

APOLLO

Functions

SL

Mathematics: analysis and approaches

First assessment 2021

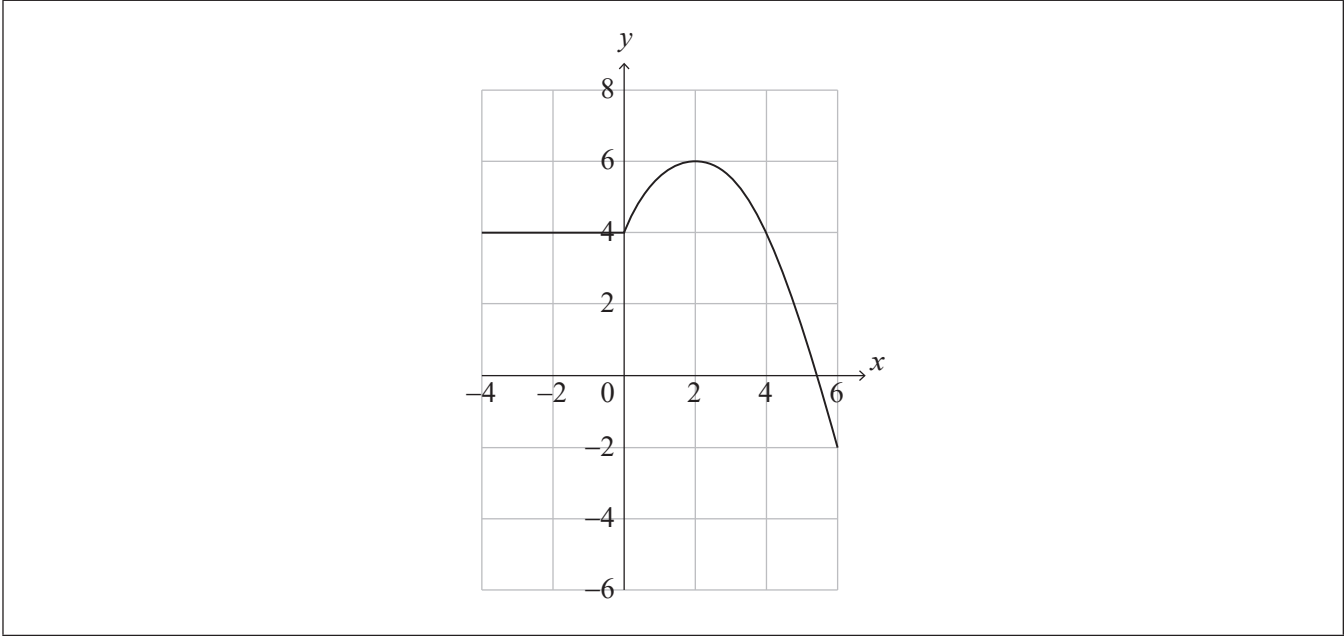
58 questions | questions

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$. [2]

(b) Let $g(x) = \frac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g . [3]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

(a) Show that $m = 4$. [6]

The function f can be expressed in the form $f(x) = 4(x - p)(x - q)$, where $p, q \in \mathbb{R}$.

(b) Find the value of p and the value of q . [2]

The function f can also be expressed in the form $f(x) = 4(x - h)^2 + k$, where $h, k \in \mathbb{R}$.

(c) Find the value of h and the value of k . [3]

(d) Hence find the values of x where the graph of f is both negative and increasing. [3]

5. [Maximum mark: 6]

(a) Find the range of f . [2]

(b) Given that $(g \circ f)(x) \leq 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c . [4]

[illegible]

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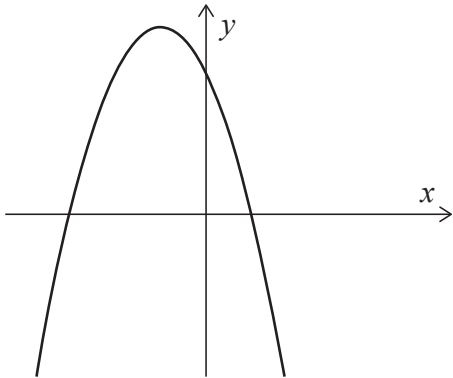
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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = -2(x - 1)(x + 3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f .



- (a) For the graph of f
 - (i) find the x -coordinates of the x -intercepts;
 - (ii) find the coordinates of the vertex. [5]

The function f can be written in the form $f(x) = -2(x - h)^2 + k$.

- (b) Write down the value of h and the value of k . [2]



3. [Maximum mark: 5]

(a) Write down the equation of

- (b) Find the coordinates where the graph of f crosses

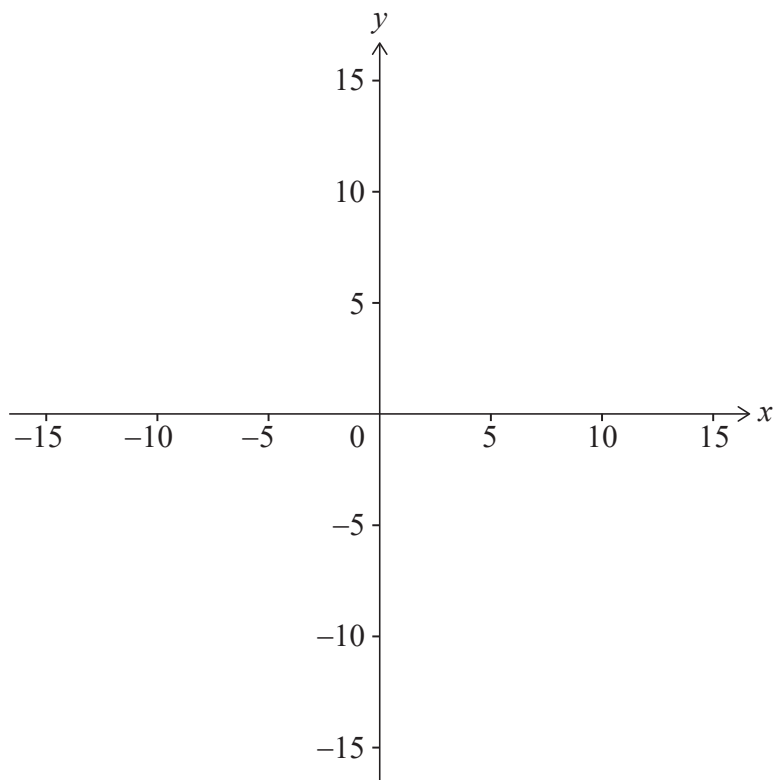
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(Question 3 continued)

(c) Sketch the graph of f on the axes below.

[1]



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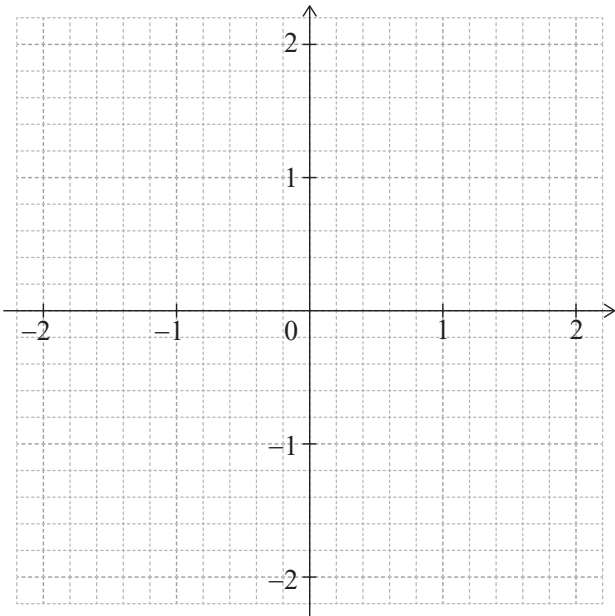
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \leq x \leq 2$.

