

General Waves

Chapter 11 / AS Level / Apollo Physics 9702



Wave and Classification of Waves

Wave

It is a disturbance in a medium.

OR

It is a mode of transferring energy from one point to another.

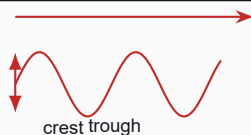
Classification of waves

1. Mechanical vs electromagnetic
2. Transverse vs longitudinal
3. Progressive vs stationary (new)

Mechanical waves	Electromagnetic waves
Waves which need a medium to travel.	No need of medium to travel.
Cannot travel in vacuum.	They can also travel in vacuum.
Sound, water waves.	Electromagnetic spectrum.

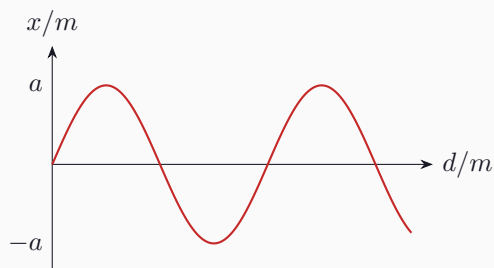
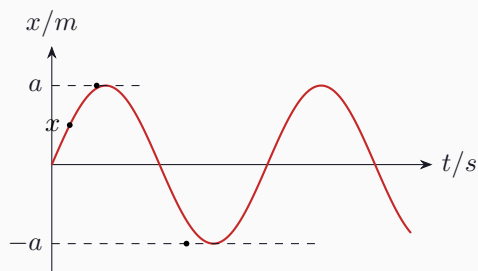
Electromagnetic spectrum

Gamma rays, X-rays, Ultraviolet, Visible light, Infrared, Microwaves, Radiowaves.

Transverse waves	Longitudinal waves
 crest trough	 compression rarefaction
Waves in which direction of vibration is perpendicular to the direction of energy transfer. * of source. Comprises of crests and troughs. Electromagnetic spectrum, water waves.	Waves in which direction of vibration is parallel to the direction of energy transfer. * of source. Comprises of compressions and rarefactions. Sound wave.

Characteristics of Waves

Wave graphs



Particles only vibrate about their mean position on the wave.

1. Displacement (x)

The distance of a point on a wave from its mean position.

2. Amplitude (a)

Maximum displacement of a point on a wave from its mean position.

3. Time period (T)

- The time taken to produce one wave.
- The time taken for one wave to pass a point.



4. Frequency (f)

- The no. of waves produced per unit time.
- The no. of waves passing a point per unit time.

Frequency and Wavespeed

Example: frequency

100 waves were produced in 20 s. Frequency = ?

$$20 \text{ s} \longrightarrow 100 \text{ waves} \quad 1 \text{ s} \longrightarrow x = \frac{100}{20} = 5 \text{ s}^{-1}$$

Let N be the number of waves in time t :

$$f = \frac{N}{t}$$

If $N = 1$, then $t = T$, hence

$$f = \frac{1}{T}$$

SI unit: Hertz (Hz).

$$v = \frac{d}{t}$$

$$d = N\lambda$$

$$v = \frac{N\lambda}{t} = f\lambda$$

$$v = f\lambda$$

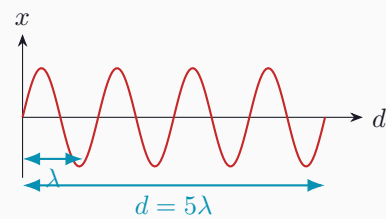
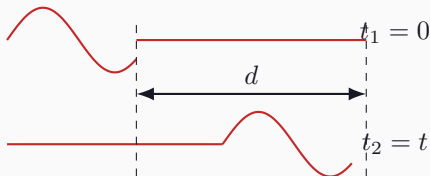
Small calculation

$$d = 10 \text{ cm}, \quad \lambda = 2 \text{ cm}$$

$$d = 5\lambda$$

5. Wavespeed (v)

The distance travelled by a waveform per unit time.



Intensity of a Wave

6. Intensity of a wave (I)

Energy delivered per unit area per unit time.

OR

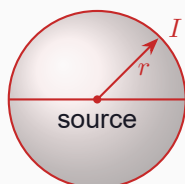
Power per unit area.

$$I = \frac{E}{At}$$

or

$$I = \frac{P}{A}$$

$$I \propto \frac{1}{A}$$



For a sphere, $A = 4\pi r^2$:

$$I = \frac{P}{4\pi r^2}$$

If P is constant:

$$I \propto \frac{1}{r^2}$$

Energy spreads out equally in all directions.

