

APOLLO

Geometry and trigonometry

HL

Mathematics: analysis and approaches

First assessment 2021

97 questions | mark schemes

5. (a)

Note: Do not award the final **A1** for proofs which work from both sides to find a common expression other than $2 \sin x \cos x - 2 \sin^2 x$.

METHOD 1 (LHS to RHS)attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ **M1**

$$\text{LHS} = 2 \sin x \cos x + \cos 2x - 1 \text{ OR}$$

$$\sin 2x + 1 - 2 \sin^2 x - 1 \text{ OR}$$

$$2 \sin x \cos x + 1 - 2 \sin^2 x - 1$$

$$= 2 \sin x \cos x - 2 \sin^2 x$$

A1

$$\sin 2x + \cos 2x - 1 = 2 \sin x (\cos x - \sin x) = \text{RHS}$$

AG**METHOD 2 (RHS to LHS)**

$$\text{RHS} = 2 \sin x \cos x - 2 \sin^2 x$$

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ **M1**

$$= \sin 2x + 1 - 2 \sin^2 x - 1$$

A1

$$= \sin 2x + \cos 2x - 1 = \text{LHS}$$

AG**[2 marks]**

(b) attempt to factorise

M1

$$(\cos x - \sin x)(2 \sin x + 1) = 0$$

A1

$$\text{recognition of } \cos x = \sin x \Rightarrow \frac{\sin x}{\cos x} = \tan x = 1 \text{ OR } \sin x = -\frac{1}{2}$$

(M1)

one correct reference angle seen anywhere, accept degrees

(A1)

$$\frac{\pi}{4} \text{ OR } \frac{\pi}{6} \text{ (accept } -\frac{\pi}{6}, \frac{7\pi}{6} \text{)}$$

Note: This **(M1)(A1)** is independent of the previous **M1A1**.

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{4}$$

A2

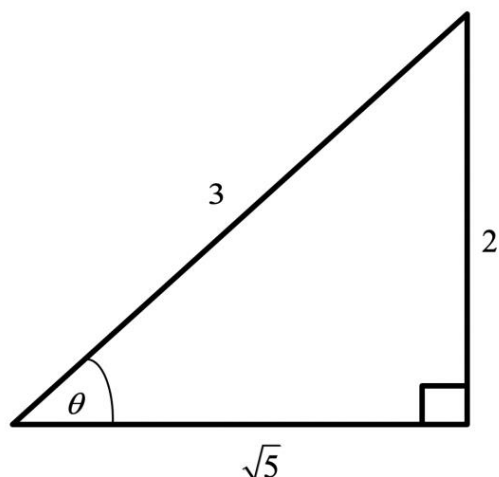
Note: Award **A1** for any two correct (radian) answers.

Award **A1A0** if additional values given with the four correct (radian) answers. Award **A1A0** for four correct answers given in degrees.

[6 marks]**Total [8 marks]**

6. METHOD 1

attempt to use a right angled triangle

M1correct placement of all three values and θ seen in the triangle**(A1)** $\cot \theta < 0$ (since $\operatorname{cosec} \theta > 0$ puts θ in the second quadrant)**R1**

$$\cot \theta = -\frac{\sqrt{5}}{2}$$

A1

Note: Award **M1A1R0A0** for $\cot \theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The **R1** should be awarded independently for a negative value only given as a final answer.

[4 marks]**METHOD 2**Attempt to use $1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$ **M1**

$$1 + \cot^2 \theta = \frac{9}{4}$$

$$\cot^2 \theta = \frac{5}{4}$$

(A1)

$$\cot \theta = \pm \frac{\sqrt{5}}{2}$$

 $\cot \theta < 0$ (since $\operatorname{cosec} \theta > 0$ puts θ in the second quadrant)**R1**

$$\cot \theta = -\frac{\sqrt{5}}{2}$$

A1

Note: Award **M1A1R0A0** for $\cot \theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The **R1** should be awarded independently for a negative value only given as a final answer.

METHOD 3

$$\sin \theta = \frac{2}{3}$$

attempt to use $\sin^2 \theta + \cos^2 \theta = 1$

M1

$$\frac{4}{9} + \cos^2 \theta = 1$$

$$\cos^2 \theta = \frac{5}{9}$$

(A1)

$$\cos \theta = \pm \frac{\sqrt{5}}{3}$$

$\cos \theta < 0$ (since $\operatorname{cosec} \theta > 0$ puts θ in the second quadrant)

R1

$$\cos \theta = -\frac{\sqrt{5}}{3}$$

$$\cot \theta = -\frac{\sqrt{5}}{2}$$

A1

Note: Award **M1A1R0A0** for $\cot \theta = \frac{\sqrt{5}}{2}$ seen as the final answer

The **R1** should be awarded independently for a negative value only given as a final answer.

11. (a) (i) $\frac{-1+1}{2} = 0 = 3-3$ **A1**

the point $(-1, 0, 3)$ lies on L_1 . **AG**

(ii) attempt to set equal to a parameter or rearrange cartesian form **(M1)**

$$\frac{x+1}{2} = y = 3-z = \lambda \Rightarrow x = 2\lambda - 1, y = \lambda, z = 3 - \lambda \text{ OR } \frac{x+1}{2} = \frac{y-0}{1} = \frac{z-3}{-1}$$

correct direction vector $\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ or equivalent seen in vector form **(A1)**

$$\mathbf{r} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \text{ (or equivalent) } \quad \mathbf{A1}$$

Note: Award **A0** if $\mathbf{r} =$ is omitted.

[4 marks]

(b) attempt to use the scalar product formula **(M1)**

$$\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix} = (\pm) \sqrt{6} \sqrt{a^2 + 2} \cos 45^\circ \quad \mathbf{(A1)(A1)}$$

Note: Award **A1** for LHS and **A1** for RHS

$$2a + 2 = \frac{(\pm) \sqrt{6} \sqrt{a^2 + 2} \sqrt{2}}{2} \left(\Rightarrow 2a + 2 = (\pm) \sqrt{3} \sqrt{a^2 + 2} \right) \quad \mathbf{A1A1}$$

Note: Award **A1** for LHS and **A1** for RHS

$$4a^2 + 8a + 4 = 3a^2 + 6$$

$$a^2 + 8a - 2 = 0$$

attempt to solve their quadratic

$$a = \frac{-8 \pm \sqrt{64 + 8}}{2} = \frac{-8 \pm \sqrt{72}}{2} (= -4 \pm 3\sqrt{2}) \quad \mathbf{A1}$$

[8 marks]

continued...

Question 11 continued

(c) **METHOD 1**attempt to equate the parametric forms of L_1 and L_2 (M1)

$$\begin{cases} 2\lambda - 1 = ta \\ \lambda = 1 + t \\ 3 - \lambda = 2 - t \end{cases} \quad \text{A1}$$

attempt to solve equations by eliminating λ or t (M1)

$$2 + 2t - 1 = ta \Rightarrow 1 = t(a - 2) \text{ or } 2\lambda - 1 = (\lambda - 1)a \Rightarrow a - 1 = \lambda(a - 2)$$

Solutions exist unless $a - 2 = 0$

$$k = 2 \quad \text{A1}$$

Note: This **A1** is independent of the following marks.

$$t = \frac{1}{a-2} \text{ or } \lambda = \frac{a-1}{a-2} \quad \text{A1}$$

$$\text{A has coordinates } \left(\frac{a}{a-2}, 1 + \frac{1}{a-2}, 2 - \frac{1}{a-2} \right) \left(= \left(\frac{a}{a-2}, \frac{a-1}{a-2}, \frac{2a-5}{a-2} \right) \right) \quad \text{A2}$$

Note: Award **A1** for any two correct coordinates seen or final answer in vector form.

METHOD 2no unique point of intersection implies direction vectors of L_1 and L_2 parallel

$$k = 2 \quad \text{A1}$$

Note: This **A1** is independent of the following marks.

attempt to equate the parametric forms of L_1 and L_2 (M1)

$$\begin{cases} 2\lambda - 1 = ta \\ \lambda = 1 + t \\ 3 - \lambda = 2 - t \end{cases} \quad \text{A1}$$

attempt to solve equations by eliminating λ or t (M1)

$$2 + 2t - 1 = ta \Rightarrow 1 = t(a - 2) \text{ or } 2\lambda - 1 = (\lambda - 1)a \Rightarrow a - 1 = \lambda(a - 2)$$

$$t = \frac{1}{a-2} \text{ or } \lambda = \frac{a-1}{a-2} \quad \text{A1}$$

$$\text{A has coordinates } \left(\frac{a}{a-2}, 1 + \frac{1}{a-2}, 2 - \frac{1}{a-2} \right) \left(= \left(\frac{a}{a-2}, \frac{a-1}{a-2}, \frac{2a-5}{a-2} \right) \right) \quad \text{A2}$$

continued...

Question 11 continued

Note: Award **A1** for any two correct coordinates seen or final answer in vector form.

[7 marks]

Total [19 marks]

2. (a) attempt to use $\cos^2 x = 1 - \sin^2 x$ **M1**

$$2\sin^2 x - 5\sin x + 2 = 0$$
A1

EITHER

attempting to factorise

$$(2\sin x - 1)(\sin x - 2)$$
M1

A1

OR

attempting to use the quadratic formula

M1

$$\sin x = \frac{5 \pm \sqrt{5^2 - 4 \times 2 \times 2}}{4} \left(= \frac{5 \pm 3}{4} \right)$$
A1

THEN

$$\sin x = \frac{1}{2}$$
(A1)

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$
A1A1

Total [7 marks]

5. METHOD 1

use of $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}|\sin \theta$ on the LHS **(M1)**

$$|\mathbf{a} \times \mathbf{b}|^2 = |\mathbf{a}|^2 |\mathbf{b}|^2 \sin^2 \theta \quad \text{A1}$$

$$= |\mathbf{a}|^2 |\mathbf{b}|^2 (1 - \cos^2 \theta) \quad \text{M1}$$

$$= |\mathbf{a}|^2 |\mathbf{b}|^2 - |\mathbf{a}|^2 |\mathbf{b}|^2 \cos^2 \theta \quad \text{OR} = |\mathbf{a}|^2 |\mathbf{b}|^2 - (|\mathbf{a}||\mathbf{b}|\cos \theta)^2 \quad \text{A1}$$

$$= |\mathbf{a}|^2 |\mathbf{b}|^2 - (\mathbf{a} \cdot \mathbf{b})^2 \quad \text{AG}$$

METHOD 2

use of $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos \theta$ on the RHS **(M1)**

$$= |\mathbf{a}|^2 |\mathbf{b}|^2 - |\mathbf{a}|^2 |\mathbf{b}|^2 \cos^2 \theta \quad \text{A1}$$

$$= |\mathbf{a}|^2 |\mathbf{b}|^2 (1 - \cos^2 \theta) \quad \text{M1}$$

$$= |\mathbf{a}|^2 |\mathbf{b}|^2 \sin^2 \theta \quad \text{OR} = (|\mathbf{a}||\mathbf{b}|\sin \theta)^2 \quad \text{A1}$$

$$= |\mathbf{a} \times \mathbf{b}|^2 \quad \text{AG}$$

Note: If candidates attempt this question using cartesian vectors, e.g

$$\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix},$$

award full marks if fully developed solutions are seen.
Otherwise award no marks.

[4 marks]

6. METHOD 1

attempt to use the cosine rule to find the value of x **(M1)**

$$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right) \quad \text{A1}$$

$$2x^2 = 100$$

$$x^2 = 50 \text{ OR } x = \sqrt{50} \left(= 5\sqrt{2}\right) \quad \text{A1}$$

attempt to find $\sin \hat{C}$ (seen anywhere) **(M1)**

$$\sin^2 \hat{C} + \left(\frac{3}{4}\right)^2 = 1 \text{ or } x^2 + 3^2 = 4^2 \text{ OR right triangle with side 3 and hypotenuse 4}$$

$$\sin \hat{C} = \frac{\sqrt{7}}{4} \quad \text{(A1)}$$

Note: The marks for finding $\sin \hat{C}$ may be awarded independently of the first three marks for finding x .

correct substitution into the area formula using their value of x (or x^2)
and their value of $\sin \hat{C}$ **(M1)**

$$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4} \text{ or } A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$$

$$A = \frac{25\sqrt{7}}{2} \quad \text{A1}$$

continued...

METHOD 2attempt to find the height, h , of the triangle in terms of x **(M1)**

$$h^2 + \left(\frac{3}{4}x\right)^2 = x^2 \text{ OR } h^2 + \left(\frac{5}{4}x\right)^2 = 10^2 \text{ OR } h = \frac{\sqrt{7}}{4}x$$

A1equating their expressions for either h^2 or h **(M1)**

$$x^2 - \left(\frac{3}{4}x\right)^2 = 10^2 - \left(\frac{5}{4}x\right)^2 \text{ OR } \sqrt{100 - \frac{25}{16}x^2} = \frac{\sqrt{7}}{4}x \text{ (or equivalent)}$$

A1

$$x^2 = 50 \text{ OR } x = \sqrt{50} \text{ (= } 5\sqrt{2} \text{)}$$

A1correct substitution into the area formula using their value of x (or x^2)**(M1)**

$$A = \frac{1}{2} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4} \sqrt{50} \text{ OR } A = \frac{1}{2} (2 \times 5\sqrt{2}) \left(\frac{\sqrt{7}}{4} 5\sqrt{2} \right)$$

$$A = \frac{25\sqrt{7}}{2}$$

A1**Total [7 marks]**

8. (a) METHOD 1setting at least two components of l_1 and l_2 equal**M1**

$$3 + 2\lambda = 2 + \mu \quad (1)$$

$$2 - 2\lambda = -\mu \quad (2)$$

$$-1 + 2\lambda = 4 + \mu \quad (3)$$

attempt to solve two of the equations eg. adding (1) and (2)

M1gives a contradiction (no solution), eg $5 = 2$ **R1**so l_1 and l_2 do not intersect**AG****Note:** For an error within the equations award **M0M1R0**.**Note:** The contradiction must be correct to award the **R1**.**[3 marks]****METHOD 2** l_1 and l_2 are parallel, so l_1 and l_2 are either identical or distinct.**R1**

Attempt to subtract two position vectors from each line,

$$\text{e.g. } \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -5 \end{pmatrix}$$

M1

$$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \neq k \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

A1**[3 marks]****(b) METHOD 1** l_1 and l_2 are parallel (as $\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ is a multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)let A be $(3, 2, -1)$ on l_1 and let B be $(2, 0, 4)$ on l_2

$$\text{Attempt to find vector } \vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix}$$

(M1)

$$\text{Distance required is } \frac{|\mathbf{v} \times \vec{AB}|}{|\mathbf{v}|}$$

M1*continued...**Question 8 continued*

$$= \frac{1}{\sqrt{3}} \left| \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix} \right| \quad (\text{A1})$$

$$= \frac{1}{\sqrt{3}} \left| \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix} \right| \quad \text{A1}$$

minimum distance is $\sqrt{18} (= 3\sqrt{2})$ A1

[5 marks]

METHOD 2

l_1 and l_2 are parallel (as $\begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$ is a multiple of $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$)

let A be a fixed point on l_1 eg $(3, 2, -1)$ and let B be a general point on l_2 $(2 + \mu, -\mu, 4 + \mu)$

attempt to find vector $\vec{AB}$ (M1)

$$\vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (\mu \in \mathbb{R}) \quad \text{A1}$$

$$|\vec{AB}| = \sqrt{(-1 + \mu)^2 + (-2 - \mu)^2 + (5 + \mu)^2} \quad (= \sqrt{3\mu^2 + 12\mu + 30}) \quad \text{M1}$$

EITHER

$$\frac{d}{d\mu} \left(|\vec{AB}|^2 \right) = 0 \Rightarrow 6\mu + 12 = 0 \Rightarrow \mu = -2 \quad \text{A1}$$

OR

$$|\vec{AB}| = \sqrt{3(\mu + 2)^2 + 18} \text{ to obtain } \mu = -2 \quad \text{A1}$$

THEN

minimum distance is $\sqrt{18} (= 3\sqrt{2})$ A1

[5 marks]

continued...

Question 8 continued

METHOD 3

let A be $(3, 2, -1)$ on l_1 and let B be $(2 + \mu, -\mu, 4 + \mu)$ on l_2 **(M1)**

(or let A be $(2, 0, 4)$ on l_2 and let B be $(3 + 2\lambda, 2 - 2\lambda, -1 + 2\lambda)$ on l_1)

$$\vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 5 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad (\mu \in \mathbb{R}) \quad \text{(or } \vec{AB} = \begin{pmatrix} 2\lambda + 1 \\ -2\lambda + 2 \\ 2\lambda - 5 \end{pmatrix}) \quad \textbf{A1}$$

$$\begin{pmatrix} \mu - 1 \\ -\mu - 2 \\ \mu + 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 0 \quad \text{(or } \begin{pmatrix} 2\lambda + 1 \\ -2\lambda + 2 \\ 2\lambda - 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = 0) \quad \textbf{M1}$$

$$\mu = -2 \quad \text{or} \quad \lambda = 1 \quad \textbf{A1}$$

$$\text{minimum distance is } \sqrt{18} (= 3\sqrt{2}) \quad \textbf{A1}$$

[5 marks]
Total [8 marks]

10. (a) $6 + 6 \cos x = 0$ (or setting their $f'(x) = 0$) **(M1)**

$$\cos x = -1 \text{ (or } \sin x = 0)$$

$$x = \pi, x = 3\pi$$

A1A1

[3 marks]

(b) attempt to integrate $\int_{\pi}^{3\pi} (6 + 6 \cos x) dx$ **(M1)**

$$= [6x + 6 \sin x]_{\pi}^{3\pi}$$

A1A1

substitute their limits into their integrated expression and subtract **(M1)**

$$= (18\pi + 6 \sin 3\pi) - (6\pi + 6 \sin \pi)$$

$$= (6(3\pi) + 0) - (6\pi + 0) \quad (= 18\pi - 6\pi)$$

A1

$$\text{area} = 12\pi$$

AG

[5 marks]

(c) attempt to substitute into the formula for surface area (including base) **(M1)**

$$\pi(2^2) + \pi(2)(l) = 12\pi$$

(A1)

$$4\pi + 2\pi l = 12\pi$$

$$2\pi l = 8\pi$$

$$l = 4$$

A1

[3 marks]

continued...

Question 10 continued

- (d) valid attempt to find the height of the cone

(M1)

e.g. $2^2 + h^2 = (\text{their } l)^2$

$$h = \sqrt{12} \quad (= 2\sqrt{3})$$

(A1)attempt to use $V = \frac{1}{3}\pi r^2 h$ with their values substituted**M1**

$$\left(\frac{1}{3}\pi(2^2)(\sqrt{12}) \right)$$

$$\text{volume} = \frac{4\pi\sqrt{12}}{3} \left(= \frac{8\pi\sqrt{3}}{3} = \frac{8\pi}{\sqrt{3}} \right)$$

A1**[4 marks]****Total [15 marks]**

12. (a) EITHERhorizontal stretch/scaling with scale factor $\frac{1}{2}$ **Note:** Do not allow 'shrink' or 'compression'followed by a horizontal translation/shift $\frac{1}{2}$ units to the left**A2****Note:** Do not allow 'move'**OR**

horizontal translation/shift 1 unit to the left

followed by horizontal stretch/scaling with scale factor $\frac{1}{2}$ **A2****THEN**vertical translation/shift up by $\frac{\pi}{4}$ (or translation through $\begin{pmatrix} 0 \\ \frac{\pi}{4} \end{pmatrix}$)**A1**

(may be seen anywhere)

[3 marks](b) let $\alpha = \arctan p$ and $\beta = \arctan q$ **M1** $p = \tan \alpha$ and $q = \tan \beta$ **(A1)**

$$\tan(\alpha + \beta) = \frac{p+q}{1-pq}$$

A1

$$\alpha + \beta = \arctan\left(\frac{p+q}{1-pq}\right)$$

A1so $\arctan p + \arctan q \equiv \arctan\left(\frac{p+q}{1-pq}\right)$ where $p, q > 0$ and $pq < 1$ **AG****[4 marks]***continued...*

Question 12 continued(c) **METHOD 1**

$$\frac{\pi}{4} = \arctan 1 \text{ (or equivalent)}$$

A1

$$\arctan\left(\frac{x}{x+1}\right) + \arctan 1 = \arctan\left(\frac{\frac{x}{x+1} + 1}{1 - \frac{x}{x+1}(1)}\right)$$

A1

$$= \arctan\left(\frac{\frac{x+x+1}{x+1}}{\frac{x+1-x}{x+1}}\right)$$

A1

$$= \arctan(2x+1)$$

AG**[3 marks]****METHOD 2**

$$\tan \frac{\pi}{4} = 1 \text{ (or equivalent)}$$

A1

$$\text{Consider } \arctan(2x+1) - \arctan\left(\frac{x}{x+1}\right) = \frac{\pi}{4}$$

$$\tan\left(\arctan(2x+1) - \arctan\left(\frac{x}{x+1}\right)\right)$$

$$= \arctan\left(\frac{2x+1 - \frac{x}{x+1}}{1 + \frac{x(2x+1)}{x+1}}\right)$$

A1

$$= \arctan\left(\frac{(2x+1)(x+1) - x}{x+1 + x(2x+1)}\right)$$

A1

$$= \arctan 1$$

AG**[3 marks]***continued...*

*Question 12 continued***METHOD 3**

$$\tan\left(\arctan(2x+1)\right) = \tan\left(\arctan\left(\frac{x}{x+1}\right) + \frac{\pi}{4}\right)$$

$$\tan\frac{\pi}{4} = 1 \text{ (or equivalent)}$$

A1

$$\text{LHS} = 2x+1$$

A1

$$\text{RHS} = \frac{\frac{x}{x+1} + 1}{1 - \frac{x}{x+1}} (= 2x+1)$$

A1**[3 marks]**

Question 12 continued

(d) let $P(n)$ be the proposition that $\sum_{r=1}^n \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{n}{n+1}\right)$ for $n \in \mathbb{Z}^+$

consider $P(1)$:

when $n=1$, $\sum_{r=1}^1 \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{1}{2}\right) = \text{RHS}$ and so $P(1)$ is true **R1**

assume $P(k)$ is true, ie. $\sum_{r=1}^k \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{k}{k+1}\right)$ ($k \in \mathbb{Z}^+$) **M1**

Note: Award **M0** for statements such as “let $n=k$ ”.

Note: Subsequent marks after this **M1** are independent of this mark and can be awarded.

consider $P(k+1)$:

$$\sum_{r=1}^{k+1} \arctan\left(\frac{1}{2r^2}\right) = \sum_{r=1}^k \arctan\left(\frac{1}{2r^2}\right) + \arctan\left(\frac{1}{2(k+1)^2}\right) \quad (\text{M1})$$

$$= \arctan\left(\frac{k}{k+1}\right) + \arctan\left(\frac{1}{2(k+1)^2}\right) \quad \text{A1}$$

$$= \arctan\left(\frac{\frac{k}{k+1} + \frac{1}{2(k+1)^2}}{1 - \left(\frac{k}{k+1}\right)\left(\frac{1}{2(k+1)^2}\right)}\right) \quad \text{M1}$$

$$= \arctan\left(\frac{(k+1)(2k^2 + 2k + 1)}{2(k+1)^3 - k}\right) \quad \text{A1}$$

Note: Award **A1** for correct numerator, with $(k+1)$ factored. Denominator does not need to be simplified

$$= \arctan\left(\frac{(k+1)(2k^2 + 2k + 1)}{2k^3 + 6k^2 + 5k + 2}\right) \quad \text{A1}$$

Note: Award **A1** for denominator correctly expanded. Numerator does not need to be simplified. These two **A** marks may be awarded in any order

$$= \arctan\left(\frac{(k+1)(2k^2 + 2k + 1)}{(k+2)(2k^2 + 2k + 1)}\right) = \arctan\left(\frac{k+1}{k+2}\right) \quad \text{A1}$$

Note: The word ‘arctan’ must be present to be able to award the last three A marks

continued...

Question 12 continued

$P(k+1)$ is true whenever $P(k)$ is true and $P(1)$ is true, so

$P(n)$ is true for $n \in \mathbb{Z}^+$

R1

Note: Award the final **R1** mark provided at least four of the previous marks have been awarded.

Note: To award the final **R1**, the truth of $P(k)$ must be mentioned. ' $P(k)$ implies $P(k+1)$ ' is insufficient to award the mark.

[9 marks]**Total [19 marks]**

3. Amplitude is $\frac{110}{2} = 55$ **(A1)**

$a = -55$ **A1**

$c = 65$ **A1**

$\frac{2\pi}{b} = 20$ OR $-55\cos(20b) + 65 = 10$ **(M1)**

$b = \frac{\pi}{10} (= 0.314)$ **A1**

Total [5 marks]

- 6 (a) attempt to find a vector perpendicular to Π_1 and Π_2
using a cross product

(M1)

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = (2 - (-2))\mathbf{i} + (1 - 3)\mathbf{j} + (-6 - 2)\mathbf{k}$$

$$= \begin{pmatrix} 4 \\ -2 \\ -8 \end{pmatrix} = 2 \begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix}$$

(A1)

equation is $4x - 2y - 8z = 0 (\Rightarrow 2x - y - 4z = 0)$

A1**[3 marks]**

- (b) attempt to solve 3 simultaneous equations in 3 variables

(M1)

$$\left(\frac{41}{21}, -\frac{10}{21}, \frac{23}{21} \right) = (1.95, -0.476, 1.10)$$

A1**[2 marks]****Total [5 marks]**

2. (a) $A = s^2$ and $P = 4s$ (A1)
- $$A = P \Rightarrow s^2 = 4s$$
- (M1)
- $$s(s - 4) = 0$$
- $$\Rightarrow s = 4 (s > 0)$$
- A1

Note: Award **A1M1A0** if both $s = 4$ and $s = 0$ are stated as final answers.

[3 marks]

- (b) $A_T = \frac{1}{2}x^2 \sin \frac{2\pi}{n}$ A1

Note: Award **A1** for a correct alternative form expressed in terms of x and n only.

For example, using Pythagoras' theorem, $A_T = x \sin \frac{\pi}{n} \sqrt{x^2 - x^2 \sin^2 \frac{\pi}{n}}$ or

$$A_T = 2 \left(\frac{1}{2} \left(x \sin \frac{\pi}{n} \right) \left(x \cos \frac{\pi}{n} \right) \right) \text{ or } A_T = x^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n}.$$

[1 mark]

continued...

Question 2 continued(c) **METHOD 1**

uses $\sin \theta = \frac{\text{opp}}{\text{hyp}}$ **(M1)**

$$\frac{\frac{y}{2}}{x} = \sin \frac{\pi}{n} \quad \text{A1}$$

$$y = 2x \sin \frac{\pi}{n} \quad \text{AG}$$

[2 marks]**METHOD 2**

uses Pythagoras' theorem $\left(\frac{y}{2}\right)^2 + h^2 = x^2$ and $h = x \cos \frac{\pi}{n}$ **(M1)**

$$\left(\frac{y}{2}\right)^2 + \left(x \cos \frac{\pi}{n}\right)^2 = x^2 \quad \left(y^2 = 4x^2 \left(1 - \cos^2 \frac{\pi}{n}\right)\right)$$

$$= 4x^2 \sin^2 \frac{\pi}{n} \quad \text{A1}$$

$$y = 2x \sin \frac{\pi}{n} \quad \text{AG}$$

[2 marks]*continued...*

*Question 2 continued***METHOD 3**

uses the cosine rule

(M1)

$$y^2 = 2x^2 - 2x^2 \cos \frac{2\pi}{n} \left(= 2x^2 \left(1 - \cos \frac{2\pi}{n} \right) \right)$$

$$= 4x^2 \sin^2 \frac{\pi}{n}$$

A1

$$y = 2x \sin \frac{\pi}{n}$$

AG**[2 marks]****METHOD 4**

uses the sine rule

(M1)

$$\frac{y}{\sin \frac{2\pi}{n}} = \frac{x}{\sin \left(\frac{\pi}{2} - \frac{\pi}{n} \right)}$$

$$y \cos \frac{\pi}{n} = 2x \sin \frac{\pi}{n} \cos \frac{\pi}{n}$$

A1

$$y = 2x \sin \frac{\pi}{n}$$

AG**[2 marks]***continued...*

Question 2 continued

$$(d) \quad A = P \Rightarrow nA_T = ny \quad (M1)$$

Note: Award **M1** for equating correct expressions for A and P .

$$\frac{1}{2}nx^2 \sin \frac{2\pi}{n} = 2nx \sin \frac{\pi}{n} \left(nx^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} = 2nx \sin \frac{\pi}{n} \right)$$

$$\frac{1}{2}x^2 \sin \frac{2\pi}{n} = 2x \sin \frac{\pi}{n} \left(x^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} = 2x \sin \frac{\pi}{n} \right) \quad A1$$

$$\text{uses } \sin \frac{2\pi}{n} = 2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} \text{ (seen anywhere in part (d) or in part (b))} \quad (M1)$$

$$x^2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} = 2x \sin \frac{\pi}{n}$$

attempts to either factorise or divide their expression (M1)

$$x \sin \frac{\pi}{n} \left(x \cos \frac{\pi}{n} - 2 \right) = 0$$

$$x = \frac{2}{\cos \frac{\pi}{n}}, \left(x \sin \frac{\pi}{n} \neq 0 \right) \text{ (or equivalent)} \quad A1$$

continued...

Question 2 continued

EITHER

substitutes $x = \frac{2}{\cos \frac{\pi}{n}}$ (or equivalent) into $P = ny$ **(M1)**

$$P = 2n \left(\frac{2}{\cos \frac{\pi}{n}} \right) \left(\sin \frac{\pi}{n} \right) \quad \textbf{A1}$$

Note: Other approaches are possible. For example, award **A1** for $P = 2nx \cos \frac{\pi}{n} \tan \frac{\pi}{n}$ and **M1** for substituting $x = \frac{2}{\cos \frac{\pi}{n}}$ into P .

OR

substitutes $x = \frac{2}{\cos \frac{\pi}{n}}$ (or equivalent) into $A = nA_T$ **(M1)**

$$A = \frac{1}{2}n \left(\frac{2}{\cos \frac{\pi}{n}} \right)^2 \left(\sin \frac{2\pi}{n} \right)$$

$$A = \frac{1}{2}n \left(\frac{2}{\cos \frac{\pi}{n}} \right)^2 \left(2 \sin \frac{\pi}{n} \cos \frac{\pi}{n} \right) \quad \textbf{A1}$$

THEN

$$A = P = 4n \tan \frac{\pi}{n} \quad \textbf{AG}$$

[7 marks]

continued...

Question 2 continued

- (e) (i) attempts to use the Maclaurin series for $\tan x$ with $x = \frac{\pi}{n}$ **(M1)**

$$\tan \frac{\pi}{n} = \frac{\pi}{n} + \frac{\left(\frac{\pi}{n}\right)^3}{3} + \frac{2\left(\frac{\pi}{n}\right)^5}{15} (+\dots)$$

$$4n \tan \frac{\pi}{n} = 4n \left(\frac{\pi}{n} + \frac{\pi^3}{3n^3} + \frac{2\pi^5}{15n^5} (+\dots) \right) \text{ (or equivalent)} \quad \textbf{A1}$$

$$= 4 \left(\pi + \frac{\pi^3}{3n^2} + \frac{2\pi^5}{15n^4} + \dots \right)$$

$$\Rightarrow \lim_{n \rightarrow \infty} \left(4n \tan \frac{\pi}{n} \right) = 4\pi \quad \textbf{A1}$$

Note: Award a maximum of **M1A1A0** if $\lim_{n \rightarrow \infty}$ is not stated anywhere.

[3 marks]

- (ii) (as $n \rightarrow \infty$, $P \rightarrow 4\pi$ and $A \rightarrow 4\pi$)
the polygon becomes a circle of radius 2

R1

Note: Award **R1** for alternative responses such as:

the polygon becomes a circle of area 4π OR

the polygon becomes a circle of perimeter 4π OR

the polygon becomes a circle with $A = P = 4\pi$.

Award **R0** for polygon becomes a circle.

[1 mark]

$$(f) \quad A = \frac{1}{2}ab \text{ and } P = a + b + \sqrt{a^2 + b^2} \quad (A1)(A1)$$

equates their expressions for A and P **M1**

$$A = P \Rightarrow a + b + \sqrt{a^2 + b^2} = \frac{1}{2}ab$$

$$\sqrt{a^2 + b^2} = \frac{1}{2}ab - (a + b) \quad \mathbf{M1}$$

Note: Award **M1** for isolating $\sqrt{a^2 + b^2}$ or $\pm 2\sqrt{a^2 + b^2}$. This step may be seen later.

$$a^2 + b^2 = \left(\frac{1}{2}ab - (a + b) \right)^2$$

$$a^2 + b^2 = \frac{1}{4}a^2b^2 - 2\left(\frac{1}{2}ab\right)(a + b) + (a + b)^2 \quad \mathbf{M1}$$

$$\left(= \frac{1}{4}a^2b^2 - a^2b - ab^2 + a^2 + 2ab + b^2 \right)$$

Note: Award **M1** for attempting to expand their RHS of either $a^2 + b^2 = \dots$
or $4(a^2 + b^2) = \dots$.

