

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

1. Question | 9709 Feb-March 2016 P3 V2 Q5

5 Let $I = \int_0^{\frac{1}{2}} \frac{9}{(3+x^2)^2} dx$.

(i) Using the substitution $x = (\sqrt{3}) \tan \theta$, show that $I = \sqrt{3} \int_0^{\frac{1}{6}\pi} \cos^2 \theta d\theta$. [3]

(ii) Hence find the exact value of I . [4]

2. Question | 9709 Feb-March 2016 P3 V2 Q9

9 Let $f(x) = \frac{3x^2 + 6x - 8}{x(x^2 + 2)}$.

(i) Express $f(x)$ in the form $A + \frac{B}{x} + \frac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\int_1^2 f(x) dx = 3 - \ln 4$. [5]

3. Question | 9709 May-June 2016 P3 V1 Q2

2 Find the exact value of $\int_0^{\frac{1}{2}} xe^{-2x} dx$. [5]

4. Question | 9709 May-June 2016 P3 V2 Q3

3 Find the exact value of $\int_0^{\frac{1}{2}\pi} x^2 \sin 2x dx$. [5]

5. Question | 9709 May-June 2016 P3 V2 Q7

7 Let $f(x) = \frac{4x^2 + 7x + 4}{(2x + 1)(x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_0^4 f(x) dx = 8 - \ln 3$. [5]

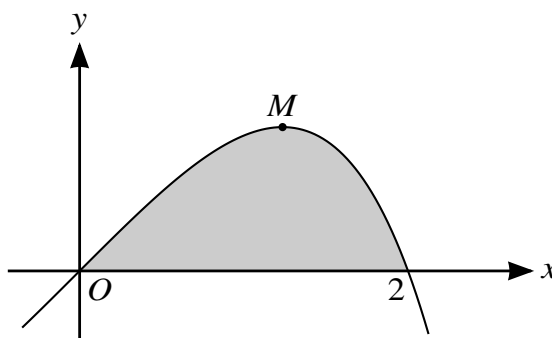
6. Question | 9709 May-June 2016 P3 V3 Q7

7 Let $I = \int_0^{\infty} \frac{x^3}{(1+x^2)^3} dx$.

(i) Using the substitution $u = 1 + x^2$, show that $I = \int_1^2 \frac{(u-1)^2}{2u^3} du$. [3]

(ii) Hence find the exact value of I . [5]

7. Question | 9709 Oct-Nov 2016 P3 V1 Q7

7

The diagram shows part of the curve $y = (2x - x^2)e^{\frac{1}{2}x}$ and its maximum point M .

(i) Find the exact x -coordinate of M . [4]

(ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x -axis. [5]

8. Question | 9709 Oct-Nov 2016 P3 V2 Q5

5 (i) Prove the identity $\tan 2\theta - \tan \theta \equiv \tan \theta \sec 2\theta$. [4]

(ii) Hence show that $\int_0^{\frac{1}{6}\pi} \tan \theta \sec 2\theta d\theta = \frac{1}{2} \ln \frac{3}{2}$. [4]

9. Question | 9709 Oct-Nov 2016 P3 V3 Q6

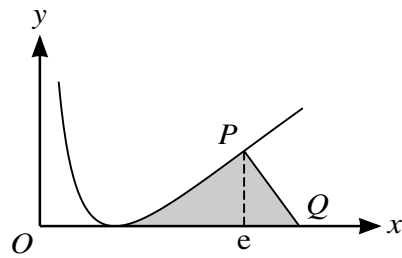
6 Let $I = \int_1^2 \frac{(\sqrt{x}) - 1}{2(x + \sqrt{x})} dx$.

(i) Using the substitution $u = \sqrt{x}$, show that $I = \int_1^2 \frac{u-1}{u+1} du$. [3]

(ii) Hence show that $I = 1 + \ln \frac{4}{9}$. [6]

10. Question part 1/2 | 9709 Feb-March 2017 P3 V2 Q10

10



The diagram shows the curve $y = (\ln x)^2$. The x -coordinate of the point P is equal to e , and the normal to the curve at P meets the x -axis at Q .

