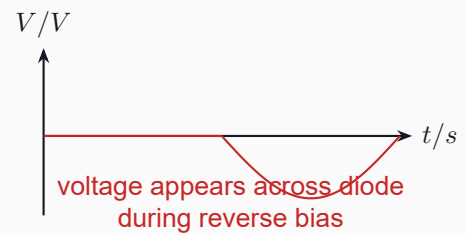
Half-Wave Rectification

Half-wave rectifier



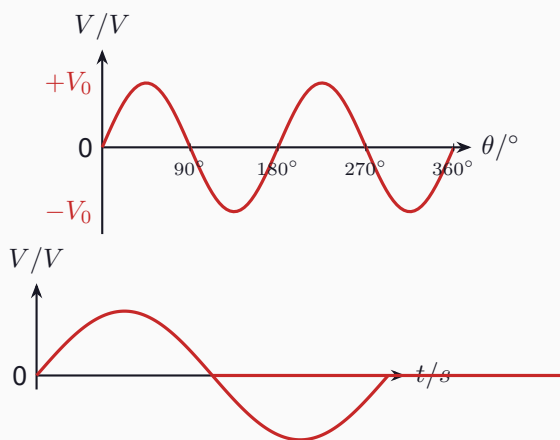
A diode allows current to pass only when it is forward biased.

Forward bias	voltage on load / lamp on
Reverse bias	no voltage on load / lamp off



During the positive half-cycle the diode conducts, so almost all the voltage appears across the load. During the negative half-cycle it blocks, so the load has no output voltage.

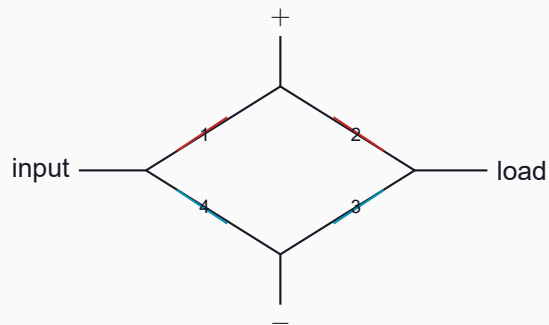
Input and output



The output is **rippling d.c.** because it has one direction but its magnitude changes with time. **Voltage on diode**

Full-Wave Rectification

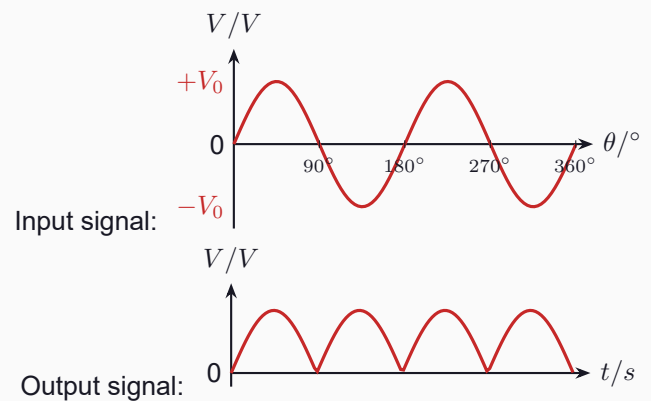
Bridge rectifier



In a full-wave rectifier, both halves of the input signal are used. The negative half cycle is turned into a positive output across the load.

- During the positive half cycle, one pair of diodes conducts.
- During the negative half cycle, the other pair of diodes conducts.
- The current through the load remains in the same direction.

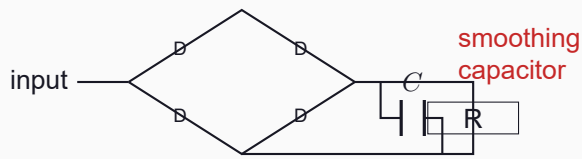
Waveforms



Full-wave rectification gives a larger mean output than half-wave rectification, because both half cycles contribute to the output.

Smoothing with a Capacitor

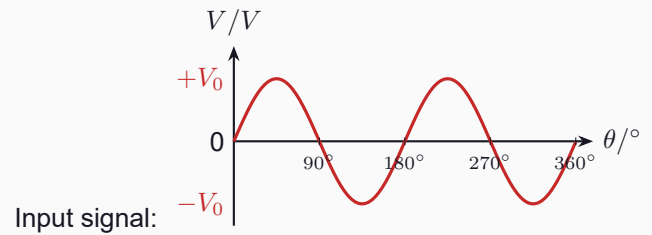
Why a smoothing capacitor is used



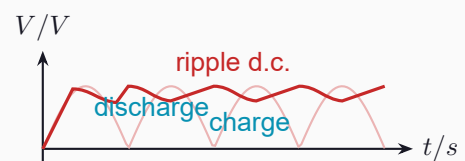
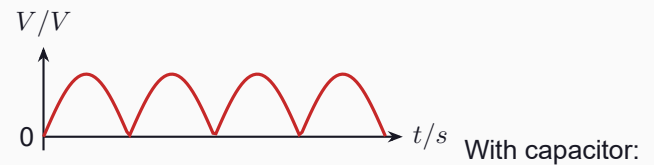
During the drop in output voltage, capacitor maintains current in the resistor.

- The capacitor charges when the rectified voltage rises.
- It discharges through the load when the rectified voltage falls.
- This reduces the ripple and increases the mean output voltage.

