

APOLLO

Number and algebra

HL

Mathematics: applications and interpretation

First assessment 2021

54 questions | mark schemes

3. (a) (i) attempt to find u_{20} using an arithmetic sequence **(M1)**
 e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** 500, 600, 700, ...

 (Charlie ran) 2400 m **A1**
- (ii) ($r =$) 1.02 **(A1)**
 attempt to find u_{20} using a geometric sequence **(M1)**
 e.g. identifying $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** 500, 510, 520.2, ...

 (Daniella ran) 728 m (728.405...) **A1**
[5 marks]
- (b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ **(M1)**

 attempt to solve inequality **(M1)**
 $n > 184.215...$
 $n = 185$ **A1**
[3 marks]
- Total [8 marks]**

9. (a) (i) $i^2 = -1$ (M1)
 $w = -2 + 1 = -1$ A1
- (ii) $w = -1 + i + 1 = i$ A1
 [3 marks]
- (b) EITHER
 rotation of 90° (anticlockwise, centre at the origin) A1A1
- Note: Award A1 for “rotation” and A1 for “ 90° ”.
- followed by a translation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ A1
- OR
- translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ A1
- followed by rotation of 90° (anticlockwise, centre at the origin) A1A1
- Note: Award A1 for “rotation” and A1 for “ 90° ”.
- [3 marks]
- (c) EITHER
- move 1 to left to $1 - i$ (M1)
- then rotate by -90° to
 $-1 - i$ A1
- OR
- $iz + 1 = 2 - i$
- $iz = 1 - i$
- $z = \frac{1 - i}{i}$ (M1)
- $-1 - i$ A1
 [2 marks]
- Total [8 marks]

5. (a) $£495 \times 0.9^5 = £292$ (£292.292...)

(M1)A1

[2 marks]

(b) $495 \times 0.9^k = 2200 \times 0.85^k$
 $k = 26.1$ (26.0968...)

(M1)

A1

Note: Award **M1A0** for $k - 1$ in place of k .

[2 marks]

(c) depreciation rates unlikely to be constant (especially over a long time period)

R1

Note: Accept reasonable answers based on the magnitude of k or the fact that “value” depends on factors other than time.

[1 mark]

Total [5 marks]

7. (a) recognition of geometric sequence *eg* $r = 0.82$ **(M1)**

$$S_{10} = \frac{450(1 - 0.82^{10})}{1 - 0.82}$$

(A1)

$$= 2160 \text{ m (2156.37...)}$$

A1

[3 marks]

(b) $S_{\infty} = \frac{450}{1 - 0.82}$ **(M1)**

$= 2500 < 2520$ so the balloon will not reach the required height. **A1**

[2 marks]

- (c) horizontal motion not taken into account,
rate of cooling will not likely be linear,
balloon is considered a point mass / size of balloon not considered,
effects of wind/weather unlikely to be consistent,
a discrete model has been used, whereas a continuous one may
offer greater accuracy

R1

Note: Accept any other sensible answer.

[1 mark]

Total [6 marks]

12. (a) (i) $z_1^3 = 27e^{\frac{i\pi}{4}} (= 27e^{0.785398\dots i})$

A1A1

Note: Award **A1** for 27 and **A1** for the angle in the correct form.

(ii) $\left(\frac{z_1}{z_2}\right)^4 = \left(\frac{81}{16}\right)e^{\frac{i\pi}{2}} (= 5.0625e^{1.57079\dots i})$

A1A2

Note: Award **A1** for $\frac{81}{16}$, **A2** for the angle in the correct form and

A1 for the angle in incorrect form e.g. $\text{cis } \frac{\pi}{2}$ and/or $\frac{5\pi}{2}$.

Award **A1** if i is given in place of $\text{cis } \frac{\pi}{2}$.

[5 marks]

(b) $z_1 z_2 = 6 \text{cis} \left(\frac{3\pi}{4} + \frac{n\pi}{16} \right)$

(M1)

$$= 6 \text{cis} \left(\frac{12\pi + n\pi}{16} \right)$$

$$12\pi + n\pi = 32\pi$$

$$n = 20$$

(M1)**A1****[3 marks]****Total [8 marks]**

1. (a) **EITHER**

$$N = 2$$

$$PV = -37\,000$$

$$I\% = 6.4$$

$$P/Y = 1$$

$$C/Y = 4$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct.

OR

$$N = 8$$

$$PV = -37\,000$$

$$I\% = 6.4$$

$$P/Y = 4$$

$$C/Y = 4$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct.

OR

$$FV = 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4 \times 2}$$

(M1)(A1)

Note: Award **M1** for substitution into compound interest formula, **(A1)** for correct substitution.

$$= 42\,010 \text{ AUD}$$

A1

Note: Award **(M1)(A1)A0** for unsupported 42009.87.

[3 marks](b) **EITHER**

$$PV = -37\,000$$

$$FV = 50\,000$$

$$I\% = 6.4$$

$$P/Y = 1$$

$$C/Y = 4$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct. The final mark can still be awarded for the correct number of months (multiple of 3).

continued...

Question 1 continued**OR**

$$PV = -37\,000$$

$$FV = 50\,000$$

$$I\% = 6.4$$

$$P/Y = 4$$

$$C/Y = 4$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct.

OR

$$50\,000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4 \times n} \quad \text{OR} \quad 50\,000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^n \quad \textbf{(M1)(A1)}$$

Note: Award **M1** for the correct inequality, 50 000 and substituted compound interest formula. Allow an equation. Award **A1** for correct substitution.

THEN

$$N = 4.74 \text{ (years) (4.74230...)} \quad \text{OR} \quad N = 18.9692... \text{ (quarters)} \quad \textbf{(A1)}$$

$$m = 57 \text{ months} \quad \textbf{A1}$$

Note: Award **A1** for rounding their m to the correct number of months. The final answer must be a multiple of 3. Follow through within this part.

[4 marks]

(c) 150 000 AUD **A1**

[1 mark]*continued...*

Question 1 continued

(d) (i) $120 \times 1700 - 150\,000$ (M1)
 $= 54\,000 \text{ AUD}$ A1

(ii) $N = 120$
 $PV = -150\,000$
 $PMT = 1700$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ (M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology or an attempt to use an annuity formula or $FV = 0$ seen. If a compound interest formula is equated to zero, award **M1**, otherwise award **M0** for a substituted compound interest formula.
 Award **A1** for all entries correct in financial app or correct substitution in annuity formula, but award **A0** for a substituted compound interest formula. Follow through marks in part (d)(ii) are contingent on working seen.

$r = 6.46 \text{ (\%)} (6.45779\dots)$ A1

[5 marks]

(e) $N = 60$
 $I = 6.46 (6.45779\dots)$
 $PV = -150\,000$
 $PMT = 1700$
 $P/Y = 12$
 $C/Y = 12$ (M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology or an attempt to use an annuity formula. Award **(M0)** for a substituted compound interest formula. Award **A1** for all entries correct. Follow through marks in part (e) are contingent on working seen.

$FV = 86\,973 \text{ AUD}$ A1

[3 marks]

continued...

Question 1 continued

(f) $204\,000 - (60 \times 1700 + 86\,973)$ **OR** $204\,000 - 188\,973$

(M1)(M1)

Note: Award **M1** for 60×1700 . Award **M1** for subtracting their $(60 \times 1700 + 86\,973)$ from their $(204\,000)$. Award at most **M1M0** for their $204\,000 - (60 \times 1700)$ or **MOM0** for their $204\,000 - (86\,973)$. Follow through from parts (d)(i) and (e). Follow through marks in part (f) are contingent on working seen.

15 027 AUD

A1**[3 marks]****Total [19 marks]**

5. (a) $N = 360$
 $I\% = 3.8$
 $PV = (\pm)170\,000$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ **(M1)(A1)**

Note: Award **(M1)** for an attempt to use a financial app in their technology with at least two entries seen, award **A1** for all entries correct. Accept a positive or negative value for PV .

$(PMT =) 792.13 \text{ AUD}$ **A1**

Note: Accept an answer of -792.13 . Do not award final **A1** if answer is not given correct to 2 dp

[3 marks]

- (b) (i) $N = 120$
 $I\% = 3.8$
 $PV = (\pm)170\,000$
 $PMT = (\mp)792.13$
 $P/Y = 12$
 $C/Y = 12$ **(M1)(A1)**

Note: Award **(M1)** for an attempt to use a financial app in their technology with a least two entries seen, award **A1** for all entries correct. PV and PMT must have opposite signs.

$(FV =) 133\,019.94 \text{ AUD}$ **A1**

Note: Do not award final **A1** if answer is not given correct to 2 dp, unless already penalized in part (a). Accept 133020.30 from use of exact value for PMT .

- (ii) amount of money paid: $120 \times 792.13 (= 95\,055.60)$ **(M1)**
 loan paid off: $170\,000 - 133\,019.94 (= 36\,980.06)$ **(M1)**
 interest paid: $(95\,055.60 - 36\,980.06 =) 58\,075.54 \text{ AUD}$ **A1**

Note: Allow 58075.60 or 58075.90 from use of some exact values from parts (a) and (b)(i). If their answer to part (b)(i) is greater than 170 000 then award at most **(M1)(M1)(A0)** for follow through in part (b)(ii).

[6 marks]

Total: [9 marks]

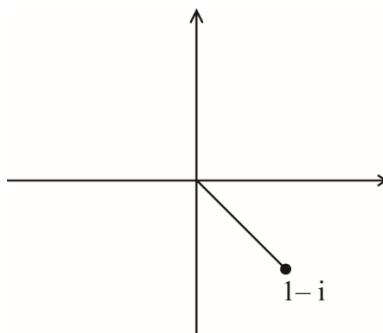
6. (a) $10 = \frac{2}{1-r}$ (M1)
 $r = 0.8$ A1
 [2 marks]

(b) $2 \times (0.8)^{n-1} < 0.5$ OR $2 \times (0.8)^{n-1} = 0.5$ (M1)
 $(n >) 7.212.....$ (A1)
 $n = 8$ A1

Note: If $n = 7$ is seen, with or without seeing the value 7.212... then award **M1A1A0**.

[3 marks]
 Total: [5 marks]

5. (a) (i)

**A1**

(ii) $z = \sqrt{2}e^{-\frac{i\pi}{4}}$

A1A1

Note: Accept an argument of $\frac{7\pi}{4}$. Do **NOT** accept answers that are not exact.

