

Superposition of Waves

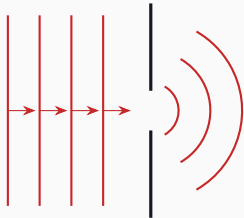
Chapter 12 / AS Level / Apollo Physics 9702



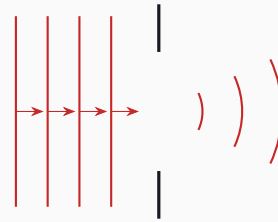
Diffraction of Waves

Diffraction of waves

- The spreading of the wave as it passes through a slit or over an edge.
- During diffraction, energy of the wave spreads out and its intensity falls.



slit size $\approx$ wavelength
Max diffraction



slit size $>$ wavelength
Less diffraction

narrower aperture $\Rightarrow$ greater spreading

Interference and Superposition

Interference

When two or more waves meet at a point, they overlap one another to produce a new resultant wave.

Conditions for interference

1. Waves must meet at a point.
2. Waves must have the same nature, i.e. both mechanical or both electromagnetic.
3. Waves should have the same frequency. A small difference in frequency can still cause interference.

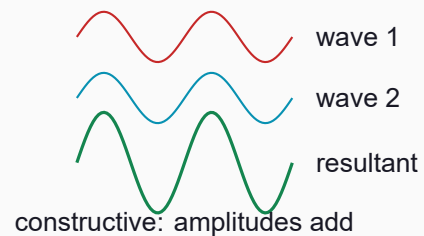
Principle of superposition

When two waves meet at a point in space, the resultant displacement of the new wave is the algebraic sum of displacements of interfering waves.

$$y_{\text{result}} = y_1 + y_2$$

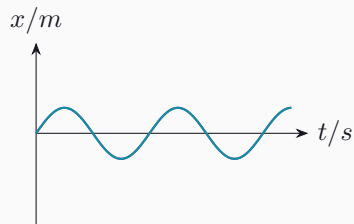
3 types of interference

1. Constructive interference
2. Destructive interference
3. Intermediate interference

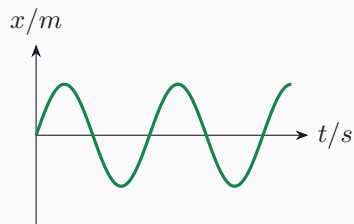


Constructive and Destructive Interference

Constructive interference

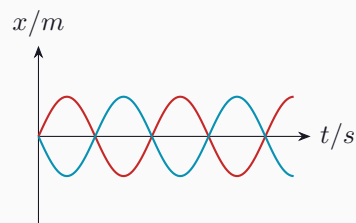


+2 and +2 add.



$$a + a = 2a$$

Destructive interference

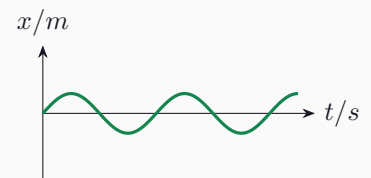
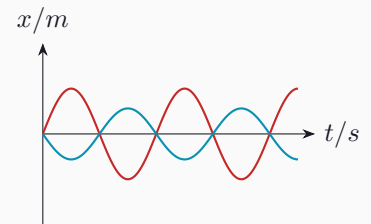


Pure destructive interference.



$$a - a = 0$$

Partially destructive



If unequal opposite displacements meet, the resultant is not zero but has a smaller amplitude.

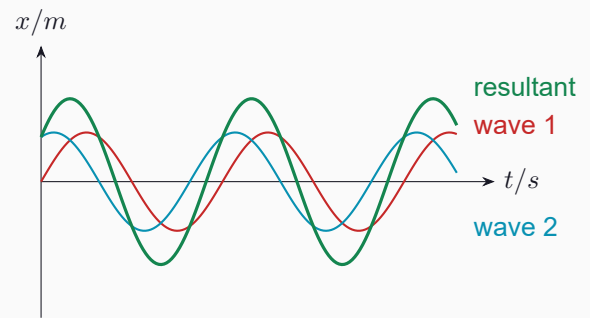
Intermediate Interference

Intermediate interference

Intermediate interference occurs when waves meet with a phase difference that is neither 0° nor 180° .