Ripple d.c. and smoothed output



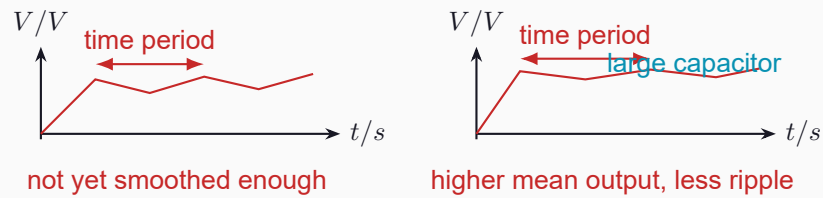
Output signal before smoothing:



Time Period and Effect of Capacitance

Effect of adding capacitance

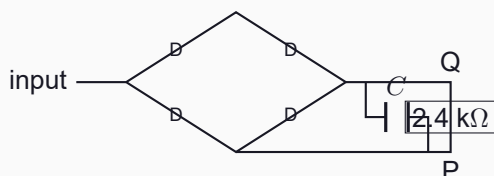
- Adding capacitance increases the mean output voltage.
- It increases the time taken for the output to fall, so the ripple becomes smaller.
- The time period of a rectified full-wave signal is smaller than the time period of the original a.c. signal because both half cycles appear as positive pulses.



Large capacitor $\Rightarrow$ slower discharge $\Rightarrow$ smaller ripple $\Rightarrow$ smoother d.c. output.

Exam-Style Rectifier Question

Question setup



A bridge rectifier contains four ideal diodes and a capacitor. The output of the rectifier is connected to a load of resistance 2.4 k.

1. Calculate the mean-square current in the load.
2. Explain how the potential difference across the load is smoothed.

Calculation Using the rms output voltage from the rectified sinusoidal input,

$$I_{rms} = \frac{V_{rms}}{R}.$$

For the values shown in the note,

$$I_{rms} = \frac{8.5}{2400} = 3.5 \times 10^{-3} \text{ A}.$$

Therefore,

$$I_{rms}^2 = (3.5 \times 10^{-3})^2 = 1.3 \times 10^{-5} \text{ A}^2$$

Smoothing explanation

- The capacitor is connected in parallel with the load.
- It charges when the rectified output is increasing.
- It discharges through the load when the rectified output is decreasing.
- This keeps the p.d. across the load nearly constant.

Summary: A.C., Rectification and Smoothing

Core equations

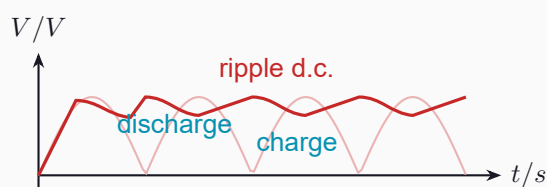
Quantity	Equation
Alternating voltage	$V = V_0 \sin \omega t$
Alternating current	$I = I_0 \sin \omega t$
Angular displacement	$\theta = \omega t$
RMS voltage	$V_{rms} = V_0 / \sqrt{2}$
RMS current	$I_{rms} = I_0 / \sqrt{2}$
Mean power	$P_{mean} = I_{rms} V_{rms}$
Resistor power	$P_{mean} = I_{rms}^2 R = V_{rms}^2 / R$

Rectification

- Half-wave rectification uses one half cycle only.
- Full-wave rectification uses both half cycles.
- A diode conducts in forward bias and blocks in reverse bias.

Smoothing

- The capacitor is placed parallel to the load.
- It charges to the peak of the rectified output.
- It discharges when the rectified voltage drops.
- Larger capacitance gives smaller ripple and greater mean output p.d.



Astronomy and Cosmology

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Basic Astronomy Vocabulary

Galaxy

A huge collection of dust, gases and stars bounded by gravitational force.

- Spiral galaxy
- Barred spiral
- Elliptical galaxy
- Lenticular galaxy
- Irregular galaxy
- Peculiar galaxy

Star

A huge cloud of gas, mostly hydrogen and helium, that produces light and heat using fusion reactions.

Meteoroid

A small rock or stone moving in space. When it enters Earth's atmosphere, it becomes a meteor.

Comet

A collection of frozen gas, dust, rock and ice. Due to heat, a gas and dust trail is left behind forming a tail.

Asteroid

A large-size stone that floats in space. Meteoroids are small stones broken out of asteroids. They can

have an orbit around a star.

Black hole

A place in space where gravity is so strong that even light cannot escape. It is produced when a star dies.

Planet

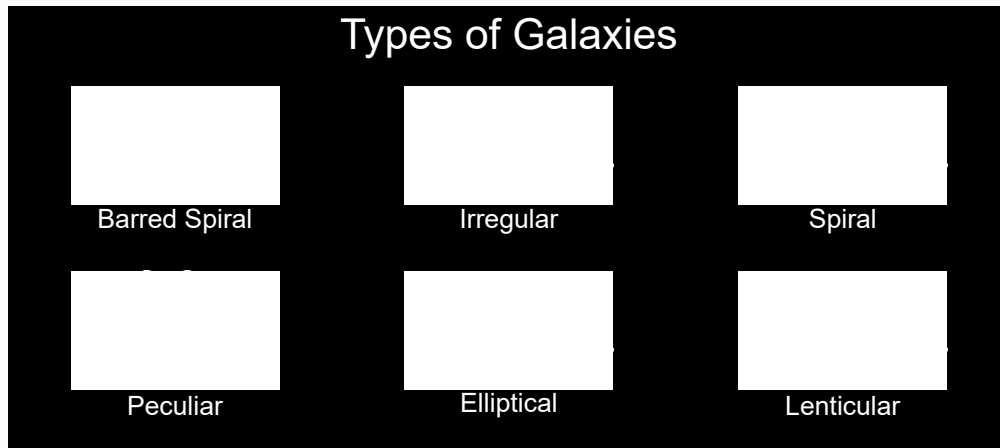
A celestial body that:

1. orbits a star,
2. is spherical due to sufficient gravity,
3. has no debris around it; it has a clear orbit.

inkgreen

Pluto is a dwarf planet because it orbits along with other masses in the Kuiper belt.