[3 marks]

$$\begin{aligned}
 \text{(b) (i)} \quad w_1 + w_2 &= e^{ix} + e^{i\left(x - \frac{\pi}{2}\right)} \\
 &= e^{ix} \left(1 + e^{-\frac{i\pi}{2}} \right) \\
 &= e^{ix} (1 - i)
 \end{aligned}$$

(M1)**A1**

$$\begin{aligned}
 \text{(ii)} \quad w_1 + w_2 &= e^{ix} \times \sqrt{2}e^{-\frac{i\pi}{4}} \\
 &= \sqrt{2}e^{i\left(x - \frac{\pi}{4}\right)}
 \end{aligned}$$

M1

attempt extract real part using cis form

(A1)**(M1)**

$$\operatorname{Re}(w_1 + w_2) = \sqrt{2} \cos\left(x - \frac{\pi}{4}\right) \quad \text{OR} \quad 1.4142... \cos(x - 0.785398...)$$

A1**[6 marks]***continued...*

Question 5 continued

$$(c) \quad (i) \quad I_t = 12 \cos(bt) + 12 \cos\left(bt - \frac{\pi}{2}\right) \quad (M1)$$

$$I_t = 12 \operatorname{Re} \left(e^{ibt} + e^{i\left(bt - \frac{\pi}{2}\right)} \right) \quad (M1)$$

$$I_t = 12\sqrt{2} \cos\left(bt - \frac{\pi}{4}\right)$$

$$\max = 12\sqrt{2} \quad (=17.0) \quad A1$$

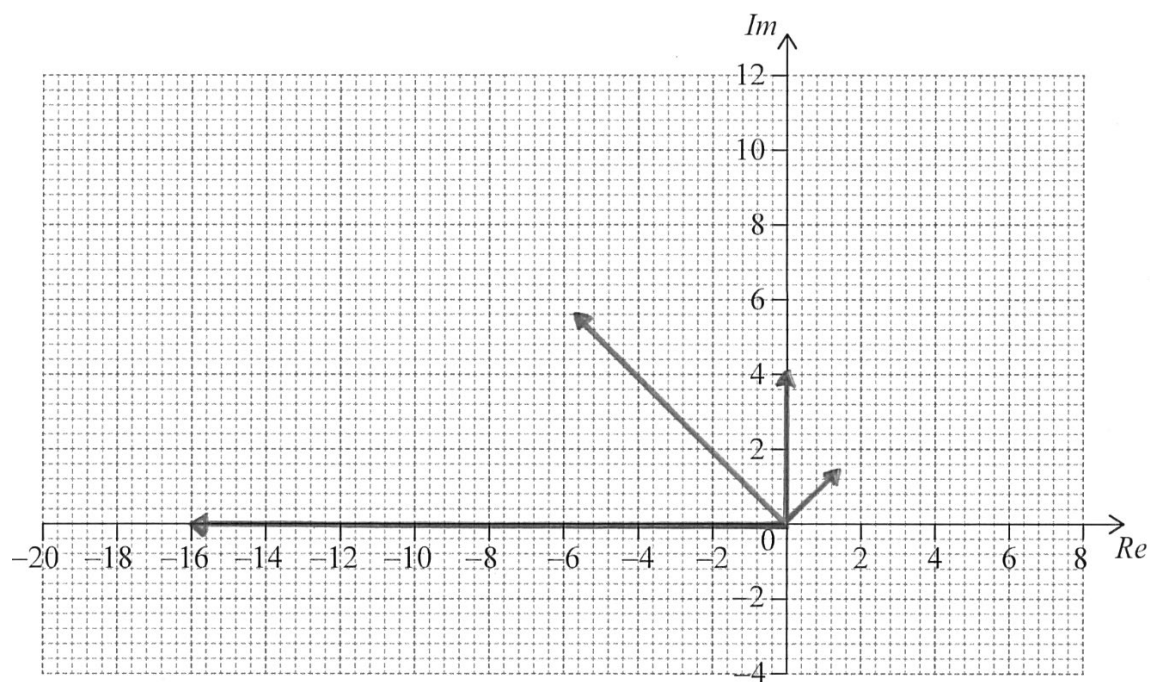
$$(ii) \quad \text{phase shift} = \frac{\pi}{4} (=0.785) \quad A1$$

[4 marks]**Total: [13 marks]**

15. (a) (i) $4e^{\frac{\pi}{2}i}, 8e^{\frac{5\pi}{4}i}, 16e^{\pi i} \quad (= 4i, -4\sqrt{2} + 4\sqrt{2}i, -16)$

(M1)A1

(ii)

**A3**

Note: Award **A1** for correct arguments, award **A1** for $4i$ and -16 clearly indicated, award **A1** for $|w| < 4$ and $4 < |w^3| < 16$.

[5 marks]

(b) $2^2 + 1^2 = a^2$

M1

$a = \sqrt{5} \quad (= 2.24)$

A1**[2 marks]****Total [7 marks]**

18. $\log A = x \log B + y \log C + \log k$ **(M1)**

$$\log 5.74 = x \log 2.1 + y \log 3.4 + \log k$$

$$\log 2.88 = x \log 1.5 + y \log 2.4 + \log k$$

$$\log 0.980 = x \log 0.8 + y \log 1.9 + \log k$$

M1A1

Note: Allow any consistent base, allow numerical equivalents.

attempting to solve their system of equations

(M1)

$$x = 1.53, y = 0.505$$

A1

$$k = 0.997$$

A1

Total [6 marks]

3. (a) (i) $N = 24$

$$I\% = 14$$

$$PV = -14000$$

$$FV = 0$$

$$P/Y = 4$$

$$C/Y = 4$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct. Accept $PV = 14000$.

$$(\text{€})871.82$$

A1

(ii) $4 \times 6 \times 871.82$

(M1)

$$(\text{€})20923.68$$

A1

(iii) $20923.68 - 14000$

(M1)

$$(\text{€})6923.68$$

A1

[7 marks]

(b) (i) $0.9 \times 14000 (= 14000 - 0.10 \times 14000)$

M1

$$(\text{€})12600.00$$

A1

(ii) $N = 72$

$$PV = 12600$$

$$PMT = -250$$

$$FV = 0$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology, award **A1** for all entries correct. Accept $PV = -12600$ provided $PMT = 250$.

$$12.56(\%)$$

A1

[5 marks]

continued...

Question 3 continued

(c) **EITHER**

Bryan should choose Option A

A1

no deposit is required

R1

Note: Award **R1** for stating that no deposit is required. Award **A1** for the correct choice from that fact. Do not award **R0A1**.

OR

Bryan should choose Option B

A1cost of Option A (6923.69) > cost of Option B ($72 \times 250 - 12\,600 = 5400$)**R1**

Note: Award **R1** for a correct comparison of costs. Award **A1** for the correct choice from that comparison. Do not award **R0A1**.

[2 marks](d) real interest rate is $0.4 - 0.1 = 0.3\%$ **(M1)**value of other payments $250 + 250 \times 1.003 + \dots + 250 \times 1.003^{71}$

use of sum of geometric sequence formula or financial app on a GDC

(M1) $= 20\,058.43$

value of deposit at the end of 6 years

 $1400 \times (1.003)^{72} = 1736.98$ **(A1)**

Total value is (€) 21 795.41

A1

Note: Both **M** marks can awarded for a correct use of the GDC's financial app:

 $N = 72 \text{ (} 6 \times 12 \text{)}$ $I\% = 3.6 \text{ (} 0.3 \times 12 \text{)}$ $PV = 0$ $PMT = -250$ $FV =$ $P/Y = 12$ $C/Y = 12$ **OR** $N = 72 \text{ (} 6 \times 12 \text{)}$ $I\% = 0.3$ $PV = 0$ $PMT = -250$ $FV =$ $P/Y = 1$ $C/Y = 1$ **[4 marks]****Total [18 marks]**

$(30) \quad (-150)$

2. (a) $x + y + z = 600$ **A1**
 $15x + 10y + 12z = 7816$ **A1**
 $x = 2y$ **A1**

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student).
 Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

- (b) $x = 308, y = 154, z = 138$ **A1A1**

Note: Award **A1** for all three correct values seen, **A1** for correctly labelled as x, y or z .
 Accept answers written in words: e.g. 308 adult tickets.

[2 marks]

[Total 5 marks]

5. (a) use of geometric sequence with $r = 0.85$ **M1**

EITHER

$$(0.85)^6(1.8) \quad \text{OR} \quad 0.678869... \quad \text{OR} \quad (0.85)^5(1.53) \quad \text{A1}$$

$$= 0.68 \text{ m}$$

$$= 68 \text{ cm} \quad \text{AG}$$

OR

$$(0.85)^6(180) \quad \text{OR} \quad (0.85)^5(153) \quad \text{A1}$$

$$= 68 \text{ cm} \quad \text{AG}$$

[2 marks]

- (b) **EITHER**

$$(0.85)^n(1.8) > 0.1 \quad \text{OR} \quad (0.85)^{n-1}(1.53) > 0.1 \quad \text{(M1)}$$

Note: If 1.8 m (or 180 cm) is used then **(M1)** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.

If 1.53 m (or 153 cm) is used then **(M1)** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

$$17 \quad \text{A1}$$

OR

$$(0.85)^{17}(1.8) = 0.114 \text{ m and } (0.85)^{18}(1.8) = 0.0966 \text{ m} \quad \text{(M1)}$$

$$17 \quad \text{A1}$$

OR

$$\text{solving } (0.85)^n(1.8) = 0.1 \text{ to find } n = 17.8 \quad \text{(M1)}$$

$$17 \quad \text{A1}$$

Note: Evidence of solving may be a graph **OR** the “solver” function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

continued...

