Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

10 November 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

31 pages

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Under controlled driving conditions, Jacob investigated the fuel efficiency of his car when using premium fuel compared to standard fuel.

Jacob recorded the distance travelled per litre ($\mathrm{km\,L^{-1}}$) using standard fuel for six days and then using premium fuel for seven days. This information is shown in the following tables.

	Standard fuel						
Distance travelled per litre ($\mathrm{km\,L^{-1}}$)	8.1	8.2	7.9	8.2	7.9	8.0	

	Premium fuel						
Distance travelled per litre ($\mathrm{km\,L^{-1}}$)	8.3	8.4	8.1	8.3	8.2	8.0	8.2

At the 5% significance level, Jacob performs a t-test to determine whether there is sufficient evidence that his car travels further using premium fuel compared to standard fuel.

(a) State one mathematical assumption made for this test to be valid. [1]

(b) Write down the

(i) null hypothesis.

(ii) alternative hypothesis. [2]

(c) (i) Find the p-value.

(ii) State your conclusion to the test in context. Justify your answer. [4]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 5]

A bakery sells egg tarts in boxes. Each box contains 6 tarts. On a particular morning, the bakery has 400 freshly baked tarts. The number of tarts **remaining**, after n boxes have been sold, forms an arithmetic sequence, $u_n = u_1 + (n-1)d$.

(a) Write down the value of d. [1]

(b) Find the value of u_1. [1]

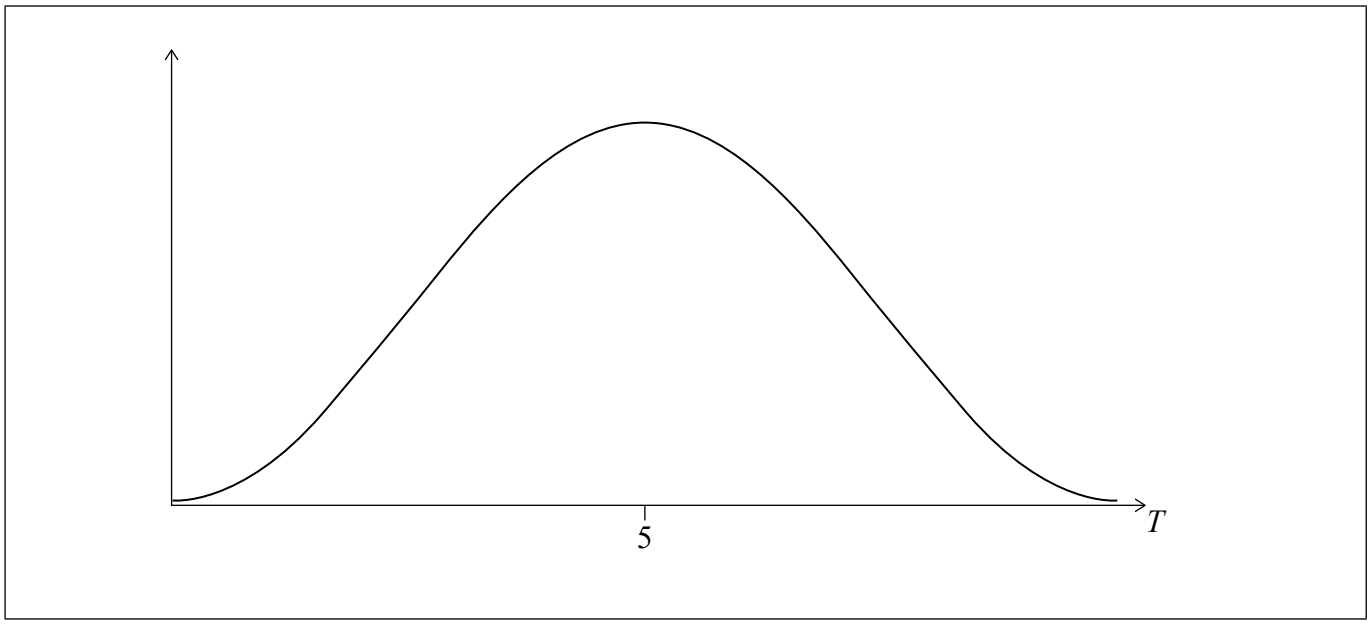
(c) Find the greatest number of boxes the bakery can sell that morning. [3]

3. [Maximum mark: 5]

In a reaction test, the time, T seconds, taken to complete the test can be modelled by a normal distribution with a mean of 5 seconds and a standard deviation of 1 second.