- (a) Find the values of x for which $f(x) = 0$. [2]
- (b) Sketch the graph of f on the following grid. [3]



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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 5]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x+5$.

- (a) Show that $(g \circ f)(x) = 2x + 11$. [2]
- (b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a . [3]

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the points $A(-2, 20)$, $B(4, 6)$ and $C(-14, 12)$. The line L passes through the point A and is perpendicular to $[BC]$.

(a) Find the equation of L . [3]

The line L passes through the point $(k, 2)$.

(b) Find the value of k . [2]

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6. [Maximum mark: 7]

(a) Describe these two transformations. [2]

(b) Given that $g(x) \geq 7$, find the smallest value of r . [5]

[illegible]

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1. [Maximum mark: 5]

Both f and g are one-to-one functions.

x	-2	0	3	4
$f(x)$	8	4	0	-3
$g(x)$	-5	-2	4	0

- Find $g(0)$. [1]
- Find $(f \circ g)(0)$. [2]
- Find the value of x such that $f(x) = 0$. [2]

[illegible]

Turn over

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R}$, $x \neq -1$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(i) the vertical asymptote;

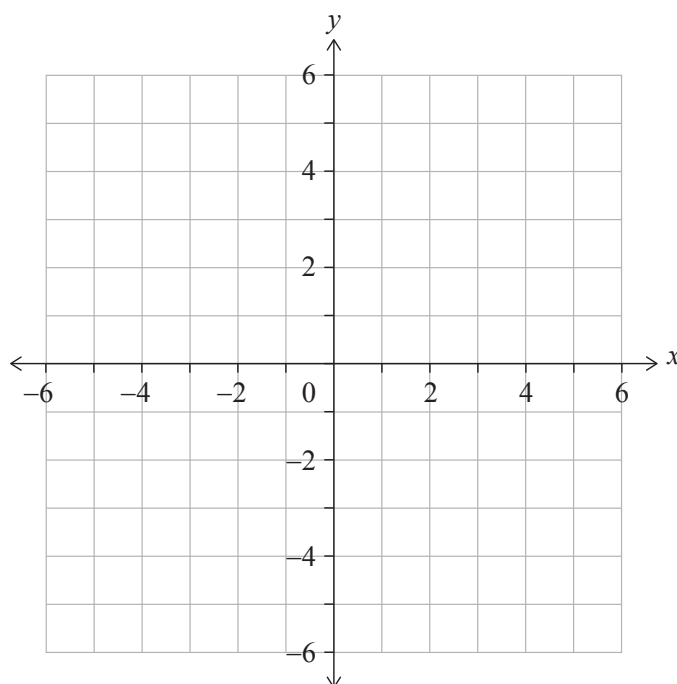
(ii) the horizontal asymptote.

[2]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.

[3]



(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$.

[1]

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[illegible]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R}$, $x \neq -4$.

- (a) For the graph of f
- (i) write down the equation of the vertical asymptote;
 - (ii) find the equation of the horizontal asymptote. [3]
- (b) (i) Find $f^{-1}(x)$.
- (ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y -axis followed by a reflection in the x -axis. [8]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

- (c) (i) Find the value of p and the value of q .
- (ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1} . [5]



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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$.

- (a) The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$.

[2]
- (b) The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1, 2)$. Find an expression for $h(x)$.

[3]
- (c) Find $(g \circ h)(0)$.

[2]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

- (a) The graph of a quadratic function f has its vertex at the point $(3, 2)$ and it intersects the x -axis at $x = 5$. Find f in the form $f(x) = a(x - h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p, t \in \mathbb{R}, p \neq 0$.

- (b) In the case where $g(-3) = g(1) = 4$,

(i) find the value of p and the value of t ;

(ii) find the range of g . [7]

- (c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t - 1)x - p$ have two distinct points of intersection for every possible value of p and t . [6]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

(a) Calculate the value of each of the following logarithms:

(i) $\log_2 \frac{1}{16};$

(ii) $\log_9 3;$

(iii) $\log_{\sqrt{3}} 81.$

[7]

(b) It is given that $\log_{ab} a = 3$, where $a, b \in \mathbb{R}^+$, $ab \neq 1$.

(i) Show that $\log_{ab} b = -2$.

(ii) Hence find the value of $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}.$

[8]



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5. [Maximum mark: 7]

$$P(t) = 15\,000e^{kt}, \text{ where } k < 0 \text{ and } t \geq 0.$$

Use this model to estimate the population of this town on 1 January 2041.

[illegible]

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of $[PQ]$.

(a) Find the coordinates of M. [2]

Line L is perpendicular to $[PQ]$ and passes through M .

(b) Find the gradient of L . [2]

(c) Hence, write down the equation of L . [1]

[illegible]

2. [Maximum mark: 7]

(a) Find the zero of $f(x)$. [2]

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

[illegible]

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3. [Maximum mark: 5]

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

(i) the vertical asymptote;

(ii) the horizontal asymptote.

[2]

(b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

(i) the y -axis;

(ii) the x -axis.

[2]

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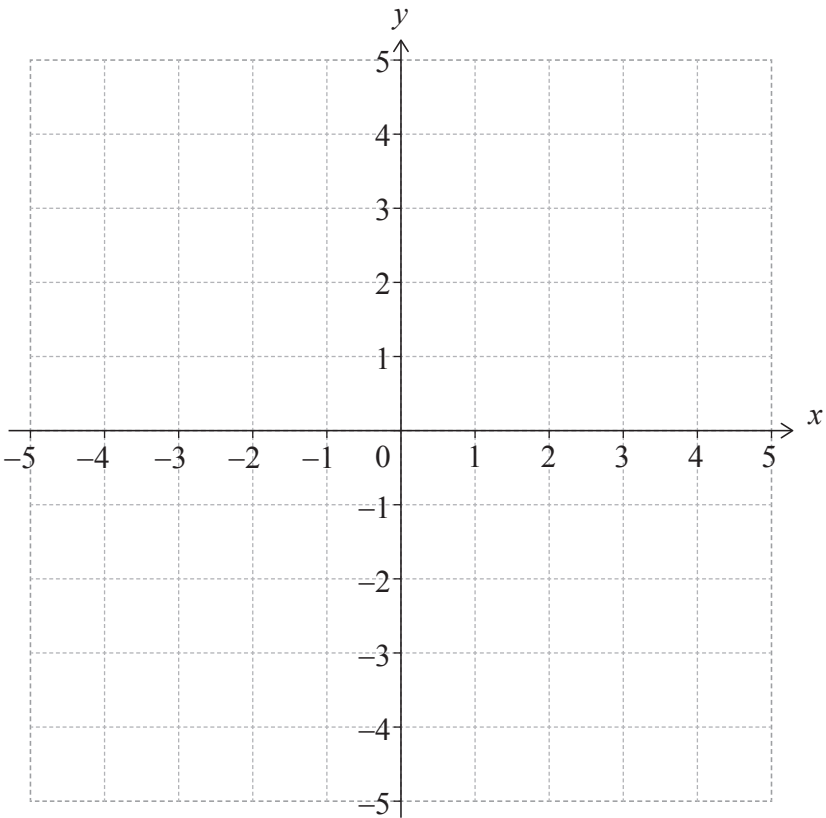
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(Question 3 continued)

- (c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).

