

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

1. Question | 9709 Feb-March 2016 P1 V2 Q8

8 The function f is such that $f(x) = a^2x^2 - ax + 3b$ for $x \leq \frac{1}{2a}$, where a and b are constants.

(i) For the case where $f(-2) = 4a^2 - b + 8$ and $f(-3) = 7a^2 - b + 14$, find the possible values of a and b . [5]

(ii) For the case where $a = 1$ and $b = -1$, find an expression for $f^{-1}(x)$ and give the domain of f^{-1} . [5]

2. Question | 9709 May-June 2016 P1 V1 Q11

11 The function f is defined by $f : x \mapsto 4 \sin x - 1$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

- (i) State the range of f . [2]
- (ii) Find the coordinates of the points at which the curve $y = f(x)$ intersects the coordinate axes. [3]
- (iii) Sketch the graph of $y = f(x)$. [2]
- (iv) Obtain an expression for $f^{-1}(x)$, stating both the domain and range of f^{-1} . [4]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

3. Question | 9709 May-June 2016 P1 V2 Q1

1 Functions f and g are defined by

$$f : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R}, \quad x \neq \frac{3}{2}.$$

Solve the equation $ff(x) = gf(2)$. [3]

4. Question | 9709 May-June 2016 P1 V2 Q11

11 The function f is defined by $f : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) \leq 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = f(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $g : x \mapsto 6x - x^2 - 5$ for $x \geq k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k , find an expression for $g^{-1}(x)$. [2]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

5. Question | 9709 May-June 2016 P1 V3 Q10

10 The function f is such that $f(x) = 2x + 3$ for $x \geq 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leq q$, where a , b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leq q$.

(i) Find the values of a and b . [3]

(ii) Find the greatest possible value of q . [2]

It is now given that $q = -3$.

(iii) Find the range of fg . [1]

(iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [3]

6. Question | 9709 Oct-Nov 2016 P1 V1 Q8

8 The functions f and g are defined by

$$f(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$

$$g(x) = \frac{4}{5x + 2} \quad \text{for } x \geq 0.$$

(i) Find and simplify an expression for $fg(x)$ and state the range of fg . [3]

(ii) Find an expression for $g^{-1}(x)$ and find the domain of g^{-1} . [5]

[Questions 9, 10 and 11 are printed on the next page.]

7. Question | 9709 Oct-Nov 2016 P1 V2 Q9

9 Relative to an origin O , the position vectors of the points A , B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB . [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and of magnitude 15 units. [3]

(iii) Find the value of the constant p for which $p\overrightarrow{OA} + \overrightarrow{OC}$ is perpendicular to $\overrightarrow{OB}$. [3]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

8. Question | 9709 Oct-Nov 2016 P1 V2 Q10

10 A function f is defined by $f : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq \pi$.

(i) Find the range of f . [2]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Solve the equation $f(x) = 6$, giving answers in terms of π . [3]

The function g is defined by $g : x \mapsto 5 - 2 \sin 2x$ for $0 \leq x \leq k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k , find an expression for $g^{-1}(x)$. [3]

9. Question | 9709 Oct-Nov 2016 P1 V3 Q4

4 The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n . [4]

10. Question | 9709 Oct-Nov 2016 P1 V3 Q8

8 (i) Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a , b and c are constants. [3]

(ii) Functions f and g are both defined for $x > 0$. It is given that $f(x) = x^2 + 1$ and $fg(x) = 4x^2 + 12x + 10$. Find $g(x)$. [1]

(iii) Find $(fg)^{-1}(x)$ and give the domain of $(fg)^{-1}$. [4]

Topical questions | 96 questions

8 The functions f and g are defined for $x \geq 0$ by

$$g : x \mapsto 3x + 2.$$

- [illegible]

-
- This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

11. Question part 2/2 | 9709 Feb-March 2017 P1 V2 Q8

(iii) Solve the equation $gf(2x) = fg(x)$. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

12. Question part 1/2 | 9709 May-June 2017 P1 V1 Q9

9 The function f is defined by $f : x \mapsto \frac{2}{3-2x}$ for $x \in \mathbb{R}$, $x \neq \frac{3}{2}$.

(i) Find an expression for $f^{-1}(x)$. [3]

[illegible]

Topical questions | 96 questions

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

- [illegible]

- [illegible]

Topical questions | 96 questions

10 The function f is defined by $f(x) = 3 \tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

(i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

[illegible]

(ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1} . [5]

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

13. Question part 2/2 | 9709 May-June 2017 P1 V2 Q10

(iii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

[3]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

14. Question | 9709 May-June 2017 P1 V3 Q3

3 Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

15. Question | 9709 Oct-Nov 2017 P1 V1 Q2

- 2 A function f is defined by $f : x \mapsto x^3 - x^2 - 8x + 5$ for $x < a$. It is given that f is an increasing function. Find the largest possible value of the constant a . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

Topical questions | 96 questions

9 Functions f and g are defined for $x > 3$ by

$$\begin{aligned} f &: x \mapsto \frac{1}{x^2 - 9}, \\ g &: x \mapsto 2x - 3. \end{aligned}$$

-
- This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

- [illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

16. Question part 2/2 | 9709 Oct-Nov 2017 P1 V1 Q9

[illegible]

(iii) Solve the equation $\lg(x) = \frac{1}{7}$.

[4]

[illegible]

2 A function f is defined by $f : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

- (i)** Find an expression for $f^{-1}(x)$ and find the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii)** Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

Topical questions | 96 questions

6 The functions f and g are defined by

$$\begin{aligned} f(x) &= \frac{2}{x^2 - 1} \text{ for } x < -1, \\ g(x) &= x^2 + 1 \text{ for } x > 0. \end{aligned}$$

- (i) Find an expression for $f^{-1}(x)$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

18. Question part 2/2 | 9709 Oct-Nov 2017 P1 V3 Q6

(ii) Solve the equation $gf(x) = 5$.

[4]

[illegible]

10 Functions f and g are defined by

$$f(x) = \frac{8}{x-2} + 2 \quad \text{for } x > 2,$$
$$g(x) = \frac{8}{x-2} + 2 \quad \text{for } 2 < x < 4.$$

(i) (a) State the range of the function f . [1]

.....

.....

.....

(b) State the range of the function g . [1]

.....

.....

.....

(c) State the range of the function fg . [1]

.....

.....

.....

(ii) Explain why the function gf cannot be formed. [1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

19. Question part 2/2 | 9709 Feb-March 2018 P1 V2 Q10

(iii) Find the set of values of x satisfying the inequality $6f'(x) + 2f^{-1}(x) - 5 < 0$.

[6]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

$$g : x \mapsto 4 + x - \frac{1}{2}x^2.$$

- [illegible]

- [illegible]

Topical questions | 96 questions

(iii) Find an expression for $fg(x)$ and deduce the range of fg . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(iv) Find the smallest value of k for which h has an inverse. [2]

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

21. Question part 1/2 | 9709 May-June 2018 P1 V3 Q10

- 10** The one-one function f is defined by $f(x) = (x - 2)^2 + 2$ for $x \geq c$, where c is a constant.

- (i) State the smallest possible value of c . [1]

.....

.....

.....

In parts **(ii)** and **(iii)** the value of c is 4.

- (ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

21. Question part 2/2 | 9709 May-June 2018 P1 V3 Q10

(iii) Solve the equation $\text{ff}(x) = 51$, giving your answer in the form $a + \sqrt{b}$.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

22. Question part 1/3 | 9709 Oct-Nov 2018 P1 V1 Q11

- 11 (a)** The one-one function f is defined by $f(x) = (x - 3)^2 - 1$ for $x < a$, where a is a constant.

(i) State the greatest possible value of a .

[1]

.....

.....

.....

- (ii)** It is given that a takes this greatest possible value. State the range of f and find an expression for $f^{-1}(x)$. [3]

[illegible]

Topical questions | 96 questions

(b) The function g is defined by $g(x) = (x - 3)^2$ for $x \geq 0$.

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge International Examinations Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cie.org.uk after the live examination series.

