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FBISE Mathematics Class 11
Topical workbook - Volume 2 of 3

140 questions arranged by official framework topic.
National Curriculum 2022-23; supported examinations from 2027.

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Mathematical Induction and Binomial Theorem

Domain A: Numbers and Algebra

FBISE model paper | Section B Q2(ix) | 4 marks

ix.	Find the last two digits of a number $(23)^{14}$	4

FBISE model paper | Section B Q2(xi) | 4 marks

xi.	Use Binomial Theorem to find the remainder when 5^{99} is divided by 13.	4

FBISE model paper | Section B Q2(xii) OR | 4 marks

OR	Apply the principle of Mathematical Induction to prove that $7^{2n} + 7$ is divisible by 8 for all positive integral values of n.	4

First Annual 2025 | Section B Q2(xi) OR | 4 marks

OR	By applying mathematical induction prove that $a+b$ is a factor of $a^n - b^n$ for all positive integer n.	**04**

Q.5	If x^4 and higher powers are neglected such that $(1+x)^{\frac{1}{4}}+(1-x)^{\frac{1}{4}}=a-bx^2$ find a and b.	08

Second Annual 2025 | Section B Q2(viii) OR | 4 marks

OR	Find the coefficient of x^3 in the binomial expansion of $(1+x)^5$.	**04**

Second Annual 2025 | Section B Q2(xii) OR | 4 marks

04	**OR**	Verify by Mathematical Induction: $1+2+3+\cdots+n=\frac{n(n+1)}{2}$	**04**

First Annual 2026 | Section B Q2(iv) OR | 4 marks

OR	Find the term independent of x in the expansion of $\left(\frac{a}{x}+\frac{x}{a}\right)^{10}$	04

First Annual 2026 | Section B Q2(vii) | 4 marks

(vii)	Prove by mathematical induction $3+7+11+....+(4n-1)=n(2n+1)$	04

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Division of polynomial

Domain A: Numbers and Algebra

FBISE model paper | Section A Q15 | 0.75 marks

15.	When dividing $x^3 + 2x^2 - 5x + 1$ by $x - 2$, the degree of the remainder is:	0	1	2	7

FBISE model paper | Section A Q16 | 0.75 marks

16.	Dividing $x^4 - 3x^2 + 2$ by $x^2 - 1$, the quotient is:	$x^2 - 2$	$x^2 + 2$	$x^2 - 1$	$x^2 + 1$

FBISE model paper | Section A Q17 | 0.75 marks

17.	For what value of k, $x - 2$ is the factor of $x^3 - kx^2 + 4x - 4$?	-3	0	3	4

FBISE model paper | Section B Q3 OR | 4 marks

OR	Divide $3x^4 - 2x^3 + 4x^2 - x + 5$ by $x^2 - x + 1$. Find the quotient and remainder.	4

First Annual 2025 | Section B Q2(iv) | 4 marks

If $P(x) = x^4 - 6x^3 + 11x^2 - 6x$, then (a) Divide ***P(x)*** by ***(x–1)*** using synthetic division (b) Solve the resulting depressed equation. Without drawing graph find range, amplitude, period	**04**

First Annual 2025 | Section B Q2(xi) OR | 4 marks

sunlight received.	
If $P(x) = ax^3 + bx^2 + 2x - 1$, leaves remainder 5 when divided by $(x-1)$ and leaves remainder 3 when divided by $(x+1)$. Find values of ***a*** and ***b*** using remainder theorem.	**04**

If $P(x) = 2x^3 + x^2 - 13x + 6$, then **a)** Verify that $(x + 3)$ is a factor of $P(x)$. **b)** Factorize $P(x)$ using synthetic division. **c)** Find all the roots of $P(x)$. **d)** Write complete factorized form of $P(x)$ and confirm by expanding.	**08**

First Annual 2025 | Section B Q2(iv) | 4 marks

(iv)	If $P(x)=3x^3-5x^2-16x+12$, then **(a)** Use factor theorem to verify that $(x+2)$ is a factor of $P(x)$. **(b)** Apply synthetic division to factorize $P(x)$ completely.

First Annual 2025 | Section B Q2(xi) OR | 4 marks

sandwich each day without repetition?

A checksum polynomial $C(x)=x^4+px^2+qx+6$ is used to detect errors in transmitted data.

(a) If $C(1)=10$ and $C(-1)=4$, set equations for p and q.

(b) Solve the equations to find values of p and q.