Question 5 continued**(c) EITHER**

distance (in one direction) travelled between first and fourth bounce

$$= \frac{(1.8 \times 0.85)(1 - 0.85^3)}{1 - 0.85} (= 3.935925) \quad \textbf{(A1)}$$

recognizing distances are travelled twice except first distance **(M1)**

$$1.8 + 2(3.935925)$$

$$= 9.67 \text{ m } (9.67185... \text{ m}) \quad \textbf{A1}$$

OR

distance (in one direction) travelled between drop and fourth bounce

$$= \frac{(1.8)(1 - 0.85^4)}{1 - 0.85} (= 5.735925) \quad \textbf{(A1)}$$

recognizing distances are travelled twice except first distance **(M1)**

$$2(5.735925) - 1.8$$

$$= 9.67 \text{ m } (9.67185... \text{ m}) \quad \textbf{A1}$$

OR

distance (in one direction) travelled between first and fourth bounce

$$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8) (= 3.935925...) \quad \textbf{(A1)}$$

recognizing distances are travelled twice except first distance **(M1)**

$$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$$

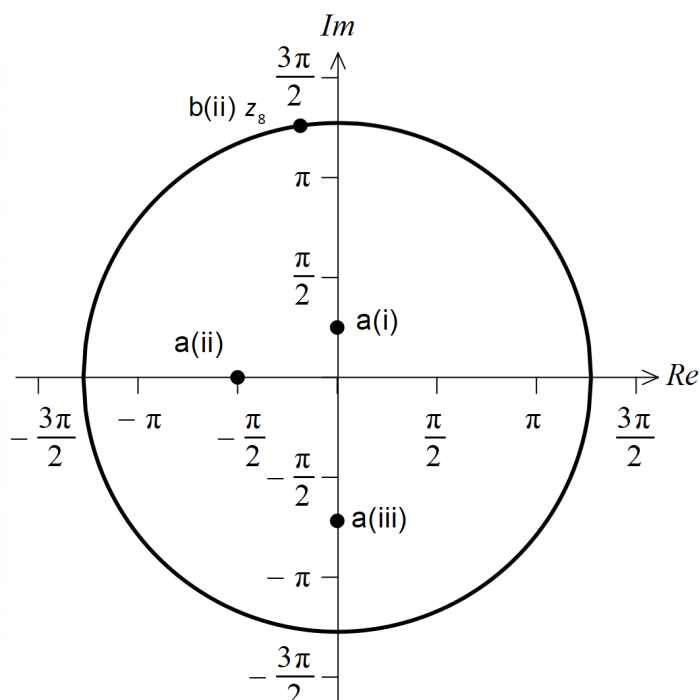
$$= 9.67 \text{ m } (9.67185... \text{ m}) \quad \textbf{A1}$$

Note: Answers may be given in cm.

[3 marks]
[Total 7 marks]

(-3.2)

10. (a)

**A1A1A1**

Note: Award **A1** for correct modulus and **A1** for correct argument for part (a)(i), and **A1** for other two points correct.
The points may not be labelled, and they may be shown by line segments.

[3 marks]

$$(b) (i) \quad \frac{1}{2}\theta = 4$$

$$\Rightarrow \theta = 8$$

(M1)**A1**(ii) z_8 is shown in the diagram above**A1A1**

Note: Award **A1** for a point plotted on the circle and **A1** for a point plotted in the second quadrant.

[4 marks]**[Total 7 marks]**

11. (a) $\lambda = 1$

$$\begin{pmatrix} -0.8 & 0.7 \\ 0.8 & -0.7 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \quad (M1)$$

$$0.8x = 0.7y \quad (A1)$$

an eigenvector is $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ (or equivalent with integer values) A1

[3 marks]

(b) **EITHER**

(the long-term probability matrix is given by the eigenvector corresponding to the eigenvalue equal to 1, scaled so that the sum of the entries is 1)

$$8 + 7 = 15 \quad (M1)$$

OR

$$\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix} \begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix} \quad (M1)$$

OR

considering high powers of the matrix e.g. $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}^{50}$ (M1)

$$\begin{pmatrix} \frac{7}{15} & \frac{7}{15} \\ \frac{8}{15} & \frac{8}{15} \end{pmatrix}$$

THEN

probability of being in state A is $\frac{7}{15}$ A1

[2 marks]

[Total 5 marks]

12. (a) $\log_{10} 100 = a - 3$ (M1)
 $a = 5$ A1
 [2 marks]
- (b) **EITHER**
 $N = 10^{5-M}$ (M1)
 $= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$
- OR**
 $100 = \frac{b}{10^3}$ (M1)
- THEN**
 $b = 100000 (= 10^5)$ A1
 [2 marks]
- (c) $N = \frac{10^5}{10^{7.2}} = 0.00631 \text{ (0.0063095...)}$ A1
- Note:** Do not accept an answer of $10^{-2.2}$.
- [1 mark]
- (d) **METHOD 1**
 $Y > 100 \Rightarrow$ no earthquakes in the first 100 years (M1)
- EITHER**
 let X be the number of earthquakes of at least magnitude 7.2 in a year
 $X \sim \text{Po}(0.0063095...)$
 $(P(X = 0))^{100}$ (M1)
- OR**
 let X be the number of earthquakes in 100 years
 $X \sim \text{Po}(0.0063095... \times 100)$ (M1)
 $P(X = 0)$
- THEN**
 $0.532 \text{ (0.532082...)}$ A1
- METHOD 2**
 $Y > 100 \Rightarrow$ no earthquakes in the first 100 years (M1)
 let X be the number of earthquakes in 100 years
 since n is large and p is small
 $X \sim \text{B}(100, 0.0063095...)$ (M1)
 $P(X = 0)$
 $0.531 \text{ (0.531019...)}$ A1
 [3 marks]
 [Total 8 marks]

.....

7. METHOD 1

$$\frac{u_1}{1-r} = 9 \quad \text{A1}$$

$$\text{therefore } u_1 = 9 - 9r$$

$$u_1 = 4 + u_1 r \quad \text{A1}$$

substitute or solve graphically: M1

$$9 - 9r = 4 + (9 - 9r)r \quad \text{OR} \quad \frac{4}{(1-r)^2} = 9$$

$$9r^2 - 18r + 5 = 0$$

$$r = \frac{1}{3} \text{ or } r = \frac{5}{3}$$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists R1

$$\text{then } u_1 = 9 - \left(9 \times \frac{1}{3}\right) = 6$$

$$u_3 = 6 \times \frac{1^2}{3} = \frac{2}{3} \quad \text{A1}$$

METHOD 2

$$\frac{u_1}{1-r} = 9 \quad \text{A1}$$

$$r = \frac{u_1 - 4}{u_1} \quad \text{A1}$$

attempt to solve M1

$$\frac{u_1}{1 - \left(\frac{u_1 - 4}{u_1}\right)} = 9$$

$$\frac{u_1}{\left(\frac{4}{u_1}\right)} = 9$$

$$(u_1)^2 = 36$$

$$u_1 = \pm 6$$

attempting to solve both possible sequences

6, 2, ... or -6, -10 ...

$$r = \frac{1}{3} \quad \text{or} \quad r = \frac{5}{3}$$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists R1

$$u_3 = 6 \times \left(\frac{1}{3}\right)^2 = \frac{2}{3} \quad \text{A1}$$

Total [5 marks]

11. (a) METHOD 1 – (With $FV=4000$)**EITHER**

$$N=10$$

$$I=1.5$$

$$FV=4000$$

$$P/Y=1$$

$$C/Y=1$$

(A1)(M1)**Note:** Award **A1** for $(3.5 - 2 =) 1.5$ seen and **M1** for all other entries correct.**OR**

$$4000 = A(1 + 0.015)^{10}$$

(A1)(M1)**Note:** Award **A1** for 1.5 or 0.015 seen, **M1** for attempt to substitute into compound interest formula **and** equating to 4000.**THEN**

$$(PV =) \$3447$$

A1**Note:** Award **A0** if not rounded to a whole number or a negative sign given.**METHOD 2 – (With FV including inflation)**calculate FV with inflation

$$4000 \times 1.02^{10}$$

$$(=4875.977\dots)$$

(A1)**EITHER**

$$4000 \times 1.02^{10} = PV \times 1.035^{10}$$

(A1)**OR**

$$N=10$$

$$I=3.5$$

$$FV=4875.977\dots$$

$$P/Y=1$$

$$C/Y=1$$

(M1)**Note:** Award **M1** for *their* FV and all other entries correct.**THEN**

$$(PV =) \$3457$$

A1**Note:** Award **A0** if not rounded to a whole number or a negative sign given.*continued...*

Question 11 continued

METHOD 3 – (Using formula to calculate real rate of return)

(real rate of return =) 1.47058...(%)

(A1)**EITHER**

$$4000 = PV \times 1.0147058...^{10}$$

(A1)**OR**

$$N = 10$$

$$I = 1.47058...$$

$$FV = 4000$$

$$P/Y = 1$$

$$C/Y = 1$$

(M1)**Note:** Award **M1** for all entries correct.**THEN**

$$(PV =) \$3457$$

A1**[3 marks]**

- (b)
- METHOD 1 – (Finding the future value of the investment using PV from part (a))**

$$N = 10$$

$$I = 3.5$$

$$PV = 3446.66...(from Method 1) \text{ OR } 3456.67...(from Methods 2, 3)$$

$$P/Y = 1$$

$$C/Y = 1$$

(M1)**Note:** Award **M1** for interest rate 3.5 **and** answer to part (a) as PV.

$$(FV =) \$4861.87 \text{ OR } \$4875.97$$

(A1)so payment required (from TVM) will be \$294 **OR** \$295**A1****Note:** Award **A0** if a negative sign given, unless already penalized in part (a).**METHOD 2 – (Using FV)**

$$N = 10$$

$$I = 3.5$$

$$PV = -1000$$

$$FV = 4875.977...$$

$$P/Y = 1$$

$$C/Y = 1$$

(A1)(M1)**Note:** Award **A1** for $I = 3.5$ **and** $FV = \pm 4875.977...$, **M1** for all other entries correct **and** opposite PV and FV signs.

$$(PMT =) \$295 \text{ (295.393)}$$

A1**Note:** Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award **A0** if a negative sign given, unless already penalized in part (a).**[3 marks]****Total [6 marks]**

13. (a) METHOD 1

recognizing that the real part is distributive

(M1)

$$V_T = \operatorname{Re}\left(2e^{3ti} + 5e^{3ti+4i}\right)$$

$$= \operatorname{Re}\left(e^{3ti}\left(2 + 5e^{4i}\right)\right)$$

(A1)

$$(\text{from the GDC}) \quad 2 + 5e^{4i} = 3.99088\dots e^{-1.89418\dots i}$$

(A1)

Note: Accept arguments differing by 2π e.g. $4.38900\dots$.

$$\text{therefore } V_T = 3.99 \cos(3t - 1.89) \quad (3.99088\dots \cos(3t - 1.89418\dots))$$

A1

Note: Award the last **A1** for the correct values of A , B and C seen either in the required form or not. If method used is unclear and answer is partially incorrect, assume Method 2 and award appropriate marks eg. **(M1)A1A0A0** if only A value is correct.

continued...