Types of Galaxies



Luminosity and Radiant Flux Intensity

Stellar and interstellar

- **Stellar:** related to stars.
- **Interstellar:** between stars.

Luminosity

The word luminous means glowing or emitting energy.

inkred

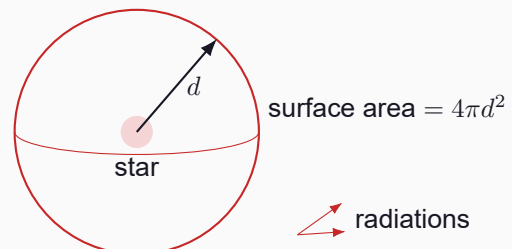
Luminosity is the total power of the radiation emitted by the star.

- The radiation includes light and all electromagnetic radiations.
- Luminosity is affected by surface area of the radiant body and the surface temperature.
- Sun's luminosity: approximately 4×10^{26} W.
- Proxima Centauri: approximately 4×10^{23} W.

Radiant flux intensity

Radiant flux intensity is the luminosity incident per unit surface area normal to that surface.

$$I = \frac{L}{S.A.} = \frac{L}{4\pi d^2}$$



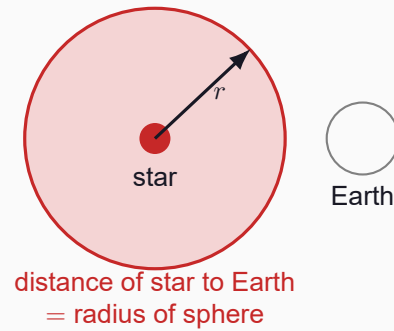
Where L is luminosity in W and d is the distance from the centre of the star to the surface.

Standard Candles

A body of known luminosity is called a **standard candle**. Luminosity is not just for light but for any radiation, for example heat.

inkgreen

Common standard candles: Type Ia supernovae and Cepheid variables.



Worked example: luminosity of the Sun

The radiant flux intensity of solar radiation at Earth's surface is about 1400 W.m^{-2} . The mean Earth-Sun distance is $1 \text{ AU} = 1.50 \times 10^{11} \text{ m}$. Estimate the luminosity of the Sun.

$$I = 1400 \text{ W.m}^{-2},$$

$$I = \frac{L}{4\pi r^2},$$

$$1400 = \frac{L}{4\pi(1.5 \times 10^{11})^2},$$

$$L \approx 3.96 \times 10^{26} \text{ W}.$$

Black Body Radiation

$$\alpha + \rho + \tau = 1$$

Symbol	Meaning
α	absorbed fraction
ρ	reflected fraction
τ	transmitted fraction

- A body appears transparent because $\tau = 1$, $\alpha = 0$, $\rho = 0$. All radiation can pass through.
- A body is opaque because $\tau = 0$ and $\alpha + \rho = 1$. Some radiation gets absorbed and some gets reflected.
- Coloured opaque objects absorb all radiations and reflect only the colour which appears to be the colour of the object. Example: a red box appears red because all colours are absorbed and red is reflected.
- A black object appears black because all radiations are absorbed and no radiation is reflected back.

Definition and properties of a black body

A black body is therefore considered a perfect absorber of all radiations and does not reflect any radiation.

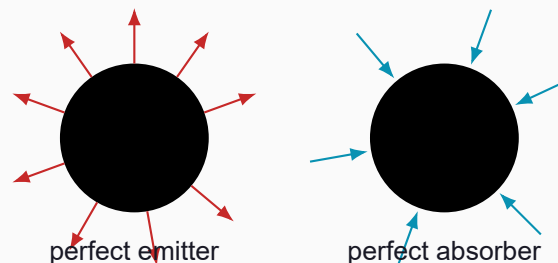
1. It completely absorbs incident radiation irrespective of wavelength. A black body is black because it does not reflect any radiation.
2. It is a perfect emitter. No other surface can emit more radiation than a black body at the same temperature.
3. It emits in all possible directions. A black body is a perfectly diffused emitter.

A black body acts either as a perfect absorber or a perfect emitter at a specific time. It does not act as both at the same time.

inkblue

A black body is basically a reference surface with which real surfaces are compared to determine how good an absorber or emitter that object is.

- Black objects are treated as black bodies.
- A tie is a black body for infrared radiations as a perfect absorber.
- A star is also a black body for infrared radiations as a perfect emitter.
- Stars are close to black-body objects.