Use binomial theorem to approximate the value of

Q.3	A polynomial $P(x) = x^4 + ax^3 - 5x^2 + bx + c$ **(a)** Leaves remainder 0 when divided by $(x - 0)$ **(b)** Leaves remainder –3 when divided by $(x - 2)$ **(c)** Leaves remainder –4 when divided by $(x + 1)$ Use remainder theorem to find values of a, b and c	**08**

Second Annual 2025 | Section B Q2(vii) OR | 4 marks

OR	Use factor theorem to factorize $x^3 - 3x^2 + 2x$	**04**

Second Annual 2025 | Section B Q2(viii) | 4 marks

(viii)	Divide $x^3 + 6x^2 + 11x + 10$ *by* $x + 2$ and find quotient and remainder.	**04**

First Annual 2026 | Section B Q2(iv) OR | 4 marks

OR	Find all roots of: $x^3-3x^2-4x+12$ by Factor theorem.	**04**

First Annual 2026 | Section B Q2(ix) | 4 marks

(ix)	Determine whether $(x+2)$ is a factor of: $f(x)=x^3+2x^2-4x-8$	**04**

First Annual 2026 | Section B Q2(iv) | 4 marks

(iv)	Find the value(s) of K if $x-1$ is a factor of: $f(x)=x^3+K^2-2K+1$	**04**

First Annual 2026 | Section B Q2(xi) OR | 4 marks

Factorize the polynomial: x^3-4x^2+x+6	**04**

First Annual 2026 | Section C Q3 | 8 marks

a.	A company's profit over the last few years can be modeled by the polynomial $P(x)=x^3+6x^2-x-6$. If x represents years since 2010, calculate the company's profit in 2012 by dividing $P(x)$ and $x-2$ finding the remainder.	08
b.	Verify your answer by long division.	
	Find the term independent of x in the expansion of	

FBISE model paper | Section A Q1(ix) | 1 mark

ix.	$\frac{x^2+3x-16}{x+4} = x-4-\frac{?}{x+4}$	3	-3	$3x$	$-3x$

FBISE model paper | Section A Q1(x) | 1 mark

x.	If length of one side of the box having volume x^3-2x^2-x+2 is $(x-2)$, then the remaining two sides are:	$(x-1)^2$	$(x+1)^2$	x^2-1	x^2+1

FBISE model paper | Section B Q2(iv) | 4 marks

iv.	If $h(x) = 7x^4 - 10x^3 + 3x^2 + 3x - 3$ and one zero of $h(x)$ is 1, then find remaining zeros.	4

FBISE model paper | Section B Q2(xi) OR | 4 marks

OR	Find roots of the cubic polynomial $P(x) = 3x^3 - 5x^2 - 11x - 3$	4

Q3	(a) Factorize $x^3 - x^2 + 4x - 12$ (b) Solve $x^3 - x^2 + 4x - 12 = 0$ and identify real and complex roots.	8

OR	Check whether $1+2i$ is the factor of polynomial $f(x)=x^2-2x+5$	04

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Vectors in Space

Domain B: Geometry

FBISE model paper | Section A Q18 | 0.75 marks

18.	Two direction angles of vector $\vec{a}$ are 30° and 60°. What is the value of third direction angle?	135°	90°	45°	30°

FBISE model paper | Section A Q19 | 0.75 marks

19.	The vector parallel to $2\hat{\imath} + 6\hat{\jmath} - 5\hat{k}$ and with magnitude 3 is:	$\left[\frac{12}{\sqrt{65}}, \frac{-18}{\sqrt{65}}, \frac{75}{\sqrt{65}}\right]$	$\left[\frac{12}{\sqrt{65}}, \frac{18}{\sqrt{65}}, \frac{25}{\sqrt{65}}\right]$	$\left[\frac{6}{\sqrt{65}}, \frac{18}{\sqrt{65}}, \frac{-15}{\sqrt{65}}\right]$	$\left[\frac{6}{\sqrt{65}}, \frac{-18}{\sqrt{65}}, \frac{8}{\sqrt{65}}\right]$

FBISE model paper | Section A Q20 | 0.75 marks

20.	The area of a parallelogram having adjacent sides $\vec{a} = [2, 5, -2]$ and $\vec{b} = [1, 4, -2]$ is:	$\sqrt{17}$	$\frac{\sqrt{17}}{2}$	$\frac{\sqrt{17}}{6}$	$2\sqrt{17}$

FBISE model paper | Section A Q21 | 0.75 marks

21.	At which angle the work done is zero?	0°	45°	90°	180°

FBISE model paper | Section A Q22 | 0.75 marks

22.	In navigation the velocity of the air-plane is calculated using:	Dot product	Vector addition	Vector subtraction	Cross product

FBISE model paper | Section A Q23 | 0.75 marks

23.	If $\vec{a} = \hat{\imath} + \hat{2}\jmath - 2\hat{k}$ and $\vec{b} = \hat{2}\imath + 3\hat{\jmath} - 3\hat{k}$ then $\hat{\imath}.\vec{a} + \hat{\jmath}.\vec{b} =$?	1	-2	2	4

FBISE model paper | Section A Q53 | 0.75 marks

Scenario for MCQs (53-56)	**A drone is flying in space. Its motion is represented by the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$. An engineer wants to analyze the relationship between these two directions using dot and cross products.**

53.	The engineer computes $\vec{a}\cdot\vec{b}$. What does this quantity help to determine?	The area of parallelogram formed by the vectors	The angle between the vectors	Law of sines	The area of a triangle

