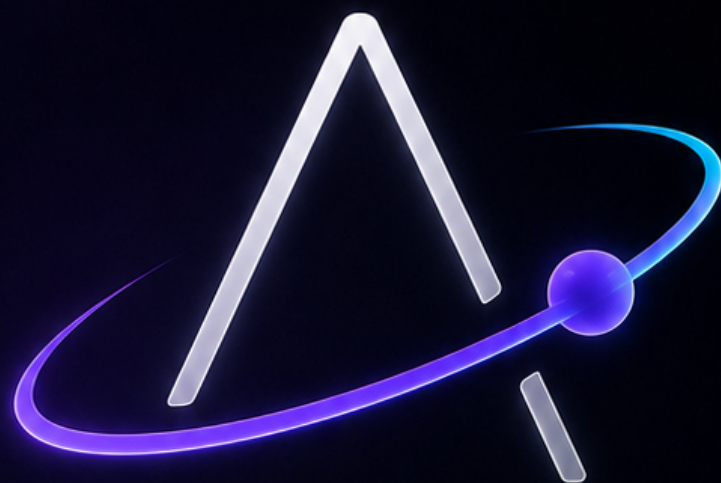




Apollo Physics 9702

Cambridge International AS Level Notes

Apollo vertical notes edition

Physical Quantities and Kinematics

Chapter 01 / AS Level / Apollo Physics 9702



Physical Quantities P1

Course structure and recommended resources.

AS Level papers

P1	MCQ	40 marks, 1 hr 15 min
P2	Theory	60 marks, 1 hr 15 min
P3	Practical	40 marks, 2 hours

A Level / A2 papers

P4	Theory	100 marks
P5	Planning and Analysis	30 marks

Reference books written in the notes

- Advanced Physics by David Sang.
- Physics by Mike Crundell.
- Pacific Physics by Poh Liong Yong and others.
- Understanding Physics by Jim Breithaupt.

Physical Quantities P2

Definitions, seven base quantities, SI base units, derived quantities and first derived-unit conversions.

Physical vs non-physical quantities

Physical quantities: quantities that can be measured/quantified.

Non-physical quantities: quantities that cannot be measured, e.g. feelings, pain, taste and spirituality.

Seven basic physical quantities

No.	Quantity	SI base unit
1	length	metre, m
2	mass	kilogram, kg
3	time	second, s
4	thermodynamic temperature	kelvin, K
5	electric current	ampere, A
6	luminous intensity	candela, cd
7	amount of substance	mole, mol

Derived quantities

Quantities made up of base quantities are called **derived quantities**; their units are called **derived units**.

Examples: force $\rightarrow$ newton, energy $\rightarrow$ joule, power $\rightarrow$ watt.

Representing derived units in SI base units

$$\text{Area: } A = l \times b = m \times m = m^2, \quad A = \pi r^2 = m^2,$$

$$\text{Volume: } V = lbh = m^3, \quad V = \pi r^2 h = m^3,$$

$$\text{Density: } \rho = \frac{m}{V} = \frac{kg}{m^3} = kg\,m^{-3},$$

$$\text{Speed: } v = \frac{d}{t} = m\,s^{-1},$$

$$\text{Acceleration: } a = \frac{\Delta v}{\Delta t} = \frac{m\,s^{-1}}{s} = m\,s^{-2}.$$

Physical Quantities P3

Mechanics/electricity derived units: force, pressure, work, energy, power, charge, voltage and resistance.

6. Force

$$F = ma = (kg)(m s^{-2}) = kg m s^{-2}.$$

$$N = kg m s^{-2}.$$

7. Pressure

$$P = \frac{F}{A} = \frac{kg m s^{-2}}{m^2} = kg m^{-1} s^{-2}$$

$$\text{Also } P = \rho gh = (kg m^{-3})(m s^{-2})(m).$$

8. Work

$$W = F \times s = (kg m s^{-2})(m) = kg m^2 s^{-2}$$

9. Energy

$$E_p = mgh = kg m^2 s^{-2}, \quad E_k = \frac{1}{2}mv^2$$

10. Power

$$P = \frac{W}{t} = kg m^2 s^{-3} = W.$$

11. Charge

$$Q = It = A s.$$

12. Voltage

$$V = \frac{W}{Q} = \frac{kg m^2 s^{-2}}{A s} = kg m^2 s^{-3} A^{-1}.$$

13. Resistance

$$R = \frac{V}{I} = kg m^2 s^{-3} A^{-2}.$$

Physical Quantities P4

Finding SI units of unknown variables in equations.

$$Q = mc\Delta T$$

Given Q = heat energy, m = mass, c = specific heat capacity, ΔT = change in temperature.

$$c = \frac{Q}{m\Delta T} = \frac{kg\,m^2s^{-2}}{kg\,K} = m^2s^{-2}K^{-1}.$$

$$R = \rho l / A$$

Given R = resistance, ρ = resistivity, l = length, A = cross-sectional area.

$$\rho = \frac{RA}{l} = \frac{(kg\,m^2s^{-3}A^{-2})(m^2)}{m} = kg\,m^3s^{-3}A^{-2}.$$

Generic force equation

For any equation, first write SI base units of the known variables. Then rearrange for the unknown constant/variable and cancel powers.

$$[F] = kg\,m\,s^{-2}, \quad [m] = kg,$$

then substitute and solve powers exactly.

Coulomb-style equation

For $F = kq_1q_2/r^2$,

$$k = \frac{Fr^2}{q_1q_2} = \frac{(kg\,m\,s^{-2})(m^2)}{(A\,s)^2} = kg\,m^3s^{-4}A^{-2}.$$

Physical Quantities P5

Proving a constant/variable is unitless.

$$P = k\rho v^2$$

Given: P = pressure, ρ = density, v = speed. Prove k is unitless.

$$k = \frac{P}{\rho v^2} = \frac{kg\,m^{-1}s^{-2}}{(kg\,m^{-3})(m\,s^{-1})^2} = \frac{kg\,m^{-1}s^{-2}}{kg\,m^{-1}s^{-2}} = 1.$$

$$P = k\rho Av^3$$

Given: P = power, ρ = density, A = area, v = velocity. Prove k is unitless.

$$k = \frac{P}{\rho Av^3} = \frac{kg\,m^2s^{-3}}{(kg\,m^{-3})(m^2)(m\,s^{-1})^3} = 1.$$

Hydrostatic pressure

For $p = \rho gh$,

$$[\rho gh] = (kg\,m^{-3})(m\,s^{-2})(m) = kg\,m^{-1}s^{-2} = [p].$$

The equation is homogeneous.

General rule

A constant is dimensionless/unitless when the SI base units of the numerator and denominator are identical after substitution.

Physical Quantities P6

Homogeneous equations and extracting unknown powers.

Homogeneous equation

An equation is **homogeneous** if every term has the same SI base units.

$$v = u + at$$

Units of u and v are $m\,s^{-1}$, and

$$[at] = (m\,s^{-2})(s) = m\,s^{-1}.$$

So the equation is homogeneous.

Example: pressure relation

$$P = \frac{F}{A} \Rightarrow [P] = kg\,m^{-1}s^{-2}.$$

If another term is added to pressure, it must also have units $kg\,m^{-1}s^{-2}$.

Finding powers

Suppose $T = k\rho^a l^b v^c$. Replace each variable by SI base units and equate powers of kg , m , s , and A on both sides.

Key method

1. Write units of LHS.
2. Write units of RHS in powers.
3. Equate powers of each base quantity.
4. Solve simultaneous equations.

Physical Quantities P7

Worked power-of-variables examples.

Example form

If $P = k\rho^x v^y A^z$, then

$$kg\,m^2\,s^{-3} = (kg\,m^{-3})^x (m\,s^{-1})^y (m^2)^z.$$

So

$$kg^1 m^2 s^{-3} = kg^x m^{-3x+y+2z} s^{-y}.$$

Equate: $x = 1$, $-y = -3 \Rightarrow y = 3$, and $-3(1) + 3 + 2z = 2 \Rightarrow z = 1$.

$$P = k\rho A v^3$$

Pendulum-style relation

For $T = kl^x g^y$,

$$s = m^x (m\,s^{-2})^y = m^{x+y} s^{-2y}.$$

Equating powers:

$$-2y = 1 \Rightarrow y = -\frac{1}{2}, \quad x + y = 0 \Rightarrow x = \frac{1}{2}.$$

$$T = k\sqrt{\frac{l}{g}}$$

Physical Quantities P8

Measuring instruments, range and precision.

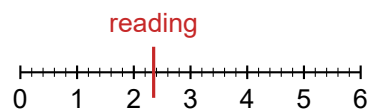
Length instruments

Instru- ment	Range	Precision
Trundle wheel	several metres	1 cm
Mea- suring tape	several metres	0.1 cm
Metre rule	1 m	0.1 cm
Vernier caliper	20–25 cm	0.01 cm (0.1 mm)
Microme- ter screw gauge	0–2.5 cm	0.001 cm (0.01 mm)

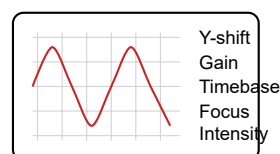
Other instruments

Mass	electronic balance, beam balance
Weight	Newton meter, spring balance, compression balance
Time	stopwatch, clock, CRO
Temperature	liquid-in-glass thermometer, thermocouple, resistance thermometer
Electric current	ammeter, galvanometer, multi-meter
Voltage	voltmeter, CRO
Resistance	ohmmeter, multimeter
Volume	measuring cylinder, burette

Reading scale diagrams



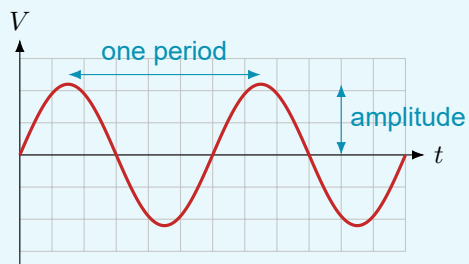
CRO layout



Physical Quantities P9

Determining information from a CRO using settings.

CRO wave reading



Example calculation

If the timebase is 50 ms div^{-1} and one complete cycle occupies 4 divisions,

$$T = 4(50 \text{ ms}) = 200 \text{ ms} = 0.20 \text{ s}, \quad f = \frac{1}{T} = 5 \text{ Hz}.$$

If the gain is 2 V div^{-1} and amplitude is 3 divisions,

$$V_{\text{peak}} = 3(2 \text{ V}) = 6 \text{ V}.$$

From the topic

The handwritten page links CRO reading to: frequency, time period, amplitude and voltage gain. The same idea is used for wave measurements: count divisions, multiply by the setting, and then use $f = 1/T$.

Cathode rays

Cathode rays are electron beams. In a CRO, the signal makes the spot move vertically while the timebase sweeps it horizontally.

Physical Quantities P10

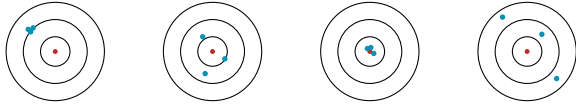
Precision and accuracy.

Definitions

Precision: how close repeated values are to each other.

Accuracy: how close a value is to the true value.

Target diagrams



Precise not accurate Accurate not precise Precise and accurate Neither

Reducing random error

Take repeated readings and average them. Random error is reduced because readings scatter above and below the true value.

Reducing systematic error

Remove or correct the cause: zero error, faulty calibration, parallax, reaction-time bias, etc.

Physical Quantities P11

Errors: systematic error and random error.

Error

Error is obtained by:

$$\text{error} = \text{experimental value} - \text{true value}.$$

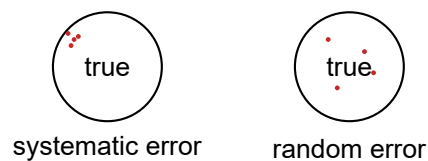
Systematic error

This occurs due to a fault in apparatus, method, environment or assumptions. It causes all values to be too high or too low. Examples: zero error, miscalibrated instrument, parallax error.

Random error

Random error causes scatter. It may be due to human reaction time, vibrations, fluctuating readings or difficulty judging a scale.

Original-style error diagram



Percentage error

$$\% \text{ error} = \frac{|\text{measured} - \text{true}|}{\text{true}} \times 100.$$

Physical Quantities P12

Random errors, uncertainty, and gradients.

Uncertainty

Uncertainty tells the possible range in which the actual value may lie.

$$x = (x_0 \pm \Delta x) \text{ unit}$$

Repeated readings

If repeated readings are available,

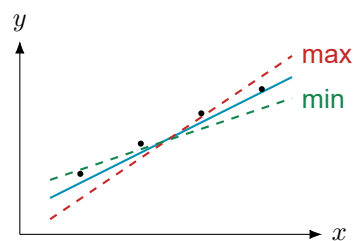
$$\Delta x = \frac{\text{maximum value} - \text{minimum value}}{2}.$$

The best value is usually the mean.

Single reading

For a single analogue reading, uncertainty is usually half the smallest division. For a digital instrument, uncertainty is often ± 1 in the last displayed digit.

Maximum and minimum gradients



Fractional and percentage uncertainty

$$\frac{\Delta x}{x}, \quad \frac{\Delta x}{x} \times 100\%.$$

Physical Quantities P13

Writing values with uncertainty, significant figures and arithmetic.

Rules

- Round uncertainty to one significant figure, unless the first digit is 1 or 2.
- Round the principal value to the same decimal place as the uncertainty.
- Final answers should not imply more precision than the measurement permits.

Example

If a length is read as 4.4 cm on a ruler with 0.1 cm divisions,

$$l = (4.4 \pm 0.1)\text{ cm}.$$

Addition/subtraction

For $y = a + b$ or $y = a - b$,

$$\Delta y = \Delta a + \Delta b.$$

Absolute uncertainties add.

Multiplication/division

For $y = ab$ or $y = a/b$,

$$\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}.$$

Percentage uncertainties add.

Powers

For $y = a^n$,

$$\frac{\Delta y}{y} = n \frac{\Delta a}{a}, \quad \Delta y\% = n\Delta a\%.$$

Calculator vs final answer

The calculator may show many digits, but the answer must be rounded using uncertainty and significant-figure rules.

Physical Quantities P14

Compound uncertainties and worked examples.

Coefficient multiplied by a variable

If

$$y = ka,$$

where k is an exact number, then

$$\Delta y = k\Delta a.$$

Example: circumference

$$C = \pi d, \quad \Delta C = \pi \Delta d.$$

If $d = (4.6 \pm 0.1) \text{ cm}$, then

$$C = 14.5 \text{ cm}, \quad \Delta C = 0.3 \text{ cm}.$$

$$C = (14.5 \pm 0.3) \text{ cm}$$

Area of a rectangle

If $A = lb$, then

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}.$$

Difference example

If $y = a - b$, then absolute uncertainty adds:

$$\Delta y = \Delta a + \Delta b.$$

This is why subtracting similar readings can produce large percentage uncertainty.

Physical Quantities P15

Uncertainty examples with cylinder and sphere.

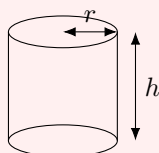
Cylinder

For a cylinder,

$$V = \pi r^2 h.$$

If r and h have uncertainties,

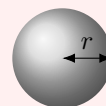
$$\frac{\Delta V}{V} = 2 \frac{\Delta r}{r} + \frac{\Delta h}{h}.$$



Sphere

For a sphere,

$$V = \frac{4}{3} \pi r^3, \quad \frac{\Delta V}{V} = 3 \frac{\Delta r}{r}.$$



Physical Quantities P16

Power-over-variable uncertainty and final examples.

Power over a variable

For $y = a^n$,

$$\Delta y\% = n\Delta a\%.$$

For area of a circle,

$$A = \pi r^2 \Rightarrow \Delta A\% = 2\Delta r\%.$$

Diameter to area

If $d = (2.56 \pm 0.01) \text{ cm}$, then

$$A = \pi \left(\frac{d}{2}\right)^2 = 5.15 \text{ cm}^2.$$

Since $A \propto d^2$,

$$\Delta A\% = 2 \left(\frac{0.01}{2.56} \times 100 \right) = 0.78\%.$$

$$\Delta A = 0.0078(5.15) = 0.04 \text{ cm}^2.$$

$$A = (5.15 \pm 0.04) \text{ cm}^2$$

Pendulum method for g

$$T = 2\pi\sqrt{\frac{l}{g}} \Rightarrow g = \frac{4\pi^2 l}{T^2}.$$

Thus

$$\Delta g\% = \Delta l\% + 2\Delta T\%.$$

Rounding final value

The uncertainty and value are finally rounded to consistent decimal places, for example

$$g = (10.3 \pm 0.3) \text{ m s}^{-2}.$$

Kinematics K1

Distance/displacement, speed/velocity, sign convention and cliff-position diagram.

Distance vs displacement

Distance	Displacement
Length of any path between two points.	Shortest distance between two points.
SI unit: metre (m).	SI unit: metre (m).
Scalar.	Vector.
Never negative; can only increase or stay constant.	Can be positive, negative or zero; can increase or decrease.

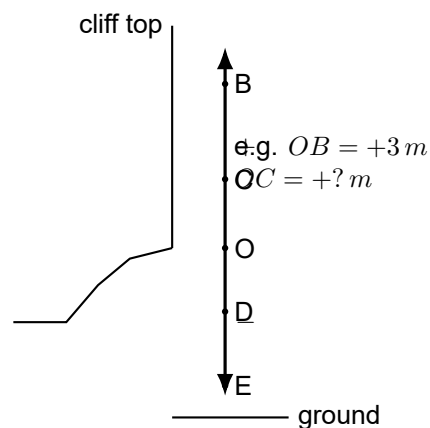
Sign of displacement

The sign tells the position of the object with respect to the origin/reference point.

Speed vs velocity

Speed	Velocity
Rate of change of distance.	Rate of change of displacement.
$v = d/t$.	$v = s/t$.
SI unit: $m\ s^{-1}$.	SI unit: $m\ s^{-1}$.
Scalar.	Vector.

Cliff reference diagram



Sign of velocity

The sign of velocity tells the direction of motion of the object.

Kinematics K2

Acceleration and graph meanings: displacement-time, velocity-time and acceleration-time.

Acceleration

Acceleration is the rate of change of velocity.

$$a = \frac{\Delta v}{\Delta t} = \frac{v - u}{t}, \quad [a] = m s^{-2}.$$

It is a vector. Acceleration may be caused by change in magnitude of velocity or change in direction.

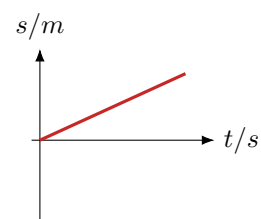
Collision example

A ball hits the floor with velocity $-10 m s^{-1}$ and rebounds with velocity $+8 m s^{-1}$ in $0.1 s$.

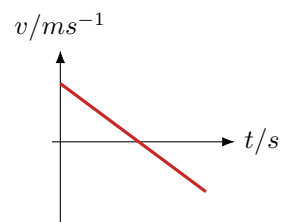
$$a = \frac{v - u}{t} = \frac{8 - (-10)}{0.1} = 180 m s^{-2}.$$

$180 m s^{-2}$ upward

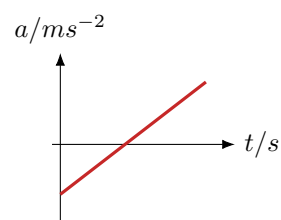
Graph rules



Displacement-time: gradient = v , area meaningless



Velocity-time: gradient = a , area = s



Acceleration-time: area = Δv

Kinematics K3

Area under acceleration-time graph, types of graph gradients, first displacement-time cases.

Example: area under $a-t$ graph

A car travelling at 5 m s^{-1} undergoes non-uniform acceleration. Area under the graph gives change in velocity.

$$\Delta v = \frac{1}{2}(10)(4) = 20 \text{ m s}^{-1}.$$

Since $\Delta v = v - u$,

$$20 = v - 5 \Rightarrow \boxed{v = 25 \text{ m s}^{-1}}.$$

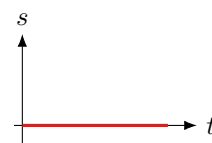
Types of graphs

Line graph: constant/uniform gradient.

Curve graph: variable gradient.

More vertical $\Rightarrow$ greater gradient; more horizontal $\Rightarrow$ smaller gradient.

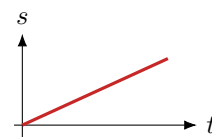
Displacement-time graph cases



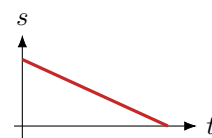
At rest at origin: $m = v = 0$



At rest ahead/behind origin



Constant velocity away from origin

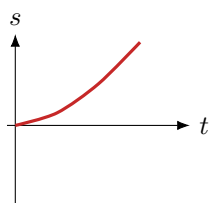


Constant velocity towards origin

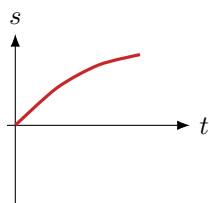
Kinematics K4

More displacement-time shapes and object thrown upwards.

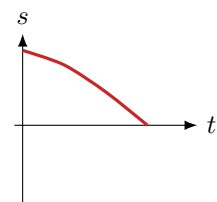
Changing velocity from s - t graphs



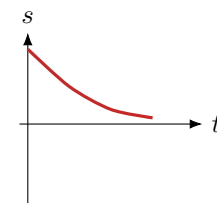
Gradient increases: object speeding up away



Gradient decreases: object slowing down away



More negative gradient: speeding up towards origin

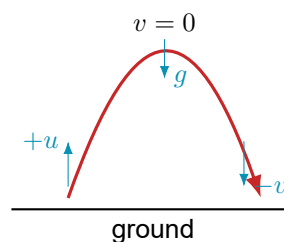


Less negative gradient: slowing down towards origin

Object thrown upwards

At release, velocity is upward and positive. During upward motion, acceleration is downward due to gravity. At maximum height, velocity is zero but acceleration is still g downward. Then the object moves downward with negative velocity and increasing speed.

Thrown-up sketch



Kinematics K5

Thrown-up graphs and rebound/height-time sketches.

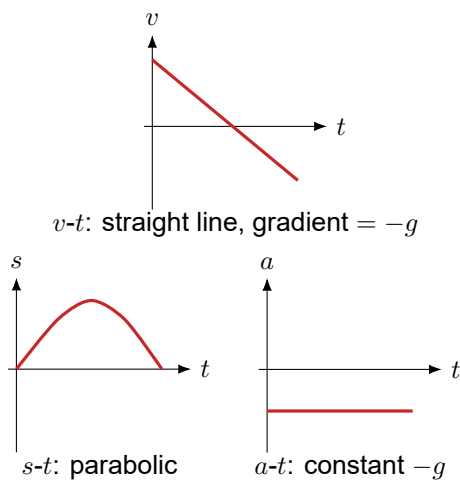
Vertical throw graphs

With upward positive:

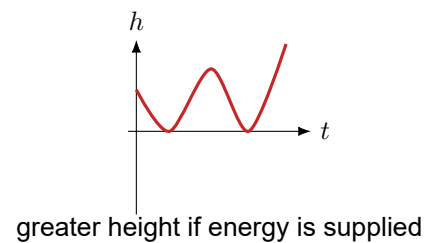
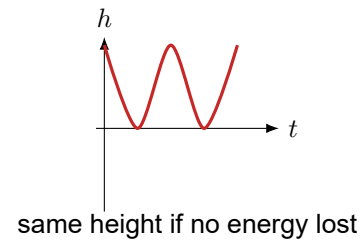
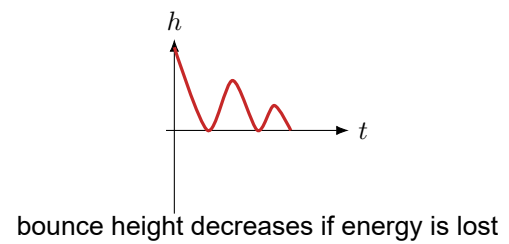
$$v = u - gt, \quad a = -g.$$

The v - t graph is a straight line with negative gradient.
The s - t graph rises with decreasing gradient, reaches a maximum, then curves downward.

Graph set



Bouncing ball height-time sketches



Kinematics K6

Further displacement-time examples and calculator style worked examples.

Reading velocity from s - t

Velocity is the gradient of a displacement-time graph:

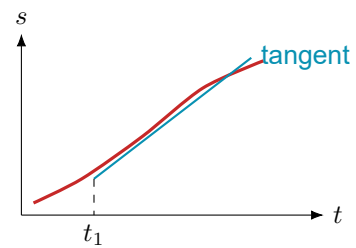
$$v = \frac{\Delta s}{\Delta t}.$$

For a curve, draw a tangent at the point and calculate the tangent gradient.

Common interpretation

- Horizontal line: object at rest.
- Straight line with positive gradient: constant positive velocity.
- Straight line with negative gradient: constant negative velocity.
- Curve getting steeper: speed increasing.
- Curve getting flatter: speed decreasing.

Tangent on a curve



Kinematics K7

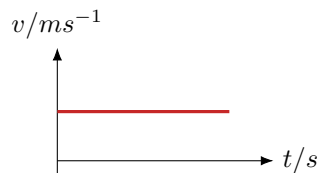
Velocity-time graph cases.

Velocity-time facts

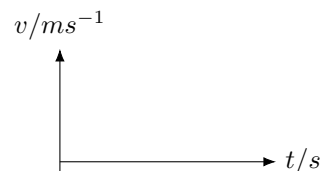
gradient = a , area under graph = displacement.

Positive area is positive displacement; area below the time axis is negative displacement.

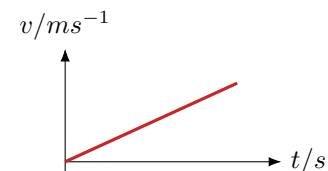
Core v - t cases



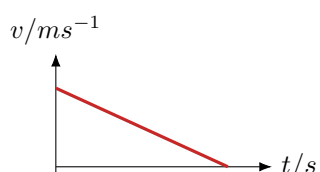
constant positive velocity



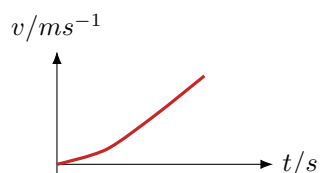
constant negative velocity



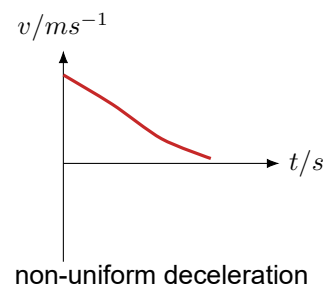
uniform positive acceleration



uniform deceleration



non-uniform acceleration



Kinematics K8

Area under velocity-time graph and non-uniform examples.

Area gives displacement

For a straight-line v - t graph:

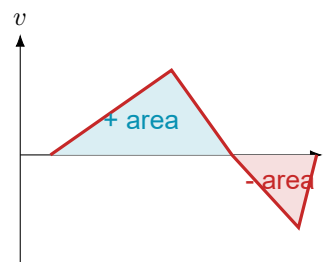
$$s = \text{area} = \frac{1}{2}(a + b)h \quad \text{or} \quad s = \frac{1}{2} \times \text{base} \times \text{height}.$$

Acceleration from gradient

$$a = \frac{\Delta v}{\Delta t}.$$

A steeper line gives greater magnitude of acceleration.

Positive and negative displacement



Lift/ball idea

When an object is released or thrown inside a moving lift, velocities are relative to the ground. Always set a sign convention first.

Kinematics K9

Equations of uniformly accelerated motion.

Equation 1

$$a = \frac{v - u}{t} \Rightarrow \boxed{v = u + at}.$$

Equation 2

From average velocity under uniform acceleration:

$$s = \left(\frac{u + v}{2} \right) t.$$

Using $v = u + at$,

$$\boxed{s = ut + \frac{1}{2}at^2}.$$

Equation 3

Using $u = v - at$ in $s = (u + v)t/2$:

$$\boxed{s = vt - \frac{1}{2}at^2}.$$

Equation 4

Eliminate t :

$$\boxed{v^2 = u^2 + 2as}.$$

No acceleration

If $a = 0$,

$$s = vt = ut, \quad v = u.$$

Sign convention

Choose one direction positive. All velocities, accelerations and displacements must be assigned signs accordingly.

Kinematics K10

SUVAT examples for vertical motion.