Question 13 continued**METHOD 2**

converting given expressions to cos form

(M1)

$$V_T = 2 \cos 3t + 5 \cos(3t + 4)$$

(from graph) $A = 3.99$ (3.99088...)**A1**

$$V_T = 3.99 \cos(Bt + C)$$

either by considering transformations or inserting points

$$B = 3$$

A1

$$C = -1.89 \text{ } (-1.89418\dots)$$

A1

Note: Accept arguments differing by 2π e.g. 4.38900....

$$(\text{so, } V_T = 3.99 \cos(3t - 1.89) \text{ } (3.99088\dots \cos(3t - 1.89418\dots)))$$

Note: It is possible to have $A = 3.99$, $B = -3$ with $C = 1.89$ **OR** $A = -3.99$, $B = 3$ with $C = 1.25$
OR $A = -3.99$, $B = -3$ with $C = -1.25$ due to properties of the cosine curve.

[4 marks]

(b) maximum voltage is 3.99 (3.99088...) (units)

A1**[1 mark]****Total [5 marks]**

1. (a) (i) **EITHER**

$$115.5 = u_1 + (3-1) \times d \quad (115.5 = u_1 + 2d)$$

$$108 = u_1 + (8-1) \times d \quad (108 = u_1 + 7d) \quad \textbf{(M1)(A1)}$$

Note: Award **M1** for attempting to use the arithmetic sequence term formula, **A1** for both equations correct. Working for **M1** and **A1** can be found in parts (i) or (ii).

$$(d = -1.5)$$

$$1.5 \text{ (cups/day)}$$

A1

Note: Answer must be written as a positive value to award **A1**.

OR

$$(d =) \frac{115.5 - 108}{5}$$

M1A1

Note: Award **M1** for attempting a calculation using the difference between term 3 and term 8; **A1** for a correct substitution.

$$(d =) 1.5 \text{ (cups/day)}$$

A1

$$(ii) \quad (u_1 =) 118.5 \text{ (cups)}$$

A1**[4 marks]**

(b) attempting to substitute their values into the term formula for arithmetic sequence equated to zero

(M1)

$$0 = 118.5 + (n-1) \times (-1.5)$$

$$(n =) 80 \text{ days}$$

A1

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

$$(c) \quad (t_5 =) 625 \times 1.064^{(5-1)}$$

(M1)(A1)

Note: Award **M1** for attempting to use the geometric sequence term formula; **A1** for a correct substitution.

$$\text{\$ } 801$$

A1

Note: The answer must be rounded to a whole number to award the final **A1**.

[3 marks]*continued...*

Question 1 continued(d) (i) $(S_{10} =) (\$) 8390 \text{ (8394.39...)}$ **A1**(ii) **EITHER**

the total cost (of dog food)

R1for 10 years beginning in 2021 **OR** 10 years before 2031**R1****OR**

the total cost (of dog food)

R1from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031**R1****[3 marks]**(e) **EITHER**

According to the model, the cost of dog food per year will eventually be too high to keep a dog.

OR

The model does not necessarily consider changes in inflation rate.

OR

The model is appropriate as long as inflation increases at a similar rate.

OR

The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.

OR

The model is appropriate since dog food bags can only be bought in discrete quantities.

R1

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either “model” is mentioned specifically, or other mathematical terms such as “increasing” or “discrete quantities” are seen. Do not accept a contextual argument in isolation, e.g. “The dog will eventually die”.

[1 mark]**Total [13 marks]**

7. (a) $I\% = 7.5$
 $PV = \mp 800$
 $PMT = \mp 500$
 $FV = \pm 10\,000$
 $P/Y = 12$
 $C/Y = 12$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award **A1** for $PMT = -500$ or $PMT = 500$, with same sign to PV and opposite sign to FV .

17.3070...

(A1) $(k =) 18$ **A1**

Note: Award **(M0)(A0)(A0)A0** for a final answer of 17 with no working. The final answer must be an integer.

[4 marks]

- (b) $10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$

(A1)(M1)

Note: Award **(A1)** for 10389 (10389.38...) seen. Award **(M1)** for subtraction of their $(18 \times 500 + 800)$ from FV . **FT** from their value of k . Award **A0M1A0** for $10\,000 - (18 \times 500 + 800)$. Do not award the final **A1FT** if their answer is negative.

589 EUR

A1

Note: Final answer must be to the nearest euro.

[3 marks]**Total [7 marks]**

1. (a) $N = 24$

$$I = 4$$

$$PV = \pm 1000$$

$$PMT = \pm 100$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).

Approaches that use the compound interest formula receive no marks.

Award **A1** for correct values of PV and PMT (signs must be the same) **and** a correct value of N .

$$FV = (\$)3577.43$$

A1

Note: Award at most **(M1)(A1)A0** if the final answer is negative or not rounded to 2 dp.

[3 marks]

(b) $N = 36.5$ (36.4689...)

(A1)

$$N = 37 \text{ (months)}$$

A1

Note: Allow **FT** from incorrect GDC inputs seen in part (a) for the first **A1** providing that PV and FV have opposite signs and the resulting value of N is positive.

[2 marks]

[Total: 5 marks]

12. METHOD 1 Analytical approach

attempt to express V_1 or V_2 in exponential form **(M1)**

e.g. $V_1 = \text{Im}(6e^{i(at+\frac{\pi}{6})})$, $V_2 = \text{Im}(6e^{i(at+\frac{\pi}{2})})$

Note: Accept angles in radians or degrees.

$$(V_1 + V_2 \Rightarrow) 6e^{i \times \frac{\pi}{6}} + 6e^{i \times \frac{\pi}{2}} \quad \textbf{(A1)}$$

Note: This mark can be awarded even if seen as part of a correct larger expression.

$$= 10.4e^{1.05i} \left(6\sqrt{3}e^{\frac{i\pi}{3}} \right) \quad \textbf{(A1)}$$

so V is 10.4 (10.3923..., $6\sqrt{3}$) and θ is 60 (degrees) **A1A1**

Note: Accept any value for θ that rounds to a 2sf answer of 60.

Do **not** accept a final answer for an angle in radians.

Do **not** award **A1** for answer of 60° resulting from incorrect working.

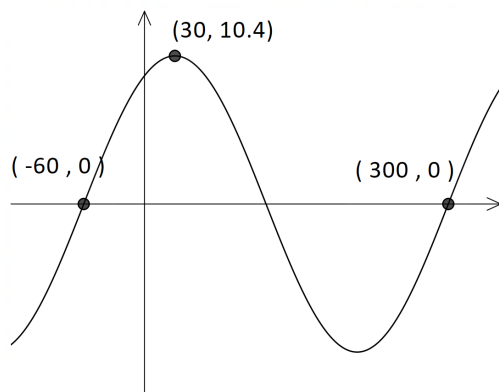
continued...

Question 12 continued

METHOD 2 Graphical approach

let $at = x$ and plot $V_1 + V_2$ curves on GDC

(M1)



attempt to find maximum

(M1)

$$V = 10.4$$

A1

attempt to find any x -axis intercept (either -60 or 300)

(M1)

$$\theta = 60 \text{ (degrees)} \quad (\theta = -300 \text{ (degrees)})$$

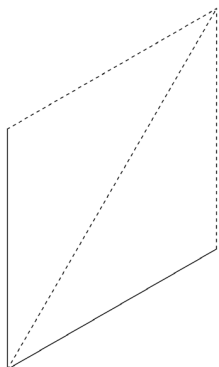
A1

continued...

Question 12 continued

METHOD 3 Geometric approach

considering the rhombus



(M1)

$$V = \sqrt{6^2 + 6^2 - 2 \times 6 \times 6 \cos 120^\circ}$$

(M1)

$$\left(= \sqrt{108} = 6\sqrt{3} \right) = 10.4 \text{ (10.3923...)}$$

A1

$$\theta = 60 \text{ (degrees)}$$

A2

Note: An answer of $\theta = -300^\circ$ is most likely to be seen in METHOD 2, but should be condoned in METHODS 1 and 3 if seen there.

[Total: 5 marks]

4. (a) **METHOD 1 (use of financial app in GDC)**

$$\begin{array}{ll}
 N = 5 & \text{OR} & N = 20 \\
 I\% = 1.2 & & I\% = 1.2 \\
 PV = \pm 520 & & PV = \pm 520 \\
 P/Y = 1 & & P/Y = 4 \\
 C/Y = 4 & & C/Y = 4
 \end{array}$$

(M1)(A1)

Note: Award **M1** for evidence of using the financial app on the calculator, **A1** for all correct entries.

(\$) 552.11

A1

Note: Award at most **(M1)(A1)A0** if correct answer is not given to two decimal places.

METHOD 2 (use of formula)

attempt to substitute into compound interest formula

(M1)

$$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4}$$

(A1)

(\$) 552.11

A1

Note: Award at most **(M1)(A1)A0** if correct answer is not given to two decimal places.

[3 marks](b) **EITHER**

$$\begin{array}{l}
 N = 5 \\
 I\% = 43.5 \text{ (43.4772...(\%))} \\
 PV = \pm 520 \\
 FV = \mp 30
 \end{array}$$

(M1)(A1)A1

Note: Award **M1** for evidence of using the finance app on the calculator, **A1** for all correct entries, **A1** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

OR

$$30 = 520 \left(1 - \frac{r}{100}\right)^5 \text{ (or equivalent)}$$

(M1)(A1)

$$(r =) 43.5\% \text{ (43.477...(\%))}$$

A1

Note: Award **M1** for using the compound interest formula, **A1** for correct substitutions and for equating to 30, **A1** for correct final answer. Accept $(r =) -43.5\%$.
Award **M1A1A0** for a final answer of 56.5%.

[3 marks]**Total [6 marks]**

11. (a) $30 \sin(t + 60^\circ) + 60 \sin(t + 10^\circ)$ (M1)
 finding maximum graphically
 $82.5 \text{ (V)} \text{ } (82.5471\dots)$ A1

Note: Award **M1A0** for 83.

[2 marks]

- (b) recognizing that a is still 1 A1
 $V_0 = 82.5$ A1
 attempt to find an x -intercept of combined voltage (M1)
 $b = 26.2^\circ \text{ } (26.1643\dots^\circ)$ **OR** any other correct x -intercept A1

Note: May be seen in the final answer. Award **M1A0** for $b = 26$ with no working.

$$\left(V_{\text{TOT}} = 82.5 \sin(t + 26.2^\circ) \text{ } (82.5471\dots \sin(t + 26.1643\dots^\circ)) \right)$$

Note: Award at most **(M1)A1(A1)A0** if phase shift of $-153.835\dots$ is seen in the final answer.
 In part (b), candidates may use $\arg(30e^{60i} + 60e^{10i})$ to determine the new phase shift,
 and hence could be awarded **M1** for this valid method.