FBISE model paper | Section A Q54 | 0.75 marks

Scenario for MCQs (53-56)	**A drone is flying in space. Its motion is represented by the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$. An engineer wants to analyze the relationship between these two directions using dot and cross products.**

54.	In terms of vectors $\vec{a}$ and $\vec{b}$, how would you express the work done on the drone?	$\lvert\vec{a}\rvert + \lvert\vec{b}\rvert$	$\lvert\vec{a} - \vec{b}\rvert$	$\vec{a} \times \vec{b}$	$\vec{a}.\vec{b}$

FBISE model paper | Section A Q55 | 0.75 marks

Scenario for MCQs (53-56)	**A drone is flying in space. Its motion is represented by the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$. An engineer wants to analyze the relationship between these two directions using dot and cross products.**

55.	If $\vec{a}\times\vec{b} = 0$. What can be concluded about the drone's angle of direction?	0°	45°	60°	90°

FBISE model paper | Section A Q56 | 0.75 marks

Scenario for MCQs (53-56)	**A drone is flying in space. Its motion is represented by the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$. An engineer wants to analyze the relationship between these two directions using dot and cross products.**

56.	An engineer wants to position a communication antenna perpendicular to both $\vec{a}$ and $\vec{b}$. Which vector gives its orientation?	$-3\hat{\imath} + 3\hat{k}$	$3\hat{\imath} + 3\hat{k}$	$-3\hat{\imath} - 3\hat{k}$	$3\hat{\imath} - 3\hat{k}$

FBISE model paper | Section B Q3 | 4 marks

3.	If the vectors $\vec{u} = p\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{v} = \hat{\imath} + q\hat{\jmath} + \hat{k}$ and $\vec{w} = \hat{\imath} + \hat{\jmath} + r\hat{k}$ are coplanar, then prove that $\frac{1}{1-p} + \frac{1}{1-q} + \frac{1}{1-r} = 1$	4

OR	Express the vector $\vec{a} = 3\hat{\imath} + 7\hat{\jmath} + 2\hat{k}$ as a sum of two vectors one of which is parallel and other is perpendicular to the vector $\vec{b} = 5\hat{\imath} + \hat{\jmath} + 8\hat{k}$.	4

Q10	If $\vec{a} = 3\underline{i} - \underline{j} + 5\underline{k}$, $\vec{b} = 4\underline{i} + 3\underline{j} - 2\underline{k}$ and $\vec{c} = 2\underline{i} + 5\underline{j} + \underline{k}$, then verify that $\vec{a}.\vec{b} \times \vec{c} = \vec{b}.\vec{c} \times \vec{a} = \vec{c}.\vec{a} \times \vec{b}$.	8

First Annual 2025 | Section B Q2(i) OR | 4 marks

A triangular park has its three corners located at points **A (2, 3, 1)**, **B (5, 7, 2)**, and **C (1, 4, 3)**. Find area of the park using cross product of vectors.	**04**

First Annual 2025 | Section B Q2(iii) | 4 marks

Find the value of p such that vectors $\hat{i}+2\hat{j}+p\hat{k}$, $3\hat{i}+p\hat{j}+4\hat{k}$ and $2\hat{i}+3\hat{j}+4\hat{k}$ are coplanar.	**04**

A force $\vec{F} = 6\hat{i} + 8\hat{j} + 4\hat{k}$ acts on an object. The object moves from A (1,2,3) to point B(4,6,5). (a) Find the displacement vector $\vec{d}$. (b) Calculate work done by the force $\vec{F}$. Solve $x + y + z = 3$; $y + z = 2$; $z = 2$ using Cramer's	04

If $\vec{a} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, then (a) Find $\vec{a} \times \vec{b}$. (b) Find $\left(\vec{a} \times \vec{b}\right) \cdot \vec{a}$ and verify that $\vec{a} \times \vec{b} \perp \vec{a}$. (c) Find $\left(\vec{a} \times \vec{b}\right) \cdot \vec{b}$ and verify that $\vec{a} \times \vec{b} \perp \vec{b}$	**08**

First Annual 2025 | Section B Q2(i) OR | 4 marks

A mechanic applies a force $\vec{F} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ at point $A\ (14, -3, 6)$ to turn a bolt with pivot at $P\ (2, -2, 1)$. Find:

(a) the moment arm $\vec{r}$ and

(b) the moment vector $\vec{M}$ about the bolt.

First Annual 2025 | Section B Q2(iii) | 4 marks

(iii)	Find the value of λ such that vectors $\hat{i} + 4\hat{j} + 2\hat{k}$, $2\hat{i} - \hat{j} + 3\hat{k}$ and $\lambda\hat{i} + 3\hat{j} + 5\hat{k}$ are coplanar.	**04**

First Annual 2025 | Section B Q2(v) OR | 4 marks

A force $\vec{F} = 5\hat{i} + 3\hat{j} + 4\hat{k}$ is applied to move an object from $A(2,3,1)$ to $B(5,7,4)$. Find work done by the force.