EITHER

$$ab\left(\frac{1}{4}ab - a - b + 2\right) = 0 \quad (ab \neq 0) \quad \mathbf{A1}$$

$$\frac{1}{4}ab - a - b + 2 = 0$$

$$ab - 4a = 4b - 8$$

OR

$$\frac{1}{4}a^2b^2 - a^2b - ab^2 + 2ab = 0$$

$$a\left(\frac{1}{4}b^2 - b\right) + (2b - b^2) = 0 \quad (a(b^2 - 4b) + (8b - 4b^2) = 0) \quad \mathbf{A1}$$

$$a = \frac{4b^2 - 8b}{b^2 - 4b}$$

continued...

Question 2 continued

THEN

$$\Rightarrow a = \frac{4b-8}{b-4}$$

A1

$$a = \frac{4b-16+8}{b-4}$$

$$a = \frac{8}{b-4} + 4$$

AG

Note: Award a maximum of **A1A1M1M1M0A0A0** for attempting to verify.

For example, verifying that $A = P = \frac{16}{b-4} + 2b + 4$ gains 4 of the 7 marks.

[7 marks]

continued...

Question 2 continued

- (g) (i) using an appropriate method **(M1)**
 eg substituting values for b or using divisibility properties
 $(5,12,13)$ and $(6,8,10)$ **A1A1**

Note: Award **A1A0** for either one set of three correct side lengths or two sets of two correct side lengths.

- [3 marks]**
- (ii) $A = P = 30$ and $A = P = 24$ **A1**

Note: Do not award **A1FT**.

[1 mark]
Total [28 marks]

6. (a) METHOD 1

attempt to write all LHS terms over a common denominator of $x-1$ **(M1)**

$$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1}\text{ OR }\frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$$

$$=\frac{2x^2-2x-3x+3-6}{x-1}\text{ OR }\frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$$
 A1

$$=\frac{2x^2-5x-3}{x-1}$$
 AG

[2 marks]

METHOD 2

attempt to use algebraic division on RHS **(M1)**

correctly obtains quotient of $2x-3$ and remainder -6 **A1**

$$=2x-3-\frac{6}{x-1}\text{ as required.}$$
 AG

[2 marks]

continue...

Question 6 continued.

(b) consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ (M1)

$$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$$

EITHER

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ (M1)

Note: Accept any variable in place of $\sin 2\theta$.

$$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$$

OR

attempt to substitute into quadratic formula (M1)

$$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$$

THEN

$$\sin 2\theta = -\frac{1}{2} \text{ or } \sin 2\theta = 3$$
 (A1)

Note: Award **A1** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) (A1)

$$\theta = \frac{7\pi}{12}, \frac{11\pi}{12} \text{ (must be in radians)}$$
 (A1)

Note: Award **A0** if additional answers given.

[5 marks]

Total [7 marks]

2. EITHER

attempt to use cosine rule

(M1)

$$12^2 + AB^2 - 2 \times 12 \times \cos 25^\circ \times AB = 7^2 \quad \text{OR} \quad AB^2 - 21.7513...AB + 95 = 0$$

(A1)

at least one correct value for AB

(A1)

$$AB = 6.05068... \quad \text{OR} \quad AB = 15.7007...$$

using their smaller value for AB to find minimum perimeter

(M1)

$$12 + 7 + 6.05068...$$

OR

attempt to use sine rule

(M1)

$$\frac{\sin B}{12} = \frac{\sin 25^\circ}{7} \quad \text{OR} \quad \sin B = 0.724488... \quad \text{OR} \quad \hat{B} = 133.573...^\circ \quad \text{OR} \quad \hat{B} = 46.4263...^\circ$$

(A1)at least one correct value for $\hat{C}$

$$\hat{C} = 21.4263...^\circ \quad \text{OR} \quad \hat{C} = 108.573...^\circ$$

(A1)using their acute value for $\hat{C}$ to find minimum perimeter**(M1)**

$$12 + 7 + \sqrt{12^2 + 7^2 - 2 \times 12 \times 7 \cos 21.4263...^\circ} \quad \text{OR} \quad 12 + 7 + \frac{7 \sin 21.4263...^\circ}{\sin 25^\circ}$$

THEN

$$25.0506...$$

minimum perimeter = 25.1.

A1**Total [5 marks]**

4. (a) attempt to find the area of either shaded region in terms of r and θ (M1)

Note: Do not award **M1** if they have only copied from the booklet and not applied to the shaded area.

$$\text{Area of segment} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta \quad \text{A1}$$

$$\text{Area of triangle} = \frac{1}{2}r^2\sin(\pi - \theta) \quad \text{A1}$$

correct equation in terms of θ only (A1)

$$\theta - \sin\theta = \sin(\pi - \theta)$$

$$\theta - \sin\theta = \sin\theta \quad \text{A1}$$

$$\theta = 2\sin\theta \quad \text{AG}$$

Note: Award a maximum of **M1A1A0A0A0** if a candidate uses degrees

(i.e., $\frac{1}{2}r^2\sin(180^\circ - \theta)$), even if later work is correct.

Note: If a candidate directly states that the area of the triangle is

$\frac{1}{2}r^2\sin\theta$, award a maximum of **M1A1A0A1A1**.

[5 marks]

- (b) $\theta = 1.89549\dots$

$$\theta = 1.90 \quad \text{A1}$$

Note: Award **A0** if there is more than one solution. Award **A0** for an answer in degrees.

[1 mark]

Total [6 marks]

9. (a) $12 = \frac{2\pi}{b}$ OR $b = \frac{2\pi}{12}$

A1

$$b = \frac{\pi}{6}$$

AG

[1 mark]

(b) $a = \frac{6.8 - 2.2}{2}$ OR $a = \frac{\text{max} - \text{min}}{2}$

(M1)

$$= 2.3 \text{ (m)}$$

A1

[2 marks]

(c) $d = \frac{6.8 + 2.2}{2}$ OR $d = \frac{\text{max} + \text{min}}{2}$

(M1)

$$= 4.5 \text{ (m)}$$

A1

[2 marks]

continued...

Question 9 continued.

(d) **METHOD 1**

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for H (A1)

$$6.8 = 2.3 \sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$$

attempt to solve their equation (M1)

$$c = 1.5 \quad \text{A1}$$

METHOD 2

using horizontal translation of $\frac{12}{4}$ (M1)

$$4.5 - c = 3 \quad \text{(A1)}$$

$$c = 1.5 \quad \text{A1}$$

METHOD 3

$$H'(t) = (2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(t - c)\right) \quad \text{(A1)}$$

attempts to solve their $H'(4.5) = 0$ for c (M1)

$$(2.3)\left(\frac{\pi}{6}\right)\cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$$

$$c = 1.5 \quad \text{A1}$$

[3 marks]

(e) attempt to find H when $t = 12$ or $t = 0$, graphically or algebraically (M1)

$$H = 2.87365\dots$$

$$H = 2.87(\text{m}) \quad \text{A1}$$

[2 marks]

continue...

Question 9 continued.

(f) attempt to solve $5 = 2.3\sin\left(\frac{\pi}{6}(t-1.5)\right) + 4.5$ (M1)

times are $t = 1.91852\dots$ and $t = 7.08147\dots$, ($t = 13.9185\dots, t = 19.0814\dots$) (A1)

total time is $2 \times (7.081\dots - 1.919\dots)$

$10.3258\dots$

$= 10.3$ (hours)

A1

Note: Accept 10.

[3 marks]

(g) **METHOD 1**

substitutes $t = \frac{11}{3}$ and $H = 6.8$ into their equation for H and attempts to solve for c (M1)

$$6.8 = 2.3\sin\left(\frac{\pi}{6}\left(\frac{11}{3} - c\right)\right) + 4.5 \Rightarrow c = \frac{2}{3}$$

$$H(t) = 2.3\sin\left(\frac{\pi}{6}\left(t - \frac{2}{3}\right)\right) + 4.5$$

A1

METHOD 2

uses their horizontal translation $\left(\frac{12}{4} = 3\right)$ (M1)

$$\frac{11}{3} - c = 3 \Rightarrow c = \frac{2}{3}$$

$$H(t) = 2.3\sin\left(\frac{\pi}{6}\left(t - \frac{2}{3}\right)\right) + 4.5$$

A1

[2 marks]

Total [15 marks]

11. (a) (i) attempts to find either $\vec{AB}$ or $\vec{AC}$ (M1)

$$\vec{AB} = \begin{pmatrix} -3 \\ -2 \\ 0 \end{pmatrix} \text{ and } \vec{AC} = \begin{pmatrix} -2 \\ 1 \\ -7 \end{pmatrix} \quad \text{A1}$$

- (ii) **METHOD 1**

attempts to find $\vec{AB} \times \vec{AC}$ (M1)

$$\vec{AB} \times \vec{AC} = \begin{pmatrix} 14 \\ -21 \\ -7 \end{pmatrix} \quad \text{A1}$$

EITHER

equation of plane is of the form $14x - 21y - 7z = d$ ($2x - 3y - z = d$) (A1)

substitutes a valid point e.g. $(3, 0, 0)$ to obtain a value of d (M1)

$$d = 42 \quad (d = 6)$$

OR

attempts to use $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$ (M1)

$$\mathbf{r} \cdot \begin{pmatrix} 14 \\ -21 \\ -7 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 14 \\ -21 \\ -7 \end{pmatrix} \quad \left(\mathbf{r} \cdot \begin{pmatrix} 14 \\ -21 \\ -7 \end{pmatrix} = 42 \right) \quad \text{A1}$$

$$\mathbf{r} \cdot \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} \quad \left(\mathbf{r} \cdot \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = 6 \right)$$

THEN

$$14x - 21y - 7z = 42 \quad (2x - 3y - z = 6) \quad \text{A1}$$

METHOD 2

$$\text{equation of plane is of the form } \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} + s \begin{pmatrix} -3 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \\ -7 \end{pmatrix} \quad \text{A1}$$

attempts to form equations for x, y, z in terms of their parameters (M1)

$$x = 3 - 3s - 2t, \quad y = -2s + t, \quad z = -7t$$

A1

eliminates at least one of their parameters (M1)

$$\text{for example, } 2x - 3y = 6 - 7t \quad (\Rightarrow 2x - 3y = 6 + z)$$

$$2x - 3y - z = 6$$

A1

[7 marks]

(b) **METHOD 1**

substitutes $\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ into their Π_1 and Π_2 (given) **(M1)**

$$\Pi_1: 2\lambda - 3(-2 + \lambda) - (-\lambda) = 6 \text{ and } \Pi_2: 3\lambda - (-2 + \lambda) + 2(-\lambda) = 2 \quad \textbf{A1}$$

Note: Award **(M1)A0** for correct verification using a specific value of λ .

so the vector equation of L can be written as $\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ **AG**

METHOD 2
EITHER

attempts to find $\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} \times \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ **M1**

$$= \begin{pmatrix} -7 \\ -7 \\ 7 \end{pmatrix}$$

OR

$$\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = (2 - 3 + 1) = 0 \text{ and } \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = (3 - 1 - 2) = 0 \quad \textbf{M1}$$

THEN

substitutes $(0, -2, 0)$ into Π_1 and Π_2

$$\Pi_1: 2(0) - 3(-2) - (0) = 6 \text{ and } \Pi_2: 3(0) - (-2) + 2(0) = 2 \quad \textbf{A1}$$

so the vector equation of L can be written as $\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ **AG**

METHOD 3

attempts to solve $2x - 3y - z = 6$ and $3x - y + 2z = 2$ **(M1)**

for example, $x = -\lambda, y = -2 - \lambda, z = \lambda$ **A1**

Note: Award **A1** for substituting $x = 0$ (or $y = -2$ or $z = 0$) into Π_1 and Π_2 and solving simultaneously. For example, solving $-3y - z = 6$ and $-y + 2z = 2$ to obtain $y = -2$ and $z = 0$.

so the vector equation of L can be written as $\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

AG**[2 marks]**

- (c) (i) substitutes the equation of L into the equation of Π_3

(M1)

$$2\lambda + 2\lambda = 3 \Rightarrow 4\lambda = 3$$

A1

$$\lambda = \frac{3}{4}$$

AG

- (ii) P has coordinates $\left(\frac{3}{4}, -\frac{5}{4}, -\frac{3}{4}\right)$

A1**[3 marks]**

- (d) (i) normal to Π_3 is $\mathbf{n} = \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$

(A1)

Note: May be seen or used anywhere.

considers the line normal to Π_3 passing through $B(0, -2, 0)$

(M1)

$$\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix}$$

A1**EITHER**

finding the point on the normal line that intersects Π_3

attempts to solve simultaneously with plane $2x - 2z = 3$

(M1)

$$4\mu + 4\mu = 3$$

$$\mu = \frac{3}{8}$$

A1

point is $\left(\frac{3}{4}, -2, -\frac{3}{4}\right)$

OR

$$\left(\begin{pmatrix} 2\mu \\ -2 \\ -2\mu \end{pmatrix} - \begin{pmatrix} \frac{3}{4} \\ -\frac{5}{4} \\ -\frac{3}{4} \end{pmatrix} \right) \cdot \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} = 0 \quad (M1)$$

$$4\mu - \frac{3}{2} + 4\mu - \frac{3}{2} = 0$$

$$\mu = \frac{3}{8} \quad A1$$

OR

attempts to find the equation of the plane parallel to Π_3 containing B' ($x - z = 3$) and solve simultaneously with L (M1)

$$2\mu' + 2\mu' = 3$$

$$\mu' = \frac{3}{4} \quad A1$$

THEN

so, another point on the reflected line is given by

$$\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \frac{3}{4} \begin{pmatrix} 2 \\ 0 \\ -2 \end{pmatrix} \quad (A1)$$

$$\Rightarrow B' \left(\frac{3}{2}, -2, -\frac{3}{2} \right) \quad A1$$

(ii) **EITHER**

attempts to find the direction vector of the reflected line using their P and B' (M1)

$$\vec{PB'} = \begin{pmatrix} \frac{3}{4} \\ -\frac{3}{4} \\ -\frac{3}{4} \end{pmatrix}$$

OR

attempts to find their direction vector of the reflected line using a vector approach (M1)

$$\vec{PB'} = \vec{PB} + \vec{BB'} = -\frac{3}{4} \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} + \frac{3}{2} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

THEN

$$\mathbf{r} = \begin{pmatrix} \frac{3}{2} \\ -2 \\ -\frac{3}{2} \end{pmatrix} + \lambda \begin{pmatrix} \frac{3}{4} \\ -\frac{3}{4} \\ -\frac{3}{4} \end{pmatrix} \text{ (or equivalent)} \quad A1$$

Note: Award **A0** for either ' $\mathbf{r} =$ ' or ' $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ ' not stated. Award **A0** for ' $\mathbf{L}' =$ '.

[9 marks]

Total [21 marks]

6. (a) attempting to use the change of base rule

M1

$$\log_9(\cos 2x + 2) = \frac{\log_3(\cos 2x + 2)}{\log_3 9}$$

A1

$$= \frac{1}{2} \log_3(\cos 2x + 2)$$

A1

$$= \log_3 \sqrt{\cos 2x + 2}$$

AG**[3 marks]**

(b) $\log_3(2 \sin x) = \log_3 \sqrt{\cos 2x + 2}$

$$2 \sin x = \sqrt{\cos 2x + 2}$$

M1

$$4 \sin^2 x = \cos 2x + 2 \text{ (or equivalent)}$$

A1

$$\text{use of } \cos 2x = 1 - 2 \sin^2 x$$

(M1)

$$6 \sin^2 x = 3$$

$$\sin x = (\pm) \frac{1}{\sqrt{2}}$$

A1

$$x = \frac{\pi}{4}$$

A1

Note: Award **A0** if solutions other than $x = \frac{\pi}{4}$ are included.

[5 marks]**Total [8 marks]**

8. recognition that the angle between the normal and the line is 60° (seen anywhere) **R1**
 attempt to use the formula for the scalar product **M1**

$$\cos 60^\circ = \frac{\left| \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ p \end{pmatrix} \right|}{\sqrt{9} \times \sqrt{1+4+p^2}} \quad \textbf{A1}$$

$$\frac{1}{2} = \frac{|2p|}{3\sqrt{5+p^2}} \quad \textbf{A1}$$

$$3\sqrt{5+p^2} = 4|p|$$

attempt to square both sides **M1**

$$9(5+p^2) = 16p^2 \Rightarrow 7p^2 = 45$$

$$p = \pm 3\sqrt{\frac{5}{7}} \text{ (or equivalent)} \quad \textbf{A1A1}$$

Total [7 marks]

1. (a) METHOD 1

attempt to use the cosine rule

(M1)

$$\cos \theta = \frac{4^2 + 4^2 - 5^2}{2 \times 4 \times 4} \quad (\text{or equivalent})$$

A1

$$\theta = 1.35$$

A1**[3 marks]****METHOD 2**

attempt to split triangle AOB into two congruent right triangles

(M1)

$$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$$

A1

$$\theta = 1.35$$

A1**[3 marks]****(b) attempt to find the area of the shaded region****(M1)**

$$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35\dots)$$

A1

$$= 39.5 \text{ (cm}^2\text{)}$$

A1**[3 marks]****Total [6 marks]**

7. attempting to find $\mathbf{r}_B - \mathbf{r}_A$ for example

(M1)

$$\mathbf{r}_B - \mathbf{r}_A = \begin{pmatrix} 3 \\ -6 \end{pmatrix} + t \begin{pmatrix} -5 \\ 4 \end{pmatrix}$$

attempting to find $|\mathbf{r}_B - \mathbf{r}_A|$

M1

$$\text{distance } d(t) = \sqrt{(3-5t)^2 + (4t-6)^2} \left(= \sqrt{41t^2 - 78t + 45} \right)$$

A1

using a graph to find the d – coordinate of the local minimum

M1

$$\text{the minimum distance between the ships is } 2.81 \text{ (km)} \left(= \frac{11\sqrt{41}}{41} \text{ (km)} \right)$$

A1

Total [5 marks]

12. (a) stating the relationship between $\cot$ and $\tan$ and stating the identity for $\tan 2\theta$

M1

$$\cot 2\theta = \frac{1}{\tan 2\theta} \text{ and } \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\Rightarrow \cot 2\theta = \frac{1 - \tan^2 \theta}{2 \tan \theta}$$

AG

[1 mark]

- (b) **METHOD 1**

attempting to substitute $\tan \theta$ for x and using the result from (a)

M1

$$\text{LHS} = \tan^2 \theta + 2 \tan \theta \left(\frac{1 - \tan^2 \theta}{2 \tan \theta} \right) - 1$$

A1

$$\tan^2 \theta + 1 - \tan^2 \theta - 1 = 0 (= \text{RHS})$$

A1

so $x = \tan \theta$ satisfies the equation

AG

attempting to substitute $-\cot \theta$ for x and using the result from (a)

M1

$$\text{LHS} = \cot^2 \theta - 2 \cot \theta \left(\frac{1 - \tan^2 \theta}{2 \tan \theta} \right) - 1$$

A1

$$= \frac{1}{\tan^2 \theta} - \left(\frac{1 - \tan^2 \theta}{\tan^2 \theta} \right) - 1$$

A1

$$\frac{1}{\tan^2 \theta} - \frac{1}{\tan^2 \theta} + 1 - 1 = 0 (= \text{RHS})$$

A1

so $x = -\cot \theta$ satisfies the equation

AG

METHOD 2

let $\alpha = \tan \theta$ and $\beta = -\cot \theta$

attempting to find the sum of roots

M1

$$\alpha + \beta = \tan \theta - \frac{1}{\tan \theta}$$

$$= \frac{\tan^2 \theta - 1}{\tan \theta}$$

A1

$$= -2 \cot 2\theta \text{ (from part (a))}$$

A1

attempting to find the product of roots

M1

$$\alpha\beta = \tan \theta \times (-\cot \theta)$$

A1

$$= -1$$

A1

the coefficient of x and the constant term in the quadratic are $2 \cot 2\theta$ and -1 respectively

R1

hence the two roots are $\alpha = \tan \theta$ and $\beta = -\cot \theta$

AG

[7 marks]

continued...

Question 12 continued

(c) **METHOD 1**

$$x = \tan \frac{\pi}{12} \text{ and } x = -\cot \frac{\pi}{12} \text{ are roots of } x^2 + \left(2 \cot \frac{\pi}{6}\right)x - 1 = 0 \quad \mathbf{R1}$$

Note: Award **R1** if only $x = \tan \frac{\pi}{12}$ is stated as a root of $x^2 + \left(2 \cot \frac{\pi}{6}\right)x - 1 = 0$.

$$x^2 + 2\sqrt{3}x - 1 = 0 \quad \mathbf{A1}$$

attempting to solve **their** quadratic equation **M1**

$$x = -\sqrt{3} \pm 2 \quad \mathbf{A1}$$

$$\tan \frac{\pi}{12} > 0 \quad (-\cot \frac{\pi}{12} < 0) \quad \mathbf{R1}$$

$$\text{so } \tan \frac{\pi}{12} = 2 - \sqrt{3} \quad \mathbf{AG}$$

METHOD 2

attempting to substitute $\theta = \frac{\pi}{12}$ into the identity for $\tan 2\theta$ **M1**

$$\tan \frac{\pi}{6} = \frac{2 \tan \frac{\pi}{12}}{1 - \tan^2 \frac{\pi}{12}}$$

$$\tan^2 \frac{\pi}{12} + 2\sqrt{3} \tan \frac{\pi}{12} - 1 = 0 \quad \mathbf{A1}$$

attempting to solve **their** quadratic equation **M1**

$$\tan \frac{\pi}{12} = -\sqrt{3} \pm 2 \quad \mathbf{A1}$$

$$\tan \frac{\pi}{12} > 0 \quad \mathbf{R1}$$

$$\text{so } \tan \frac{\pi}{12} = 2 - \sqrt{3} \quad \mathbf{AG}$$

[5 marks]

(d) $\tan \frac{\pi}{24} - \cot \frac{\pi}{24}$ is the sum of the roots of $x^2 + \left(2 \cot \frac{\pi}{12}\right)x - 1 = 0$ **R1**

$$\tan \frac{\pi}{24} - \cot \frac{\pi}{24} = -2 \cot \frac{\pi}{12} \quad \mathbf{A1}$$

$$= \frac{-2}{2 - \sqrt{3}} \quad \mathbf{A1}$$

attempting to rationalise **their** denominator **(M1)**

$$= -4 - 2\sqrt{3} \quad \mathbf{A1A1}$$

[6 marks]**Total [19 marks]**

1. (a) METHOD 1

consider right-angled triangle OCX where $CX = \frac{x}{2}$

$$\sin \frac{\pi}{3} = \frac{\frac{x}{2}}{1}$$

M1A1

$$\Rightarrow \frac{x}{2} = \frac{\sqrt{3}}{2} \Rightarrow x = \sqrt{3}$$

A1

$$P_i = 3 \times x = 3\sqrt{3}$$

AG**METHOD 2**

eg use of the cosine rule $x^2 = 1^2 + 1^2 - 2(1)(1)\cos \frac{2\pi}{3}$

M1A1

$$x = \sqrt{3}$$

A1

$$P_i = 3 \times x = 3\sqrt{3}$$

AG

Note: Accept use of sine rule.

[3 marks]

(b) $\sin \frac{\pi}{4} = \frac{1}{x}$ where x = side of square

M1

$$x = \sqrt{2}$$

A1

$$P_i = 4\sqrt{2}$$

A1**[3 marks]**

(c) 6 equilateral triangles $\Rightarrow x = 1$

A1

$$P_i = 6$$

A1**[2 marks]**

(d) in right-angled triangle $\sin\left(\frac{\pi}{n}\right) = \frac{\frac{x}{2}}{1}$

M1

$$\Rightarrow x = 2 \sin\left(\frac{\pi}{n}\right)$$

A1

$$P_i = n \times x$$

$$P_i = n \times 2 \sin\left(\frac{\pi}{n}\right)$$

M1

$$P_i = 2n \sin\left(\frac{\pi}{n}\right)$$

AG**[3 marks]***continued...*

Question 1 continued

(e) consider $\lim_{n \rightarrow \infty} 2n \sin\left(\frac{\pi}{n}\right)$

use of $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

M1

$$2n \sin\left(\frac{\pi}{n}\right) = 2n \left(\frac{\pi}{n} - \frac{\pi^3}{6n^3} + \frac{\pi^5}{120n^5} - \dots \right)$$

(A1)

$$= 2 \left(\pi - \frac{\pi^3}{6n^2} + \frac{\pi^5}{120n^4} - \dots \right)$$

A1

$$\Rightarrow \lim_{n \rightarrow \infty} 2n \sin\left(\frac{\pi}{n}\right) = 2\pi$$

A1

as $n \rightarrow \infty$ polygon becomes a circle of radius 1 and $P_i = 2\pi$

R1**[5 marks]**

(f) consider an n -sided polygon of side length x

$2n$ right-angled triangles with angle $\frac{2\pi}{2n} = \frac{\pi}{n}$ at centre

M1A1

opposite side $\frac{x}{2} = \tan\left(\frac{\pi}{n}\right) \Rightarrow x = 2 \tan\left(\frac{\pi}{n}\right)$

M1A1

Perimeter $P_c = 2n \tan\left(\frac{\pi}{n}\right)$

AG**[4 marks]**

(g) consider $\lim_{n \rightarrow \infty} 2n \tan\left(\frac{\pi}{n}\right) = \lim_{n \rightarrow \infty} \left(\frac{2 \tan\left(\frac{\pi}{n}\right)}{\frac{1}{n}} \right)$

$$= \lim_{n \rightarrow \infty} \left(\frac{2 \tan\left(\frac{\pi}{n}\right)}{\frac{1}{n}} \right) = \frac{0}{0}$$

R1

attempt to use L'Hopital's rule

M1

$$= \lim_{n \rightarrow \infty} \left(\frac{-\frac{2\pi}{n^2} \sec^2\left(\frac{\pi}{n}\right)}{-\frac{1}{n^2}} \right)$$

A1A1

$$= 2\pi$$

A1**[5 marks]**

continued...

Question 1 continued

(h) $P_i < 2\pi < P_c$

$$2n \sin\left(\frac{\pi}{n}\right) < 2\pi < 2n \tan\left(\frac{\pi}{n}\right)$$

M1

$$n \sin\left(\frac{\pi}{n}\right) < \pi < n \tan\left(\frac{\pi}{n}\right)$$

A1**[2 marks]**

- (i) attempt to find the lower bound and upper bound approximations within
0.005 of π
 $n = 46$

(M1)**A2****[3 marks]****Total [30 marks]**

3. (a) $(f \circ g)(x) = f(2x)$ **(A1)**

$$f(2x) = \sqrt{3} \sin 2x + \cos 2x$$
A1

[2 marks]

(b) $\sqrt{3} \sin 2x + \cos 2x = 2 \cos 2x$

$$\sqrt{3} \sin 2x = \cos 2x$$

recognizing to use $\tan$ or $\cot$ **M1**

$$\tan 2x = \frac{1}{\sqrt{3}} \text{ OR } \cot 2x = \sqrt{3} \text{ (values may be seen in right triangle)}$$
(A1)

$$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6} \text{ (seen anywhere) (accept degrees)}$$
(A1)

$$2x = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12}$$
A1A1

Note: Do not award the final **A1** if any additional solutions are seen.

Award **A1A0** for correct answers in degrees.

Award **A0A0** for correct answers in degrees with additional values.

[5 marks]

Total [7 marks]

11. (a) METHOD 1

attempt to eliminate a variable

M1

obtain a pair of equations in two variables

EITHER

$-3x + z = -3$ and

A1

$-3x + z = 44$

A1**OR**

$-5x + y = -7$ and

A1

$-5x + y = 40$

A1**OR**

$3x - z = 3$ and

A1

$3x - z = -\frac{79}{5}$

A1**THEN**the two lines are parallel ($-3 \neq 44$ or $-7 \neq 40$ or $3 \neq -\frac{79}{5}$)**R1****Note:** There are other possible pairs of equations in two variables.To obtain the final **R1**, at least the initial **M1** must have been awarded.

hence the three planes do not intersect

AG**[4 marks]***continued...*

*Question 11 continued***METHOD 2**

$$\text{vector product of the two normals} = \begin{pmatrix} -1 \\ -5 \\ -3 \end{pmatrix} \quad (\text{or equivalent}) \quad \mathbf{A1}$$

$$\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} \quad (\text{or equivalent}) \quad \mathbf{A1}$$

Note: Award **A0** if “ $r =$ ” is missing. Subsequent marks may still be awarded.

$$\text{Attempt to substitute } (1 + \lambda, -2 + 5\lambda, 3\lambda) \text{ in } l_3 \quad \mathbf{M1}$$

$$-9(1 + \lambda) + 3(-2 + 5\lambda) - 2(3\lambda) = 32$$

$$-15 = 32, \text{ a contradiction} \quad \mathbf{R1}$$

hence the three planes do not intersect **AG**

[4 marks]

continued...

*Question 11 continued***METHOD 3**

attempt to eliminate a variable

M1

$$-3y + 5z = 6$$

A1

$$-3y + 5z = 100$$

A1 $0 = 94$, a contradiction**R1****Note:** Accept other equivalent alternatives. Accept other valid methods.To obtain the final **R1**, at least the initial **M1** must have been awarded.

hence the three planes do not intersect

AG**[4 marks]***continued...*

Question 11 continued

(b) (i) $\Pi_1 : 2 + 2 + 0 = 4$ and $\Pi_2 : 1 + 4 + 0 = 5$ **A1**

(ii) **METHOD 1**

attempt to find the vector product of the two normals **M1**

$$\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 \\ -5 \\ -3 \end{pmatrix}$$

A1

$$\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$

A1A1

Note: Award **A1A0** if “ $r =$ ” is missing.
Accept any multiple of the direction vector.

Working for (b)(ii) may be seen in part (a) Method 2. In this case penalize lack of “ $r =$ ” only once.

continued...

*Question 11 continued***METHOD 2**attempt to eliminate a variable from Π_1 and Π_2 **M1**

$$3x - z = 3 \quad \text{OR} \quad 3y - 5z = -6 \quad \text{OR} \quad 5x - y = 7$$

Let $x = t$ substituting $x = t$ in $3x - z = 3$ to obtain

$$z = -3 + 3t \quad \text{and} \quad y = 5t - 7 \quad (\text{for all three variables in parametric form})$$

A1

$$\mathbf{r} = \begin{pmatrix} 0 \\ -7 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}$$

A1A1

Note: Award **A1A0** if “ $r =$ ” is missing.
 Accept any multiple of the direction vector. Accept other position vectors which satisfy both the planes Π_1 and Π_2 .

[5 marks]*continued...*

*Question 11 continued***(c) METHOD 1**

the line connecting L and Π_3 is given by L_1

attempt to substitute position and direction vector to form L_1 **(M1)**

$$\mathbf{s} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + t \begin{pmatrix} -9 \\ 3 \\ -2 \end{pmatrix} \quad \mathbf{A1}$$

substitute $(1-9t, -2+3t, -2t)$ in Π_3 **M1**

$$-9(1-9t) + 3(-2+3t) - 2(-2t) = 32$$

$$94t = 47 \Rightarrow t = \frac{1}{2} \quad \mathbf{A1}$$

attempt to find distance between $(1, -2, 0)$ and their point $\left(-\frac{7}{2}, -\frac{1}{2}, -1\right)$ **(M1)**

$$= \left| \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} -9 \\ 3 \\ -2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} \right| = \frac{1}{2} \sqrt{(-9)^2 + 3^2 + (-2)^2}$$

$$= \frac{\sqrt{94}}{2} \quad \mathbf{A1}$$

[6 marks]

continued...