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

9 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) State the range of f . [1]

.....

.....

.....

.....

.....

Topical questions | 96 questions

(iii) State the largest value of k for which g has an inverse. [1]

.....

.....

.....

.....

(iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

11 **(i)** Express $2x^2 - 12x + 11$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The function f is defined by $f(x) = 2x^2 - 12x + 11$ for $x \leq k$.

(ii) State the largest value of the constant k for which f is a one-one function. [1]

.....

.....

.....

(iii) For this value of k find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

.....

.....

.....

.....

.....

.....

.....

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

24. Question part 2/3 | 9709 Oct-Nov 2018 P1 V3 Q11

[illegible]

The function g is defined by $g(x) = x + 3$ for $x \leq p$.

- (iv) With k now taking the value 1, find the largest value of the constant p which allows the composite function fg to be formed, and find an expression for $fg(x)$ whenever this composite function exists. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]

Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.

Topical questions | 96 questions

8 (i) Express $x^2 - 4x + 7$ in the form $(x + a)^2 + b$. [2]

[illegible]

(ii) State the largest value of k for which f is a decreasing function. [1]

.....

.....

.....

(iii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

25. Question part 2/2 | 9709 Feb-March 2019 P1 V2 Q8

(iv) The function g is defined by $g(x) = \frac{2}{x-1}$ for $x > 1$. Find an expression for $gf(x)$ and state the range of gf . [4]

[illegible]

26. Question part 1/2 | 9709 May-June 2019 P1 V1 Q9

9 The function f is defined by $f(x) = 2 - 3 \cos x$ for $0 \leq x \leq 2\pi$.

(i) State the range of f . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Sketch the graph of $y = f(x)$. [2]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

26. Question part 2/2 | 9709 May-June 2019 P1 V1 Q9

The function g is defined by $g(x) = 2 - 3 \cos x$ for $0 \leq x \leq p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

.....

.....

.....

.....

.....

(iv) For this value of p , find an expression for $g^{-1}(x)$. [2]

[illegible]

$$\begin{aligned} f &: x \mapsto 3x - 2, & x \in \mathbb{R}, \\ g &: x \mapsto \frac{2x + 3}{x - 1}, & x \in \mathbb{R}, x \neq 1. \end{aligned}$$

-
- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

27. Question part 2/2 | 9709 May-June 2019 P1 V2 Q7

(ii) Solve the equation $\text{fg}(x) = \frac{7}{3}$.

[3]

[illegible]

28. Question | 9709 May-June 2019 P1 V3 Q4

4 The function f is defined by $f(x) = \frac{48}{x-1}$ for $3 \leq x \leq 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leq x \leq b$, where a and b are constants.

- (i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf . [2]

.....

.....

.....

.....

.....

.....

.....

It is now given that the conditions for the formation of gf are satisfied.

- (ii) Find an expression for $gf(x)$. [1]

.....

.....

.....

.....

- (iii) Find an expression for $(gf)^{-1}(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

Topical questions | 96 questions

7 Functions f and g are defined by

$$\begin{aligned} f : x &\mapsto \frac{3}{2x+1} && \text{for } x > 0, \\ g : x &\mapsto \frac{1}{x} + 2 && \text{for } x > 0. \end{aligned}$$

(i) Find the range of f and the range of g . [3]

[illegible]

Topical questions | 96 questions

(ii) Find an expression for $fg(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a , b and c are integers. [2]

This image shows a full page of white paper with ten evenly spaced horizontal dashed lines, typical of primary school handwriting practice paper. The lines extend across the entire width of the page, leaving small margins at the top and bottom. There are no other markings, text, or illustrations present.[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

30. Question | 9709 Oct-Nov 2019 P1 V3 Q2

- 2** The function g is defined by $g(x) = x^2 - 6x + 7$ for $x > 4$. By first completing the square, find an expression for $g^{-1}(x)$ and state the domain of g^{-1} . [5]

[illegible]

Topical questions | 96 questions

2 The graph of $y = f(x)$ is transformed to the graph of $y = 1 + f(\frac{1}{5}x)$.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Topical questions | 96 questions

9 (a) Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

[illegible]

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

[illegible]

The function g is defined by $g(x) = 2x - 3$ for $x \leq k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) State the largest value of k possible for the composition fg to be defined. [1]

.....

.....

.....

.....

.....

6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\begin{aligned} f &: x \mapsto \frac{1}{2}x - a, \\ g &: x \mapsto 3x + b, \end{aligned}$$

where a and b are constants.

(a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Using these values of a and b , find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

.....

.....

.....

.....

.....

.....

.....

Topical questions | 96 questions

5 The function f is defined for $x \in \mathbb{R}$ by

where a is a constant.

-
- This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

-
- This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

35. Question | 9709 May-June 2020 P1 V2 Q9

9 Functions f and g are such that

$$\begin{aligned} f(x) &= 2 - 3 \sin 2x \quad \text{for } 0 \leq x \leq \pi, \\ g(x) &= -2f(x) \quad \text{for } 0 \leq x \leq \pi. \end{aligned}$$

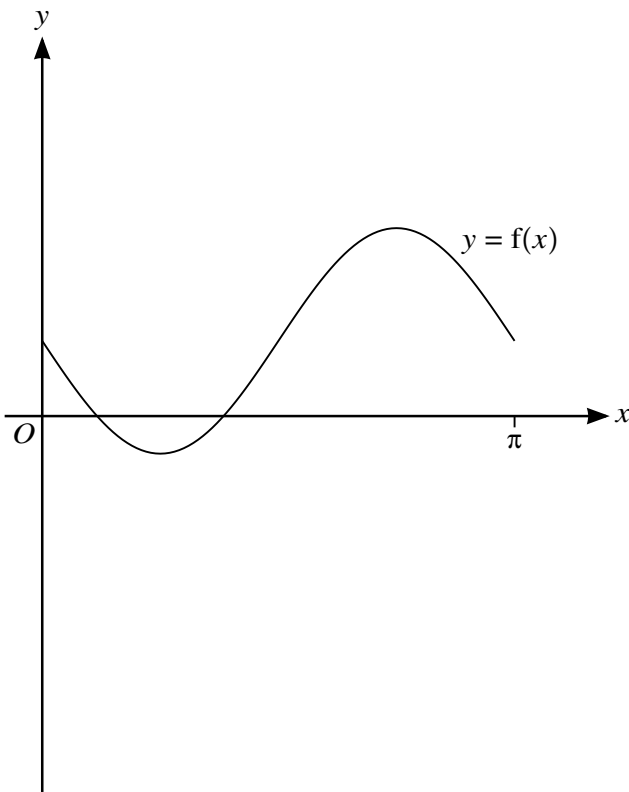
(a) State the ranges of f and g . [3]

.....

.....

.....

The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leq x \leq 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

.....

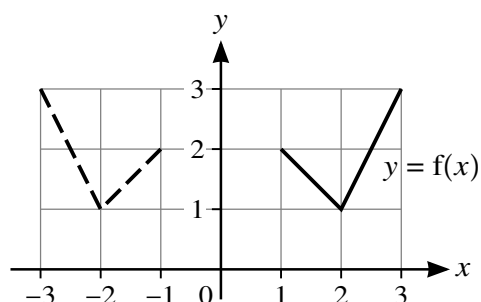
.....

.....

36. Question | 9709 May-June 2020 P1 V3 Q3

- 3 In each of parts (a), (b) and (c), the graph shown with solid lines has equation $y = f(x)$. The graph shown with broken lines is a transformation of $y = f(x)$.

(a)

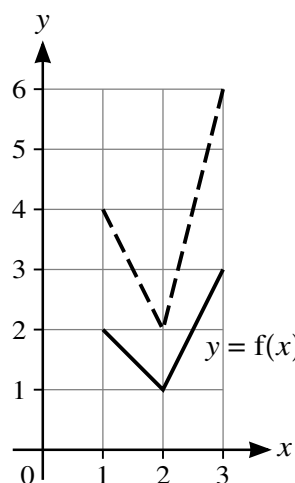


State, in terms of f , the equation of the graph shown with broken lines.

