

APOLLO

Number and algebra SL

Mathematics: applications and interpretation

First assessment 2021

52 questions | questions

Katya approximates π , correct to four decimal places, by using the following expression.

$$3 + \frac{1}{6 + \frac{13}{16}}$$

- Calculate Katya's approximation of π , correct to four decimal places. [2]
- Calculate the percentage error in using Katya's four decimal place approximation of π , compared to the exact value of π in your calculator. [2]

[illegible]

8. [Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran 500 m. On each subsequent day, Charlie ran 100 m more than the previous day whereas Daniella increased her distance by 2 % of the distance ran on the previous day.

- (a) Calculate how far
 - (i) Charlie ran on day 20 of his fitness programme.
 - (ii) Daniella ran on day 20 of her fitness programme. [5]

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

- (b) Find the value of n . [3]



10. [Maximum mark: 6]

Tommaso and Pietro have each been given 1500 euro to save for college.

Pietro invests his money in an account that pays a nominal annual interest rate of 2.75%, **compounded half-yearly**.

- (a) Calculate the amount Pietro will have in his account after 5 years. Give your answer correct to 2 decimal places.

[3]

Tommaso wants to invest his money in an account such that his investment will increase to 1.5 times the initial amount in 5 years. Assume the account pays a nominal annual interest of $r\%$ **compounded quarterly**.

- (b) Determine the value of r .

[3]

This image shows a full page of primary-ruled paper. It features ten sets of horizontal lines across the page. Each set consists of three lines: a solid top line, a dashed middle line, and a dotted bottom line. The lines are evenly spaced and extend from the left margin to the right edge of the page. There are no margins or other markings present.

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3. [Maximum mark: 19]

Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested 37 000 Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

- (a) Calculate the value of Daisy's investment after 2 years. [3]

After m months, the amount of money in the fixed deposit account has appreciated to more than 50 000 AUD.

- (b) Find the minimum value of m , where $m \in \mathbb{N}$. [4]

Daisy is saving to purchase a new apartment. The price of the apartment is 200 000 AUD.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

- (c) Write down the amount of the loan. [1]

The loan is for 10 years, compounded monthly, with equal monthly payments of 1700 AUD made by Daisy at the end of each month.

- (d) For this loan, find

(i) the amount of interest paid by Daisy.

(ii) the annual interest rate of the loan. [5]

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

- (e) Find the amount of Daisy's final payment. [3]

- (f) Find how much money Daisy saved by making one final payment after 5 years. [3]

3. [Maximum mark: 16]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

(a) Find:

(i) the number of seats in the last row.

(ii) the total number of seats in the concert hall.

[5]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

(b) Find the average number of visitors per concert in 2020.

[2]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

(c) Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall.

[5]

It is assumed that the concert hall will host 50 concerts each year.

(d) Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025.

[4]

Turn over

10. [Maximum mark: 9]

In this question, give all answers correct to 2 decimal places.

Raul and Rosy want to buy a new house and they need a loan of 170 000 Australian dollars (AUD) from a bank. The loan is for 30 years and the annual interest rate for the loan is 3.8 %, compounded monthly. They will pay the loan in fixed monthly instalments at the end of each month.

- (a) Find the amount they will pay the bank each month. [3]
- (b) (i) Find the amount Raul and Rosy will still owe the bank at the end of the first 10 years.
- (ii) Using your answers to parts (a) and (b)(i), calculate how much interest they will have paid in total during the first 10 years. [6]



Turn over

2. [Maximum mark: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

- (a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n .

- (b) (i) Write down the common ratio of the sequence.
- (ii) Find an expression for u_n .
- (iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were 10 380 places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n .

- (c) Write down an expression for v_n . [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an \$80 acceptance fee.

- (d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

- (e) Find k . [3]
- (f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

Turn over

The Osaka Tigers basketball team play in a multilevel stadium.



Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

- Write down the value of the common difference, d [1]
- Calculate the price of a ticket in the 16th row. [2]
- Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

[illegible]

1. [Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**. No deposit required and repayments are made each quarter.

- (a) (i) Find the repayment made each quarter.
- (ii) Find the total amount paid for the car.
- (iii) Find the interest paid on the loan.

[7]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\%$ **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

- (b) (i) Find the amount to be borrowed for this option.
- (ii) Find the annual interest rate, r .
- (c) State which option Bryan should choose. Justify your answer.

[5]

[2]

Bryan's car depreciates at an annual rate of 25% per year.

- (d) Find the value of Bryan's car six years after it is purchased.

[3]

Turn over

5. [Maximum mark: 5]

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, \$)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was \$7816.
- There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x , the number of child tickets sold be y , and the number of student tickets sold be z .

