

Stationary Waves

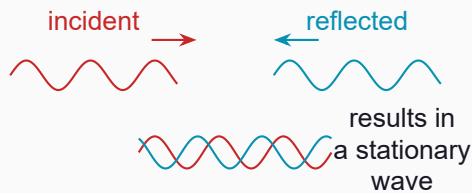
Chapter 13 / AS Level / Apollo Physics 9702



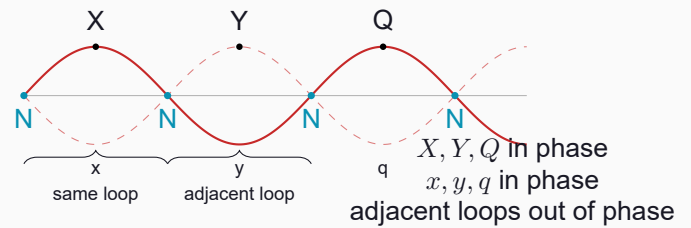
Stationary Waves / Standing Waves

Formation of a stationary wave

- 2 waves
- same frequency
- travelling in opposite direction
- interfere

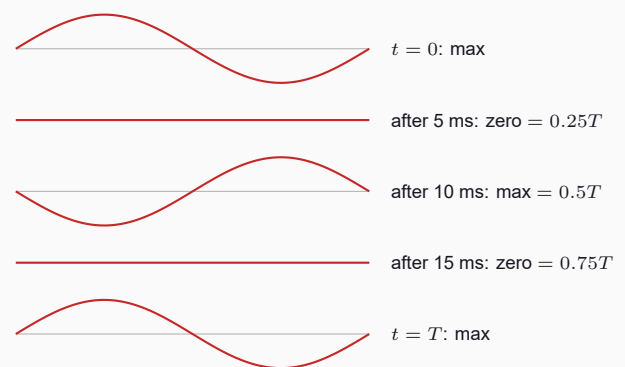


Nodes, antinodes and phase



Time period for a stationary wave

If $T = 20$ ms:



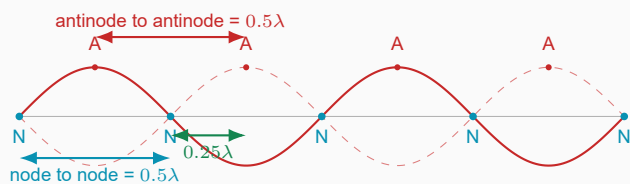
The pattern goes **max** → **zero** → **max inverted** → **zero** → **max**.

Progressive vs stationary

Progressive	Stationary
Travelling waveform.	Standing waveform.
Net transfer of energy.	No net transfer of energy.
Crest/trough or compression/rarefactions travel through the medium.	Nodes : no vibration due to destructive interference.
Every point has same amplitude.	Antinodes : maximum vibration due to constructive interference.
Points have different phase angles.	Points have different amplitudes. Points between nodes are in phase, $\Delta\phi = 0^\circ$. Points adjacent to nodes are out of phase, $\Delta\phi = 180^\circ$.

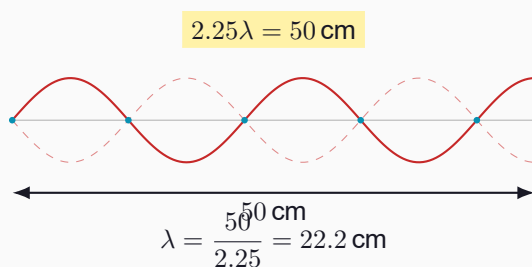
Wavelength of a Stationary Wave

Distances in the pattern



- **Node to node** = 0.5λ .
- **Antinode to antinode** = 0.5λ for consecutive antinodes.
- **Node to antinode** = 0.25λ .

Counting quarters



Reflection at an end

Fixed / closed end	Free / open end
Node Out of phase	Antinode In phase

Rule to remember

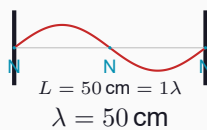
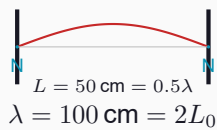
Fixed/closed end $\Rightarrow$ node.
Free/open end $\Rightarrow$ antinode.

Phase on reflection

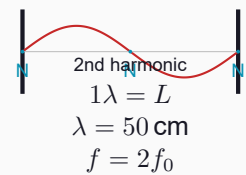
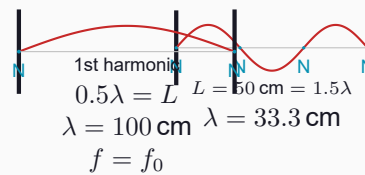
Closed end: reflected wave is inverted.
Open end: reflected wave is not inverted.