[1]

.....

(b)

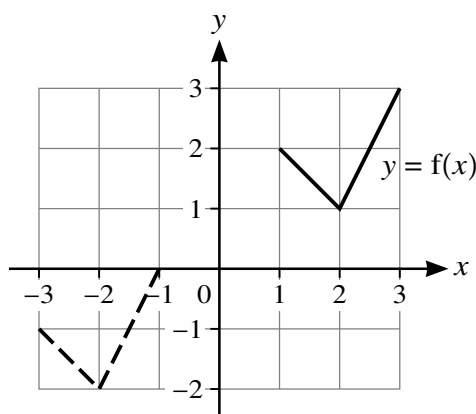


State, in terms of f , the equation of the graph shown with broken lines.

[1]

.....

(c)



State, in terms of f , the equation of the graph shown with broken lines.

[2]

.....

9 The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$
$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

It is given that f is a one-one function.

(b) State the smallest possible value of c . [1]

.....

.....

.....

.....

.....

.....

.....

.....

Topical questions | 96 questions

—

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$

$$g(x) = 2x + 1 \quad \text{for } x > -\frac{1}{2}.$$

- (a)** Find an expression for $\text{fg}(x)$. [1]

[illegible]

- (b)** Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings present.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

38. Question part 2/2 | 9709 Oct-Nov 2020 P1 V1 Q11

(c) Solve the equation $fg(x) - 3 = gf(x)$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The lines are evenly spaced across the entire page, providing a guide for consistent letter formation. There is no text or other markings on the page.

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R}, x \neq -1.$$

-
-
-
-
-

- [illegible]

40. Question | 9709 Oct-Nov 2020 P1 V3 Q6

6 The function f is defined by $f(x) = \frac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that $\frac{2}{3} + \frac{2}{3(3x-1)}$ can be expressed as $\frac{2x}{3x-1}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

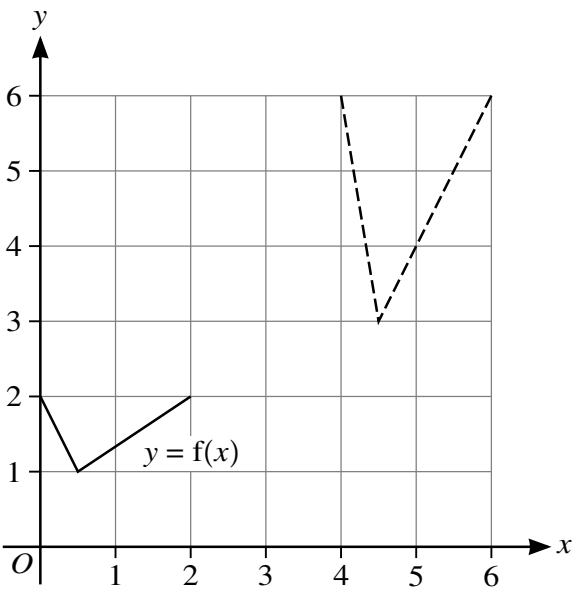
(c) State the range of f . [1]

.....

.....

.....

5



In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

- (a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

.....

.....

.....

.....

.....

.....

.....

.....

- (b) State in terms of y , f and x , the equation of the graph shown with broken lines. [2]

.....

.....

.....

.....

.....

Topical questions | 96 questions

7 Functions f and g are defined as follows:

$$g : x \mapsto 2x + 1 \text{ for } x \geq -1.$$

- (a) Express $f(x)$ in the form $(x + a)^2 + b$ and state the range of f . [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Topical questions | 96 questions

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[illegible]

9 Functions f and g are defined as follows:

$$f(x) = (x - 2)^2 - 4 \text{ for } x \geq 2,$$
$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f . [1]

.....

.....

.....

(b) Find $f^{-1}(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

(c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

43. Question part 2/2 | 9709 May-June 2021 P1 V1 Q9

(d) Given instead that $\text{ggf}^{-1}(12) = 62$, find the possible values of a .

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

44. Question | 9709 May-June 2021 P1 V2 Q2

- 2 (a)** The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

[illegible]

- (b)** The curve $y = \sin 2x - 5x$ is reflected in the y -axis and then stretched by scale factor $\frac{1}{3}$ in the x -direction.

Write down the equation of the transformed curve. [2]

[illegible]

Topical questions | 96 questions

5 The function f is defined by $f(x) = 2x^2 + 3$ for $x \geq 0$.

[illegible]

Topical questions | 96 questions

2 The function f is defined by $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a . [4]

[illegible]

6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

- (a)** By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b)** Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

.....

.....

.....

.....

.....

.....

Topical questions | 96 questions

8 Functions f and g are defined as follows:

$$\begin{aligned} f &: x \mapsto x^2 - 1 \text{ for } x < 0, \\ g &: x \mapsto \frac{1}{2x+1} \text{ for } x < -\frac{1}{2}. \end{aligned}$$

(a) Solve the equation $\text{fg}(x) = 3$. [4]

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

48. Question part 2/2 | 9709 May-June 2021 P1 V3 Q8

(b) Find an expression for $(fg)^{-1}(x)$.

[3]

[illegible]

8 **(a)** Express $-3x^2 + 12x + 2$ in the form $-3(x - a)^2 + b$, where a and b are constants. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leq k$.

(b) State the largest possible value of the constant k . [1]

.....

.....

.....

.....

It is now given that $k = -1$.

(c) State the range of f . [1]

.....

.....

.....

.....

Topical questions | 96 questions

-
- This image shows a full page of white paper with ten horizontal dashed lines, evenly spaced from top to bottom. There are no margins, text, or other markings on the page.

[illegible]

- [illegible]

(b) State the coordinates of the corresponding point on the original curve $y = f(x)$. [2]

This image shows a full page of primary-ruled paper. It features ten sets of horizontal dashed lines, each set consisting of three parallel lines. These lines are evenly spaced vertically across the entire page, providing a guide for handwriting practice. The background is white, and there are no margins or other markings present.

Topical questions | 96 questions

3 The function f is defined as follows:

(a) Find the value of $\text{ff}(5)$. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

[illegible]

9709 Mathematics Paper 1 - Functions

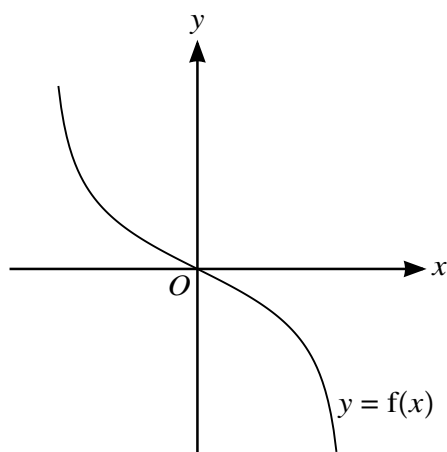
Topical questions | 96 questions

52. Question | 9709 Oct-Nov 2021 P1 V3 Q1

- 1** The graph of $y = f(x)$ is transformed to the graph of $y = 3 - f(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

[illegible]



(a) On this diagram sketch the graph of $y = f^{-1}(x)$. [1]

(b) Find an expression for $f^{-1}(x)$. [4]

[illegible]

Topical questions | 96 questions

[illegible]

(c) State the maximum possible value of a for which fg can be formed. [1]

[illegible][illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

54. Question | 9709 Feb-March 2022 P1 V2 Q5

- 5** (a) Express $2x^2 - 8x + 14$ in the form $2[(x - a)^2 + b]$. [2]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

The functions f and g are defined by

$$f(x) = x^2 \quad \text{for } x \in \mathbb{R},$$

$$g(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

- (b)** Describe fully a sequence of transformations that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$, making clear the order in which the transformations are applied. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

Topical questions | 96 questions

9 Functions f , g and h are defined as follows:

$$h : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geq 0.$$

- [illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

55. Question part 2/2 | 9709 Feb-March 2022 P1 V2 Q9

(b) Given that $f(x) \equiv gh(x)$, find the values of m and n .