(a) Find the probability that a randomly selected person completes the reaction test in less than 4 seconds. [2]

The graph of the distribution of T is shown in the following diagram.



The slowest 15% of participants will be given training to reduce their time to complete the test.

(b) (i) Shade the region on the given diagram that corresponds to these participants.

(ii) Find the least time taken to complete the test by a person who will be given training. [3]

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(Question 3 continued)

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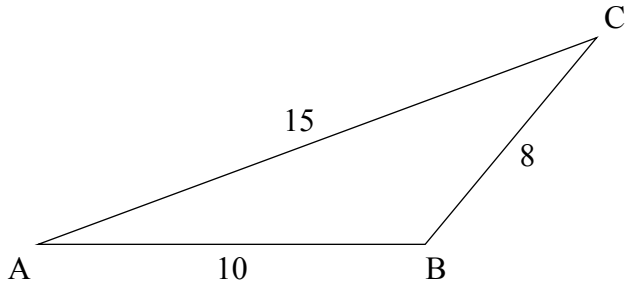
4. [Maximum mark: 8]

Triangle ABC has sides of length $AB = 10\,\text{cm}$, $BC = 8\,\text{cm}$ and $AC = 15\,\text{cm}$, as shown in the diagram.

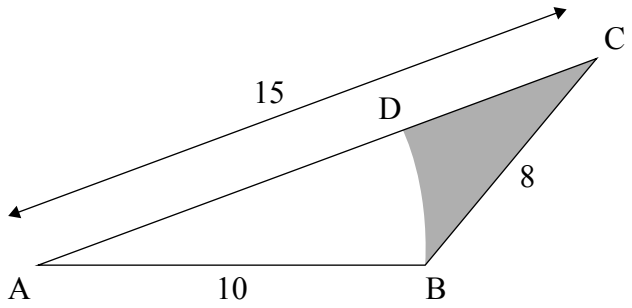
diagram not to scale



(a) Find the size of $B\hat{A}C$. [3]

The circular arc BD, with centre A, is drawn inside the triangle, such that D lies on AC.

diagram not to scale



(b) Find the area of the shaded region. [5]

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(Question 4 continued)

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5. [Maximum mark: 5]

Two complex numbers are $z = 2 + a\mathrm{i}$ and $w = b + 4\mathrm{i}$, where $a, b \in \mathbb{R}$.

(a) Find an expression for the real part of z^2, in terms of a. [2]

(b) Find the value of a and of b, given that $z^2 + 2w = 3 + 6\mathrm{i}$. [3]

6. [Maximum mark: 7]

At 12:05 pm, Navam starts draining water from a small reservoir into an empty pond.