*Question 11 continued***METHOD 2**

$$\text{unit normal vector equation of } \Pi_3 \text{ is given by } \frac{\begin{pmatrix} -9 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}}{\sqrt{81+9+4}} \quad (\text{M1})$$

$$= \frac{32}{\sqrt{94}} \quad \text{A1}$$

let Π_4 be the plane parallel to Π_3 and passing through P,

then the normal vector equation of Π_4 is given by

$$\begin{pmatrix} -9 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -9 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = -15 \quad \text{M1}$$

unit normal vector equation of Π_4 is given by

$$\frac{\begin{pmatrix} -9 \\ 3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix}}{\sqrt{81+9+4}} = \frac{-15}{\sqrt{94}} \quad \text{A1}$$

$$\text{distance between the planes is } \frac{32}{\sqrt{94}} - \frac{-15}{\sqrt{94}} \quad (\text{M1})$$

$$= \frac{47}{\sqrt{94}} \left(= \frac{\sqrt{94}}{2} \right) \quad \text{A1}$$

[6 marks]

Total [15 marks]

4. determines $\frac{\pi}{4}$ (or 45°) as the first quadrant (reference) angle **(A1)**

attempts to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}$ **(M1)**

Note: Award **M1** for attempting to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}, \frac{7\pi}{4} (, \dots)$

$\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow x < 0$ and so $\frac{\pi}{4}$ is rejected **(R1)**

$\frac{x}{2} + \frac{\pi}{3} = 2\pi - \frac{\pi}{4} \left(= \frac{7\pi}{4} \right)$ **A1**

$x = \frac{17\pi}{6}$ (must be in radians) **A1**

[5 marks]

2. (a) valid approach to find area of segment by finding area of sector – area of triangle (M1)

$$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2\sin\theta \quad (A1)$$

$$\text{area} = 2\theta - 2\sin\theta \quad A1$$

[3 marks]

- (b) EITHER

$$\text{area of logo} = \text{area of rectangle} - \text{area of segments} \quad (M1)$$

$$5 \times 4 - 2 \times (2\theta - 2\sin\theta) = 13.4 \quad (A1)$$

OR

$$\text{area of one segment} = \frac{20 - 13.4}{2} (= 3.3) \quad (M1)$$

$$2\theta - 2\sin\theta = 3.3 \quad (A1)$$

THEN

$$\theta = 2.35672\dots$$

$$\theta = 2.36 \text{ (do not accept an answer in degrees)} \quad A1$$

Note: Award (M1)(A1)A0 if there is more than one solution.
Award (M1)(A1FT)A0 if the candidate works in degrees and obtains a final answer of 135.030...

[3 marks]

Total [6 marks]

7. (a) $|a| = \sqrt{12^2 + (-5)^2} (=13)$ (A1)

$2 \leq |a + b| \leq 28$ (accept min 2 and max 28) A1

Note: Award (A1)A0 for 2 and 28 seen with no indication that they are the endpoints of an interval.

[2 marks]

(b) recognition that p or b is a negative multiple of a (M1)

$$p = -2a \text{ OR } b = -\frac{15}{13}a = -\frac{15}{13}\begin{pmatrix} 12 \\ -5 \end{pmatrix}$$

$$p = -\frac{2}{13}\begin{pmatrix} 12 \\ -5 \end{pmatrix} = \begin{pmatrix} -1.85 \\ 0.769 \end{pmatrix} \quad \text{A1}$$

[2 marks]

(c) **METHOD 1**

q is perpendicular to $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$

$\Rightarrow q$ is in the direction $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$ (M1)

$$q = k \begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad \text{(A1)}$$

$$(|q| =) \sqrt{(5k)^2 + (12k)^2} = 15 \quad \text{(M1)}$$

$$k = \frac{15}{13} \quad \text{(A1)}$$

$$q = \frac{15}{13} \begin{pmatrix} 5 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{75}{13} \\ \frac{180}{13} \end{pmatrix} = \begin{pmatrix} 5.77 \\ 13.8 \end{pmatrix} \quad \text{A1}$$

[5 marks]

continued...

*Question 7 continued***METHOD 2**

$\mathbf{q}$ is perpendicular to $\begin{pmatrix} 12 \\ -5 \end{pmatrix}$

attempt to set scalar product $\mathbf{q} \cdot \mathbf{a} = 0$ OR product of gradients $= -1$ **(M1)**

$$12x - 5y = 0 \quad \text{A1}$$

$$(|\mathbf{q}| =) \sqrt{x^2 + y^2} = 15$$

attempt to solve simultaneously to find a quadratic in x or in y **(M1)**

$$x^2 + \left(\frac{12x}{5}\right)^2 = 15^2 \quad \text{OR} \quad \left(\frac{5y}{12}\right)^2 + y^2 = 15^2$$

$$\mathbf{q} = \begin{pmatrix} \frac{75}{13} \\ \frac{180}{13} \end{pmatrix} \left(= \begin{pmatrix} 5.77 \\ 13.8 \end{pmatrix} \right) \quad \text{A1A1}$$

Note: Award **A1** independently for each value. Accept values given as $x = \frac{75}{13}$

and $y = \frac{180}{13}$ or equivalent.

[5 marks]

Total [9 marks]

1. (a) **EITHER**

uses the cosine rule

(M1)

$$AB^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 1.9$$

(A1)**OR**

uses right-angled trigonometry

(M1)

$$\frac{AB}{5} = \sin 0.95$$

(A1)**OR**

uses the sine rule

(M1)

$$\alpha = \frac{1}{2}(\pi - 1.9) (= 0.6207\dots)$$

$$\frac{AB}{\sin 1.9} = \frac{5}{\sin 0.6207\dots}$$

(A1)**THEN**

$$AB = 8.1341\dots$$

$$AB = 8.13 \text{ (m)}$$

A1**[3 marks]***continued...*

Question 1 continued

- (b) let the shaded area be
- A

METHOD 1

Attempt at finding reflex angle

(M1)

$$\hat{AOB} = 2\pi - 1.9 \quad (= 4.3831\dots)$$

substitution into area formula

(A1)

$$A = \frac{1}{2} \times 5^2 \times 4.3831\dots \quad \text{OR} \quad \left(\frac{2\pi - 1.9}{2\pi} \right) \times \pi(5^2)$$

$$= 54.7898\dots$$

$$= 54.8 \text{ (m}^2\text{)}$$

A1**METHOD 2**let the area of the circle be A_c and the area of the unshaded sector be A_u

$$A = A_c - A_u$$

(M1)

$$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9 \quad (= 78.5398\dots - 23.75)$$

(A1)

$$= 54.7898\dots$$

$$= 54.8 \text{ (m}^2\text{)}$$

A1**[3 marks]****Total [6 marks]**

11. (a) let ϕ be the required angle (bearing)

EITHER

$$\phi = 90^\circ - \arctan \frac{1}{2} \quad (= \arctan 2) \quad \textbf{(M1)}$$

Note: Award **M1** for a labelled sketch.

OR

$$\cos \phi = \frac{\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 2 \end{pmatrix}}{\sqrt{1} \times \sqrt{20}} \quad \left(= 0.4472\dots, = \frac{1}{\sqrt{5}} \right) \quad \textbf{(M1)}$$

$$\phi = \arccos(0.4472\dots)$$

THEN

$$063^\circ$$

A1

Note: Do not accept 063.4° or 63.4° or 1.10° .

[2 marks]

- (b) **Method 1**

let $|\mathbf{b}_A|$ be the speed of A and let $|\mathbf{b}_B|$ be the speed of B

attempts to find the speed of one of A or B

(M1)

$$|\mathbf{b}_A| = \sqrt{(-6)^2 + 2^2 + 4^2} \quad \text{or} \quad |\mathbf{b}_B| = \sqrt{4^2 + 2^2 + (-2)^2}$$

Note: Award **M0** for $|\mathbf{b}_A| = \sqrt{19^2 + (-1)^2 + 1^2}$ and $|\mathbf{b}_B| = \sqrt{1^2 + 0^2 + 12^2}$.

$$|\mathbf{b}_A| = 7.48\dots \quad (= \sqrt{56}) \quad (\text{km min}^{-1}) \quad \text{and} \quad |\mathbf{b}_B| = 4.89\dots \quad (= \sqrt{24}) \quad (\text{km min}^{-1})$$

A1

$|\mathbf{b}_A| > |\mathbf{b}_B|$ so A travels at a greater speed than B

AG

[2 marks]

continued...

*Question 11 continued***Method 2**

attempts to use $\text{speed} = \frac{\text{distance}}{\text{time}}$

$$\text{speed}_A = \frac{|r_A(t_2) - r_A(t_1)|}{t_2 - t_1} \text{ and } \text{speed}_B = \frac{|r_B(t_2) - r_B(t_1)|}{t_2 - t_1} \quad \textbf{(M1)}$$

for example:

$$\text{speed}_A = \frac{|r_A(1) - r_A(0)|}{1} \text{ and } \text{speed}_B = \frac{|r_B(1) - r_B(0)|}{1}$$

$$\text{speed}_A = \frac{\sqrt{(-6)^2 + 2^2 + 4^2}}{1} \text{ and } \text{speed}_B = \frac{\sqrt{4^2 + 2^2 + 2^2}}{1}$$

$$\text{speed}_A = 7.48... \left(2\sqrt{14} \right) \text{ and } \text{speed}_B = 4.89... \left(\sqrt{24} \right) \quad \textbf{A1}$$

$$\text{speed}_A > \text{speed}_B \text{ so } A \text{ travels at a greater speed than } B \quad \textbf{AG}$$

[2 marks]

continued...

Question 11 continued

- (c) attempts to use the angle between two direction vectors formula **(M1)**

$$\cos \theta = \frac{(-6)(4) + (2)(2) + (4)(-2)}{\sqrt{(-6)^2 + 2^2 + 4^2} \sqrt{4^2 + 2^2 + (-2)^2}} \quad \text{**(A1)**}$$

$$\cos \theta = -0.7637... \left(= -\frac{7}{\sqrt{84}} \right) \text{ or } \theta = \arccos(-0.7637...) (= 2.4399...)$$

attempts to find the acute angle $180^\circ - \theta$ using their value of θ **(M1)**

$$= 40.2^\circ \quad \text{**A1**}$$

[4 marks]

continued...

Question 11 continued

- (d) (i) for example, sets $\mathbf{r}_A(t_1) = \mathbf{r}_B(t_2)$ and forms at least two equations (M1)

$$19 - 6t_1 = 1 + 4t_2$$

$$-1 + 2t_1 = 2t_2$$

$$1 + 4t_1 = 12 - 2t_2$$

Note: Award **M0** for equations involving t only.

EITHER

attempts to solve the system of equations for one of t_1 or t_2 (M1)

$$t_1 = 2 \text{ or } t_2 = \frac{3}{2} \quad \text{A1}$$

OR

attempts to solve the system of equations for t_1 and t_2 (M1)

$$t_1 = 2 \text{ and } t_2 = \frac{3}{2} \quad \text{A1}$$

THEN

substitutes their t_1 or t_2 value into the corresponding $\mathbf{r}_A$ or $\mathbf{r}_B$ (M1)

$$P(7, 3, 9) \quad \text{A1}$$

Note: Accept $\vec{OP} = \begin{pmatrix} 7 \\ 3 \\ 9 \end{pmatrix}$. Accept 7 km east of O, 3 km north of O and 9 km above sea level.

continued...

Question 11 continued

- (ii) attempts to find the value of $t_1 - t_2$ **(M1)**

$$t_1 - t_2 = 2 - \frac{3}{2}$$

0.5 minutes (30 seconds)

A1

[7 marks]

continued...

Question 11 continued

(e) **EITHER**attempts to find $\mathbf{r}_B - \mathbf{r}_A$ **(M1)**

$$\mathbf{r}_B - \mathbf{r}_A = \begin{pmatrix} -18 \\ 1 \\ 11 \end{pmatrix} + t \begin{pmatrix} 10 \\ 0 \\ -6 \end{pmatrix}$$

attempts to find their $D(t)$ **(M1)**

$$D(t) = \sqrt{(10t - 18)^2 + 1 + (11 - 6t)^2}$$

A1**OR**attempts to find $\mathbf{r}_A - \mathbf{r}_B$ **(M1)**

$$\mathbf{r}_A - \mathbf{r}_B = \begin{pmatrix} 18 \\ -1 \\ -11 \end{pmatrix} + t \begin{pmatrix} -10 \\ 0 \\ 6 \end{pmatrix}$$

attempts to find their $D(t)$ **(M1)**

$$D(t) = \sqrt{(18 - 10t)^2 + (-1)^2 + (-11 + 6t)^2}$$

A1

Note: Award **MOM0A0** for expressions using two different time parameters.

THEN

either attempts to find the local minimum point of $D(t)$ or attempts to find the value of t such that $D'(t) = 0$ (or equivalent)

(M1)

$$t = 1.8088... \left(= \frac{123}{68} \right)$$

$$D(t) = 1.01459...$$

minimum value of $D(t)$ is $1.01 \left(= \frac{\sqrt{1190}}{34} \right)$ (km)

A1**[5 marks]**

Note: Award **M0** for attempts at the shortest distance between two lines.

Total [20 marks]

2.(a) (i) attempt to find midpoint of A and B **(M1)**centre $(-1, 3, -2)$ (accept vector notation and/or missing brackets) **A1**(ii) attempt to find AB or half of AB or distance between the centre and A (or B) **(M1)**

$$\frac{\sqrt{4^2 + 2^2 + 4^2}}{2} \text{ or } \sqrt{2^2 + 1^2 + 2^2}$$

$$= 3 \quad \text{A1}$$

[4 marks](b) attempt to find the distance between their centre and V
(the perpendicular height of the cone) **(M1)**

$$\sqrt{0^2 + 4^2 + 2^2} \quad \text{OR} \quad \sqrt{(\text{their slant height})^2 - (\text{their radius})^2}$$

$$= \sqrt{20} (= 2\sqrt{5}) \quad \text{(A1)}$$

$$\text{Volume} = \frac{1}{3} \pi 3^2 \sqrt{20}$$

$$= 3\pi\sqrt{20} (= 6\pi\sqrt{5}) \quad \text{A1}$$

[3 marks]**Total [7 marks]**

10. (a) $\cos^2 x - 3\sin^2 x = 0$

valid attempt to reduce equation to one involving one trigonometric function (M1)

$$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3} \quad \text{OR} \quad 1 - \sin^2 x - 3\sin^2 x = 0 \quad \text{OR} \quad \cos^2 x - 3(1 - \cos^2 x) = 0$$

$$\text{OR} \quad \cos 2x - 1 + \cos 2x = 0$$

correct equation (A1)

$$\tan^2 x = \frac{1}{3} \quad \text{OR} \quad \cos^2 x = \frac{3}{4} \quad \text{OR} \quad \sin^2 x = \frac{1}{4} \quad \text{OR} \quad \cos 2x = \frac{1}{2}$$

$$\tan x = \pm \frac{1}{\sqrt{3}} \quad \text{OR} \quad \cos x = \pm \frac{\sqrt{3}}{2} \quad \text{OR} \quad \sin x = (\pm) \frac{1}{2} \quad \text{OR} \quad 2x = \frac{\pi}{3} \left(\frac{5\pi}{3} \right) \quad \text{(A1)}$$

$$x = \frac{\pi}{6}, x = \frac{5\pi}{6} \quad \text{A1A1}$$

Note: Award **M1A1A0A1A0** for candidates who omit the $\pm$ (for tan or cos) and give only $x = \frac{\pi}{6}$.

Award **M1A1A0A0A0** for candidates who omit the $\pm$ (for tan or cos) and give only $x = 30^\circ$.

Award **M1A1A1A1A0** for candidates who give both answers in degrees.

Award **M1A1A1A1A0** for candidates who give both correct answers in radians, but who include additional solutions outside the domain.

Award a maximum of **M1A0A0A1A1** for correct answers with no working.

[5 marks]

continued...

Question 10 continued

- (b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) **(M1)**

$$f'(x) = -2 \cos x \sin x - 6 \sin x \cos x (= -8 \sin x \cos x = -4 \sin 2x) \quad \mathbf{A1}$$

- (ii) valid attempt to solve their $f'(x) = 0$ **(M1)**

at least 2 correct x -coordinates (may be seen in coordinates) **(A1)**

$$x = 0, x = \frac{\pi}{2}, x = \pi$$

Note: Accept additional correct solutions outside the domain.

Award **A0** if any additional incorrect solutions are given.

correct coordinates (may be seen in graph for part (c))

A1A1A1

$$(0, 1), (\pi, 1), \left(\frac{\pi}{2}, -3\right)$$

Note: Award a maximum of **M1A1A1A1A0** if any additional solutions are given.

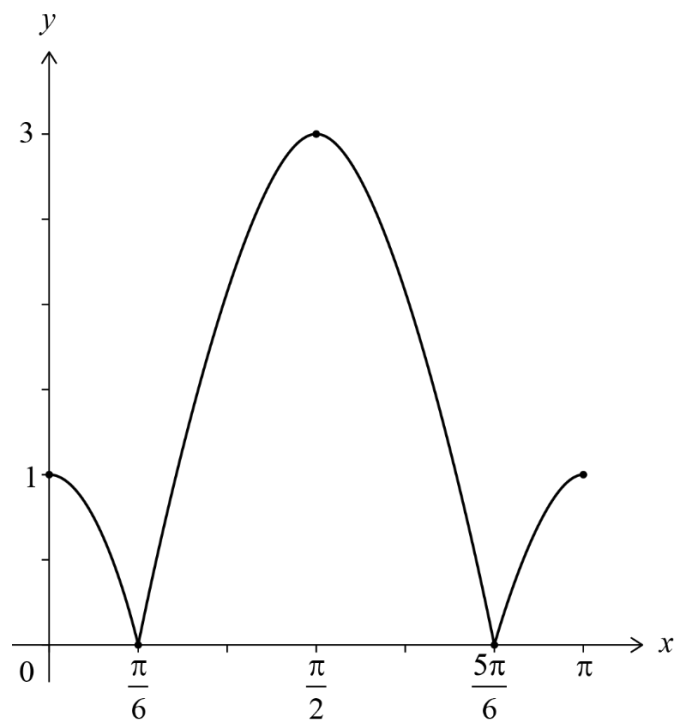
Note: If candidates do not find at least two correct x -coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as **M1A0A0A1A0**.

[7 marks]

continued...

Question 10 continued

(c)

attempt to reflect the negative part of the graph of f in the x -axis**M1**endpoints have coordinates $(0,1)$, $(\pi,1)$ **A1**smooth maximum at $\left(\frac{\pi}{2}, 3\right)$ **A1**sharp points (cusps) at x -intercepts $\frac{\pi}{6}$, $\frac{5\pi}{6}$ **A1****[4 marks]***continued...*

Question 10 continued

- (d) considers points of intersection of $y = |f(x)|$ and $y = 1$ on graph or algebraically **(M1)**

$$-(\cos^2 x - 3\sin^2 x) = 1 \text{ or } -(1 - 4\sin^2 x) = 1 \text{ or } -(4\cos^2 x - 3) = 1 \text{ or } -(2\cos 2x - 1) = 1$$

$$\tan^2 x = 1 \text{ or } \sin^2 x = \frac{1}{2} \text{ or } \cos^2 x = \frac{1}{2} \text{ or } \cos 2x = 0 \quad \textbf{(A1)}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4} \quad \textbf{(A1)}$$

$$\text{For } |f(x)| > 1$$

$$\frac{\pi}{4} < x < \frac{3\pi}{4} \quad \textbf{A1}$$

[4 marks]

Total [20 marks]

7. (a) METHOD 1

attempt to use scalar product or formula for angle between two vectors **(M1)**

$$\mathbf{u} \cdot \mathbf{v} = \cos \frac{1}{n} + \sin \frac{1}{n} \text{ (seen anywhere)} \quad \textbf{(A1)}$$

$$\cos \theta = \frac{\cos \frac{1}{n} + \sin \frac{1}{n}}{\sqrt{2} \sqrt{\left(\cos^2 \frac{1}{n} + \sin^2 \frac{1}{n} \right)}} \left(\frac{\cos \frac{1}{n} + \sin \frac{1}{n}}{\sqrt{2}} \right) \quad \textbf{A1}$$

METHOD 2

attempt to use an Argand diagram showing two complex numbers in the first quadrant with the angle between them marked as θ **(M1)**

$$\arg(\mathbf{u}) = \frac{\pi}{4} \text{ (accept } 45^\circ \text{ or } \arctan(1) \text{) and } \arg(\mathbf{v}) = \frac{1}{n} \quad \textbf{(A1)}$$

$$\cos \theta = \cos \left| \frac{\pi}{4} - \frac{1}{n} \right| \quad \textbf{A1}$$

continued...

Question 7 continued

(b) use of $\frac{1}{n} \rightarrow 0$ as $n \rightarrow \infty$ **(M1)**

EITHER

$(\cos \theta \rightarrow) \frac{1}{\sqrt{2}}$ **(A1)**

OR

$(v \rightarrow) i$ **(A1)**

THEN

the limit is $\frac{\pi}{4}$ **A1**

Note: Accept 45° . Do not accept rounded values such as 0.785.

[3 marks]

Total [6 marks]

9. (a) (i) attempt to use the cosine rule (M1)

$$AC = \sqrt{2^2 + 4^2 - 2(2)(4)\cos\alpha} \left(= \sqrt{20 - 16\cos\alpha} = 2\sqrt{5 - 4\cos\alpha} \right) \quad \text{A1}$$

(ii) $AC = \sqrt{6^2 + 8^2 - 2(6)(8)\cos\beta} \left(= \sqrt{100 - 96\cos\beta} = 2\sqrt{25 - 24\cos\beta} \right) \quad \text{A1}$

(iii) $5 - 4\cos\alpha = 25 - 24\cos\beta$

$$\alpha = \arccos(6\cos\beta - 5) \quad \text{A1}$$

[4 marks]

- (b) attempt to find the sum of two triangle areas using $A = \frac{1}{2}ab\sin C$ (M1)

Note: Do not award this **M1** if the triangle is assumed to be right angled.

$$\text{Area} = \frac{1}{2}(8)\sin\alpha + \frac{1}{2}(48)\sin\beta \quad \text{(A1)}$$

attempt to express the area in terms of one variable only (M1)

$$= 4\sqrt{1 - (6\cos\beta - 5)^2} + 24\sin\beta \text{ or } 4\sin(\arccos(6\cos\beta - 5)) + 24\sin\beta \text{ OR}$$

$$4\sin\alpha + 24\sqrt{1 - \left(\frac{5 + \cos\alpha}{6}\right)^2} \text{ or } 4\sin\alpha + 24\sin\left(\arccos\left(\frac{5 + \cos\alpha}{6}\right)\right)$$

$$\text{Max area} = 19.5959\dots$$

$$= 19.6 \quad \text{A1}$$

[4 marks]

Total [8 marks]

Section B

12.

$$(a) \quad \vec{AB} = \begin{pmatrix} k-1 \\ -4 \\ -2 \end{pmatrix}, \vec{AC} = \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix}$$

A1A1**[2 marks]****(b) METHOD 1**

$$k - 1 = 2 \times 4$$

M1

$$k = 9$$

AG**METHOD 2**

in order by y or z -ordinate, the points are $(k, -2, 1), (5, 0, 2), (1, 2, 3)$

$$k - 5 = 5 - 1$$

M1

$$k = 9$$

AG**[1 mark]***continued...*

Question 12 continued

- (c) (i) attempt to set up a vector equation using a point, a parameter and a direction vector

(M1)

$$\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} \text{ (or equivalent)}$$

A1

Note: “ $\mathbf{r} =$ ” or equivalent must be seen for A1 .
--

continued...

Question 12 continued

(ii) **METHOD 1**

point on line L_1 has coordinates $(1+4\lambda, 2-2\lambda, 3-\lambda)$

attempt to use a different parameter for L_2

(M1)

$$\frac{x-1}{2} = \frac{y}{3} = 1-z = \mu \text{ or } \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

point on line L_2 has coordinates $(1+2\mu, 3\mu, 1-\mu)$

(A1)

Note: This A1 may be implied by $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$.
--

$$1+4\lambda = 1+2\mu$$

$$2-2\lambda = 3\mu$$

$$3-\lambda = 1-\mu$$

any two of the above equations

A1

attempt to solve two simultaneous equations with two parameters

(M1)

eg $\lambda = 0.25$, $\mu = 0.5$ or $\lambda = 1.6$, $\mu = -0.4$ or $\lambda = -2$, $\mu = -4$

A1

substitute into third equation or solve a different pair of simultaneous equations

M1

obtain contradiction eg $3-0.25 \neq 1-0.5$ or $1+4(1.6) \neq 1+2(-0.4)$ or

$$2-2(-2) \neq 3(-4) \text{ (so the lines do not intersect)}$$

R1

Note: Do not award this R1 if it is based on incorrect values.

lines are not parallel

R1

so lines are skew

AG

continued...

Question 12 continued

METHOD 2

point on line L_1 has coordinates $(1+4\lambda, 2-2\lambda, 3-\lambda)$

attempt to use the equation of L_2 to generate at least two equations in λ **(M1)**

if the two lines intersect,

$$\frac{(1+4\lambda)-1}{2} = \frac{2-2\lambda}{3} \left(\Rightarrow 2\lambda = \frac{2-2\lambda}{3} \right)$$

$$\frac{(1+4\lambda)-1}{2} = 1-(3-\lambda) \left(\Rightarrow 2\lambda = \lambda - 2 \right)$$

$$\frac{2-2\lambda}{3} = 1-(3-\lambda) \Rightarrow \left(\frac{2-2\lambda}{3} = \lambda - 2 \right)$$

any two of the above equations **A1A1**

attempt to solve at least one equation in λ **(M1)**

one of $\lambda = \frac{1}{4}$, $\lambda = -2$, $\lambda = \frac{8}{5}$ seen **A1**

substitute into second equation or solve second equation **M1**

obtain contradiction eg $\lambda = \frac{1}{4} \neq -2$ or $2\left(\frac{1}{4}\right) \neq \frac{1}{4} - 2$ (so the lines do not

intersect) **R1**

Note: Do not award this **R1** if it is based on incorrect values.

lines are not parallel **R1**

so lines are skew **AG**

continued...

Question 12 continued

METHOD 3

attempt to use a find Cartesian equation for L_1 **(M1)**

$$\frac{x-1}{4} = \frac{y-2}{-2} = \frac{z-3}{-1} \quad \text{A1}$$

attempt to isolate one variable in both equations **(M1)**

$$L_1 : z = \frac{1-x}{4} + 3 = \frac{y-2}{2} + 3 \quad L_2 : z = \frac{1-x}{2} + 1 = \frac{-y}{3} + 1 \quad \text{OR}$$

$$L_1 : y = \frac{1-x}{2} + 2 = 2(z-3) + 2 \quad L_2 : y = \frac{3(x-1)}{2} = 3(1-z) \quad \text{OR}$$

$$L_1 : x = 1 - 2(y-2) = 1 - 4(z-3) \quad L_2 : x = \frac{2y}{3} + 1 = 1 - 2(z-1) \quad \text{A1}$$

attempt to solve for each of the other two variables **(M1)**

$$\text{e.g. } \frac{1-x}{2} + 1 = \frac{1-x}{4} + 3 \quad \text{and} \quad \frac{-y}{3} + 1 = \frac{y-2}{2} + 3$$

$$x = -7, y = -1.2 \quad \text{OR} \quad x = 2, z = 1.4 \quad \text{OR} \quad y = 1.5, z = 5 \quad \text{A1}$$

obtain contradiction eg $z = 5 \neq 1.4$ OR $y = 1.5 \neq -1.2$ OR $x = 2 \neq -7$

(so the lines do not intersect) **R1**

Note: Do not award this **R1** if it is based on incorrect values.

lines are not parallel **R1**

so lines are skew **AG**

[10 marks]

continued...

Question 12 continued

(d) (i) **METHOD 1**attempt to find cross product of two of $\vec{AB}$, $\vec{AC}$ and $\vec{BC}$ or their opposites**M1**

$$\text{eg } \vec{AB} \times \vec{AC} = \begin{pmatrix} 0 \\ k-9 \\ 18-2k \end{pmatrix} = (k-9) \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

A1

attempt to substitute their cross product and a point into the equation of a plane

(M1)

$$(k-9)y + 2(9-k)z = 2(k-9) + 6(9-k)$$

$$(k-9)y + 2(9-k)z = 36 - 4k \quad (\Rightarrow y - 2z = -4 \text{ since } k \neq 9)$$

A1**METHOD 2**attempt to find vector equation of Π and write x, y and z in parametric form**M1**

$$\left(\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} k-1 \\ -4 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -2 \\ -1 \end{pmatrix} \Rightarrow x = 1 + \lambda(k-1) + 4\mu, \quad y = 2 - 4\lambda - 2\mu, \right.$$

$$z = 3 - 2\lambda - \mu \text{ or equivalent}$$

A1

attempt to eliminate both parameters to work towards Cartesian form

M1

$$(k-9)y + 2(9-k)z = 36 - 4k \quad (\Rightarrow y - 2z = -4 \text{ since } k \neq 9)$$

A1*continued...*

*Question 12 continued***(ii) METHOD 1**

attempt to find the equation of the line through (0, 0, 0) perpendicular
to the plane

(M1)**EITHER**

$$(r =) t \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

(A1)

attempt to find the point where the line and plane intersect

(M1)

$$t + 4t + 4 = 0$$

$$t = -\frac{4}{5}$$

(A1)**OR**

$$(r =) t(k-9) \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}$$

(A1)

attempt to find the point where the line and plane intersect

(M1)

$$t(k-9)^2 + 4t(k-9)^2 + 4(k-9) = 0$$

$$t = -\frac{4}{5(k-9)}$$

(A1)**THEN**

so the point on the plane closest to the origin is (0, -0.8, 1.6)

A1*continued...*

*Question 12 continued***METHOD 2**

choose a point on the plane (p, q, r)

$$q - 2r + 4 = 0 \text{ OR } q(k - 9) - 2r(k - 9) + 4(k - 9) = 0 \Rightarrow q = 2r - 4$$

distance to the origin is $\sqrt{p^2 + (2r - 4)^2 + r^2}$ **(A1)**

since p is independent of r , distance is minimised when $p = 0$ **(R1)**

attempt to find the value of r for which their $\sqrt{(2r - 4)^2 + r^2}$ is minimised **(M1)**

$$r = 1.6 \quad \text{span style="float: right;">**(A1)**}$$

so the point on the plane closest to the origin is $(0, -0.8, 1.6)$ **A1**

continued...

*Question 12 continued***METHOD 3**

attempt to find a vector from the origin to the closest point on the plane **(M1)**

EITHER

$$(r =) t \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \quad (A1)$$

$$\text{distance to the origin} = \left(\frac{4}{\sqrt{1^2 + (-2)^2}} = \frac{4}{\sqrt{5}} \right) = \frac{4\sqrt{5}}{5} \quad (A1)$$

$$t = \pm \frac{4}{5}$$

check in equation of plane $y - 2z = -4$ to get $t = -\frac{4}{5}$ **(R1)**

OR

$$(r =) t(k-9) \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \quad (A1)$$

$$\text{distance to the origin} = \left(\frac{4}{\sqrt{1^2 + (-2)^2}} = \frac{4}{\sqrt{5}} \right) = \frac{4\sqrt{5}}{5} \quad (A1)$$

$$t = \pm \frac{4}{5(k-9)}$$

check in equation of plane $y - 2z = -4$ to get $t = -\frac{4}{5(k-9)}$ **(R1)**

THEN

so the point on the plane closest to the origin is $(0, -0.8, 1.6)$ **A1**

[9 marks]

Total [22 marks]

3. $1 - 2\sin^2 x = \sin x$ **A1**

$$2\sin^2 x + \sin x - 1 = 0$$

valid attempt to solve quadratic **(M1)**

$$(2\sin x - 1)(\sin x + 1) \text{ OR } \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)}$$

recognition to solve for $\sin x$ **(M1)**

$$\sin x = \frac{1}{2} \text{ OR } \sin x = -1$$

any correct solution from $\sin x = -1$ **A1**

any correct solution from $\sin x = \frac{1}{2}$ **A1**

Note: The previous two marks may be awarded for degree or radian values, irrespective of domain.

$$x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$

A1

Note: If no working shown, award no marks for a final value(s).

Award **A0** for $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ if additional values also given.

Total [6 marks]

5. (a) recognition that period is $4m$ OR substitution of a point on f (except the origin) **(M1)**

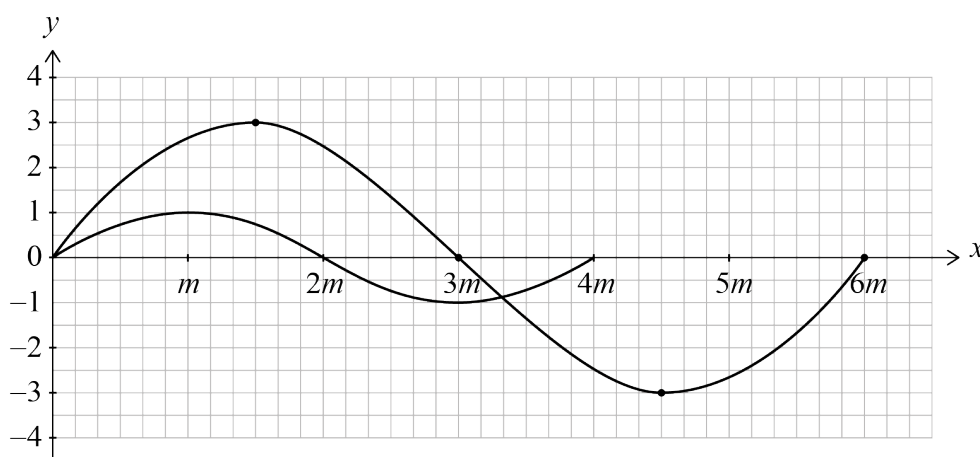
$$4m = \frac{2\pi}{q} \text{ OR } 1 = \sin qm$$

$$m = \frac{\pi}{2q}$$

A1**[2 marks]**

- (b) horizontal scale factor is $\frac{3}{2}$ (seen anywhere) **(A1)**

Note: This **(A1)** may be earned by seeing a period of $6m$, half period of $3m$ or the correct x -coordinate of the maximum/minimum point.

