APOLLO

Mathematics: analysis and approaches SL Workbook

First assessment 2021
Whole questions arranged by syllabus topic.
Original question and scheme crops preserve source diagrams.

APOLLO

Number and algebra

47 questions

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 4]

The diameter of a spherical planet is $6 \times 10^4 \text{km}$.

(a) Write down the radius of the planet. [1]

The volume of the planet can be expressed in the form $\pi(a \times 10^k)\text{km}^3$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Find the value of a and the value of k. [3]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d.

Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 4]

Consider two consecutive positive integers, n and $n+1$.

Show that the difference of their squares is equal to the sum of the two integers.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

In the expansion of $(x+k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63.

Find the possible values of k.

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

Consider the expansion of $(3+x^2)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of x^4 is 20412, find the value of n.

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will not be marked.

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving $\$20\,000$. They each have $\$9000$ to invest.

(a) Amelia invests her $\$9000$ in an account that offers an interest rate of 7% per annum compounded **annually**.

(i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars.

(ii) Determine the number of years required for Amelia's investment to reach the target. [5]

(b) Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

(c) A third friend Chris also wants to reach the $\$20\,000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

(i) Show that Chris will never reach the target if his initial deposit is $\$9000$.

(ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [8]



Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Let S_n denote the sum of the first n terms of the sequence.

(b) Find the maximum value of S_n. [3]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0, a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

(a) Show that $a = 8$. [2]

(b) Write down an expression for $f^{-1}(x)$. [1]

(c) Find the value of $f^{-1}\left(\sqrt{32}\right)$. [3]

(d) Consider the arithmetic sequence $\log_8 27, \log_8 p, \log_8 q, \log_8 125$, where $p > 1$ and $q > 1$.

(i) Show that $27, p, q$ and 125 are four consecutive terms in a geometric sequence.

(ii) Find the value of p and the value of q. [9]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 9]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. [2]

(b) Find S_∞. [3]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. [4]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

(a) Show that $(2n-1)^2+(2n+1)^2=8n^2+2$, where $n \in \mathbb{Z}$. [2]

(b) Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5%, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

Laurie makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the series $\ln x + p\ln x + \frac{1}{3}\ln x + \ldots$, where $x \in \mathbb{R}, x > 1$ and $p \in \mathbb{R}, p \neq 0$.

(a) Consider the case where the series is geometric.

(i) Show that $p = \pm\frac{1}{\sqrt{3}}$.

(ii) Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x. [5]

(b) Now consider the case where the series is arithmetic with common difference d.

(i) Show that $p = \frac{2}{3}$.

(ii) Write down d in the form $k\ln x$, where $k \in \mathbb{Q}$.

(iii) The sum of the first n terms of the series is $-3\ln x$.

Find the value of n. [10]

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

The nth term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

(b) Given that the nth term of this sequence is -33, find the value of n. [2]

(c) Find the common difference, d. [2]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

Consider any three consecutive integers, $n-1$, n and $n+1$.

(a) Prove that the sum of these three integers is always divisible by 3. [2]

(b) Prove that the sum of the squares of these three integers is never divisible by 3. [4]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

Consider the binomial expansion $(x+1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + ... + 1$ where $x \neq 0$ and $a, b \in \mathbb{Z}^+$.

(a) Show that $b = 21$. [2]

The third term in the expansion is the mean of the second term and the fourth term in the expansion.

(b) Find the possible values of x. [5]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

In this question, give all answers correct to two decimal places.

Sam invests $\$1700$ in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

(a) Find the amount that Sam will have in his account after 10 years. [3]

David also invests $\$1700$ in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or withdrawals from this account.

(b) Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. [2]

(c) Find the interest David will earn over the 10 years. [1]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

Let a be a constant, where $a > 1$.

(a) Show that $a^2 + \left(\frac{a^2-1}{2}\right)^2 = \left(\frac{a^2+1}{2}\right)^2$. [3]

Consider a right-angled triangle with sides of length a, $\left(\frac{a^2-1}{2}\right)$ and $\left(\frac{a^2+1}{2}\right)$.

(b) Find an expression for the area of the triangle in terms of a. [2]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

Consider the expansion of $\frac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\frac{8}{7}a^5$.

Find the value of a.

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a) (i) Find the sum of the first five terms.

(ii) Given that $S_6 = 60$, find u_6. [4]

(b) Find u_1. [2]

(c) Hence or otherwise, write an expression for u_n in terms of n. [3]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r. [3]

(e) Given that $v_{99} < 0$, find v_5. [2]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in $\text{USD}\,(\$)$.

(a) Write down the annual rate of depreciation of the car. [1]

(b) Find the value of the car on 1 January 2028. [2]

Alvie wants to buy this car. On 1 January 2023, he invested $\$15\,000$ in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra $\$M$.

(c) Find the value of M. [3]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

The coefficient of x^6 in the expansion of $(ax^3 + b)^8$ is 448.

The coefficient of x^6 in the expansion of $(ax^3 + b)^{10}$ is 2880.

Find the value of a and the value of b, where $a, b > 0$.

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

The expansion of $(x+h)^8$, where $h>0$, can be written as $x^8+ax^7+bx^6+cx^5+dx^4+...+h^8$, where $a, b, c, d, ... \in \mathbb{R}$.

(a) Find an expression, in terms of h, for

(i) a;

(ii) b;

(iii) d. [4]

(b) Given that a, b, and d are the first three terms of a geometric sequence, find the value of h. [3]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is $\$4200$.

Option B The first payment is $\$1500$. In each month which follows, the payment is 4% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(i) Option A;

(ii) Option B. [5]

Sorin received an inheritance of $\$160\,000$. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 5% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years. [1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c) (i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month.

(ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least $\$257\,000$. [6]

At the end of the three years, Daniela invested $\$30\,000$ for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to $\$41\,000$ after six years. [3]

Do **not** write solutions on this page.

Question 8 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is $\$5500$.

Option B The first payment is $\$2000$. In each month which follows, the payment is 6% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(i) Option A;

(ii) Option B. [5]

Sorin received an inheritance of $\$120\,000$. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 4% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years. [1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c) (i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month.

(ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least $\$250\,000$. [6]

At the end of the three years, Daniela invested $\$40\,000$ for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to $\$53\,000$ after six years. [3]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

(b) Find an expression for u_n, the nth term. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

It is given that $\log_{10} a = \frac{1}{3}$, where $a > 0$.

Find the value of

(a) $\log_{10}\left(\frac{1}{a}\right)$; [2]

(b) $\log_{1000} a$. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

(a) Find an expression for S_n in the form $\frac{a^n - 1}{b}$, where $a, b \in \mathbb{Z}^+$. [1]

(b) Hence, show that $S_1 + S_2 + S_3 + \ldots + S_n = \frac{10\left(10^n - 1\right) - 9n}{81}$. [4]

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

Consider the arithmetic sequence $a, p, q \ldots$, where $a, p, q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence $a, s, t \ldots$, where $a, s, t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(i) arithmetic sequence;

(ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

(ii) Show that $\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]

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Answers written on this page
will not be marked.

Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 7]

Darren buys a car for $\$35\,000$. The value of the car decreases by 15% in the first year.

(a) Find the value of the car at the end of the first year. [2]

After the first year, the value of the car decreases by 11% in each subsequent year.

(b) Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar. [2]

When Darren has owned the car for n complete years, the value of the car is less than 10% of its original value.

(c) Find the least value of n. [3]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 4]

Prove that $(2n+5)^2-(2n-5)^2$ is a multiple of 10 for all $n \in \mathbb{Z}^+$.

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$.

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 4]

Prove that $(3n+2)^2-(3n-2)^2$ is a multiple of 12 for all $n \in \mathbb{Z}^+$.

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$.

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 4]

Find the coefficient of x^8 in the expansion of $(2x - 5)^{11}$.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 4]

Find the coefficient of x^6 in the expansion of $(2x - 5)^9$.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 7]

Bob invests 1000 dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1000(1+k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k. [1]

(b) Expand and simplify $(1+x)^4$. [2]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 8]

Consider a sequence of ten rectangular picture frames $F_1, F_2, \ldots, F_9, F_{10}$.

Picture frame F_1 has width $4\,\text{cm}$ and height $5\,\text{cm}$.

The width and height of picture frame F_n, are each increased by $50\,\%$ to generate the width and height of the next picture frame F_{n+1}, for $n \in \mathbb{Z}^+,\ 1 \leq n \leq 9$.

(a) (i) Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1}\,\text{cm}^2$.

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^a - 1\right)\text{cm}^2$, where $p \in \mathbb{Q}^+,\ a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^4\,\text{cm}^2$, where $q \in \mathbb{Q}^+$. [3]

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Consider the sequence $\{u_n\}$, with $n\text{th}$ term given by u_n. The first three terms are

$$u_1 = k - 5,\ u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

(i) Find the value of k.

(ii) Hence, or otherwise, find u_3. [5]

(b) Consider the case where $k = 12$.

(i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

(ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k.

(i) Show that $k^2 - 10k - 24 = 0$.

(ii) Find the first three terms of $\{u_n\}$ for this second value of k.

(iii) Hence, write down the value of S_{2m}, the sum of the first $2m$ terms, for this second value of k. [7]

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Question 8 - working space

Section B

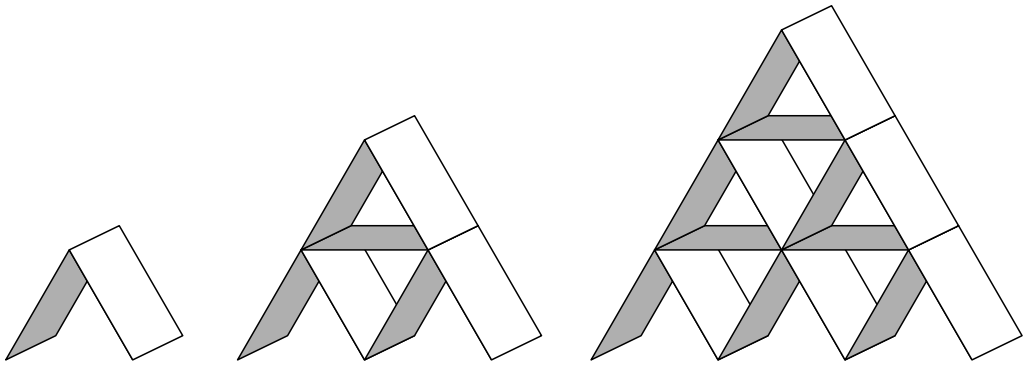
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of $60°$ to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



$n = 1$ $n = 2$ $n = 3$

Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

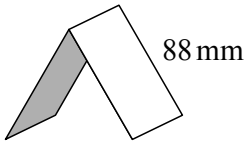
(c) Show that $t_n = \frac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

The long edge of each playing card measures $88\,\text{mm}$ as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

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Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

Consider the expansion of $(2x + k)^{10}$, where $k \in \mathbb{Z}$.

Given that the coefficient of x^6 is 8.4×10^6, find the possible values of k.

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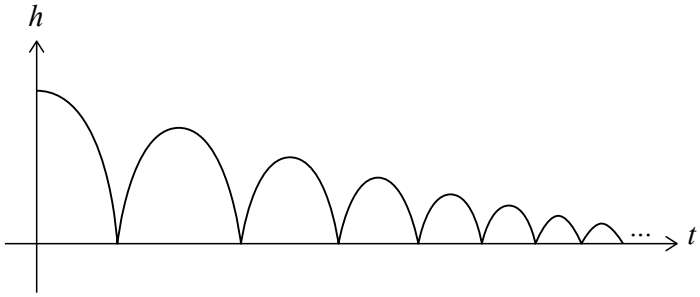
Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

A tennis ball is dropped from a height. After each bounce the maximum height reached by the ball is $\frac{2}{3}$ of its previous maximum height. This can be seen in the diagram below where h, in metres, is the height of a ball after t seconds.



A box contains tennis balls. Each ball satisfies the condition of rebounding to $\frac{2}{3}$ of their previous maximum height. The tennis balls are numbered Ball $1, 2, 3, ...$

Ball 1 is dropped from a height of 10 metres.

(a) Find the maximum height of Ball 1 after the 5th bounce. [3]

(b) Find the total distance travelled by Ball 1 immediately before the 5th bounce. [3]

Let δ be the total distance travelled by any of these balls.

(c) A ball is dropped from a height of x metres. Show that $\delta = 5x$ metres. [3]

Let δ_1 be the total distance travelled by Ball 1.

(d) Write down the value of δ_1. [1]

Ball 2 is dropped from a height of 9.56 metres.

Let δ_2 be the total distance travelled by Ball 2, and so on for each ball in the box.

It is given that $\delta_1, \delta_2, \delta_3, ...$ form an arithmetic sequence.

(e) Determine which tennis ball is the first ball to travel less than 25 metres. [4]

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will not be marked.

Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider the expansion of $(x + k)^{11}$, where $k > 0$.

(a) Write down the number of terms in the expansion. [1]

In the expansion, the coefficient of x^7 is 1320.

(b) Find the value of k. [4]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The 1st and 5th terms of an arithmetic sequence are 36 and 12 respectively.

(a) Find the 13th term of this arithmetic sequence. [4]

The sum of the first n terms of this arithmetic sequence is zero.

(b) Find the value of n. [2]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The 7th term of an arithmetic sequence is 6.

The sum of the 6th term and the 12th term is 24.

Find the first term and the common difference.

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

In the expansion of $\left(\sqrt{x}+k\right)^{12}$ where $k \in \mathbb{Z}$, the coefficient of the term in $x^{\frac{3}{2}}$ is -112640.

Find the value of k.

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Andy and Jess each have $\$5000$.

Andy invests the money in a new savings plan that will pay interest at the end of each month.

Andy will receive a fixed amount of interest each month. The amount received is 0.315% of the initial investment.

(a) (i) Determine the amount of interest Andy will receive at the end of each month. Give this answer correct to two decimal places.

(ii) Hence, determine the amount of interest Andy will receive each year.

(iii) Write down an expression in the form $5000 + pn$, where $p \in \mathbb{Z}^+$, for the amount of money Andy will have in his savings plan at the end of n years.

(iv) Hence, show that Andy will have $\$5945$ in his savings plan at the end of 5 years. [4]

In this part, where appropriate, give all answers to the nearest dollar.

Jess invests her $\$5000$ in a new account that pays 3% interest compounded annually.

(b) (i) Determine the amount of money that will be in Jess's account at the end of 5 years.

(ii) Hence, find the amount of interest Jess will receive in the 5 years. [3]

(c) (i) Write an expression in the form $5000 \times q^n$, where $q \in \mathbb{R}^+$, for the amount of money that Jess will have in her account at the end of n years.

(ii) Hence, determine the smallest number of complete years that it will take for Jess to have more money in her account than Andy has in his savings plan. [5]

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Question 7 - working space

APOLLO

Functions

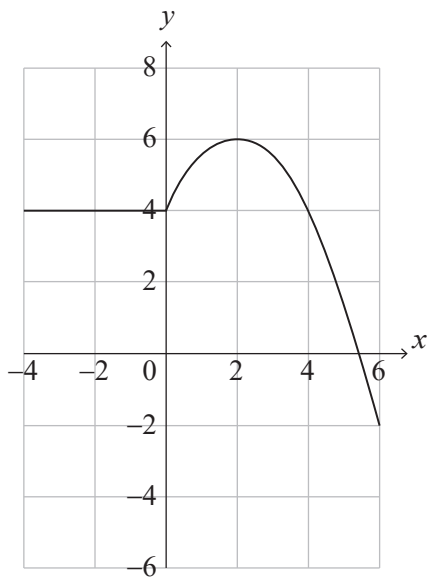
58 questions

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \le x \le 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$. [2]

(b) Let $g(x) = \frac{1}{2} f(x) + 1$ for $-4 \le x \le 6$. On the axes above, sketch the graph of g. [3]

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Question 1 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

(a) Show that $m = 4$. [6]

The function f can be expressed in the form $f(x) = 4(x - p)(x - q)$, where $p, q \in \mathbb{R}$.

(b) Find the value of p and the value of q. [2]

The function f can also be expressed in the form $f(x) = 4(x - h)^2 + k$, where $h, k \in \mathbb{R}$.

(c) Find the value of h and the value of k. [3]

(d) Hence find the values of x where the graph of f is both negative and increasing. [3]

Question 7 - working space

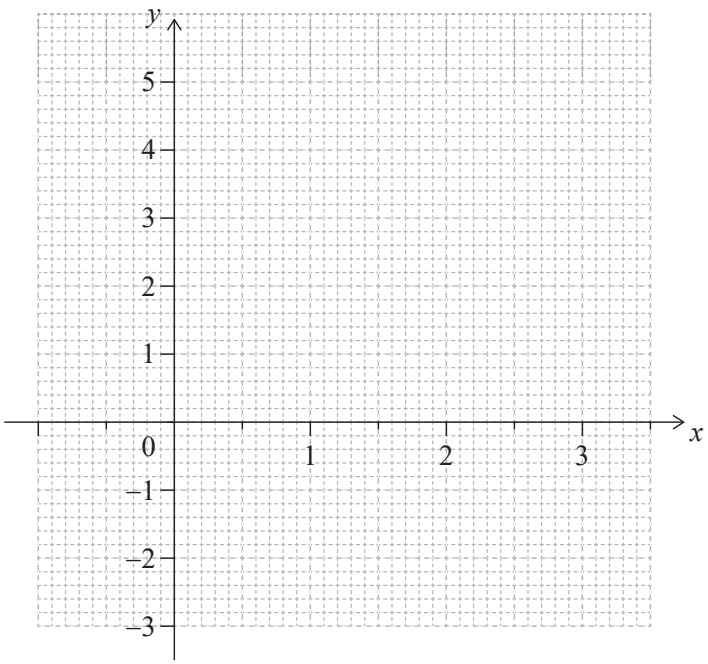
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Let $f(x) = 3x - 4^{0.15x^2}$ for $0 \le x \le 3$.

(a) Sketch the graph of f on the grid below. [3]



(b) Find the value of x for which $f'(x) = 0$. [2]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f. [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c. [4]

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount, A, of carbon-14 present in a plant t years after its death can be modelled by $A = A_0 e^{-kt}$ where $t \geq 0$ and A_0, k are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that $A_0 = 100$. [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that $k = \frac{\ln 2}{5730}$. [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

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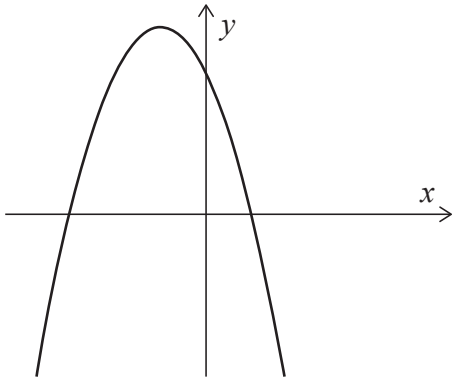
Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = -2(x-1)(x+3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.



(a) For the graph of f

(i) find the x-coordinates of the x-intercepts;

(ii) find the coordinates of the vertex. [5]

The function f can be written in the form $f(x) = -2(x-h)^2 + k$.

(b) Write down the value of h and the value of k. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The function f is defined by $f(x)=\frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

(ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

(i) the x-axis;

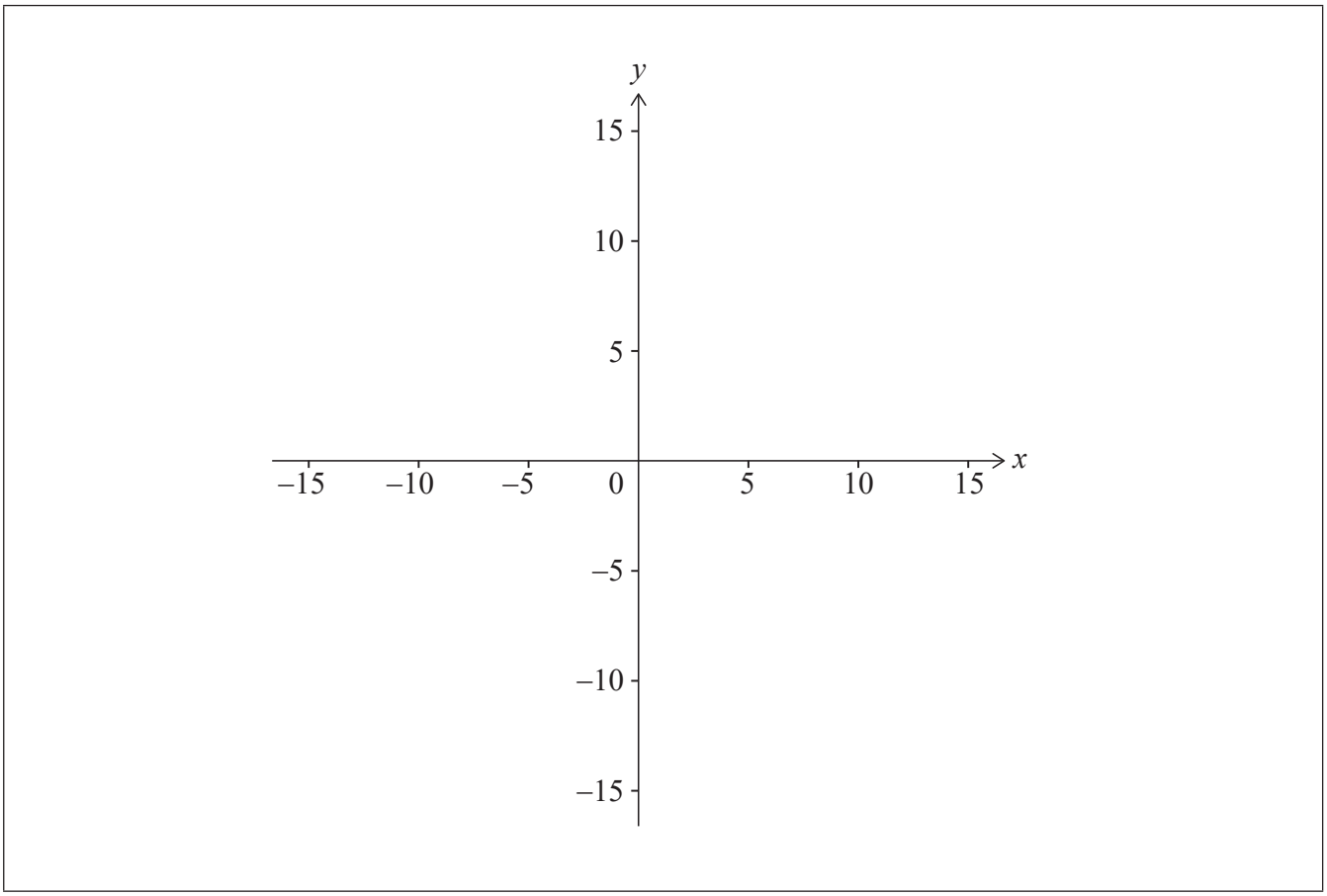
(ii) the y-axis. [2]

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(Question 3 continued)

(c) Sketch the graph of f on the axes below. [1]



Question 3 - working space

Section A

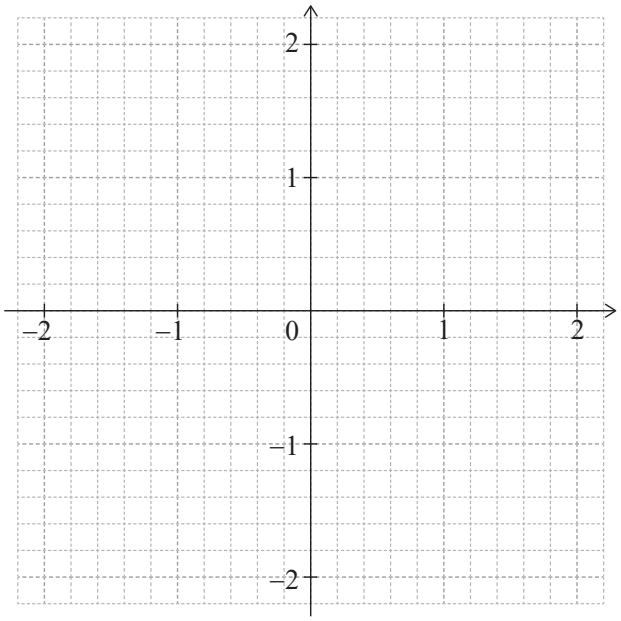
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \le x \le 2$.

(a) Find the values of x for which $f(x) = 0$. [2]

(b) Sketch the graph of f on the following grid. [3]



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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x + 5$.

(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the points $\mathrm{A}(-2, 20)$, $\mathrm{B}(4, 6)$ and $\mathrm{C}(-14, 12)$. The line L passes through the point A and is perpendicular to $[\mathrm{BC}]$.

(a) Find the equation of L. [3]

The line L passes through the point $(k, 2)$.

(b) Find the value of k. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a) Describe these two transformations. [2]

The y-intercept of the graph of g is at $(0, r)$.

(b) Given that $g(x) \geq 7$, find the smallest value of r. [5]

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Do **not** write solutions on this page.

Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following table shows values of $f(x)$ and $g(x)$ for different values of x.

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

(a) Find $g(0)$. [1]

(b) Find $(f \circ g)(0)$. [2]

(c) Find the value of x such that $f(x) = 0$. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

A function f is defined by $f(x)=\frac{2x-1}{x+1}$, where $x \in \mathbb{R}$, $x \neq -1$.

(a) The graph of $y=f(x)$ has a vertical asymptote and a horizontal asymptote.

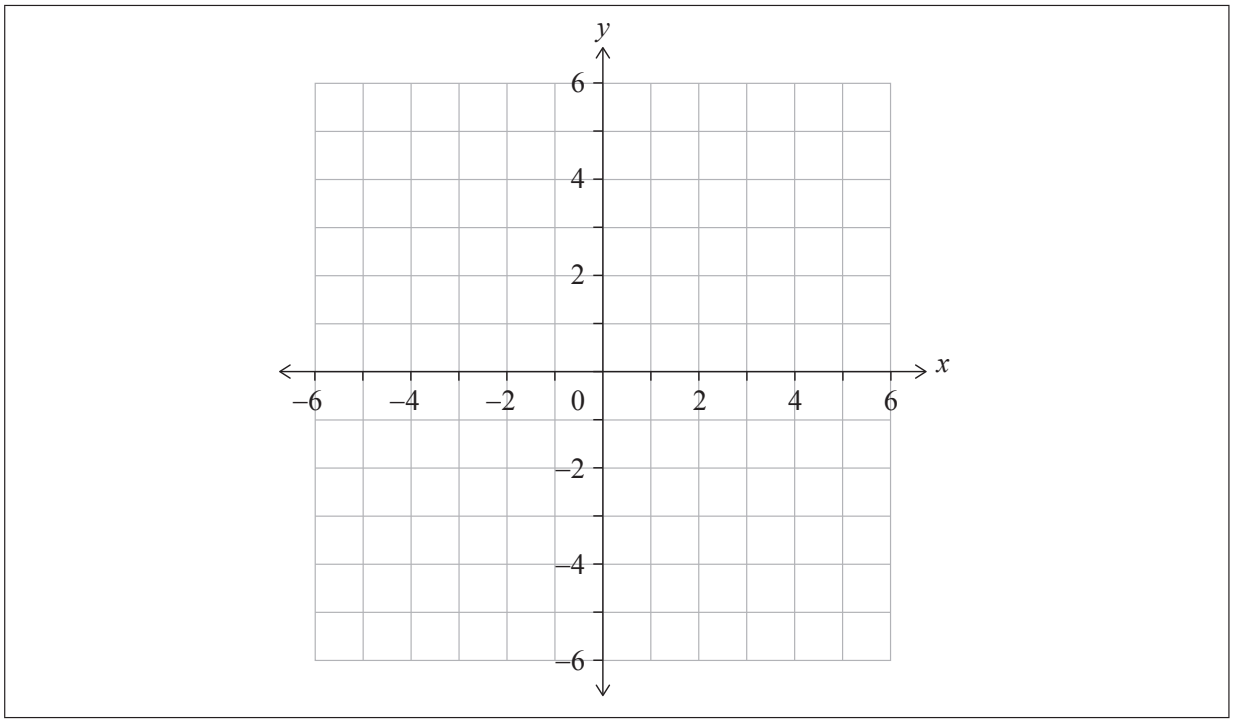
Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(b) On the set of axes below, sketch the graph of $y=f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes. [3]



(c) Hence, solve the inequality $0<\frac{2x-1}{x+1}<2$. [1]

(This question continues on the following page)

(Question 4 continued)

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R}$, $x \neq -4$.

(a) For the graph of f

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

(b) (i) Find $f^{-1}(x)$.

(ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. [8]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

(c) (i) Find the value of p and the value of q.

(ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1}. [5]

Do **not** write solutions on this page

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$.

(a) The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$. [2]

(b) The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1, 2)$. Find an expression for $h(x)$. [3]

(c) Find $(g \circ h)(0)$. [2]

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Question 1 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

(a) The graph of a quadratic function f has its vertex at the point $(3, 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f(x) = a(x - h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p, t \in \mathbb{R}, p \neq 0$.

(b) In the case where $g(-3) = g(1) = 4$,

(i) find the value of p and the value of t;

(ii) find the range of g. [7]

(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t - 1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

(a) Calculate the value of each of the following logarithms:

(i) $\log_2 \frac{1}{16}$;

(ii) $\log_9 3$;

(iii) $\log_{\sqrt{3}} 81$. [7]

(b) It is given that $\log_{ab} a = 3$, where a, $b \in \mathbb{R}^+$, $ab \neq 1$.

(i) Show that $\log_{ab} b = -2$.

(ii) Hence find the value of $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}$. [8]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

The population of a town t years after 1 January 2014 can be modelled by the function

$$P(t) = 15\,000\mathrm{e}^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by $11\,\%$.

Use this model to estimate the population of this town on 1 January 2041.

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of $[\mathrm{PQ}]$.

(a) Find the coordinates of M. [2]

Line L is perpendicular to $[\mathrm{PQ}]$ and passes through M.

(b) Find the gradient of L. [2]

(c) Hence, write down the equation of L. [1]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 7]

The function f is defined by $f(x)=\frac{7x+7}{2x-4}$ for $x \in \mathbb{R}$, $x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y=f(x)$, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^{x}$ has at least one real solution.

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(i) the y-axis;

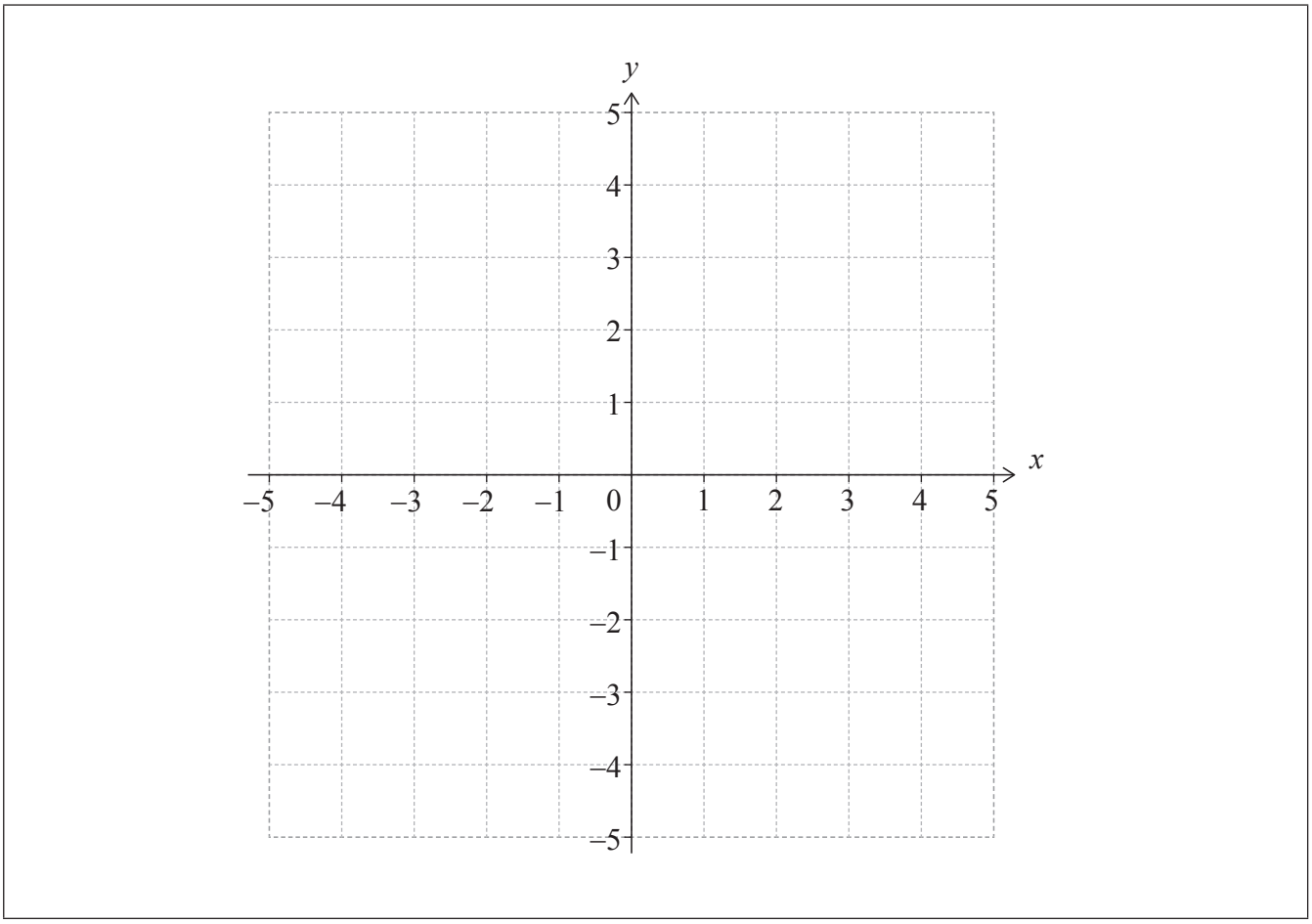
(ii) the x-axis. [2]

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(Question 3 continued)

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b). [1]



Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f(x) = ax + b, \text{ where } a, b \in \mathbb{Z}$$

$$g(x) = x^2 + x + 3.$$

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$.

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Question 6 - working space

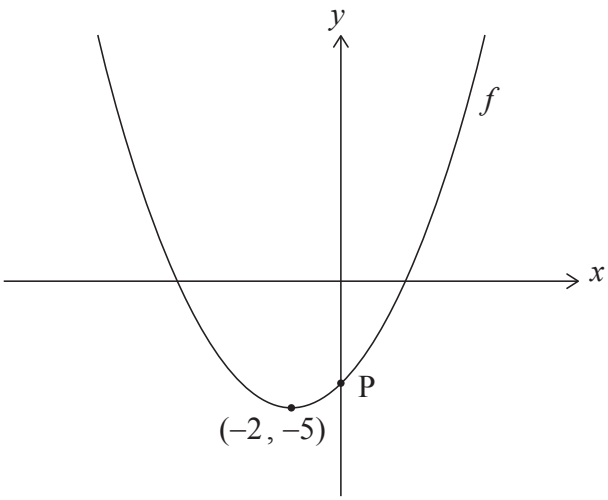
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

The following diagram shows part of the graph of a quadratic function f.

The vertex of the parabola is $(-2, -5)$ and the y-intercept is at point P.



(a) Write down the equation of the axis of symmetry. [1]

The function can be written in the form $f(x) = \frac{1}{4}(x-h)^2 + k$, where $h, k \in \mathbb{Z}$.

(b) Write down the values of h and k. [2]

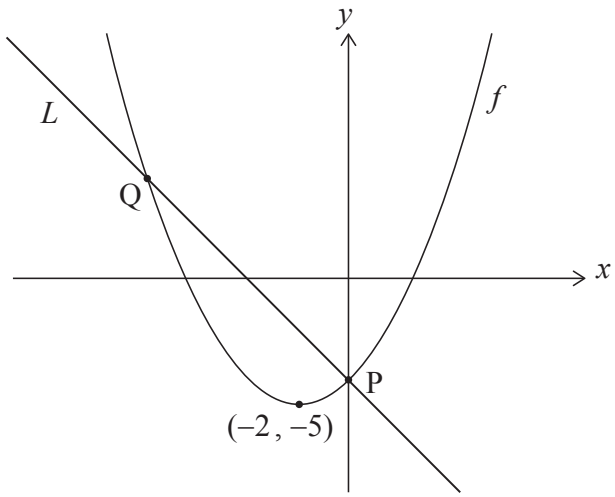
(c) Find the y-coordinate of P. [2]

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Do **not** write solutions on this page.

(Question 7 continued)

In the following diagram, the line L is normal to the graph of f at point P.



(d) Find the equation of the line L, in the form $y = ax + b$. [4]

The line L intersects the graph of f at a second point, Q, as shown above.

(e) Calculate the distance between P and Q. [8]

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Question 7 - working space

Section A

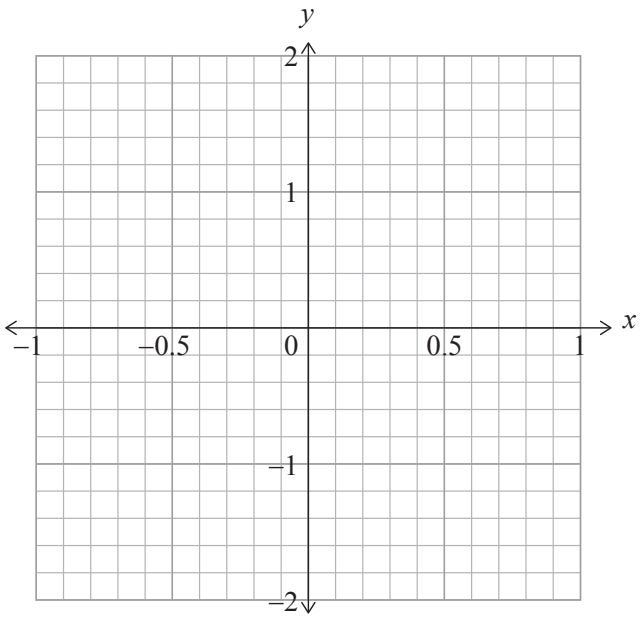
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The functions f and g are defined by $f(x) = 2x - x^3$ and $g(x) = \tan x$.

(a) Find $(f \circ g)(x)$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \leq x \leq 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]



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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The amount of a drug, in milligrams (mg), in a patient's body can be modelled by the function $A(t) = 500\text{e}^{-kt}$, where k is a positive constant and t is the time in hours after the initial dose is given.

(a) Write down the amount of the drug in the patient's body when $t = 0$. [1]

After three hours, the amount of the drug in the patient's body has decreased to $280\,\text{mg}$.

(b) Find the value of k. [2]

The second dose is given T hours after the initial dose, when the amount of the drug in the patient's body is $140\,\text{mg}$.

(c) Find the value of T. [2]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Consider the function $h(x) = \sqrt{4x-2}$, for $x \geq \frac{1}{2}$.

(a) (i) Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain.

(ii) Write down the range of $h^{-1}(x)$. [5]

(b) The graph of h intersects the graph of h^{-1} at two points.

Find the x-coordinates of these two points. [3]

(c) Find the area enclosed by the graph of h and the graph of h^{-1}. [2]

(d) Find $h'(x)$. [2]

(e) Find the value of x for which the graph of h and the graph of h^{-1} have the same gradient. [3]

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Question 7 - working space

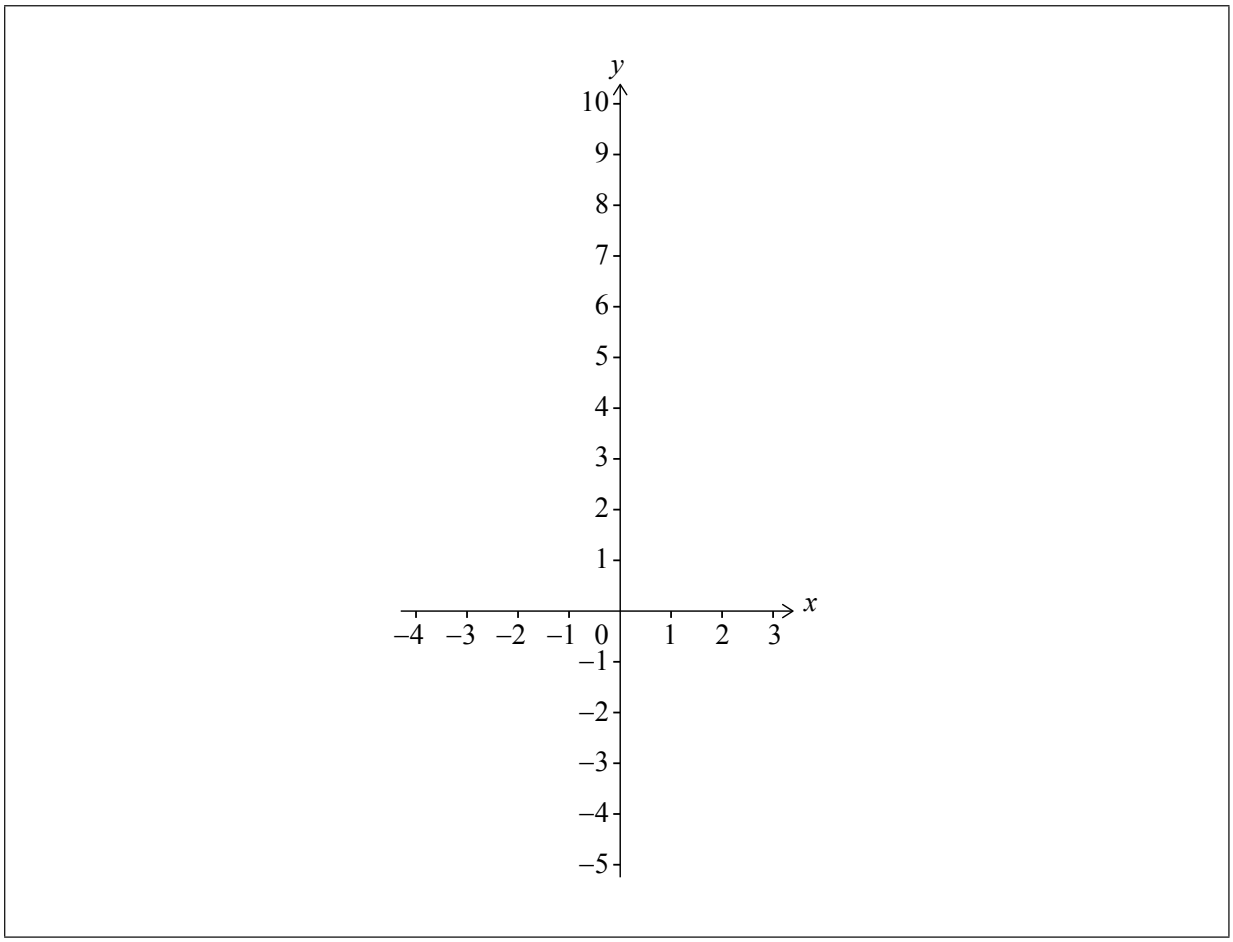
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider the function $f(x) = e^x - 2x - 5$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$. [3]



The function g is defined by $g(x) = e^{3x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k, followed by a vertical translation of c units.