- It is not purely constructive.
- It is not purely destructive.
- The resultant amplitude lies between maximum and minimum values.

A_{result} depends on phase difference



Path Difference for Sources in Phase

Path difference

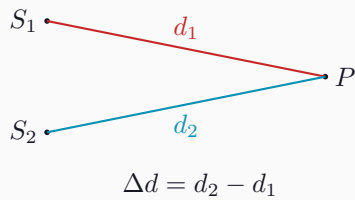
$$\text{path difference} = d_2 - d_1$$

For two in-phase sources:

$$\text{Constructive: path difference} = n\lambda \quad (n = 0, 1, 2, 3, \dots)$$

$$\text{Destructive: path difference} = \left(n + \frac{1}{2}\right)\lambda \quad (n = 0, 1, 2, \dots)$$

Equivalently, destructive happens for $0.5\lambda, 1.5\lambda, 2.5\lambda, \dots$



Example 1

$$\lambda = 2 \text{ cm}, d_1 = 10 \text{ cm}, d_2 = 14 \text{ cm}.$$

$$\Delta d = 14 - 10 = 4 \text{ cm} = 2\lambda$$

$$\text{Constructive interference: } a + a = 2a.$$

Example 2

$$\lambda = 2 \text{ cm}, d_1 = 10 \text{ cm}, d_2 = 15 \text{ cm}.$$

$$\Delta d = 15 - 10 = 5 \text{ cm} = 2.5\lambda$$

$$\text{Destructive interference: } a - a = 0.$$

Sources Vibrating Out of Phase

If the sources vibrate out of phase

Rules flip

When the sources are 180° out of phase:

Destructive: $\Delta d = n\lambda$ ($n = 0, 1, 2, 3, \dots$)

Constructive: $\Delta d = \left(n + \frac{1}{2}\right)\lambda$ ($n = 0, 1, 2, \dots$)



crest from one source is emitted with trough from the other

Example 3

$\lambda = 2 \text{ cm}$, $d_1 = 10 \text{ cm}$, $d_2 = 14 \text{ cm}$; sources vibrate out of phase.

$$\Delta d = 14 - 10 = 4 \text{ cm} = 2\lambda$$

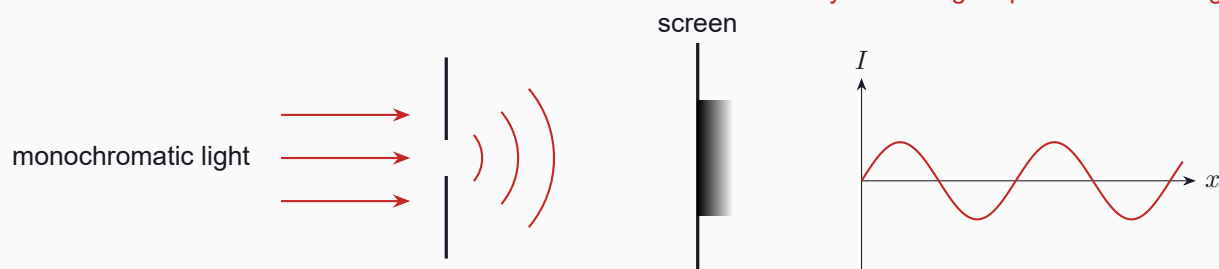
Because the sources are out of phase, an integral number of wavelengths gives destructive interference.

Destructive interference

Single Slit Diffraction

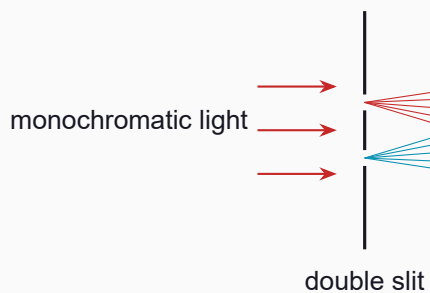
Single slit diffraction

- With no slit, the light continues without spreading.
- With a single slit, light spreads out.
- Intensity falls as light spreads over a larger area.



