

Dynamics and Momentum

Chapter 03 / AS Level / Apollo Physics 9702

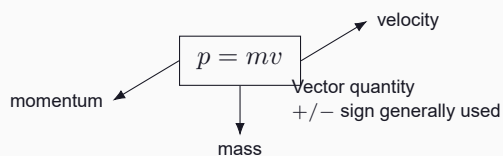


Topic 1 – Momentum, Change in Momentum, and Force

Momentum

“It is the product of mass and **velocity** of a body.”

$$p = mv$$



SI unit: kg m s^{-1} or Ns .

⇒ Momentum tells about the amount (strength) of motion.

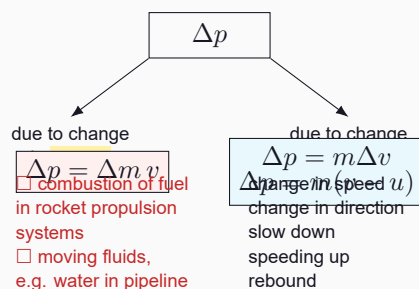


$$p = mv = 0.2 \times 100 = 20 \text{ N s},$$

$$p = mv = 4 \times 5 = 20 \text{ N s}.$$

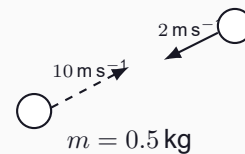
Change in Momentum

$$\Delta p = p_f - p_i$$



Example #1

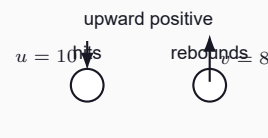
Find Δp .



$$\Delta p = m(v - u) = 0.5(10 - 2) = \boxed{4 \text{ N s}}.$$

Example #2

$m = 0.5 \text{ kg}$, find Δp for a ball that hits the ground at 10 m s^{-1} and rebounds at 8 m s^{-1} .



$$\Delta p = m(v - u) = 0.5\{8 - (-10)\} = \boxed{9 \text{ N s}}.$$

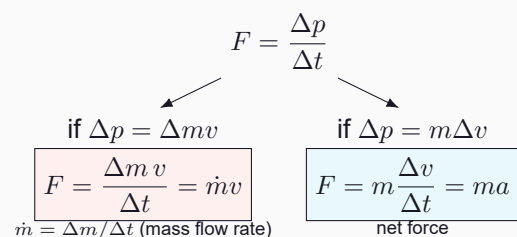
With the opposite sign convention, $\Delta p = 0.5(-8 - 10) = -9 \text{ N s}$; the magnitude is 9 N s .

Force

“Rate of change of momentum is called force.”

$$F_{\text{net}} = \frac{\Delta p}{\Delta t}$$

- **Vector**; SI unit: **newton**.
- The greater the Δt , the lesser the force, and vice versa.



Topic 2 – Mass Flow Rate, Jet Force, and Graphs

$$F_{\text{net}} = ma \rightarrow \text{generally on rigid bodies}$$

$$F_{\text{net}} = \dot{m}v \rightarrow \text{jet of liquid / moving stream of air}$$

Mass Flow Rate

The mass of liquid or gas that passes a point per unit time.

$$\dot{m} = \frac{\Delta m}{\Delta t}$$

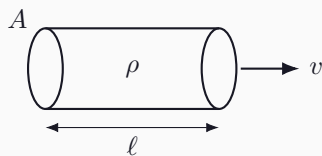
Example: $\dot{m} = 4 \text{ kg min}^{-1}$, i.e. 4 kg of liquid passes a point in 1 min.

$$\dot{m} = \frac{\Delta m}{\Delta t} \begin{cases} \text{mass that passed} \\ \text{time taken} \end{cases}$$

or

$$\dot{m} = \rho Av \begin{cases} \text{density} \\ \text{velocity} \\ \text{cross-sectional area} \end{cases}$$

$$\dot{m} = \frac{\Delta m}{\Delta t}, \quad \dot{m} = \rho \frac{\Delta V}{\Delta t}, \quad \dot{m} = \rho \frac{A\ell}{\Delta t} = \rho Av.$$



Exam-style example: platform and water jet

A man standing on a platform is attached to a flexible pipe. Water is pumped through the pipe so that he remains at a constant height. The combined mass of the man and platform is 96 kg. The mass of water discharged vertically downwards is 40 kg s^{-1} .

Since resultant vertical force is zero,

$$F_{\text{water}} = W = 96g.$$

Using $F = \dot{m}v$,

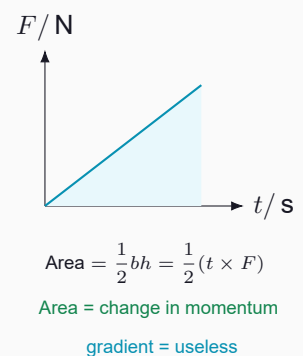
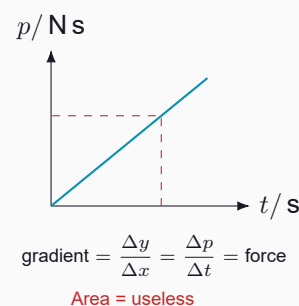
$$40v = 96 \times 9.81 \Rightarrow v = \frac{96 \times 9.81}{40} = 23.5 \approx \boxed{24 \text{ m s}^{-1}}.$$

Hose-pipe example

Water is pumped through a hose-pipe at a rate of 90 kg min^{-1} . Water emerges horizontally from the hose-pipe with a speed of 20 m s^{-1} . What is the minimum force required to prevent the hose-pipe moving backwards?