Find the value of k and the value of c. [2]

(This question continues on the following page)

(Question 3 continued)

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Question 3 - working space

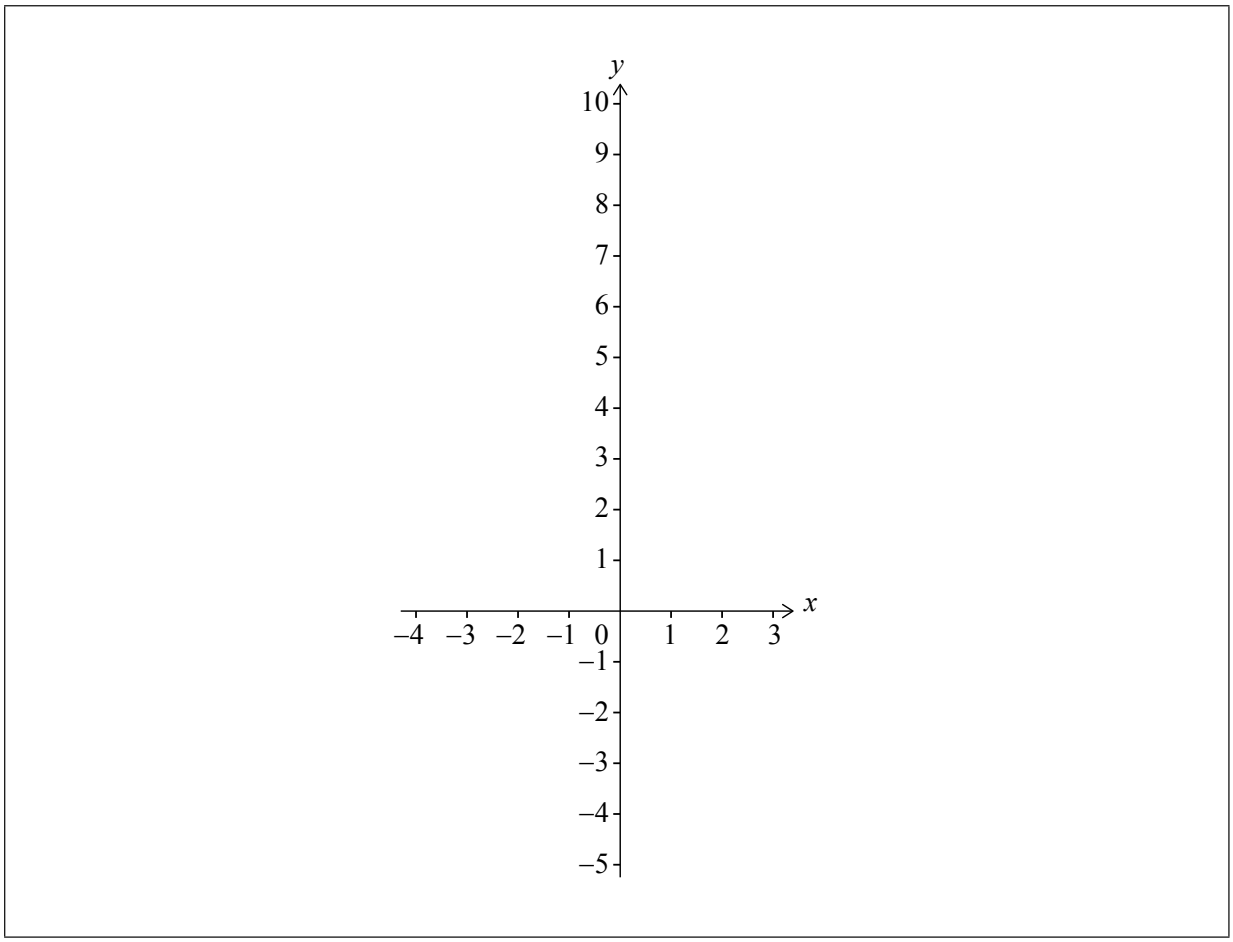
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider the function $f(x) = e^x - 3x - 4$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$. [3]



The function g is defined by $g(x) = e^{2x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k, followed by a vertical translation of c units.

Find the value of k and the value of c. [2]

(This question continues on the following page)

(Question 3 continued)

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the number of solutions of the equation $f(x) = g(x)$. [2]

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Question 8 - working space

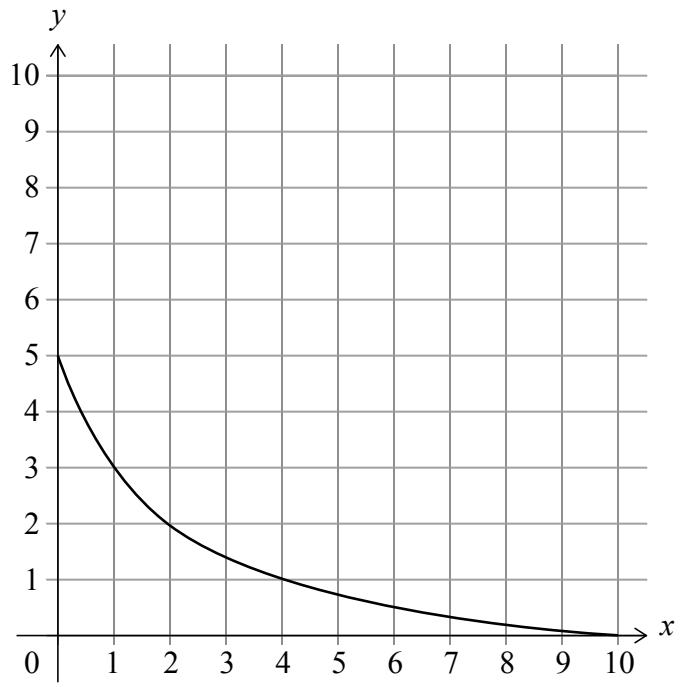
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $0 \le x \le 10$ is shown in the following diagram.

The graph intercepts the axes at $(10, 0)$ and $(0, 5)$.



(a) Write down the value of

(i) $f(4)$;

(ii) $f \circ f(4)$;

(iii) $f^{-1}(3)$. [3]

(b) On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes. [2]

(This question continues on the following page)

(Question 1 continued)

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

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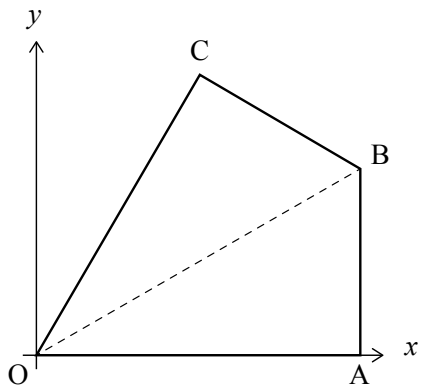
Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

Quadrilateral OABC is shown on the following set of axes.



OABC is symmetrical about $[\mathrm{OB}]$.

A has coordinates $(6, 0)$ and C has coordinates $\left(3, 3\sqrt{3}\right)$.

(a) (i) Write down the coordinates of the midpoint of $[\mathrm{AC}]$.

(ii) Hence or otherwise, find the equation of the line passing through the points O and B. [4]

(b) Given that $[\mathrm{OA}]$ is perpendicular to $[\mathrm{AB}]$, find the area of the quadrilateral OABC. [3]

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Question 4 - working space

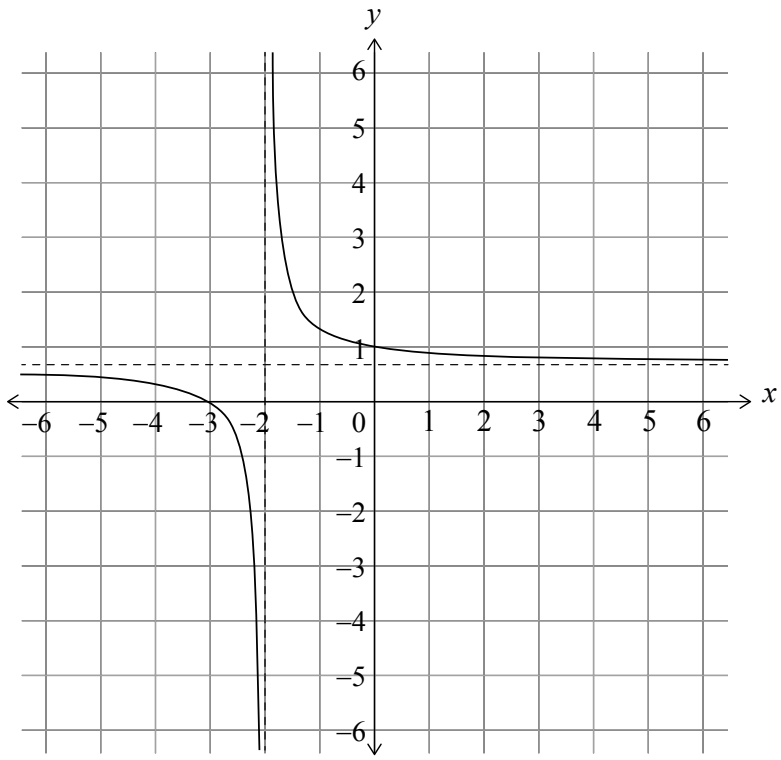
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 8]

A function f is defined by $f(x)=\frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y=f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x)=mx+1$, where $m \in \mathbb{R}, m \neq 0$.

(b) (i) Write down the number of solutions to $f(x)=g(x)$ for $m>0$.

(ii) Determine the value of m such that $f(x)=g(x)$ has only one solution for x.

(iii) Determine the range of values for m, where $f(x)=g(x)$ has two solutions for $x \geq 0$. [7]

(This question continues on the following page)

(Question 6 continued)

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Question 6 - working space

Section B

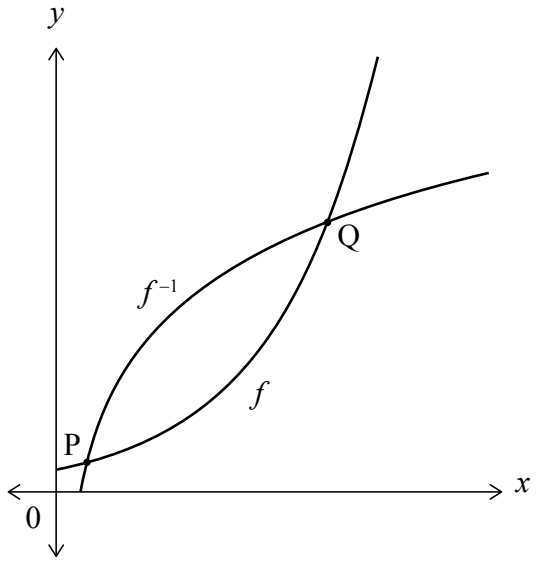
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Consider the function defined by $f(x) = \frac{3}{2}e^{x-2}$, $0 \le x \le 4$.

(a) Show that the inverse function is given by $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$. [3]

The graphs of f and f^{-1} intersect at two points P and Q, as shown on the following diagram.



(b) Find PQ. [3]

The graph of f is reflected in the x-axis and then translated parallel to the y-axis by 5 units in the positive direction to give the graph of a function g.

(c) Write down

(i) an expression for $g(x)$;

(ii) the domain of g. [3]

(d) Solve the equation $f(x) = g(x)$. Give your answer in the form $x = a + \ln b$, where $a, b \in \mathbb{Q}$. [3]

Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

The loudness of a sound, L, measured in decibels, is related to its intensity, I units, by $L = 10 \log_{10}(I \times 10^{12})$.

Consider two sounds, S_1 and S_2.

S_1 has an intensity of 10^{-6} units and a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1.

(a) State the intensity of S_2. [1]

(b) Determine the loudness of S_2. [2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I, of the thunder. [3]

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(i) $f(2)$;

(ii) $f\left(\frac{1}{8}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

Do **not** write solutions on this page.

Question 8 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\frac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = 7\sqrt{x} - x - 7$, where $0 \le x \le 60$.

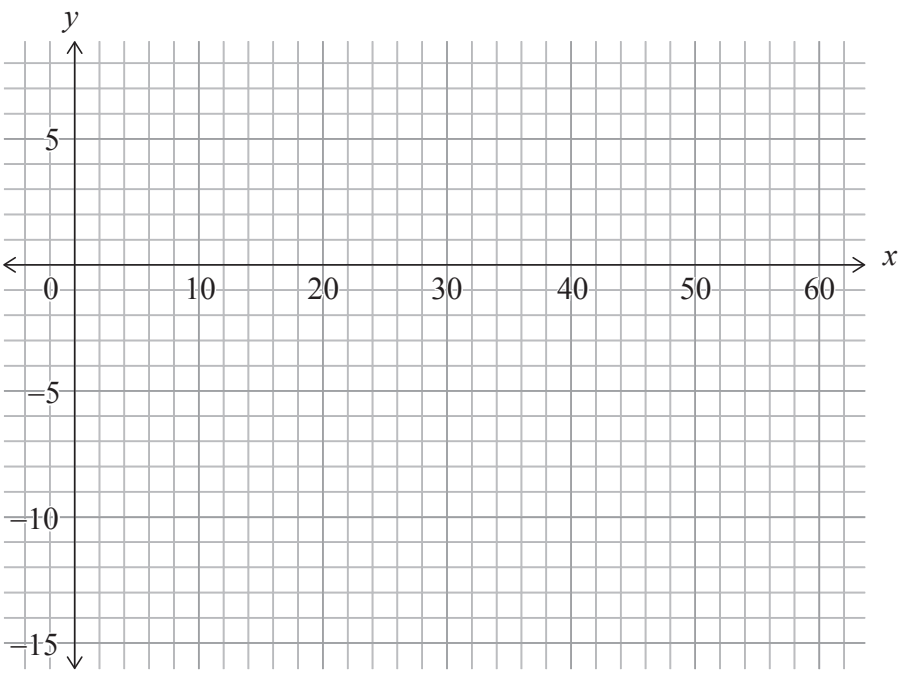
(a) Find the value of

(i) $f(0)$;

(ii) $f(60)$. [2]

(b) Find the two roots of $f(x) = 0$. [2]

(c) Sketch the graph of $y = f(x)$ on the following grid. [3]



(This question continues on the following page)

(Question 1 continued)

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

Consider the function $h(x) = \log_{10}(3x^2 - rx + r - 2)$, where $x \in \mathbb{R}$.

Find the possible values of r. [5]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = 11\sqrt{x} - 2x - 11$, where $0 \le x \le 20$.

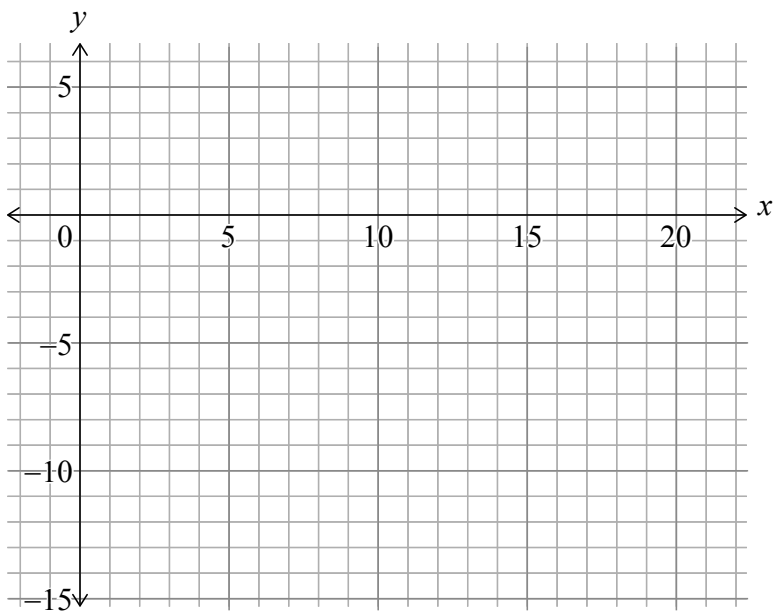
(a) Find the value of

(i) $f(0)$;

(ii) $f(20)$. [2]

(b) Find the two roots of $f(x) = 0$. [2]

(c) Sketch the graph of $y = f(x)$ on the following grid. [3]



(This question continues on the following page)

(Question 1 continued)

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

Consider the function $h(x) = \log_{10}(4x^2 - rx + r - 1)$, where $x \in \mathbb{R}$.

Find the possible values of r. [5]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Write each of the following expressions in the form $\ln k$, where $k \in \mathbb{Z}^+$.

(a) $\ln 3 + \ln 4$ [1]

(b) $3\ln 2$ [2]

(c) $-\ln\frac{1}{2}$ [2]

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Question 1 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The function f is defined by $f(x) = 5(x+1)(x+3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]

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Question 8 - working space

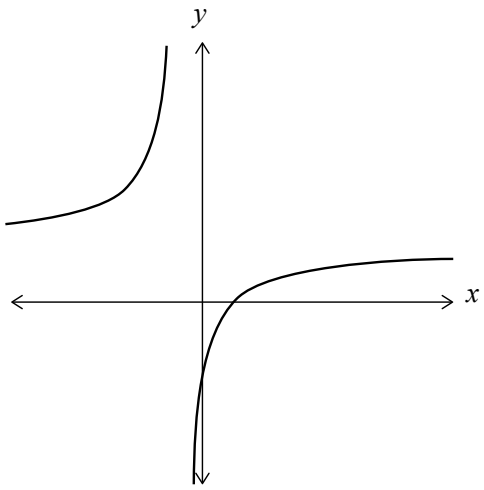
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The function f is defined by $f(x)=\frac{3x-2}{2x+1}$ for $x \in \mathbb{R}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y=f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x)=-f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

The quadratic equation $x^2 + kx + 15 - k = 0$ has two distinct real roots.

(a) Find the possible values of k. [5]

(b) Find the possible values of k in the case where the two distinct real roots are both positive or both negative. [2]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

(a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

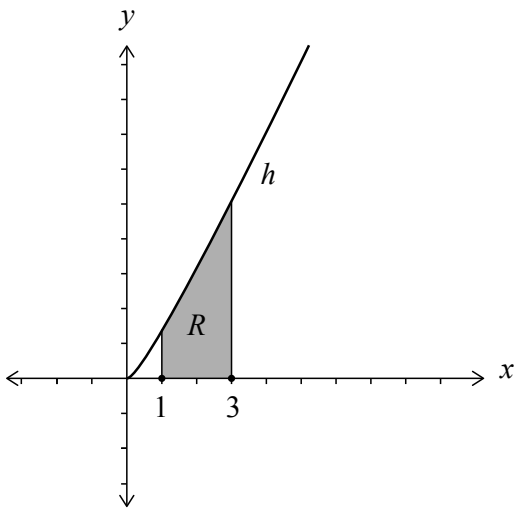
(b) (i) Find an expression for $g^{-1}(x)$.

(ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]

(c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h. Let R be the region enclosed by the graph of h and the x-axis, between the lines $x = 1$ and $x = 3$.



(d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.

(ii) Hence or otherwise, find the area of R, giving your answer in the form $p + q \ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Let $\log_{10} 2 = p$ and $\log_{10} 3 = q$.

(a) Find an expression for $\log_{10} 24$ in terms of p and q. [3]

(b) Find an expression for $\log_3 8$ in terms of p and q. [2]

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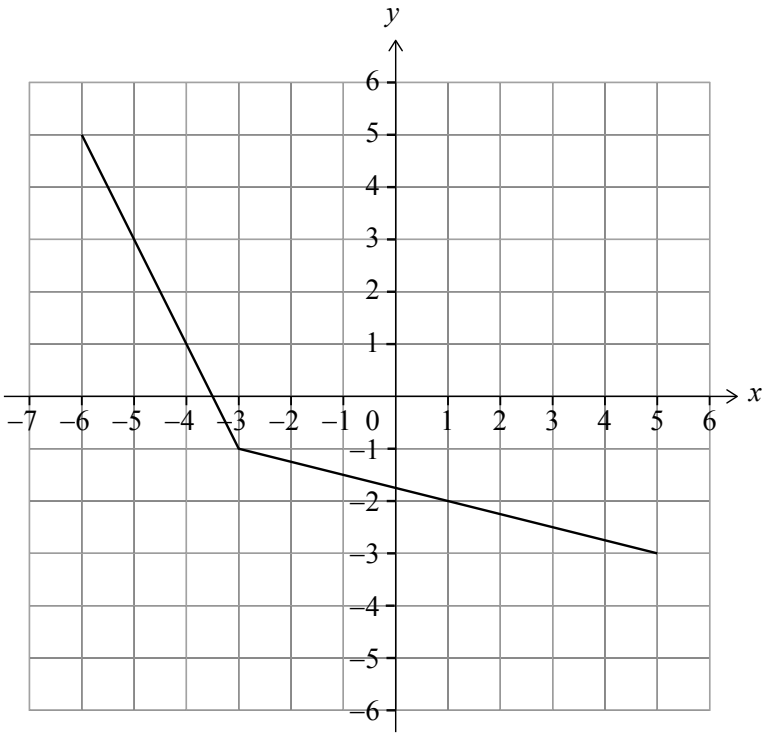
Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$, for $-6 \leq x \leq 5$.



(a) Write down the value of $f(-3)$. [1]

(b) State the domain of f^{-1}, the inverse function of f. [1]

(c) Find the value of x that satisfies $f^{-1}(2x - 7) = -3$. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

Consider the curve $y = x^2 - x - 1$ and the line $y = mx - 3$, where $m \in \mathbb{R}$.

(a) Show that the curve and the line meet when $x^2 - (m+1)x + 2 = 0$. [2]

(b) Hence, find the values of m when the line is tangent to the curve. [5]

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Question 5 - working space

Section B

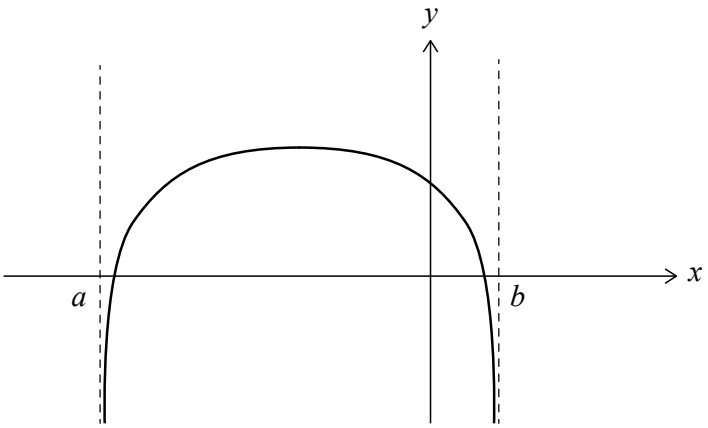
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

(a) (i) Solve $5 - 4x - x^2 = 0$.

(ii) Hence, find the values of x such that $5 - 4x - x^2 > 0$. [4]

Consider the function $f(x) = \log_k(5 - 4x - x^2)$, where $a < x < b$ and $k > 1$.
Part of the graph of f is shown in the following diagram.



The graph of f has vertical asymptotes at $x = a$ and $x = b$.

(b) Write down the value of

(i) a;

(ii) b. [2]

(c) Find the exact values of x such that $f(x) = 0$. [4]

The graph of f has a maximum value of 2.

(d) Find the value of k. [6]

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Question 9 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R}$, $x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $\mathrm{P}\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0, 0)$.

(b) (i) Find the value of k.

(ii) Hence, write down the coordinates of P. [4]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$, and $c, d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d, write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d. [2]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]

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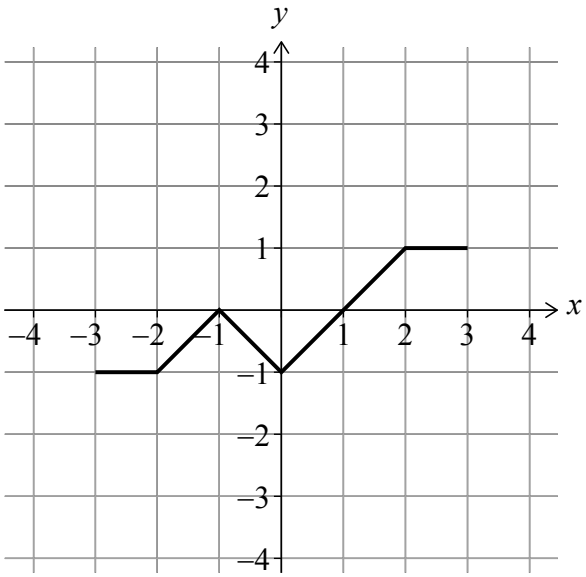
Question 9 - working space

Section A

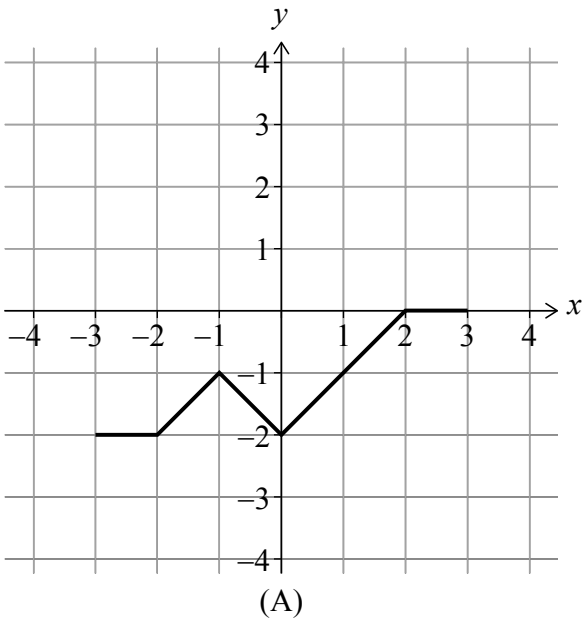
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

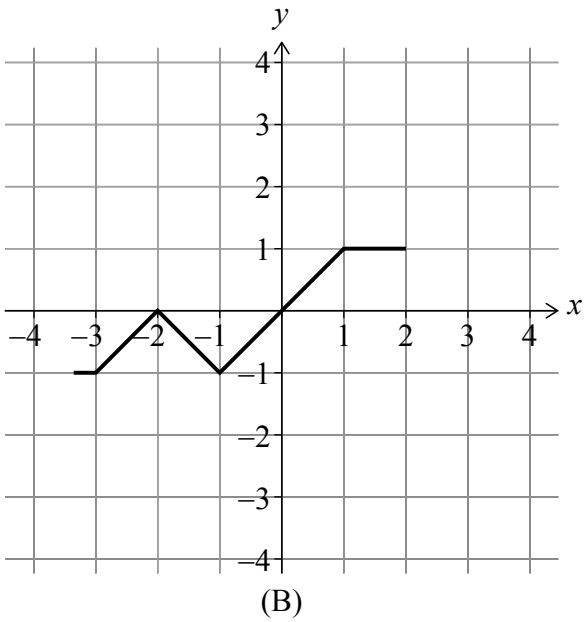
The graph of a function f for $-3 \leq x \leq 3$ is shown in the following diagram.



The following graphs are transformations of the graph of $y = f(x)$.



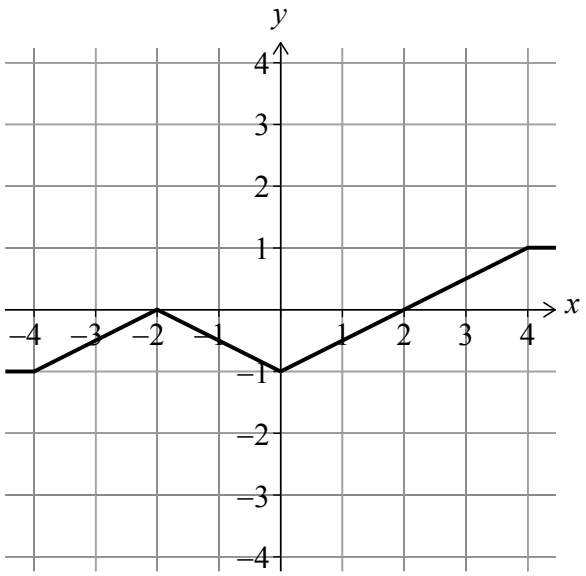
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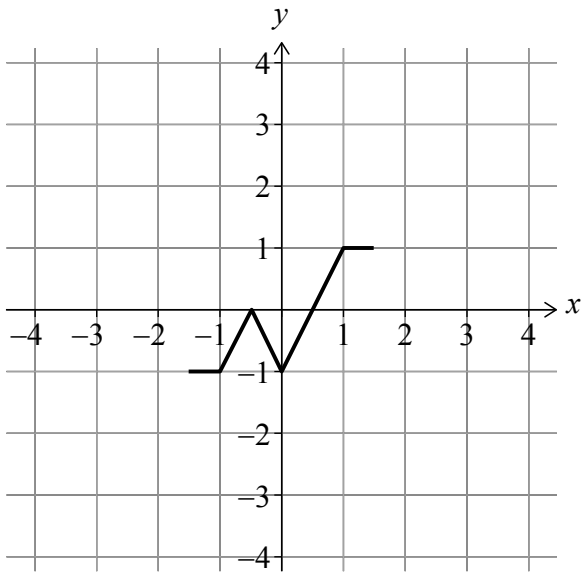
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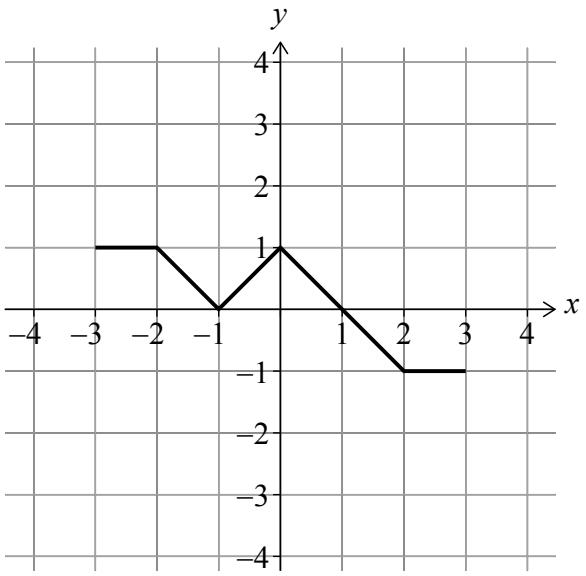
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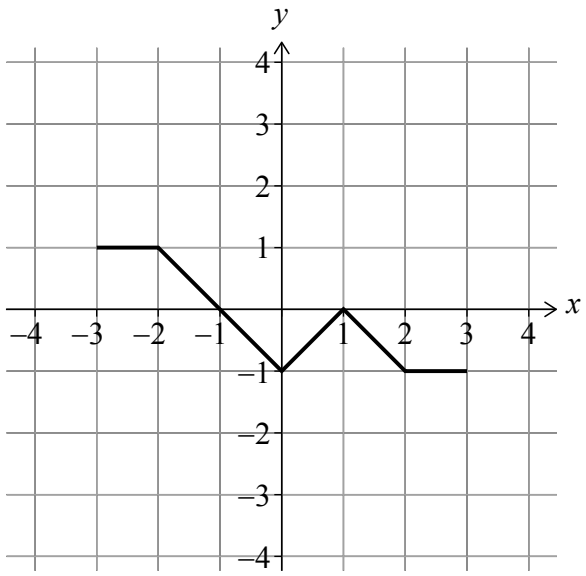
(C)



(D)



(E)



(F)

Consider the following table of transformations of $y = f(x)$.

Next to each transformed function, write down the letter that corresponds to its graph.

Function	Letter
$y = f(x + 1)$	
$y = f(x) - 1$	
$y = f(-x)$	
$y = f(2x)$	

Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

(a) Given that $x > 7$, show that $\frac{x}{x^2-8x+7} \times \frac{x^2-1}{x+1} = \frac{x}{x-7}$. [2]

(b) Hence, or otherwise, solve $\log_2[x(x^2-1)] - 1 = \log_2[(x^2-8x+7)(x+1)]$. [4]

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 8]

For $x \in \mathbb{R}$, the functions f and g are defined by

$$f(x) = -x^2 + 4x + p \text{ and}$$

$$g(x) = x^2 + qx - 1,$$

where p and q are non-zero constants.

The graph of $y = f(x)$ has a vertex at A and the graph of $y = g(x)$ has a vertex at B.

Points A and B both lie on the line $y = 2x - 1$.

(a) Show that $p = -1$. [3]

(b) Find the value of q. [5]

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Consider the function $f(x) = 2x + 3$ where $x \in \mathbb{R}$.

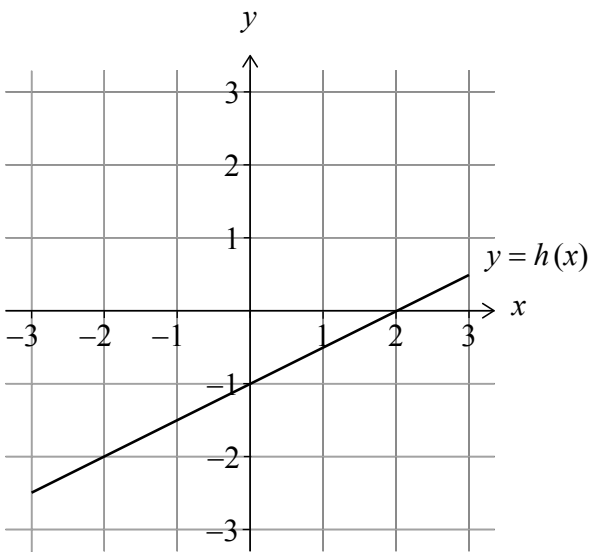
(a) Find

(i) $f(2)$;

(ii) $f^{-1}(x)$. [3]

The following diagram shows part of the graph of a linear function h.

The graph has x-intercept 2 and y-intercept -1.



(b) Write down an expression for $h(x)$. [2]

(c) Solve the equation $h^{-1}(x) = -2$. [2]

(This question continues on the following page)

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(Question 7 continued)

Consider the function $g(x) = mx + c$, where $x \in \mathbb{R}$ and $m, c \in \mathbb{Q}$.

(d) Given that $h(x) = (f^{-1} \circ g)(x)$, find the value of m and the value of c. [4]

A function k exists such that $h\big(k(x)\big) = x$.

(e) Find an expression for $k(x)$. [2]

(f) State the single transformation that maps the graph of $y = k(x)$ onto the graph of $y = h(x)$. [1]

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Question 7 - working space

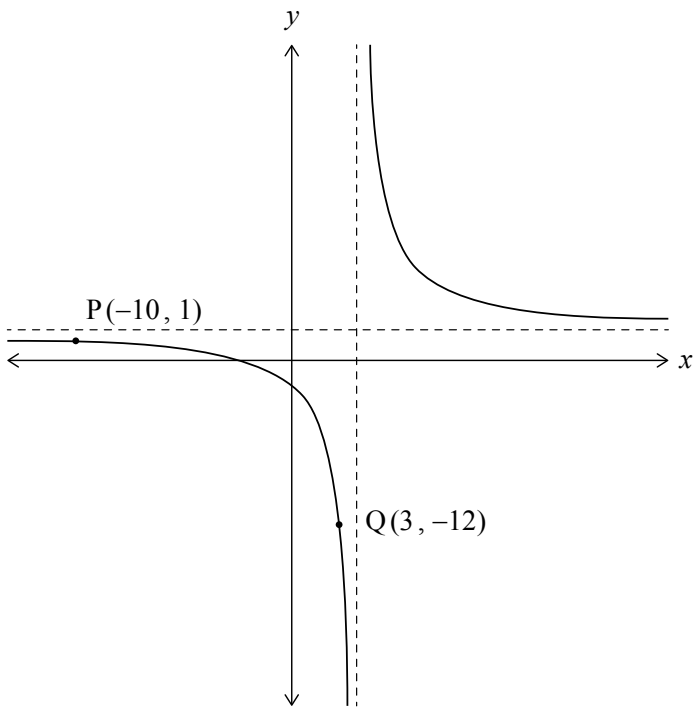
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 8]

The following diagram shows the graph of $y = \frac{Ax + B}{x - 4}$, where $x \in \mathbb{R}, x \neq 4$ and $A, B \in \mathbb{Z}$.

The graph passes through the points $\mathrm{P}(-10, 1)$ and $\mathrm{Q}(3, -12)$.



(a) Determine the value of A and the value of B. [3]

(b) Describe a sequence of transformations that would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{Ax + B}{x - 4}$. [5]

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

Solve the equation $3\log_8 10x - \log_4 x = 1$ for $x > 0$.

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Question 6 - working space

APOLLO

Geometry and trigonometry

72 questions

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 8]

(a) Show that $\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$. [2]

(b) Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$. [6]

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Question 6 - working space

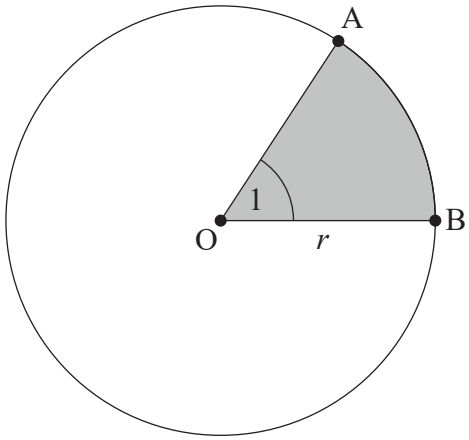
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius r.

diagram not to scale



Points A and B lie on the circumference of the circle, and $\mathrm{A\hat{O}B} = 1$ radian.

The perimeter of the shaded region is 12.

(a) Find the value of r. [3]

(b) Hence, find the exact area of the **non-shaded** region. [3]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

(a) Show that the equation $2\cos^2 x + 5\sin x = 4$ may be written in the form $2\sin^2 x - 5\sin x + 2 = 0$. [1]

(b) Hence, solve the equation $2\cos^2 x + 5\sin x = 4,\ 0 \le x \le 2\pi$. [5]

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Question 3 - working space

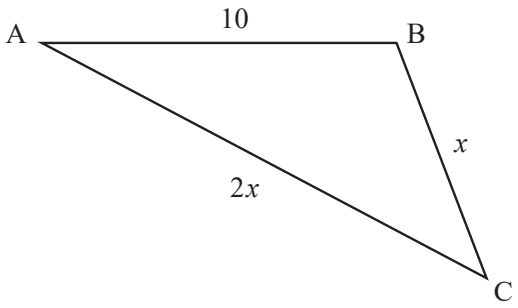
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

The following diagram shows triangle ABC, with $\mathrm{AB} = 10$, $\mathrm{BC} = x$ and $\mathrm{AC} = 2x$.

diagram not to scale



Given that $\cos \hat{\mathrm{C}} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

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Question 6 - working space

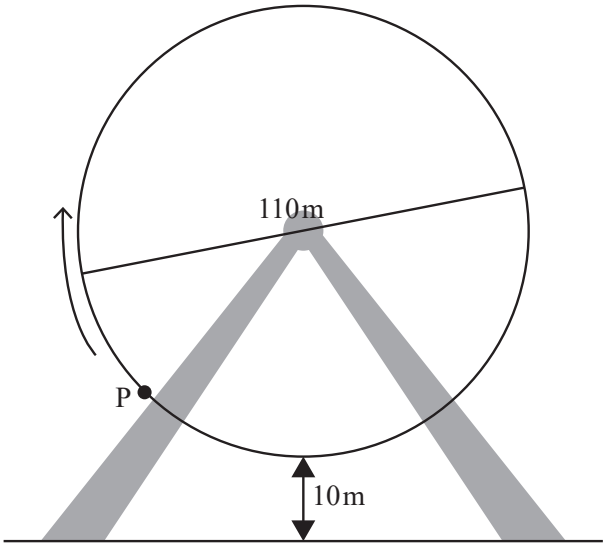
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes.

diagram not to scale



The height, h metres, of P above the ground after t minutes is given by $h(t) = a\cos(bt) + c$, where $a, b, c \in \mathbb{R}$.

Find the values of a, b and c.

