Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

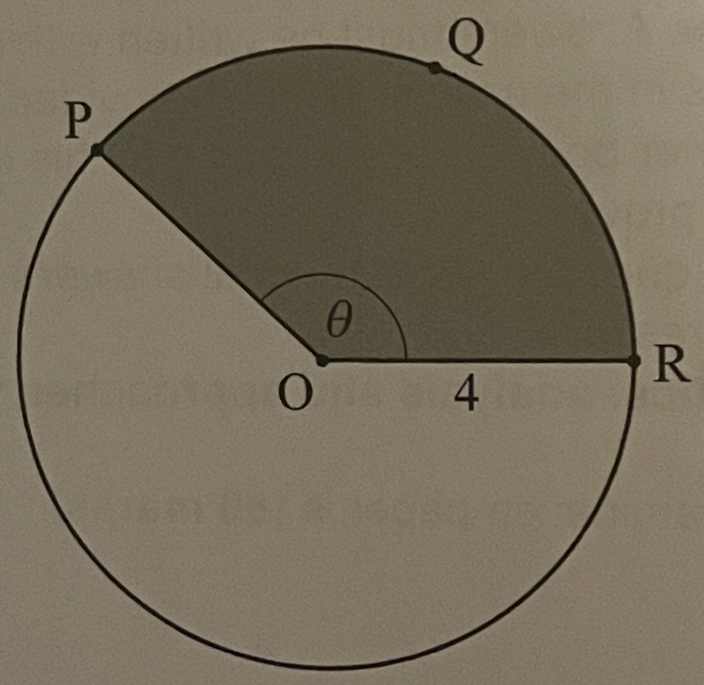
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius $4\,\mathrm{cm}$.

diagram not to scale



The points P, Q and R lie on the circumference of the circle and $\mathrm{P\hat{O}R} = \theta$, where θ is measured in radians.

The length of arc PQR is $10\,\mathrm{cm}$.

(a) Find the perimeter of the shaded sector. [2]

(b) Find θ. [2]

(c) Find the area of the shaded sector. [2]

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Please **do not** write on this page.

Answers written on this page
will not be marked.

2. [Maximum mark: 6]

Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

(a) Find the common difference, d. [2]

The following table shows the probability distribution of a discrete random variable X such that $\mathrm{P}(X = n) = \frac{u_n}{k}$, where $n \in \mathbb{Z}^+$, $1 \leq n \leq 4$ and $k \in \mathbb{R}^+$.

n	1	2	3	4
$\mathrm{P}(X = n)$	$\frac{0.6}{k}$	$\frac{u_2}{k}$	$\frac{u_3}{k}$	$\frac{0.15}{k}$

(b) Find the value of k. [4]

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3. [Maximum mark: 5]

A function f is defined by $f(x)=1-\frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

(a) The graph of $y=f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(i) the vertical asymptote;

(ii) the horizontal asymptote. [2]

(b) Find the coordinates of the point where the graph of $y=f(x)$ intersects

(i) the y-axis;

(ii) the x-axis. [2]