[1]



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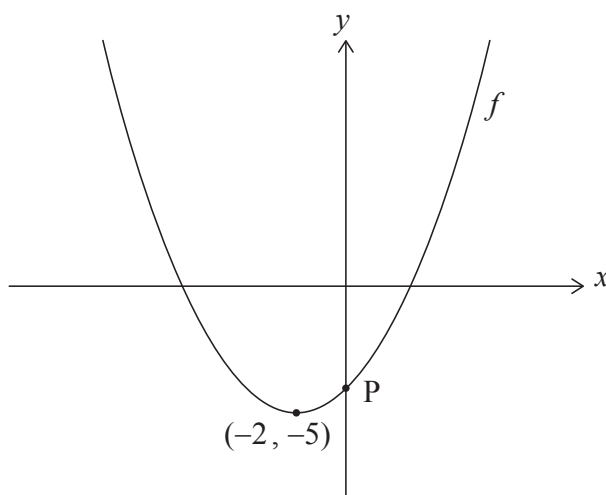
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

The following diagram shows part of the graph of a quadratic function f .

The vertex of the parabola is $(-2, -5)$ and the y -intercept is at point P.



(a) Write down the equation of the axis of symmetry.

[1]

The function can be written in the form $f(x) = \frac{1}{4}(x-h)^2 + k$, where $h, k \in \mathbb{Z}$.

(b) Write down the values of h and k .

[2]

(c) Find the y -coordinate of P.

[2]

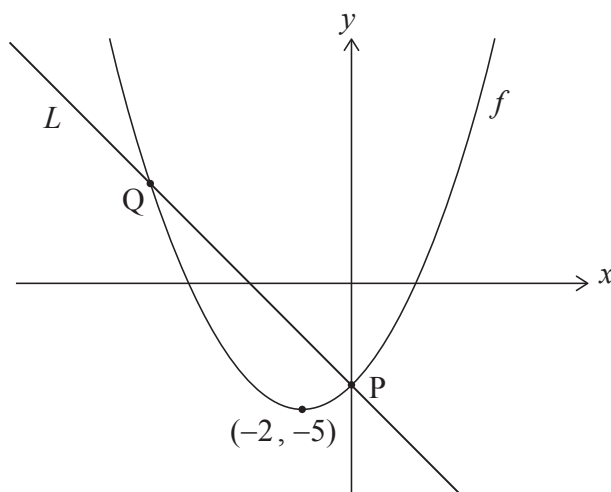
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(Question 7 continued)

In the following diagram, the line L is normal to the graph of f at point P .



- (d) Find the equation of the line L , in the form $y = ax + b$. [4]

The line L intersects the graph of f at a second point, Q , as shown above.

- (e) Calculate the distance between P and Q . [8]



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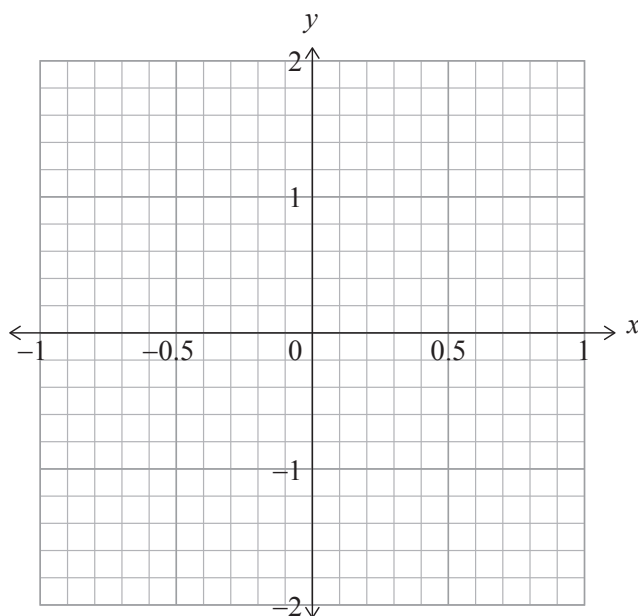
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3. [Maximum mark: 5]

(a) Find $(f \circ g)(x)$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \leq x \leq 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Consider the function $h(x) = \sqrt{4x-2}$, for $x \geq \frac{1}{2}$.

- (a) (i) Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain.
(ii) Write down the range of $h^{-1}(x)$. [5]
- (b) The graph of h intersects the graph of h^{-1} at two points.
Find the x -coordinates of these two points. [3]
- (c) Find the area enclosed by the graph of h and the graph of h^{-1} . [2]
- (d) Find $h'(x)$. [2]
- (e) Find the value of x for which the graph of h and the graph of h^{-1} have the same gradient. [3]



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Section A

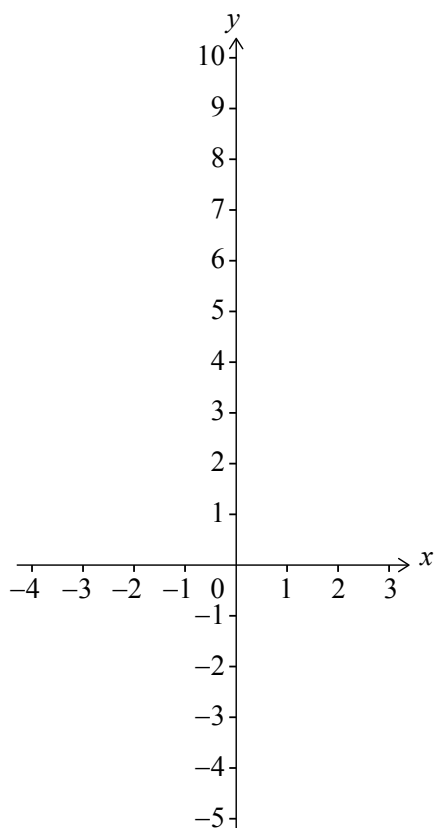
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider the function $f(x) = e^x - 2x - 5$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$.

[3]



The function g is defined by $g(x) = e^{3x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k , followed by a vertical translation of c units.

Find the value of k and the value of c .

[2]

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(Question 3 continued)

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Section A

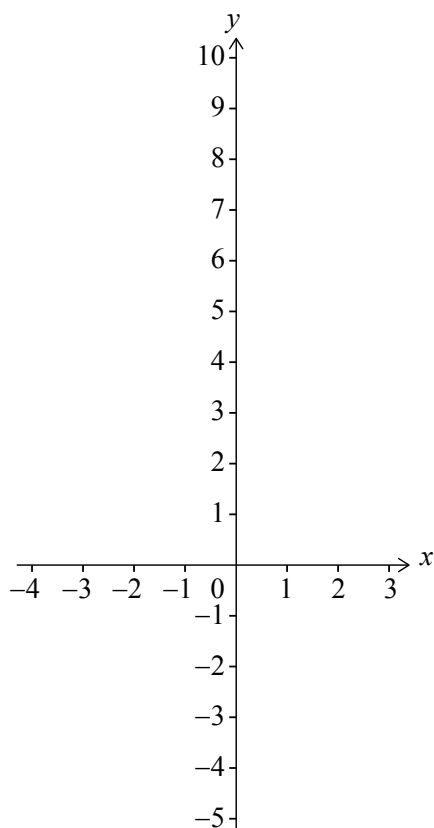
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

Consider the function $f(x) = e^x - 3x - 4$.

(a) On the following axes, sketch the graph of f for $-4 \leq x \leq 3$.

[3]



The function g is defined by $g(x) = e^{2x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k , followed by a vertical translation of c units.