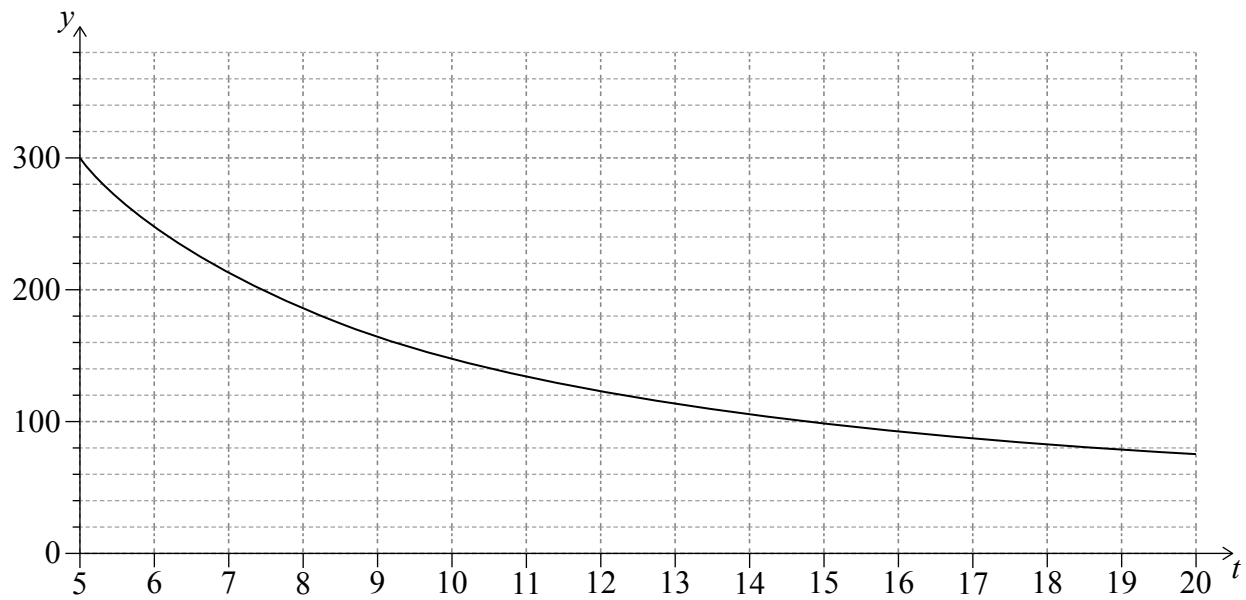
He controls the rate the water is drained so that the volume of water **remaining** in the reservoir, $V\,\mathrm{m}^3$, varies inversely with the time, t, where t is the number of minutes after 12:00 pm.

At 12:05 pm, the volume of water in the reservoir is $300\,\mathrm{m}^3$.

(a) Show that $V = \dfrac{1500}{t}$. [2]

(b) Find the value of t when the volume of water remaining in the reservoir equals the volume of water in the pond. [2]

The following diagram shows part of the graph of $y = V(t)$.



(c) On the same diagram, sketch the graph of $y = P(t)$, where P is the volume of water in the pond. [2]

(d) Write down an expression for $P(t)$. [1]

(This question continues on the following page)

(Question 6 continued)

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7. [Maximum mark: 8]

Consider the function $B(x) = 20 + \frac{32}{x} + 2x$, where $x \geq 1$, $x \in \mathbb{R}$.

(a) (i) Find $B'(x)$.

(ii) Use your answer to part (a)(i) to find the minimum value of $B(x)$. You may assume that $B(x)$ has no local maximum point over the given domain. [6]

Kim goes to a restaurant that offers a large family banquet. The time, T, in minutes, to prepare the family banquet depends on the number of chefs, n.

To predict the value of T, Kim uses the model $T(n) = 20 + \frac{32}{n} + 2n$, where $n \in \mathbb{Z}^+$.

The restaurant informs Kim that the time taken to prepare the family banquet is less than 40 minutes. There are k chefs preparing the family banquet.

(b) Find the possible values of k. [2]

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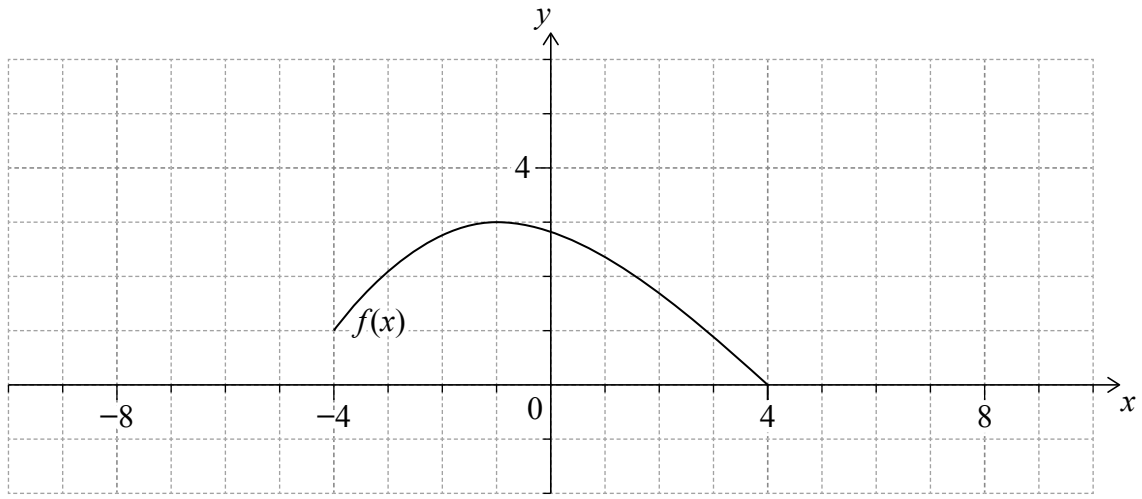
8. [Maximum mark: 4]