Relationships in the notes

- Intensity $\propto$ frequency for electromagnetic radiation:

$$I \propto f$$

- Intensity $\propto$ (amplitude)²: $I \propto a^2$.

- Intensity $\propto \frac{1}{(\text{distance})^2}$: $I \propto \frac{1}{d^2}$.

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{I_1}{I_2} = \frac{d_2^2}{d_1^2}$$

Since

$$I \propto a^2, I \propto \frac{1}{d^2}$$

we get

$$a^2 \propto \frac{1}{d^2} \Rightarrow a \propto \frac{1}{d}$$

$$a_1 d_1 = a_2 d_2$$

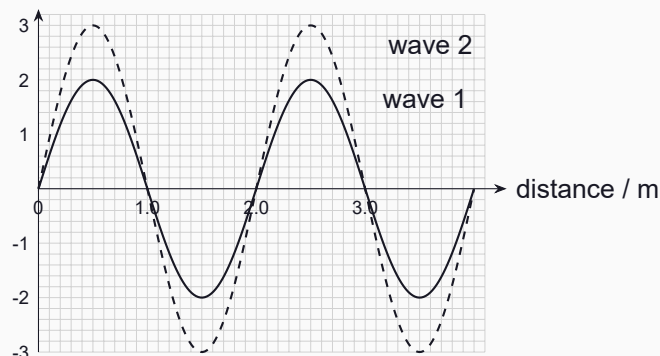
Exam Question: Intensity Ratio

Question 23

A sound wave consists of a series of moving pressure variations from the normal, constant air pressure. The graph shows these pressure variations for two waves at one instant in time.

Wave 1 has an intensity of $1.6 \times 10^{-6} \text{ W m}^{-2}$. What is the intensity of wave 2?

pressure variation / 10^{-2} Pa



- A $2.4 \times 10^{-6} \text{ W m}^{-2}$ B $3.0 \times 10^{-6} \text{ W m}^{-2}$ C $3.6 \times 10^{-6} \text{ W m}^{-2}$ D $4.5 \times 10^{-6} \text{ W m}^{-2}$

Solution: ratio method

$$I \propto a^2$$

From the graph:

$$a_1 = 2, a_2 = 3$$

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2} \right)^2 = \left(\frac{2}{3} \right)^2 = \frac{4}{9}$$

$$\frac{1.6 \times 10^{-6}}{I_2} = \frac{4}{9}$$

$$I_2 = \frac{(1.6 \times 10^{-6}) \times 9}{4}$$

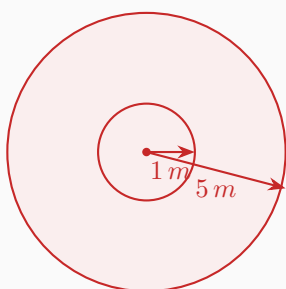
$$I_2 = 3.6 \times 10^{-6} \text{ W m}^{-2}$$

Exam Question: Sound Spreading from a Source

M/J 1983 P1 Q4

A small source of sound radiates energy equally in all directions. At a particular frequency, the intensity of the sound 1.0 m from the source is $1.0 \times 10^{-5} \text{ W m}^{-2}$, corresponding to an amplitude of oscillation of the air molecules of $70 \mu\text{m}$. Assuming that the sound is propagated without energy loss, what will be

- the intensity of the sound,
- the amplitude of oscillation of the air molecules at a distance of 5.0 m from the source?



Given

$$d_1 = 1 \text{ m}, \quad I_1 = 1.0 \times 10^{-5} \text{ W m}^{-2}, \quad a_1 = 70 \mu\text{m}$$

