

APOLLO

Functions

SL

Mathematics: applications and interpretation

First assessment 2021

64 questions | questions

4. [Maximum mark: 7]

The price of gas at Leon’s gas station is \$1.50 per litre. If a customer buys a minimum of 10 litres, a discount of \$5 is applied.

This can be modelled by the following function, L , which gives the total cost when buying a minimum of 10 litres at Leon’s gas station.

$$L(x) = 1.50x - 5, \, x \geq 10$$

where x is the number of litres of gas that a customer buys.

(a) Find the total cost of buying 40 litres of gas at Leon’s gas station. [2]

(b) Find $L^{-1}(70)$. [2]

The price of gas at Erica’s gas station is \$1.30 per litre. A customer must buy a minimum of 10 litres of gas. The total cost at Erica’s gas station is cheaper than Leon’s gas station when $x > k$.

(c) Find the minimum value of k . [3]



Turn over

7. [Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R , by the function $R(t) = 100e^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

- Find the value of p . [2]
- Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

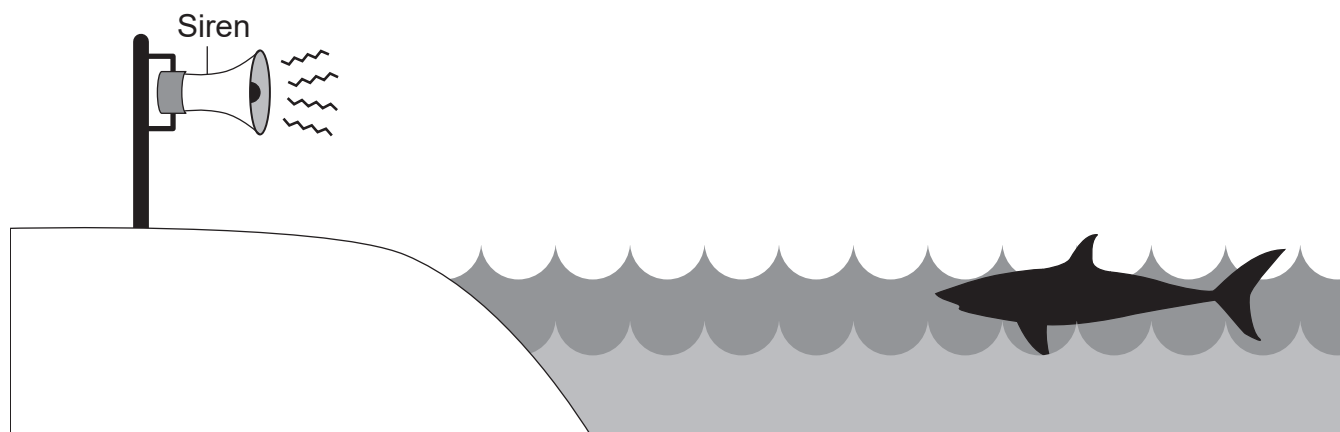
- (c) State a mathematical reason why Professor Wei might believe this. [1]
- (d) Write down one possible limitation of the **domain** of the model. [1]

[illegible]

Turn over

11. [Maximum mark: 6]

If a shark is spotted near to Brighton beach, a lifeguard will activate a siren to warn swimmers.

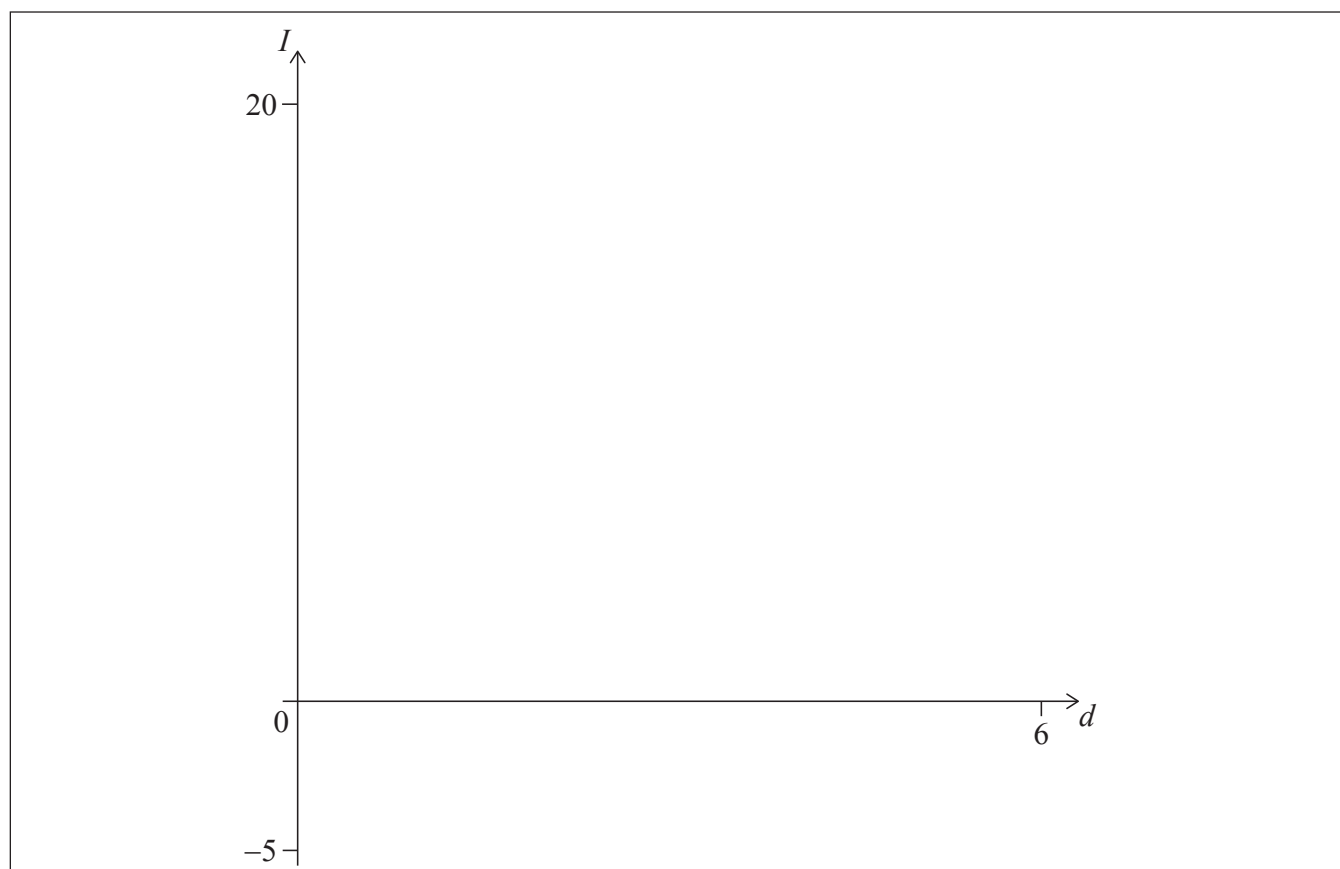


The sound intensity, I , of the siren varies inversely with the square of the distance, d , from the siren, where $d > 0$.

It is known that at a distance of 1.5 metres from the siren, the sound intensity is 4 watts per square metre (W m^{-2}).

(a) Show that $I = \frac{9}{d^2}$. [2]

(b) Sketch the curve of I on the axes below showing clearly the point (1.5, 4). [2]



(This question continues on the following page)



(Question 11 continued)

Whilst swimming, Scarlett can hear the siren only if the sound intensity at her location is greater than $1.5 \times 10^{-6} \text{ Wm}^{-2}$.

- (c) Find the values of d where Scarlett cannot hear the siren. [2]



Turn over

1. [Maximum mark: 5]

The amount, in milligrams, of a medicinal drug in the body t hours after it was injected is given by $D(t) = 23(0.85)^t$, $t \geq 0$. Before this injection, the amount of the drug in the body was zero.

- (a) Write down
 - (i) the initial dose of the drug.
 - (ii) the percentage of the drug that leaves the body each hour. [3]
- (b) Calculate the amount of the drug remaining in the body 10 hours after the injection. [2]



Turn over

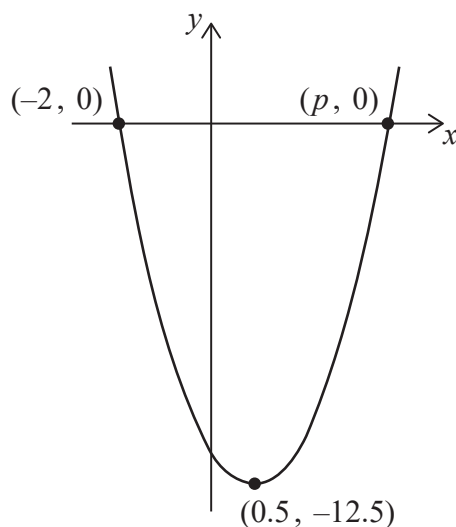
A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \leq x \leq 7$, $x \neq -5$.

- (a) Find the range of f . [3]
- (b) Find the value of $f^{-1}(0)$. [2]

[illegible]

12. [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x -axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale

- (a) Find the value of p . [1]
- (b) Find the value of
- (i) a .
 - (ii) b .
 - (iii) c . [5]
- (c) Write down the equation of the axis of symmetry of the graph. [1]

(This question continues on the following page)

[illegible]

Apollo | Original source crop | 9

3. [Maximum mark: 4]

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = Ae^{kt},$$

where P is the area covered by mould in mm^2 , t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is 112mm^2 at the start of the experiment and 360mm^2 after 5 days.

- (a) Write down the value of A . [1]
- (b) Find the value of k . [3]

[illegible]

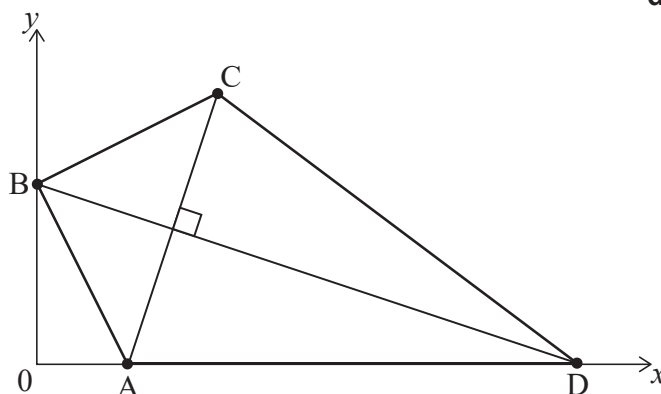
Turn over

4. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm.

The coordinates of A, B and C are (2, 0), (0, 4) and (4, 6) respectively. Point D lies on the x -axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



- (a) Find the gradient of the line through A and C. [2]
- (b) Write down the gradient of the line through B and D. [1]
- (c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a , b and d are integers. [2]
- (d) Write down the x -coordinate of point D. [1]

(This question continues on the following page)



(Question 4 continued)

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Turn over

5. [Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter x cm.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \leq x \leq 14.$$

- (a) Find the range of h .
- [3]

The function h^{-1} is the inverse function of h .

- (b) (i) Find $h^{-1}(10)$.
- (ii) In the context of the question, interpret your answer to part (b)(i).
- (iii) Write down the range of h^{-1} .
- [4]

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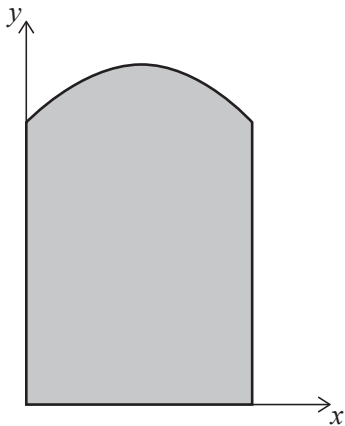
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13. [Maximum mark: 8]

Irina uses a set of coordinate axes to draw her design of a window. The base of the window is on the x -axis, the upper part of the window is in the form of a quadratic curve and the sides are vertical lines, as shown on the diagram. The curve has end points $(0, 10)$ and $(8, 10)$ and its vertex is $(4, 12)$. Distances are measured in centimetres.



The quadratic curve can be expressed in the form $y = ax^2 + bx + c$ for $0 \leq x \leq 8$.