Double Slit Diffraction – Young's Double Slit Experiment

Young's double slit experiment

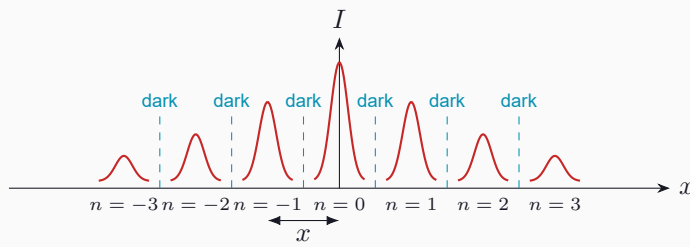


Why is the pattern produced?

- Light coming from two slits meets at different points on the screen.
 - At some points, light rays are in phase, so a **bright spot** is seen.
 - At some points, light rays are out of phase, so a **dark region** is seen.
- min
max

Bright fringe / maxima	constructive interference
Dark fringe / minima	destructive interference

Fringe Pattern and Formula



Bright spot

- Constructive interference.
- Path difference = $n\lambda$, where $n = 0, 1, 2, 3, \dots$
- Phase difference = 0° .

Dark spot

- Destructive interference.
- Path difference = $(n + \frac{1}{2})\lambda$, where $n = 0, 1, 2, \dots$
- Phase difference = 180° .

Pattern

1. Intensity falls sideways.
2. Symmetric pattern.
3. Equidistant spots.

Double slit formula

$$x = \frac{\lambda D}{a}$$

- a slit separation
 λ wavelength
 x fringe separation
 D slit-to-screen distance

Effects on Fringe Pattern I

1. Wavelength of light increased

Frequency decreases, e.g. blue colour to red.

- Fringe separation: *increases* because $x \propto \lambda$.
- Brightness of bright fringe: unchanged.
- Darkness of dark fringe: unchanged.

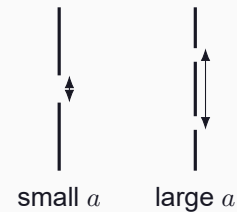
2. Slit-to-screen distance increased

- Fringe separation: *increases* because $x \propto D$.
- Brightness of bright fringe: decreases because the spot becomes large.
- Darkness of dark fringe: unchanged.

3. Slit separation increased

- Fringe separation: *decreases* because $x \propto \frac{1}{a}$.
- Brightness of bright fringe: unchanged.
- Darkness of dark fringe: unchanged.

$$x = \frac{\lambda D}{a}$$



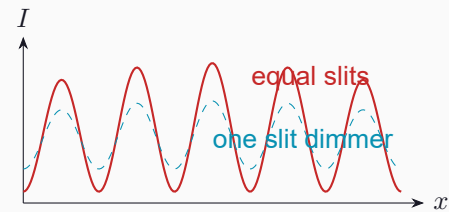
Effects on Fringe Pattern II

4. Intensity of light decreased

Examples: lamp dim, smaller slit size, lamp far away from the slits.

- Fringe separation: unchanged.
- Brightness of bright fringe: decreases.
- Darkness of dark fringe: unchanged.

If light coming from one of the two slits is decreased, overall light is reduced. The dark fringe becomes less dark because no pure destructive interference occurs.



5. Light from one slit decreased

- Fringe separation: unchanged.
- Brightness of bright fringe: decreases.
- Darkness of dark fringe: decreases.