(i) Find the x -coordinate of Q . [4]

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(ii) Show that $\int \ln x \, dx = x \ln x - x + c$, where c is a constant. [1]

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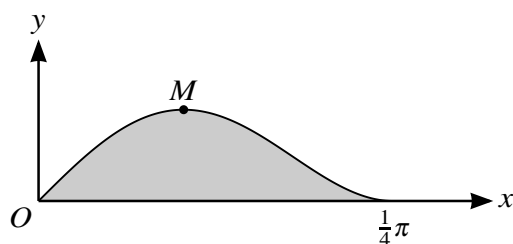
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10. Question part 2/2 | 9709 Feb-March 2017 P3 V2 Q10

- (iii) Using integration by parts, or otherwise, find the exact value of the area of the shaded region between the curve, the x -axis and the normal PQ . [5]

[illegible]



(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x -axis. [6]

[illegible]

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11. Question part 2/2 | 9709 May-June 2017 P3 V1 Q10

(ii) Find the x -coordinate of M . Give your answer correct to 2 decimal places.

[6]

[illegible]

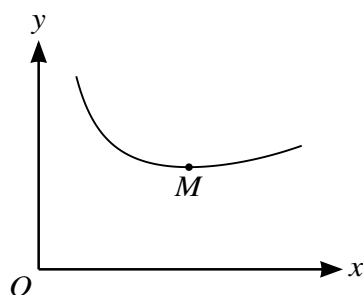
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12. Question | 9709 May-June 2017 P3 V3 Q4

4 Find the exact value of $\int_0^{\frac{\pi}{2}} \theta \sin \frac{1}{2} \theta \, d\theta$. [4]

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(i) Find the x -coordinate of M .

[4]

[illegible]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_1^3 \frac{e^{\frac{1}{2}x}}{x} \, dx,$$

giving your answer correct to 2 decimal places. [3]

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(iii) The estimate found in part (ii) is denoted by E . Explain, without further calculation, whether another estimate found using the trapezium rule with four intervals would be greater than E or less than E . [1]

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14. Question part 1/2 | 9709 May-June 2017 P3 V3 Q9

9 Let $f(x) = \frac{3x^2 - 4}{x^2(3x + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

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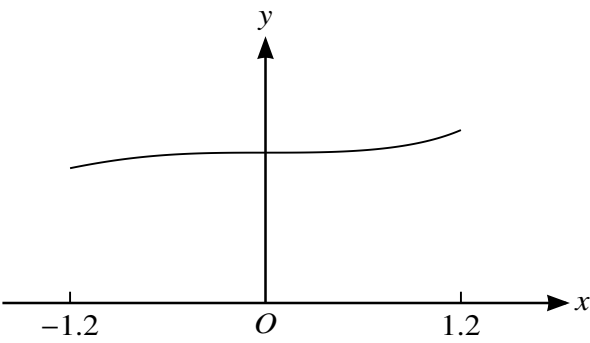
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14. Question part 2/2 | 9709 May-June 2017 P3 V3 Q9

(ii) Hence show that $\int_1^2 f(x) \, dx = \ln\left(\frac{25}{8}\right) - 1$. [5]

[illegible]

1



The diagram shows a sketch of the curve $y = \frac{3}{\sqrt{9 - x^3}}$ for values of x from -1.2 to 1.2 .

(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-1.2}^{1.2} \frac{3}{\sqrt{9 - x^3}} \, dx,$$

giving your answer correct to 2 decimal places. [3]

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(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

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16. Question part 1/2 | 9709 Oct-Nov 2017 P3 V2 Q9

9 It is given that $\int_1^a x^{\frac{1}{2}} \ln x \, dx = 2$, where $a > 1$.

(i) Show that $a^{\frac{3}{2}} = \frac{7 + 2a^{\frac{3}{2}}}{3 \ln a}$. [5]

[illegible]