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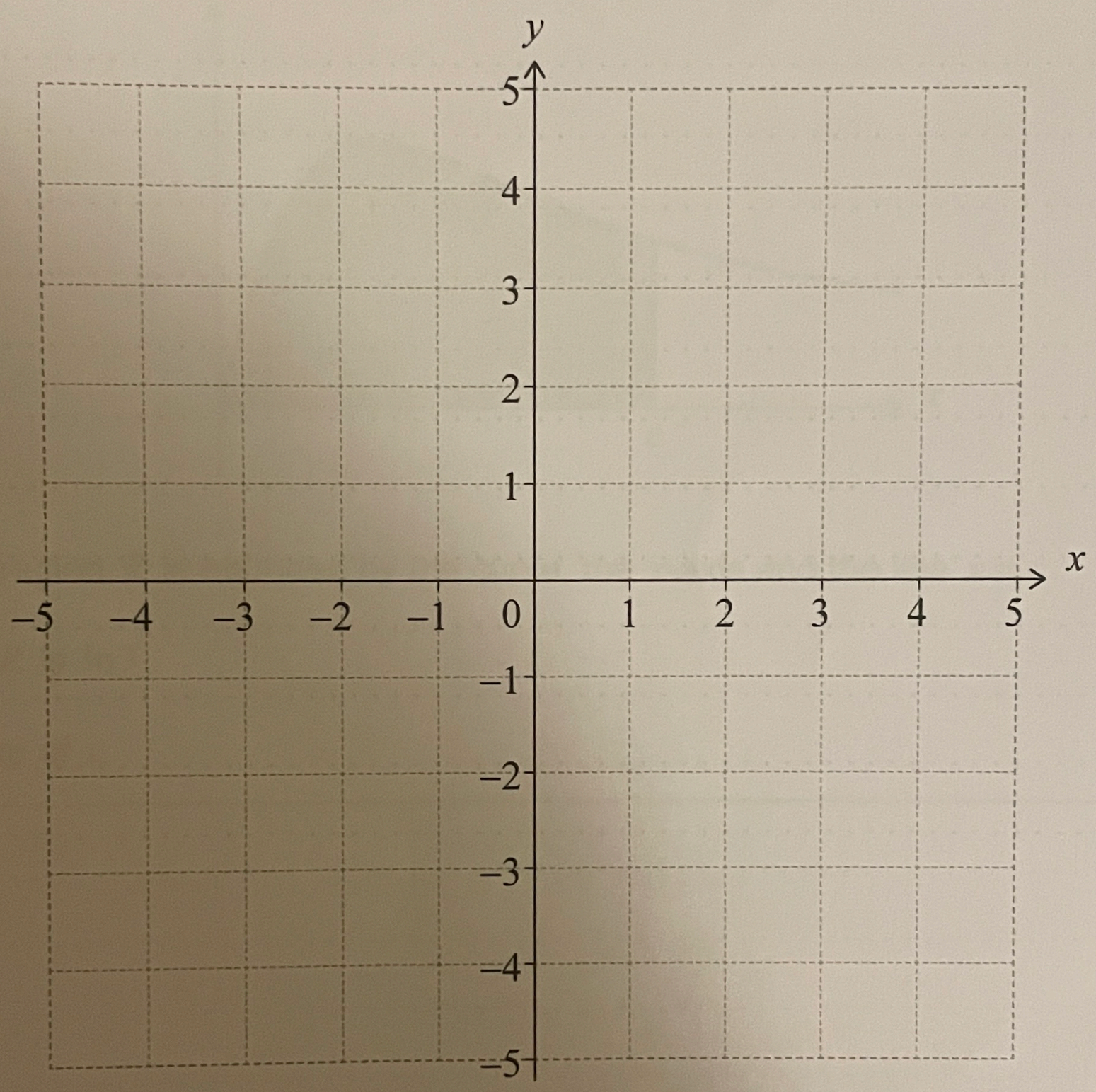
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(Question 3 continued)

(c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).



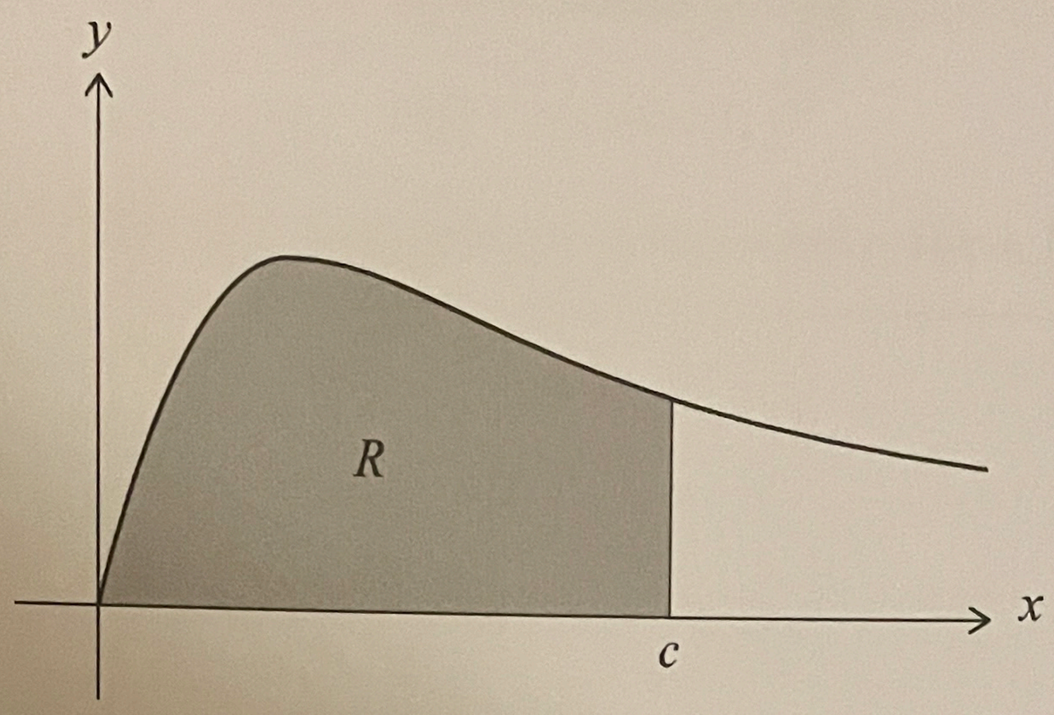
4. [Maximum mark: 5]

Events A and B are such that $\mathrm{P}(A) = 0.4$, $\mathrm{P}(A|B) = 0.25$ and $\mathrm{P}(A \cup B) = 0.55$.