[4]

[illegible]

Topical questions | 96 questions

6 The function f is defined as follows:

(a) Find an expression for $f^{-1}(x)$. [3]

[illegible]

Topical questions | 96 questions

(b) Show that $1 - \frac{8}{x^2 + 4}$ can be expressed as $\frac{x^2 - 4}{x^2 + 4}$ and hence state the range of f . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- [illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

57. Question part 1/2 | 9709 May-June 2022 P1 V1 Q8

- 8 (a)** The curve $y = \sin x$ is transformed to the curve $y = 4 \sin(\frac{1}{5}x - 30^\circ)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

[illegible]

Topical questions | 96 questions

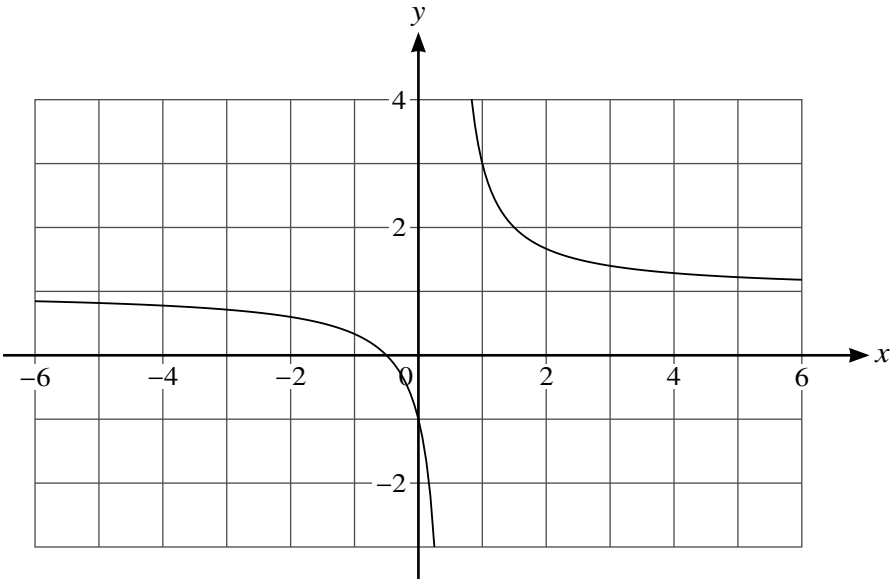
—

[illegible]

10 Functions f and g are defined as follows:

$$f(x) = \frac{2x + 1}{2x - 1} \quad \text{for } x \neq \frac{1}{2},$$
$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1} . [1]

.....

.....

(b) Find an expression for $f^{-1}(x)$. [3]

.....

.....

.....

.....

.....

(c) Find $gf^{-1}(3)$. [2]

.....

.....

Topical questions | 96 questions

(d) Explain why $g^{-1}(x)$ cannot be found. [1]

- (e) Show that $1 + \frac{2}{2x-1}$ can be expressed as $\frac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x - and y -axes. [6]

[illegible]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- [illegible]

Topical questions | 96 questions

6 The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

Topical questions | 96 questions

(c) Find an expression for $f^{-1}(x)$. [3]

[illegible]

(d) Find and simplify an expression for $fg(x)$. [2]

[illegible]

Topical questions | 96 questions

9 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$g(x) = 2x^2 + 4x + 12.$$

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

[illegible]

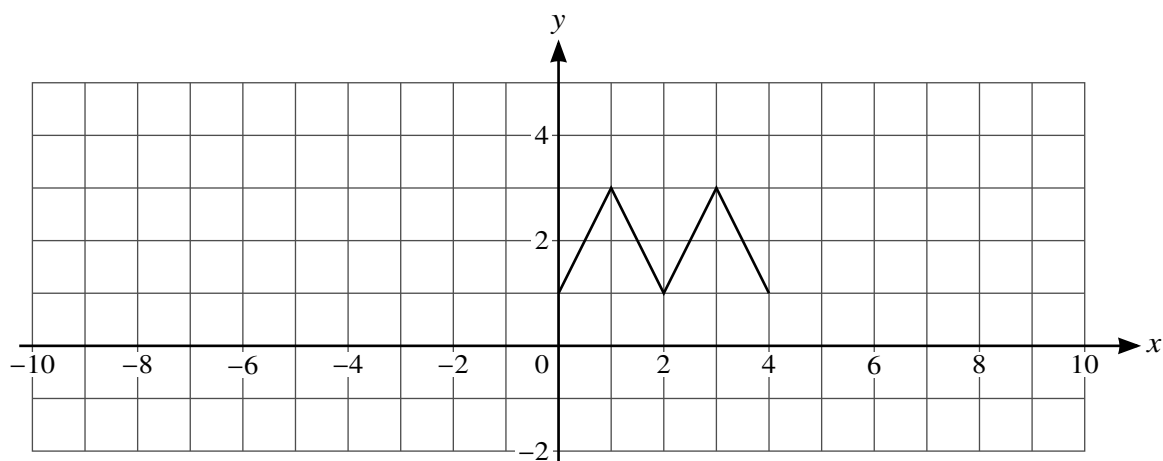
Topical questions | 96 questions

—

- [illegible]

- [illegible]

On the diagram sketch the graph of $y = g(x)$. [3]



(b) Find an expression for $g(x)$ in terms of $f(x)$. [2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

9 Functions f and g are defined by

$$\begin{aligned} f(x) &= x + \frac{1}{x} \quad \text{for } x > 0, \\ g(x) &= ax + 1 \quad \text{for } x \in \mathbb{R}, \end{aligned}$$

where a is a constant.

(a) Find an expression for $gf(x)$. [1]

.....

.....

.....

.....

.....

(b) Given that $gf(2) = 11$, find the value of a . [2]

.....

.....

.....

.....

.....

.....

(c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

.....

.....

.....

.....

.....

.....

.....

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

63. Question part 2/2 | 9709 Oct-Nov 2022 P1 V2 Q9

It is given instead that $a = 5$.

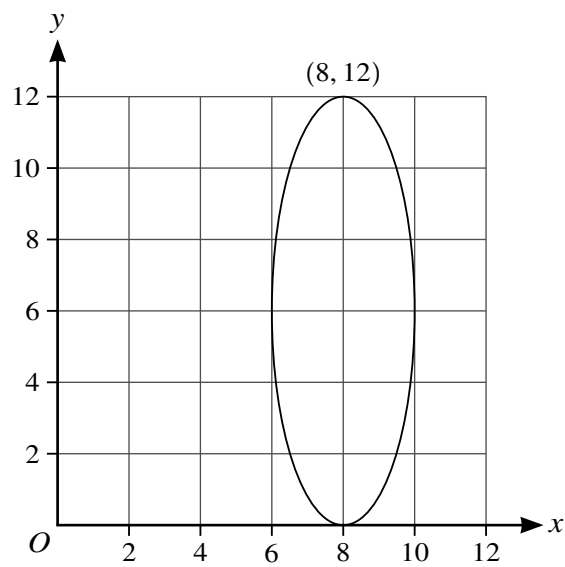
(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

[illegible]

(e) Explain why the composite function fg cannot be formed. [1]

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

5



The diagram shows a curve which has a maximum point at (8, 12) and a minimum point at (8, 0). The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y-direction.

(a) State the scale factor of the stretch. [1]

.....

.....

(b) State the radius of the original circle. [1]

.....

.....

(c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

.....

.....

.....

(d) State the coordinates of the centre of the original circle. [2]

.....

.....

.....

Topical questions | 96 questions

2 A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Reflection in the y -axis

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a , b and c are constants. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

Topical questions | 96 questions

9 The function f is defined by $f(x) = -3x^2 + 2$ for $x \leq -1$.

.....

.....