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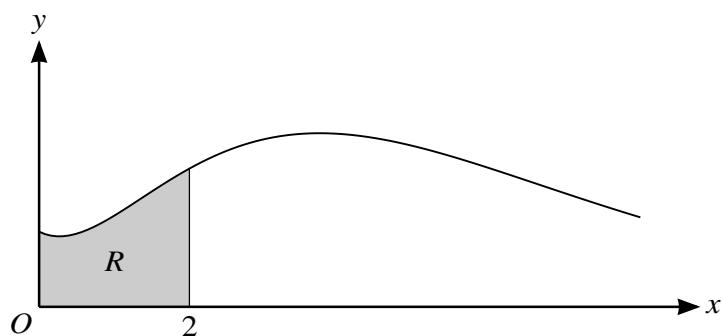
(ii) Show by calculation that a lies between 2 and 4.

[2]

[illegible]
$$a_{n+1} = \left(\frac{7 + 2a_n^{\frac{3}{2}}}{3 \ln a_n} \right)^{\frac{2}{3}}$$

[3]

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(i) Find the exact values of the x -coordinates of the stationary points of the curve. [4]

[illegible]

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17. Question part 2/2 | 9709 Oct-Nov 2017 P3 V3 Q9

(ii) Show that the exact value of the area of R is $18 - \frac{42}{e}$. [5]

[illegible]

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18. Question | 9709 Feb-March 2018 P3 V2 Q1

1 Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{1 - \tan x} \, dx,$$

giving your answer correct to 3 decimal places.

[3]

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19. Question part 1/2 | 9709 May-June 2018 P3 V1 Q5

5 Let $I = \int_{\frac{1}{4}}^4 \sqrt{\left(\frac{x}{1-x}\right)} dx$.

(i) Using the substitution $x = \cos^2 \theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2 \cos^2 \theta \, d\theta$. [4]

[illegible]

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19. Question part 2/2 | 9709 May-June 2018 P3 V1 Q5

(ii) Hence find the exact value of I .

[4]

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8 The positive constant a is such that $\int_0^a x e^{-\frac{1}{2}x} dx = 2$.

[illegible]

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(ii) Verify by calculation that a lies between 3 and 3.5. [2]

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(iii) Use an iteration based on the equation in part **(i)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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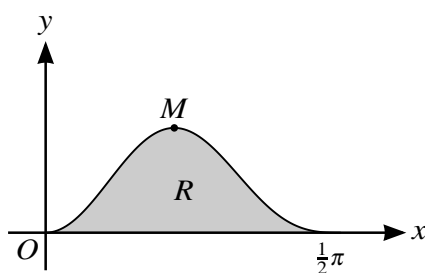
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21. Question | 9709 May-June 2018 P3 V3 Q3

3 Showing all necessary working, find the value of $\int_0^6 x \cos 3x \, dx$, giving your answer in terms of π . [5]

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(i) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [5]

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22. Question part 2/2 | 9709 Oct-Nov 2018 P3 V1 Q7

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R . [4]

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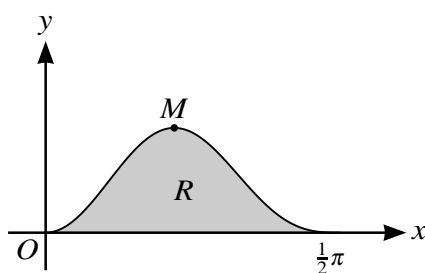
3

(i)

[3]

(ii) Hence show that $\int_1^2 \frac{\ln x}{x^3} dx = \frac{1}{16}(3 - \ln 4)$.

[2]



(i) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [5]

[illegible]

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24. Question part 2/2 | 9709 Oct-Nov 2018 P3 V3 Q7

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R . [4]

[illegible]

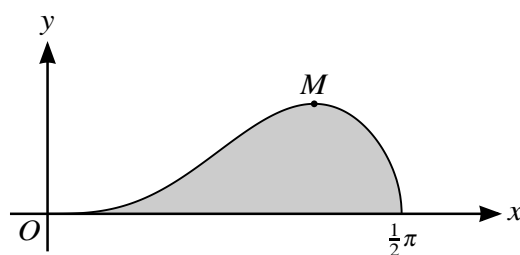
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25. Question | 9709 Feb-March 2019 P3 V2 Q4