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Question 4 - working space

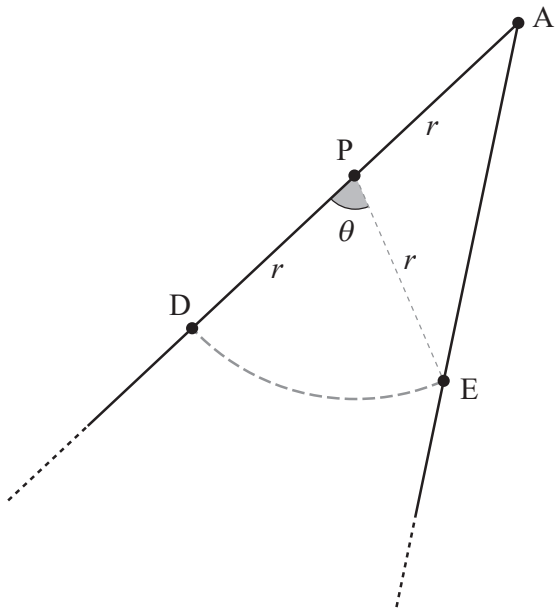
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

Two straight fences meet at point A and a field lies between them.

A horse is tied to a post, P, by a rope of length r metres. Point D is on one fence and point E is on the other, such that $\mathrm{PD} = \mathrm{PE} = \mathrm{PA} = r$ and $\mathrm{D\hat{P}E} = \theta$ radians. This is shown in the following diagram.



The length of the arc DE shown in the diagram is $28\,\mathrm{m}$.

(a) Write down an expression for r in terms of θ. [1]

(b) Show that the area of the field that the horse can reach is $\frac{392}{\theta^2}(\theta + \sin\theta)$. [4]

(c) The area of field that the horse can reach is $460\,\mathrm{m}^2$. Find the value of θ. [2]

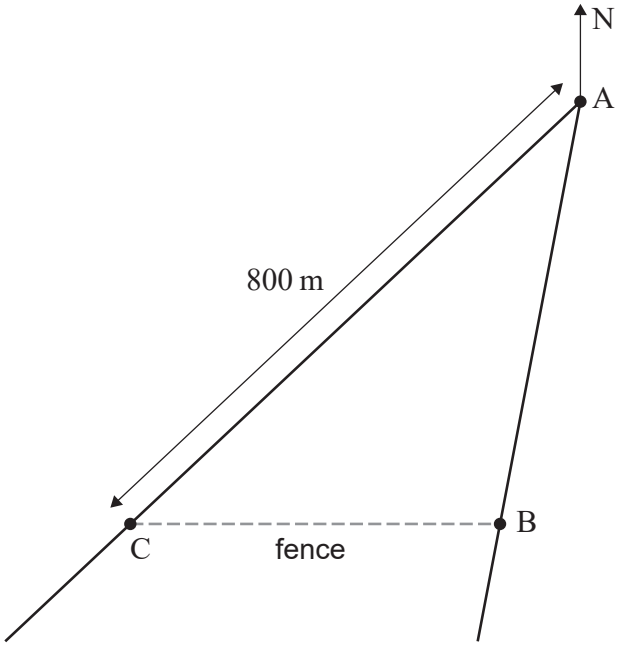
(d) Hence, find the size of $\mathrm{D\hat{A}E}$. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

A new fence is to be constructed between points B and C which will enclose the field, as shown in the following diagram.



Point C is due west of B and $AC = 800\,\text{m}$. The bearing of B from A is $195°$.

(e) (i) Find the size of $A\hat{B}C$.

(ii) Find the length of new fence required. [5]

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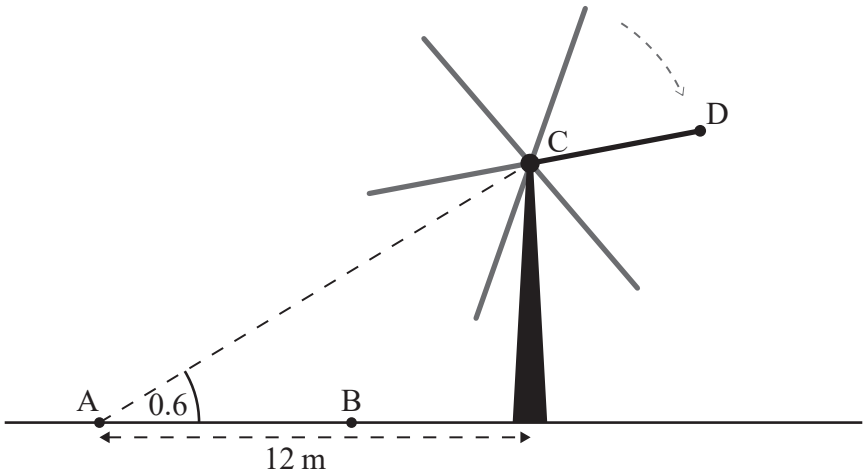
Question 8 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

(a) Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. [2]

An observer walks 7 metres from point A to point B.

(b) Find the angle of elevation of point C from point B. [2]

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

(c) Find the length of each blade of the windmill. [2]

One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height h, in metres, of point D above the ground can be modelled by the function $h(t) = p\cos\left(\frac{3\pi}{10}t\right) + q$, where t is in seconds and $p, q \in \mathbb{R}$. When $t = 0$, point D is at its maximum height.

(d) Find the value of p and the value of q. [4]

If the observer stands directly under point C for one minute, point D will pass over his head n times.

(e) Find the value of n. [3]

Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

(a) Show that $2x-3-\frac{6}{x-1}=\frac{2x^2-5x-3}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$. [2]

(b) Hence or otherwise, solve the equation $2\sin 2\theta - 3 - \frac{6}{\sin 2\theta - 1} = 0$ for $0 \leq \theta \leq \pi$, $\theta \neq \frac{\pi}{4}$. [5]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider a triangle ABC, where $AC = 12$, $CB = 7$ and $B\hat{A}C = 25°$.

Find the smallest possible perimeter of triangle ABC.

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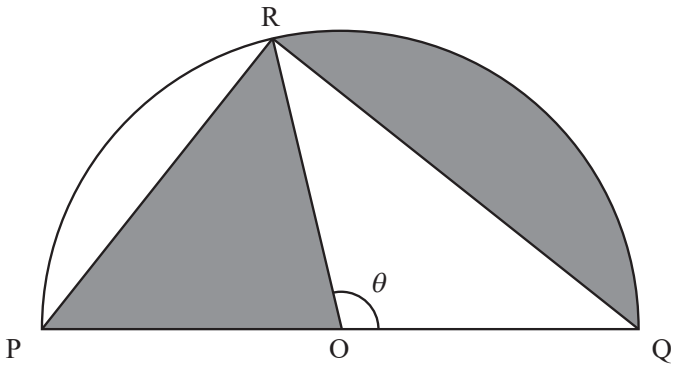
Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

The following diagram shows a semicircle with centre O and radius r. Points P, Q and R lie on the circumference of the circle, such that $\mathrm{PQ} = 2r$ and $\mathrm{R\hat{O}Q} = \theta$, where $0 < \theta < \pi$.



(a) Given that the areas of the two shaded regions are equal, show that $\theta = 2\sin\theta$. [5]

(b) Hence determine the value of θ. [1]

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Question 5 - working space

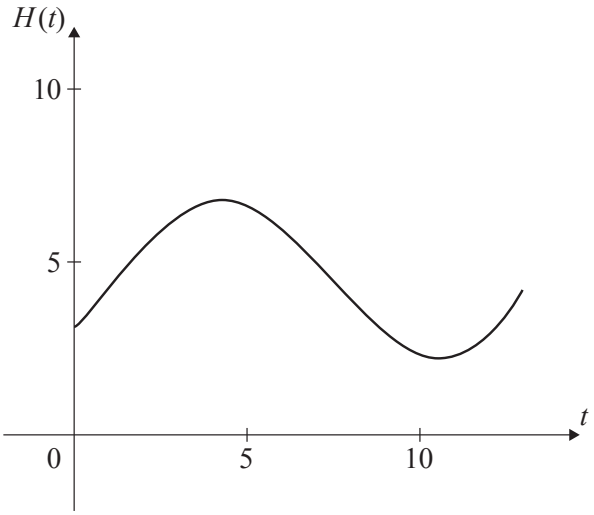
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin(b(t-c)) + d$, where t is the number of hours after midnight, and a, b, c and d are constants, where $a > 0$, $b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at 04:30 and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\,\text{m}$ and $6.8\,\text{m}$.

All heights are given correct to one decimal place.

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Find the value of a. [2]

(c) Find the value of d. [2]

(d) Find the smallest possible value of c. [3]

(e) Find the height of the water at 12:00. [2]

(f) Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. [3]

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Question 8 - working space

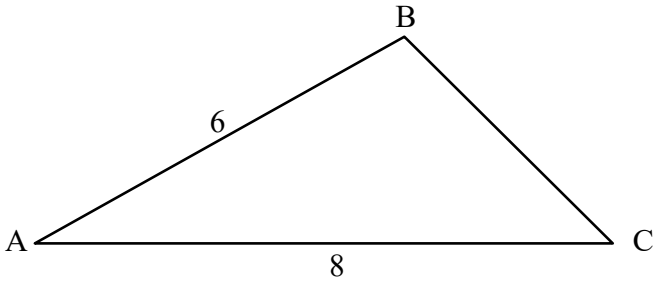
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

diagram not to scale



(a) Given that $\cos \hat{A} = \frac{5}{6}$, find the value of $\sin \hat{A}$. [3]

(b) Find the area of triangle ABC. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 8]

(a) Show that $\log_9(\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$. [3]

(b) Hence or otherwise solve $\log_3(2\sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. [5]

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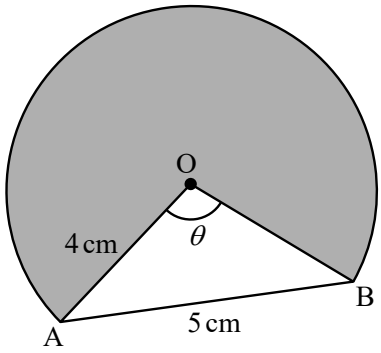
Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius $4\,\mathrm{cm}$.



Chord AB has a length of $5\,\mathrm{cm}$ and $\mathrm{A\hat{O}B} = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

(b) Find the area of the shaded region. [3]

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Question 2 - working space

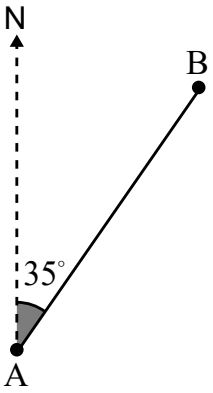
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

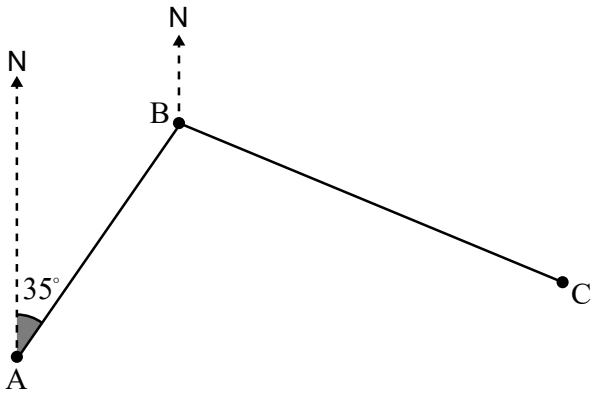
7. [Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of $4.2\,\mathrm{km/h}$ for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a) Find the distance from point A to point B. [2]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of $4.6\,\mathrm{km}$ until he reaches point C.



(b) (i) Show that $\mathrm{A\hat{B}C}$ is 101°.

(ii) Find the distance from the camp to point C. [5]

(c) Find $\mathrm{B\hat{C}A}$. [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C.

(d) Find the bearing that Jacob must take to point C. [3]

Jacob hikes at an average speed of $3.9\,\mathrm{km/h}$.

(e) Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Do **not** write solutions on this page.

Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 13]

Consider a function f, such that $f(x) = 5{,}8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

(a) Find the period of f. [2]

The function f has a local maximum at the point $(2, 21.8)$, and a local minimum at $(8, 10.2)$.

(b) (i) Find the value of b.

(ii) Hence, find the value of $f(6)$. [4]

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q$, $0 \le x \le 10$; $p, q \in \mathbb{R}$.

The function g passes through the points $(3, 2.5)$ and $(6, 15.1)$.

(c) Find the value of p and the value of q. [5]

(d) Find the value of x for which the functions have the greatest difference. [2]

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will not be marked.



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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3}\sin x + \cos x$ where $0 \le x \le \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

(a) Find $(f \circ g)(x)$. [2]

(b) Solve the equation $(f \circ g)(x) = 2\cos 2x$ where $0 \le x \le \pi$. [5]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

Find the least positive value of x for which $\cos\left(\frac{x}{2}+\frac{\pi}{3}\right)=\frac{1}{\sqrt{2}}$.

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Question 5 - working space

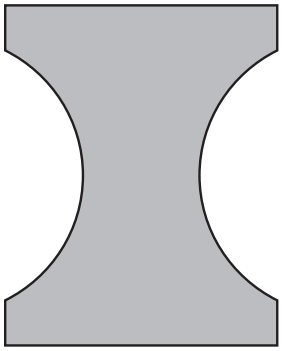
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

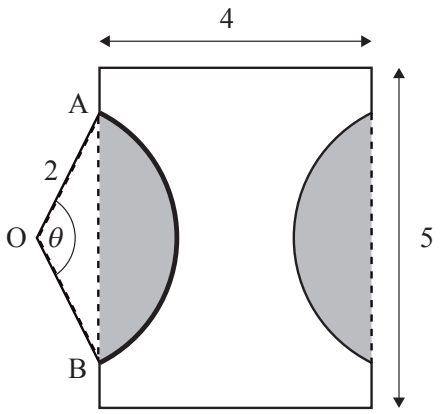
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of θ. [3]

(b) Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ. [3]

(This question continues on the following page)

(Question 3 continued)

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Question 3 - working space

Section B

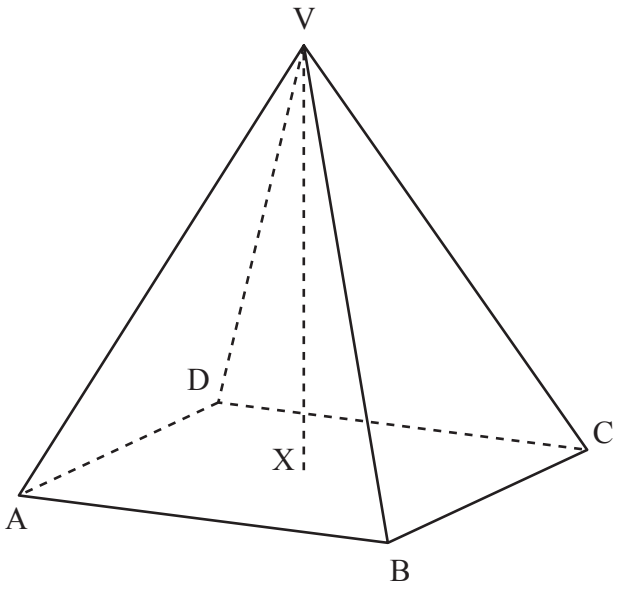
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

All lengths in this question are in centimetres.

A solid metal ornament is in the shape of a right pyramid, with vertex V and square base ABCD. The centre of the base is X. Point V has coordinates $(1, 5, 0)$ and point A has coordinates $(-1, 1, 6)$.

diagram not to scale



(a) Find AV. [2]

(b) Given that $\mathrm{A\hat{V}B} = 40^\circ$, find AB. [3]

The volume of the pyramid is $57.2\,\mathrm{cm}^3$, correct to three significant figures.

(c) Find the height of the pyramid, VX. [3]

A second ornament is in the shape of a cuboid with a rectangular base of length $2x\,\mathrm{cm}$, width $x\,\mathrm{cm}$ and height $y\,\mathrm{cm}$. The cuboid has the same volume as the pyramid.

(d) The cuboid has a minimum surface area of $S\,\mathrm{cm}^2$. Find the value of S. [5]

Do **not** write solutions on this page.

Question 7 - working space

Section A

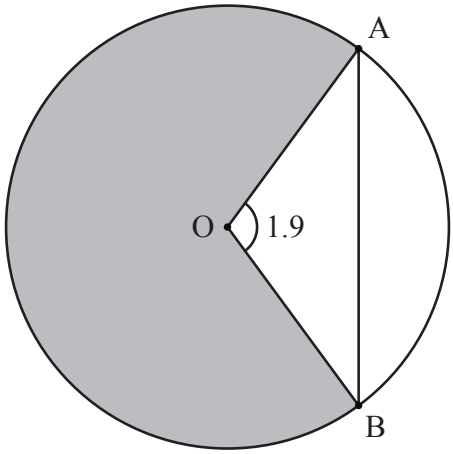
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $A\hat{O}B = 1.9$ radians.

diagram not to scale



(a) Find the length of the chord $[AB]$. [3]

(b) Find the area of the shaded sector. [3]

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Question 1 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

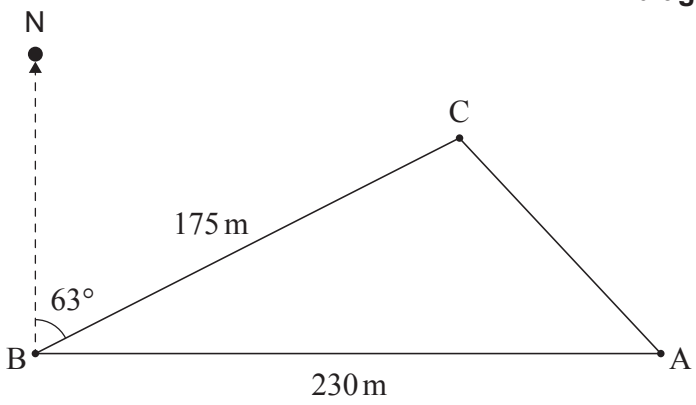
7. [Maximum mark: 14]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of $063°$ to reach point C.

This is shown in the following diagram.

diagram not to scale



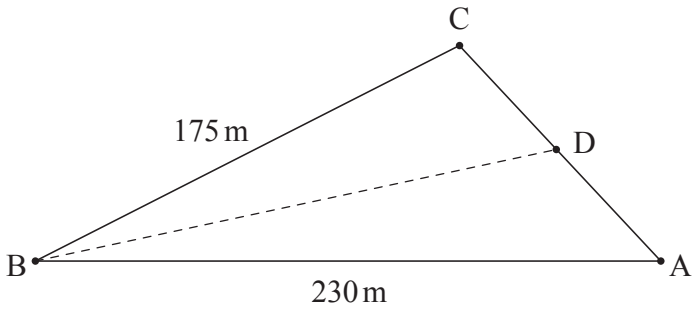
(a) Find the distance from point A to point C. [4]

(b) Find the area of this piece of land. [2]

(c) Find $C\hat{A}B$. [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale



(d) Find the distance from point B to point D. [5]

Turn over

Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 7]

Consider a circle with a diameter AB, where A has coordinates $(1, 4, 0)$ and B has coordinates $(-3, 2, -4)$.

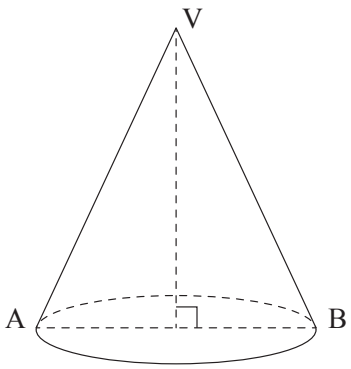
(a) Find

(i) the coordinates of the centre of the circle;

(ii) the radius of the circle. [4]

The circle forms the base of a right cone whose vertex V has coordinates $(-1, -1, 0)$.

diagram not to scale



(b) Find the exact volume of the cone. [3]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

The function f is defined by $f(x) = \cos^2 x - 3\sin^2 x,\ 0 \le x \le \pi$.

(a) Find the roots of the equation $f(x) = 0$. [5]

(b) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]

(c) Sketch the graph of $y = f(x)$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [3]

References:

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Question 9 - working space

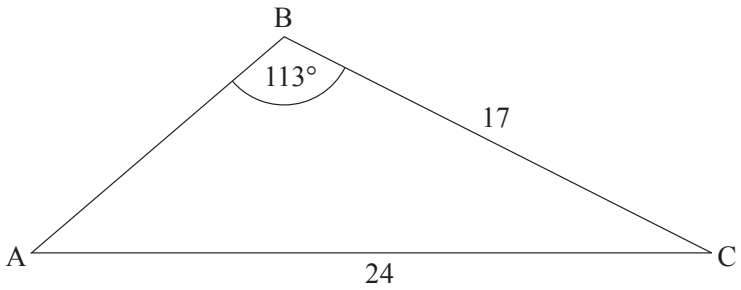
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows triangle ABC, with $AC = 24$, $BC = 17$, and $A\hat{B}C = 113°$.

diagram not to scale



(a) Find $B\hat{A}C$. [3]

(b) Find AB. [3]

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Question 2 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

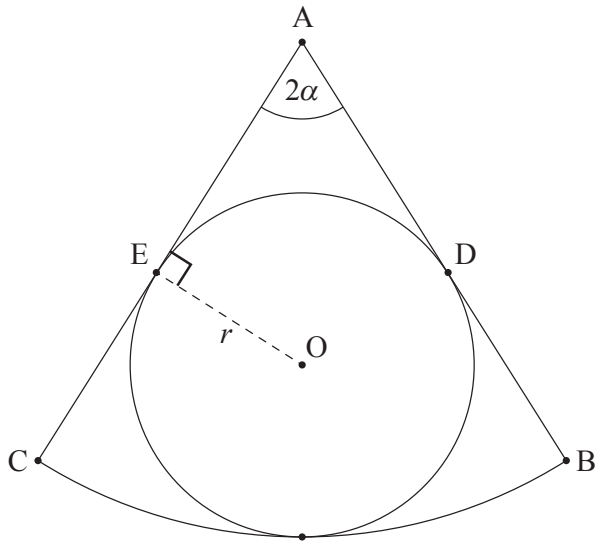
8. [Maximum mark: 13]

The following diagram shows a sector ABC of a circle with centre A. The angle $\mathrm{B\hat{A}C} = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $\mathrm{O\hat{E}A} = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale



(a) Show that the area of the quadrilateral ADOE is $\frac{r^2}{\tan\alpha}$. [4]

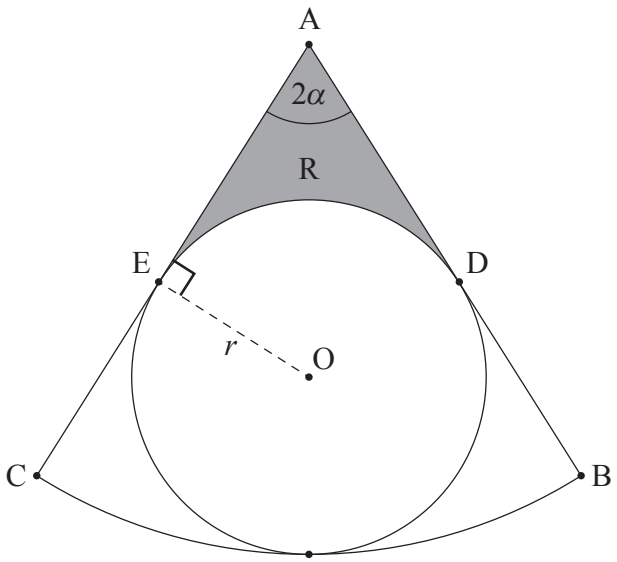
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(Question 8 continued)

R represents the shaded region shown in the following diagram.

diagram not to scale



(b) (i) Find $\mathrm{D\hat{O}E}$ in terms of α.

(ii) Hence or otherwise, find an expression for the area of R. [5]

(c) Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r. [4]

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Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

(a) Show that the equation $\cos 2x = \sin x$ can be written in the form $2\sin^2 x + \sin x - 1 = 0$. [1]

(b) Hence, solve $\cos 2x = \sin x$, where $-\pi \leq x \leq \pi$. [5]

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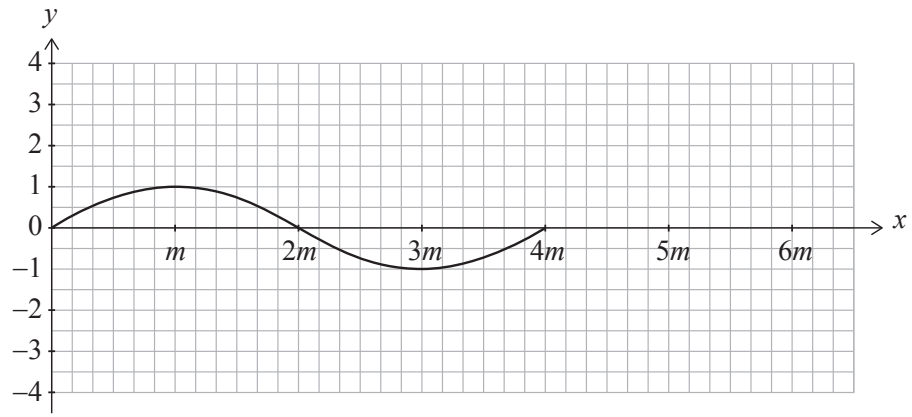
Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \le x \le 4m$, where x is in radians. There are x-intercepts at $x = 0$, $2m$ and $4m$.



(a) Find an expression for m in terms of q. [2]

The function g is defined by $g(x) = 3\sin\frac{2qx}{3}$, for $0 \le x \le 6m$.

(b) On the axes above, sketch the graph of g. [4]

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Question 6 - working space

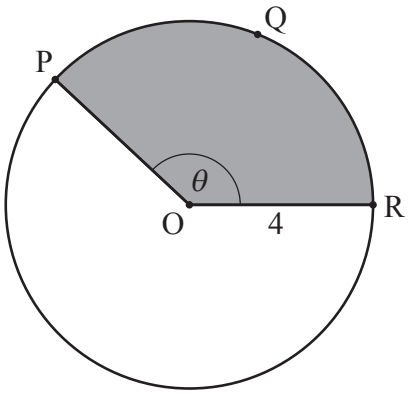
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius $4\,\mathrm{cm}$.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\mathrm{P\hat{O}R} = \theta$, where θ is measured in radians.

The length of arc PQR is $10\,\mathrm{cm}$.

(a) Find the perimeter of the shaded sector. [2]

(b) Find θ. [2]

(c) Find the area of the shaded sector. [2]

(This question continues on the following page)

(Question 1 continued)

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Question 1 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

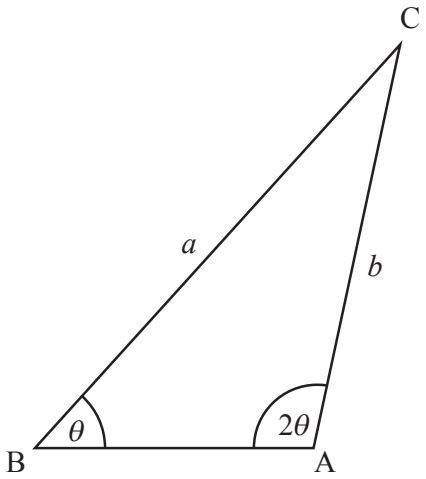
Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$.

(a) Find the value of

(i) $\sin\theta$;

(ii) $\sin 2\theta$. [4]

The following diagram shows triangle ABC, with $\hat{\mathrm{B}} = \theta$, $\hat{\mathrm{A}} = 2\theta$, $\mathrm{BC} = a$ and $\mathrm{AC} = b$.



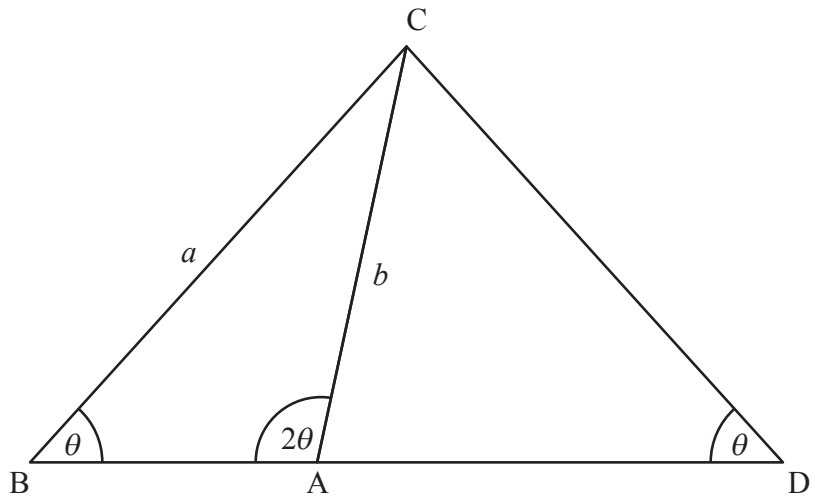
(b) Show that $b = \frac{3a}{4}$. [2]

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Do **not** write solutions on this page.

(Question 8 continued)

$[BA]$ is extended to form an isosceles triangle DAC, with $\hat{D} = \theta$, as shown in the following diagram.



(c) Find the value of $\sin C\hat{A}D$. [3]

(d) Find the area of triangle DAC, in terms of a. [5]

Do **not** write solutions on this page.

Question 8 - working space

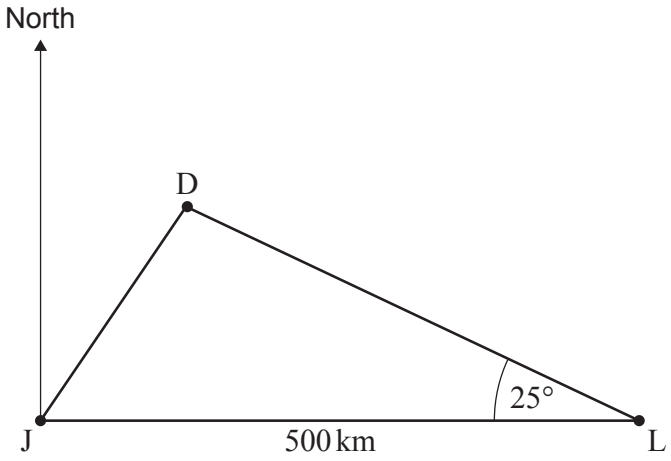
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The cities Lucknow (L), Jaipur (J) and Delhi (D) are represented in the following diagram. Lucknow lies $500\,\mathrm{km}$ directly east of Jaipur, and $\mathrm{J\hat{L}D} = 25°$.

diagram not to scale



The bearing of D from J is $034°$.

(a) Find $\mathrm{J\hat{D}L}$. [2]

(b) Find the distance between Lucknow and Delhi. [3]

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Question 1 - working space

Section B

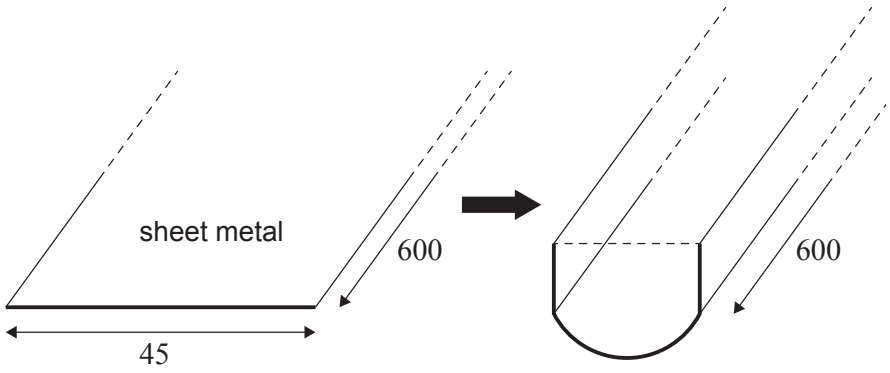
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

An engineer is designing a gutter to catch rainwater from the roof of a house.

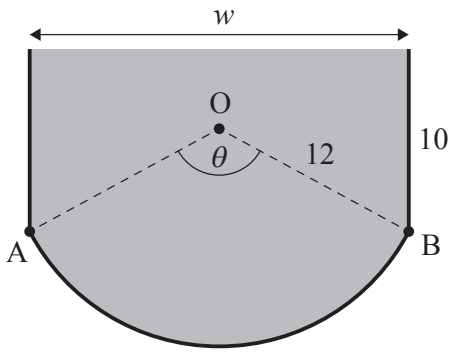
The gutter will be open at the top and is made by folding a piece of sheet metal $45\,\text{cm}$ wide and $600\,\text{cm}$ long.

diagram not to scale



The cross-section of the gutter is shaded in the following diagram.

diagram not to scale



The height of both vertical sides is $10\,\text{cm}$. The width of the gutter is $w\,\text{cm}$.

Arc AB lies on the circumference of a circle with centre O and radius $12\,\text{cm}$.

(This question continues on the following page)

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(Question 9 continued)

Let $\mathrm{A\hat{O}B} = \theta$ radians, where $0 < \theta < \pi$.

(a) Show that $\theta = 2.08$, correct to three significant figures. [3]

(b) Find the area of the cross-section of the gutter. [7]

In a storm, the total volume, in cm^3, of rainwater that enters the gutter can be modelled by a function $R(t)$, where t is the time, in seconds, since the start of the storm.

It was determined that the **rate** at which rainwater entered the gutter could be modelled by

$$R'(t) = 50\cos\left(\frac{2\pi t}{5}\right) + 3000\,,\ t \geq 0\,.$$

During any 60-second period, if the volume of rainwater entering the gutter is greater than the volume of the gutter, it will overflow.

(c) Determine whether the gutter overflowed in this storm. Justify your answer. [5]

References:

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Question 9 - working space

Section A

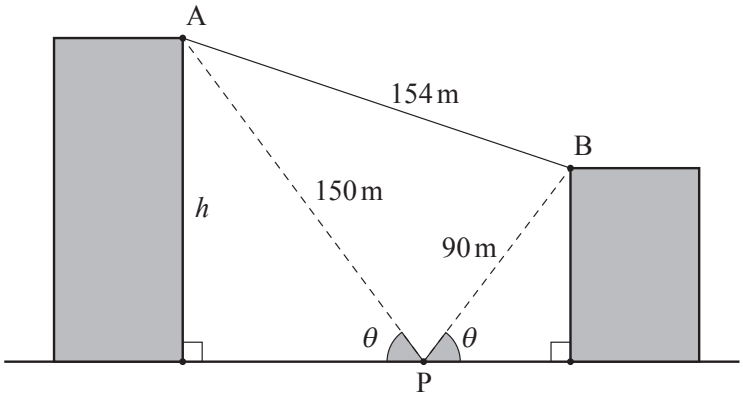
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows two buildings situated on level ground.

From point P on the ground directly between the two buildings, the angle of elevation to the top of each building is θ.

diagram not to scale



The distance from point P to point A at the top of the taller building is 150 metres.

The distance from point P to point B at the top of the shorter building is 90 metres.

The distance between A and B is 154 metres.

(a) Find the measure of $\mathrm{A\hat{P}B}$. [3]

(b) Find the height, h, of the taller building. [3]

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Question 2 - working space

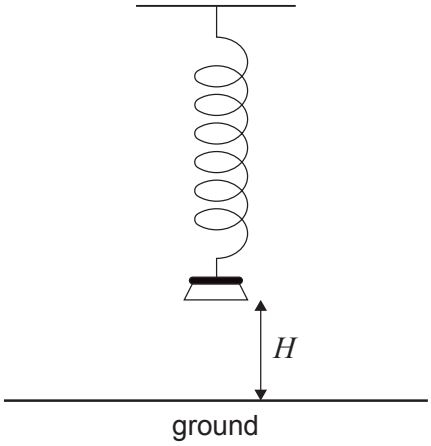
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

A weight suspended on a spring is pulled down and released, so that it moves up and down vertically.

The height, H metres, of the base of the weight above the ground can be modelled by the function $H(t) = a\cos(7.8t) + b$, for $a, b \in \mathbb{R}$ and $0 \leq t \leq 10$, where t is the time in seconds after the weight is released.



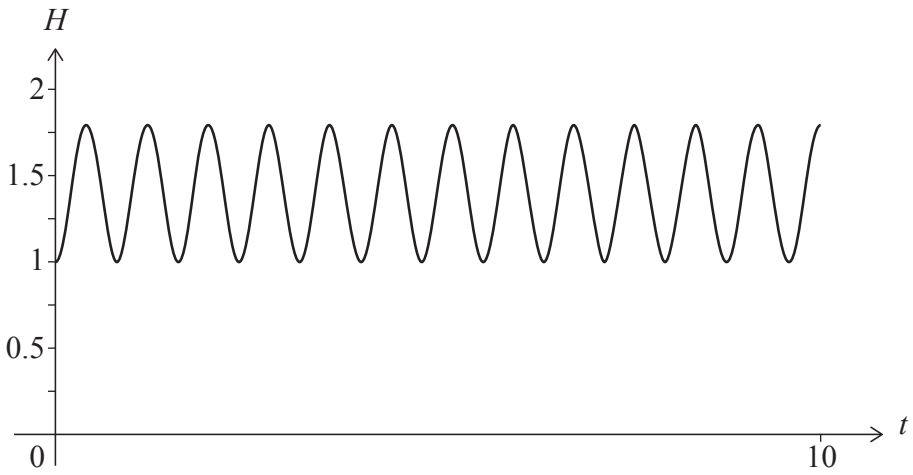
(a) Find the period of the function. [2]

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(Question 8 continued)

The weight is released when its base is at a minimum height of 1 metre above the ground, and it reaches a maximum height of 1.8 metres above the ground. The graph of H is shown in the following diagram.



(b) Find the value of

(i) a;

(ii) b. [3]

(c) Find the number of times that the weight reaches its maximum height in the first five seconds of its motion. [2]

(d) Find the first time that the base of the weight reaches a height of 1.5 metres. [2]

A camera is set to take a picture of the weight at a random time during the first five seconds of its motion.

(e) Find the probability that the height of the base of the weight is greater than 1.5 metres at the time the picture is taken. [4]

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Question 8 - working space

Section A

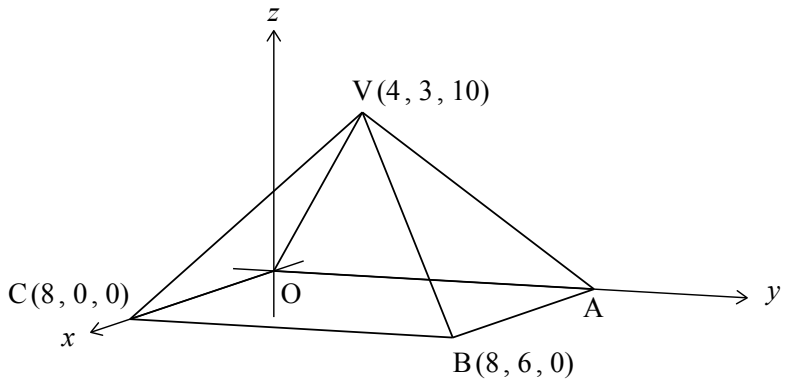
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows a pyramid with vertex V and rectangular base OABC.

Point B has coordinates $(8, 6, 0)$, point C has coordinates $(8, 0, 0)$ and point V has coordinates $(4, 3, 10)$.

diagram not to scale



(a) Find BV. [2]

(b) Find the size of $\mathrm{B\hat{V}C}$. [4]

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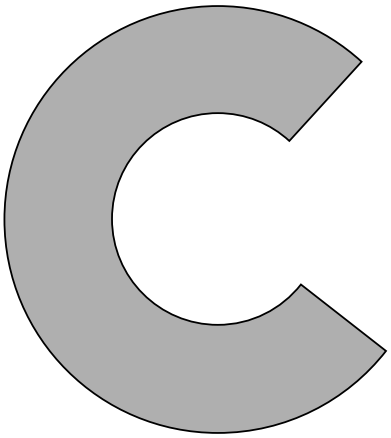
Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

A company is designing a new logo in the shape of a letter “C”.



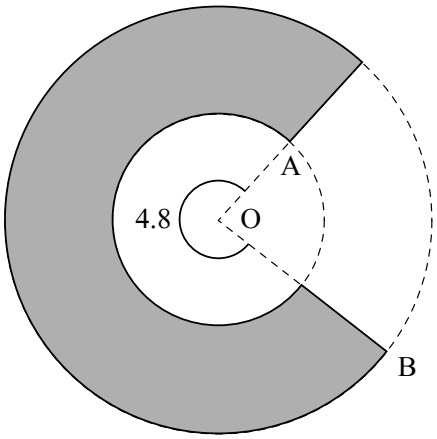
The letter “C” is formed between two circles with centre O.

The point A lies on the circumference of the inner circle with radius $r\,\mathrm{cm}$, where $r < 10$.

The point B lies on the circumference of the outer circle with radius $10\,\mathrm{cm}$.

The reflex angle $\mathrm{A\hat{O}B}$ is 4.8 radians. The letter “C” is shown by the shaded area in the following diagram.

diagram not to scale



(This question continues on the following page)

(Question 4 continued)

(a) Show that the area of the "C" is given by $240 - 2.4r^2$. [2]

The area of the "C" is $176\,\text{cm}^2$.

(b) (i) Find the value of r.