- (a) Write down three equations that express the information given above. [3]
- (b) Find the number of each type of ticket sold. [2]

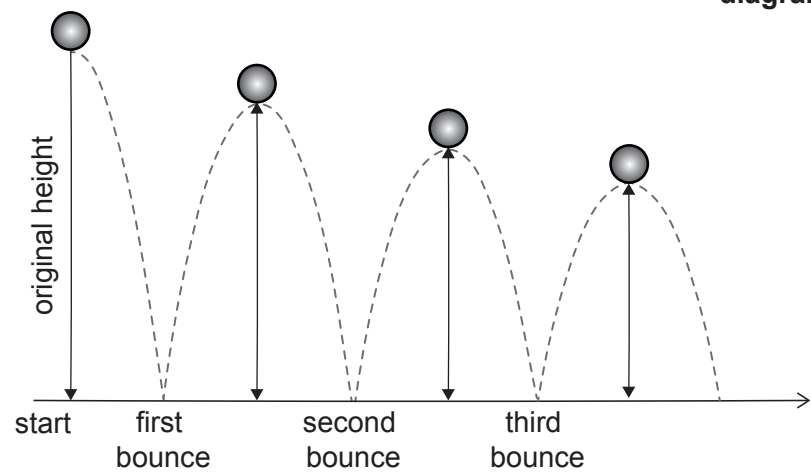


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13. [Maximum mark: 7]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85 % of the previous maximum height.

diagram not to scale



- (a) Show that the maximum height reached by the ball after it has bounced for the sixth time is 68 cm, to the nearest cm.

[2]
- (b) Find the number of times, after the first bounce, that the maximum height reached is greater than 10 cm.

[2]
- (c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce.

[3]

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References:

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13. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying 3.5 % interest, compounded annually. She expects the annual inflation rate to be 2 % per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of \$4000, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest \$1000 at the start of the 10-year period and then invest \$ x into the account at the end of each year (including the first and last years).

- (a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar.

[3]
- (b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar.

[3]

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References:

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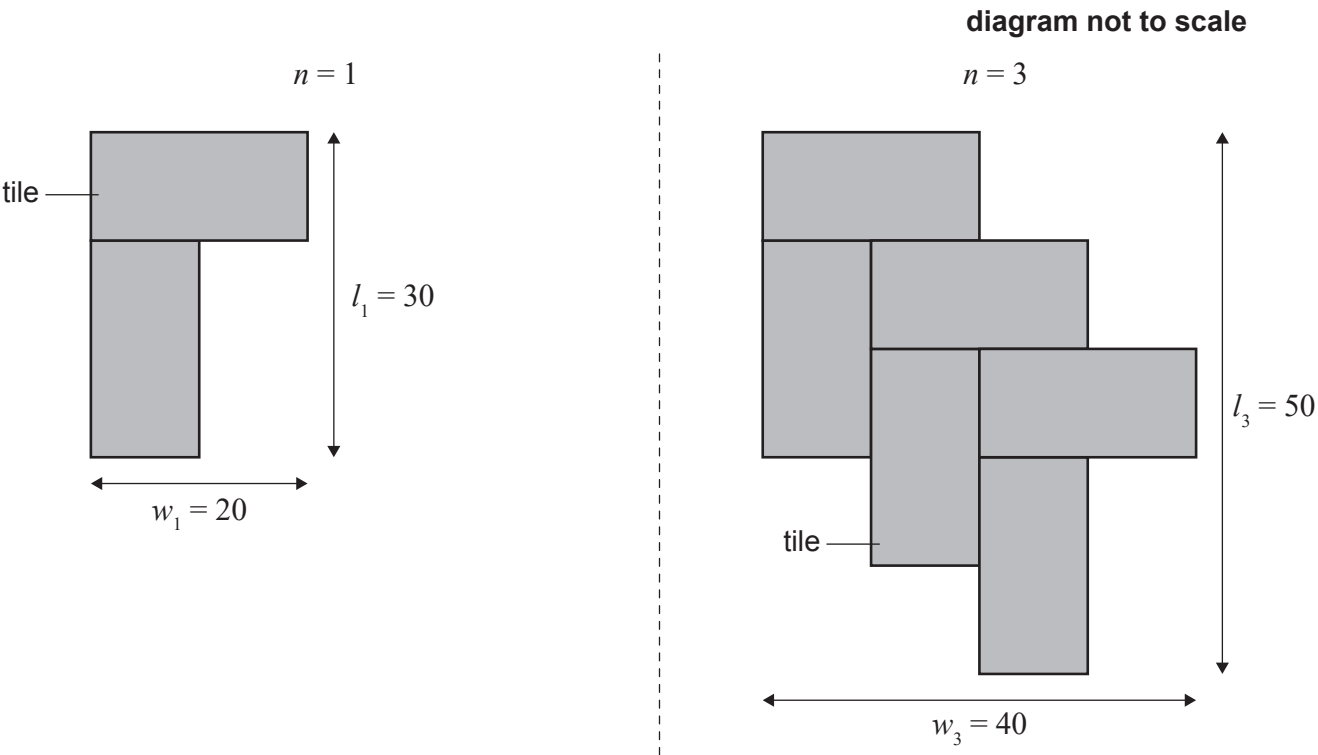


2. [Maximum mark: 19]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n .

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

(This question continues on the following page)

(Question 2 continued)

(a) Find the value of

(i) a .

(ii) b .

[2]

(b) Write down an expression in terms of n for

(i) w_n .

(ii) l_n .

[3]

Eddie's lawn has a length 740 cm.

(c) (i) Show that Eddie needs 144 tiles.

(ii) Find the value of w_n for this path.

[3]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer.

[3]

The tiles cost \$24.50 per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles.

[3]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order.

[3]

There is a fixed delivery cost of \$35.

(g) Find the total cost for Eddie's order.