Diffraction Grating

Problems with double slit

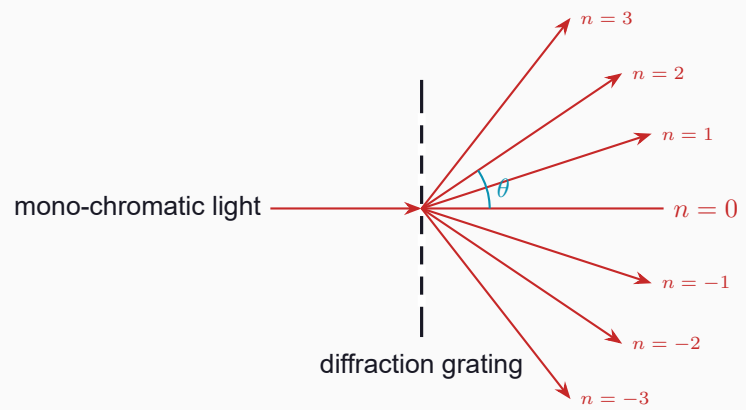
1. Very low intensity of spots.
2. Very small gap between spots.

Diffraction grating resolves these problems.

Diffraction grating

Grating comprises of hundreds of slits per unit length so that more light can pass and bright spots are produced on the screen.

- Maxima: bright fringe, constructive interference.
- Minima: dark fringe, destructive interference.



Grating Equation

Diffraction grating formula

$$n\lambda = d \sin \theta$$

n order of maximum

λ wavelength

d slit separation / grating spacing

θ angle between zero-order and n th-order maximum

Finding grating spacing

If a grating has 4000 lines in 0.01 m:

$$d = \frac{0.01}{4000} = 2.5 \times 10^{-6} \text{ m}$$

Example

First-order maximum at 10° with $d = 2.5 \times 10^{-6} \text{ m}$.

$$n\lambda = d \sin \theta$$

$$(1)\lambda = (2.5 \times 10^{-6}) \sin 10^\circ$$

$$\lambda = 4.34 \times 10^{-7} \text{ m} = 434 \text{ nm}$$

Bluelight.



Grating Calculations and Orders

Angle of third-order maximum

Using $\lambda = 4.34 \times 10^{-7} \text{ m}$ and $d = 2.5 \times 10^{-6} \text{ m}$:

$$3(4.34 \times 10^{-7}) = (2.5 \times 10^{-6}) \sin \theta$$

$$\theta = 31.4^\circ$$

Angle between 1st and 3rd order maxima

$$\theta_3 - \theta_1 = 31.4^\circ - 10^\circ = 21.4^\circ$$

Angle between both third-order maxima

$$2\theta_3 = 2(31.4^\circ) = 62.8^\circ$$

Highest order maximum visible

For the maximum possible angle, $\theta_{\max} = 90^\circ$:

$$n\lambda = d \sin 90^\circ$$

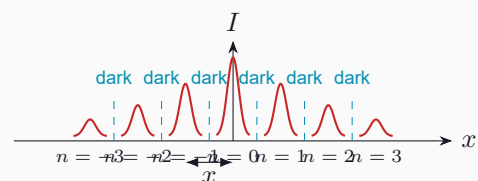
$$n = \frac{2.5 \times 10^{-6}}{4.34 \times 10^{-7}} = 5.76$$

So $n = 5$ is the highest visible order.

Total bright spots on screen

$$5 + 5 + 1 = 11$$

11 bright fringes, including zeroth order.



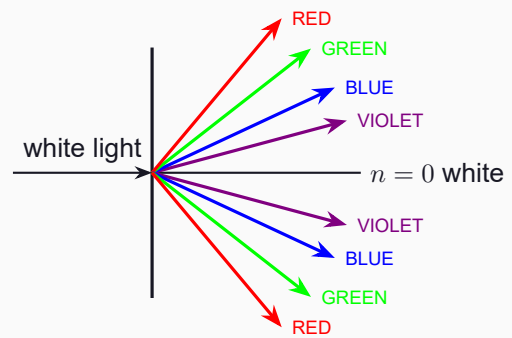
Diffraction of White Light

White light through a grating

When white light is passed through a diffraction grating, spectra are formed on either side of the central maximum.

- White light comprises colours from red to blue/violet.
- Every colour has its own wavelength.
- Upon diffraction, colours spread based on their wavelengths.

$$n\lambda = d \sin \theta \quad \Rightarrow \quad \theta \propto \lambda$$

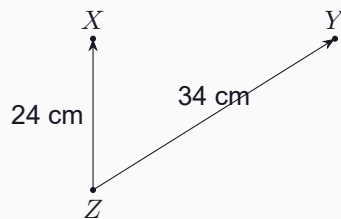


$n = 0$ is a white spot because all colours have zero path difference and zero phase difference there. The colours combine to form white light.