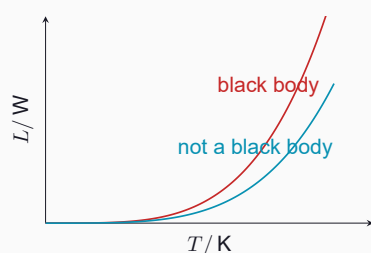


Stefan-Boltzmann Law

Luminosity of a black body at temperature T is directly proportional to its surface area and to T^4 .

$$L = \sigma AT^4$$

Quantity	Meaning
σ	Stefan-Boltzmann constant, $5.67 \times 10^{-8} \text{ W.m}^{-2}.\text{K}^{-4}$
A	surface area, $A = 4\pi r^2$
T	absolute temperature in K
L	luminosity in W



- The equation is applied considering surrounding temperature as 0 K.
- If surrounding temperature is higher, luminosity decreases.
- If two stars have the same luminosity, $L \propto A/T^4$; a hotter star must have a smaller radius and vice versa.

$$L = \sigma A (T^4 - T_s^4)$$

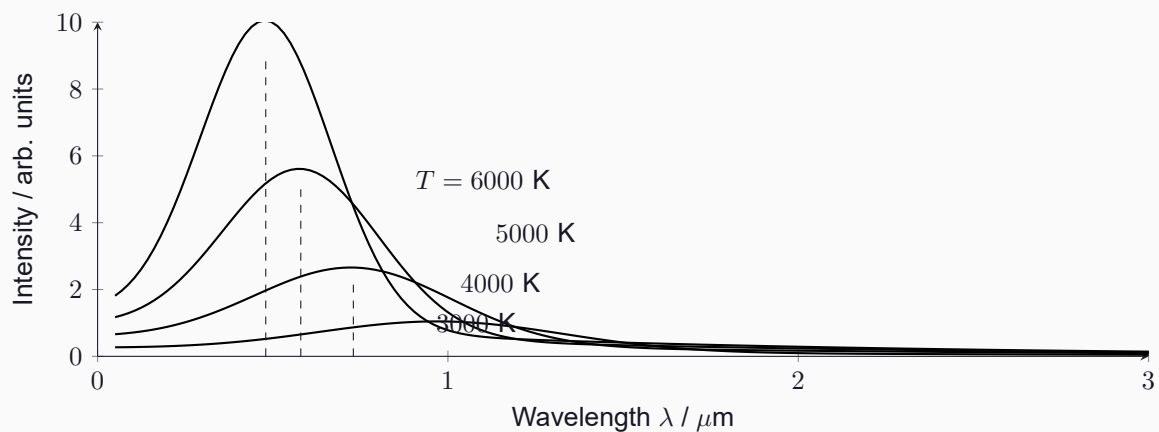
if the surroundings are at temperature T_s , not zero kelvin.

Wien's Displacement Law

The wavelength at peak intensity is inversely proportional to thermodynamic temperature.

$$\lambda_{\max} \propto \frac{1}{T}, \quad \lambda_{\max} = \frac{b}{T}, \quad \lambda_{\max} T = 2.9 \times 10^{-3} \text{ m.K}$$

- T is surface temperature in kelvin.
- $\lambda_{\max}$ is the wavelength having maximum intensity.
- At higher temperature, photon energy is higher, so intensity and the number of photons are higher.
- Higher temperature produces shorter wavelength.
- Colour of light from a hot object moves from red to blue and then to white.
- Brightness of light increases due to increasing intensity.



Red Shift and Hubble's Law

Red shift

When stars or galaxies move away from the observer, the observed frequency of light coming from it decreases. Hence a red shift is seen in its colour. The shift in frequency or wavelength is directly proportional to speed of recession.

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}, \quad \frac{\Delta f}{f} = \frac{v}{c}$$

This formula is applicable when v is much smaller than c . It is used to calculate the speed of the star or galaxy.

Hubble's Law

The speed of recession v of a star is directly proportional to the distance d of the star from Earth.

$$v = Hd$$

where:

- v is speed of recession,
- H is Hubble's constant,
- d is distance between source and observer at that instant.

$$H = 2.2 \times 10^{-18} \text{ s}^{-1}$$

inkblue

This idea of galaxies receding proves the Big Bang Theory: the universe is expanding. The presence of cosmic microwave background radiation also supports the Big Bang Theory.



Age of the Universe

How old is the universe?

$$\begin{aligned} v &= Hd, \\ c &= \frac{d}{T} \quad \text{for the fastest galaxy moving at } v = c, \\ c &= Hd, \\ T &= \frac{1}{H}. \end{aligned}$$

Using $H = 2.2 \times 10^{-18} \text{ s}^{-1}$,

$$T = \frac{1}{2.2 \times 10^{-18}} = 4.55 \times 10^{17} \text{ s}.$$

Converting into years,

$$T = \frac{4.55 \times 10^{17}}{365 \times 24 \times 3600} = 1.44 \times 10^{10} \text{ years}.$$

Worked Examples: Stefan-Boltzmann and Wien's Law

Example 1: luminosity and radius of a star

Given $\sigma = 5.67 \times 10^{-8} \text{ W.m}^{-2}.\text{K}^{-4}$, a star has luminosity $6.0 \times 10^{28} \text{ W}$ and surface temperature 3400 K.