[2]

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2. [Maximum mark: 13]

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

- (a) Find the number of cups of dog food
- (i) fed to the dog per day;
 - (ii) remaining in the bag at the end of the first day. [4]
- (b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]
- In 2021, Scott spent \$625 on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.
- (c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]
- (d) (i) Calculate the value of $\sum_{n=1}^{10} (625 \times 1.064^{(n-1)})$.
- (ii) Describe what the value in part (d)(i) represents in this context. [3]
- (e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

2. [Maximum mark: 7]

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

- Find the number of trees to be planted in the 15th month. [3]
- Find the total number of trees to be planted in the first 15 months. [2]
- Find the mean number of trees planted per month during the first 15 months. [2]

[illegible]

7. [Maximum mark: 7]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5 % compounded monthly. At the end of each month, Laviola deposits an additional EUR 500 into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw EUR 10 000.

- (a) Find the smallest possible value of k , for $k \in \mathbb{Z}^+$.

[4]
- (b) For this value of k , find the interest earned in the savings account.
Express your answer correct to the nearest EUR.

[3]

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10. [Maximum mark: 6]

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m , of another star can be modelled as a function of its brightness, b , relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

- (a) Find the magnitude of Acubens.
- [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

- (b) Find the brightness of Ceres.
- [2]
- (c) Find how many times brighter Acubens is compared to Ceres.
- [2]

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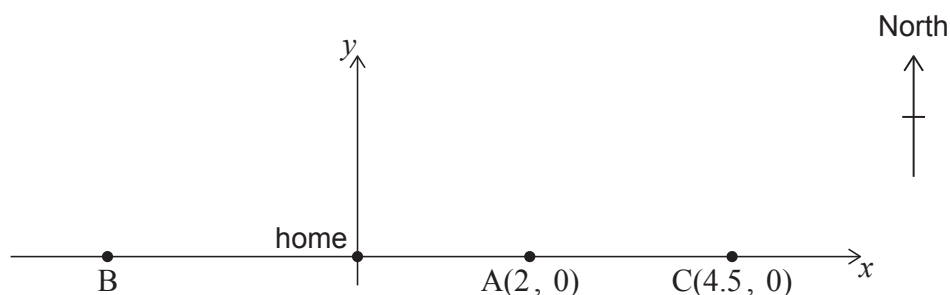
11. [Maximum mark: 7]

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0, 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $A(2, 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $C(4.5, 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x -coordinate. These x -coordinates form a geometric sequence.

- (a) Show that the common ratio, r , is -1.5 . [2]

On the 6th day, Kristi runs to point F.

- (b) Find the location of point F. [2]
- (c) Find the total distance Kristi runs during the first 7 days of training. [3]

(This question continues on the following page)



(Question 11 continued)

[illegible]

Turn over

3. [Maximum mark: 5]

On 1 January 2022, Mina deposited \$ 1000 into a bank account with an annual interest rate of 4% , compounded monthly. At the end of January, and the end of every month after that, she deposits \$ 100 into the same account.

- (a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]
- (b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than \$ 5000 in her account. [2]



8. [Maximum mark: 7]

“Password entropy” is a measure of the predictability of a computer password. The higher the entropy, the more difficult it is to guess the password.

The relationship between the password entropy, p , (measured in bits) and the number of guesses, G , required to decode the password is given by $0.301p = \log_{10} G$.

- (a) Calculate the value of p for a password that takes 5000 guesses to decode. [2]
- (b) Write down G as a function of p . [1]
- (c) Find the number of guesses required to decode a password that has an entropy of 28 bits. Write your answer in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

There is a point on the graph of the function $G(p)$ with coordinates $(0, 1)$.

- (d) Explain what these coordinate values mean in the context of computer passwords. [1]



1. [Maximum mark: 5]

Zaha is designing a bridge to cross a river. She believes that the weight of the steel needed for this bridge is approximately 53 632 000 kg.

The exact weight of the steel needed for the bridge is 55 625 000 kg.

- (a) Find the percentage error in Zaha’s approximation. [2]

Zaha’s design is used to build five identical bridges.

- (b) (i) Find the weight of the steel needed for these five bridges, **to three significant figures**. [2]
- (ii) Write down your answer to part (b)(i) in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

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2. [Maximum mark: 6]

Angel has \$ 520 in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2 % compounded quarterly.

- (a) Calculate the amount of money Angel would have at the end of 5 years with the bank.
Give your answer correct to two decimal places. [3]

Instead of investing the money, Angel decides to buy a phone that costs \$ 520. At the end of 5 years, the phone will have a value of \$ 30. It may be assumed that the depreciation rate per year is constant.

- (b) Calculate the annual depreciation rate of the phone. [3]

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Turn over

2. [Maximum mark: 15]

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

- (a) Calculate the number of pendulums she would make on the 12th day. [3]