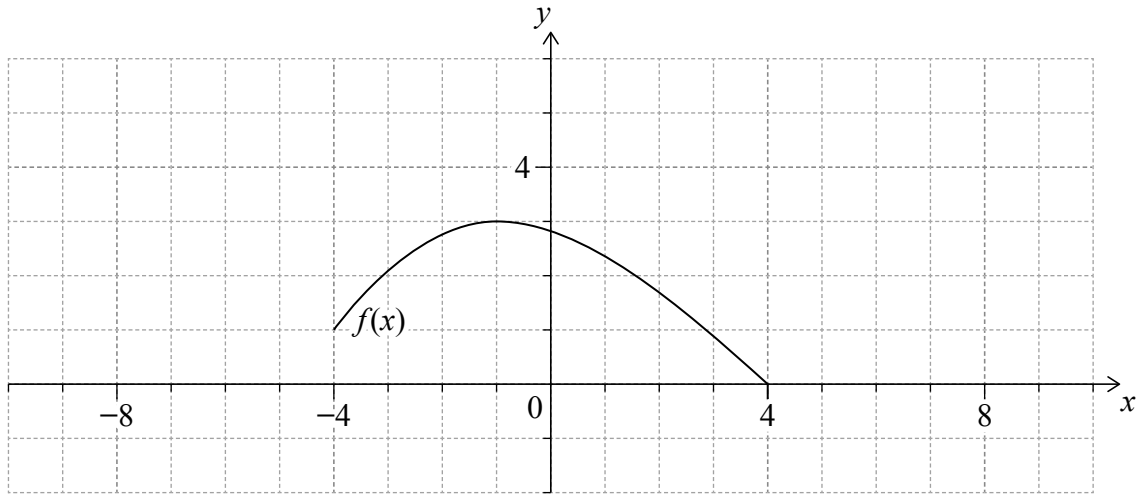
The function f has domain $-4 \le x \le 4$.

The graph of $y = f(x)$ has endpoints $(-4, 1)$ and $(4, 0)$, and a local maximum at $(-1, 3)$.
This is shown in each of the following diagrams.

(a) On the axes provided, sketch the graph of $y = 2f(x) - 1$. [2]



(b) On the axes provided, sketch the graph of $y = f(2x - 2)$. [2]



9. [Maximum mark: 6]

A help desk receives an average of 34 calls each hour. The number of calls received can be modelled by a Poisson distribution.

The help desk staff can answer a maximum of 22 calls in any 30-minute period.

(a) Find the probability that some calls are not answered in a 30-minute period. [3]

A supervisor is deciding whether to employ extra staff for the help desk. They monitor the phone calls for a random 30-minute period each day for 10 days.

(b) Find the probability that all the monitored calls are answered on at least 9 of these 10 days. [3]

10. [Maximum mark: 5]

The function f is defined by $f(x) = \dfrac{6}{2x^2 + 5x + 4}$.

(a) Find $f'(x)$. [2]

(b) Hence or otherwise, find the x-coordinates of the points of inflexion of f. [3]