[4 marks]

Total [6 marks]

1. (a) attempt to use the term formula for a geometric sequence **(M1)**
 $4 \times (0.8)^2$
 $= 2.56 \text{ (mm)} \left(\frac{64}{25} \right)$ **A1**
[2 marks]
- (b) attempt to use the sum formula for a geometric sequence **(M1)**
 $\frac{4 \times (1 - 0.8^{12})}{1 - 0.8}$
 $= 18.6 \text{ (mm)} \text{ (18.6256...)}$ **A1**
[2 marks]
- (c) attempt to use infinite geometric sum **(M1)**
e.g. $\frac{4}{1 - 0.8}$ **OR** 20
adding 25 to their expression or value **(M1)**
(maximum width = $25 + 20$)
45 (mm) **A1**
[3 marks]
[Total 7 marks]

8. (a) $z = 2e^{-0.524i} \left(= 2e^{-\frac{\pi i}{6}} \right)$

A1A1

Note: Award **A1** for the correct modulus and **A1** for the correct argument.

[2 marks](b) **METHOD 1**

$$z_1 + z_2 = e^{2ti} \left(1 + 2e^{-\frac{\pi i}{6}} \right)$$

(M1)

$$= e^{2ti} \times 2.90931...e^{-0.350879...i}$$

(A1)

$$\text{Im}(z_1 + z_2) = 2.91 \sin(2t - 0.351)$$

A1A1**METHOD 2**

attempt to find the modulus of $\left(1 + 2e^{-\frac{\pi i}{6}} \right)$

(M1)

$$p = \left| 1 + 2e^{-\frac{\pi i}{6}} \right|$$

attempt to find the argument of $\left(1 + 2e^{-\frac{\pi i}{6}} \right)$

(M1)

$$q = \arg \left(1 + 2e^{-\frac{\pi i}{6}} \right)$$

$$\text{Im}(z_1 + z_2) = 2.91 \sin(2t - 0.351)$$

A1A1**METHOD 3**

sketching $\text{Im}(z_1 + z_2)$

(M1)

max is (0.961, 2.91)

(A1)

first root is 0.1754

(A1)

$$\text{Im}(z_1 + z_2) = 2.91 \sin(2t - 0.351)$$

A1**[4 marks]****[Total 6 marks]**

3. (a) (i) attempt to find 15% or 85% of 285000 (M1)
 285000×0.85
 242 250 (USD) A1

Note: Do not award **A1** if answer is not given exact.

- (ii) $N = 360$
 $I\% = 4$
 $PV = (\pm) 242\,250$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ (M1)(A1)

Note: Award **M1** for an attempt to use a financial app in their technology with at least two entries seen, award **A1** for all entries correct.

$(PMT =) 1156.54$ (USD) A1

Note: Do not award final **A1** if answer is not given to 2 dp.

[5 marks]

- (b) 1156.54×360 (M1)
 416354 (USD) A1

Note: Do not award **A1** if answer is not given to the nearest dollar, unless already penalized in part (a)(ii).

[2 marks]

- (c) $I\% = 4$
 $PV = (\pm) 242\,250$
 $PMT = (\mp) 1300$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ (A1)

Note: Award **A1** for $PMT = (\mp) 1300$ seen.

- $(N =) 292$ A1
 [2 marks]

- (d) **METHOD 1**
 $N = 291$
 $I\% = 4$
 $PV = (\pm) 242\,250$
 $PMT = (\mp) 1300$
 $P/Y = 12$
 $F/Y = 12$ (A1)

Note: Award **A1** for $N = 291$ seen.

- $(FV =) 871.91$ (871.908...) A1

valid attempt to find interest in final month (e.g. $N = 1$ **OR** $PV = 871.91$) (M1)

$$\begin{aligned} N &= 1 \\ I\% &= 4 \\ PV &= 871.91 \text{ (871.908...)} \\ FV &= 0 \\ P/Y &= 12 \\ F/Y &= 12 \end{aligned}$$

- $(PMT =) 874.82$ (USD) A1

Note: Do not award **A1** if answer is not given correct to 2dp, unless already penalized previously.

METHOD 2

$$N = 292$$

$$I\% = 4$$

$$PV = (\pm) 242\,250$$

$$PMT = (\mp) 1300$$

$$P/Y = 12$$

$$F/Y = 12$$

(A1)**Note:** Award **A1** for $N = 292$ seen.

$$(FV \Rightarrow) 425.185\dots$$

A1

$$1300 - 425.185\dots$$

(A1)

$$(PMT \Rightarrow) 874.82 \text{ (USD)}$$

A1**Note:** Accept 874.81. Do not award **A1** if answer is not given correct to 2dp, unless already penalized previously.**[4 marks]**

(e) $291 \times 1300 + 874.82$

(M1)

$$379174.82$$

attempt to find difference between their value and their part (b)
 $(416354 - 379174.82)$

(M1)

$$37179 \text{ (USD)}$$

A1**Note:** Accept 37180 (USD) from using the 2 dp. answer from part (b). Do not penalize for not rounding to nearest dollar if this has already been penalized in part (b).**[3 marks]****[Total 16 marks]**

1. (a) **EITHER**

$$N = 96$$

$$PV = \mp 100000$$

$$FV = \pm 150000$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)**OR**

$$N = 8$$

$$PV = \mp 100000$$

$$FV = \pm 150000$$

$$P/Y = 1$$

$$C/Y = 12$$

(M1)(A1)**OR**

$$150000 = 100000 \left(1 + \frac{I}{100 \times 12} \right)^{12 \times 8}$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use a compound interest formula.
Award **A1** for all entries correct in financial app or correct substitution in compound interest formula.

THEN

$$I = 5.08 \text{ (5.07903...)}$$

A1**[3 marks]**(b) $N = 120$

$$I\% = 6.1$$

$$PV = \mp 150000$$

$$PMT = \pm 1000$$

$$P/Y = 12$$

$$C/Y = 12$$

(M1)(A1)

Note: Award **M1** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award **A1** for all entries correct in financial app (condone missing +/- sign if the correct final answer is seen).

$$FV = (\$) 110867$$

A1

Note: Answer must be correct to nearest dollar to award the final **A1**. Award **(M1)(A1)A0** for an unsupported final answer to a greater degree of accuracy eg. (\$) 110866.70...
Award **M1A1A0** for a truncated answer of 110866 if no working is shown.

[3 marks]**[Total: 6 marks]**

6. (a) attempt to substitute into geometric sequence formula for twelfth term **OR**
at least three correct terms of the sequence **(M1)**

$$u_{12} = 40 \times 1.1^{12-1} \text{ OR } 40, 44, 48.4, \dots$$

$$114 \quad (114.124\dots)$$

A1**[2 marks]**

- (b) (i) attempt to substitute into geometric series formula **OR**
a sum of at least the first three terms **(M1)**

$$S_{12} = \frac{40(1.1^{12} - 1)}{1.1 - 1} \quad \text{OR} \quad \sum_1^{12} (40 \times 1.1^{n-1}) \quad \text{OR} \quad 40 + 44 + 48.4 + \dots$$

Note: Award **M1** for $u_1 = 40$ and $r = 1.1$ seen as part of a geometric series formula, or **M1** for sigma notation and their u_n formula (condone missing limits), or **M1** for the sum of at least the **correct** first three terms of the sequence.

$$S_{12} = 855 \quad (855.371\dots)$$

A1

- (ii) finding $S_{24} = 3539.89\dots$ or attempt to find the sum between u_{13} and u_{24} **(M1)**

Note: Award **M1** for $S_{24} = 3539.89\dots$ or sigma notation that includes correct limits and their u_n formula or a substituted geometric series formula that includes $125.537\dots$ and $n = 12$ or a list of terms that includes at least the 13th term and the 24th term.

$$3539.89\dots - 855.371\dots \quad \text{OR} \quad \sum_{13}^{24} (40 \times 1.1^{n-1}) \quad \text{OR}$$

$$(S_{13 \text{ to } 24} =) \frac{125.537\dots(1.1^{12} - 1)}{1.1 - 1} \quad \text{OR} \quad 125.537 + \dots + 358.172\dots \quad \text{(A1)}$$

Note: Accept a calculation using $u_{13} = 125$ or 126.

$$2680 \quad (2684.52\dots, 2684, 2685)$$

A1

Note: For $u_{13} = 125$, the sum is $2673.03\dots$ and for $u_{13} = 126$, the sum is $2694.41\dots$

[5 marks]**[Total: 7 marks]**

13. (a) (i) $\left(|z_1| = \sqrt{4^2 + 5^2} =\right) 6.40 \text{ (} 6.40312\dots, \sqrt{41}\text{)}$ **A1**

(ii) $\left(\arg(z_1) =\right) 0.896 \left(0.896055\dots, 51.3401\dots^\circ, \arctan\left(\frac{5}{4}\right)\right)$ **A1**

[2 marks]

(b) angle in triangle is $2 - 0.896055\dots$ **OR** $114.591^\circ - 51.3401\dots^\circ$ **(A1)**
use of area of triangle formula **(M1)**

$\frac{1}{2} \times 6.40312\dots \times 3 \times \sin(2 - 0.896055\dots)$ **(A1)**

8.58 (8.57688...) **A1**

Note: Accept methods that use Cartesian form or vector product.

[4 marks]

[Total 6 marks]

1. (a) **EITHER**

$$\begin{array}{ll} N = 72 & N = 6 \\ I = 1.25 & I = 1.25 \\ PV = -3000 & \text{OR} \quad PV = -3000 \end{array}$$

$$\begin{array}{ll} P/Y = 12 & P/Y = 1 \\ C/Y = 12 & C/Y = 12 \end{array}$$

(M1)(A1)

Note: Award **M1** for attempt to use finance app on GDC, **A1** for all entries correct.

OR

$$3000 \left(1 + \frac{1.25}{1200} \right)^{72}$$

(M1)(A1)

Note: Award **M1** for using CI formula, **A1** for correct entries.

THEN

$$FV = (\$) 3233.53$$

A1

Note: Answer must be correct to 2 dp for the final **A1** to be awarded.

[3 marks](b) **EITHER**

$$\begin{array}{ll} I = 1.25 & I = 1.25 \\ PV = -3000 & PV = -3000 \\ FV = 3550 & \text{OR} \quad FV = 3550 \end{array}$$

$$\begin{array}{ll} P/Y = 12 & P/Y = 1 \\ C/Y = 12 & C/Y = 12 \end{array}$$

(A1)

Note: Award **A1** for all entries correct and opposite signs for PV and FV values.