(a) Show that its surface area is about $7.9 \times 10^{21} \text{ m}^2$.

$$L = \sigma AT^4$$

$$6.0 \times 10^{28} = 5.67 \times 10^{-8} \times A \times (3400)^4$$

$$A \approx 7.8 \times 10^{21} \text{ m}^2.$$

(b) Calculate its radius and the ratio of its radius to the radius of the Sun, using $r_{\text{Sun}} = 7.0 \times 10^8 \text{ m}$.

$$A = 4\pi r^2 \Rightarrow 7.8 \times 10^{21} = 4\pi r^2$$

$$r \approx 2.5 \times 10^{10} \text{ m}$$

$$\frac{r_{\text{star}}}{r_{\text{Sun}}} = \frac{2.5 \times 10^{10}}{7.0 \times 10^8} \approx 35.7.$$

Example 2: peak wavelength

If a star has peak intensity at wavelength 620 nm,

$$\lambda_{\text{max}}T = 2.9 \times 10^{-3}$$

$$T = \frac{2.9 \times 10^{-3}}{620 \times 10^{-9}} \approx 4677 \text{ K}.$$

Example 3: larger diameter

If a star has twice the Sun's surface temperature and its diameter is four times the Sun's diameter:

$$L \propto AT^4.$$

Because diameter is four times, area is $4^2 = 16$ times. Because temperature is twice, T^4 is $2^4 = 16$ times. Therefore:

$$L_{\text{star}} = 16 \times 16 L_{\text{Sun}} = 256 L_{\text{Sun}}.$$

Worked Examples: Red Shift and Hubble's Law

Example 1: shifted spectral line

A spectral line of a star is displaced from its laboratory value of 434 nm by +0.087 nm. Determine whether it is moving towards or away from Earth and find the speed.

$$\Delta\lambda = +0.087 \text{ nm} \Rightarrow \text{red shift, so the star recedes.}$$

$$\begin{aligned}\frac{\Delta\lambda}{\lambda} &= \frac{v}{c} \\ \frac{0.087}{434} &= \frac{v}{3.0 \times 10^8} \\ v &\approx 6.0 \times 10^4 \text{ m.s}^{-1}.\end{aligned}$$

Example 2: recession speed and distance of a galaxy

The same spectral line is measured at 398.6 nm in a distant galaxy and 393.3 nm in the laboratory. Given $H = 2.2 \times 10^{-18} \text{ s}^{-1}$:

$$\begin{aligned}\Delta\lambda &= 398.6 - 393.3 = 5.3 \text{ nm} \\ \frac{5.3}{393.3} &= \frac{v}{3.0 \times 10^8} \\ v &\approx 4.04 \times 10^6 \text{ m.s}^{-1}.\end{aligned}$$

Using Hubble's law:

$$\begin{aligned}v &= Hd \Rightarrow d = \frac{v}{H} \\ d &= \frac{4.04 \times 10^6}{2.2 \times 10^{-18}} \approx 1.8 \times 10^{24} \text{ m}.\end{aligned}$$

Example 3: light year

$$\begin{aligned}1 \text{ light year} &= c \times (365 \times 24 \times 3600) \\ &= 3.0 \times 10^8 \times 365 \times 24 \times 3600 = 9.46 \times 10^{15} \text{ m} \\ \frac{1.76 \times 10^{24}}{9.46 \times 10^{15}} &\approx 1.86 \times 10^8 \text{ light years}.\end{aligned}$$

inkgreen

Cosmic microwave background radiation is the relic radiation left from the early hot universe and is taken as evidence for the Big Bang model.

Electromagnetic Induction

Chapter 16 / A2 Level / Apollo Physics 9702

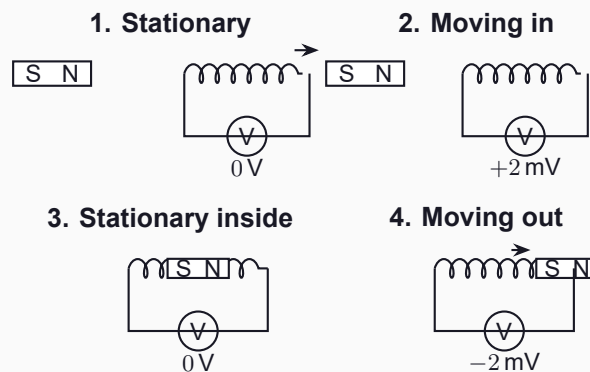


Core idea

In the DC motor, a force was produced using **current and magnetic field**. In electromagnetic induction, the reverse question is asked: can **current/voltage** be produced using **motion/force and magnetic field**?

An induced e.m.f. appears when the magnetic flux linkage through a circuit changes.

Case	Situation	Voltmeter reading
1	Magnet stationary outside the coil	0
2	Magnet moving towards the coil	positive pulse
3	Magnet stationary inside the coil	0
4	Magnet being pulled out of the coil	negative pulse
5	Increasing number of turns of coil	larger induced e.m.f.
6	Increasing strength of magnet	larger induced e.m.f.
7	Increasing speed of motion	larger induced e.m.f.
8	Field lines interacting with coil	magnetic flux linkage

**Important pattern**

No induced e.m.f. is produced merely by the presence of a magnet. The e.m.f. is produced only when the magnetic flux linkage changes.

Magnetic flux linkage, Faraday's law and Lenz's law

From cases 1 to 4

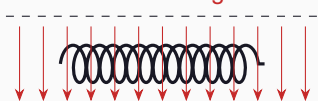
To induce e.m.f. in a conductor, a **change in the interaction of magnetic field lines with the coil** is needed.

$$\mathcal{E} \propto N, \quad \mathcal{E} \propto \Delta\Phi, \quad \mathcal{E} \propto \frac{\Delta\Phi}{\Delta t}$$

Therefore

$$\mathcal{E} \propto \frac{N\Delta\Phi}{\Delta t}$$

field lines interacting with coil



magnetic flux linkage

Magnetic flux linkage

Magnetic flux linkage means magnetic field lines interacting with the coil.

$$\Phi = BA \quad (\text{when field is normal to the area}), \quad N\Phi = NBA$$

where N is the number of turns.

