

APOLLO

Geometry and trigonometry

SL

Mathematics: applications and interpretation

First assessment 2021

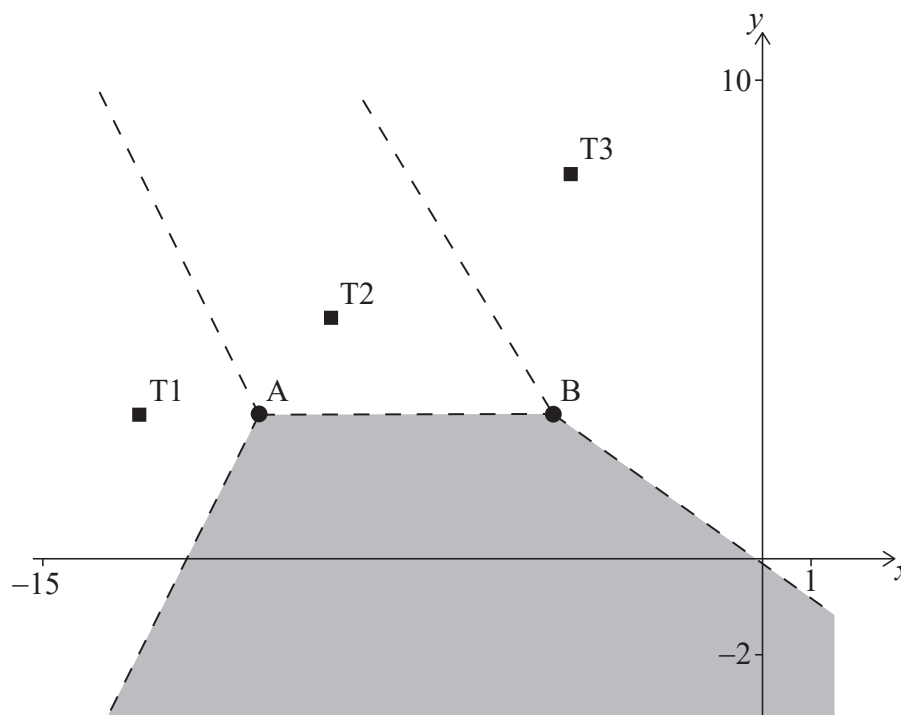
70 questions | questions

5. [Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

- (a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

- (b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates $(-13, 3)$.

- (c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]

(This question continues on the following page)



(Question 5 continued)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

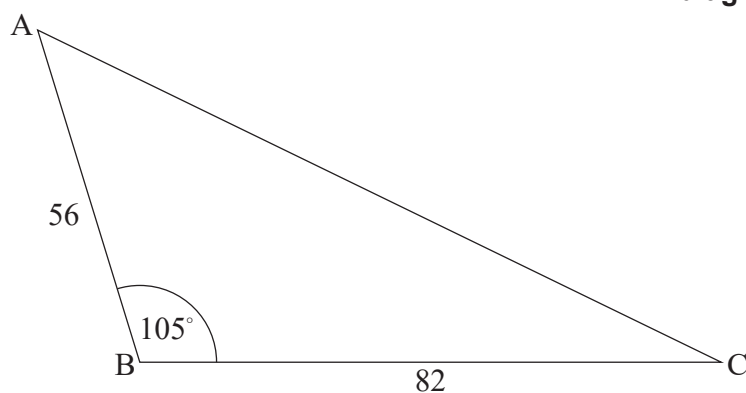


Turn over

9. [Maximum mark: 5]

A triangular field ABC is such that $AB = 56\text{ m}$ and $BC = 82\text{ m}$, each measured correct to the nearest metre, and the angle at B is equal to 105° , measured correct to the nearest 5° .

diagram not to scale



Calculate the maximum possible area of the field.

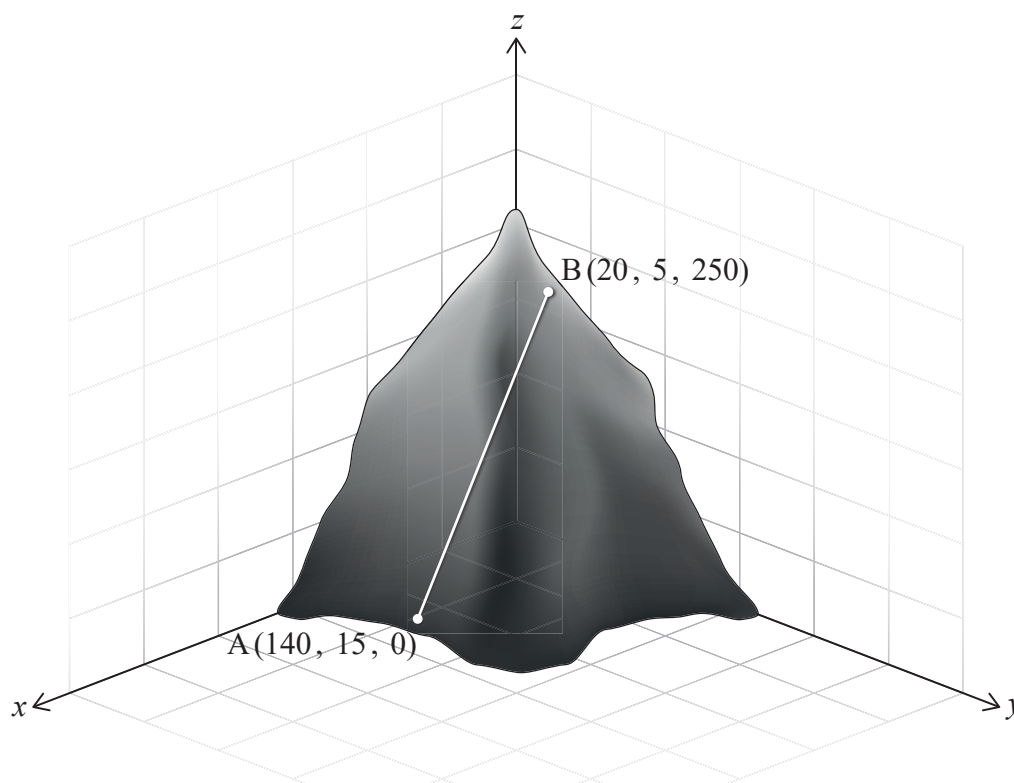
[illegible]

Turn over

2. [Maximum mark: 5]

An inclined railway travels along a straight track on a steep hill, as shown in the diagram.

diagram not to scale



The locations of the stations on the railway can be described by coordinates in reference to x , y , and z -axes, where the x and y axes are in the horizontal plane and the z -axis is vertical.

The ground level station A has coordinates $(140, 15, 0)$ and station B, located near the top of the hill, has coordinates $(20, 5, 250)$. All coordinates are given in metres.

- (a) Find the distance between stations A and B. [2]

Station M is to be built halfway between stations A and B.

- (b) Find the coordinates of station M. [2]

- (c) Write down the height of station M, in metres, above the ground. [1]

(This question continues on the following page)



(Question 2 continued)

[illegible]

Turn over



Please **do not** write on this page.

Answers written on this page
will not be marked.

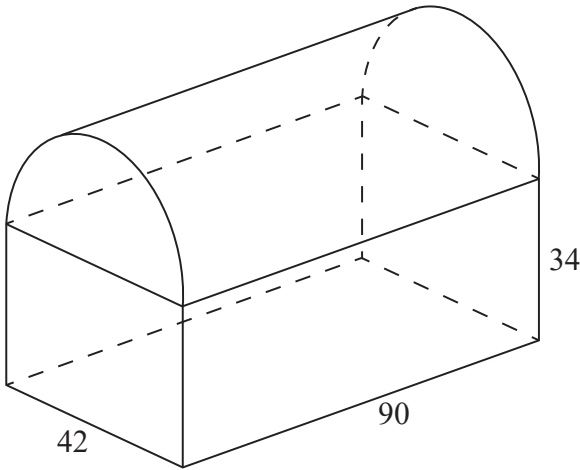


3. [Maximum mark: 7]

A storage container consists of a box of length 90 cm, width 42 cm and height 34 cm, and a lid in the shape of a half-cylinder, as shown in the diagram. The lid fits the top of the box exactly. The total exterior surface of the storage container is to be painted.

Find the area to be painted.

diagram not to scale



.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

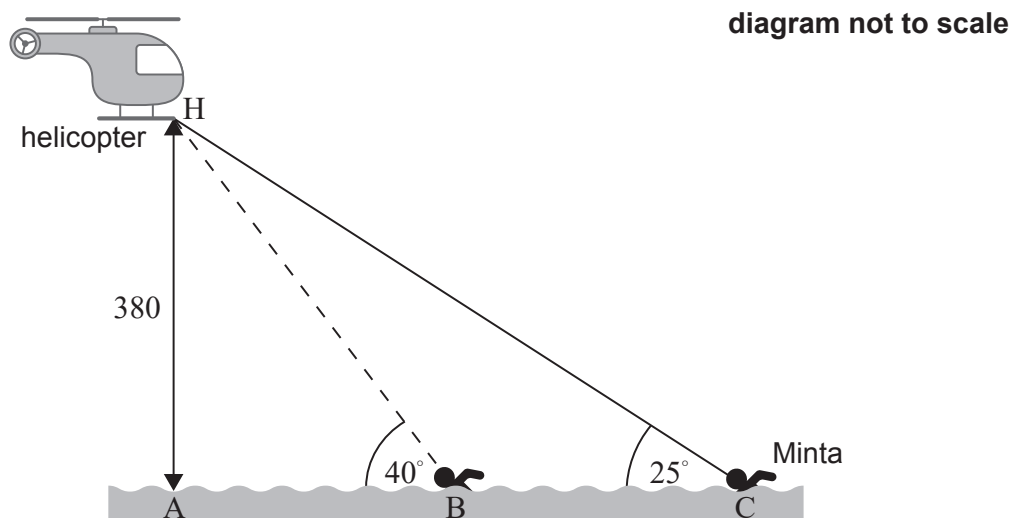
.....



Turn over

4. [Maximum mark: 7]

The diagram below shows a helicopter hovering at point H, 380m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.



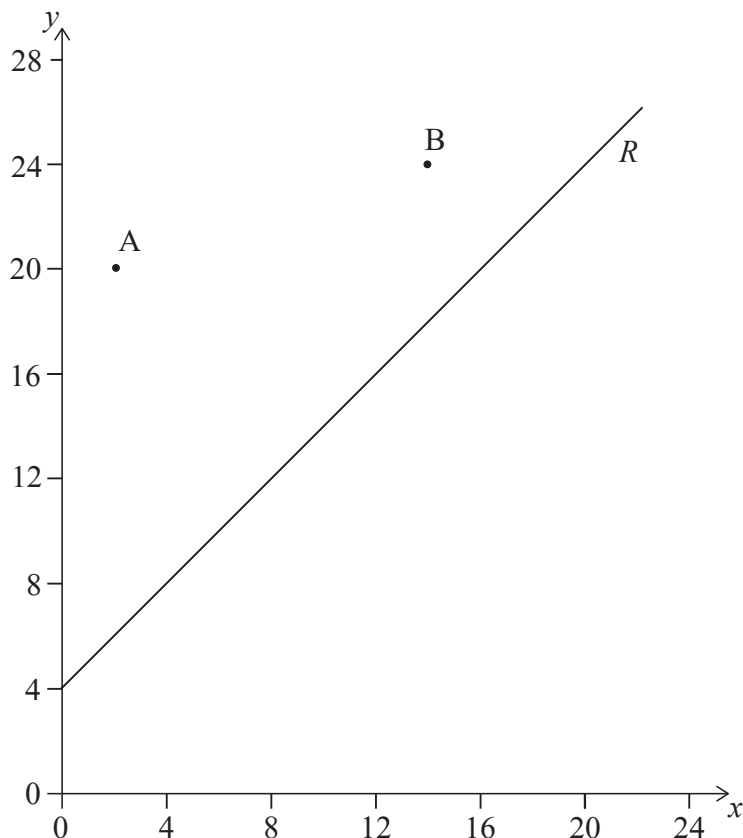
Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25° . After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40° .

- Write down the size of the angle of depression from H to C. [1]
- Find the distance from A to C. [2]
- Find the distance from B to C. [3]
- Find Minta's speed, in metres per hour. [1]

[illegible]

6. [Maximum mark: 7]

Two schools are represented by points $A(2, 20)$ and $B(14, 24)$ on the graph below. A road, represented by the line R with equation $-x + y = 4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



- (a) Find the equation of the perpendicular bisector of $[AB]$. Give your equation in the form $y = mx + c$. [5]
- (b) Determine the coordinates of the point on R where the bus stop should be located. [2]

(This question continues on the following page)



(Question 6 continued)

[illegible]

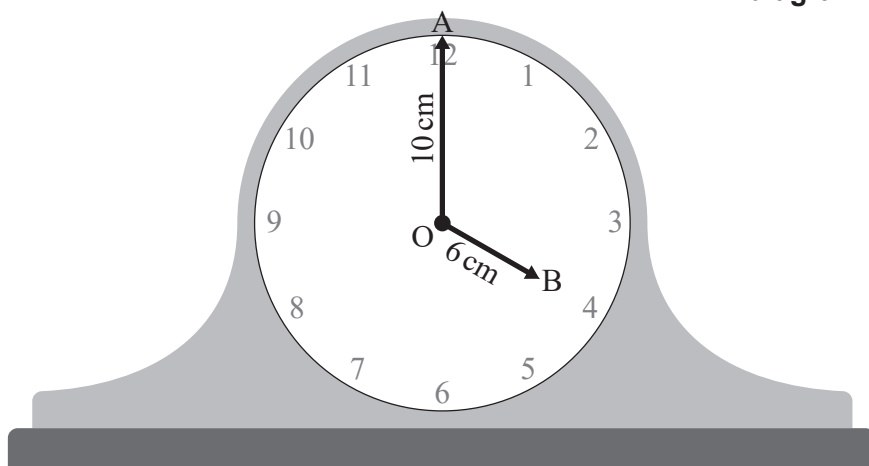
Turn over

2. [Maximum mark: 17]

The diagram below shows a circular clockface with centre O . The clock's minute hand has a length of 10 cm. The clock's hour hand has a length of 6 cm.

At 4:00 pm the endpoint of the minute hand is at point A and the endpoint of the hour hand is at point B .

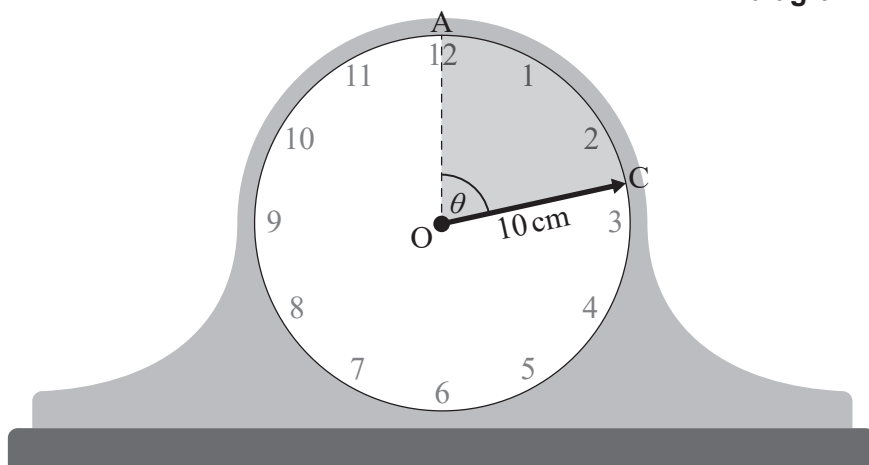
diagram not to scale



- (a) Find the size of angle $\text{A}\hat{\text{O}}\text{B}$ in degrees. [2]
- (b) Find the distance between points A and B . [3]

Between 4:00 pm and 4:13 pm, the endpoint of the **minute hand** rotates through an angle, θ , from point A to point C . This is illustrated in the diagram.

diagram not to scale

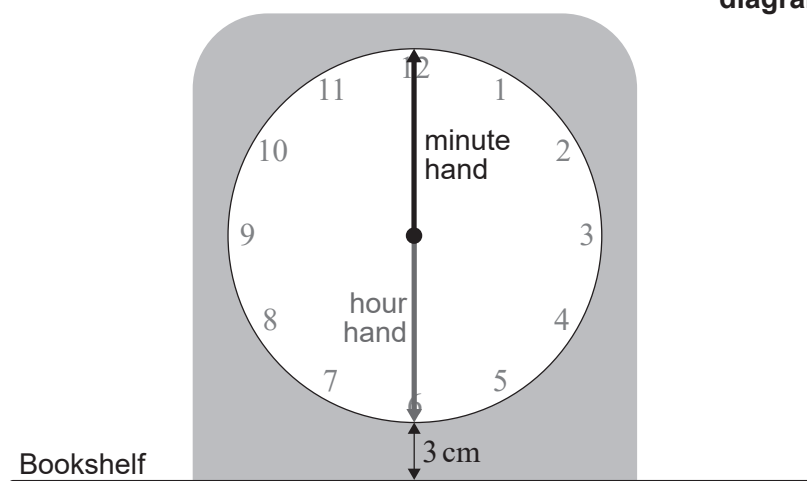


- (c) Find the size of angle θ in degrees. [2]
- (d) Calculate the length of arc AC . [2]
- (e) Calculate the area of the shaded sector, AOC . [2]

(This question continues on the following page)

(Question 2 continued)

A **second** clock is illustrated in the diagram below. The clock face has radius 10 cm with minute and hour hands both of length 10 cm. The time shown is 6:00 am. The bottom of the clock face is located 3 cm above a horizontal bookshelf.

diagram not to scale

- (f) Write down the height of the endpoint of the minute hand above the bookshelf at 6:00 am.

[1]

The height, h centimetres, of the endpoint of the minute hand above the bookshelf is modelled by the function

$$h(\theta) = 10 \cos \theta + 13, \theta \geq 0,$$

where θ is the angle rotated by the minute hand from 6:00 am.

- (g) Find the value of h when $\theta = 160^\circ$.

[2]

The height, g centimetres, of the endpoint of the **hour hand** above the bookshelf is modelled by the function

$$g(\theta) = -10 \cos \left(\frac{\theta}{12} \right) + 13, \theta \geq 0,$$

where θ is the angle in degrees rotated by the minute hand from 6:00 am.

- (h) Write down the amplitude of $g(\theta)$.

[1]

The endpoints of the minute hand and hour hand meet when $\theta = k$.

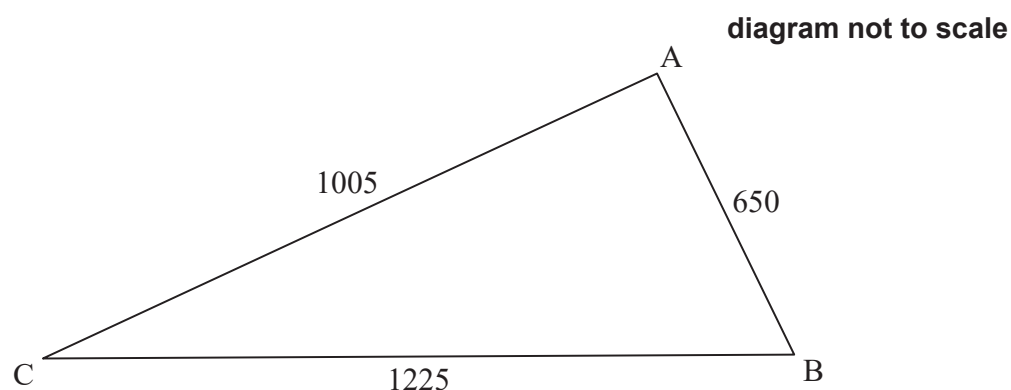
- (i) Find the smallest possible value of k .