- (a) (i) Write down the value of c .
 - (ii) Hence form two equations in terms of a and b .
 - (iii) Hence find the equation of the quadratic curve. [5]
- (b) Find the area of the shaded region in Irina's design. [3]

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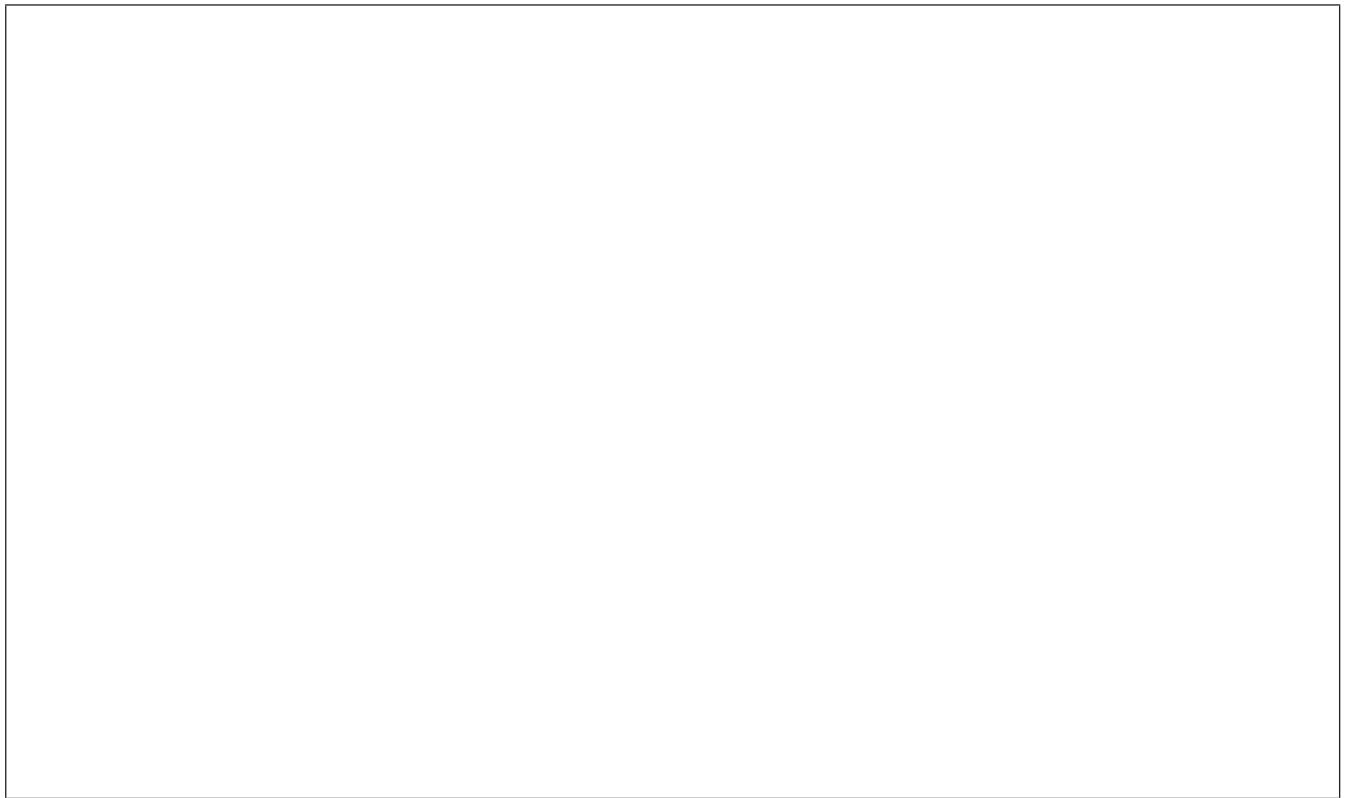
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Turn over

(Question 13 continued)



References:

2. Bermuda Triangle map [online] Available at: [https://commons.wikimedia.org/wiki/File:Bermuda_Triangle_map_\(de\).svg](https://commons.wikimedia.org/wiki/File:Bermuda_Triangle_map_(de).svg)
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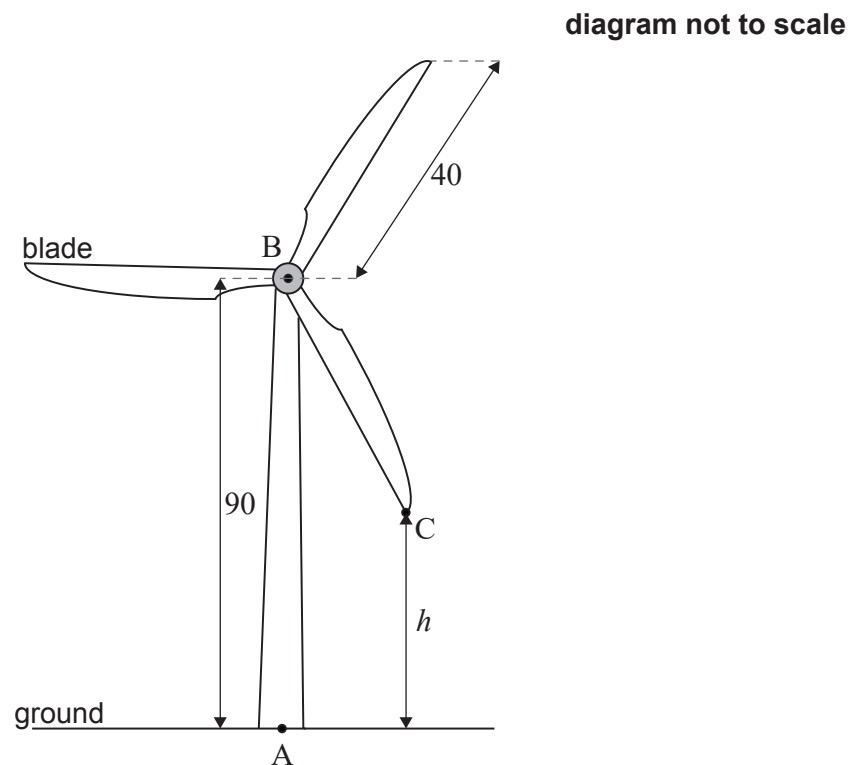
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3. [Maximum mark: 20]

A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is 90 m. The blades of the turbine are centred at B and are each of length 40 m. This is shown in the following diagram.



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

(a) Find the

(i) maximum value of h .

(ii) minimum value of h .

[2]

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

(b) (i) Find the time, in seconds, it takes for the blade [BC] to make one complete rotation under these conditions.

(ii) Calculate the angle, in degrees, that the blade [BC] turns through in one second.

[3]

(This question continues on the following page)

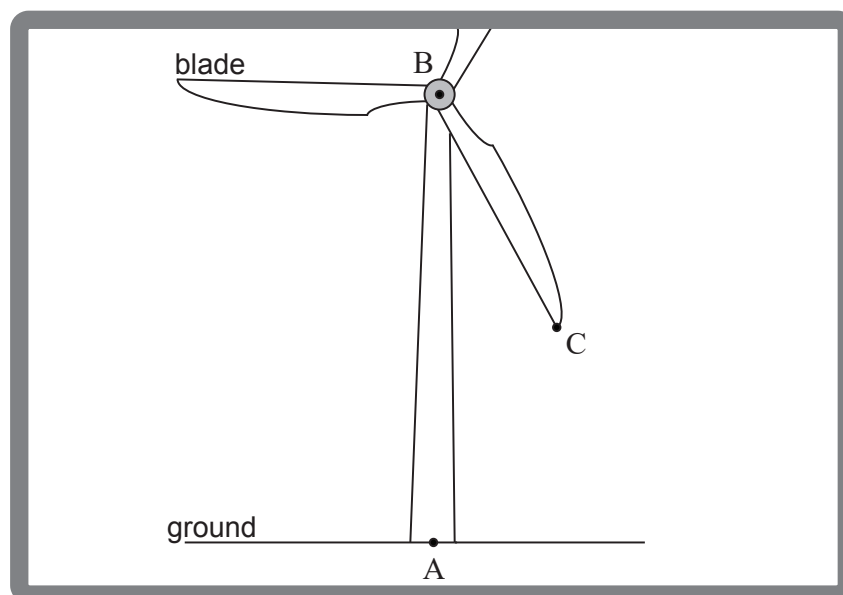
(Question 3 continued)

The height, h , of point C can be modelled by the following function. Time, t , is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40 \cos(72t^\circ), \quad t \geq 0$$

- (c) (i) Write down the amplitude of the function.
- (ii) Find the period of the function. [2]
- (d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points. [3]
- (e) (i) Find the height of C above the ground when $t = 2$.
- (ii) Find the time, in seconds, that point C is above a height of 100 m, during each complete rotation. [5]

Looking through his window, Tim has a partial view of the rotating wind turbine. The position of his window means that he cannot see any part of the wind turbine that is **more than 100 m** above the ground. This is illustrated in the following diagram.



- (f) (i) At any given instant, find the probability that point C is visible from Tim's window.

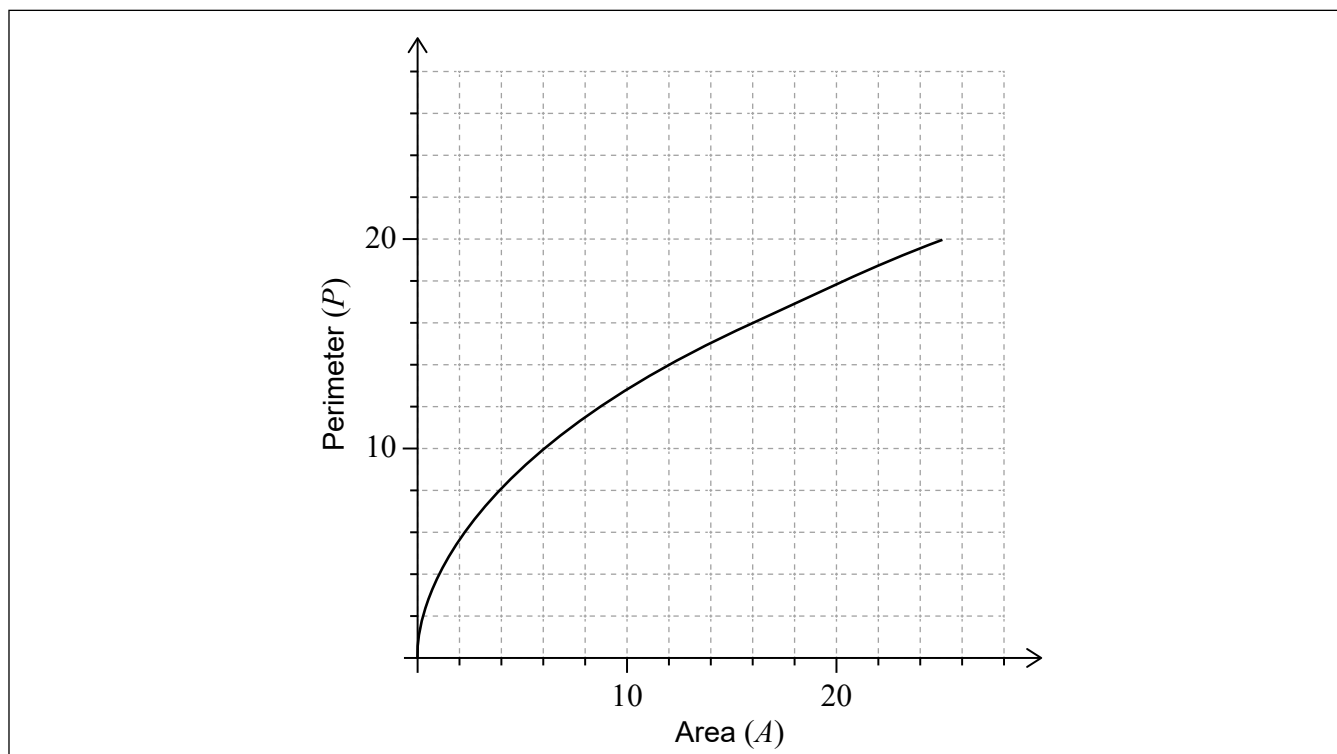
The wind speed increases. The blades rotate at twice the speed, but still at a constant rate.

- (ii) At any given instant, find the probability that Tim can see point C from his window. Justify your answer. [5]

Turn over

4. [Maximum mark: 6]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



- (a) Write down the value of $P(25)$.

[1]

The range of $P(A)$ is $0 \leq P(A) \leq n$.

- (b) Hence write down the value of n .

[1]

- (c) On the axes above, draw the graph of the inverse function, P^{-1} .

[3]

- (d) In the context of the question, explain the meaning of $P^{-1}(8) = 4$.

[1]

(This question continues on the following page)



[illegible]

5. [Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

- (a) Find the population of the Bulbul birds,
 - (i) at the start of the migration season.
 - (ii) in the wetlands after 5 days. [3]
- (b) Calculate the time taken for the population to decrease below 1400. [2]
- (c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

[illegible]

8. [Maximum mark: 4]

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre (W m^{-2}). The intensity level is given by the following formula.

$$L = 10\log_{10}(S \times 10^{12}), S \geq 0$$

- (a) An orchestra has a sound intensity of $6.4 \times 10^{-3} \text{ W m}^{-2}$. Calculate the intensity level, L of the orchestra.

[2]
- (b) A rock concert has an intensity level of 112 dB. Find the sound intensity, S .

[2]

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5. [Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \text{ m s}^{-1}$, and braking distance, $d \text{ m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s \text{ m s}^{-1}$	0	6	10
Braking distance, $d \text{ m}$	0	12	60

This information was used to create Model A, where d is a function of s , $s \geq 0$.

$$\text{Model A: } d(s) = ps^2 + qs, \text{ where } p, q \in \mathbb{Z}$$

At a speed of 6 m s^{-1} , Model A can be represented by the equation $6p + q = 2$.