Find the value of k and the value of c .

[2]

(This question continues on the following page)



(Question 3 continued)

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Turn over

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f . [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p , where $p \in \mathbb{Q}$.

(c) Show that $p = \frac{9}{2}$. [1]

(d) Find the value of b and the value of c . [3]

(e) Find the y -coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the number of solutions of the equation $f(x) = g(x)$. [2]



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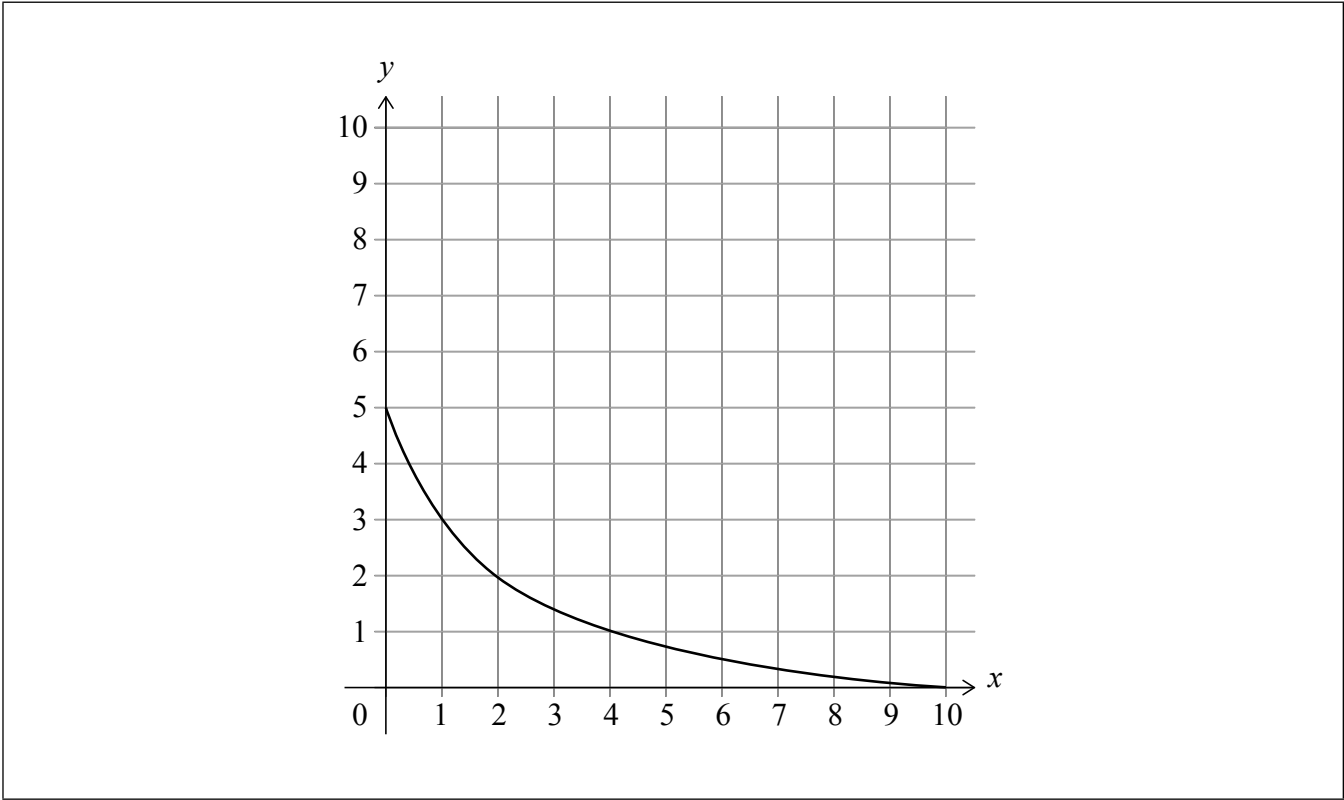
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $0 \leq x \leq 10$ is shown in the following diagram.

The graph intercepts the axes at $(10, 0)$ and $(0, 5)$.



(a) Write down the value of

(i) $f(4)$;

(ii) $f \circ f(4)$;

(iii) $f^{-1}(3)$.

[3]

(b) On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes.

[2]

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will not be marked.



4. [Maximum mark: 7]

A has coordinates $(6, 0)$ and C has coordinates $(3, 3\sqrt{3})$.

- (a) (i) Write down the coordinates of the midpoint of $[AC]$.
- (ii) Hence or otherwise, find the equation of the line passing through the points O and B.
- (b) Given that $[OA]$ is perpendicular to $[AB]$, find the area of the quadrilateral OABC.

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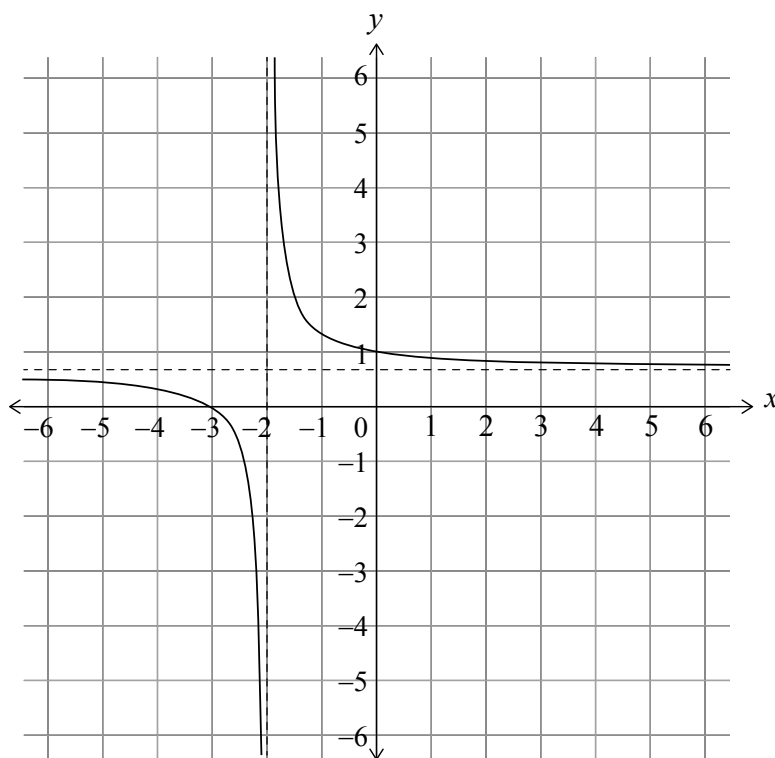
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 8]

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}$, $x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote.

[1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R}$, $m \neq 0$.

(b) (i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.

(ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x .

(iii) Determine the range of values for m , where $f(x) = g(x)$ has two solutions for $x \geq 0$.

[7]

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Section B

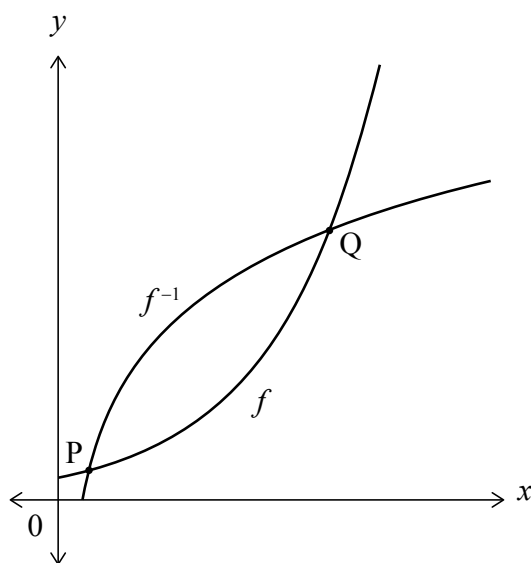
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Consider the function defined by $f(x) = \frac{3}{2}e^{x-2}$, $0 \leq x \leq 4$.