(ii) Find the perimeter of the "C". [5]

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Question 4 - working space

Section A

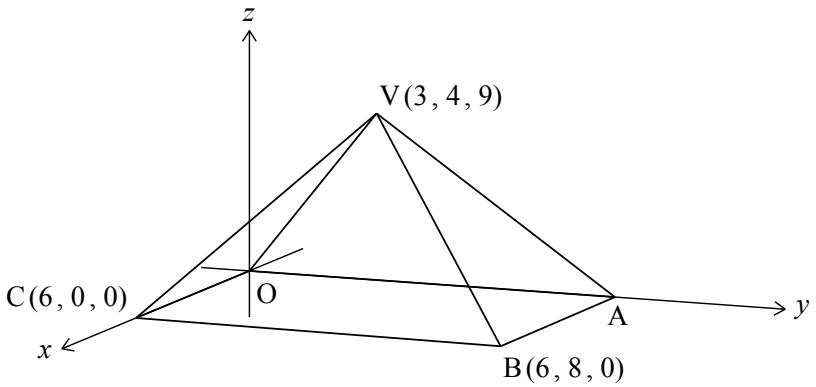
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows a pyramid with vertex V and rectangular base OABC.

Point B has coordinates $(6, 8, 0)$, point C has coordinates $(6, 0, 0)$ and point V has coordinates $(3, 4, 9)$.

diagram not to scale



(a) Find BV. [2]

(b) Find the size of $\mathrm{B\hat{V}C}$. [4]

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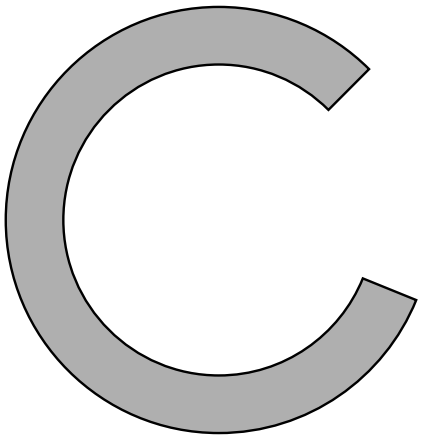
Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

A company is designing a new logo in the shape of a letter “C”.



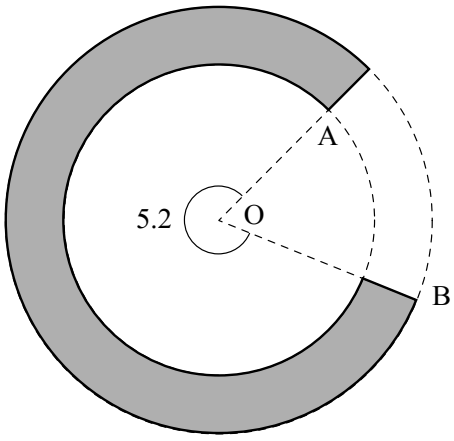
The letter “C” is formed between two circles with centre O.

The point A lies on the circumference of the inner circle with radius $r\,\mathrm{cm}$, where $r < 10$.

The point B lies on the circumference of the outer circle with radius $10\,\mathrm{cm}$.

The reflex angle $\mathrm{A\hat{O}B}$ is 5.2 radians. The letter “C” is shown by the shaded area in the following diagram.

diagram not to scale



(This question continues on the following page)

(Question 4 continued)

(a) Show that the area of the "C" is given by $260 - 2.6r^2$. [2]

The area of the "C" is $64\,\text{cm}^2$.

(b) (i) Find the value of r.

(ii) Find the perimeter of the "C". [5]

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Question 4 - working space

Section A

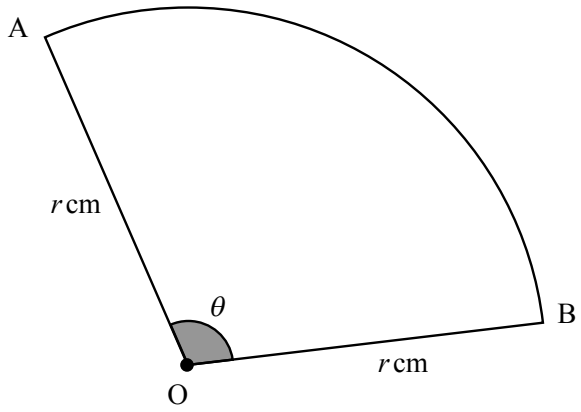
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius $r\,\text{cm}$ with centre at O.

The sector OAB is shown on the following diagram. The angle $A\hat{O}B$ is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is $10\,\text{cm}$ and the area of the sector is $6.25\,\text{cm}^2$.

(a) Show that $4r^2 - 20r + 25 = 0$. [4]

(b) Hence, or otherwise, find the value of r and the value of θ. [4]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark:4]

Solve $\tan(2x - 5°) = 1$ for $0° \leq x \leq 180°$.

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Question 2 - working space

Section B

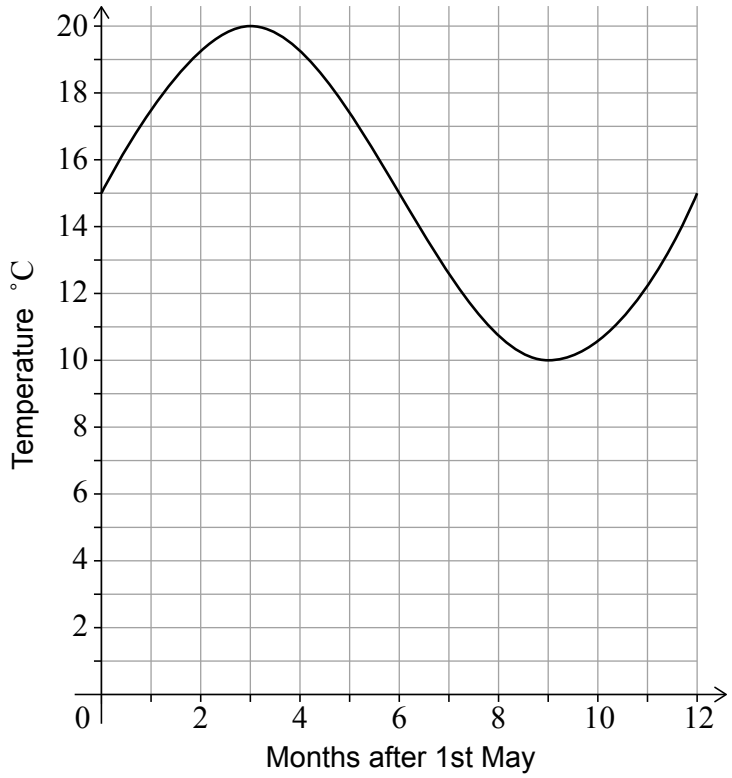
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Alex only swims in the sea if the water temperature is at least $15°\,\mathrm{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a\sin bx + \mathrm{c}$ for $0 \le x \le 12$, where x is the number of months after 1st May and where $a,\ b,\ c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Write down the value of

(i) a;

(ii) c. [2]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5\sin\frac{\pi}{6}x + 11$, where $0 \le x \le 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(i) find the midday water temperature on 1st October, five months after 1st May.

(ii) show that the midday water temperature is never warm enough for Alex to swim. [6]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q. [3]

Do **not** write solutions on this page.

Question 7 - working space

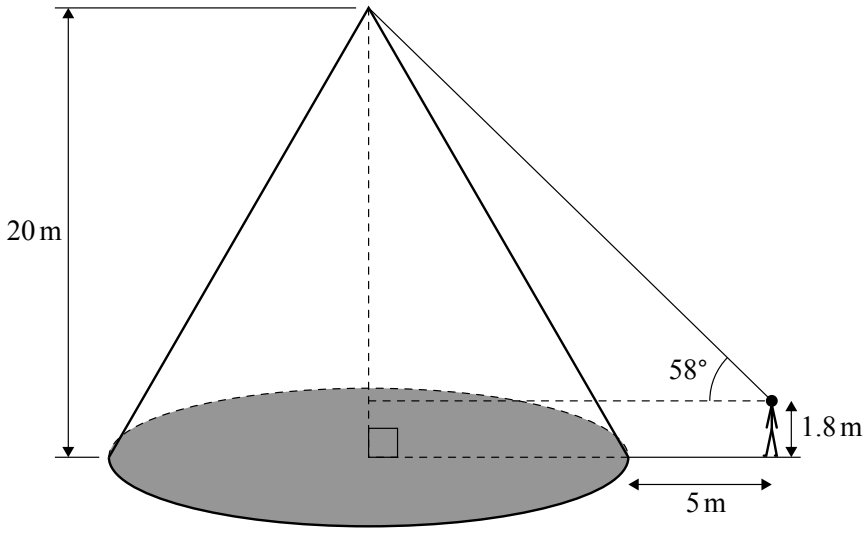
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

A monument is in the shape of a right cone with a vertical height of 20 metres. Oliver stands 5 metres from the base of the monument. His eye level is 1.8 metres above the ground and the angle of elevation from Oliver's eye level to the vertex of the cone is $58°$, as shown on the following diagram.

diagram not to scale



(a) Find the radius of the base of the cone. [3]

(b) Find the volume of the monument. [2]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

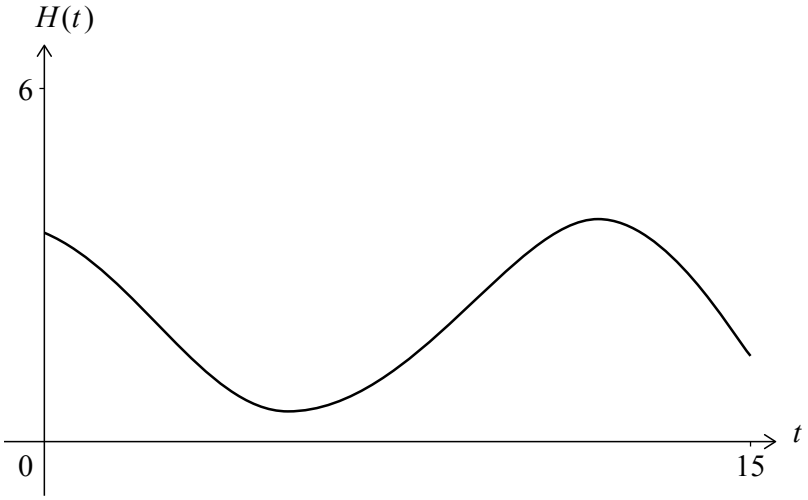
Sule Skerry and Rockall are small islands in the Atlantic Ocean, in the same time zone.

On a given day, the height of water in metres at Sule Skerry is modelled by the function $H(t) = 1.63\sin\left(0.513\left(t - 8.20\right)\right) + 2.13$, where t is the number of hours after midnight.

The following graph shows the height of the water for 15 hours, starting at midnight.

At low tide the height of the water is $0.50\,\text{m}$. At high tide the height of the water is $3.76\,\text{m}$.

All heights are given correct to two decimal places.



(a) The length of time between the first low tide and the first high tide is 6 hours and m minutes. Find the value of m to the nearest integer. [3]

(b) Between two consecutive high tides, determine the length of time, in hours, for which the height of the water is less than 1 metre. [2]

(c) Find the rate of change of the height of the water when $t = 13$, giving your answer in metres per hour. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

On the same day, the height of water at the second island, Rockall, is modelled by the function $h(t) = a\sin(b(t-c)) + d$, where t is the number of hours after midnight, and $a, b, c, d > 0$.

The first low tide occurs at 02:41 when the height of the water is $0.40\,\text{m}$.

The first high tide occurs at 09:02 when the height of the water is $2.74\,\text{m}$.

(d) Find the values of a, b, c and d. [7]

When $t = T$, the height of the water at Sule Skerry is the same as the height of the water at Rockall for the first time.

(e) Find the value of T. [2]

Do **not** write solutions on this page.

Question 8 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

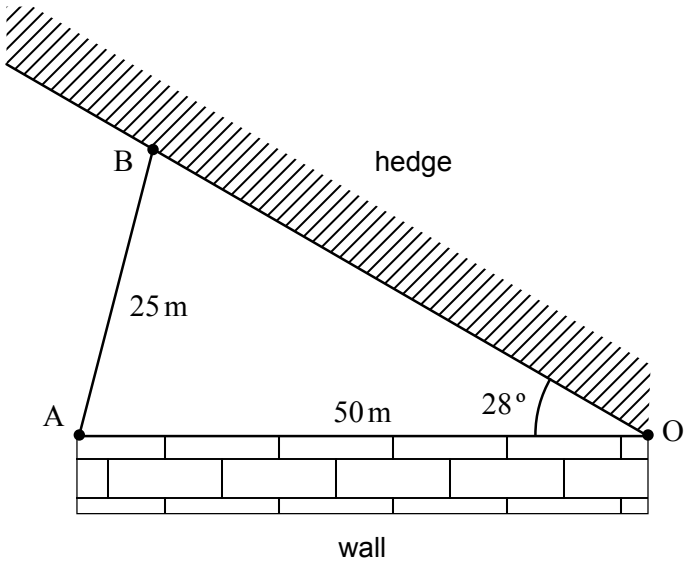
All angles in this question are given in degrees.

A farmer owns land which lies between a wall and a hedge. The wall has a length of $50\,\text{m}$ and lies between points O and A. The hedge meets the wall at O and the angle between the wall and the hedge is $28°$.

The farmer plans to form a triangular field for her prizewinning goats by placing a fence with a fixed length of 25 metres from point A to the hedge. The fence meets the hedge at a point B.

The information is shown on the following diagram.

diagram not to scale



(a) (i) Find the two possible sizes of $\text{O}\hat{\text{A}}\text{B}$, giving your answers in degrees.

(ii) Hence, find the two possible areas of the triangular field. [8]

(This question continues on the following page)

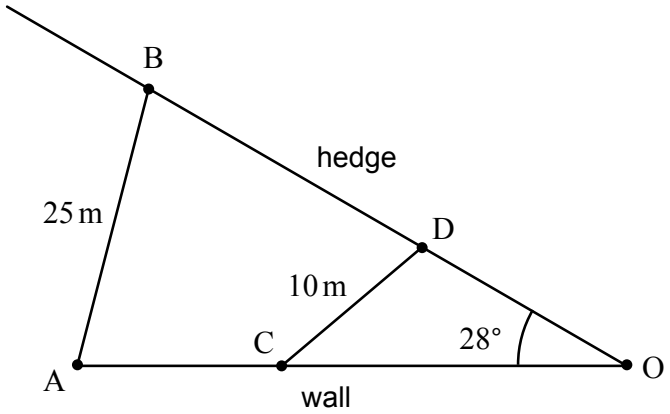
Do **not** write solutions on this page.

(Question 9 continued)

One of the goats, Brenda, fights with the other goats. The farmer plans to place a second fence with a fixed length of 10 metres between the wall and the hedge to form a small triangular field inside OAB for Brenda.

The information is shown on the following diagram.

diagram not to scale



The small triangular field OCD has an area of $60\,\mathrm{m}^2$.

Let x be the distance OC and let y be the distance OD.

(b) Show that $x^2 + y^2 = 100 + \dfrac{240}{\tan 28°}$. [5]

(c) Hence, determine the two possible lengths of OC. [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Question 9 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

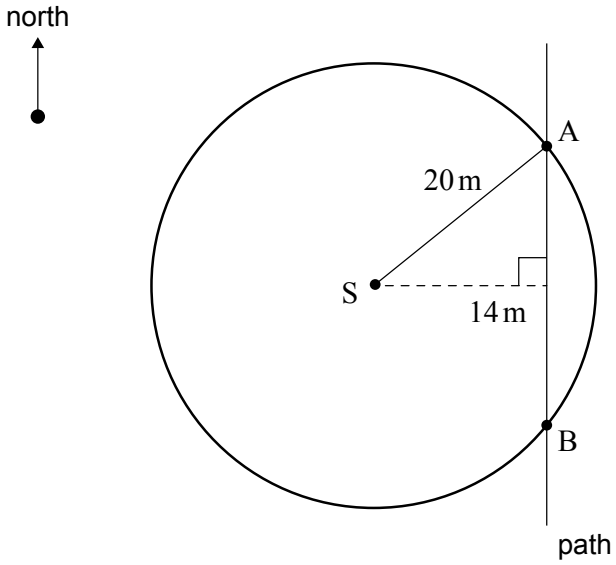
A rotating sprinkler is at a fixed point S.

It waters all points inside and on a circle of radius 20 metres.

Point S is 14 metres from the edge of a path which runs in a north-south direction.

The edge of the path intersects the circle at points A and B.

This information is shown in the following diagram.



(a) Show that $\mathrm{AB} = 28.57$, correct to four significant figures. [3]

The sprinkler rotates at a constant rate of one revolution every 16 seconds.

(b) Show that the sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second. [1]

Let T seconds be the time that $[\mathrm{AB}]$ is watered in each revolution.

(c) Find the value of T. [4]

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Do **not** write solutions on this page.

(Question 9 continued)

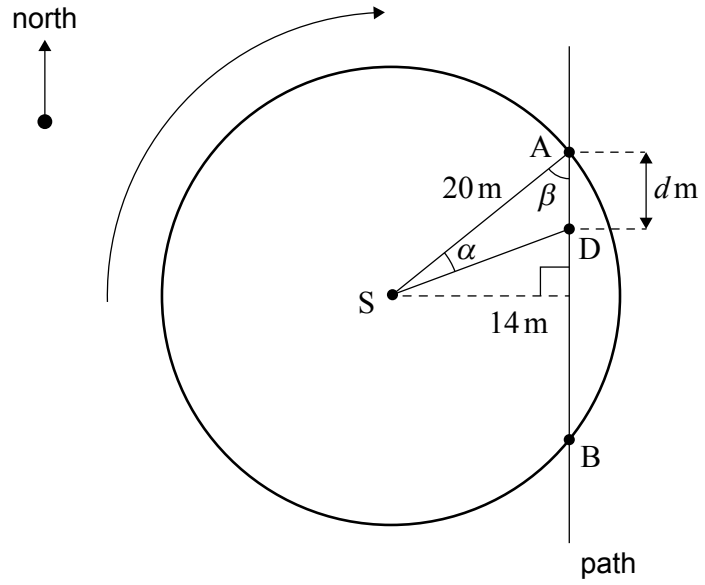
Consider one clockwise revolution of the sprinkler.

At $t = 0$, the water crosses the edge of the path at A.

At time t seconds, the water crosses the edge of the path at a movable point D which is a distance d metres south of point A.

Let $\alpha = \mathrm{A\hat{S}D}$ and $\beta = \mathrm{S\hat{A}B}$, where α, β are measured in radians.

This information is shown in the following diagram.



(d) Write down an expression for α in terms of t. [1]

It is known that $\beta = 0.7754$ radians, correct to four significant figures.

(e) By using the sine rule in $\Delta\mathrm{ASD}$, show that the distance, d, at time t, can be modelled by

$$d(t) = \frac{20\sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}.$$ [3]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 9 continued)

A turtle walks south along the edge of the path.

At time t seconds, the turtle's distance, g metres south of A, can be modelled by

$$g(t) = 0.05t^2 + 1.1t + 18, \text{ where } t \geq 0.$$

(f) At $t = 0$, state how far south the turtle is from A. [1]

Let w represent the distance between the turtle and point D at time t seconds.

(g) (i) Use the expressions for $g(t)$ and $d(t)$ to write down an expression for w in terms of t.

(ii) Hence find when and where on the path the water first reaches the turtle. [4]

Question 9 - working space

Section A

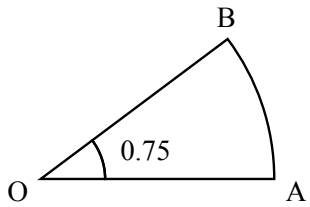
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius $r\,\text{cm}$, where $A\hat{O}B = 0.75$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $6\,\text{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

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Question 2 - working space

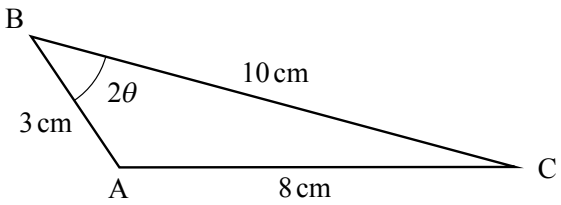
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 3\,\text{cm}$, $BC = 10\,\text{cm}$, $AC = 8\,\text{cm}$ and $A\hat{B}C = 2\theta$.

diagram not to scale



Find the exact value of $\cos\theta$, giving your answer in the form $\frac{\sqrt{p}}{q}$, where $p, q \in \mathbb{Z}^+$.

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Question 5 - working space

Section A

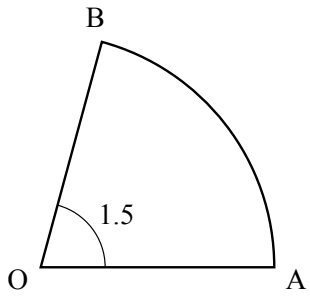
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius $r\,\text{cm}$, where $A\hat{O}B = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is $48\,\text{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

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Question 2 - working space

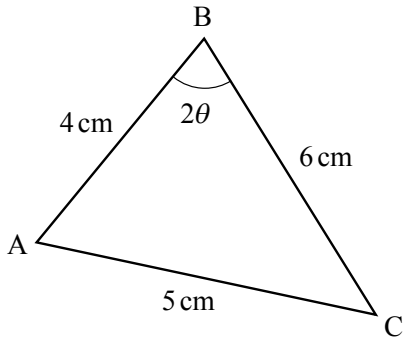
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 4\,\text{cm}$, $BC = 6\,\text{cm}$, $AC = 5\,\text{cm}$ and $A\hat{B}C = 2\theta$.

diagram not to scale



Find the exact value of $\cos\theta$, giving your answer in the form $\frac{p\sqrt{2}}{q}$, where $p, q \in \mathbb{Z}^+$.

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Question 5 - working space

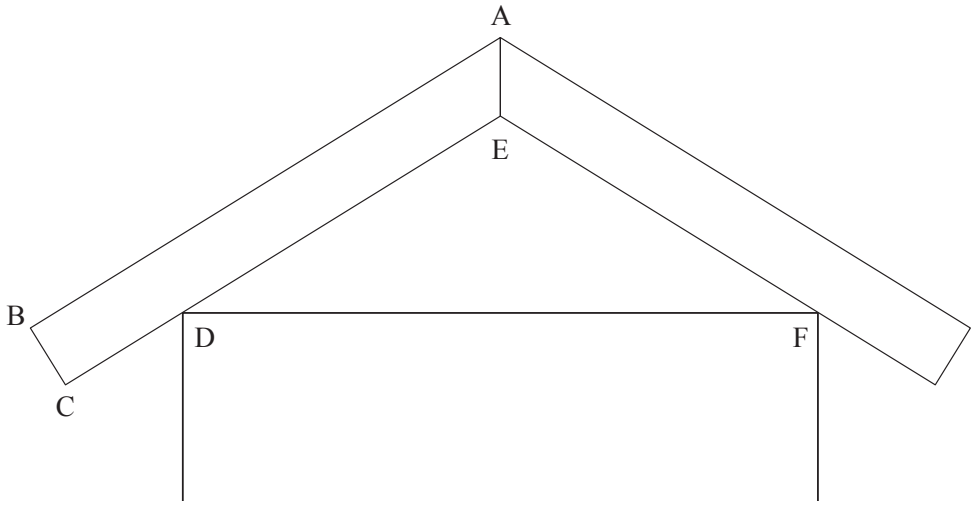
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 8]

The following diagram shows the cross section of the roof of a house. The cross section is symmetrical about the vertical line through points A and E.

diagram not to scale



The gradient of $[\mathrm{BA}]$ is $\frac{4}{15}$.

(a) Find the size of $\mathrm{B\hat{A}E}$, expressing your answer in degrees. [3]

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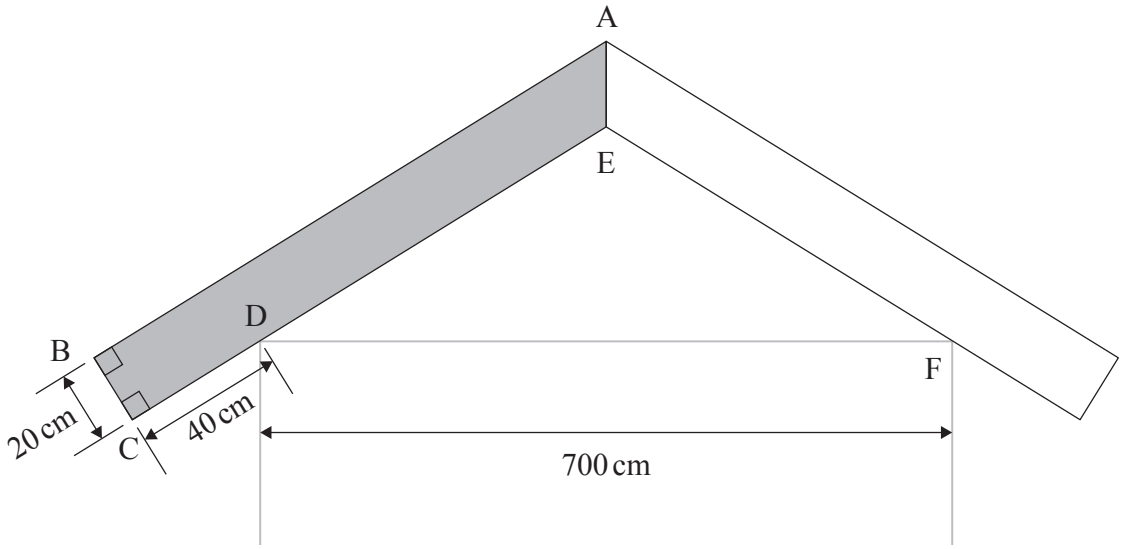
(Question 5 continued)

A builder requires the lengths of the sides $[BA]$ and $[CE]$.

The builder has the following measurements:

$A\hat{B}C = B\hat{C}E = 90°$, $DC = 40\,\text{cm}$, $BC = 20\,\text{cm}$, and $DF = 700\,\text{cm}$.

diagram not to scale



(b) Find

(i) CE;

(ii) BA. [5]

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Answers written on this page will not be marked.

Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

In this question all values of x and t are in radians.

Consider the function $f(x) = 3\sin(4\pi x)$.

(a) (i) Write down the amplitude of the graph of f.

(ii) Find the period of f. [3]

Consider a second function $g(x) = -4\cos(4\pi x)$.

The sum of these functions can be expressed in the form $f(x) + g(x) = a\cos\big(b(x-c)\big)$, where $a, b, c > 0$.

(b) By considering the graph of $y = f(x) + g(x)$, determine

(i) the value of a;

(ii) the value of b;

(iii) the smallest possible value of c. [4]

A car is travelling along a straight residential street with speed bumps placed at regular intervals on the road to encourage safer driving. The car travels at a minimum velocity when passing over speed bumps and reaches a maximum velocity in between speed bumps.

Its velocity, in m s^{-1}, can be modelled by the function $v(t) = -3.5\cos\left(\frac{\pi}{14}(t-5)\right) + 9$, where t is measured in seconds.

(c) Find the time at which the car first reaches its maximum velocity. [1]

(d) Find the number of speed bumps the car passes over in the first two minutes of motion. [1]

(e) (i) Find $v'(t)$.

(ii) Hence, or otherwise, write down the maximum acceleration of the car. [4]

(f) Find the distance, in metres, between consecutive speed bumps. [3]

Question 9 - working space

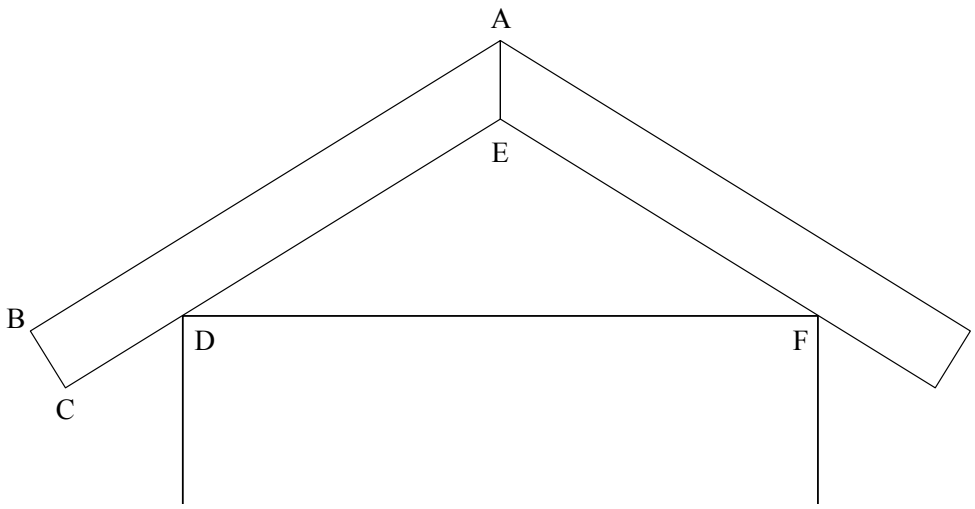
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 8]

The following diagram shows the cross section of the roof of a house. The cross section is symmetrical about the vertical line through points A and E.

diagram not to scale



The gradient of $[\mathrm{BA}]$ is $\frac{7}{12}$.

(a) Find the size of $\mathrm{B\hat{A}E}$, expressing your answer in degrees. [3]

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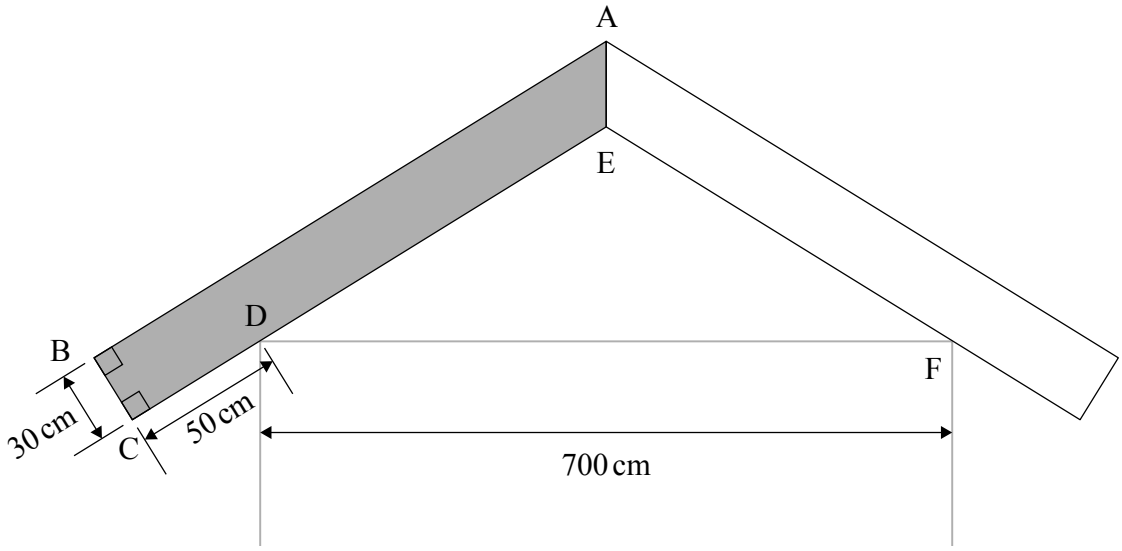
(Question 5 continued)

A builder requires the lengths of the sides $[BA]$ and $[CE]$.

The builder has the following measurements:

$A\hat{B}C = B\hat{C}E = 90°$, $DC = 50\,\text{cm}$, $BC = 30\,\text{cm}$, and $DF = 700\,\text{cm}$.

diagram not to scale



(b) Find

(i) CE;

(ii) BA. [5]

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Answers written on this page
will not be marked.

Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

In this question all values of x and t are in radians.

Consider the function $f(x) = 6\cos(\pi x)$.

(a) (i) Write down the amplitude of the graph of f.

(ii) Find the period of f. [3]

Consider a second function $g(x) = -8\sin(\pi x)$.

The sum of these functions can be expressed in the form $f(x) + g(x) = a\cos\big(b(x-c)\big)$, where $a, b, c > 0$.

(b) By considering the graph of $y = f(x) + g(x)$, determine

(i) the value of a;

(ii) the value of b;

(iii) the smallest possible value of c. [4]

A car is travelling along a straight residential street with speed bumps placed at regular intervals on the road to encourage safer driving. The car travels at a minimum velocity when passing over speed bumps and reaches a maximum velocity in between speed bumps.

Its velocity, in m s^{-1}, can be modelled by the function $v(t) = -4.5\cos\left(\frac{\pi}{19}(t-4)\right) + 10.5$, where t is measured in seconds.

(c) Find the time at which the car first reaches its maximum velocity. [1]

(d) Find the number of speed bumps the car passes over in the first two minutes of motion. [1]

(e) (i) Find $v'(t)$.

(ii) Hence, or otherwise, write down the maximum acceleration of the car. [4]

(f) Find the distance, in metres, between consecutive speed bumps. [3]

Question 9 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

A solid cylinder has height $h\,\text{cm}$ and base radius $R\,\text{cm}$.

The cylinder fits exactly inside a hollow sphere of radius $r\,\text{cm}$.

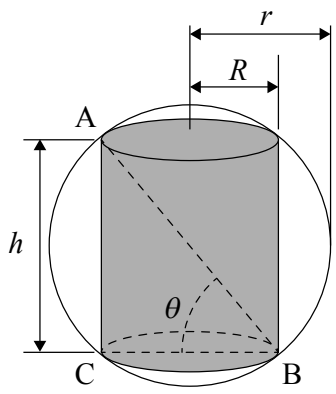
Points A, B and C are points where the surface of the cylinder touches the surface of the sphere.

The line segment $[\text{AB}]$ is a diameter of the sphere.

The line segment $[\text{BC}]$ is a diameter of the base of the cylinder and $\text{A}\hat{\text{B}}\text{C} = \theta$.

This information is shown on the following diagram.

diagram not to scale



(a) (i) By considering triangle ABC, show that $R = r\cos\theta$.

(ii) Find an expression for h in terms of r and θ. [4]

(b) Hence or otherwise, show that the total surface area, $S\,\text{cm}^2$, of the cylinder is given by $S = 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$. [4]

The external surface area of the sphere is $2S$.

(c) Show that $\tan\theta = 2$. [4]

The volume of the cylinder is $V\,\text{cm}^3$.

(d) Find V, giving your answer in the form $p\pi r^3\sqrt{5}$, where $p \in \mathbb{Q}^+$. [5]

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Question 9 - working space

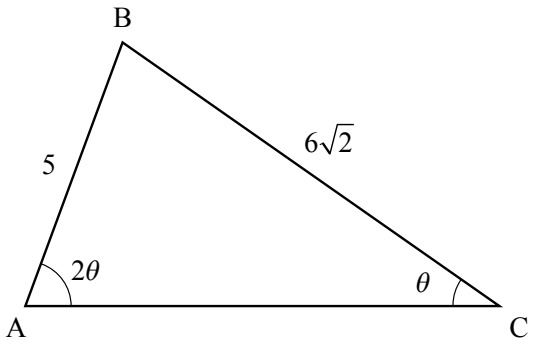
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

The following diagram shows a non-right angled triangle ABC.

diagram not to scale



$\mathrm{AB} = 5$, $\mathrm{BC} = 6\sqrt{2}$, $\mathrm{A\hat{C}B} = \theta$ and $\mathrm{B\hat{A}C} = 2\theta$, where $0 < \theta < \frac{\pi}{2}$.

(a) Using the sine rule, show that $\cos\theta = \frac{3\sqrt{2}}{5}$. [3]

(b) Hence, find $\sin\theta$. [2]

Point D is located on $[\mathrm{AC}]$ such that the area of triangle BCD is $2\sqrt{14}$.

(c) Find DC. [2]

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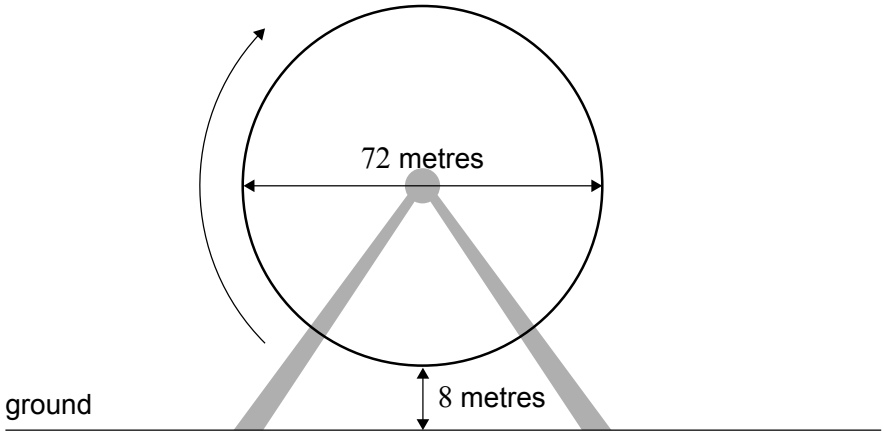
Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A Ferris wheel with diameter 72 metres rotates at a constant speed. The lowest point on the wheel is 8 metres above the ground, as shown on the following diagram. A seat starts at the lowest point.



The wheel completes one revolution in 12 minutes.

After t minutes, the height, h metres, of the seat above the ground is given by $h(t) = a\cos(bt) + 44$, where $a, b \in \mathbb{R}$.

(a) Find the value of

(i) a;

(ii) b. [4]

(b) Sketch the graph of h, for $0 \leq t \leq 12$. Clearly label the coordinates of the maximum point and the end points. [2]

Consider one revolution of the Ferris wheel.

(c) The seat is at least $26\,\text{m}$ above the ground for T minutes. Find the value of T. [5]

(d) (i) Find $h'(t)$.

(ii) Hence or otherwise, find the time at which the height of the seat above the ground is decreasing at its fastest rate. [4]

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Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

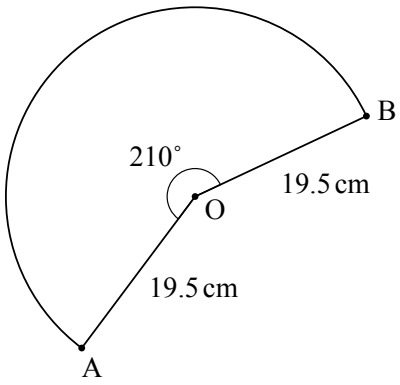
2. [Maximum mark: 6]

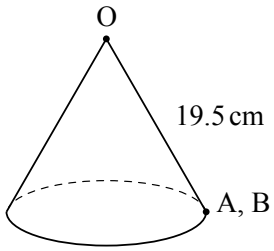
The points A and B lie on a circle, with centre O and radius $19.5\,\text{cm}$, such that $\widehat{BOA} = 210^\circ$.

A piece of paper is cut into the shape of the sector BOA.

A hollow cone with no base is constructed from the sector by joining the points A and B. The sector forms the curved surface of the cone.

This is shown in the following diagrams.





Find

(a) the area of the sector BOA; [3]

(b) the radius of the cone. [3]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 4]

Consider the function $f(x) = a\tan(2x) + b$, where $x \neq \frac{(2n+1)\pi}{4}$, $n \in \mathbb{Z}$ and $a, b \in \mathbb{R}$.

(a) Write down the period of f. [1]

The graph of $y = f(x)$ passes through the points $\left(\frac{\pi}{12}, 5\right)$ and $\left(\frac{\pi}{3}, 7\right)$.

(b) Find the value of a and the value of b. [3]

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Question 3 - working space

Section A

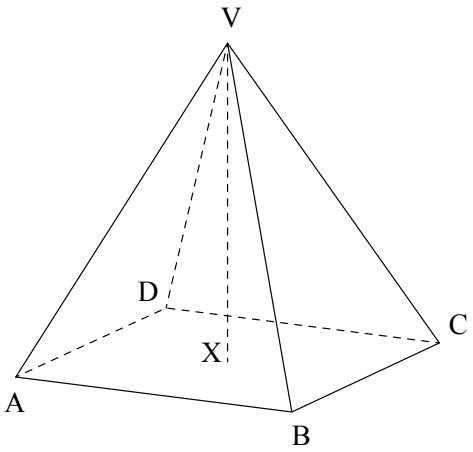
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows a square-based right-pyramid with vertex $\mathrm{V}(1, 7, 0)$.

Point $\mathrm{X}(-3, 4, 2)$ is the centre of the base ABCD.

diagram not to scale



(a) Find VX. [2]

The square base has side length $5\,\mathrm{cm}$.

(b) Find AC. [2]

(c) Find the size of the angle between the edge $[\mathrm{VC}]$ and the base of the pyramid. [2]

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Question 2 - working space

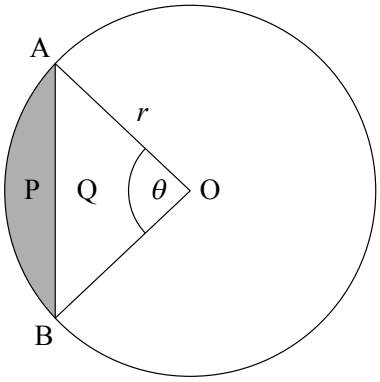
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius $r\,\mathrm{cm}$. Points A and B lie on the circle and $\mathrm{A\hat{O}B} = \theta$ radians.

Sector OAB is divided into two regions, a shaded segment P and a triangle Q.



The area of the shaded segment P is $12.8\,\mathrm{cm}^2$.

The areas of P and Q are in the ratio $3:5$.

Find the value of r.

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Question 6 - working space

Section B

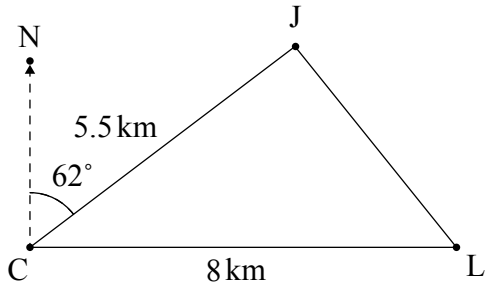
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

A lighthouse, L, is located 8 kilometres due East of a coastguard station, C, on a straight stretch of coastline.

