

Current Electricity

Chapter 09 / AS Level / Apollo Physics 9702



Circuit Symbols

Name	Symbol	Name	Symbol
Cell		Switch: one-way	
Battery		Switch: two-way	
Power supply: DC		Ammeter	
Power supply: AC		Galvanometer	
Fixed resistor		Voltmeter	
Rheostat / variable resistor		Ohmmeter	
LDR		Diode	
Thermistor		LED	
Filament lamp		Capacitors	

Current, Conventional Current, Electron Flow, DC and AC

Current

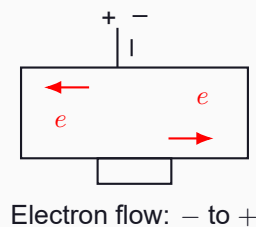
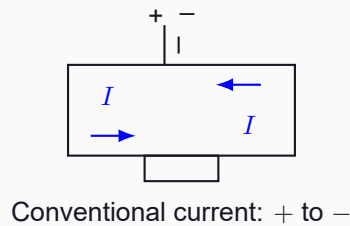
Rate of flow of electric charges

OR

Rate of flow of electrons

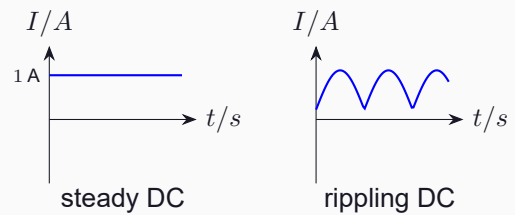
$$I = \frac{q}{t} \quad \text{or} \quad I = \frac{\Delta q}{\Delta t}$$

- **SI unit:** ampere (A)
- **Nature:** scalar quantity
- Δq : charge passing a point; Δt : time taken.



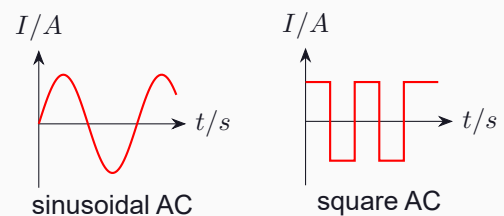
1. Direct Current (DC)

It flows in one direction.



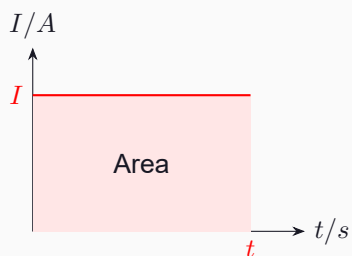
2. Alternating Current (AC)

Current changes direction periodically.



Graph: $I - t$, Charge and Number of Electrons

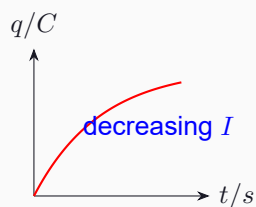
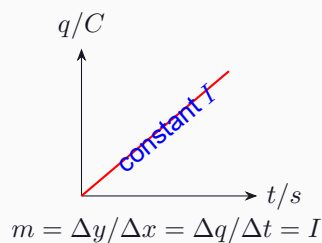
Area under an $I - t$ graph



$$\text{Area} = l \times b = I \times t = q$$

$$1e = 1.6 \times 10^{-19} \text{ C}$$

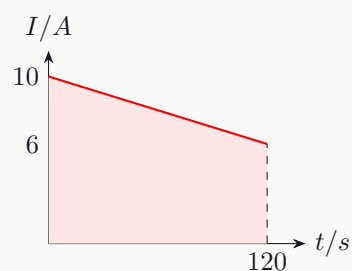
Graph: $q - t$



Example 1

A current in a circuit decreases **linearly** from 10 A to 6 A in a time of 2 minutes. Calculate:

- the total charge passing a point,
- the number of electrons passing a point.



$$q = \frac{1}{2}(10 + 6)(120) = 960 \text{ C}$$

$$1.6 \times 10^{-19} \text{ C} \equiv 1e, \quad 960 \text{ C} \equiv x$$

$$x = \frac{960 \times 1}{1.6 \times 10^{-19}} = 6.0 \times 10^{21} \text{ electrons.}$$

Current Equation and Number Density

Current through a conductor

$$I = nAve$$

I :	current
n :	number density
A :	cross-sectional area
v :	mean drift speed of charge carriers
e :	elementary charge

Number density, n

Number of conduction electrons available per unit volume:

$$n = \frac{N}{V}$$

where N is the number of free mobile electrons and V is the volume of conductor.