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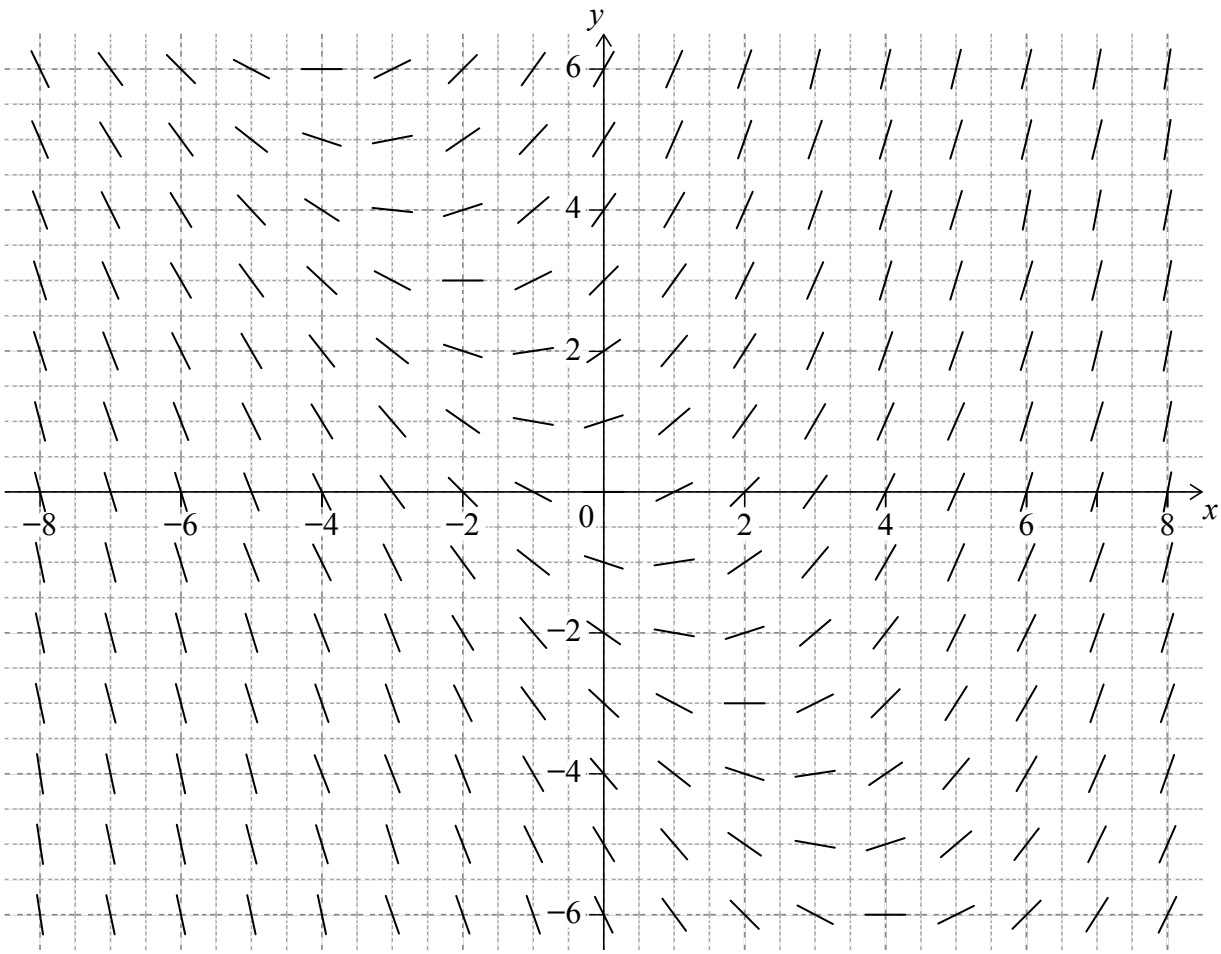
11. [Maximum mark: 4]

Consider the differential equation $\frac{dy}{dx} = \frac{x}{2} + \frac{y}{3}$.

The slope field for the differential equation is shown in the diagram.



All the solutions to $\frac{dy}{dx} = 0$ lie on a straight line.

(a) (i) Find the equation of this line.

(ii) Sketch this line on the slope field. [2]

One solution to $\frac{dy}{dx} = \frac{x}{2} + \frac{y}{3}$ passes through the point $(6, 4)$.

(b) Sketch this solution on the slope field. [2]

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(Question 11 continued)

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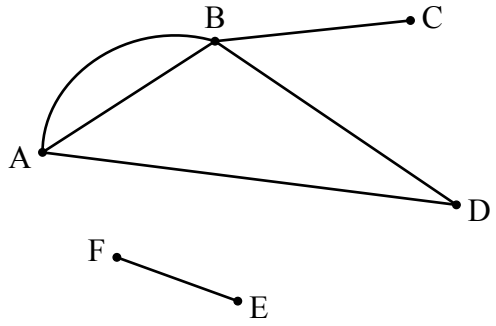
12. [Maximum mark: 8]

The graph G has six vertices and is shown in the diagram.