The coastguard station sees a Jet Ski, J, on a bearing of $062°$ and at a distance of 5.5 kilometres. This is shown on the following diagram.

diagram not to scale



(a) Find JL. [4]

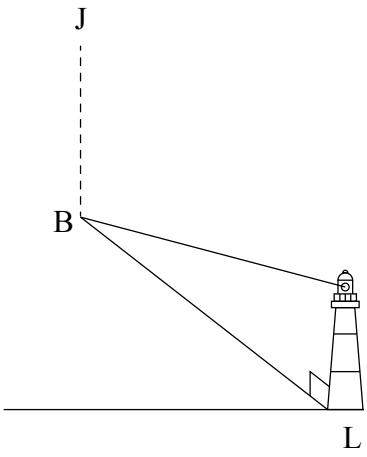
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(Question 7 continued)

While travelling due South, the Jet Ski breaks down at point B, before it reaches the coastline. The position of the Jet Ski at B and the lighthouse are shown in the following diagram.

diagram not to scale



From the top of the 60-metre-tall lighthouse, the angle of depression to the Jet Ski at B, is measured to be $0.94°$.

(b) Find BL. [3]

The bearing from the Jet Ski at B to the lighthouse is $121°$.

(c) Find the bearing from L to B. [2]

The jet-skier sets off a distress flare which is seen at the lighthouse and the coastguard station at the same time.

The lighthouse has a small rescue boat which travels at a speed of $48\,\mathrm{kmh}^{-1}$.

(d) Find the time, in minutes, for the lighthouse rescue boat to reach the Jet Ski at B. [2]

The coastguard rescue boat travels at a speed of $55\,\mathrm{kmh}^{-1}$ and sets out at the same time as the lighthouse rescue boat.

(e) Determine which rescue boat reaches the Jet Ski first. Justify your answer. [4]

Turn over

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Question 7 - working space

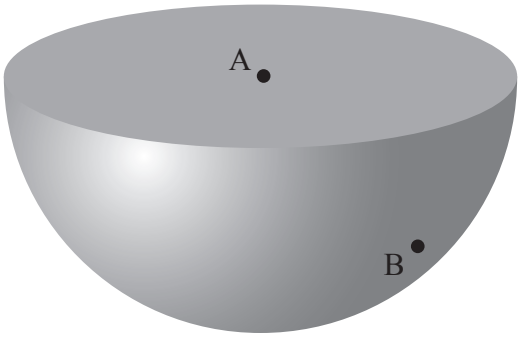
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows a solid hemisphere with centre $\mathrm{A}(6, -1, -3)$.

Point $\mathrm{B}(4, -5, -9)$ lies on the curved surface.



(a) Find AB, the radius of the hemisphere. [2]

(b) Hence, find the total surface area of the solid hemisphere. [3]

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Question 1 - working space

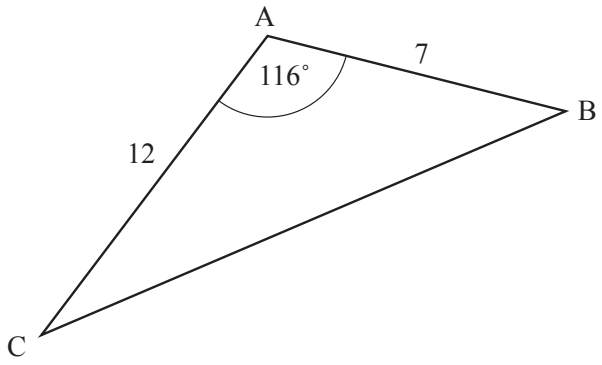
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows a triangle ABC, with $\mathrm{AB} = 7$, $\mathrm{AC} = 12$ and $\mathrm{B\hat{A}C} = 116°$.

diagram not to scale



(a) Find BC. [3]

(b) Find $\mathrm{A\hat{C}B}$. [3]

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Question 2 - working space

Section A

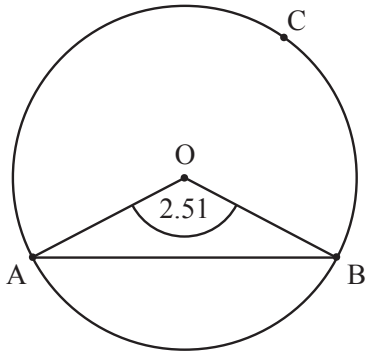
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

The following diagram shows a circle with centre O.

Points A, B and C lie on the circle.

diagram not to scale



The area of triangle AOB is $26\,\text{cm}^2$ and $A\hat{O}B = 2.51$ radians.

Find the length of arc ACB.

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

(a) Show that $\frac{2\tan\theta}{1+\tan^2\theta}$ can be expressed as $\sin 2\theta$. [3]

(b) Hence, solve the equation $\frac{2\tan\theta}{1+\tan^2\theta}=\frac{1}{2}$ for $0° \leq \theta \leq 180°$. [5]

Let $x=\tan\left(22\frac{1}{2}^{\circ}\right)$.

(c) Use the result from part (a) to show that $x^2-2\sqrt{2}x+1=0$. [4]

(d) Hence, find the exact value of $\tan\left(22\frac{1}{2}^{\circ}\right)$. [3]

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Question 9 - working space

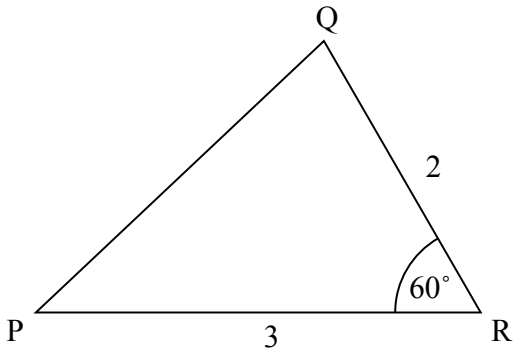
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

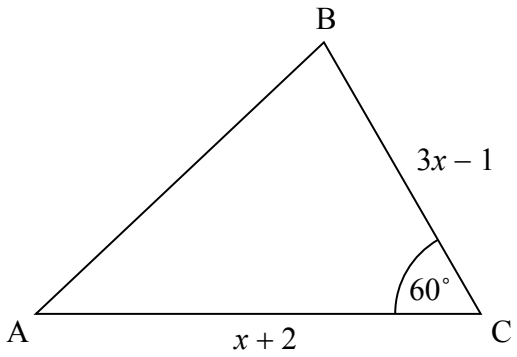
Consider a triangle PQR where $\mathrm{PR} = 3$, $\mathrm{QR} = 2$ and $\mathrm{P\hat{R}Q} = 60°$.

diagram not to scale



(a) Find PQ. [3]

Now consider a general triangle ABC where $\mathrm{AC} = x + 2$, $\mathrm{BC} = 3x - 1$ and $\mathrm{A\hat{C}B} = 60°$.



(b) Show that $\mathrm{AB} = \sqrt{7x^2 - 7x + 7}$. [5]

(c) Given that $\mathrm{AB} = 7$,

(i) find the value of x;

(ii) find the area of triangle ABC. [6]

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Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

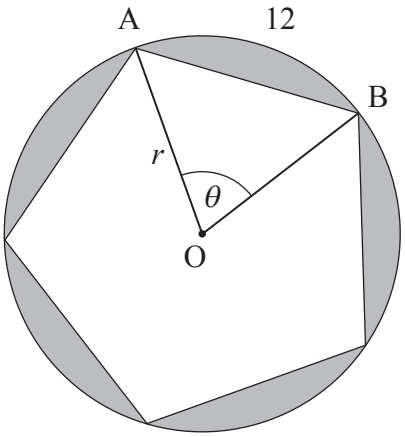
1. [Maximum mark: 6]

The following diagram shows a regular pentagon inscribed in a circle with centre O and radius $r\,\mathrm{cm}$.

The angle $\mathrm{A\hat{O}B}$ is θ, where θ is measured in radians.

The arc AB is $12\,\mathrm{cm}$.

diagram not to scale



(a) Find

(i) θ;

(ii) r. [3]

(b) Find the area of the shaded region. [3]

(This question continues on the following page)

(Question 1 continued)

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Question 1 - working space

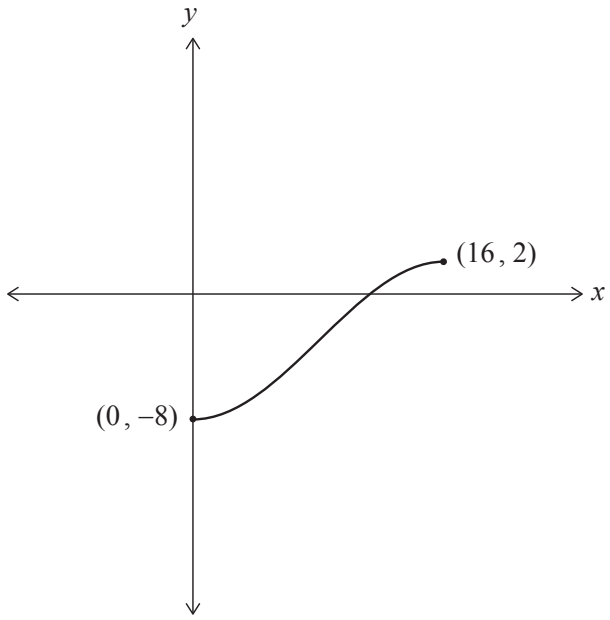
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

Consider the function $g(x) = a\cos(bx) - 3$, where $x \in \mathbb{R}$ and $a, b \in \mathbb{R}$.

The following diagram shows part of the graph of g.



The graph of g has a local minimum at $(0, -8)$ and a local maximum at $(16, 2)$.

(a) Find the value of

(i) a;

(ii) b, where $b > 0$. [3]

(b) Write down the smallest positive value of the constant k such that $g(x + k) = g(x)$ for all x. [1]

(c) The function $g(x)$ can be written in the form $f(x) = p\sin b(x - q) - 3$ where $p, q \in \mathbb{Z}^+$.

(i) Find the smallest positive value of q.

(ii) For this value of q, write down the value of p. [3]

(This question continues on the following page)

(Question 5 continued)

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

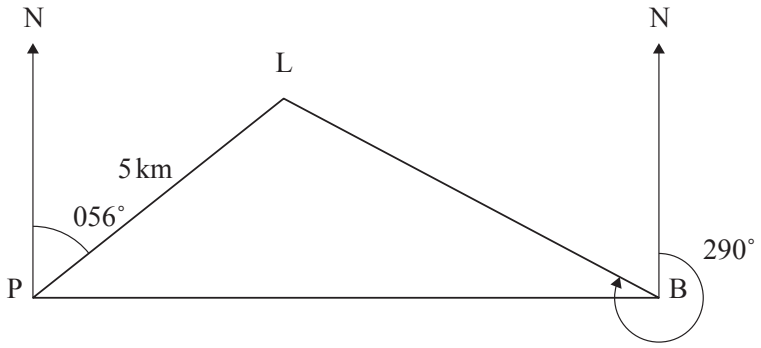
A boat sets off from a port P and travels on a bearing of $090°$ until reaching point B.

A lighthouse L is $5\,\mathrm{km}$ from P on a bearing of $056°$.

The bearing of the lighthouse from B is $290°$.

This is shown in the following diagram.

diagram not to scale



(a) Find the angle $\mathrm{L\hat{P}B}$. [1]

(b) Find LB, the distance from point L to point B. [4]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

(a) Show that $3\cos 2x + 11\sin x = 3 + 11\sin x - 6\sin^2 x$. [2]

(b) Hence, or otherwise, solve the equation $3\cos 2x + 11\sin x - 6 = 0$ for $0° \leq x \leq 180°$. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

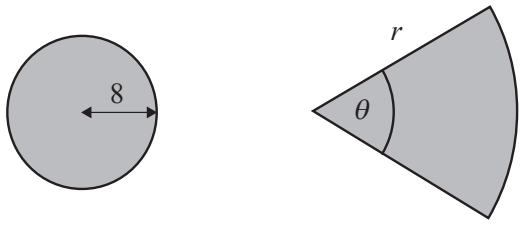
Consider a circle and a sector.

The radius of the circle is $8\,\text{mm}$.

The radius of the sector is $r\,\text{mm}$ and the acute angle at the centre is θ radians.

This is shown in the following diagram.

diagram not to scale



The perimeter of the sector is 1.5 times the circumference of the circle.

(a) Show that $r = \dfrac{24\pi}{2+\theta}$. [3]

It is given that the area of the circle is the same as the area of the sector.

(b) Determine the value of θ. [3]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

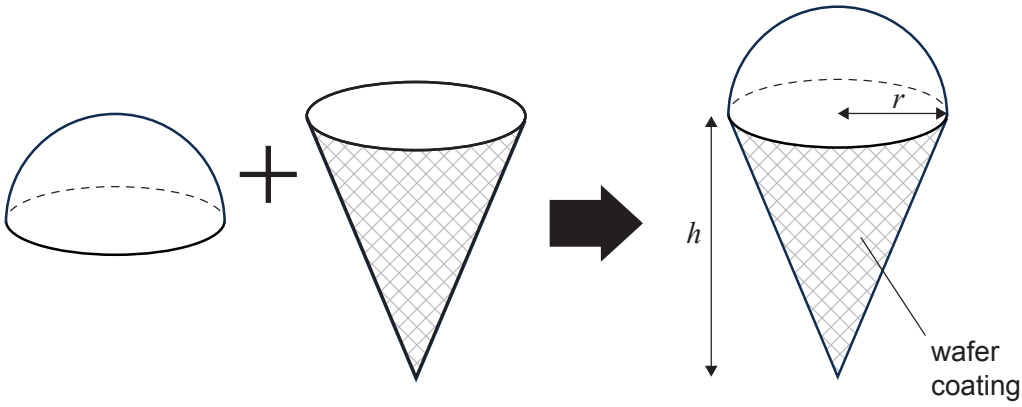
8. [Maximum mark: 15]

A manufacturer creates an ice cream model consisting of two parts.

The top part consists of a hemisphere of ice cream with radius $r\,\text{cm}$.

The bottom part consists of a cone of ice cream with radius $r\,\text{cm}$ and height $h\,\text{cm}$, which is wrapped in a wafer coating.

This is shown in the following diagram.



(a) Consider the case where $r = 3$ and $h = 8$.

(i) Show that the total volume, V, of ice cream is $132\,\text{cm}^3$, correct to 3 significant figures.

(ii) Determine the curved surface area, S, of the wafer coating. [7]

The manufacturer changes the dimensions of the model to ensure that the total volume, V, of ice cream is $120\,\text{cm}^3$.

(b) Show that $h = \dfrac{360 - 2\pi r^3}{\pi r^2}$. [3]

(c) Hence, show that the curved surface area of the wafer coating is given by

$$S = \pi r\sqrt{r^2 + \left(\frac{360 - 2\pi r^3}{\pi r^2}\right)^2}\,.$$ [2]

The manufacturer wants to use the minimum possible value of S.

(d) Determine the minimum value of S and the corresponding value of r. [3]

Do **not** write solutions on this page.

Question 8 - working space

APOLLO

Statistics and probability

81 questions

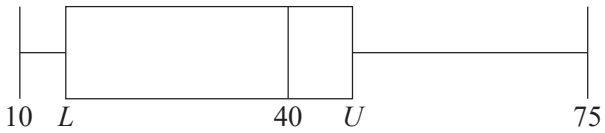
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.

diagram not to scale



The interquartile range is 20 grams and there are no outliers in the results.

(a) Find the minimum possible value of U. [3]

(b) Hence, find the minimum possible value of L. [2]

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$\mathrm{P}(X=x)$	p	p	p	$\frac{1}{2}p$

(a) Find the value of p. [2]

(b) Hence, find the value of $\mathrm{E}(X)$. [2]

A second biased four-sided die, B, is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given in the following table.

y	1	2	3	4
$\mathrm{P}(Y=y)$	q	q	q	r

(c) (i) State the range of possible values of r.

(ii) Hence, find the range of possible values of q. [3]

(d) Hence, find the range of possible values for $\mathrm{E}(Y)$. [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is $\frac{1}{2}$.

(e) Find the value of $\mathrm{E}(Y)$. [6]

References:

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Question 9 - working space

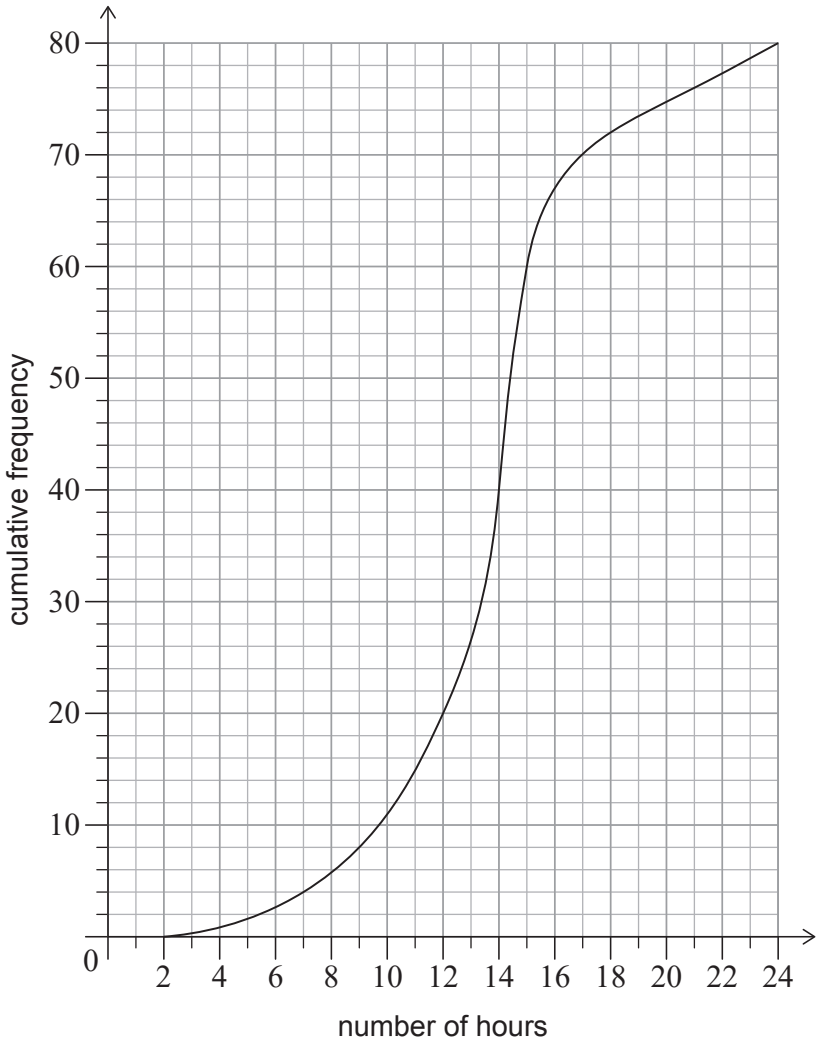
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A large school has students from Year 6 to Year 12.

A group of 80 students in Year 12 were randomly selected and surveyed to find out how many hours per week they each spend doing homework. Their results are represented by the following cumulative frequency graph.



(This question continues on the following page)

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(Question 7 continued)

(a) Find the median number of hours per week these Year 12 students spend doing homework. [2]

(b) Given that 10% of these Year 12 students spend more than k hours per week doing homework, find the value of k. [3]

This same information is represented by the following table.

Hours (h) spent doing homework	$2 < h \leq 7$	$7 < h \leq 15$	$15 < h \leq 21$	$21 < h \leq 24$
Frequency	4	p	16	q

(c) Find the value of p and the value of q. [4]

There are 320 students in Year 12 at this school.

(d) Estimate the number of Year 12 students that spend more than 15 hours each week doing homework. [3]

(e) (i) Explain why this sampling method might not provide an accurate representation of the amount of time **all** of the students in the school spend doing homework.

(ii) Suggest a more appropriate sampling method. [2]

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Question 7 - working space

Section A

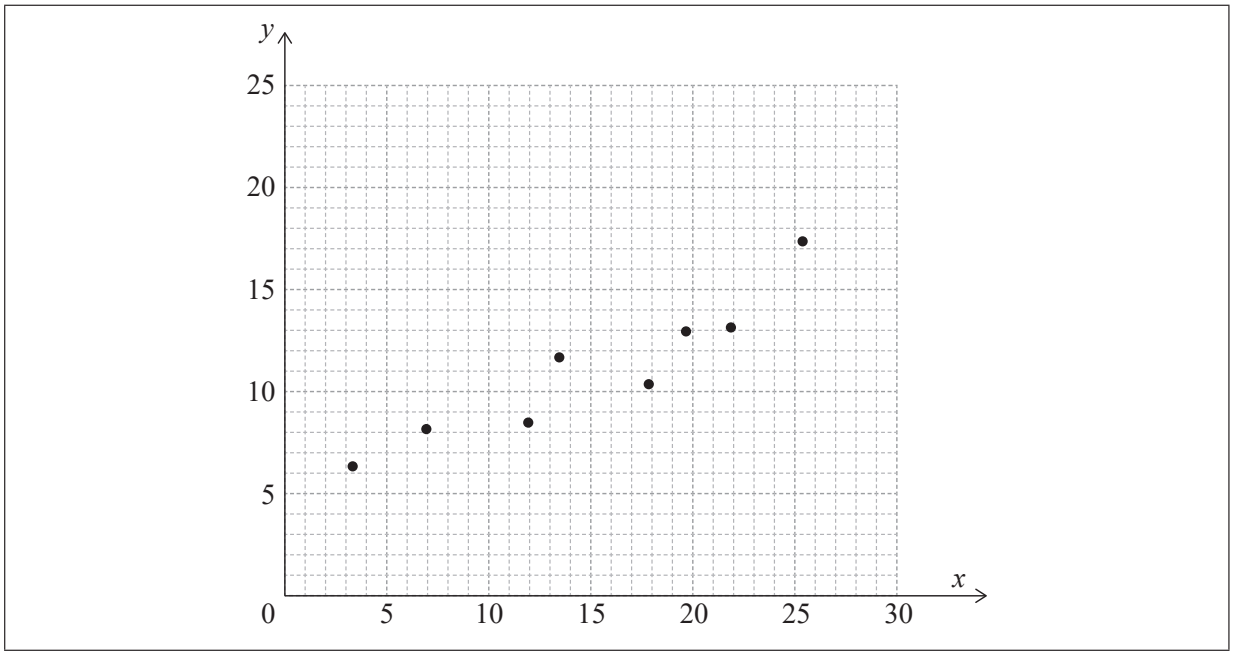
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 7]

The following table shows the data collected from an experiment.

x	3.3	6.9	11.9	13.4	17.8	19.6	21.8	25.3
y	6.3	8.1	8.4	11.6	10.3	12.9	13.1	17.3

The data is also represented on the following scatter diagram.



The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$, where $a, b \in \mathbb{R}$.

(a) Write down the value of a and the value of b. [2]

(b) Use this model to predict the value of y when $x = 18$. [2]

(c) Write down the value of $\overline{x}$ and the value of $\overline{y}$. [1]

(d) Draw the line of best fit on the scatter diagram. [2]

(This question continues on the following page)

(Question 2 continued)

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

(a) Find the probability that a bag selected at random is rejected. [2]

(b) Estimate the number of bags which will be rejected from a random sample of 100 bags. [1]

(c) Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

At a café, the waiting time between ordering and receiving a cup of coffee is dependent upon the number of customers who have already ordered their coffee and are waiting to receive it.

Sarah, a regular customer, visited the café on five consecutive days. The following table shows the number of customers, x, ahead of Sarah who have already ordered and are waiting to receive their coffee and Sarah's waiting time, y minutes.

Number of customers (x)	3	9	11	10	5
Sarah's waiting time (y)	6	10	12	11	6

The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$.

(a) (i) Find the value of a and the value of b.

(ii) Write down the value of Pearson's product-moment correlation coefficient, r. [3]

(b) Interpret, in context, the value of a found in part (a)(i). [1]

On another day, Sarah visits the café to order a coffee. Seven customers have already ordered their coffee and are waiting to receive it.

(c) Use the result from part (a)(i) to estimate Sarah's waiting time to receive her coffee. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 8]

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

(a) Find the probability that the student plays a sport and is involved in theatre. [2]

(b) Find the probability that the student is involved in theatre, but does not play a sport. [2]

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let G be the event "the student is a girl" and let T be the event "the student is involved in theatre".

(c) Find $\mathrm{P}(G \cap T)$. [2]

(d) Determine if the events G and T are independent. Justify your answer. [2]

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The flight times, T minutes, between two cities can be modelled by a normal distribution with a mean of 75 minutes and a standard deviation of σ minutes.

(a) Given that 2% of the flight times are longer than 82 minutes, find the value of σ. [3]

(b) Find the probability that a randomly selected flight will have a flight time of more than 80 minutes. [2]

(c) Given that a flight between the two cities takes longer than 80 minutes, find the probability that it takes less than 82 minutes. [4]

On a particular day, there are 64 flights scheduled between these two cities.

(d) Find the expected number of flights that will have a flight time of more than 80 minutes. [3]

(e) Find the probability that more than 6 of the flights on this particular day will have a flight time of more than 80 minutes. [3]

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Question 8 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

All answers in this question should be given to four significant figures.

In a local weekly lottery, tickets cost $\$2$ each.

In the first week of the lottery, a player will receive $\$D$ for each ticket, with the probability distribution shown in the following table. For example, the probability of a player receiving $\$10$ is 0.03. The grand prize in the first week of the lottery is $\$1000$.

d	0	2	10	50	Grand Prize
$\mathrm{P}(D = d)$	0.85	c	0.03	0.002	0.0001

(a) Find the value of c. [2]

(b) Determine whether this lottery is a fair game in the first week. Justify your answer. [4]

If nobody wins the grand prize in the first week, the probabilities will remain the same, but the value of the grand prize will be $\$2000$ in the second week, and the value of the grand prize will continue to double each week until it is won. All other prize amounts will remain the same.

(c) Given that the grand prize is not won and the grand prize continues to double, write an expression in terms of n for the value of the grand prize in the nth week of the lottery. [2]

The wth week is the first week in which the player is expected to make a profit. Ryan knows that if he buys a lottery ticket in the wth week, his expected profit is $\$p$.

(d) Find the value of p. [7]

References:

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

Box 1 contains 5 red balls and 2 white balls.
Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

(b) Determine whether events A and R are independent. [2]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

In Lucy's music academy, eight students took their piano diploma examination and achieved scores out of 150. For her records, Lucy decided to record the average number of hours per week each student reported practising in the weeks prior to their examination. These results are summarized in the table below.

Average weekly practice time (h)	28	13	45	33	17	29	39	36
Diploma score (D)	115	82	120	116	79	101	110	121

(a) Find Pearson's product-moment correlation coefficient, r, for these data. [2]

(b) The relationship between the variables can be modelled by the regression equation $D = ah + b$. Write down the value of a and the value of b. [1]

(c) One of these eight students was disappointed with her result and wished she had practised more. Based on the given data, determine how her score could have been expected to alter had she practised an extra five hours per week. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

A factory manufactures lamps. It is known that the probability that a lamp is found to be defective is 0.05. A random sample of 30 lamps is tested.

(a) Find the probability that there is at least one defective lamp in the sample. [3]

(b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. [4]

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

The random variable X follows a normal distribution with mean μ and standard deviation σ.

(a) Find $\mathrm{P}(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$. [3]

The avocados grown on a farm have weights, in grams, that are normally distributed with mean μ and standard deviation σ. Avocados are categorized as small, medium, large or premium, according to their weight. The following table shows the probability an avocado grown on the farm is classified as small, medium, large or premium.

Category	Small	Medium	Large	Premium
Probability	0.04	0.576	0.288	0.096

The maximum weight of a small avocado is 106.2 grams.

The minimum weight of a premium avocado is 182.6 grams.

(b) Find the value of μ and of σ. [5]

A supermarket purchases all the avocados from the farm that weigh more than 106.2 grams.

(c) Find the probability that an avocado chosen at random from this purchase is categorized as

(i) medium;

(ii) large;

(iii) premium. [4]

The selling prices of the different categories of avocado at this supermarket are shown in the following table:

Category	Medium	Large	Premium
Selling price ($) per avocado	1.10	1.29	1.96

The supermarket pays the farm $\$200$ for the avocados and assumes it will then sell them in exactly the same proportion as purchased from the farm.

(d) According to this model, find the minimum number of avocados that must be sold so that the net profit for the supermarket is at least $\$438$. [4]

References:

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Let A and B be events such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cup B) = 0.6$.
Find $\mathrm{P}(A \mid B)$.

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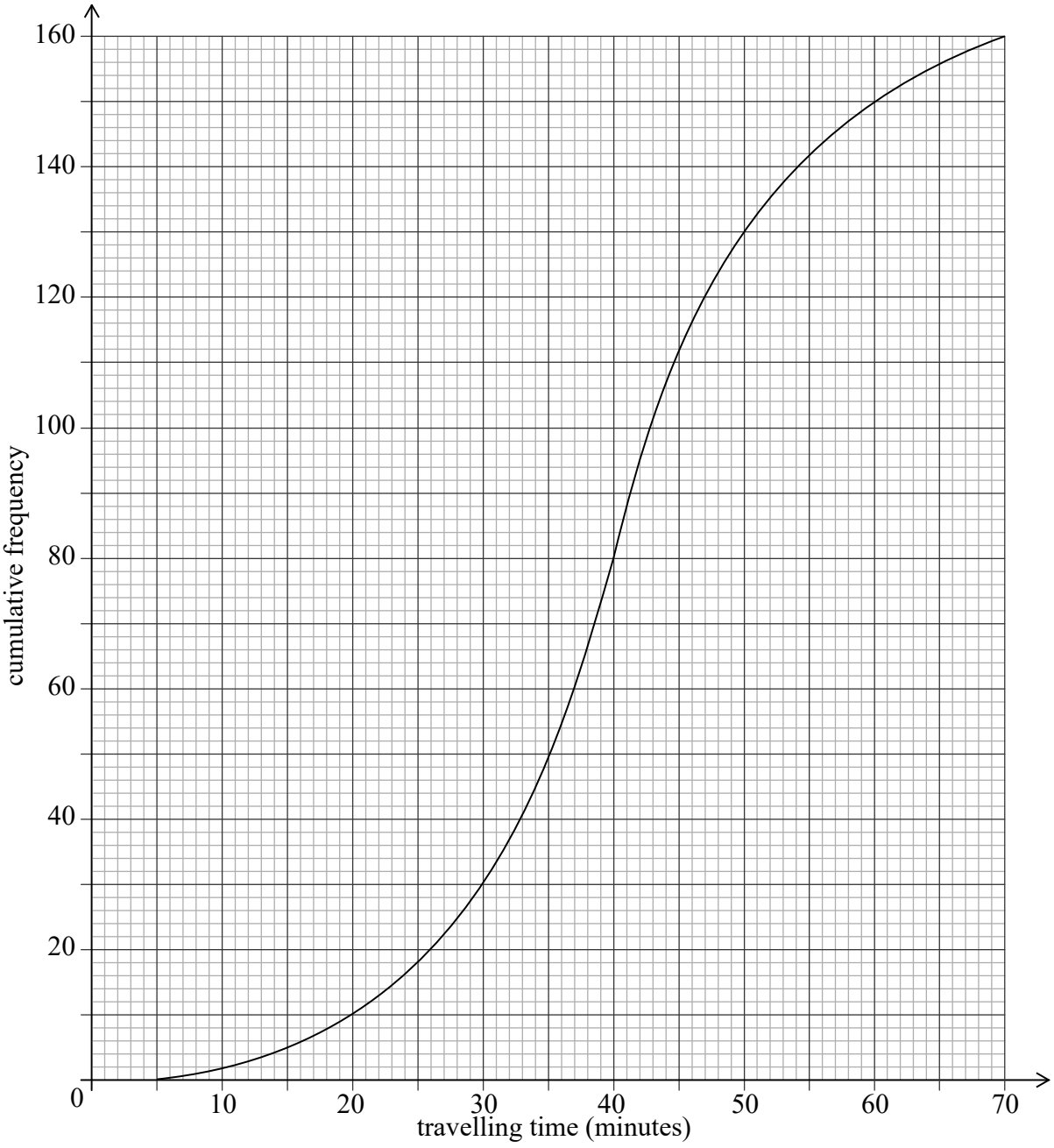
Question 2 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



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(Question 7 continued)

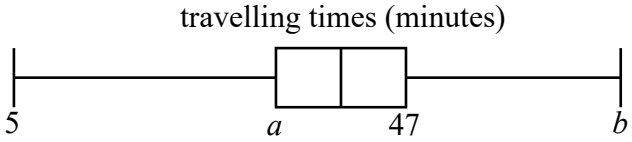
(a) Find the median number of minutes spent traveling to work. [2]

(b) Find the number of employees whose travelling time is within 15 minutes of the median. [3]

Only 10% of the employees spent more than k minutes traveling to work.

(c) Find the value of k. [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.



(d) Write down the value of b. [1]

(e) (i) Find the value of a.

(ii) Hence, find the interquartile range. [4]

Travelling times of less than p minutes are considered outliers.

(f) Find the value of p. [2]

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

A six-sided biased die is weighted in such a way that the probability of obtaining a "six" is $\frac{7}{10}$.

The die is tossed five times. Find the probability of obtaining

(a) at most three "sixes". [3]

(b) the third "six" on the fifth toss. [3]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

The following table below shows the marks scored by seven students on two different mathematics tests.

Test 1 (x)	15	23	25	30	34	34	40
Test 2 (y)	20	26	27	32	35	37	35

Let L_1 be the regression line of x on y. The equation of the line L_1 can be written in the form $x = ay + b$.

(a) Find the value of a and the value of b. [2]

Let L_2 be the regression line of y on x. The lines L_1 and L_2 pass through the same point with coordinates (p, q).

(b) Find the value of p and the value of q. [3]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The length, $X\,\text{mm}$, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

(a) Find $\text{P}(24.15 < X < 25)$. [2]

(b) (i) Find σ, the standard deviation of X.

(ii) Hence, find the probability that a seashell selected at random has a length greater than $26\,\text{mm}$. [5]

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than $26\,\text{mm}$.

(c) Find $\text{E}(Y)$. [3]

(d) Find the probability that exactly three of these seashells have a length greater than $26\,\text{mm}$. [2]

A seashell selected at random has a length less than $26\,\text{mm}$.

(e) Find the probability that its length is between $24.15\,\text{mm}$ and $25\,\text{mm}$. [3]

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Question 8 - working space

Section A

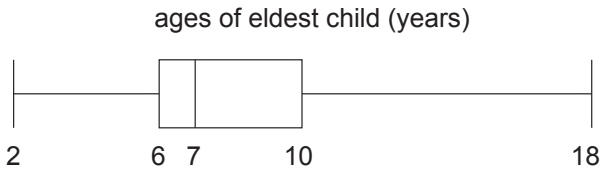
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 7]

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.

diagram not to scale



(a) Find the largest value of c that would not be considered an outlier. [3]

The regression line of a on c is $a = \frac{7}{4}c + 20$. The regression line of c on a is $c = \frac{1}{2}a - 9$.

(b) (i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child.

(ii) Find the mean age of all the adults surveyed. [4]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$\mathrm{P}(X=x)$	p	0.3	q	0.1

For this probability distribution, it is known that $\mathrm{E}(X)=2$.

(a) Show that $p=0.4$ and $q=0.2$. [5]

(b) Find $\mathrm{P}(X>2)$. [2]

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

(c) Assuming that rolls of the die are independent, find the probability that Nicky wins the game. [5]

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.

Both yellow dice have faces labelled 1, 2, 3 and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$\mathrm{P}(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

The first red die has faces labelled 1, 2, 2 and 3. The second red die has faces labelled 1, a, a and b, where $a<b$ and $a, b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

(d) Determine the value of b. [2]

(e) Find the value of a, providing evidence for your answer. [2]

References:

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 4]

The number of hours spent exercising each week by a group of students is shown in the following table.

Exercising time (in hours)	**Number of students**
2	5
3	1
4	4
5	3
6	x

The median is 4.5 hours.

(a) Find the value of x. [2]

(b) Find the standard deviation. [2]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

A discrete random variable, X, has the following probability distribution:

x	0	1	2	3
$\mathrm{P}(X=x)$	0.41	$k-0.28$	0.46	$0.29-2k^2$

(a) Show that $2k^2-k+0.12=0$. [1]

(b) Find the value of k, giving a reason for your answer. [3]

(c) Hence, find $\mathrm{E}(X)$. [2]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

Let A and B be two independent events such that $\mathrm{P}(A \cap B') = 0.16$ and $\mathrm{P}(A' \cap B) = 0.36$.

(a) Given that $\mathrm{P}(A \cap B) = x$, find the value of x. [4]

(b) Find $\mathrm{P}(A' \mid B')$. [2]

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of $62\,\text{g}$ and standard deviation of $2.9\,\text{g}$.

(a) Find the probability that a randomly selected chocolate muffin weighs less than $61\,\text{g}$. [2]

(b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than $61\,\text{g}$. [2]

The weights, B grams, of the banana muffins are normally distributed with a mean of $68\,\text{g}$ and standard deviation of $3.4\,\text{g}$.

Each day $60\,\%$ of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

(c) (i) Find the probability that the randomly selected muffin weighs less than $61\,\text{g}$.

(ii) Given that a randomly selected muffin weighs less than $61\,\text{g}$, find the probability that it is chocolate. [7]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to $\sigma\,\text{g}$. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than $61\,\text{g}$ is now 0.157.

(d) Find the value of σ. [5]

References:

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Answers written on this page
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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

Gemma and Kaia started working for different companies on January 1st 2011.

Gemma's starting annual salary was $\$45\,000$, and her annual salary increases $2\,\%$ on January 1st each year after 2011.

(a) Find Gemma's annual salary for the year 2021, to the nearest dollar. [3]

Kaia's annual salary is based on a yearly performance review. Her salary for the years 2011, 2013, 2014, 2018, and 2022 is shown in the following table.

year (x)	2011	2013	2014	2018	2022
annual salary $(\$S)$	45 000	47 200	48 500	53 000	57 000

(b) Assuming Kaia's annual salary can be approximately modelled by the equation $S = ax + b$, show that Kaia had a higher salary than Gemma in the year 2021, according to the model. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

Events A and B are independent and $\mathrm{P}(A) = 3\mathrm{P}(B)$.

Given that $\mathrm{P}(A \cup B) = 0.68$, find $\mathrm{P}(B)$.

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Question 4 - working space

Section A

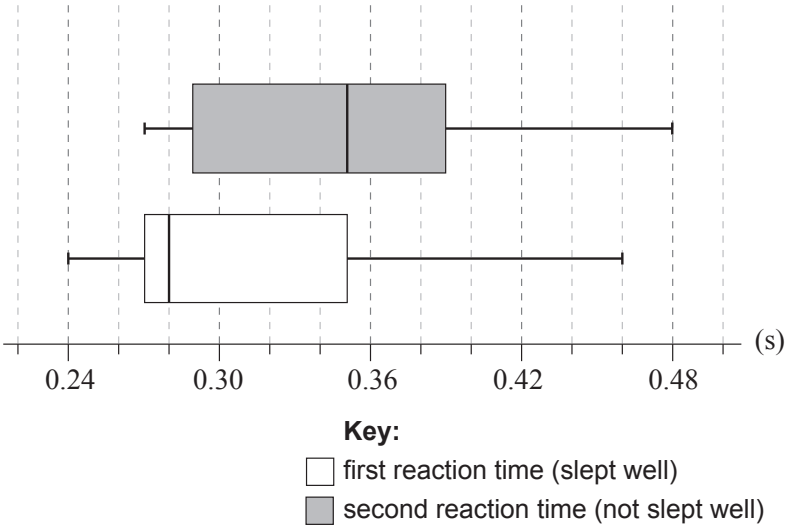
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

A random sample of nine adults were selected to see whether sleeping well affected their reaction times to a visual stimulus. Each adult's reaction time was measured twice.

The first measurement for reaction time was taken on a morning after the adult had slept well. The second measurement was taken on a morning after the same adult had not slept well.

The box and whisker diagrams for the reaction times, measured in seconds, are shown below.



Consider the box and whisker diagram representing the reaction times after sleeping well.

(a) State the median reaction time after sleeping well. [1]

(b) Verify that the measurement of 0.46 seconds is not an outlier. [3]

(c) State why it appears that the mean reaction time is greater than the median reaction time. [1]

Now consider the two box and whisker diagrams.

(d) Comment on whether these box and whisker diagrams provide any evidence that might suggest that not sleeping well causes an increase in reaction time. [1]

(This question continues on the following page)

(Question 5 continued)

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 18]

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

(a) Find the value of σ. [4]

(b) On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. [2]

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

(c) Find the probability that she will be late to work at least one day next week. [3]

(d) Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. [5]

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

(e) Find the probability that Suzi will receive a bonus. [4]

References:

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.8$.

(a) Determine the value of $\mathrm{P}(A \cap B)$ in the case where the events A and B are independent. [1]

(b) Determine the minimum possible value of $\mathrm{P}(A \cap B)$. [3]

(c) Determine the maximum possible value of $\mathrm{P}(A \cap B)$, justifying your answer. [2]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following table shows the Mathematics test scores (x) and the Science test scores (y) for a group of eight students.

Mathematics scores (x)	64	68	72	75	80	82	85	86
Science scores (y)	67	72	77	76	84	83	89	91

The regression line of y on x for this data can be written in the form $y = ax + b$.

(a) Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use the equation of your regression line to predict the Science test score for a student who has a score of 78 on the Mathematics test. Express your answer to the nearest integer. [2]

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Question 1 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

The time worked, T, in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

(a) Find the probability that an employee selected at random works more than 40 hours per week. [2]

(b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week. [3]

(c) A large group of employees work more than 40 hours per week.