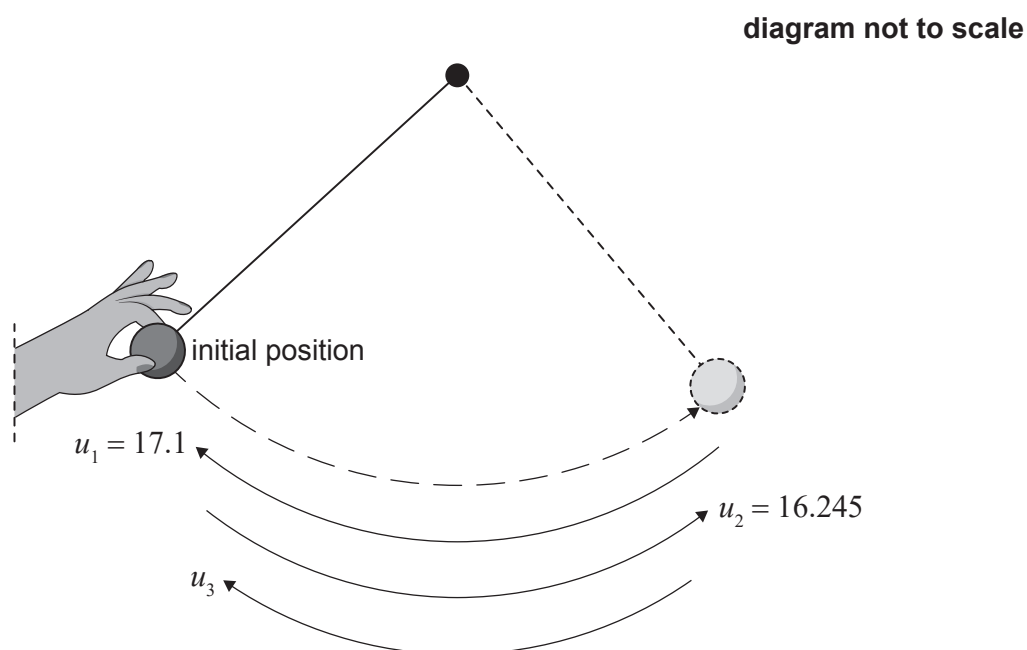
She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

- (b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

- (c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.



(This question continues on the following page)

(Question 2 continued)

On each successive swing, the distance that the ball travelled was 95 % of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

- (d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]
- (e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]
- (f) Calculate the distance that the ball travelled before Daina started recording. [2]

Turn over

4. [Maximum mark: 16]

Ruben wants to buy a car for a price of 285 000 South African rand (ZAR). He goes to a bank to get a loan to buy the car. To be eligible for the loan, Ruben must make an initial down payment equal to 25 % of the price of the car.

The bank offers him a 5-year loan for the remaining balance, with a 4.5 % nominal interest rate per annum, compounded monthly. Ruben will pay the loan in fixed payments at the end of each month.

- (a) (i) Find the original amount of the loan after the down payment is paid.
Give the exact answer.
- (ii) Calculate Ruben's monthly payment for this loan, to two decimal places. [5]
- (b) Using your answer from part (a)(ii), calculate the total amount Ruben will pay over the life of the loan, to the nearest ZAR. Do **not** include the initial down payment. [2]

Ruben would like to repay the loan faster and increases his payments such that he pays 4600 ZAR each month.

- (c) Find the total number of monthly payments he will need to make to pay off the loan. [2]

This strategy will result in Ruben's final payment being less than 4600 ZAR.

- (d) Determine the amount of Ruben's final payment, to two decimal places. [4]
- (e) Hence, determine the total amount Ruben will save, to the nearest ZAR, by making the higher monthly payments. [3]

4. [Maximum mark: 16]

Tiffany wants to buy a house for a price of 285 000 US Dollars (USD). She goes to a bank to get a loan to buy the house. To be eligible for the loan, Tiffany must make an initial down payment equal to 15 % of the price of the house.

The bank offers her a 30-year loan for the remaining balance, with a 4 % nominal interest rate per annum, compounded monthly. Tiffany will pay the loan in fixed payments at the end of each month.

- (a) (i) Find the original amount of the loan after the down payment is paid.
Give the exact answer.
- (ii) Calculate Tiffany's monthly payment for this loan, to two decimal places. [5]
- (b) Using your answer from part (a)(ii), calculate the total amount Tiffany will pay over the life of the loan, to the nearest dollar. Do **not** include the initial down payment. [2]

Tiffany would like to repay the loan faster and increases her payments such that she pays 1300 USD each month.

- (c) Find the total number of monthly payments she will need to make to pay off the loan. [2]

This strategy will result in Tiffany's final payment being less than 1300 USD.

- (d) Determine the amount of Tiffany's final payment, to two decimal places. [4]
- (e) Hence, determine the total amount Tiffany will save, to the nearest dollar, by making the higher monthly payments. [3]

3. [Maximum mark: 7]

On 1 January 2025, the Faber Car Company will release a new car to global markets. The company expects to sell 40 cars in January 2025. The number of cars sold each month can be modelled by a geometric sequence where $r = 1.1$.

- (a) Use this model to find the number of cars that will be sold in December 2025. [2]
- (b) Use this model to find the total number of cars that will be sold in the year
(i) 2025.
(ii) 2026. [5]

[illegible]

Turn over

Maan deposited \$100 000 into a savings account with a nominal annual interest rate of $I\%$ **compounded monthly**. At the end of the eighth year, the amount in the account had increased to \$150 000.

[3]

[3]

[illegible]

Turn over

5. [Maximum mark: 9]

Imani invests \$3000 in a bank that pays a nominal annual interest rate of 1.25% compounded monthly.

- (a) Calculate the amount of money Imani will have in the bank at the end of 6 years. Give your answer correct to two decimal places. [3]
- (b) Calculate the number of months it takes until Imani has at least \$3550 in the bank. [2]

Imani uses the \$3550 as a partial payment for a used car costing \$22 000. For the remainder she takes out a loan from a bank.