Exam Example: Water Waves and Path Difference

Question

Wave generators at points X and Y produce water waves of the same wavelength. At point Z , the waves from X have the same amplitude as the waves from Y . Distances $XZ = 24\text{ cm}$ and $YZ = 34\text{ cm}$. When the generators operate in phase, the amplitude at Z is zero. What could be the wavelength?



Solution

For zero amplitude with sources in phase:

$$\Delta d = \left(n + \frac{1}{2}\right) \lambda$$

$$\Delta d = 34 - 24 = 10\text{ cm}$$

Try options:

$$\lambda = 2\text{ cm} \Rightarrow n = 5 \quad (\text{constructive})$$

$$\lambda = 3\text{ cm} \Rightarrow n = 3.33$$

$$\lambda = 4\text{ cm} \Rightarrow 10 = 2.5\lambda \quad (\text{destructive})$$

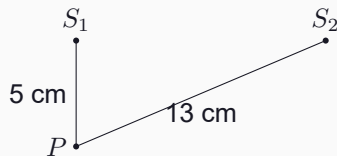
$$\lambda = 6\text{ cm} \Rightarrow n = 1.67$$

Answer : 4 cm.

Exam Example: Sources Out of Phase

Question

Two sources S_1 and S_2 oscillate with a phase difference of 180° . At P , distances are $S_1P = 5 \text{ cm}$ and $S_2P = 13 \text{ cm}$. Each source produces wavelength 2.0 cm and amplitude x_0 at P . What is the amplitude at P ?



Solution

Path difference:

$$\Delta d = 13 - 5 = 8 \text{ cm} = 4\lambda$$

Since the sources are out of phase, the rules flip. Integral path difference gives destructive interference.

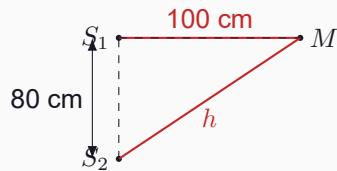
$$\boxed{\text{Amplitude at } P = 0}$$

If sources are out of phase, one source makes crest while the other makes trough at the same time, so the path-difference rule flips.

Two Sound Sources: Geometry

Setup

Two sources S_1 and S_2 of sound are situated 80 cm apart. A microphone M is 100 cm from S_1 along a line normal to S_1S_2 . The sources always vibrate in phase but have different amplitudes.



Path lengths

$$h^2 = p^2 + b^2$$

$$h = \sqrt{80^2 + 100^2} = 128 \text{ cm}$$

$$\Delta d = 128 - 100 = 28 \text{ cm} = 0.28 \text{ m}$$

As frequency is varied, maxima and minima of sound intensity are detected at M .

Two Sound Sources: Minima as Frequency Changes

Conditions for zero intensity at M

1. Phase difference between interfering waves must be 180° or π rad.
2. Waves must have the same amplitude.
3. Path difference between waves must be $(n + \frac{1}{2})\lambda$ where $n = 0, 1, 2, \dots$

Given

Speed of sound $v = 330 \text{ m s}^{-1}$ and path difference $\Delta d = 0.28 \text{ m}$.

$$\Delta d = \left(n + \frac{1}{2}\right) \lambda = \left(n + \frac{1}{2}\right) \frac{v}{f}$$

Frequencies for minima

For $n = 0.5$ notation from the notes:

$$0.28 = \frac{(0.5)(330)}{f} \Rightarrow f = 589 \text{ Hz}$$

For $n = 1.5$:

$$0.28 = \frac{(1.5)(330)}{f} \Rightarrow f = 1767 \text{ Hz}$$

For $n = 2.5$:

$$0.28 = \frac{(2.5)(330)}{f} \Rightarrow f = 2946 \text{ Hz}$$

For $n = 3.5$:

$$f = 4125 \text{ Hz} > 4.0 \text{ kHz}$$

Between 1.0 and 4.0 kHz, minima occur at 1767 Hz and 2946 Hz.