4 Show that $\int_1^4 x^{-\frac{3}{2}} \ln x \, dx = 2 - \ln 4$. [5]

[illegible]



(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x -axis. [6]

[illegible]

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26. Question part 2/3 | 9709 Feb-March 2019 P3 V2 Q10

- (ii) Showing all your working, find the x -coordinate of M , giving your answer correct to 3 decimal places. [6]

[illegible]

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26. Question part 3/3 | 9709 Feb-March 2019 P3 V2 Q10

Additional Page

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27. Question | 9709 May-June 2019 P3 V1 Q1

- 1** Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 |2^x - 4| dx. \quad [3]$$

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28. Question part 1/2 | 9709 May-June 2019 P3 V2 Q8

8 Let $f(x) = \frac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

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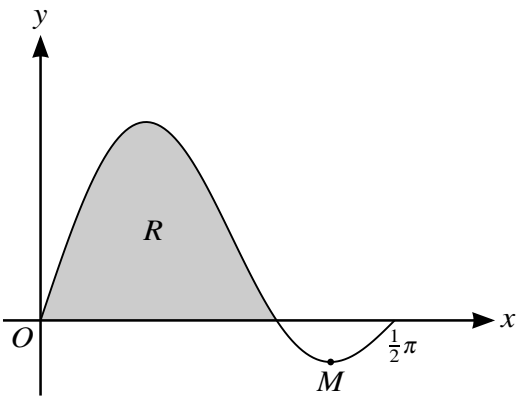
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28. Question part 2/2 | 9709 May-June 2019 P3 V2 Q8

(ii) Hence show that $\int_0^1 f(x) \, dx = \frac{1}{2} \ln \frac{9}{5} + \frac{1}{5}$. [5]

[illegible]

10



The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leq x \leq \frac{1}{2}\pi$ and its minimum point M . The shaded region R is bounded by the curve and the x -axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \frac{1}{2}(\sin 4x + \sin 2x).$$
 [3]

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(ii) Using the result of part (i) and showing all necessary working, find the exact area of the region R . [4]

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29. Question part 2/3 | 9709 May-June 2019 P3 V2 Q10

[illegible]

- (iii) Using the result of part (i), express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x -coordinate of M , giving your answer correct to 2 decimal places. [5]

[illegible]

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30. Question | 9709 May-June 2019 P3 V3 Q2

2 Show that $\int_0^{\frac{3}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

[illegible]

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[2]

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[6]

[illegible]

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31. Question part 2/2 | 9709 Oct-Nov 2019 P3 V1 Q6

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

32. Question part 1/2 | 9709 Oct-Nov 2019 P3 V2 Q8

8 Let $f(x) = \frac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

[illegible]

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32. Question part 2/2 | 9709 Oct-Nov 2019 P3 V2 Q8

(ii) Hence, showing full working, find $\int_1^5 f(x) \, dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

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9 It is given that $\int_0^a x \cos \frac{1}{3}x \, dx = 3$, where the constant a is such that $0 < a < \frac{3}{2}\pi$.

$$a = \frac{4 - 3 \cos \frac{1}{3}a}{\sin \frac{1}{3}a}. \quad [5]$$
[illegible]

Topical questions | 70 questions

(ii) Verify by calculation that a lies between 2.5 and 3.

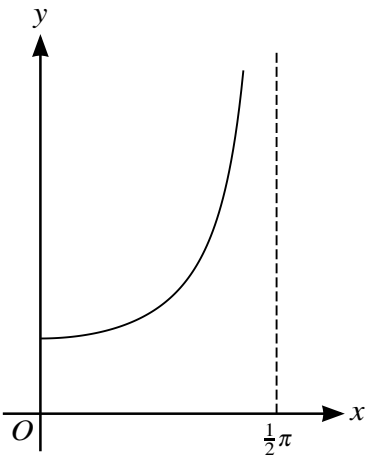
[2]

[illegible]

- [3]

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8



The diagram shows the graph of $y = \sec x$ for $0 \leq x < \frac{1}{2}\pi$.