- (c) Write down the amount of money that Imani takes out as a loan. [1]

The loan is for 8 years and the nominal annual interest rate is 12.6% compounded monthly. Imani will pay the loan in fixed monthly instalments at the end of each month.

- (d) Calculate the amount, correct to the nearest dollar, that Imani will have to pay the bank each month. [3]



Turn over

7. [Maximum mark: 7]

The pH scale is a measure of the acidity of a solution. Its value is given by the formula

$$\text{pH} = -\log_{10} [\text{H}^+],$$

where $[\text{H}^+]$ is the concentration of hydrogen ions in the solution (measured in moles per litre).

- (a) Calculate the pH value if the concentration of hydrogen ions is 0.0003. [2]

The pH of milk is 6.6.

- (b) Calculate the concentration of hydrogen ions in milk. [2]

The strength of an acid is measured by its concentration of hydrogen ions.

A lemon has a pH value of 2 and a tomato has a pH value of 4.5.

- (c) Calculate how many times stronger the acid in a lemon is when compared to the acid in a tomato. [3]



Turn over

9. [Maximum mark: 6]

A marathon is a race over a distance of 42.195 km. Two runners, Eefje and Shumay, are training to run a marathon.

The two runners train in different ways:

- Eefje runs 5 km on the first day of training and then increases the distance she runs by 2 km on each subsequent day.
- Shumay runs 5 km on the first day of training and then increases the distance she runs by 13% on each subsequent day.

Determine which runner will be the first to run the distance of a marathon on a particular day of their training, and state on which day of their training this will occur.

[illegible]

Turn over

3. [Maximum mark: 5]

Tickets to enter a museum are priced at a dollars for adults and c dollars for children.

A school group of 7 adults and 60 children paid a total of \$832.

A family of 3 adults and 5 children paid a total of \$108.

(a) Write down **two** equations that represent this information. [2]

(b) Hence, find the price of

(i) an adult ticket

(ii) a child ticket. [2]

Rounded to the **nearest thousand**, there were 203 000 visitors at the museum last year.

(c) Write down the lower bound for the number of visitors last year. [1]



8. [Maximum mark: 5]

On 1 January in a particular year, Anton invests \$18 000 in a new bank account. The account earns 4% simple interest, on the original \$18 000, at the start of each subsequent year.

The amounts in the account at the start of each year form an arithmetic sequence.

(a) Find the common difference of this sequence. [2]

After k complete years, the amount in Anton’s account will be greater than \$32 000 for the first time.

(b) Find the value of k . [3]



Turn over

10. [Maximum mark: 6]

When Humberto retires, he invests \$300 000 in an annuity fund that earns interest at a nominal rate of 3.8% per year, compounded monthly.

Humberto then withdraws \$2800 at the end of every month to pay for his living expenses.

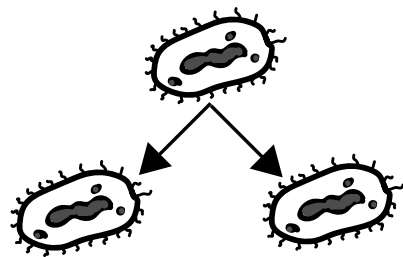
- (a) Find how much is in the annuity fund after 8 years. [3]
- (b) Calculate how many times Humberto is able to make these withdrawals. [3]



Turn over

13. [Maximum mark: 7]

A type of bacteria reproduces by **dividing in two** every 10 minutes.



There were 1250 bacteria in a colony 10 minutes after the start of an experiment.

The following table is used to estimate the number of bacteria, u_n , for this colony.

The values of u_n form the terms of a sequence.

n	1	2	3		9		k
Time in minutes	10	20	30		90		$10k$
Number of bacteria, u_n	1250	2500	5000		...		...

- (a) Complete the table by adding the two missing values. [3]

As the number of bacteria increases from 1250 to 2500, the total number of **bacterial divisions** is 1250.

- (b) (i) Find the value of n when the number of bacteria is 1.28×10^6 .
(ii) Hence or otherwise, find the total number of bacterial divisions as the number of bacteria increases from 1250 to 1.28×10^6 . Give your answer correct to the nearest thousand bacterial divisions. [4]

(This question continues on the following page)



Please **do not** write on this page.

Answers written on this page
will not be marked.



3. [Maximum mark: 5]

Tickets to enter a theme park are priced at a dollars for adults and c dollars for children.

A school group of 6 adults and 50 children paid a total of \$1292.

A family of 2 adults and 3 children paid a total of \$130.

(a) Write down **two** equations that represent this information. [2]

(b) Hence, find the price of

(i) an adult ticket

(ii) a child ticket. [2]

Rounded to the **nearest thousand**, there were 101 000 visitors at the theme park last year.

(c) Write down the lower bound for the number of visitors last year. [1]



8. [Maximum mark: 5]

On 1 January in a particular year, Eva invests \$25 000 in a new bank account. The account earns 5% simple interest, on the original \$25 000, at the start of each subsequent year.

The amounts in the account at the start of each year form an arithmetic sequence.

(a) Find the common difference of this sequence. [2]

After k complete years, the amount in Eva’s account will be greater than \$44 000 for the first time.

(b) Find the value of k . [3]



Turn over

10. [Maximum mark: 6]

When Daniel retires, he invests \$400 000 in an annuity fund that earns interest at a nominal rate of 4.5 % per year, compounded monthly.