**A1A1A1**

Note: Curve must be an approximate sinusoidal shape (sine or cosine).
 Only in this case, award the following:
A1 for correct amplitude.
A1 for correct domain.
A1 for correct max and min points **and** correct x -intercepts.

[4 marks]**Total [6 marks]**

12. (a) $2t + 1 \times 0 + 0 \times (3 + t) (= 2t)$ (seen anywhere) (A1)

one correct magnitude $\sqrt{1^2 + 1^2 + 0^2}, \sqrt{(2t)^2 + (3 + t)^2}$ (A1)

correct substitution of their magnitudes and scalar product M1

$$2t = \sqrt{2} \times \sqrt{(2t)^2 + (3 + t)^2} \times \cos \frac{\pi}{3} \quad \text{OR} \quad \cos \frac{\pi}{3} = \frac{2t}{\sqrt{2} \times \sqrt{5t^2 + 6t + 9}}$$

$$4t = \sqrt{2(4t^2 + 9 + 6t + t^2)} \quad \text{OR} \quad \frac{1}{2} = \frac{2t}{\sqrt{2(5t^2 + 6t + 9)}} \quad (\text{or equivalent}) \quad \text{A1}$$

$$4t = \sqrt{10t^2 + 12t + 18} \quad \text{AG}$$

[4 marks]

(b) correct quadratic equation A1

$$16t^2 = 10t^2 + 12t + 18, \quad 6t^2 - 12t - 18 = 0, \quad t^2 - 2t - 3 = 0$$

valid attempt to solve their quadratic set =0 (M1)

$$(t + 1)(t - 3) \quad \text{OR} \quad \frac{12 \pm \sqrt{(-12)^2 - 4 \times 6 \times (-18)}}{12} \quad \text{OR} \quad (t - 1)^2 - 4 \quad \text{(A1)}$$

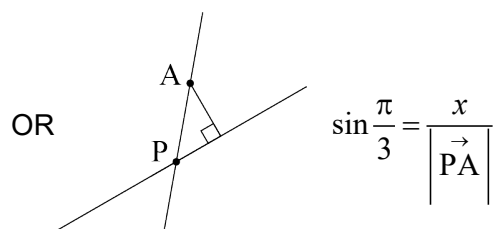
$$t = 3 \quad \text{A1}$$

Note: Award **A0** if additional answer(s) given.

[4 marks]

continued...

Question 12 continued

(c) **METHOD 1**recognizing shortest distance from A is perpendicular to L_1 **(M1)**

$$|\vec{PA}| = \sqrt{6^2 + 6^2} \quad (= \sqrt{72}, 6\sqrt{2}) \quad (\text{seen anywhere}) \quad \textbf{(A1)}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{72}} \quad \textbf{(A1)}$$

$$x = \frac{\sqrt{216}}{2} \quad (= \sqrt{54}, 3\sqrt{6})$$

$$\text{shortest distance is } \frac{\sqrt{216}}{2} \quad (= \sqrt{54}, 3\sqrt{6}) \quad \textbf{A1}$$

[4 marks]*continued...*

*Question 12 continued***METHOD 2**

recognition that the distance required is $\frac{|\vec{v} \times \vec{PA}|}{|\vec{v}|}$ **(M1)**

$$= \frac{1}{\sqrt{2}} \left| \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} \right| \quad \text{span style="float: right;">**(A1)**$$

$$= \frac{1}{\sqrt{2}} \left| \begin{pmatrix} 6 \\ -6 \\ -6 \end{pmatrix} \right| \quad \text{span style="float: right;">**(A1)**$$

shortest distance is $\sqrt{54} (=3\sqrt{6})$ **A1**

[4 marks]*continued...*

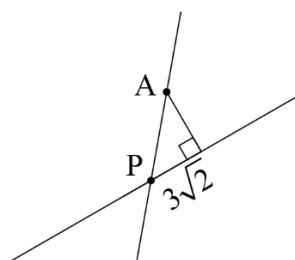
Question 12 continued

METHOD 3

recognition that the base of the triangle is $\frac{|\vec{v} \cdot \vec{PA}|}{|\vec{v}|}$ **(M1)**

$$\frac{1}{\sqrt{2}} \left| \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix} \right|$$

$$= \frac{6}{\sqrt{2}} (=3\sqrt{2}) \text{ OR}$$

**(A1)**

$$|\vec{PA}| = \sqrt{6^2 + 6^2} \quad (= \sqrt{72}, 6\sqrt{2}) \quad (\text{seen anywhere}) \quad \textbf{(A1)}$$

Note: The value of $|\vec{PA}| = \sqrt{6^2 + 6^2}$ may be seen as part of the working

$$\text{of their shortest distance, } d = \sqrt{|\vec{PA}|^2 - b^2} = \sqrt{(\sqrt{72})^2 - (3\sqrt{2})^2}$$

$$\text{shortest distance is } \sqrt{54} (=3\sqrt{6})$$

A1**[4 marks]***continued...*

Question 12 continued

METHOD 4

Let B be a general point on L_1 $(\lambda, 8 + \lambda, -3)$ such that AB is perpendicular to L_1

attempt to find vector $\vec{AB}$ OR $\left| \vec{AB} \right|$ (the shortest distance from A to L_1) **(M1)**

$$\vec{AB} = \vec{OB} - \vec{OA} = \vec{OP} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \vec{OA} = \begin{pmatrix} 0 \\ 8 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} = \vec{AP} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad (\lambda \in \mathbb{R})$$

$$\vec{AB} = \begin{pmatrix} -6 \\ 0 \\ -6 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \quad \text{OR} \quad \left| \vec{AB} \right| = \sqrt{(\lambda - 6)^2 + (8 + \lambda - 8)^2 + (-3 - 3)^2} \quad \mathbf{A1}$$

$$\left| \vec{AB} \right| = \sqrt{(\lambda - 6)^2 + \lambda^2 + (-6)^2} \quad (= \sqrt{2\lambda^2 - 12\lambda + 72})$$

EITHER

$$\frac{d}{d\lambda} \left(\left| \vec{AB} \right|^2 \right) = 0 \Rightarrow 4\lambda - 12 = 0 \Rightarrow \lambda = 3 \quad \mathbf{A1}$$

OR

$$\left| \vec{AB} \right| = \sqrt{2(\lambda - 3)^2 + 54} \quad \text{to obtain } \lambda = 3 \quad \mathbf{A1}$$

OR

$$\begin{pmatrix} -6 + \lambda \\ \lambda \\ -6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = 0 \Rightarrow -6 + \lambda + \lambda = 0 \Rightarrow \lambda = 3 \quad \mathbf{A1}$$

THEN

$$\text{shortest distance is } \sqrt{54} \quad (= 3\sqrt{6}) \quad \mathbf{A1}$$

[4 marks]

continued...

Question 12 continued

- (d) attempt to find the vector product of two direction vectors

(M1)

$$\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 6 \\ 0 \\ 6 \end{pmatrix}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \text{ (or any scalar multiple of this) (accept } \mathbf{n} = \langle 1, -1, -1 \rangle \text{ or equivalent)}$$

A1

Note: Award **A0** for a final answer given in coordinate form.

[2 marks]*continued...*

Question 12 continued

- (e) substituting their x into volume formula and equating (M1)

$$\frac{1}{3}\pi(3\sqrt{6})^2 h = 90\sqrt{3}\pi$$

$$h = 5\sqrt{3} \text{ (seen anywhere)} \quad \text{A1}$$

recognition that the position vector of vertex is given by $\vec{OA} + \mu \mathbf{n}$ OR $\vec{OA} + h \times \hat{\mathbf{n}}$ (M1)

$$\begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \text{ OR } (6 + \mu, 8 - \mu, 3 - \mu)$$

EITHER

recognition that $\mu|\mathbf{n}| = h$ (where μ is a parameter) (M1)

$$\mu|\mathbf{n}| = 5\sqrt{3} \text{ OR } \sqrt{\mu^2 + (-\mu)^2 + (-\mu)^2} = 5\sqrt{3} \text{ OR } 3\mu^2 = 75 \left(\Rightarrow \sqrt{3}\mu = 5\sqrt{3} \right)$$

$$\mu = \pm 5 \text{ (accept } \mu = 5) \quad \text{(A1)}$$

OR

attempt to find cone's height vector $h \times \hat{\mathbf{n}}$ (M1)

$$\hat{\mathbf{n}} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} \quad \text{(A1)}$$

$$5\sqrt{3} \times \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

THEN

$$= \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} \pm 5 \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \\ 3 \end{pmatrix} \pm \begin{pmatrix} 5 \\ -5 \\ -5 \end{pmatrix}$$

vertex = (11, 3, -2) and (1, 13, 8) (accept position vectors) **A1A1**

Note: Award a maximum of **(M0)A0(M1)(M1)(A1)A1A1FT** for $\left(\frac{39}{4}, \frac{17}{4}, -\frac{3}{4}\right)$ and $\left(\frac{9}{4}, \frac{47}{4}, \frac{27}{4}\right)$ obtained using $x = \left| \vec{PA} \right|$ from part (c).

[7 marks]

Total [21 marks]

1. (a) attempts to find perimeter (M1)
 $\text{arc} + 2 \times \text{radius}$ OR $10 + 4 + 4$
 $= 18 \text{ (cm)}$ A1

[2 marks]

- (b) $10 = 4\theta$ (A1)
 $\theta = \frac{10}{4} \left(= \frac{5}{2}, 2.5 \right)$ A1

[2 marks]

- (c) $\text{area} = \frac{1}{2} \left(\frac{10}{4} \right) (4^2)$ (A1)
 $= 20 \text{ (cm}^2\text{)}$ A1

[2 marks]

Total [6 marks]

8. (a) $\cos k = \frac{\sin k}{\cos k}$

A1

$$\cos^2 k = \sin k$$

AG**[1 mark]**

(b) $f'(k) = -\sin k$ and $g'(k) = \sec^2 k$

A1

Note: Award **A1** for $f'(x) = -\sin x$ and $g'(x) = \sec^2 x$.

EITHER

$$f'(k)g'(k) = -\frac{\sin k}{\cos^2 k}$$

M1

$$\cos^2 k = \sin k \Rightarrow f'(k)g'(k) \left(= -\frac{\sin k}{\sin k} \right) = -1$$

R1**OR**

$$g'(k) = \frac{1}{\cos^2 k}$$

M1

$$\cos^2 k = \sin k \Rightarrow g'(k) = \frac{1}{\sin k} = -\frac{1}{f'(k)}$$

R1

Note: Accept showing that $f'(k) = -\frac{1}{g'(k)}$.

Note: Allow 'backwards methods' such as starting with $f'(k) = -\frac{1}{g'(k)}$ leading to

$$\cos^2 k = \sin k$$

THEN

$\Rightarrow$ the two tangents intersect at right angles at P

AG

Note: To obtain the final **R1**, all of the previous marks must have been awarded.

[3 marks]*continued...*

Question 8 continued

(c) $1 - \sin^2 k = \sin k$ (from part (a))

A1

$$\sin^2 k + \sin k - 1 = 0$$

attempts to solve for $\sin k$ **(M1)**

$$\sin k = \frac{-1 \pm \sqrt{1^2 - 4(1)(-1)}}{2}$$

$$\left(\text{for } 0 < k < \frac{\pi}{2}, \sin k > 0\right) \Rightarrow \sin k = \frac{-1 + \sqrt{5}}{2}$$

A1

$$(a = -1, b = 5, c = 2)$$

Note: Award **A0** if more than one solution is given**[3 marks]****Total [7 marks]**

9. (a) $\vec{OM} = \mathbf{a} + k\mathbf{c}$ A1

$\vec{MC} = (1-k)\mathbf{c} - \mathbf{a}$ A1

[2 marks]

(b) attempts to expand their dot product $\vec{OM} \cdot \vec{MC} = (\mathbf{a} + k\mathbf{c}) \cdot ((1-k)\mathbf{c} - \mathbf{a})$ M1

$$= (1-2k)(\mathbf{a} \cdot \mathbf{c}) - |\mathbf{a}|^2 + k(1-k)|\mathbf{c}|^2 \text{ (or equivalent)}$$

uses $|\mathbf{c}| = 2|\mathbf{a}|$ and $\mathbf{a} \cdot \mathbf{c} = 2|\mathbf{a}|^2 \cos \theta$ M1

$$= 2(1-2k)|\mathbf{a}|^2 \cos \theta - |\mathbf{a}|^2 + 4k(1-k)|\mathbf{a}|^2$$

$$= 2(1-2k)|\mathbf{a}|^2 \cos \theta - (1-2k)^2 |\mathbf{a}|^2$$
 A1

$$|\mathbf{a}|^2 (1-2k)(2 \cos \theta - (1-2k)) = 0$$
 AG

[3 marks]

(c) attempts to solve $|\mathbf{a}|^2 (1-2k)(2 \cos \theta - (1-2k)) = 0$ for k (M1)

$$k = \frac{1}{2} \text{ or } k = \frac{1}{2} - \cos \theta \quad (|\mathbf{a}|^2 > 0)$$

Note: Award **(M1)** for their ' $k =$ ' or their ' $\cos \theta =$ '. For example, $\cos \theta = \frac{1-2k}{2}$ or equivalent.

$$\text{as } 0 \leq k \leq 1, \quad 0 \leq \frac{1}{2} - \cos \theta \leq 1$$

$$-\frac{1}{2} \leq \cos \theta \leq \frac{1}{2}$$
 A1

$$\frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3}, \quad \theta \neq \frac{\pi}{2}$$
 A1A1

$$(\theta = \frac{\pi}{2} \text{ corresponds to only one possible position for M when } k = \frac{1}{2})$$

[4 marks]

Total [9 marks]

Section B

1. (a) Let N be North

$$\hat{N}\hat{J}D = 34^\circ \text{ OR } \hat{D}\hat{J}L = 56^\circ \text{ (must be labelled or indicated in diagram):} \quad (\text{A1})$$

$$\hat{J}\hat{D}L = 99^\circ \quad \text{A1}$$

Note: Accept $\frac{11\pi}{20}$, 1.73 (radians).

[2 marks]

- (b) attempt to apply the sine rule (M1)

$$\frac{DL}{\sin 56^\circ} = \frac{500}{\sin 99^\circ} \text{ OR } \frac{DL}{\sin 0.977384\dots} = \frac{500}{\sin 1.72787\dots} \quad (\text{A1})$$

$$419.685\dots$$

$$DL = 420 \text{ (km)} \quad \text{A1}$$

Note: Award **M1A1A0** for 261 (km) from use of degrees with GDC set in radians (with or without working).

[3 marks]

Total [5 marks]

8. direction vector of the line is $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ (seen anywhere) **(A1)**

normal vector of the plane is $\begin{pmatrix} 4 \\ \cos \alpha \\ \sin \alpha \end{pmatrix}$ (seen anywhere) **(A1)**

EITHER

correct scalar product $12 + 2 \cos \alpha - \sin \alpha$ (seen anywhere) **(A1)**

one correct magnitude (seen anywhere) **(A1)**

$$\sqrt{16 + \cos^2 \alpha + \sin^2 \alpha} (= \sqrt{17}), \sqrt{9 + 4 + 1} (= \sqrt{14})$$

recognizing angle between normal and direction vector is $\frac{\pi}{2} - \alpha$ (seen anywhere) **(M1)**

Note: angle $\frac{\pi}{2} - \alpha$ may be implied by use of $\sin \alpha$ on the RHS of the step below

attempt to substitute into the formula for the angle between two vectors to form an equation in α **(M1)**

$$12 + 2 \cos \alpha - \sin \alpha = \sqrt{17} \sqrt{14} \cos \left(\frac{\pi}{2} - \alpha \right) \text{ OR } 12 + 2 \cos \alpha - \sin \alpha = \sqrt{17} \sqrt{14} \sin \alpha$$

continued...

Question 8 continued

OR

correct expression for the magnitude of the vector product

$$\left| \begin{pmatrix} 2 \sin \alpha + \cos \alpha \\ -4 - 3 \sin \alpha \\ 3 \cos \alpha - 8 \end{pmatrix} \right| = \sqrt{(2 \sin \alpha + \cos \alpha)^2 + (-4 - 3 \sin \alpha)^2 + (3 \cos \alpha - 8)^2} \quad (\text{seen anywhere}) \quad (\mathbf{A1})$$

one correct magnitude (seen anywhere)

(A1)

$$\sqrt{16 + \cos^2 \alpha + \sin^2 \alpha} (= \sqrt{17}), \sqrt{9 + 4 + 1} (= \sqrt{14})$$

recognizing angle between normal and direction vector is $\frac{\pi}{2} - \alpha$ (seen anywhere)**(M1)**

Note: angle $\frac{\pi}{2} - \alpha$ may be implied by use of $\cos \alpha$ on the RHS of the step below

attempt to substitute into the formula for the angle between two vectors to form an equation in α **(M1)**

$$\sqrt{(2 \sin \alpha + \cos \alpha)^2 + (-4 - 3 \sin \alpha)^2 + (3 \cos \alpha - 8)^2} = \sqrt{17} \sqrt{14} \sin \left(\frac{\pi}{2} - \alpha \right) \quad \text{OR}$$

$$\sqrt{(2 \sin \alpha + \cos \alpha)^2 + (-4 - 3 \sin \alpha)^2 + (3 \cos \alpha - 8)^2} = \sqrt{17} \sqrt{14} \cos \alpha$$

THEN

$$\alpha = 0.932389\dots$$

$$\alpha = 0.932$$

A1

Note: Award maximum **(A1)(A1)(A1)(A1)(M1)(M1)A0** for a correct answer given in degrees $\alpha = 54.4219\dots^\circ$.

[7 marks]

10. (a) recognition that $45 = 10 + 10 + \text{arc length}$ (M1)
 arc length = 25 (cm) (A1)
 $25 = 12\theta$ A1
 $\theta = 2.08$ correct to 3 significant figures AG

[3 marks]

(b)

Note: There are many different ways to dissect the cross-section to determine its area. In all approaches, candidates will need to find w or $\frac{w}{2}$. Award the first three marks for work seen anywhere.

EITHER

evidence of using the cosine rule OR sine rule (M1)

$$w^2 = 12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cos(2.08) \text{ OR } \frac{w}{\sin(2.08)} = \frac{12}{\sin(0.530796...)} \quad (A1)$$

$$w = 20.6977... \text{ or } \frac{w}{2} = 10.3488... \quad (A1)$$

ORusing trig ratios in a right triangle with angle $\frac{2.08}{2}$ and side length $\frac{w}{2}$ (M1)

$$\sin\left(\frac{2.08}{2}\right) = \frac{\frac{w}{2}}{12} \quad (A1)$$

$$w = 20.6977... \text{ or } \frac{w}{2} = 10.3488... \quad (A1)$$

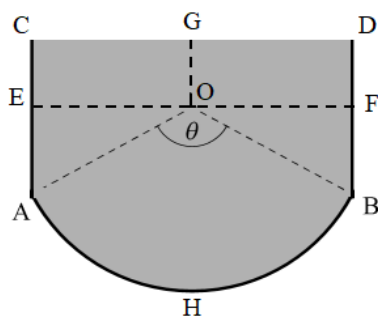
Note: Accept $w = 20.7179...$ from use of $\frac{\theta}{2} = \frac{25}{24}$.

continued...

Question 10 continued

THEN

Let the points A, B, C, D, E, F, G, H lie on the figure as follows:



EITHER

(segment AHB =) sector OAB – triangle OAB

(M1)

$$= \frac{1}{2} \times 12^2 \times 2.08 - \frac{1}{2} \times 12^2 \times \sin 2.08 (= 149.76 - 62.8655... = 86.8944...)$$

(A1)

valid approach to find total cross-sectional area (seen anywhere)

(M1)

sector OAB – triangle OAB + rectangle CDBA

$$= 86.8944... + 10w (= 86.8944... + 206.977...)$$

Note: Use of $\theta = \frac{25}{12}$ throughout leads to segment OAB = 87.2517... and cross-sectional area = 87.2517... + 207.179....

continued...

*Question 10 continued***OR**trapezium CGOA (= rectangle CGOE + triangle EOA) **(M1)**

$$= \frac{1}{2} \times (10 + (10 - 12 \cos(1.04))) \times \frac{20.6977...}{2} (= 72.0557) \quad \textbf{(A1)}$$

valid approach to find total cross-sectional area (seen anywhere) **(M1)**

2 × trapezium CGOA + sector OAB

$$= 2(72.0557...) + \frac{1}{2} \times 12^2 \times 2.08 (= 144.111... + 149.76)$$

Note: Use of $\theta = \frac{25}{12}$ leads to area of trapezium CGOA = 72.2154... and cross-sectional area = 144.430... + 150.

OR2 x area of trapezium CGOA (= area of rectangle CDFE + 2 x triangle EOA) **(M1)**

$$20.6977... \times (10 - 12 \cos(1.04)) + 2 \times \frac{1}{2} \times 12 \cos(1.04) \times 12 \sin(1.04) \quad \textbf{(A1)}$$

$$(= 81.2458... + 62.8655...)$$

valid approach to find total cross-sectional area (seen anywhere) **(M1)**

2 x trapezium CGOA + sector OAB

$$= 144.111... + \frac{1}{2} \times 12^2 \times 2.08 (= 144.111... + 149.76)$$

Note: Use of $\theta = \frac{25}{12}$ leads to 2 x area of trapezium CGOA = 144.430... and cross-sectional area = 144.430... + 150.

THEN

area of cross-section = 293.871... (294.430... from exact answer)

$$= 294 \text{ (cm}^2\text{)}$$

A1**[7 marks]***continued...*

Question 10 continued

(c) **METHOD 1**

–4.71976... volume of gutter = 176323 OR 176658 (OR $600 \times$ their area) (seen anywhere) **A1**

recognising rainfall can be represented by an integral **(M1)**

$$\int_0^{60} R'(t) dt \left(= \frac{250}{2p} \sin\left(\frac{2p \times 60}{5}\right) + 3000 \times 60 \right) \quad \textbf{(A1)}$$

Note: Accept any 60 second interval or any interval which is a multiple of 5 seconds (one period) scaled up to 60 seconds e.g. $12 \int_0^5 R'(t) dt$.

rainfall over 60 seconds = 180000 (cm^3) **A1**

the gutter will overflow because the rainfall > gutter volume **A1**

METHOD 2

volume of gutter = 176323 OR 176658 (OR $600 \times$ their area) (seen anywhere) **A1**

recognition that cosine has a minimum value of -1 **(M1)**

$$R'(t) \geq -1 \times 50 + 3000 (\text{cm}^3 \text{s}^{-1}) \quad \textbf{(A1)}$$

rainfall over 60 seconds ≥ 177000 **(A1)**

the gutter will overflow because the rainfall > gutter volume **A1**

METHOD 3

volume of gutter = 176323 OR 176658 (OR $600 \times$ their area) (seen anywhere) **A1**

recognising rainfall can be represented by an integral **(M1)**

$$\text{attempt to solve } 60 > 58.8 \text{ OR } \int_0^T R'(t) dt = 176658 \quad \textbf{(M1)}$$

time to reach overflow point = 58.7875... OR 58.8990... **A1**

the gutter will overflow because $60 > 58.8$ OR $60 > 58.9$ **A1**

[5 marks]

Total [15 marks]

2. METHOD 1

attempt to substitute into cosine rule

(M1)

$$154^2 = 150^2 + 90^2 - 2(150)(90)\cos \hat{A}PB \quad \text{OR} \quad \cos \hat{A}PB = \frac{150^2 + 90^2 - 154^2}{2(150)(90)} \quad \text{(A1)}$$

$$\hat{A}PB = 75.2286...^\circ \quad \text{OR} \quad 1.31298... \text{ radians}$$

$$\hat{A}PB = 75.2^\circ \quad \text{OR} \quad 1.31 \text{ radians} \quad \text{(A1)}$$

valid approach to find θ **(M1)**

$$\theta = \frac{180^\circ - \hat{A}PB}{2} \quad \text{OR} \quad \theta = \frac{180^\circ - 75.2286...^\circ}{2} \quad (= 52.3856...) \quad \text{OR}$$

$$\theta = \frac{\pi - 1.31298...}{2} \quad (= 0.914302...)$$

valid approach to express h in terms of θ **(M1)**

$$\sin \theta = \frac{h}{150} \quad \text{OR} \quad h = 150 \sin 52.3856...^\circ$$

$$h = 118.820...$$

$$h = 119 \text{ (m)}$$

A1**[6 marks]****METHOD 2**attempts to find either the distance between the buildings or the difference in height between the buildings in terms of θ **(M1)**distance between the buildings is $(150 + 90)\cos \theta$ and the difference in height between the buildings is $(150 - 90)\sin \theta$ **(A1)**uses Pythagoras and attempts to solve for θ **(M1)**

$$(60\sin \theta)^2 + (240\cos \theta)^2 = 154^2$$

$$\theta = 0.914302... \quad (= 52.3856...^\circ) \quad \text{(A1)}$$

$$\frac{h}{150} = \sin(0.914302...) \quad \text{(M1)}$$

$$h = 118.820...$$

$$h = 119 \text{ (m)}$$

A1**[6 marks]**

6. (a) METHOD 1

the general point on L has coordinates $(\lambda, 2 - 2\lambda, 4 - 2\lambda)$

substitutes this general point into both Π_1 and Π_2

(M1)

$$2\lambda - (2 - 2\lambda) + 2(4 - 2\lambda) (= 2\lambda - 2 + 2\lambda + 8 - 4\lambda)$$

A1

$$= 6$$

AG

$$4\lambda + 3(2 - 2\lambda) - (4 - 2\lambda) (= 4\lambda + 6 - 6\lambda - 4 + 2\lambda)$$

A1

$$= 2$$

AG

so the vector equation of L can be written as $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

AG

Note: Award **(M1)A0A0** for correct verification using a specific value of λ .

continued...

*Question 6 continued***METHOD 2**

substitutes $(0, 2, 4)$ into both Π_1 and Π_2 and shows that

$$0 - 2 + 8 = 6 \text{ and } 0 + 6 - 4 = 2$$

A1

hence $(0, 2, 4)$ lies in both Π_1 and Π_2

AG**EITHER**

attempts to find $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$

M1

$$= \begin{pmatrix} -5 \\ 10 \\ 10 \end{pmatrix}$$

A1**OR**

attempts to find $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$

M1

$$(2 + 2 - 4) = 0 \text{ and } (4 - 6 + 2) = 0$$

A1**THEN**

(so $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$ is perpendicular to both normal vectors)

so the vector equation of L can be written as $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

AG

Note: Award **M1** for substituting $x = 0$ (or $y = 2$ or $z = 4$) into Π_1 and Π_2 and solving simultaneously, for example, solving $-y + 2z = 6$ and $3y - z = 2$. Award **A1** for $y = 2$ and $z = 4$, for example.

continued...

*Question 6 continued***METHOD 3**

attempts row reduction to obtain eg,

$$x + \frac{z}{2} = 2 \text{ and } y - z = -2 \quad (M1)$$

$$\text{substitutes } x = \lambda \text{ into } x + \frac{z}{2} = 2, \text{ solves for } z \text{ and obtains } z = 4 - 2\lambda \quad A1$$

$$\text{substitutes } z = 4 - 2\lambda \text{ into } y - z = -2, \text{ solves for } y \text{ and obtains } y = 2 - 2\lambda \quad A1$$

$$\text{so the vector equation of } L \text{ can be written as } \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \quad AG$$

METHOD 4

$$\text{attempts to solve } 2x - y + 2z = 6 \text{ and } 4x + 3y - z = 2 \quad (M1)$$

$$\text{for example, } x = \lambda, y = 2 - 2\lambda, z = 4 - 2\lambda \quad A2$$

$$\text{so the vector equation of } L \text{ can be written as } \mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} \quad AG$$

Note: Only award marks for convincing use of a GDC.

[3 marks]*continued...*

Question 6 continued**(b) EITHER**

the position vector for point P nearest to the origin is perpendicular to the direction of L

$$\begin{pmatrix} \lambda \\ 2-2\lambda \\ 4-2\lambda \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = 0 \quad (M1)$$

$$\lambda - 2(2-2\lambda) - 2(4-2\lambda) = 0 \quad (A1)$$

$$9\lambda - 12 = 0 \quad (A1)$$

OR

let s be the distance from the origin to a point P on L , then

$$s^2 = \lambda^2 + (2-2\lambda)^2 + (4-2\lambda)^2 \quad (A1)$$

attempts to find λ such that $\frac{d(s^2)}{d\lambda} = 0 \quad (M1)$

either $\frac{d(s^2)}{d\lambda} = 18\lambda - 24 (=0)$ or a graph of s^2 versus $\lambda \quad (A1)$

Note: Award as above for use of $s = \sqrt{\lambda^2 + (2-2\lambda)^2 + (4-2\lambda)^2}$.

THEN

$$\lambda = \frac{4}{3} \quad A1$$

$$P\left(\frac{4}{3}, -\frac{2}{3}, \frac{4}{3}\right) (P(1.33, -0.667, 1.33)) \quad A1$$

[5 marks]**Total [8 marks]**

$$10. \quad (a) \quad 7.8 = \frac{2\pi}{\text{period}} \quad (M1)$$

$$\frac{2\pi}{7.8} = 0.805536\dots$$

$$\text{period} = 0.806 \left(= \frac{20\pi}{78} \right) \quad A1$$

[2 marks]

(b) **METHOD 1**

$$(i) \quad \text{amplitude} = \frac{\text{max} - \text{min}}{2} \quad (M1)$$

$$\frac{1.8 - 1}{2}$$

$$a = -0.4 \quad A1$$

$$(ii) \quad b = 1.4 \quad A1$$

METHOD 2attempt to form two simultaneous equations in a and b (M1)

$$H(0) = 1 \Rightarrow a + b = 1, \quad H\left(\frac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$$

$$a = -0.4, b = 1.4 \quad A1A1$$

[3 marks]

continued...

Question 10 continued(c) **EITHER**

$$\frac{5}{\text{period}} = 6.207... < 6\frac{1}{2} \quad (\text{A1})$$

OR

consideration of number of maximums on graph in first 5 seconds (A1)

OR

maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ (A1)

THEN

6 times A1
[2 marks]

(d) recognizing that $H(t) = 1.5$ (M1)

$$-0.4 \cos(7.8t) + 1.4 = 1.5$$

$$0.233779...$$

$$t = 0.234 \text{ (seconds)} \quad (\text{A1})$$

[2 marks]*continued...*

Question 10 continued

(e) finding second time height is 1.5 metres **(M1)**

$$t = 0.571757...$$

in each period, height is greater than 1.5 metres for 0.337978... seconds **(A1)**

Note: Award **(M1)(A1)** for total time 2.02787... seen.

multiplying their value by 6 and divide by 5 **(M1)**

$$\frac{0.337978... \times 6}{5} \text{ OR } \frac{2.02787...}{5}$$

$$= 0.405574...$$

$P(\text{height is greater than 1.5 m}) = 0.406$ **A1**

[4 marks]

Total [13 marks]

4. METHOD 1**EITHER**

attempt to use Pythagoras' theorem in a right-angled triangle.