[2]

Turn over

2. [Maximum mark: 15]

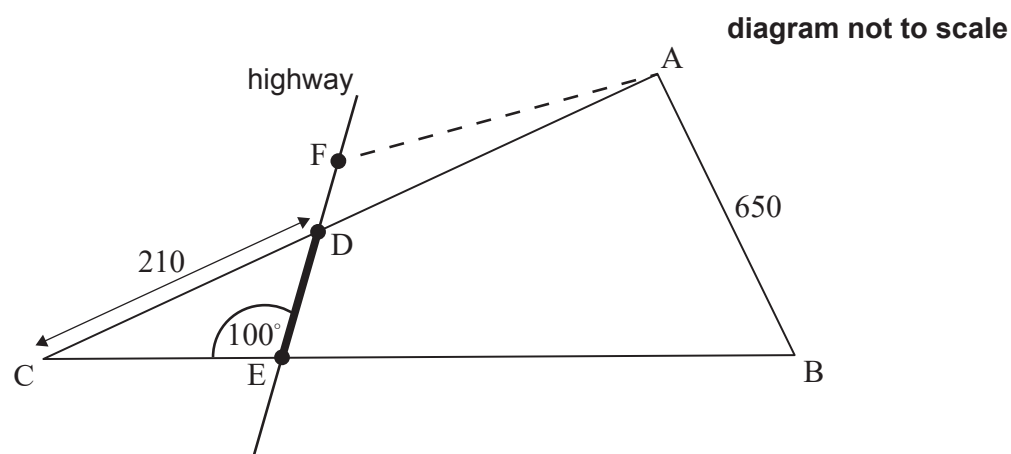
A farmer owns a field in the shape of a triangle ABC such that $AB = 650$ m, $AC = 1005$ m and $BC = 1225$ m.



(a) Find the size of $\hat{ACB}$.

[3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $DC = 210$ m and $\hat{CED} = 100^\circ$, as shown in the diagram below.



(b) Find DE.

[3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE.

[5]

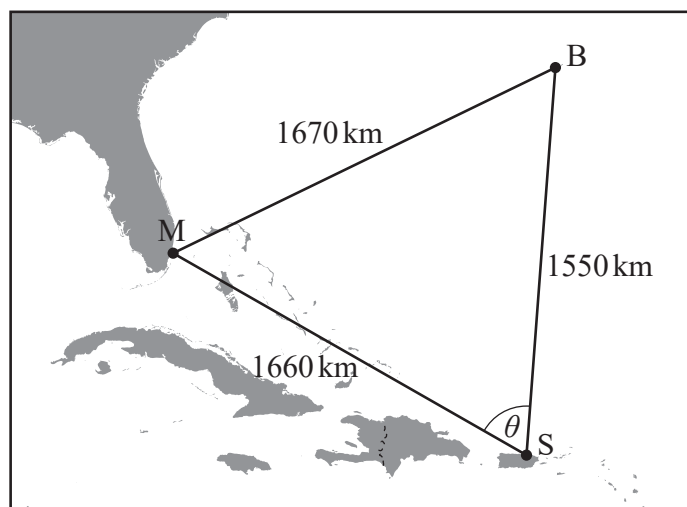
(d) Estimate DF. You may assume the highway has a width of zero.

[4]

2. [Maximum mark: 5]

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

(a) Calculate the value of θ , the measure of angle $\widehat{MSB}$. [3]

(b) Find the area of the Bermuda Triangle. [2]

(This question continues on the following page)



(Question 2 continued)

[illegible]

Turn over

Please **do not** write on this page.

Answers written on this page
will not be marked.

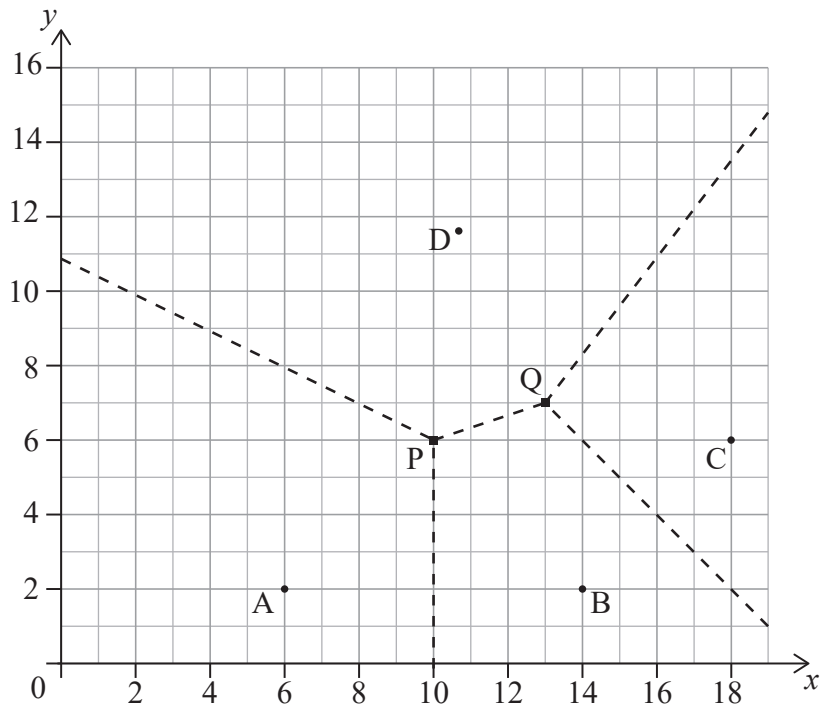


7. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral $ABCD$. The new station will be located so that it is as far as possible from the nearest existing station.

- (a) Show that the new station should be built at P . [3]

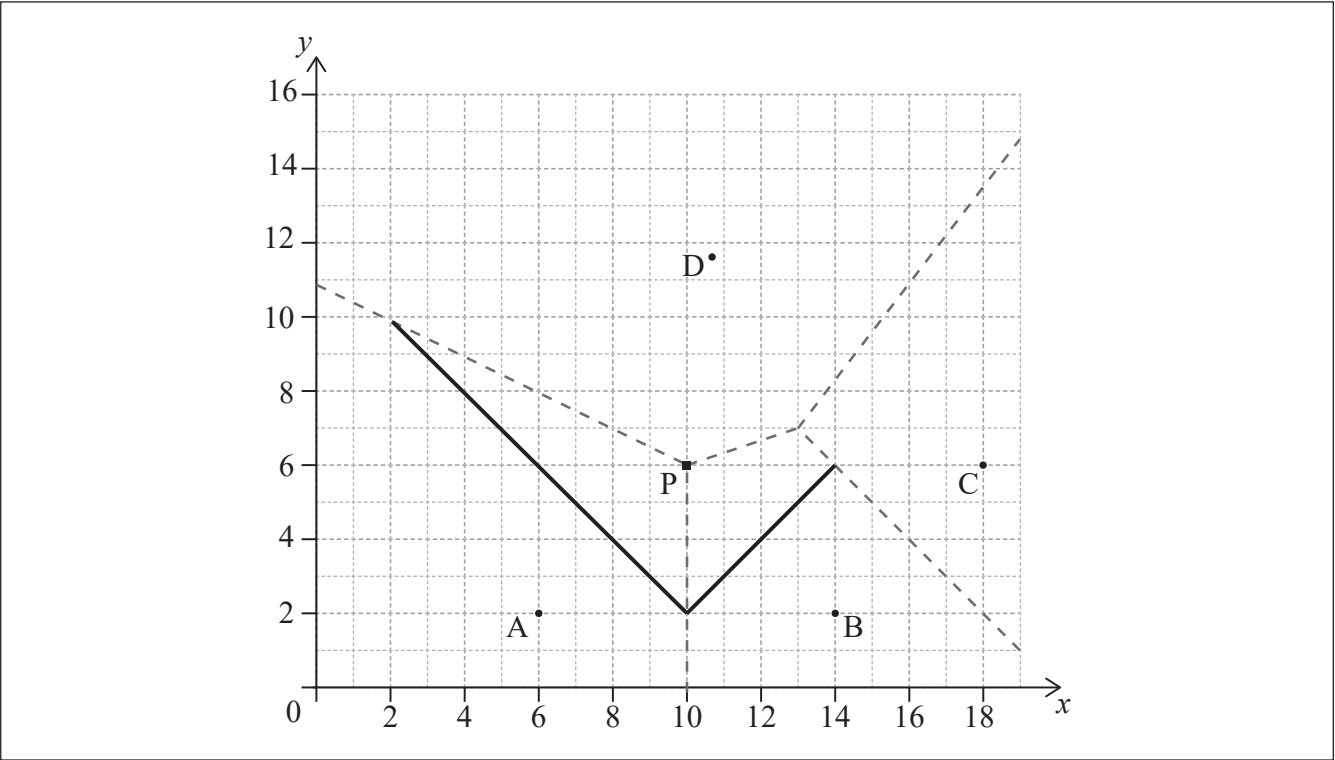
The Voronoi diagram is to be updated to include the region around the new station at P . The edges defined by the perpendicular bisectors of $[AP]$ and $[BP]$ have been added to the following diagram.

- (b) (i) Write down the equation of the perpendicular bisector of $[PC]$.
(ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

(This question continues on the following page)



(Question 7 continued)



.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

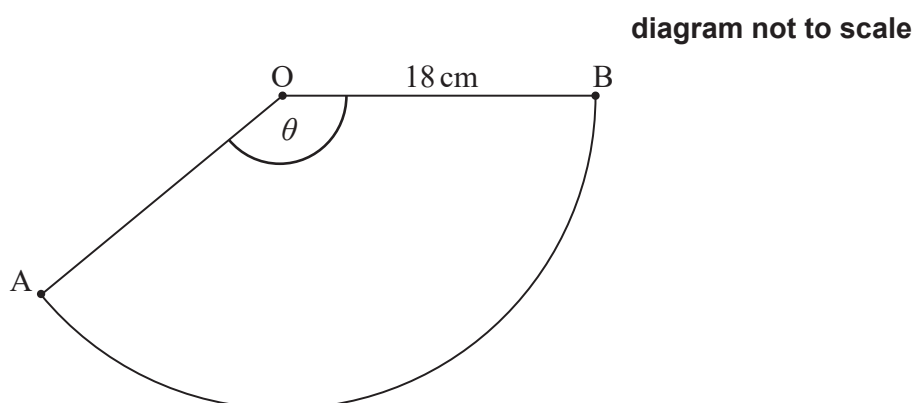
.....



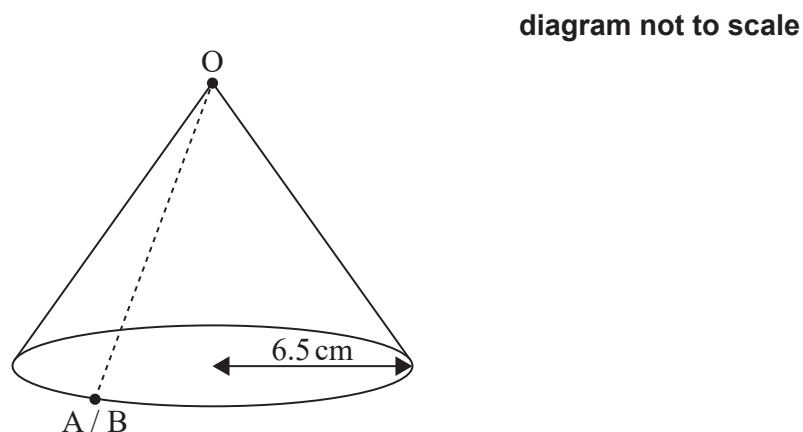
Turn over

8. [Maximum mark: 5]

Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB , of a circular piece of paper with a radius of 18 cm and $\angle AOB = \theta$ as shown in the diagram.



To make the hat, sides $[OA]$ and $[OB]$ are joined together. The hat has a base radius of 6.5 cm.



(a) (i) Write down the perimeter of the base of the hat in terms of π .

(ii) Find the value of θ .

[3]

(b) Find the surface area of the outside of the hat.

[2]

(This question continues on the following page)



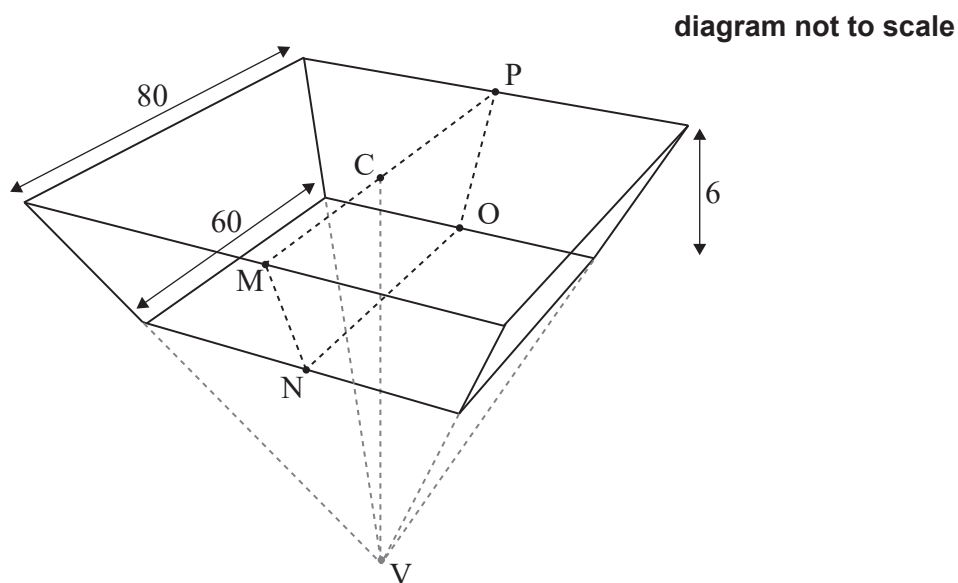
(Question 8 continued)

[illegible]

Turn over

4. [Maximum mark: 14]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.

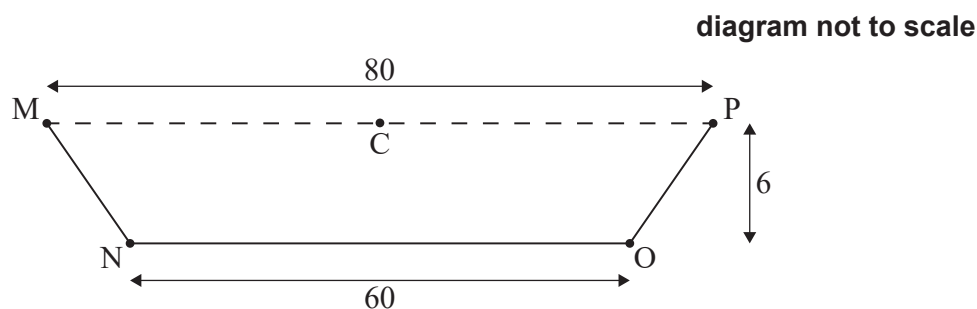


The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

(This question continues on the following page)

(Question 4 continued)

The second diagram shows a vertical cross section, $MNOPC$, of the reservoir.



- (a) Find the angle of depression from M to N . [2]
- (b) (i) Find CV . [5]
- (ii) Hence or otherwise, show that the volume of the reservoir is $29\,600\text{ m}^3$.

Every day 80 m^3 of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

- (c) By finding an appropriate value, determine whether Joshua is correct. [2]

To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

- (d) Find the area that was painted. [5]

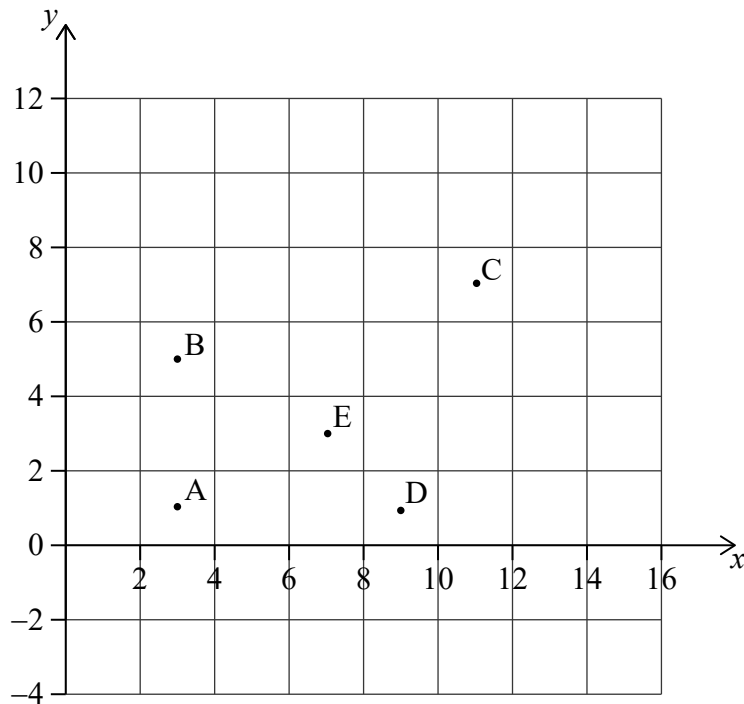
Turn over

7. [Maximum mark: 6]

Points A(3, 1), B(3, 5), C(11, 7), D(9, 1) and E(7, 3) represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.



- (a) Calculate the gradient of the line segment AE.

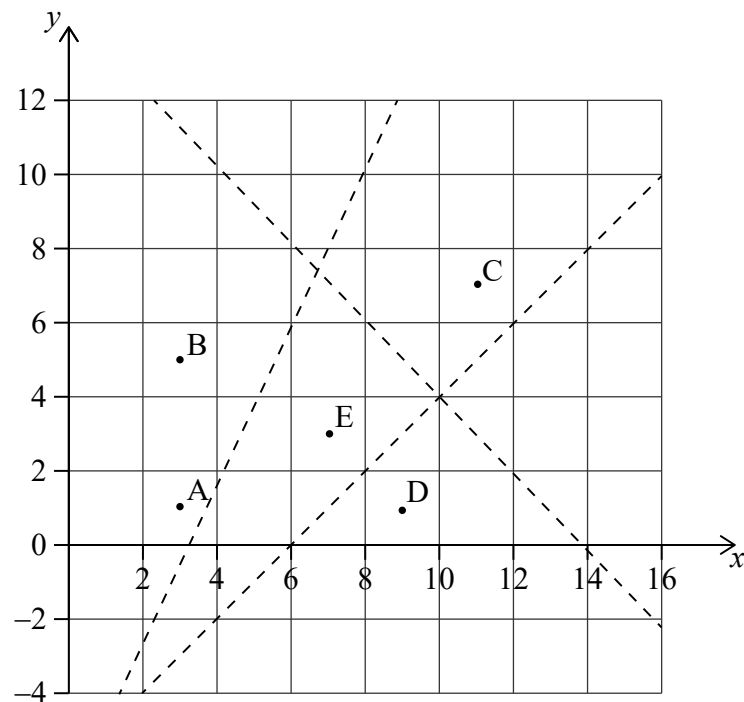
[2]

(This question continues on the following page)



(Question 7 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



- (b)

Find the equation of the line which would complete the Voronoi cell containing site E.
Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$.

[3]
- (c)

In the context of the question, explain the significance of the Voronoi cell containing site E.

[1]

.....

.....

.....

.....

.....

.....

.....

.....

.....