Faraday's law of electromagnetic induction

The e.m.f. induced in a conductor is directly proportional to the rate of change of magnetic flux linkage.

$$\mathcal{E} = \frac{\Delta(N\Phi)}{\Delta t}$$

For calculus form:

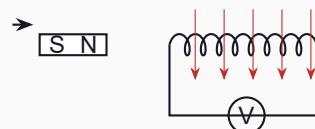
$$\mathcal{E} = \frac{d(N\Phi)}{dt}$$

Lenz's law

The induced e.m.f. is in such a direction that it **opposes the change which produces it**.

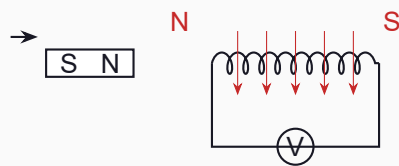
$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

The negative sign gives the opposition predicted by Lenz's law.

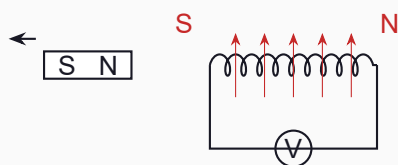


Approaching magnet causes a change in magnetic flux linkage

Lenz's law and conservation of energy



Induced current creates a field that opposes the approach.



When the magnet withdraws, the induced field opposes withdrawal.

Why this agrees with conservation of energy

Lenz's law also relates to the law of conservation of energy:

Energy is neither created nor destroyed; it is converted from one form to another.

For electrical energy to be produced, the magnet must lose kinetic energy. The opposition faced by the magnet achieves this goal. Hence work must be done continuously to keep the coil moving relative to the magnet.

Physical meaning of the minus sign

$$\mathcal{E} = - \frac{d(N\Phi)}{dt}$$

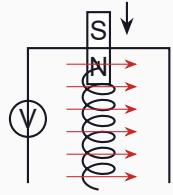
The sign does not make the e.m.f. "negative energy". It tells the direction of the induced e.m.f. and current: the induced current makes a magnetic field that resists the change in flux linkage.

No free energy

If the induced current helped the change, the magnet would accelerate and produce more electrical energy without any external work. Lenz's law prevents this by making the induced effect oppose the cause.

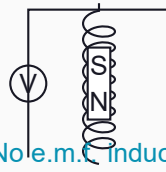
Dropping a small magnet through a long coil

Magnet enters



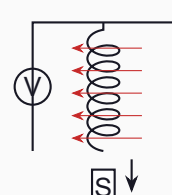
Flux linkage increases;
hence e.m.f. is induced.

Inside coil



No e.m.f. induced.
Equal interaction of field with
both sides gives no change in
magnetic flux linkage.

Magnet exits

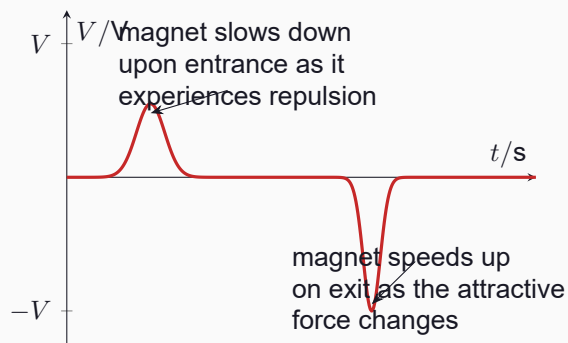


Flux linkage decreases;
e.m.f. is induced again,
with opposite sign.

Lenz force during the drop

As the magnet enters the coil, it experiences repulsion: the magnet slows down upon entrance. Repulsion increases as it approaches closer. As the magnet exits the coil, it experiences attraction in the opposite direction: the magnet may speed up, but the attractive force decreases as its distance from the coil increases.

Induced e.m.f. graph for a falling magnet



Interpreting the graph

- A pulse is produced on entry because flux linkage changes.
- No e.m.f. is produced while flux linkage is not changing appreciably.
- A pulse of the opposite sign is produced on exit because the direction of change of flux linkage reverses.
- The narrower/larger pulse can correspond to faster motion and a greater rate of change of flux linkage.

Rate matters

$$|\mathcal{E}| = \left| \frac{\Delta(N\Phi)}{\Delta t} \right|$$

Greater speed, stronger magnet, or more turns means a larger induced e.m.f. because the rate of change of flux linkage is larger.

Exam-style example: oscillating magnet and coil

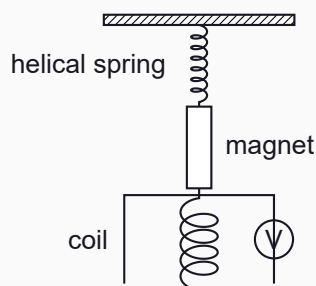


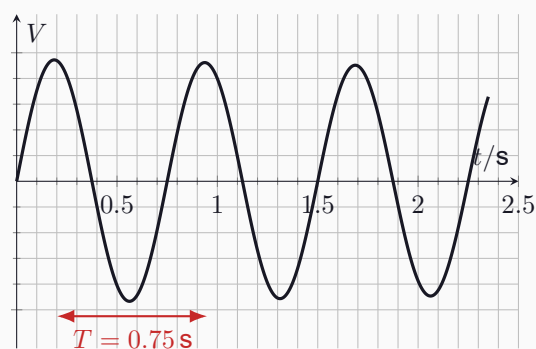
Fig. 5.1

A bar magnet is suspended vertically from the free end of a helical spring. One pole of the magnet is situated in a coil. The coil is connected in series with a high-resistance voltmeter. The magnet is displaced vertically and then released.