(M1)

$$\left(\sqrt{5^2 - 1^2} =\right)\sqrt{24}$$

(A1)**OR**attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ **(M1)**

$$\sin^2 \hat{BAC} = 1 - \left(\frac{1}{5}\right)^2$$

(A1)**THEN**

$$\sin \hat{BAC} = \frac{\sqrt{24}}{5} \quad (\text{may be seen in area formula})$$

A1attempt to use 'Area = $\frac{1}{2}ab \sin C$ ' (must include their calculated value of $\sin \hat{BAC}$)**(M1)**

$$= \frac{1}{2} \times 10 \times \sqrt{6} \times \frac{\sqrt{24}}{5}$$

(A1)

$$= 12 \text{ (cm}^2\text{)}$$

A1**[6 marks]***continued...*

*Question 4 continued***METHOD 2**

attempt to find perpendicular height of triangle BAC (M1)

EITHER

$$\text{height} = \sqrt{6} \times \sin \hat{BAC}$$

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ (M1)

$$\text{height} = \sqrt{6} \times \sqrt{1 - \left(\frac{1}{5}\right)^2} \quad \text{(A1)}$$

$$= \sqrt{6} \times \frac{\sqrt{24}}{5} \left(= \frac{12}{5} \right) \quad \text{A1}$$

OR

$$\text{adjacent} = \frac{\sqrt{6}}{5} \quad \text{(A1)}$$

attempt to use Pythagoras' theorem in a right-angled triangle. (M1)

$$\text{height} = \sqrt{6 - \frac{6}{25}} \left(= \frac{12}{5} \right) \quad \text{(may be seen in area formula)} \quad \text{(A1)}$$

THEN

attempt to use 'Area = $\frac{1}{2}$ base $\times$ height' (must include their calculated value for height) (M1)

$$= \frac{1}{2} \times 10 \times \frac{12}{5}$$

$$= 12 \text{ (cm}^2\text{)} \quad \text{A1}$$

[6 marks]

12. (a) attempt to expand using binomial theorem:

(M1)

Note: Award **(M1)** for seeing at least one term with a product of a binomial coefficient, power of $i \sin \theta$ and a power of $\cos \theta$.

$$(\cos \theta + i \sin \theta)^5 = \cos^5 \theta + {}^5C_1 i \cos^4 \theta \sin \theta + {}^5C_2 i^2 \cos^3 \theta \sin^2 \theta$$

$$+ {}^5C_3 i^3 \cos^2 \theta \sin^3 \theta + {}^5C_4 i^4 \cos \theta \sin^4 \theta + i^5 \sin^5 \theta$$

A1

$$= (\cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta) + i(5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta)$$

A1A1

Note: Award **A1** for correct real part and **A1** for correct imaginary part.

[4 marks]

(b) $(\cos \theta + i \sin \theta)^5 = \cos 5\theta + i \sin 5\theta$

(A1)

equate imaginary parts:

(M1)

$$\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$$

A1substitute $\cos^2 \theta = 1 - \sin^2 \theta$ **(M1)**

$$\sin 5\theta = 5(1 - \sin^2 \theta)^2 \sin \theta - 10 \sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta$$

A1

$$\sin 5\theta = 5(1 - 2 \sin^2 \theta + \sin^4 \theta) \sin \theta - 10 \sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta$$

A1

$$= 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$$

AG

Note: Some of this working may be seen in part (a). Allow for awarding marks in part (b).

[6 marks]*continued...*

Question 12 continued

- (c) (i) factorising $16\sin^5 \theta - 20\sin^3 \theta + 5\sin \theta$ **M1**
 $(\sin 5\theta =) \sin \theta (16\sin^4 \theta - 20\sin^2 \theta + 5)$

EITHER

$$\sin 5\left(\frac{\pi}{5}\right) = 0 \text{ and } \sin 5\left(\frac{3\pi}{5}\right) = 0$$
R1

Note: The **R1** is independent of the **M1**.

ORsolving $\sin 5\theta = 0$

$$\theta = \frac{k\pi}{5} \text{ where } k \in \mathbb{Z}$$
R1

Note: The **R1** is independent of the **M1**.

THENtherefore either $\sin \theta = 0$ OR $16\sin^4 \theta - 20\sin^2 \theta + 5 = 0$

$$\sin \frac{\pi}{5} \neq 0 \text{ and } \sin \frac{3\pi}{5} \neq 0 \text{ (or only solution to } \sin \theta = 0 \text{ is } \theta = 0)$$
R1

$$\text{therefore } \frac{\pi}{5}, \frac{3\pi}{5} \text{ are solutions of } 16\sin^4 \theta - 20\sin^2 \theta + 5 = 0$$
AG

Note: The final **R1** is dependent on both previous marks.

continued...

Question 12 continued

(ii) **METHOD 1**

attempt to use quadratic formula:

(M1)

$$\sin^2 \theta = \frac{20 \pm \sqrt{80}}{32}$$

A1

$$= \frac{5 \pm \sqrt{5}}{8}$$

$$\sin \theta = \sqrt{\frac{5 \pm \sqrt{5}}{8}}$$

$$\Rightarrow \sin \frac{\pi}{5} \sin \frac{3\pi}{5} = \sqrt{\frac{5 + \sqrt{5}}{8}} \sqrt{\frac{5 - \sqrt{5}}{8}}$$

M1

$$= \sqrt{\frac{20}{64}}$$

A1

$$= \frac{\sqrt{5}}{4}$$

AG**METHOD 2**

$$\text{roots of quartic are } \sin \frac{\pi}{5}, \sin \frac{2\pi}{5}, \sin \frac{3\pi}{5}, \sin \frac{4\pi}{5}$$

A1

$$\text{attempt to set product of roots equal to } \pm \frac{5}{16}$$

M1

$$\sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5} = \frac{5}{16}$$

A1

$$\text{recognition that } \sin \frac{\pi}{5} = \sin \frac{4\pi}{5} \text{ and } \sin \frac{2\pi}{5} = \sin \frac{3\pi}{5}$$

$$\sin^2 \frac{\pi}{5} \sin^2 \frac{3\pi}{5} = \frac{5}{16}$$

A1

$$\sin \frac{\pi}{5} \sin \frac{3\pi}{5} = \frac{\sqrt{5}}{4}$$

AG*continued...*

*Question 12 continued***METHOD 3**

Consider $16\sin^4 \theta - 20\sin^2 \theta + 5 = 0$ as a quadratic in $\sin^2 \theta$ **M1**

$(\theta = \frac{\pi}{5}, \frac{3\pi}{5} \text{ are roots})$, so $\sin^2 \frac{\pi}{5}$ and $\sin^2 \frac{3\pi}{5}$ are roots of the quadratic. **A1**

Consider product of roots: **M1**

$$\Rightarrow \sin^2 \frac{\pi}{5} \sin^2 \frac{3\pi}{5} = \frac{5}{16} \quad \text{A1}$$

$$\sin \frac{\pi}{5} \sin \frac{3\pi}{5} = \frac{\sqrt{5}}{4} \quad \text{AG}$$

[7 marks]

Total [17 marks]

4. METHOD 1**EITHER**

attempt to use Pythagoras' theorem in a right-angled triangle.

(M1)

$$\left(\sqrt{5^2 - 1^2} =\right)\sqrt{24}$$

(A1)**OR**attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ **(M1)**

$$\sin^2 \hat{BAC} = 1 - \left(\frac{1}{5}\right)^2$$

(A1)**THEN**

$$\sin \hat{BAC} = \frac{\sqrt{24}}{5} \quad (\text{may be seen in area formula})$$

A1attempt to use 'Area = $\frac{1}{2}ab \sin C$ ' (must include their calculated value of $\sin \hat{BAC}$)**(M1)**

$$= \frac{1}{2} \times 10 \times \sqrt{6} \times \frac{\sqrt{24}}{5}$$

(A1)

$$= 12 \text{ (cm}^2\text{)}$$

A1**[6 marks]***continued...*

*Question 4 continued***METHOD 2**

attempt to find perpendicular height of triangle BAC (M1)

EITHER

$$\text{height} = \sqrt{6} \times \sin \hat{BAC}$$

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ (M1)

$$\text{height} = \sqrt{6} \times \sqrt{1 - \left(\frac{1}{5}\right)^2} \quad (\text{A1})$$

$$= \sqrt{6} \times \frac{\sqrt{24}}{5} \left(= \frac{12}{5} \right) \quad \text{A1}$$

OR

$$\text{adjacent} = \frac{\sqrt{6}}{5} \quad (\text{A1})$$

attempt to use Pythagoras' theorem in a right-angled triangle. (M1)

$$\text{height} = \sqrt{6 - \frac{6}{25}} \left(= \frac{12}{5} \right) \quad (\text{may be seen in area formula}) \quad (\text{A1})$$

THEN

attempt to use 'Area = $\frac{1}{2}$ base $\times$ height' (must include their calculated value for height) (M1)

$$= \frac{1}{2} \times 10 \times \frac{12}{5}$$

$$= 12 \text{ (cm}^2\text{)} \quad \text{A1}$$

[6 marks]

12. (a) attempt to expand using binomial theorem:

(M1)

Note: Award **(M1)** for seeing at least one term with a product of a binomial coefficient, power of $i \sin \theta$ and a power of $\cos \theta$.

$$(\cos \theta + i \sin \theta)^5 = \cos^5 \theta + {}^5C_1 i \cos^4 \theta \sin \theta + {}^5C_2 i^2 \cos^3 \theta \sin^2 \theta$$

$$+ {}^5C_3 i^3 \cos^2 \theta \sin^3 \theta + {}^5C_4 i^4 \cos \theta \sin^4 \theta + i^5 \sin^5 \theta$$

A1

$$= (\cos^5 \theta - 10 \cos^3 \theta \sin^2 \theta + 5 \cos \theta \sin^4 \theta) + i(5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta)$$

A1A1

Note: Award **A1** for correct real part and **A1** for correct imaginary part.

[4 marks]

(b) $(\cos \theta + i \sin \theta)^5 = \cos 5\theta + i \sin 5\theta$

(A1)

equate imaginary parts:

(M1)

$$\sin 5\theta = 5 \cos^4 \theta \sin \theta - 10 \cos^2 \theta \sin^3 \theta + \sin^5 \theta$$

A1substitute $\cos^2 \theta = 1 - \sin^2 \theta$ **(M1)**

$$\sin 5\theta = 5(1 - \sin^2 \theta)^2 \sin \theta - 10 \sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta$$

A1

$$\sin 5\theta = 5(1 - 2 \sin^2 \theta + \sin^4 \theta) \sin \theta - 10 \sin^3 \theta (1 - \sin^2 \theta) + \sin^5 \theta$$

A1

$$= 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$$

AG

Note: Some of this working may be seen in part (a). Allow for awarding marks in part (b).

[6 marks]*continued...*

Question 12 continued

- (c) (i) factorising $16\sin^5 \theta - 20\sin^3 \theta + 5\sin \theta$ **M1**
 $(\sin 5\theta =) \sin \theta (16\sin^4 \theta - 20\sin^2 \theta + 5)$

EITHER

$$\sin 5\left(\frac{\pi}{5}\right) = 0 \text{ and } \sin 5\left(\frac{3\pi}{5}\right) = 0$$
R1

Note: The **R1** is independent of the **M1**.

ORsolving $\sin 5\theta = 0$

$$\theta = \frac{k\pi}{5} \text{ where } k \in \mathbb{Z}$$
R1

Note: The **R1** is independent of the **M1**.

THENtherefore either $\sin \theta = 0$ OR $16\sin^4 \theta - 20\sin^2 \theta + 5 = 0$

$$\sin \frac{\pi}{5} \neq 0 \text{ and } \sin \frac{3\pi}{5} \neq 0 \text{ (or only solution to } \sin \theta = 0 \text{ is } \theta = 0)$$
R1

$$\text{therefore } \frac{\pi}{5}, \frac{3\pi}{5} \text{ are solutions of } 16\sin^4 \theta - 20\sin^2 \theta + 5 = 0$$
AG

Note: The final **R1** is dependent on both previous marks.

continued...

Question 12 continued

(ii) **METHOD 1**

attempt to use quadratic formula:

(M1)

$$\sin^2 \theta = \frac{20 \pm \sqrt{80}}{32}$$

A1

$$= \frac{5 \pm \sqrt{5}}{8}$$

$$\sin \theta = \sqrt{\frac{5 \pm \sqrt{5}}{8}}$$

$$\Rightarrow \sin \frac{\pi}{5} \sin \frac{3\pi}{5} = \sqrt{\frac{5 + \sqrt{5}}{8}} \sqrt{\frac{5 - \sqrt{5}}{8}}$$

M1

$$= \sqrt{\frac{20}{64}}$$

A1

$$= \frac{\sqrt{5}}{4}$$

AG**METHOD 2**

$$\text{roots of quartic are } \sin \frac{\pi}{5}, \sin \frac{2\pi}{5}, \sin \frac{3\pi}{5}, \sin \frac{4\pi}{5}$$

A1

$$\text{attempt to set product of roots equal to } \pm \frac{5}{16}$$

M1

$$\sin \frac{\pi}{5} \sin \frac{2\pi}{5} \sin \frac{3\pi}{5} \sin \frac{4\pi}{5} = \frac{5}{16}$$

A1

$$\text{recognition that } \sin \frac{\pi}{5} = \sin \frac{4\pi}{5} \text{ and } \sin \frac{2\pi}{5} = \sin \frac{3\pi}{5}$$

$$\sin^2 \frac{\pi}{5} \sin^2 \frac{3\pi}{5} = \frac{5}{16}$$

A1

$$\sin \frac{\pi}{5} \sin \frac{3\pi}{5} = \frac{\sqrt{5}}{4}$$

AG*continued...*

*Question 12 continued***METHOD 3**

Consider $16\sin^4 \theta - 20\sin^2 \theta + 5 = 0$ as a quadratic in $\sin^2 \theta$ **M1**

($\theta = \frac{\pi}{5}, \frac{3\pi}{5}$ are roots), so $\sin^2 \frac{\pi}{5}$ and $\sin^2 \frac{3\pi}{5}$ are roots of the quadratic. **A1**

Consider product of roots: **M1**

$$\Rightarrow \sin^2 \frac{\pi}{5} \sin^2 \frac{3\pi}{5} = \frac{5}{16} \quad \textbf{A1}$$

$$\sin \frac{\pi}{5} \sin \frac{3\pi}{5} = \frac{\sqrt{5}}{4} \quad \textbf{AG}$$

[7 marks]

Total [17 marks]

1. (a) $BV = \sqrt{(6-3)^2 + (8-4)^2 + (0-9)^2}$ (A1)
 $= 10.2956...$
 $= 10.3 (= \sqrt{106})$ A1

Note: Award **SC(A0)A1** for $BV = \begin{pmatrix} -3 \\ -4 \\ 9 \end{pmatrix}$ where a candidate has misinterpreted notation.

[2 marks]

(b) **METHOD 1**

$BV = VC$ AND $BC = 8$ (seen anywhere) (A1)

attempt to use the cosine rule on triangle BVC for any angle (M1)

Note: Recognition must be shown in context either in terms of labelled sides or in side lengths.

$$\cos \hat{BVC} = \frac{10.2...^2 + 10.2...^2 - 8^2}{2 \times 10.2... \times 10.2...} \text{ OR}$$

$$8^2 = 10.2...^2 + 10.2...^2 - 2 \times 10.2... \times 10.2... \cos \hat{BVC} \quad (A1)$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ) \quad A1$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

METHOD 2

let M be the midpoint of BC

$BM = 4$ (seen anywhere) (A1)

attempt to use sine or cosine in triangle BMV or CMV (M1)

$$\arcsin \frac{4}{\sqrt{106}} \text{ OR } \frac{\pi}{2} - \arccos \frac{4}{\sqrt{106}} \text{ OR } 0.399018 \quad (A1)$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ) \quad A1$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

METHOD 3

$$\overrightarrow{VC} = \begin{pmatrix} 3 \\ -4 \\ -9 \end{pmatrix} \text{ and } \overrightarrow{VB} = \begin{pmatrix} 3 \\ 4 \\ -9 \end{pmatrix} \quad (\text{A1})$$

attempt to use the cosine formula for an angle using two vectors representing the edges of triangle BVC (M1)

$$\cos \hat{BVC} = \frac{3(3) - 4(4) - 9(-9)}{\sqrt{3^2 + (-4)^2 + (-9)^2} \sqrt{3^2 + 4^2 + (-9)^2}} \left(= \frac{74}{106} \right) \quad (\text{A1})$$

$$\hat{BVC} = 0.798037\dots$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ \text{)} \quad \text{A1}$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

METHOD 4

$$\overrightarrow{VC} = \begin{pmatrix} 3 \\ -4 \\ -9 \end{pmatrix} \text{ and } \overrightarrow{VB} = \begin{pmatrix} 3 \\ 4 \\ -9 \end{pmatrix} \quad (\text{A1})$$

attempt to use the cross product formula for an angle using two vectors representing the edges of triangle BVC (M1)

$$\sin \hat{BVC} = \frac{\left| \begin{pmatrix} 72 \\ 0 \\ 24 \end{pmatrix} \right|}{\sqrt{3^2 + (-4)^2 + (-9)^2} \sqrt{3^2 + 4^2 + (-9)^2}} \left(= \frac{\sqrt{5760}}{106} \right) \quad (\text{A1})$$

$$\hat{BVC} = 0.798037\dots$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ \text{)} \quad \text{A1}$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

Award **SC(A1)(M1)(A0)A0** for area = $\frac{1}{2} \left| \begin{pmatrix} 72 \\ 0 \\ 24 \end{pmatrix} \right| = \frac{\sqrt{5760}}{2} (= 37.9)$ where a candidate has

misinterpreted notation.

[4 marks]

Total [6 marks]

3. (a) use of sector area formula to find area of at least one sector (M1)

$$\frac{1}{2} \times 5.2 \times 100 - \frac{1}{2} \times 5.2 \times r^2 \quad \text{OR} \quad 10^2 \pi - \frac{1}{2} 10^2 \times (2\pi - 5.2) - \left(\pi r^2 - \frac{1}{2} \times (2\pi - 5.2) \times r^2 \right) \quad \text{A1}$$

$$(\text{area}) = 260 - 2.6r^2 \quad \text{AG}$$

Note: There are many different ways to find the area of the “C”. In all methods, the **A** mark is awarded for working which leads directly to the **AG**.
Many candidates are working with rounded intermediate values. Award the **A** mark to correct work with values that round to the 3sf value of 260 and the 2sf value of 2.6 eg
 $259.99 - 2.6015r^2$.

[2 marks]

- (b) (i) $260 - 2.6r^2 = 64$ (A1)

$$r = 8.68243\dots$$

$$= 8.68(\text{cm}) \left(\frac{14\sqrt{65}}{13} \text{ exact} \right) \quad \text{A1}$$

- (ii) 10×5.2 OR $8.68\dots \times 5.2$ (A1)

substituting their value of r into $10 \times 5.2 + r \times 5.2 + 2(10 - r)$ (or equivalent) (M1)

$$\text{Perimeter} = 10 \times 5.2 + 8.68\dots \times 5.2 + 2(10 - 8.68\dots) \quad (= 52 + 45.1486\dots + 2.63513\dots)$$

$$= 99.7837\dots$$

$$= 99.8 \text{ (cm)} \quad \text{A1}$$

[5 marks]

Total [7 marks]

8. (a) $\overrightarrow{AB} = \begin{pmatrix} 1-p \\ -1 \end{pmatrix}$ **A1**

$\overrightarrow{AC} = \begin{pmatrix} p \\ -p \\ 2 \end{pmatrix}$ **A1**

attempt to evaluate their $\overrightarrow{AB} \times \overrightarrow{AC}$ by use of formula or determinant **M1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2(1-p)-p \\ -(2+p) \\ -p-p(1-p) \end{pmatrix}$ OR $(2(1-p)-p)\mathbf{i} - (2+p)\mathbf{j} + (-p-p(1-p))\mathbf{k}$ **A1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$ **AG**

[4 marks]

(b) $|\overrightarrow{AB} \times \overrightarrow{AC}|^2$

$= (2-3p)^2 + (-2-p)^2 + (p^2-2p)^2 (= p^4 - 4p^3 + 14p^2 - 8p + 8)$ **(A1)**

attempt to find minimum of their $|\overrightarrow{AB} \times \overrightarrow{AC}|^2$ **(M1)**

6.75257... OR $p = 0.3264...$

min value is 6.75 **A1**

[3 marks]

continued...

Question 8 continued(c) **METHOD 1**

valid attempt to find $\text{area} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$ using their answer to part b) **(M1)**

$$\text{area} = \frac{1}{2} \sqrt{6.75257...}$$

$$= 1.299285...$$

$$= 1.30 \text{ (units}^2\text{)} \quad \mathbf{A1}$$

[2 marks]**Total [9 marks]**

✓ ✓

12. (a) attempt to set at least two components of L and M equal

M1

$$1 + 2s = 9 + 4t$$

$$2 + 3s = 9 + t$$

$$-3 + 6s = 11 + 2t$$

attempt to solve two of their equations simultaneously

(M1)

$$s = 2 \text{ OR } t = -1$$

A1**EITHER**

substitute $s = 2$ and $t = -1$ into remaining component e.g. $-3 + 6(2) = 11 + 2(-1)$

R1**OR**

recognition that 2nd and 3rd equations are equivalent

R1**THEN**

position vector of A is $\begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix}$

A1

Note: Accept a row vector and/or coordinates.

The final **A1** is independent of **R1**.

[5 marks]

- (b) **METHOD 1**

attempt to substitute at least one line into the equation of the plane

(M1)

$$\begin{pmatrix} 1+2s \\ 2+3s \\ -3+6s \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 2(2+3s) - 1(-3+6s) = 7$$

A1

$$\begin{pmatrix} 9+4t \\ 9+t \\ 11+2t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 2(9+t) - 1(11+2t) = 7$$

A1*continued...*

*Question 12 continued***METHOD 2**

consideration the direction of one line and a point on that line

(M1)

$$\text{direction } \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ AND point } \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \quad \text{or} \quad \begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$$

A1

$$\text{direction } \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ AND point } \begin{pmatrix} 9 \\ 9 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \quad \text{or} \quad \begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$$

A1**METHOD 3**

consideration of direction of both lines

(M1)**EITHER**

$$\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ and } \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ (hence } L \text{ and } M \text{ are parallel to the plane)}$$

A1**OR**

$$\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \times \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ -10 \end{pmatrix} = k \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \text{ (hence } L \text{ and } M \text{ are parallel to the plane)}$$

A1**THEN**

$$\begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \text{ OR } \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \text{ OR } \begin{pmatrix} 9 \\ 9 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$$

A1**[3 marks]***continued...*

Question 12 continued

(c) (i) position vector of point on the line is $(\mathbf{r} = \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 12 + 2\lambda \\ 2 - \lambda \end{pmatrix})$ **(A1)**

attempt to substitute position vector into equation of plane Π **(M1)**

meets Π when $\begin{pmatrix} -3 \\ 12 + 2\lambda \\ 2 - \lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$

$$2(12 + 2\lambda) - (2 - \lambda) = 7$$

$$22 + 5\lambda = 7$$

$$\lambda = -3$$
 (A1)

position vector of $\mathbf{r} = \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} - 3 \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix}$ **A1**

Note: Accept a row vector and/or coordinates.

(ii) **METHOD 1**

attempt to find $\overrightarrow{BC}$ using $\overrightarrow{OC} - \overrightarrow{OB}$ **(M1)**

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix}$$

attempt to use distance formula to find $|\overrightarrow{BC}|$ **(M1)**

$$|\overrightarrow{BC}| = \sqrt{(-6)^2 + 3^2}$$

$$= 6.71 (= \sqrt{45} = 3\sqrt{5})$$
 A1

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METHOD 2

recognition that $|\overrightarrow{BC}| = 3 \times \begin{vmatrix} 0 \\ 2 \\ -1 \end{vmatrix}$ **(M1)**

attempt to use distance formula to find $\begin{vmatrix} 0 \\ 2 \\ -1 \end{vmatrix}$ **(M1)**

$$|\overrightarrow{BC}| = 3\sqrt{2^2 + (-1)^2}$$

$$= 6.71 \left(= \sqrt{45} = 3\sqrt{5} \right) \quad \textbf{A1}$$

[7 marks]*continued...*

Question 12 continued

- (d) let
- B'
- be the image of
- B

METHOD 1

$$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \quad (\text{A1})$$

recognition that $\mu = 2\lambda (= -6)$ OR $|BC| = |CB'|$ (may be seen in a diagram) (M1)

$$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix}$$

so coordinates are $B'(-3, 0, 8)$ A1

METHOD 2

$$\overrightarrow{BC} = \overrightarrow{CB'} = \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix} \quad (\text{A1})$$

Note: This may come from $\overrightarrow{BC} = -3\sqrt{5}\mathbf{n}$ using the unit normal vector $\mathbf{n} = \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

$$\overrightarrow{OB'} = \overrightarrow{OC} + \overrightarrow{CB'} \quad \text{OR} \quad \overrightarrow{OB'} = \overrightarrow{OB} + 2\overrightarrow{BC} \quad \text{OR} \quad \overrightarrow{OB'} = 2\overrightarrow{OC} - \overrightarrow{OB} \quad (\text{M1})$$

$$= \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} -6 \\ 12 \\ 10 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix}$$

$$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix} \quad (\text{so coordinates are } B'(-3, 0, 8)) \quad \text{A1}$$

[3 marks]**Total [18 marks]**

1. (a) $BV = \sqrt{(6-3)^2 + (8-4)^2 + (0-9)^2}$ (A1)
 $= 10.2956...$
 $= 10.3 (= \sqrt{106})$ A1

Note: Award **SC(A0)A1** for $BV = \begin{pmatrix} -3 \\ -4 \\ 9 \end{pmatrix}$ where a candidate has misinterpreted notation.

[2 marks]

(b) **METHOD 1**

$BV = VC$ AND $BC = 8$ (seen anywhere) (A1)

attempt to use the cosine rule on triangle BVC for any angle (M1)

Note: Recognition must be shown in context either in terms of labelled sides or in side lengths.

$$\cos \hat{BVC} = \frac{10.2...^2 + 10.2...^2 - 8^2}{2 \times 10.2... \times 10.2...} \text{ OR}$$

$$8^2 = 10.2...^2 + 10.2...^2 - 2 \times 10.2... \times 10.2... \cos \hat{BVC} \quad (A1)$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ) \quad A1$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

METHOD 2

let M be the midpoint of BC

$BM = 4$ (seen anywhere) (A1)

attempt to use sine or cosine in triangle BMV or CMV (M1)

$$\arcsin \frac{4}{\sqrt{106}} \text{ OR } \frac{\pi}{2} - \arccos \frac{4}{\sqrt{106}} \text{ OR } 0.399018 \quad (A1)$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ) \quad A1$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

METHOD 3

$$\overrightarrow{VC} = \begin{pmatrix} 3 \\ -4 \\ -9 \end{pmatrix} \text{ and } \overrightarrow{VB} = \begin{pmatrix} 3 \\ 4 \\ -9 \end{pmatrix} \quad (\text{A1})$$

attempt to use the cosine formula for an angle using two vectors representing the edges of triangle BVC (M1)

$$\cos \hat{BVC} = \frac{3(3) - 4(4) - 9(-9)}{\sqrt{3^2 + (-4)^2 + (-9)^2} \sqrt{3^2 + 4^2 + (-9)^2}} \left(= \frac{74}{106} \right) \quad (\text{A1})$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ \text{)} \quad \text{A1}$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

METHOD 4

$$\overrightarrow{VC} = \begin{pmatrix} 3 \\ -4 \\ -9 \end{pmatrix} \text{ and } \overrightarrow{VB} = \begin{pmatrix} 3 \\ 4 \\ -9 \end{pmatrix} \quad (\text{A1})$$

attempt to use the cross product formula for an angle using two vectors representing the edges of triangle BVC (M1)

$$\sin \hat{BVC} = \frac{\left| \begin{pmatrix} 72 \\ 0 \\ 24 \end{pmatrix} \right|}{\sqrt{3^2 + (-4)^2 + (-9)^2} \sqrt{3^2 + 4^2 + (-9)^2}} \left(= \frac{\sqrt{5760}}{106} \right) \quad (\text{A1})$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ \text{)} \quad \text{A1}$$

Note: If no working shown, award **(A0)(M1)(A0)A0** for $\hat{BVC} = 0.80$ (or 46°) (2sf).

Award **SC(A1)(M1)(A0)A0** for area = $\frac{1}{2} \left| \begin{pmatrix} 72 \\ 0 \\ 24 \end{pmatrix} \right| = \frac{\sqrt{5760}}{2} (= 37.9)$ where a candidate has

misinterpreted notation.

[4 marks]

Total [6 marks]

3. (a) use of sector area formula to find area of at least one sector (M1)

$$\frac{1}{2} \times 5.2 \times 100 - \frac{1}{2} \times 5.2 \times r^2 \quad \text{OR} \quad 10^2 \pi - \frac{1}{2} 10^2 \times (2\pi - 5.2) - \left(\pi r^2 - \frac{1}{2} \times (2\pi - 5.2) \times r^2 \right) \quad \text{A1}$$

$$(\text{area}) = 260 - 2.6r^2 \quad \text{AG}$$

Note: There are many different ways to find the area of the “C”. In all methods, the **A** mark is awarded for working which leads directly to the **AG**.
Many candidates are working with rounded intermediate values. Award the **A** mark to correct work with values that round to the 3sf value of 260 and the 2sf value of 2.6 eg $259.99 - 2.6015r^2$.

[2 marks]

- (b) (i) $260 - 2.6r^2 = 64$ (A1)

$$r = 8.68243\dots$$

$$= 8.68(\text{cm}) \left(\frac{14\sqrt{65}}{13} \text{ exact} \right) \quad \text{A1}$$

- (ii) 10×5.2 OR $8.68\dots \times 5.2$ (A1)

substituting their value of r into $10 \times 5.2 + r \times 5.2 + 2(10 - r)$ (or equivalent) (M1)

$$\text{Perimeter} = 10 \times 5.2 + 8.68\dots \times 5.2 + 2(10 - 8.68\dots) (= 52 + 45.1486\dots + 2.63513\dots)$$

$$= 99.7837\dots$$

$$= 99.8 (\text{cm}) \quad \text{A1}$$

[5 marks]

Total [7 marks]

8. (a) $\overrightarrow{AB} = \begin{pmatrix} 1-p \\ -1 \end{pmatrix}$ **A1**

$\overrightarrow{AC} = \begin{pmatrix} p \\ -p \\ 2 \end{pmatrix}$ **A1**

attempt to evaluate their $\overrightarrow{AB} \times \overrightarrow{AC}$ by use of formula or determinant **M1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2(1-p)-p \\ -(2+p) \\ -p-p(1-p) \end{pmatrix}$ OR $(2(1-p)-p)\mathbf{i} - (2+p)\mathbf{j} + (-p-p(1-p))\mathbf{k}$ **A1**

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$ **AG**

[4 marks]

(b) $|\overrightarrow{AB} \times \overrightarrow{AC}|^2$

$= (2-3p)^2 + (-2-p)^2 + (p^2-2p)^2 (= p^4 - 4p^3 + 14p^2 - 8p + 8)$ **(A1)**

attempt to find minimum of their $|\overrightarrow{AB} \times \overrightarrow{AC}|^2$ **(M1)**

6.75257... OR $p = 0.3264...$

min value is 6.75 **A1**

[3 marks]

continued...

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Question 8 continued(c) **METHOD 1**

valid attempt to find $\text{area} = \frac{1}{2} |\overrightarrow{AB} \times \overrightarrow{AC}|$ using their answer to part b) **(M1)**

$$\text{area} = \frac{1}{2} \sqrt{6.75257...}$$

$$= 1.299285...$$

$$= 1.30 \text{ (units}^2\text{)} \quad \mathbf{A1}$$

[2 marks]**Total [9 marks]**

✓ ✓

12. (a) attempt to set at least two components of L and M equal

M1

$$1 + 2s = 9 + 4t$$

$$2 + 3s = 9 + t$$

$$-3 + 6s = 11 + 2t$$

attempt to solve two of their equations simultaneously

(M1)

$$s = 2 \text{ OR } t = -1$$

A1**EITHER**

substitute $s = 2$ and $t = -1$ into remaining component e.g. $-3 + 6(2) = 11 + 2(-1)$

R1**OR**

recognition that 2nd and 3rd equations are equivalent

R1**THEN**

position vector of A is $\begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix}$

A1

Note: Accept a row vector and/or coordinates.

The final **A1** is independent of **R1**.

[5 marks]

- (b) **METHOD 1**

attempt to substitute at least one line into the equation of the plane

(M1)

$$\begin{pmatrix} 1 + 2s \\ 2 + 3s \\ -3 + 6s \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 2(2 + 3s) - 1(-3 + 6s) = 7$$

A1

$$\begin{pmatrix} 9 + 4t \\ 9 + t \\ 11 + 2t \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 2(9 + t) - 1(11 + 2t) = 7$$

A1*continued...*

*Question 12 continued***METHOD 2**

consideration the direction of one line and a point on that line

(M1)

$$\text{direction } \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ AND point } \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \quad \text{or} \quad \begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$$

A1

$$\text{direction } \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ AND point } \begin{pmatrix} 9 \\ 9 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \quad \text{or} \quad \begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$$

A1**METHOD 3**

consideration of direction of both lines

(M1)**EITHER**

$$\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ and } \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 0 \text{ (hence } L \text{ and } M \text{ are parallel to the plane)}$$

A1**OR**

$$\begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \times \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ -10 \end{pmatrix} = k \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \text{ (hence } L \text{ and } M \text{ are parallel to the plane)}$$

A1**THEN**

$$\begin{pmatrix} 5 \\ 8 \\ 9 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \text{ OR } \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7 \text{ OR } \begin{pmatrix} 9 \\ 9 \\ 11 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$$

A1**[3 marks]***continued...*

Question 12 continued

(c) (i) position vector of point on the line is $(\mathbf{r} =) \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 12 + 2\lambda \\ 2 - \lambda \end{pmatrix}$ **(A1)**

attempt to substitute position vector into equation of plane Π **(M1)**

meets Π when $\begin{pmatrix} -3 \\ 12 + 2\lambda \\ 2 - \lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$

$$2(12 + 2\lambda) - (2 - \lambda) = 7$$

$$22 + 5\lambda = 7$$

$$\lambda = -3$$
 (A1)

position vector of $\mathbf{r} = \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} - 3 \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix}$ **A1**

Note: Accept a row vector and/or coordinates.