Daniel then withdraws \$3600 at the end of every month to pay for his living expenses.

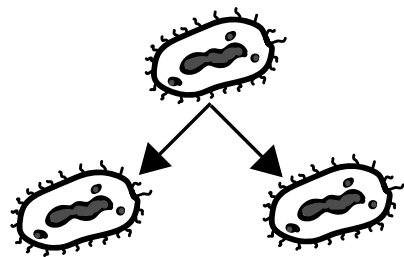
- (a) Find how much is in the annuity fund after 5 years. [3]
- (b) Calculate how many times Daniel is able to make these withdrawals. [3]

[illegible]

Turn over

13. [Maximum mark: 7]

A type of bacteria reproduces by **dividing in two** every 10 minutes.



There were 1250 bacteria in a colony 10 minutes after the start of an experiment.

The following table is used to estimate the number of bacteria, u_n , for this colony.

The values of u_n form the terms of a sequence.

n	1	2	3		9		k
Time in minutes	10	20	30		90		$10k$
Number of bacteria, u_n	1250	2500	5000		...		...

- (a) Complete the table by adding the two missing values. [3]

As the number of bacteria increases from 1250 to 2500, the total number of **bacterial divisions** is 1250.

- (b) (i) Find the value of n when the number of bacteria is 1.28×10^6 .
(ii) Hence or otherwise, find the total number of bacterial divisions as the number of bacteria increases from 1250 to 1.28×10^6 . Give your answer correct to the nearest thousand bacterial divisions. [4]

(This question continues on the following page)



(Question 13 continued)

[illegible]

Please **do not** write on this page.

Answers written on this page
will not be marked.



3. [Maximum mark: 5]

A bakery sells boxes of cupcakes at the following prices:

- a small box, containing 1 cupcake, costs \$4.50
- a medium box, containing 6 cupcakes, costs \$23.50
- a large box, containing 12 cupcakes, costs \$44.75.

Medium box



In a particular week, the bakery sold 88 boxes containing a total of 383 cupcakes. The bakery collected \$1486 from cupcake sales for the week.

The number of boxes of cupcakes sold during the week can be represented by

$$x + y + z = 88,$$

where x is the number of small boxes sold, y is the number of medium boxes sold, and z is the number of large boxes sold.

(a) Write down an equation in terms of x , y , and z to represent

- the number of cupcakes sold during the week.
- the money collected from sales of cupcakes for the week.

[3]

(b) Hence, use your graphic display calculator to determine the number of medium boxes that were sold during the week.

[2]

(This question continues on the following page)



(Question 3 continued)

[illegible]

Turn over

1. [Maximum mark: 7]

Give answers to this question correct to two decimal places.

Pierre invests 1500 euros (EUR) at the end of each month for 10 years into a savings plan that pays a nominal annual interest rate of 3.6% compounded monthly.

(a) Calculate the value of Pierre's savings plan at the end of the 10 years. [3]

At the end of the 10 years, Pierre withdraws 100 000 EUR from the savings plan to use as a deposit on a house.

Pierre invests the remainder into another account for 15 years at a nominal annual interest rate of 4.5% compounded quarterly.

(b) Calculate the amount in Pierre's account at the end of this time. [4]

(This question continues on the following page)



(Question 1 continued)

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Turn over

3. [Maximum mark: 6]

As part of an experiment, a colony of ants was created and observed. At the start of week one, 200 ants were observed in the colony. The number of ants in the colony increased by 15% each week.

(a) Calculate the number of ants in the colony at the start of week 11. [3]

The number of ants in the colony was first observed to exceed 3000 in week k .

(b) Find the value of k . [3]



Turn over

6. [Maximum mark: 6]

A speed camera is used to determine whether a car is exceeding a speed limit of 8.3 m s^{-1} .

An exact distance of 10 m is marked out.

The car travels this 10 m distance in 1.2 seconds, measured to the nearest 0.1 second.

Determine whether it is certain that the car was exceeding the speed limit of 8.3 m s^{-1} .

Justify your answer.

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Turn over

7. [Maximum mark: 5]

The loudness of a sound, L , measured in decibels (dB) is determined by the intensity of the sound, I , measured in watts per square metre (Wm^{-2}). The relationship between loudness and intensity can be expressed using the logarithmic function

$$L = 10 \log_{10} \left(\frac{I}{I_0} \right), \quad I > 0$$

where I_0 is the reference intensity (the intensity of the least audible sound to the human ear).

The reference intensity I_0 is 10^{-12}Wm^{-2} .

The intensity of sound on a busy street is 10^{-5}Wm^{-2} .

- (a) Calculate the loudness of the sound. [2]

The sound of a jet engine reaches a loudness of 185 dB.

- (b) Determine the intensity of its sound. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10, k \in \mathbb{Z}$. [3]

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Turn over

11. [Maximum mark: 8]

Sofia takes out a loan for 50 000 euros. She pays a nominal interest rate of 4.2 % per year, compounded monthly. She must pay back the loan in 25 years through regular monthly payments, made at the end of each month.

- (a) Find the amount Sofia must pay back each month. Give your answer correct to two decimal places.

[2]

At the end of eight years, Sofia wins the lottery and pays back the remainder of the loan early in one single payment.