Object thrown vertically upwards

At the highest point, $v = 0$. With upward positive, acceleration $a = -g$.

$$v^2 = u^2 + 2as \Rightarrow 0 = u^2 - 2gh,$$

$$h = \frac{u^2}{2g}.$$

Time to highest point

$$v = u + at \Rightarrow 0 = u - gt, \quad t = \frac{u}{g}.$$

Object dropped from rest

For downward positive, $u = 0$, $a = g$:

$$s = \frac{1}{2}gt^2, \quad v = gt, \quad v^2 = 2gs.$$

Common errors

- Forgetting that velocity is zero only at the top, not acceleration.
- Mixing signs for g and displacement.
- Using distance when displacement is required.

Kinematics K11

Exam-style graph/kinematics question: falling object and graph reading.

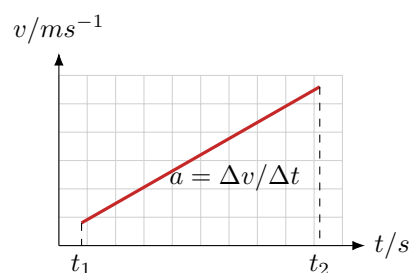
Question form

A small object is released and its motion is recorded.
Use the graph to find acceleration or displacement.

Solution method

1. Identify whether the graph is s - t , v - t , or a - t .
2. If s - t , gradient gives velocity.
3. If v - t , gradient gives acceleration and area gives displacement.
4. If a - t , area gives change in velocity.

Typical graph calculation



Kinematics K12

Exam-style vector/graph question with velocity and displacement.

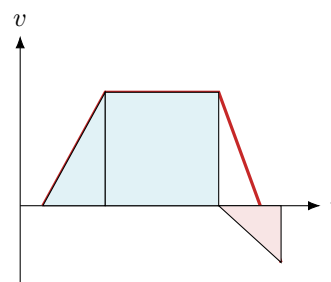
Question form

A body changes velocity as shown on a graph. Determine acceleration, displacement and/or average velocity over a given interval.

Formulae used

$$a = \frac{\Delta v}{\Delta t}, \quad s = \text{area under } v\text{-}t \text{ graph}, \quad \bar{v} = \frac{s}{t}.$$

Area decomposition



Kinematics K13

Past-paper style graph: calculating time, acceleration and displacement.

Worked graph logic

Given a graph with marked points:

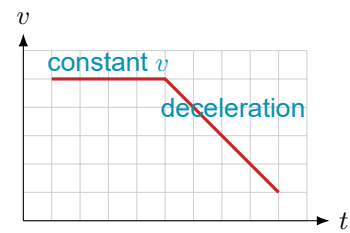
- Find gradients using $\Delta y / \Delta x$.
- Use geometry for areas: rectangles, triangles and trapezia.
- Keep direction signs if the graph crosses below zero velocity.

Example calculation

If an object decelerates uniformly from 12 m s^{-1} to 0 in 4 s,

$$a = \frac{0 - 12}{4} = -3 \text{ m s}^{-2}, \quad s = \frac{1}{2}(4)(12) = 24 \text{ m}.$$

Graph with annotations



Kinematics K14

Past-paper style: train/braking and slope/graph problem.

Train braking at a signal

A train moving with speed u decelerates uniformly to rest over distance s .

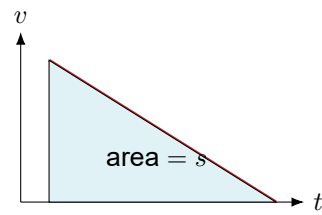
$$v^2 = u^2 + 2as, \quad 0 = u^2 + 2as, \quad a = -\frac{u^2}{2s}.$$

Time to stop

$$v = u + at \quad \Rightarrow \quad t = \frac{-u}{a}.$$

The distance may also be checked by area under the v - t graph.

Braking graph



Kinematics K15

Past-paper style: displacement-time graph and tangent gradients.

Finding instantaneous velocity

For a displacement-time graph, instantaneous velocity at a point is obtained by drawing the tangent and calculating its gradient.

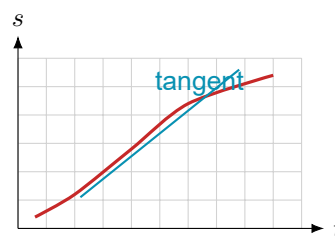
$$v = \frac{\Delta s}{\Delta t}.$$

Finding acceleration from a velocity-time graph

If a later graph of velocity is available, acceleration is the gradient:

$$a = \frac{\Delta v}{\Delta t}.$$

Tangent calculation



Kinematics K16

Projectile motion theory.

Projectile motion

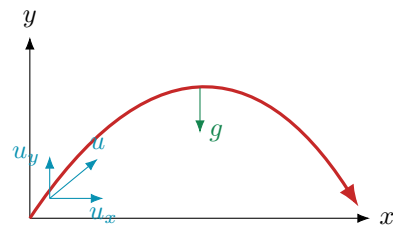
Horizontal and vertical components are treated independently.

- Horizontal acceleration is zero (neglecting air resistance).
- Vertical acceleration is g downward.
- Time is common to both components.

Component table

Horizontal	Vertical
$a_x = 0$	$a_y = -g$ or $+g$ depending on sign
$v_x = u_x$ constant	$v_y = u_y + a_y t$
$x = u_x t$	$y = u_y t + \frac{1}{2} a_y t^2$

Projectile path



Kinematics K17

Projectile example: launched at angle.

Resolving velocity

For launch speed u at angle θ :

$$u_x = u \cos \theta, \quad u_y = u \sin \theta.$$

At maximum height, $v_y = 0$.

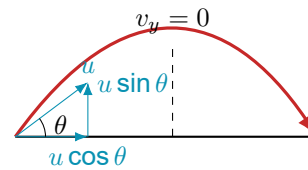
Time of flight for same level

$$0 = u_y t - \frac{1}{2} g t^2 \quad \Rightarrow \quad t = \frac{2u_y}{g} = \frac{2u \sin \theta}{g}.$$

Range:

$$R = u_x t = \frac{u^2 \sin 2\theta}{g}.$$

Projectile components



Kinematics K18

Past-paper style projectile graph.

Question form

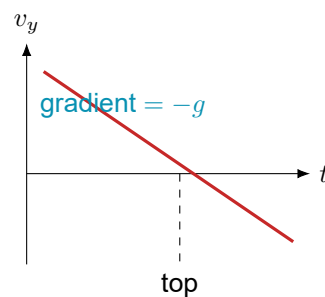
A ball is thrown from a point above the ground. Use the velocity-time graph or path diagram to find velocity components, time, range, or height.

Vertical component method

Use

$$y = u_y t - \frac{1}{2} g t^2, \quad v_y = u_y - g t, \quad v_y^2 = u_y^2 - 2 g y.$$

Vertical velocity graph



Kinematics K19

Resolving projectiles and graph areas.

Horizontal motion

$$x = u_x t.$$

Since $a_x = 0$, horizontal velocity is constant.

Vertical motion

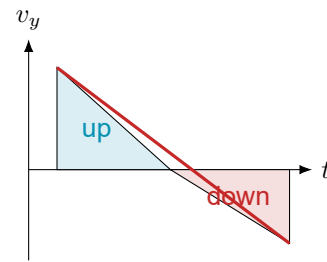
$$y = u_y t + \frac{1}{2} a_y t^2.$$

Set $a_y = -g$ if upward is positive.

Graph area

Area under a v_y - t graph gives vertical displacement.
Positive area above the axis minus negative area below the axis gives net vertical displacement.

Area on vertical velocity graph



Kinematics K20

Projectile past-paper: horizontal range and final speed.

Range

Once time of flight t is known from vertical motion,

$$R = u_x t.$$

Final velocity

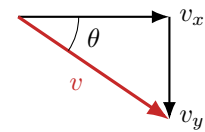
Horizontal component remains $v_x = u_x$. Vertical component is

$$v_y = u_y - gt.$$

Resultant speed:

$$v = \sqrt{v_x^2 + v_y^2}, \quad \theta = \tan^{-1} \left(\frac{|v_y|}{v_x} \right).$$

Final velocity vector



Kinematics K21

Experimental/free-fall question with markers.

Marker setup

Two markers M_1 and M_2 are a vertical distance h apart. A steel ball is released from rest a distance x above M_1 . It reaches M_1 at time t_1 and M_2 at time t_2 .

Derivation

Taking downward positive, from release:

$$x = \frac{1}{2}gt_1^2, \quad x + h = \frac{1}{2}gt_2^2.$$

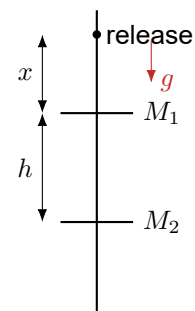
Subtract:

$$h = \frac{1}{2}g(t_2^2 - t_1^2).$$

Thus

$$g = \frac{2h}{t_2^2 - t_1^2}.$$

Marker diagram



Kinematics K22

Graph experiment: displacement and gradient.

Using the graph

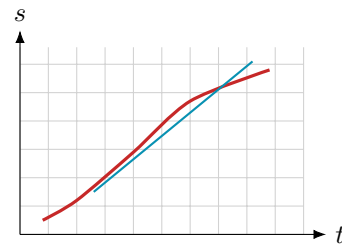
For a displacement-time curve, draw tangents to find velocities at different times. A graph that curves upward indicates increasing velocity; a graph that curves downward indicates decreasing velocity.

Example tangent result

If the tangent changes from 4 m s^{-1} to 10 m s^{-1} over 3 s ,

$$a = \frac{10 - 4}{3} = 2.0 \text{ m s}^{-2}.$$

Curve with tangent



Kinematics K23

Final graph question: velocity-time graph and displacement.

Displacement from graph

Break the graph into simple shapes and add signed areas.

$$s = s_1 + s_2 + s_3 + \dots$$

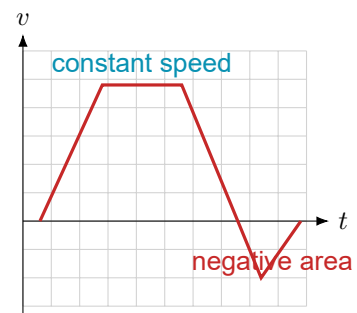
Use positive area above the axis and negative area below the axis.

Acceleration from straight segment

For any straight segment,

$$a = \frac{v_2 - v_1}{t_2 - t_1}.$$

Composite v - t graph



Dynamics and Momentum

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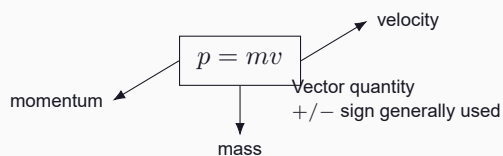


Topic 1 – Momentum, Change in Momentum, and Force

Momentum

“It is the product of mass and **velocity** of a body.”

$$p = mv$$



SI unit: kg m s^{-1} or Ns .

⇒ Momentum tells about the amount (strength) of motion.

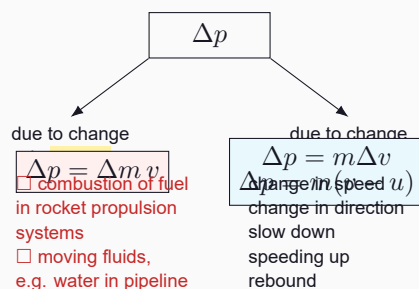


$$p = mv = 0.2 \times 100 = 20 \text{ N s},$$

$$p = mv = 4 \times 5 = 20 \text{ N s}.$$

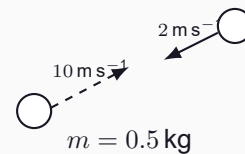
Change in Momentum

$$\Delta p = p_f - p_i$$



Example #1

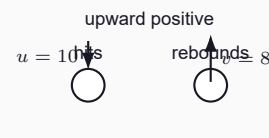
Find Δp .



$$\Delta p = m(v - u) = 0.5(10 - 2) = \boxed{4 \text{ N s}}.$$

Example #2

$m = 0.5 \text{ kg}$, find Δp for a ball that hits the ground at 10 m s^{-1} and rebounds at 8 m s^{-1} .



$$\Delta p = m(v - u) = 0.5\{8 - (-10)\} = \boxed{9 \text{ N s}}.$$

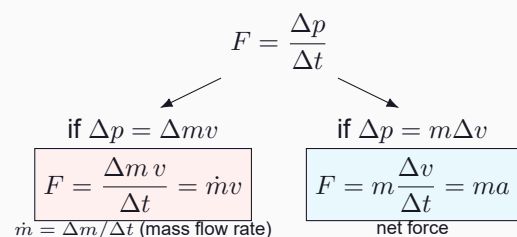
With the opposite sign convention, $\Delta p = 0.5(-8 - 10) = -9 \text{ N s}$; the magnitude is 9 N s .

Force

“Rate of change of momentum is called force.”

$$F_{\text{net}} = \frac{\Delta p}{\Delta t}$$

- **Vector**; SI unit: **newton**.
- The greater the Δt , the lesser the force, and vice versa.



Topic 2 – Mass Flow Rate, Jet Force, and Graphs

$$F_{\text{net}} = ma \rightarrow \text{generally on rigid bodies}$$

$$F_{\text{net}} = \dot{m}v \rightarrow \text{jet of liquid / moving stream of air}$$

Mass Flow Rate

The mass of liquid or gas that passes a point per unit time.

$$\dot{m} = \frac{\Delta m}{\Delta t}$$

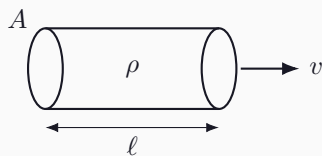
Example: $\dot{m} = 4 \text{ kg min}^{-1}$, i.e. 4 kg of liquid passes a point in 1 min.

$$\dot{m} = \frac{\Delta m}{\Delta t} \begin{cases} \text{mass that passed} \\ \text{time taken} \end{cases}$$

or

$$\dot{m} = \rho Av \begin{cases} \text{density} \\ \text{velocity} \\ \text{cross-sectional area} \end{cases}$$

$$\dot{m} = \frac{\Delta m}{\Delta t}, \quad \dot{m} = \rho \frac{\Delta V}{\Delta t}, \quad \dot{m} = \rho \frac{A\ell}{\Delta t} = \rho Av.$$



Exam-style example: platform and water jet

A man standing on a platform is attached to a flexible pipe. Water is pumped through the pipe so that he remains at a constant height. The combined mass of the man and platform is 96 kg. The mass of water discharged vertically downwards is 40 kg s^{-1} .

Since resultant vertical force is zero,

$$F_{\text{water}} = W = 96g.$$

Using $F = \dot{m}v$,

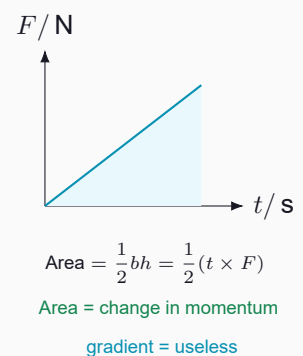
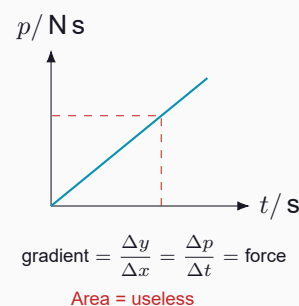
$$40v = 96 \times 9.81 \Rightarrow v = \frac{96 \times 9.81}{40} = 23.5 \approx \boxed{24 \text{ m s}^{-1}}.$$

Hose-pipe example

Water is pumped through a hose-pipe at a rate of 90 kg min^{-1} . Water emerges horizontally from the hose-pipe with a speed of 20 m s^{-1} . What is the minimum force required to prevent the hose-pipe moving backwards?

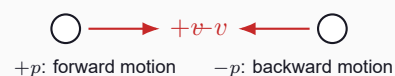
$$F = \dot{m}v = \frac{90}{60} \times 20 = \boxed{30 \text{ N}}.$$

Graphs of $F-t$ and $p-t$



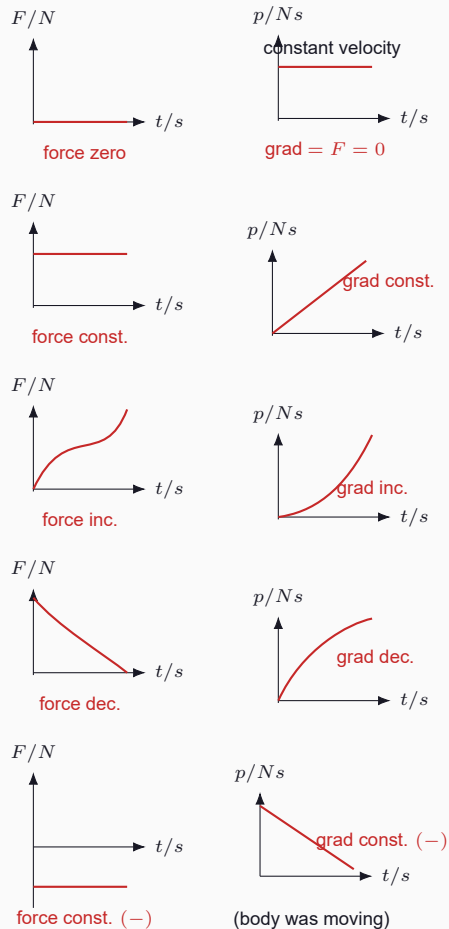
$$F = \frac{\Delta p}{\Delta t}, \quad \Delta p = F\Delta t.$$

Direction sign convention



Topic 3 – Graph Shapes and Types of Collisions

Force–time and momentum–time graph patterns



Types of collisions

Elastic Collision

1. **Ideal collision.**
2. No energy lost upon collision.
3. E.g. collision between small masses such as atoms, neutrons, molecules.

Momentum: conserved

Kinetic energy: conserved

Total energy: conserved

Inelastic Collision

1. **Real collision.**
2. Energy lost upon collision as heat.
3. E.g. collision between large masses, explosion, bodies sticking upon collision.

Momentum: conserved

Kinetic energy: NOT conserved

Total energy: conserved

How to check if a collision is elastic?

A collision is elastic if:

1. **Total K.E before collision = Total K.E after collision**

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2.$$



2. **Relative speed of approach = relative speed of separation.**

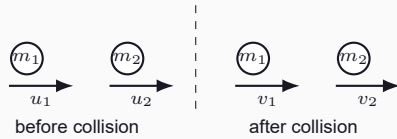
- If bodies move in the **same direction**: subtract / minus speeds.
- If bodies move in **opposite directions**: add speeds.

→ → same direction ⇒ minus speeds

→ ← opposite direction ⇒ add speeds

Topic 4 – Law of Conservation of Momentum

Law of Conservation of Momentum



- **For an isolated system**, the total momentum of bodies before collision is equal to their total momentum after collision.

$$(p_1 + p_2)_{\text{before}} = (p_1 + p_2)_{\text{after}}$$

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2,$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2),$$

$$-m_1(v_1 - u_1) = m_2(v_2 - u_2).$$

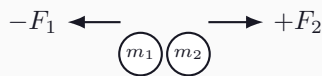
Since $\Delta p = m(v - u)$,

$$-\Delta p_1 = +\Delta p_2.$$

- For an isolated system, **the momentum lost by one body is equal to that gained by the other.**

Dividing both sides by Δt , i.e. the time of collision,

$$-\frac{\Delta p_1}{\Delta t} = +\frac{\Delta p_2}{\Delta t} \Rightarrow -F_1 = +F_2.$$



Newton's 3rd Law of Motion

- Both bodies had the same Δp , but in opposite directions.
- The time of collision was the same for both.
- Since $F = \Delta p / \Delta t$, $-F_1 = +F_2$.

"During collision, momentum of a body is not conserved."

⇒ Body always undergoes a change in momentum!

Important distinction

- The momentum of **one body** changes during collision.
- The **total momentum of the isolated system** is conserved.

Exam-style example: equal masses on an air track

Two equal masses X and Y are moving towards each other on a frictionless air track. The masses make an elastic collision.



For equal masses in an elastic collision, the bodies exchange velocities:

$$v_X = 30 \text{ cm s}^{-1} \text{ to the left,}$$

$$v_Y = 50 \text{ cm s}^{-1} \text{ to the right.}$$

This corresponds to option **D** in the handwritten page.

If masses are equal and the collision is elastic, bodies exchange velocities.

Chapter checkpoint

Momentum of individual bodies changes during collisions; total momentum of an isolated system remains constant.

Forces

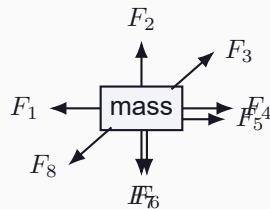
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Newton's First Law and Equilibrium Examples

Newton's 1st Law of Motion

- a.k.a. **law of equilibrium**
- a.k.a. **law of inertia**

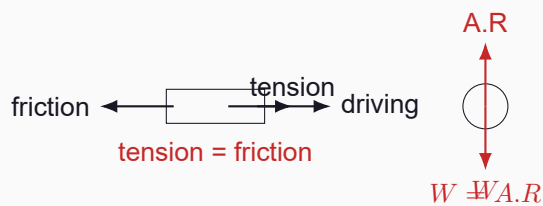


An object continues in a state of rest or uniform motion unless acted upon by a resultant force.

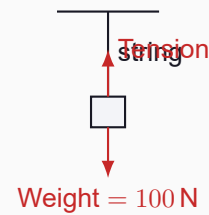
- **Rest:** speed = 0.
- **Uniform motion:** constant speed + straight line.

Keywords: rest, hanging, constant speed, terminal velocity.

$$\boxed{\sum F = 0} \quad \Rightarrow \quad \boxed{\sum F_x = 0}, \quad \boxed{\sum F_y = 0}$$



Example #1



Box is in equilibrium:

$$\boxed{\text{Tension} = \text{Weight} = 100 \text{ N}}$$

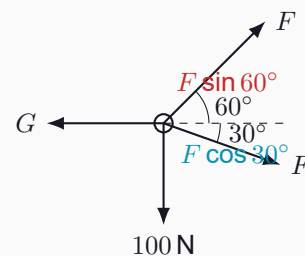
Tension force acts away from the point and towards the centre of the string.

Similarly, if a lift is moving upwards at **constant speed**,

$$\boxed{\text{Tension in rope} = \text{weight of man} + \text{lift} .}$$

Example #2

The ball is in equilibrium. Find F and G .



$$x\text{-axis: } F \cos 60^\circ + F \cos 30^\circ = G,$$

$$y\text{-axis: } F \sin 60^\circ - F \sin 30^\circ = 100.$$

$$F = \frac{100}{\sin 60^\circ - \sin 30^\circ} = 273.2 \approx \boxed{273 \text{ N}},$$

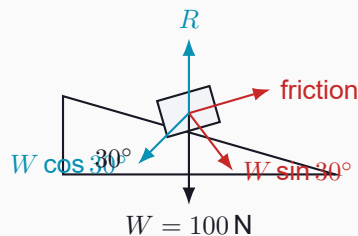
$$G = 273.2(\cos 60^\circ + \cos 30^\circ) = 373.2 \approx \boxed{373 \text{ N}}.$$

Inclined Plane and Newton's Second Law

Example #3

A box of weight 100 N rests on a rough inclined surface. Calculate:

- the friction force between box and surface,
- the normal contact force.



$$W \sin \theta = \text{friction}, \quad 100 \sin 30^\circ = \text{friction}$$

$$\boxed{\text{friction} = 50 \text{ N}}$$

$W \sin \theta$ is responsible to make the object slip down the plane.

$$R = W \cos \theta = 100 \cos 30^\circ = \boxed{86.6 \text{ N}}$$

- smooth surface: no friction;
- rough surface: friction exists.

Newton's 2nd Law of Motion

Rate of change of momentum is directly proportional to resultant force.

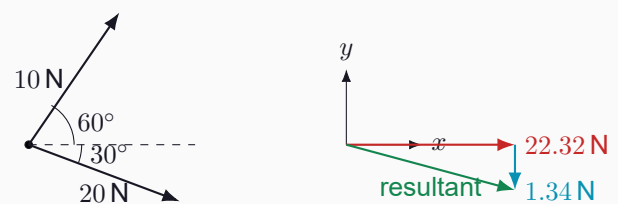
$$F_{\text{net}} \propto \frac{\Delta p}{\Delta t} \quad \Rightarrow \quad \boxed{F_{\text{net}} = ma}$$

$$\boxed{\text{forward force} - \text{backward force} = ma}$$

Keywords: speeding up, slowing down, accelerate, decelerate.

Example #1

Find the resultant of the two forces.



$$R_x = 10 \cos 60^\circ + 20 \cos 30^\circ = 22.32 \text{ N}$$

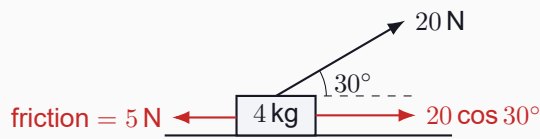
$$R_y = 10 \sin 60^\circ - 20 \sin 30^\circ = -1.34 \text{ N}$$

$$R = \sqrt{(1.34)^2 + (22.32)^2} = \boxed{22.36 \text{ N}}$$

Systems, Pulleys, and Tension

Example #2

A 4 kg block is pulled by 20 N at 30° and friction is 5 N. Find acceleration.



$$F_{\text{net}} = ma, \quad 20 \cos 30^\circ - 5 = 4a$$

$$a = 3.08 \approx 3.1 \text{ m s}^{-2}$$

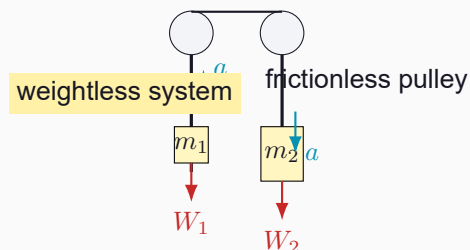
Example #3

A transport system contains a powered vehicle of mass 200 kg and four baggage carts, each of mass 400 kg. The system accelerates at 2.0 m s^{-2} . Find the tension between carts 1 and 2, ignoring friction on the carts.



$$F_{\text{net}} = ma, \quad T = (400 \times 3)(2) = \boxed{2400 \text{ N}}.$$

Example #4



For the acceleration of the system:

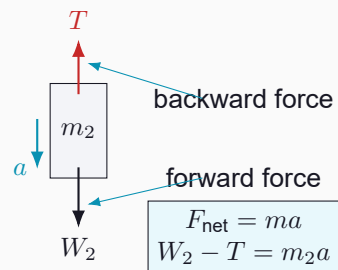
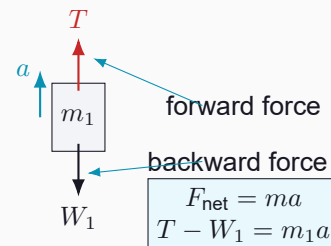
$$F_{\text{net}} = ma, \quad W_2 - W_1 = (m_2 + m_1)a.$$

$$W_2 - W_1 = m_2g - m_1g = (m_2 + m_1)a$$

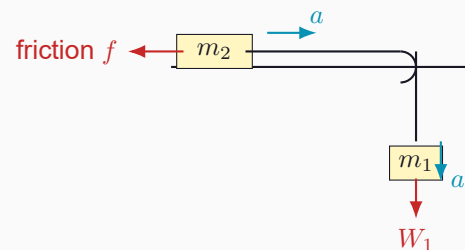
$$(m_2 - m_1)g = (m_2 + m_1)a$$

$$a = \frac{(m_2 - m_1)g}{m_2 + m_1}$$

For finding the tension in the string



Example #5



If $W_1 > f$, the system accelerates even if $m_2 > m_1$.

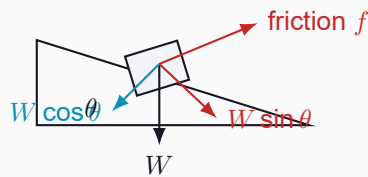
$$F_{\text{net}} = ma, \quad W_1 - f = (m_1 + m_2)a.$$

If friction = 0,

$$W_1 = (m_1 + m_2)a.$$

Inclined Planes and Newton's Third Law

Example #6



If the surface is smooth:

$$F_{\text{net}} = ma, \quad W \sin \theta = ma.$$

Since $W = mg$,

$$mg \sin \theta = ma \quad \Rightarrow \quad a = g \sin \theta.$$

For smooth surfaces, acceleration depends only on the angle of the ramp.

If the surface is rough:

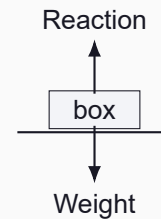
$$F_{\text{net}} = ma, \quad W \sin \theta - f = ma.$$

Newton's 3rd Law of Motion

If body A exerts a force on body B, B exerts the same force on A, equal in magnitude but opposite in direction.