- (a) (i) Write down a second equation to represent Model A, when the speed is 10 m s^{-1} .
(ii) Find the values of p and q [4]
- (b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]
- (c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]
- (d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

$$\text{Model B: } d(s) = 0.95s^2 - 3.92s$$

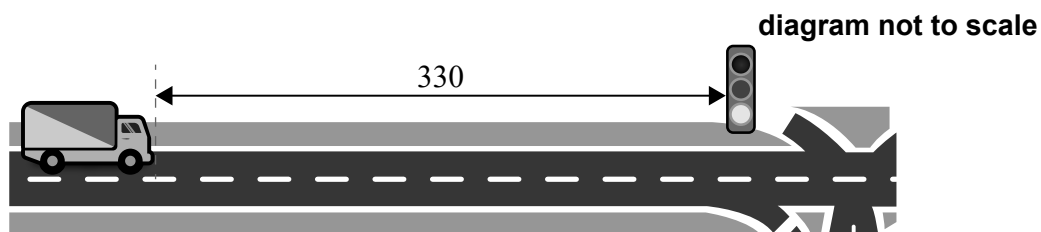
- (e) Use Model B to calculate an estimate for the braking distance at a speed of 20 m s^{-1} . [2]
- The actual braking distance at 20 m s^{-1} is 320 m .
- (f) Calculate the percentage error in the estimate in part (e). [2]

(This question continues on the following page)

(Question 5 continued)

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s \text{ m s}^{-1}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of 330 m.



- (g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection.

[3]

3. [Maximum mark: 5]

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

- (a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]
- (b) Find the value of t when the ball hits the ground. [2]
- (c) State an appropriate domain for t in this model. [2]



Turn over

11. [Maximum mark: 8]

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N , which have a magnitude of at least M . For a particular region the equation is

$$\log_{10} N = a - M, \text{ for some } a \in \mathbb{R}.$$

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

- (a) Find the value of a . [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

- (b) Find the value of b . [2]

- (c) Given $0 < M < 8$, find the range for N . [2]

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

- (d) Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year. [2]

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4. [Maximum mark: 5]

The pH of a solution measures its acidity and can be determined using the formula $\text{pH} = -\log_{10} C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

- (a) Calculate the pH of the coffee.
- [2]

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

- (b) Determine whether the unknown liquid is more or less acidic than the coffee.
Justify your answer mathematically.
- [3]

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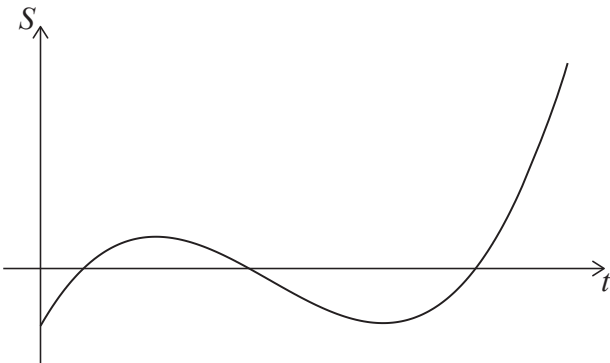
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9. [Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t , the number of years after graduating from university.



- (a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario. [1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a , b , c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	-5	3	-1	-5

- (b) (i) Write down the value of d .
(ii) Write down three simultaneous equations for a , b and c .
(iii) Hence, or otherwise, find the values of a , b and c . [4]

A negative value of S indicates that a graduate is expected to be in debt.

- (c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university. [3]

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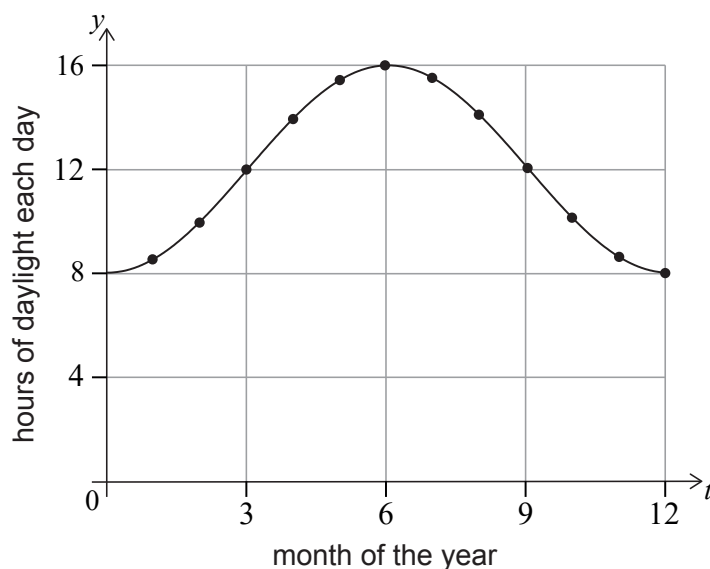


[illegible]

1. [Maximum mark: 15]

Boris recorded the number of daylight hours on the first day of each month in a northern hemisphere town.

This data was plotted onto a scatter diagram. The points were then joined by a smooth curve, with minimum point $(0, 8)$ and maximum point $(6, 16)$ as shown in the following diagram.



Let the curve in the diagram be $y = f(t)$, where t is the time, measured in months, since Boris first recorded these values.

Boris thinks that $f(t)$ might be modelled by a quadratic function.

- (a) Write down one reason why a quadratic function would not be a good model for the number of hours of daylight per day, across a number of years.

[1]

(This question continues on the following page)

(Question 1 continued)

Paula thinks that a better model is $f(t) = a \cos(bt) + d$, $t \geq 0$, for specific values of a , b and d .

(b) For Paula's model, use the diagram to write down

(i) the amplitude.

(ii) the period.

(iii) the equation of the principal axis. [4]

(c) Hence or otherwise find the equation of this model in the form: [3]

$$f(t) = a \cos(bt) + d$$

(d) For the first year of the model, find the length of time when there are more than 10 hours and 30 minutes of daylight per day. [4]

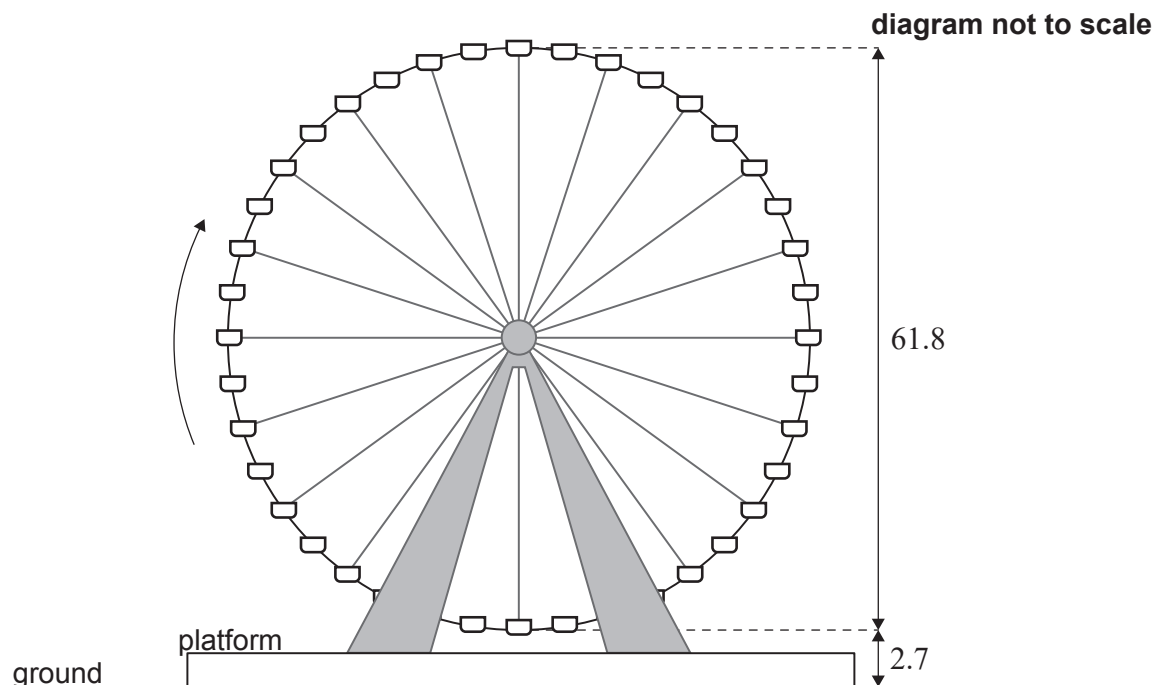
The true maximum number of daylight hours was 16 hours and 14 minutes.

(e) Calculate the percentage error in the maximum number of daylight hours Boris recorded in the diagram. [3]

Turn over

4. [Maximum mark: 17]

The Texas Star is a Ferris wheel at the state fair in Dallas. The Ferris wheel has a diameter of 61.8 m. To begin the ride, a passenger gets into a chair at the lowest point on the wheel, which is 2.7 m above the ground, as shown in the following diagram. A ride consists of multiple revolutions, and the Ferris wheel makes 1.5 revolutions per minute.



The height of a chair above the ground, h , measured in metres, during a ride on the Ferris wheel can be modelled by the function $h(t) = -a \cos(bt) + d$, where t is the time, in seconds, since a passenger began their ride.

(a) Calculate the value of

(i) a ;

(ii) b ;

(iii) d .

[6]

A ride on the Ferris wheel lasts for 12 minutes in total.

(b) Calculate the number of revolutions of the Ferris wheel per ride.

[2]

(c) For exactly one ride on the Ferris wheel, suggest

(i) an appropriate domain for $h(t)$;

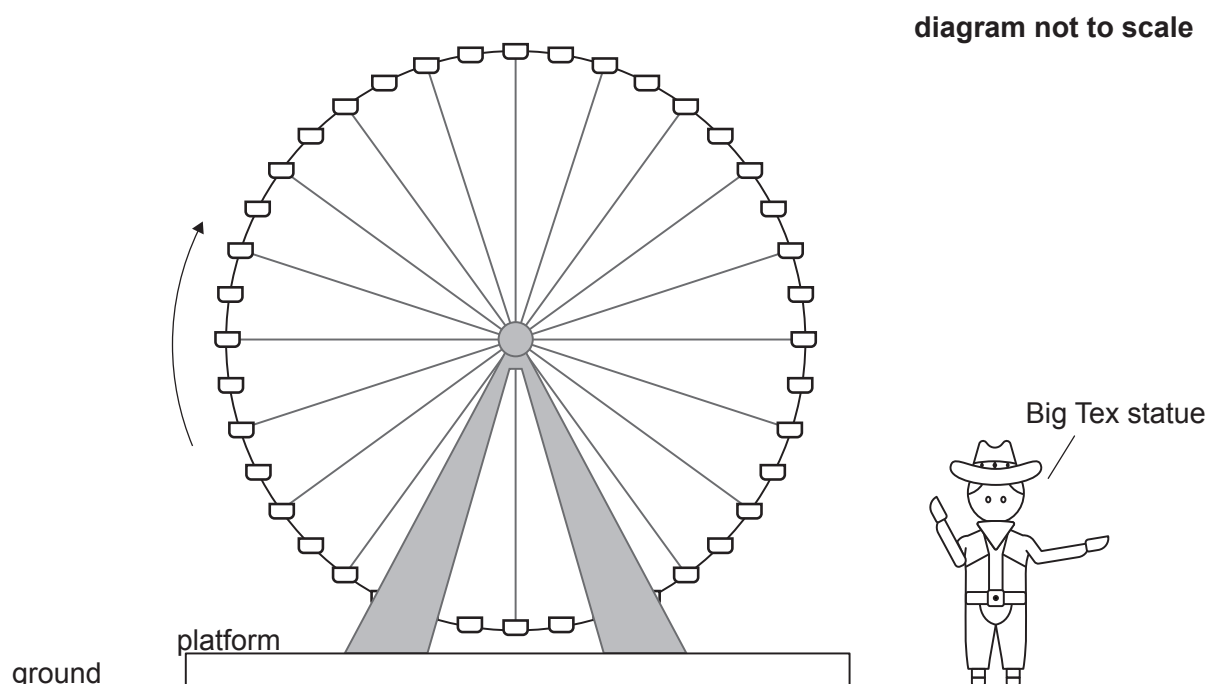
(ii) an appropriate range for $h(t)$.

[3]

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(Question 4 continued)

Big Tex is a 16.7 metre-tall cowboy statue that stands on the horizontal ground next to the Ferris wheel.



- (d) By considering the graph of $h(t)$, determine the length of time during one revolution of the Ferris wheel for which the chair is higher than the cowboy statue. [3]

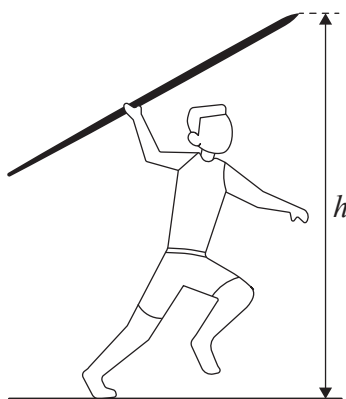
There is a plan to relocate the Texas Star Ferris wheel onto a taller platform which will increase the maximum height of the Ferris wheel to 65.2 m. This will change the value of one parameter, a , b or d , found in part (a).

- (e) (i) Identify which parameter will change.
- (ii) Find the new value of the parameter identified in part (e)(i). [3]

Turn over

3. [Maximum mark: 5]

DeVaughn throws a javelin in a school track and field competition.



The height, h , of the front tip of the javelin above the ground, in metres, is modelled by the following quadratic function,

$$h(t) = -3.6t^2 + 10.8t + 1.8, \quad t \geq 0$$

where t is the time in seconds after the javelin is thrown.

- Write down the height of the front tip of the javelin at the time it is thrown. [1]
- Find the value of t when the front tip of the javelin reaches its maximum height. [2]
- Find the value of t when the front tip of the javelin strikes the ground. [2]

[illegible]

Turn over

5. [Maximum mark: 6]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee’s temperature, T , measured in $^{\circ}\text{C}$, could be modelled by the following function,

$$T(t) = 71\text{e}^{-0.0514t} + 23, \; t \geq 0,$$

where t is the time, in minutes, after the coffee started to cool.