- (i) Use the trapezium rule with 2 intervals to estimate the value of $\int_0^{1.2} \sec x \, dx$, giving your answer correct to 2 decimal places. [3]

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- (ii) Explain, with reference to the diagram, whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part (i). [1]

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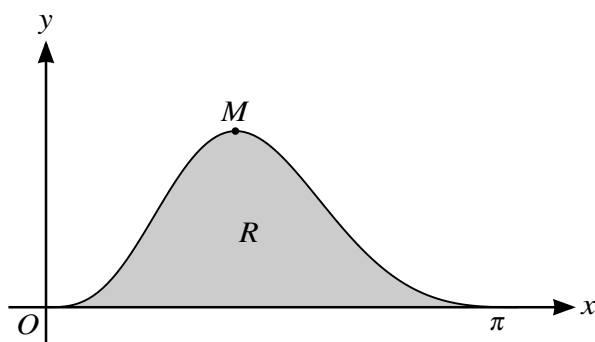
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34. Question part 2/2 | 9709 Oct-Nov 2019 P3 V3 Q8

(iii) P is the point on the part of the curve $y = \sec x$ for $0 \leq x < \frac{1}{2}\pi$ at which the gradient is 2. By first differentiating $\frac{1}{\cos x}$, find the x -coordinate of P , giving your answer correct to 3 decimal places.

[6]

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(i) Find the x -coordinate of M . Show all necessary working and give your answer correct to 2 decimal places. [5]

[illegible]

Topical questions | 70 questions

(ii) By first using the substitution $u = \cos x$, find the exact value of the area of R . [7]

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[illegible]

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36. Question | 9709 Feb-March 2020 P3 V2 Q4

4 Find $\int_{\frac{1}{6}\pi}^{\frac{5}{3}\pi} x \sec^2 x \, dx$. Give your answer in a simplified exact form. [7]

[illegible]

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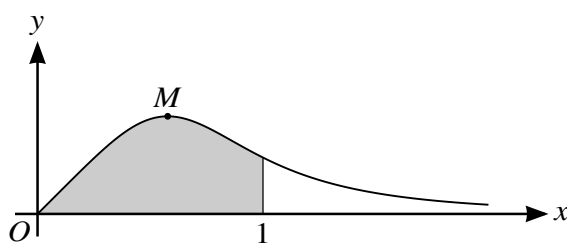
37. Question | 9709 May-June 2020 P3 V2 Q3

3 Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$

[5]

[illegible]



(a) Find the x -coordinate of M , giving your answer correct to 3 decimal places. [4]

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38. Question part 2/2 | 9709 May-June 2020 P3 V2 Q6

- (b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 1$. [5]

[illegible]

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39. Question | 9709 May-June 2020 P3 V3 Q2

2 Find the exact value of $\int_0^1 (2-x)e^{-2x} dx$. [5]

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Topical questions | 70 questions

7 Let $f(x) = \frac{2}{(2x-1)(2x+1)}$.

[illegible]
$$(f(x))^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}. \quad [2]$$
[illegible]

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40. Question part 2/2 | 9709 May-June 2020 P3 V3 Q7

(c) Hence show that $\int_1^2 (f(x))^2 dx = \frac{2}{5} + \frac{1}{2} \ln\left(\frac{5}{9}\right)$. [5]

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41. Question part 1/2 | 9709 Oct-Nov 2020 P3 V2 Q9

9 Let $f(x) = \frac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions.

[5]

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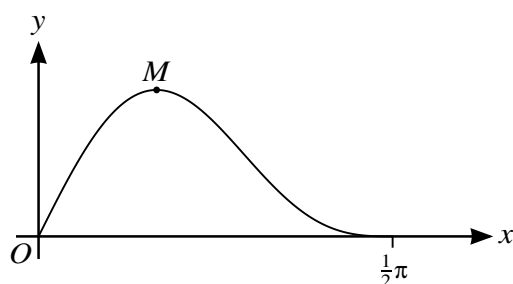
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41. Question part 2/2 | 9709 Oct-Nov 2020 P3 V2 Q9

(b) Hence find the exact value of $\int_0^2 f(x) \, dx$.