(a) Show that the inverse function is given by $f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$. [3]

The graphs of f and f^{-1} intersect at two points P and Q, as shown on the following diagram.



(b) Find PQ. [3]

The graph of f is reflected in the x -axis and then translated parallel to the y -axis by 5 units in the positive direction to give the graph of a function g .

(c) Write down

(i) an expression for $g(x)$;

(ii) the domain of g . [3]

(d) Solve the equation $f(x) = g(x)$. Give your answer in the form $x = a + \ln b$, where $a, b \in \mathbb{Q}$. [3]



Turn over

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 6]

The loudness of a sound, L , measured in decibels, is related to its intensity, I units, by $L = 10 \log_{10}(I \times 10^{12})$.

Consider two sounds, S_1 and S_2 .

S_1 has an intensity of 10^{-6} units and a loudness of 60 decibels.

S_2 has an intensity that is twice that of S_1 .

(a) State the intensity of S_2 . [1]

(b) Determine the loudness of S_2 . [2]

The maximum loudness of thunder in a thunderstorm was measured to be 115 decibels.

(c) Find the corresponding intensity, I , of the thunder. [3]



Turn over

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(i) $f(2)$;

(ii) $f\left(\frac{1}{8}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\frac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]



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Section A

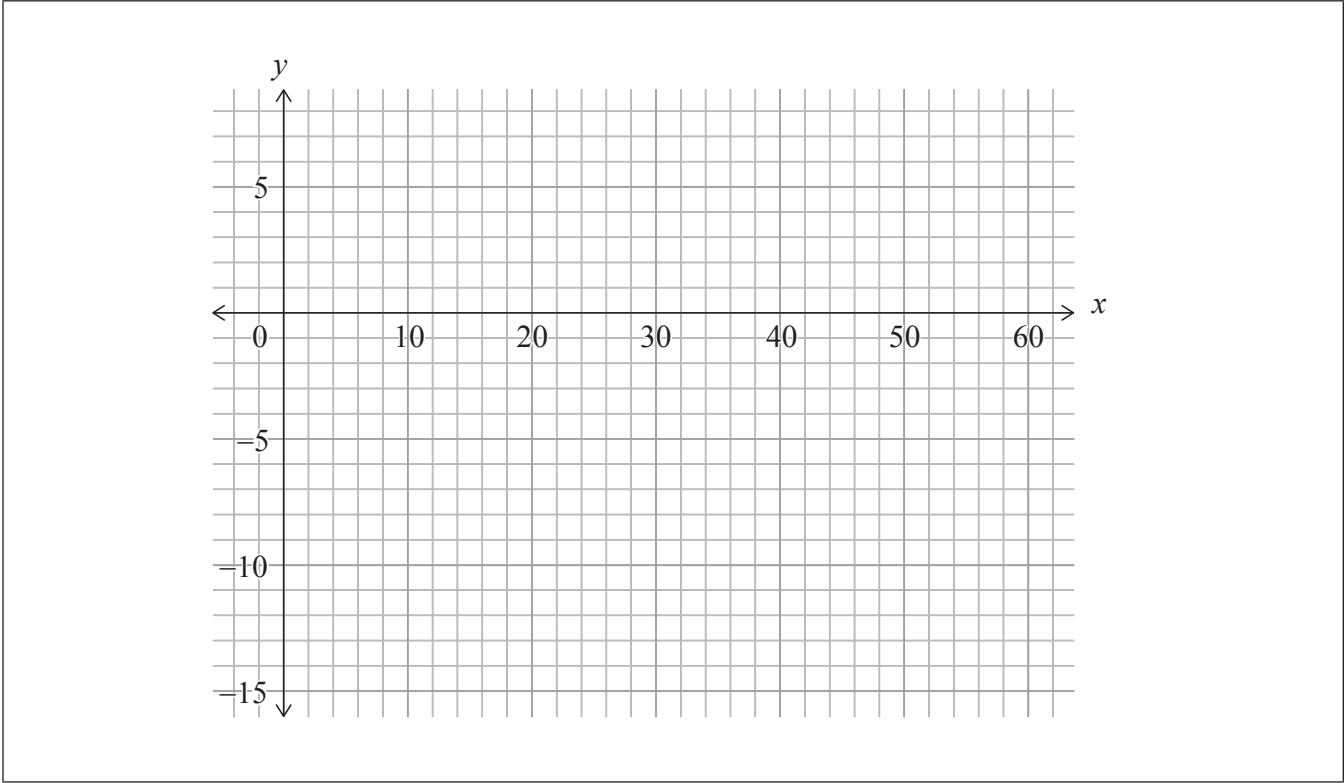
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = 7\sqrt{x} - x - 7$, where $0 \leq x \leq 60$.

- (a) Find the value of
- (i) $f(0)$;

(ii) $f(60)$.
- [2]
- (b) Find the two roots of $f(x) = 0$.
- [2]
- (c) Sketch the graph of $y = f(x)$ on the following grid.
- [3]



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(Question 1 continued)

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Section A

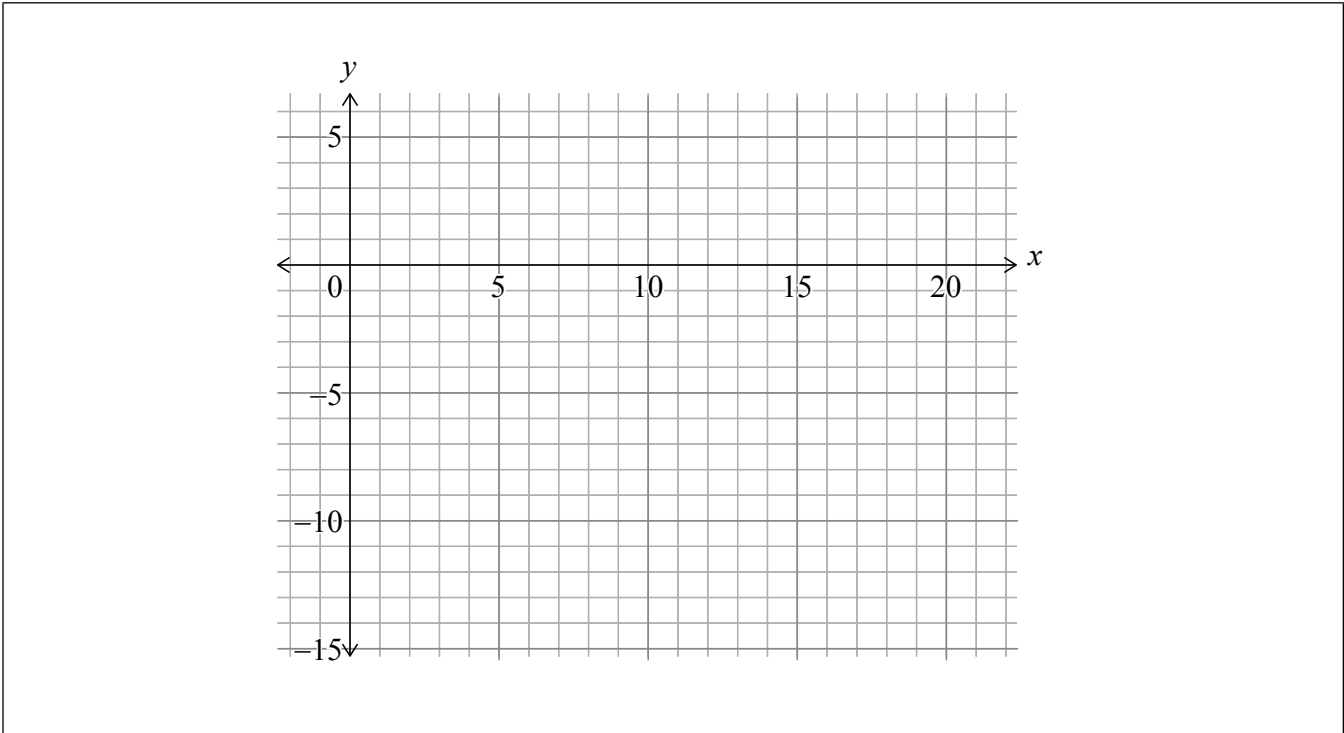
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = 11\sqrt{x} - 2x - 11$, where $0 \leq x \leq 20$.