$$d_2 = 5 \text{ m}, \quad I_2 = ?, \quad a_2 = ?$$

(a) Intensity

$$I \propto \frac{1}{d^2}$$

$$I_1 d_1^2 = I_2 d_2^2$$

$$(1 \times 10^{-5})(1)^2 = I_2(5)^2$$

$$I_2 = 4.0 \times 10^{-7} \text{ W m}^{-2}$$

(b) Amplitude

$$a \propto \frac{1}{d} \quad \Rightarrow \quad a_1 d_1 = a_2 d_2$$

$$70(1) = a_2(5)$$

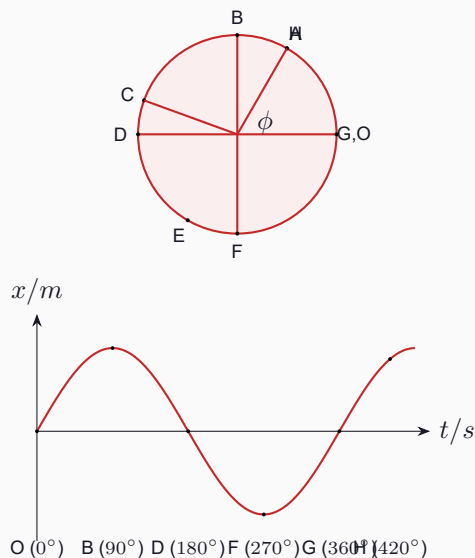
$$a_2 = 14 \mu\text{m}$$

Phase

7. Phase (ϕ)

It is used to tell about the state of motion of a point on a wave.

It is represented in “angles”.



$$\begin{aligned}\phi_O &= 0^\circ, & \phi_A &= 60^\circ, & \phi_B &= 90^\circ \\ \phi_C &= 160^\circ, & \phi_D &= 180^\circ, & \phi_E &= 240^\circ \\ \phi_F &= 270^\circ, & \phi_G &= 360^\circ \text{ or } 0^\circ, & \phi_H &= 420^\circ \text{ or } 60^\circ\end{aligned}$$

Degree to radian

$$180^\circ = \pi \text{ rad}, \quad 360^\circ = 2\pi \text{ rad}, \quad 90^\circ = \frac{\pi}{2} \text{ rad}$$

Phase angle using ratio method

$$T \longrightarrow 360^\circ, \quad t \longrightarrow \phi$$

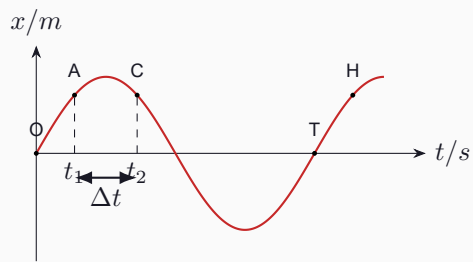
$$\phi = \frac{t}{T} \times 360^\circ \quad (\text{in degree})$$

$$\phi = \frac{t}{T} \times 2\pi \quad (\text{in radians})$$

$$\lambda \longrightarrow 360^\circ, \quad d \longrightarrow \phi$$

$$\phi = \frac{d}{\lambda} \times 360^\circ \quad \text{or} \quad \phi = \frac{d}{\lambda} \times 2\pi$$

Phase Difference Between Two Points



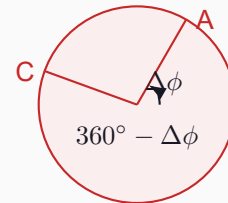
$$T \longrightarrow 360^\circ, \quad \Delta t \longrightarrow \Delta\phi$$

$$\Delta\phi = \frac{\Delta t}{T} \times 360^\circ$$

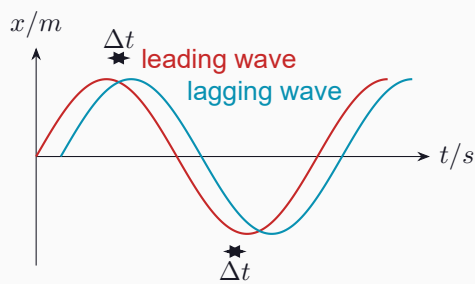
Similarly for distance

$$\lambda \longrightarrow 360^\circ, \quad \Delta d \longrightarrow \Delta\phi$$

$$\Delta\phi = \frac{\Delta d}{\lambda} \times 360^\circ$$



Phase Difference Between Two Waves and Coherence



$$T \rightarrow 360^\circ, \quad \Delta t \rightarrow \Delta\phi$$

$$\Delta\phi = \frac{\Delta t}{T} \times 360^\circ$$

where Δt is the time difference between corresponding points of both waves.

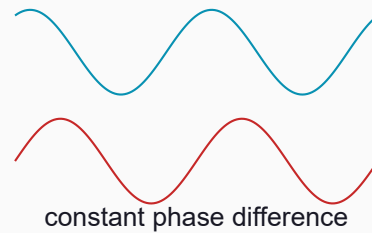
Meaning of lead and lag

The **leading wave** has its crest/trough produced earlier in time than the other. The **lagging wave** is delayed.

Coherent waves

Waves are said to be coherent if they have the same phase difference throughout.

“For waves to be coherent, they must have the same frequency.”