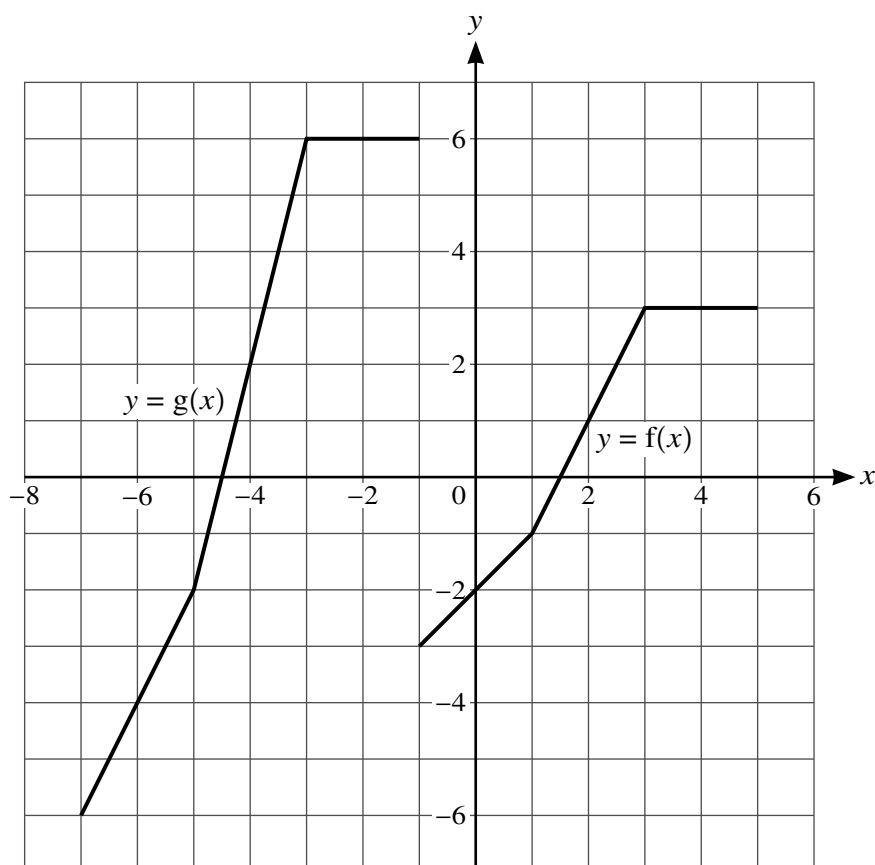
.....

[illegible]

Topical questions | 96 questions

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

[illegible]



Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical questions | 96 questions

8 The functions f and g are defined as follows, where a and b are constants.

$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

- (a) Given that $f(7) = \frac{5}{2}$ and $gf(5) = 4$, find the values of a and b . [4]

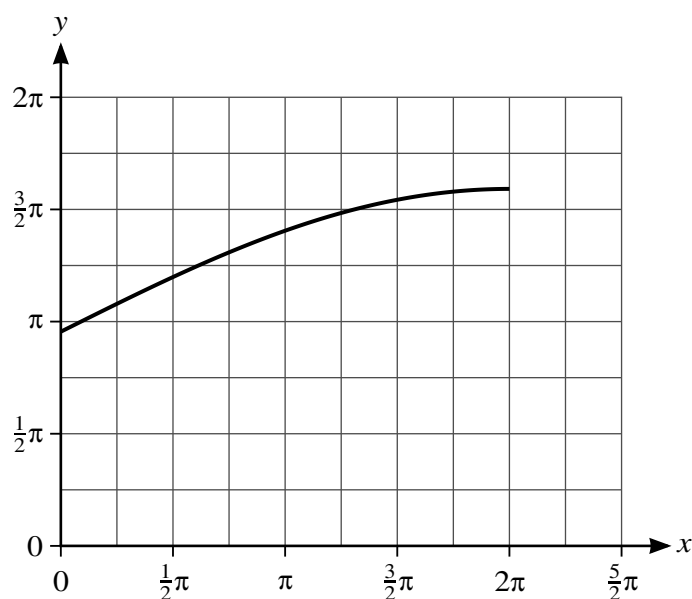
[illegible]

Topical questions | 96 questions

68. Question part 2/2 | 9709 May-June 2023 P1 V1 Q8

(b) Find the domain of f^{-1} . [1]

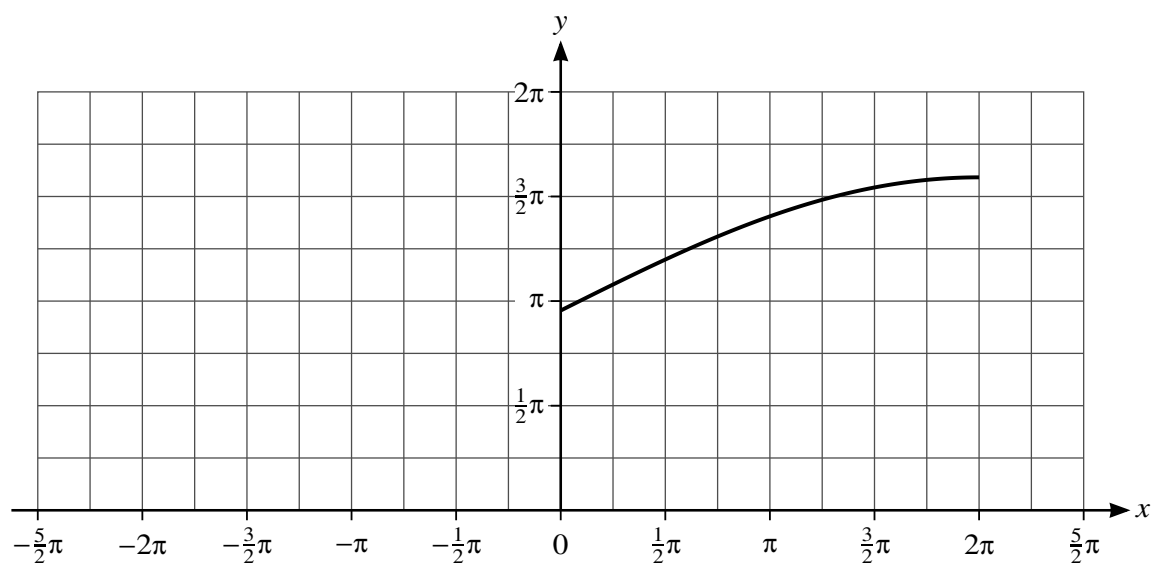
This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank.


$$f(x) = 3 + 2 \sin \frac{1}{4}x \text{ for } 0 \leq x \leq 2\pi.$$

- (b)** Find an expression for $f^{-1}(x)$. [2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2 \sin \frac{1}{4}x$ for $-2\pi \leq x \leq 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

.....

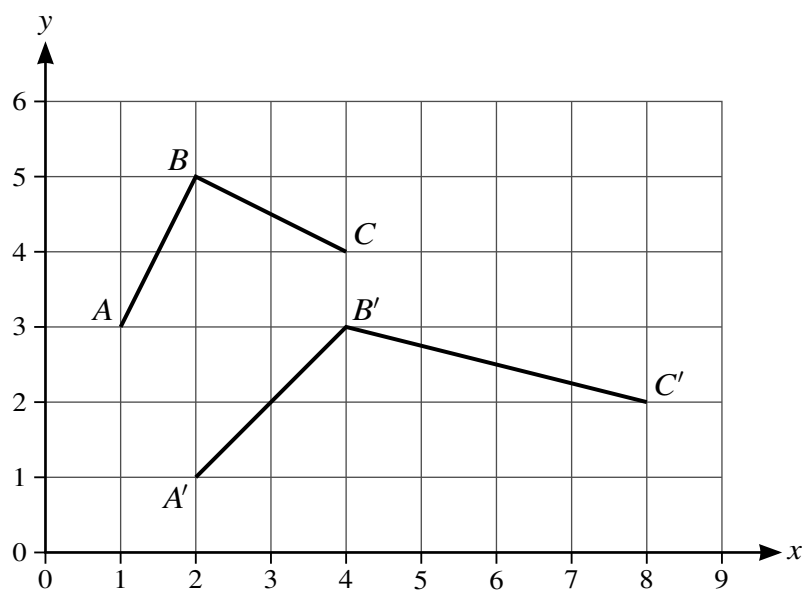
.....