- (a) Find the value of
- (i) $f(0)$;

(ii) $f(20)$.
- [2]
- (b) Find the two roots of $f(x) = 0$.
- [2]
- (c) Sketch the graph of $y = f(x)$ on the following grid.
- [3]



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(Question 1 continued)

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6. [Maximum mark: 5]

Find the possible values of r .

[5]

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Write each of the following expressions in the form $\ln k$, where $k \in \mathbb{Z}^+$.

(a) $\ln 3 + \ln 4$

[1]

(b) $3 \ln 2$

[2]

(c) $-\ln \frac{1}{2}$

[2]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The function f is defined by $f(x) = 5(x + 1)(x + 3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]



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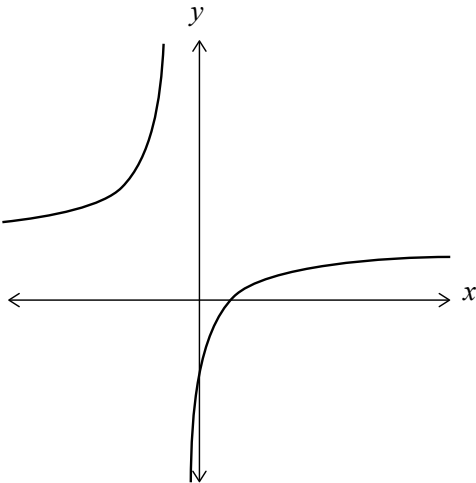
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The function f is defined by $f(x) = \frac{3x-2}{2x+1}$ for $x \in \mathbb{R}, x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.



(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g . [3]

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5. [Maximum mark: 7]

(a) Find the possible values of k . [5]

(b) Find the possible values of k in the case where the two distinct real roots are both positive or both negative. [2]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

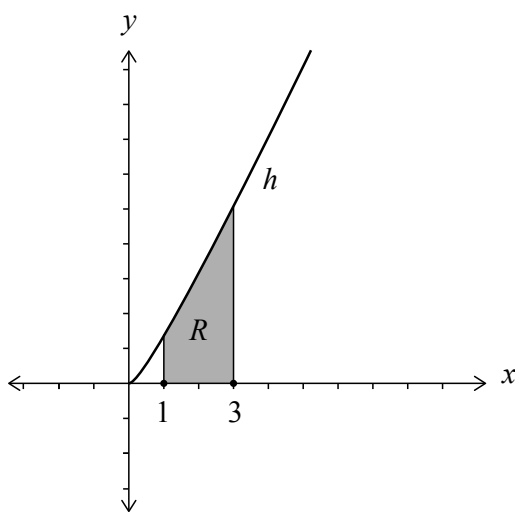
- (a) Find $f^{-1}(8)$. Express your answer in the form $\frac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.

- (b) (i) Find an expression for $g^{-1}(x)$.
 (ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]
 (c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \frac{4x^2}{2x+1}$, $x \neq -\frac{1}{2}$.

The following diagram shows part of the graph of h . Let R be the region enclosed by the graph of h and the x -axis, between the lines $x = 1$ and $x = 3$.



- (d) (i) Show that $2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$.
 (ii) Hence or otherwise, find the area of R , giving your answer in the form $p + q \ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]



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2. [Maximum mark: 5]

(a) Find an expression for $\log_{10} 24$ in terms of p and q . [3]

(b) Find an expression for $\log_3 8$ in terms of p and q . [2]

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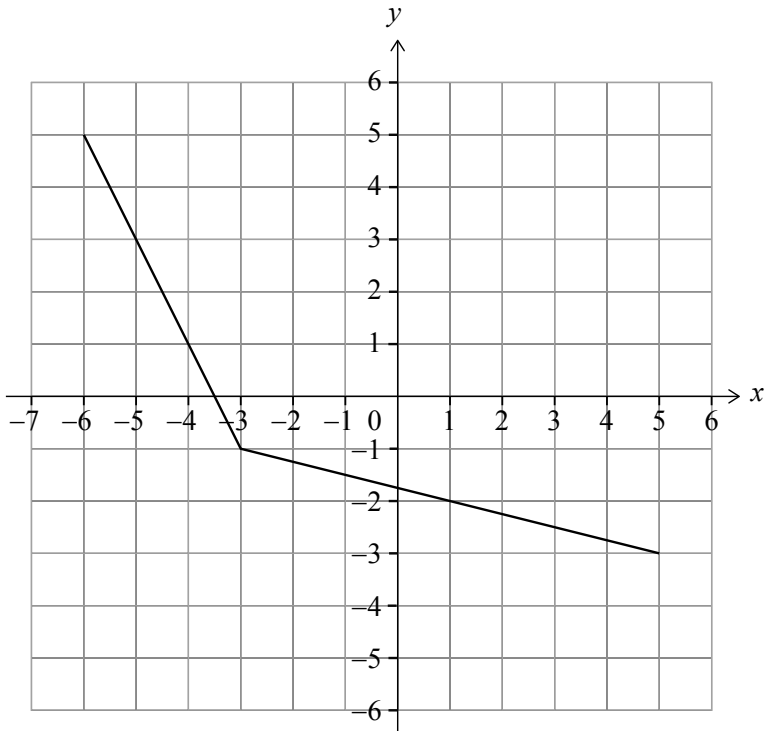
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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$, for $-6 \leq x \leq 5$.



- (a) Write down the value of $f(-3)$. [1]
- (b) State the domain of f^{-1} , the inverse function of f . [1]
- (c) Find the value of x that satisfies $f^{-1}(2x - 7) = -3$. [3]

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5. [Maximum mark: 7]

(a) Show that the curve and the line meet when $x^2 - (m + 1)x + 2 = 0$. [2]

(b) Hence, find the values of m when the line is tangent to the curve. [5]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

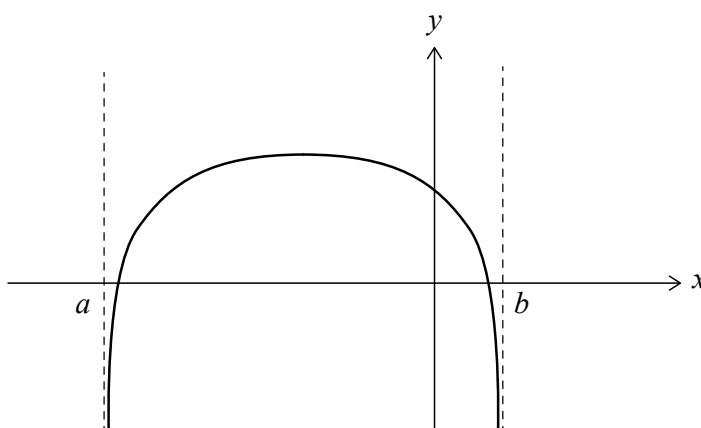
9. [Maximum mark: 16]

(a) (i) Solve $5 - 4x - x^2 = 0$.