[6]

[illegible]



(a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x -axis. [5]

[illegible]

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42. Question part 2/3 | 9709 Feb-March 2021 P3 V2 Q10

(b) Find the exact x -coordinate of M .

[6]

[illegible]

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43. Question | 9709 May-June 2021 P3 V2 Q4

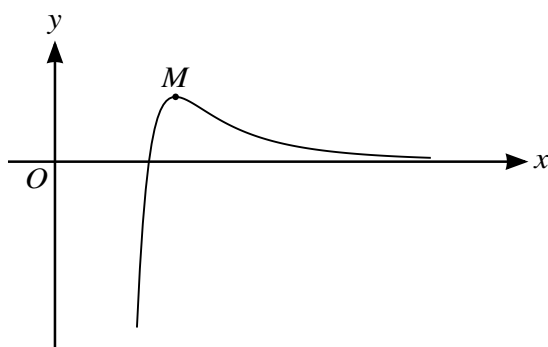
4

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Topical questions | 70 questions

4 Let $f(x) = \frac{15 - 6x}{(1 + 2x)(4 - x)}$.

[illegible][illegible]



(a) Find the exact coordinates of M . [4]

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9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

45. Question part 2/2 | 9709 May-June 2021 P3 V3 Q8

(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4} dx < \frac{1}{9}$. [6]

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

46. Question | 9709 Oct-Nov 2021 P3 V1 Q4

4 Using the substitution $u = \sqrt{x}$, find the exact value of

$$\int_3^{\infty} \frac{1}{(x+1)\sqrt{x}} dx. \quad [6]$$

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Topical questions | 70 questions

8 The constant a is such that $\int_1^a \frac{\ln x}{\sqrt{x}} dx = 6$.

[$\exp(x)$ is an alternative notation for e^x .]

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- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

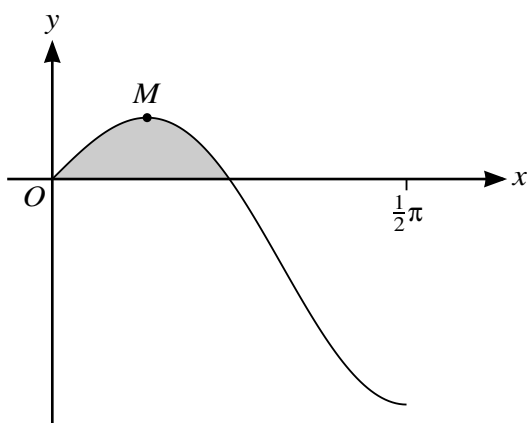
9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

48. Question | 9709 Oct-Nov 2021 P3 V3 Q4

4 Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \frac{1}{2}x \, dx$. [5]

[illegible]



(a) Find the x -coordinate of M , giving your answer correct to 3 significant figures. [6]

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

49. Question part 2/3 | 9709 Feb-March 2022 P3 V2 Q11

- (b) Using the substitution $u = \cos x$, find the area of the shaded region enclosed by the curve and the x -axis in the first quadrant, giving your answer in a simplified exact form. [5]

[illegible]

Topical questions | 70 questions

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Topical questions | 70 questions

6 Let $I = \int_0^1 \frac{27}{(9+x^2)^2} dx$.

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

50. Question part 2/2 | 9709 May-June 2022 P3 V1 Q6

(b) Hence find the exact value of I .

[4]

[illegible]

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Topical questions | 70 questions

51. Question part 1/3 | 9709 May-June 2022 P3 V3 Q10

10 The constant a is such that $\int_1^a x^2 \ln x \, dx = 4$.

(a) Show that $a = \left(\frac{35}{3 \ln a - 1} \right)^{\frac{1}{3}}$. [5]

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Topical questions | 70 questions

(b) Verify by calculation that a lies between 2.4 and 2.8.