$n = \frac{N}{V}$	→ No. of free mobile electrons → Volume of conductor
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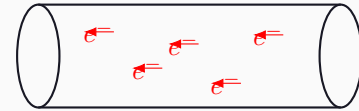
Conductor > semi-conductor > insulator: **number density decreases.**

$e \ e \ e \ e \ e \ e \rightarrow e \ e \ e \rightarrow e$
 conductor semi-conductor insulator

Mean Drift Speed, v

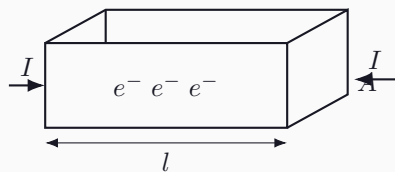
Mean: not all electrons have the same speed. The calculated value is an average value.

Drift: electrons do not flow smoothly; they drift from their places. Their motion is hindered by atoms and other electrons.



Deriving $I = nAve$ and Drift-Speed Proportions

Derivation



$$I = \frac{q}{t}, \quad q = Ne \Rightarrow I = \frac{Ne}{t}.$$

Since $n = N/V$, $N = nV$:

$$I = \frac{nVe}{t}.$$

For a length l of wire, $V = Al$, and since $v = l/t$,

$$I = \frac{n(Al)e}{t} = nA \left(\frac{l}{t} \right) e = \boxed{nAve}.$$

Mean drift speed

$$\boxed{v = \frac{I}{nAe}}, \quad v \propto I, \quad v \propto \frac{1}{A}, \quad v \propto \frac{1}{n}.$$

Physical meanings

- $v \propto I$: more current means more electrons passing a point, so electrons must travel faster.
- $v \propto 1/A$: where the cross-sectional area is smaller, electrons travel faster to establish the same rate of flow.
- $v \propto 1/n$: lower number density means fewer conduction electrons are available, so they must travel faster to establish the same rate of flow.



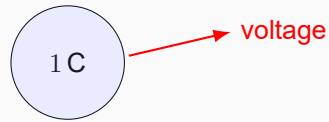
Same current through a smaller area $\Rightarrow$ faster drift.

Voltage, EMF and Potential Difference

Voltage

Energy possessed by a unit charge.

SI unit: volt (V) or J C^{-1} Scalar quantity



Electromotive Force (EMF)	Potential Difference (PD)
Definition: It is the work done per unit charge to move electrons around the complete circuit. - It is the voltage of a source e.g. cell, battery, power supply, generator, etc.	Definition: It is the energy converted per unit charge when passing between two points in a circuit. - It is a voltage across any component of the circuit e.g. resistor, lamp, cell, battery, etc. P.D. across the source is called EMF.

Resistance

Definition

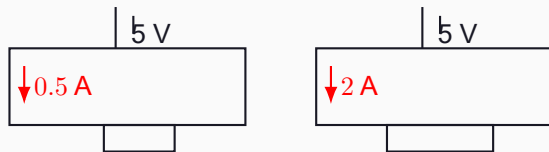
Resistance is the ratio of voltage across a resistor to the current through it.

$$R = \frac{V}{I}$$

SI unit: ohm (Ω)

It is the hindrance in the flow of electrons.

Two examples at the same voltage



$$R = V/I = 5/0.5 = 10\ \Omega \quad R = V/I = 5/2 = 2.5\ \Omega$$

Meaning

- $R = 10\ \Omega$ means 10 V per ampere.
- $R = 2.5\ \Omega$ means 2.5 V per ampere.

For the same work done, the first resistor had less current, so it has more resistance.

OR for the same value of current, more work needs to be done in the first resistor; hence it has larger resistance.

Factors Affecting Resistance

1. Temperature

Temperature $\uparrow \rightarrow$ lattice vibrations increase $\rightarrow$ mean drift speed $\downarrow \rightarrow I \downarrow \rightarrow R \uparrow$

2. Length of conductor, l

$$R \propto l$$

Length $\uparrow \rightarrow$ more work needs to be done for same current $\rightarrow V \uparrow \rightarrow R \uparrow$.