First Annual 2025 | Section B Q2(viii) OR | 4 marks

A boat is sailing north at 10 km/h but is also being pushed east by ocean currents at 5 km/h. Find:

(a) actual speed (V) of the boat.

(b) direction angle (θ) of the boat relative to due north.

If $\vec{a} = 5\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + 6\hat{j} + 4\hat{k}$, then **(a)** Find $\vec{a} \times \vec{b}$ **(b)** Find $\left(\vec{a} \times \vec{b}\right) \cdot \vec{a}$ and verify that $\vec{a} \times \vec{b} \perp \vec{a}$ **(c)** Find $\left(\vec{a} \times \vec{b}\right) \cdot \vec{b}$ and verify that $\vec{a} \times \vec{b} \perp \vec{b}$	**08**

Second Annual 2025 | Section B Q2(iii) OR | 4 marks

inverse of given matrix using row operations. $\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$	04	OR	Find the value of m if vectors $\underline{a} = \hat{\imath} + m\hat{\jmath} + 2\hat{k}$, $\underline{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$, $\underline{c} = 3\hat{\imath} + 2\hat{\jmath} + \hat{k}$ are coplanar.	04

Second Annual 2025 | Section B Q2(ix) OR | 4 marks

OR	If $\underline{a} = 5\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\underline{b} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, then find projection of $\underline{a}$ along $\underline{b}$ and projection of $\underline{b}$ along $\underline{a}$.	04

A geometric sequence has first term $a = 3$ and

OR	Apply cross product to find angle between two vectors $\underline{a} = \hat{\imath}$ and $\underline{b} = \frac{1}{2}\hat{\imath} + \frac{\sqrt{3}}{2}\hat{\jmath}$	**04**

The vertices of a tetrahedron are $A(1,2,3), B(2,31), C(3,1,2)$ and $D(4,2,1)$ **a.** Find volume of the tetrahedron $ABCD$ **b.** Using same vectors, find the volume of the parallelepiped formed by $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$.	**08**

First Annual 2026 | Section B Q2(vi) OR | 4 marks

OR	A force $\overline{F} = 3i - 2j + k$ acts at point $(1,2,3)$. Find the moment about the origin.	04

First Annual 2026 | Section B Q2(x) OR | 4 marks

OR	Find the value of "λ" so that vectors $\vec{v} = \hat{i} - \hat{j} + \hat{k}$, $\vec{u} = 2\hat{i} + \hat{j} + \hat{k}$ and $\overline{w} = -\hat{i} + \lambda\hat{j} + 2\hat{k}$ are coplanar.	04

First Annual 2026 | Section B Q2(i) OR | 4 marks

A force $\vec{F} = 2i + 3j + k$ moves an object through displacement $\vec{d} = i + 2j + 2k$. find the work done.	04

First Annual 2026 | Section B Q2(iii) | 4 marks

(iii)	Find the volume of a tetrahedron formed by vectors: $\vec{a} = 2i + j, \vec{b} = i + 3j + k, \vec{c} = j + k$	04

First Annual 2026 | Section B Q2(v) OR | 4 marks

Find the angle between vectors: $\vec{a} = 4i + j - 8k \ , \ \vec{b} = i + 4j + k$	04

FBISE model paper | Section A Q1(xi) | 1 mark

xi.	If the dot product of vector $\underline{a} = \underline{i} - 2\underline{j} + 2\underline{k}$ and $\underline{b} = \alpha\,\underline{i} - \underline{j} + 2\underline{k}$ is 10 then value of α is:	0	1	2	3

FBISE model paper | Section A Q1(xii) | 1 mark

xii.	If $\underline{a} = \underline{i} - 2\underline{j}$ and $\underline{b} = 2\underline{j} + \underline{k}$, then $\underline{a} \times \underline{b}$ is:	$-2\,\underline{i} - \underline{j} - 2\underline{k}$	$-2\,\underline{i} + \underline{j} - 2\underline{k}$	$2\,\underline{i} - \underline{j} + 2\underline{k}$	$-2\,\underline{i} - \underline{j} + 2\underline{k}$

FBISE model paper | Section A Q1(xiii) | 1 mark

xiii.	Which of the vector pairs is orthogonal?	$\underline{i} + 2\underline{j} - \underline{k}$ and $\underline{i} + j + \underline{k}$	$\underline{i} - 2\underline{j} - \underline{k}$ and $\underline{i} + j - \underline{k}$	$-\underline{i} + 2\underline{j} + k$ and $\underline{i} + j + \underline{k}$	$-\underline{i} + 2\underline{j} - \underline{k}$ and $-\underline{i} + j + \underline{k}$

FBISE model paper | Section B Q2(i) OR | 4 marks

If the angle between two vectors $\underline{a} = 2\underline{i} - 3\underline{j} + 4\underline{k}$ and $\underline{b} = \underline{i} + 2\underline{j} + 2\underline{k}$ is θ, then find the values of $cos\ \theta$ and $sin\ \theta$.	4