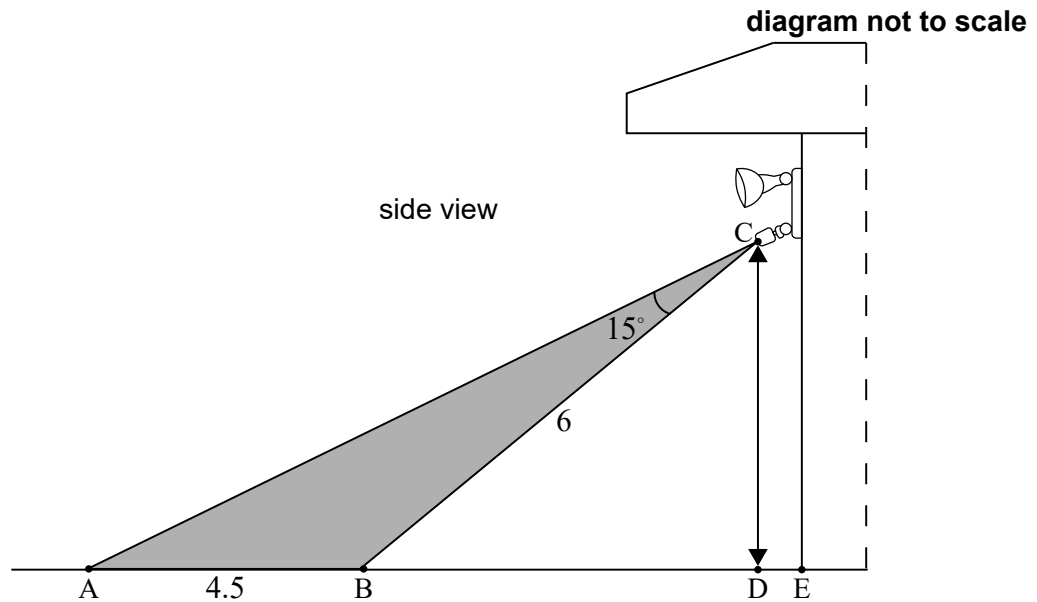
.....

.....



14. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D . The area covered by the sensor is shown by the shaded region enclosed by triangle ABC . The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $\widehat{ACB}$ is 15° .



- (a) Find $\widehat{CAB}$. [3]

Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D , below the sensor. He walks towards point B .

- (b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



Please **do not** write on this page.

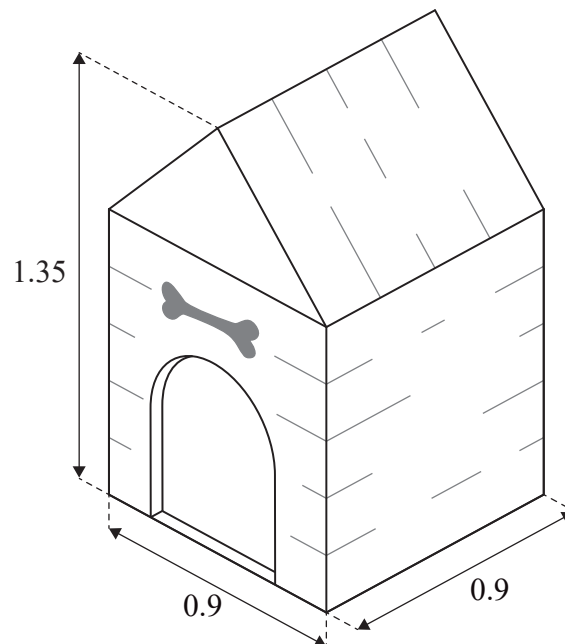
Answers written on this page
will not be marked.



1. [Maximum mark: 5]

The front view of a doghouse is made up of a square with an isosceles triangle on top. The doghouse is 1.35 m high and 0.9 m wide, and sits on a square base.

diagram not to scale



The top of the rectangular surfaces of the roof of the doghouse are to be painted.

Find the area to be painted.

(This question continues on the following page)



(Question 1 continued)

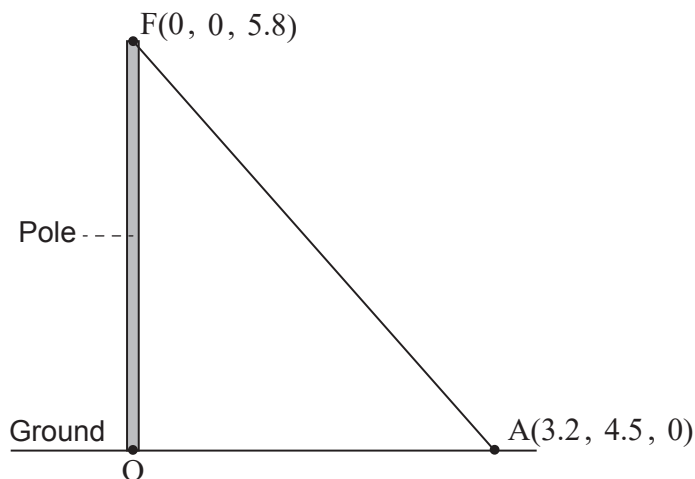
[illegible]

Turn over

2. [Maximum mark: 4]

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates (0, 0, 5.8). All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F.

One of the ropes is attached to the ground at a point A with coordinates (3.2, 4.5, 0). The rope forms a straight line from A to F.

- (a) Find the length of the rope connecting A to F. [2]
- (b) Find $\hat{FAO}$, the angle the rope makes with the ground. [2]

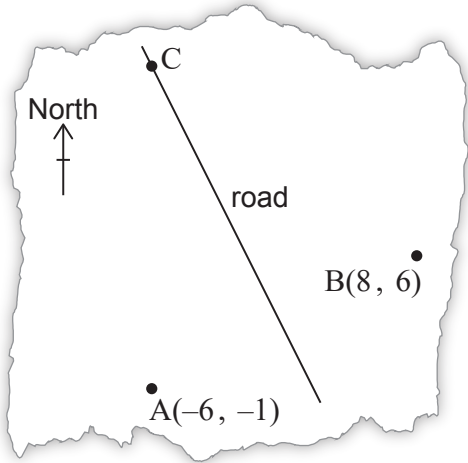
[illegible]

4. [Maximum mark: 7]

Three towns, A, B and C are represented as coordinates on a map, where the x and y axes represent the distances east and north of an origin, respectively, measured in kilometres.

Town A is located at $(-6, -1)$ and town B is located at $(8, 6)$. A road runs along the perpendicular bisector of $[AB]$. This information is shown in the following diagram.

diagram not to scale



(a) Find the equation of the line that the road follows. [5]

Town C is due north of town A and the road passes through town C.

(b) Find the y -coordinate of town C. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

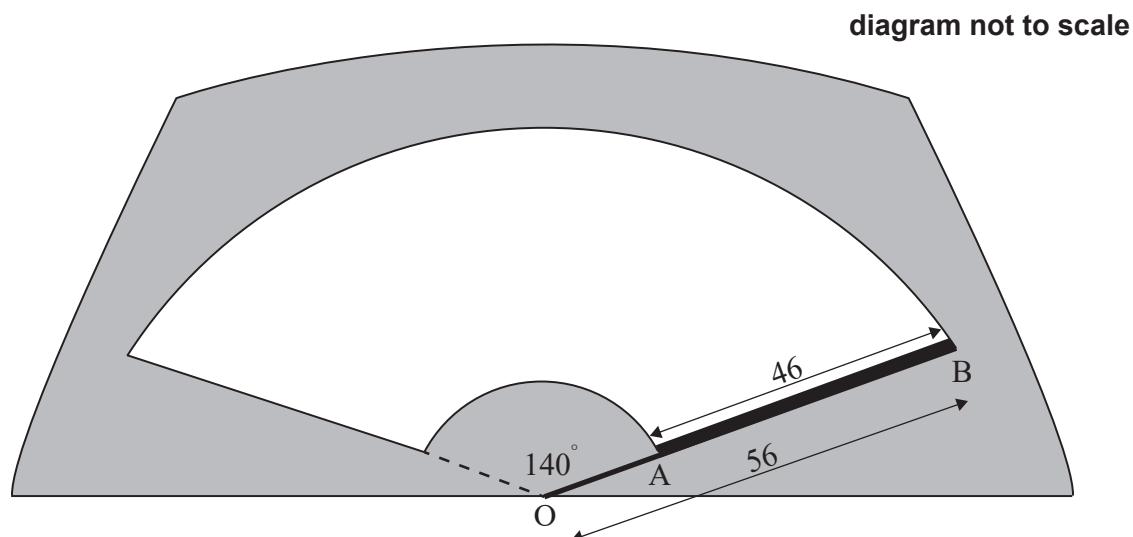
.....



1. [Maximum mark: 5]

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O , through an angle of 140° . The windscreen is cleared by a rubber blade of length 46 cm that is attached to the metal arm between points A and B . The total length of the metal arm, OB , is 56 cm.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.



- (a) Calculate the length of the arc made by B , the end of the rubber blade. [2]
- (b) Determine the area of the windscreen that is cleared by the rubber blade. [3]

(This question continues on the following page)





4444-1217

(Question 1 continued)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

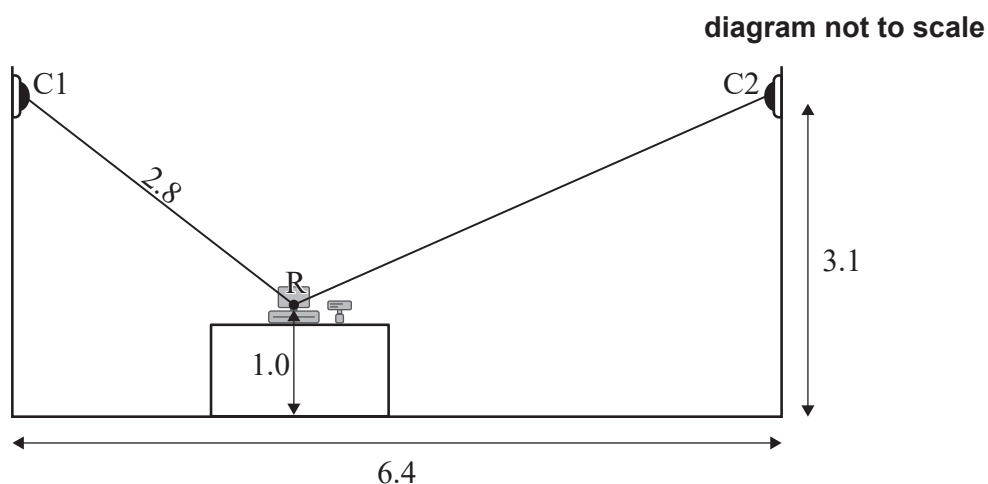


Turn over

3. [Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points C1 and C2. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.



The cameras are positioned at a height of 3.1 m, and the horizontal distance between the cameras is 6.4 m. The cash register is sitting on a counter so that its centre, R, is 1.0 m above the floor.

The distance from Camera 1 to the centre of the cash register is 2.8 m.

- Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]
- Calculate the distance from Camera 2 to the centre of the cash register. [4]
- Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

(This question continues on the following page)

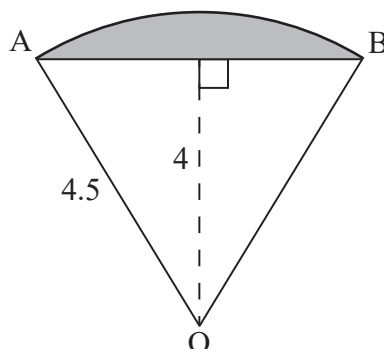


[illegible]

4. [Maximum mark: 15]

A sector of a circle, centre O and radius 4.5 m, is shown in the following diagram.

diagram not to scale



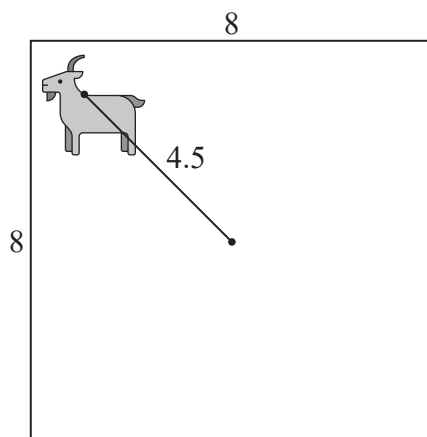
(a) (i) Find the angle $\hat{AOB}$.

(ii) Find the area of the shaded segment.

[8]

A square field with side 8 m has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to 4.5 m from the post.

diagram not to scale



(b) (i) Find the area of a circle with radius 4.5 m.

(ii) Find the area of the field that can be reached by the goat.

[5]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

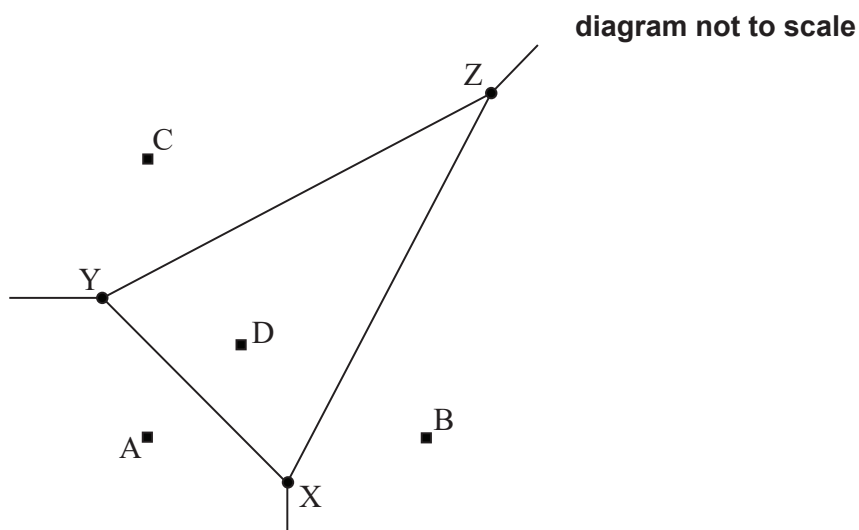
The goat eats grass at the rate of $\frac{dV}{dt} = 0.3te^{-t}$.

(c) Find the value of t at which the goat is eating grass at the greatest rate.

[2]

3. [Maximum mark: 18]

The Voronoi diagram below shows four supermarkets represented by points with coordinates $A(0, 0)$, $B(6, 0)$, $C(0, 6)$ and $D(2, 2)$. The vertices X , Y , Z are also shown. All distances are measured in kilometres.



(a) Find the midpoint of $[BD]$. [2]

(b) Find the equation of (XZ) . [4]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X . [3]

The coordinates of Y are $(-1, 3)$ and the coordinates of Z are $(7, 7)$.

(d) Determine the exact length of $[YZ]$. [2]

(e) Given that the exact length of $[XY]$ is $\sqrt{32}$, find the size of $\hat{XYZ}$ in degrees. [4]

(f) Hence find the area of triangle XYZ . [2]

A town planner believes that the larger the area of the Voronoi cell XYZ , the more people will shop at supermarket D .

(g) State one criticism of this interpretation. [1]

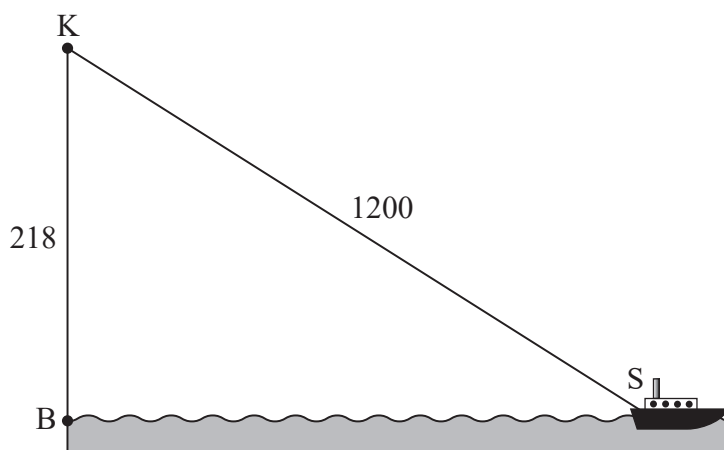
Turn over

1. [Maximum mark: 6]

Kacheena stands at point K, the top of a 218 m vertical cliff. The base of the cliff is located at point B. A ship is located at point S, 1200 m from Kacheena.

This information is shown in the following diagram.

diagram not to scale



- (a) Find the angle of elevation from the ship to Kacheena. [2]
- (b) Find the horizontal distance from the base of the cliff to the ship. [2]
- (c) Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [2]

(This question continues on the following page)



(Question 1 continued)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



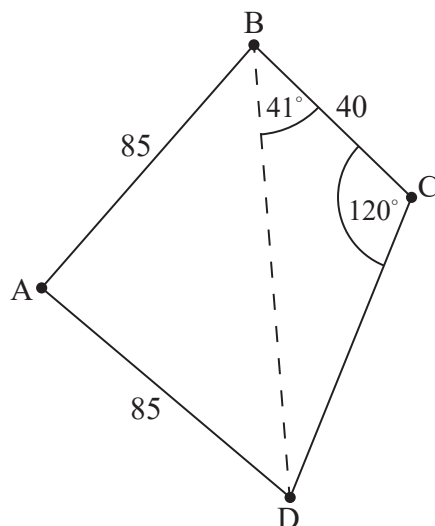
Turn over

2. [Maximum mark: 17]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$AB = 85 \text{ m}, AD = 85 \text{ m}, BC = 40 \text{ m}, \hat{CBD} = 41^\circ, \hat{BCD} = 120^\circ$$

diagram not to scale



- (a) (i) Write down the value of angle BDC. [4]
- (ii) Hence use triangle BDC to find the length of path BD. [4]
- (b) Calculate the size of angle $\hat{BAD}$, correct to five significant figures. [3]

The size of angle $\hat{BAD}$ rounds to 77° , correct to the nearest degree. Use $\hat{BAD} = 77^\circ$ for the rest of this question.