Doppler Effect

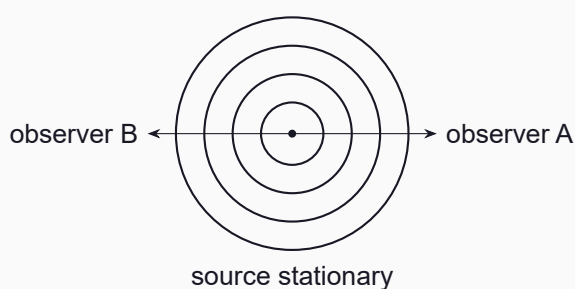
Definition

The change in the observed frequency of the wave when the source moves relative to observer.

$$f_o = \frac{v}{v \pm v_s} \times f$$

- f_o = observed frequency
- v = speed of wave
- v_s = speed of source
- f = actual/source frequency

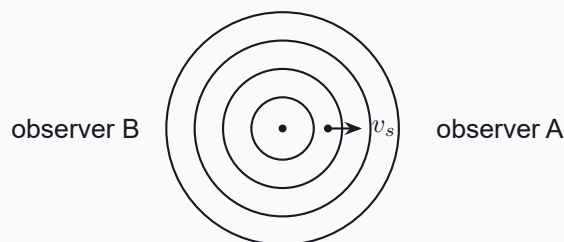
Stationary source



Observed frequency is same as source frequency as source is stationary:

$$f_o = f$$

Source receding



For source receding, $f_o < f$:

$$f_o = \frac{v}{v + v_s} \times f$$

Distance plus in recede, so use +.

Source approaching

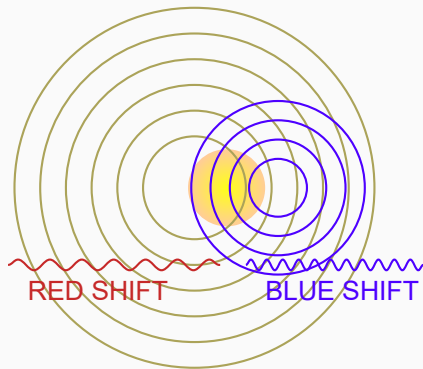
For source approaching, $f_o > f$:

$$f_o = \frac{v}{v - v_s} \times f$$

Distance minus in approach, so use -.

Doppler Effect: Red Shift and Blue Shift

Visual idea



As the star moves away, there is a **red shift** in its colour. This is due to the Doppler effect.

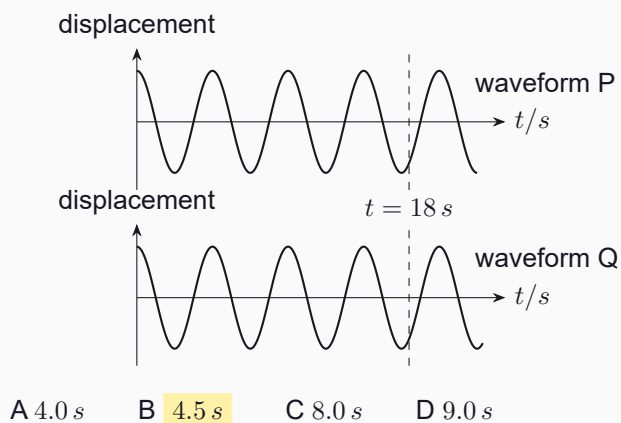
- Due to star moving away, the observed frequency is less than the actual frequency.
- Red colour has a lower frequency, hence the light coming from the star has a red shift in it.
- This also explains that the universe is expanding.

When a source approaches, wavefronts are closer together in front of the source; the wavelength is smaller and the observed frequency increases. This gives a blue shift for light.

Exam Question: Phase Difference of Waveforms

Question 26

The diagram shows two sinusoidal waveforms. At time $t = 0$ the waves are in phase. At the dotted line, $t = 18\text{ s}$. At which time is the phase difference between the two oscillations $\frac{1}{8}$ of a cycle?



Solution

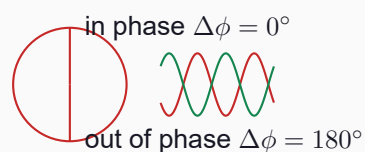
$$\frac{1}{8} \text{ of cycle} = \frac{1}{8} \times 360^\circ = 45^\circ$$

The notes compare 18 s to 180° :

$$18\text{ s} \longrightarrow 180^\circ, \quad x \longrightarrow 45^\circ$$

$$x = \frac{18 \times 45}{180} = 4.5\text{ s}$$

$$x = 4.5\text{ s}$$



Exam Question: Doppler Frequency

Question 26

A car travelling at a steady speed in a straight line passes close to a stationary observer. The observer measures the frequency of the sound from the engine.