(ii) Hence, find the values of x such that $5 - 4x - x^2 > 0$.

[4]

Consider the function $f(x) = \log_k(5 - 4x - x^2)$, where $a < x < b$ and $k > 1$.
Part of the graph of f is shown in the following diagram.



The graph of f has vertical asymptotes at $x = a$ and $x = b$.

(b) Write down the value of

(i) a ;

(ii) b .

[2]

(c) Find the exact values of x such that $f(x) = 0$.

[4]

The graph of f has a maximum value of 2.

(d) Find the value of k .

[6]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider the function $f(x) = \frac{2-2x}{x+2}$, where $x \in \mathbb{R}$, $x \neq -2$.

(a) Show that $f^{-1}(x) = f(x)$. [3]

The point $P\left(k, \frac{2-2k}{k+2}\right)$ is the point on the graph of $y = f(x)$ that is closest to the origin $(0, 0)$.

(b) (i) Find the value of k .

(ii) Hence, write down the coordinates of P . [4]

Consider the function $g(x) = \frac{2-3x}{cx+d}$, where $x \in \mathbb{R}$, $x \neq -\frac{d}{c}$, and $c, d \neq 0$.

The graph of $y = g(x)$ has a vertical asymptote and a horizontal asymptote.

(c) In terms of c and d , write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

It is given that $g^{-1}(x) = g(x)$.

(d) Find the value of d . [2]

Consider the case where $c = 1$.

(e) Sketch the graph of $y = \frac{1}{g(x)}$, showing the values of any intercepts with the axes and including any asymptotes, labelled with their equations. [4]



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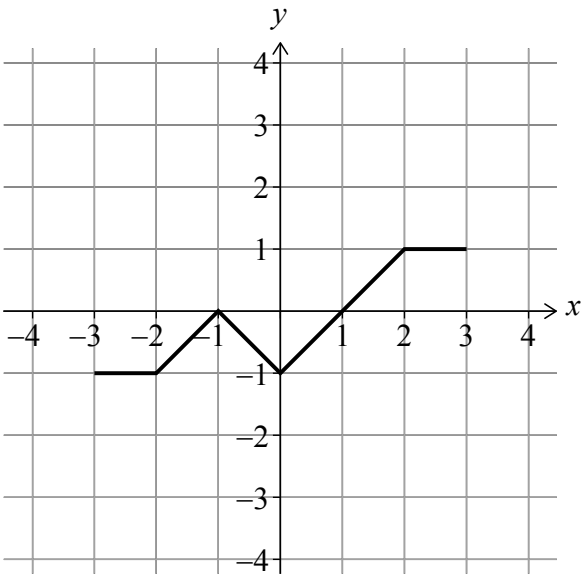


Section A

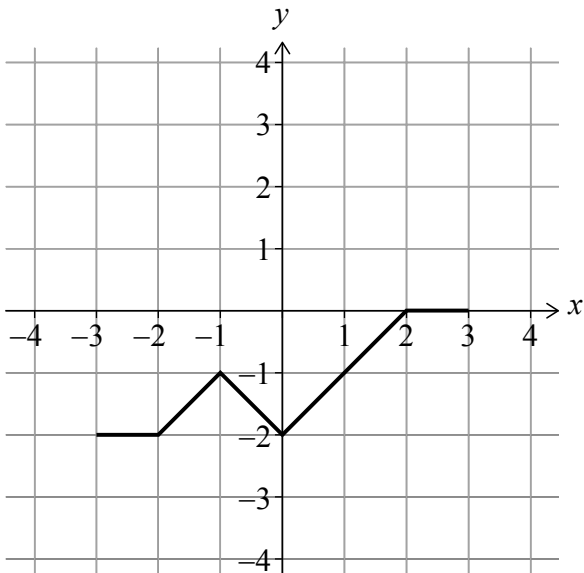
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

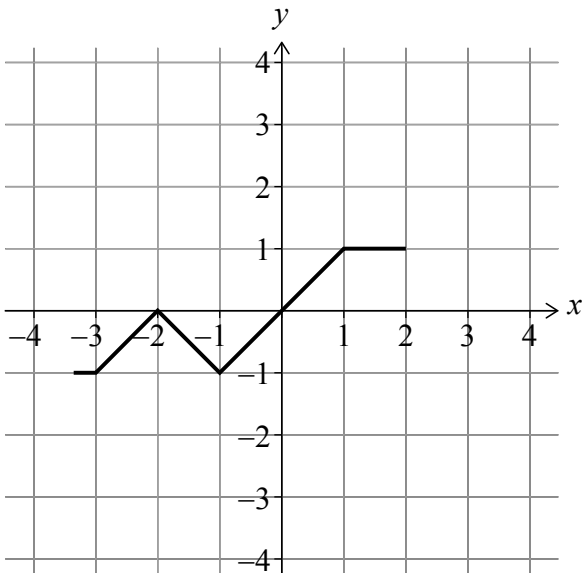
The graph of a function f for $-3 \leq x \leq 3$ is shown in the following diagram.



The following graphs are transformations of the graph of $y = f(x)$.



(A)