- (a) Find the coffee’s temperature 16 minutes after it started to cool. [2]

The graph of T has a horizontal asymptote.

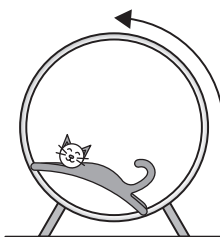
- (b) Write down the equation of the horizontal asymptote. [1]
(c) Write down the room temperature. [1]
(d) Given that $T^{-1}(50) = k$, find the value of k . [2]



Turn over

12. [Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a \sin(bt) + c$ where t is the time in seconds and $a, b, c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of 78 cm.

- (a) Write down the value of c . [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

- (b) Find the value of
- (i) a .
- (ii) b . [3]
- (c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

- (d) Write down the new value of b . [1]

[illegible]

5. [Maximum mark: 5]

Line L_1 is tangent to the graph of a function $f(x)$ at the point $P(3, -1)$. Line L_2 is given by the equation $y = -\frac{1}{2}x - \frac{5}{2}$ and is perpendicular to L_1 .

- (a) Write down the gradient of L_1 . [1]
- (b) Find the equation of L_1 in the form $y = mx + c$. [2]
- (c) Show that L_2 is not the line that is normal to $f(x)$ at point P . [2]



6. [Maximum mark: 5]

When the brakes of a car are fully applied the car will continue to travel some distance before it completely stops. This stopping distance, d , in metres is directly proportional to the square of the speed of the car, v , in kilometres per hour (km h^{-1}).

When a car is travelling at a speed of 50 km h^{-1} it will travel 12.3 m after the brakes are fully applied before it completely stops.

- (a) Determine an equation for d in terms of v . [2]

The police can use this equation to estimate if cars are exceeding the speed limit.

A car is found to have travelled 33 m, after fully applying its brakes, before it completely stopped.

- (b) Use your equation from part (a) to estimate the speed at which this car was travelling before the brakes were applied. [2]

- (c) After the brakes have been fully applied, identify one other variable besides speed that could affect stopping distance. [1]



Turn over

10. [Maximum mark: 7]

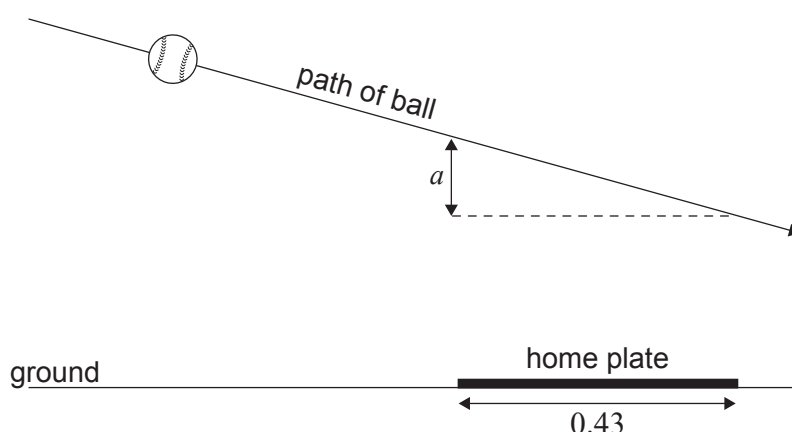
In a baseball game, Sakura is the batter standing beside home plate. The ball is thrown towards home plate along a path that can be modelled by the following function

$$y = -0.045x + 2.$$

In this model, x is the horizontal distance of the ball from the point the ball is thrown and y is the vertical height of the ball above the ground. Both measured in metres.

The outcome of the throw is called a strike if the height of the ball is between 0.53 m and 1.24 m at some point while it travels over home plate. The length of home plate is 0.43 m.

diagram not to scale



When the ball reaches the front of home plate, the height of the ball above the ground is 1.25 m. The height of the ball changes by a metres as the ball travels over the length of home plate.

(a) (i) Find the value of a .

(ii) Justify why this throw is a strike.

[4]

On the next throw, Sakura hits the ball towards a wall that is 5 metres high. The horizontal distance of the wall from the point where the ball was hit is 96 metres. The path of the ball after it is hit can be modelled by the function $h(d)$.

$$h(d) = -0.01d^2 + 1.04d + 0.66, \text{ for } h, d > 0$$

In this model, h is the height of the ball above the ground and d is the horizontal distance of the ball from the point where it was hit. Both h and d are measured in metres.

(b) Determine whether the ball will go over the wall. Justify your answer.

[3]

(This question continues on the following page)



[illegible]

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10. [Maximum mark: 8]

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5, \, t \geq 0,$$

where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

- (a) Find how long it takes for the basketball to reach its maximum height. [2]
- (b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

Another player catches the basketball when it is at a height of 1.2 metres.

- (c) Find the value of t when this player catches the basketball. [2]
- (d) Write down two limitations of using $h(t)$ to model the height of the basketball. [2]

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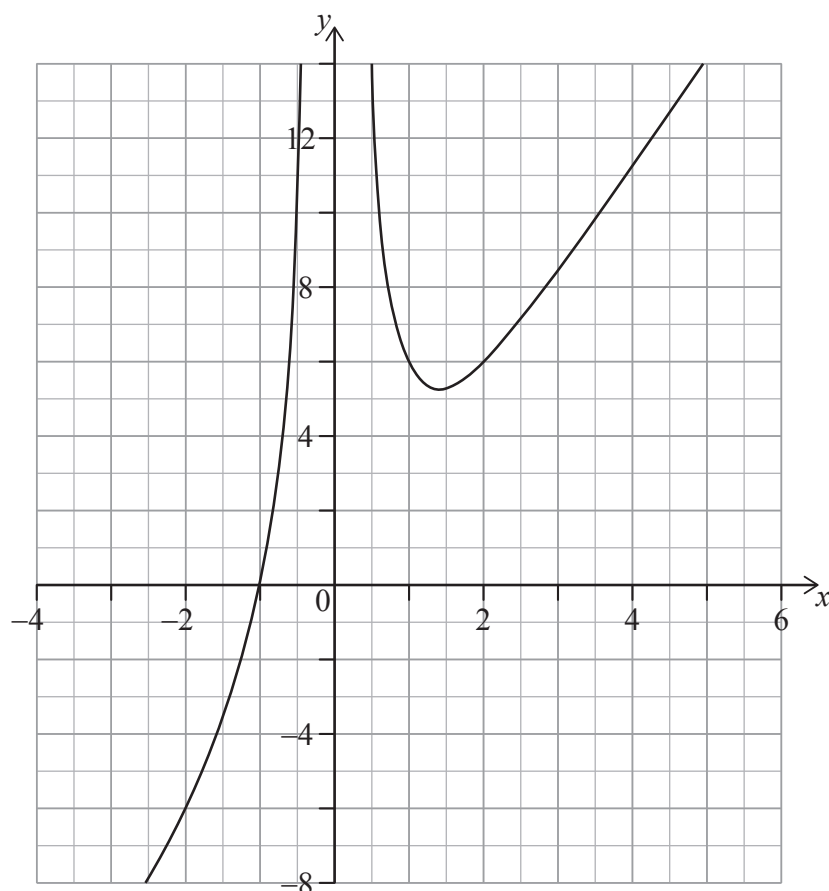
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2. [Maximum mark: 16]

Consider the function $f(x) = 3x - 1 + 4x^{-2}$. Part of the graph of $y = f(x)$ is shown below.



The function is defined for all values of x except for $x = a$.

- (a) Write down the value of a . [1]
- (b) Use your graphic display calculator to find the coordinates of the local minimum. [2]

The equation $f(x) = w$, where $w \in \mathbb{R}$, has three solutions.

- (c) Identify one possible value for w . [1]

The line $y = mx - \frac{1}{4}$ is tangent to $f(x)$ when $x = -4$.

- (d) Write down whether the value of m is positive or negative. Justify your answer. [2]

(This question continues on the following page)

(Question 2 continued)

A second function is given by $g(x) = kp^x - 9$, where $p > 0$. The graph of $y = g(x)$ intersects the y -axis at point A(0, -5) and passes through point B(3, 4.5).

(e) Find the value of

(i) k ;

(ii) p .

[4]

(f) Write down the equation of the horizontal asymptote of $y = g(x)$.

[2]

(g) Find the solution of $f(x) = g(x)$ when $x > 0$.

[2]

Consider a third function, h , where $h(x) = f(x) + g(x)$. The point C(-1, q) lies on the graph of $g(x)$.

(h) State whether C also lies on the graph of $h(x)$. Justify your answer.

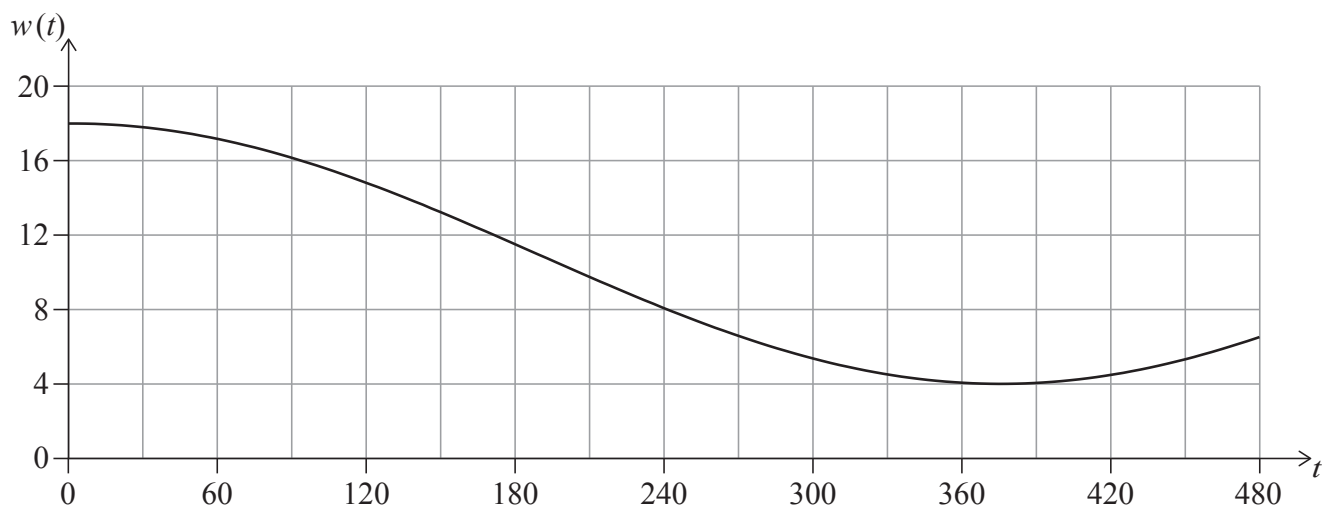
[2]

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3. [Maximum mark: 15]

The depth of water, w metres, in a particular harbour can be modelled by the function $w(t) = a \cos(bt^\circ) + d$ where t is the length of time, in minutes, after 06:00.

On 20 January, the first high tide occurs at 06:00, at which time the depth of water is 18 m. The following low tide occurs at 12:15 when the depth of water is 4 m. This is shown in the diagram.



- (a) Find the value of a . [2]
- (b) Find the value of d . [2]
- (c) Find the period of the function in minutes. [3]
- (d) Find the value of b . [2]

Naomi is sailing to the harbour on the morning of 20 January. Boats can enter or leave the harbour only when the depth of water is at least 6 m.

- (e) Find the latest time before 12:00, to the nearest minute, that Naomi can enter the harbour. [4]
- (f) Find the length of time (in minutes) between 06:00 and 15:00 on 20 January during which Naomi **cannot** enter or leave the harbour. [2]

3. [Maximum mark: 15]

A scientist is conducting an experiment on the growth of a certain species of bacteria.

The population of the bacteria, P , can be modelled by the function

$$P(t) = 1200 \times k^t, \quad t \geq 0,$$

where t is the number of hours since the experiment began, and k is a positive constant.

(a) (i) Write down the value of $P(0)$.

(ii) Interpret what this value means in this context. [2]

3 hours after the experiment began, the population of the bacteria is 18 750.

(b) Find the value of k . [2]

(c) Find the population of the bacteria 1 hour and 30 minutes after the experiment began. [2]

The scientist conducts a second experiment with a different species of bacteria.

The population of this bacteria, S , can be modelled by the function

$$S(t) = 5000 \times 1.65^t, \quad t \geq 0,$$

where t is the number of hours since both experiments began.

(d) Find the value of t when the two populations of bacteria are equal. [2]

It takes 2 hours and m minutes for the number of bacteria in the second experiment to reach 19 000.

(e) Find the value of m , giving your answer as an integer value. [4]

The bacteria in the second experiment are growing inside a container. The scientist models the volume of each bacterium in the second experiment to be $1 \times 10^{-18} \text{ m}^3$, and the available volume inside the container is $2.1 \times 10^{-5} \text{ m}^3$.