As the car approaches, the observed frequency is 220 Hz. When the car moves away, the observed frequency is 180 Hz.

The speed of sound in air is 340 m s^{-1} .

What is the speed of the car?

- A 8.5 m s^{-1} B 31 m s^{-1} C 34 m s^{-1} D 38 m s^{-1}

Solution

For approach:

$$220 = \frac{340}{340 - v_s} f$$

For recede:

$$180 = \frac{340}{340 + v_s} f$$

Divide the equations to eliminate f :

$$\frac{220}{180} = \frac{340 + v_s}{340 - v_s}$$

$$220(340 - v_s) = 180(340 + v_s)$$

$$74800 - 220v_s = 61200 + 180v_s$$

$$13600 = 400v_s$$

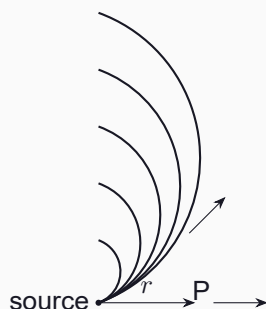
$$v_s = 34 \text{ m s}^{-1}$$

Spherical Waves and Diffraction MCQ

Question 27

A small source emits spherical waves. The wave intensity I at any point P , a distance r from the source, is inversely proportional to r^2 .

What is the relationship between the wave amplitude a and the distance r ?



- A $a^2 \propto \frac{1}{r}$ B $a \propto \frac{1}{r}$ C $a \propto \frac{1}{r^2}$ D $a \propto \frac{1}{r^4}$

Solution

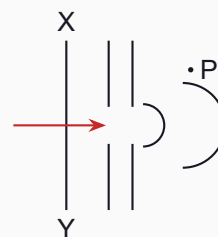
$$I \propto \frac{1}{r^2}, \quad I \propto a^2$$

$$a^2 \propto \frac{1}{r^2} \quad \Rightarrow \quad a \propto \frac{1}{r}$$

Question 29

A monochromatic plane wave of speed c and wavelength λ is diffracted at a small aperture. The diagram illustrates successive wavefronts.

After what time will some portion of the wavefront XY reach point P ?



- A $\frac{3\lambda}{2c}$ B $\frac{2\lambda}{c}$ C $\frac{3\lambda}{c}$ D $\frac{4\lambda}{c}$

$$v = f\lambda, \quad c = \frac{\lambda}{T} \quad \Rightarrow \quad T = \frac{\lambda}{c}$$

Since three wavefront spacings are involved:

$$t = 3T = \frac{3\lambda}{c}$$

Power per Unit Area and Diffraction

Question 24: Power per unit area

A light wave of amplitude A is incident normally on a surface of area S . The power per unit area reaching the surface is P .

The amplitude of the light wave is increased to $2A$. The light is then focused on to a smaller area $\frac{1}{3}S$.

What is the power per unit area on this smaller area?

A $36P$ B $18P$ C $12P$ D $6P$

Solution

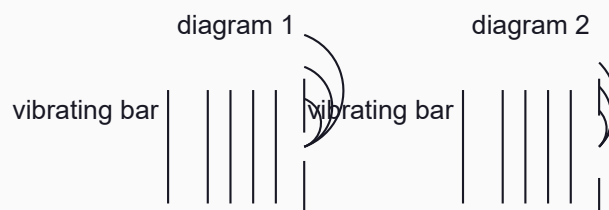
$$I = \frac{P}{A}, \quad I \propto a^2$$

Amplitude: $A \rightarrow 2A$, so intensity becomes $4P$. Area is reduced to $\frac{1}{3}S$, so power per unit area increases by another factor of 3.

$$P \times 4 \times 3 = 12P$$

Question 24: Diffraction in a ripple tank

Diagram 1 shows a ripple tank experiment in which plane waves are diffracted through a narrow slit. Diagram 2 shows the same tank with a slit of greater width.



Which action would cause the waves in diagram 1 to be diffracted less and so produce an emergent pattern closer to that shown in diagram 2?

A *increasing the frequency of vibration of the bar*

B increasing the speed of the waves by making the water in the tank deeper

C reducing the amplitude of vibration of the bar

D reducing the length of the vibrating bar

$\lambda \simeq$ slit size gives maximum diffraction. To diffract less, make λ smaller. Since $v = f\lambda$, increasing f decreases λ when v is constant.