3. Cross-sectional area, A

$$R \propto \frac{1}{A}$$

Cross-sectional area $\uparrow \rightarrow$ more electrons pass due to more space $\rightarrow I \uparrow \rightarrow R \downarrow$.

$$R \propto \frac{l}{A} \quad \Rightarrow \quad R = \rho \frac{l}{A}$$

Changing Length, Diameter, Area and Resistance; Resistivity

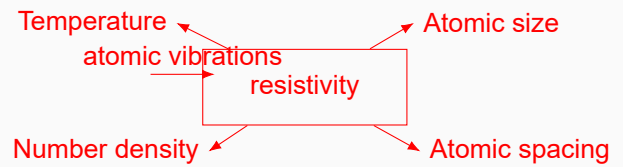
l/m	d/m	A/m^2	R/Ω
l	d	A	R
$2l$	d	A	$2R$
$l/2$	d	A	$R/2$
l	$2d$	$4A$	$R/4$
l	$d/3$	$A/9$	$9R$
$2l$	$3d$	$9A$	$\frac{R \times 2}{9} = \frac{2R}{9}$
$l/2$	$2d$	$4A$	$\frac{R}{2 \times 4} = \frac{R}{8}$
$2l$	$d/2$	$A/4$	$R \times 2 \times 4 = 8R$
$l/3$	$d/3$	$A/9$	$\frac{R \times 9}{3} = 3R$

$$A = \pi r^2 = \frac{\pi d^2}{4}, \quad A \propto d^2.$$

Resistivity, ρ

It is a material property and depends upon the element used to make the conductor.

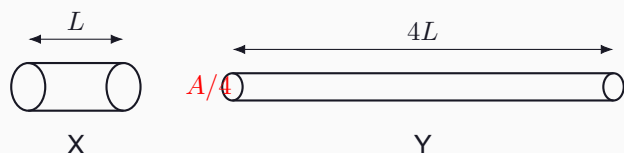
$$R = \rho \frac{l}{A} \quad \Rightarrow \quad \rho = \frac{RA}{l}.$$



Exam Practice: Resistance and Resistivity

Q34. Same volume wires

Two copper wires X and Y have the same volume. Wire Y is four times as long as wire X.



What is the ratio $\frac{\text{resistance of wire Y}}{\text{resistance of wire X}}$?

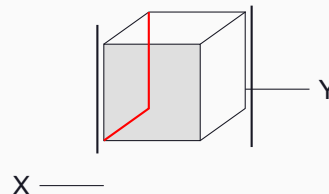
- A 4 B 8 C 16 D 64

Solution:

$$R_Y = R_X \times 4 \times 4 = 16R_X \quad \Rightarrow \quad \frac{R_Y}{R_X} = 16.$$

Q34. Metal cube between plates

The resistance of a metal cube is measured by placing it between two parallel plates. The cube has volume V and is made of a material with resistivity ρ .



For a cube, $V = l^3$, so $l = V^{1/3}$ and $A = l^2$. Hence

$$R = \rho \frac{l}{A} = \rho \frac{l}{l^2} = \frac{\rho}{l} = \frac{\rho}{V^{1/3}}.$$

Correct option: C.

Ohm's Law

Statement

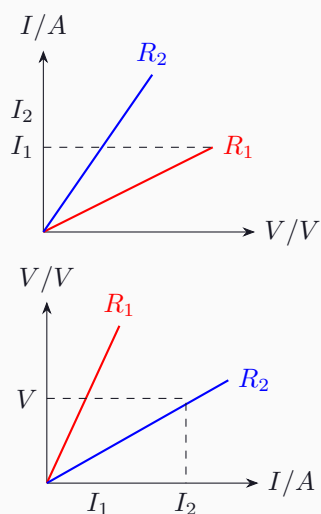
At constant temperature, the current flowing through a conductor is directly proportional to the potential difference across it.

$$I \propto V$$

$$I = GV, \quad G = \frac{1}{R}, \quad I = \frac{1}{R}V, \quad V = IR.$$

where G is conductance and $G = 1/R$.