.....

(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leq x \leq \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

1



The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC . The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations.

[4]

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

71. Question part 1/2 | 9709 May-June 2023 P1 V3 Q7

7 The function f is defined by $f(x) = 2 - \frac{5}{x+2}$ for $x > -2$.

(a) State the range of f . [1]

.....

.....

.....

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

This image shows a full page of white paper designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines running from left to right across the entire width of the page. There are no margins, text, or other markings present.

Topical questions | 96 questions

The function g is defined by $g(x) = x + 3$ for $x > 0$.

-
- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

4 The transformation R denotes a reflection in the x -axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(a) Find the equation, $y = g(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

.....

.....

.....

.....

.....

.....

(b) Find the equation, $y = h(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

.....

.....

.....

.....

.....

.....

(c) State fully the transformation that maps the curve $y = g(x)$ onto the curve $y = h(x)$. [2]

.....

.....

.....

.....

.....

Topical questions | 96 questions

9 (a) Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

(b) Given that it is possible to form the composite function gf , find the least possible value of p and the greatest possible value of q . [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical questions | 96 questions

- [illegible]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

Topical questions | 96 questions

6 The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve. [2]

This image shows a full page of a handwriting practice worksheet. It consists of ten sets of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve. [2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

74. Question part 2/2 | 9709 Oct-Nov 2023 P1 V2 Q6

- (c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a , b and c are integers to be found. [4]

[illegible]

$$g(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

-
- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

-
-

-

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

75. Question part 2/2 | 9709 Oct-Nov 2023 P1 V2 Q8

(c) Given that $a = \frac{7}{2}$, solve the equation $gf(x) = 0$.

[3]

[illegible]

76. Question | 9709 Oct-Nov 2023 P1 V3 Q7

7 The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(a) State the range of f . [1]

.....

.....

.....

(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$. [2]

.....

.....

.....

.....

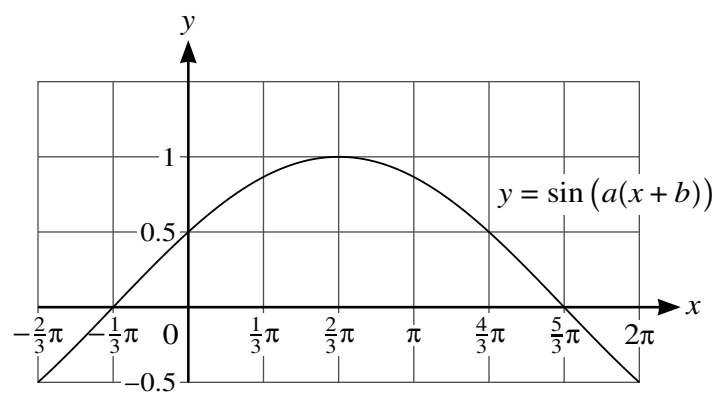
.....

.....

.....

.....

8



The diagram shows part of the graph of $y = \sin(a(x + b))$, where a and b are positive constants.

- (a) State the value of a and one possible value of b . [2]

.....

.....

.....

.....

Another curve, with equation $y = f(x)$, has a single stationary point at the point (p, q) , where p and q are constants. This curve is transformed to a curve with equation

$$y = -3f\left(\frac{1}{4}(x + 8)\right).$$

- (b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

Topical questions | 96 questions

9 The functions f and g are defined for all real values of x by

where k is a constant.

-
- This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- [illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

78. Question part 2/2 | 9709 Feb-March 2024 P1 V2 Q9

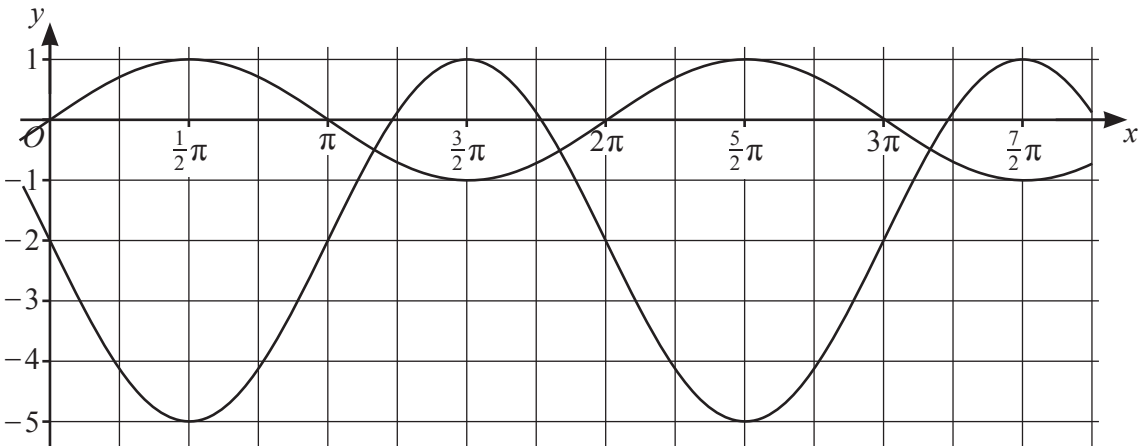
This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) The function h is defined for all real values of x and is such that $gh(x) = 35x + 19$.

Find an expression for $g^{-1}(x)$ and hence, or otherwise, find an expression for $h(x)$. [3]

[illegible]

2



The diagram shows two curves. One curve has equation $y = \sin x$ and the other curve has equation $y = f(x)$.

- (a) In order to transform the curve $y = \sin x$ to the curve $y = f(x)$, the curve $y = \sin x$ is first reflected in the x -axis.

Describe fully a sequence of two further transformations which are required. [4]

.....

.....

.....

.....

.....

.....

.....

- (b) Find $f(x)$ in terms of $\sin x$. [2]

.....

.....

.....

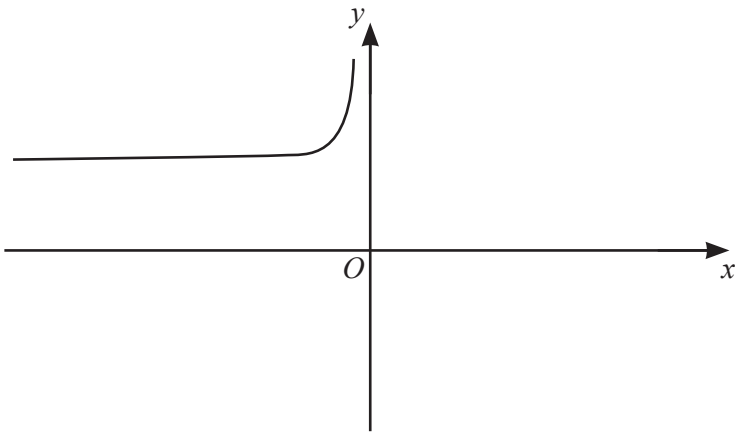
.....

.....

.....

.....

6



The function f is defined by $f(x) = \frac{2}{x^2} + 4$ for $x < 0$. The diagram shows the graph of $y = f(x)$.

(a) On this diagram, sketch the graph of $y = f^{-1}(x)$. Show any relevant mirror line. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Solve the equation $f(x) = 4.5$. [1]

.....

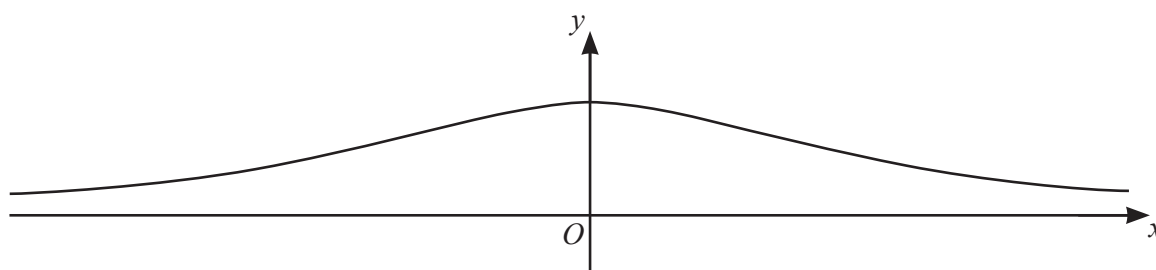
.....

