

Density and Pressure

Chapter 06 / AS Level / Apollo Physics 9702



Mass, Volume, Density and Mixtures

Mass

The measurement of inertia is called mass.

- SI unit: kg.
- Scalar quantity.
- Measured usually by electronic balance.

Inertia is a property of matter such that it resists changes in its state of rest or motion.

more inertia $\rightarrow$ more mass

Weight

Effect of gravitational force on mass is called weight.

$$W = mg$$

- Vector quantity.
- Spring balance is used to measure weight.

Gravitational Field

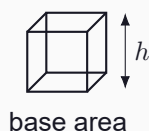
A region in space around a mass where another mass experiences gravitational force.

Volume

Space occupied by a body.

General formula: $V = \text{base area} \times \text{height}$

Shape	Volume
Cube	$V = L^3$
Cuboid	$V = L \times b \times h$
Cylinder	$V = \pi r^2 h$
Cone	$V = \frac{1}{3} \pi r^2 h$
Sphere	$V = \frac{4}{3} \pi r^3$

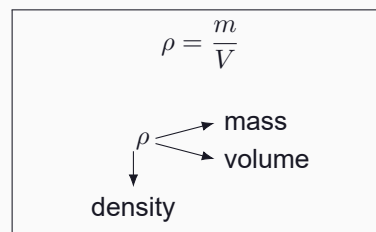


Density

Mass per unit volume.

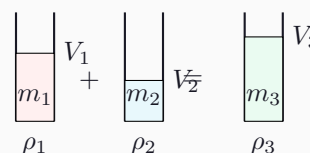
Understanding: it tells about the concentration of mass in a given volume.

more dense $\rightarrow$ more mass $\rightarrow$ heavier



SI unit: kg m^{-3}

Finding Density of Mixture



$$\rho_{mix} = \frac{m_{mix}}{V_{mix}}, \quad \rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

Example 19

1.5 m^3 of water is mixed with 0.50 m^3 of alcohol. The density of water is 1000 kg m^{-3} and the density of alcohol is 800 kg m^{-3} . Find density of mixture with volume 2.0 m^3 .

$$\rho_{mix} = \frac{m_1 + m_2}{V_{mix}} = \frac{(1000 \times 1.5) + (800 \times 0.5)}{2} = \boxed{950 \text{ kg m}^{-3}}$$

Example 10

Density of water = 1.0 g cm^{-3} , density of glycerine = 1.3 g cm^{-3} . Water is added to 40 cm^3 glycerine so that density of mixture is 1.1 g cm^{-3} .

$$1.1 = \frac{1.0(V_1) + 1.3(40)}{V_1 + 40} \Rightarrow V_1 = 80 \text{ cm}^3$$

$$\boxed{80 \text{ cm}^3}$$

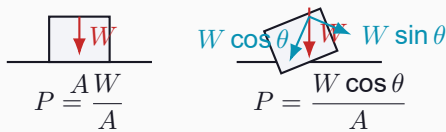
Pressure, Liquid Pressure and Atmospheric Pressure

Pressure

Force per unit (surface) area.

$$P = \frac{F}{A}$$

- Force is at 90° angle with the surface.
- It is a contact force.
- Scalar quantity.
- SI unit: Pascal, Pa.



Applied force example

applied force = 20 N



contact force = 80 N

$$P = \frac{F}{A} \Rightarrow P = \frac{80}{A}$$

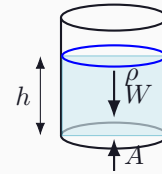
Atmospheric Pressure

- It is due to the collision of air molecules around us.
- Air molecules, due to their continuous random motion, collide with one another and with walls of objects, exerting a force. This force generates pressure.

$$P_{atm} = 101325 \text{ Pa} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ mmHg} = 14.7 \text{ psi}$$

- P_{atm} decreases with altitude.
- Reason 1: low density of air, i.e. fewer molecules.
- Reason 2: low temperature of air, i.e. molecules are slow.

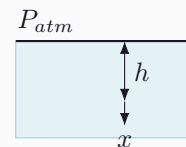
Pressure in Liquids



$$P = \frac{F}{A} = \frac{W}{A} = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho (Ah) g}{A}$$

$$P = \rho g h$$

- ρ is density.
- h is depth taken from top surface.



$$P_T = P_L + P_{atm}, \quad P_T = \rho g h + P_{atm}$$

Total pressure: atmospheric pressure gets transmitted throughout the liquid.