[2]

[illegible]

- [3]

[illegible]

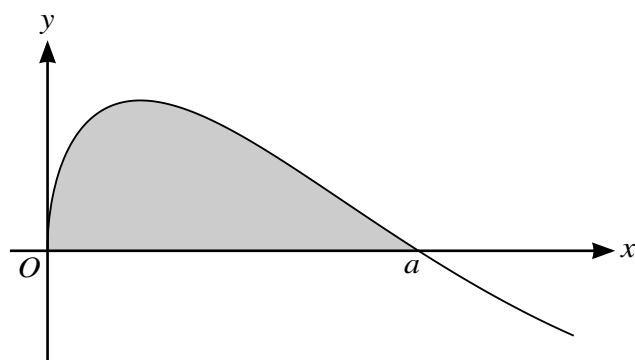
Topical questions | 70 questions

51. Question part 3/3 | 9709 May-June 2022 P3 V3 Q10

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

[illegible]

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(a) State the exact value of a . [1]

.....

.....

.....

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9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

52. Question part 2/2 | 9709 Oct-Nov 2022 P3 V2 Q8

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Topical questions | 70 questions

53. Question | 9709 Oct-Nov 2022 P3 V3 Q3

3

Find the exact value of $\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx$.

[5]

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Topical questions | 70 questions

9 The constant a is such that $\int_0^a x e^{-2x} dx = \frac{1}{8}$.

[illegible]

Topical questions | 70 questions

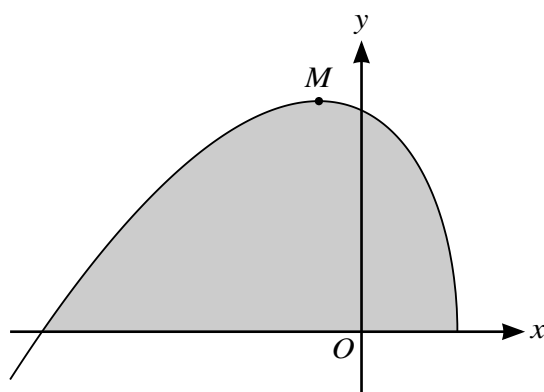
54. Question part 2/2 | 9709 May-June 2023 P3 V1 Q9

[2]

[illegible]

[3]

[illegible]



(a) Find the exact coordinates of M .

[5]

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9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

55. Question part 2/2 | 9709 May-June 2023 P3 V2 Q10

- (b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x -axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

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Topical questions | 70 questions

7 (a) Use the substitution $u = \cos x$ to show that

$$\int_0^\pi \sin 2x e^{2\cos x} dx = \int_{-1}^1 2ue^{2u} du. \quad [4]$$

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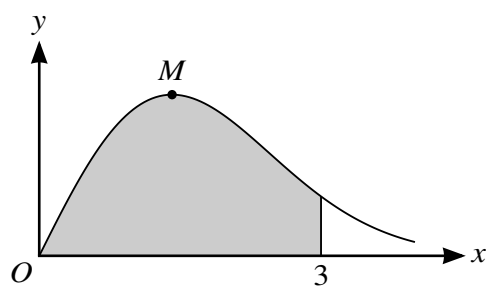
9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

56. Question part 2/2 | 9709 May-June 2023 P3 V3 Q7

(b) Hence find the exact value of $\int_0^\pi \sin 2x e^{2 \cos x} dx$. [4]

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(a) Find the exact coordinates of M . [4]

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

57. Question part 2/2 | 9709 Oct-Nov 2023 P3 V1 Q9

(b) Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 3$. [5]

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

58. Question | 9709 Oct-Nov 2023 P3 V2 Q5

5 Find the exact value of $\int_0^1 \frac{x(x+1)}{x^2+4} dx$. [6]

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Topical questions | 70 questions

8 Use the substitution $u = 1 - \sin x$ to find the exact value of

Give your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

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Topical questions | 70 questions

59. Question part 2/2 | 9709 May-June 2024 P3 V1 Q8

[illegible]

Topical questions | 70 questions

10 (a) Given that $2x = \tan y$, show that $\frac{dy}{dx} = \frac{2}{1+4x^2}$. [3]

[illegible]

(b) Hence find the exact value of $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x \tan^{-1}(2x) dx$. [7]