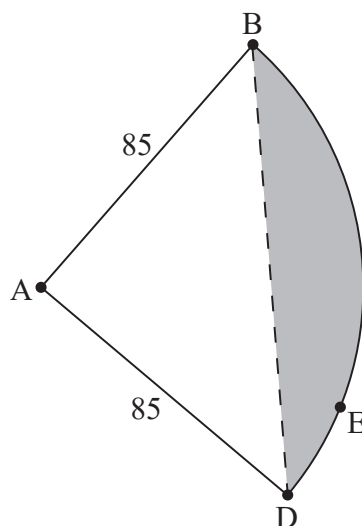
- (c) Find the area bounded by the path BD, and fences AB and AD. [3]

(This question continues on the following page)

(Question 2 continued)

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



- (d) Write down the distance from A to E. [1]
- (e) Find the perimeter of the proposed park, ABED. [3]
- (f) Find the area of the shaded region in the proposed park. [3]

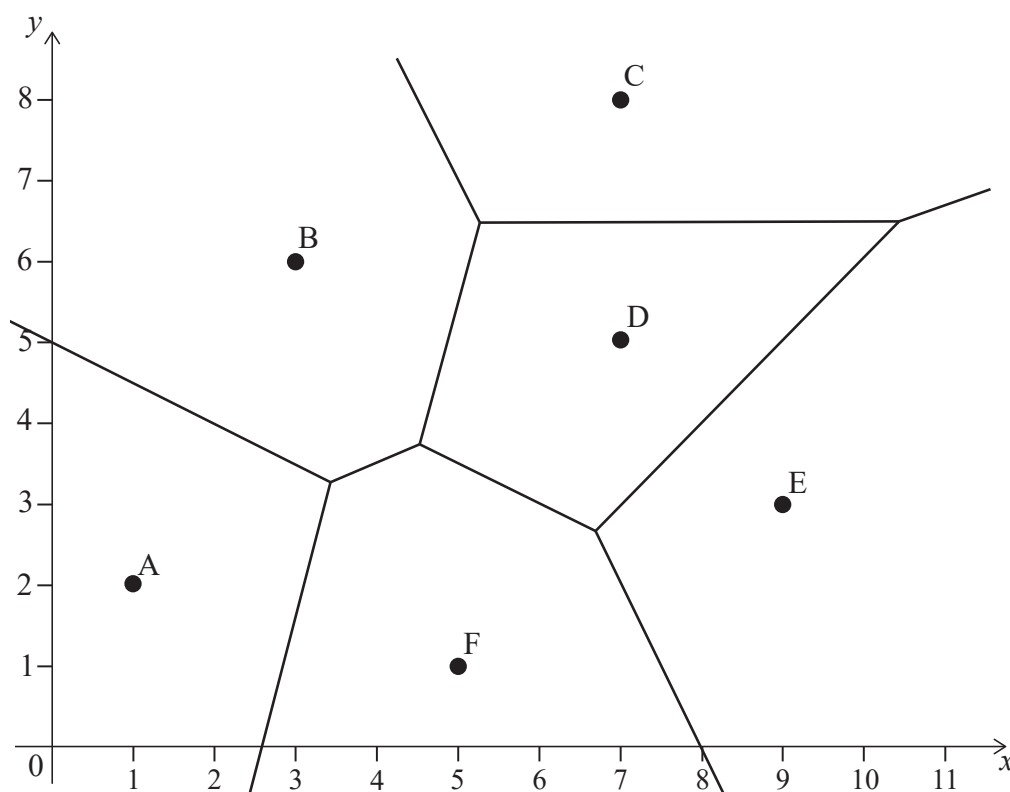
Turn over



Blank page

3. [Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



- (a) Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(i) $(2, 7)$.

(ii) $(0, 1)$, when restaurant A is closed.

[2]

Restaurant C is at $(7, 8)$ and restaurant D is at $(7, 5)$.

- (b) Find the equation of the perpendicular bisector of CD.

[2]

Restaurant B is at $(3, 6)$.

- (c) Find the equation of the perpendicular bisector of BC.

[5]

- (d) Hence find

(i) the coordinates of the point which is of equal distance from B, C and D.

(ii) the distance of this point from D.

[4]

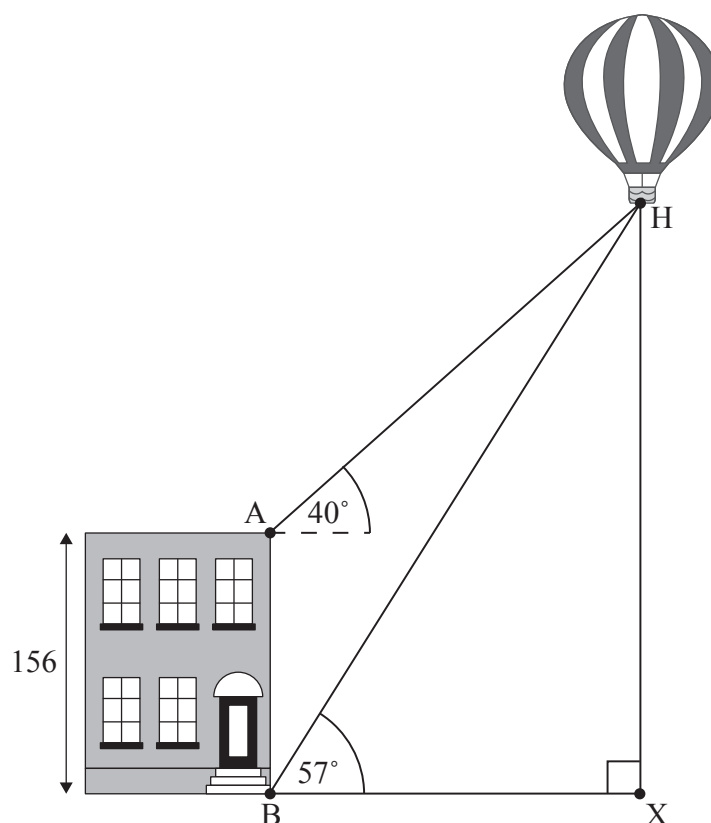
Turn over

2. [Maximum mark: 6]

Point H on a hot-air balloon is sighted at the same time by two observers. One observer is at the top of a vertical building that is 156 metres tall. The other observer is at the base of the building.

The angle of elevation from point A (at the top of the building) to H is 40° , and the angle of elevation from point B (at the base of the building) to H is 57° . Point X is the ground directly below point H. This information is shown in the diagram.

diagram not to scale



(a) Find the size of angle $\hat{A}HB$. [2]

(b) Calculate the distance from point B to point H. [3]

The hot-air balloon remains at a constant height as it moves further away from the building.

(c) Describe, in words, the change in the angle of depression from point H to point B as the horizontal distance between the balloon and the building increases. [1]

(This question continues on the following page)



(Question 2 continued)

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



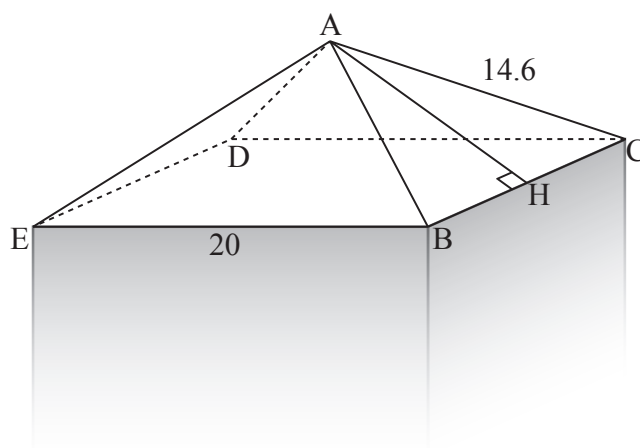
Turn over

11. [Maximum mark: 7]

Vertical posts are to be placed around the outer edge of a children's park. Each post is formed from a cuboid with a right square-based pyramid on top.

The cuboid part of the post is machine-made such that its width, and hence the width of the pyramid, is exactly 20 cm. The length from the apex of the pyramid, A , to any corner of the base of the pyramid is 14.6 cm, **but** this is only accurate to the nearest tenth of a centimetre. The post is shown in the diagram.

diagram not to scale



- (a) Write down the upper bound and lower bound for the possible lengths of edge AC . [2]

Point H is the midpoint of BC .

- (b) Determine the upper bound and lower bound for AH , the slant height of the pyramid. [3]

For the post to be safe for children, the angle between the slant height and the base of the pyramid must be less than 22° .

- (c) Show that this post is safe for children. Justify your answer. [2]

(This question continues on the following page)



(Question 11 continued)

[illegible]

Turn over

13. [Maximum mark: 5]

A boat travels 8 km on a bearing of 315° and then a further 6 km on a bearing of 045° .
Find the bearing on which the boat should travel to return directly to the starting point.

References:
© International Baccalaureate Organization 2023



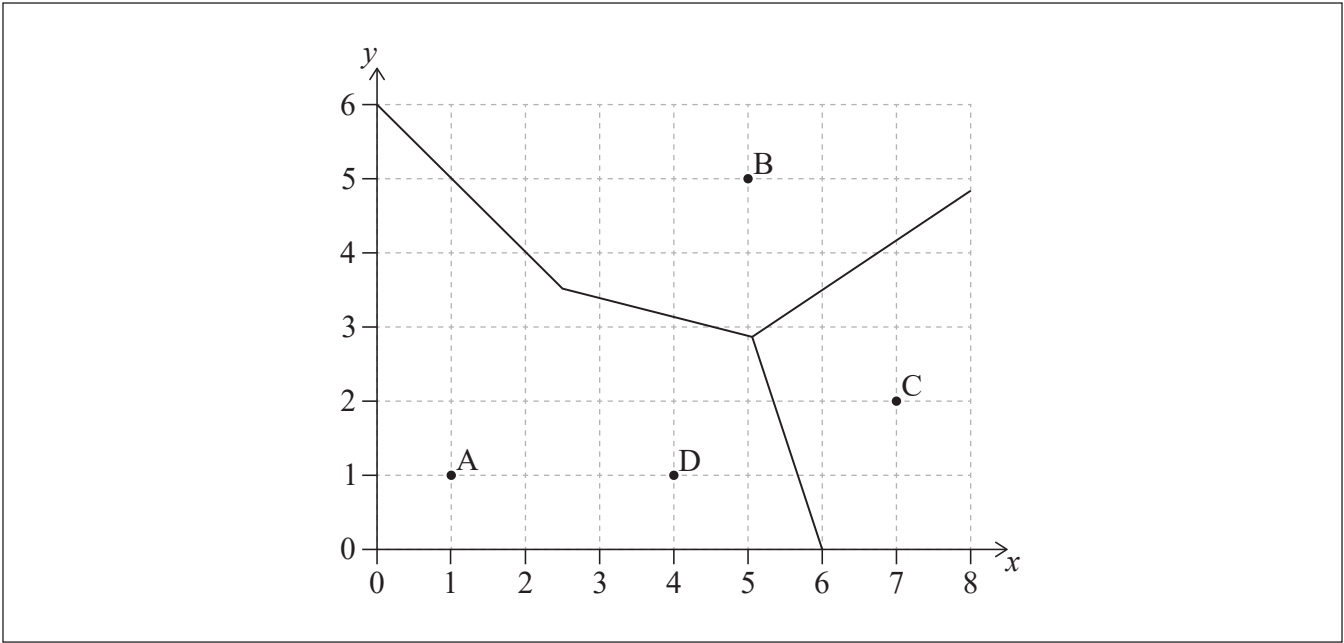
Please **do not** write on this page.

Answers written on this page
will not be marked.



7. [Maximum mark: 6]

Ani owns four cafes represented by points A, B, C and D. Ani wants to divide the area into delivery regions. This process has been started in the following incomplete Voronoi diagram, where 1 unit represents 1 kilometre.



The midpoint of CD is (5.5, 1.5).

- (a) Show that the equation of the perpendicular bisector of [CD] is $y = -3x + 18$. [3]
- (b) Complete the Voronoi diagram shown above. [1]

Ani opens an office equidistant from three of the cafes, B, C and D. The equation of the perpendicular bisector of [BC] is $3y = 2x - 1.5$.

- (c) Find the coordinates of the office. [2]

(This question continues on the following page)



(Question 7 continued)

[illegible]

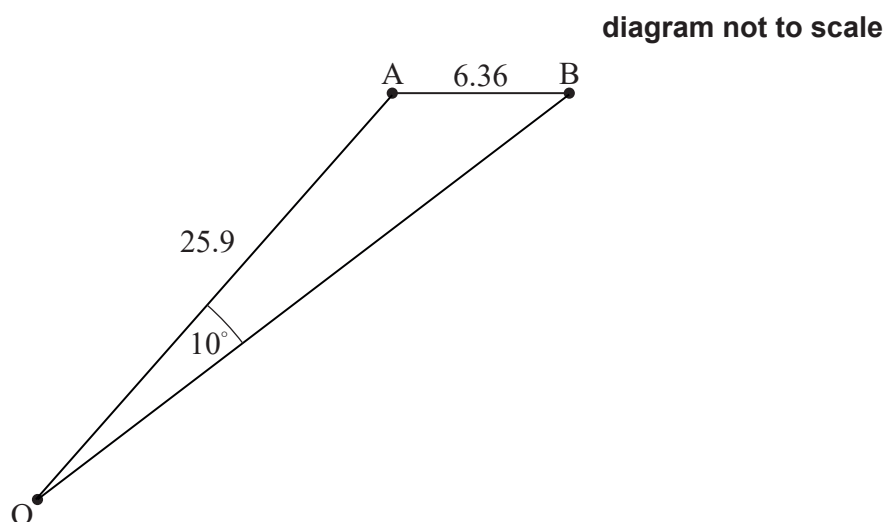
Turn over

1. [Maximum mark: 17]

The diagram shows points in a park viewed from above, at a specific moment in time.

The distance between two trees, at points A and B, is 6.36 m.

Odette is playing football in the park and is standing at point O, such that $\hat{AOB} = 10^\circ$, $OA = 25.9$ m and $\hat{OAB}$ is obtuse.



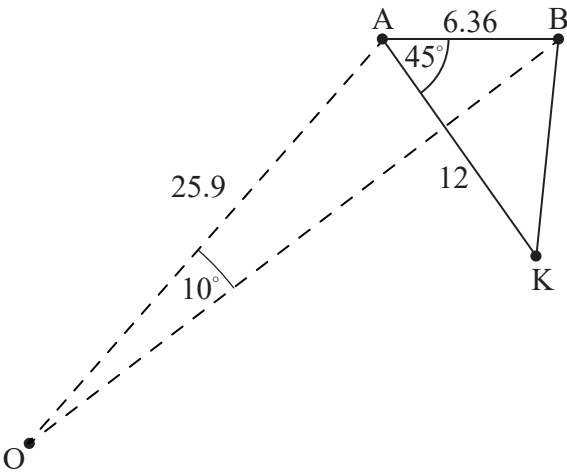
- (a) Calculate the size of $\hat{ABO}$. [3]
- (b) Calculate the area of triangle AOB. [4]

(This question continues on the following page)

(Question 1 continued)

Odette’s friend, Khemil, is standing at point K such that he is 12 m from A and $\widehat{KAB} = 45^\circ$.

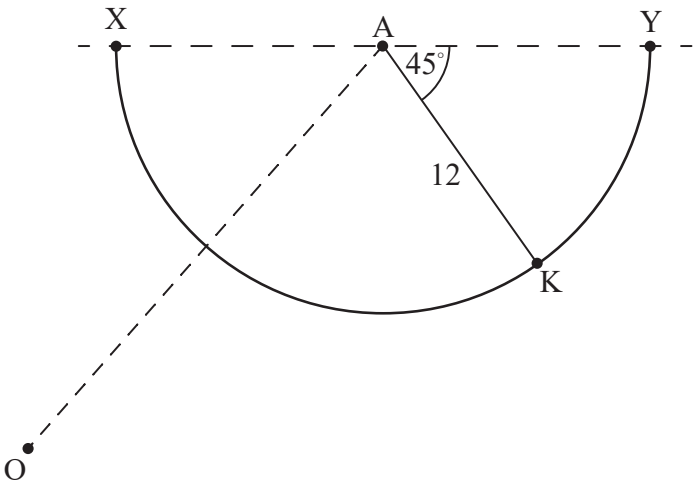
diagram not to scale



- (c) Calculate Khemil’s distance from B. [3]

XY is a semicircular path in the park with centre A, such that $\widehat{KAY} = 45^\circ$. Khemil is standing on the path and Odette’s football is at point X. This is shown in the diagram below.

diagram not to scale



The length $KX = 22.2\text{ m}$, $\widehat{KOX} = 53.8^\circ$ and $\widehat{OKX} = 51.1^\circ$.

- (d) Find whether Odette or Khemil is closer to the football. [4]

Khemil runs along the semicircular path to pick up the football.

- (e) Calculate the distance that Khemil runs. [3]

2. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).



[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 <https://creativecommons.org/licenses/by/2.5/#>. Image adapted.]

Viktor reads online that 1 REC is equal to 0.52 metres, rounded to two decimal places.

- (a) Write down the upper and lower bounds of 1 REC in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of 440 REC and a height of 280 REC. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

- (b) Find the minimum possible volume of the pyramid in cubic metres. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

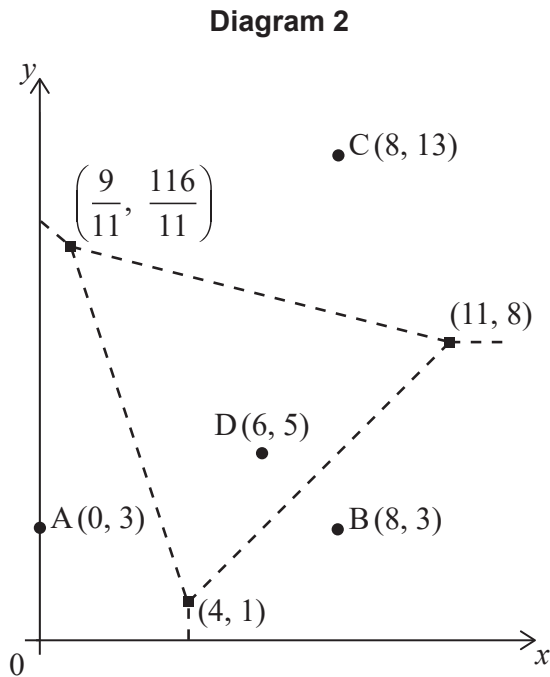
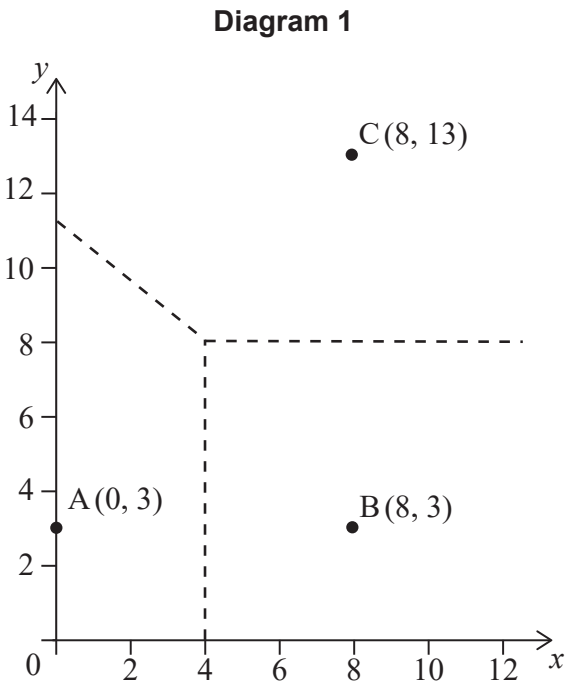
.....

.....



4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

- (a) Write down the coordinates of this point. [1]
- (b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

- (c) The wind turbine should be as far from the nearest farmhouses as possible.
 - (i) By calculating appropriate distances, find the location of the wind turbine.
 - (ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

(This question continues on the following page)



(Question 4 continued)

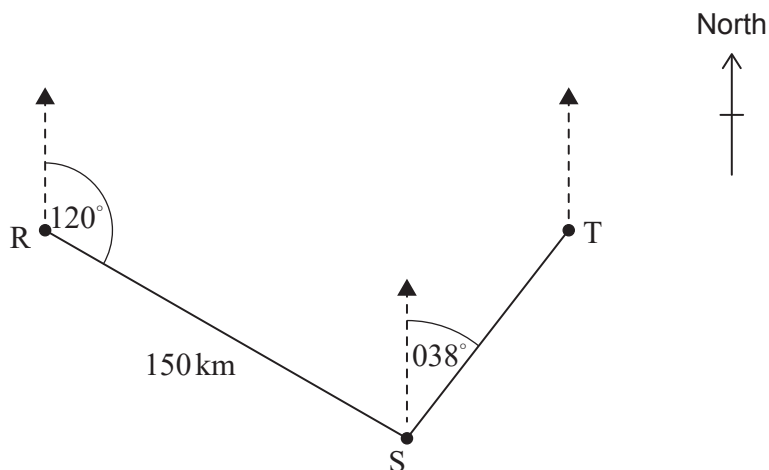
[illegible]

Turn over

5. [Maximum mark: 6]

Ron sails his boat from point R for a distance of 150 km, on a bearing of 120° , to arrive at point S. He then sails on a bearing of 038° to point T. Ron's journey is shown in the diagram.

diagram not to scale



- (a) Find $R\hat{S}T$. [2]

Point T is directly east of point R.