(i) An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week.

(ii) Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week. [7]

It is known that $\mathrm{P}(a \leq T \leq b) = 0.904$ and that $\mathrm{P}(T > b) = 2\mathrm{P}(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

(d) Find the maximum time, in hours per week, that an employee can work and still be considered part-time. [4]

References:

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

On a Monday at an amusement park, a sample of 40 visitors was randomly selected as they were leaving the park. They were asked how many times that day they had been on a ride called *The Dragon*. This information is summarized in the following frequency table.

Number of times on *The Dragon*	**Frequency**
0	6
1	16
2	13
3	2
4	3

It can be assumed that this sample is representative of all visitors to the park for the following day.

(a) For the following day, Tuesday, estimate

(i) the probability that a randomly selected visitor will ride *The Dragon*;

(ii) the expected number of times a visitor will ride *The Dragon*. [4]

It is known that 1000 visitors will attend the amusement park on Tuesday. *The Dragon* can carry a maximum of 10 people each time it runs.

(b) Estimate the minimum number of times *The Dragon* must run to satisfy demand. [2]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

(a) Find the common difference, d. [2]

The following table shows the probability distribution of a discrete random variable X such that $\mathrm{P}(X = n) = \frac{u_n}{k}$, where $n \in \mathbb{Z}^+$, $1 \leq n \leq 4$ and $k \in \mathbb{R}^+$.

n	1	2	3	4
$\mathrm{P}(X = n)$	$\frac{0.6}{k}$	$\frac{u_2}{k}$	$\frac{u_3}{k}$	$\frac{0.15}{k}$

(b) Find the value of k. [4]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

Events A and B are such that $\mathrm{P}(A) = 0.4$, $\mathrm{P}(A \mid B) = 0.25$ and $\mathrm{P}(A \cup B) = 0.55$.

Find $\mathrm{P}(B)$.

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

The total number of children, y, visiting a park depends on the highest temperature, T, in degrees Celsius ($°C$). A park official predicts the total number of children visiting his park on any given day using the model $y = -0.6T^2 + 23T + 110$, where $10 \leq T \leq 35$.

(a) Use this model to estimate the number of children in the park on a day when the highest temperature is $25\,°C$. [2]

An ice cream vendor investigates the relationship between the total number of children visiting the park and the number of ice creams sold, x. The following table shows the data collected on five different days.

Total number of children (y)	81	175	202	346	360
Ice creams sold (x)	15	27	23	35	46

(b) Find an appropriate regression equation that will allow the vendor to predict the number of ice creams sold on a day when there are y children in the park. [3]

(c) Hence, use your regression equation to predict the number of ice creams that the vendor sells on a day when the highest temperature is $25\,°C$. [2]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

A company manufactures metal tubes for bicycle frames. The diameters of the tubes, $D\,\text{mm}$, are normally distributed with mean 32 and standard deviation σ. The interquartile range of the diameters is 0.28.

Find the value of σ.

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 17]

In a large city, 160 people were surveyed. Of those, 60 were children (C) and the rest adults (A).

Each person in the survey was asked whether they preferred milk chocolate (M) or dark chocolate (D). It was found that 48 of the children preferred milk chocolate. This information is shown in the following table.

	M **(milk chocolate)**	***D*** **(dark chocolate)**	**Total**
C **(children)**	48	p	60
A **(adults)**	x	y	q

(a) Find the value of

(i) p;

(ii) q. [2]

(b) Three people are chosen at random from those surveyed. Find the probability that all three are adults. [4]

(c) (i) Given that $\mathrm{P}(A \mid M) = \frac{1}{3}$, find the value of x.

(ii) A person is chosen at random from those surveyed. Write down the probability that they are an adult who prefers milk chocolate. [4]

(d) Determine if the events A and M are independent. Justify your answer. [3]

It can be assumed that the survey results are representative of the population of the city.

(e) Ten people in the city are chosen at random. Find the probability that at least five of them prefer dark chocolate. [4]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A botanist is conducting an experiment which studies the growth of plants.

The heights of the plants are measured on seven different days.

The following table shows the number of days, d, that the experiment has been running and the average height, $h\,\text{cm}$, of the plants on each of those days.

Number of days (d)	2	5	13	24	33	37	42
Average height (h)	10	16	30	59	76	79	82

(a) The regression line of h on d for this data can be written in the form $h = ad + b$.

Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use your regression line to estimate the average height of the plants when the experiment has been running for 20 days. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 8]

The weights, W grams, of bags of rice packaged in a factory can be modelled by a normal distribution with mean 204 grams and standard deviation 5 grams.

(a) A bag of rice is selected at random.

Find the probability that it weighs more than 210 grams. [2]

According to this model, $80\,\%$ of the bags of rice weigh between w grams and 210 grams.

(b) Find the probability that a randomly selected bag of rice weighs less than w grams. [2]

(c) Find the value of w. [2]

(d) Ten bags of rice are selected at random.

Find the probability that exactly one of the bags weighs less than w grams. [2]

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

A bag contains n balls. It is known that ten of the balls are green, and the rest of the balls are red. Balls are drawn from the bag, one after the other, without replacement.

(a) Find, in terms of n, the probability that

(i) the first ball drawn is green;

(ii) the first two balls are green. [3]

For the following parts of this question, let $n = 25$.

(b) Show that the probability that the first two balls are red is 0.35. [2]

(c) Find the probability that the first three balls are all red. [2]

(d) Find the probability that at least one of the first three balls is green. [2]

A game is played where **four** balls are drawn, one after the other, from the bag of 25 balls, without replacement. A player earns points based on when the first green ball is drawn. At the end of each game, the four balls are put back in the bag.

A player earns zero points if no green ball is picked, or if the first green ball is picked on the first or second draw.

A player earns 10 points if the first green ball is picked on the third draw and earns 50 points if the first green ball is picked on the fourth draw.

Millie plays this game k times. She finds her score by adding together her points from each game.

(e) Find the least value of k such that Millie's expected score is greater than 100. [6]

References:

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X, where $a, k \in \mathbb{R}^+$.

x	1	2	3	4
$\mathrm{P}(X = x)$	k	k^2	a	k^3

Given that $\mathrm{E}(X) = 2.6$, find the value of a.

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

A farmer is growing a field of rice plants. The height, $H\,\text{cm}$, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\text{P}(H < 82.4) = 0.213$ and $\text{P}(H > 87.3) = 0.409$.

(a) Find the probability that the height of a randomly selected plant is between $82.4\,\text{cm}$ and $87.3\,\text{cm}$. [2]

(b) Find the value of μ and the value of σ. [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than $87.3\,\text{cm}$ is considered ready to harvest. Heights of plants are independent of each other.

(c) (i) Find the probability that exactly 32 plants are ready to harvest.

(ii) Given that fewer than 44 plants are ready to harvest, find the probability that exactly 32 plants are ready to harvest. [6]

In another field, the farmer is growing the same variety of rice, but is using a different fertilizer. The heights of these plants, $F\,\text{cm}$, are normally distributed with mean 92.8 and standard deviation d. The farmer finds the interquartile range to be $4.52\,\text{cm}$.

(d) Find the value of d. [3]

Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X, where $a, k \in \mathbb{R}^+$.

x	1	2	3	4
$\mathrm{P}(X = x)$	k	k^2	a	k^3

Given that $\mathrm{E}(X) = 2.3$, find the value of a.

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

A farmer is growing a field of wheat plants. The height, $H\,\text{cm}$, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\text{P}(H < 94.6) = 0.288$ and $\text{P}(H > 98.1) = 0.434$.

(a) Find the probability that the height of a randomly selected plant is between $94.6\,\text{cm}$ and $98.1\,\text{cm}$. [2]

(b) Find the value of μ and the value of σ. [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than $98.1\,\text{cm}$ is considered ready to harvest. Heights of plants are independent of each other.

(c) (i) Find the probability that exactly 34 plants are ready to harvest.

(ii) Given that fewer than 49 plants are ready to harvest, find the probability that exactly 34 plants are ready to harvest. [6]

In another field, the farmer is growing the same variety of wheat, but is using a different fertilizer. The heights of these plants, $F\,\text{cm}$, are normally distributed with mean 98.6 and standard deviation d. The farmer finds the interquartile range to be $4.82\,\text{cm}$.

(d) Find the value of d. [3]

Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

Claire rolls a six-sided die 16 times.

The scores obtained are shown in the following frequency table.

Score	Frequency
1	p
2	q
3	4
4	2
5	0
6	3

It is given that the mean score is 3.

(a) Find the value of p and the value of q. [5]

Each of Claire's scores is multiplied by 10 in order to determine the final score for a game she is playing.

(b) Write down the mean final score. [1]

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Question 2 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

A bag contains buttons which are either red or blue.

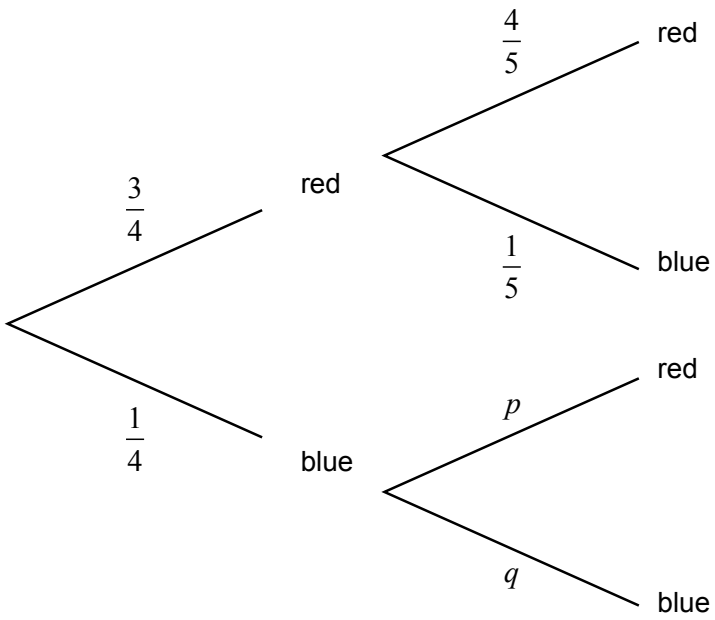
Initially, the bag contains three red buttons and one blue button.

Francine randomly selects one button from the bag. She then replaces the button and adds one extra button of the same colour.

For example, if she selects a red button, she then replaces it and adds one extra red button so that the bag then contains four red buttons and one blue button.

Francine then randomly selects a second button from the bag.

The following tree diagram represents the probabilities of the first two selections.



(a) Find the value of p and the value of q. [2]

(b) Show that the probability that Francine selects two buttons of the same colour is $\frac{7}{10}$. [2]

(c) Given that Francine selects two buttons of the same colour, find the probability that she selects two red buttons. [3]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 9 continued)

The random variable X is defined as the number of red buttons selected by Francine.

The following table shows the probability distribution of X.

x	0	1	2
$\mathrm{P}(X = x)$	$\frac{1}{10}$	a	b

(d) Find the value of a and the value of b. [2]

(e) Hence, find the expected number of red buttons selected by Francine. [2]

Francine restarts the process with three red buttons and one blue button in the bag. She selects buttons as before, replacing the button and adding one extra button of the same colour each time. She repeats this until she selects a blue button.

(f) Given that the first two buttons she selects are red, write down the probability that the next button she selects is blue. [1]

The probability that she selects the first blue button after n selections in total is $\frac{3}{56}$.

(g) Find the value of n. [5]

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

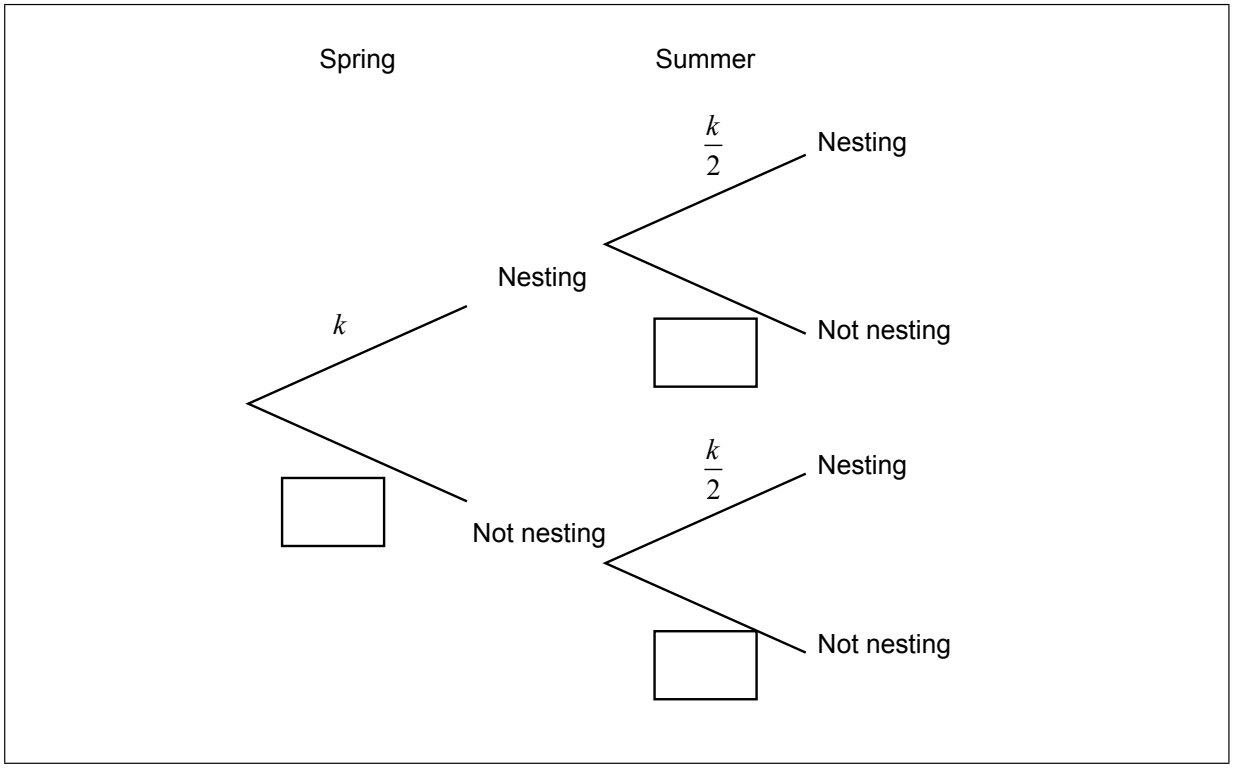
5. [Maximum mark: 6]

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k.

The probability of nesting in Summer is $\frac{k}{2}$.

This is shown in the following tree diagram.



(a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k. [2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\frac{5}{9}$.

(b) (i) Show that $9k^2 - 27k + 8 = 0$.

(ii) Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution. [4]

(This question continues on the following page)

(Question 5 continued)

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

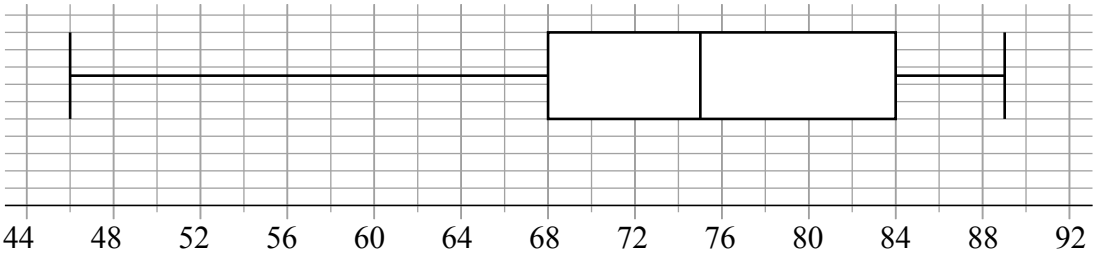
Janie claims that rabbits in Australia have longer ears than rabbits in Spain.

To test her claim, a randomly selected sample of rabbits was collected in each country.

The length of one ear of each rabbit was measured and the value recorded correct to the nearest millimetre (mm).

In the Australian sample, the median recorded value was $80\,\text{mm}$ and the interquartile range was $11\,\text{mm}$.

The recorded values for the Spanish sample are shown in the following box and whisker diagram.



(a) Complete the following table for the recorded values of the lengths of the rabbits' ears in each sample. [3]

	Australia	**Spain**
Median (mm)	80	
Interquartile range (mm)	11	

(This question continues on the following page)

(Question 1 continued)

(b) Justifying your answers, compare the distributions of the lengths of rabbits' ears in Australia and Spain using

(i) the median;

(ii) the interquartile range. [4]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 4]

The random variable X is normally distributed with mean 10 and standard deviation 2.

(a) Find the probability that X is more than 1.5 standard deviations above the mean. [2]

The probability that X is more than k standard deviations above the mean is 0.1, where $k \in \mathbb{R}$.

(b) Find the value of k. [2]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

A class is given two tests, Test A and Test B. Each test is scored out of a total of 100 marks. The scores of the students are shown in the following table.

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Test A	52	71	100	93	81	80	88	100	70	61
Test B	58	80	92	98	90	82	100	100	65	74

Let x be the score on Test A and y be the score on Test B.

The teacher finds that the equation of the regression line of y on x for these scores is $y = 0.822x + 18.4$.

(a) Find the value of Pearson's product-moment correlation coefficient, r. [2]

Giovanni was absent for Test A and Paulo was absent for Test B.

The teacher uses the regression line of y on x to estimate the missing scores.

Paulo scored 10 on Test A.

The teacher estimated his score on Test B to be 27 to the nearest integer using the following calculation:

$$y = 0.822(10) + 18.4 \approx 27$$

(b) Give a reason why this method is not appropriate for Paulo. [1]

Giovanni scored 90 on Test B.

The teacher estimated his score on Test A to be 87 to the nearest integer using the following calculation:

$$90 = 0.822x + 18.4, \text{ so } x = \frac{90 - 18.4}{0.822} \approx 87$$

(c) (i) Give a reason why this method is not appropriate for Giovanni.

(ii) Use an appropriate method to show that the estimated Test A score for Giovanni is 86 to the nearest integer. [4]

(This question continues on the following page)

(Question 6 continued)

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will not be marked.

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

In a study, the mobile phone usage of a random sample of ten students was examined on a particular day.

The length of time, t hours, that the ten students used their phones for are listed below.

$$0.7 \quad 1.2 \quad 1.9 \quad 4.0 \quad 4.4 \quad 4.5 \quad 4.9 \quad 5.7 \quad 6.5 \quad 11.7$$

(a) For these data, find the

(i) median;

(ii) interquartile range. [3]

An outlier is a value that is less than $Q_1 - 1.5 \times \text{IQR}$ or greater than $Q_3 + 1.5 \times \text{IQR}$.

(b) Show that 11.7 is an outlier. [3]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider the following bivariate data set where $p, q \in \mathbb{Z}^+$.

x	5	6	6	8	10
y	9	13	p	q	21

The regression line of y on x has equation $y = 2.1875x + 0.6875$.

The regression line passes through the mean point $(\overline{x}, \overline{y})$.

(a) Given that $\overline{x} = 7$, verify that $\overline{y} = 16$. [1]

(b) Given that $q - p = 3$, find the value of p and the value of q. [4]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 5]

Consider a random variable X such that $X \sim \mathrm{B}(n, 0.25)$.

Determine the least value of n such that $\mathrm{P}(X \geq 1) > 0.99$.

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A lake contains a type of fish called carp. The lengths, $L\,\text{cm}$, of the carp can be modelled by a normal distribution with mean $45.6\,\text{cm}$ and standard deviation $4.2\,\text{cm}$.

According to this model, carp with a length between $41.4\,\text{cm}$ and $k\,\text{cm}$ lie within one standard deviation of the mean.

(a) Write down the value of k. [2]

(b) Find the probability that a randomly selected carp is greater than $48\,\text{cm}$ in length. [2]

(c) It is known that 99% of carp in the lake have a length greater than $x\,\text{cm}$. Find the value of x. [2]

(d) Consider a random sample of 100 carp from the lake.

(i) Find the expected number of carp with lengths between $40\,\text{cm}$ and $56\,\text{cm}$.

(ii) Find the probability that in this sample, exactly 95 carp have a length between $40\,\text{cm}$ and $56\,\text{cm}$. [5]

A large sample of carp from the lake is studied. The length of each fish is measured and recorded correct to the nearest $0.1\,\text{cm}$.

(e) Find the probability that a randomly selected carp has a length recorded as $45.6\,\text{cm}$. [3]

Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the following set of ordered data.

27	27	31	32	33	37	41	46	54	57

(a) Write down

(i) the mode;

(ii) the range;

(iii) the median. [3]

(b) Find the interquartile range. [3]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.45$, $\mathrm{P}(B) = 0.65$ and $\mathrm{P}(A \cup B) = 0.8$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Find $\mathrm{P}(A' | B')$. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the following set of ordered data.

22	22	26	27	28	32	36	38	39	42

(a) Write down

(i) the mode;

(ii) the range;

(iii) the median. [3]

(b) Find the interquartile range. [3]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.45$ and $\mathrm{P}(A \cup B) = 0.85$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Find $\mathrm{P}(A' \mid B')$. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

A discrete random variable, X, has the following probability distribution:

$$\mathrm{P}(X = x) = \frac{kx}{15} \text{ for } x \in \{2, 4, 7, 10\}.$$

(a) Find the value of k. [2]

(b) Find $\mathrm{E}(X)$. [3]

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

MyLife is a social media platform with 89.8 million users, all aged 12 years old and above. The following frequency table shows the number of users by age group.

Age, a (years)	Millions of users
$12 \leq a < 18$	5.8
$18 \leq a < 35$	42.7
$35 \leq a < 55$	26.3
$55 \leq a < 75$	12.9
$75 \leq a < 95$	2.1
Total	**89.8**

(a) Find the percentage of *MyLife* users aged 55 years or older. [1]

(b) A sample of 1000 *MyLife* users is chosen at random. Find the probability that fewer than 150 of them are aged 55 years or older. [3]

(c) Given that a *MyLife* user chosen at random is 55 years or older, find the probability that they are 75 years or older. [4]

(d) List the mid-interval value for each class interval. [1]

(e) Hence, for *MyLife* users, estimate

(i) the mean age;

(ii) the variance of the ages. [3]

(This question continues on the following page)

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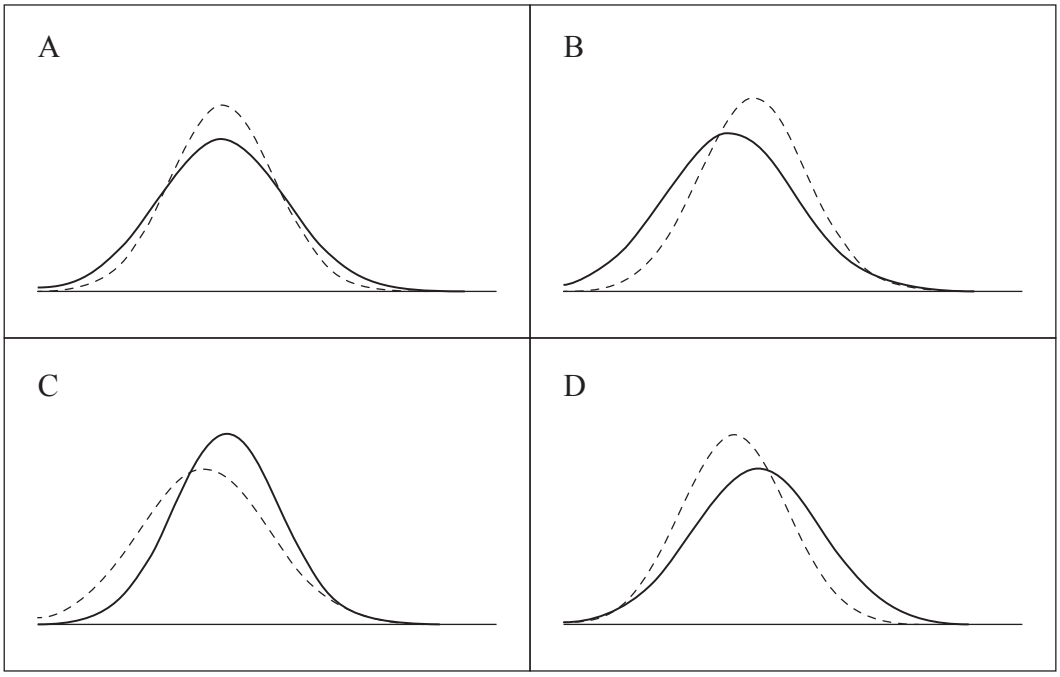
(Question 7 continued)

A different social media platform, *SmallTalk*, reports that its users have a mean age of 29.9 years and a variance of 137 years^2.

The following four diagrams represent age distributions.

(f) (i) Identify the diagram which best represents the age distributions for the users of *MyLife* and *SmallTalk*.

(ii) In your chosen diagram, identify which social media platform is represented by the dotted line. [2]



Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

A discrete random variable, X, has the following probability distribution:

$$\mathrm{P}(X = x) = \frac{kx}{20} \text{ for } x \in \{3, 5, 8, 11\}.$$

(a) Find the value of k. [2]

(b) Find $\mathrm{E}(X)$. [3]

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

MyLife is a social media platform with 93.6 million users, all aged 13 years old and above. The following frequency table shows the number of users by age group.

Age, a (years)	Millions of users
$13 \leq a < 18$	5.5
$18 \leq a < 25$	23.6
$25 \leq a < 45$	43.5
$45 \leq a < 65$	17.3
$65 \leq a < 85$	3.7
Total	**93.6**

(a) Find the percentage of *MyLife* users aged 45 years or older. [1]

(b) A sample of 1000 *MyLife* users is chosen at random. Find the probability that fewer than 200 of them are aged 45 years or older. [3]

(c) Given that a *MyLife* user chosen at random is 45 years or older, find the probability that they are 65 years or older. [4]

(d) List the mid-interval value for each class interval. [1]

(e) Hence, for *MyLife* users, estimate

(i) the mean age;

(ii) the variance of the ages. [3]

(This question continues on the following page)

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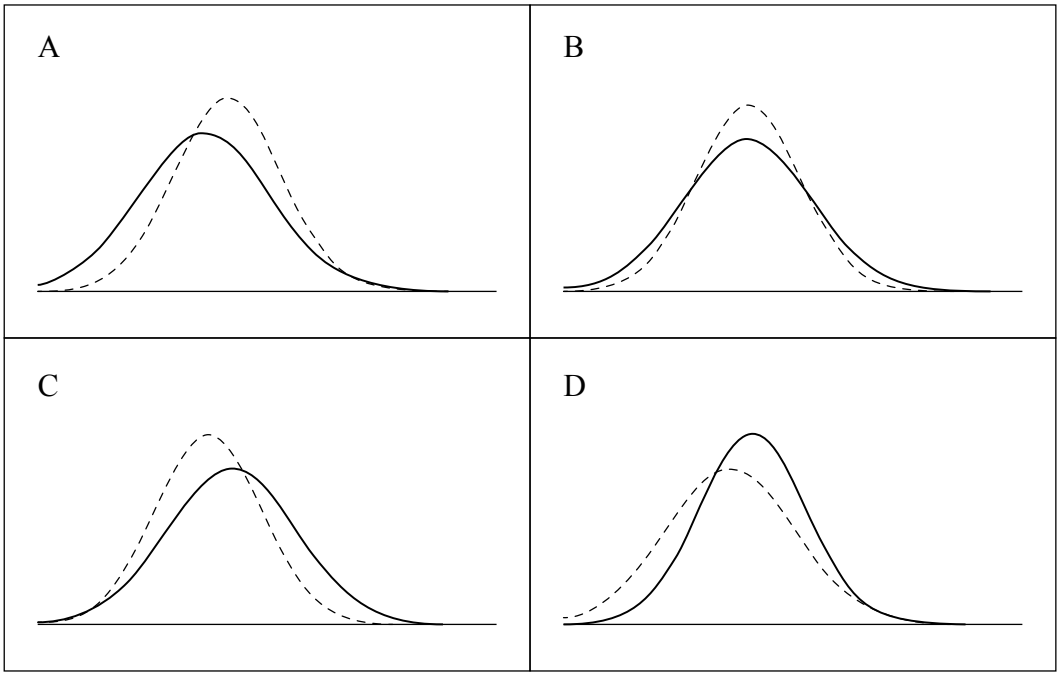
(Question 7 continued)

A different social media platform, *SmallTalk*, reports that its users have a mean age of 29.9 years and a variance of 137 years^2.

The following four diagrams represent age distributions.

(f) (i) Identify the diagram which best represents the age distributions for the users of *MyLife* and *SmallTalk*.

(ii) In your chosen diagram, identify which social media platform is represented by the solid line. [2]



Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

Consider events A and B such that $\mathrm{P}(A') = \mathrm{P}(A \cup B) = \frac{3}{4}$ and $\mathrm{P}(B|A) = \frac{2}{3}$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Show that events A and B are independent. [3]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

A discrete random variable, X, has the following probability distribution, where $a > 0$ and k is a constant.

x	0	a	$2a$	$3a$
$\mathrm{P}(X = x)$	k	$3k^2$	$2k^2$	k^2

(a) Show that $k = \frac{1}{3}$. [5]

(b) Find $\mathrm{P}(X < 3a)$. [2]

(c) Find $\mathrm{P}(X \geq a \mid X < 3a)$. [3]

(d) Given that $\mathrm{E}(X) = 20$, find the value of a. [3]

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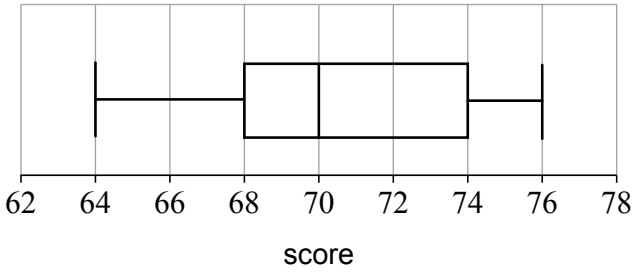
Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The scores achieved by 80 golfers in a competition are summarized in the following box and whisker diagram.



(a) Find the interquartile range. [2]

(b) Find the number of golfers that scored between 70 and 74. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

The random variables X and Y are normally distributed with $X \sim \mathrm{N}(7, a^2)$ and $Y \sim \mathrm{N}(19, a^2)$, where $a > 0$.

(a) Find b such that $\mathrm{P}(X > b) = \mathrm{P}(Y > 22)$. [2]

(b) Write down the approximate value of $\mathrm{P}(7 - a < X < 7 + a)$, correct to two significant figures. [1]

(c) Given that $a = 3$, calculate the approximate value of $\mathrm{P}(Y < 22)$, correct to two significant figures. [3]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 8]

In a study, measurements for arm span, $A\,\text{cm}$, and foot length, $F\,\text{cm}$, are taken from a large group of adults.

For this group, the regression line of F on A is found to be $F = 0.335A - 32.6$, and the regression line of A on F is found to be $A = 2.89F + 99.3$. Each regression line passes through the mean point.

(a) By using an appropriate regression line, find an estimate of the arm span for an adult with a foot length of $19.8\,\text{cm}$. [2]

(b) For this group of adults, find the mean arm span and the mean foot length. [3]

The heights, $H\,\text{cm}$, of adults in the group can be modelled by a normal distribution with mean $163\,\text{cm}$ and standard deviation $\sigma\,\text{cm}$.

It is found that 88% of the group have a height between $153\,\text{cm}$ and $173\,\text{cm}$.

(c) Find the value of σ. [3]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Lynn is playing a game with two unbiased six-sided dice, each with faces marked with the integers from 1 to 6.

In each round, she throws both dice once. The outcomes can be displayed in the following sample space diagram, which has been partially completed:

Die 1 \ Die 2	1	2	3	4	5	6
1						
2		2,2				
3				3,4		
4						
5			5,3			
6						

Lynn scores points according to the following rules.

- If the two dice show the same score, she scores 10 points.
- If the two dice show scores which have a difference of one, for example the scores 4 and 5 in any order, she scores 5 points.
- Otherwise, she scores 0 points.

(a) Show that the probability that Lynn scores 5 points in one round is $\frac{5}{18}$. [2]

(b) Find the probability that Lynn scores no points in one round. [2]

The random variable X represents the number of points Lynn scores in one round.

(c) Find $\mathrm{E}(X)$. [4]

(d) Hence, estimate the total number of points that Lynn scores if she plays 90 rounds. [2]

A prize is awarded to any player who scores more than 40 points in total.

Lynn plays exactly five rounds.

(e) Find the probability that Lynn wins a prize. [4]

Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The following table shows the number of hours of play time, x, and sleep time, y, for a group of six children, over the period of one week.

Play time (x)	11	13	14	17	22	24
Sleep time (y)	62	65	68	75	84	87

The regression line of y on x for this data can be written in the form $y = ax + b$.

(a) Find the value of a and the value of b. [2]

(b) Use the equation of the regression line to estimate the sleep time of a child whose weekly play time is 20 hours. [2]

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Question 1 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 17]

At Adam's Apple Orchard the weights of apples, W, in grams, are normally distributed with a mean 175 grams and standard deviation 8 grams.

(a) Find the probability that a randomly chosen apple weighs less than 170 grams. [2]

(b) It is found that 20% of the apples weigh more than w grams. Find w, correct to four significant figures. [2]

All orchards classify an apple as premium when its weight is between 170 and 185 grams.

(c) Find the percentage of apples that are classified as premium at Adam's Apple Orchard. [2]

After orders are completed, there are many apples left over. Boxes are filled with randomly chosen left-over apples. Each box contains 40 apples.

(d) Find the probability that a randomly chosen box contains at least 30 premium apples. [3]

(e) If 10 of these boxes are randomly selected, find the probability that exactly 4 boxes have at least 30 premium apples. [2]

At a neighbouring orchard the weights of apples, M, in grams, are normally distributed with mean μ and standard deviation σ. It is known that:

- 82% of their apples are classified as premium
- the percentage of apples that weigh less than 170 grams is twice the percentage of apples that weigh more than 185 grams.

(f) Find the value of μ. [6]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

A supermarket analyses the shopping habits of its customers.

The number of times, X, each customer visits the supermarket in a week is given by the following probability distribution.

x	1	2	3	4	5	≥ 6
$\mathrm{P}(X = x)$	$1.5a$	$2a$	0.281	a	0.026	0

(a) (i) Find the value of a.

(ii) Write down the mode of X. [3]

(b) Find the mean of X. [2]

The manager wants to know why customers come to their supermarket. They survey the first 50 customers to arrive at the supermarket on a particular day.

(c) Identify which one of the following best describes the manager's sampling method. Circle your answer. [1]

Simple random / Systematic / Convenience / Quota / Stratified

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 17]

Consider a discrete random variable X.

(a) State two conditions required for X to be modelled by a binomial distribution. [2]

A water theme park has two rides: *Daifong* and *Torbellino*. Each visitor's decision to ride on either *Daifong* or *Torbellino* is made independently of any other person.

From previous records, it is expected that 37% of the visitors on any particular day will ride *Daifong*.

On Saturday, 1900 people will visit the theme park.

(b) Find the number of people that are expected to ride *Daifong*. [2]

(c) Find the probability that

(i) 712 people will ride *Daifong*;

(ii) between 684 and 712 people, inclusive, will ride *Daifong*. [4]

(d) Given that between 684 and 712 people, inclusive, will ride *Daifong*, find the probability that at most 692 people will ride *Daifong*. [4]

The ride *Torbellino* is more popular at the theme park. It is expected that 61% of the visitors on any particular day will ride *Torbellino*.

It can be assumed that the probability a person will ride *Daifong* is independent of them riding *Torbellino*.

(e) Find the probability that a person will ride both *Daifong* and *Torbellino*. [2]

Next Tuesday n people will visit the theme park. The probability that at most 500 people will ride *Torbellino* is approximately 0.693.

(f) Find the value of n. [3]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cap B) = 0.4$.

(a) Find $\mathrm{P}(A \cup B)$. [2]

(b) Show that A and B are **not** independent. [1]

(c) Find $\mathrm{P}(A \mid B')$. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The discrete random variable X has the following probability distribution:

x	1	2	3	4
$\mathrm{P}(X = x)$	0.2	0.3	k	0.1

(a) Find the value of k. [2]

(b) Find $\mathrm{P}(X > 1.5)$. [2]

(c) Find $\mathrm{E}(X)$. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A)=\frac{1}{2}$, $\mathrm{P}(B)=\frac{1}{3}$ and $\mathrm{P}(A\,|\,B)=\frac{1}{4}$.

(a) Find the probability that both events A and B occur. [2]

(b) Find $\mathrm{P}(A\,|\,B')$. [4]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

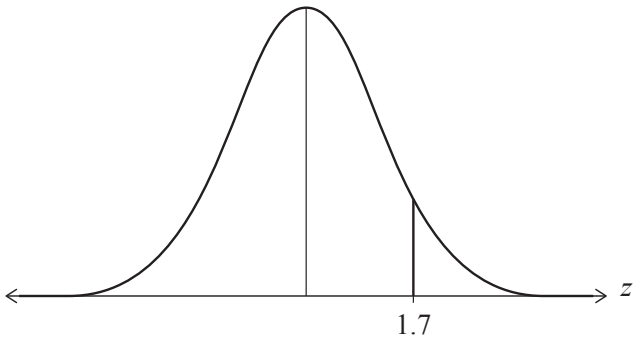
2. [Maximum mark: 4]

The random variable Z has a standard normal distribution with mean 0 and standard deviation 1.

(a) Find $\mathrm{P}(\mathrm{Z} < 1.7)$. [1]

It is known that $\mathrm{P}(a < \mathrm{Z} < 1.7) = 0.7865$, correct to four significant figures.

(b) Sketch this information on the following diagram. [1]



(c) Hence, or otherwise, find the value of a. [2]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 7]

Data was collected on two variables, x and y, from eight members of a population. The data is shown in the table below.

x	3.8	6.3	2.5	2.4	8.9	10.7	6.7	10.3
y	11.8	9.7	6.1	4.1	p	23.3	13.7	24.5

(a) Find $\overline{x}$. [2]

The regression line of y on x for this data is $y = 0.014375 + 2.1625x$.

(b) Find

(i) $\overline{y}$;

(ii) the value of p. [4]

(c) Hence, find Pearson's product-moment correlation coefficient, r. [1]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

Two machines are used in a factory to manufacture semiconductors. Machine A manufactures defective semiconductors 10% of the time, while machine B manufactures defective semiconductors 5% of the time.

A randomly selected machine manufactures ten semiconductors.

Both machines are equally likely to be selected.

(a) Find the probability that exactly two semiconductors are defective. [4]

(b) Given that exactly two semiconductors are defective, find the probability that they were manufactured by machine A. [3]

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Question 6 - working space

Section A

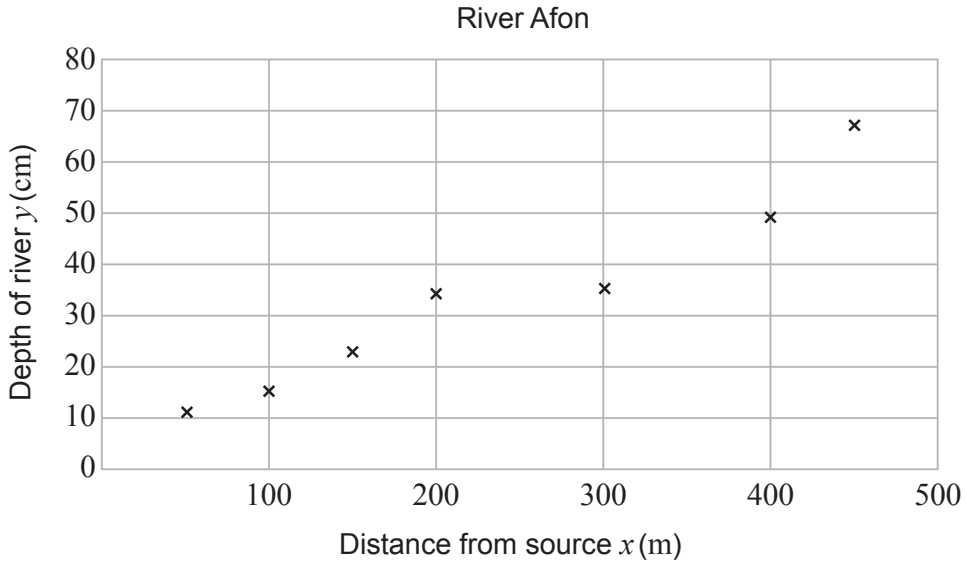
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Kiran and Logan collect the following data about the river Afon, where x is the distance in metres from the source and y is the depth in centimetres.

Distance from the source x (m)	50	100	150	200	300	400	450
Depth y (cm)	11	15	23	34	35	49	67

This data is represented in the following scatter diagram.



(This question continues on the following page)

(Question 1 continued)

Kiran knows that the depth of the river is $0\,\text{cm}$ at the source.

Kiran calculates $\bar{x}$ and $\bar{y}$ for the seven points given in the table on page 2 and draws a line on the scatter diagram through the mean point $(\bar{x}, \bar{y})$ and the point $(0, 0)$.

(a) Find

(i) the value of $\bar{x}$ and the value of $\bar{y}$;

(ii) the equation of Kiran's line. [3]

For the seven points given in the table Logan finds the regression line of y on x with equation $y = ax + b$, where $a, b \in \mathbb{R}$.