$Number = 2$

Fringe Spacing Scaling Question

Question

Light of wavelength 700 nm is incident on a pair of slits, forming fringes 3.0 mm apart on a screen. What is the fringe spacing when light of wavelength 350 nm is used and the slit separation is doubled?

$$\lambda = 700 \text{ nm}, \quad x = 3.0 \text{ mm}, \quad a = a$$

Final:

$$\lambda = 350 \text{ nm} = \frac{700}{2} \text{ nm}, \quad a = 2a$$

Solution

So

$$x = \frac{\lambda D}{a}$$
$$x \propto \lambda, \quad x \propto \frac{1}{a}$$

$$x_{\text{new}} = x \times \frac{1}{2} \times \frac{1}{2} = \frac{x}{4} = 0.75 \text{ mm}$$

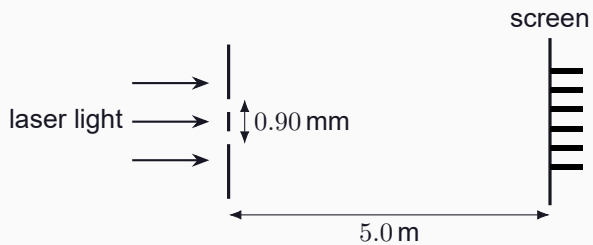
Initial:

0.75 mm

Young's Slits: Finding Wavelength

Question

A Young's slits experiment uses slit separation $a = 0.90 \text{ mm}$ and screen distance $D = 5.0 \text{ m}$. The pattern on the screen has $x = 3.0 \text{ mm}$ for the fringe separation. Find the wavelength.



Solution

$$x = \frac{\lambda D}{a}$$

From the ruler in the notes:

$$x = 30 \text{ mm} - 27 \text{ mm} = 3 \text{ mm}$$

$$3 \times 10^{-3} = \frac{\lambda(5)}{0.90 \times 10^{-3}}$$

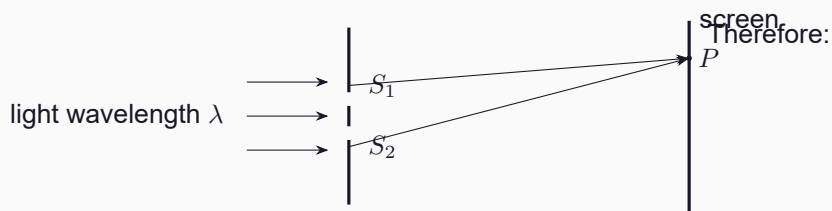
$$\lambda = 5.4 \times 10^{-7} \text{ m}$$

$$\boxed{5.4 \times 10^{-7} \text{ m}}$$

n th Dark Fringe in Young's Experiment

Question

Monochromatic light of wavelength λ is incident on two narrow slits S_1 and S_2 . A series of bright and dark fringes is observed on a distant screen. The n th dark fringe from the central bright fringe is observed at P . Which expression is correct for $S_2P - S_1P$?



Solution

For a dark fringe:

$$\Delta d = \left(m + \frac{1}{2}\right) \lambda, \quad m = 0, 1, 2, \dots$$

Counting the first dark fringe as $n = 1$:

$$n = 1 \Rightarrow \Delta d = \frac{1}{2} \lambda$$

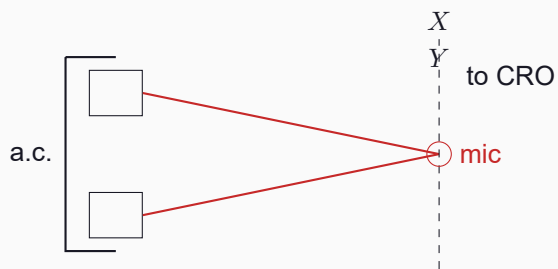
$$n = 2 \Rightarrow \Delta d = \frac{3}{2} \lambda$$

$$S_2P - S_1P = \left(n - \frac{1}{2}\right) \lambda$$

Correct option : C

Double Slit Experiment with Sound

Sound double slit / two speaker experiment

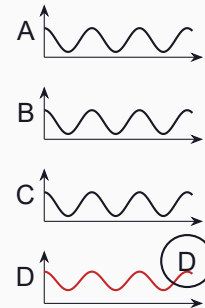


For an interference pattern to be produced, the waves must be coherent, i.e. they must have a constant phase difference.