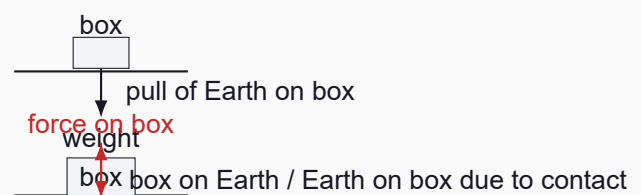
- Forces act on different bodies.
- Type of force is the same.
- Magnitude is equal.
- Direction is opposite.

Weight and reaction are not a third-law pair



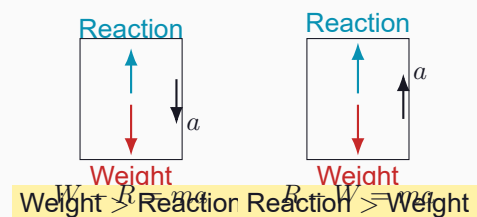
Both forces act on the box, so they are different types of forces and are not a Newton's 3rd-law pair.

Action–reaction pair



Lift examples

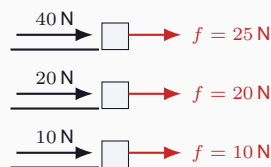
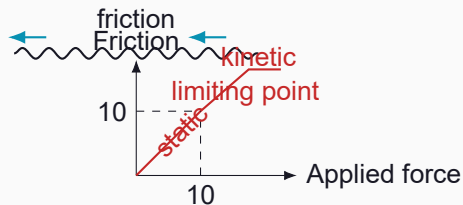
If a body is at rest or moving at constant speed, **weight = reaction**.



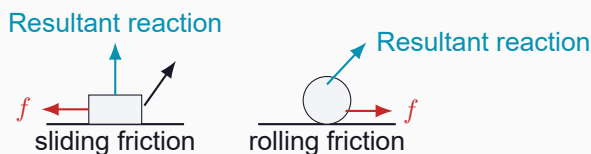
Friction, Mass, Inertia, and Drag Force

Friction Force

- It is a **resistive force** that tends to slow down moving objects.
- It is a **contact** and a **reaction force**.



Normal contact/resultant reaction:



Mass

It is the measurement of inertia.

More inertia $\Rightarrow$ More mass in the body.

Inertia is a property of matter such that it resists sudden changes in its state of rest or motion.

SI unit: kg Scalar

Drag Force

When an object moves in a fluid, it experiences a drag force.

- drag force $\rightarrow$ viscous drag
- air resistance
- viscous drag: water, oil, glycerine

Factors affecting drag force

1. Viscosity of fluid

more viscosity $\rightarrow$ thicker fluid $\rightarrow$ more drag force.

$$\text{drag force} \propto \text{viscosity}$$

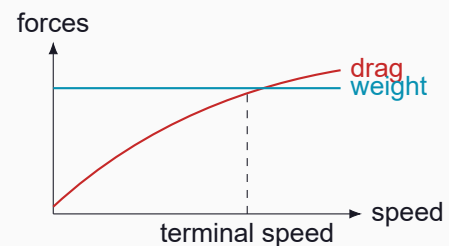
2. Surface area

greater surface area $\rightarrow$ more drag force. More molecules collide with the body, so cumulative force increases.

3. Speed of object

greater speed $\rightarrow$ greater drag force.

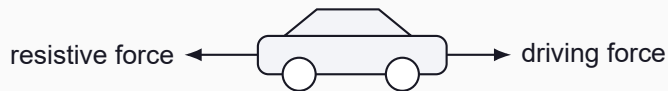
Air resistance increases on a falling body until the body reaches terminal velocity, when weight = air resistance.



Multiple Choice: Driving Force, Resistive Force, Terminal Speed

9702/01 May/June 2008 – Paper 1 Multiple Choice

11. A car of mass 750 kg has a horizontal driving force of 2.0 kN acting on it. It has a forward horizontal acceleration of 2.0 m s^{-2} . What is the resistive force acting horizontally?



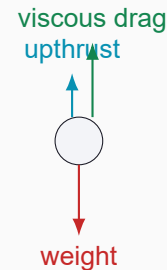
$$F_{\text{net}} = ma, \quad F_D - F_R = ma.$$

$$2000 - F_R = 750 \times 2, \quad 2000 - F_R = 1500, \quad F_R = 500 \text{ N}.$$

$$F_R = 0.5 \text{ kN}$$

Answer: A.

12. A ball is falling at terminal speed in still air. The forces on the ball are upthrust, viscous drag and weight. What is the order of increasing magnitude of these three forces?



At terminal speed, acceleration is zero, so

$$W = \text{upthrust} + \text{viscous drag}.$$

The upthrust is very small for a ball in air, and viscous drag is larger than upthrust but smaller than the total weight.

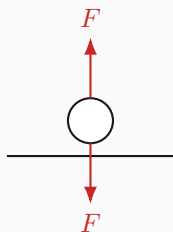
upthrust $\rightarrow$ viscous drag $\rightarrow$ weight

Answer: A.

Multiple Choice: Third Law and Pulley Acceleration

9702/01 October/November 2008 – Paper 1 Multiple Choice

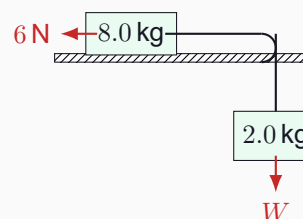
9. A ball falls vertically and bounces on the ground. Which statement about the forces acting while the ball is in contact with the ground is correct?



The force the ball exerts on the ground is always equal in magnitude and opposite in direction to the force the ground exerts on the ball.

Answer: B

11. A box of mass 8.0 kg rests on a horizontal rough surface. A string passes over a smooth pulley and supports a 2.0 kg mass. Friction on the box is 6.0 N. Find the acceleration of the box.



$$F_{\text{net}} = ma, \quad W - f = (m_1 + m_2)a.$$

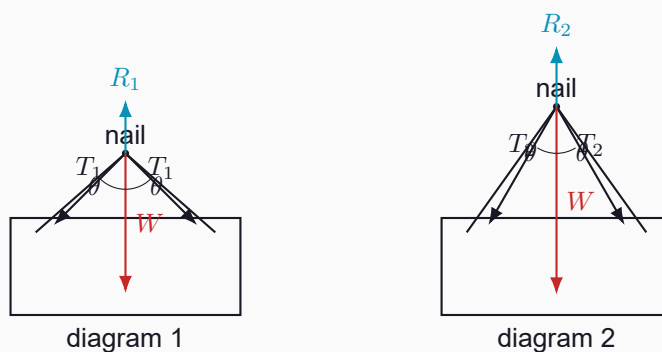
$$2(9.81) - 6 = (8 + 2)a, \quad a = 1.36 \text{ m s}^{-2} \approx \boxed{1.4 \text{ m s}^{-2}}.$$

Answer: A.

Multiple Choice: Hanging Picture Tension

9702/11 October/November 2009 – Paper 1 Multiple Choice

12. The diagrams show two ways of hanging the same picture. In both cases, a string is attached to the same points and looped symmetrically over a nail. Diagram 1 has the shorter loop. Forces shown are those that act on the nail.



For the nail, horizontal components cancel. The vertical components support the picture:

$$T \cos \theta + T \cos \theta = W$$

$$T = \frac{W}{2 \cos \theta}$$

A wider angle gives more tension. A smaller angle gives less tension.

In diagram 1 the string is shorter and the angle is wider, so

$$T_1 > T_2.$$

The vertical resultant on the nail equals the same weight in both cases:

$$R_1 = R_2.$$

$$R_1 = R_2, \quad T_1 > T_2$$

Answer: B.

Multiple Choice: Definition of Force and Pulley Motion

9702/11 May/June 2012 – Paper 1 Multiple Choice

10. Each option gives a correct word equation involving force. Which option gives the definition of force?

A. force = energy divided by displacement

B. force = mass $\times$ acceleration

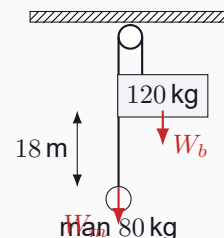
C. force = pressure $\times$ area

D. force = rate of change of momentum

Definition of force: rate of change of momentum

Answer: D.

12. A barrel of mass 120 kg hangs from a frictionless pulley. The rope is secured to a stake, and a man of mass 80 kg holds the rope after untying it. The bottom of the barrel is 18 m above the man's head. Find the man's upward speed when his head is level with the barrel. Take $g = 10 \text{ m s}^{-2}$.



For the system:

$$F_{\text{net}} = ma, \quad W_b - W_m = (120 + 80)a.$$

$$1200 - 800 = 200a \Rightarrow a = 2 \text{ m s}^{-2}.$$

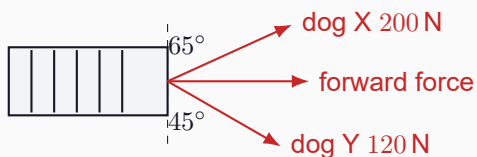
$$v^2 - u^2 = 2as, \quad v^2 = 2(2)(9), \quad v = \boxed{6 \text{ m s}^{-1}}.$$

Answer: A.

Multiple Choice: Resultant Components and Weight Graph

9702/11 May/June 2018 – Paper 1 Multiple Choice

3. Two dogs pull a sledge along an icy track. Dog X pulls with 200 N at 65° to the front edge. Dog Y pulls with 120 N at 45° .



Forward component:

$$200 \sin 65^\circ + 120 \sin 45^\circ = 266.1 \approx \boxed{270 \text{ N}}.$$

Answer: C.

7. The graph shows the variation with mass of the weight of objects on a particular planet. What is the acceleration of free fall?



Since $W = mg$, gradient of the weight–mass graph equals g .

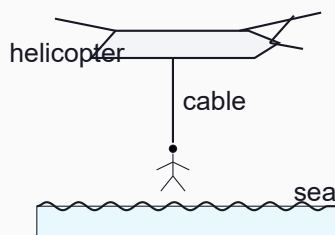
$$g = \frac{4}{2.5} = \boxed{1.6 \text{ m s}^{-2}}.$$

Answer: B.

Structured Question: Helicopter Cable Lift, Part 1

9702/23 May/June 2011 – Question 3

A helicopter has a cable hanging from it towards the sea below. A man of mass 80 kg rescues a child of mass 50.5 kg. The two are attached to the cable and are lifted from the sea to the helicopter. The lifting process consists of initial uniform acceleration, constant velocity, and final uniform deceleration.

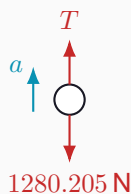


- (a) Calculate the combined weight of the man and child.

$$W = mg = (80 + 50.5)(9.81) = 1280.205 \text{ N} \approx \boxed{1300 \text{ N}}.$$

- (b) Calculate the tension in the cable during:

- (i) the initial acceleration of 0.570 m s^{-2} .



$$F_{\text{net}} = ma, \quad T - W = ma.$$

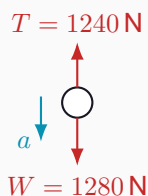
$$T - 1280.205 = 130.5(0.570), \quad T = 1354.6 \text{ N} \approx \boxed{1400 \text{ N}}.$$

- (ii) the period of constant velocity of 2.00 m s^{-1} .

$$F_{\text{net}} = 0 \Rightarrow T = W = 1280 \text{ N} \approx \boxed{1300 \text{ N}}.$$

Structured Question: Helicopter Cable Lift, Part 2

(c) During the final deceleration, the tension in the cable is 1240 N. Calculate the deceleration.



$$F_{\text{net}} = ma, \quad T - W = ma.$$

$$1240 - 1280 = 130.5a, \quad a = -0.31 \text{ m s}^{-2}.$$

The negative sign shows the acceleration is downward, so the **deceleration** is

$$0.31 \text{ m s}^{-2}.$$

(d) Calculate the time over which the man and child are:

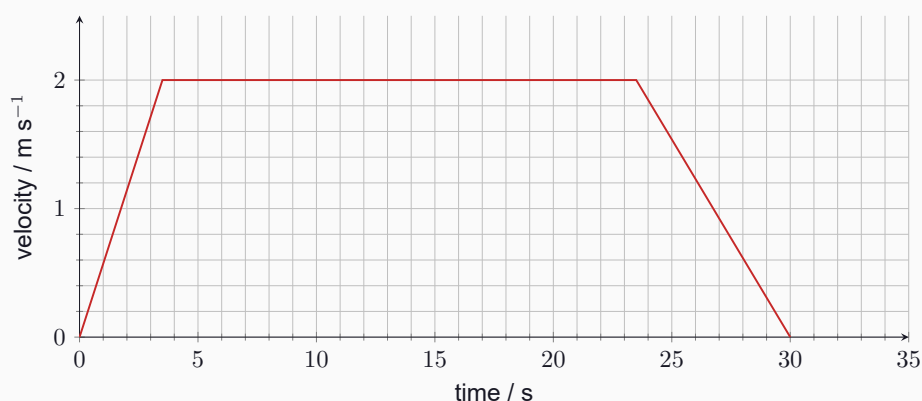
- moving with uniform acceleration from rest to 2.0 m s^{-1} ,

$$v = u + at, \quad 2 = 0 + 0.57t, \quad t = 3.5 \text{ s}.$$

- moving with uniform deceleration from 2.0 m s^{-1} to rest,

$$0 = 2 + (-0.306)t, \quad t = 6.5 \text{ s}.$$

The time at constant velocity is 20 s. Sketch the velocity–time graph for the complete lifting process.



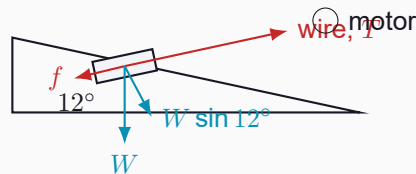
Structured Question: Log Pulled Up a Slope

9702/22 May/June 2012 – Question 3

- (a) State Newton's first law.

An object continues in a state of rest or uniform motion unless acted upon by a resultant force.

- (b) A log of mass 450 kg is pulled up a slope by a wire attached to a motor. The slope makes an angle of 12° with the horizontal. The frictional force acting on the log is 650 N. The log travels with constant velocity.



- (i) Discuss whether the log is in equilibrium.

Yes, it is in equilibrium because it is moving at constant velocity. Hence resultant force and acceleration are zero.

- (ii) Calculate the tension in the wire.

$$T = f + W \sin 12^\circ.$$

$$T = 650 + (450 \times 9.81) \sin 12^\circ = 1567.8 \text{ N} \approx \boxed{1600 \text{ N}}.$$

- (iii) Is the gain in potential energy per unit time equal to the output power of the motor?

No. The motor output must supply both the gain in gravitational potential energy and work done against friction. Hence motor output power is greater than the rate of gain of GPE.

Structured Question: Mass, Weight, and Rope Equilibrium

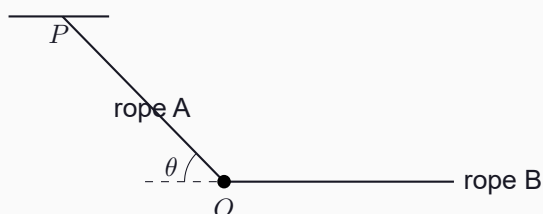
9702/21 May/June 2013 – Question 2

- (a) Distinguish between mass and weight.

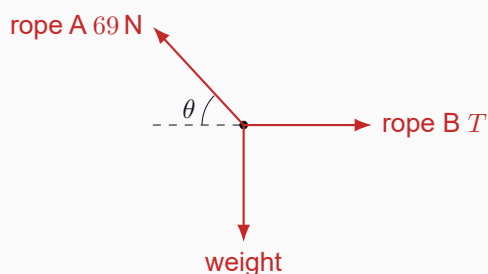
mass: measurement of inertia; scalar; SI unit kg.

weight: effect of gravitational force on a mass; vector; $W = mg$; SI unit N.

- (b) An object O of mass 4.9 kg is suspended by rope A fixed at point P . It is pulled to one side and held in equilibrium by a second rope B. Rope A is at angle θ to the horizontal, rope B is horizontal, tension in rope A is 69 N, and tension in rope B is T .



- (i) Draw arrows showing all forces on O .



- (ii) Calculate:

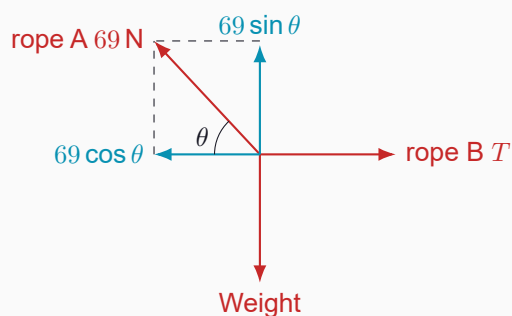
1. the angle θ .

$$W = mg = 4.9 \times 9.81 = 48.069 \text{ N.}$$

$$69 \sin \theta = W, \quad \sin \theta = \frac{48.069}{69}, \quad \theta = \boxed{44.1^\circ \approx 44^\circ}.$$

Structured Question: Resolving Rope Forces

Continuing the rope equilibrium example:



1. Angle θ

Horizontal equilibrium gives:

$$W = mg = 4.9 \times 9.81 = 48.069 \text{ N}$$

$$69 \sin \theta = W$$

$$\sin \theta = \frac{48.069}{69}$$

$$\theta = 44.1^\circ \approx 44^\circ$$

$$69 \cos \theta = T.$$

$$T = 69 \cos 44.1^\circ = \boxed{49.5 \text{ N}}.$$

2. Tension T

Chapter 4 conversion note: All handwritten theory, worked examples, diagrams and exam-style questions from the Forces notes have been converted into live LaTeX text, equations, tables and TikZ/PGFPlots diagrams.

Turning Effect of Forces

Chapter 05 / AS Level / Apollo Physics 9702

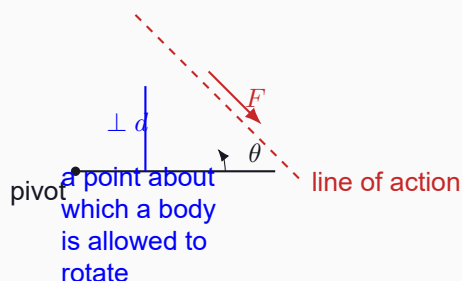


Moment of Force, Resultant Moment, Principle of Moments

Moment of Force / Torque

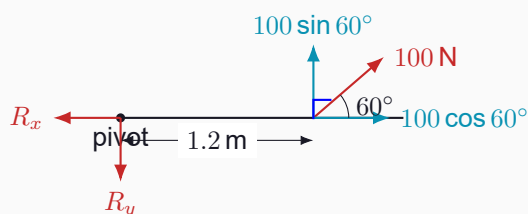
It is the product of force and perpendicular distance from the pivot to the line of action of force.

$$\tau = F \times \perp d$$



- Vector quantity.
- SI unit: N m.
- Horizontal force $\Rightarrow$ use vertical distance.
- Vertical force $\Rightarrow$ use horizontal distance.

Example #1



(a) Calculate the moment produced.

$100 \cos 60^\circ$ does not produce any moment about pivot as it passes through the pivot. It does not have any perpendicular distance from it.

$$\tau = F \times \perp d, \quad \tau = 100 \sin 60^\circ \times 1.2$$

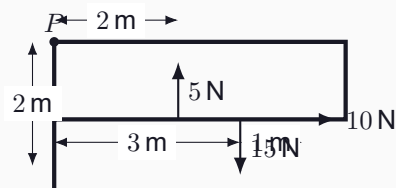
$$\tau = 104 \text{ N m}$$

(b) Force exerted by pivot.

R_x and R_y are reaction/contact forces that act on the rod by the pivot and keep the rod in translational equilibrium.

$$\sum F_x = 0 \Rightarrow R_x = 100 \cos 60^\circ, \quad \sum F_y = 0 \Rightarrow R_y = 100 \sin 60^\circ$$

Resultant Moment



$$\text{ACW} = (5 \times 2) + (10 \times 2) = 30 \text{ N m}$$

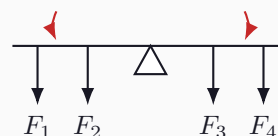
$$\text{CW} = 15 \times 3 = 45 \text{ N m}$$

$$\text{Resultant} = 45 - 30 = 15 \text{ N m clockwise}$$

Principle of Moments

For an object to be in rotational equilibrium, sum of clockwise moments must be equal to sum of anticlockwise moments.

$$\sum \tau_{cw} = \sum \tau_{acw}$$



Centre of Gravity

The point at which the whole weight of a body appears to act.

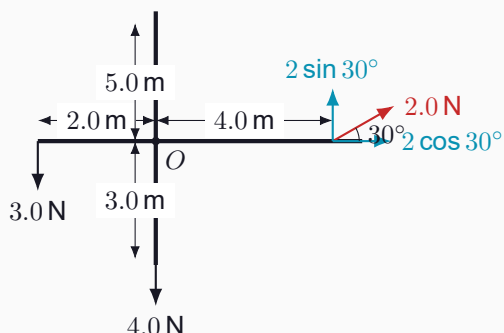


For uniform bodies, the C.O.G. is found at the geometric centre, i.e. at the middle of the body.

Exam Examples: Rotational Equilibrium and Common Point Rule

Example DN 2015/P13

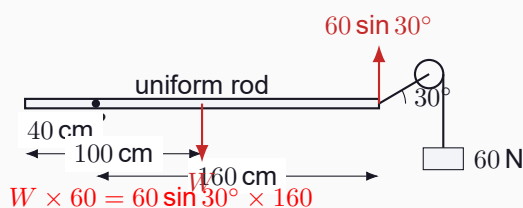
A cross-shaped structure is pivoted at O . It is in rotational equilibrium.



$$\begin{aligned}\tau_{cw} &= \tau_{acw} \\ F \times 5 &= (2 \sin 30^\circ \times 4) + (3 \times 2) \\ F &= \frac{4 + 6}{5} = \boxed{2 \text{ N}}\end{aligned}$$

Example DN 2018/P11

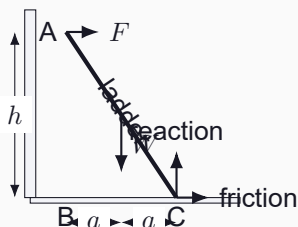
A uniform rod of length 200 cm is pivoted at P . A 60 N weight passes over a frictionless pulley. Find the weight of the rod.



$$\begin{aligned}W \times 60 &= 60 \sin 30^\circ \times 160 \\ W &= \boxed{80 \text{ N}}\end{aligned}$$

Example DN 2014/P11: Ladder

A uniform ladder rests against a smooth wall and rough ground. Which moment equation is formed?

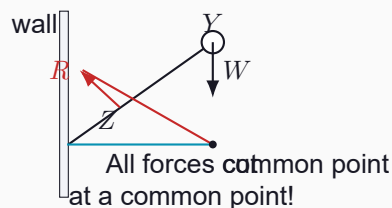


Taking A as pivot:

$$\begin{aligned}Wa + Fh &= W(2a) \\ \boxed{Wa + Fh} &= \boxed{2Wa}\end{aligned}$$

When a body is in rotational equilibrium, all forces acting on that

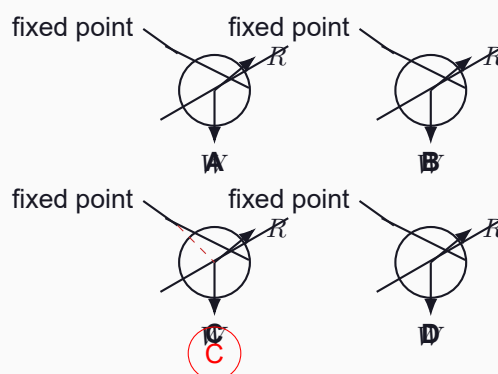
Example: rigid rod resting at wall and roller



Direction of force at X : the force at X must pass through the same common point. The matching option is

☐ D.

Cylinder on a smooth slope with thread

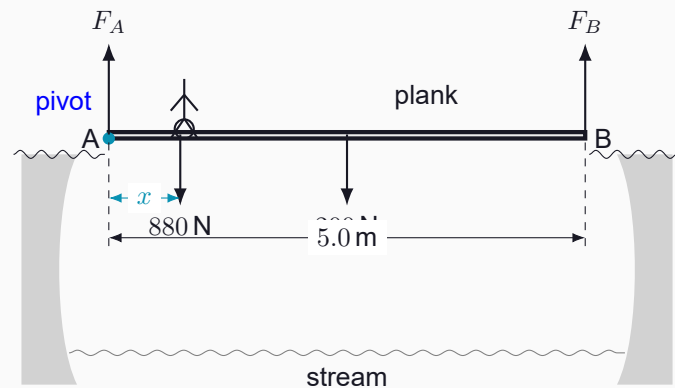


For equilibrium, the three forces W , R and the tension in the thread must pass through one common point. The correct diagram is

☐ C.

Double Pivot: Uniform Plank Across a Stream

DOUBLE PIVOT



A uniform plank AB of length 5.0 m and weight 200 N is placed across a stream. A man of weight 880 N stands a distance x from end A .

The ground exerts vertical forces F_A and F_B at the two ends. As the man moves along the plank, the plank is always in equilibrium.

(a)(i) Why is $F_A + F_B$ constant?

For plank to be in equilibrium, total downward and upward force must be equal. As downward force is constant, so would the sum of $F_A + F_B$.

$$F_A + F_B = 880 + 200 = 1080\text{ N}$$

Pivot choice note

When taking any point as pivot, consider that point which helps reduce the unknown forces from the moment equation.

A taken as pivot so that F_A does not become part of moment equation.

(a)(ii) Man at $x = 0.50\text{ m}$: find F_B

Take moments about A .

$$\tau_{cw} = \tau_{acw}$$

$$(880 \times 0.5) + (200 \times 2.5) = F_B \times 5$$

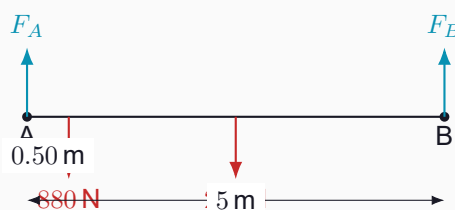
$$F_B = \frac{440 + 500}{5} = \boxed{188\text{ N}}$$

For F_A :

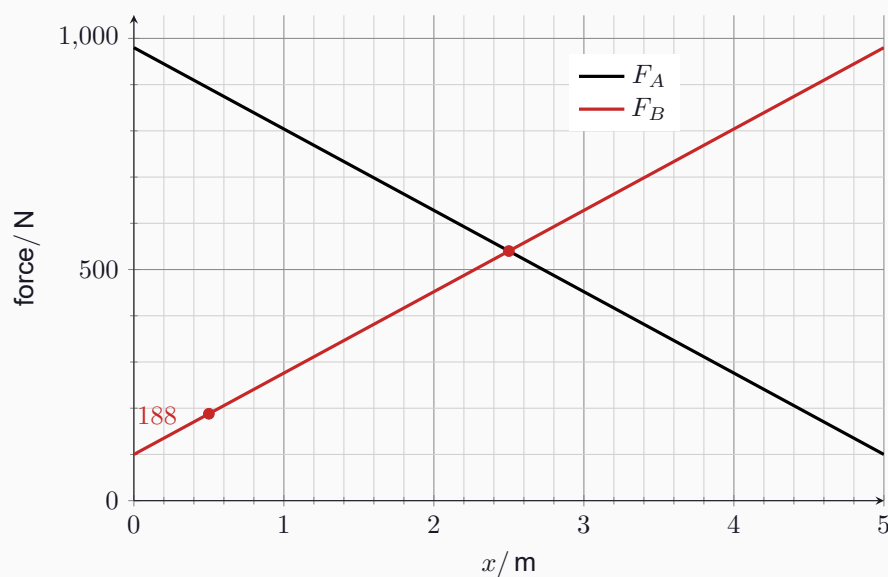
$$F_A + F_B = 1080$$

$$F_A + 188 = 1080$$

$$F_A = \boxed{892\text{ N}}$$



Graph of F_A and F_B Against Position x



Reasoning from the previous page

Total upward force is constant:

$$F_A + F_B = 1080 \text{ N.}$$

Taking moments about A:

$$F_B(5) = 880x + 200(2.5)$$

$$F_B = 176x + 100.$$

Therefore,

$$F_A = 1080 - F_B = 980 - 176x.$$

Sketching F_B

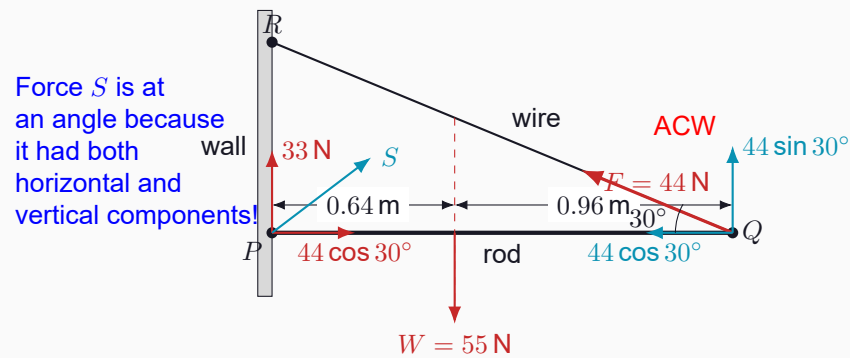
The graph of F_B is a straight line increasing with x .