OR

$$3550 = 3000 \left(1 + \frac{1.25}{1200} \right)^{12N}$$

(A1)

Note: Award **A1** for all entries correct.

THEN

$$N = 162 \text{ (161.686...)} \text{ (months)}$$

A1**[2 marks]**

(c) 18450

A1**[1 mark]**

- (d) EITHER
- N = 96

I = 12.6

PV = −18450

FV = 0

P/Y = 12

C/Y = 12

OR

N = 96

I = 1.05

PV = −18450

FV = 0

P/Y = 1

C/Y = 1
- (M1)(A1)

Note: Award **M1** for attempt to use finance app on GDC, **A1** for N=96 , PV = −18450 and FV=0,

THEN

PMT = (\$) 306

A1

Note: The answer must be correct to the nearest dollar for the final **A1** to be awarded.

[3 marks]

[Total: 9 marks]

5. (a) recognizing that the growth in one year is the difference in the two heights (M1)
 $5 \times 0.8 = 4$ (m) A1

[2 marks]

- (b) recognizing geometric sequence, $r = 0.8$ (M1)
 attempt to find total height by adding initial height to a term in series (M1)

EITHER

$$42 + \frac{4(1 - (0.8)^6)}{1 - (0.8)} \quad (A1)$$

OR

$$37 + \frac{5(1 - (0.8)^7)}{1 - (0.8)} \quad (A1)$$

THEN

$$= 56.7571\dots$$

$$= 56.76 \text{ (m)} \quad \text{OR} \quad 5676 \text{ (cm)} \quad A1$$

Note: Accept an answer in cm or in m to two decimal places.

[4 marks]

- (c) attempt to use infinite geometric series to find the total growth of the tree (M1)
 e.g. $\frac{5}{1 - 0.8}$ OR $\frac{4}{1 - 0.8}$

$$\left(S_{\infty} = 37 + \frac{5}{1 - 0.8}, S_{\infty} = 42 + \frac{4}{1 - 0.8} \right)$$

$$k = 62 \quad A1$$

[2 marks]

[Total: 8 marks]

15. (a) $\sqrt{2} e^{\frac{5\pi i}{4}}$

A1A1

[2 marks]

(b) (i) enlargement SF $\sqrt{2}$, (1.41, 1.41421...), (centre (0, 0))

A1

rotation of $\frac{3\pi}{4}$, (2.36, 2.35619...), (centre (0, 0))

A1

(ii)
$$\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} \cos\left(\frac{3\pi}{4}\right) & -\sin\left(\frac{3\pi}{4}\right) \\ \sin\left(\frac{3\pi}{4}\right) & \cos\left(\frac{3\pi}{4}\right) \end{pmatrix}$$

(A1)(A1)

Note: The order of matrix multiplication does not matter in this case.

$$= \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$$

A1

Note: Do not **FT** if their matrices in part (b)(i) are not representative of a dilation AND a rotation.

[5 marks]

(c) 8

A1

Note: Since $\frac{3\pi n}{4}$ must be a multiple of 2π .

[1 mark]

[Total: 8 marks]

4. (a) recognition that % increase gives the common difference **(M1)**
 $25000 \times 0.05 = 1250$ **A1**
[2 marks]
- (b) $25000 + 1250n > 44000$ **OR** $25000 + 1250(n-1) > 44000$ (accept equality) **(A1)**
 attempt to solve their inequality **OR** equality (sketch **OR** cross-over values
OR algebraic manipulation) **(M1)**
 $k=16$ **A1**

Note: Award **(A1)(M1)A0** for 15.2, 16.2 or 17 seen.

[3 marks]
[Total: 5 marks]

7. (a) $N = 60$, $I = 4.5$, $PV = +400\,000$, $PMT = -3600$, $P/Y = 12$, $C/Y = 12$ (M1)(A1)

Note: Award **M1** for use of Finance solver with any 2 entries correct, Award **A1** for all correct.

$$FV = \$259\,000 (\$258\,994)$$

A1
[3 marks]

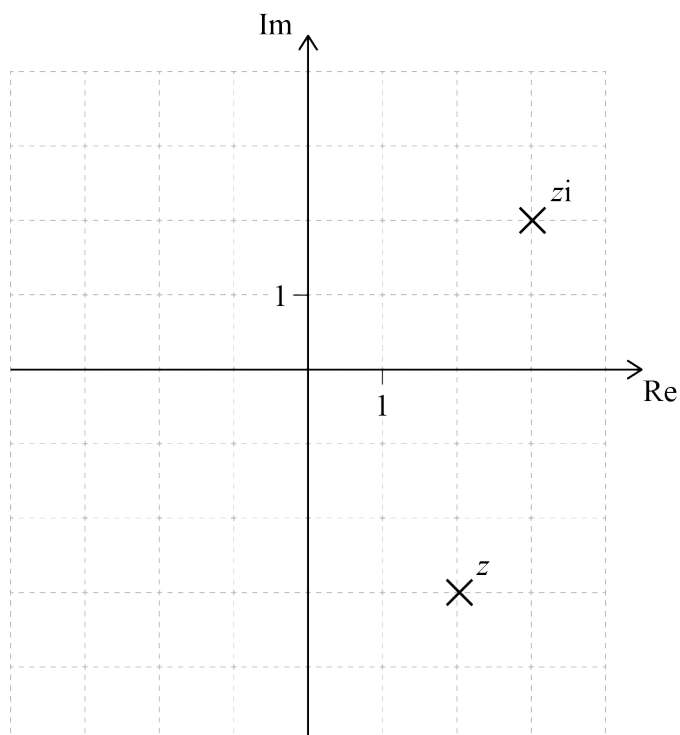
- (b) $I = 4.5$, $PV = +400\,000$, $PMT = -3600$, $FV = 0$, $P/Y = 12$, $C/Y = 12$ (A1)(A1)

Note: Award **A1** for $FV = 0$, award **A1** for all correct.

$$N = 144$$

A1
[3 marks]
[Total 6 marks]

9. (a)



A1
[1 mark]

(b) (i) 3.61 ($3.60555\dots$, $\sqrt{13}$)

A1

(ii) -0.983 ($-0.982793\dots$)

A1
[2 marks]

(c) $3 + 2i$

A1
[1 mark]

(d) **EITHER**

rotation, (anticlockwise) $\frac{\pi}{2}$ (or 90°) (about the origin)

A1

OR

translation $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$

A1

[1 mark]
[Total 5 marks]

11. (a) attempt to find $|M - \lambda I|$ (M1)
 $(-4 - \lambda)(3 - \lambda) + 6 (= 0)$ (A1)
 $2, -3$ A1
[3 marks]

(b) (i) $\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$ OR $\begin{pmatrix} -3 & 0 \\ 0 & 2 \end{pmatrix}$ A1

(ii) EITHER

attempt to solve $M \begin{pmatrix} x \\ y \end{pmatrix} = 2 \begin{pmatrix} x \\ y \end{pmatrix}$ OR $M \begin{pmatrix} x \\ y \end{pmatrix} = -3 \begin{pmatrix} x \\ y \end{pmatrix}$ (M1)

OR

attempt to use $(M - \lambda I) \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ (M1)

THEN

$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ (or any multiple) (A1)(A1)

$P = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ OR $P = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ A1

Note: Award **A1** for correct eigenvectors in a matrix in the correct position for their **D**.

[5 marks]
[Total 8 marks]

3. (a) recognition of a geometric sequence (M1)

$$3\left(\frac{2}{3}\right)^4 \text{ OR } 3, 2, 4/3 \dots$$

$$= 0.593 \text{ (0.592592..., 16/27) (cm)}$$

A1
[2 marks]

- (b) attempt to sum a geometric sequence (M1)

$$\frac{3\left(1-\left(\frac{2}{3}\right)^5\right)}{1-\frac{2}{3}} \text{ OR } 3+2+\frac{4}{3} \dots$$

$$= 7.81 \text{ (7.81481..., 211/27) (cm)}$$

A1
[2 marks]

- (c) recognition of need to find sum to infinity (M1)

$$\frac{3}{1-\frac{2}{3}}$$

$$= 9 \text{ (cm)}$$

(A1)

A1
[3 marks]

- (d) Comparing the sum of the widths greater than (or equal to) 8.5 (M1)

$$\text{e.g. } \frac{3\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\frac{2}{3}} \geq 8.5 \text{ OR sketch OR list of values with cross-over values}$$

$$7.13 \text{ (7.1285338740543...) seen}$$

$$n \geq 7.13$$

$$n = 8$$

(A1)

A1
[3 marks]

- (e) attempt to divide two adjacent areas OR $\left(\frac{2}{3}\right)^2$ (M1)

$$\frac{4}{9} \text{ (0.444, 0.444444...)}$$

A1
[2 marks]

continue...

Question 3 continued.

(f) $u_1 = 13.5$ (may be seen in part (e))

(A1)

attempt to find the sum of their n terms, with their u_1 and their r

M1

$$S_8 = \frac{13.5 \left(1 - \left(\frac{4}{9} \right)^8 \right)}{1 - \frac{4}{9}}$$

Note: Do not award **M1** if 5 or infinity used for n or if $\frac{2}{3}$ used for r .

$$= 24.3 \text{ (24.2630...) (cm}^2\text{)}$$

A1

Note: Accept 24.2 (24.2439...) from using 0.444.

[3 marks]

[Total: 15 marks]

3. (a) (i) $N = 72$
 $I\% = 7.15$
 $PV = \pm 42695$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ **(M1)(A1)**

Note: Award **(M1)** for attempt to use finance app in GDC and **(A1)** for the first five entries correct.

$(PMT =) 730.99$ (USD) **A1**

Note: Do not award final **A1** if answer is not given to 2 d.p. or if given as a negative value.

- (ii) attempt to find 80% of 42 695 **(M1)**

$N = 72$
 $I\% = 6.40$
 $PV = \pm 34156$
 $FV = 0$
 $P/Y = 12$
 $C/Y = 12$ **(A1)**

$(PMT =) 572.54$ (USD) **A1**

Note: Do not award final **A1** if answer is not given to 2 d.p. or if given as a negative value.