- One light source and two slits are used so light from both slits has the same phase difference.
- Two loudspeakers and one power source are used so both speakers produce coherent waves.

Intensity graph choice

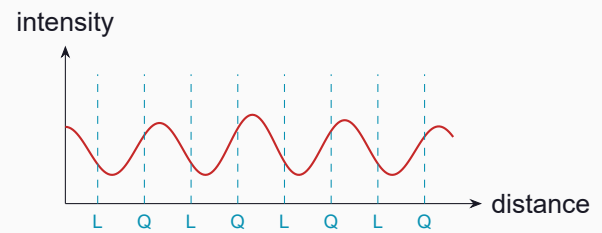
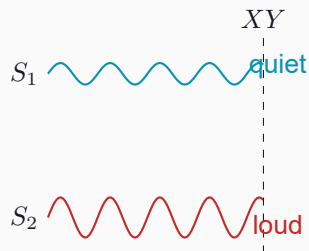
The notes mark the graph where intensity never falls to zero because the waves from the two speakers can have unequal amplitudes at the microphone.



Sound Interference: Loud and Quiet Positions

Unequal amplitudes at the detector

- Sound coming from source 1 travels less distance than sound from source 2.
- Hence the wave from S_2 has smaller amplitude than the wave from S_1 .
- At Y , no pure destructive interference occurs and some intensity is still left.



- Loud positions: constructive interference.
- Quiet positions: partially destructive interference.
- Middle constructive interference has $n = 0$.

Microwave Interference Exam Question

Conditions for maxima in a microwave interference pattern

1. Phase difference = 0 or waves in phase.
2. Path difference = $n\lambda$, where $n = 0, 1, 2, 3, \dots$
3. Waves must meet at a point.

Microwave wavelength

$$v = f\lambda$$

$$3.0 \times 10^8 = (12 \times 10^9)\lambda$$

$$\lambda = 0.025 \text{ m}$$

Detector moved from O to P

At O , path difference is zero, so maximum. At P :

$$\Delta d = 0.90 - 0.75 = 0.15 \text{ m}$$

$$\frac{0.15}{0.025} = 6$$

So the detector passes through alternating maxima and minima. The notes describe the reading as increasing and decreasing periodically, including a maximum at O and at P .

Replacing microwaves with visible light

To observe the interference pattern with coherent light:

1. Decrease the size of the slit to the size of wavelength of light.
2. Decrease the gap between the slits.

Coherence, Interference, Frequency and Laser Fringes

Coherence and interference

- **Coherence:** waves have a constant phase difference throughout their motion.
- **Interference:** when two waves meet at a point, they produce a new wave which is the resultant of both.

Frequency from graph

If the speed of waves is 240 m s^{-1} and the wavelength from the graph is 3.2 m :

$$v = f\lambda$$

$$240 = f(3.2)$$

$$f = 75 \text{ Hz}$$

Laser double slit calculation

Given $\lambda = 546 \text{ nm}$, slit separation $a = 0.13 \text{ mm}$, screen distance $D = 85 \text{ cm}$.

$$x = \frac{\lambda D}{a}$$

$$x = \frac{546 \times 10^{-9} \times 0.85}{0.13 \times 10^{-3}} = 3.57 \times 10^{-3} \text{ m}$$

If B corresponds to path difference 2.5λ and A to path difference zero:

$$AB = 2.5x = 8.93 \times 10^{-3} \text{ m}$$

$$AB \approx 8.9 \times 10^{-3} \text{ m}$$

Laser changed to red light

Red light has a larger wavelength than 546 nm , therefore fringe separation increases because $x \propto \lambda$.