This formula is applicable for all 3 states of matter.

Cylinder on a surface

A solid metal cylinder of length x and cross-sectional area A exerts pressure p on a surface.

$$P = \rho g h = \rho g x \Rightarrow \rho = \frac{p}{gx}$$

Altitude example

Air density decreases almost linearly from 1.2 kg m^{-3} at sea level to 0.74 kg m^{-3} at altitude 5000 m. Given $g = 9.7 \text{ m s}^{-2}$ and surface pressure 100000 Pa:

$$\rho_{avg} = \frac{1.2 + 0.74}{2} = 0.97 \text{ kg m}^{-3}$$

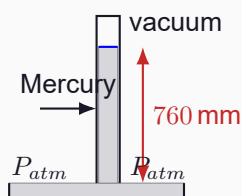
$$\Delta P = \rho g h = 0.97(9.7)(5000) = 47050 \text{ Pa}$$

$$P_x = 100000 - 47050 \approx 52950 \text{ Pa}$$

Barometer, Manometer and Upthrust

Barometer

- Atmospheric pressure on the outer surface of mercury balances the liquid pressure on the inside of the tube.
- 760 mm of mercury column was able to balance P_{atm} .
- Shape and size of tube does not matter on the level of mercury.



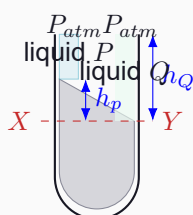
water: $\rho = 1000 \text{ kg m}^{-3}$, $P = \rho gh$

$$101325 = (1000)(9.81)h \Rightarrow h = 10.3 \text{ m}$$

mercury: $\rho = 13600 \text{ kg m}^{-3} \Rightarrow 13.6 \text{ times heavier.}$

Manometer

- Used to compare pressures on both sides of tube.
- When manometer is in equilibrium, pressure on both sides above a common point is equal.
- For comparison, pick the level that has different liquids above and below.



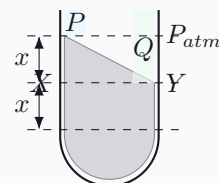
$$P_X = P_Y$$

$$P_P + P_{atm} = P_Q + P_{atm}$$

$$\rho_P gh_P + P_{atm} = \rho_Q gh_Q + P_{atm}$$

Open U-tube example

Two liquids labelled P and Q do not mix. The liquids are in equilibrium in an open U-tube.



$$P_X = P_Y$$

$$\rho_P g(2x) + P_{atm} = \rho_Q g(x) + P_{atm}$$

$$\rho_P(2x) = \rho_Q(x) \Rightarrow \boxed{\frac{\rho_P}{\rho_Q} = \frac{1}{2}}$$

Closed U-tube with trapped air

A U-tube closed at one end contains mercury. Air at pressure $5.0 \times 10^4 \text{ Pa}$ is trapped at the closed end. Atmospheric pressure is $1.01 \times 10^5 \text{ Pa}$. A piston of mass 5.0 kg and area $5.0 \times 10^{-4} \text{ m}^2$ acts on the other side.

$$P_p + P_{atm} = P_{air} + \rho gh$$

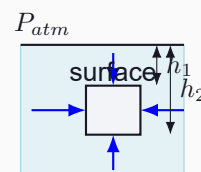
$$\frac{F}{A} + 1.01 \times 10^5 = (5.0 \times 10^4) + (13600)(9.8)h$$

$$\frac{5 \times 9.81}{5 \times 10^{-4}} + 1.01 \times 10^5 = 5.0 \times 10^4 + (13600)(9.8)h$$

$$h = 1.1 \text{ m} = \boxed{110 \text{ cm}}$$

Upthrust

An upward force acting on the body due to the difference in pressures at the top and bottom surface of the body submerged in a fluid.



$P = \rho gh$, $P \times h$ idea: pressure increases with depth.

$$F_1 = P_1 A, F_2 = P_2 A$$

$$F_{up} = F_2 - F_1$$

$$\boxed{F_{up} = (P_2 - P_1)A}$$

Upthrust Derivation, Floating, Sinking and Drag

- Upthrust force is not water resistance / drag force.
- Drag force appears on a body when it starts to move in a fluid.
- Stationary body: no drag force.

$$F_{up} = (P_2 - P_1)A$$

P_2 : total pressure at 2, P_1 : total pressure at 1

$$F_{up} = \{(\rho gh_2 + P_{atm}) - (\rho gh_1 + P_{atm})\}A$$

$$F_{up} = (\rho gh_2 - \rho gh_1)A$$

$$F_{up} = \rho g(h_2 - h_1)A$$

Since $H = h_2 - h_1$,

$$F_{up} = \rho gHA$$

As $V = A \times H$,

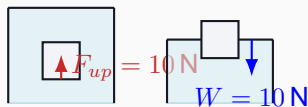
$$F_{up} = \rho gV$$

$$F_{up} = m_L g$$

m_L : mass of liquid displaced.