- (b) (i) Find how much Sofia pays back in this single payment. Give your answer correct to two decimal places.

- (ii) Find the amount of money saved by Sofia by paying back the loan early.

[6]

[illegible]

Turn over

2. [Maximum mark: 15]

Give all answers in this question to the nearest dollar.

Anika invested 10 000 Canadian dollars (CAD) in an account which earns a nominal annual interest rate of 3.4%, compounded monthly.

- (a) Calculate the value of Anika's investment after 6 years. [3]

Anika wants to purchase a house that costs 225 000 CAD. Anika uses the value obtained in part (a) to make an initial payment on the house and takes out a loan to pay the rest.

- (b) Determine the amount of Anika's loan. [1]

Anika amortizes her loan over 15 years. She arranges to make a loan payment at the end of each month. The nominal annual interest rate for the loan is 6.4%, compounded half-yearly.

- (c) Determine Anika's monthly loan payment. [5]

After 2 complete years, Anika decides to increase her monthly loan payment to 2200 CAD. At the time, the remaining amount of the loan is 194 572 CAD.

- (d) (i) Calculate the amount by which the loan will decrease during the next 3 complete years.
(ii) Determine the total interest that Anika will pay over this 3-year period. [6]

3. [Maximum mark: 17]

Felix is training for a competition consisting of running and swimming. He plans to train for 12 weeks to prepare for the competition.

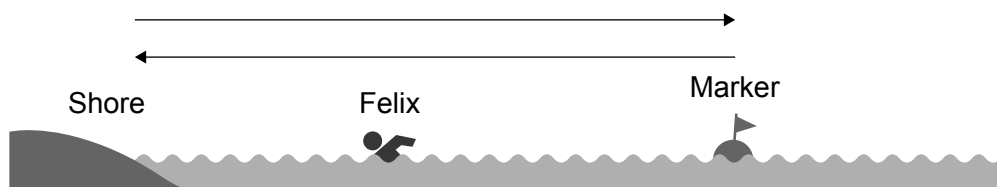
The running part of the competition is a 5 km race. During the 12 weeks, Felix runs on **3 days each week**. On the first day of training, Felix runs 2 km. Each subsequent day that he runs, he increases his distance by 5%.

- (a) Explain why the distances of each of Felix's runs form a geometric sequence and state the common ratio. [2]
- (b) Find the total distance of Felix's first 15 runs. [2]

A published recommendation states that, on the last day of training, an athlete should run a distance that is at least twice as far as the race distance.

- (c) Show that Felix's final run of his 12-week training plan meets this recommendation. [3]

For the swimming part of the competition, competitors must swim 375 m out to a marker in the water and then 375 m back to the shore.



Felix plans to swim **2 times per week** to train for the swimming portion of the competition. On each day he swims in the 1st week, Felix swims out to a marker that is 80 m from the shore, and then swims back to shore.

At the start of each subsequent week, the marker is moved to increase the distance between the marker and the shore by a constant amount. Felix swims a total distance of 704 m in the 4th week.

The distances that Felix swims each week form an arithmetic sequence, where each term, u_n , is the **total distance** he swims in the n th week.

- (d) (i) Find u_1 . [7]
- (ii) Determine the increase in the total distance Felix swims each week.
- (iii) Hence calculate the distance between the marker and the shore in the 2nd week of training.
- (e) Calculate the distance of Felix's final swim in his 12th week of training. [3]

Turn over

1. [Maximum mark: 16]

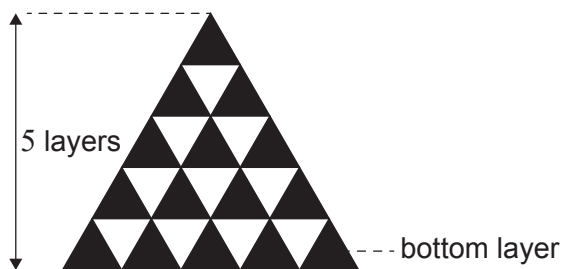
Thai cushions are designed with a triangular cross-section and are made from layers of smaller cushions. These cushions can be modelled as triangular prisms.

This is shown in the diagram.

diagram not to scale



Thai cushion with 4 layers



Cross-section of Thai cushion with 5 layers

(a) Write down the number of triangular prisms in the bottom layer of the cushion with

(i) 4 layers.

(ii) 5 layers.

[2]

Mayumi notices that the number of triangular prisms in the bottom layer of the cushions forms an arithmetic sequence.

(b) (i) Write down the common difference of this sequence.

(ii) Find an expression for the number of triangular prisms in the bottom layer of a cushion with n layers.

[3]

(This question continues on the following page)

(Question 1 continued)

Mayumi wants to extend this design to create a cushion with 9 layers.

- (c) (i) Find the number of triangular prisms in the bottom layer of Mayumi's cushion. [3]
- (ii) Calculate the **total** number of triangular prisms in Mayumi's cushion. [3]
- (d) Find an expression for the **total** number of triangular prisms in a cushion with n layers, giving your answer in its simplest form. [2]

The cross-section of the cushion consists of black triangles and white triangles.



This cushion with 4 layers has a total of 6 white triangles.



This cushion with 5 layers has 4 white triangles in its bottom layer.

- (e) Write down the total number of black triangles in a cushion with 4 layers. [1]

The number of black triangles in each layer forms an arithmetic sequence.