- (b) Calculate the distance that Ron sails to return directly from point T to point R. [4]

[illegible]

2. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).



[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 <https://creativecommons.org/licenses/by/2.5/#>. Image adapted.]

Viktor reads online that 1 REC is equal to 0.52 metres, rounded to two decimal places.

- (a) Write down the upper and lower bounds of 1 REC in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of 440 REC and a height of 280 REC. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

- (b) Find the minimum possible volume of the pyramid in cubic metres. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

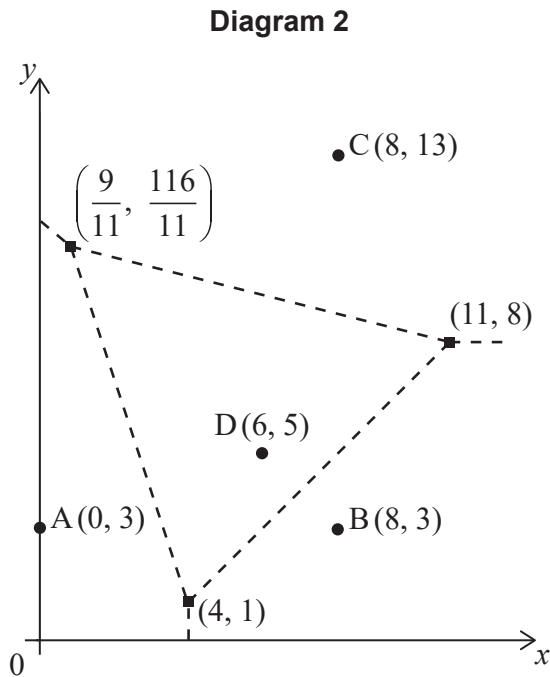
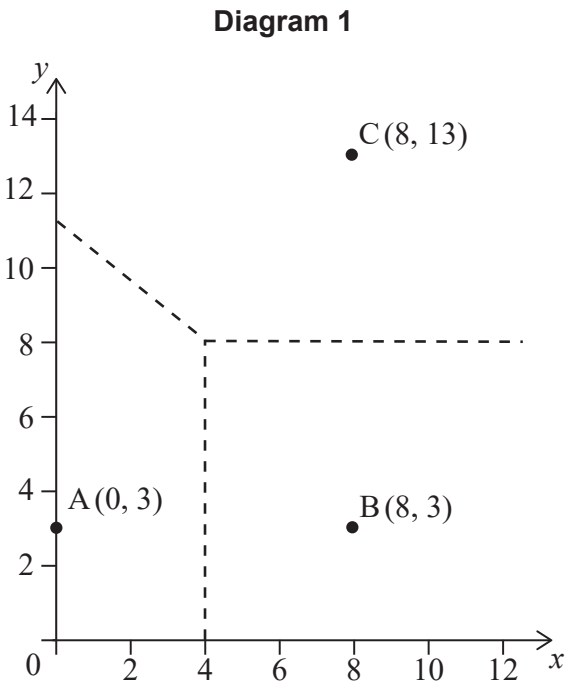
.....

.....



4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

- (a) Write down the coordinates of this point. [1]
- (b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

- (c) The wind turbine should be as far from the nearest farmhouses as possible.
 - (i) By calculating appropriate distances, find the location of the wind turbine.
 - (ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

(This question continues on the following page)

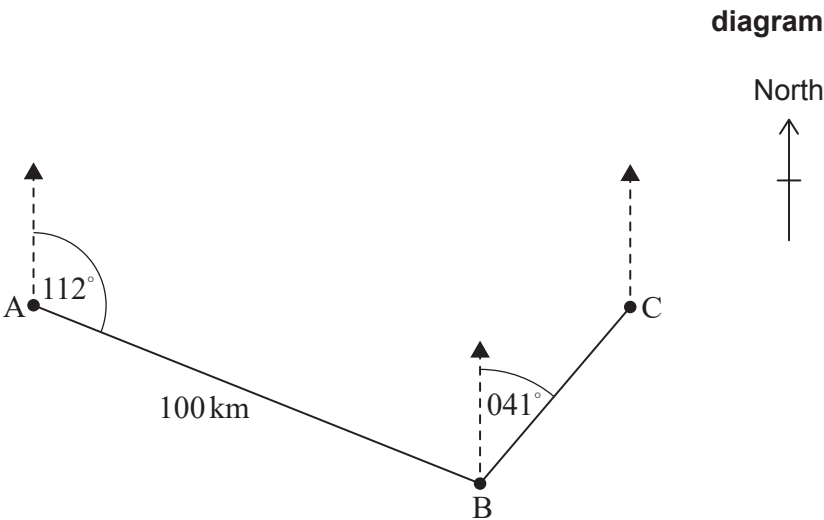


[illegible]

Turn over

5. [Maximum mark: 6]

Jason sails his boat from point A for a distance of 100 km, on a bearing of 112° , to arrive at point B. He then sails on a bearing of 041° to point C. Jason's journey is shown in the diagram.



- (a) Find $\hat{ABC}$. [2]

Point C is directly east of point A.

- (b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

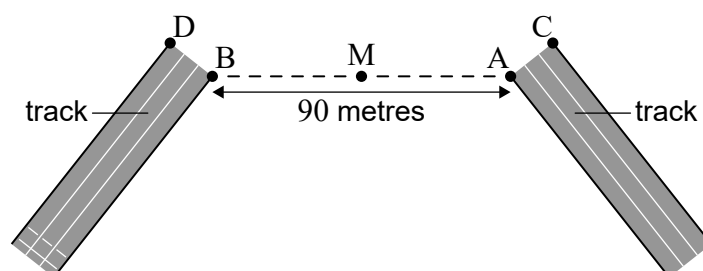


2. [Maximum mark: 15]

Ansel is designing a racing track for a local bicycle club. The following diagram shows an incomplete portion of the track.

Ansel wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 90 metres.

diagram not to scale



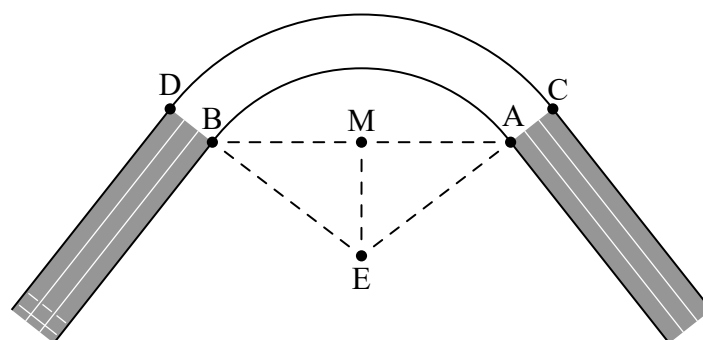
To create a smooth curve, Ansel first walks to M, the midpoint of [AB].

- (a) Write down the length of [BM].

[1]

Ansel then walks 32 metres in a direction perpendicular to [AB] to get from point M to point E. Point E is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



- (b) (i) Find the length of [BE].
 (ii) Find $\hat{BEM}$.
 (c) Hence, find the length of arc AB.

[4]

[3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre E, such that the track is 4 metres wide.

- (d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 15 cm deep.

- (e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

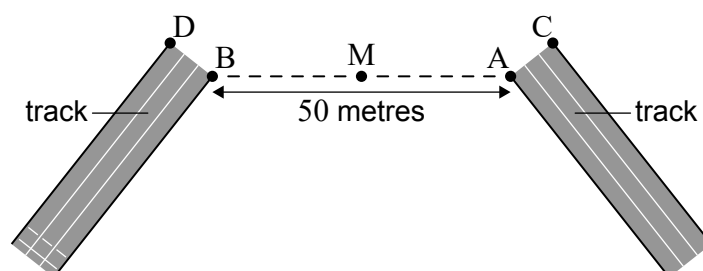
Turn over

2. [Maximum mark: 15]

Madhu is designing a jogging track for the campus of her school. The following diagram shows an incomplete portion of the track.

Madhu wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 50 metres.

diagram not to scale



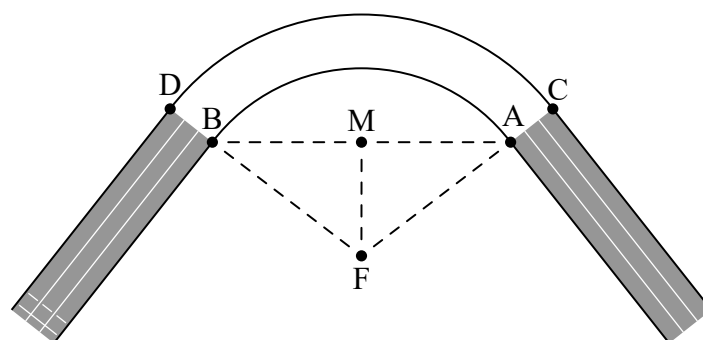
To create a smooth curve, Madhu first walks to M, the midpoint of [AB].

- (a) Write down the length of [BM].

[1]

Madhu then walks 20 metres in a direction perpendicular to [AB] to get from point M to point F. Point F is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



- (b) (i) Find the length of [BF].
 (ii) Find $\hat{BFM}$.
 (c) Hence, find the length of arc AB.

[4]

[3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre F, such that the track is 2 metres wide.

- (d) Calculate the area of the curved portion of the track, ABDC. [4]

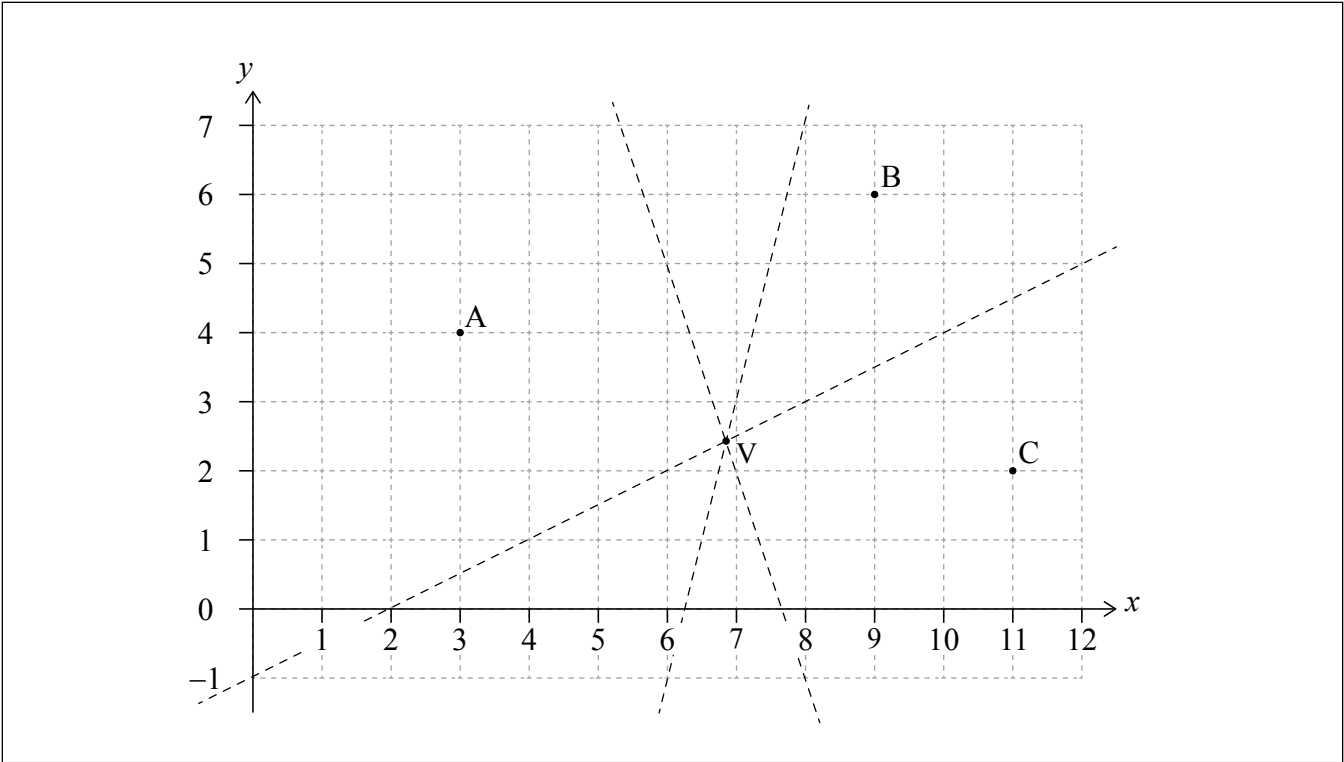
The base of the track will be made of concrete that is 12 cm deep.

- (e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

Turn over

6. [Maximum mark: 6]

Points A(3, 4), B(9, 6) and C(11, 2) are shown on the following diagram, along with the perpendicular bisectors of [AB], [AC] and [BC].



The perpendicular bisector of [BC] intercepts the axes at coordinates (0, −1) and (2, 0).

- (a) Write down the equation of the perpendicular bisector of [BC]. [2]

The equation of the perpendicular bisector of [AB] is $y = -3x + 23$.

- (b) Find the coordinates of point V where the perpendicular bisectors meet. Give your answer to four significant figures. [2]

A Voronoi diagram is constructed with points A, B and C as the three sites.

- (c) Draw, clearly, the edges of the Voronoi diagram on the given diagram. [2]

(This question continues on the following page)



(Question 6 continued)

[illegible]

Turn over

11. [Maximum mark: 6]

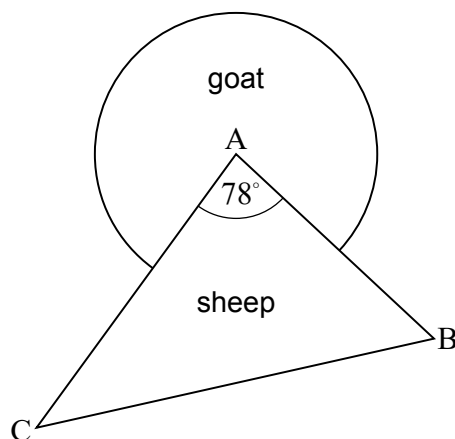
A sheep is in a field in the shape of a triangle, ABC.

AC = 21 metres, AB = 15 metres and $\hat{CAB} = 78^\circ$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

The fields are shown in the diagram.

diagram not to scale



Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field.

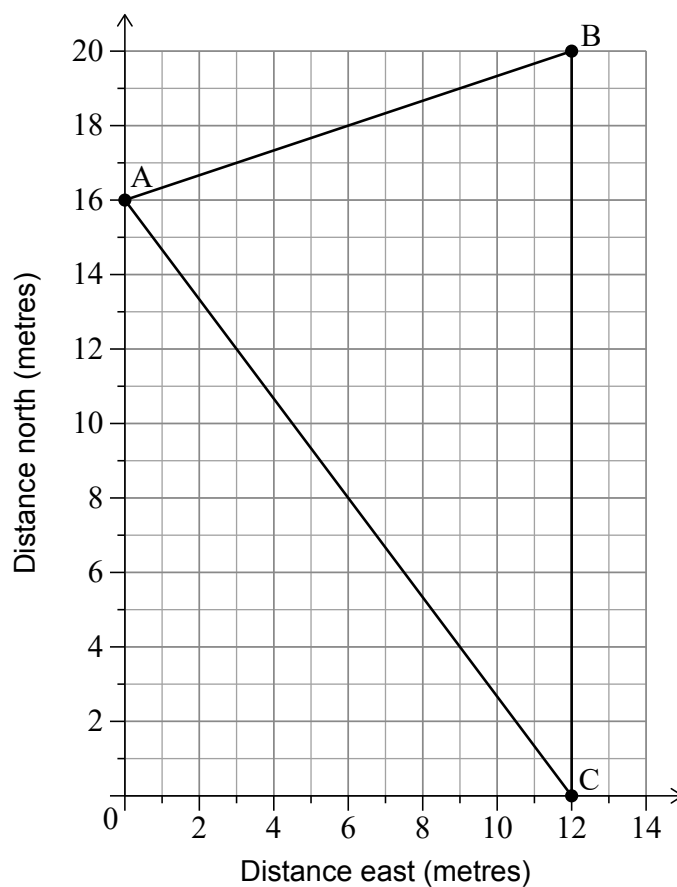
[illegible]

Turn over

2. [Maximum mark: 14]

Mai is at an amusement park. A map of part of the amusement park is represented on the following coordinate axes.

Mai's favourite three attractions are positioned at $A(0, 16)$, $B(12, 20)$ and $C(12, 0)$. All measurements are in metres.



(a) Write down the distance between B and C.

[1]

(b) Calculate the distance between A and B.

[2]

(This question continues on the following page)

(Question 2 continued)

Mai is standing at the attraction at B and wants to walk directly to the attraction at A.

- (c) Calculate the bearing of A from B. [3]

A drinking fountain is to be installed at a point that is an equal distance from each of the attractions at A, B and C.

- (d) (i) Write down the gradient of [AC].
(ii) Write down the mid-point of [AC].
(iii) Hence calculate the coordinates of the drinking fountain. [8]

Turn over



2227-1210

Blank page

5. [Maximum mark: 16]

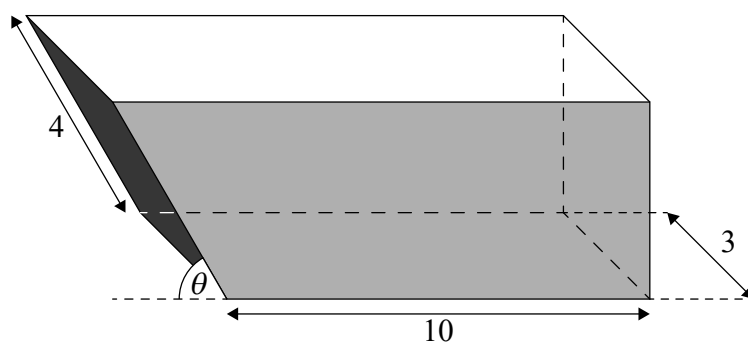
A skip is a container used to carry garbage away from a construction site. For safety reasons the garbage must not extend beyond the top of the skip. The maximum volume of garbage to be removed is therefore equal to the volume of the skip.



A particular design of skip can be modelled as a prism with a trapezoidal cross section. For the skip to be transported, it must have a rectangular base of length 10m and width 3m. The length of the sloping edge is fixed at 4m, and makes an angle of θ with the horizontal.