[6 marks]

- (b) (i) does not have (enough) money for initial payment / does not want to make initial payment **R1**
- (ii) lower monthly payments / will pay less over the life of the loan / less interest will be paid over life of loan **R1**

[2 marks]

[Total 8 marks]

10. (a) correct use of one log law (M1)
 x^{-2} OR x^2 OR $(2-x)(-2-x)$

$$(g(x) =) \frac{x^2 - 4}{x^2} \text{ OR } \frac{(2-x)(-2-x)}{x^2} \text{ OR } 1 - 4x^{-2} \text{ OR equivalent} \quad \text{A1}$$

[2 marks]

- (b) $f^{-1}(x) < -2$ OR $(-\infty, -2)$ A2

Note: Award **A1** for strictness with -2 using either $<$ or parenthesis and **A2** for completely correct inequality. Award **A2** for $y < -2$.

[2 marks]

- (c) attempt to exchange variables (seen anywhere) (M1)

$$x = \ln\left(\frac{y^2 - 4}{y^2}\right) \text{ OR } x = \ln(1 - 4y^{-2})$$

$$e^x = \frac{y^2 - 4}{y^2} \text{ OR } e^x = 1 - 4y^{-2} \quad \text{(A1)}$$

$$y^2(e^x - 1) = -4 \quad \text{(A1)}$$

$$(y =) \pm \frac{2}{\sqrt{1 - e^x}} \quad \text{A1}$$

(since domain of $f(x)$ is $x < -2$) $a = -2$ A1

[5 marks]

[Total 9 marks]

11. (a) (i) $(u_2 =) 6 + 6i$ A1

(ii) $(u_3 =) 8i$ A1
[2 marks]

(b) attempt to find $|r|$ (M1)

$$|r| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2} \quad \text{A1}$$

$$|r| = \frac{2\sqrt{2}}{3} \quad \text{A1}$$

statement that multiplying a number by a complex number changes the modulus of the number by a factor of $|r|$ (may be seen at any stage) R1

Note: Award at most **(M1)A1A0R1** if common ratio is given as 0.942809.....

therefore it is a geometric sequence AG
[4 marks]

(c) substitution of their r and 9 into the sum formula for infinite sequence (A1)

$$S_{\infty} = \frac{9}{1 - \frac{2\sqrt{2}}{3}}$$

$(S_{\infty} =) 157 \ (157.367\dots, 81 + 54\sqrt{2})$ A1

Note: No **FT** if their $|r| \geq 1$.

[2 marks]
 [Total 8 marks]

14. (a) $L_1(x) = \text{Im}\left(14.8e^{0.35xi}e^{1.27i}\right)$ **A1**
 $L_2(x) = \text{Im}\left(18.5e^{0.35xi}e^{-2.36i}\right)$ **A1**

[2 marks]

(b) $L_3 = \text{Im}\left(14.8e^{0.35xi}e^{1.27i} + 18.5e^{0.35xi}e^{-2.36i}\right)$

$L_3 = \text{Im}\left(e^{0.35xi}\left(14.8e^{1.27i} + 18.5e^{-2.36i}\right)\right)$ **(A1)**

$L_3 = \text{Im}\left(e^{0.35xi}\left(8.81559\dots e^{3.01604\dots i}\right)\right)$ **(A1)**

$L_3 = 8.81559\dots \sin(0.35x + 3.01604\dots)$

$q = 8.82 (8.81559\dots), r = 3.02 (3.01604\dots)$ **A1A1**

Note: Award both marks if last line missing, but values are seen in equation for L_3 .

[4 marks]

[Total 6 marks]

✓ - ✓

1. (a) after 10 years

 N : 120 $I\%$: 3.6 PV : 0 PMT : (-)1500 P/Y : 12 C/Y : 12**(M1)(A1)**

Note: Award **(M1)** for an attempt to use a financial app in their technology with at least two entries seen, award **A1** for all entries correct except PV.

 FV : 216 278.58 (euros)**A1**

Note: Award **A0** for a negative answer or answer not given to 2 decimal places.

[3 marks](b) **METHOD 1 (C.I. formula)**

invests: 116 278.58

(M1)after 15 years: $116278.58 \times \left(1 + \frac{4.5}{400}\right)^{60}$ **(M1)(A1)** FV : 227 515.92 (euros)**A1**

Note: Award the first **M1** for subtracting *their* FV from part (a) even if the value of the investment is negative.

Award the second **M1** for use of the C.I. formula.Award first **A1** for all entries correct except PV.

Allow full follow through from part (a).

Award final **A0** for FV not given to 2 decimal places unless this has already been penalized in (a).

Accept 227 515.93 from use of unrounded value.

METHOD 2 (financial app)

invests: 116 278.58

(M1)**THEN****EITHER** N : 60 $I\%$: 4.5 PV : 116 278.58 PMT : 0 P/Y : 4 C/Y : 4**OR** N : 15 $I\%$: 4.5 PV : 116 278.58 PMT : 0 P/Y : 1 C/Y : 4**OR** N : 180 $I\%$: 4.5 PV : 116 278.58 PMT : 0 P/Y : 12 C/Y : 4**(M1)(A1)**

continued...

Question 1 continued

Note: Award the first **M1** for subtracting *their FV* from part (a) even if the value of the investment is negative.

Award **(M1)** for an attempt to use a financial app in their technology with a least two entries seen, award **A1** for all entries correct except *PV*.

THEN

FV: 227 515.92 (euros)

A1

Note: Award **A0** for a negative answer or answer not given to 2 decimal places unless already penalized in part (a).

Accept 227 515.93 from use of unrounded value.

[4 marks]

Total [7 marks]

5. use of $\text{speed} = \frac{\text{distance}}{\text{time}}$ **OR** $\text{speed} \times \text{time} = \text{distance}$ **OR** $\text{time} = \frac{\text{distance}}{\text{speed}}$

seen anywhere

(M1)

attempt to find a bounding value of t (**OR** use appropriate values of t that support and contradict the statement that the car is speeding)

(M1)

$(t =) 1.25(\text{s})$ **OR** $(t =) 1.15(\text{s})$

(A1)

THEN

EITHER

use of their upper bound for t to find minimum speed (**OR** of any value of t where $1.20481... < t \leq 1.25$ to find speed)

$(\text{speed} =) \frac{10}{1.25}$ (or $\frac{10}{\text{their time}}$)

A1

$(\text{speed} =) 8(\text{ms}^{-1})$ (or a value where $8 \leq \text{speed} < 8.3$) seen

hence $\text{speed} < 8.3(\text{ms}^{-1})$

A1

OR

use of their upper bound for t to find maximum distance (**OR** of any value of t where $1.20481... < t \leq 1.25$ to find distance)

$(\text{distance} =) 8.3 \times 1.25$ (or $8.3 \times \text{their time}$)

A1

$(\text{distance} =) 10.375 \text{ m}$ (or a value where $10 < \text{distance} \leq 10.375$) seen

hence $\text{distance} > 10(\text{m})$

A1

OR

time taken at speed limit $(\text{time} =) \frac{10}{8.3}$

A1

$(\text{time} =) 1.20481...(\text{s})$

hence time of $1.2(\text{s})$ to the nearest $0.1(\text{s})$ could be greater than or less than this

A1

THEN

conclusion: it is not certain that the car was exceeding the speed limit.

A1

Note: The conclusion must have been justified by further showing that for some appropriate value of t the car is speeding

Award at most **M1M1A0A1A0A1** for the use of a value of t where $t > 1.25$

Award at most **M1M0A0A0A0A1** for $t = 1.2$ used with a corresponding justified conclusion.

Total [6 marks]

16. (a) an attempt to solve the equation for $t = 1$ showing at least one of the solutions $z = -1 + i\sqrt{7}$ or $z = -1 - i\sqrt{7}$ **M1**
 required solution $= -1 + 2.65i$ $(= -1 + 2.64575...i)$ $(= -1 + i\sqrt{7})$ **A1**
[2 marks]
- (b) using the quadratic formula **M1**
 $z = -t \pm \sqrt{t^2 - 8t}$
 $= -t \pm i\sqrt{8t - t^2}$ **A1**
 attempt to find $|z|^2$ **M1**
 $|z|^2 = t^2 + 8t - t^2 = 8t$ **A1**
 this is maximised when $t = 2$ **A1**
[5 marks]
Total [7 marks]
-

Q2.**(a)** attempt to substitute the given values in the function**(M1)**

$$(L =) 10 \log_{10} \left(\frac{10^{-5}}{10^{-12}} \right)$$

$$= 70(\text{dB})$$

A1**[2 marks]**

$$(b) \quad 185 = 10 \log_{10} \left(\frac{I}{10^{-12}} \right)$$

(A1)

attempt to solve their logarithmic equation using graph, solver or analytically

(M1)

$$I = 3160000 (\text{Wm}^{-2}) (3162277.66)$$

$$I = 3.16 \times 10^6 (3.16227... \times 10^6)$$

A1

Note: Award at most **A1M1A0** or **A0M1A1FT** for an incorrect answer written correctly in the required form.

[3 marks]**Total [5 marks]**

Q5.

(a) evidence of correct method

(M1)

$$N = 300, I\% = 4.2, PV = \mp 50000, FV = 0, P/Y = 12, C/Y = 12 \text{ OR } 269.4711 \text{ seen}$$

Note: Award **M1** for using the financial app on their GDC with at least 2 correct entries.

269.47(euros)

A1

Note: Award at most **(M1)A0** for -269.47 . Accept 269.48.

[2 marks](b) (i) evidence of correct method $N = 96$ **(M1)**

$$N = 96, I\% = 4.2, PV = \mp 50000, PMT = \pm 269.47, P/Y = 12, C/Y = 12$$

OR

$$N = 96, I\% = 0.35, PV = \mp 50000, PMT = \pm 269.47, P/Y = 1, C/Y = 1$$

Note: Award **M1** for using the financial app on their GDC with at least 2 correct entries.

39, 243.42

A1

Note: Accept 39,243.29 which results from the full calculator value for the payment in part (a). Accept 39,243.28 which results from the use of 269.48.

Note: If two decimal place accuracy was penalised in part (a) do not penalise again.