- (f) Find and simplify an expression for the total number of black triangles in a cushion with n layers. [2]

The total number of white triangles in a cushion with n layers is $\frac{n(n-1)}{2}$.

- (g) Using both the given expression and your answer to part (f), find and simplify an expression for the total number of black and white triangles in a cushion with n layers. [3]

Turn over

1. [Maximum mark: 18]

Cathie is a financial analyst studying the growth of two investment accounts, Account 1 and Account 2, for a new client.

Account 1 has an initial amount of 5000 US Dollars (USD). Interest is added to the amount in Account 1 at the end of each year in the following manner: 200USD at the end of the first year, 260USD at the end of the second year, 320USD at the end of the third year, 380USD at the end of the fourth year and 440USD at the end of the fifth year.

Assume the amount of interest continues to increase each year so that it follows an arithmetic sequence.

(a) Find

(i) the common difference.

(ii) the amount of interest, in USD, added at the end of the 10th year. [3]

(b) Show that the amount of money in Account 1 after n years may be expressed as

$$5000 + \frac{n}{2}(340 + 60n). \quad [3]$$

(c) Hence or otherwise, find the amount of money in Account 1 at the end of 10 years. [2]

Account 2 has the same initial amount of 5000USD. Account 2 pays 6.5% interest compounded annually. The interest is added to the amount in the account at the end of each year.

The amount in Account 2 after n years can be expressed as $5000 \times B^n$ where $B \in \mathbb{R}$.

(d) (i) Write down the value of B .

(ii) Hence or otherwise, show that Account 1 will have more money than Account 2 at the end of 10 years. [4]

The client is interested in a longer-term investment. Cathie finds that it will take at least m complete years for the amount in Account 2 to exceed the amount in Account 1.

(e) Find the value of m . [3]

(f) Determine the total interest added to Account 2 at the end of m years. Give your answer correct to the nearest dollar. [3]

The first three terms of a geometric sequence are 2, 6 and 18.

- Write down the common ratio, r . [1]
- Find the 8th term of the sequence. [2]
- Find the sum of the first 10 terms of the sequence. [2]

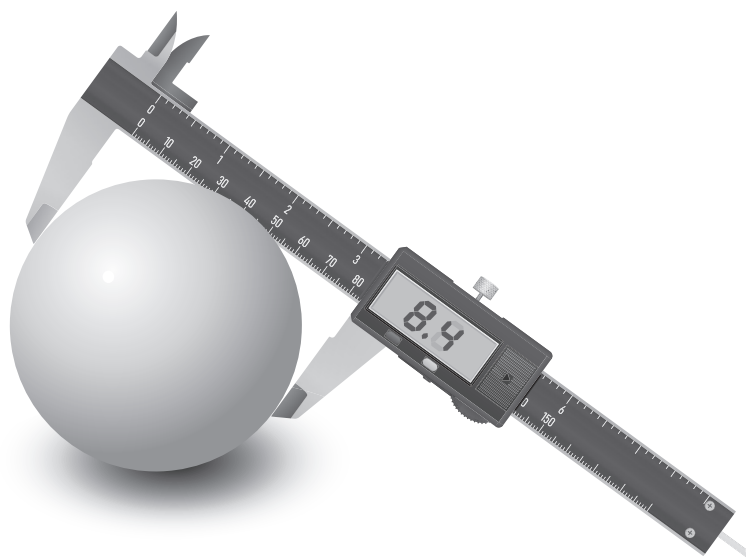
[illegible]

Turn over

2. [Maximum mark: 6]

A digital caliper is used to measure the diameter of a spherical ball.

diagram not to scale



The digital caliper displays the diameter of the ball as 8.4 cm, and is accurate to one decimal place.

- (a) (i) Write down the smallest possible diameter of the ball.
- (ii) Hence, calculate the smallest possible volume of the ball.

[4]

The exact diameter of the ball is 8.38 cm.

- (b) Calculate the percentage error in the measurement of 8.4 cm, as shown on the digital caliper.

[2]

(This question continues on the following page)



(Question 2 continued)

[illegible]

Turn over

5. [Maximum mark: 6]

Mikko deposits \$10 000 into a savings account with a fixed nominal annual interest rate of $r\%$ compounded monthly. At the end of each month, he deposits \$800 into the account.

After 10 years, the value in the savings account is \$160 000.

- (a) Find the value of r . [3]
- (b) Find the amount of interest earned over the 10 years. [3]

[illegible]

Turn over

1. [Maximum mark: 15]

A marathon is a race over a distance of 42.19 km.

- (a) (i) Write down 42.19 km correct to the nearest kilometre.
- (ii) Find the percentage error if 42.19 km is rounded to the nearest kilometre. [3]

Rolando and Davi are training to run a marathon. They both start their training in the same week.

Rolando runs 6 km in the first week, 8 km in the second week and 10 km in the third week. In each subsequent week he runs 2 km further than the previous week.

- (b) Find the week during which Rolando runs 42 km. [3]
- (c) Find the **total** distance that Rolando runs in the first 10 weeks. [2]

Davi runs 3 km in the first week. In each subsequent week he runs 20 % further than the previous week. These distances form the terms of a geometric sequence.

- (d) Find the first week during which Davi runs more than 42 km. [4]
- (e) Find the first week that the **total** distance that Davi has run, since the start of training, exceeds 150 km. [3]

Turn over