(a) State, giving a reason, whether G is

(i) simple;

(ii) connected. [2]

(b) Complete the adjacency matrix for G. [2]

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & \square & 0 & 1 & 0 & 0 \\ \square & 0 & \square & 1 & 0 & 0 \\ 0 & \square & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

(c) Hence, determine the number of walks from A to B with fewer than four edges. [4]

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(Question 12 continued)

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13. [Maximum mark: 8]

Air traffic control begins to monitor two airplanes.

The position of the first airplane is given by

$$\boldsymbol{r}_1 = \begin{pmatrix} 29 \\ -32 \\ 5.8 \end{pmatrix} + t\begin{pmatrix} -320 \\ 600 \\ 0.5 \end{pmatrix},$$

where t is the time, in hours, after air traffic control begins to monitor the airplane.

Distances are in kilometres and are relative to an origin at air traffic control.

The second airplane is initially at $\begin{pmatrix} -13 \\ 50 \\ 7.2 \end{pmatrix}$ and is travelling at a speed of $800\,\mathrm{km\,h^{-1}}$ in the direction $\begin{pmatrix} 7 \\ -24 \\ 0 \end{pmatrix}$.

(a) Find an expression for $\boldsymbol{r}_2$, the position of the second airplane at time t. [3]

(b) Find the least distance between the two airplanes. [5]

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14. [Maximum mark: 6]

Consider the system of coupled differential equations given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2.2x - 2.6y$$

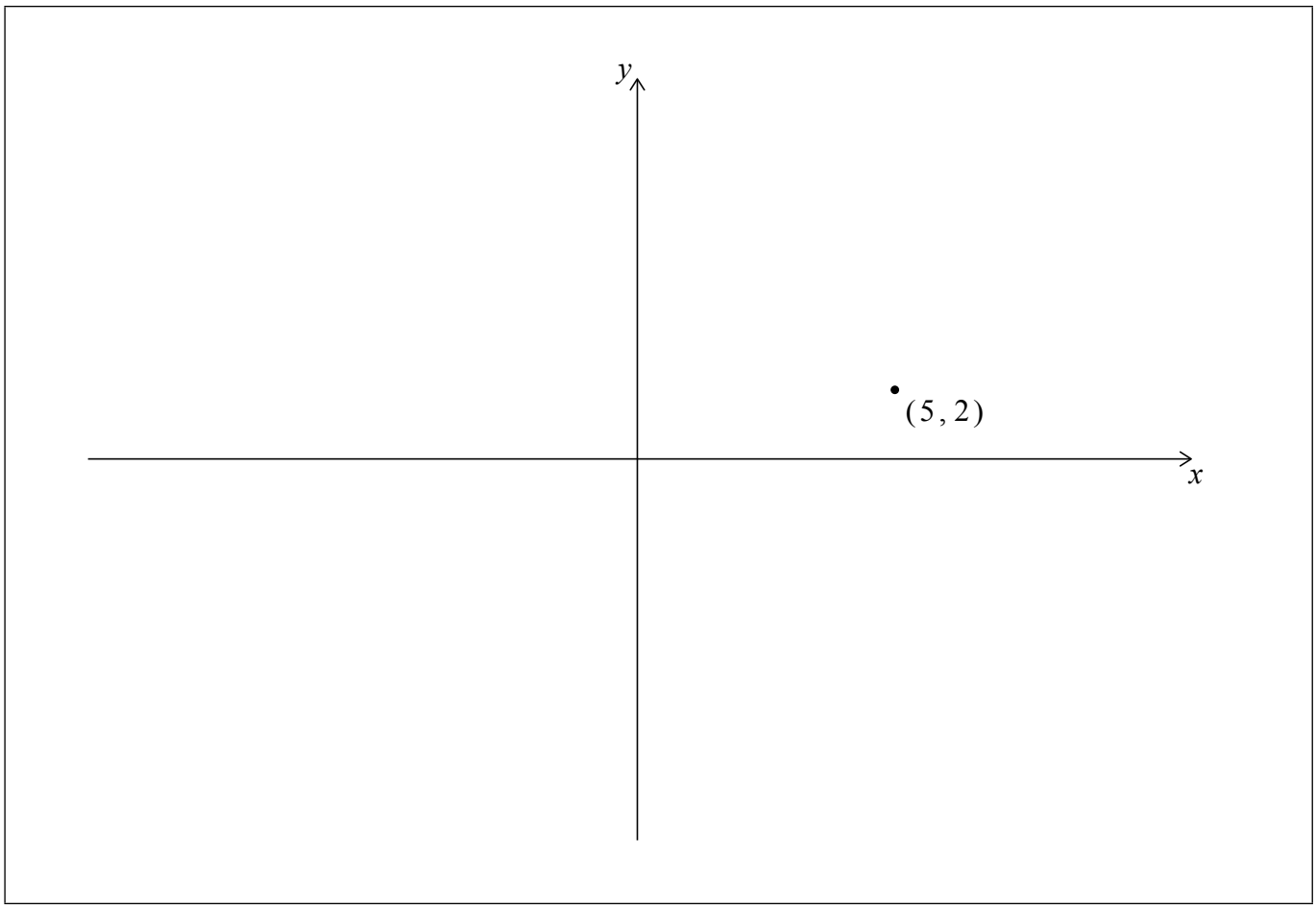
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 3.4x - 2.2y\,.$$