(ii) **METHOD 1**

attempt to find $\overrightarrow{BC}$ using $\overrightarrow{OC} - \overrightarrow{OB}$ **(M1)**

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix}$$

attempt to use distance formula to find $|\overrightarrow{BC}|$ **(M1)**

$$|\overrightarrow{BC}| = \sqrt{(-6)^2 + 3^2}$$

$$= 6.71 (= \sqrt{45} = 3\sqrt{5})$$
 A1

METHOD 2

recognition that $|\overrightarrow{BC}| = 3 \times \begin{vmatrix} 0 \\ 2 \\ -1 \end{vmatrix}$ **(M1)**

attempt to use distance formula to find $\begin{vmatrix} 0 \\ 2 \\ -1 \end{vmatrix}$ **(M1)**

$$|\overrightarrow{BC}| = 3\sqrt{2^2 + (-1)^2}$$

$$= 6.71 (= \sqrt{45} = 3\sqrt{5})$$
A1

[7 marks]*continued...*

Question 12 continued

- (d) let
- B'
- be the image of
- B

METHOD 1

$$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} \quad (\text{A1})$$

recognition that $\mu = 2\lambda (= -6)$ OR $|BC| = |CB'|$ (may be seen in a diagram) (M1)

$$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix}$$

so coordinates are $B'(-3, 0, 8)$ A1

METHOD 2

$$\overrightarrow{BC} = \overrightarrow{CB'} = \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix} \quad (\text{A1})$$

Note: This may come from $\overrightarrow{BC} = -3\sqrt{5}\mathbf{n}$ using the unit normal vector $\mathbf{n} = \frac{1}{\sqrt{5}} \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

$$\overrightarrow{OB'} = \overrightarrow{OC} + \overrightarrow{CB'} \quad \text{OR} \quad \overrightarrow{OB'} = \overrightarrow{OB} + 2\overrightarrow{BC} \quad \text{OR} \quad \overrightarrow{OB'} = 2\overrightarrow{OC} - \overrightarrow{OB} \quad (\text{M1})$$

$$= \begin{pmatrix} -3 \\ 6 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 0 \\ -6 \\ 3 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} -6 \\ 12 \\ 10 \end{pmatrix} - \begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix}$$

$$\overrightarrow{OB'} = \begin{pmatrix} -3 \\ 0 \\ 8 \end{pmatrix} \quad (\text{so coordinates are } B'(-3, 0, 8)) \quad \text{A1}$$

[3 marks]**Total [18 marks]**

3. (a) $2r + r\theta = 10$ **A1**

$$\frac{1}{2}r^2\theta = 6.25$$
 A1

attempt to eliminate θ to obtain an equation in r **M1**

correct intermediate equation in r **A1**

$$10 - 2r = \frac{25}{2r} \quad \text{OR} \quad \frac{10}{r} - 2 = \frac{25}{2r^2} \quad \text{OR} \quad \frac{1}{2}r^2\left(\frac{10}{r} - 2\right) = 6.25 \quad \text{OR} \quad 12.5 + 2r^2 = 10r$$

$$4r^2 - 20r + 25 = 0$$
 AG

[4 marks]

(b) attempt to solve quadratic by factorizing or use of formula or completing the square **(M1)**

$$(2r - 5)^2 = 0 \quad \text{OR} \quad r = \frac{20 \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} \left(= \frac{20 \pm \sqrt{400 - 400}}{8} \right)$$

$$r = \frac{5}{2}$$
 A1

attempt to substitute their value of r into their perimeter or area equation **(M1)**

$$\theta = \frac{10 - 2\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)} \quad \text{or} \quad \theta = \frac{25}{2\left(\frac{5}{2}\right)^2}$$

$$\theta = 2$$
 A1

[4 marks]

Total [8 marks]

1. $\tan^{-1} 1 = 45^\circ$ or equivalent **(A1)**

attempt to equate $2x - 5^\circ$ to their reference angle **(M1)**

Note: Do not accept $2x - 5^\circ = 1$.

$$2x - 5^\circ = 45^\circ, (225^\circ)$$

$$x = 25^\circ, 115^\circ \quad \textbf{A1A1}$$

Note: Do not award the final **A1** if any additional solutions are seen.

[4 marks]

11.

$$(a) \quad 2 \times 2 + 6 \times \frac{1}{2} - 2 \times 1 = 5$$

A1**[1 mark]**

$$(b) \quad 2(k^2 - 6) + 6(2k + 3) + 12 (= 0)$$

(A1)

equating their scalar product of the direction normals to zero

(M1)

$$2(k^2 - 6) + 6(2k + 3) + 12 = 0$$

$$k^2 - 6 + 6k + 9 + 6 = 0 \quad \text{OR} \quad (k + 3)^2 = 0$$

$$k = -3$$

A1attempt to substitute k, p and coordinates of A into Π_2 **(M1)**

$$q = 3 \times 2 - 3 \times \frac{1}{2} - 6 \times 1$$

$$q = -\frac{3}{2}$$

A1**[5 marks]***continued...*

Question 11 continued.

- (c) attempt to equate a pair of ratios or equate vector product to zero vector **(M1)**

$$\begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} = \mu \begin{pmatrix} 3 \\ 9 \\ p \end{pmatrix} \Rightarrow \mu = \frac{2}{3} \text{ OR } \frac{2}{-2} = \frac{3}{p} \text{ OR } \frac{6}{-2} = \frac{9}{p} \text{ OR } 6p + 18 = 0 \text{ OR } -6 - 2p = 0$$

$$p = -3$$

A1**[2 marks]**

- (d) (i) attempt to find the vector equation of the line through A perpendicular to Π_1 **(M1)**

$$(\mathbf{r} =) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix}$$

attempt to substitute their vector equation into Π_2 **(M1)**

$$3(2 + 2\lambda) + 9\left(\frac{1}{2} + 6\lambda\right) - 3(1 - 2\lambda) = -\frac{51}{2}$$

$$6 + 6\lambda + \frac{9}{2} + 54\lambda - 3 + 6\lambda = -\frac{51}{2} \quad \text{(or equivalent)} \quad \textbf{(A1)}$$

$$66\lambda = -\frac{51}{2} - 6 - \frac{9}{2} + 3 \quad \text{(or equivalent)}$$

$$\lambda = -\frac{1}{2} \quad \textbf{A1}$$

$$\left(1, -\frac{5}{2}, 2\right) \quad \textbf{A1}$$

continued...

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Question 11 continued.

$$(ii) \quad \text{distance AB or } \left| \lambda \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} \right| \quad (M1)$$

$$\sqrt{(2-1)^2 + \left(\frac{1}{2} + \frac{5}{2}\right)^2 + (1-2)^2} \quad \text{OR} \quad \sqrt{1+9+1} \quad A1$$

$$= \sqrt{11} \quad AG$$

[7 marks]

- (e) Valid method to find a point C on Π_3 using $\overrightarrow{AC} = \overrightarrow{BA}$ or $\overrightarrow{BC} = 2\overrightarrow{BA}$ or A as the midpoint of BC. (M1)

$$\lambda = \frac{1}{2}, \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ \frac{1}{2} \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -\frac{5}{2} \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix}, \quad \overrightarrow{OA} = \frac{1}{2}(\overrightarrow{OB} + \overrightarrow{OC})$$

point on Π_3 is $\left(3, \frac{7}{2}, 0\right)$ A1

attempt to substitute their $\left(3, \frac{7}{2}, 0\right)$ into $\Pi_3 : x + 3y - z = d$ (or equivalent) (M1)

$$1 \times 3 + 3 \times \frac{7}{2} - 1 \times 0 \left(= \frac{27}{2} \right)$$

$$\Pi_3 : x + 3y - z = \frac{27}{2} \quad (2x + 6y - 2z = 27) \text{ (or equivalent)} \quad A1$$

[4 marks]

Total [19 marks]

2. (a) attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r + 5)$ **(M1)**

$$\tan 58^\circ = \frac{18.2}{r + 5} \quad \text{OR} \quad \frac{r + 5}{\sin 32^\circ} = \frac{18.2}{\sin 58^\circ} \quad \text{OR} \quad (r + 5) = 11.3726... \quad \text{(A1)}$$

$$r = 6.37262... \text{ (m)}$$

$$(r =) 6.37 \text{ (m)} \quad \text{A1}$$

[3 marks]

- (b) attempt to substitute $h = 20$ and their radius into the correct volume of cone formula **(M1)**

$$V = \frac{\pi(6.37262...)^2(20)}{3}$$

$$= 850.540...$$

$$= 851 \text{ (m}^3\text{)} \quad \text{A1}$$

Note: Accept 849.840... (850) obtained from using $r = 6.37$.

[2 marks]

Total [5 marks]

10. (a) EITHER

attempt to find value of t for the first low tide OR the first high tide **(M1)**

$$11.2619... - 5.13801...$$

$$= 6.12396... \quad \text{A1}$$

OR

attempt to find half of the period **(M1)**

$$\frac{1}{2} \times \frac{2\pi}{0.513}$$

$$= 6.12396... \quad \text{A1}$$

THEN

$$m = (6.12396... - 6) \times 60 = 7.43773...$$

$$m = 7 \quad \text{A1}$$

[3 marks]

(b) attempt to solve $H(t) = 1$ **(M1)**

$$3.56919... \text{ OR } 6.70684... \text{ OR } 15.8171... \text{ OR } 18.9547...$$

$$(6.70684... - 3.56919...) = 3.13764...$$

$$= 3.14 \text{ (hours)} \quad \text{A1}$$

[2 marks]

(c) recognition that $H'(13)$ is required **(M1)**

$$= -0.650622...$$

$$= -0.651 \text{ (m/h)} \quad \text{A1}$$

[2 marks]*continued...*

Question 10 continued

(d)

Note: In part (d), award the marks for a , b , c and d independent of each other.

METHOD 1

$$a = 1.17 \quad \text{A1}$$

$$d = 1.57 \quad \text{A1}$$

attempt to find time between low and high tide in hours (M1)

6 hours and 21 minutes = 6.35 hours

(period =) 12.7 (A1)

$$b = \frac{2\pi}{12.7} = 0.494739\dots$$

$$b = 0.495 \left(= \frac{60\pi}{381} \right) \quad \text{A1}$$

attempt to find mean of low and high tide times OR substitute values of a known point (M1)

$$c = \frac{1}{2} \left(2 \frac{41}{60} + 9 \frac{2}{60} \right) \text{ OR eg } 0.40 = 1.17 \sin(0.495(2.68333\dots - c)) + 1.57$$

$$c = 5.85833\dots$$

$$c = 5.86 \quad \text{A1}$$

Note: Award **(M1)A1** for $c = 18.6$.
Award **(M1)A0** for $c = -6.84$.

[7 marks]

continued...

Question 10 continued

METHOD 2

$$a = 1.17 \quad \text{A1}$$

$$d = 1.57 \quad \text{A1}$$

substituting at least one point into $h(t)$ (M1)

$$1.17 \sin \left(b \left(2 \frac{41}{60} - c \right) \right) + 1.57 = 0.4 \quad \text{OR} \quad 1.17 \sin \left(b \left(9 \frac{2}{60} - c \right) \right) + 1.57 = 2.74$$

$$b \left(2 \frac{41}{60} - c \right) = -\frac{\pi}{2} (= -1.57) \quad \text{AND} \quad b \left(9 \frac{2}{60} - c \right) = \frac{\pi}{2} (= 1.57) \quad \text{(A1)}$$

Note: accept any angles of the form $-\frac{\pi}{2} + c\pi k$ and $\frac{\pi}{2} + c\pi k$.

EITHER

use of graph or table to find their intersection (M1)

OR

attempt to solve their equations simultaneously (M1)

$$\frac{2 \frac{41}{60} - c}{9 \frac{2}{60} - c} = -1$$

THEN

$$c = 5.85833\dots$$

$$c = 5.86 \quad \text{A1}$$

$$b = 0.494739\dots$$

$$b = 0.495 \quad \text{A1}$$

[7 marks]

(e) attempt to find point of intersection of two graphs (M1)

$$T = 4.16292\dots \quad \text{OR} \quad T = 4.16417\dots \text{ (using 3 sf)}$$

$$T = 4.16 \quad \text{A1}$$

[2 marks]

Total [16 marks]

12. (a) Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$.

$$\begin{aligned}
 & (\mathbf{u} \cdot \mathbf{v})^2 + |\mathbf{u} \times \mathbf{v}|^2 \\
 &= (|\mathbf{u}||\mathbf{v}|\cos\theta)^2 + (|\mathbf{u}||\mathbf{v}|\sin\theta)^2 \quad \text{OR} \quad |\mathbf{u}|^2|\mathbf{v}|^2\cos^2\theta + |\mathbf{u}|^2|\mathbf{v}|^2\sin^2\theta & (A1) \\
 &= |\mathbf{u}|^2|\mathbf{v}|^2(\cos^2\theta + \sin^2\theta) & A1 \\
 &= |\mathbf{u}|^2|\mathbf{v}|^2 & AG
 \end{aligned}$$

[2 marks]

(b) (i) $|\mathbf{u} \times \mathbf{v}| = 2\sqrt{6} (= 4.89897... = 4.90)$ A1

(ii) $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}$ (A1)

$$|\mathbf{v}| = \sqrt{3^2 + 1^2 + (-1)^2} (= \sqrt{11} = 3.31662...) \quad (A1)$$

substitution of values $\mathbf{u} \cdot \mathbf{v}$, $|\mathbf{u} \times \mathbf{v}|$ and $|\mathbf{v}|$ into $(\mathbf{u} \cdot \mathbf{v})^2 + |\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2|\mathbf{v}|^2$ M1

$$3^2 + (2\sqrt{6})^2 = |\mathbf{u}|^2 |\sqrt{11}|^2$$

$$|\mathbf{u}| = \sqrt{3} (= 1.73205... = 1.73) \quad A1$$

continued...

Question 12 continued

$$(iii) \quad \mathbf{u} = \begin{pmatrix} p \\ q-1 \\ 1 \end{pmatrix} \quad (A1)$$

attempt to use $\mathbf{u} \cdot \mathbf{v} = 3$ (M1)

$$\begin{pmatrix} p \\ q-1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} = 3$$

$$3p + q = 5 (\Rightarrow q = 5 - 3p) \quad A1$$

attempt to use $|\mathbf{u}| = \sqrt{3}$ (M1)

$$p^2 + (q-1)^2 + 1^2 = \sqrt{3}^2 (\Rightarrow p^2 + q^2 - 2q + 1 + 1 = 3)$$

Note: Award **M1** for use of $|\mathbf{u} \times \mathbf{v}|^2 = q^2 + (p+3)^2 + (p-3q+3)^2 = (2\sqrt{6})^2$.

attempt to form quadratic in one variable, p or q (M1)

$$p^2 + (4-3p)^2 = 2 \quad \text{OR} \quad p^2 + (5-3p)^2 - 2p = 1 \quad \text{OR} \quad 10p^2 - 24p + 14 = 0 \quad \text{OR}$$

$$\left(\frac{5-q}{3}\right)^2 + (q-1)^2 = 2 \quad \text{OR} \quad \left(\frac{5-q}{3}\right)^2 + q^2 - 2q = 1 \quad \text{OR} \quad 10q^2 - 28q + 16 = 0 \quad (A1)$$

$$p = 1 \quad \text{or} \quad p = 1.4 \left(= \frac{7}{5} \right) \quad A1$$

$$q = 2 \quad \text{or} \quad q = 0.8 \left(= \frac{4}{5} \right) \quad A1$$

Note: Award final **A1** marks for correct values, even if the p values and q values are not explicitly paired.

[13 marks]

continued...

Question 12 continued

(c) **METHOD 1**

attempt to express $w = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in terms of one variable **(M1)**

$$v \cdot w = 3x + y - z = 0, \quad u \cdot w = x + y + z = 0$$

$$y = -2x \text{ and } z = x \text{ OR } x = z = -\frac{1}{2}y \text{ OR } x = z \text{ and } y = -2z \quad \textbf{(A1)}$$

attempt to use area of a triangle = $\frac{\text{base} \times \text{height}}{2} = \frac{|w||v|}{2}$ **(M1)**

$$\frac{\sqrt{x^2 + y^2 + z^2} \sqrt{11}}{2} = 5$$

$$6x^2 = \frac{100}{11} \text{ or } \frac{3y^2}{2} = \frac{100}{11} \text{ or } 6z^2 = \frac{100}{11}$$

$$x = \pm 1.2309... \left(= \pm \frac{5\sqrt{66}}{33} \right) \text{ OR } y = \mp 2.4618... \left(\mp \frac{10\sqrt{66}}{33} \right) \text{ OR}$$

$$z = \pm 1.2309... \left(= \pm \frac{5\sqrt{66}}{33} \right) \quad \textbf{A1}$$

$$w = \pm \begin{pmatrix} 1.23091... \\ -2.46182... \\ 1.23091... \end{pmatrix}$$

$$w = \pm \begin{pmatrix} 1.23 \\ -2.46 \\ 1.23 \end{pmatrix} \text{ or } w = \pm \frac{5}{\sqrt{66}} \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} \quad \textbf{A1}$$

Note: If no working shown, award **M1A1(M1)A1A0** for $w = \pm \begin{pmatrix} 1.2 \\ -2.5 \\ 1.2 \end{pmatrix}$ (2 sf)

continued...

Question 12 continued

METHOD 2

attempt to write $\mathbf{u} \times \mathbf{v}$ as a multiple of $\mathbf{w}$ or recognizing that $\mathbf{w}$ is normal to $\mathbf{u}$ and $\mathbf{v}$ **(M1)**

$$\mathbf{w} = \mathbf{u} \times \mathbf{v} = \lambda \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} -2\lambda \\ 4\lambda \\ -2\lambda \end{pmatrix} \quad \textbf{(A1)}$$

attempt to use area of a triangle = $\frac{\text{base} \times \text{height}}{2}$ OR area = $\frac{1}{2} |\mathbf{v} \times \mathbf{w}|$ **(M1)**

$$\frac{\sqrt{(-2\lambda)^2 + (4\lambda)^2 + (-2\lambda)^2} \sqrt{11}}{2} = 5 \quad \text{OR} \quad \frac{1}{2} |\mathbf{v} \times \mathbf{w}| = \lambda \begin{vmatrix} 1 \\ 4 \\ 7 \end{vmatrix} = 5$$

$$(-2\lambda)^2 + (4\lambda)^2 + (-2\lambda)^2 = \frac{100}{11} \quad \text{OR} \quad \lambda^2 + (4\lambda)^2 + (7\lambda)^2 = 25$$

$$24\lambda^2 = \frac{100}{11} \quad \text{OR} \quad 66\lambda^2 = 25$$

$$\lambda = \pm 0.61545... \left(= \pm \frac{5\sqrt{66}}{66} \right) \quad \textbf{A1}$$

$$\mathbf{w} = \pm \begin{pmatrix} 1.23091... \\ -2.46182... \\ 1.23091... \end{pmatrix}$$

$$\mathbf{w} = \pm \begin{pmatrix} 1.23 \\ -2.46 \\ 1.23 \end{pmatrix} \quad \text{or} \quad \mathbf{w} = \pm \frac{5}{\sqrt{66}} \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} \quad \textbf{A1}$$

Note: If no working shown, award **M1A1(M1)A1A0** for $\mathbf{w} = \pm \begin{pmatrix} 1.2 \\ -2.5 \\ 1.2 \end{pmatrix}$ (2 sf)

continued...

Question 12 continued

METHOD 3

$$\text{recognising } \frac{1}{2}|\mathbf{u} \times \mathbf{w}| = 5 \times \frac{|\mathbf{u}|}{|\mathbf{v}|} \left(= \frac{5\sqrt{3}}{\sqrt{11}} \right) \quad (\text{M1})$$

since $\mathbf{w}$ is perpendicular to both $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{w}$ is a multiple of their normal

$$\Rightarrow \mathbf{w} = \lambda \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} \quad (\text{A1})$$

attempt to find $\mathbf{u} \times \mathbf{w}$ (M1)

$$\mathbf{u} \times \mathbf{w} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \times \lambda \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} = \lambda \begin{pmatrix} -6 \\ 0 \\ 6 \end{pmatrix}$$

$$|\mathbf{u} \times \mathbf{w}| = \lambda\sqrt{72}$$

$$\frac{1}{2}\lambda\sqrt{72} = 5\frac{\sqrt{3}}{\sqrt{11}} \Rightarrow \lambda = \frac{10}{\sqrt{264}} (= 0.615457...) \quad \text{A1}$$

$$\mathbf{w} = \pm \begin{pmatrix} 1.23091... \\ -2.46182... \\ 1.23091... \end{pmatrix}$$

$$\mathbf{w} = \pm \begin{pmatrix} 1.23 \\ -2.46 \\ 1.23 \end{pmatrix} \text{ or } \mathbf{w} = \pm \frac{5}{\sqrt{66}} \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} \quad \text{A1}$$

Note: If no working shown, award **M1A1(M1)A1A0** for $\mathbf{w} = \pm \begin{pmatrix} 1.2 \\ -2.5 \\ 1.2 \end{pmatrix}$ (2 sf)

[5 marks]

Total [20 marks]

11. (a) METHOD 1

let M be the midpoint of $[AB]$ and so $AB = 2AM$

attempts to use Pythagoras' theorem to find AM^2 OR AM **(M1)**

$$AM^2 = 20^2 - 14^2 (= 204) \text{ OR } AM = \sqrt{20^2 - 14^2} (= 14.2828... = \sqrt{204} = 2\sqrt{51})$$

recognizes that $AB = 2AM$ **(A1)**

$$AB = 2 \times 14.2828... (= 28.5657...) (= 2\sqrt{204} = 4\sqrt{51}) \quad \textbf{A1}$$

$$AB = 28.5657...$$

$$AB = 28.57 \text{ (m)} \quad \textbf{AG}$$

METHOD 2

let M be the midpoint of $[AB]$ and so $AB = 2AM$

let $\theta = \hat{ASM}$

$$\theta = 0.795398... \left(= \cos^{-1} \frac{14}{20} \right) \quad \textbf{(A1)}$$

attempts to use a valid trigonometric ratio **M1**

EITHER

$$AM = 14 \tan(0.795398...) \left(= 14.2828... = 14 \tan \left(\cos^{-1} \frac{14}{20} \right) \right) \quad \textbf{A1}$$

OR

$$AM = 20 \sin(0.795398...) \left(= 14.2828... = 20 \sin \left(\cos^{-1} \frac{14}{20} \right) \right) \quad \textbf{A1}$$

THEN

$$AB = 28.5657...$$

$$AB = 28.57 \text{ (m)} \quad \textbf{AG}$$

[3 marks]

continued...

Question 11 continued.

(b) **EITHER**

the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and

hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second

A1

OR

$$\left(\frac{2\pi}{n} = 16 \Rightarrow n = \right) \frac{2\pi}{16} \left(= \frac{\pi}{8} \right)$$

A1

THEN

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second

AG

[1 mark]

continued...

Question 11 continued.

(c)

Note: For candidates that used Method 2 in part (a) apply full FT from their value of θ .

attempts to find 2θ where $\theta = \hat{A}\hat{S}\hat{M}$ **(M1)**

$$= 2(0.795398...) \left(1.59079... = 2\cos^{-1}\frac{14}{20} \right)$$

uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving T **(M1)**

$$\frac{2\pi}{16} = \frac{1.59079...}{T} \left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T} \right) \quad \text{A1}$$

$$T = 4.05093... \left(= \frac{1.59079...}{\frac{2\pi}{16}} \right) \left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}} \right)$$

$$T = 4.05 \text{ (s)} \quad \text{A1}$$

[4 marks]

continued...

Question 11 continued.

(d) $\alpha = \frac{\pi t}{8}$

A1

[1 mark]

(e) applies sine rule in $\triangle ASD$

A1

$$\frac{d}{\sin \alpha} = \frac{20}{\sin \hat{ADS}}$$

attempts to find $\hat{ADS}$ in terms of α

M1

$$\hat{ADS} = \pi - \beta - \alpha \quad (= \pi - 0.7754 - \alpha) \quad (= 2.366... - \alpha) \quad (= 2.37 - \alpha)$$

$$d = \frac{20 \sin \alpha}{\sin(2.366... - \alpha)} \left(= \frac{20 \sin \alpha}{\sin(2.37 - \alpha)} \right) \text{ (accept } d = \frac{20 \sin \alpha}{\sin(\pi - \beta - \alpha)} \text{)}$$

A1

$$d = \frac{20 \sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}$$

AG

[3 marks]

(f) 18 (m)

A1

[1 mark]

continued...

Question 11 continued.

(g) (i) $w = \left| 0.05t^2 + 1.1t + 18 - \frac{20 \sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)} \right|$ **A1**

(ii) attempts to solve $w = 0$ for t **(M1)**

$$t = 3.34880\dots(12.7765\dots)$$

$$t = 3.35 \text{ (s)} \quad \textbf{A1}$$

$$22.2444\dots$$

$$22.2 \text{ (m) (south of A)} \quad \textbf{A1}$$

[4 marks]

Total [17 marks]

1. (a) $\frac{1}{2}r^2\theta = 48$ OR $\frac{1}{2}r^2(1.5) = 48$ (A1)

attempt to solve their equation to find r or r^2 (M1)

Note: To award the **M1**, candidate's equation must include r^2 and $\theta = 1.5$, and they must attempt to isolate r^2 or r .

$$r^2 = 64$$

$$r = 8 \text{ (cm)}$$

A1

[3 marks]

(b) evidence of summing the two radii and the arc length (M1)

$$\text{perimeter} = 2r + r\theta$$

$$= 16 + 8(1.5)$$

$$= 28 \text{ (cm)}$$

A1

[2 marks]

Total [5 marks]

4. attempt to substitute into cosine rule (M1)

$$(\cos 2\theta =) \frac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6} \text{ OR } 5^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 2\theta$$

$$(\cos 2\theta =) \frac{27}{48} \left(= \frac{9}{16} \right) \quad \text{(A1)}$$

attempt to use $\cos 2\theta = 2\cos^2 \theta - 1$ (M1)

$$\cos^2 \theta = \frac{1 + \frac{27}{48}}{2} \left(= \frac{1 + \frac{9}{16}}{2} \right)$$

$$\cos^2 \theta = \frac{75}{96} \left(= \frac{25}{32} \right) \quad \text{A1}$$

$$\cos \theta = (\pm) \sqrt{\frac{75}{96}} \left(= \sqrt{\frac{25}{32}} = \frac{5}{\sqrt{32}} \right) \quad \text{(A1)}$$

$$= \frac{5}{4\sqrt{2}}$$

$$= \frac{5\sqrt{2}}{8} \quad (p=5, q=8) \quad \text{A1}$$

Note: The final answer must be positive.

[6 marks]

8. (a) $\vec{OP} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a})$ **A1**
 $= (1 - \lambda)\mathbf{a} + \lambda\mathbf{b}$ **AG**
[1 mark]

(b) **METHOD 1**

recognition that $\vec{OP} \cdot \vec{AB} = 0$ (may be seen anywhere) **(M1)**

$$[(1 - \lambda)\mathbf{a} + \lambda\mathbf{b}] \cdot [\mathbf{b} - \mathbf{a}] (= 0) \quad \text{A1}$$

attempt to multiply out scalar product **M1**

$$(1 - \lambda)\mathbf{a} \cdot \mathbf{b} + \lambda\mathbf{b} \cdot \mathbf{b} - (1 - \lambda)\mathbf{a} \cdot \mathbf{a} - \lambda\mathbf{b} \cdot \mathbf{a} (= 0) \quad \text{(A1)}$$

attempt to substitute for $\mathbf{a} \cdot \mathbf{b}$ and $|\mathbf{a}|$ and $|\mathbf{b}|$ **(M1)**

$$\frac{1}{4}(1 - \lambda) + 4\lambda - (1 - \lambda) - \frac{\lambda}{4} (= 0) \quad \text{(A1)}$$

$$1 - \lambda + 16\lambda - 4 + 4\lambda - \lambda = 0$$

$$18\lambda - 3 = 0$$

$$\lambda = \frac{1}{6} \quad \text{A1}$$

continued...

Question 8 continued.

METHOD 2

$$\cos AOB = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{1}{8} \quad \mathbf{A1}$$

attempt to use cosine rule to find AB **M1**

$$|AB|^2 = 1^2 + 2^2 - 2(1)(2)\left(\frac{1}{8}\right)$$

$$AB = \frac{3\sqrt{2}}{2} \quad \mathbf{A1}$$

attempt to apply Pythagoras' Theorem twice: **M1**

$$|OP|^2 + \left(\frac{3\sqrt{2}}{2}\lambda\right)^2 = 1 \text{ and}$$

$$|OP|^2 + \left(\frac{3\sqrt{2}}{2}(1-\lambda)\right)^2 = 4 \quad \mathbf{A1}$$

attempt to solve simultaneously: **M1**

$$\frac{9}{2}(1-\lambda)^2 - \frac{9}{2}\lambda^2 = 3$$

$$\lambda = \frac{1}{6} \quad \mathbf{A1}$$

[7 marks]

Total [8 marks]

9. (a) METHOD 1

attempt to use identity $\tan(A - B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B}$ **M1**

$$\tan\left(\theta - \frac{\pi}{4}\right) = \frac{\tan \theta - 1}{1 + \tan \theta} \quad \textbf{A1}$$

attempt to write their RHS in terms of $\sin \theta$ and $\cos \theta$ **M1**

$$\frac{\frac{\sin \theta}{\cos \theta} - 1}{1 + \frac{\sin \theta}{\cos \theta}} \quad \text{OR} \quad \frac{\sin \theta - \cos \theta}{\cos \theta + \sin \theta}$$

multiply through by the conjugate of the denominator $\frac{\cos \theta - \sin \theta}{\cos \theta - \sin \theta}$ **M1**

$$= \frac{-\sin^2 \theta - \cos^2 \theta + 2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} \quad \textbf{A1}$$

$$= \frac{-(\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta)}{\cos^2 \theta - \sin^2 \theta}$$

$$= \frac{-(1 - \sin 2\theta)}{\cos^2 \theta - \sin^2 \theta} \quad \textbf{A1}$$

$$= \frac{\sin 2\theta - 1}{\cos 2\theta} \quad \textbf{AG}$$

continued...

Question 9 continued.

METHOD 2

attempt to write $\tan\left(\theta - \frac{\pi}{4}\right)$ in terms of sin and cos: **M1**

$$\left(\tan\left(\theta - \frac{\pi}{4}\right)\right) = \frac{\sin\left(\theta - \frac{\pi}{4}\right)}{\cos\left(\theta - \frac{\pi}{4}\right)}$$

attempt to use both sin and cos addition formulae: **M1**

$$= \frac{\sin \theta \cos \frac{\pi}{4} - \cos \theta \sin \frac{\pi}{4}}{\cos \theta \cos \frac{\pi}{4} + \sin \theta \sin \frac{\pi}{4}}$$

$$= \frac{\sin \theta - \cos \theta}{\cos \theta + \sin \theta} \quad \textbf{A1}$$

multiply through by the conjugate of the denominator $\frac{\cos \theta - \sin \theta}{\cos \theta - \sin \theta}$ **M1**

$$= \frac{-\sin^2 \theta - \cos^2 \theta + 2 \sin \theta \cos \theta}{\cos^2 \theta - \sin^2 \theta} \quad \textbf{A1}$$

$$= \frac{-(\sin^2 \theta + \cos^2 \theta - 2 \sin \theta \cos \theta)}{\cos^2 \theta - \sin^2 \theta}$$

$$= \frac{-(1 - \sin 2\theta)}{\cos^2 \theta - \sin^2 \theta} \quad \textbf{A1}$$

$$= \frac{\sin 2\theta - 1}{\cos 2\theta} \quad \textbf{AG}$$

continued...