(a) Faraday's law

The e.m.f. induced in the conductor is directly proportional to the rate of change of magnetic flux linkage:

$$\mathcal{E} \propto \frac{\Delta(N\Phi)}{\Delta t}.$$



From the graph

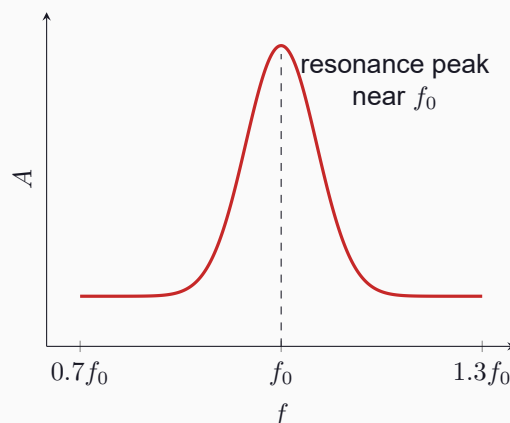
The voltmeter reading alternates between positive and negative values because the direction of motion of the magnet is reversed after every half oscillation.

$$T = 0.75 \text{ s}, \quad f_0 = \frac{1}{T} = \frac{1}{0.75} = 1.33 \text{ Hz}.$$

Resonance of the magnet-coil system

Why the graph has positive and negative values

As the magnet oscillates, the direction of motion changes after every half oscillation. This reverses the direction of change of magnetic flux linkage through the coil. By Lenz's law, the induced e.m.f. also reverses sign.



Frequency calculation

$$T = 0.75 \text{ s}$$
$$f_0 = \frac{1}{T} = \frac{1}{0.75} = 1.33 \text{ Hz.}$$

Sketch requirement

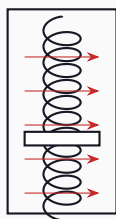
When an alternating supply is connected to the coil, maximum amplitude occurs when the driving frequency equals the natural frequency of the magnet-spring system. Thus the amplitude-frequency curve peaks at $f = f_0$.

Magnetic flux density and flux linkage in a solenoid

Definition of the tesla

One tesla is the magnetic flux density when a wire of length 1 m, carrying a current of 1 A normal to the magnetic field, experiences a force of 1 N.

$$B = \frac{F}{IL}$$



solenoid
area = 28 cm^2
coil C, 160 turns

For a long solenoid,

$$B = \mu_0 n I.$$

Given $n = 1.5 \times 10^3 \text{ m}^{-1}$, $I = 3.5 \text{ A}$, and $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$,

$$B = (4\pi \times 10^{-7})(1.5 \times 10^3)(3.5) = 6.6 \times 10^{-3} \text{ T}.$$

Flux linkage

Area:

$$A = 28 \text{ cm}^2 = 28 \times 10^{-4} = 2.8 \times 10^{-3} \text{ m}^2.$$

$$N\Phi = NBA = 160(6.6 \times 10^{-3})(2.8 \times 10^{-3})$$

$$N\Phi \approx 2.96 \times 10^{-3} \text{ Wb}.$$

Average e.m.f. when current is reversed

Faraday's law

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}.$$

The magnitude of the average induced e.m.f. is

$$|\mathcal{E}| = \left| \frac{(N\Phi)_2 - (N\Phi)_1}{\Delta t} \right|.$$

Worked example

If the flux linkage changes from

$$+2.6 \times 10^{-3} \text{ Wb}$$

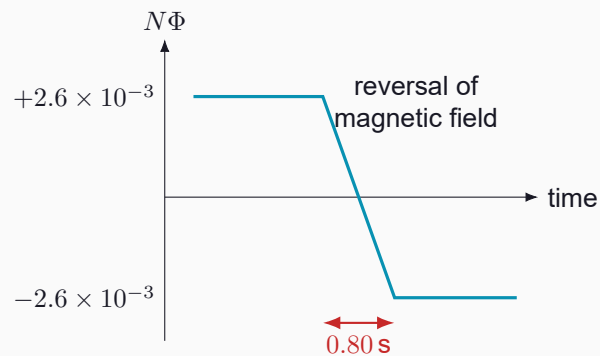
to

$$-2.6 \times 10^{-3} \text{ Wb}$$

in 0.80 s, then

$$\Delta(N\Phi) = -2.6 \times 10^{-3} - (+2.6 \times 10^{-3}) = -5.2 \times 10^{-3} \text{ Wb}.$$

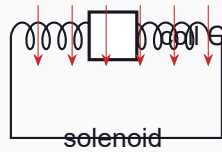
$$|\mathcal{E}| = \frac{5.2 \times 10^{-3}}{0.80} = 6.5 \times 10^{-3} \text{ V}.$$



Sign convention

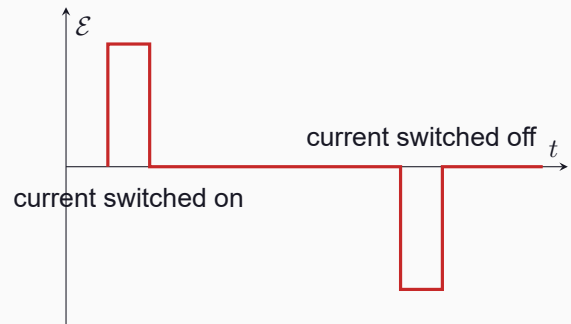
The sign of the induced e.m.f. depends on the chosen direction of the coil and flux. Exam calculations often ask for the magnitude unless a sign is explicitly required.