(f) Determine how long it would take for the bacteria to fill the container. [3]

6. [Maximum mark: 4]

Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}, \text{ for } x > 1.$$

- (a) Find $h^{-1}(1)$. [2]
- (b) Find the domain of $h^{-1}(x)$. [2]

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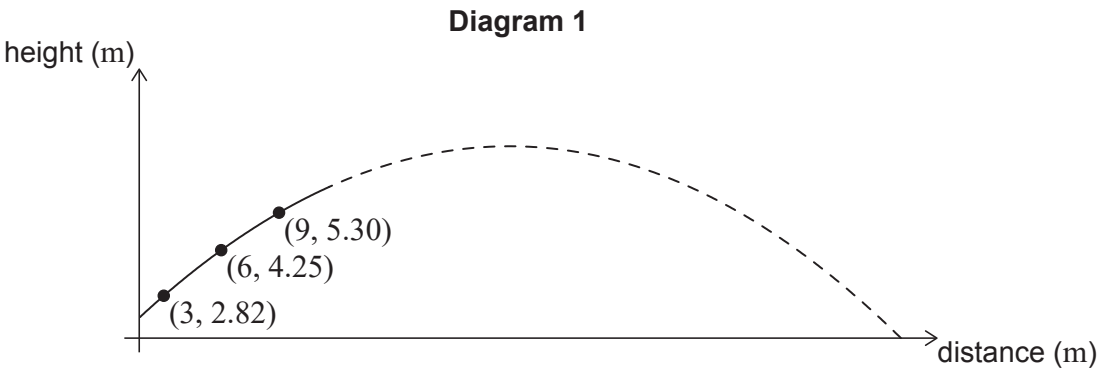
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7. [Maximum mark: 9]

An athlete on a horizontal athletic field throws a discus. The height of the discus above the field, in metres, after it is thrown can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the discus has travelled from the athlete.

A specialized camera tracks the initial path of the discus after it is thrown by the athlete. The camera records that the discus travels through the three points (3, 2.82), (6, 4.25) and (9, 5.30), as shown in **Diagram 1**.

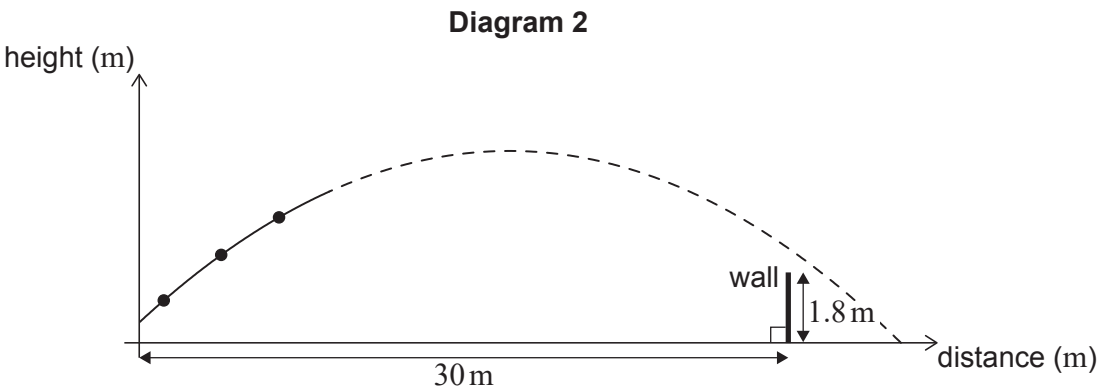
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- (a) Use the coordinates (3, 2.82) to write down an equation in terms of a , b and c . [1]
- (b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the discus. [3]

A 1.8-metre-high wall is 30 metres from where the athlete threw the discus, as shown in **Diagram 2**.

diagram not to scale



- (c) Show that the model predicts that the discus will go over the wall. [3]
- (d) Find the horizontal distance that the discus will travel, from the athlete until it first hits the ground, according to this model. [2]

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(Question 7 continued)

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Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}, \text{ for } x > 1.$$

- (a) Find $h^{-1}(1)$. [2]
- (b) Find the domain of $h^{-1}(x)$. [2]

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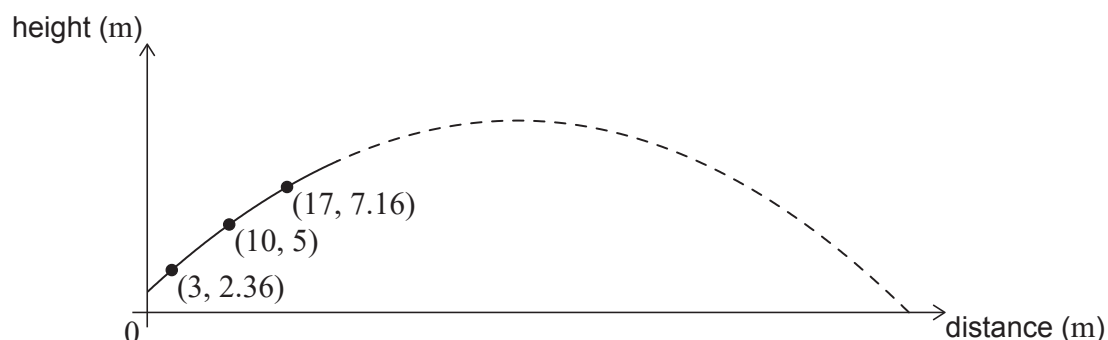
7. [Maximum mark: 9]

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3, 2.36)$, $(10, 5)$ and $(17, 7.16)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1

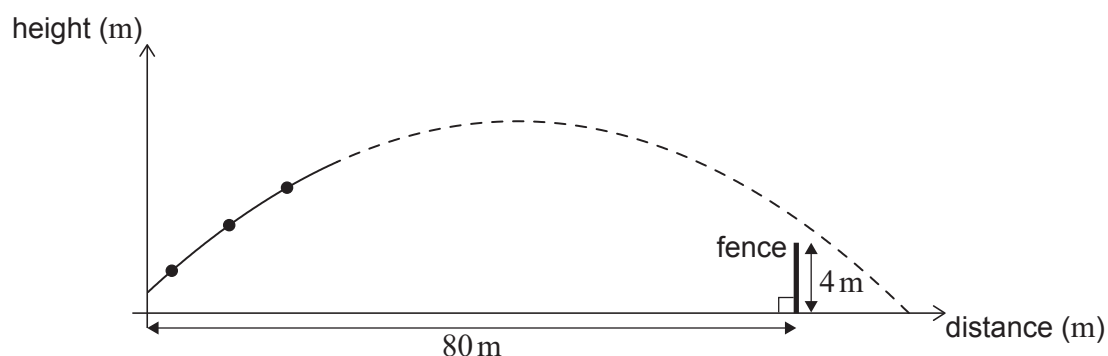


- (a) Use the coordinates $(3, 2.36)$ to write down an equation in terms of a , b , and c . [1]
- (b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



- (c) Show that the model predicts that the ball will go over the fence. [3]
- (d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

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(Question 7 continued)

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4. [Maximum mark: 7]

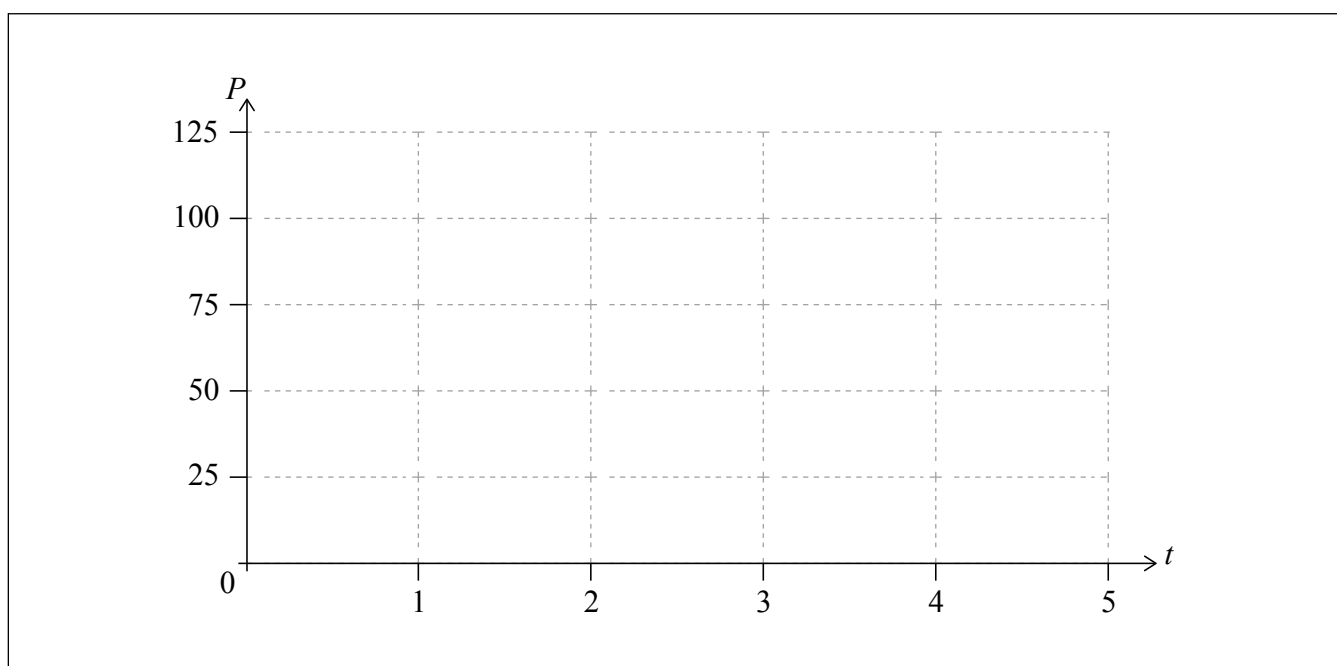
A cell phone starts charging at 07:00. While being charged, the percentage of power, P , in the phone is modelled by the function $P = 100 - 60 \times a^{-t}$, where t is the number of hours after 07:00.

- (a) Find the percentage of power in the phone at 07:00. [2]

The percentage of power in the phone reaches 75% at 08:00.

- (b) Find the value of a . [2]

- (c) Draw the graph of $P = 100 - 60 \times a^{-t}$ on the following set of axes. [2]



- (d) State a mathematical reason why the model predicts the percentage of power in the phone will never reach 100%. [1]

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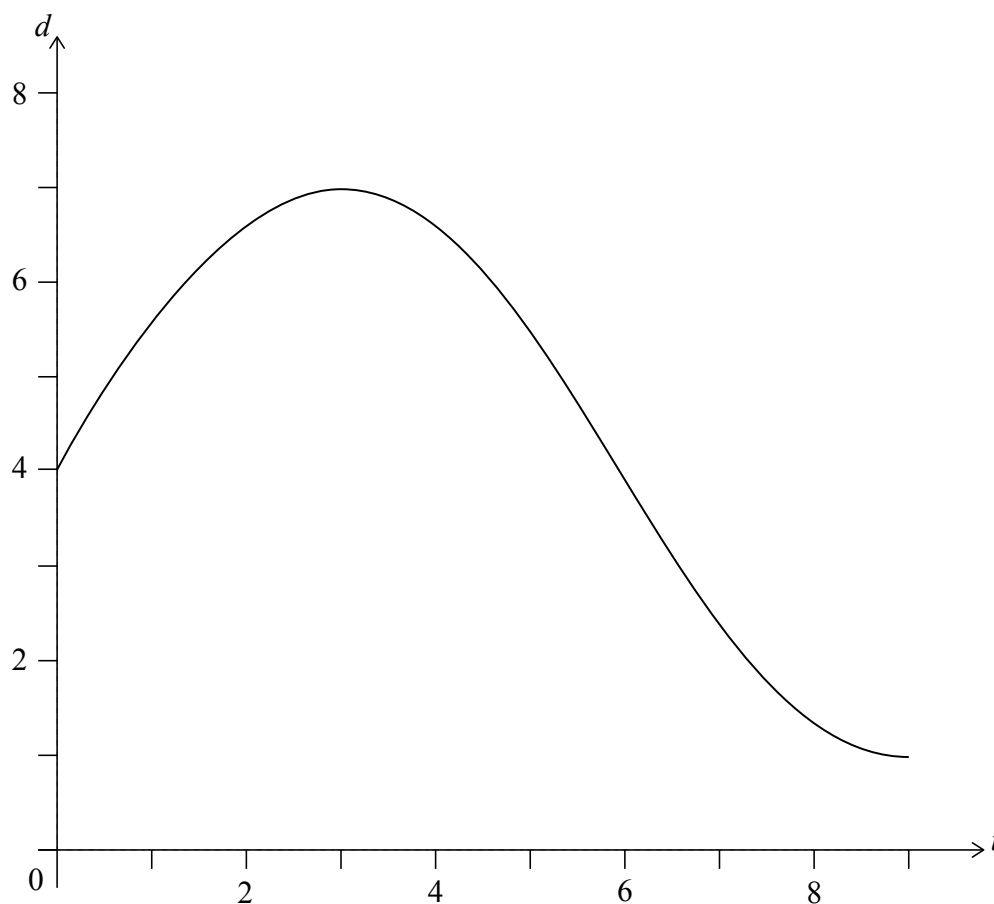


7. [Maximum mark: 6]

The following graph shows the depth of water, d metres, in a river at t hours after 12:00.

At 15:00, the depth of water reaches 7 m, its highest level. At 21:00, the depth of water drops to 1 m, its lowest level.

The depth can be modelled by the function $d(t) = a \sin(bt) + 4$.



- (a) Find the value of a . [1]
- (b) Find the value of b . [2]
- (c) Find the first time after 12:00 when the depth is equal to 3 m. Give your answer to the nearest minute. [3]

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4. [Maximum mark: 6]

At a particular building site, the number of square metres of bricks, n , that can be laid in one working day varies directly with the number of bricklayers, B .