When $t = 0$, $x = 5$ and $y = 2$.

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $t = 0$. [3]

The eigenvalues for this system are $\pm 2\mathrm{i}$.

(b) On the following phase portrait, sketch the trajectory that passes through the point $(5, 2)$. Clearly indicate the direction of this trajectory. [3]



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(Question 14 continued)

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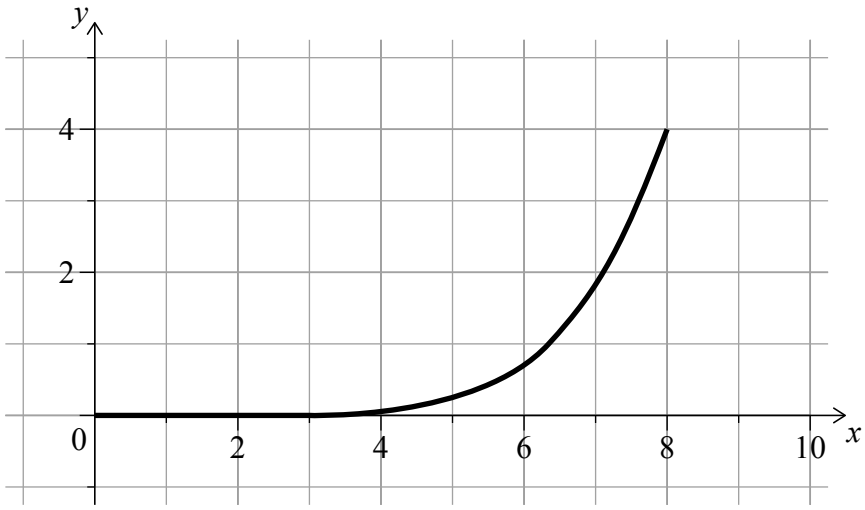
15. [Maximum mark: 8]

Jun Ho models the cross-section of a bowl, in order to calculate its volume.

His model for part of the cross-section is $y = \frac{x^6}{65536}$, where $0 \le x \le 8$, as shown in the following graph. One unit represents one centimetre.



Let R be the region enclosed by this graph, the line $y = 4$ and the line $x = 0$.

He obtains the volume of the bowl by rotating R, 2π radians about the y-axis.

(a) Find the volume of the bowl. [5]

Jun Ho pours $250\,\text{cm}^3$ of water into the bowl.