- At $x = 0$, $F_B = 100 \text{ N}$.
- At $x = 0.50 \text{ m}$, $F_B = 188 \text{ N}$.
- At $x = 2.5 \text{ m}$, the man stands in the middle and $F_A = F_B$.
- At $x = 5.0 \text{ m}$, $F_B = 980 \text{ N}$.

$F_A = F_B$ at $x = 2.5 \text{ m}$.

When $x = 2.5 \text{ m}$, i.e. the man stands in the middle, $F_A = F_B$.

Rod Attached to Wall by a Wire



The length of the rod is 1.60 m. The weight W of the rod acts 0.64 m from P. The rod is kept horizontal and in equilibrium by a wire attached to Q and the wall at R. The wire provides a force F on the rod of 44 N at 30° to the horizontal.

(a)(i) Vertical component of F

$$F_y = 44 \sin 30^\circ$$

$$F_y = \boxed{22 \text{ N}}$$

(b) Taking moments about P: find W

Only the vertical component of F produces a moment about P.

$$\tau_{cw} = \tau_{acw}$$

(a)(ii) Horizontal component of F

$$F_x = 44 \cos 30^\circ = 38.1 \text{ N}$$

$$F_x \approx \boxed{38 \text{ N}}$$

$$W \times 0.64 = 22 \times 1.6$$

$$W = \frac{22 \times 1.6}{0.64} = \boxed{55 \text{ N}}$$

Force at Wall and Direction of Reaction S

(c) Explain why the wall must exert a force on the rod at P

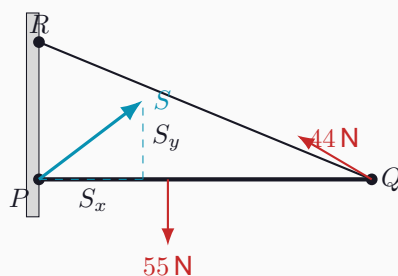
Wall must exert a force to balance $44 \cos 30^\circ$ and the extra 33 N of weight that is acting downwards.

Horizontal balance: $S_x = 44 \cos 30^\circ \approx 38 \text{ N}$

Vertical balance: $S_y + 22 = 55 \Rightarrow S_y = 33 \text{ N}$

So the wall reaction is not purely horizontal or vertical; it has both components.

(d) Direction of force S at P



Component summary

force	x -component	y -component
F	$44 \cos 30^\circ$ left	$44 \sin 30^\circ$ up
W	0	55 N down
S	38 N right	33 N up

$$|S| = \sqrt{38^2 + 33^2} \approx 50 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{33}{38} \right) \approx 41^\circ \text{ above the horizontal.}$$

The arrow labelled S should be drawn from P upwards and to the right.

Density and Pressure

Chapter 06 / AS Level / Apollo Physics 9702



Mass, Volume, Density and Mixtures

Mass

The measurement of inertia is called mass.

- SI unit: kg.
- Scalar quantity.
- Measured usually by electronic balance.

Inertia is a property of matter such that it resists changes in its state of rest or motion.

more inertia $\rightarrow$ more mass

Weight

Effect of gravitational force on mass is called weight.

$$W = mg$$

- Vector quantity.
- Spring balance is used to measure weight.

Gravitational Field

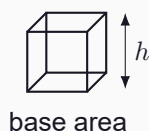
A region in space around a mass where another mass experiences gravitational force.

Volume

Space occupied by a body.

General formula: $V = \text{base area} \times \text{height}$

Shape	Volume
Cube	$V = L^3$
Cuboid	$V = L \times b \times h$
Cylinder	$V = \pi r^2 h$
Cone	$V = \frac{1}{3} \pi r^2 h$
Sphere	$V = \frac{4}{3} \pi r^3$

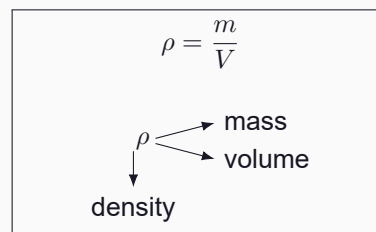


Density

Mass per unit volume.

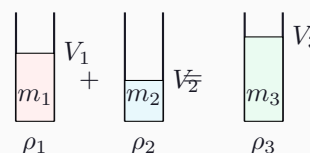
Understanding: it tells about the concentration of mass in a given volume.

more dense $\rightarrow$ more mass $\rightarrow$ heavier



SI unit: kg m^{-3}

Finding Density of Mixture



$$\rho_{mix} = \frac{m_{mix}}{V_{mix}}, \quad \rho_{mix} = \frac{m_1 + m_2}{V_1 + V_2} = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$$

Example 19

1.5 m^3 of water is mixed with 0.50 m^3 of alcohol. The density of water is 1000 kg m^{-3} and the density of alcohol is 800 kg m^{-3} . Find density of mixture with volume 2.0 m^3 .

$$\rho_{mix} = \frac{m_1 + m_2}{V_{mix}} = \frac{(1000 \times 1.5) + (800 \times 0.5)}{2} = \boxed{950 \text{ kg m}^{-3}}$$

Example 10

Density of water = 1.0 g cm^{-3} , density of glycerine = 1.3 g cm^{-3} . Water is added to 40 cm^3 glycerine so that density of mixture is 1.1 g cm^{-3} .

$$1.1 = \frac{1.0(V_1) + 1.3(40)}{V_1 + 40} \Rightarrow V_1 = 80 \text{ cm}^3$$

$$\boxed{80 \text{ cm}^3}$$

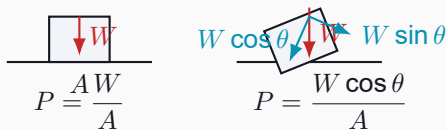
Pressure, Liquid Pressure and Atmospheric Pressure

Pressure

Force per unit (surface) area.

$$P = \frac{F}{A}$$

- Force is at 90° angle with the surface.
- It is a contact force.
- Scalar quantity.
- SI unit: Pascal, Pa.



Applied force example

applied force = 20 N



contact force = 80 N

$$P = \frac{F}{A} \Rightarrow P = \frac{80}{A}$$

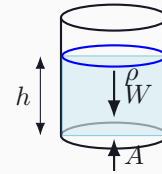
Atmospheric Pressure

- It is due to the collision of air molecules around us.
- Air molecules, due to their continuous random motion, collide with one another and with walls of objects, exerting a force. This force generates pressure.

$$P_{atm} = 101325 \text{ Pa} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ mmHg} = 14.7 \text{ psi}$$

- P_{atm} decreases with altitude.
- Reason 1: low density of air, i.e. fewer molecules.
- Reason 2: low temperature of air, i.e. molecules are slow.

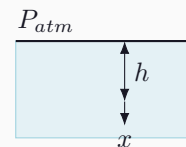
Pressure in Liquids



$$P = \frac{F}{A} = \frac{W}{A} = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho (Ah) g}{A}$$

$$P = \rho g h$$

- ρ is density.
- h is depth taken from top surface.



$$P_T = P_L + P_{atm}, \quad P_T = \rho g h + P_{atm}$$

Total pressure: atmospheric pressure gets transmitted throughout the liquid.

This formula is applicable for all 3 states of matter.

Cylinder on a surface

A solid metal cylinder of length x and cross-sectional area A exerts pressure p on a surface.

$$P = \rho g h = \rho g x \Rightarrow \rho = \frac{p}{gx}$$

Altitude example

Air density decreases almost linearly from 1.2 kg m^{-3} at sea level to 0.74 kg m^{-3} at altitude 5000 m. Given $g = 9.7 \text{ m s}^{-2}$ and surface pressure 100000 Pa:

$$\rho_{avg} = \frac{1.2 + 0.74}{2} = 0.97 \text{ kg m}^{-3}$$

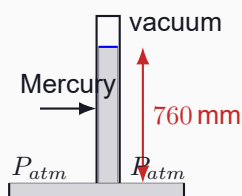
$$\Delta P = \rho g h = 0.97(9.7)(5000) = 47050 \text{ Pa}$$

$$P_x = 100000 - 47050 \approx 52000 \text{ Pa}$$

Barometer, Manometer and Upthrust

Barometer

- Atmospheric pressure on the outer surface of mercury balances the liquid pressure on the inside of the tube.
- 760 mm of mercury column was able to balance P_{atm} .
- Shape and size of tube does not matter on the level of mercury.



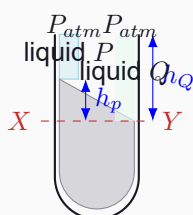
water: $\rho = 1000 \text{ kg m}^{-3}$, $P = \rho gh$

$$101325 = (1000)(9.81)h \Rightarrow h = 10.3 \text{ m}$$

mercury: $\rho = 13600 \text{ kg m}^{-3} \Rightarrow 13.6 \text{ times heavier.}$

Manometer

- Used to compare pressures on both sides of tube.
- When manometer is in equilibrium, pressure on both sides above a common point is equal.
- For comparison, pick the level that has different liquids above and below.



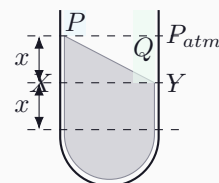
$$P_X = P_Y$$

$$P_P + P_{atm} = P_Q + P_{atm}$$

$$\rho_P gh_P + P_{atm} = \rho_Q gh_Q + P_{atm}$$

Open U-tube example

Two liquids labelled P and Q do not mix. The liquids are in equilibrium in an open U-tube.



$$P_X = P_Y$$

$$\rho_P g(2x) + P_{atm} = \rho_Q g(x) + P_{atm}$$

$$\rho_P(2x) = \rho_Q(x) \Rightarrow \boxed{\frac{\rho_P}{\rho_Q} = \frac{1}{2}}$$

Closed U-tube with trapped air

A U-tube closed at one end contains mercury. Air at pressure $5.0 \times 10^4 \text{ Pa}$ is trapped at the closed end. Atmospheric pressure is $1.01 \times 10^5 \text{ Pa}$. A piston of mass 5.0 kg and area $5.0 \times 10^{-4} \text{ m}^2$ acts on the other side.

$$P_p + P_{atm} = P_{air} + \rho gh$$

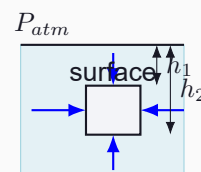
$$\frac{F}{A} + 1.01 \times 10^5 = (5.0 \times 10^4) + (13600)(9.8)h$$

$$\frac{5 \times 9.81}{5 \times 10^{-4}} + 1.01 \times 10^5 = 5.0 \times 10^4 + (13600)(9.8)h$$

$$h = 1.1 \text{ m} = \boxed{110 \text{ cm}}$$

Upthrust

An upward force acting on the body due to the difference in pressures at the top and bottom surface of the body submerged in a fluid.



$P = \rho gh$, $P \times h$ idea: pressure increases with depth.

$$F_1 = P_1 A, F_2 = P_2 A$$

$$F_{up} = F_2 - F_1$$

$$\boxed{F_{up} = (P_2 - P_1)A}$$

Upthrust Derivation, Floating, Sinking and Drag

- Upthrust force is not water resistance / drag force.
- Drag force appears on a body when it starts to move in a fluid.
- Stationary body: no drag force.

$$F_{up} = (P_2 - P_1)A$$

P_2 : total pressure at 2, P_1 : total pressure at 1

$$F_{up} = \{(\rho gh_2 + P_{atm}) - (\rho gh_1 + P_{atm})\}A$$

$$F_{up} = (\rho gh_2 - \rho gh_1)A$$

$$F_{up} = \rho g(h_2 - h_1)A$$

Since $H = h_2 - h_1$,

$$F_{up} = \rho gHA$$

As $V = A \times H$,

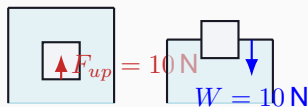
$$F_{up} = \rho gV$$

$$F_{up} = m_L g$$

m_L : mass of liquid displaced.

Key result

Upthrust force is equal to the weight of liquid displaced.

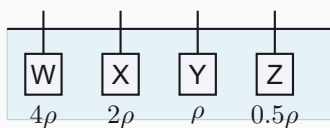


Volume/size of object decides the volume of water displaced and hence the upthrust force.

Cuboids held at same depth

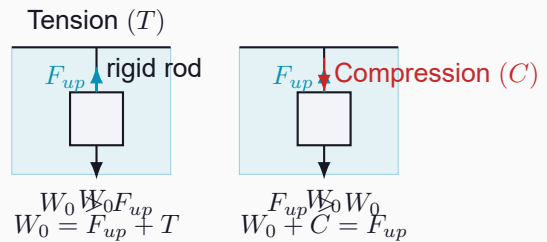
Four cuboids with identical length, breadth and height are immersed in water. They are at the same depth and in identical orientations. Therefore each displaces the same volume of water:

$$F_{up} = \rho gV \Rightarrow \text{same upthrust for all.}$$



The upthrust of the water on each cuboid is the same.

Rigid Rod: Tension and Compression

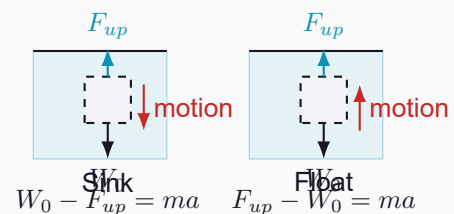


Equilibrium: rest.

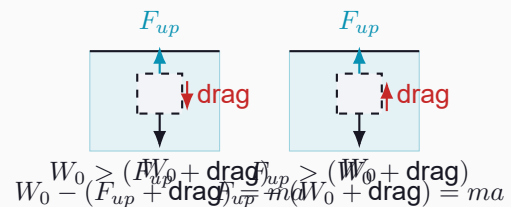
If the rigid rod is disconnected

The body accelerates.

$$F_{net} = ma$$



With drag after motion starts



Exam Question: Balance Reading and Immersion

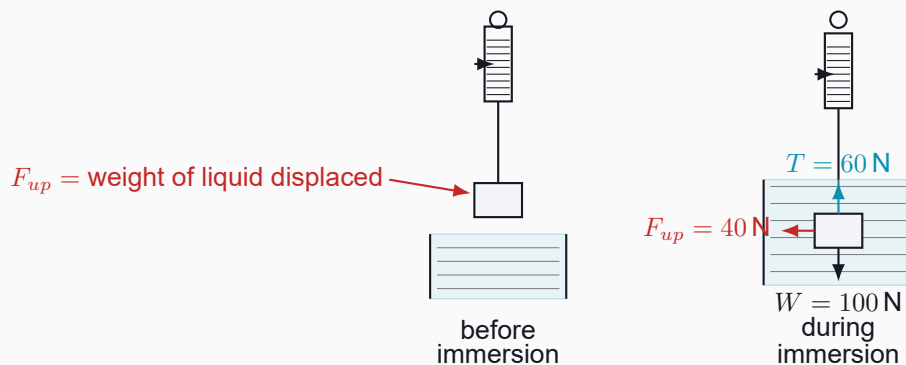
PHYSICS

Paper 1 Multiple Choice

1 hour

9702/12**October/November 2010**

- 13 The diagrams show a metal cube suspended from a spring balance before and during immersion in water.



A reduction in the balance reading occurs as a consequence of the immersion.

Which statement is correct?

- A The balance reading will be further reduced if the cube is lowered further into the water.
- B The balance reading during immersion corresponds to the upthrust of the water on the cube.
- C The forces acting on the vertical sides of the cube contribute to the change in the balance reading.
- D The gravitational pull on the cube is unchanged by the immersion.

Explanation

During immersion the balance reading is reduced because the water provides an upward upthrust. If the cube is already completely submerged, lowering it further does not increase the displaced volume, so the upthrust is unchanged. Side forces cancel horizontally. The weight/gravitational pull on the cube remains $W = mg$, so the correct option is **D**.

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Work, Energy and Power

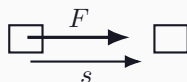
Chapter 08 / AS Level / Apollo Physics 9702



Work Done and Work Done by Gas

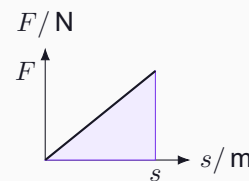
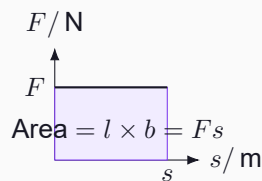
Work

It is the product of force and displacement parallel to force.



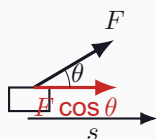
$$W = F \times s$$

- Scalar quantity.
- SI unit: Joule (J).



Force at an angle

Only the component of force parallel to displacement does work.



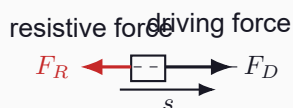
$$W = (F \cos \theta)s$$

Workdone by driving force

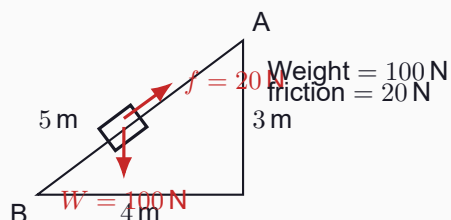
$$W = F_D \times s$$

Workdone against resistive force

$$W = F_R \times s$$



Worked example: block on a slope



(a) Workdone by gravity/weight:

$$W = 100 \times 3 = 300 \text{ J}$$

(b) Workdone against friction:

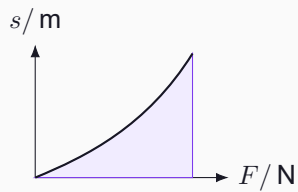
$$W = 20 \times 5 = 100 \text{ J}$$

Area under a force-displacement graph

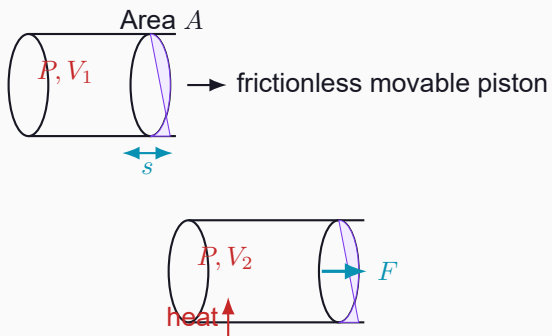
Area under $F - s$ graph = workdone.

Area under force-displacement graph

For variable force, **workdone is the area below the graph and displacement axis.**



Workdone by gas



“Piston moves at constant pressure.”

$$W = F \times s, \quad F = PA$$

$$W = PA \times s$$

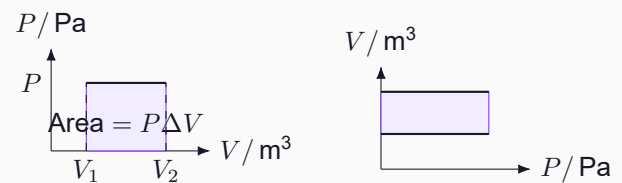
Since $A \times s = \Delta V$,

$$W = P\Delta V$$

$$W = P(V_2 - V_1)$$

Workdone by gas due to pressure P .

Area under a $P - V$ graph



Area under $P - V$ graph = workdone by gas.

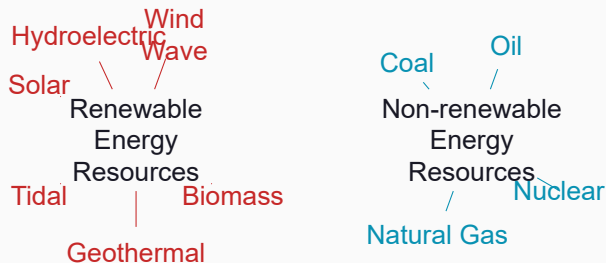
Energy, GPE and Kinetic Energy

Energy

Ability of a body to do work.

- Scalar quantity.
- SI unit: Joules (J).

Energy resources



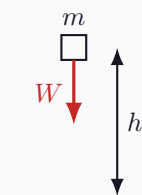
Types of energy

1. Gravitational potential energy
2. Elastic potential energy
3. Kinetic energy
4. Heat
5. Light
6. Sound
7. Nuclear energy
8. Chemical energy
9. Total energy
10. Internal energy

Gravitational Potential Energy

It is the energy possessed by a body at a point in a gravitational field.

Gain in GPE = Workdone against gravity

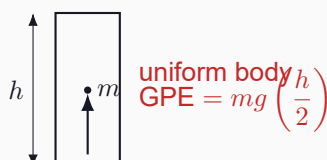


$$\Delta \text{GPE} = F \times s = W \times h$$

$$\Delta \text{GPE} = mgh$$

Taking height for GPE

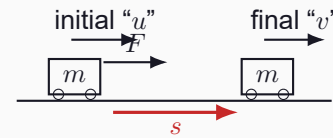
To determine GPE, take height till the C.O.G. of body.



Uniform body: $\text{GPE} = mgh, \quad \text{GPE} = mg \left(\frac{H}{2} \right)$

Kinetic Energy

It is the energy possessed by a body due to its motion.



Change in K.E. = Workdone by force

$$\Delta K.E. = F \times s = ma \times s$$

Using $v^2 - u^2 = 2as$, so $s = \frac{v^2 - u^2}{2a}$,

$$\Delta K.E. = ma \left(\frac{v^2 - u^2}{2a} \right)$$

$$\Delta K.E. = \frac{1}{2}m(v^2 - u^2)$$

If $u = 0$, then $K.E_i = 0$ and

$$K.E. = \frac{1}{2}mv^2$$

K.E. in terms of mass and momentum

$$p = mv, \quad v = \frac{p}{m}, \quad K.E. = \frac{1}{2}mv^2$$

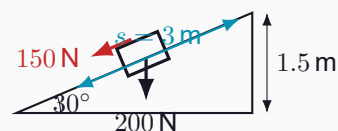
$$K.E. = \frac{1}{2}m \left(\frac{p}{m} \right)^2 = \frac{1}{2}m \frac{p^2}{m^2}$$

$$K.E. = \frac{p^2}{2m}$$

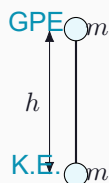
Work-Energy Relationship, Conservation and Power

Work-Energy relationship and Law of Conservation of Energy

"Energy can neither be created nor be destroyed. It is only converted from one form to another."



Example #1: ball falls under gravity



Ignoring air resistance:

loss in GPE = gain in K.E.

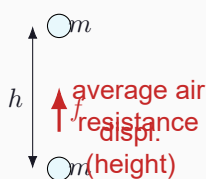
$$mgh = \frac{1}{2}m(v^2 - u^2)$$

With air resistance:

loss in GPE = gain in K.E. + heat

$$mgh = \frac{1}{2}m(v^2 - u^2) + (f \times s)$$

Work done by person = GPE + heat



Exam-style example: block sliding down slope

A block of mass 2.0 kg is released from rest on a slope. It travels 7.0 m down the slope and falls a vertical distance of 3.0 m. The block experiences a frictional force parallel to the slope of 5.0 N. What is the speed after falling this distance?



$$W_D = mgh + (f \times s) = 200(1.5) + 150(3) = \boxed{750 \text{ J}}$$

loss in GPE = gain in K.E. + work done against friction

$$2(9.81)(3) = \frac{1}{2}(2)(v^2 - 0) + (5)(7)$$

$$v = 4.88 \text{ m s}^{-1} \approx \boxed{4.9 \text{ m s}^{-1}}$$

Choices: **A** 4.9 m s⁻¹ **B** 6.6 m s⁻¹ **C** 6.8 m s⁻¹
D 10.1 m s⁻¹

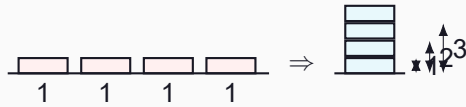
Exam-style example: pushing a box up a ramp

A box of weight 200 N is pushed so that it moves at steady speed along a ramp through a height of 1.5 m. The ramp angle is 30° and the frictional force is 150 N.

Choices: **A** 150 J **B** 300 J **C** 450 J **D** 750 J

Stacking blocks

Initially, four identical uniform blocks, each of mass m and thickness h , are spread on a table. How much work is done in stacking them one on another?

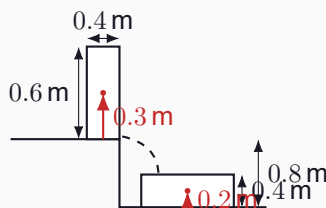


Workdone = gain in GPE = $mgh + 2mgh + 3mgh = 6mgh$

Choices: **A** $3mgh$ **B** $6mgh$ **C** $8mgh$ **D** $10mgh$

Block rotating over a step

A uniform solid block has weight 500 N, width 0.4 m and height 0.6 m. The block rests on the edge of a step of depth 0.8 m. It rotates through 90° before coming to rest with the 0.6 m edge horizontal.



C.O.G. is in the middle.

$$h_i = 0.3 + 0.8 = 1.1 \text{ m}, \quad h_f = 0.2 \text{ m}$$

$$\Delta \text{GPE} = mg\Delta h = 500(0.2 - 1.1) = -450 \text{ J}$$

Magnitude of change: 450 J.

Power

Rate at which work is done.

Rate at which energy is converted.

$$P = \frac{W}{t} \rightarrow \frac{\text{workdone}}{\text{time}}, \quad P = \frac{E}{t} \rightarrow \frac{\text{energy}}{\text{time}}$$

- SI unit: Watt (W).
- Scalar quantity.

$$P = \frac{W}{t}, \quad W = F \times s$$

Since $\frac{s}{t} = v$,

$$P = Fv$$

Force	Power
Driving force	useful power output
Resistive force (friction, air resistance, drag)	power loss as heat

power loss $\rightarrow$ rate of heat loss (same w.r.t. time)

Power Formulae, Efficiency and Exam Questions

Power from energy formulae

$$P = \frac{E}{t}$$

If $E = mgh$

$$P = \frac{mgh}{t}$$

Since $\frac{m}{t} = \dot{m}$ is mass flow rate,

$$P = \dot{m}gh$$

Examples: water pumped to a height.

$$P = \frac{mgh}{t}$$

Since $\frac{h}{t} = v$ is speed,

$$P = mgv$$

If $E = \frac{1}{2}mv^2$

$$P = \frac{\frac{1}{2}mv^2}{t}$$

$$P = \frac{1}{2}\dot{m}v^2$$

Example: liquid flowing horizontally.

$$\dot{m} = \rho Av$$

Efficiency

It is the ratio of useful output to the required input.

$$\text{efficiency} = \frac{\text{useful output}}{\text{required input}} \times 100$$

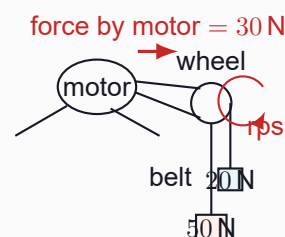
Valid ratios:

$$\frac{E_o}{E_i}, \quad \frac{W_o}{W_i}, \quad \frac{P_o}{P_i}, \quad \frac{W_o}{E_i}, \quad \frac{E_o}{W_i}$$

Ratio of work or energy with power is not valid.

Electric motor and belt

The wheel attached to the motor axle has a circumference of 0.5 m. The belt passes over the wheel without slipping. The weights are 50 N and 20 N. The wheel makes 20 revolutions per second.



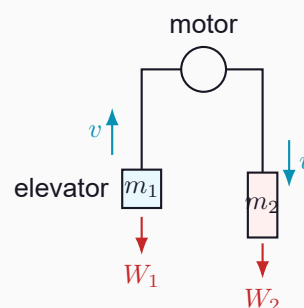
$$P = Fv = (50 - 20) \times \left(\frac{0.5 \times 20}{1} \right)$$

$$P = 30 \times 10 = 300 \text{ W}$$

Choices: **A** 300 W **B** 500 W **C** 600 W **D** 700 W

Counterbalanced lift

An elevator of mass m_1 is partly counterbalanced by a heavy weight of mass m_2 . The elevator rises at steady speed v .