FBISE model paper | Section B Q2(iii) | 4 marks

iii.	Find volume of the tetrahedron if $a = 2\underline{i} - 3\underline{j} + \underline{k}$, $b = \underline{i} + 2\underline{j} - \underline{k}$ and $c = -3\underline{i} - \underline{j} + 5\underline{k}$ are its coterminous edges.	4

A force $\vec{F} = 3\underline{i} - 2\underline{j} + 5\underline{k}$ acts on a particle at point $P\ (3, -4, 2)$. Find moment of the force about origin and a point $(1, -1, -1)$.	4

If $\underline{a} = 2\underline{i} - \underline{j} + 3\underline{k}$, $\underline{b} = 3\underline{i} + 2\underline{j} + 4\underline{k}$ and $\underline{c} = \underline{i} + 3\underline{j} - 5\underline{k}$, then verify that $\underline{a} \cdot \underline{b} \times \underline{c} = \underline{b} \cdot \underline{c} \times \underline{a} = \underline{c} \cdot \underline{a} \times \underline{b}$.	8

First Annual 2025 | Section B Q2(iii) | 4 marks

(iii)	Find projection of vector $\vec{a} = 9\hat{i} - \hat{j} + \hat{k}$ along the vector $\vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$.	04

First Annual 2025 | Section B Q2(viii) OR | 4 marks

OR	Find $\cos\theta$ if: $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$	04

Q.4	Find the volume of parallelepiped whose vertices are the following points by fixing point "A". $A(2,3,1), B(-1,-2,0), C(0,2,-5), D(0,1,-2)$	**08**

Second Annual 2025 | Section B Q2(i) OR | 4 marks

OR	Find the value of λ for which vectors $\underline{a} = \underline{i} + 2\underline{j} + 3\underline{k}$, $\underline{b} = 2\underline{i} - \underline{j} + \underline{k}$ and $\underline{c} = 3\underline{i} + 2\underline{j} + 5\underline{k}$ are coplanar.	04

Second Annual 2025 | Section B Q2(xi) OR | 4 marks

OR	Find angle between two vectors $\underline{a} = 2\hat{i}$ and $\underline{b} = \hat{i} + \sqrt{3}\hat{j}$ using cross product method.	04

First Annual 2026 | Section B Q2(ii) OR | 4 marks

OR	In triangle OPR, M is the mid-point of PR. If $\overrightarrow{OP} = p$, $\overrightarrow{OR} = r$ then find the following in terms of p and r. a. $\overrightarrow{PR}$ b. $\overrightarrow{PM}$ c. $\overrightarrow{OM}$	04

First Annual 2026 | Section B Q2(iii) OR | 4 marks

(iii)	For $A = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$ and $B = [-2 -1 -4]$, verify that	04	OR	For what value of λ, the vectors $\underline{a} = \underline{i} + 3\underline{j} + \underline{k}$, $\underline{b} = 2\underline{i} - \underline{j} - \underline{k}$ and $\underline{c} = \lambda\underline{j} + 3\underline{k}$ are coplanar.

First Annual 2026 | Section B Q2(vi) OR | 4 marks

OR	Find length of the vector $\overrightarrow{AB}$ from $A(-3,5)$ to $B(7,9)$. Also find a unit vector in the direction of $\overrightarrow{AB}$ with magnitude 4.	04

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Fundamental Law of Trigonometry

Domain B: Geometry

FBISE model paper | Section A Q24 | 0.75 marks

24.	If $\sin\theta = \frac{3}{5}$ and θ is acute, what is the value of $\sin 2\theta$?	$\frac{24}{25}$	$\frac{12}{25}$	$\frac{7}{25}$	$\frac{16}{25}$

FBISE model paper | Section A Q25 | 0.75 marks

25.	If $\cos\theta = -\frac{1}{3}$ and θ lies in quadrant II, the value of $\sin\left(\frac{\theta}{2}\right)$ is:	$\frac{\sqrt{6}}{3}$	$-\frac{\sqrt{6}}{3}$	$\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3}$

FBISE model paper | Section A Q26 | 0.75 marks

26.	Simplifying $2(\sin 3x)(\cos x)$ into a sum or difference we get:	$\sin 4x + \sin 2x$	$\sin 4x - \sin 2x$	$\sin 2x - \sin x$	$\sin 2x + \sin x$

FBISE model paper | Section A Q27 | 0.75 marks

27.	If $\tan\alpha = \frac{1}{2}$ and $\tan\beta = \frac{1}{3}$, (α, β are acute), the exact value of $(\alpha + \beta)$ is:	$0°$	$8°$	$45°$	$90°$