(ii) **METHOD 1**

$$\text{full payment } 12 \times 25 \times 269.47 = 80,841.00$$

(A1)

$$\text{actual payment } 12 \times 8 \times 269.47 + 39243.42$$

(M1)

$$= 65,112.54$$

(A1)

$$\text{money saved} = 15,700 \text{ (15,728.46) (euros)}$$

A1

Note: Accept 15,728.59 from using 269.47 from part (a) and 39,243.29 from part (b)(i).
Accept 15,730.64 from using 269.48 from part (a) and 39,243.28 from part b(i).

continued

*Question 5 continued***METHOD 2**

if Sophie did not pay back the loan early her subsequent payments
would be $269.47 \times 12 \times 17$

(M1)

Note: Award **M1** for attempting to find the remaining amount to pay if loan not paid back.

$$= 54971.88 \text{ (euros)}$$

(A1)

money saved is therefore
 $54971.88 - 39243.42$

(M1)

$$= 15,700(15,728.46) \text{ (euros)}$$

A1**[6 marks]****Total [8 marks]**

1. (a) (i) 7 A1
 (ii) 9 A1
[2 marks]
 (b) (i) 2 A1
 (ii) $1 + 2(n-1) (= 2n-1)$ (M1)A1

Note: (M1) for substituting their (b)(i) into the arithmetic sequence formula.

- (c) (i) 17 [3 marks] A1
 (ii) $\frac{9}{2}(1+17)$ OR $\frac{9}{2}(2 \times 1 + (9-1) \times 2)$ OR $1 + 3 + \dots + 15 + 17$ (M1)
 $= 81$ A1

- (d) $\frac{n}{2}(1 + (2n-1))$ OR $\frac{n}{2}(2 \times 1 + (n-1) \times 2)$ [3 marks] (M1)
 $= n^2$ A1

- (e) 10 [2 marks] A1

- (f) $1 + 2 + 3 + \dots + n$ [1 mark] (M1)
 $= \frac{n}{2}(1+n)$ A1

Note: Award (M1) for attempt for use of sum to n terms of an arithmetic sequence.

- (g) $\frac{n(1+n)}{2} + \frac{n(n-1)}{2}$ [2 marks] M1

Note: Award M1 for addition of answers to parts (f) and given expression.

$$= \frac{n}{2}[(1+n) + (n-1)] \quad \text{OR} \quad \frac{n + n^2 + n^2 - n}{2} \quad \text{M1}$$

Note: Award M1 for valid non-trivial attempt at further simplification of their sum.

$$= \frac{n}{2}(2n) \quad \text{OR} \quad \frac{2n^2}{2} \\ = n^2 \quad \text{A1}$$

Note: Award a maximum of M1M1A0 for ft from an incorrect part (f).

[3 marks]
Total [16 marks]

- Q7.** (a) argument is 1.31 $(\frac{5\pi}{12}, 1.30899\dots)$ **(A1)**
 modulus is 5 **(A1)**
 $5e^{1.31i} (5e^{\frac{5\pi}{12}i})$ **A1**

Note: Award **A1A1A0** for correct answer $1.29409\dots + 4.82962\dots i$ in incorrect form

[3 marks]

- (b) (i) length of branches in next iteration = 0.7×5 **(M1)**
 $2 \times 5 + 4(0.7 \times 5)$ **A1**
 $= 24$ **AG**

Note: Award **M1A0** for $2 \times 5i + 4(0.7 \times 5i)$.

- (ii) **EITHER**
 recognition of a geometric sequence / series: for example a value given for r , or consideration of the lengths of branches added. **(M1)**
 $r = 2 \times 0.7 (= 1.4)$ **(A1)**
 use of formula for geometric series for 10 terms **(M1)**

$$\frac{10((0.7 \times 2)^{10} - 1)}{0.7 \times 2 - 1}$$
 (A1)
OR
 attempt to find the formula for n^{th} term, for example correct first three terms found
 $2 \times 5 + 2^2 \times 5 \times 0.7 + 2^3 \times 5 \times 0.7^2 + \dots$ **(M1)**
 $n^{\text{th}} \text{ term} = 5 \times 0.7^{n-1} \times 2^n$ **(A1)**
 attempt to find a summation for 10 terms clearly shown **(M1)**

$$\sum_{n=1}^{10} 5 \times 0.7^{n-1} \times 2^n$$
 (A1)
THEN
 $= 698(698.136\dots)$ **A1**

continued.

Question 7 continued.

(iii) **EITHER**as common ratio of the geometric series = 1.4 (> 1)**R1****OR**as the **total** length added at each stage is increasing**R1****THEN**

there is no limit

A1**Note:** Do not award **R0A1**.**[9 marks]**(c) (i) correct moduli 5, 5×0.7 , 5×0.7^2 (5, 3.5, 2.45)**A1**attempt to calculate the argument by adding or subtracting $\frac{\pi}{12}$ (0.261799...)(**M1**)correct arguments 1.31, 1.05, 1.31 $\left(\frac{5\pi}{12}, \frac{4\pi}{12}, \frac{5\pi}{12}\right)$ **A1**

$$b = 5e^{1.31i} + 3.5 \times e^{1.05i} + 2.45 \times e^{1.31i} \left(= 5e^{\frac{5\pi}{12}i} + 3.5 \times e^{\frac{\pi}{3}i} + 2.45 \times e^{\frac{5\pi}{12}i} \right)$$

A1(ii) $(b =) 3.68 + 10.2i(3.67820... + 10.2272...i)$ **A1****Note:** Accept $3.66 + 10.2i$ (from using their 3 sf values).**[5 marks]****Total [17 marks]**

2. (a) -6

A1
[1 mark]

(b) 394

A1
[1 mark]

(c) attempt to solve $394 - 6(n - 1) \geq 0$ **OR** $400 \div 6$

(M1)

Note: Accept equality or any inequality with zero seen, graphical approach to find zero, use of table/list to find zero.

($n <$) $66.6666 \dots$

(A1)

($n =$) 66

A1
[3 marks]

Note: If 400 is given as the answer in (b) then $n = 67$ in part (c).

Total [5 marks]

5. (a) attempt to expand the brackets

(M1)

$$z^2 = 4 + 4ai - a^2$$

$$\operatorname{Re}(z^2) = 4 - a^2$$

A1

[2 marks]

- (b) attempt to equate Re or Im parts

(M1)

$$a = -0.5 \quad \left(= -\frac{1}{2} \right)$$

A1

$$b = -0.375 \quad \left(= -\frac{3}{8} \right)$$

A1

Note: Do not follow through if imaginary part is missing in (a) as the resulting equation has no solution.

[3 marks]

Total [5 marks]

7. (a) $\begin{pmatrix} 8 \\ 5 \end{pmatrix}, \begin{pmatrix} 13 \\ 8 \end{pmatrix}$

A1A1
[2 marks]

(b) attempt to write LHS as a product of matrices or expand the RHS

M1

$$\begin{pmatrix} f_n + f_{n-1} \\ f_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} f_n \\ f_{n-1} \end{pmatrix}$$

A1

$$\begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} f_n \\ f_{n-1} \end{pmatrix}$$

AG
[2 marks]

(c) attempt to find the determinant of $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} - \lambda I$

(M1)

$$(1 - \lambda)(-\lambda) - 1 = 0$$

A1

$$\lambda^2 - \lambda - 1 = 0$$

AG

Note: The **AG** line must be seen in order to award the final **A1**.
Accept finding the trace and determinant of $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and
substituting into $\lambda^2 - (\text{Trace})\lambda + \text{det} = 0$.

[2 marks]

(d) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \lambda_1 \\ 1 \end{pmatrix} = \begin{pmatrix} \lambda_1 + 1 \\ \lambda_1 \end{pmatrix}$

A1

$$= \begin{pmatrix} \lambda_1^2 \\ \lambda_1 \end{pmatrix} \quad (\text{since } \lambda^2 - \lambda - 1 = 0)$$

A1

$$= \lambda_1 \begin{pmatrix} \lambda_1 \\ 1 \end{pmatrix}$$

AG

[2 marks]

(e) (i) $\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$

A1

(ii) $\begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix}$

A1

(iii) $\frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} 1 & -\lambda_2 \\ -1 & \lambda_1 \end{pmatrix}$

A1

Note: Accept $\begin{pmatrix} \lambda_2 & 0 \\ 0 & \lambda_1 \end{pmatrix}$ in part (i) and FT for parts (ii) and (iii) and part (f).

[3 marks]

(f) attempt to expand $\begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1^{n-1} & 0 \\ 0 & \lambda_2^{n-1} \end{pmatrix} \frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} 1 & -\lambda_2 \\ -1 & \lambda_1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **(M1)**

EITHER (right to left)

$$\frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1^{n-1} & 0 \\ 0 & \lambda_2^{n-1} \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \textbf{A1}$$

$$\frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} \lambda_1 & \lambda_2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \lambda_1^{n-1} \\ -\lambda_2^{n-1} \end{pmatrix} \quad \textbf{A1}$$

OR (Left to right)

$$\frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} \lambda_1^n & \lambda_2^n \\ \lambda_1^{n-1} & \lambda_2^{n-1} \end{pmatrix} \begin{pmatrix} 1 & -\lambda_2 \\ -1 & \lambda_1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \textbf{A1}$$

$$\frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} \lambda_1^n - \lambda_2^n & \lambda_1 \lambda_2^n - \lambda_2 \lambda_1^n \\ \lambda_1^{n-1} - \lambda_2^{n-1} & \lambda_1 \lambda_2^{n-1} - \lambda_2 \lambda_1^{n-1} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \textbf{A1}$$

THEN

$$\begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix} = \frac{1}{\lambda_1 - \lambda_2} \begin{pmatrix} \lambda_1^n - \lambda_2^n \\ \lambda_1^{n-1} - \lambda_2^{n-1} \end{pmatrix} \quad \textbf{A1}$$

Note: To award the final **A1** the RHS must be correct and must be equated to

$\begin{pmatrix} f_{n+1} \\ f_n \end{pmatrix}$ at some point during the proof.

Award marks as appropriate if candidates do a combination of expanding from the left and the right.

so that $f_n = \frac{1}{\lambda_1 - \lambda_2} ((\lambda_1)^{n-1} - (\lambda_2)^{n-1})$ **AG**
[4 marks]

(g) (i) attempt to use quadratic formula or complete the square **(M1)**
 $\frac{1 \pm \sqrt{5}}{2}$ **A1**

(ii) $f_n = \frac{1}{\frac{1+\sqrt{5}}{2} - \frac{1-\sqrt{5}}{2}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^{n-1} - \left(\frac{1-\sqrt{5}}{2} \right)^{n-1} \right) \left(= \frac{1}{\sqrt{5}} \left(\left(\frac{1+\sqrt{5}}{2} \right)^{n-1} - \left(\frac{1-\sqrt{5}}{2} \right)^{n-1} \right) \right)$ **A1**

Note: Exact values required.

[3 marks]
Total [18 marks]