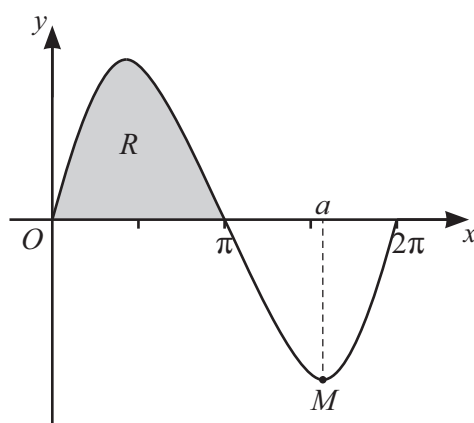
[illegible]

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Topical questions | 70 questions

60. Question part 2/2 | 9709 May-June 2024 P3 V1 Q10

[illegible]



(a) Find the value of a correct to 2 decimal places.

[5]

[illegible]

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Topical questions | 70 questions

61. Question part 2/3 | 9709 Oct-Nov 2024 P3 V3 Q11



(b) Use the substitution $u = 2 + \cos x$ to find the exact area of the shaded region R .

[6]

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Topical questions | 70 questions

61. Question part 3/3 | 9709 Oct-Nov 2024 P3 V3 Q11



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Topical questions | 70 questions

62. Question part 1/3 | 9709 Feb-March 2025 P3 V2 Q11

11 Find the exact value of $\int_0^\pi x^2 \cos \frac{1}{3}x \, dx$.

[6]

[illegible]

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Topical questions | 70 questions

62. Question part 2/3 | 9709 Feb-March 2025 P3 V2 Q11

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[illegible]

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9 The constant a is such that $\int_1^a 6x \ln x \, dx = 4$.

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Topical questions | 70 questions

63. Question part 2/2 | 9709 May-June 2025 P3 V1 Q9



- (b)** Verify by calculation that a lies between 2 and 2.1.

[2]

[illegible]

- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[3]

[illegible]

Topical questions | 70 questions

10 (a) Find the quotient and remainder when x^2 is divided by $1 + 4x^2$. [2]

[illegible][illegible]

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Topical questions | 70 questions

64. Question part 2/2 | 9709 May-June 2025 P3 V2 Q10

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

65. Question | 9709 May-June 2025 P3 V3 Q3

3 Find the exact value of $\int_{\frac{1}{5}\pi}^{\frac{4}{5}\pi} 3 \cos^2 5x \, dx$. [4]

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Topical questions | 70 questions

66. Question | 9709 Oct-Nov 2025 P3 V1 Q1

- 1 Find the exact value of $\int_1^2 \ln 3x \, dx$. Give your answer in the form $a + \ln b$, where a and b are integers. [4]

[illegible]

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Topical questions | 70 questions

67. Question | 9709 Oct-Nov 2025 P3 V2 Q4

4 Find the exact value of $\int_0^1 x \tan^{-1} x \, dx$. [6]

[illegible]

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Topical questions | 70 questions

68. Question part 1/2 | 9709 Oct-Nov 2025 P3 V3 Q6

6 Find the exact value of $\int_0^{\frac{\pi}{6}} x^2 \sin 2x \, dx$. [6]

[illegible]

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Topical questions | 70 questions

68. Question part 2/2 | 9709 Oct-Nov 2025 P3 V3 Q6

[illegible]

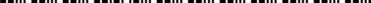
Topical questions | 70 questions

7 (a) Use the substitution $u = x^2 - 3$ to show that

where a and b are values to be found. [4]

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Topical questions | 70 questions



[4]

[illegible]

9709 Mathematics Paper 3 - Integration

Topical questions | 70 questions

70. Question part 1/2 | 9709 Oct-Nov 2025 P3 V5 Q10

10 The constant a is such that $\int_0^a x e^{\frac{1}{2}x} dx = 6$.

(a) Show that $a = 2 + e^{-\frac{1}{2}a}$.

[5]

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Topical questions | 70 questions

70. Question part 2/2 | 9709 Oct-Nov 2025 P3 V5 Q10



- (b)** Verify by calculation that a lies between 2.2 and 2.4.

[2]

[illegible]

- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.