(b) Determine the value of a and the value of b. [2]

(c) By using the equation of Logan's regression line, estimate the depth of the river $350\,\text{m}$ from its source. [2]

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Question 1 - working space

Section A

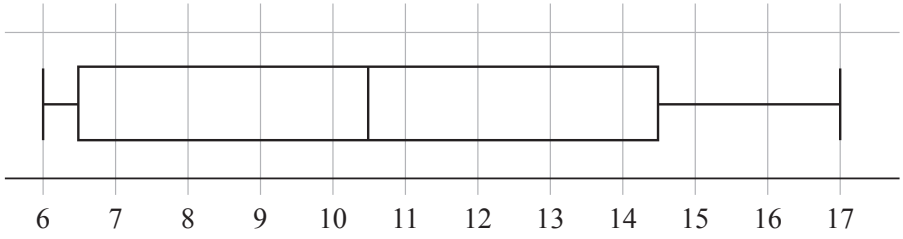
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

A teacher sets her class of 30 pupils a quiz.

Aiden and Brett were absent on the day of the quiz.

The following box and whisker diagram shows the results of the 28 pupils who took the quiz on the day.



Aiden and Brett take the quiz when they return.

Aiden scores less than 6.

Brett scores more than 17.

(a) Explain, briefly, why the median score for all 30 pupils would still be 10.5. [1]

The mean score of the 28 pupils was 10.5.

The mean score for all 30 pupils is now 10.6.

The range of scores for all 30 pupils is 14.

(b) Determine Aiden's score and Brett's score. [5]

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Question 4 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

A garden centre sells seeds for two varieties of sunflowers: large and giant.

The heights of large sunflowers are normally distributed with mean μ and standard deviation σ.

It is known that $25\,\%$ of large sunflowers have a height of over $180\,\text{cm}$.

(a) Given that $\mu + A\sigma = 180$, where $A \in \mathbb{R}$, find the value of A. [3]

The heights of giant sunflowers are also normally distributed.

The giant sunflowers have a mean height that is $35\,\text{cm}$ more than that of the large sunflowers and a standard deviation that is double that of the large sunflowers.

It is known that $98\,\%$ of giant sunflowers have a height greater than $180\,\text{cm}$.

(b) Determine μ and σ. [4]

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Question 6 - working space

APOLLO

Calculus

83 questions

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

Consider the functions $f(x) = -(x-h)^2 + 2k$ and $g(x) = \mathrm{e}^{x-2} + k$ where $h, k \in \mathbb{R}$.

(a) Find $f'(x)$. [1]

The graphs of f and g have a common tangent at $x = 3$.

(b) Show that $h = \frac{\mathrm{e}+6}{2}$. [3]

(c) Hence, show that $k = \mathrm{e} + \frac{\mathrm{e}^2}{4}$. [3]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $y = \frac{\ln x}{x^4}$ for $x > 0$.

(a) Show that $\frac{dy}{dx} = \frac{1 - 4\ln x}{x^5}$. [3]

Consider the function defined by $f(x) = \frac{\ln x}{x^4}$ for $x > 0$ and its graph $y = f(x)$.

(b) The graph of f has a horizontal tangent at point P. Find the coordinates of P. [5]

(c) Given that $f''(x) = \frac{20\ln x - 9}{x^6}$, show that P is a local maximum point. [3]

(d) Solve $f(x) > 0$ for $x > 0$. [2]

(e) Sketch the graph of f, showing clearly the value of the x-intercept and the approximate position of point P. [3]

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Question 8 - working space

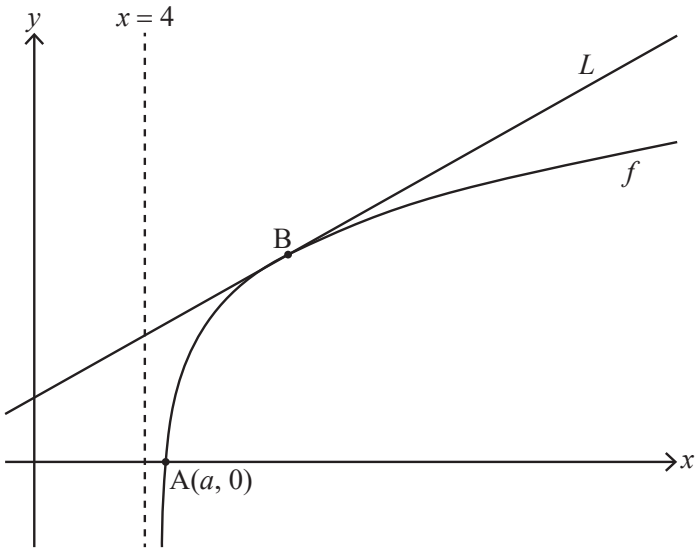
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 9]

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.



(a) Find the exact value of a. [3]

(b) Given that the gradient of L is $\frac{1}{3}$, find the x-coordinate of B. [6]

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Question 5 - working space

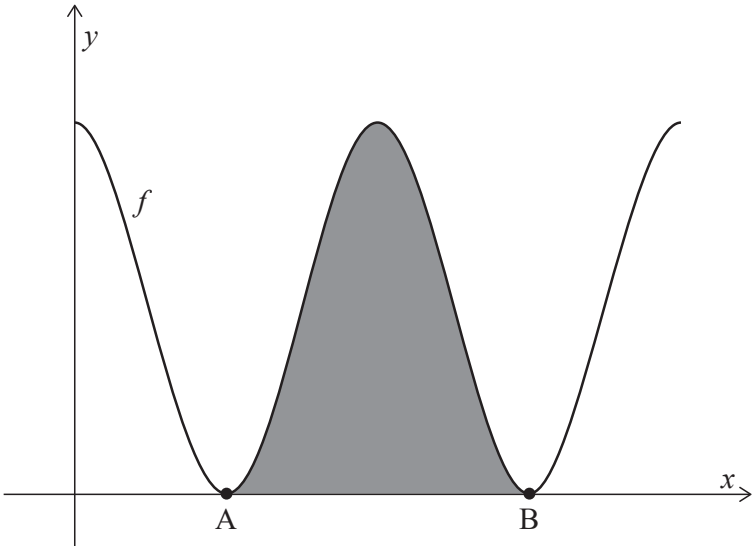
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the function f defined by $f(x) = 6 + 6\cos x$, for $0 \le x \le 4\pi$.

The following diagram shows the graph of $y = f(x)$.



The graph of f touches the x-axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x-axis, between the points A and B.

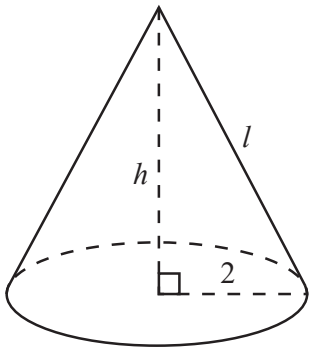
(a) Find the x-coordinates of A and B. [3]

(b) Show that the area of the shaded region is 12π. [5]

The right cone in the following diagram has a total surface area of 12π, equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h, and slant height l.

diagram not to scale



(c) Find the value of l. [3]

(d) Hence, find the volume of the cone. [4]

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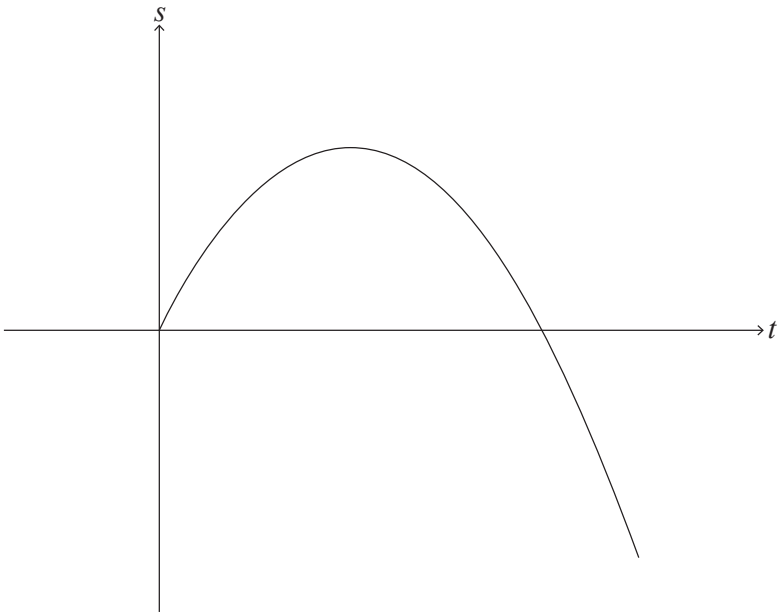
Question 8 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \le t \le 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

(a) Find the value of p. [2]

Particle A changes direction when $t = q$.

(b) (i) Find the value of q.

(ii) Find the displacement of particle A from the origin when $t = q$. [4]

(c) Find the distance of particle A from the origin when $t = 10$. [2]

The total distance travelled by particle A is given by d.

(d) Find the value of d. [2]

A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \ge 0$.

When $t = k$, the distance travelled by particle B is equal to d.

(e) Find the value of k. [4]

References:

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will not be marked.

Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

(a) Find $\int (6x+7)\mathrm{d}x$. [3]

(b) Given $f'(x) = 6x + 7$ and $f(1.2) = 7.32$, find $f(x)$. [3]

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Question 1 - working space

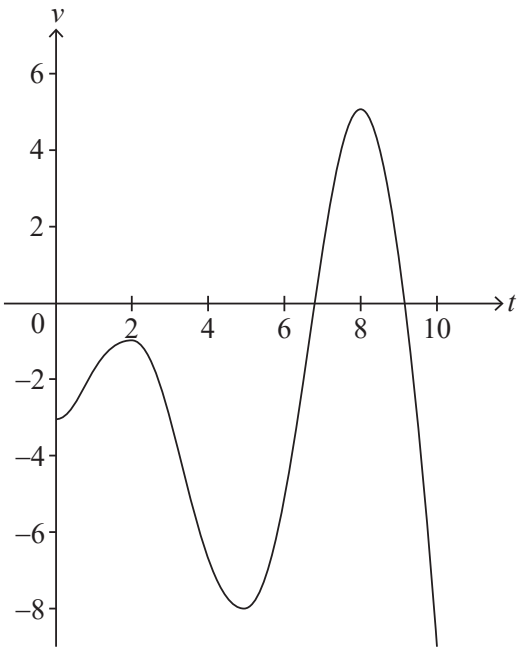
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\,\text{ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of v.



(a) Find the smallest value of t for which the particle is at rest. [2]

(b) Find the total distance travelled by the particle. [2]

(c) Find the acceleration of the particle when $t = 7$. [2]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider the function f defined by $f(x) = 90\,e^{-0.5x}$ for $x \in \mathbb{R}^+$.

The graph of f and the line $y = x$ intersect at point P.

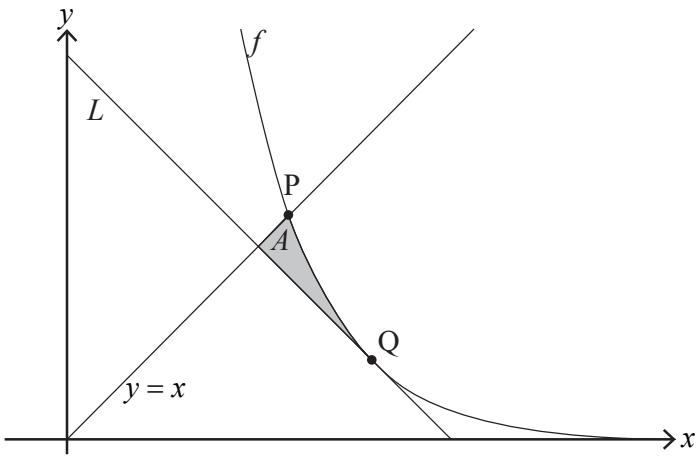
(a) Find the x-coordinate of P. [2]

The line L has a gradient of -1 and is a tangent to the graph of f at the point Q.

(b) Find the exact coordinates of Q. [4]

(c) Show that the equation of L is $y = -x + 2\ln 45 + 2$. [2]

The shaded region A is enclosed by the graph of f and the lines $y = x$ and L.



(d) (i) Find the x-coordinate of the point where L intersects the line $y = x$.

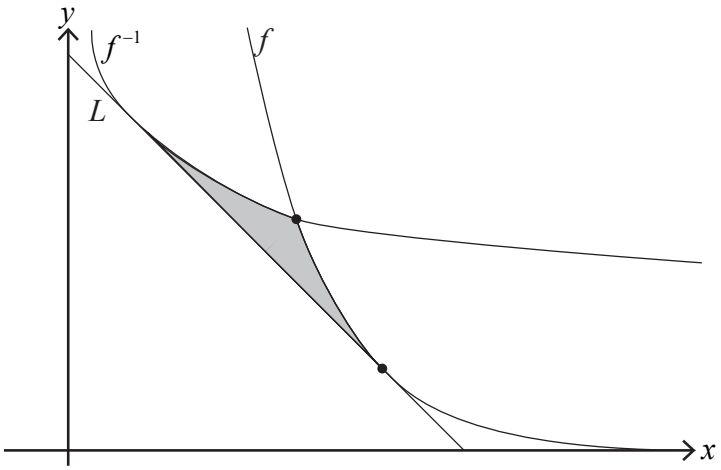
(ii) Hence, find the area of A. [5]

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(Question 9 continued)

The line L is tangent to the graphs of both f and the inverse function f^{-1}.



(e) Find the shaded area enclosed by the graphs of f and f^{-1} and the line L. [2]

References:

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 4]

Given that $\frac{\mathrm{d}y}{\mathrm{d}x} = \cos\left(x - \frac{\pi}{4}\right)$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

(a) Write down the value of $f'(4)$. [1]

(b) Find $f(4)$. [1]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f\big(g(x)\big)$.

(c) Find $h(4)$. [2]

(d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\text{m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \le t \le 3$. When $t = 0$, P is at the origin O.

(a) (i) Find the value of t when P reaches its maximum velocity.

(ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [7]

(b) Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

(c) Find the total distance travelled by P. [5]

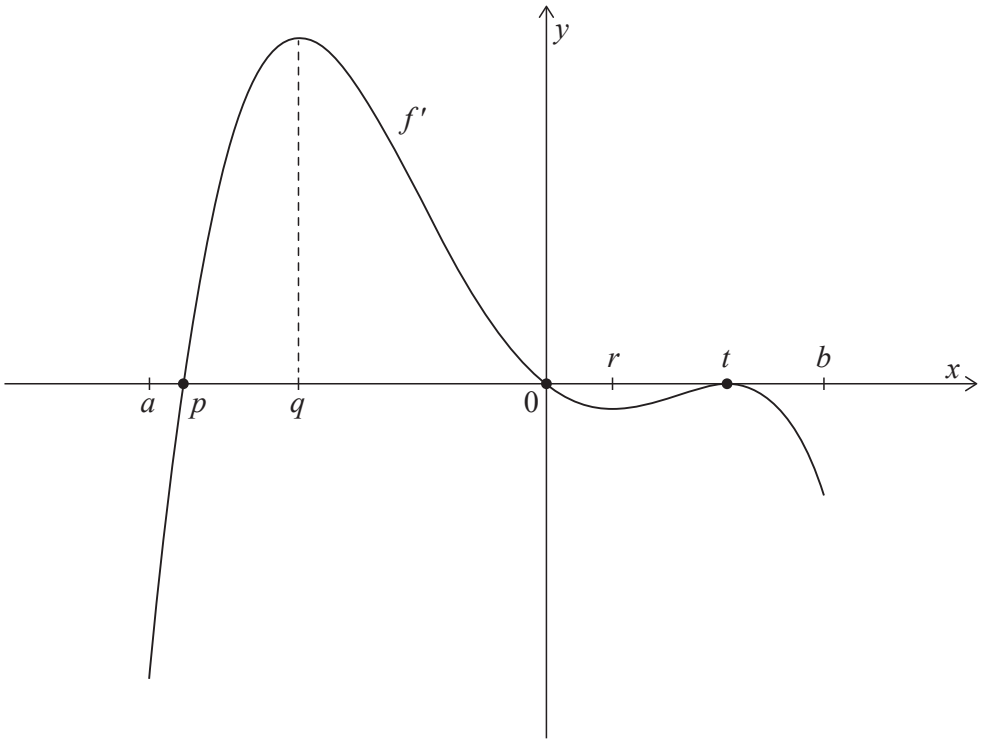
Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

Consider a function f with domain $a < x < b$. The following diagram shows the graph of f', the derivative of f.



The graph of f', the derivative of f, has x-intercepts at $x = p$, $x = 0$ and $x = t$. There are local maximum points at $x = q$ and $x = t$ and a local minimum point at $x = r$.

(a) Find all the values of x where the graph of f is increasing. Justify your answer. [2]

(b) Find the value of x where the graph of f has a local maximum. [1]

(c) (i) Find the value of x where the graph of f has a local minimum. Justify your answer.

(ii) Find the values of x where the graph of f has points of inflexion. Justify your answer. [5]

(d) The total area of the region enclosed by the graph of f', the derivative of f, and the x-axis is 20.

Given that $f(p) + f(t) = 4$, find the value of $f(0)$. [6]

References:

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Question 9 - working space

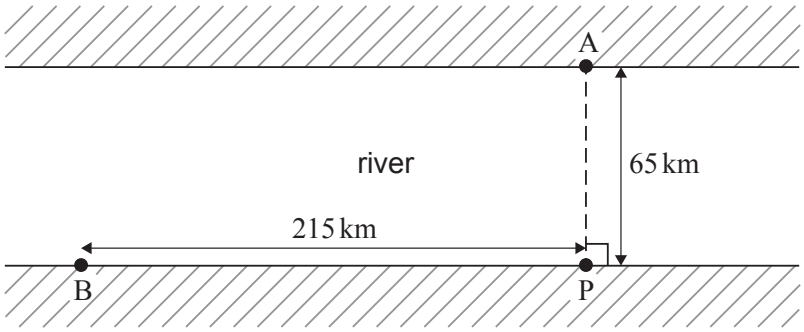
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Points A and P lie on opposite banks of a river, such that AP is the shortest distance across the river. Point B represents the centre of a city which is located on the riverbank. $\mathrm{PB} = 215\,\mathrm{km}$, $\mathrm{AP} = 65\,\mathrm{km}$ and $\mathrm{A\hat{P}B} = 90^{\circ}$.

The following diagram shows this information.



A boat travels at an average speed of $42\,\mathrm{km\,h^{-1}}$. A bus travels along the straight road between P and B at an average speed of $84\,\mathrm{km\,h^{-1}}$.

(a) Find the travel time, in hours, from A to B given that

(i) the boat is taken from A to P, and the bus from P to B;

(ii) the boat travels directly to B. [4]

There is a point D, which lies on the road from P to B, such that $\mathrm{BD} = x\,\mathrm{km}$. The boat travels from A to D, and the bus travels from D to B.

(b) (i) Find an expression, in terms of x for the travel time T, from A to B, passing through D.

(ii) Find the value of x so that T is a minimum.

(iii) Write down the minimum value of T. [6]

(c) An excursion involves renting the boat and the bus. The cost to rent the boat is $\$200$ per hour, and the cost to rent the bus is $\$150$ per hour.

(i) Find the new value of x so that the total cost C to travel from A to B via D is a minimum.

(ii) Write down the minimum total cost for this journey. [4]

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Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 5]

Let $f'(x) = \frac{8x}{\sqrt{2x^2 + 1}}$. Given that $f(0) = 5$, find $f(x)$.

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

(a) Find $f'(x)$. [2]

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(b) Find the value of a and the value of b. [3]

(c) (i) Sketch the graph of $y = f'(x)$.

(ii) Hence explain why the graph of f has a local maximum point at $x = a$. [2]

(d) (i) Find $f''(b)$.

(ii) Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$. [4]

The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q).

(e) Find the value of p and the value of q. [5]

Do **not** write solutions on this page.

Question 8 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

Let $f(x) = \frac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

(a) Show that $f'(x) = \frac{1 - \ln 5x}{kx^2}$. [3]

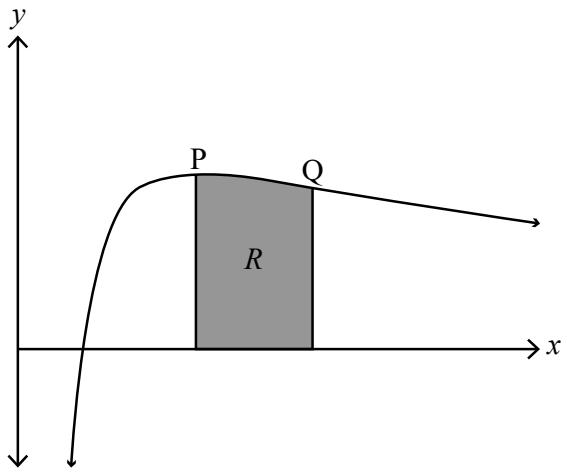
The graph of f has exactly one maximum point P.

(b) Find the x-coordinate of P. [3]

The second derivative of f is given by $f''(x) = \frac{2\ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

(c) Show that the x-coordinate of Q is $\frac{1}{5}\mathrm{e}^{\frac{3}{2}}$. [3]

The region R is enclosed by the graph of f, the x-axis, and the vertical lines through the maximum point P and the point of inflexion Q.



(d) Given that the area of R is 3, find the value of k. [7]

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \leq t \leq 5$.

(a) Find the maximum distance of the particle from O. [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

(a) The expression $\frac{3\sqrt{x}-5}{\sqrt{x}}$ can be written as $3-5x^p$. Write down the value of p. [1]

(b) Hence, find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}}\right)\mathrm{d}x$. [4]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

Consider the curve with equation $y = (2x - 1)e^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$.

The tangent to the curve at the point where $x = 1$ is parallel to the line $y = 5e^{k}x$.

Find the value of k.

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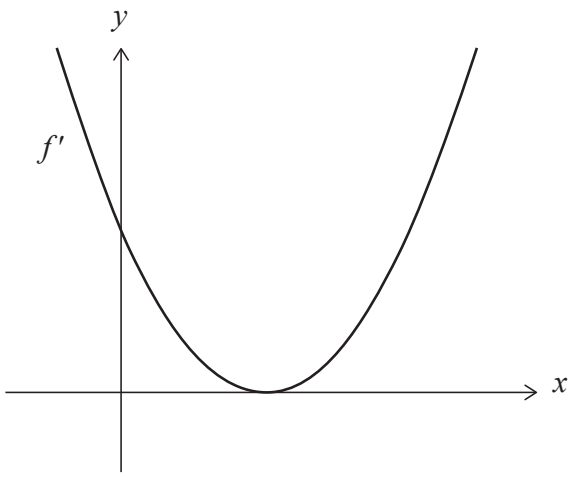
Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A function, f, has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f'.



The graph of f' has an axis of symmetry $x = q$.

(a) Find the value of q. [2]

The vertex of the graph of f' lies on the x-axis.

(b) (i) Write down the value of the discriminant of f'.

(ii) Hence or otherwise, find the value of p. [4]

(c) Find the value of the gradient of the graph of f' at $x = 0$. [3]

(d) Sketch the graph of f'', the second derivative of f. Indicate clearly the x-intercept and the y-intercept. [2]

The graph of f has a point of inflexion at $x = a$.

(e) (i) Write down the value of a.

(ii) Find the values of x for which the graph of f is concave-down. Justify your answer. [3]

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Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

(a) (i) Expand and simplify $(1-a)^3$ in ascending powers of a.

(ii) By using a suitable substitution for a, show that $1-3\cos 2x+3\cos^2 2x-\cos^3 2x=8\sin^6 x$. [6]

Consider $f(x)=4\cos x\left(1-3\cos 2x+3\cos^2 2x-\cos^3 2x\right)$.

(b) (i) Show that $\int_0^m f(x)\,\mathrm{d}x=\frac{32}{7}\sin^7 m$, where m is a positive real constant.

(ii) It is given that $\int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x=\frac{127}{28}$, where $0\le m\le\frac{\pi}{2}$. Find the value of m. [9]

References:

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Question 9 - working space

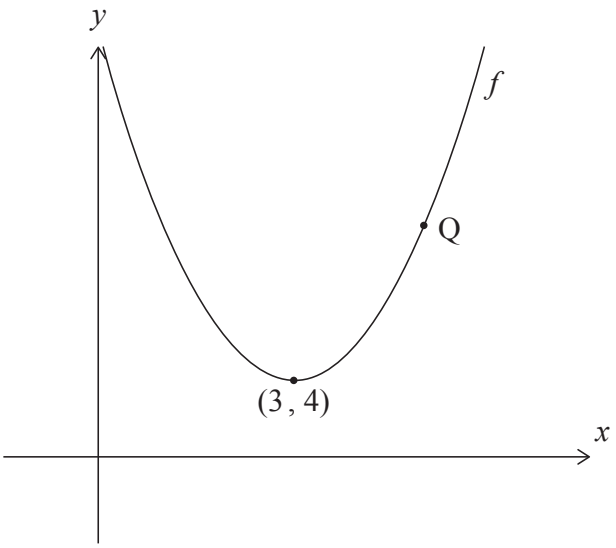
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

The following diagram shows part of the graph of a quadratic function f.

The graph of f has its vertex at $(3, 4)$, and it passes through point Q as shown.



(a) Write down the equation of the axis of symmetry. [1]

(b) The function can be written in the form $f(x) = a(x - h)^2 + k$.

(i) Write down the values of h and k.

(ii) Point Q has coordinates $(5, 12)$. Find the value of a. [4]

The line L is tangent to the graph of f at Q.

(c) Find the equation of L. [4]

Now consider another function $y = g(x)$. The derivative of g is given by $g'(x) = f(x) - d$, where $d \in \mathbb{R}$.

(d) Find the values of d for which g is an increasing function. [3]

(e) Find the values of x for which the graph of g is concave-up. [3]

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Question 7 - working space

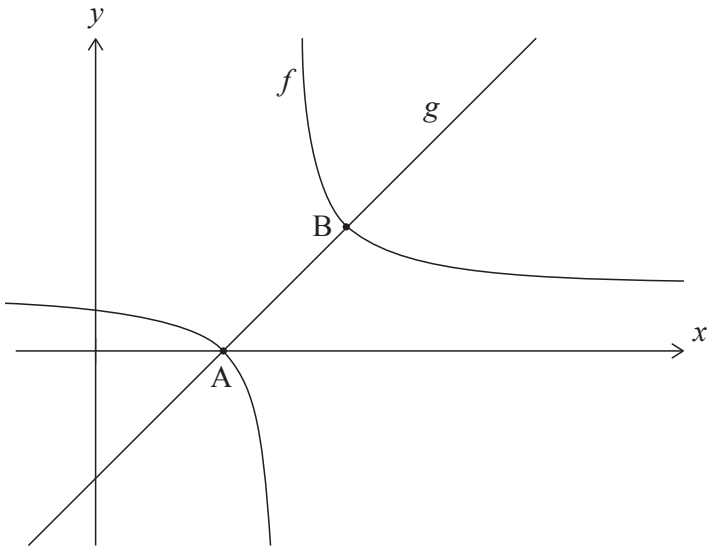
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the functions $f(x) = \frac{1}{x-4} + 1$, for $x \neq 4$, and $g(x) = x - 3$ for $x \in \mathbb{R}$.

The following diagram shows the graphs of f and g.



The graphs of f and g intersect at points A and B. The coordinates of A are $(3, 0)$.

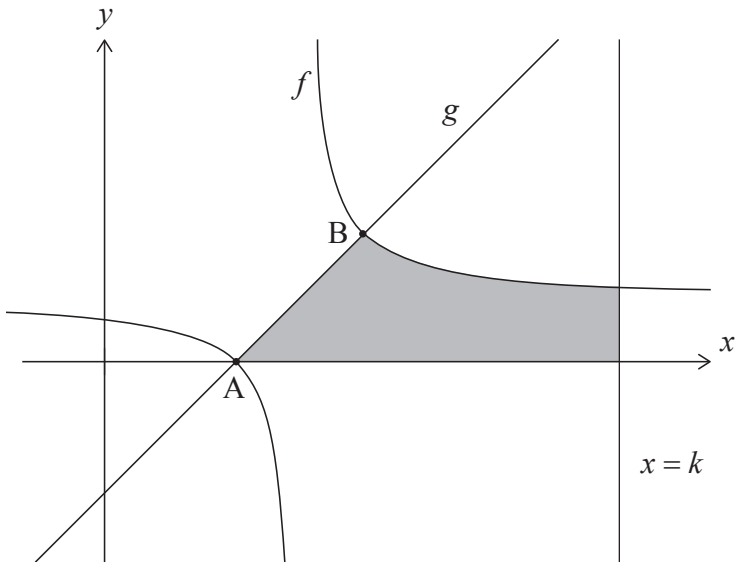
(a) Find the coordinates of B. [5]

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(Question 8 continued)

In the following diagram, the shaded region is enclosed by the graph of f, the graph of g, the x-axis, and the line $x = k$, where $k \in \mathbb{Z}$.



The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

(b) Find the value of k and the value of p. [10]

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Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\text{m s}^{-1}$, after t seconds is given by $v(t) = e^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

(a) Find the value of t when the particle is at rest. [2]

(b) Find the acceleration of the particle when it changes direction. [3]

(c) Find the total distance travelled by the particle. [2]

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

The derivative of a function g is given by $g'(x) = 3x^2 + 5e^x$, where $x \in \mathbb{R}$. The graph of g passes through the point $(0, 4)$. Find $g(x)$.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v = \frac{\left(t^2+1\right)\cos t}{4}$, $0 \le t \le 3$.

(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is $-1.9\,\text{m s}^{-2}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

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Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 12]

A scientist conducted a nine-week experiment on two plants, A and B, of the same species. He wanted to determine the effect of using a new plant fertilizer. Plant A was given fertilizer regularly, while Plant B was not.

The scientist found that the height of Plant A, $h_A \text{cm}$, at time t weeks can be modelled by the function $h_A(t) = \sin(2t + 6) + 9t + 27$, where $0 \leq t \leq 9$.

The scientist found that the height of Plant B, $h_B \text{cm}$, at time t weeks can be modelled by the function $h_B(t) = 8t + 32$, where $0 \leq t \leq 9$.

(a) Use the scientist's models to find the initial height of

(i) Plant B;

(ii) Plant A correct to three significant figures. [3]

(b) Find the values of t when $h_A(t) = h_B(t)$. [3]

(c) For $0 \leq t \leq 9$, find the total amount of time when the rate of growth of Plant B was greater than the rate of growth of Plant A. [6]

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Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 4]

The function g is defined by $g(x) = e^{x^2+1}$, where $x \in \mathbb{R}$.

Find $g'(-1)$.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

The derivative of the function f is given by $f'(x) = \frac{6x}{x^2 + 1}$.

The graph of $y = f(x)$ passes through the point $(1, 5)$. Find an expression for $f(x)$.

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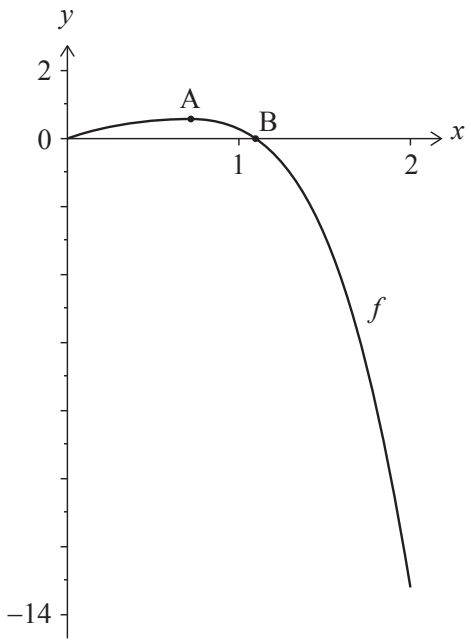
Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \le x \le 2$. The graph of f is shown in the following diagram.



The graph of f has a local maximum at point A. The graph intersects the x-axis at the origin and at point B.

(a) Find the coordinates of A. [2]

(b) Find the x-coordinate of B. [1]

(c) Find the total area enclosed by the graph of f, the x-axis and the line $x = 2$. [3]

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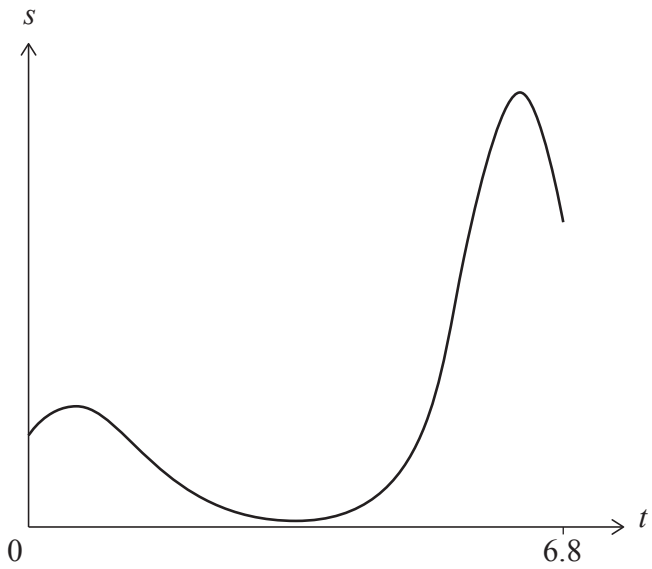
Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s(t) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.



(a) Find the particle's initial displacement from the point P. [2]

(b) Find the particle's velocity when $t = 2$. [2]

(c) Determine the intervals of time when the particle is moving away from the point P. [5]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(d) Find the value of b and the value of c. [4]

(e) Find the total distance travelled by the particle for $b \le t \le c$. [3]

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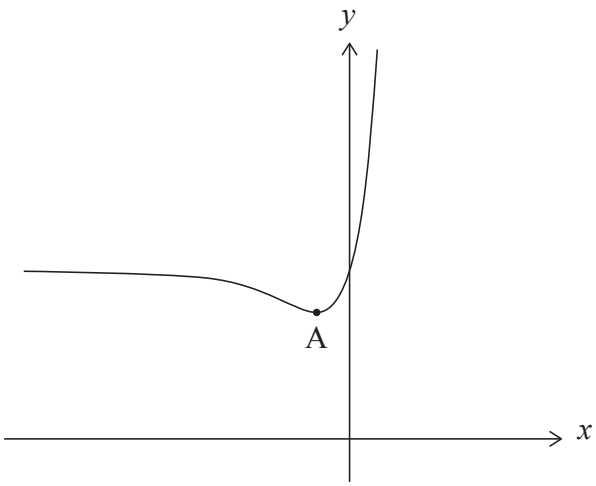
Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The function h is defined by $h(x) = 2xe^x + 3$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of h, which has a local minimum at point A.



(a) Find the value of the y-intercept. [2]

(b) Find $h'(x)$. [2]

(c) Hence, find the coordinates of A. [5]

(d) (i) Show that $h''(x) = (2x + 4)e^x$.

(ii) Find the values of x for which the graph of h is concave-up. [4]

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Question 7 - working space

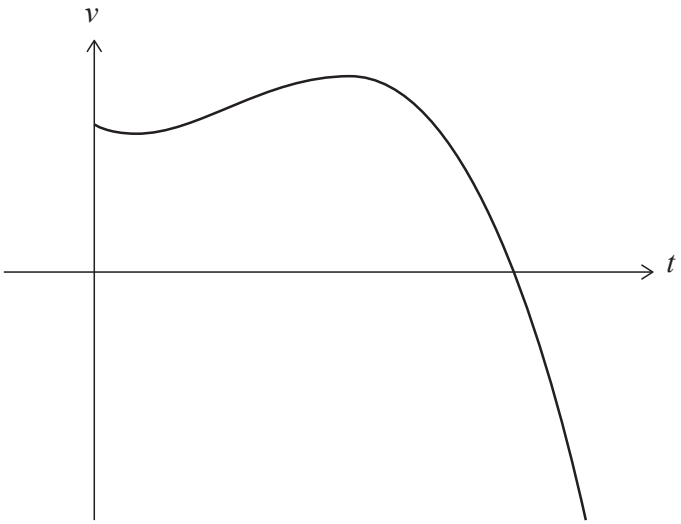
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6$, for $0 \le t \le 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

(a) Find the displacement of the object from the origin at $t = 1$. [5]

(b) Find an expression for the acceleration of the object. [2]

(c) Hence, find the greatest speed reached by the object before it comes to rest. [5]

(d) Find the greatest speed reached by the object for $0 \le t \le 4$. [2]

(e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

References:

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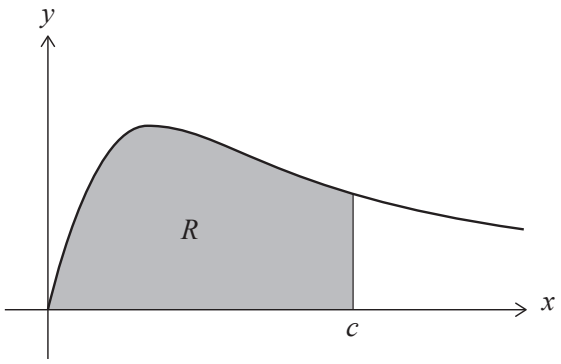
Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

The following diagram shows part of the graph of $y = \frac{x}{x^2 + 2}$ for $x \geq 0$.



The shaded region R is bounded by the curve, the x-axis and the line $x = c$.

The area of R is $\ln 3$.

Find the value of c.

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Question 5 - working space

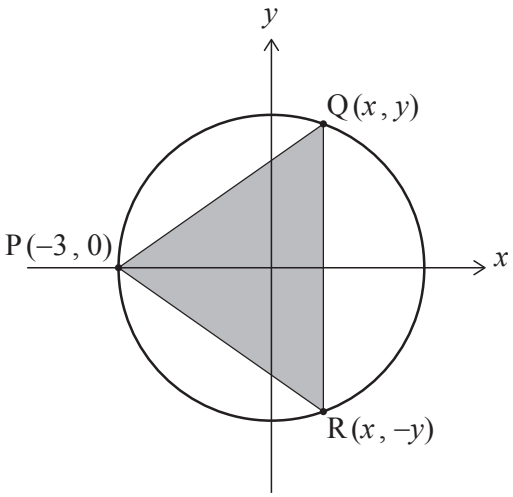
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $\mathrm{P}(-3, 0)$, $\mathrm{Q}(x, y)$ and $\mathrm{R}(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



(a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]

(b) Hence, find an expression for A, the area of triangle PQR, in terms of x. [3]

(c) Show that $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]

(d) Hence or otherwise, find the y-coordinate of R such that A is a maximum. [6]

References:

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Question 9 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The temperature of a cup of tea, t minutes after it is poured, can be modelled by $H(t) = 21 + 75e^{-0.08t}$, $t \geq 0$. The temperature is measured in degrees Celsius (°C).

(a) (i) Find the initial temperature of the tea.

(ii) Find the temperature of the tea three minutes after it is poured. [2]

(b) Write down the value of $H'(3)$. [2]

(c) Interpret the meaning of your answer to part (b) in the given context. [2]

(d) After k minutes, the tea will be below $67\,°C$ and cool enough to drink.

Find the least possible value of k, where $k \in \mathbb{Z}^+$. [3]

As the tea cools, $H(t)$ approaches the temperature of the room, which is constant.

(e) Find the temperature of the room. [2]

(f) Find the limit of $H'(t)$ as t approaches infinity. [2]

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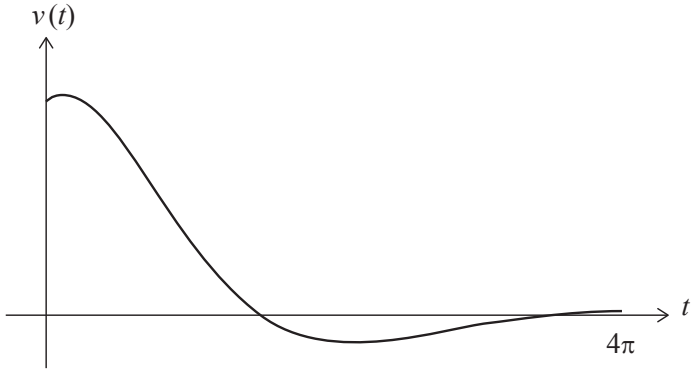
Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v(t) = 4e^{-\frac{t}{3}}\cos\left(\frac{t}{2} - \frac{\pi}{4}\right)$, for $0 \le t \le 4\pi$. The graph of v is shown in the following diagram.



Let t_1 be the first time when the particle's **acceleration** is zero.

(a) Find the value of t_1. [2]

Let t_2 be the **second** time when the particle is instantaneously at rest.

(b) Find the value of t_2. [2]

(c) Find the distance travelled by the particle between $t = t_1$ and $t = t_2$. [2]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function defined by $f(x) = x^2 - 10x$. The graph of f passes through the point $\mathrm{A}(4, -24)$.

(a) (i) Find the gradient of the tangent to the graph of f at the point A.

(ii) Hence, write down the gradient of the normal to the graph of f at point A. [3]

(b) Write down the equation of the normal to the graph of f at point A. [1]

The normal to the graph of f at point A intersects the graph of f again at a second point B.

(c) Find the coordinates of B. [3]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 5.2\sin\left(\sqrt{4t+6}\right)$, where $0 \le t \le 10$.

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

(b) Find the total distance that the particle travels in the first q seconds. [3]

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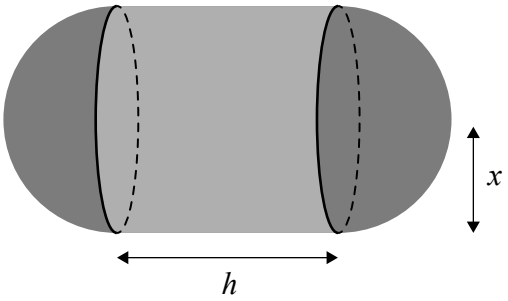
Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height $h\,\text{cm}$ and radius $x\,\text{cm}$. The hemispheres fit exactly onto either end of the cylinder.