Both Ends Fixed

Changing length



Changing frequency for $L = 50 \text{ cm}$



Every next stationary wave is half wavelength (0.5λ) extra.

Example

Two consecutive stationary waves were formed at 48 cm and 56 cm respectively in one wire.

$$56 - 48 = 0.5\lambda, \quad 8 = 0.5\lambda, \quad \boxed{\lambda = 16 \text{ cm}}.$$

Whole number multiples

For both ends fixed:

$$\boxed{L_n = \frac{n\lambda}{2}}, \quad \boxed{f_n = nf_0} \quad (n = 1, 2, 3, \dots)$$

Examples of allowed lengths/frequencies:

Mode	Length pattern	Frequency pattern	Harmonic
1	L_0	f_0	1 st
2	$2L_0$	$2f_0$	2 nd
3	$3L_0$	$3f_0$	3 rd
4	$4L_0$	$4f_0$	4 th

A similar pattern is obtained for both ends open.

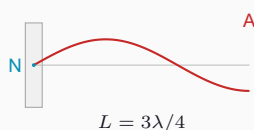
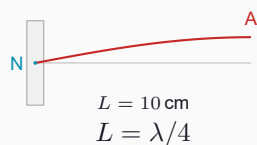
WHOLE NUMBER MULTIPLE

$$L_n = nL_0, \quad f_n = nf_0, \quad \lambda_n = \frac{\lambda_0}{n}.$$

Length	6 cm	12 cm	18 cm	24 cm
Frequency	15 Hz	30 Hz	45 Hz	60 Hz
Mode	f_0	$2f_0$	$3f_0$	$4f_0$

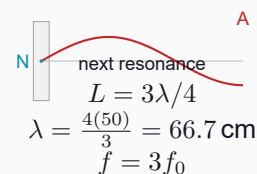
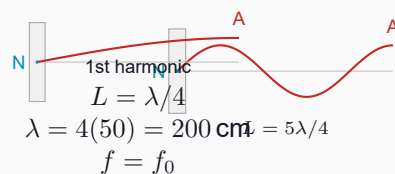
One End Closed, One End Open

Changing length



Every next stationary wave is half wavelength (0.5λ) extra.

Changing frequency for $L = 50 \text{ cm}$



Only odd harmonics occur. If $f_0 = 150 \text{ Hz}$:

150 Hz, 450 Hz, 750 Hz, ...

Odd number multiples

For one end closed and one end open:

$$L_n = \frac{(2n-1)\lambda}{4},$$

$$f_n = (2n-1)f_0$$

where $n = 1, 2, 3, \dots$

Length	8 cm	24 cm	40 cm	56 cm
Pattern	L_0	$3L_0$	$5L_0$	$7L_0$
Frequency	f_0	$3f_0$	$5f_0$	$7f_0$

ODD NUMBER MULTIPLES

Closed end: node

Air cannot vibrate freely at the closed end.

Open end: antinode

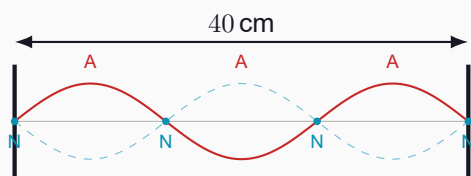
Air vibrates with maximum displacement at the open end.

Speed of a Stationary Wave

Speed rule

- When the incident wave is reflected at one end and they both interfere, they form a stationary wave.
- Speed of the stationary wave is either the speed of the incident or reflected wave.

$$v = f\lambda$$



The diagram contains 1.5λ in 40 cm.

$$1.5\lambda = 40 \text{ cm} \Rightarrow \lambda = 26.6 \text{ cm.}$$

If the generator frequency is f , then

$$v = f\lambda.$$

Quick method summary

Setup	Allowed modes
Both ends fixed	$L = \frac{n\lambda}{2}$, so $f_n = nf_0$. Whole number multiples.
Both ends open	Same pattern as both ends fixed: $f_n = nf_0$.
One end closed, one end open	$L = \frac{(2n-1)\lambda}{4}$, so $f_n = (2n-1)f_0$. Odd number multiples only.

Node/antinode spacing checklist

$$\text{Node to node} = \lambda/2$$

$$\text{Antinode to antinode} = \lambda/2$$

Exam trigger words

- stationary / standing wave
- node, antinode
- fixed end / closed end / open end
- resonance, harmonics
- successive resonant lengths or frequencies