*Question 9 continued.***METHOD 3**

$$\text{attempt to use given identity } \tan(A - B) \equiv \frac{\tan A - \tan B}{1 + \tan A \tan B} \quad \mathbf{M1}$$

$$\tan\left(\theta - \frac{\pi}{4}\right) = \frac{\tan \theta - 1}{1 + \tan \theta} \quad \mathbf{A1}$$

$$\text{multiply through by the conjugate of the denominator } \frac{1 - \tan \theta}{1 - \tan \theta} \quad \mathbf{M1}$$

$$\frac{(\tan \theta - 1)(1 - \tan \theta)}{(1 + \tan \theta)(1 - \tan \theta)} \quad \mathbf{A1}$$

$$\frac{2 \tan \theta - \tan^2 \theta - 1}{1 - \tan^2 \theta} \left(= \frac{2 \tan \theta - \sec^2 \theta}{1 - \tan^2 \theta} \right)$$

$$\text{attempt to write their RHS in terms of } \sin \theta \text{ and } \cos \theta \quad \mathbf{M1}$$

$$= \frac{2 \frac{\sin \theta}{\cos \theta} - \frac{1}{\cos^2 \theta}}{1 - \frac{\sin^2 \theta}{\cos^2 \theta}}$$

$$= \frac{2 \sin \theta \cos \theta - 1}{\cos^2 \theta - \sin^2 \theta} \quad \mathbf{A1}$$

$$= \frac{\sin 2\theta - 1}{\cos 2\theta} \quad \mathbf{AG}$$

[6 marks]*continued...*

Question 9 continued.

(b) recognition that $x = 2\theta \Rightarrow \theta = \frac{x}{2}$

M1

$$\tan\left(\frac{x}{2} - \frac{\pi}{4}\right) = \sqrt{3}$$

$$\frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{3}, \left(\frac{4\pi}{3}\right)$$

A1

$$x = \frac{7\pi}{6}$$

A1

Note: Award **A0** if extra solutions outside the domain are seen.

[3 marks]**Total [9 marks]**

SECTION B

4. (a) METHOD 1

attempt to use right triangle trigonometry (M1)

$$\tan \hat{BAE} = \frac{12}{7} \text{ OR } \tan(90^\circ - \hat{BAE}) = \frac{7}{12} \quad (\text{A1})$$

59.7435...

$$\hat{BAE} = 59.7^\circ \quad \text{A1}$$

Note: Award (M1)(A1)A0 for the equivalent radian value of 1.04.
METHOD 2attempt to find $\hat{BAE}$ using sine rule OR cosine rule (M1)

$$\frac{\sin \hat{BAE}}{12} = \frac{\sin 90}{\sqrt{12^2 + 7^2}} \text{ OR } 12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \hat{BAE} \quad (\text{A1})$$

 $\hat{BAE} = 59.7435...$

$$\hat{BAE} = 59.7^\circ \quad \text{A1}$$

Note: Award (M1)(A1)A0 for the equivalent radian value of 1.04.
[3 marks]**(b) (i) METHOD 1**

attempt to find DE using right angle trigonometry (M1)

$$\sin 59.7435...^\circ = \frac{350}{DE} \text{ OR equivalent} \quad (\text{A1})$$

$$DE = 405.196...$$

$$CE = 405.196... + 50$$

$$= 455.196...$$

$$= 455 \text{ (cm)} \quad \text{A1}$$

METHOD 2

$$\text{Let } DE = EF = x$$

attempt to find DE using their $\hat{DEF}$ and the sine rule OR cosine rule (M1)

$$\frac{700}{\sin(119.487...)} = \frac{DE}{\sin(30.2564...)} \text{ OR } x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564... \quad (\text{A1})$$

$$DE = 405.196...$$

$$CE = 405.196... + 50$$

$$= 455.196...$$

$$= 455 \text{ (cm)} \quad \text{A1}$$

continued...

Question 4 continued.

METHOD 3

Let G be the midpoint of DF

$$EG = \frac{7}{12} \times 350 \left(= \frac{1225}{6} = 204.166... \right) \quad (\text{A1})$$

use of Pythagoras' with their EG to find DE (M1)

$$DE = \sqrt{204.166...^2 + 350^2} \quad (= 405.196...)$$

$$CE = 405.196... + 50$$

$$= 455.196...$$

$$= 455 \text{ (cm)} \quad \text{A1}$$

$$(ii) \quad \tan(59.7435...^\circ) = \frac{30}{x} \text{ OR } \frac{12}{7} = \frac{30}{x} \quad (\text{A1})$$

$$x = 17.5$$

$$BA = 455.196... + 17.5$$

$$= 472.696...$$

$$= 473 \text{ (cm)} \quad \text{A1}$$

[5 marks]

Total [8 marks]

11. (a) attempt to set equal to a parameter or rearrange cartesian form (M1)

$$-\frac{x}{2} + 1 = \lambda \Rightarrow x = 2 - 2\lambda, \quad y + 4 = \lambda \Rightarrow y = -4 + \lambda, \quad \frac{z}{3} = \lambda \Rightarrow z = 3\lambda \quad \text{OR}$$

$$\frac{x-2}{-2} = \frac{y+4}{1} = \frac{z-0}{3}$$

correct direction vector x, y, z or equivalent seen in vector form (A1)

$$\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix}$$

A1

Note: Award (M1)(A1)A0 if " $\mathbf{r} =$ " is omitted.

[3 marks]

- (b) **METHOD 1**

L passes through $P(2 - 2\lambda, -4 + \lambda, 3\lambda)$ (A1)

attempt to apply distance formula to find their $|\vec{OP}|$ or their $|\vec{OP}|^2$ (M1)

$$|\vec{OP}|^2 = (2 - 2\lambda)^2 + (-4 + \lambda)^2 + (3\lambda)^2 \quad (A1)$$

attempt to find their minimum value of $|\vec{OP}|$ or $|\vec{OP}|^2$ using GDC (M1)

$$\lambda = 0.571428... \left(\Rightarrow |\vec{OP}| = \sqrt{15.4285...} \quad \text{OR} \quad |\vec{OP}|^2 = 15.4285... \right)$$

$$3.92792...$$

$$3.93 \left(= \frac{6\sqrt{21}}{7} \right) \quad (A1)$$

continued...

Question 11 continued.

METHOD 2

setting the scalar product of their line and their direction vector to zero

(M1)

$$\begin{pmatrix} 2-2\lambda \\ -4+\lambda \\ 3\lambda \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = 0$$

$$-4 + 4\lambda - 4 + \lambda + 9\lambda = 0$$

$$-8 + 14\lambda = 0$$

$$\lambda = \frac{4}{7} (= 0.571428\dots)$$

(A1)

attempt to substitute their value for λ into their $\mathbf{r}$ to find the position vector of the closest point and find $|\mathbf{r}|$

(M1)

$$\mathbf{r} = \begin{pmatrix} 2 \\ -4 \\ 0 \end{pmatrix} + \frac{4}{7} \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{6}{7} \\ -\frac{24}{7} \\ \frac{12}{7} \end{pmatrix}$$

$$|\mathbf{r}| = \sqrt{\left(\frac{6}{7}\right)^2 + \left(-\frac{24}{7}\right)^2 + \left(\frac{12}{7}\right)^2}$$

(A1)

$$= 3.92792\dots = \frac{6\sqrt{21}}{7}$$

$$= 3.93 \left(= \frac{6\sqrt{21}}{7} \right)$$

A1

continued...

Question 11 continued.

METHOD 3

Let P be a point on L

attempt to find the cross product between $\vec{OP}$ and the direction, $\mathbf{b}$, of L (M1)

$$\begin{pmatrix} 2 \\ -4 \\ -0 \end{pmatrix} \times \begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} = (-12 - 0)\mathbf{i} + (0 - 6)\mathbf{j} + (2 - 8)\mathbf{k}$$

$$= \begin{pmatrix} -12 \\ -6 \\ -6 \end{pmatrix} = -6 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \quad (A1)$$

attempt to find shortest distance using $\frac{|\vec{OP} \times \mathbf{b}|}{|\mathbf{b}|}$ (M1)

$$\frac{\sqrt{216}}{\sqrt{14}} \quad (A1)$$

3.92792...

$$3.93 \left(= \frac{6\sqrt{21}}{7} \right) \quad A1$$

[5 marks]

continued...

Question 11 continued.

(c) **METHOD 1**

substitute their x , y , z into equation of plane $6x - 3y + 5z (= 24)$

M1

$$6(2 - 2\lambda) - 3(-4 + \lambda) + 5(3\lambda)$$

(A1)

$$= 12 - 12\lambda + 12 - 3\lambda + 15\lambda$$

A1

$$= 24$$

so the line is contained in the plane

AG

Note: For FT from an incorrect part a), award **M1A0A0**.

METHOD 2

consider the direction of the line **and** a point on the line

M1

$$\begin{pmatrix} -2 \\ 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix} = -12 - 3 + 15 = 0 \text{ (so line is parallel to plane)}$$

A1

$$6(2) - 3(-4) + 5(0) = 12 + 12 = 24 \text{ (so line lies in the plane)}$$

A1

so the line is contained in the plane

AG

Note: Both **A** marks are dependent on the **M** mark.

Note: For FT from an incorrect part a), award **M1A0A0**.

[3 marks]

continued...

Question 11 continued.

(d) **METHOD 1**

recognition that the direction M is from the point $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ and a point on the z -axis $\begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}$ **(M1)**

direction of M is $(\pm) \begin{pmatrix} 4 \\ 1 \\ 2-z \end{pmatrix}$ **(A1)**

the direction of the normal of Π is $\begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix}$ (seen anywhere) **(A1)**

attempt to use the scalar product with their normal and their direction vector and equate to 0 **(M1)**

$$\begin{pmatrix} 4 \\ 1 \\ 2-z \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow 24 - 3 + 10 - 5z = 0$$

$$z = \frac{31}{5}$$
 (A1)

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ -\frac{21}{5} \end{pmatrix}$$

attempt to express equation in the form $s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ **(M1)**

$$s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ -\frac{21}{5} \end{pmatrix} \text{ OR } s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 20 \\ 5 \\ -21 \end{pmatrix} \quad \mathbf{A1}$$

continued...

Question 11 continued.

METHOD 2

let the equation of M be $M : s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} p \\ q \\ r \end{pmatrix}$

Note: Consideration of the z-axis intersection and consideration of direction may be done in either order and marks should be awarded independently.

recognition M intersects the z -axis where $x = y = 0$ (M1)

$$4 + \mu p = 0, 1 + \mu q = 0$$

$$\frac{-4}{p} = \frac{-1}{q} \Rightarrow p = 4q$$
 (A1)

the direction of the normal of Π is $\begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix}$ (seen anywhere) (A1)

attempt to use the scalar product with their normal and their direction vector and equate to 0 (M1)

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -3 \\ 5 \end{pmatrix} = 0 \Rightarrow 6p - 3q + 5r = 0$$

$$6(4q) - 3q + 5r = 0 \Rightarrow 21q = -5r$$

$$\begin{pmatrix} p \\ q \\ r \end{pmatrix} = \begin{pmatrix} 20 \\ 5 \\ -21 \end{pmatrix} \text{ (or any multiple of } \begin{pmatrix} 20 \\ 5 \\ -21 \end{pmatrix} \text{)}$$
 (A1)

attempt to express equation in the form $s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} p \\ q \\ r \end{pmatrix}$ (M1)

$$s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ 1 \\ -\frac{21}{5} \end{pmatrix} \text{ OR } s = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 20 \\ 5 \\ -21 \end{pmatrix}$$
 A1

[7 marks]

Total [18 marks]

6. direction vectors are $aj + k \left(= \begin{pmatrix} a \\ 1 \end{pmatrix} \right)$ and $i + 2j + 3k \left(= \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right)$ **(A1)**

recognition that the scalar product of the direction vectors is 0 **(M1)**

$$\begin{pmatrix} 0 \\ a \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} (= 2a + 3) = 0$$

$$a = -\frac{3}{2}$$
 A1

at point of intersection $4 = 1 + \mu$, $-\frac{3}{2}\lambda = 2\mu$ and $-1 + \lambda = -b + 3\mu$ **(A1)**

attempt to solve 3 equations in μ , λ and b , derived from the point of intersection, to find μ , λ and b **(M1)**

$$\mu = 3, \lambda = -4$$

$$b = 14$$
 A1

Total [6 marks]

11. (a) normals of Π_1 and Π_2 are $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$ (A1)

attempt to use the scalar product for the angle between two vectors M1

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \left| \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \right| \left| \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \right| \cos \theta$$

$$-3 = \sqrt{6}\sqrt{6} \cos \theta \quad (\text{so } -3 = 6 \cos \theta) \quad \text{A1A1}$$

Note: Award **A1** for correct scalar product and **A1** for correct magnitudes of normals and $\cos \theta$.

$$\cos \theta = -\frac{1}{2} \quad \text{A1}$$

$$\theta = 120^\circ \text{ OR } \cos(180^\circ - \theta) = \frac{1}{2} \quad \text{A1}$$

acute angle is 60° AG

[6 marks]

(b) attempt to find vector product of their normal vectors M1

$$\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -4+1 \\ 1+2 \\ -1-2 \end{pmatrix}$$

$$= \begin{pmatrix} -3 \\ 3 \\ -3 \end{pmatrix} = 3 \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \quad \text{(A1)}$$

equation of Π_3 is $-3x + 3y - 3z = d$ OR $-x + y - z = d$ or equivalent

attempt to substitute $x = 5$, $y = -5$, $z = 5$ into their equation (M1)

equation of Π_3 is $-3x + 3y - 3z = -45$ (so $-x + y - z = -15$) A1

[4 marks]

continued...

Question 11 continued.

- (c) (i) attempt to use the identity for $\sin(30^\circ + 45^\circ)$ (M1)

$$\sin(30^\circ + 45^\circ) = \sin(30^\circ)\cos(45^\circ) + \cos(30^\circ)\sin(45^\circ)$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

A1A1

$$= \frac{\sqrt{2} + \sqrt{6}}{4}$$

AG

Note: A1 for each term. Award A1 A0 for correct answers where the denominator has not been rationalized such as $= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$.

- (ii) let $x = QR$

attempt to use the sine rule with the angles $75^\circ, 45^\circ$

(M1)

$$\frac{5}{\sin 75^\circ} = \frac{x}{\sin 45^\circ}$$

$$x = \frac{20}{\sqrt{2}(\sqrt{2} + \sqrt{6})} \left(= \frac{20}{2(1 + \sqrt{3})} \right) \text{ or equivalent}$$

A1

attempt to rationalise a denominator of the form $(\sqrt{a} + \sqrt{b})$ by multiplying numerator and denominator by $\pm(\sqrt{a} - \sqrt{b})$ or equivalent

(M1)

$$x = \frac{20(\sqrt{2} - \sqrt{6})}{\sqrt{2}(\sqrt{2} + \sqrt{6})(\sqrt{2} - \sqrt{6})} \left(= \frac{20(1 - \sqrt{3})}{2(1 + \sqrt{3})(1 - \sqrt{3})} \right)$$

$$x = 5(\sqrt{3} - 1) \text{ (cm)} \text{ (} p = 5, q = 3 \text{)}$$

A1

[7 marks]

Total [17 marks]

2. (a) attempt to set equal to a parameter or to add detail to cartesian form (M1)

$$\frac{x-1}{2} = \frac{y+2}{3} = z = \lambda \Rightarrow x = 2\lambda + 1, y = 3\lambda - 2, z = \lambda \quad \text{OR} \quad \frac{x-1}{2} = \frac{y-(-2)}{3} = \frac{z-0}{1}$$

$$\mathbf{r} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad (\text{or equivalent}) \quad \text{A1}$$

Note: Award **A0** if $\mathbf{r} =$ OR $\begin{pmatrix} x \\ y \\ z \end{pmatrix} =$ is omitted.

[2 marks]

- (b) attempt to equate at least one component (M1)

$$1 + 2\lambda = t \quad \text{OR} \quad -2 + 3\lambda = 4 \quad \text{OR} \quad \lambda = -8 + 2t$$

$$\lambda = 2 \quad \text{OR} \quad t = 5 \quad \text{(A1)}$$

$$\text{intersection point} = (5, 4, 2) \quad \text{A1}$$

Note: Condone coordinates given in vector form.

[3 marks]

Total [5 marks]

3.

(a) correct substitution in sine rule

(A1)

$$\frac{\sin \theta}{5} = \frac{\sin 2\theta}{6\sqrt{2}} \text{ (or equivalent)}$$

attempt to use double angle rule for $\sin 2\theta$ **(M1)**

$$\frac{\sin \theta}{5} = \frac{2 \sin \theta \cos \theta}{6\sqrt{2}}$$

$$6\sqrt{2} \sin \theta = 10 \sin \theta \cos \theta \quad \text{OR} \quad \frac{1}{5} = \frac{2 \cos \theta}{6\sqrt{2}} \quad \text{OR equivalent}$$

A1

$$\cos \theta = \frac{3\sqrt{2}}{5}$$

AG**[3 marks]**(b) valid attempt to find $\sin \theta$ **(M1)**

$$\sin^2 \theta + \left(\frac{3\sqrt{2}}{5} \right)^2 = 1 \quad \text{OR right triangle with adjacent side and hypotenuse labelled}$$

$$\sin \theta = \frac{\sqrt{7}}{5}$$

A1**[2 marks]**

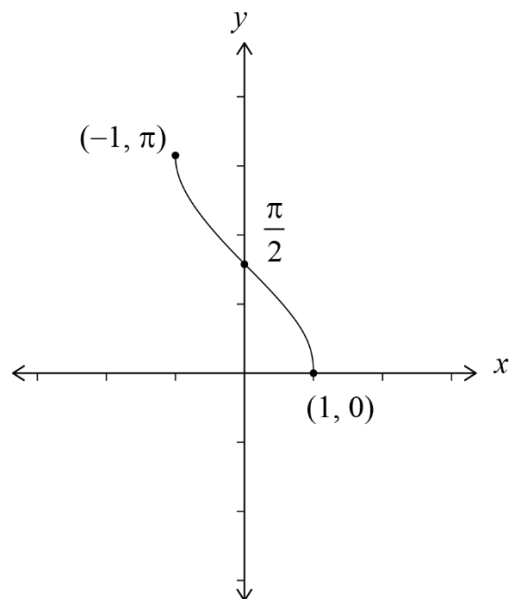
$$(c) \quad \frac{1}{2} \times 6\sqrt{2} \times DC \times \frac{\sqrt{7}}{5} = 2\sqrt{14}$$

(A1)

$$DC = \frac{10}{3}$$

A1**[2 marks]****Total [7 marks]**

8. (a)



For a curve with correct shape within the domain, correct labelled y -intercept and end points. **A2**

Note: Award **A1** for any two of the following provided a curve is seen:

- correct shape within the domain
- correctly labelled y - intercept
- correctly labelled end points.

[2 marks]

continued...

Question 8 continued.

(b) **METHOD 1**

recognizing to use compound angle rule for $\cos(A + B)$ **(M1)**

$$\cos(\arccos x)\cos(\arccos x\sqrt{3}) - \sin(\arccos x)\sin(\arccos x\sqrt{3})$$

valid attempt to find $\sin(\arccos x)$ or $\sin(\arccos x\sqrt{3})$ in terms of x **(M1)**

use of the identity $\sin \theta = \sqrt{1 - \cos^2 \theta}$ OR use of a right-angled triangle
($\sin A$ and $\sin B$ are both positive)

$$x \times x\sqrt{3} - \sqrt{1 - x^2} \times \sqrt{1 - 3x^2} \left(= \cos \frac{3\pi}{2} \right) \quad \textbf{A1}$$

$$\cos \frac{3\pi}{2} = 0 \quad (\text{seen anywhere}) \quad \textbf{(A1)}$$

attempt to write an equation in x without the square roots **(M1)**

$$3x^4 = 1 - 4x^2 + 3x^4$$

$$x = \pm \frac{1}{2}$$

(as seen in part (a) graph, when $x = \frac{1}{2}$, $\arccos(x) + \arccos(x\sqrt{3}) < \pi \Rightarrow$ no solution)

$$x = -\frac{1}{2} \quad \textbf{A1}$$

continued...

Question 8 continued.

METHOD 2

recognizing to use compound angle rule for $\sin(A + B)$ (M1)

$$\sin(\arccos x) \cos(\arccos x \sqrt{3}) + \cos(\arccos x) \sin(\arccos x \sqrt{3})$$

valid attempt to find $\sin(\arccos x)$ or $\sin(\arccos x \sqrt{3})$ in terms of x (M1)

use of the identity $\sin \theta = \sqrt{1 - \cos^2 \theta}$ OR use of a right-angled triangle
($\sin A$ and $\sin B$ are both positive)

$$\sqrt{1 - x^2} \times x \sqrt{3} + x \times \sqrt{1 - 3x^2} \left(= \sin \frac{3\pi}{2} \right) \quad \text{A1}$$

$$\sin \frac{3\pi}{2} = -1 \quad (\text{seen anywhere}) \quad \text{(A1)}$$

attempt to write an equation in x without the square roots (M1)

$$4x^4 - 4x^2 + 1 = 4x^2 - 12x^4 \left(\Rightarrow 16x^4 - 8x^2 + 1 = 0 \Rightarrow (4x^2 - 1)^2 = 0 \right)$$

$$x = \pm \frac{1}{2}$$

(as seen in part (a) graph, when $x = \frac{1}{2}$, $\arccos(x) + \arccos(x\sqrt{3}) < \pi \Rightarrow$ no solution)

$$x = -\frac{1}{2} \quad \text{A1}$$

[6 marks]

Total [8 marks]

3. recognizing to use $\cos 2\theta = 2\cos^2 \theta - 1$ (M1)
- $$2(2\cos^2 \theta - 1) - 5\cos \theta + 2 (= 0) \quad \text{A1}$$
- $$4\cos^2 \theta - 5\cos \theta (= 0)$$
- choosing an appropriate method to solve their quadratic equation (M1)
- $$\cos \theta (4\cos \theta - 5) \quad \text{OR} \quad \frac{5 \pm \sqrt{(-5)^2 - 4 \times 4 \times 0}}{2 \times 4} \quad \text{(A1)}$$
- $$\cos \theta = 0$$
- $$\theta = \frac{3\pi}{2} \quad \text{A1}$$

Note: Do not award final A1 if any extra solutions given.
--

[5 marks]

11. (a) $\vec{BC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **OR** $\vec{BA} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ (A1)

$(3, 0, 2)$

A1

[2 marks]

(b) **EITHER**

recognizing to find midpoint of [AC] or [BD]

(M1)

$\left(\frac{-1+1}{2}, \frac{-2-4}{2}, \frac{4-0}{2} \right)$ **OR** $\left(\frac{-3+3}{2}, \frac{-6+0}{2}, \frac{2+2}{2} \right)$

OR

Let $E(a, b, c)$

$\vec{AE} = \vec{EC} \Rightarrow \begin{pmatrix} a-1 \\ b+4 \\ c \end{pmatrix} = \begin{pmatrix} -1-a \\ -2-b \\ 4-c \end{pmatrix}$ **OR** $\vec{AE} = \frac{1}{2}\vec{AC} \Rightarrow \begin{pmatrix} a-1 \\ b+4 \\ c \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$ (M1)

THEN

$(0, -3, 2)$

A1

[2 marks]

continued...

Question 11 continued

$$(c) \quad (i) \quad \vec{AB} = \begin{pmatrix} -4 \\ -2 \\ 2 \end{pmatrix} \text{ AND } \vec{AD} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$$

A1

Note: Award **A1** for finding both vectors, one of which may have been seen in (a).

$$\begin{pmatrix} -4 \\ -2 \\ 2 \end{pmatrix} \times \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -12 \\ 12 \\ -12 \end{pmatrix}$$

$$m = 12$$

A1

$$(ii) \quad \sqrt{(-12)^2 + 12^2 + (-12)^2} \text{ OR } 12\sqrt{(-1)^2 + 1^2 + (-1)^2}$$

$$= 12\sqrt{3}$$

(A1)**A1****[4 marks]**

(d) substituting cross product and a point into the equation of a plane

(M1)

$$-x + y - z = -5$$

A1**[2 marks]***continued...*

Question 11 continued

- (e) let a normal to
- Π_1
- be
- $\mathbf{n}_1$
- and a normal to
- Π_2
- be
- $\mathbf{n}_2$

$$\text{Eg. } \mathbf{n}_1 = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \text{ and } \mathbf{n}_2 = \begin{pmatrix} 5 \\ 1 \\ -7 \end{pmatrix}$$

recognizing angle between $\mathbf{n}_1$ and $\mathbf{n}_2$ is required

(M1)

$$|\mathbf{n}_2| = \sqrt{75}$$

A1

$$\cos \theta = \frac{-1 \times 5 + 1 \times 1 + (-1) \times (-7)}{\sqrt{3} \times \sqrt{75}} \left(= \frac{3}{\sqrt{3} \times \sqrt{75}} \right)$$

A1

Note: The negative of this may be found but to obtain the final **A1** there needs to be a recognition that this obtains the obtuse angle for the minus to be dropped.

$$\cos \theta = \frac{1}{5}$$

AG

[3 marks]

- (f) attempt to substitute normal to
- Π_1
- and their E into vector equation of line

(M1)

$$(\mathbf{r} =) \begin{pmatrix} 0 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$$

A1

substitutes for λ into Π_2

(M1)

$$5(-\lambda) + (-3 + \lambda) - 7(2 - \lambda) = 1$$

$$\lambda = 6$$

A1

$$(-6, 3, -4)$$

A1

[5 marks]

Total [18 marks]

1. (a) EITHER

$$210^\circ = \frac{210\pi}{180} \left(= \frac{7\pi}{6} = 3.66519... \right) \text{ radians} \quad (\text{A1})$$

attempt to use radian formula for area of sector (M1)

$$\text{area} = \frac{1}{2}(19.5)^2 \left(\frac{7\pi}{6} \right)$$

OR

attempt to use degree formula for area of sector (M1)

$$\text{area} = \frac{210}{360} \pi (19.5)^2 \quad (\text{A1})$$

THEN

$$\text{area} = \frac{3549\pi}{16} = 696.844...$$

$$= 697 \left(= \frac{3549\pi}{16} \right) (\text{cm}^2) \quad \text{A1}$$

[3 marks]**(b) EITHER**

$$\text{arc length} = 19.5 \left(\frac{7\pi}{6} \right) \text{ OR } = \frac{210}{360} (2\pi(19.5)) \left(= \frac{91\pi}{4} = 71.4712... \right) \quad (\text{A1})$$

attempt to set $2\pi r$ equal to arc length (M1)

$$2\pi r = 71.4712...$$

OR

attempt to set $\pi r l$ equal to their area from (a) (M1)

$$19.5\pi r = 696.844... \quad (\text{A1})$$

THEN

$$r = 11.4 \left(= \frac{91}{8} = 11.375 \right) (\text{cm}) \quad \text{A1}$$

[3 marks]**Total[6 marks]**

2. (a) period is $\frac{\pi}{2} (= 1.57079... = 1.57)$ **A1**

[1 mark]

- (b) attempt to substitute $x = \frac{\pi}{12}, f(x) = 5$ and $x = \frac{\pi}{3}, f(x) = 7$ to obtain two equations **(M1)**

Note: accept work where x values have been converted into degrees

$$a \tan\left(\frac{\pi}{6}\right) + b = 5 \text{ and } a \tan\left(\frac{2\pi}{3}\right) + b = 7 \left(\Rightarrow \frac{a}{\sqrt{3}} + b = 5 \text{ and } -a\sqrt{3} + b = 7 \right)$$

$$a = -\frac{\sqrt{3}}{2} (= -0.866025... = -0.866) \quad \textbf{A1}$$

$$b = \frac{11}{2} (= 5.5) \quad \textbf{A1}$$

Note: These **A1** marks may be awarded independently.

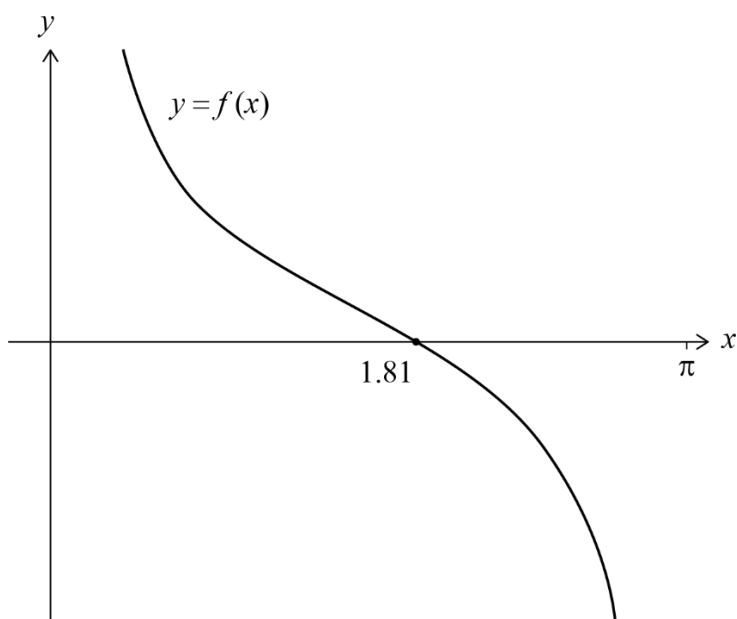
[3 marks]

Total [4 marks]

6. (a) (i) $(f(x) = \frac{4 \cos x}{\sin x} + \sin x$

A1

(ii)



correct domain and curvature

A1

Note: The correct domain may be implied by a single branch of the function.
Condone the absence of π on the x -axis.

Award **A0** if the graph is not a function.

correct x -intercept at $x = 1.80911... = 1.81$ **A1**

Note: Accept coordinates $(1.81, 0)$. This mark is independent of the first **A1**.

[3 marks]

(b) $f^{-1}(2) = 1.31837...$
 $= 1.32$

A1**[1 mark]***continued...*

Question 6 continued.

(c) **METHOD 1**

EITHER

recognition that $\sec x = \frac{1}{\cos x}$ (M1)

$$\cos \alpha = \frac{2}{3} (= 0.666666\dots)$$

$$\alpha = \arccos \frac{2}{3} (= 0.841068\dots) \quad (A1)$$

OR

$$\alpha = \operatorname{arcsec} 1.5 (= 0.841068\dots) \quad (A2)$$

THEN

$$f(\alpha) = f(0.841068\dots) = 4.32306\dots$$

$$= 4.32 \left(= \frac{29\sqrt{5}}{15} \right) \quad A1$$

METHOD 2

recognition that $\sec x = \frac{1}{\cos x}$ (M1)

$$\cos \alpha = \frac{2}{3}, \sin \alpha = \frac{\sqrt{5}}{3} \left(\cot x = \frac{2}{\sqrt{5}} \right) \quad (A1)$$

$$f(\alpha) = \frac{4\left(\frac{2}{3}\right)}{\frac{\sqrt{5}}{3}} + \frac{\sqrt{5}}{3} \left(= 4\left(\frac{2}{\sqrt{5}}\right) + \frac{\sqrt{5}}{3} \right)$$

$$= \frac{29\sqrt{5}}{15} (= 4.32306\dots = 4.32) \quad A1$$

[3 marks]

Total [7 marks]

7. (a) recognition that the speed is the magnitude of the velocity

(M1)

$$\text{speed} = 4\sqrt{10^2 + (-25)^2}$$

$$= 107.703\dots$$

$$= 108 \quad (= 20\sqrt{29}) \quad (\text{km/h})$$

A1

[2 marks]

- (b) **METHOD 1**

attempt to use right-angled triangle with the horizontal speed found in (a)

(M1)

$$\tan \beta = \frac{16}{20\sqrt{29}} \left(= \frac{4}{5\sqrt{29}} \right)$$

(A1)

$$\beta = 8.44984(^{\circ})$$

$$\beta = 8.45(^{\circ})$$

A1

Note: Award **M1A1A0** for answer of 0.147 radians.

METHOD 2

attempt to find angle between $\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix}$

(M1)

$$\cos \beta = \frac{\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix}}{\sqrt{10^2 + (-25)^2 + (-4)^2} \sqrt{10^2 + (-25)^2}} \left(= \frac{725}{\sqrt{741}\sqrt{725}} = 0.989144\dots \right) \text{ OR}$$

$$\sin \beta = \frac{\left| \begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix} \times \begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix} \right|}{\sqrt{10^2 + (-25)^2 + (-4)^2} \sqrt{10^2 + (-25)^2}} = \frac{\left| \begin{pmatrix} -100 \\ -40 \\ 0 \end{pmatrix} \right|}{\sqrt{741}\sqrt{725}} \left(= \frac{4}{\sqrt{741}} = 0.146943\dots \right)$$

(A1)

$$\beta = 8.44984(^{\circ})$$

$$\beta = 8.45^{\circ}$$

A1

continued...

Question 7 continued.

METHOD 3

attempt to find angle between $\begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix}$ and a plane parallel to $z = 0$ **(M1)**

$$\sin \beta = \frac{\left| \begin{pmatrix} 10 \\ -25 \\ -4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right|}{\sqrt{10^2 + (-25)^2 + (-4)^2}} \left(= \frac{4}{\sqrt{741}} = 0.146943... \right) \quad \textbf{(A1)}$$

Note: This could also be written as $\cos(90^\circ - \beta) = \dots$

$$\beta = 8.44984(^{\circ})$$

$$\beta = 8.45^{\circ}$$

A1

[3 marks]

Total [5 marks]

2.