Force by motor:

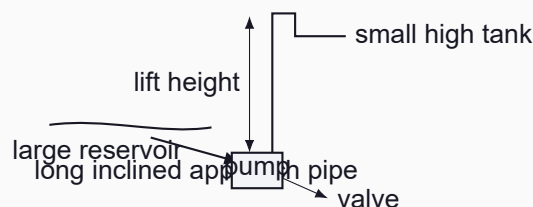
$$W_1 - W_2 = m_1g - m_2g$$

$$P = Fv = (m_1 - m_2)gv$$

Answer: $(m_1 - m_2)gv$.

Hydraulic ram

A pump called a hydraulic ram uses the kinetic energy of moving water in a long approach pipe to lift a small quantity of water to a height of 15 m. The pipe holds 500 kg of water moving at 2.0 m s^{-1} . Efficiency is 10%.



$$10\% \text{ of K.E.} = \text{GPE}$$

$$m = \boxed{0.68 \text{ kg}}$$

$$0.1 \left(\frac{1}{2} (500) (2.0)^2 \right) = m (9.81) (15)$$

Choices: **A** 0.15 kg

B 0.68 kg

C 1.5 kg

D 6.8 kg

Current Electricity

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Circuit Symbols

Name	Symbol	Name	Symbol
Cell		Switch: one-way	
Battery		Switch: two-way	
Power supply: DC		Ammeter	
Power supply: AC		Galvanometer	
Fixed resistor		Voltmeter	
Rheostat / variable resistor		Ohmmeter	
LDR		Diode	
Thermistor		LED	
Filament lamp		Capacitors	

Current, Conventional Current, Electron Flow, DC and AC

Current

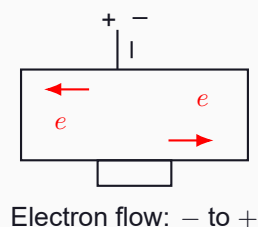
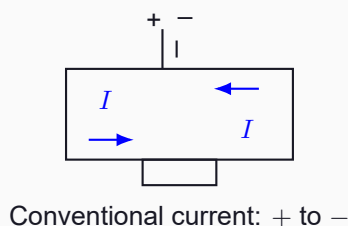
Rate of flow of electric charges

OR

Rate of flow of electrons

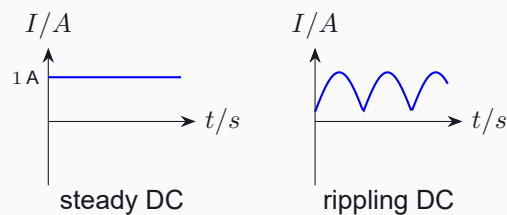
$$I = \frac{q}{t} \quad \text{or} \quad I = \frac{\Delta q}{\Delta t}$$

- **SI unit:** ampere (A)
- **Nature:** scalar quantity
- Δq : charge passing a point; Δt : time taken.



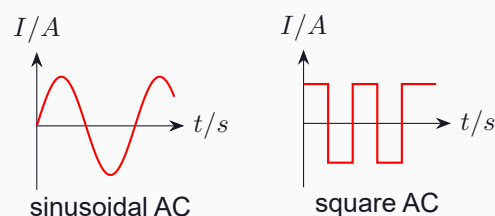
1. Direct Current (DC)

It flows in one direction.



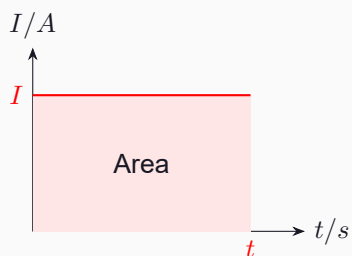
2. Alternating Current (AC)

Current changes direction periodically.



Graph: $I - t$, Charge and Number of Electrons

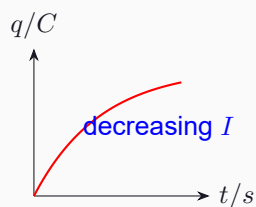
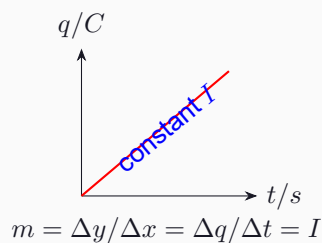
Area under an $I - t$ graph



$$\text{Area} = l \times b = I \times t = q$$

$$1e = 1.6 \times 10^{-19} \text{ C}$$

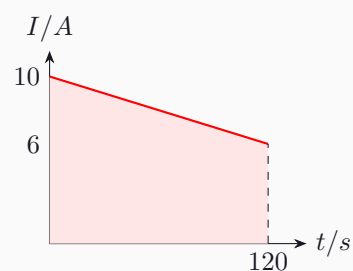
Graph: $q - t$



Example 1

A current in a circuit decreases **linearly** from 10 A to 6 A in a time of 2 minutes. Calculate:

- the total charge passing a point,
- the number of electrons passing a point.



$$q = \frac{1}{2}(10 + 6)(120) = 960 \text{ C}$$

$$1.6 \times 10^{-19} \text{ C} \equiv 1e, \quad 960 \text{ C} \equiv x$$

$$x = \frac{960 \times 1}{1.6 \times 10^{-19}} = 6.0 \times 10^{21} \text{ electrons.}$$

Current Equation and Number Density

Current through a conductor

$$I = nAve$$

I :	current
n :	number density
A :	cross-sectional area
v :	mean drift speed of charge carriers
e :	elementary charge

Number density, n

Number of conduction electrons available per unit volume:

$$n = \frac{N}{V}$$

where N is the number of free mobile electrons and V is the volume of conductor.

$n = \frac{N}{V}$	→ No. of free mobile electrons → Volume of conductor
-------------------	--

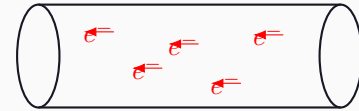
Conductor > semi-conductor > insulator: number density decreases.

$e \ e \ e \ e \ e \ \longrightarrow \ e \ e \ e \ \longrightarrow \ e$
 conductor semi-conductor insulator

Mean Drift Speed, v

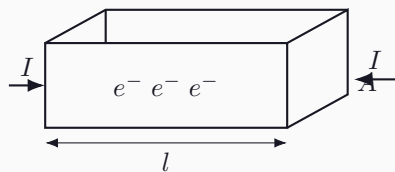
Mean: not all electrons have the same speed. The calculated value is an average value.

Drift: electrons do not flow smoothly; they drift from their places. Their motion is hindered by atoms and other electrons.



Deriving $I = nAve$ and Drift-Speed Proportions

Derivation



$$I = \frac{q}{t}, \quad q = Ne \Rightarrow I = \frac{Ne}{t}.$$

Since $n = N/V$, $N = nV$:

$$I = \frac{nVe}{t}.$$

For a length l of wire, $V = Al$, and since $v = l/t$,

$$I = \frac{n(Al)e}{t} = nA \left(\frac{l}{t} \right) e = \boxed{nAve}.$$

Mean drift speed

$$\boxed{v = \frac{I}{nAe}}, \quad v \propto I, \quad v \propto \frac{1}{A}, \quad v \propto \frac{1}{n}.$$

Physical meanings

- $v \propto I$: more current means more electrons passing a point, so electrons must travel faster.
- $v \propto 1/A$: where the cross-sectional area is smaller, electrons travel faster to establish the same rate of flow.
- $v \propto 1/n$: lower number density means fewer conduction electrons are available, so they must travel faster to establish the same rate of flow.



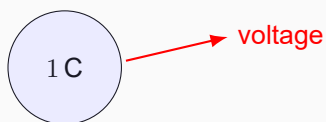
Same current through a smaller area $\Rightarrow$ faster drift.

Voltage, EMF and Potential Difference

Voltage

Energy possessed by a unit charge.

SI unit: volt (V) or J C^{-1} Scalar quantity



Electromotive Force (EMF)	Potential Difference (PD)
Definition: It is the work done per unit charge to move electrons around the complete circuit. - It is the voltage of a source e.g. cell, battery, power supply, generator, etc.	Definition: It is the energy converted per unit charge when passing between two points in a circuit. - It is a voltage across any component of the circuit e.g. resistor, lamp, cell, battery, etc. P.D. across the source is called EMF.

Resistance

Definition

Resistance is the ratio of voltage across a resistor to the current through it.

$$R = \frac{V}{I}$$

SI unit: ohm (Ω)

It is the hindrance in the flow of electrons.

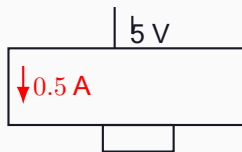
Meaning

- $R = 10\ \Omega$ means 10 V per ampere.
- $R = 2.5\ \Omega$ means 2.5 V per ampere.

For the same work done, the first resistor had less current, so it has more resistance.

OR for the same value of current, more work needs to be done in the first resistor; hence it has larger resistance.

Two examples at the same voltage



$$R = V/I = 5/0.5 = 10\ \Omega \quad R = V/I = 5/2 = 2.5\ \Omega$$

Factors Affecting Resistance

1. Temperature

Temperature $\uparrow \rightarrow$ lattice vibrations increase $\rightarrow$ mean drift speed $\downarrow \rightarrow I \downarrow \rightarrow R \uparrow$

2. Length of conductor, l

$$R \propto l$$

Length $\uparrow \rightarrow$ more work needs to be done for same current $\rightarrow V \uparrow \rightarrow R \uparrow$.

3. Cross-sectional area, A

$$R \propto \frac{1}{A}$$

Cross-sectional area $\uparrow \rightarrow$ more electrons pass due to more space $\rightarrow I \uparrow \rightarrow R \downarrow$.

$$R \propto \frac{l}{A} \quad \Rightarrow \quad R = \rho \frac{l}{A}$$

Changing Length, Diameter, Area and Resistance; Resistivity

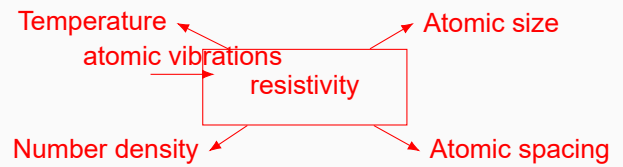
l/m	d/m	A/m^2	R/Ω
l	d	A	R
$2l$	d	A	$2R$
$l/2$	d	A	$R/2$
l	$2d$	$4A$	$R/4$
l	$d/3$	$A/9$	$9R$
$2l$	$3d$	$9A$	$\frac{R \times 2}{9} = \frac{2R}{9}$
$l/2$	$2d$	$4A$	$\frac{R}{2 \times 4} = \frac{R}{8}$
$2l$	$d/2$	$A/4$	$R \times 2 \times 4 = 8R$
$l/3$	$d/3$	$A/9$	$\frac{R \times 9}{3} = 3R$

$$A = \pi r^2 = \frac{\pi d^2}{4}, \quad A \propto d^2.$$

Resistivity, ρ

It is a material property and depends upon the element used to make the conductor.

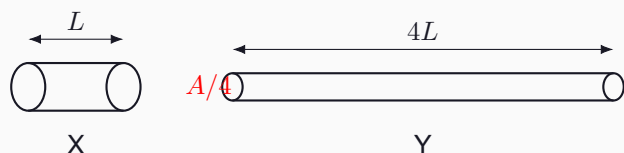
$$R = \rho \frac{l}{A} \quad \Rightarrow \quad \rho = \frac{RA}{l}.$$



Exam Practice: Resistance and Resistivity

Q34. Same volume wires

Two copper wires X and Y have the same volume. Wire Y is four times as long as wire X.



What is the ratio $\frac{\text{resistance of wire Y}}{\text{resistance of wire X}}$?

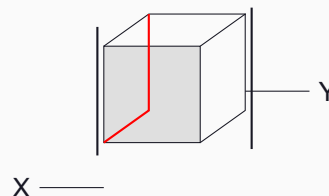
- A 4 B 8 C 16 D 64

Solution:

$$R_Y = R_X \times 4 \times 4 = 16R_X \quad \Rightarrow \quad \frac{R_Y}{R_X} = 16.$$

Q34. Metal cube between plates

The resistance of a metal cube is measured by placing it between two parallel plates. The cube has volume V and is made of a material with resistivity ρ .



For a cube, $V = l^3$, so $l = V^{1/3}$ and $A = l^2$. Hence

$$R = \rho \frac{l}{A} = \rho \frac{l}{l^2} = \frac{\rho}{l} = \boxed{\frac{\rho}{V^{1/3}}}.$$

Correct option: C.

Ohm's Law

Statement

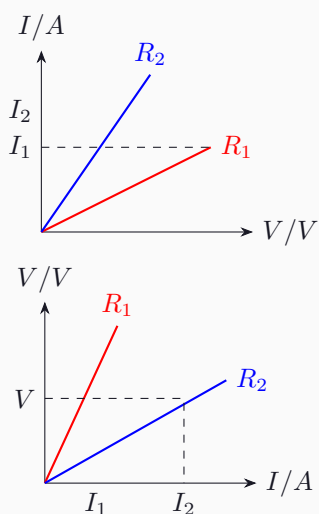
At constant temperature, the current flowing through a conductor is directly proportional to the potential difference across it.

$$I \propto V$$

$$I = GV, \quad G = \frac{1}{R}, \quad I = \frac{1}{R}V, \quad V = IR.$$

where G is conductance and $G = 1/R$.

$I - V$ graphs



Important note

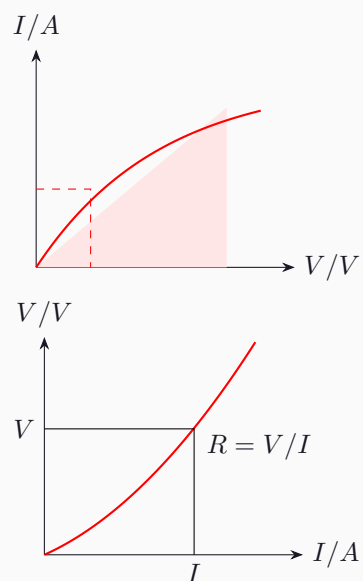
Do not explain using gradient as $R \neq \Delta V / \Delta I$ for every graph; use $R = V/I$.

Pick a value of V and check the corresponding value of I . The one which has less current has more resistance.

- R_1 has more resistance than R_2 .
- Resistors that obey Ohm's law are called **Ohmic conductors**.

Filament Lamp

Characteristic curves

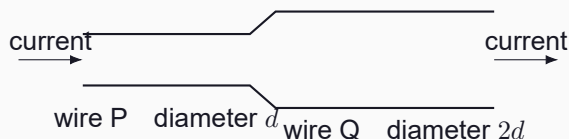


As the temperature of the filament lamp increases due to higher currents, its resistance also increases. Hence, for the same increase of voltage, the increase of current is less at higher voltages.

Exam Practice: Mean Drift Speed

Q33. Two copper wires joined together

Two copper wires are joined together and carry a current as shown. Wire P has diameter d and wire Q has diameter $2d$.



Find the ratio

average drift speed of free electrons in wire P
 average drift speed of free electrons in wire Q.

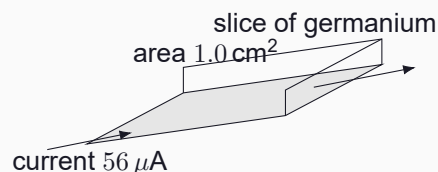
A $\frac{1}{4}$ B $\frac{1}{2}$ C 2 D 4

Solution: $v = I/(nAe)$. Same material and same current, so $v \propto 1/A$ and $A \propto d^2$.

$$v_P d^2 = v_Q (2d)^2 \Rightarrow \frac{v_P}{v_Q} = 4.$$

Q30. Germanium slice

A slice of germanium of cross-sectional area 1.0 cm^2 carries a current of $56 \mu\text{A}$. The number density of charge carriers is $2.0 \times 10^{13} \text{ cm}^{-3}$. Each charge carrier has charge equal to that of an electron.



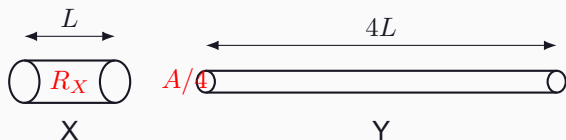
$$I = nAve \Rightarrow 56 \times 10^{-6} = 2 \times 10^{13} (1) v (1.6 \times 10^{-19})$$

$$v = 17.5 \text{ cm s}^{-1} = 0.175 \text{ m s}^{-1} \approx 0.18 \text{ m s}^{-1}.$$

Exam Practice: Resistance with Same Volume and Wire Radius

Q34. Same volume: resistance ratio

Two copper wires X and Y have the same volume. Wire Y is four times as long as wire X.



$$R = \rho \frac{l}{A}, \quad R \propto l, \quad R \propto \frac{1}{A}.$$

Same volume gives $V = Al$ constant, so if l increases by 4, area decreases by 4.

$$R_Y = R_X \times 4 \times 4 = 16R_X \quad \Rightarrow \quad \frac{R_Y}{R_X} = 16.$$

Q32. Radius of wire

A cylindrical wire 4.0 m long has resistance $31 \, \Omega$ and is made of metal of resistivity $1.0 \times 10^{-6} \, \Omega \text{ m}$. Find the radius of cross-section of the wire.

$$R = \rho \frac{l}{A} = \frac{\rho l}{\pi r^2}$$

$$31 = \frac{(1.0 \times 10^{-6})(4.0)}{\pi r^2}$$

$$r = \sqrt{\frac{(1.0 \times 10^{-6})(4.0)}{31\pi}} \approx 2.0 \times 10^{-4} \text{ m}.$$

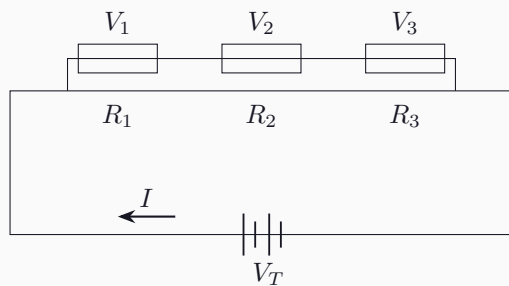
Correct option: **D $2.0 \times 10^{-4} \text{ m}$** .

DC Circuits and Kirchhoff's Laws

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Series Combination



Current

- It is the same everywhere in a series circuit irrespective of resistance.
- If one resistor is replaced by another resistor of a different value, a new current flows in the circuit.

$$I = I_1 = I_2 = I_3$$

Voltage

- Voltage is divided among resistors in the ratio of their resistances.
- Larger resistor consumes more voltage and smaller resistor consumes less.

Example: $R_1 = 100\ \Omega$, $R_2 = 500\ \Omega$, $R_3 = 300\ \Omega$, $V_T = 9\text{ V}$.

$$R_1 : R_2 : R_3 = 100 : 500 : 300 = 1 : 5 : 3$$

$$V_1 = 1\text{ V}, \quad V_2 = 5\text{ V}, \quad V_3 = 3\text{ V}$$

Sum of all potential drops across resistors is equal to EMF:

$$V_T = V_1 + V_2 + V_3$$

Resistance

$$V_T = V_1 + V_2 + V_3, \quad IR_T = IR_1 + IR_2 + IR_3$$

Since $I_T = I_1 = I_2 = I_3$,

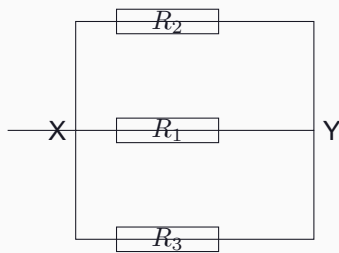
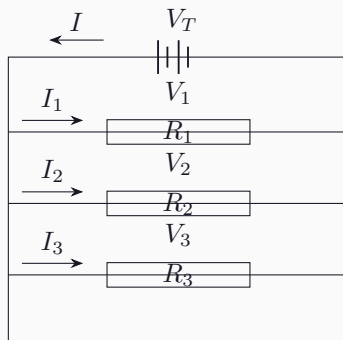
$$R_T = R_1 + R_2 + R_3$$

If all resistors are same:

$$R_T = nR$$

Parallel Combination

Standard parallel circuit



Voltage

The voltage across each resistor is the same irrespective of resistance, as all resistors are attached across the same terminals.

$$V_T = V_1 = V_2 = V_3$$

Current

Current in each resistor depends on the resistance of each resistor.

- Larger resistance receives less current.
- Smaller resistance receives more current.

Several circuit layouts may still be electrically equivalent if their resistors share the same two terminals.

Parallel Combination: Resistance Formula

However, the sum of currents in each resistor equals the current passing through the battery.

$$I_T = I_1 + I_2 + I_3$$

Resistance derivation

$$I_T = I_1 + I_2 + I_3$$

$$\frac{V_T}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

As $V_T = V_1 = V_2 = V_3$,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

If all parallel resistors have the same value

$$\frac{1}{R_T} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

$$R_T = \frac{R}{3}$$

In general,

$$R_T = \frac{R}{n}$$

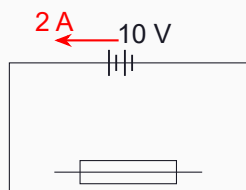
where n is the number of resistors.

Example: 3 resistors of $30\ \Omega$ in parallel:

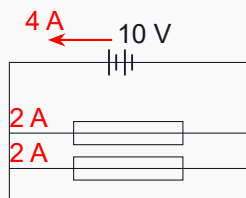
$$R_T = \frac{30}{3} = 10\ \Omega.$$

Attaching resistors in parallel decreases resistance. The total resistance is smaller than the smallest resistor.

Current Drawn in Series and Parallel: Worked Examples



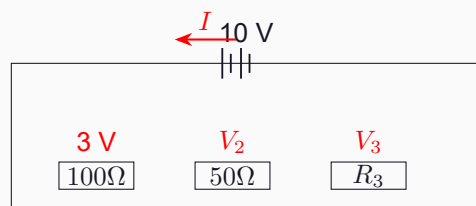
Series: $10 = 2R$, $R = 5\ \Omega$



Parallel: $10 = 4R$, $R = 2.5\ \Omega$

For the same voltage applied, the current drawn from the battery is more in case of parallel. Since $R = V/I$, effective resistance decreases.

Series voltage example



Determine I , V_2 , V_3 , R_3 .

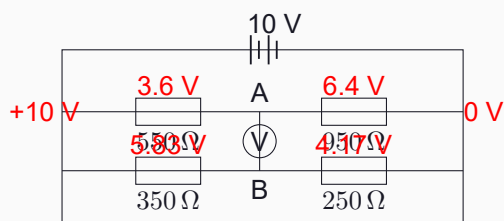
$$3 = I(100) \quad \Rightarrow \quad I = 0.03\text{ A} \quad \square$$

$$V_2 = IR = 0.03(50) = 1.5\text{ V} \quad \square$$

$$V_3 = 10 - 3 - 1.5 = 5.5\text{ V} \quad \square$$

$$5.5 = 0.03R_3 \quad \Rightarrow \quad R_3 = 183\ \Omega \quad \square$$

Determining Potential Difference Between Two Points



In a series circuit, I is constant and $V \propto R$.

$$\frac{R_T}{R} = \frac{V_T}{V}, \quad V = \frac{R}{R_T} V_T$$

$$V_{AB} = V_A - V_B.$$

For the upper branch:

$$1500\Omega \rightarrow 10\text{ V}, \quad 550\Omega \rightarrow x$$

$$x = \frac{550 \times 10}{1500} = 3.6\text{ V}, \quad V_A = 10 - 3.6 = 6.4\text{ V}$$

For the lower branch:

$$600\Omega \rightarrow 10\text{ V}, \quad 350\Omega \rightarrow x$$

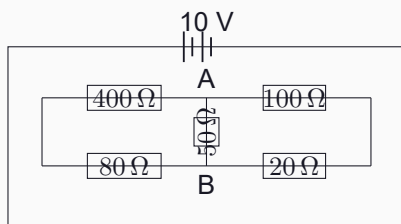
$$x = \frac{350 \times 10}{600} = 5.83\text{ V}, \quad V_B = 10 - 5.83 = 4.17\text{ V}$$

$$V_{AB} = 6.4 - 4.17 = 2.23\text{ V}$$

Another way: subtract the voltages across the resistors behind A and B.

$$V_{AB} = |3.6 - 5.83| = 2.23\text{ V}.$$

Potential Difference and Redundant Resistors



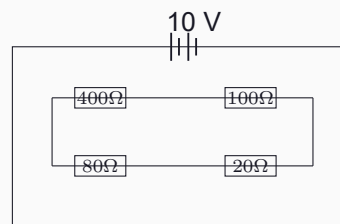
Determine the p.d. between A and B and the effective resistance of the circuit.

$$500\,\Omega \rightarrow 10\,\text{V}, \quad 400\,\Omega \rightarrow x \Rightarrow x = 8\,\text{V}, \quad V_A = 2\,\text{V}$$

$$100\,\Omega \rightarrow 10\,\text{V}, \quad 80\,\Omega \rightarrow x \Rightarrow x = 8\,\text{V}, \quad V_B = 2\,\text{V}$$

$$V_{AB} = 0\,\text{V}$$

No current flows through the $50\,\Omega$ resistor, as there is no potential difference between its ends. The $50\,\Omega$ resistor is redundant. Ignore it / see zone it:

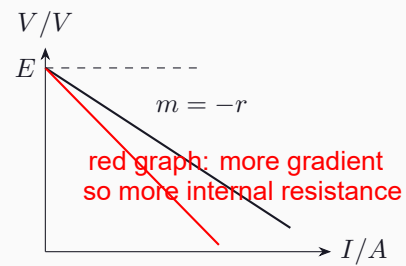
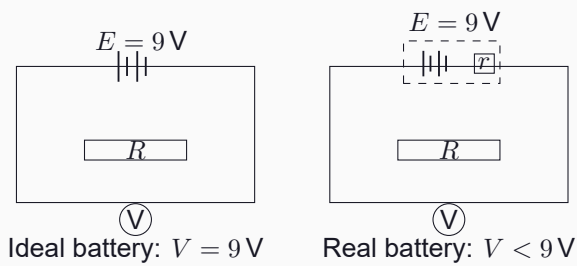


The upper branch is $500\,\Omega$ and the lower branch is $100\,\Omega$ in parallel:

$$\frac{1}{R_T} = \frac{1}{500} + \frac{1}{100} = \frac{6}{500}$$

$$R_T = 83.3\,\Omega$$

Internal Resistance and Maximum Power Transfer Theorem



Internal resistance is the resistance of the battery itself that causes a reduction in terminal potential difference across the source.

$$V = E - Ir$$

Terminal potential difference is the voltage across the terminals of the battery.

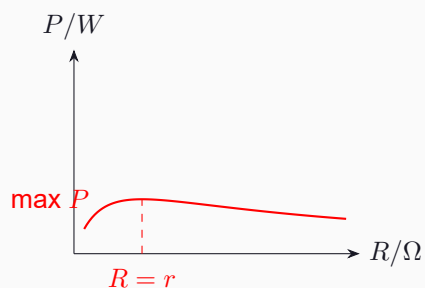
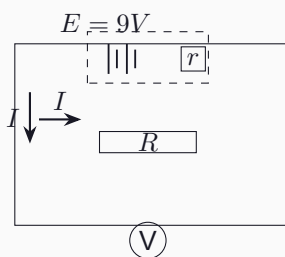
$$V = -rI + E \quad (y = mx + c)$$

Maximum power transfer

External resistor receives maximum power when the external resistance is equal to internal resistance:

$$R = r$$

Power Output at External Resistor



Power output at external resistor:

$$P = I^2 R$$

The circuit is a series circuit:

$$V_T = V_1 + V_2, E = Ir + IR = I(r + R), \quad I = \frac{E}{r + R}.$$

Replace I in $P = I^2 R$:

$$P = \left(\frac{E}{r + R} \right)^2 R = \frac{R}{(r + R)^2} E^2$$

Assume $E = 10 \text{ V}$, $r = 2 \Omega$:

$$R = 1 \Omega : P = \frac{1}{(2 + 1)^2} (10)^2 = 11.1 \text{ W}$$

$$R = 2 \Omega : P = \frac{2}{(2 + 2)^2} (10)^2 = 12.5 \text{ W}$$

$$R = 5 \Omega : P = \frac{5}{(2 + 5)^2} (10)^2 = 10.2 \text{ W}$$

If battery were ideal, $r = 0 \Omega$ and $E = 10 \text{ V}$:

$$P = \frac{V^2}{R} = \frac{10^2}{2} = 50 \text{ W}.$$

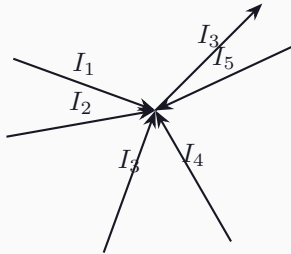
$$P_{\text{real}} = 25\% P_{\text{ideal}}$$

Kirchhoff's Laws

Kirchhoff's 1st Law

Kirchhoff's Current Law (KCL)

The total current entering a point in a circuit is equal to the total current coming out of that point.