Five bricklayers can lay an area of 60 m^2 of bricks in one working day.

- (a) Calculate the area that can be laid by 7 bricklayers in one working day.
- [3]

At another building site, the time, t hours, it takes to lay bricks varies inversely with the number of bricklayers, B .

Five bricklayers take 8 hours to lay an area of 60 m^2 .

- (b) Calculate how long it takes 12 bricklayers to lay an area of 60 m^2 .
- [3]

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5. [Maximum mark: 7]

The total cost, $C(d)$, in Canadian dollars (CAD), to hire a bicycle for d days from *Pedal Paradise* is given by the function

$$C(d) = 60d + 10, d \geq 3, d \in \mathbb{Z}.$$

The total cost includes a fixed charge to hire both a helmet and a repair kit.

- (a) State, in context, what the values 60 and 10 represent. [2]
- (b) Calculate the cost of hiring a bicycle for 5 days. [2]

Hema hires a bicycle from *Pedal Paradise*.

- (c) Write down the minimum number of days she can hire the bicycle. [1]
- (d) Given that $C^{-1}(1270) = k$, find the value of k . [2]

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6. [Maximum mark: 6]

Radioactive carbon is a material that decays over time.

The mass, $m(t)$ (in nanograms), of radioactive carbon in a fossil of a plant, after t years, can be modelled by the function

$$m(t) = 120e^{-0.000121t}$$

where t is the time since the plant died.

- (a) Write down the initial mass of the radioactive carbon. [1]
- (b) Find the mass of the radioactive carbon after 20 000 years. [2]
- (c) Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]



Turn over

5. [Maximum mark: 7]

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The total cost includes a fixed charge to hire both a helmet and a repair kit.

- (a) State, in context, what the values 60 and 10 represent. [2]
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Hema hires a bicycle from *Pedal Paradise*.

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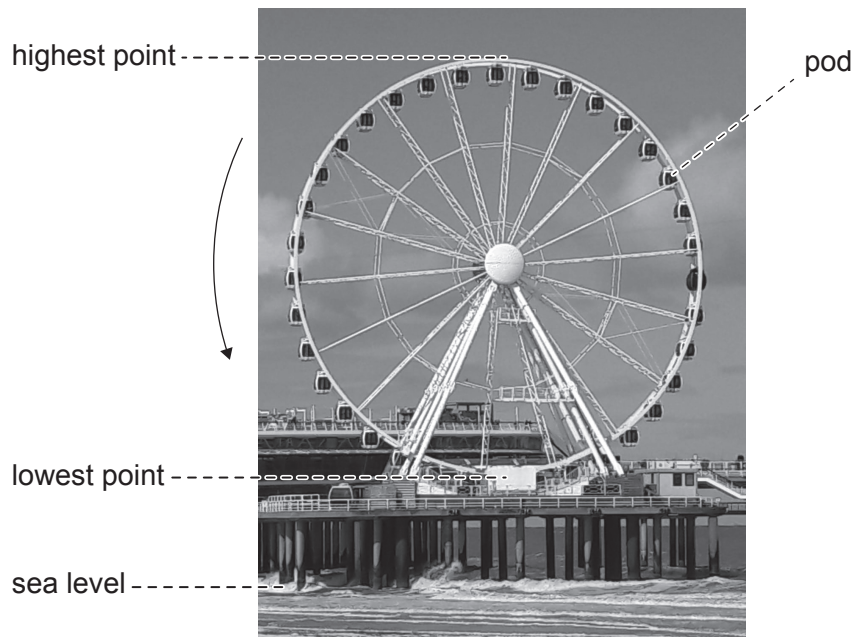
- Write down the initial mass of the radioactive carbon. [1]
- Find the mass of the radioactive carbon after 20 000 years. [2]
- Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]

[illegible]

Turn over

2. [Maximum mark: 16]

The Scheveningen Ferris wheel's lowest point is 8 m above sea level, and its highest point is 45 m above sea level.



(a) (i) Show that the radius of the Ferris wheel is 18.5 m.

(ii) Calculate the circumference of the Ferris wheel.

[2]

There are pods, equally spaced around the wheel, that carry passengers.

(b) When the wheel rotates 10° , find the distance that a pod travels along the circumference. [3]

(This question continues on the following page)

(Question 2 continued)

The height in metres, above sea level, of a particular pod is modelled by the function:

$$h(t) = a \sin(bt) + d, \text{ for } a, b > 0,$$

where t is the time, measured in minutes.

The wheel takes 15 minutes to complete 1 revolution.

- (c) (i) Find the value of b .
(ii) Find the value of d .
(iii) Hence, write down the equation of the sinusoidal model. [5]
- (d) Use the model to find the values of t when the height of this pod is 33 m above sea level for $0 \leq t \leq 15$. [3]

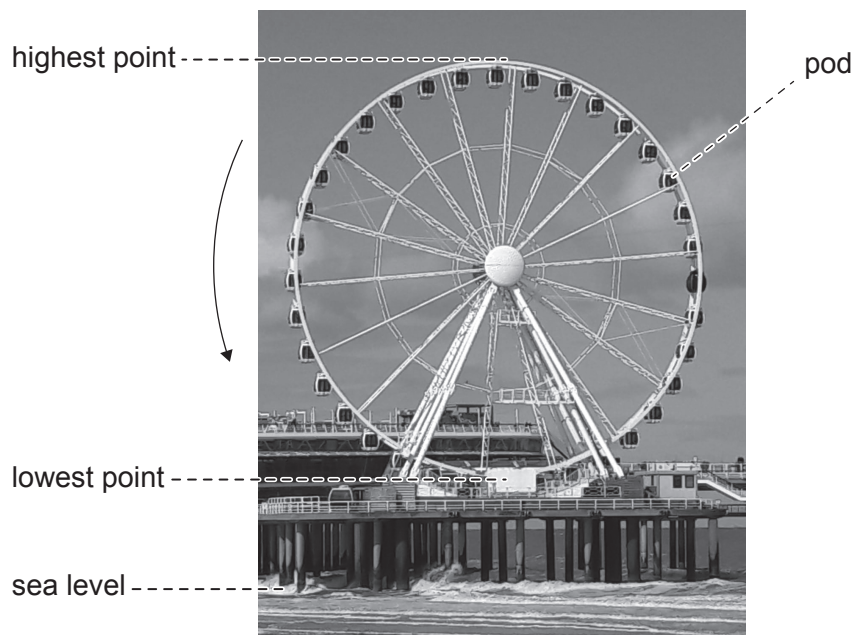
Since the Ferris wheel opened, it has been operating for 3000 days, and each day it rotates nonstop for 8 hours.

- (e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

Turn over

2. [Maximum mark: 16]

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(This question continues on the following page)

(Question 2 continued)

The height in metres, above sea level, of a particular pod is modelled by the function:

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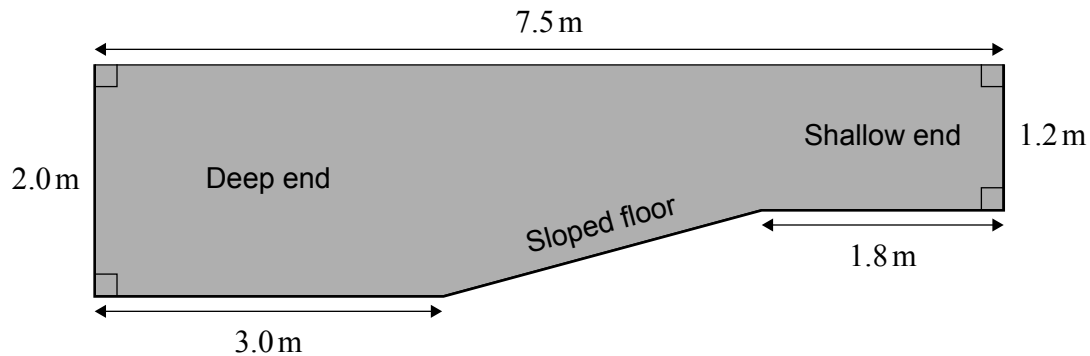
- (e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

Turn over

4. [Maximum mark: 5]

A swimming pool has a shallow end where the depth of the water is 1.2 m and a deep end where the depth of the water is 2.0 m. A cross-section of the water in the swimming pool, with additional dimensions, is shown in the diagram.

diagram not to scale



A safety regulation states that the gradient of the sloped floor of the swimming pool, as shown in the cross-section, must not be greater than $\frac{1}{3}$.

- (a) Show that the swimming pool satisfies the safety regulation.

[2]

The time to fill the pool with water is inversely proportional to the water flow rate, in litres per minute, of the hose being used. A hose with a water flow rate of 300 litres per minute will completely fill this pool from empty in 4.5 hours if the hose is run continuously.

- (b) Determine the time it takes to completely fill the pool from empty, using a hose run continuously with a water flow rate of 170 litres per minute.

[3]

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(Question 4 continued)

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5. [Maximum mark: 7]

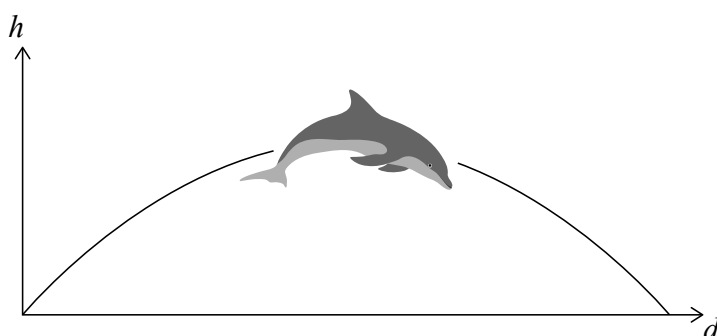
A dolphin jumps out of water. The path of its jump can be modelled by a quadratic function using the following variables:

d is the horizontal distance from the point where the dolphin leaves the water

h is the height of the dolphin above the surface of the water.

All distances are measured in metres, from the point where the dolphin leaves the water.

diagram not to scale



On a particular jump, the dolphin first reaches a height of 5.1 m above the surface of the water when the horizontal distance is 3 m and again when the horizontal distance is 8.5 m.

- (a) Write down the equation of the axis of symmetry of the quadratic function. [2]

The quadratic function that models the dolphin's jump is $h(d) = -0.2d^2 + bd$, where $d > 0$ and b is a constant.

- (b) Calculate the value of b . [2]
- (c) Find the horizontal distance when the dolphin re-enters the water. [2]
- (d) Explain what a negative h -value would mean in this context. [1]

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(Question 5 continued)

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Turn over

10. [Maximum mark: 7]

A manufacturer designs hats that businesses can order.

For an order of 15 hats or fewer, the price per hat is p euros (EUR). For larger orders, the price of **each** hat after the first 15 ordered is reduced by 5 EUR.

$C(n)$ represents the total cost of purchasing n hats from the manufacturer.

- (a) Write down an expression in terms of p for the total cost of ordering 15 hats.

[1]
- (b) Write down an equation for $C(n)$, in terms of p and n , for the total cost of ordering n hats when $n > 15$.

[3]

A company decides to order 100 hats. The mean price per hat for this order is 25.73 EUR.

- (c) (i) Calculate the exact total cost of the order.
- (ii) Hence, determine the value of p , correct to 2 decimal places.

[3]

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11. [Maximum mark: 5]

Mongolia is one of the coldest countries in the world, with the air temperature in winter as low as -40°C . It can feel even colder if a wind is blowing. This effect is called wind chill.

On a day when the air temperature is -40°C , the approximate wind chill index, W , can be calculated by the equation

$$W = -34.1 - 7.33 \ln(v),$$

where v is the wind speed, in kilometres per hour (km h^{-1}).

- (a) Find the approximate wind chill index on a day when the air temperature is -40°C and the wind speed is measured as 13 km h^{-1} . [2]

Due to errors in the measuring device, the percentage error in the **approximate wind chill index** calculated in part (a) could be as high as 6%.

- (b) Predict the maximum **and** the minimum wind chill index for this day. [3]

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9. [Maximum mark: 5]

Consider the following function, $f(x)$, defined on the domain of integers from 0 to 4 inclusive.

x	0	1	2	3	4
$f(x)$	3	1	0	4	2

- (a) Solve $f(x) = 4$.

[1]
- (b) Solve $f(x) = x$.