(b) Find the depth of the water in the bowl. [3]

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(Question 15 continued)

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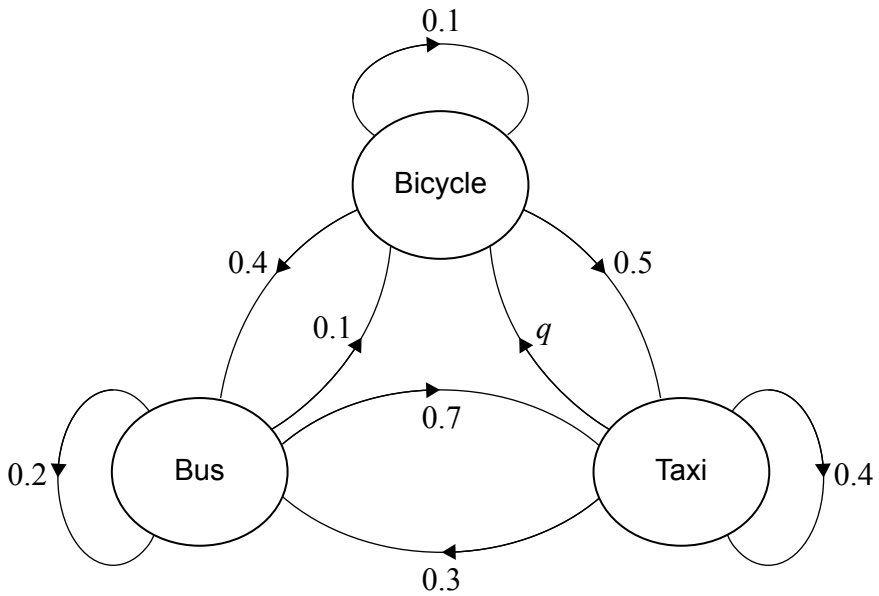
16. [Maximum mark: 8]

Each day Anirudh goes to work by bus (B) or bicycle (C) or taxi (T). The probability of him using a mode of transport depends on which mode of transport he used the previous day, and is shown in the following transition diagram.



(a) Write down the value of q. [1]

On day 1, Anirudh goes to work by bus.

(b) Write down the probability that he goes to work by taxi on day 2. [1]

(c) Find the probability that the mode of transport Anirudh uses on day 3 is the same as the one he uses on day 2. [3]

(d) Complete the following transition matrix. [1]

$$\begin{pmatrix} 0.2 & 0.4 & 0.3 \\ 0.1 & 0.1 & \square \\ 0.7 & \square & \square \end{pmatrix}$$

(e) Hence, find the long-term proportion of days that he goes to work by bicycle. [2]

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(Question 16 continued)

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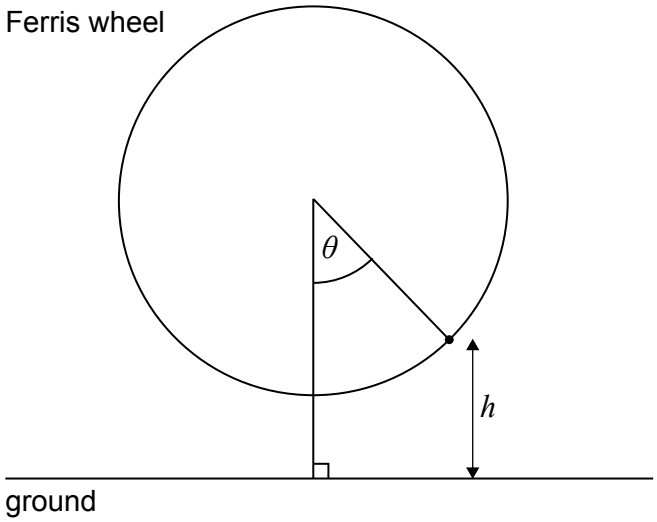
17. [Maximum mark: 8]

Younsue rides on a Ferris wheel. As the wheel rotates, her height above the ground, h metres, can be modelled in terms of the angle, θ radians, that the wheel has rotated, using $h = 16 - 15\cos\theta$. The value of θ is measured from Younsue's starting position at the bottom of the wheel.

diagram not to scale



(a) Write down the radius of the Ferris wheel. [1]

The speed at which the wheel rotates changes over time and is given by

$$\frac{d\theta}{dt} = \frac{\pi^2}{24}\left|\sin\left(\frac{\pi}{2}t\right)\right|$$

where t is the time, in minutes, after Younsue starts her ride.

(b) (i) Find the angle rotated by the Ferris wheel between $t = 0$ and $t = 4.5$.

(ii) Find the value of $\frac{dh}{dt}$, when $t = 4.5$. [7]

(This question continues on the following page)

(Question 17 continued)

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