$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

$$I_1 + I_2 = I_3 + I_4 + I_5$$

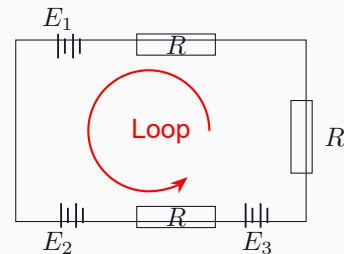
Example: $4\text{ A} + 5\text{ A} = 1\text{ A} + 3\text{ A} + 5\text{ A}$.

It is related to the law of conservation of charge.

Kirchhoff's 2nd Law

Kirchhoff's Voltage Law (KVL)

The sum of all the EMFs is equal to the sum of all the potential drops in a loop.

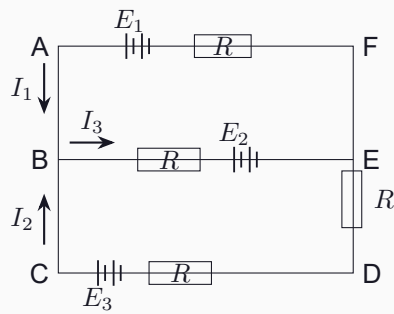


$$\sum \text{EMF} = \sum \text{PD}$$

$$E_1 + E_2 - E_3 = IR + IR + IR = 3IR$$

It is related to the law of conservation of energy.

Kirchhoff Loop Equations



Using Kirchhoff laws, write down:

- i) I_1 , I_2 and I_3 .
- ii) Loop ABEF.
- iii) Loop BCDE.
- iv) Loop ABCDEF.

i)

$$I_1 + I_2 = I_3$$

ii) For loop ABEF:

$$E_1 - E_2 = I_1 R + I_3 R$$

iii) For loop BCDE:

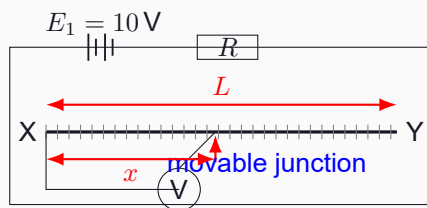
$$E_2 - E_3 = I_2 R + I_3 R + I_2 R$$

iv) For loop ABCDEF:

$$E_1 - E_3 = I_1 R + 2I_2 R$$

Note: E_2 opposes E_1 , so it is taken negative. Direction of a current opposite to the chosen convention is taken negative.

Potentiometer



If the junction T is at:

- X : $V = 0$ as $x = 0$;
- Y : $V = V_{XY}$ as $x = L$;
- moving from X to Y : voltmeter reading increases.

Uniform wire principle

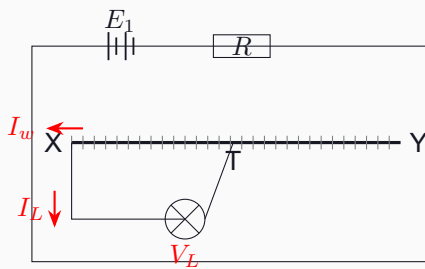
$$R \propto L, \quad V \propto R \Rightarrow V \propto L$$

The wire acts as a variable resistor. Voltage across the length of wire considered depends upon the length considered.

$$\frac{L}{x} = \frac{V_{XY}}{V}$$

$$V = \frac{xV_{XY}}{L}$$

Replacing the Voltmeter with a Lamp



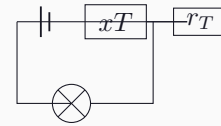
Replacing the voltmeter with the lamp shows how voltage changes from X to Y .

- When T is at X : $V_{\text{lamp}} = 0$, so $I_L = 0$ and lamp is off.
- When T is at Y : $V_{\text{lamp}} = V_{XY}$, which is maximum, so I_L is maximum and lamp is at maximum brightness.

Moving the junction from X to Y :

$$V_{\text{lamp}} \uparrow \Rightarrow I_L \uparrow$$

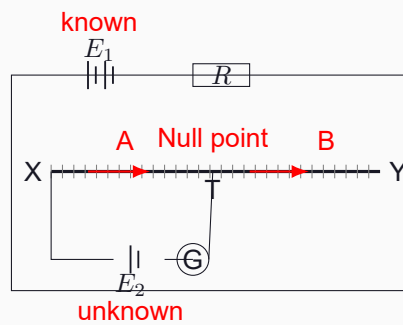
$$I = I_w + I_L$$



Lamp is in parallel to the wire xT , so:

$$V_{xT} = V_{\text{lamp}}.$$

Finding an Unknown EMF Using a Known EMF



Voltage across the wire between the two junctions X and T is used to balance out the current produced by cell E_2 .

The length between X and T is adjusted to change the potential difference across E_2 .

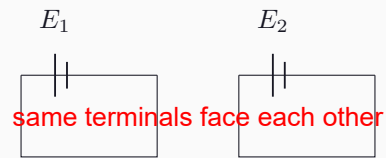
- If length XT is small, then $V_{XT} < E_2$ and current flows according to E_2 .
- If length XT is large, then $V_{XT} > E_2$ and current flows according to XT .
- At a particular length, $V_{XT} = E_2$.
- Galvanometer reads zero at the null point.
- Current only flows in the top part of the circuit and is the same value everywhere.

Find V_{XT} using the ratio method or $V = IR$ to find E_2 eventually.

Conditions for Potentiometer Method

For this method to work:

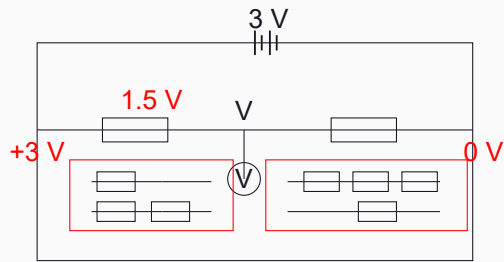
1. $E_2 < E_1$, so the voltage across the wire can balance it.
2. E_1 and E_2 must have the same terminals facing each other.



Practice and exam-question applications

MCQ: Equivalent Resistance and Voltmeter Reading

A circuit is set up as shown, supplied by a 3 V battery. All resistances are $1\text{ k}\Omega$.



What will be the reading on the voltmeter?

- A 0 B 0.5 V C 1.0 V D 1.5 V

Left network:

$$R_{\text{left}} = 1.5\text{ k}\Omega$$

Right network:

$$R_{\text{right}} = 0.75\text{ k}\Omega$$

Ratio method:

$$V = \frac{R}{R_T} \times V_T = \frac{1.5}{1.5 + 0.75} \times 3 = 2\text{ V}$$

Potential at the middle points differs by:

$$2.0\text{ V} - 1.5\text{ V} = 0.5\text{ V}$$

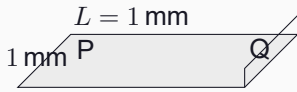
Voltmeter reading = 0.5 V

Thin Film Resistor: $R = \rho L / A$

A thin film resistor is made of nichrome of thickness $1 \mu\text{m}$. The resistivity of nichrome is $10^{-6} \Omega \text{m}$. Calculate the resistance between opposite edges of such a film of area 1 mm^2 .

a) If it is a square

$$R_s = \frac{\rho L}{A}$$



$$R = \frac{\rho L}{A} = \frac{10^{-6}(1 \times 10^{-3})}{(1 \times 10^{-3})(1 \times 10^{-6})} = 1 \Omega$$

Because breadth changes with length, the resistance can change significantly with orientation.

$$R = \frac{10^{-6}}{(1 \times 10^{-6})20x} = \frac{1}{20} \Omega$$

b) If rectangle length is 20 times its breadth

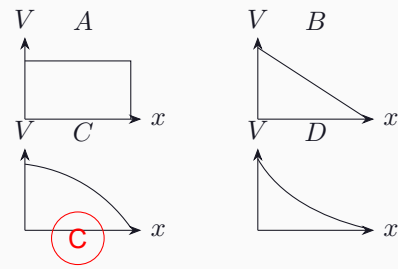
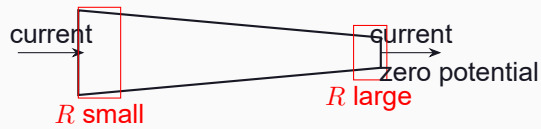


$$R = \frac{10^{-6}(20x)}{x(1 \times 10^{-6})} = 20 \Omega$$

For the same sheet area A_s :

Variable Cross-sectional Area: Potential Graph

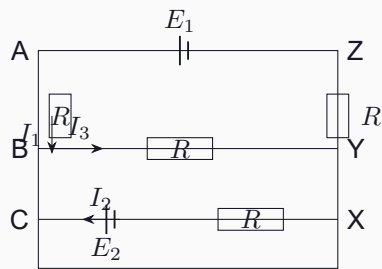
The circular cross-sectional area of a metal wire varies along its length. There is a current in the wire. The narrow end of the wire is at a reference potential of zero.



Correct option: **C**. The potential falls more steeply near the narrow end where the resistance per unit length is larger.

Where resistance is larger, the potential drop is larger.

Kirchhoff Laws: Network with Two Cells



$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

$$I_1 + I_2 = I_3$$

$$\sum \text{EMF} = \sum \text{PD}$$

Loop BCXYB:

$$E_2 = I_3 R + I_2 R$$

Loop ABCXYZA:

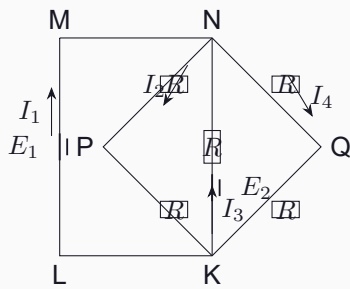
$$E_1 - E_2 = I_1 R + (-I_2) R + I_1 R$$

$$E_1 - E_2 = 2I_1 R - I_2 R$$

Use Kirchhoff's laws to state the relation:

- between currents I_1 , I_2 and I_3 ;
- between E_2 , R , I_2 and I_3 in loop BCXYB;
- between E_1 , E_2 , R , I_1 and I_2 in loop ABCXYZA.

Kirchhoff Laws: Diamond Network



At junction N/K:

$I_1 + I_3 = I_2 + I_4$

Loop NKLMN:

$$E_1 - E_2 = I_3 R$$

Loop NKQN:

$$E_2 = I_3 R + I_4 R + I_4 R$$

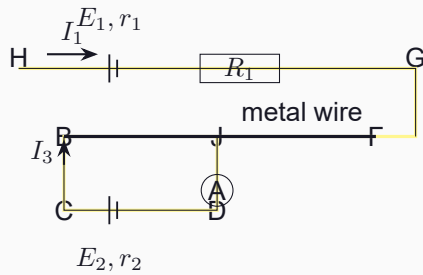
$$E_2 = I_3 R + 2I_4 R$$

Use Kirchhoff's laws to state the relation:

- i) between I_1, I_2, I_3 and I_4 ;
- ii) between E_1, E_2, R and I_3 in loop NKLMN;
- iii) between E_2, R, I_3 and I_4 in loop NKQN.

Potentiometer Comparison Circuit

A potentiometer circuit used as a means of comparing potential differences is shown.



Relations:

$$I_1 + I_3 = I_2$$

For loop HBJFGH:

$$E_1 = I_1 r_1 + I_1 R_1 + I_1 (0.5 R_2) + I_2 (0.5 R_2)$$

For loop HBCDJFGH:

$$E_1 - E_2 = I_3 r_2 + I_1 r_1 + I_1 R_1 + I_1 (0.5 R_2)$$

Moving J changes the resistance experienced by E_2 , so the current passing through E_2 changes and the sensitive ammeter reading changes.

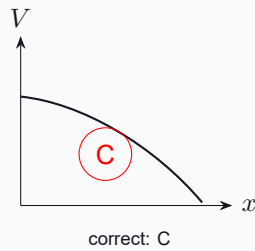
Potential Variation in Wires

Variable-area wire

For a wire with changing cross-section:

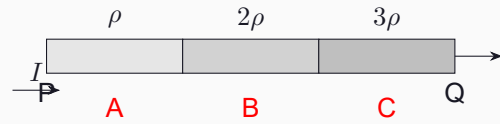
$$R = \frac{\rho L}{A}, \quad V = IR$$

Since the resistance per unit length is larger where area is smaller, the potential falls more rapidly in the narrow part.



Composite wire

A wire PQ is made of three materials with resistivities ρ , 2ρ and 3ρ . There is a current I .

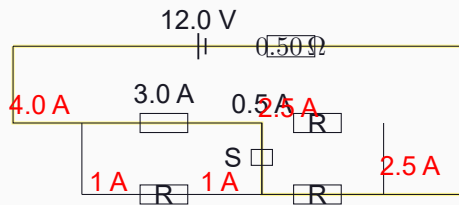


The graph with progressively larger gradient across ρ , 2ρ , 3ρ is correct.

Correct option: B

Circuit with Resistor Neither Series nor Parallel

The circuit contains a resistor S that is neither in series nor in parallel with the other resistors.



Using KCL, label currents as shown. Around the outer highlighted loop:

$$\sum \text{EMF} = \sum \text{PD}$$

$$12 = I(R) + I_S R_S + 4(0.5)$$

With the branch currents, the required identical resistor value is:

$$R = 4 \Omega$$

Correct option: B 4.0Ω

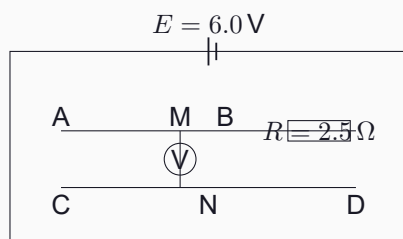
Uniform Resistance Wire and Parallel Branches

A uniform resistance wire AB has length 50 cm and diameter 0.36 mm. The resistivity is $5.1 \times 10^{-7} \Omega \text{ m}$.

a) Show that $R_{AB} = 2.5 \Omega$

$$R = \frac{\rho L}{A} = \frac{(5.1 \times 10^{-7})(0.50)}{\pi(0.36 \times 10^{-3})^2/4} = 2.5 \Omega$$

b) Circuit



The second wire CD has length 100 cm, diameter 0.18 mm and is made of the same metal. Since length doubles and area quarters,

$$R_{CD} = 2.5 \times 2 \times 4 = 20 \Omega.$$

For branch AB+R:

$$6 = I_1(5) \Rightarrow I_1 = 1.2 \text{ A}$$

For branch CD:

$$6 = I_2(20) \Rightarrow I_2 = 0.3 \text{ A}$$

$$I = I_1 + I_2 = 1.5 \text{ A}$$

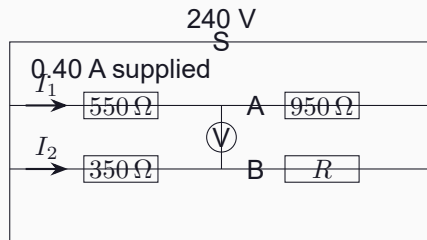
Potentials:

$$V_{AM} = 1.2(1.25) = 1.5 \text{ V}, \quad V_{CN} = 0.3(10) = 3.0 \text{ V}$$

$$V_{MN} = 3.0 - 1.5 = 1.5 \text{ V}$$

Two Parallel Series Branches: Power Ratio

A 240 V power supply with negligible internal resistance is connected to four resistors as shown.



Upper branch:

$$240 = I_1(550 + 950) \Rightarrow I_1 = 0.16 \text{ A}$$

Total current is 0.40 A, hence:

$$I_2 = 0.40 - 0.16 = 0.24 \text{ A}$$

Lower branch:

$$240 = 0.24(350 + R) \Rightarrow 350 + R = 1000$$

$$R = 650 \Omega$$

Power ratio:

$$\frac{P_{350}}{P_{550}} = \frac{I_2^2(350)}{I_1^2(550)} = \frac{0.24^2 \times 350}{0.16^2 \times 550} = 1.43$$

$$\text{ratio} = 1.43$$

General Waves

Chapter 11 / AS Level / Apollo Physics 9702



Wave and Classification of Waves

Wave

It is a disturbance in a medium.

OR

It is a mode of transferring energy from one point to another.

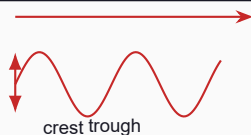
Classification of waves

1. Mechanical vs electromagnetic
2. Transverse vs longitudinal
3. Progressive vs stationary (new)

Mechanical waves	Electromagnetic waves
Waves which need a medium to travel.	No need of medium to travel.
Cannot travel in vacuum.	They can also travel in vacuum.
Sound, water waves.	Electromagnetic spectrum.

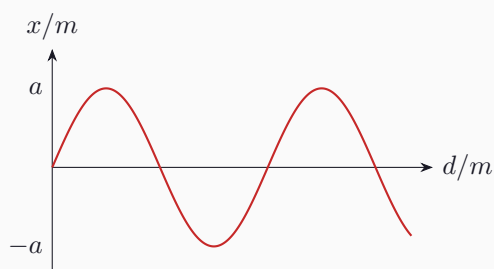
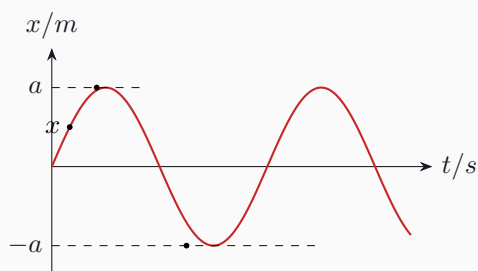
Electromagnetic spectrum

Gamma rays, X-rays, Ultraviolet, Visible light, Infrared, Microwaves, Radiowaves.

Transverse waves	Longitudinal waves
 crest trough	 compression rarefaction
Waves in which direction of vibration is perpendicular to the direction of energy transfer.	Waves in which direction of vibration is parallel to the direction of energy transfer.
* of source. Comprises of crests and troughs.	* of source. Comprises of compressions and rarefactions.
Electromagnetic spectrum, water waves.	Sound wave.

Characteristics of Waves

Wave graphs



Particles only vibrate about their mean position on the wave.

1. Displacement (x)

The distance of a point on a wave from its mean position.

2. Amplitude (a)

Maximum displacement of a point on a wave from its mean position.

3. Time period (T)

- The time taken to produce one wave.
- The time taken for one wave to pass a point.



4. Frequency (f)

- The no. of waves produced per unit time.
- The no. of waves passing a point per unit time.

Frequency and Wavespeed

Example: frequency

100 waves were produced in 20 s. Frequency = ?

$$20 \text{ s} \longrightarrow 100 \text{ waves} \quad 1 \text{ s} \longrightarrow x = \frac{100}{20} = 5 \text{ s}^{-1}$$

Let N be the number of waves in time t :

$$f = \frac{N}{t}$$

If $N = 1$, then $t = T$, hence

$$f = \frac{1}{T}$$

SI unit: Hertz (Hz).

$$v = \frac{d}{t}$$

$$d = N\lambda$$

$$v = \frac{N\lambda}{t} = f\lambda$$

$$v = f\lambda$$

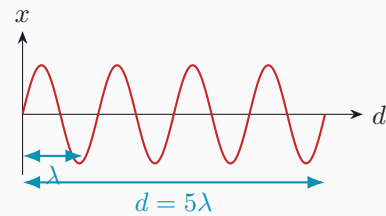
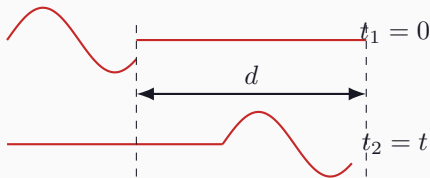
Small calculation

$$d = 10 \text{ cm}, \quad \lambda = 2 \text{ cm}$$

$$d = 5\lambda$$

5. Wavespeed (v)

The distance travelled by a waveform per unit time.



Intensity of a Wave

6. Intensity of a wave (I)

Energy delivered per unit area per unit time.

OR

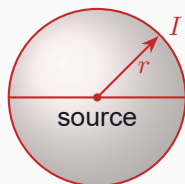
Power per unit area.

$$I = \frac{E}{At}$$

or

$$I = \frac{P}{A}$$

$$I \propto \frac{1}{A}$$



For a sphere, $A = 4\pi r^2$:

$$I = \frac{P}{4\pi r^2}$$

If P is constant:

$$I \propto \frac{1}{r^2}$$

Energy spreads out equally in all directions.

Relationships in the notes

- **Intensity** $\propto$ **frequency** for electromagnetic radiation:

$$I \propto f$$

- **Intensity** $\propto$ (**amplitude**)²: $I \propto a^2$.

- **Intensity** $\propto \frac{1}{(\text{distance})^2}$: $I \propto \frac{1}{d^2}$.

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{I_1}{I_2} = \frac{d_2^2}{d_1^2}$$

Since

$$I \propto a^2, I \propto \frac{1}{d^2}$$

we get

$$a^2 \propto \frac{1}{d^2} \Rightarrow a \propto \frac{1}{d}$$

$$a_1 d_1 = a_2 d_2$$

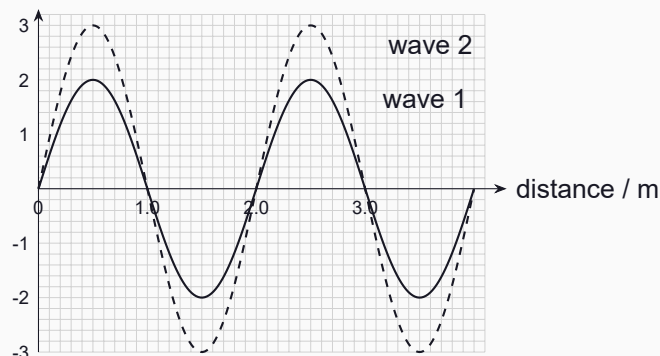
Exam Question: Intensity Ratio

Question 23

A sound wave consists of a series of moving pressure variations from the normal, constant air pressure. The graph shows these pressure variations for two waves at one instant in time.

Wave 1 has an intensity of $1.6 \times 10^{-6} \text{ W m}^{-2}$. What is the intensity of wave 2?

pressure variation / 10^{-2} Pa



- A $2.4 \times 10^{-6} \text{ W m}^{-2}$ B $3.0 \times 10^{-6} \text{ W m}^{-2}$ C $3.6 \times 10^{-6} \text{ W m}^{-2}$ D $4.5 \times 10^{-6} \text{ W m}^{-2}$

Solution: ratio method

$$I \propto a^2$$

From the graph:

$$a_1 = 2, a_2 = 3$$

$$\frac{I_1}{I_2} = \left(\frac{a_1}{a_2} \right)^2 = \left(\frac{2}{3} \right)^2 = \frac{4}{9}$$

$$\frac{1.6 \times 10^{-6}}{I_2} = \frac{4}{9}$$

$$I_2 = \frac{(1.6 \times 10^{-6}) \times 9}{4}$$

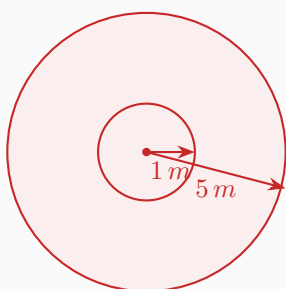
$$I_2 = 3.6 \times 10^{-6} \text{ W m}^{-2}$$

Exam Question: Sound Spreading from a Source

M/J 1983 P1 Q4

A small source of sound radiates energy equally in all directions. At a particular frequency, the intensity of the sound 1.0 m from the source is $1.0 \times 10^{-5} \text{ W m}^{-2}$, corresponding to an amplitude of oscillation of the air molecules of $70 \mu\text{m}$. Assuming that the sound is propagated without energy loss, what will be

- the intensity of the sound,
- the amplitude of oscillation of the air molecules at a distance of 5.0 m from the source?



Given

$$d_1 = 1 \text{ m}, \quad I_1 = 1.0 \times 10^{-5} \text{ W m}^{-2}, \quad a_1 = 70 \mu\text{m}$$

$$d_2 = 5 \text{ m}, \quad I_2 = ?, \quad a_2 = ?$$

(a) Intensity

$$I \propto \frac{1}{d^2}$$

$$I_1 d_1^2 = I_2 d_2^2$$

$$(1 \times 10^{-5})(1)^2 = I_2(5)^2$$

$$I_2 = 4.0 \times 10^{-7} \text{ W m}^{-2}$$

(b) Amplitude

$$a \propto \frac{1}{d} \quad \Rightarrow \quad a_1 d_1 = a_2 d_2$$

$$70(1) = a_2(5)$$

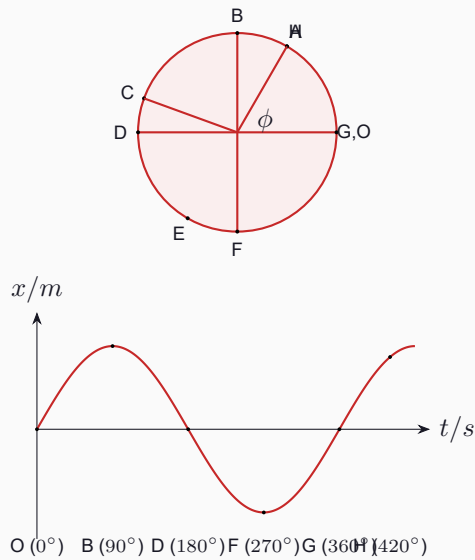
$$a_2 = 14 \mu\text{m}$$

Phase

7. Phase (ϕ)

It is used to tell about the state of motion of a point on a wave.

It is represented in “angles”.



$$\begin{aligned} \phi_O &= 0^\circ, & \phi_A &= 60^\circ, & \phi_B &= 90^\circ \\ \phi_C &= 120^\circ, & \phi_D &= 180^\circ, & \phi_E &= 240^\circ \\ \phi_F &= 270^\circ, & \phi_G &= 360^\circ \text{ or } 0^\circ, & \phi_H &= 420^\circ \text{ or } 60^\circ \end{aligned}$$

Degree to radian

$$180^\circ = \pi \text{ rad}, \quad 360^\circ = 2\pi \text{ rad}, \quad 90^\circ = \frac{\pi}{2} \text{ rad}$$

Phase angle using ratio method

$$T \longrightarrow 360^\circ, \quad t \longrightarrow \phi$$

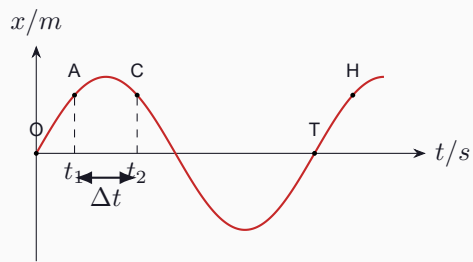
$$\phi = \frac{t}{T} \times 360^\circ \quad (\text{in degree})$$

$$\phi = \frac{t}{T} \times 2\pi \quad (\text{in radians})$$

$$\lambda \longrightarrow 360^\circ, \quad d \longrightarrow \phi$$

$$\phi = \frac{d}{\lambda} \times 360^\circ \quad \text{or} \quad \phi = \frac{d}{\lambda} \times 2\pi$$

Phase Difference Between Two Points



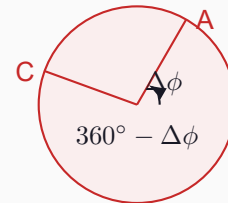
$$T \longrightarrow 360^\circ, \quad \Delta t \longrightarrow \Delta\phi$$

$$\Delta\phi = \frac{\Delta t}{T} \times 360^\circ$$

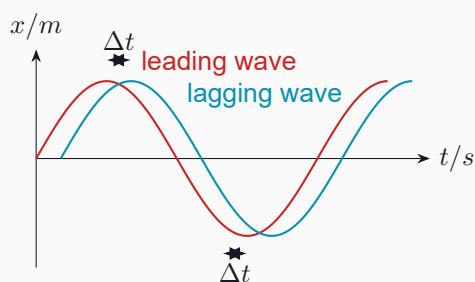
Similarly for distance

$$\lambda \longrightarrow 360^\circ, \quad \Delta d \longrightarrow \Delta\phi$$

$$\Delta\phi = \frac{\Delta d}{\lambda} \times 360^\circ$$



Phase Difference Between Two Waves and Coherence



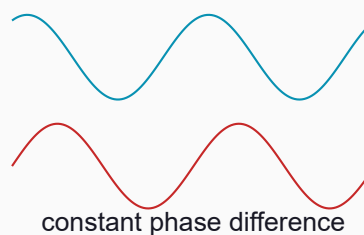
Meaning of lead and lag

The **leading wave** has its crest/trough produced earlier in time than the other. The **lagging wave** is delayed.