[1]
- (c) Complete the following table.

x	0	1	2	3	4
$f^{-1}(x)$					

[3]

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3. [Maximum mark: 6]

The water level, h , in metres, in a water tank after t hours of irrigation is modelled by the following function.

$$h(t) = \frac{20}{2t+5}, \quad t \geq 0$$

- (a) Find the value of $h(0.5)$. [2]
- (b)
 - (i) Find the value of $h^{-1}(2.5)$.
 - (ii) Interpret this value in context. [3]
- (c) Write down the range of h^{-1} . [1]

[illegible]

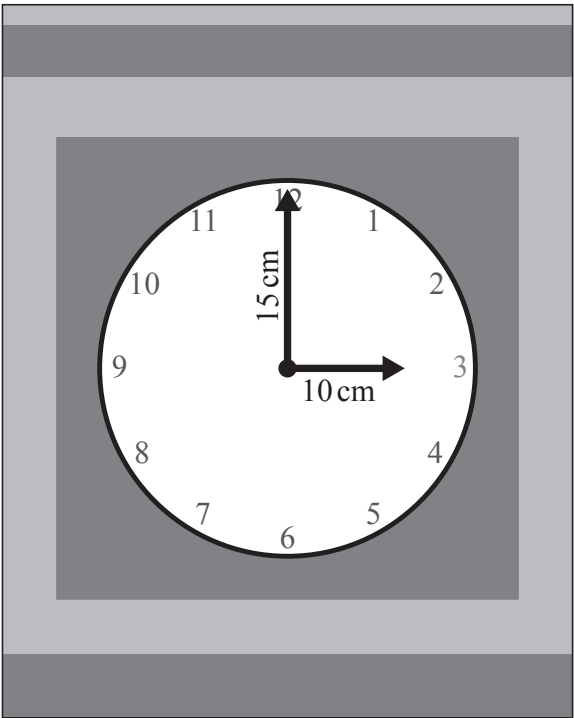
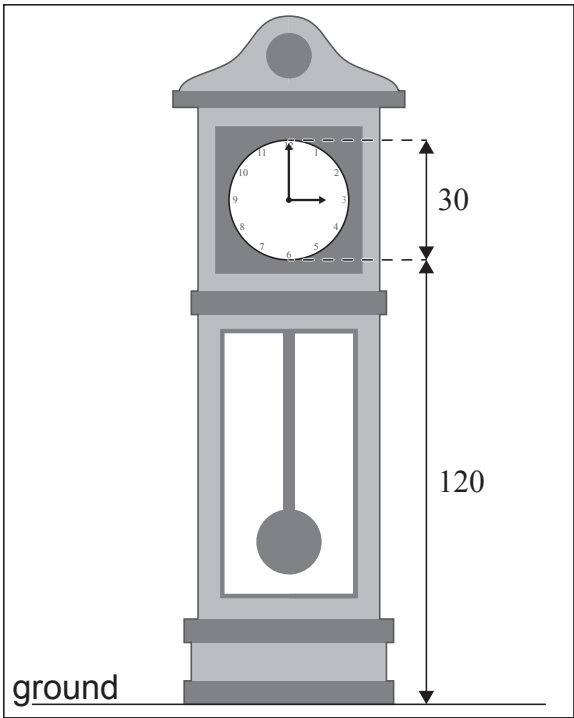
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4. [Maximum mark: 16]

The circular clock face of a grandfather clock is 30 cm in diameter and its lowest point is 120 cm above the ground. The length of the minute hand is 15 cm and the length of the hour hand is 10 cm. This is shown in the diagram.

The clock is started at 3 pm precisely.

diagram not to scale



- (a) (i) Write down the size of the angle between the hour hand and the minute hand of the clock at 3 pm.
- (ii) Calculate the distance between the tips of the two hands of the clock at 3 pm. [3]

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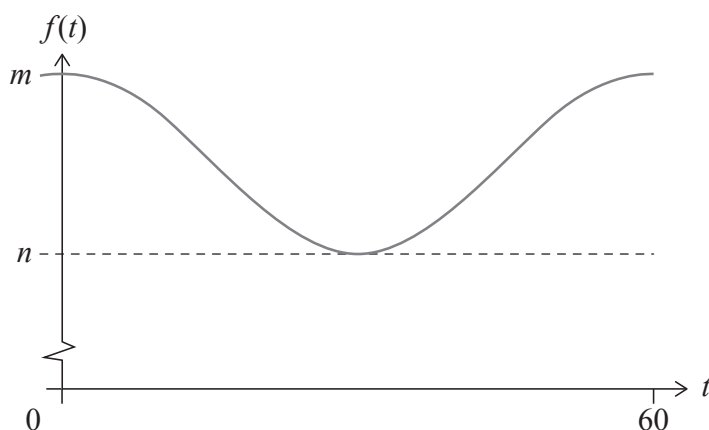
(Question 4 continued)

The height of the tip of the minute hand, $f(t)$, above the ground can be modelled by the cosine function

$$f(t) = a \cos(bt) + d, \quad a, b > 0$$

where the angle bt is measured in degrees and t is the number of minutes after 3 pm.

A sketch of the graph of $y = f(t)$ as time passes from 3 pm to 4 pm is shown.



(b) Write down the value of

(i) m .

(ii) n .

[2]

(c) Hence find the value of

(i) a .

(ii) b .

(iii) d .

[3]

(This question continues on page 9)

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(Question 4 continued)

The height of the tip of the hour hand, $g(t)$, above the ground can be modelled by the sine function

$$g(t) = p \sin(qt) + d$$

where qt is measured in degrees and t is the number of minutes after 3 pm.

- (d) (i) Show that $q = 0.5$.
(ii) Find the value of p . [3]
- (e) Find the height of the tip of the hour hand above the ground at 4 pm. [2]
- (f) Find the first time after 3 pm at which the tips of the two hands of the clock are at the same height above the ground.
Give your answer correct to the nearest minute. [3]

Turn over

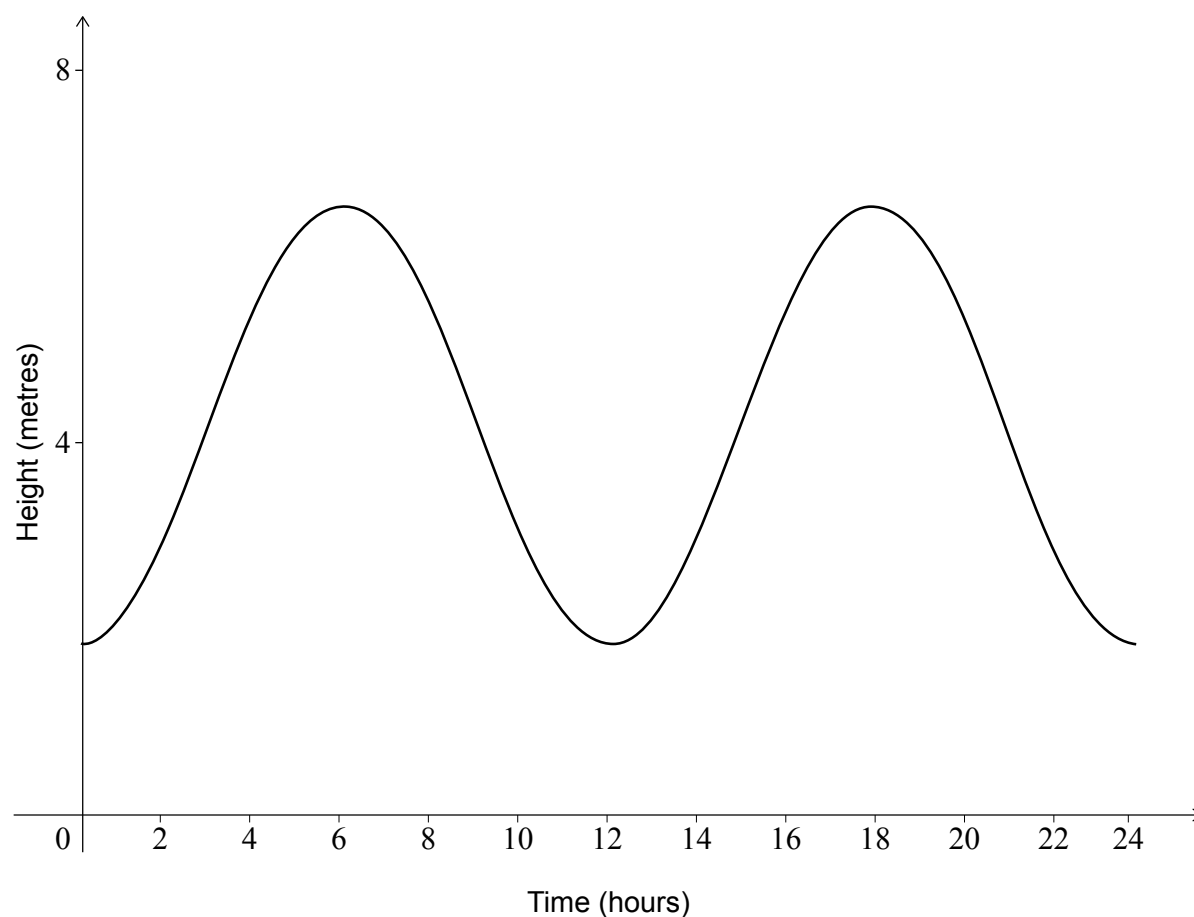
4. [Maximum mark: 13]

On a particular day the height of the tide, h , in metres, at Albion harbour can be modelled by the function

$$h(t) = -2.5 \cos(bt^\circ) + 4.5, \text{ where } b \in \mathbb{R}, 0 \leq t \leq 24$$

and t represents the number of hours after midnight.

The graph of h is shown in the following diagram.



- (a) Show that the value of b is 30. [1]
- (b) Find the height of the tide when $t = 5$. [2]
- (c) Write down
- (i) the amplitude of h .
 - (ii) the equation of the principal axis. [3]

(This question continues on the following page)

(Question 4 continued)

Boats can only leave or return to Albion harbour when $h(t) \geq 2.65$. Robin wants to leave the harbour to go fishing as soon as possible after the time is 12:00.

- (d) Determine the earliest possible time that Robin could leave the harbour.
Give your answer to the nearest minute. [3]

The boat will take 15 minutes to travel from the harbour to the fishing site. Robin intends to return to the harbour on the same day.

- (e) Determine the maximum length of time he could spend at the fishing site, in hours, and still be certain he will be able to enter the harbour on his return. [4]

Turn over

3. [Maximum mark: 6]

TurboEats is a food delivery company that charges customers \$0.80 per kilometre plus a fixed amount of \$1.50. Their delivery charge, T in \$, is represented by the function

$$T(x) = 0.80x + 1.50, \text{ for } x \geq 0,$$

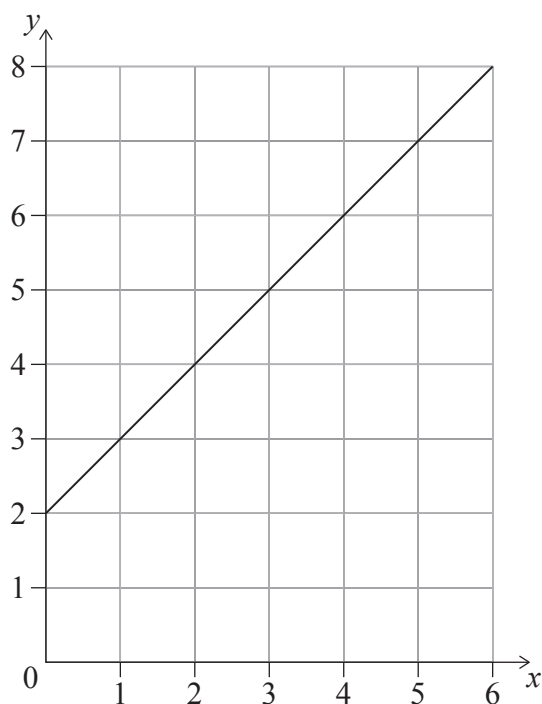
where x is the distance in kilometres from where the food is picked up to where it is delivered.

Lorena's home is 5.1 km from a pizza café.

- (a) Find the charge for TurboEats to deliver a pizza from the café to her home.

[2]

A second food delivery company, GogoDart, charges customers according to the function $G(x) = ax + b$, for $x \geq 0$. Their charges for the first 6 km are shown in the following graph of $y = G(x)$. The line passes through integer grid points.



- (b) (i) Write down the value of a .

- (ii) Write down the value of b .

[2]

- (c) Explain why TurboEats is **always** cheaper than GogoDart.

[2]

(This question continues on the following page)



(Question 3 continued)

[illegible]

Turn over

8. [Maximum mark: 8]

Sheena purchased a new mobile phone. The value of Sheena’s mobile phone, V in Philippine Pesos (PHP), can be modelled by the function

$$V(t) = 2000 + 30\,000(e^{-kt}), \text{ for } t \geq 0,$$

where t is the time, in months, after the mobile phone was purchased.

- (a) Calculate the value of the mobile phone when it was purchased. [2]

Ten months after it was purchased, the value of the mobile phone has decreased by 40%.

- (b) Find the value of k . [4]

Sheena believes the value of the mobile phone will always be more than PHP 2000.