- (a) attempt to find distance between two points **(M1)**

$$\sqrt{(-3-1)^2 + (4-7)^2 + (2-0)^2} (= 5.38516...)$$

$$VX = 5.39 (= \sqrt{29})$$

A1**[2 marks]**

- (b) attempt to use Pythagoras' theorem **(M1)**

$$\sqrt{5^2 + 5^2} (= 7.07106...)$$

$$AC = 7.07 (= 5\sqrt{2})$$

A1**[2 marks]**

- (c) valid attempt to use trig ratio in triangle VXC **(M1)**

$$\tan \theta = \frac{\frac{\sqrt{29}}{5\sqrt{2}} \left(= \frac{5.38516...}{3.53553...} \right)}{2}$$

$$56.7138...^\circ \text{ OR } 0.989842... \text{ rad}$$

$$\theta = 56.7^\circ \text{ OR } \theta = 0.990 \text{ rad (accept } \theta = 0.99)$$

A1**[2 marks]****Total [6 marks]**

6.

METHOD 1

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) **(M1)**

$\text{area}_{\text{triangle}} = 21.3333\ldots$ OR $\text{area}_{\text{sector}} = 34.1333\ldots$ **A1**

correct equation in r and θ **(A1)**

$$\frac{1}{2}r^2\theta = 34.1333\ldots \text{ OR } \frac{1}{2}r^2 \sin \theta = 21.3333\ldots \text{ OR } \frac{1}{2}r^2(\theta - \sin \theta) = 12.8 \text{ (seen anywhere)}$$

correct equation in one variable **A1**

$$\frac{1}{2}r^2 \left(\frac{68.2666\ldots}{r^2} - \sin \left(\frac{68.2666\ldots}{r^2} \right) \right) = 12.8 \text{ OR } \frac{1}{2} \left(\frac{68.2666\ldots}{\theta} \right) \sin \theta = 21.3333\ldots \text{ OR}$$

$$\frac{1}{2} \left(\frac{68.2666\ldots}{\theta} \right) (\theta - \sin \theta) = 12.8$$

attempt to solve their equation or use of graph **(M1)**

$$\theta = 1.59934\ldots$$

$$6.53330\ldots$$

$$r = 6.53 \quad \textbf{A1}$$

METHOD 2

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) **(M1)**

$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin \theta)$ (seen anywhere) **(A1)**

$$\frac{\frac{1}{2}r^2(\theta - \sin \theta)}{\frac{1}{2}r^2 \sin \theta} = \frac{3}{5}$$

correct equation without r **A1**

$$\frac{(\theta - \sin \theta)}{\sin \theta} = \frac{3}{5}$$

$$\theta = 1.59934\ldots \quad \textbf{(A1)}$$

continued...

Question 6 continuedattempt to solve for r using their θ **(M1)**

$$\frac{1}{2}r^2(1.59... - \sin 1.59...) = 12.8$$

$$6.53330...$$

$$r = 6.53$$

A1**METHOD 3**recognising $\text{area}_{\text{segment}} = \frac{3}{8} \times \text{area}_{\text{sector}}$ (seen anywhere)**(M1)**

$$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin \theta) \quad (\text{seen anywhere})$$

(A1)

$$\frac{1}{2}r^2(\theta - \sin \theta) = \frac{3}{8} \times \frac{1}{2}r^2\theta$$

correct equation without r **A1**

$$\frac{1}{2}(\theta - \sin \theta) = \frac{3}{8} \times \frac{1}{2}\theta$$

$$\theta = 1.59934...$$

(A1)attempt to solve for r using their θ **(M1)**

$$\frac{1}{2}r^2(1.59... - \sin 1.59...) = 12.8$$

$$6.53330...$$

$$r = 6.53$$

A1**[6 marks]**

9. (a) METHOD 1

correct direction vector, **e.g.** $\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ (or equivalent)

A1

attempt to find direction normal using vector product

(M1)

$$\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \text{ (or equivalent)}$$

A1

correct substitution of a point on the line to obtain d

A1

$$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = d \Rightarrow d = -2 \text{ OR } \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = d \Rightarrow d = -2 \text{ OR}$$

$$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix} = d \Rightarrow d = 2 \text{ (or equivalent)}$$

hence equation of the plane Π_1 is $x + y - z = -2$

AG**METHOD 2**

attempt to form parametric equations of a plane through point $(0,0,2)$ and two vectors in the

plane $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$

M1

$$\vec{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

A1

$$x = \lambda + 2\mu$$

$$y = \mu$$

$$z = 2 + \lambda + 3\mu$$

continued...

Question 9 continued

$$\lambda = x - 2y$$

$$z = 2 + \lambda + 3y$$

A1

$$z = 2 + x - 2y + 3y$$

A1

$$x + y - z = -2$$

AG**[4 marks]****(b) METHOD 1**

attempt to form an equation to eliminate one of the variables

M1**EITHER**

$$(b - 2)y + z = 7$$

A1

$$2y - 3z = -2 - d$$

A1**OR**

$$x + (b - 1)y = 5$$

A1

$$3x + y = d - 4$$

A1**THEN**

$$(3b - 4)y = 19 - d$$

$$y = \frac{19 - d}{3b - 4}$$

$$b = \frac{4}{3} \text{ and } d = 19$$

A1

Note: Award **M1A1** for $b = \frac{4}{3}$ seen anywhere

continued...

*Question 9 continued***METHOD 2**

$$\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & b & -1 & 3 \\ 1 & -1 & 2 & d \end{array}$$

attempt to use row reduction

M1

$$\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & b-2 & 1 & 7 \\ 0 & -2 & 3 & d+2 \end{array} \quad \text{or equivalent}$$

A1

$$\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 0 & b-2 & 1 & 7 \\ 0 & 0 & 3+\frac{2}{b-2} & d+2+\frac{14}{b-2} \end{array} \quad \text{or equivalent}$$

(A1)

$$3+\frac{2}{b-2}=0 \text{ and } d+2+\frac{14}{\frac{4}{3}-2}=0$$

$$b=\frac{4}{3} \text{ and } d=19$$

A1

Note: Award **M1A1** for $b=\frac{4}{3}$ seen anywhere

continued..

Question 9 continued**METHOD 3**

attempt to use determinant and equate it to zero

(M1)

$$\begin{array}{ccc|c} 1 & 1 & -1 & -2 \\ 2 & b & -1 & 3 \\ 1 & -1 & 2 & d \end{array}$$

$$\det \begin{pmatrix} \begin{bmatrix} 1 & 1 & -1 \end{bmatrix} \\ \begin{bmatrix} 2 & b & -1 \end{bmatrix} \\ \begin{bmatrix} 1 & -1 & 2 \end{bmatrix} \end{pmatrix} = 1 \times \begin{vmatrix} b & -1 \\ -1 & 2 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix} + (-1) \times \begin{vmatrix} 2 & b \\ 1 & -1 \end{vmatrix}$$

$$\det = 3b - 4$$

$$3b - 4 = 0$$

$$b = \frac{4}{3}$$

A1

$$\det \begin{pmatrix} \begin{bmatrix} 1 & -1 & -2 \end{bmatrix} \\ \begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \\ \begin{bmatrix} 1 & 2 & d \end{bmatrix} \end{pmatrix} = 0$$

(A1)

$$d = 19$$

A1

Note: Award **(M1)A1** for $b = \frac{4}{3}$ seen anywhere

[4 marks]**Total [8 marks]**

SECTION B

1. (a) substituting into cosine rule (M1)

$$(BC^2 =) 12^2 + 7^2 - 2 \times 12 \times 7 \times \cos(116^\circ) \quad (A1)$$

$$\sqrt{266.646...} = 16.3293...$$

$$(BC =) 16.3 \quad A1$$

[3 marks]

- (b) substituting into sine rule or cosine rule for angle $\hat{ACB}$ or angle $\hat{ABC}$ (M1)

$$\frac{\sin(\hat{ACB})}{7} = \frac{\sin(116^\circ)}{16.3293...} \quad \text{OR} \quad 7^2 = 12^2 + 16.3293...^2 - 2 \times 12 \times 16.3293... \times \cos(\hat{ACB})$$

$$\text{OR } 180^\circ - 116^\circ - 41.3^\circ \text{ from finding angle } \hat{ABC} \quad (A1)$$

$$22.6618...$$

$$(\hat{ACB} =) 22.7 (^\circ) \text{ (accept } 22.8 (^\circ) \text{ if using previous 3sf AND the cosine rule)} \quad A1$$

[3 marks]

Total [6 marks]

4. Attempts to use the area formula for triangle AOB **AND** equate it to 26 (M1)

$$\frac{1}{2}a \times b \times \sin(2.51) = 26 \text{ OR } \frac{1}{2}OA \times OB \times \sin AOB = 26$$

$$\frac{1}{2} \times r^2 \times \sin(2.51) = 26 \text{ or equivalent} \quad (A1)$$

$$r = 9.38462... \quad (A1)$$

use of arc length formula to find the major arc ACB (M1)

$$9.38462... \times (2\pi - 2.51) \text{ OR } 2\pi \times 9.38462... - 9.38462... \times 2.51 \text{ OR } 9.38462... \times 3.773185...$$

$$35.4099...$$

$$(\text{arc length} =) 35.4 \text{ (cm)} \quad A1$$

[5 marks]

6. (a) substituting into scalar product formula (M1)
- (i) $(\mathbf{a} \cdot \mathbf{c} =) -5p - 42$ A1
- (ii) $(\mathbf{b} \cdot \mathbf{c} =) -8p - 54$ A1

[3 marks]

- (b) substituting into the angle between two vectors formula to find either $\mathbf{a} \cdot \mathbf{c}$ or $\mathbf{b} \cdot \mathbf{c}$ (M1)

$$\cos \theta = \frac{-5p - 42}{\sqrt{74}\sqrt{p^2 + 36}}, \cos \theta = \frac{-8p - 54}{\sqrt{145}\sqrt{p^2 + 36}} \quad (\text{A1})(\text{A1})$$

equating their two expressions for $\cos \theta$ (M1)

$$\frac{-5p - 42}{\sqrt{74}} = \frac{-8p - 54}{\sqrt{145}}$$

$$p = 4.78727\dots$$

$$(p =) 4.79 \quad \text{A1}$$

[5 marks]

Total [8 marks]

2.

- (a) (i) attempt to use Pythagoras' theorem

(M1)

$$\sqrt{12500^2 + 6000^2}$$

$$= 13900 \text{ (km)} \text{ (13865.4...)}$$

A1**[2 marks]**

- (ii) attempt to use an appropriate inverse trig ratio

(M1)

$$\text{e.g. } \arctan\left(\frac{12500}{6000}\right) \text{ or } \arctan\left(\frac{6000}{12500}\right)$$

identifying the correct angle

(M1)

$$= (0)64.4^\circ \text{ OR } 064^\circ (64.3589...)$$

A1**Note:** The two method marks can be awarded independently.The first **(M1)** can come from any geometry leading to an inverse trigonometric ratio.The second **(M1)** requires recognition of which angle is the bearing, possibly on a diagram. 64.4 appearing in intermediate working is not sufficient.**[3 marks]**

- (b) (i)
- $p \cdot n = 0$

M1

$$\text{so the angle is } 90^\circ \text{ OR } \frac{\pi}{2} \text{ rads}$$

A1**Note:** Award **M0A0** for an answer of 90 degrees without any mention that the scalar product is zero.**[2 marks]**

- (ii) attempt to use arc length formula
- $s = r\theta$
- OR
- $\frac{90^\circ}{360}$
- of circumference

(M1)

$$= 6000\left(\frac{\pi}{2}\right) \text{ OR } \frac{1}{4} \times 2\pi \times 6000$$

A1

$$= 3\pi \text{ thousand km OR } 3000\pi \text{ km}$$

AG**[2 marks]***continued*

Question 2 continued.

- (c) (i) attempt to find cross product (e.g. correct except for sign error)

(M1)

$$\mathbf{a} \times \mathbf{p} = \begin{pmatrix} 0 \\ -36 \\ 0 \end{pmatrix}$$

A1

[2 marks]

- (ii) **METHOD 1**

$$\mathbf{a} \times \mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 36 \end{pmatrix}$$

A1

choosing to use scalar product for their $\mathbf{a} \times \mathbf{p}$ and $\mathbf{a} \times \mathbf{n}$

M1

$$(\mathbf{a} \times \mathbf{p}) \cdot (\mathbf{a} \times \mathbf{n}) = 0$$

A1

Note: The final **A1** can be awarded for seeing the dot product equals zero without incorrect working. Follow through from (i) is permitted and if they explicitly state that they are using $\mathbf{a} \times \mathbf{n}$ then allow internal follow through.

so angle is 90°

AG

METHOD 2

$$\mathbf{a} \times \mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 36 \end{pmatrix}$$

A1

attempt to use cross product for their $\mathbf{a} \times \mathbf{p}$ and $\mathbf{a} \times \mathbf{n}$. This must include both an

attempt to find the cross product and knowledge that $\sin \theta = \frac{|\mathbf{x} \times \mathbf{y}|}{|\mathbf{x}||\mathbf{y}|}$

M1

$$(\mathbf{a} \times \mathbf{n}) \times (\mathbf{a} \times \mathbf{p}) = \begin{pmatrix} 1296 \\ 0 \\ 0 \end{pmatrix} \text{ OR } (\mathbf{a} \times \mathbf{p}) \times (\mathbf{a} \times \mathbf{n}) = \begin{pmatrix} -1296 \\ 0 \\ 0 \end{pmatrix}$$

$$\sin \theta = \frac{1296}{36 \times 36} = 1$$

A1

so angle is 90°

AG

[3 marks]

continued

Question 2 continued.

(d) attempt to use $\frac{\theta}{360} \times 2\pi r = 6$ OR $s = r\theta$ **(M1)**

$$\frac{\theta}{360} \times 2\pi \times 6 = 6 \quad \text{OR} \quad 6 = 6\theta \Rightarrow \theta = 1 \text{ radian}$$

$$\theta = 57.2957...^\circ$$
 A1

Note: The fourth significant figure must be seen somewhere to award the **A1**.

$$\theta = 57.3^\circ \text{ to 3 significant figures}$$

AG
[2 marks]

continued

Question 2 continued.

(e) **METHOD 1**

EITHER (Scalar product)

attempt to use scalar product to find $\widehat{BOM} = \arccos\left(\frac{b \cdot m}{|b||m|}\right)$ (M1)

Note: This may be written as $\cos(\widehat{BOM}) = \frac{b \cdot m}{|b||m|}$.

$$= \arccos\left(\frac{(6 \cos 120^\circ)(6 \cos 57.3^\circ)}{6^2}\right)$$
 (A1)

Note: This (A1) can be awarded for $\cos \theta = \frac{6 \cos 120^\circ \times 6 \cos 57.3^\circ}{6^2}$.

$$= 106^\circ (= 105.673\dots^\circ) \text{ OR } 1.84 \text{ radians } (1.84434\dots)$$
 (A1)

OR (Cosine Rule)

$$|BM| = \sqrt{(6 \sin 120 - 0)^2 + (6 \cos 120 - 6 \sin 57.3)^2 + (0 - 6 \sin 57.3)^2}$$

$$= 9.56(9.56299\dots) \text{ OR } 9560 \text{ km}$$
 (A1)

attempt to use the cosine rule to find angle BOM (M1)

$$|BM|^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos \widehat{BOM}$$

$$\widehat{BOM} = 106^\circ (= 105.673\dots^\circ) \text{ OR } 1.84 \text{ radians } (1.84434\dots)$$
 (A1)

THEN

attempt to use $\frac{\theta}{360} \times 2\pi r$ OR $s = r\theta$ (M1)

$$= \frac{105.671^\circ}{360} \times 2\pi \times 6$$

$$= 11.1 \text{ (11.0660... thousand km)}$$
 A1

Note: Accept an answer of 11100 OR 11066.0... without units (i.e. “km” is implicit).

continued

Question 2 continued.

METHOD 2 (Cross product)

attempt to use cross product to find $\hat{BOM} = \arcsin \frac{|b \times m|}{|b||m|}$ **(M1)**

Note: This may be written as $\sin \hat{BOM} = \frac{|b \times m|}{|b||m|}$.

$$= \frac{\begin{vmatrix} 36 \cos 120 \sin \theta \\ -36 \sin 120 \sin \theta \\ 36 \sin 120 \cos \theta \end{vmatrix}}{6 \times 6}$$

$$= \frac{\sqrt{\sin^2 \theta + 3}}{2} = 0.962818... \quad \textbf{(A1)}$$

$$\hat{BOM} = 180 - \arcsin 0.962818...$$

$$= 106^\circ (105.673..., 1.844346 \text{ rads}) \quad \textbf{(A1)}$$

attempt to use $\frac{\theta}{360} \times 2\pi r$ OR $s = r\theta$ **(M1)**

$$= \frac{105.671}{360} \times 2\pi \times 6$$

$$= 11.1 \text{ (11.0660 thousand km)} \quad \textbf{A1}$$

Note: Award at most **(M1)(A1)(A0)(M1)A0** for an answer of 7.78 thousand km from finding the principal root of the arcsin.

[5 marks]

continued

Question 2 continued.

(f) attempt to calculate $\mathbf{b} \times \mathbf{m}$ (M1)

$$\mathbf{b} \times \mathbf{m} = \begin{pmatrix} 36 \cos 120^\circ \sin 57.3^\circ \\ -36 \sin 120^\circ \sin 57.3^\circ \\ 36 \sin 120^\circ \cos 57.3^\circ \end{pmatrix} = \begin{pmatrix} -15.1464... \\ -26.2344... \\ 16.8449... \end{pmatrix} \quad (\text{A1})$$

attempt to use scalar product to find angle between vectors (M1)

$$(\cos \theta) = \left(\frac{(\mathbf{b} \times \mathbf{m}) \cdot (\mathbf{b} \times \mathbf{p})}{|\mathbf{b} \times \mathbf{m}| |\mathbf{b} \times \mathbf{p}|} \right)$$

$$(\cos \theta) = \left(\frac{1090.54...}{34.6614... \times 36} \right) \quad (\text{A1})(\text{A1})$$

Note: Award (A1) for numerator, (A1) for denominator.

bearing is $(0)29.1^\circ (29.0770...^\circ)$ A1

Note: Do not penalise absence of degree symbol.

Award full marks for answer in radians = 0.507.

Award full marks for 029° .

Special Case: If a candidate calculates $\mathbf{m} \times \mathbf{b}$ instead of $\mathbf{b} \times \mathbf{m}$ then you can award at most (M1)(A0)(M1)(A1)(A1)A0.

[6 marks]

Total [27 marks]

8. (a) METHOD 1 Using the angle sum formula

attempt to use angle sum formula

M1

$$\sin 2\theta \cos \theta + \cos 2\theta \sin \theta$$

A1

attempt to use a double angle identity or the Pythagorean identity

M1

$$2 \sin \theta \cos^2 \theta + (1 - 2 \sin^2 \theta) \sin \theta \quad \text{OR} \quad 2 \sin \theta (1 - \sin^2 \theta) + \cos 2\theta \sin \theta$$

$$2 \sin \theta (1 - \sin^2 \theta) + (1 - 2 \sin^2 \theta) \sin \theta \quad (\text{or equivalent})$$

A1

$$3 \sin \theta - 4 \sin^3 \theta$$

AG**METHOD 2 Using de Moivre's Theorem and comparing coefficients**

$$(\cos \theta + i \sin \theta)^3 = \cos 3\theta + i \sin 3\theta$$

A1

attempt to expand LHS

M1

$$\cos^3 \theta - 3 \cos \theta \sin^2 \theta + i(3 \cos^2 \theta \sin \theta - \sin^3 \theta)$$

comparing imaginary parts

M1

$$\sin 3\theta = 3 \cos^2 \theta \sin \theta - \sin^3 \theta$$

A1

$$(\sin 3\theta =) 3 \sin \theta - 4 \sin^3 \theta$$

AG**[4 marks]**

(b) attempt to rewrite expression using result of part (a)

(M1)

$$y = 2 \sin 3 \left(\theta + \frac{\pi}{6} \right), \quad y = 2 \sin \left(3\theta + \frac{\pi}{2} \right)$$

vertical stretch factor 2 (seen anywhere)

A1horizontal stretch factor $\frac{1}{3}$ followed by horizontal translation $\frac{\pi}{6}$ to the left**A2****OR**horizontal translation $\frac{\pi}{2}$ to the left followed by horizontal stretch factor $\frac{1}{3}$ **A2**
Note: Award **A1A0** for correct horizontal transformations specified in the wrong order.
[4 marks]**Total [8 marks]**

Section B

10. (a) $L_1 : \mathbf{r} = \begin{pmatrix} 1 \\ -13 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

A1

Note: The $\mathbf{r}$ = must be seen.

[1 mark]

(b) $L_2 : \mathbf{s} = \begin{pmatrix} 2 \\ -4 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix}$ OR $\mathbf{s} = \begin{pmatrix} 7 \\ -6 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix}$ (or equivalent)

A1A1

Note: The $\mathbf{s}$ = must be seen, but do not penalise if same error seen in part (a).
Allow $\mathbf{r}$ = instead of $\mathbf{s}$ =.

[2 marks]

(c) $\begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix} \neq k \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ so L_1, L_2 are not parallel (may be seen anywhere)

R1

Note: Not parallel may be shown using cross product.

equating any two components from each line

M1

$$-1 + 7\lambda = 2 + 5\mu \quad (7\lambda - 5\mu = 3)$$

$$1 + \lambda = -4 - 2\mu \quad (\lambda + 2\mu = -5)$$

$$-13 + 2\lambda = 2 - \mu \quad (2\lambda + \mu = 15)$$

solve simultaneously: (1st/2nd compts OR 1st/3rd compts OR 2nd /3rd compts)

M1

$$\lambda = -1, \mu = -2 \quad \text{OR} \quad \lambda = \frac{78}{17}, \mu = \frac{99}{17} \quad \text{OR} \quad \lambda = \frac{35}{3}, \mu = -\frac{25}{3}$$

A1

check for remaining component, e.g. 3rd component: $-13 + (-1)2 \neq 2 + (-2)(-1)$

R1

$$(-15 \neq 4)$$

hence lines are not parallel and do not intersect, so are skew.

AG

Note: Condone omission of **AG** line.

[5 marks]

continued...

Question 10 continued

- (d) attempt to calculate $\vec{PN} = \vec{ON} - \vec{OP}$ **(M1)**

$$\begin{pmatrix} 2+5\mu \\ -4-2\mu \\ 2-\mu \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ -13 \end{pmatrix}$$

$$= \begin{pmatrix} 3+5\mu \\ -5-2\mu \\ 15-\mu \end{pmatrix}$$

A1

- attempt to calculate $\vec{PN} \cdot \vec{AB}$ **(M1)**

$$= \begin{pmatrix} 3+5\mu \\ -5-2\mu \\ 15-\mu \end{pmatrix} \cdot \begin{pmatrix} 5 \\ -2 \\ -1 \end{pmatrix}$$

$$= 10 + 30\mu$$

A1**[4 marks]**

- (e) setting and solving $10 + 30\mu = 0$ **(M1)**

$$\mu = -\frac{1}{3}$$

A1

$$\text{so } N\left(\frac{1}{3}, -\frac{10}{3}, \frac{7}{3}\right)$$

A1**[3 marks]***continued...*

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Question 10 continued

(f) considers the vectors $k \begin{pmatrix} -1 \\ 1 \\ -13 \end{pmatrix}$ and $s \begin{pmatrix} 1 \\ -10 \\ 7 \end{pmatrix}$ or equivalent **(A1)**

attempt to find cross product in either order: **(M1)**

e.g. $\begin{pmatrix} -1 \\ 1 \\ -13 \end{pmatrix} \times \begin{pmatrix} 1 \\ -10 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} -123 \\ -6 \\ 9 \end{pmatrix}$ **A1**

equation of plane is $-123x - 6y + 9z = 0$ (or equivalent) **A1**

$(\alpha = -123, \beta = -6, \gamma = 9, \delta = 0)$

[4 marks]**Total [19 marks]**

1. (a) (i) 1.25663...

$$\theta = 1.26 \left(= \frac{2\pi}{5} \right) \quad \text{A1}$$

$$(ii) \quad \frac{2\pi}{5} \times r = 12 \text{ or equivalent} \quad \text{(A1)}$$

9.54929...

$$r = 9.55 \text{ (cm)} \left(= \frac{60}{2\pi} = \frac{30}{\pi} \right) \quad \text{A1}$$

[3 marks](b) **METHOD 1**attempt to find area of at least one shaded segment (M1)

$$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta \quad \text{OR} \quad \frac{1}{2}r^2(\theta - \sin\theta)$$

$$\frac{1}{2}(9.54929\dots)^2 \times 1.25663\dots - \frac{1}{2}(9.54929\dots)^2 \sin 1.25663\dots \text{ (or equivalent)} \quad \text{(A1)}$$

$$\text{shaded area} = 5 \times (57.2957\dots - 43.3629\dots) (= 5 \times 13.9328\dots)$$

69.6640...

$$= 69.7 \text{ (cm}^2\text{)} \quad \text{A1}$$

[3 marks]**METHOD 2**attempt to find area of one triangle (M1)

$$\frac{1}{2} \times (9.54929\dots)^2 \times \sin 1.25663\dots$$

attempt to subtract five times their triangle area from their circle area (M1)

$$\pi \times (9.54929\dots)^2 - 5 \times \frac{1}{2} \times (9.54929\dots)^2 \times \sin 1.25663\dots$$

69.6640...

$$= 69.7 \text{ (cm}^2\text{)} \quad \text{A1}$$

[3 marks]**Total [6 marks]**

4. (a) (i) $a = -5$ **A1**

(ii) **EITHER**
period = 32 **(A1)**

OR
correct substitution of a point on the graph of g and their a **(A1)**

$$2 = -5 \cos(b \times 16) - 3 \quad \text{OR} \quad -3 = -5 \cos(b \times 8) - 3$$

Note: Award **A0** if candidate substitutes $x = 0, y = -8$.

THEN

$$0.196349\dots$$

$$b = 0.196 \left(= \frac{2\pi}{32} = \frac{\pi}{16} \right) \quad \text{A1}$$

[3 marks]

(b) $k = 32$ **A1**

[1 mark]

(c) (i) **EITHER**
attempt to use a horizontal translation **(M1)**

$$24 - q = 16 \quad \text{OR} \quad 16 - q = 8$$

OR

substituting a point on the graph of g into $f(x) = p \sin(b(x - q)) - 3$ **(M1)**

$$-8 = p \sin\left(\frac{\pi}{16}(0 - q)\right) - 3 \quad \text{OR} \quad -3 = p \sin\left(\frac{\pi}{16}(8 - q)\right) - 3 \quad \text{OR} \quad 2 = p \sin\left(\frac{\pi}{16}(16 - q)\right) - 3$$

THEN

$$q = 8 \quad \text{A1}$$

(ii) $p = 5$ **A1**

[3 marks]

Total [7 marks]

12. (a) attempt to set up a system of equations with at least one correct equation

M1

$$5 + s = -1 + 2t, \quad 4 - s = 7 + t, \quad 2 + s = -5 + 3t$$

$$t = 1 \quad \text{OR} \quad s = -4$$

A1

$$\begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} - 4 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} -1 \\ 7 \\ -5 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

A1

$$\text{point of intersection} \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix}$$

AG**[3 marks]**

$$(b) \quad (i) \quad \mathbf{r} = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \quad \text{OR} \quad \mathbf{r} = \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

$$\text{OR} \quad \mathbf{r} = \begin{pmatrix} -1 \\ 7 \\ -5 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$$

A1

Note: $\mathbf{r}$ = must be seen.

continued...

Question 12 continued.

(ii) **METHOD 1**

attempt to write equation of the plane in the form $\mathbf{r} \cdot \mathbf{n} = a$

M1

$$\mathbf{r} \cdot \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} (= -18)$$

$$-4x - y + 3z = -18$$

A1

$$4x + y - 3z = 18$$

AG

Note: Students may use $\begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 7 \\ -5 \end{pmatrix}$ in place of $\begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix}$ or other points from lines L_1 or L_2 .

METHOD 2

recognizing that $\mathbf{b} \times \mathbf{c}$ is normal to the plane.

M1

$$-4x - y + 3z = k$$

correct substitution of one of the points to find k

A1

$$k = 18$$

$$4x + y - 3z = 18$$

AG

[3 marks]

(c) (i) $q = 18$

A1

(ii) $r = -6$

A1

[2 marks]

continued...

Question 12 continued.

$$(d) \quad \vec{PQ} = \begin{pmatrix} -4.5 \\ 18 \\ 0 \end{pmatrix} \quad \text{OR} \quad \vec{PR} = \begin{pmatrix} -4.5 \\ 0 \\ -6 \end{pmatrix} \text{ seen anywhere} \quad (\text{A1})$$

attempt to find a vector product (M1)

$$\vec{PQ} \times \vec{PR}$$

$$\vec{PQ} \times \vec{PR} = \begin{pmatrix} -108 \\ -27 \\ 81 \end{pmatrix} \quad \text{A1}$$

attempt to find magnitude of their cross product (M1)

$$\sqrt{(-108)^2 + (-27)^2 + 81^2}, \sqrt{18954}, 137.673$$

$$\text{Area} = 68.8 \quad \left(= \frac{1}{2} \sqrt{18954} \right) \left(= \frac{27\sqrt{26}}{2} \right) \quad \text{A1}$$

[5 marks]

$$(e) \quad \mathbf{r}_3 = \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix} + \gamma \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix} \quad \text{A2}$$

Note: Accept any multiple of the direction vector.

Award **A1A0** if $\mathbf{r}_3$ is not seen.

Accept $\mathbf{r} =$

[2 marks]

continued...

Question 12 continued.

- (f) attempt to set their r_3 equal to the vector $\begin{pmatrix} -11 \\ 5 \\ 7 \end{pmatrix}$ (M1)

$$-11 = 1 - 4\gamma, \quad 5 = 8 - \gamma, \quad 7 = -2 + 3\gamma$$

$$\gamma = 3$$

A1

Note: Accept any correct value of γ for their direction vector. For example accept $\gamma = -3$ for a

direction vector of $\begin{pmatrix} 4 \\ 1 \\ -3 \end{pmatrix}$.

[2 marks]

- (g) attempt to subtract appropriate vectors (in any order) OR consider $3n$ (M1)

$$\begin{pmatrix} -11 \\ 5 \\ 7 \end{pmatrix} - \begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix}, \quad \begin{pmatrix} -12 \\ -3 \\ 9 \end{pmatrix}$$

(height of pyramid =) $\sqrt{(-12)^2 + (-3)^2 + 9^2}, \quad \sqrt{234}, 3\sqrt{26}, 15.2970\dots$ A1

attempt to substitute their values into the formula for the volume of a pyramid (M1)

$$\frac{1}{3} \times \frac{1}{2} \sqrt{18954} \times \sqrt{234}$$

$$V = 351 \text{ (exact) (cubic units)}$$

A1

[4 marks]

Total [21 marks]

1. (a) attempt to use double angle formula for $\cos 2x$ (seen anywhere)

M1

Note: Only award the **M1** if a double angle formula for $\cos 2x$ is used, and not only for stating it.

$$3(1 - 2\sin^2 x) = 3 - 6\sin^2 x \text{ (seen anywhere)}$$

A1

$$3\cos 2x + 11\sin x = 3 + 11\sin x - 6\sin^2 x$$

AG**[2 marks]**

- (b) **METHOD 1**

attempt to solve a quadratic equation in $\sin x$

(M1)

$$\sin x = 0.333333... \left(= \frac{1}{3} \right)$$

$$x = 19.4712..., 160.528...$$

$$x = 19.5, 161$$

A1A1**METHOD 2**

attempt to solve $3\cos 2x + 11\sin x - 6 = 0$ (may be seen in a sketch)

(M1)

$$x = 19.4712..., 160.528...$$

$$x = 19.5, 161$$

A1A1**[3 marks]****Total [5 marks]**

3. (a) perimeter of sector is $1.5 \times$ circumference of circle (seen anywhere) **A1**
 $1.5 \times 16\pi$ OR 24π OR 75.3982...

perimeter of sector $r\theta + 2r$ **(M1)**

$$24\pi = r\theta + 2r \quad \mathbf{A1}$$

$$r = \frac{24\pi}{2 + \theta} \quad \mathbf{AG}$$

[3 marks]

- (b) correct substitution of $r = \frac{24\pi}{2 + \theta}$ into $\frac{1}{2}r^2\theta$ **(A1)**

$$64\pi = \frac{1}{2} \times \theta \times \left(\frac{24\pi}{2 + \theta} \right)^2$$

attempt to solve their equation (must be quadratic) **(M1)**

$$\theta = 0.411273\dots$$

$$\theta = 0.411 \quad \mathbf{A1}$$

Note: Award (A1)(M1)A0 if additional answers are given.

[3 marks]

Total [6 marks]