Coherent waves

Waves are said to be coherent if they have the same phase difference throughout.

“For waves to be coherent, they must have the same frequency.”



$$T \rightarrow 360^\circ, \quad \Delta t \rightarrow \Delta\phi$$

$$\Delta\phi = \frac{\Delta t}{T} \times 360^\circ$$

where Δt is the time difference between corresponding points of both waves.

Doppler Effect

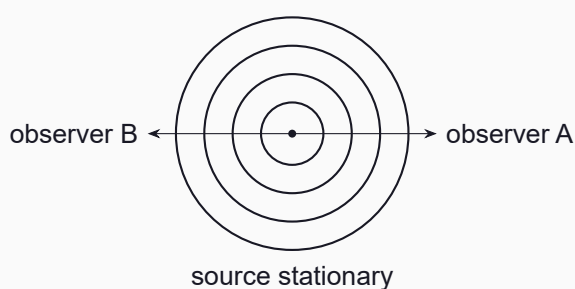
Definition

The change in the observed frequency of the wave when the source moves relative to observer.

$$f_o = \frac{v}{v \pm v_s} \times f$$

- f_o = observed frequency
- v = speed of wave
- v_s = speed of source
- f = actual/source frequency

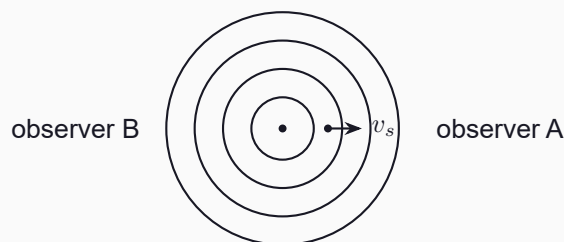
Stationary source



Observed frequency is same as source frequency as source is stationary:

$$f_o = f$$

Source receding



For source receding, $f_o < f$:

$$f_o = \frac{v}{v + v_s} \times f$$

Distance plus in recede, so use +.

Source approaching

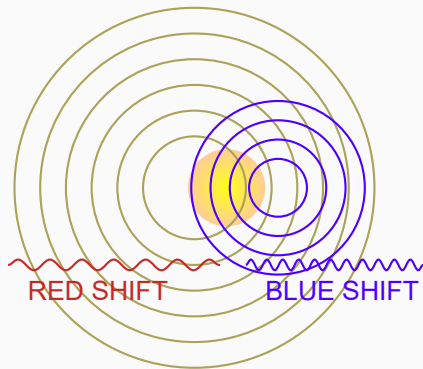
For source approaching, $f_o > f$:

$$f_o = \frac{v}{v - v_s} \times f$$

Distance minus in approach, so use -.

Doppler Effect: Red Shift and Blue Shift

Visual idea



As the star moves away, there is a **red shift** in its colour. This is due to the Doppler effect.

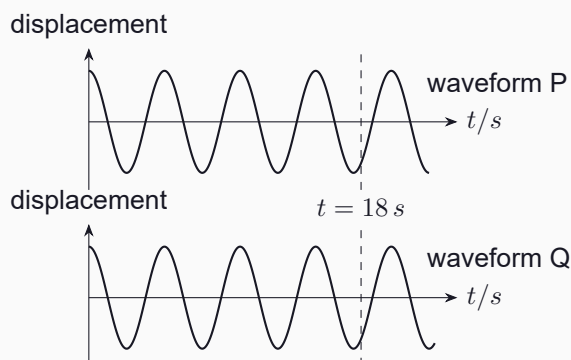
- Due to star moving away, the observed frequency is less than the actual frequency.
- Red colour has a lower frequency, hence the light coming from the star has a red shift in it.
- This also explains that the universe is expanding.

When a source approaches, wavefronts are closer together in front of the source; the wavelength is smaller and the observed frequency increases. This gives a blue shift for light.

Exam Question: Phase Difference of Waveforms

Question 26

The diagram shows two sinusoidal waveforms. At time $t = 0$ the waves are in phase. At the dotted line, $t = 18\text{ s}$. At which time is the phase difference between the two oscillations $\frac{1}{8}$ of a cycle?



- A 4.0 s B 4.5 s C 8.0 s D 9.0 s

Solution

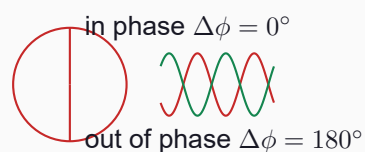
$$\frac{1}{8} \text{ of cycle} = \frac{1}{8} \times 360^\circ = 45^\circ$$

The notes compare 18 s to 180° :

$$18\text{ s} \rightarrow 180^\circ, \quad x \rightarrow 45^\circ$$

$$x = \frac{18 \times 45}{180} = 4.5\text{ s}$$

$$x = 4.5\text{ s}$$



Exam Question: Doppler Frequency

Question 26

A car travelling at a steady speed in a straight line passes close to a stationary observer. The observer measures the frequency of the sound from the engine.

As the car approaches, the observed frequency is 220 Hz. When the car moves away, the observed frequency is 180 Hz.

The speed of sound in air is 340 m s^{-1} .

What is the speed of the car?

- A 8.5 m s^{-1} B 31 m s^{-1} C 34 m s^{-1} D 38 m s^{-1}

Solution

For approach:

$$220 = \frac{340}{340 - v_s} f$$

For recede:

$$180 = \frac{340}{340 + v_s} f$$

Divide the equations to eliminate f :

$$\frac{220}{180} = \frac{340 + v_s}{340 - v_s}$$

$$220(340 - v_s) = 180(340 + v_s)$$

$$74800 - 220v_s = 61200 + 180v_s$$

$$13600 = 400v_s$$

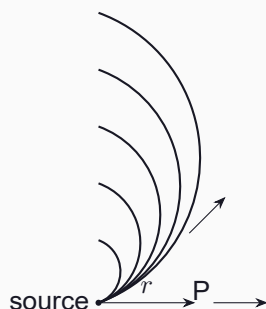
$$v_s = 34 \text{ m s}^{-1}$$

Spherical Waves and Diffraction MCQ

Question 27

A small source emits spherical waves. The wave intensity I at any point P , a distance r from the source, is inversely proportional to r^2 .

What is the relationship between the wave amplitude a and the distance r ?



- A $a^2 \propto \frac{1}{r}$ B $a \propto \frac{1}{r}$ C $a \propto \frac{1}{r^2}$ D $a \propto \frac{1}{r^4}$

Solution

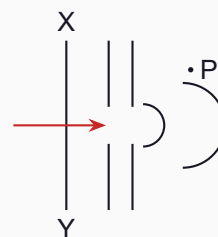
$$I \propto \frac{1}{r^2}, \quad I \propto a^2$$

$$a^2 \propto \frac{1}{r^2} \quad \Rightarrow \quad a \propto \frac{1}{r}$$

Question 29

A monochromatic plane wave of speed c and wavelength λ is diffracted at a small aperture. The diagram illustrates successive wavefronts.

After what time will some portion of the wavefront XY reach point P ?



- A $\frac{3\lambda}{2c}$ B $\frac{2\lambda}{c}$ C $\frac{3\lambda}{c}$ D $\frac{4\lambda}{c}$

$$v = f\lambda, \quad c = \frac{\lambda}{T} \quad \Rightarrow \quad T = \frac{\lambda}{c}$$

Since three wavefront spacings are involved:

$$t = 3T = \frac{3\lambda}{c}$$

Power per Unit Area and Diffraction

Question 24: Power per unit area

A light wave of amplitude A is incident normally on a surface of area S . The power per unit area reaching the surface is P .

The amplitude of the light wave is increased to $2A$. The light is then focused on to a smaller area $\frac{1}{3}S$.

What is the power per unit area on this smaller area?

- A $36P$ B $18P$ C $12P$ D $6P$

Solution

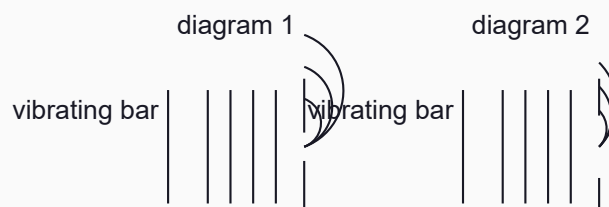
$$I = \frac{P}{A}, \quad I \propto a^2$$

Amplitude: $A \rightarrow 2A$, so intensity becomes $4P$. Area is reduced to $\frac{1}{3}S$, so power per unit area increases by another factor of 3.

$$P \times 4 \times 3 = 12P$$

Question 24: Diffraction in a ripple tank

Diagram 1 shows a ripple tank experiment in which plane waves are diffracted through a narrow slit. Diagram 2 shows the same tank with a slit of greater width.



Which action would cause the waves in diagram 1 to be diffracted less and so produce an emergent pattern closer to that shown in diagram 2?

A *increasing the frequency of vibration of the bar*

B increasing the speed of the waves by making the water in the tank deeper

C reducing the amplitude of vibration of the bar

D reducing the length of the vibrating bar

$\lambda \simeq$ slit size gives maximum diffraction. To diffract less, make λ smaller. Since $v = f\lambda$, increasing f decreases λ when v is constant.

Superposition of Waves

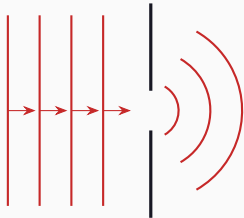
Chapter 12 / AS Level / Apollo Physics 9702



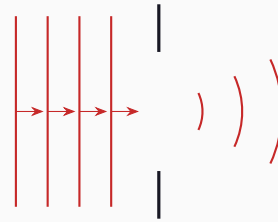
Diffraction of Waves

Diffraction of waves

- The spreading of the wave as it passes through a slit or over an edge.
- During diffraction, energy of the wave spreads out and its intensity falls.



slit size $\approx$ wavelength
Max diffraction



slit size $>$ wavelength
Less diffraction

narrower aperture $\Rightarrow$ greater spreading

Interference and Superposition

Interference

When two or more waves meet at a point, they overlap one another to produce a new resultant wave.

Conditions for interference

1. Waves must meet at a point.
2. Waves must have the same nature, i.e. both mechanical or both electromagnetic.
3. Waves should have the same frequency. A small difference in frequency can still cause interference.

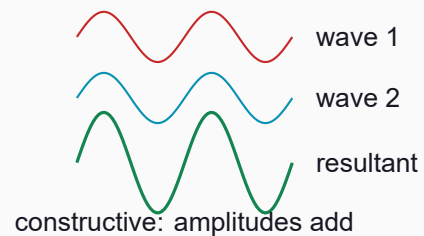
Principle of superposition

When two waves meet at a point in space, the resultant displacement of the new wave is the algebraic sum of displacements of interfering waves.

$$y_{\text{result}} = y_1 + y_2$$

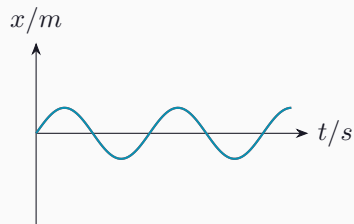
3 types of interference

1. Constructive interference
2. Destructive interference
3. Intermediate interference

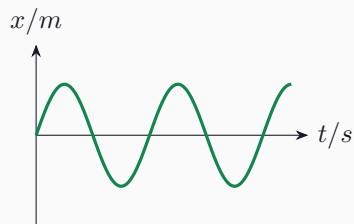


Constructive and Destructive Interference

Constructive interference

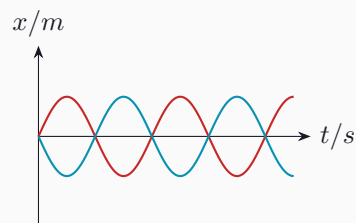


+2 and +2 add.



$$a + a = 2a$$

Destructive interference

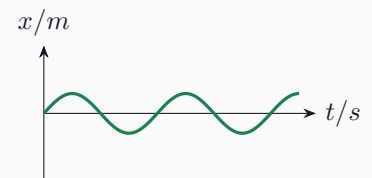
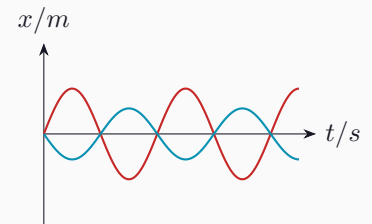


Pure destructive interference.



$$a - a = 0$$

Partially destructive



If unequal opposite displacements meet, the resultant is not zero but has a smaller amplitude.

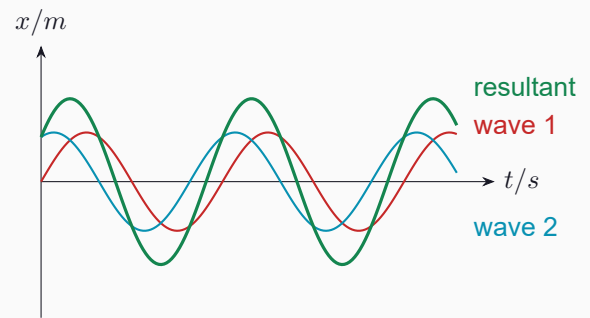
Intermediate Interference

Intermediate interference

Intermediate interference occurs when waves meet with a phase difference that is neither 0° nor 180° .

- It is not purely constructive.
- It is not purely destructive.
- The resultant amplitude lies between maximum and minimum values.

A_{result} depends on phase difference



Path Difference for Sources in Phase

Path difference

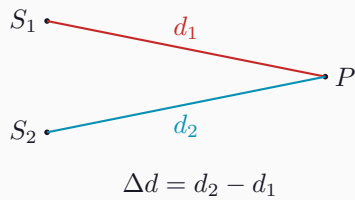
$$\text{path difference} = d_2 - d_1$$

For two in-phase sources:

$$\text{Constructive: path difference} = n\lambda \quad (n = 0, 1, 2, 3, \dots)$$

$$\text{Destructive: path difference} = \left(n + \frac{1}{2}\right)\lambda \quad (n = 0, 1, 2, \dots)$$

Equivalently, destructive happens for $0.5\lambda, 1.5\lambda, 2.5\lambda, \dots$



Example 1

$$\lambda = 2 \text{ cm}, d_1 = 10 \text{ cm}, d_2 = 14 \text{ cm}.$$

$$\Delta d = 14 - 10 = 4 \text{ cm} = 2\lambda$$

$$\text{Constructive interference: } a + a = 2a.$$

Example 2

$$\lambda = 2 \text{ cm}, d_1 = 10 \text{ cm}, d_2 = 15 \text{ cm}.$$

$$\Delta d = 15 - 10 = 5 \text{ cm} = 2.5\lambda$$

$$\text{Destructive interference: } a - a = 0.$$

Sources Vibrating Out of Phase

If the sources vibrate out of phase

Rules flip

When the sources are 180° out of phase:

Destructive: $\Delta d = n\lambda$ ($n = 0, 1, 2, 3, \dots$)

Constructive: $\Delta d = \left(n + \frac{1}{2}\right)\lambda$ ($n = 0, 1, 2, \dots$)

S_1 

S_2 

crest from one source is emitted with trough from the other

Example 3

$\lambda = 2 \text{ cm}$, $d_1 = 10 \text{ cm}$, $d_2 = 14 \text{ cm}$; sources vibrate out of phase.

$$\Delta d = 14 - 10 = 4 \text{ cm} = 2\lambda$$

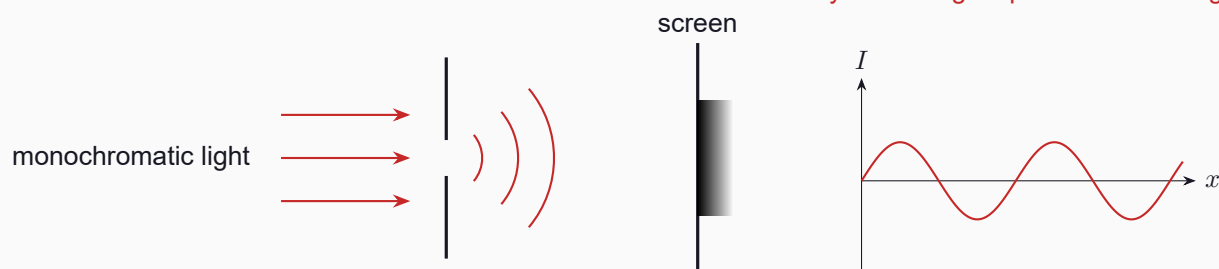
Because the sources are out of phase, an integral number of wavelengths gives destructive interference.

Destructive interference

Single Slit Diffraction

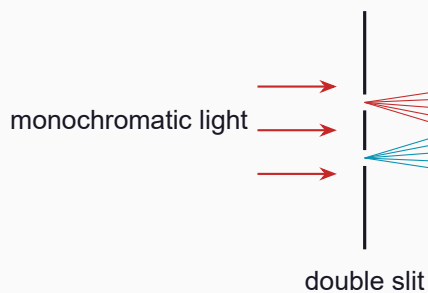
Single slit diffraction

- With no slit, the light continues without spreading.
- With a single slit, light spreads out.
- Intensity falls as light spreads over a larger area.



Double Slit Diffraction – Young's Double Slit Experiment

Young's double slit experiment

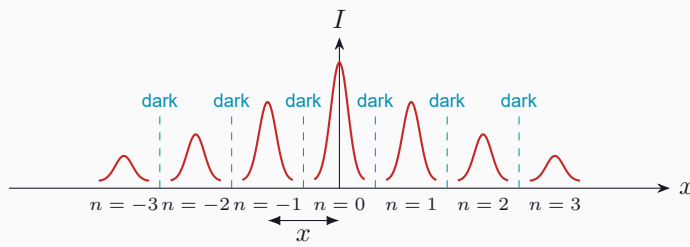


Why is the pattern produced?

- Light coming from two slits meets at different points on the screen.
- At some points, light rays are in phase, so a **bright spot** is seen.
- At some points, light rays are out of phase, so a **dark region** is seen.

Bright fringe / maxima	constructive interference
Dark fringe / minima	destructive interference

Fringe Pattern and Formula



Bright spot

- Constructive interference.
- Path difference = $n\lambda$, where $n = 0, 1, 2, 3, \dots$
- Phase difference = 0° .

Dark spot

- Destructive interference.
- Path difference = $(n + \frac{1}{2})\lambda$, where $n = 0, 1, 2, \dots$
- Phase difference = 180° .

Pattern

1. Intensity falls sideways.
2. Symmetric pattern.
3. Equidistant spots.

Double slit formula

$$x = \frac{\lambda D}{a}$$

- a slit separation
 λ wavelength
 x fringe separation
 D slit-to-screen distance

Effects on Fringe Pattern I

1. Wavelength of light increased

Frequency decreases, e.g. blue colour to red.

- Fringe separation: *increases* because $x \propto \lambda$.
- Brightness of bright fringe: unchanged.
- Darkness of dark fringe: unchanged.

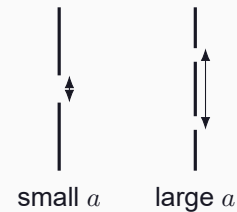
2. Slit-to-screen distance increased

- Fringe separation: *increases* because $x \propto D$.
- Brightness of bright fringe: decreases because the spot becomes large.
- Darkness of dark fringe: unchanged.

3. Slit separation increased

- Fringe separation: *decreases* because $x \propto \frac{1}{a}$.
- Brightness of bright fringe: unchanged.
- Darkness of dark fringe: unchanged.

$$x = \frac{\lambda D}{a}$$



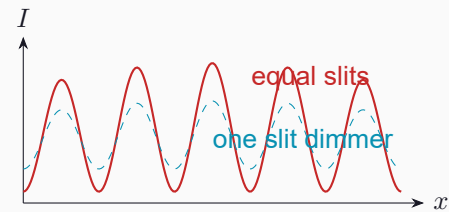
Effects on Fringe Pattern II

4. Intensity of light decreased

Examples: lamp dim, smaller slit size, lamp far away from the slits.

- Fringe separation: unchanged.
- Brightness of bright fringe: decreases.
- Darkness of dark fringe: unchanged.

If light coming from one of the two slits is decreased, overall light is reduced. The dark fringe becomes less dark because no pure destructive interference occurs.



5. Light from one slit decreased

- Fringe separation: unchanged.
- Brightness of bright fringe: decreases.
- Darkness of dark fringe: decreases.

Diffraction Grating

Problems with double slit

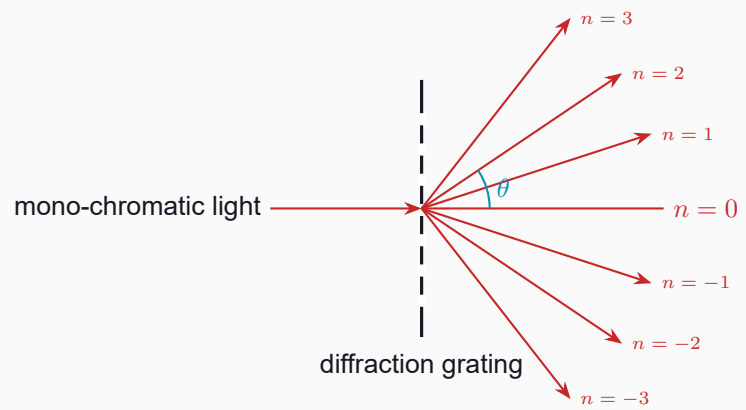
1. Very low intensity of spots.
2. Very small gap between spots.

Diffraction grating resolves these problems.

Diffraction grating

Grating comprises of hundreds of slits per unit length so that more light can pass and bright spots are produced on the screen.

- Maxima: bright fringe, constructive interference.
- Minima: dark fringe, destructive interference.



Grating Equation

Diffraction grating formula

$$n\lambda = d \sin \theta$$

n order of maximum

λ wavelength

d slit separation / grating spacing

θ angle between zero-order and n th-order maximum

Finding grating spacing

If a grating has 4000 lines in 0.01 m:

$$d = \frac{0.01}{4000} = 2.5 \times 10^{-6} \text{ m}$$

Example

First-order maximum at 10° with $d = 2.5 \times 10^{-6} \text{ m}$.

$$n\lambda = d \sin \theta$$

$$(1)\lambda = (2.5 \times 10^{-6}) \sin 10^\circ$$

$$\lambda = 4.34 \times 10^{-7} \text{ m} = 434 \text{ nm}$$

Bluelight.



Grating Calculations and Orders

Angle of third-order maximum

Using $\lambda = 4.34 \times 10^{-7} \text{ m}$ and $d = 2.5 \times 10^{-6} \text{ m}$:

$$3(4.34 \times 10^{-7}) = (2.5 \times 10^{-6}) \sin \theta$$

$$\theta = 31.4^\circ$$

Angle between 1st and 3rd order maxima

$$\theta_3 - \theta_1 = 31.4^\circ - 10^\circ = 21.4^\circ$$

Angle between both third-order maxima

$$2\theta_3 = 2(31.4^\circ) = 62.8^\circ$$

Highest order maximum visible

For the maximum possible angle, $\theta_{\max} = 90^\circ$:

$$n\lambda = d \sin 90^\circ$$

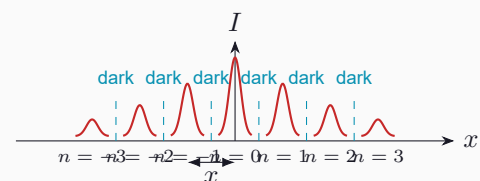
$$n = \frac{2.5 \times 10^{-6}}{4.34 \times 10^{-7}} = 5.76$$

So $n = 5$ is the highest visible order.

Total bright spots on screen

$$5 + 5 + 1 = 11$$

11 bright fringes, including zeroth order.



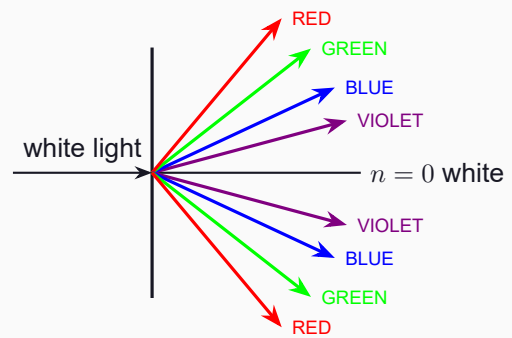
Diffraction of White Light

White light through a grating

When white light is passed through a diffraction grating, spectra are formed on either side of the central maximum.

- White light comprises colours from red to blue/violet.
- Every colour has its own wavelength.
- Upon diffraction, colours spread based on their wavelengths.

$$n\lambda = d \sin \theta \quad \Rightarrow \quad \theta \propto \lambda$$

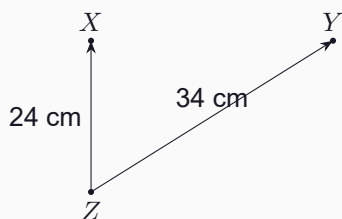


$n = 0$ is a white spot because all colours have zero path difference and zero phase difference there. The colours combine to form white light.

Exam Example: Water Waves and Path Difference

Question

Wave generators at points X and Y produce water waves of the same wavelength. At point Z , the waves from X have the same amplitude as the waves from Y . Distances $XZ = 24\text{ cm}$ and $YZ = 34\text{ cm}$. When the generators operate in phase, the amplitude at Z is zero. What could be the wavelength?



Solution

For zero amplitude with sources in phase:

$$\Delta d = \left(n + \frac{1}{2}\right) \lambda$$

$$\Delta d = 34 - 24 = 10\text{ cm}$$

Try options:

$$\lambda = 2\text{ cm} \Rightarrow n = 5 \quad (\text{constructive})$$

$$\lambda = 3\text{ cm} \Rightarrow n = 3.33$$

$$\lambda = 4\text{ cm} \Rightarrow 10 = 2.5\lambda \quad (\text{destructive})$$

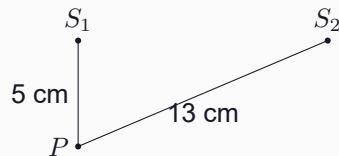
$$\lambda = 6\text{ cm} \Rightarrow n = 1.67$$

Answer : 4 cm.

Exam Example: Sources Out of Phase

Question

Two sources S_1 and S_2 oscillate with a phase difference of 180° . At P , distances are $S_1P = 5 \text{ cm}$ and $S_2P = 13 \text{ cm}$. Each source produces wavelength 2.0 cm and amplitude x_0 at P . What is the amplitude at P ?



Solution

Path difference:

$$\Delta d = 13 - 5 = 8 \text{ cm} = 4\lambda$$

Since the sources are out of phase, the rules flip. Integral path difference gives destructive interference.

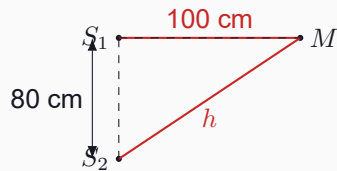
$Amplitude \text{ at } P = 0$

If sources are out of phase, one source makes crest while the other makes trough at the same time, so the path-difference rule flips.

Two Sound Sources: Geometry

Setup

Two sources S_1 and S_2 of sound are situated 80 cm apart. A microphone M is 100 cm from S_1 along a line normal to S_1S_2 . The sources always vibrate in phase but have different amplitudes.



Path lengths

$$h^2 = p^2 + b^2$$

$$h = \sqrt{80^2 + 100^2} = 128 \text{ cm}$$

$$\Delta d = 128 - 100 = 28 \text{ cm} = 0.28 \text{ m}$$

As frequency is varied, maxima and minima of sound intensity are detected at M .