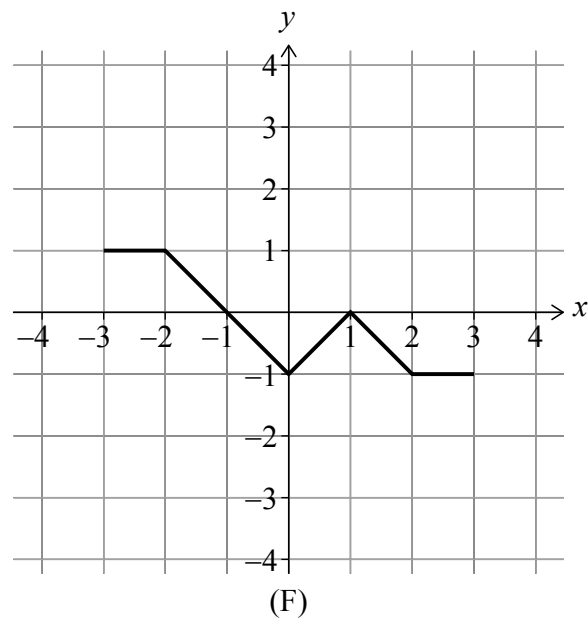
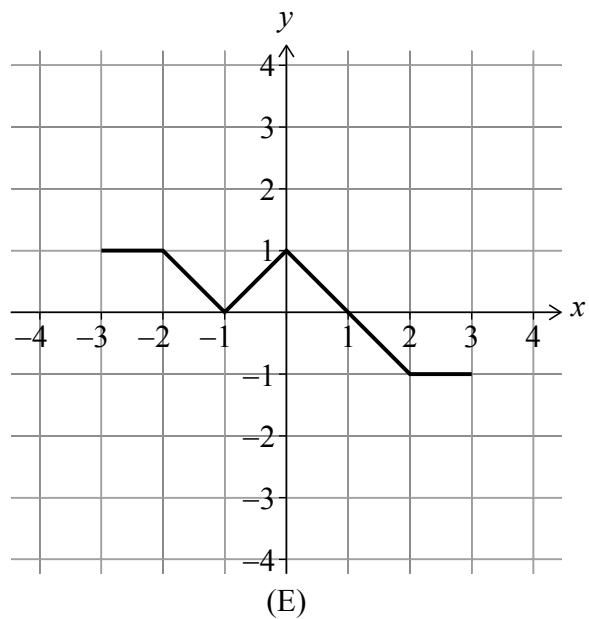
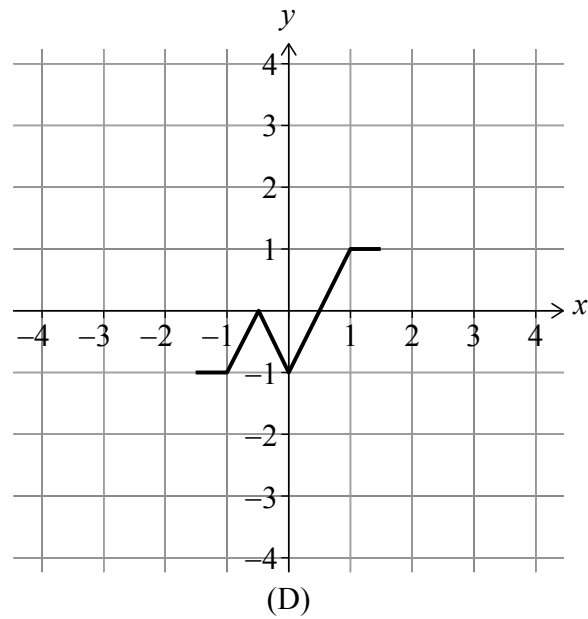
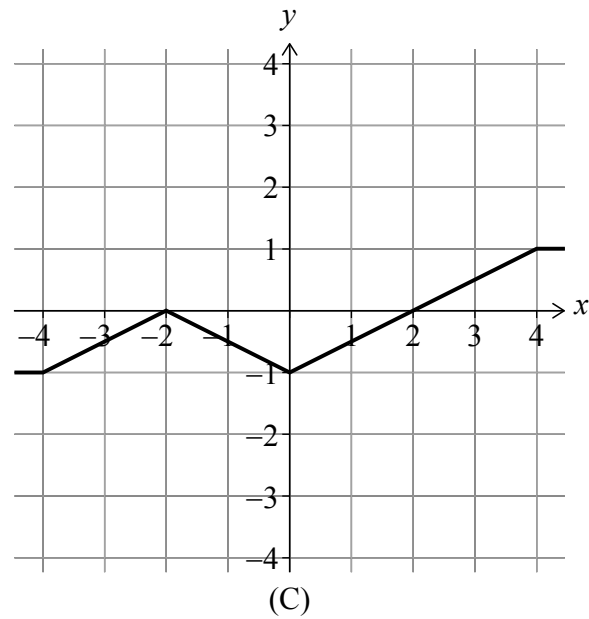


(B)

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(Question 1 continued)



Consider the following table of transformations of $y = f(x)$.

Next to each transformed function, write down the letter that corresponds to its graph.

Function	Letter
$y = f(x + 1)$	
$y = f(x) - 1$	
$y = f(-x)$	
$y = f(2x)$	



6. [Maximum mark: 8]

$$f(x) = -x^2 + 4x + p \text{ and}$$

$$g(x) = x^2 + qx - 1,$$

The graph of $y = f(x)$ has a vertex at A and the graph of $y = g(x)$ has a vertex at B.

Points A and B both lie on the line $y = 2x - 1$.

- (a) Show that $p = -1$. [3]
- (b) Find the value of q . [5]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Consider the function $f(x) = 2x + 3$ where $x \in \mathbb{R}$.

(a) Find

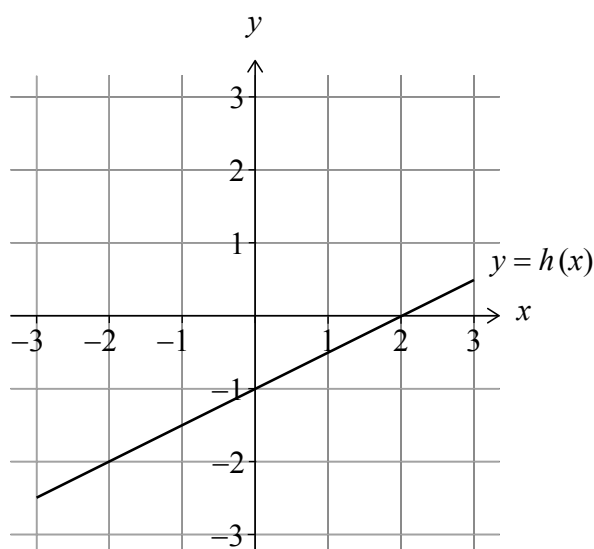
(i) $f(2)$;

(ii) $f^{-1}(x)$.

[3]

The following diagram shows part of the graph of a linear function h .

The graph has x -intercept 2 and y -intercept -1 .



(b) Write down an expression for $h(x)$.

[2]

(c) Solve the equation $h^{-1}(x) = -2$.

[2]

(This question continues on the following page)



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(Question 7 continued)

Consider the function $g(x) = mx + c$, where $x \in \mathbb{R}$ and $m, c \in \mathbb{Q}$.

- (d) Given that $h(x) = (f^{-1} \circ g)(x)$, find the value of m and the value of c . [4]

A function k exists such that $h(k(x)) = x$.

- (e) Find an expression for $k(x)$. [2]

- (f) State the single transformation that maps the graph of $y = k(x)$ onto the graph of $y = h(x)$. [1]

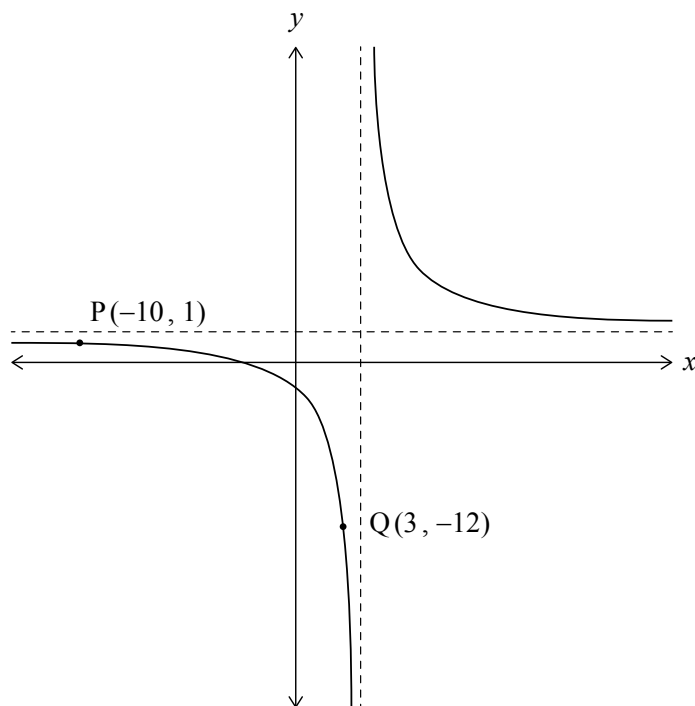


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5. [Maximum mark: 8]

The graph passes through the points P(-10, 1) and Q(3, -12).



- (a) Determine the value of A and the value of B . [3]
- (b) Describe a sequence of transformations that would map the graph of $y = \frac{1}{x}$ onto the graph of $y = \frac{Ax+B}{x-4}$. [5]

[illegible]

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