- (c) (i) State a mathematical property of the graph of $y = V(t)$ that supports Sheena’s belief.
- (ii) State one contextual reason why Sheena’s belief might be incorrect. [2]



Turn over

10. [Maximum mark: 7]

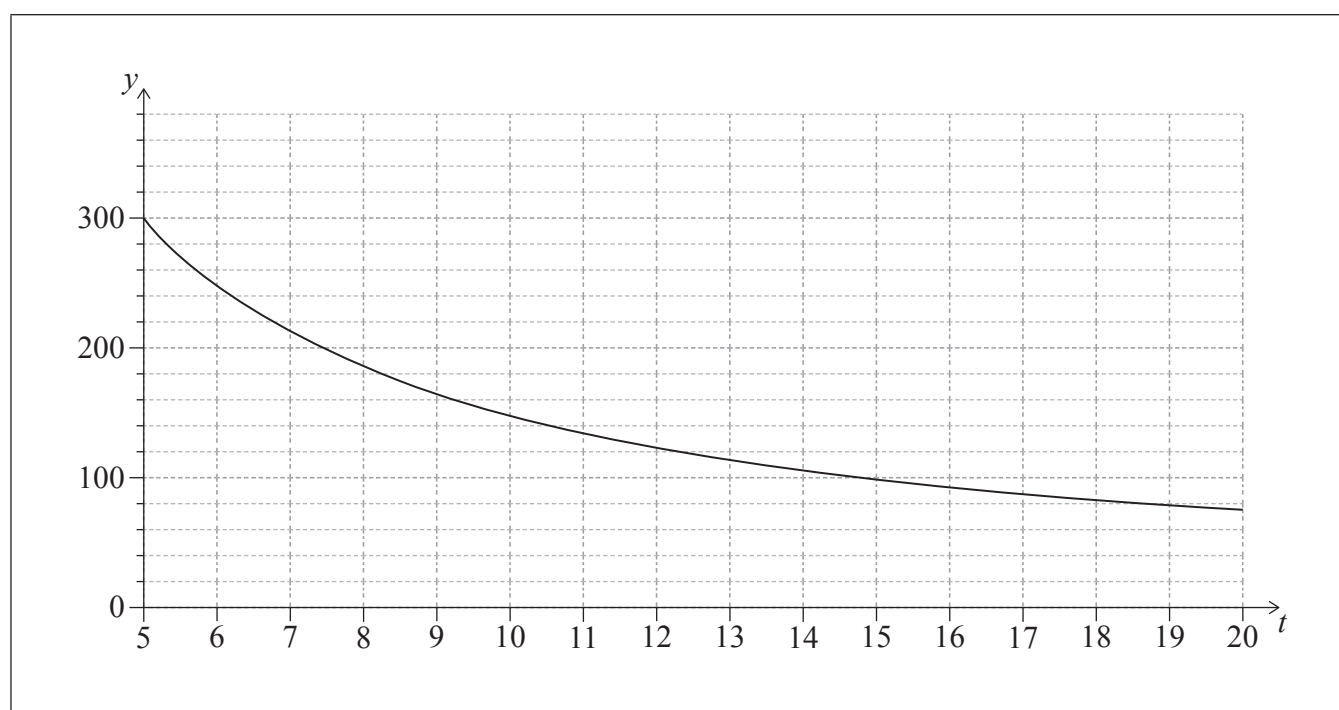
At 12:05 pm, Navam starts draining water from a small reservoir into an empty pond.

He controls the rate the water is drained so that the volume of water **remaining** in the reservoir, $V\text{m}^3$, varies inversely with the time, t , where t is the number of minutes after 12:00 pm.

At 12:05 pm, the volume of water in the reservoir is 300m^3 .

- (a) Show that $V = \frac{1500}{t}$. [2]
- (b) Find the value of t when the volume of water remaining in the reservoir equals the volume of water in the pond. [2]

The following diagram shows part of the graph of $y = V(t)$.



- (c) On the same diagram, sketch the graph of $y = P(t)$, where P is the volume of water in the pond. [2]
- (d) Write down an expression for $P(t)$. [1]

(This question continues on the following page)



(Question 10 continued)

[illegible]

Turn over

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Answers written on this page
will not be marked.



2. [Maximum mark: 6]

The number of words that a child understands is modelled by the equation

$$y = -167 + 6264 \log_{10} x, \text{ for } x \in \mathbb{R}^+,$$

where x is the number of years since the child was born, and y is the number of words the child understands.

- (a) Use the model to predict the number of words that a child understands when $x = 10$. [2]

To be fluent in a language, a child must understand at least 5000 words.

- (b) Use the model to predict how long it takes for a child to become fluent. Give your answer correct to four significant figures. [2]

- (c) Determine if the model is realistic for a child on their first birthday. Give a reason to support your decision. [2]



Turn over

6. [Maximum mark: 6]

A function is defined by $f(x) = \frac{4}{x} - 3$, for $-6 \leq x \leq 10, x \neq 0$.

- (a) Find $f(10)$. [1]
- (b) Find the range of f . [3]
- (c) Find $f^{-1}(-2)$. [2]



Turn over

8. [Maximum mark: 6]

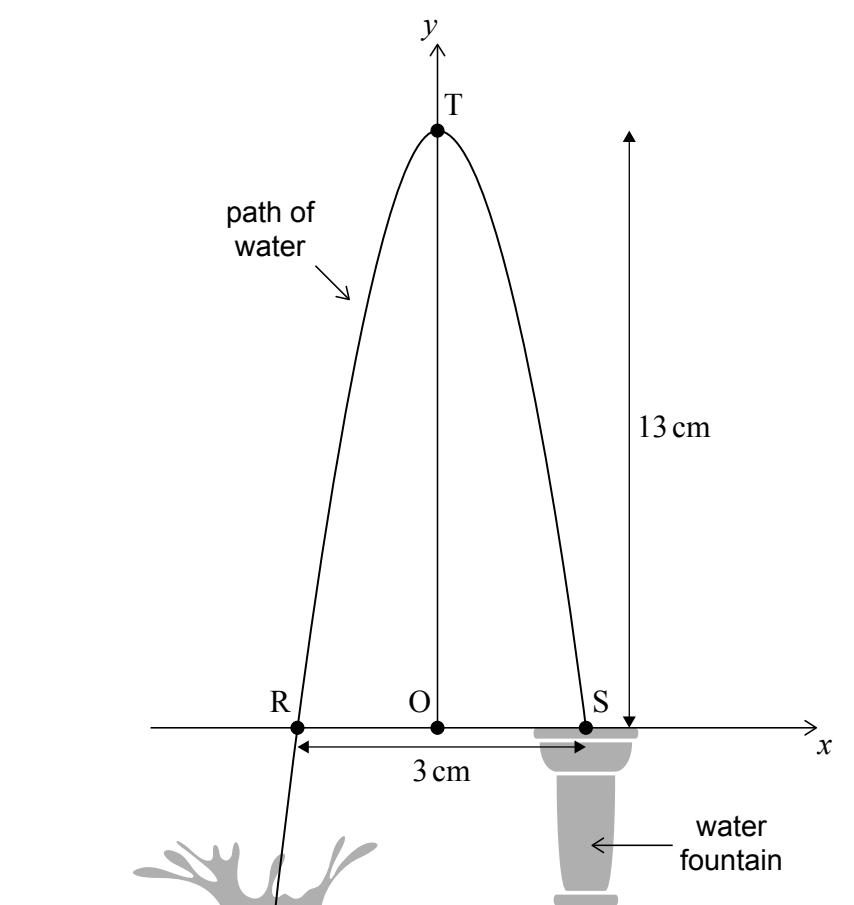
Eduardo takes a photograph of a water fountain. He notices the path of the water looks like a parabola, and he decides to model the curve using the equation

$$y = ax^2 + bx + c.$$

Eduardo takes measurements from the photograph. He finds the distance between the x -intercepts, R and S, to be 3 cm and the maximum height of the water to be 13 cm above the fountain.

These measurements are shown on the following diagram, where the point $(0, 0)$ is indicated as O and the vertex, T, is on the y -axis.

diagram not to scale



- (a) Write down the value of c . [1]
- (b) (i) Write down the coordinates of S. [2]
- (ii) Use the coordinates for S to write down an equation, in terms of only a and b . [2]
- (c) Find the value of a and the value of b in Eduardo's model. [3]

(This question continues on the following page)



[illegible]

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12. [Maximum mark: 6]

The amount of a radioactive material present in a sample, in micrograms, can be modelled by

$$f(t) = ab^t, \text{ for } t \geq 0,$$

where t represents the number of days since the sample was taken.

Paula takes two samples of this material, sample X and sample Y.

The initial amount of radioactive material in sample X is 100 micrograms, and after 138 days the amount of radioactive material present is 50 micrograms.

- (a) For sample X,
(i) write down a .
(ii) find b , given that $b > 0$. [3]

After 30 days there were 4 micrograms of the radioactive material present in sample Y.

- (b) Use your value of b found in part (a)(ii) to find the initial amount of radioactive material in sample Y. [3]

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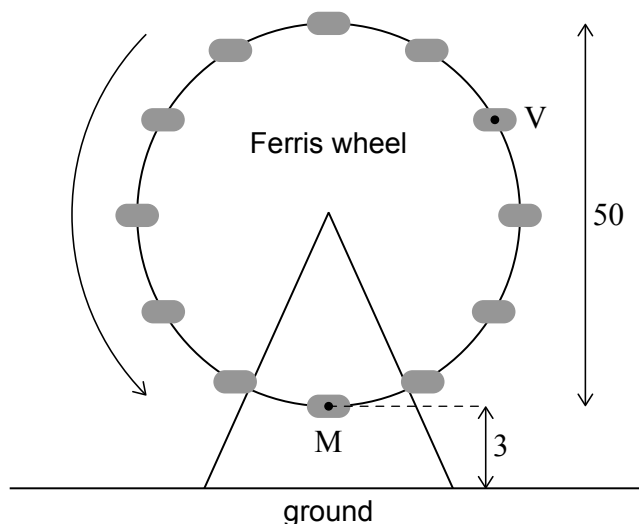
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5. [Maximum mark: 18]

The following diagram represents a Ferris wheel with 50-metre diameter and lowest point 3 metres above the ground. There are twelve evenly spaced chairs on the Ferris wheel. The Ferris wheel rotates in an anticlockwise (counterclockwise) direction.

Martz is seated in the chair labelled M, which **initially** is located at the lowest point of the Ferris wheel when it starts rotating. At the same time, Victoria is seated in the chair labelled V.

diagram not to scale



Let t be the length of time, in minutes, after the Ferris wheel started rotating.

- (a) Find the height of V above the ground when $t = 0$. [3]

The wheel completes one revolution every 12 minutes.

- (b) Find the height of M above the ground when $t = 9$. [2]
- (c) Write down the values of t when M and V have the same height above the ground, for $0 \leq t \leq 12$. You may assume that Victoria stays on the ride during this interval. [3]

The height of M above the ground is given by the function

$$h(t) = -25 \cos bt + d, \text{ for } t \geq 0.$$

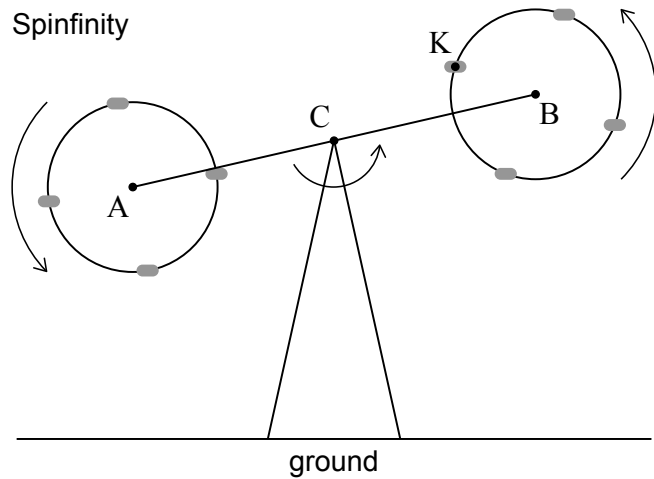
- (d) Write down the amplitude of h . [1]
- (e) Find the value of
- (i) b , given that $b > 0$;
- (ii) d . [3]

(This question continues on the following page)

(Question 5 continued)

Kofi rides “Spinfinity”, a modified Ferris wheel. The beam AB rotates in an anticlockwise (counterclockwise) direction about its midpoint C . The beam supports two circular wheels with centres A and B , each rotating in an anticlockwise (counterclockwise) direction. Kofi is in a chair labelled K . This information is illustrated in the following diagram.

diagram not to scale



The height in metres of B above the ground, T seconds after “Spinfinity” starts moving, is modelled by the function $b(T) = 20 \sin(36T) + 40$.

The height in metres, of K above B , after T seconds, is modelled by the function $k(T) = 10 \cos(72T)$.

- (f) Find the furthest distance from Kofi to point C . [2]
- (g) Find Kofi’s height above the ground after 18 seconds. [4]
-

4. [Maximum mark: 18]

A particular island is only accessible by road at times when the water level is low enough. Otherwise, the road is underwater.

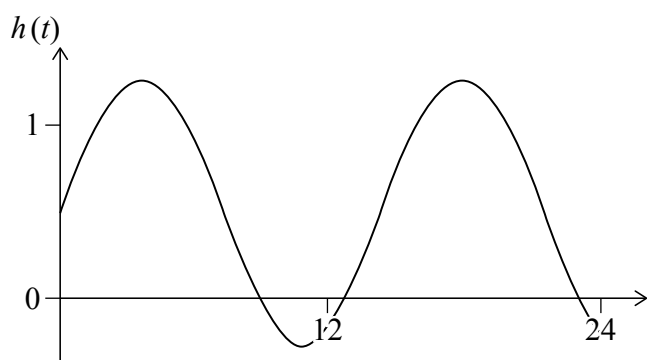
The height of the water above the road, h metres, can be modelled by

$$h(t) = a \sin(bt) + d,$$

where t is time measured in **hours** from 00:00 (midnight) on Tuesday.

The first maximum (high tide) is at 03:36 with a height of 1.26 m, and the first minimum (low tide) occurs at 10:48 with a height of -0.26 m.

This is shown in the graph.



- (a) Find the value of a , given that $a > 0$. [2]
- (b) Find the value of d . [2]
- (c) Find the period of the function in hours. [3]
- (d) Find the equation of the model. [3]
- (e) Find the value of $h(t)$ at 02:30. [2]
- (f) State the meaning of $h(t) < 0$ in context. [1]
- (g) Find the length of time on Tuesday when $h(t) < 0$. [5]

Turn over