FBISE model paper | Section A Q28 | 0.75 marks

28.	Which of the following is equivalent to $\sin(270° - \theta)$?	$-\sin\theta$	$-\cos\theta$	$\sin\theta$	$\cos\theta$

FBISE model paper | Section A Q29 | 0.75 marks

29.	If $sin25^\circ = t$, then $cos50^\circ$ in terms of t is:	$2t^2 - 1$	$2t\sqrt{1-t^2}$	$1 - 2t^2$	$\sqrt{1-2t^2}$

FBISE model paper | Section B Q5 | 4 marks

5.	Given: $y = 2\cos(5x)\cos(x) + 2\sin(3x)\sin(x)$ i. Simplify y into a single sum or difference of cosine functions. ii. Calculate the exact numerical value of y when $x = 15°$.	4

First Annual 2025 | Section B Q2(ii) | 4 marks

by completing the square method. Verify that:. $\sin\theta \cdot \sin\left(\frac{\pi}{3} - \theta\right) \cdot \sin\left(\frac{\pi}{3} + \theta\right) = \frac{1}{4}\sin 3\theta$	04

First Annual 2025 | Section B Q2(vi) OR | 4 marks

Verify that: $\frac{1+sin2\theta+cos2\theta}{1+sin2\theta-cos2\theta}=cot\theta$	**04**

First Annual 2025 | Section B Q2(vii) OR | 4 marks

If $tan\theta=\frac{3}{4}$ with $\pi<\theta<\frac{3\pi}{2}$, find the exact values of (a) $sin\left(\frac{\theta}{2}\right)$ and (b) $cos\left(\frac{\theta}{2}\right)$ without using calculator.	**04**

Verify the fundamental law of trigonometry: $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ Where α,β are real angles in standard position and $\alpha>\beta$.	**08**

First Annual 2025 | Section B Q2(ii) | 4 marks

(ii)	Verify that: $cos\theta \,.\, cos\left(\frac{\pi}{3}-\theta\right).\, cos\left(\frac{\pi}{3}+\theta\right)=\frac{1}{4}cos3\theta$

First Annual 2025 | Section B Q2(vi) OR | 4 marks

Verify that: $\frac{cos\theta+cos4\theta+cos7\theta}{sin\theta+sin4\theta+sin7\theta}=\cot 4\theta$

First Annual 2025 | Section B Q2(vii) OR | 4 marks

If $\sin\theta = -\frac{3}{5}$ and θ lies in 3rd quadrant, find the exact values of

(a) $\cos(180^\circ + \theta)$

(b) $\tan(270^\circ - \theta)$

Second Annual 2025 | Section B Q2(i) OR | 4 marks

OR	Given $tan\beta = 2$, find the values of $cos2\beta$ and $tan2\beta$	**04**

OR	Verify that $cos^2\theta - sin^2\theta = cos2\theta$ for $\theta = \frac{\pi}{6}$	**04**

	~~Candidates can make their own grid on answer book.~~	
OR	Prove the fundamental law of trigonometry $cos(\alpha - \beta) = cos\alpha\ cos\beta + sin\alpha\ sin\beta$ where α, β (real angles) are in standard position and $\alpha > \beta$.	**08**

First Annual 2026 | Section B Q2(vii) | 4 marks

(vii)	Verify the identity: $\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$	**04**

First Annual 2026 | Section B Q2(xi) OR | 4 marks

OR	Verify: $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$	**04**

he middle term(s) in the expansion of $\left(-\frac{1}{2x}\right)^{10}$. Also find the 9th term.	08	OR	If $\sin\alpha=\frac{3}{5}, \sin\beta=\frac{1}{\sqrt{5}}$, where terminal ray of α and β are in 2nd quadrant, then find the value of $\cos(\alpha-\beta)$ and $\sin(\alpha+\beta)$.	08
or what value of k the remainder of			a Prove that scalar triple product is invariant under	

First Annual 2026 | Section B Q2(ii) | 4 marks

(ii)	Verify that $\sin 4\theta = 8\sin\theta\cos^3\theta - 4\sin\theta\cos\theta$	04

First Annual 2026 | Section B Q2(vi) OR | 4 marks

Using fundamental law of trigonometry find the value of $\cos 15°$. (do not use calculator)	04

OR	Express $2sin\,12\theta\cos 6\theta$ as a sum or difference.	04

First Annual 2026 | Section C Q4 OR | 8 marks

Prove fundamental law of trigonometry: $\cos(A-B)=\cos A\cos B+\sin A\sin B$ Diagram is necessary.	08

FBISE model paper | Section A Q1(xiv) | 1 mark

xiv.	If $cos\alpha=\frac{12}{13}; 0<\alpha<\frac{\pi}{2}$ and $sin\beta=\frac{5}{13}; \frac{\pi}{2}<\beta<\pi$ then value of $\cos(\alpha+\beta)$ is:	1	-1	$\frac{144}{169}$	$-\frac{144}{169}$

FBISE model paper | Section A Q1(xv) | 1 mark

xv.	If the expression $4sin5\alpha.cos3\alpha.cos2\alpha$ is expressed as sum of three sines, then two of them are $sin4\alpha$ and $sin10\alpha$. The third one is:	$\sin8\alpha$	$\sin6\alpha$	$\sin5\alpha$	$\sin12\alpha$