Key result

Upthrust force is equal to the weight of liquid displaced.

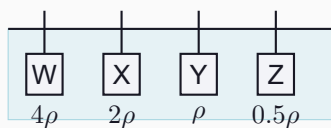


Volume/size of object decides the volume of water displaced and hence the upthrust force.

Cuboids held at same depth

Four cuboids with identical length, breadth and height are immersed in water. They are at the same depth and in identical orientations. Therefore each displaces the same volume of water:

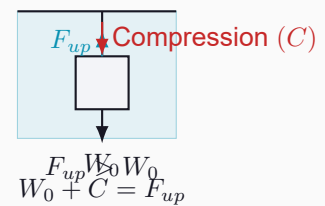
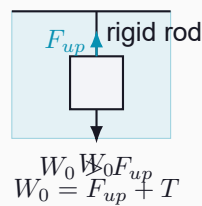
$$F_{up} = \rho gV \Rightarrow \text{same upthrust for all.}$$



The upthrust of the water on each cuboid is the same.

Rigid Rod: Tension and Compression

Tension (T)

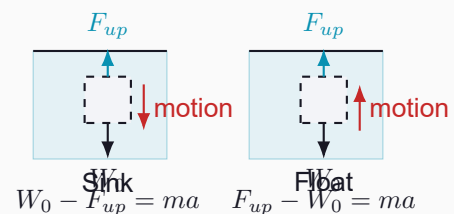


Equilibrium: rest.

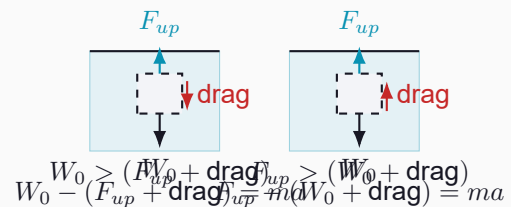
If the rigid rod is disconnected

The body accelerates.

$$F_{net} = ma$$



With drag after motion starts



Exam Question: Balance Reading and Immersion

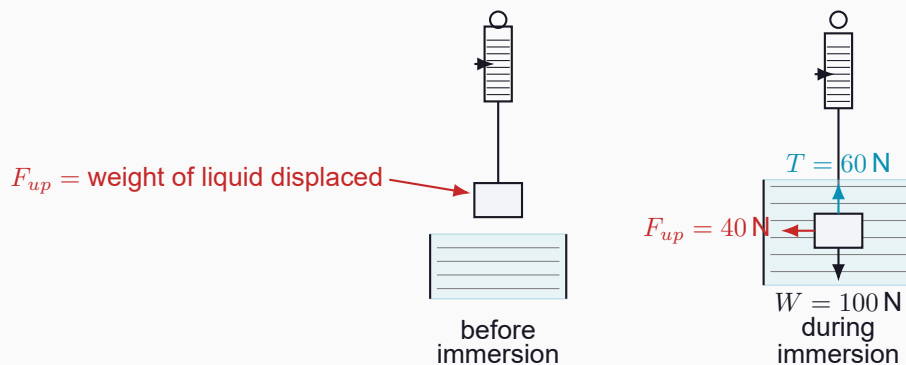
PHYSICS

Paper 1 Multiple Choice

1 hour

9702/12**October/November 2010**

- 13 The diagrams show a metal cube suspended from a spring balance before and during immersion in water.



A reduction in the balance reading occurs as a consequence of the immersion.

Which statement is correct?

- A The balance reading will be further reduced if the cube is lowered further into the water.
- B The balance reading during immersion corresponds to the upthrust of the water on the cube.
- C The forces acting on the vertical sides of the cube contribute to the change in the balance reading.
- D The gravitational pull on the cube is unchanged by the immersion.

Explanation

During immersion the balance reading is reduced because the water provides an upward upthrust. If the cube is already completely submerged, lowering it further does not increase the displaced volume, so the upthrust is unchanged. Side forces cancel horizontally. The weight/gravitational pull on the cube remains $W = mg$, so the correct option is **D**.

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