(d) Explain why the equation $f^{-1}(x) = f(x)$ has no solution. [1]

.....

.....

(a) Find an expression for $f^{-1}(x)$. [1]

[illegible]

(b) State the range of g and explain whether g^{-1} exists. [2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Topical questions | 96 questions

The function h is defined by $h(x) = \frac{1}{x^2 + 2}$ for $x \geq 0$.

- (c) Solve the equation $hf(x) = f\left(\frac{25}{16}\right)$. Give your answer in the form $a+b\sqrt{c}$, where a , b and c are integers. [4]

[illegible]

-
- This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- Give details of the reflection and translation involved. [2]

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

Topical questions | 96 questions

[illegible]

(c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]

.....

.....

.....

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

82. Question part 3/3 | 9709 Oct-Nov 2024 P1 V1 Q11



Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]

.....

.....

.....

.....

This image shows a blank sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

83. Question part 2/2 | 9709 Oct-Nov 2024 P1 V2 Q5

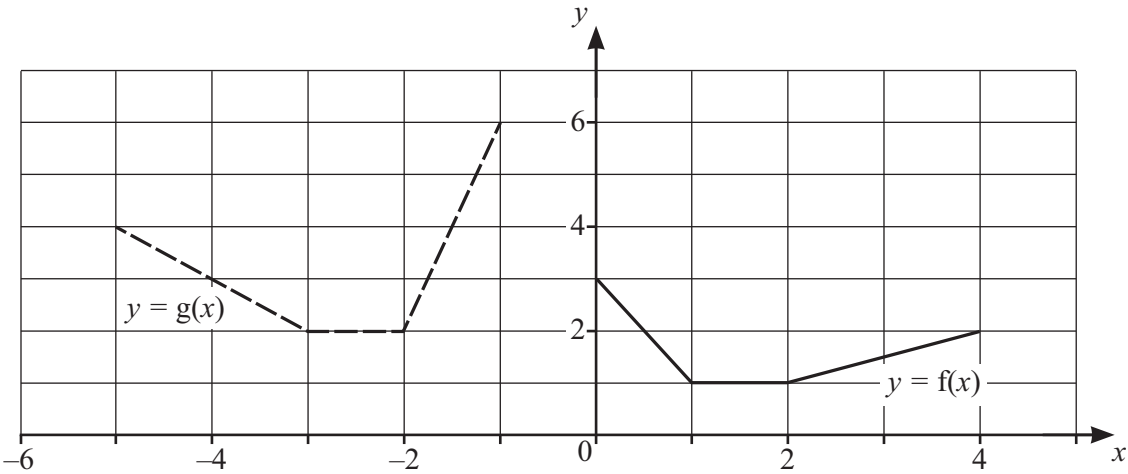
This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

The function g is defined by $g(x) = 3x + 2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

[illegible]

5



In the diagram, the graph with equation $y = f(x)$ is shown with solid lines and the graph with equation $y = g(x)$ is shown with broken lines.

- (a) Describe fully a sequence of three transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find an expression for $g(x)$ in the form $af(bx + c)$, where a , b and c are integers. [2]

.....

.....

.....

.....

.....

.....

.....

.....

Topical questions | 96 questions

8 (a) Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

(b) Find the least value of k for which the function f^{-1} exists. [1]

.....

.....

.....

(c) Find an expression for $f^{-1}(x)$. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

85. Question part 2/2 | 9709 Oct-Nov 2024 P1 V3 Q8



(d) Hence or otherwise solve the equation $\text{ff}(x) = 29$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

86. Question part 1/3 | 9709 Feb-March 2025 P1 V2 Q11

11 Functions f and g are defined for all real values of x by

$$f(x) = 4x^2 - c \quad \text{and} \quad g(x) = 2x + k,$$

where c and k are positive constants. It is given that $g^{-1}(3k+1) = c$.

(a) Show that $\text{gf}(x) = 8x^2 - k - 1$. [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- Translation of $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

Reflection in the x -axis

[illegible]

- [illegible]



If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

87. Question part 1/3 | 9709 May-June 2025 P1 V1 Q10

10 The functions f and g are defined by

$$f(x) = \sqrt{x} \qquad \text{for } x \geq 0,$$

$$g(x) = 3\sqrt{x+2} - 5 \qquad \text{for } x \geq -2.$$

- (a) Describe fully a sequence of transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. You should make clear the order in which the transformations are applied. [5]

.....

.....

.....

.....

.....

.....

.....

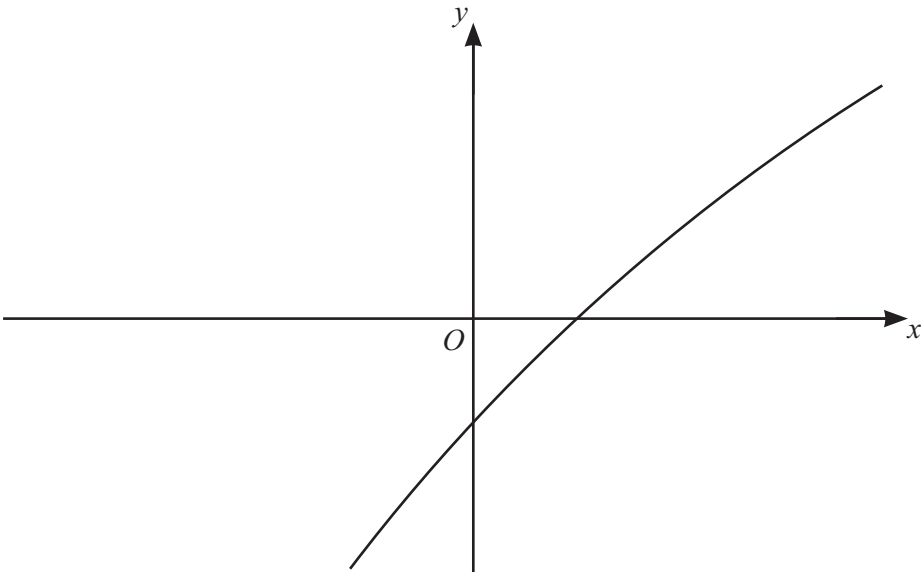
.....

.....

.....

.....

.....



The diagram shows the graph of $y = g(x)$.

- (b) On the diagram sketch the graph of $y = g^{-1}(x)$ together with any relevant mirror line. [2]

87. Question part 2/3 | 9709 May-June 2025 P1 V1 Q10



(c) Find an expression for $g^{-1}(x)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

(d) State the range of g^{-1} . [1]

.....

.....

The function h is defined by

$$h(x) = x - 2 \quad \text{for } x \geq 0.$$

(e) Find the value of $g^{-1}h(4)$. [1]

.....

.....

.....

.....

.....

.....

.....

(f) Explain why the composite function hg^{-1} cannot be formed. [1]

.....

.....

.....

.....

.....

.....

.....