FBISE model paper | Section B Q2(ii) | 4 marks

ii.	Prove that $cos\,(\frac{\pi}{3}+x)-sin\,(\frac{\pi}{6}-x)=0$	4

FBISE model paper | Section B Q2(vi) OR | 4 marks

If $A, B,$ and C are the angle measures of a triangle such that $A+B+C=\pi$, then prove that $tanA+tanB+tanC = tanA\,tanB\,tanC$	4

Prove that $\frac{sin5x - sin3x}{cos5x + 2cos4x + cos3x} = \tan\frac{x}{2}$	4

OR	Prove the fundamental law of trigonometry $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$ where α and β be any two real angles.	8

First Annual 2025 | Section B Q2(ix) | 4 marks

(ix)	Verify that: $\text{Cos}\frac{\pi}{9}\text{Cos}\frac{2\pi}{9}\text{Cos}\frac{\pi}{3}\text{Cos}\frac{4\pi}{9}=\frac{1}{16}$	04

First Annual 2025 | Section B Q2(vi) OR | 4 marks

OR	Reduce $\text{Sin}^4 x$ in terms of Cosine function raised to the first power multiple of angle.	04

(viii)	Verify that: $\text{Sin}\,4x = 8\,\text{Sin}\,x\,\text{Cos}^3\,x - 4\,\text{Sin}\,x\,\text{Cos}\,x$	04

Q.3	Verify the fundamental law of trigonometry: $\text{Cos}(\alpha - \beta) = \text{Cos}\,\alpha\,\text{Cos}\,\beta + \text{Sin}\,\alpha\,\text{Sin}\,\beta$	**08**

Second Annual 2025 | Section B Q2(iv) OR | 4 marks

OR	In triangle DEF, $DE = 5, EF = 7$ and $\angle D = 60°$. If $DF = k - 1$, find k using law of sines.	04

Second Annual 2025 | Section B Q2(ix) OR | 4 marks

OR	Without using calculator prove that $(sin75°cos15° + cos75°sin15°) + (cos75°cos15° - sin75°sin15°) = 1$	04

Second Annual 2025 | Section B Q2(v) | 4 marks

(v)	Find area of a triangle ABC with side $AB = 10$ cm, and angles $\angle A = 45°$, $\angle B = 75°$.	04

Second Annual 2025 | Section B Q2(xi) | 4 marks

(xi)	If $tan\theta = \frac{3}{4}$, then find the values of $tan2\theta$ and $cos\theta$. $\left[0 < \theta < \frac{\pi}{2}\right]$	04

First Annual 2026 | Section B Q2(ix) | 4 marks

(ix)	If $\sin\theta = \frac{4}{5}$ and terminal ray of θ is in quad-II then find the values of $\sin\frac{\theta}{2}$ and $\cos\frac{\theta}{2}$.	04

First Annual 2026 | Section B Q2(x) | 4 marks

(x)	Verify that $\sin 5\theta + \sin\theta + 2\sin 3\theta = 4\sin 3\theta\cos^2\theta$	04

OR	Verify that: $\sin\frac{\pi}{9}.\sin\frac{2\pi}{9}.\sin\frac{\pi}{3}.\sin\frac{4\pi}{9}=\frac{3}{16}$	08

APOLLO

Trigonometric Functions

Domain B: Geometry

FBISE model paper | Section A Q30 | 0.75 marks

30.	What is the range of the function $y = 3sin(2x)$?	$[-1, 1]$	$[-2, 2]$	$[-3, 3]$	$(-\infty, \infty)$

FBISE model paper | Section A Q31 | 0.75 marks

31.	The sum of $sin(-x) + sinx$ is?	0	1	$2sinx$	$sin2x$

FBISE model paper | Section A Q32 | 0.75 marks

32.	The period of $4cos\left(\frac{x}{3}\right)$ is:	$\frac{4\pi}{3}$	$\frac{2\pi}{3}$	2π	6π

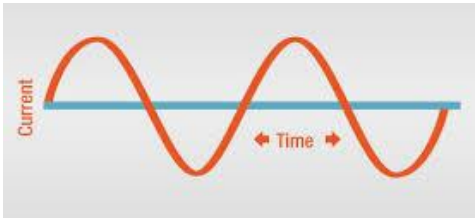
FBISE model paper | Section A Q33 | 0.75 marks

33.	What is the maximum value of $10 \sin x - 5 \cos x + 2$?	3	17	7	13

FBISE model paper | Section A Q34 | 0.75 marks

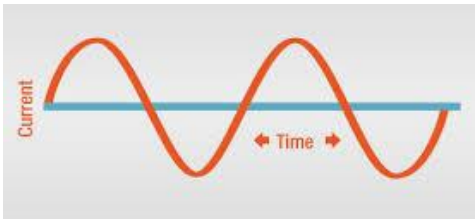
34.	Which pair of the trigonometric functions has period π?	$sinx, cosx$	$tanx, cotx$	$secx, cosecx$	$sinx, tanx$