Two Sound Sources: Minima as Frequency Changes

Conditions for zero intensity at M

1. Phase difference between interfering waves must be 180° or π rad.
2. Waves must have the same amplitude.
3. Path difference between waves must be $(n + \frac{1}{2})\lambda$ where $n = 0, 1, 2, \dots$

Given

Speed of sound $v = 330 \text{ m s}^{-1}$ and path difference $\Delta d = 0.28 \text{ m}$.

$$\Delta d = \left(n + \frac{1}{2}\right) \lambda = \left(n + \frac{1}{2}\right) \frac{v}{f}$$

Frequencies for minima

For $n = 0.5$ notation from the notes:

$$0.28 = \frac{(0.5)(330)}{f} \Rightarrow f = 589 \text{ Hz}$$

For $n = 1.5$:

$$0.28 = \frac{(1.5)(330)}{f} \Rightarrow f = 1767 \text{ Hz}$$

For $n = 2.5$:

$$0.28 = \frac{(2.5)(330)}{f} \Rightarrow f = 2946 \text{ Hz}$$

For $n = 3.5$:

$$f = 4125 \text{ Hz} > 4.0 \text{ kHz}$$

Between 1.0 and 4.0 kHz, minima occur at 1767 Hz and 2946 Hz.

$Number = 2$

Fringe Spacing Scaling Question

Question

Light of wavelength 700 nm is incident on a pair of slits, forming fringes 3.0 mm apart on a screen. What is the fringe spacing when light of wavelength 350 nm is used and the slit separation is doubled?

$$\lambda = 700 \text{ nm}, \quad x = 3.0 \text{ mm}, \quad a = a$$

Final:

$$\lambda = 350 \text{ nm} = \frac{700}{2} \text{ nm}, \quad a = 2a$$

Solution

So

$$x = \frac{\lambda D}{a}$$
$$x \propto \lambda, \quad x \propto \frac{1}{a}$$

$$x_{\text{new}} = x \times \frac{1}{2} \times \frac{1}{2} = \frac{x}{4} = 0.75 \text{ mm}$$

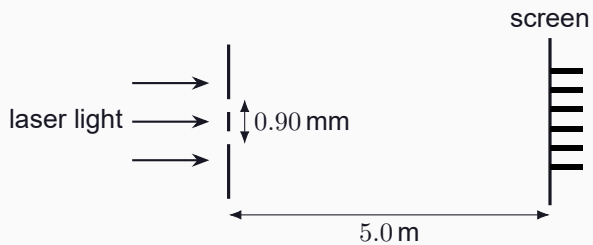
Initial:

0.75 mm

Young's Slits: Finding Wavelength

Question

A Young's slits experiment uses slit separation $a = 0.90 \text{ mm}$ and screen distance $D = 5.0 \text{ m}$. The pattern on the screen has $x = 3.0 \text{ mm}$ for the fringe separation. Find the wavelength.



Solution

$$x = \frac{\lambda D}{a}$$

From the ruler in the notes:

$$x = 30 \text{ mm} - 27 \text{ mm} = 3 \text{ mm}$$

$$3 \times 10^{-3} = \frac{\lambda(5)}{0.90 \times 10^{-3}}$$

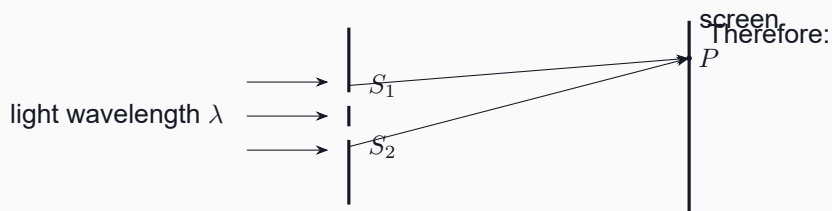
$$\lambda = 5.4 \times 10^{-7} \text{ m}$$

$$\boxed{5.4 \times 10^{-7} \text{ m}}$$

n th Dark Fringe in Young's Experiment

Question

Monochromatic light of wavelength λ is incident on two narrow slits S_1 and S_2 . A series of bright and dark fringes is observed on a distant screen. The n th dark fringe from the central bright fringe is observed at P . Which expression is correct for $S_2P - S_1P$?



Solution

For a dark fringe:

$$\Delta d = \left(m + \frac{1}{2}\right) \lambda, \quad m = 0, 1, 2, \dots$$

Counting the first dark fringe as $n = 1$:

$$n = 1 \Rightarrow \Delta d = \frac{1}{2} \lambda$$

$$n = 2 \Rightarrow \Delta d = \frac{3}{2} \lambda$$

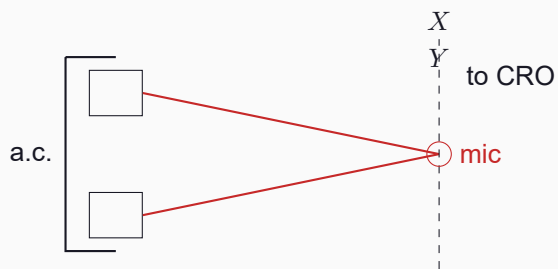
Therefore:

$$S_2P - S_1P = \left(n - \frac{1}{2}\right) \lambda$$

Correct option : C

Double Slit Experiment with Sound

Sound double slit / two speaker experiment

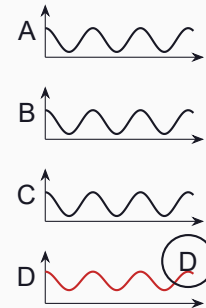


For an interference pattern to be produced, the waves must be coherent, i.e. they must have a constant phase difference.

- One light source and two slits are used so light from both slits has the same phase difference.
- Two loudspeakers and one power source are used so both speakers produce coherent waves.

Intensity graph choice

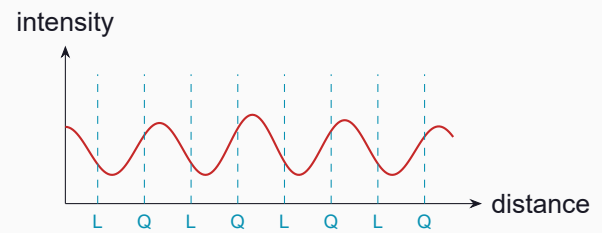
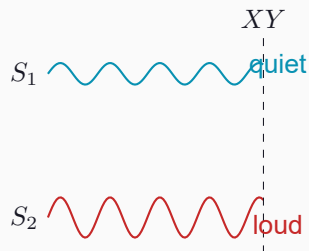
The notes mark the graph where intensity never falls to zero because the waves from the two speakers can have unequal amplitudes at the microphone.



Sound Interference: Loud and Quiet Positions

Unequal amplitudes at the detector

- Sound coming from source 1 travels less distance than sound from source 2.
- Hence the wave from S_2 has smaller amplitude than the wave from S_1 .
- At Y , no pure destructive interference occurs and some intensity is still left.



- Loud positions: constructive interference.
- Quiet positions: partially destructive interference.
- Middle constructive interference has $n = 0$.

Microwave Interference Exam Question

Conditions for maxima in a microwave interference pattern

1. Phase difference = 0 or waves in phase.
2. Path difference = $n\lambda$, where $n = 0, 1, 2, 3, \dots$
3. Waves must meet at a point.

Microwave wavelength

$$v = f\lambda$$

$$3.0 \times 10^8 = (12 \times 10^9)\lambda$$

$$\lambda = 0.025 \text{ m}$$

Detector moved from O to P

At O , path difference is zero, so maximum. At P :

$$\Delta d = 0.90 - 0.75 = 0.15 \text{ m}$$

$$\frac{0.15}{0.025} = 6$$

So the detector passes through alternating maxima and minima. The notes describe the reading as increasing and decreasing periodically, including a maximum at O and at P .

Replacing microwaves with visible light

To observe the interference pattern with coherent light:

1. Decrease the size of the slit to the size of wavelength of light.
2. Decrease the gap between the slits.

Coherence, Interference, Frequency and Laser Fringes

Coherence and interference

- **Coherence:** waves have a constant phase difference throughout their motion.
- **Interference:** when two waves meet at a point, they produce a new wave which is the resultant of both.

Frequency from graph

If the speed of waves is 240 m s^{-1} and the wavelength from the graph is 3.2 m :

$$v = f\lambda$$

$$240 = f(3.2)$$

$$f = 75 \text{ Hz}$$

Laser double slit calculation

Given $\lambda = 546 \text{ nm}$, slit separation $a = 0.13 \text{ mm}$, screen distance $D = 85 \text{ cm}$.

$$x = \frac{\lambda D}{a}$$

$$x = \frac{546 \times 10^{-9} \times 0.85}{0.13 \times 10^{-3}} = 3.57 \times 10^{-3} \text{ m}$$

If B corresponds to path difference 2.5λ and A to path difference zero:

$$AB = 2.5x = 8.93 \times 10^{-3} \text{ m}$$

$$AB \approx 8.9 \times 10^{-3} \text{ m}$$

Laser changed to red light

Red light has a larger wavelength than 546 nm , therefore fringe separation increases because $x \propto \lambda$.

Stationary Waves

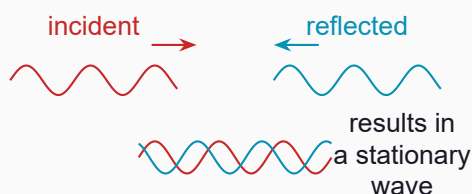
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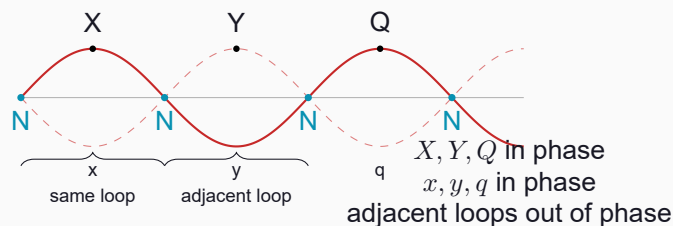
Stationary Waves / Standing Waves

Formation of a stationary wave

- 2 waves
- same frequency
- travelling in opposite direction
- interfere

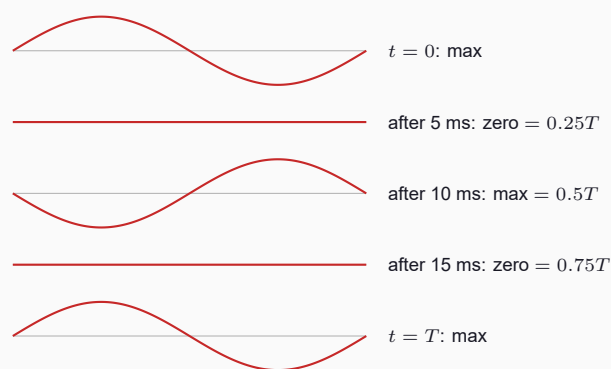


Nodes, antinodes and phase



Time period for a stationary wave

If $T = 20$ ms:



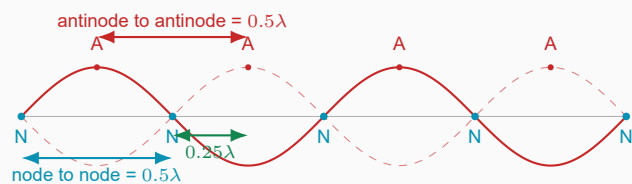
The pattern goes **max** → **zero** → **max inverted** → **zero** → **max**.

Progressive vs stationary

Progressive	Stationary
Travelling waveform.	Standing waveform.
Net transfer of energy.	No net transfer of energy.
Crest/trough or compression/rarefactions travel through the medium.	Nodes : no vibration due to destructive interference.
Every point has same amplitude.	Antinodes : maximum vibration due to constructive interference.
Points have different phase angles.	Points have different amplitudes. Points between nodes are in phase, $\Delta\phi = 0^\circ$. Points adjacent to nodes are out of phase, $\Delta\phi = 180^\circ$.

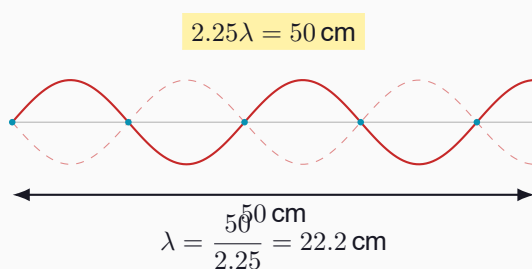
Wavelength of a Stationary Wave

Distances in the pattern



- **Node to node** = 0.5λ .
- **Antinode to antinode** = 0.5λ for consecutive antinodes.
- **Node to antinode** = 0.25λ .

Counting quarters



Reflection at an end

Fixed / closed end	Free / open end
Node Out of phase	Antinode In phase

Rule to remember

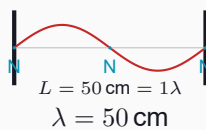
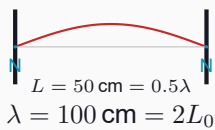
Fixed/closed end $\Rightarrow$ node.
Free/open end $\Rightarrow$ antinode.

Phase on reflection

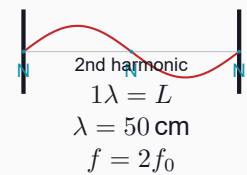
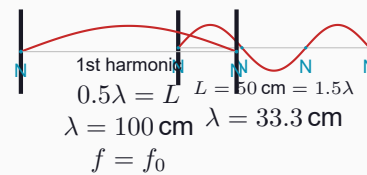
Closed end: reflected wave is inverted.
Open end: reflected wave is not inverted.

Both Ends Fixed

Changing length



Changing frequency for $L = 50 \text{ cm}$



Every next stationary wave is half wavelength (0.5λ) extra.

Example

Two consecutive stationary waves were formed at 48 cm and 56 cm respectively in one wire.

$$56 - 48 = 0.5\lambda, \quad 8 = 0.5\lambda, \quad \boxed{\lambda = 16 \text{ cm}}.$$

Whole number multiples

For both ends fixed:

$$\boxed{L_n = \frac{n\lambda}{2}}, \quad \boxed{f_n = nf_0} \quad (n = 1, 2, 3, \dots)$$

Examples of allowed lengths/frequencies:

Mode	Length pattern	Frequency pattern	Harmonic
1	L_0	f_0	1 st
2	$2L_0$	$2f_0$	2 nd
3	$3L_0$	$3f_0$	3 rd
4	$4L_0$	$4f_0$	4 th

A similar pattern is obtained for both ends open.

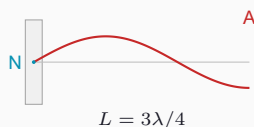
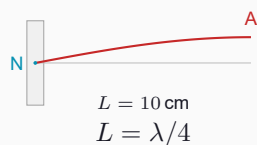
WHOLE NUMBER MULTIPLE

$$L_n = nL_0, \quad f_n = nf_0, \quad \lambda_n = \frac{\lambda_0}{n}.$$

Length	6 cm	12 cm	18 cm	24 cm
Frequency	15 Hz	30 Hz	45 Hz	60 Hz
Mode	f_0	$2f_0$	$3f_0$	$4f_0$

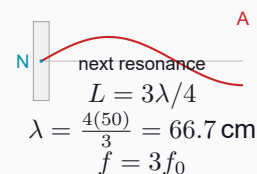
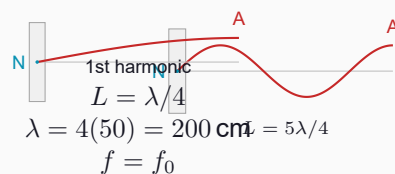
One End Closed, One End Open

Changing length



Every next stationary wave is half wavelength (0.5λ) extra.

Changing frequency for $L = 50 \text{ cm}$



Only odd harmonics occur. If $f_0 = 150 \text{ Hz}$:

150 Hz, 450 Hz, 750 Hz, ...

Odd number multiples

For one end closed and one end open:

$$L_n = \frac{(2n-1)\lambda}{4},$$

$$f_n = (2n-1)f_0$$

where $n = 1, 2, 3, \dots$

Length	8 cm	24 cm	40 cm	56 cm
Pattern	L_0	$3L_0$	$5L_0$	$7L_0$
Frequency	f_0	$3f_0$	$5f_0$	$7f_0$

ODD NUMBER MULTIPLES

Closed end: node

Air cannot vibrate freely at the closed end.

Open end: antinode

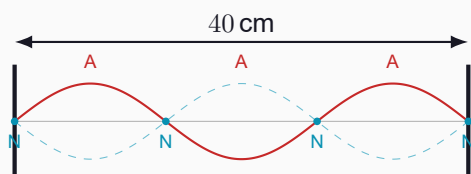
Air vibrates with maximum displacement at the open end.

Speed of a Stationary Wave

Speed rule

- When the incident wave is reflected at one end and they both interfere, they form a stationary wave.
- Speed of the stationary wave is either the speed of the incident or reflected wave.

$$v = f\lambda$$



The diagram contains 1.5λ in 40 cm.

$$1.5\lambda = 40 \text{ cm} \Rightarrow \lambda = 26.6 \text{ cm.}$$

If the generator frequency is f , then

$$v = f\lambda.$$

Quick method summary

Setup	Allowed modes
Both ends fixed	$L = \frac{n\lambda}{2}$, so $f_n = nf_0$. Whole number multiples.
Both ends open	Same pattern as both ends fixed: $f_n = nf_0$.
One end closed, one end open	$L = \frac{(2n-1)\lambda}{4}$, so $f_n = (2n-1)f_0$. Odd number multiples only.

Node/antinode spacing checklist

$$\text{Node to node} = \lambda/2$$

$$\text{Antinode to antinode} = \lambda/2$$

Exam trigger words

- stationary / standing wave
- node, antinode
- fixed end / closed end / open end
- resonance, harmonics
- successive resonant lengths or frequencies

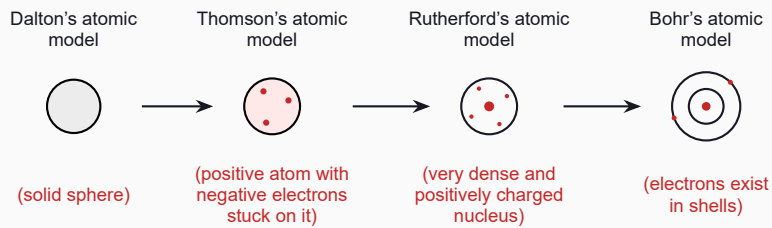
Particle Physics

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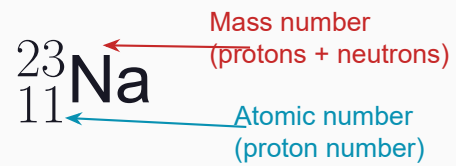


Atomic Models, Structure of the Atom, and Isotopes

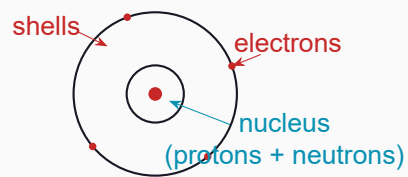
Historical atomic models



Nuclear notation



Structure of atom



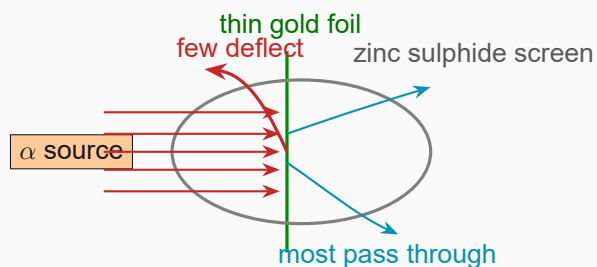
Isotopes

Atoms of the same element with same number of protons but different number of neutrons.

Mass number, A	number of protons + neutrons
Atomic number, Z	number of protons
Neutron number	$A - Z$

Geiger-Marsden Gold Foil Experiment and Nuclear Density

Geiger-Marsden gold foil experiment



- Most of the alpha particles were able to pass through.
⇒ Atom is mostly empty.
- A few deflected at large angles and very few bounced back.
- ⇒ A very small and dense positively charged nucleus exists in the middle.

Charge of alpha particle

Alpha particle: ${}^4_2\alpha$, charge = +2.

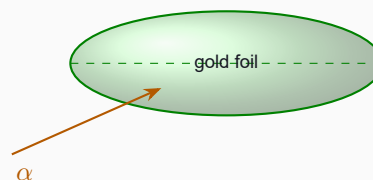
How dense is the nucleus?

Carbon atom: $6p$ and $6n$.

- mass = $12u$, where $1u = 1.66 \times 10^{-27} \text{ kg}$
- diameter of nucleus $\approx 10^{-14} \text{ m}$
- assuming nucleus as a sphere

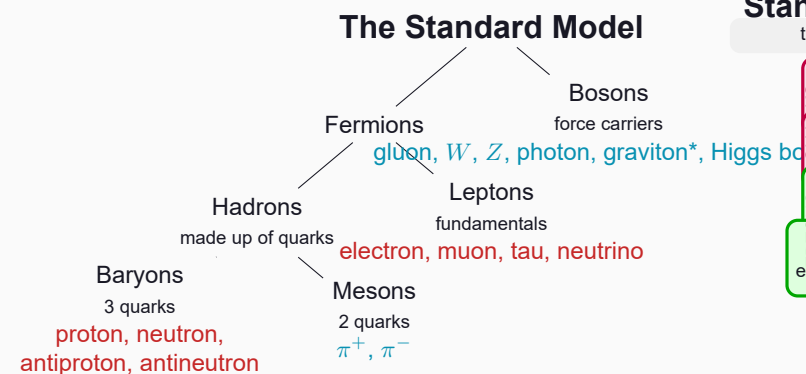
$$\rho = \frac{m}{V} = \frac{12(1.66 \times 10^{-27})}{\frac{4}{3}\pi(10^{-14})^3} \approx 4 \times 10^{15} \text{ kg m}^{-3}.$$

Extremely dense!



The Standard Model

Classification



Standard Model of Elementary Particles

Standard Model of Elementary Particles

three generations of matter (fermions)			interactions / carriers (bosons)	
QUARKS	u up	c charm	t top	g gluon
	d down	s strange	b bottom	γ photon
	e electron	μ muon	τ tau	Z Z boson
LEPTONS	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson
				H higgs

SCALAR BOSONS

GAUGE BOSONS

Four fundamental forces of universe

1. Strong nuclear force
2. Electromagnetic force
3. Weak nuclear / interaction force
4. Gravitational force

Decreasing strength of force ↓

(Experienced by quarks and leptons.)

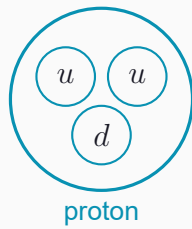
Quarks and Charge Calculations

Quarks

6 flavours of quarks

up quark (u)	$+\frac{2}{3}e$	charm quark (c)	$+\frac{2}{3}e$	top quark (t)	$+\frac{2}{3}e$
down quark (d)	$-\frac{1}{3}e$	strange quark (s)	$-\frac{1}{3}e$	bottom quark (b)	$-\frac{1}{3}e$

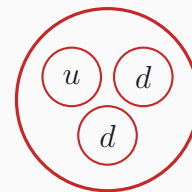
Proton: uud



proton

$$u + u + d = +\frac{2}{3}e + \frac{2}{3}e - \frac{1}{3}e = +\frac{3}{3}e = \boxed{+1e}.$$

Neutron: udd



neutron

$$u + d + d = +\frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = \boxed{0}.$$

Antimatter and the Neutrino

Antimatter

- Particle having same mass but opposite charge.
- When matter collides with its corresponding antimatter, they annihilate one another.
- Energy is released as electromagnetic radiation.

Particle	Antiparticle
+ proton (uud)	– antiproton ($\bar{u}\bar{u}\bar{d}$)
0 neutron (udd)	0 antineutron ($\bar{u}\bar{d}\bar{d}$)
– electron (e^-)	+ positron (e^+)
0 neutrino (ν)	0 antineutrino ($\bar{\nu}$)

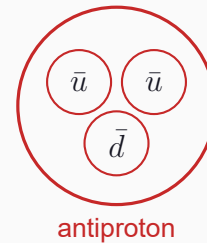
Symbol of neutrino

neutrino (ν), antineutrino ($\bar{\nu}$).

Antiproton charge

$$\bar{p} = \bar{u}\bar{u}\bar{d}$$

$$\bar{u} + \bar{u} + \bar{d} = -\frac{2}{3}e - \frac{2}{3}e + \frac{1}{3}e = \boxed{-1e}.$$



HARDER PARTICLES!

- Produced during beta decay.
- No charge.
- Negligible mass.

Radioactivity

Definition

The phenomenon of emission of radioactive radiations from the nucleus of an unstable atom.

Why it happens

Nucleus gets unstable when the number of nucleons inside it becomes more than it can hold at that size.

Random process

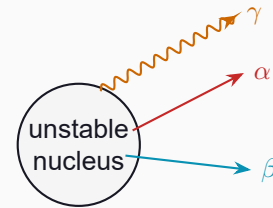
Every nucleus has equal chance to decay. It cannot be predicted which nucleus is going to decay when.

Spontaneous process

There is no control over this reaction; changing external conditions like temperature and pressure has no effect on its activity.

Types of radiations

1. Alpha rays
2. Beta rays
 - (a) beta positive (positron)
 - (b) beta negative (electron)
3. Gamma rays



Alpha Rays and Beta Negative

ALPHA RAYS

Nuclear symbol:	${}^4_2\alpha$
Nature:	Helium nucleus ($2p + 2n$)
Charge:	+2
Mass:	$4u$, where $1u = 1.66 \times 10^{-27} \text{ kg}$
Ionization power:	Greatest. Removes electrons from air molecules and produces ions.
Penetration power:	Weakest. Large particle so easily stopped by a piece of paper.
Range:	$\sim 10 \text{ cm}$, commonly 4 cm to 5 cm
Equation:	${}^{235}_{92}\text{U} \rightarrow {}^4_2\alpha + {}^{231}_{90}\text{X} + \text{energy}$

$$E = mc^2$$

BETA NEGATIVE

Nuclear symbol:	${}^0_{-1}\beta$
Nature:	fast moving electron (variable speed)
Charge:	-1
Mass:	$9.11 \times 10^{-31} \text{ kg}$
Ionization power:	Less than alpha due to less charge.
Penetration power:	Greater than alpha due to smaller size. 10 mm to 20 mm thick aluminium sheet
Range:	few metres ($\sim 10 \text{ m}$)
Equation:	—

Beta Positive and Equations of Beta Negative

BETA POSITIVE (Antimatter)

Nuclear symbol:

$${}_{+1}^0\beta$$

Nature:

fast moving positron (variable speed)

Charge:

$$+1$$

Mass:

$$9.11 \times 10^{-31} \text{ kg}$$

Ionization power:

Less than alpha due to less charge.

Penetration power:

Greater than alpha due to smaller size; stopped by 10 mm to 20 mm thick aluminium sheet.

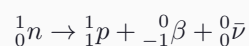
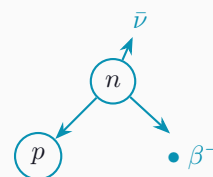
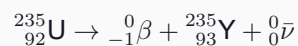
Range:

short range as it annihilates.

Equation:

—

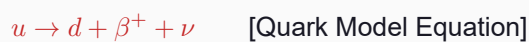
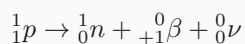
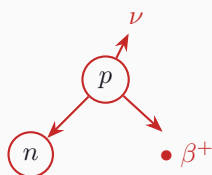
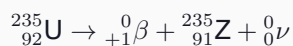
Equations of beta negative



$$d \rightarrow u + \beta^- + \bar{\nu} \quad [\text{Quark Model Equation}]$$

Equations of Beta Positive and Gamma Rays

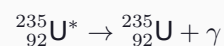
Equations of beta positive



GAMMA RAYS

Nuclear symbol:	γ
Nature:	High frequency electromagnetic radiation
Charge:	0
Mass:	0
Range:	several metres (~ 50 m)
Ionization power:	Least. It does not have charge but it can move electrons.
Penetration power:	Greatest. No size; stopped by 4 cm thick lead or 15 cm thick concrete wall.

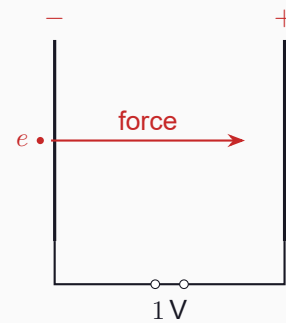
Equation



Quantities Conserved During Nuclear Reaction and Electron-Volts

Quantities conserved during nuclear reaction

1. Proton number
2. Mass number
3. Charge
4. Momentum
5. Mass-energy conserved



Electron-volts (eV)

- It is a unit of energy.
- It is the energy gained when an electron is accelerated through a potential difference of 1 V.

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$V = \frac{W}{q} \quad \text{or} \quad V = \frac{E}{q}$$

$$E = Vq = (1 \text{ V})(1e) = 1 \text{ eV}$$

$$E = (1 \text{ V})(1.6 \times 10^{-19} \text{ C}) = 1.6 \times 10^{-19} \text{ J}.$$