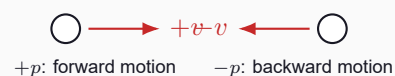
$$F = \dot{m}v = \frac{90}{60} \times 20 = \boxed{30 \text{ N}}.$$

Graphs of $F-t$ and $p-t$



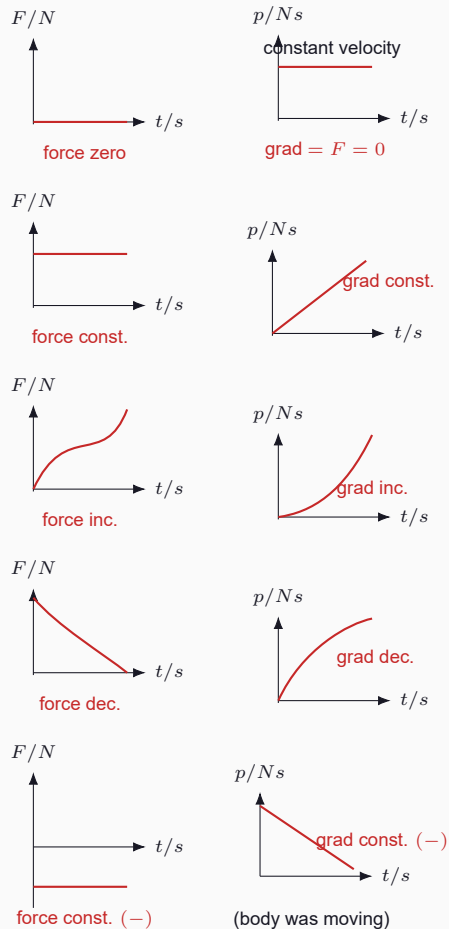
$$F = \frac{\Delta p}{\Delta t}, \quad \Delta p = F\Delta t.$$

Direction sign convention



Topic 3 – Graph Shapes and Types of Collisions

Force–time and momentum–time graph patterns



Types of collisions

Elastic Collision

1. **Ideal collision.**
2. No energy lost upon collision.
3. E.g. collision between small masses such as atoms, neutrons, molecules.

Momentum: conserved

Kinetic energy: conserved

Total energy: conserved

Inelastic Collision

1. **Real collision.**
2. Energy lost upon collision as heat.
3. E.g. collision between large masses, explosion, bodies sticking upon collision.

Momentum: conserved

Kinetic energy: NOT conserved

Total energy: conserved

How to check if a collision is elastic?

A collision is elastic if:

1. **Total K.E before collision = Total K.E after collision**

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2.$$



2. **Relative speed of approach = relative speed of separation.**

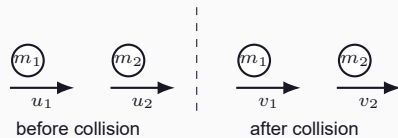
- If bodies move in the **same direction**: subtract / minus speeds.
- If bodies move in **opposite directions**: add speeds.

→ → same direction ⇒ minus speeds

→ ← opposite direction ⇒ add speeds

Topic 4 – Law of Conservation of Momentum

Law of Conservation of Momentum



- **For an isolated system**, the total momentum of bodies before collision is equal to their total momentum after collision.

$$(p_1 + p_2)_{\text{before}} = (p_1 + p_2)_{\text{after}}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2,$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2),$$

$$-m_1(v_1 - u_1) = m_2(v_2 - u_2).$$

Since $\Delta p = m(v - u)$,

$$-\Delta p_1 = +\Delta p_2.$$

- For an isolated system, **the momentum lost by one body is equal to that gained by the other.**

Dividing both sides by Δt , i.e. the time of collision,

$$-\frac{\Delta p_1}{\Delta t} = +\frac{\Delta p_2}{\Delta t} \Rightarrow -F_1 = +F_2.$$



Newton's 3rd Law of Motion

- Both bodies had the same Δp , but in opposite directions.
- The time of collision was the same for both.
- Since $F = \Delta p / \Delta t$, $-F_1 = +F_2$.

"During collision, momentum of a body is not conserved."

$\Rightarrow$ Body always undergoes a change in momentum!

Important distinction

- The momentum of **one body** changes during collision.
- The **total momentum of the isolated system** is conserved.

Exam-style example: equal masses on an air track

Two equal masses X and Y are moving towards each other on a frictionless air track. The masses make an elastic collision.



For equal masses in an elastic collision, the bodies exchange velocities:

$$v_X = 30 \text{ cm s}^{-1} \text{ to the left,}$$

$$v_Y = 50 \text{ cm s}^{-1} \text{ to the right.}$$

This corresponds to option **D** in the handwritten page.

If masses are equal and the collision is elastic, bodies exchange velocities.

Chapter checkpoint

Momentum of individual bodies changes during collisions; total momentum of an isolated system remains constant.