The volume of the cylinder is $41\,\text{cm}^3$.

(a) Show that the total surface area, $S\,\text{cm}^2$, of the solid is given by $S = \frac{82}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (i) Find an expression for $\frac{\mathrm{d}S}{\mathrm{d}x}$.

(ii) Hence, find the **exact** value of a. [5]

(c) (i) Find an expression for $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$.

(ii) Use the second derivative of S to justify that S is a minimum when $x = a$.

(iii) Find the minimum surface area of the solid. [6]

Do **not** write solutions on this page.

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function defined by $f(x) = x^2 - 8x$. The graph of f passes through the point $\mathrm{A}(3, -15)$.

(a) (i) Find the gradient of the tangent to the graph of f at the point A.

(ii) Hence, write down the gradient of the normal to the graph of f at point A. [3]

(b) Write down the equation of the normal to the graph of f at point A. [1]

The normal to the graph of f at point A intersects the graph of f again at a second point B.

(c) Find the coordinates of B. [3]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 4.3\sin\left(\sqrt{3t+5}\right)$, where $0 \le t \le 10$.

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

(b) Find the total distance that the particle travels in the first q seconds. [3]

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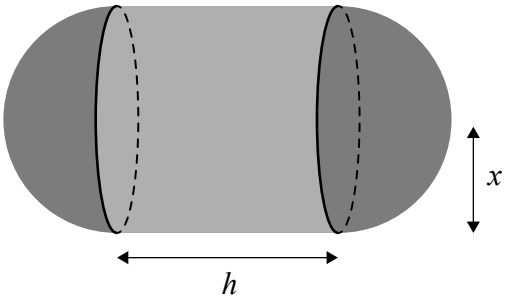
Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height $h\,\text{cm}$ and radius $x\,\text{cm}$. The hemispheres fit exactly onto either end of the cylinder.



The volume of the cylinder is $45\,\text{cm}^3$.

(a) Show that the total surface area, $S\,\text{cm}^2$, of the solid is given by $S = \frac{90}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (i) Find an expression for $\frac{\mathrm{d}S}{\mathrm{d}x}$.

(ii) Hence, find the **exact** value of a. [5]

(c) (i) Find an expression for $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$.

(ii) Use the second derivative of S to justify that S is a minimum when $x = a$.

(iii) Find the minimum surface area of the solid. [6]

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Question 7 - working space

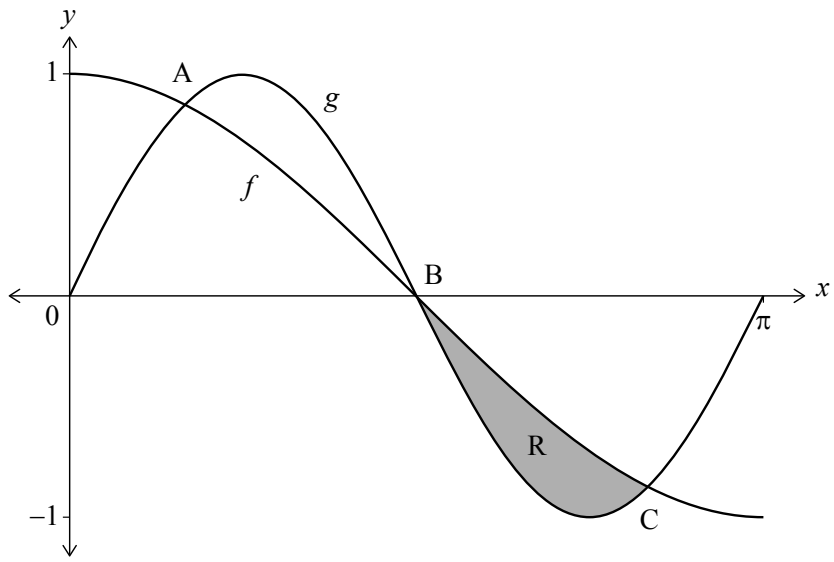
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

Consider the functions $f(x) = \cos x$ and $g(x) = \sin 2x$, where $0 \le x \le \pi$.

The graph of f intersects the graph of g at the point A, the point $\mathrm{B}\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



(a) Find the x-coordinate of point A and the x-coordinate of point C. [3]

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

(b) Find the area of R. [4]

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Question 5 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Consider the curve with equation $y = x^3 - x^2 - x + 1$.

(a) Find

(i) $\frac{\mathrm{d}y}{\mathrm{d}x}$;

(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [3]

The curve has a local maximum at A.

(b) Find the coordinates of A, using your answer to part (a)(ii) to justify your answer. [6]

The curve has a point of inflexion at B.

(c) Find the x-coordinate of B. [2]

The line L is the normal to the curve at the point $(0, 1)$.

(d) Find the equation of L. [3]

Do **not** write solutions on this page.

Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 17]

The derivative of a function f is given by $f'(x) = \frac{2x+2}{x^2+2x+2}$, for $x \in \mathbb{R}$.

(a) (i) Show that $x^2 + 2x + 2 > 0$ for all values of x.

(ii) Hence, find the values of x for which f is increasing. [3]

(b) (i) Write down the value of x for which $f'(x) = 0$.

(ii) Show that $f''(x) = \frac{-2x^2 - 4x}{(x^2+2x+2)^2}$.

(iii) Hence, justify that the value of x found in part (b)(i) corresponds to a local minimum point on the graph of f. [7]

It is given that $f(2) = 3 + \ln 10$.

(c) Find an expression for $f(x)$. [4]

(d) Find the equation of the normal to the graph of f at $(2, 3 + \ln 10)$. [3]

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Question 8 - working space

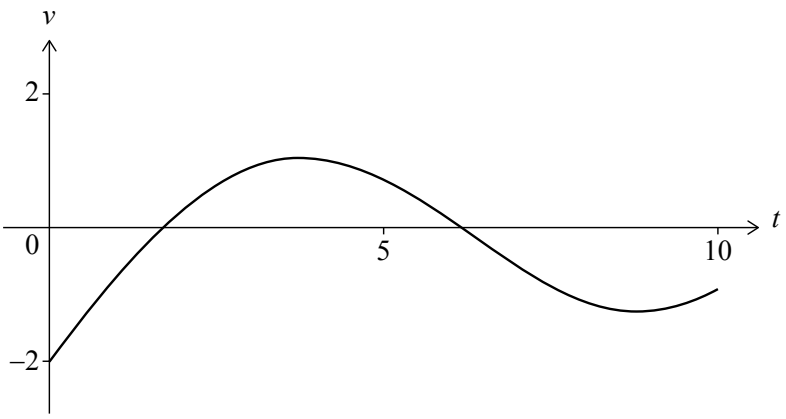
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O. For $0 \le t \le 10$ its velocity, v metres per second, is given by $v = 2\sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.



(a) Find the smallest value of t when the particle changes direction. [2]

The displacement of the particle is measured in metres from O.

(b) Find the range of values of t for which the displacement of the particle is increasing. [2]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The functions f and g are both defined for $-1 \leq x \leq 0$ by

$$f(x) = 1 - x^2$$

$$g(x) = \mathrm{e}^{2x}.$$

The graphs of f and g intersect at $x = a$ and $x = b$, where $a < b$.

(a) Find the value of a and the value of b. [3]

(b) Find the area of the region enclosed by the graphs of f and g. [3]

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by $v(t) = 1 + \text{e}^{-t} - \text{e}^{-\sin 2t}$ for $0 \leq t \leq 2$.

(a) Find the velocity of the particle at $t = 2$. [1]

(b) Find the maximum velocity of the particle. [2]

(c) Find the acceleration of the particle at the instant it changes direction. [3]

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Question 5 - working space

Section B

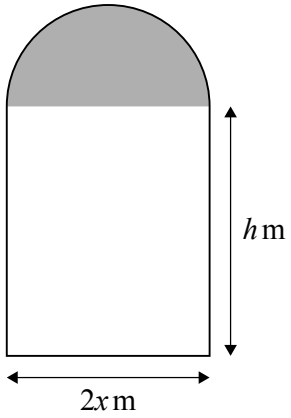
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A window is designed in the shape of a semicircle attached to a rectangle.

The rectangular section of the window has dimensions $2x$ metres by h metres.

The window consisting of its two sections is shown in the following diagram.



Let the area of the window be A square metres.

(a) Write down an expression for A in terms of x and h. [2]

Let the perimeter of the window be P metres.

(b) Given that $P = 10$, show that $h = \frac{1}{2}(10 - 2x - \pi x)$. [2]

The window is designed to let in the maximum amount of light.

The rectangular section of the window consists of clear glass and lets in three units of light per square metre.

The semicircular section of the window consists of tinted glass and lets in one unit of light per square metre.

(c) Show that the amount of light, L units, let in by the window is given by $L = 30x - 6x^2 - \frac{5}{2}\pi x^2$. [4]

(d) (i) Find an expression for $\frac{\mathrm{d}L}{\mathrm{d}x}$.

(ii) Find the value of x so that the window lets in the maximum amount of light. Justify that this value of x gives a maximum.

(iii) Find the value of h so that the window lets in the maximum amount of light. [7]

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Question 8 - working space

Section B

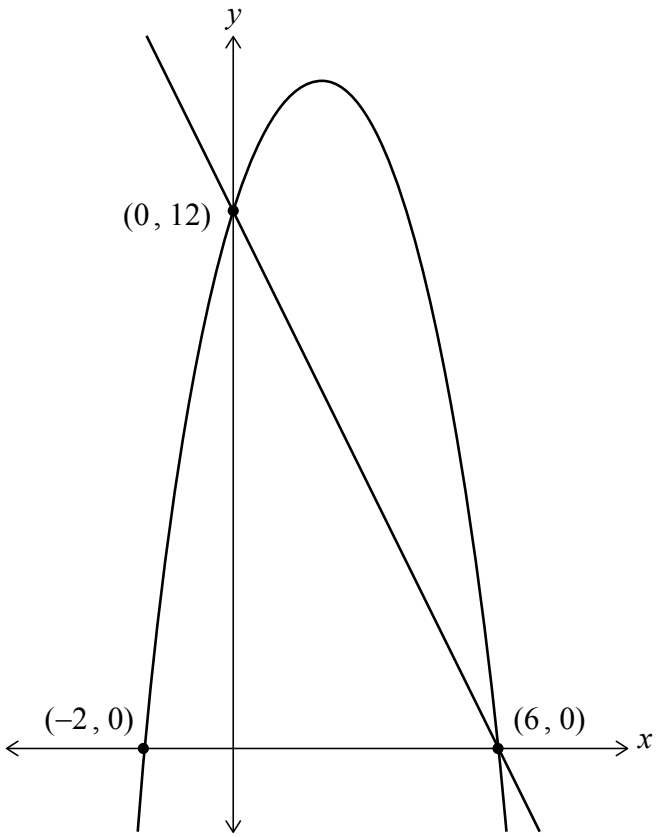
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g.

The graph of f is linear, has an x-intercept at $(6, 0)$ and a y-intercept at $(0, 12)$.

The graph of g has x-intercepts at $(-2, 0)$ and $(6, 0)$ and a y-intercept at $(0, 12)$.



(a) Write down the equation for f in the form $f(x) = mx + c$. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

The function g is given by $g(x) = -x^2 + bx + 12$, where b is a real constant.

(b) Find the value of b. [3]

(c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^6 (-x^2 + 6x)\mathrm{d}x$. [2]

(d) Hence, find the area of the region enclosed by the graph of f and the graph of g. [4]

Point P is on the graph of g. The tangent to the graph of g at P is parallel to the graph of f.

(e) Find the coordinates of P. [5]

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Question 7 - working space

Section B

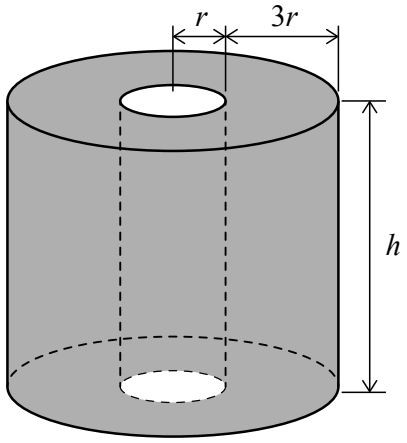
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi\,\text{cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]

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Question 9 - working space

Section B

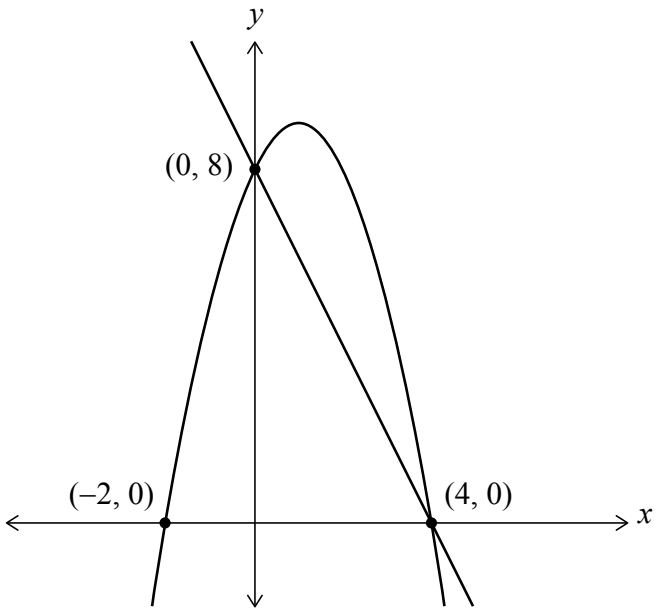
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g.

The graph of f is linear, has an x-intercept at $(4, 0)$ and a y-intercept at $(0, 8)$.

The graph of g has x-intercepts at $(-2, 0)$ and $(4, 0)$ and a y-intercept at $(0, 8)$.



(a) Write down the equation for f in the form $f(x) = mx + c$. [2]

The function g is given by $g(x) = -x^2 + bx + 8$, where b is a real constant.

(b) Find the value of b. [3]

(c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^4 (-x^2 + 4x)\mathrm{d}x$. [2]

(d) Hence, find the area of the region enclosed by the graph of f and the graph of g. [4]

Point P is on the graph of g. The tangent to the graph of g at P is parallel to the graph of f.

(e) Find the coordinates of P. [5]

Do **not** write solutions on this page.

Question 7 - working space

Section B

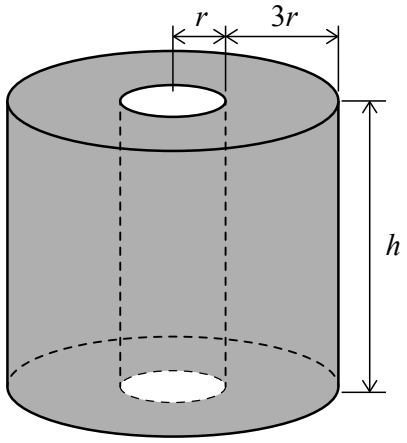
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi\,\text{cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]

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Question 9 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

(a) Find $\int \left(e^{3x} + 5\right) dx$. [3]

It is given that $h'(x) = e^{3x} + 5$ and $h(2.1) = 185$.

(b) Find $h(x)$. [3]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.6	32.3	34.1	35.6	38.0

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

(a) (i) Write down the value of a and the value of b.

(ii) Interpret, in context, the value of a. [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.4 million people.

(b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 30.6(1.007)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

(c) (i) Use Benoit's model to predict the population of Canada in the year 2100.

(ii) Interpret, in context, the value 1.007 in Benoit's model. [3]

(This question continues on the following page)

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(Question 8 continued)

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{61}{1 + e^{-0.03t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

(d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]

(e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]

(f) Find the value of

(i) $B'(40)$;

(ii) $C'(40)$. [2]

(g) Compare and interpret, in context, the values of $B'(40)$ and $C'(40)$. [2]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

(a) Find $\int \left(e^{4x} + 6\right) dx$. [3]

It is given that $h'(x) = e^{4x} + 6$ and $h(1.5) = 105$.

(b) Find $h(x)$. [3]

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Question 3 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.7	32.2	34.0	35.7	37.9

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

(a) (i) Write down the value of a and the value of b.

(ii) Interpret, in context, the value of a. [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.3 million people.

(b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 33.5(1.005)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

(c) (i) Use Benoit's model to predict the population of Canada in the year 2100.

(ii) Interpret, in context, the value 1.005 in Benoit's model. [3]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{62}{1 + e^{-0.02t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

(d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]

(e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]

(f) Find the value of

(i) $B'(75)$;

(ii) $C'(75)$. [2]

(g) Compare and interpret, in context, the values of $B'(75)$ and $C'(75)$. [2]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Consider the function $f(x) = \frac{4x^3}{3} - 16x$, where $x \in \mathbb{R}$.

The graph of $y = f(x)$ has a local minimum point at (p, q) where $p > 0$.

Find the value of p and the value of q.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 4]

Find the area completely enclosed by the curves $y = \mathrm{e}^x$, $y = -\mathrm{e}^x$, and the lines $x = -1$ and $x = 1$.

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Question 4 - working space

Section B

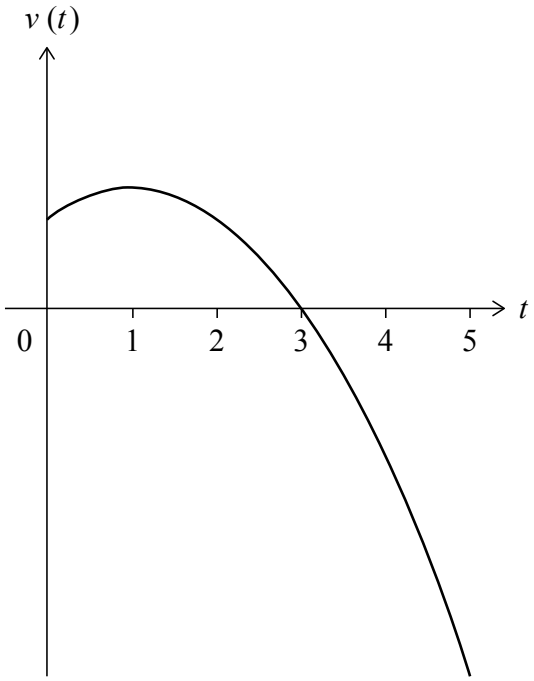
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v\,\text{m}\,\text{s}^{-1}$, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \le t \le 5$.

The graph of v is shown in the following diagram.



The graph of v has a local maximum point where $t = 1$ and intersects the t-axis at $t = 3$.

(a) Determine the object's

(i) maximum velocity;

(ii) maximum speed. [4]

At $t = T$, the object changes direction.

(b) (i) Write down the value of T.

(ii) Find the distance travelled by the object in the first T seconds. [5]

(c) Determine whether the object returns to its initial position during the time period $0 \le t \le 5$, justifying your answer. [4]

2225 – 7109

Do **not** write solutions on this page

Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Consider the function $f(x) = x^3 + 5x^2 - 8$, where $x \in \mathbb{R}$.

(a) Find $f'(1)$. [2]

(b) Find the equation of the tangent to the graph of f at $x = 1$. [2]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

The derivative of a function g is given by $g'(x) = \cos x + e^{2x}$, where $x \in \mathbb{R}$.

Given that $g(0) = 7$, find $g(x)$.

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Question 2 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

Consider the function $f(x) = \sqrt{x^2 \ln x + 4 - x^2}$, where $x \in \mathbb{R}$, $x > 0$.

(a) Show that the distance, l, between the origin and any point on the graph of f is given by $l = \sqrt{x^2 \ln x + 4}$. [1]

(b) Hence, find the x-coordinate of the point on the graph of f which is closest to the origin. [6]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

(a) Show that $\cos^4 x - \sin^4 x = \cos 2x$. [3]

(b) Hence, find $\int (\cos^4 x - \sin^4 x)\,\mathrm{d}x$. [3]

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Question 4 - working space

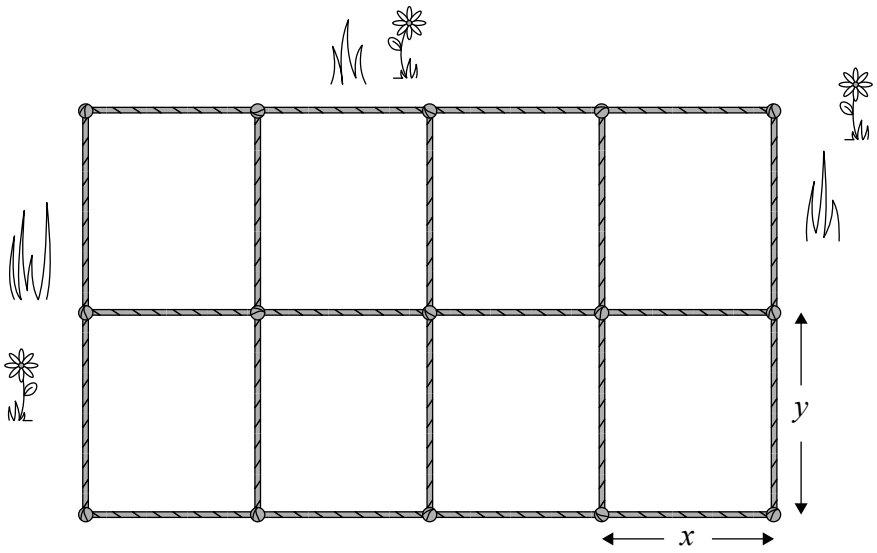
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

A gardener plans to enclose part of their garden with rope. The total area being enclosed is $60\,\text{m}^2$. This will be further divided by rope to make eight identical rectangular areas, each measuring x metres by y metres, where $x, y > 0$. This is shown in the following diagram.

diagram not to scale



garden

(a) Find an expression for y in terms of x. [2]

(b) Show that the total length, T metres, of rope required is given by

$$T = 12x + \frac{75}{x}.$$ [2]

(c) Find an expression for $\frac{\mathrm{d}T}{\mathrm{d}x}$. [2]

(This question continues on the following page)

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(Question 7 continued)

When $x = k$, $\frac{\mathrm{d}T}{\mathrm{d}x} = 0$.

(d) (i) Find the value of k.

(ii) Hence, calculate the value of T when $x = k$.

(iii) Find the value of y when $x = k$. [7]

(e) (i) Find an expression for $\frac{\mathrm{d}^2T}{\mathrm{d}x^2}$.

(ii) Hence, justify whether T has a local minimum or a local maximum when $x = k$. [2]

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Question 7 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the function $f(x) = \frac{(x-1)^2}{x}$, where $x \in \mathbb{R}$, $x \neq 0$.

(a) Show that $\frac{(x-1)^2}{x} = x - 2 + \frac{1}{x}$. [2]

(b) Hence, find $\int f(x)\,\mathrm{d}x$. [3]

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Question 1 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

A population, P, has a rate of change modelled by $\frac{\mathrm{d}P}{\mathrm{d}t} = -104000\mathrm{e}^{-0.0145t}$, where t is the time measured in years since the **start** of 2022.

At the start of 2022, the population was 6.78×10^6.

Based on this model, find the predicted population at the start of 2026.

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Question 4 - working space

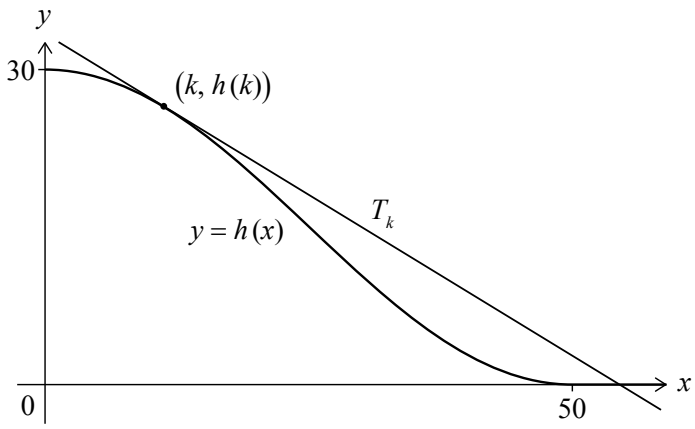
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

Consider the function $h(x) = 15\cos\left(\frac{\pi x}{50}\right) + 15$, where $0 \le x \le 50$.

The tangent, T_k, to the curve $y = h(x)$ at the point $(k, h(k))$ is shown on the following diagram.



(a) Find the gradient of T_k in terms of k. [3]

Consider the case where the angle between T_k and the x-axis is $\frac{\pi}{8}$ radians.

(b) Find the possible values of k. [3]

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Question 6 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

The derivative of a function f is given by $f'(x) = 4 + 2x - 3e^x$, where $x \in \mathbb{R}$.

(a) Find the values of x for which f is decreasing. [3]

(b) Find the values of x for which the graph of f is concave-up. [3]

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Question 3 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 7]

A particle P moves in a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P, at time t seconds is given by $v(t) = \mathrm{e}^{-\sin t}\cos(2t)$, for $0 \le t \le 5$.

(a) Find the maximum speed of P. [2]

(b) Find the total distance travelled by P. [2]

(c) Find the acceleration when P changes direction for the **second** time. [3]

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Question 5 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum marks: 7]

Consider the function $f(x) = \frac{(2x+a)^3}{(x+5)^2}$, where $x \neq -5$ and $a \in \mathbb{R}^+$.

(a) Find an expression for $f'(x)$, in terms of a. [3]

When $x = 1$, the tangent to the graph of f makes an angle of $70°$ to the horizontal.

(b) Find the two possible values of a. [4]

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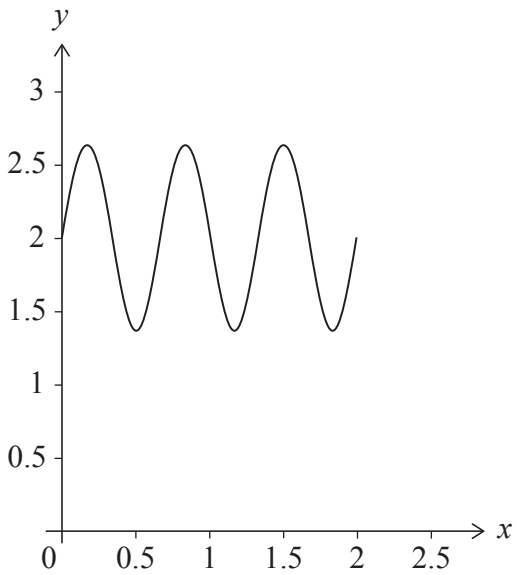
Question 6 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Consider the function $f(x) = \frac{2}{\pi}\sin(3\pi x) + 2$, where $0 \le x \le 2$. The following diagram shows the graph of f.



(a) (i) Write down the amplitude of f.

(ii) Find the period of f. [3]

(b) The point P has coordinates $(1.63, 2.16)$. State whether P lies above, below or on the graph of f. Justify your answer. [3]

The line L_1 has equation $x - 6y + 11 = 0$.

(c) Write down the gradient of the line L_1. [1]

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(Question 7 continued)

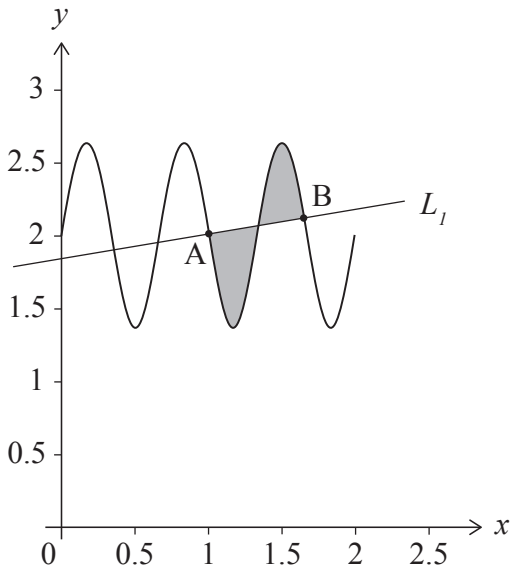
The line L_1 is normal to the graph of f at point $\mathrm{A}(1, 2)$.

The line L_2 is tangent to the graph of f at A.

(d) (i) Find the gradient of L_2.

(ii) Hence, or otherwise, find the equation of L_2. [3]

The line L_1 intersects the graph of f at another point B, where the x-coordinate of B is greater than 1.5. This is shown in the following diagram.



(e) Find the coordinates of B. [2]

The shaded region is enclosed by the graph of f and the line L_1 between A and B.

(f) Find the area of the shaded region. [3]

Do **not** write solutions on this page.

Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in m s^{-1}, during the race can be modelled by $v(t) = \dfrac{8.14\,t}{\sqrt{t^2 + 0.2}}$, where $t \geq 0$. Time, t, is measured in seconds from when the race starts.

(a) (i) Write down the value of $v(1)$.

(ii) Find the time when Fiona's velocity is $5\,\text{m s}^{-1}$. [3]

(b) Find the time when Fiona's acceleration is $4\,\text{m s}^{-2}$. [2]

(c) (i) Write down the limit of $v(t)$ as t approaches infinity.

(ii) State a reason why the value in part (c)(i) is not valid in the context of this question. [3]

Lucy's velocity, in m s^{-1}, during the race can be modelled by $w(t) = \dfrac{8\,t}{\sqrt{t^2 + 0.3}}$, where $t \geq 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

(d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres. [6]

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Question 9 - working space

Section A

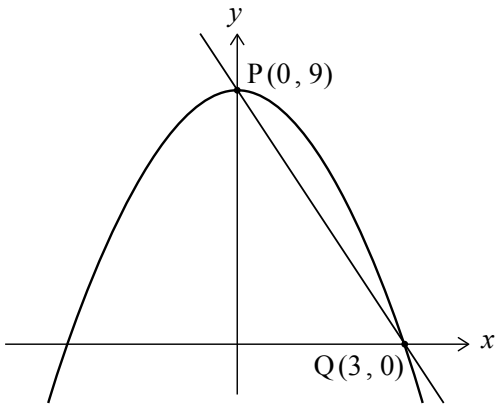
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

A line with equation $y = -3x + 9$ intersects the axes at the points $\mathrm{P}(0, 9)$ and $\mathrm{Q}(3, 0)$.

A parabola of the form $y = ax^2 + c$, where $a, c \in \mathbb{Z}$, also passes through the points P and Q.

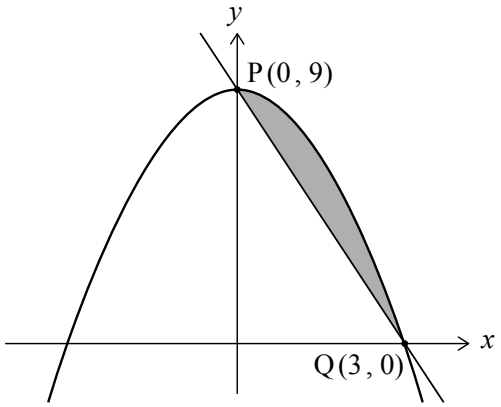
This is shown in the following diagram.



(a) (i) Write down the value of c.

(ii) Find the value of a. [3]

The region enclosed by the line and the parabola is shaded in the following diagram.



(b) Find the area of the shaded region. [4]

(This question continues on the following page)

(Question 4 continued)

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Question 4 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

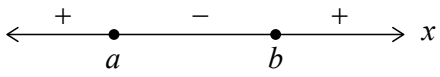
8. [Maximum mark: 14]

The function f has a derivative given by $f'(x) = 3x^2 + 12x - 15$.

The graph of $y = f(x)$ has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(a) Find the value of a and the value of b. [3]

The following diagram shows three intervals along the x-axis defined by a and b. The sign of the **first derivative** of f is shown in each interval.

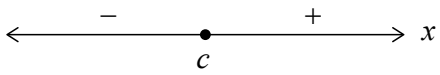


(b) State, with a reason, whether there is a local maximum point or a local minimum point on the graph of $y = f(x)$ at $x = a$. [2]

The second derivative $f''(x)$ is zero at $x = c$.

(c) Find the value of c. [3]

The following diagram shows two intervals along the x-axis defined by c. The sign of the **second derivative** is shown in each interval.



(d) State, with a reason, whether there is a point of inflexion on the graph of $y = f(x)$ at $x = c$. [2]

(e) Given that $f(-2) = 36$, find $f(x)$. [4]

Do **not** write solutions on this page.

Question 8 - working space

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

The gradient of a curve is given by $\frac{\mathrm{d}y}{\mathrm{d}x} = 4\cos x - \sin x$. The curve passes through the point $(\pi, 3)$. Find the equation of the curve.

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Question 2 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the function defined by $f(x) = \frac{1}{2}x^2 + kx + 13$, where $x \in \mathbb{R}$ and $k \in \mathbb{Z}^+$.

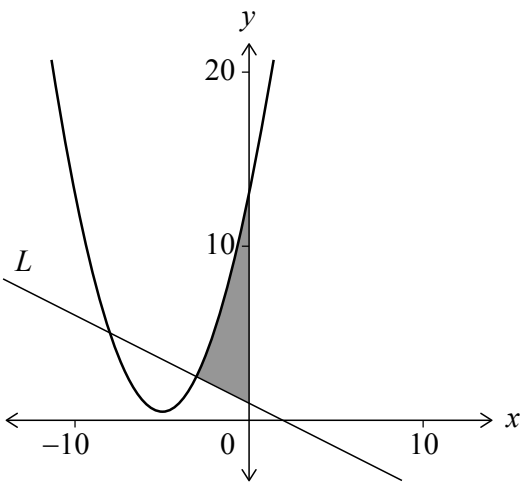
(a) Given that the equation $f(x) = 0$ has no real roots, show that the greatest possible value of k is 5. [2]

For the remainder of this question, consider the case $k = 5$.

(b) (i) Write down the equation of the axis of symmetry of the graph of f.

(ii) Hence, or otherwise, determine the coordinates of the minimum point on the graph of f. [3]

The following diagram shows the graph of f and a line L which is normal to the curve at $x = -3$. The shaded area shown is bounded by the curve, the line L and the y-axis.



(c) Show that the equation of L is given by $y = -\frac{1}{2}x + 1$. [5]

(d) Hence, find the shaded area. [5]

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Question 8 - working space

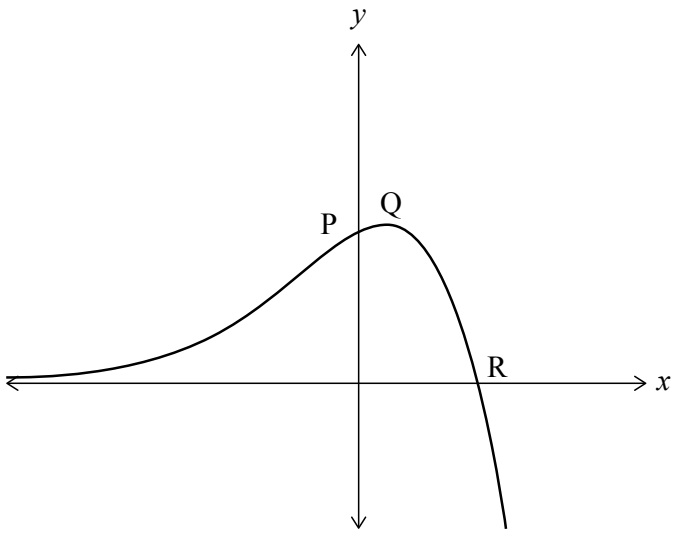
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

The function f is defined by $f(x) = \frac{5e^x - e^{2x}}{e^x + 1}$. The following diagram shows part of the graph of f. The graph intersects the y-axis at point P and intersects the x-axis at point R.

Point Q is a local maximum point with coordinates $\left(q, 7 - 2\sqrt{6}\right)$.



(a) (i) Find the coordinates of point P.

(ii) Find the coordinates of point R. [6]

(b) State the range of f. [1]

(c) Show that $f'(x) = \frac{-e^{3x} - 2e^{2x} + 5e^x}{\left(e^x + 1\right)^2}$. [3]

(d) Hence, show that $q = \ln\left(\sqrt{6} - 1\right)$. [6]

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Question 9 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

Consider the function $f(x) = \frac{3x+1}{2x-4}$ where $x \neq 2$.

(a) Write down the equation of

(i) the vertical asymptote of the graph of f;

(ii) the horizontal asymptote of the graph of f. [2]

(b) Find $f'(x)$. [4]

(c) Hence, explain briefly why f is never increasing. [2]

The point $\mathrm{P}(x,\ f(x))$ is on the graph of f such that $x < 2$.

Let d be the distance between the points P and $\mathrm{Q}(3, 5)$.

(d) Find an expression for d in terms of x. [2]

(e) Hence, find the coordinates of P such that d is a minimum. [3]

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Do **not** write solutions on this page.

Question 7 - working space

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

A population of frogs, F, in a swamp after t months, can be modelled by the function

$$F(t) = 1850 \times 1.105^t \text{ where } t \geq 0.$$

(a) Find the population of frogs after one year. [2]

(b) After x complete months, the population will be at least $35\,000$ frogs. Find the value of x. [3]

The function F can be written in the form $F(t) = 1850\mathrm{e}^{kt}$.

(c) Find the exact value of k. [2]

(d) Find the rate at which the population of frogs is growing after 15 months. [2]

A more realistic model describing the population of frogs, G, after t months is given by

$$G(t) = \frac{35000}{1 + A\mathrm{e}^{-0.0998t}} \text{ where } t \geq 0.$$

(e) After 15 months, this model predicts a population of 6995 frogs. Find the value of A. [2]

(f) Find the value of t when the rate of population growth is the greatest. [2]

(g) By considering the graph of G or otherwise, state one reason why $G(t)$ is a more appropriate long-term model than $F(t)$. [1]

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Question 8 - working space

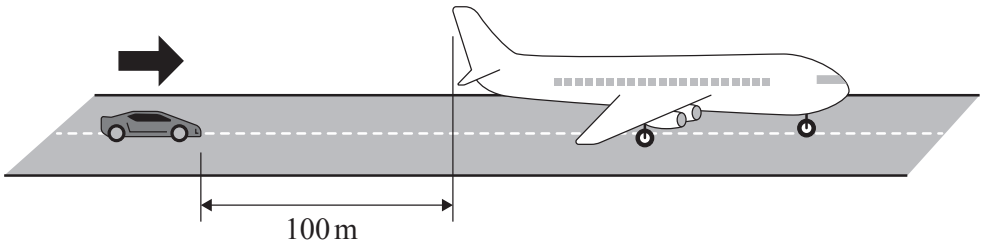
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

An airplane lands on a runway 100 metres in front of a stationary car. At the instant the airplane lands, the car begins to travel in the same direction towards the airplane.

diagram not to scale



Let t represent the number of seconds after the airplane lands. For $t \geq 0$, the velocities of the airplane and the car in m s^{-1}, can be modelled by the following equations:

$$v_{air} = 60\text{e}^{-0.1t}$$

$$v_{car} = 5t$$

(a) When the airplane lands, write down the speed of

(i) the airplane;

(ii) the car. [2]

(b) Find

(i) the value of t when the airplane and the car have the same speed;

(ii) the speed at this time. [3]

Let $d(t)$ represent the distance, in metres, between the car and the back of the airplane after t seconds.

(c) If $d(0) = 100$, find $d(t)$. [7]

(d) Hence, find how long it takes for the car to reach the back of the airplane. [2]

(e) Find the distance travelled by the car when it reaches the back of the airplane. [3]

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Question 9 - working space

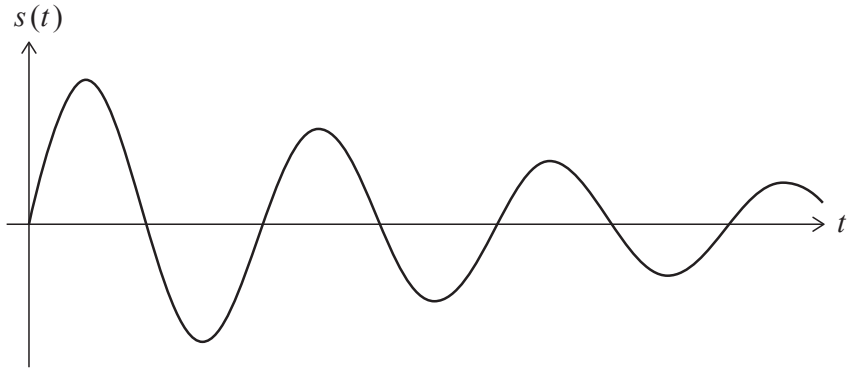
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

A particle P moves in a straight line so that its displacement, $s\,\text{cm}$, from a fixed point O at time t seconds is given by $s(t) = 2^{\left(1-\frac{t}{5}\right)}\sin\left(\frac{2\pi t}{3}\right)$, where $t \geq 0$.

The following diagram shows part of the graph of $y = s(t)$.



(a) Find

(i) the maximum displacement of P from O;

(ii) the maximum velocity of P. [5]

(b) Find

(i) the minimum value of the displacement function $s(t)$;

(ii) the displacement of P from O when $t = 3.5$. [3]

(c) Hence, determine the **total distance** travelled by P in the first 3.5 seconds. [3]

The first time that P returns and passes through O is when $t = T$.

(d) Write down the value of T. [1]

(This question continues on the following page)

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(Question 9 continued)

The particle passes through O every T seconds.

A sequence $u_1, u_2, u_3 \ldots$ is formed where $u_1, u_2, u_3 \ldots$ are the largest **distances** from O in each of the intervals $0 < t < T$, $T < t < 2T$, $2T < t < 3T \ldots$ respectively.

It is known that $u_1, u_2, u_3 \ldots$ form a geometric sequence.

(e) (i) Determine the value of the common ratio r of this geometric sequence.

(ii) Calculate the **total distance** travelled by the particle if it were to continue to move in this way indefinitely. [5]

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Question 9 - working space