FBISE model paper | Section A Q57 | 0.75 marks

Scenario for MCQs (57-60)	**Current in the circuit with the passage of time follows the wave pattern.** 

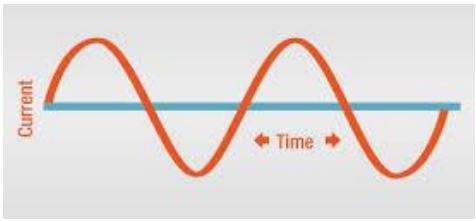
57.	Wave shown in the figure is often known as:	secant wave	tangent wave	sine wave	longitudinal wave
58.	If the function $y = 2sinx$ represents the wave, then what is the domain of the function?	$(-\infty, 0]$	$[0, \infty)$	$(-\infty, \infty)$	$[1, \infty)$

FBISE model paper | Section A Q58 | 0.75 marks

Scenario for MCQs (57-60)	**Current in the circuit with the passage of time follows the wave pattern.** 

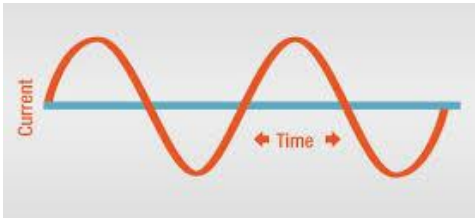
58.	If the function $y = 2sinx$ represents the wave, then what is the domain of the function?	$(-\infty, 0]$	$[0, \infty)$	$(-\infty, \infty)$	$[1, \infty)$

FBISE model paper | Section A Q59 | 0.75 marks

Scenario for MCQs (57-60)	**Current in the circuit with the passage of time follows the wave pattern.** 

59.	What is the maximum value of $y = 2sinx$?	0	1	2	∞

FBISE model paper | Section A Q60 | 0.75 marks

Scenario for MCQs (57-60)	**Current in the circuit with the passage of time follows the wave pattern.** 

60.	If $y = 2sinx$ is replaced by $y = 3cosx$, then what is the range of the new function?	$(-3, 3)$	$[-3, 3]$	$(-3, 3]$	$[-3, 3)$

OR	A Ferris wheel has a radius of 10 meters and its centre is 12 meters above the ground. It completes one revolution in 40 seconds. A passenger starts at the lowest point at time $t = 0$. i. Model the passenger's height $h(t)$ above the ground using a cosine function of the form $h(t) = a\cos(kt) + c$. Find the values of a, k, and c. ii. Calculate the passenger's height after 10 seconds.	4

FBISE model paper | Section B Q7 | 4 marks

7.	Determine whether the function $f(x) = x\sin x$ is even, odd, or neither. Also, check whether it is periodic; if yes, find its fundamental period.	4

First Annual 2025 | Section B Q2(iii) OR | 4 marks

Find the maximum and minimum values of the function $f(\theta) = \dfrac{1}{3+5\cos(2\theta+\pi)}$	04

First Annual 2025 | Section B Q2(v) | 4 marks

(b) Solve the resulting depressed equation. Without drawing graph find range, amplitude, period and frequency of the function $y = -4cos(7x - \pi)$	04

First Annual 2025 | Section B Q2(x) OR | 4 marks

A solar panel's sunlight reception is modeled as $P(\theta) = 100\ cos(3\theta - 90^{\circ})$, with $0^{\circ} \leq \theta \leq 180^{\circ}$ (a) Find angle for the maximum sunlight and the max sunlight received. (b) Find angle for the minimum sunlight and the min sunlight received.	04

$y = 3cos2x \ ; -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ **08**

a) Make table of values for given interval
b) Draw the graph of the function for given interval

A company has 7 engineers and 5 managers. In how

First Annual 2025 | Section B Q2(iii) OR | 4 marks

Find the maximum and minimum values of the function: $f(\theta) = \frac{1}{3 + 4sin(5\theta + \pi)}$

First Annual 2025 | Section B Q2(v) | 4 marks

(v) Without drawing graph find range, amplitude, period and frequency of the function $y = -3sin\left(5x + \frac{\pi}{3}\right)$

$y = 2\sin(2x)\ ;\ -\frac{\pi}{2} \le x \le \frac{\pi}{2}$ **a)** Make table of values for given interval **b)** Draw the graph of the function for given interval	**08**
A basketball club has 7 forwards and 5 guards. In how	

Second Annual 2025 | Section B Q2(vii) | 4 marks

(vii)	Find maximum and minimum values of $y = 5 + 2sin\theta$	**04**

Second Annual 2025 | Section B Q2(xi) | 4 marks

(xi)	Given $f(\theta) = cos2\theta$, **a.** Determine whether $f(\theta)$ is even, odd or neither **b.** Find the period of $f(\theta)$.	**04**

Q.6	Draw the graph of $y = 2sin2x$; where $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	**08**