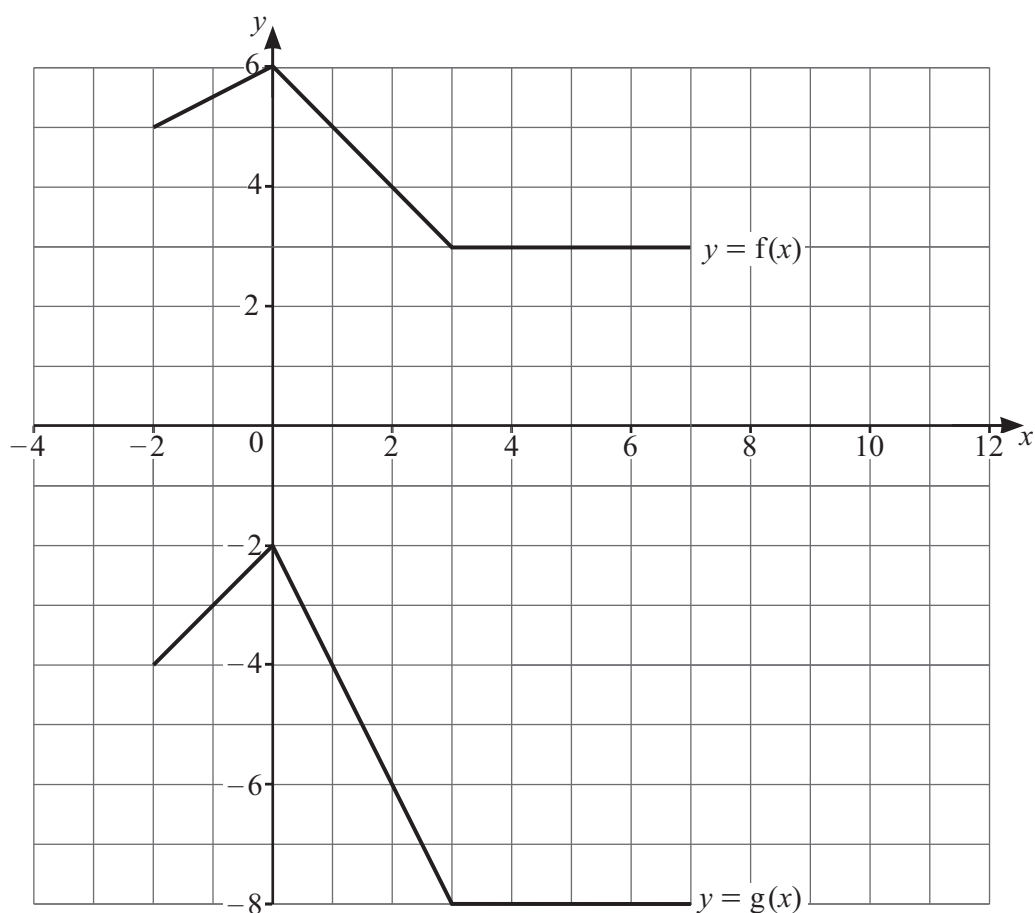
Topical questions | 96 questions



If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

1



Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. Make clear the order in which the transformations should be applied. [4]

[illegible]

Topical questions | 96 questions

11 (a) Express $x^2 + 4x + 2$ in the form $(x + a)^2 + b$, where a and b are integers. [2]

This image shows a single page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, leaving a small margin at the top. There is no handwriting or printed text on the page.
$$f(x) = x^2 + 4x + 2 \quad \text{for } x \leq -2$$

$$g(x) = -x - 4 \quad \text{for } x \geq -2$$

(b) (i) Find an expression for $f^{-1}(x)$. [3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

89. Question part 2/3 | 9709 May-June 2025 P1 V2 Q11



(ii) Find an expression for $(gf)^{-1}(x)$.

[4]

[illegible]

Topical questions | 96 questions



If you use the following page to complete the answer to any question, the question number must be clearly shown.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Topical questions | 96 questions

11 The function f is defined by $f(x) = x^2 + 4ax + a$ for $x \in \mathbb{R}$, where a is a constant.

The function g is such that $g^{-1}(x) = \sqrt[3]{2x-4}$ for $x \in \mathbb{R}$.

- (a) Given that the range of f is $f(x) \geq -33$, find the possible values of a . [4]

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

90. Question part 2/3 | 9709 May-June 2025 P1 V3 Q11



(b) Given instead that $\text{fgg}(0) = 96$, find the value of a .

[6]

[illegible]

Topical questions | 96 questions

90. Question part 3/3 | 9709 May-June 2025 P1 V3 Q11



If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Topical questions | 96 questions

4 (a) Express $1 - 6x - x^2$ in the form $a - (x + b)^2$, where a and b are constants. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- (b)** The graph of $y = x^2$ is transformed to the graph of $y = 1 - 6x - x^2$ by a reflection followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$. Give details of the reflection and determine the values of m and n . [3]

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

92. Question | 9709 Oct-Nov 2025 P1 V1 Q6

6 Functions f and g are defined by

$$f(x) = (x+3)^2 - 12 \quad \text{for } x \geq 0,$$

$$g(x) = 2x - 5 \quad \text{for } x \in \mathbb{R}.$$

- (a) State the range of f . [1]

.....

.....

.....

.....

- (b)** Find an expression for $f^{-1}(x)$. [2]

[illegible]

- (c) Solve the equation $gf(x) = 69$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

93. Question | 9709 Oct-Nov 2025 P1 V2 Q3

- 3 (a)** The graph of $y = f(x)$ is transformed to the graph of $y = f(3x) + 2$.

Describe fully the two transformations which have been combined to give the resulting graph. [3]

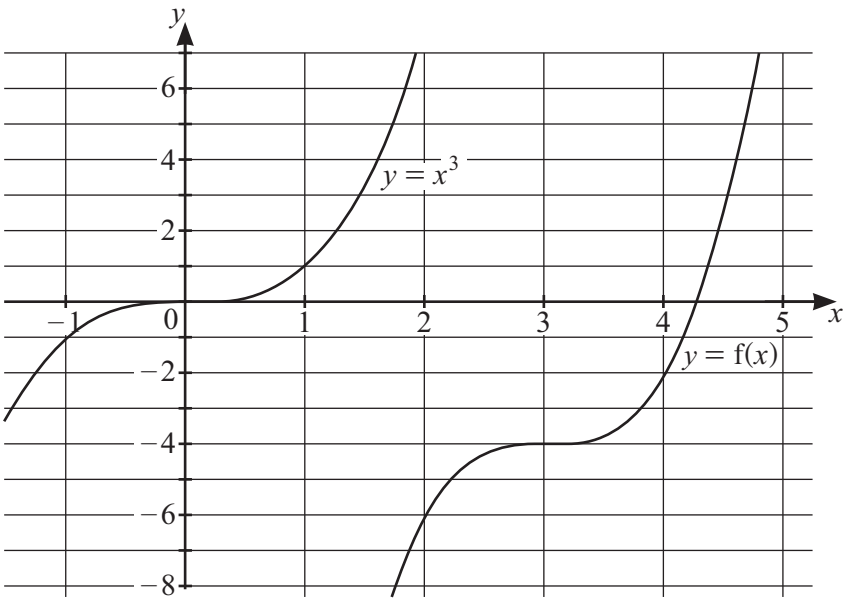
[illegible]

- (b)** A different graph has equation $y = g(x)$. This graph is stretched by scale factor 3 in the y -direction and then reflected in the y -axis.

Write down the equation of the transformed graph in terms of the function g . [2]

[illegible]

6



The diagram shows the graphs of $y = x^3$ and $y = f(x)$. The graph of $y = x^3$ is transformed to the graph of $y = f(x)$ by a sequence of transformations.

- (a) Describe fully a suitable sequence of transformations. Make clear the order in which the transformations are applied. [5]

.....

.....

.....

.....

.....

.....

- (b) You are given that $f(x) = a(x+b)^3 + c$.

State the values of the constants a , b and c . [3]

.....

.....

.....

.....

.....

.....

Topical questions | 96 questions

7 The function g is defined by $g(x) = \frac{2}{ax-3} + \frac{1}{2}$ for $x > \frac{3}{a}$, where a is a positive constant.

[illegible]

9709 Mathematics Paper 1 - Functions

Topical questions | 96 questions

96. Question part 1/2 | 9709 Oct-Nov 2025 P1 V5 Q10

10 The function f is defined by

$$f(x) = 3 + \frac{7}{x-2}$$

for $x > 2$.

(a) It is given that $f(a) = 4$.

Find the value of a .

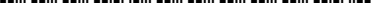
[2]

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

[illegible]

Topical questions | 96 questions


$$g(x) = \frac{1+4x}{2x-3}$$

Show that $\text{fg}(x) \equiv kx$, where k is a constant to be determined.

[3]

[illegible]