Switching current on and off: induced e.m.f. pulse



What happens

When current in the solenoid is switched on, magnetic flux linkage through coil C changes from zero to a value, so an e.m.f. pulse is induced. When the current becomes steady, $\Delta(N\Phi)/\Delta t = 0$, so no e.m.f. is induced. When switched off, the flux linkage decreases to zero and an opposite pulse is produced.

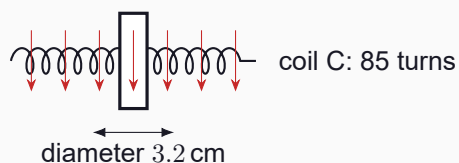


Graph rule

$$\mathcal{E} \propto -\frac{\Delta I}{\Delta t}$$

for a fixed solenoid-coil geometry, because $B \propto I$ and $N\Phi \propto B$.

Solenoid question: flux linkage and mean e.m.f.



Show flux linkage

Diameter $d = 3.2 \text{ cm} = 3.2 \times 10^{-2} \text{ m}$, so

$$A = \pi \left(\frac{d}{2} \right)^2 = \pi (1.6 \times 10^{-2})^2.$$

If

$$B = 3.1 \times 10^{-3} I,$$

then for $I = 2.8 \text{ A}$,

$$B = 8.68 \times 10^{-3} \text{ T}.$$

$$N\Phi = NBA = 85(8.68 \times 10^{-3})\pi(1.6 \times 10^{-2})^2 \approx 6.0 \times 10^{-4} \text{ Wb}.$$

Mean e.m.f. when current is reversed

If current is reversed, flux linkage changes from

$$+6.0 \times 10^{-4} \text{ Wb} \quad \text{to} \quad -6.0 \times 10^{-4} \text{ Wb}.$$

$$\Delta(N\Phi) = 1.2 \times 10^{-3} \text{ Wb}.$$

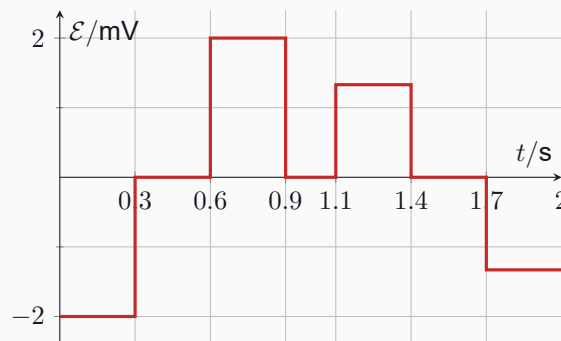
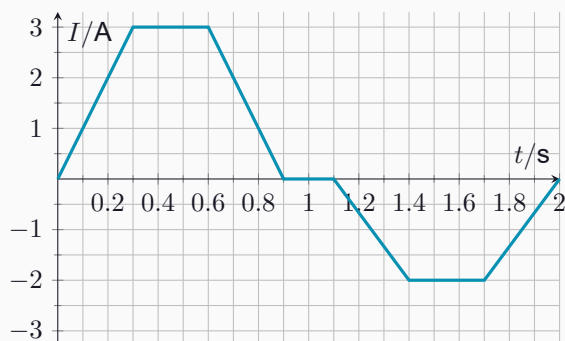
In 0.30 s,

$$|\mathcal{E}| = \frac{1.2 \times 10^{-3}}{0.30} = 4.0 \times 10^{-3} \text{ V} = \boxed{4.0 \text{ mV}}.$$

Reasoning shortcut

Because $N\Phi \propto I$, the sign of the flux linkage reverses when the current is reversed. The change is therefore twice the initial magnitude.

Induced e.m.f. from a changing current waveform



How to sketch $\mathcal{E}$

$$\mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

Since $N\Phi \propto I$,

$$\mathcal{E} \propto -\frac{\Delta I}{\Delta t}.$$

Thus the induced e.m.f. is constant wherever the current graph has a constant gradient, zero wherever the current is constant, and changes sign when the gradient changes sign.

Signs

If ΔI is positive, $\mathcal{E}$ is negative. If ΔI is negative, $\mathcal{E}$ is positive. If $\Delta I = 0$, $\mathcal{E} = 0$.

Final practice summary

Common calculation chain

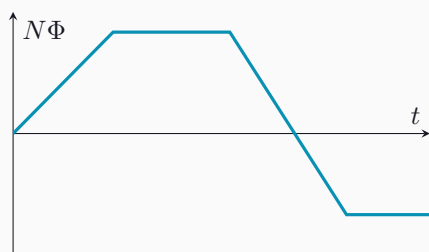
$$B \rightarrow \Phi = BA \rightarrow N\Phi = NBA \rightarrow \mathcal{E} = -\frac{\Delta(N\Phi)}{\Delta t}$$

For a solenoid where $B \propto I$,

$$N\Phi \propto I, \quad \mathcal{E} \propto -\frac{\Delta I}{\Delta t}.$$

If a graph of $N\Phi$ is given

- Gradient of $N\Phi$ -time graph gives $\Delta(N\Phi)/\Delta t$.
- Induced e.m.f. is the negative of this gradient.
- A horizontal section gives zero e.m.f.
- A steeper section gives larger magnitude e.m.f.



What must be retained in answers

1. State Faraday's law using "rate of change of magnetic flux linkage".
2. State Lenz's law using "opposes the change producing it".
3. Use the correct number of turns N when computing flux linkage.
4. Convert cm^2 to m^2 .
5. When current reverses, flux linkage changes from $+N\Phi$ to $-N\Phi$, so total change is $2N\Phi$.