$I - V$ graphs



Important note

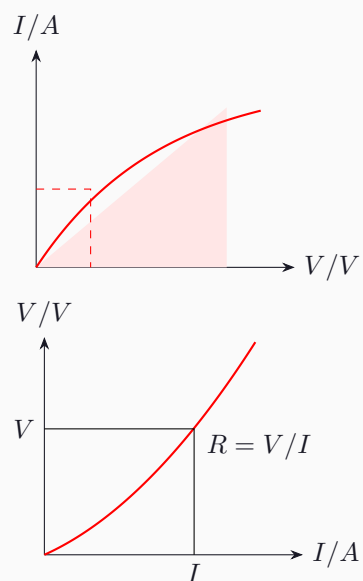
Do not explain using gradient as $R \neq \Delta V / \Delta I$ for every graph; use $R = V/I$.

Pick a value of V and check the corresponding value of I . The one which has less current has more resistance.

- R_1 has more resistance than R_2 .
- Resistors that obey Ohm's law are called **Ohmic conductors**.

Filament Lamp

Characteristic curves

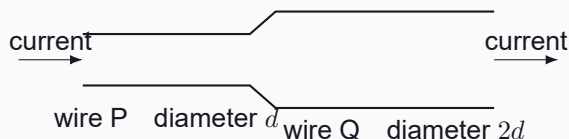


As the temperature of the filament lamp increases due to higher currents, its resistance also increases. Hence, for the same increase of voltage, the increase of current is less at higher voltages.

Exam Practice: Mean Drift Speed

Q33. Two copper wires joined together

Two copper wires are joined together and carry a current as shown. Wire P has diameter d and wire Q has diameter $2d$.



Find the ratio

average drift speed of free electrons in wire P
 average drift speed of free electrons in wire Q.

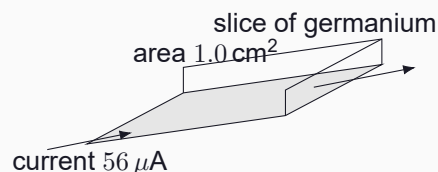
A $\frac{1}{4}$ B $\frac{1}{2}$ C 2 D 4

Solution: $v = I/(nAe)$. Same material and same current, so $v \propto 1/A$ and $A \propto d^2$.

$$v_P d^2 = v_Q (2d)^2 \Rightarrow \frac{v_P}{v_Q} = 4.$$

Q30. Germanium slice

A slice of germanium of cross-sectional area 1.0 cm^2 carries a current of $56 \mu\text{A}$. The number density of charge carriers is $2.0 \times 10^{13} \text{ cm}^{-3}$. Each charge carrier has charge equal to that of an electron.



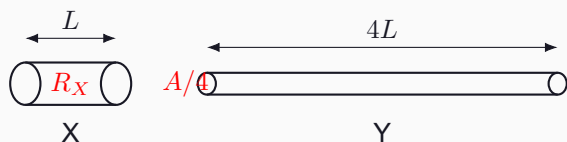
$$I = nAve \Rightarrow 56 \times 10^{-6} = 2 \times 10^{13} (1) v (1.6 \times 10^{-19})$$

$$v = 17.5 \text{ cm s}^{-1} = 0.175 \text{ m s}^{-1} \approx 0.18 \text{ m s}^{-1}.$$

Exam Practice: Resistance with Same Volume and Wire Radius

Q34. Same volume: resistance ratio

Two copper wires X and Y have the same volume. Wire Y is four times as long as wire X.



$$R = \rho \frac{l}{A}, \quad R \propto l, \quad R \propto \frac{1}{A}.$$

Same volume gives $V = Al$ constant, so if l increases by 4, area decreases by 4.

$$R_Y = R_X \times 4 \times 4 = 16R_X \quad \Rightarrow \quad \frac{R_Y}{R_X} = 16.$$

Q32. Radius of wire

A cylindrical wire 4.0 m long has resistance $31 \, \Omega$ and is made of metal of resistivity $1.0 \times 10^{-6} \, \Omega \text{ m}$. Find the radius of cross-section of the wire.

$$R = \rho \frac{l}{A} = \frac{\rho l}{\pi r^2}$$

$$31 = \frac{(1.0 \times 10^{-6})(4.0)}{\pi r^2}$$

$$r = \sqrt{\frac{(1.0 \times 10^{-6})(4.0)}{31\pi}} \approx 2.0 \times 10^{-4} \text{ m}.$$

Correct option: **D $2.0 \times 10^{-4} \text{ m}$** .