The following diagram shows such a skip.

diagram not to scale



(This question continues on the following page)

(Question 5 continued)

- (a) Find the volume of this skip,
- (i) if the length of the top edge of the skip is 11 m.
 - (ii) if the height of the skip is 3.2 m.
 - (iii) if θ is 60° . [9]
- (b) Show that the volume, $V \text{ m}^3$, of the skip is given by
- $$24 \sin(\theta)(5 + \cos(\theta)).$$
- [2]
- (c) Explain, in context, why $\theta \neq 0$. [1]
- (d) (i) Sketch the graph of $V = 24 \sin(\theta)(5 + \cos(\theta))$, $0 < \theta < 90^\circ$.
- (ii) Find the maximum volume of the skip and the value of θ for which this maximum volume occurs. [4]
-

Disclaimer:

Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

References:

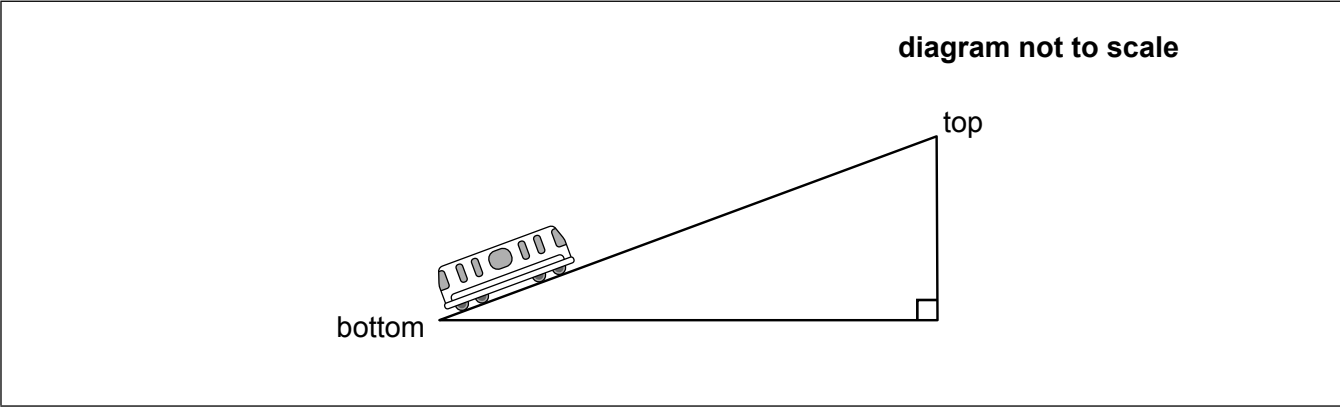
5. Andyqwe, n.d. *Dumpster truck* [image online] Available at: <https://www.gettyimages.co.uk/detail/photo/dumpster-truck-royalty-free-image/157611454> [Accessed 18 April 2023] Source adapted.

All other texts, graphics and illustrations © International Baccalaureate Organization 2024

1. [Maximum mark: 4]

One of the steepest train tracks in the world is in Tennessee, USA.

This track is 1.52 km long, and the angle of elevation from the bottom of the track to the top is 36.1° .



- (a) Label the diagram with the given values for the track length and the angle of elevation. [2]
- (b) Find the vertical change in height from the bottom of the track to the top. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

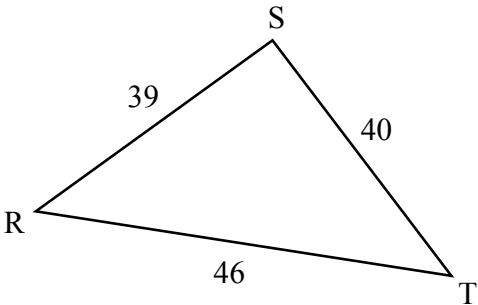
.....



4. [Maximum mark: 5]

Consider the following triangle, RST, such that $RS = 39\text{ cm}$, $ST = 40\text{ cm}$, and $TR = 46\text{ cm}$.

diagram not to scale



- (a) Find the value of $\hat{TRS}$.

[3]
- (b) Find the area of the triangle RST.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

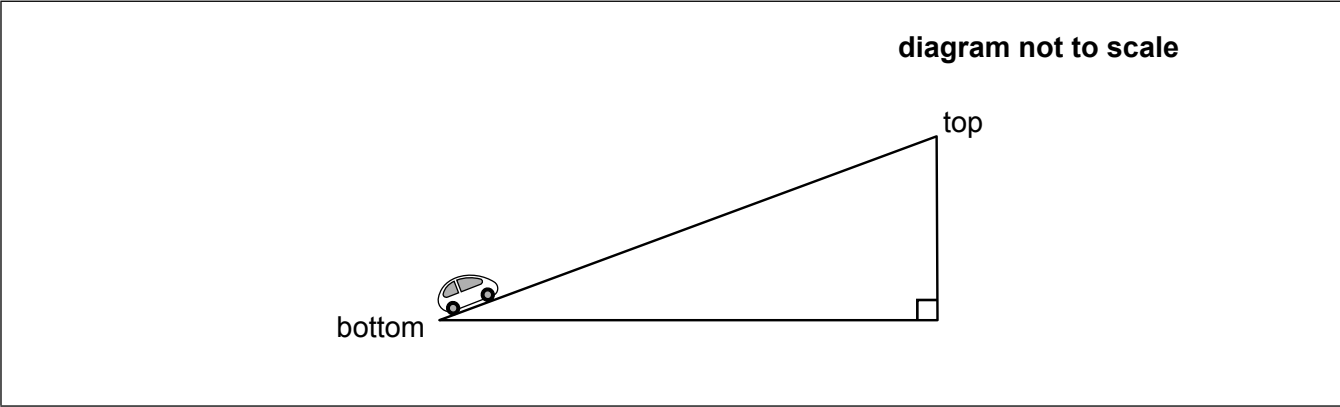


Turn over

1. [Maximum mark: 4]

One of the steepest straight roads in the world is in Dunedin, New Zealand.

This road is 161 m long, and the angle of elevation from the bottom of the road to the top is 16.3° .



- (a) Label the diagram with the given values for the road length and the angle of elevation. [2]
- (b) Find the vertical change in height from the bottom of the road to the top. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

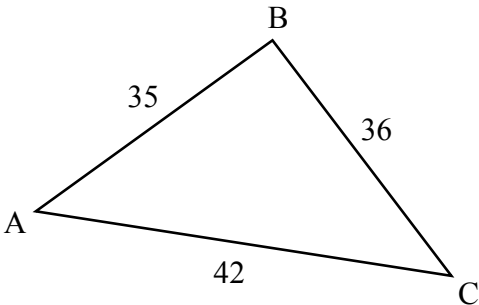
.....



4. [Maximum mark: 5]

Consider the following triangle, ABC , such that $AB = 35\text{ cm}$, $BC = 36\text{ cm}$, and $CA = 42\text{ cm}$.

diagram not to scale



- (a) Find the value of $\hat{CAB}$.

[3]
- (b) Find the area of the triangle ABC .

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

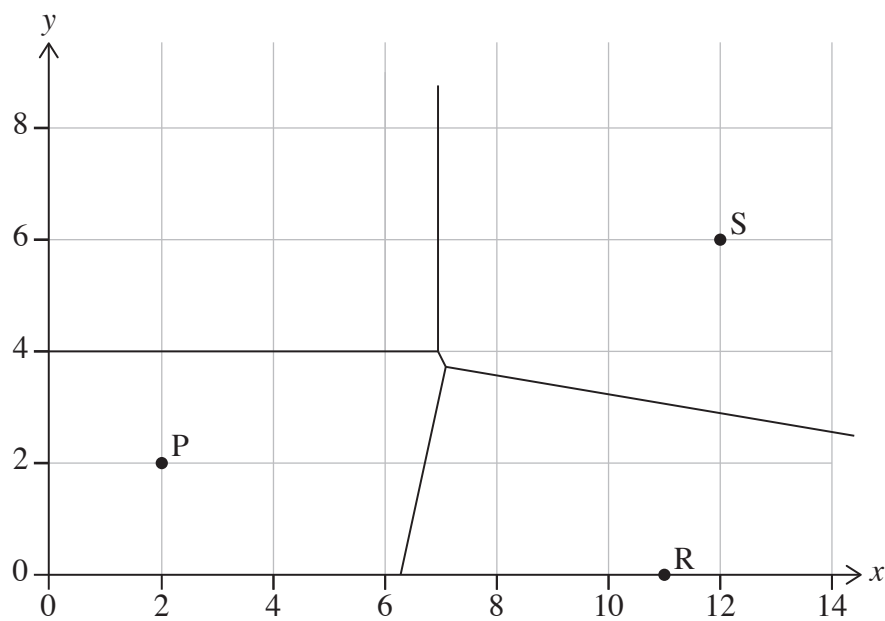
.....



Turn over

1. [Maximum mark: 13]

The following grid shows a restaurant’s floorplan. There are four food stations centred at points P, Q, R and S. The Voronoi diagram for these four points is shown. Point Q is not shown.



One unit represents 1 metre.

Point P is located at (2 , 2) .

The equation of the perpendicular bisector of [PQ] is $y = 4$.

(a) Write down the coordinates of Q . [1]

Points R and S are located at (11 , 0) , and (12 , 6) , respectively.

(b) Find

- (i) the coordinates of the midpoint of [RS]
- (ii) the equation of the perpendicular bisector of [RS] . Give your answer in the form $y = mx + c$. [6]

(This question continues on the following page)

(Question 1 continued)

Customers in the restaurant take their food from the nearest food station.

The following table shows the average waiting time, in minutes, for each food station.

Food station	P	Q	R	S
Average waiting time (minutes)	8	4	9	6

- (c) Using nearest-neighbour interpolation, find the average waiting time for a customer at point (6, 8). [1]

The restaurant owner wishes to determine whether customers spend more money during the weekend. She decides to use a two-sample t -test at a 5% level of significance.

For this test, the null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

where μ_1 is the mean amount of money spent by all customers on weekdays, and μ_2 is the mean amount of money spent by all customers on weekends.

- (d) State the alternative hypothesis. [1]

The owner considers collecting her sample data from 2 weekdays and 1 weekend by recording the amount spent on the first 12 orders on each of these days.

- (e) (i) State which one of the following methods best describes the owner’s planned sampling:

convenience, systematic, or stratified.

- (ii) State one disadvantage of this sampling technique. [2]

The restaurant owner instead collects the data using simple random sampling.

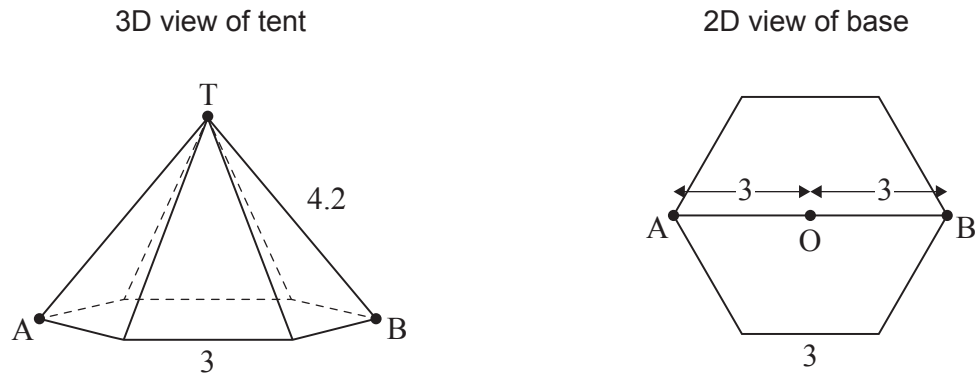
The p -value is 0.0897 for the sampled data.

- (f) State the conclusion for the test. Justify your answer. [2]

5. [Maximum mark: 17]

Jamali is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre O . The length of each side of the base is 3 m , the length of each sloping edge is 4.2 m , and the distance between each vertex on the base and O is 3 m , as shown in the diagrams.

diagrams not to scale

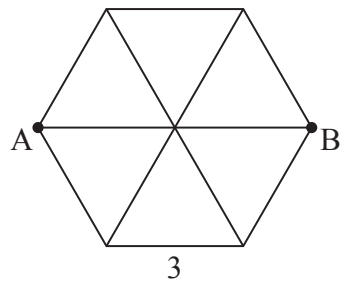


The top of the tent, T , will be supported by a vertical pole from O .

- (a) Find the length of the pole, OT . [2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



- (b) Find
- (i) the area of the base
 - (ii) the volume of the tent. [5]
- (c) Find the value of $\hat{OAT}$. [2]

(This question continues on the following page)

(Question 5 continued)

For extra support, Jamali decides to attach a rope, with length 3.5 m, to the tent at a point, P, on the edge AT.

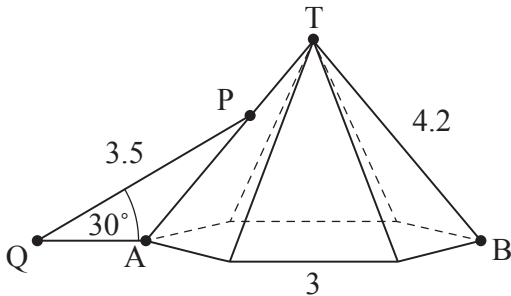
The rope will be fixed to the ground at point Q, such that:

- the rope, [PQ], is straight
- points Q, A and B lie on a straight line
- $\angle QPA = 30^\circ$.

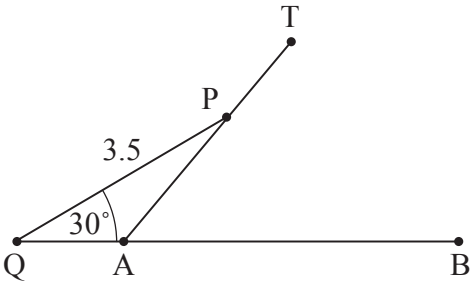
This is shown in the diagrams.

diagrams not to scale

3D view of tent and rope



2D view of triangle APQ



(d) Find AQ. [4]

For decoration at night, Jamali wants to fix a strip of lights from point A to a point, R, along the rope [PQ].

The strip of lights, [AR], is straight and has length 0.8 m.

(e) Find the two possible values of QR. [4]

Disclaimer:

Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

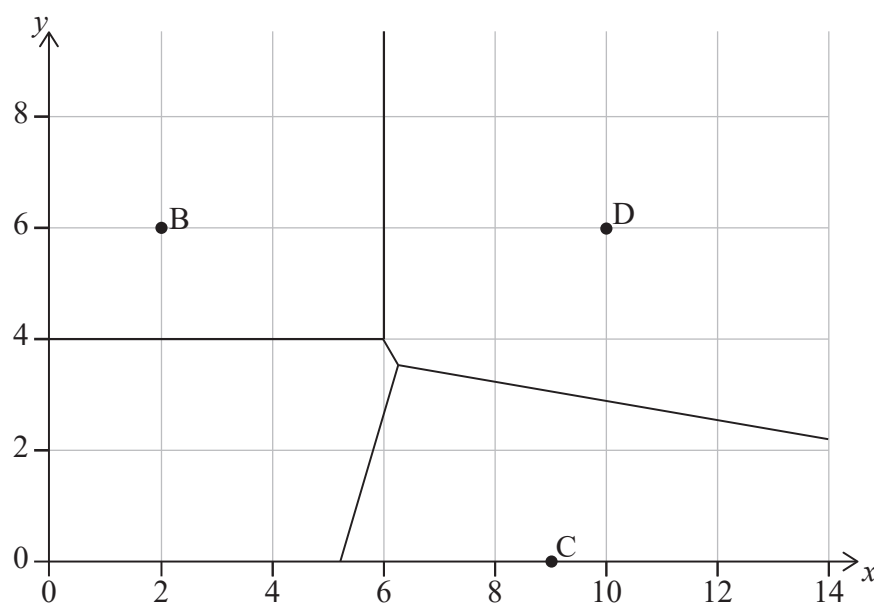
References:

3. InterNations. 2022. *Expatriate Insider 2022*. [online] Available at: <https://www.internations.org/expat-insider/2022/>. Source adapted.
- Heliwell, J. F., Huang, H., Wang, S. and Norton, M., 2022. Statistical Appendix for “Happiness, benevolence, and trust during COVID-19 and beyond,” Chapter 2 of *World Happiness Report 2022*. [pdf online] Available at: <https://worldhappiness.report/ed/2022/happiness-benevolence-and-trust-during-covid-19-and-beyond/> [Accessed 13 November 2023]. Source adapted.

All other texts, graphics and illustrations © International Baccalaureate Organization 2024

1. [Maximum mark: 13]

The following grid shows a restaurant's floorplan. There are four food stations centred at points A, B, C and D. The Voronoi diagram for these four points is shown. Point A is not shown.



One unit represents 1 metre.

Point B is located at $(2, 6)$.

The equation of the perpendicular bisector of $[AB]$ is $y = 4$.

(a) Write down the coordinates of A.

[1]

Points C and D are located at $(9, 0)$, and $(10, 6)$, respectively.

(b) Find

(i) the coordinates of the midpoint of $[CD]$

(ii) the equation of the perpendicular bisector of $[CD]$. Give your answer in the form $y = mx + c$.

[6]

(This question continues on the following page)

(Question 1 continued)

Customers in the restaurant take their food from the nearest food station.

The following table shows the average waiting time, in minutes, for each food station.

Food station	A	B	C	D
Average waiting time (minutes)	9	5	10	7

- (c) Using nearest-neighbour interpolation, find the average waiting time for a customer at point (4, 6). [1]

The restaurant owner wishes to determine whether customers spend more money during the weekend. She decides to use a two-sample t -test at a 5% level of significance.

For this test, the null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

where μ_1 is the mean amount of money spent by all customers on weekdays, and μ_2 is the mean amount of money spent by all customers on weekends.

- (d) State the alternative hypothesis. [1]

The owner considers collecting her sample data from 2 weekdays and 1 weekend by recording the amount spent on the first 12 orders on each of these days.

- (e) (i) State which one of the following methods best describes the owner's planned sampling:

convenience, systematic, or stratified.

(ii) State one disadvantage of this sampling technique. [2]

The restaurant owner instead collects the data using simple random sampling.

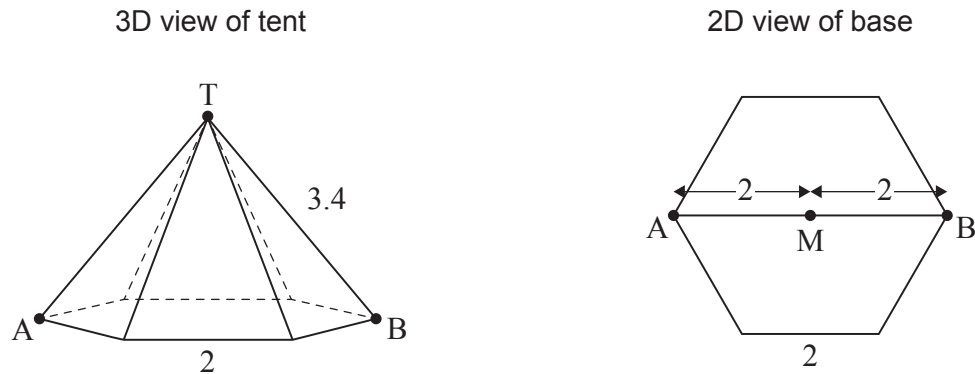
The p -value is 0.0897 for the sampled data.

- (f) State the conclusion for the test. Justify your answer. [2]

5. [Maximum mark: 17]

Gaurika is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre M . The length of each side of the base is 2 m , the length of each sloping edge is 3.4 m , and the distance between each vertex on the base and M is 2 m , as shown in the diagrams.

diagrams not to scale

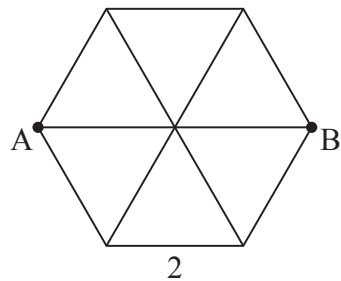


The top of the tent, T , will be supported by a vertical pole from M .

- (a) Find the length of the pole, MT .
- [2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



- (b) Find
- (i) the area of the base

(ii) the volume of the tent.
- [5]
- (c) Find the value of $\hat{MT}$.
- [2]

(This question continues on the following page)

(Question 5 continued)

For extra support, Gaurika decides to attach a rope, with length 2.6 m, to the tent at a point, X, on the edge AT.

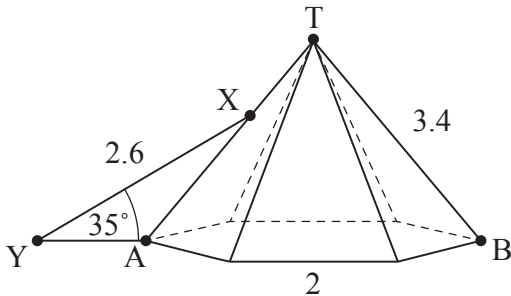
The rope will be fixed to the ground at point Y, such that:

- the rope, [XY], is straight
- points Y, A and B lie on a straight line
- $\widehat{AYX} = 35^\circ$.

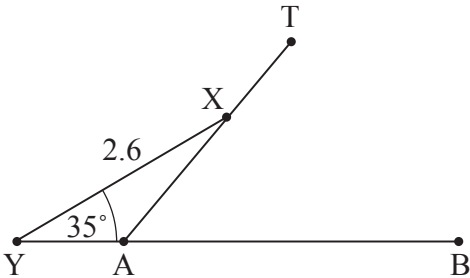
This is shown in the diagrams.

diagrams not to scale

3D view of tent and rope



2D view of triangle AXY



(d) Find AY. [4]

For decoration at night, Gaurika wants to fix a strip of lights from point A to a point, Z, along the rope [XY].

The strip of lights, [AZ], is straight and has length 0.9 m.

(e) Find the two possible values of YZ. [4]

Disclaimer:

Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

References:

3. InterNations. 2022. *Expatriate Insider 2022*. [online] Available at: <https://www.internations.org/expat-insider/2022/>. Source adapted.
- Heliwell, J. F., Huang, H., Wang, S. and Norton, M., 2022. Statistical Appendix for “Happiness, benevolence, and trust during COVID-19 and beyond,” Chapter 2 of *World Happiness Report 2022*. [pdf online] Available at: <https://worldhappiness.report/ed/2022/happiness-benevolence-and-trust-during-covid-19-and-beyond/> [Accessed 13 November 2023]. Source adapted.

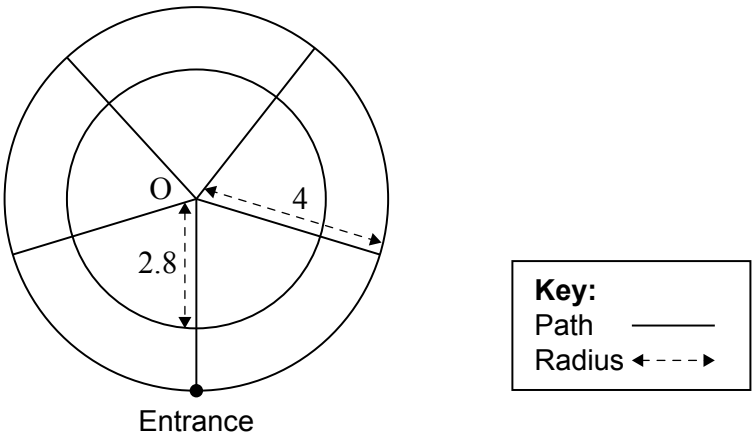
All other texts, graphics and illustrations © International Baccalaureate Organization 2024

6. [Maximum mark: 6]

A child's game is played by making paths in the snow. First, two circular paths are made using the same centre, O . The radius of the smaller circle is 2.8 m , and the radius of the larger circle is 4 m . Additional paths are then made from O to the outer edge of the larger circle, dividing each circle into 5 equal sectors, as shown in the following diagram.

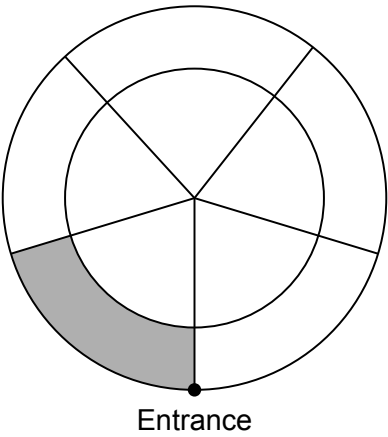
For your calculations, ignore the widths of the paths.

diagram not to scale



Each region **between** paths will be coloured with environmentally friendly dye. The shaded region below will be coloured orange.

diagram not to scale



Maureen has enough orange dye to cover an area of 6 m^2 .

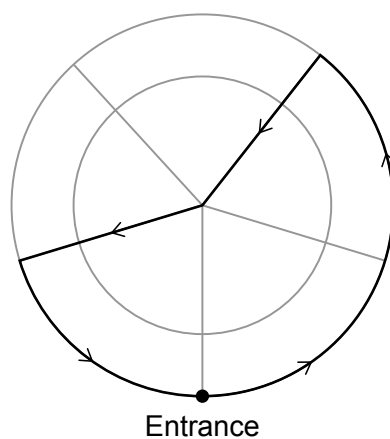
(a) Show that Maureen has enough orange dye to cover the shaded region. [3]

(This question continues on the following page)



During the game, the players start at the entrance and must travel only along the paths made in the snow. Maureen travels from the entrance along the path shown in the following diagram.

diagram not to scale



- (b) Calculate the distance Maureen travels along this path, starting from the entrance and returning to the entrance.

[3]

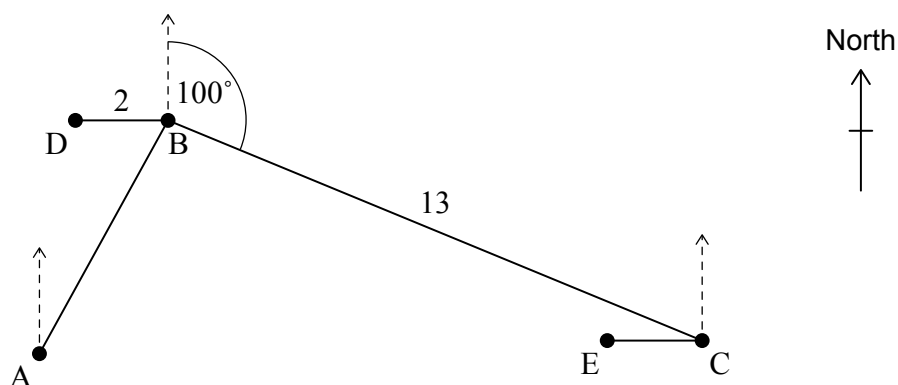
[illegible]

Turn over

13. [Maximum mark: 9]

Ticky is sailing a boat in the ocean. For the first part of her trip, she plans to start at point A and travel to point B. She then plans to turn and travel 13 km on a bearing of 100° to point C as shown in the following diagram.

diagram not to scale



Due to navigational error in the first part of the trip, Ticky arrives at point D instead of point B. Point D is 2 km due west of point B.

(a) Determine

(i) the size of $\hat{DBC}$.

(ii) the distance from point D to point C.

(iii) the bearing Ticky must use to travel directly from point D to point C.

[7]

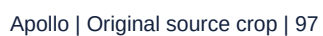
Due to navigational error in the second part of the trip, Ticky arrives at point E instead of point C. Point E is 2 km due west of point C.

(b) Write down the distance between point D and point E. Justify your answer.

[2]

(This question continues on the following page)



[illegible]

Disclaimer:

Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

References:

3. ONYXprj, 2017. *Realistic cupcakes and cookies. Biscuits muffins packaging, creamy and chocolate bakery products in white box vector illustration* [image online] Available at: <https://www.gettyimages.co.uk/detail/illustration/realistic-cupcakes-and-cookies-biscuits-royalty-free-illustration/1257757782> [Accessed 19 April 2024]
Source adapted.



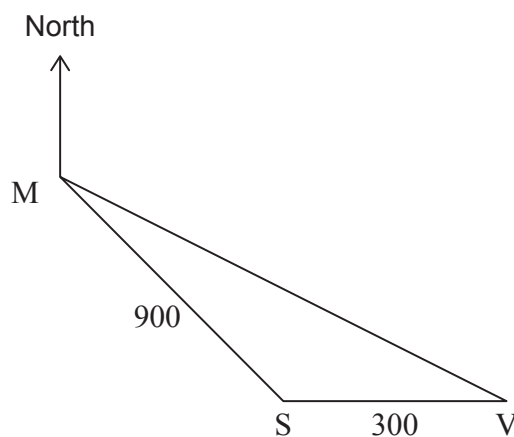
5. [Maximum mark: 8]

A ship sails 900 km on a bearing of 135° (south-east) from Lomé, M, to São Tomé, S, where she unloads her cargo.

The ship then sails 300 km due east from São Tomé to Libreville, V. From Libreville she then returns directly to Lomé.

These journeys are shown in the diagram.

diagram not to scale



- | | | |
|-----|---|-----|
| (a) | Find the size of $\angle VSM$. | [1] |
| (b) | Calculate MV. | [3] |
| (c) | Calculate the bearing of Lomé, M, from Libreville, V. | [4] |

(This question continues on the following page)



(Question 5 continued)

[illegible]

Turn over

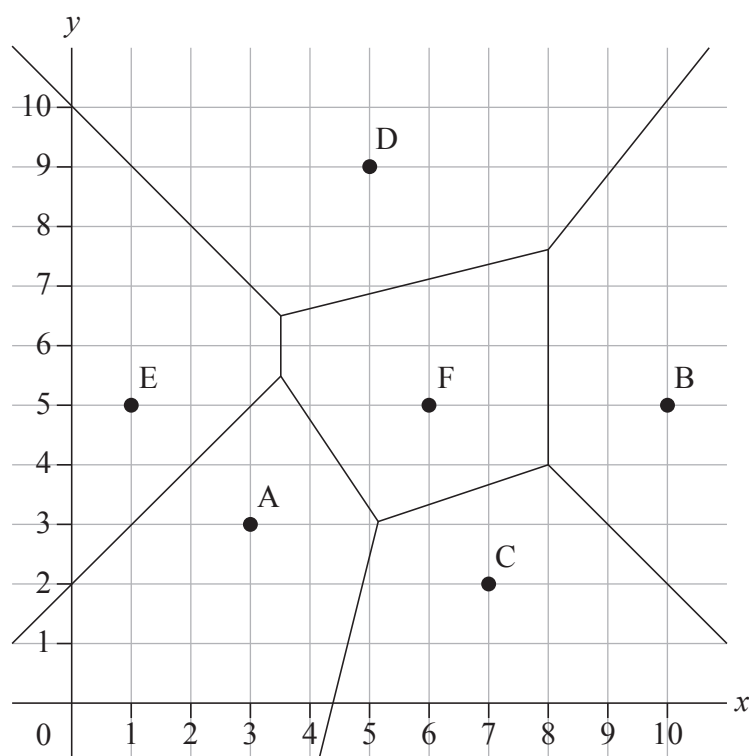
Please **do not** write on this page.

Answers written on this page
will not be marked.



7. [Maximum mark: 9]

Consider the Voronoi diagram which shows the sites $A(3, 3)$, $B(10, 5)$, $C(7, 2)$, $D(5, 9)$, $E(1, 5)$ and $F(6, 5)$. The diagram also shows the cells formed by each site and their boundaries.



- (a) Find the equation of the boundary between sites D and E. [2]

Vertex X is equidistant from sites B, C and F.

- (b) (i) Write down the coordinates of X.
- (ii) The exact value of BX is $\sqrt{n}$. Write down the value of n . [2]

Vertex Y(a , b) is equidistant from sites B, D and F.

- (c) (i) Write down the value of a .
- (ii) Find the exact value of b . [5]

(This question continues on the following page)



(Question 7 continued)

[illegible]

Turn over

11. [Maximum mark: 9]

The point A has coordinates $(1, 2, 1)$ and the point B has coordinates $(3, 5, 2)$.

(a) Find AB. [2]

Triangle ABC is right-angled with its right angle at B. The point C has coordinates $(2, 8, k)$.

(b) Find the value of k . [4]

(c) Calculate the size of $\hat{BAC}$. [3]

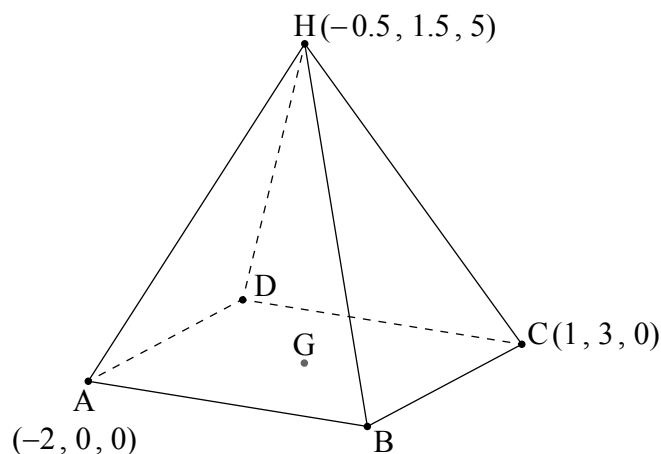


1. [Maximum mark: 5]

A metal structure on a flat surface is in the form of a right-pyramid with rectangular base $ABCD$ and vertex $H(-0.5, 1.5, 5)$. Point A has coordinates $(-2, 0, 0)$ and point C has coordinates $(1, 3, 0)$. This is shown in the following diagram.

All units are in centimetres.

diagram not to scale



The centre of the base, G , is the midpoint of AC .

- | | |
|---|-----|
| (a) Find the coordinates of G . | [2] |
| (b) Write down the vertical height HG . | [1] |
| (c) Find the distance between C and H . | [2] |

(This question continues on the following page)



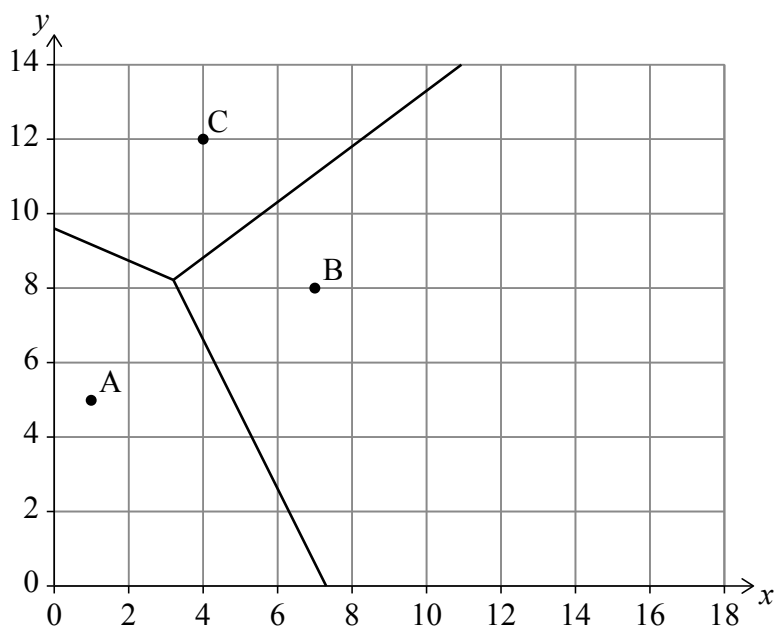
(Question 1 continued)

[illegible]

Turn over

5. [Maximum mark: 9]

A telecommunications company has identical cell towers in a rural area. They are located at the points $A(1, 5)$, $B(7, 8)$ and $C(4, 12)$. The coverage areas are divided as shown in the Voronoi diagram. All distances are in kilometres.



- (a) Find the equation of the perpendicular bisector of $[AB]$.

[4]

(This question continues on the following page)



Turn over

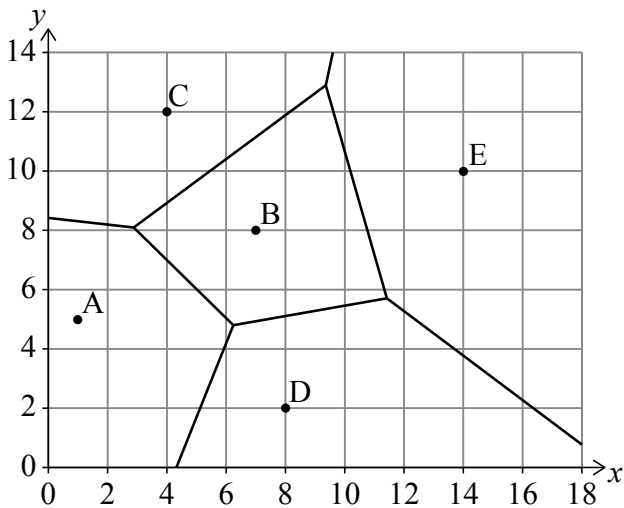
(Question 5 continued)

The company is planning to improve the coverage of its cellular network in the area by adding two new towers. It identifies potential locations at the points $D(8, 2)$ and $E(14, 10)$.

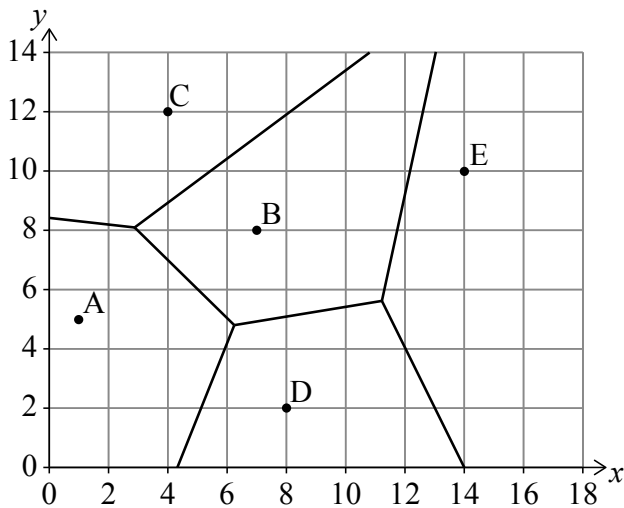
The company reviews the coverage areas and draws a new Voronoi diagram.

(b) Identify the correct Voronoi diagram from the options shown in the following diagrams. [2]

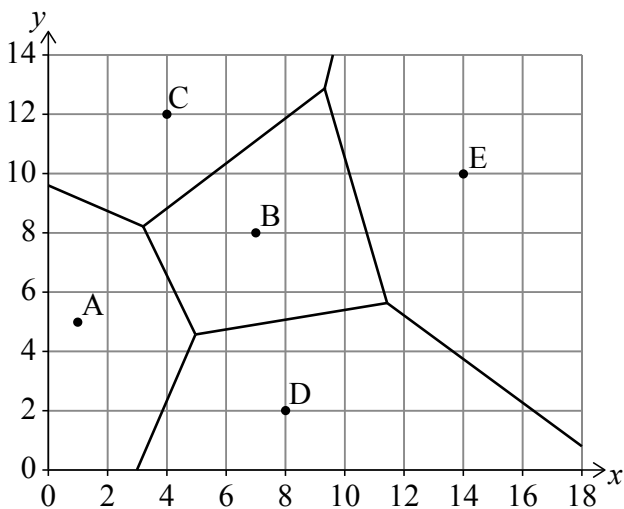
Option 1



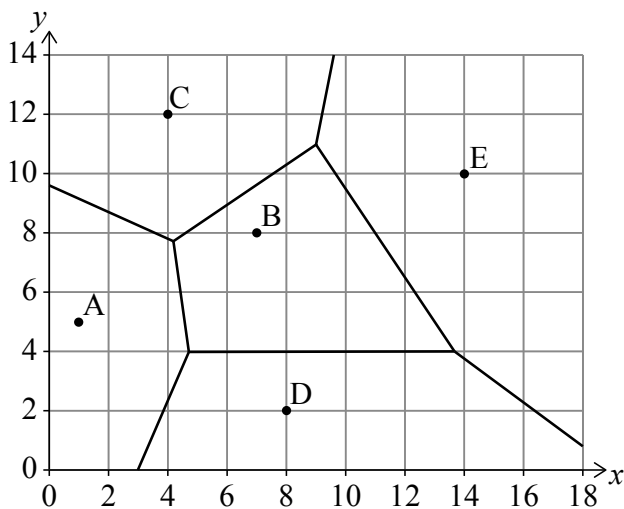
Option 2



Option 3



Option 4



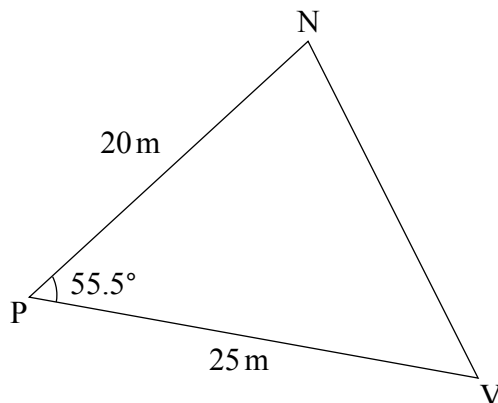
(This question continues on the following page)



6. [Maximum mark: 9]

Three points N, P, and V are shown on the following diagram. NP is 20 metres, PV is 25 metres and $\angle VP\hat{N}$ is 55.5° .

diagram not to scale



- (a) Find NV. [3]
- (b) Find $\angle P\hat{N}V$. [3]
- (c) Hence or otherwise, find the shortest distance between P and [NV]. [3]

(This question continues on the following page)



(Question 6 continued)

This image shows a full page of a document template. It consists of a series of evenly spaced, horizontal dotted lines extending across the width of the page. These lines are intended to guide handwriting or typing. There are no margins, headers, footers, or other markings present on the page.

Turn over

— IT —

2225-1319

Please **do not** write on this page.
Answers written on this page
will not be marked.

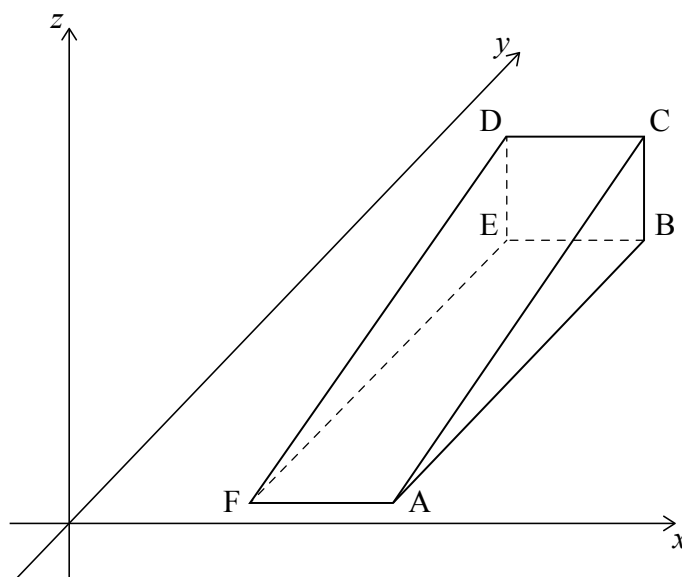


4. [Maximum mark: 14]

AirFlow Industries is designing a doorstop in the shape of a triangular prism, $ABCDEF$.

The points $A(13, 1, 0)$, $B(13, 25, 0)$ and $C(13, 25, 7)$ are the coordinates of the vertices of one of the triangular faces of the prism. Point $D(4, 25, 7)$ is another vertex as shown in the following diagram. All the measurements are in centimetres.

diagram not to scale



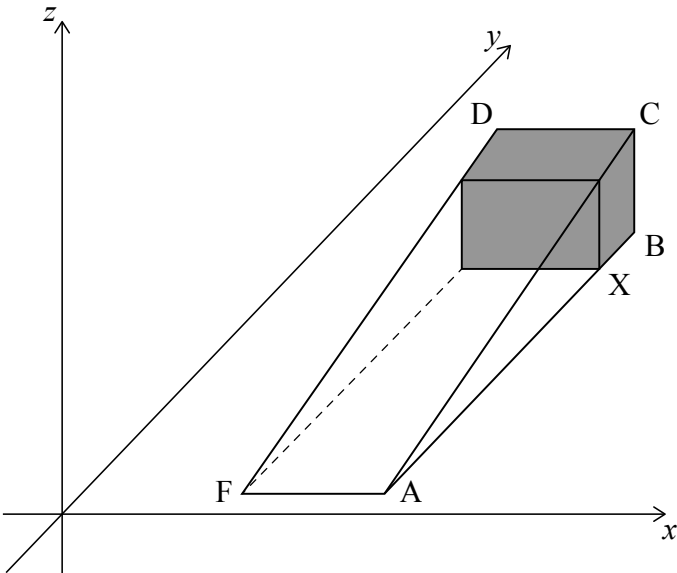
- (a) Write down the coordinates of F . [2]
- (b) Find AC . [2]
- (c) Show that triangle ABC is a right-angled triangle. [2]
- (d) Find the volume of material needed to make the doorstop. [3]

(This question continues on the following page)

(Question 4 continued)

To lower the cost of the doorstop, AirFlow decides to reduce its volume to 625 cm^3 by removing the shaded section shown in the following diagram. The remaining triangular faces are still right-angled triangles.

diagram not to scale



The length of the smaller doorstop, AX , can be calculated by

$$AX = AB \times \sqrt{\frac{V_s}{V_o}}$$

where V_s is the volume of the smaller doorstop, and V_o is the volume of the original doorstop, both in cubic centimetres.

- (e) Find AX . [2]

The material needed to make the doorstop costs 0.025 US dollars (USD) per cubic centimetre. The production of each doorstop requires 10% more material than its final volume, due to wastage in the production process.

To make a profit, the company will sell the doorstop for 20% more than the cost of the material.

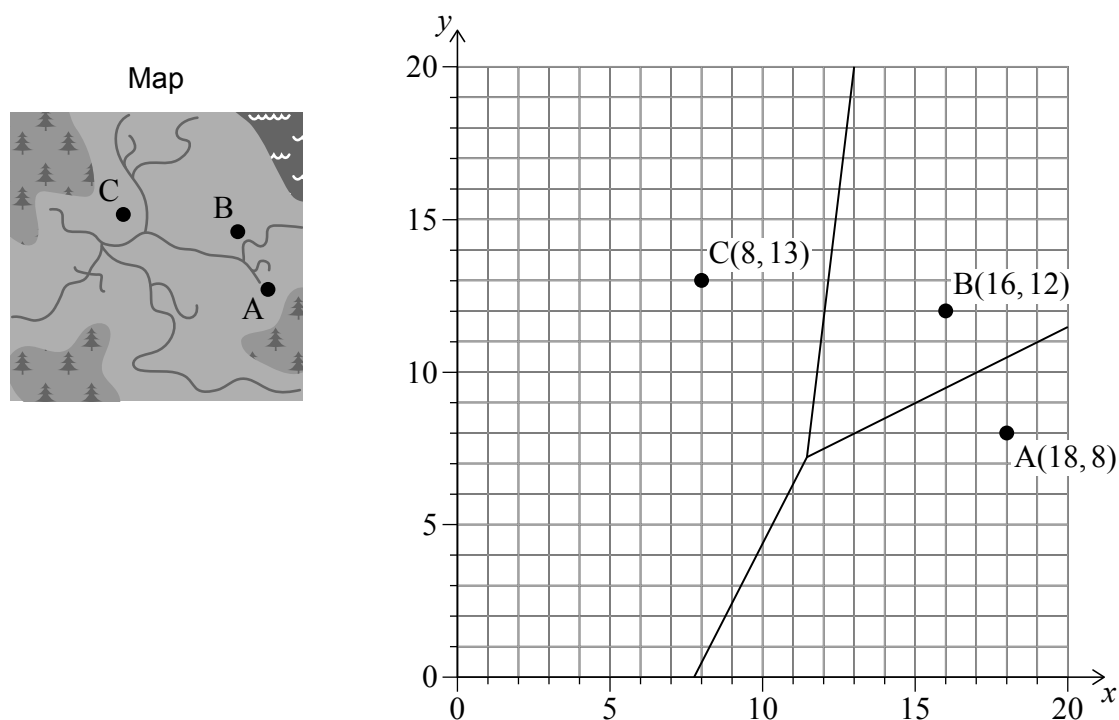
- (f) Determine the price, to two decimal places, the company will charge for the **new** doorstop. [3]

Turn over

5. [Maximum mark: 19]

The locations of three fire stations within a 50 by 50 kilometre rural area of Japan are shown on the following map.

A Voronoi diagram can be used to determine the part of the rural area for which each fire station is responsible. The Voronoi diagram for the three fire stations is shown on the coordinate grid beside the map, where 1 unit represents 2.5 km.



- (a) (i) Find the midpoint of $[BC]$.
- (ii) Find the gradient of the perpendicular bisector of $[BC]$.
- (iii) Hence find the equation of the edge between sites B and C. [6]
- (b) (i) Identify the fire station that is expected to respond, based on the Voronoi diagram, if a fire is reported at a location with coordinates $(14, 10)$. Justify your response.
- (ii) Suggest a reason why a different fire station might respond to this fire. [2]

(This question continues on the following page)

(Question 5 continued)

A fire is reported at a location D with coordinates $\left(11\frac{1}{3}, 7\frac{1}{6}\right)$. The distance of D from A on the Voronoi diagram, to six significant figures, is 6.71855 units (16.7964 km on the map).

- (c) (i) Show that the distance of D from B, to six significant figures, is also 6.71855 units.
- (ii) Show that any of the three fire stations would be expected to respond to the fire reported at D. [4]

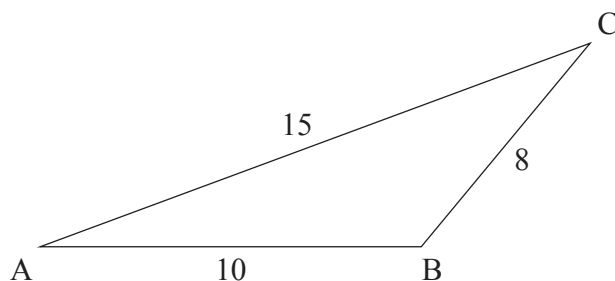
The equation of the edge between fire stations A and B is $y = 0.5x + 1.5$. The edge between fire stations A and C has an x -intercept of 7.75.

- (d) (i) Determine the area, to the nearest square unit, of the region within the given Voronoi diagram for which fire station A is responsible.
- (ii) Hence, calculate the actual area, in square kilometres, of this region. [7]
-

6. [Maximum mark: 8]

Triangle ABC has sides of length $AB = 10$ cm, $BC = 8$ cm and $AC = 15$ cm, as shown in the diagram.

diagram not to scale

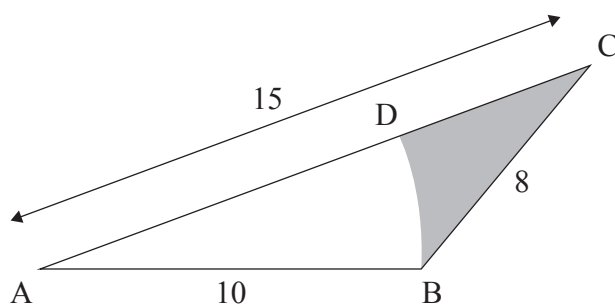


(a) Find the size of $\hat{BAC}$.

[3]

The circular arc BD, with centre A, is drawn inside the triangle, such that D lies on AC.

diagram not to scale



(b) Find the area of the shaded region.

[5]

(This question continues on the following page)



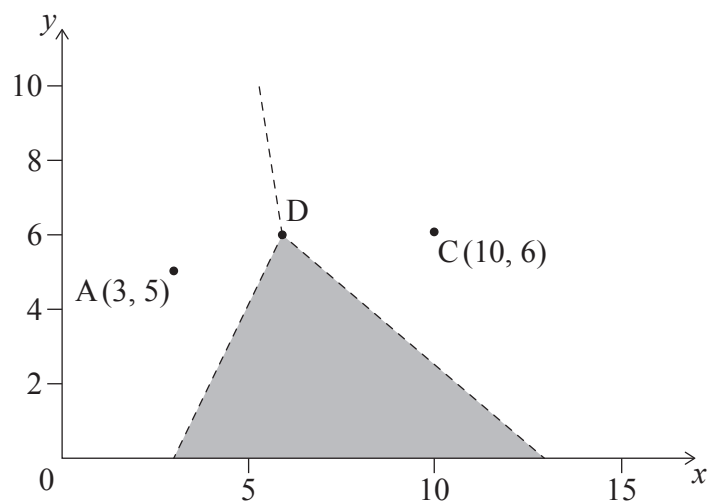
(Question 6 continued)

[illegible]

Turn over

12. [Maximum mark: 7]

The city of Benoulli has three police stations, located at $A(3, 5)$, $C(10, 6)$ and B . In the following Voronoi diagram, A and C represent sites. The dashed lines represent the edges and the cell for site B is shaded.



(a) State the geometrical significance of

(i) the shaded region;

(ii) point D .

[2]

The equation of the edge between A and B is $y = 2x - 6$. The equation of the edge between C and B is $y = -x + 13$.

(b) Calculate the coordinates of B .

[5]

(This question continues on the following page)



(Question 12 continued)

[illegible]

Disclaimer:

Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB. Any trademarks™ or registered® trademarks included are used for illustrative purposes only, and use does not imply any affiliation with or endorsement by the International Baccalaureate.

References:

2. Pepifoto, 2021. *Digital caliper measuring screw*. [image online] Available at: <https://www.gettyimages.co.uk/detail/photo/digital-caliper-measuring-screw-royalty-free-image/1302481493?phrase=digital+caliper+hand&adppopup=true> Images [Accessed 20 November 2024]. Source adapted.
- Lesikvit, 2024. *White round sphere or 3d ball*. [image online] Available at: <https://www.gettyimages.co.uk/detail/illustration/white-round-sphere-or-3d-ball-royalty-free-illustration/2157157426?phrase=grey+sphere&adppopup=true> Images [Accessed 20 November 2024]. Source adapted.



Please **do not** write on this page.

Answers written on this page
will not be marked.



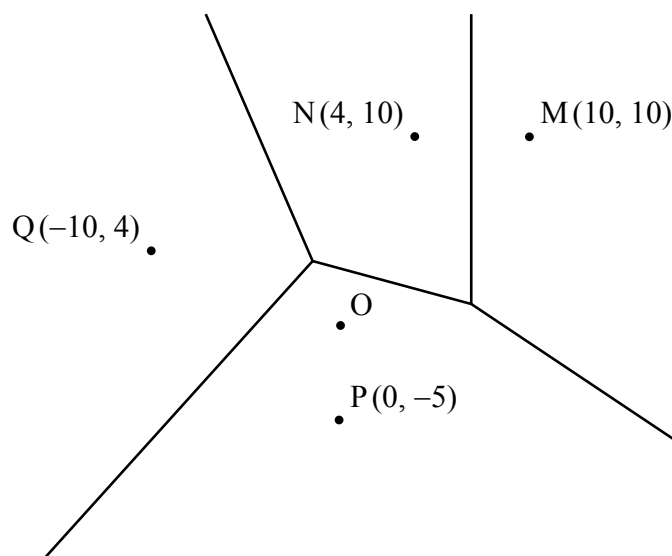
Please **do not** write on this page.

Answers written on this page
will not be marked.



5. [Maximum mark: 9]

The following Voronoi diagram represents a map of a town. It shows the location of four bakeries, at points $M(10, 10)$, $N(4, 10)$, $P(0, -5)$ and $Q(-10, 4)$. The town centre is at location O with coordinates $(0, 0)$.



Olga is at the town centre and wants to go to the nearest bakery.

- (a) Write down the nearest bakery to Olga. [1]

Before going to the nearest bakery, Olga checks on her phone and finds that it is closed.

- (b) Find which bakery or bakeries Olga should go to instead. Justify your answer. [3]
- (c) Write down the equation of the line containing all the points that are of equal distance from bakery M and bakery N . [1]
- (d) Find the equation of the perpendicular bisector of $[MP]$. [4]

(This question continues on the following page)



(Question 5 continued)

[illegible]

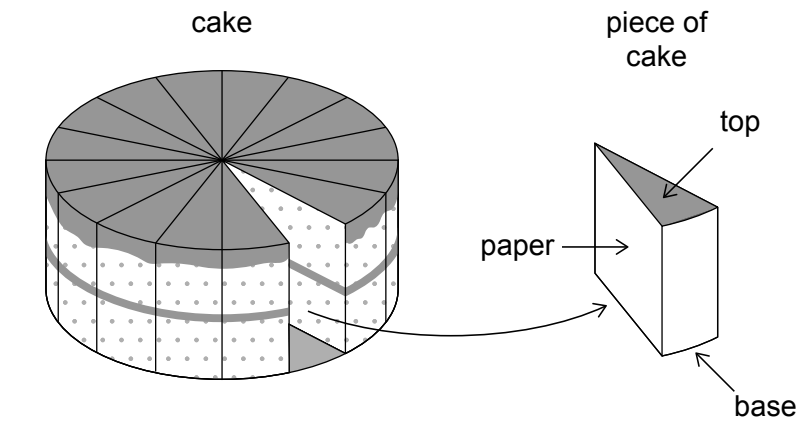
Turn over

Please **do not** write on this page.
Answers written on this page will not be marked.



11. [Maximum mark: 6]

A cylindrical-shaped cake has diameter 22 cm and height 10 cm . Coco cuts the cake into 16 pieces of equal size, as shown in the diagram.



- (a) Find the volume of one piece of cake. [3]

Paper is wrapped around all three **vertical** sides of a piece of cake.

- (b) Find the area of the paper used to wrap around one piece of cake, given that the paper fits exactly. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

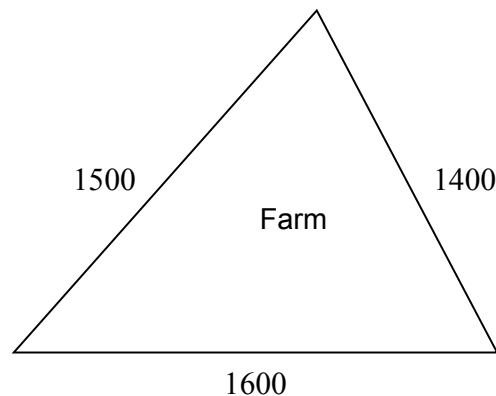
.....

.....



3. [Maximum mark: 15]

Inola has a triangular-shaped farm with boundaries of length 1600 m, 1500 m and 1400 m.



- (a) Find the area of the farm. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[6]

Inola's farm has a barn and a farmhouse.

Inola is constructing a 2 m-wide straight path on the level ground between the barn and the farmhouse. The barn is located 300 m east and 200 m north of the farmhouse.

- (b) Find the distance from the barn to the farmhouse.

[2]

The path costs \$ 120 per square metre.

- (c) Find the cost, c , of the path, to the nearest dollar.

[2]

Inola takes out a loan to pay for supplies to make the path. Inola borrows c dollars for five years, compounded monthly. To repay the loan, she makes a payment of \$ 1750 at the end of each month.

- (d) (i) Find the total amount of interest paid.
(ii) Find the nominal annual interest rate of the loan.

[5]