Find $\mathrm{P}(B)$.

5. [Maximum mark: 6]

The following diagram shows part of the graph of $y = \frac{x}{x^2 + 2}$ for $x \geq 0$.



The shaded region R is bounded by the curve, the x-axis and the line $x = c$.

The area of R is $\ln 3$.

Find the value of c.

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Answers written on this page
will not be marked.

6. [Maximum mark: 7]

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f(x) = ax + b, \text{ where } a, b \in \mathbb{Z}$$

$$g(x) = x^2 + x + 3.$$

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$.

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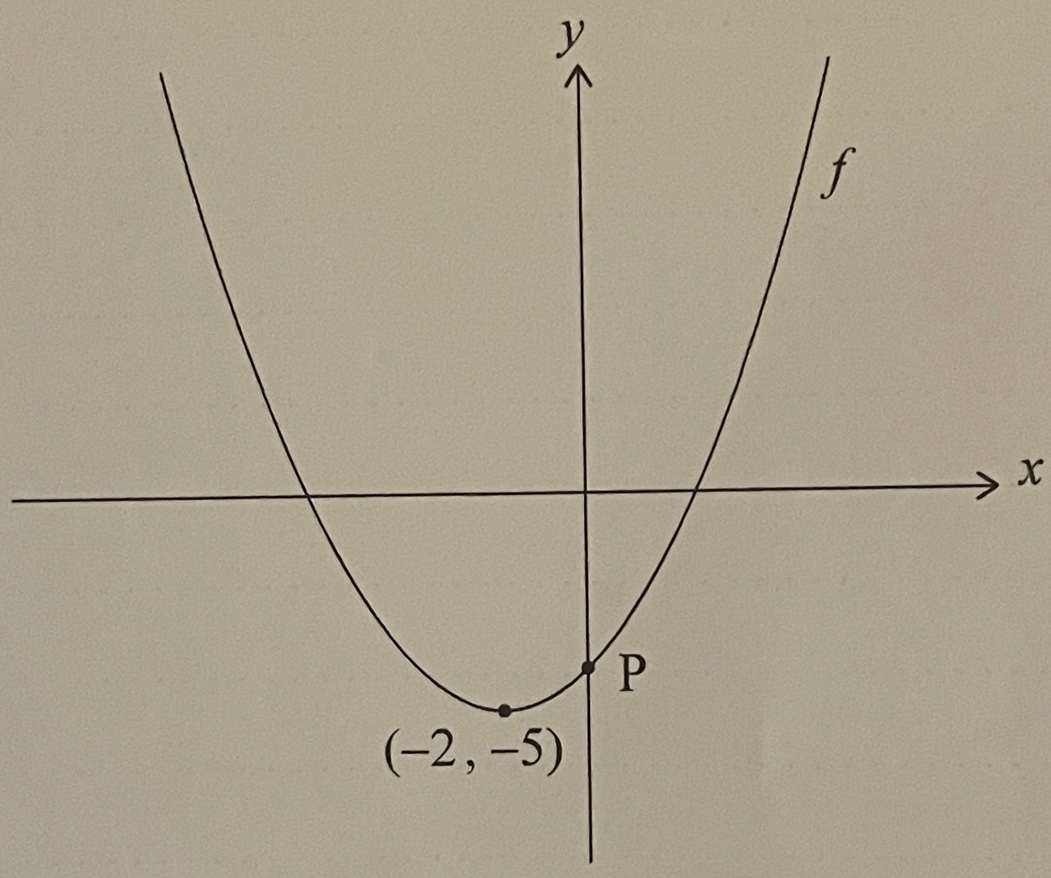
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

The following diagram shows part of the graph of a quadratic function f.

The vertex of the parabola is $(-2, -5)$ and the y-intercept is at point P.



(a) Write down the equation of the axis of symmetry. [1]

The function can be written in the form $f(x) = \frac{1}{4}(x-h)^2 + k$, where $h, k \in \mathbb{Z}$.

(b) Write down the values of h and k. [2]

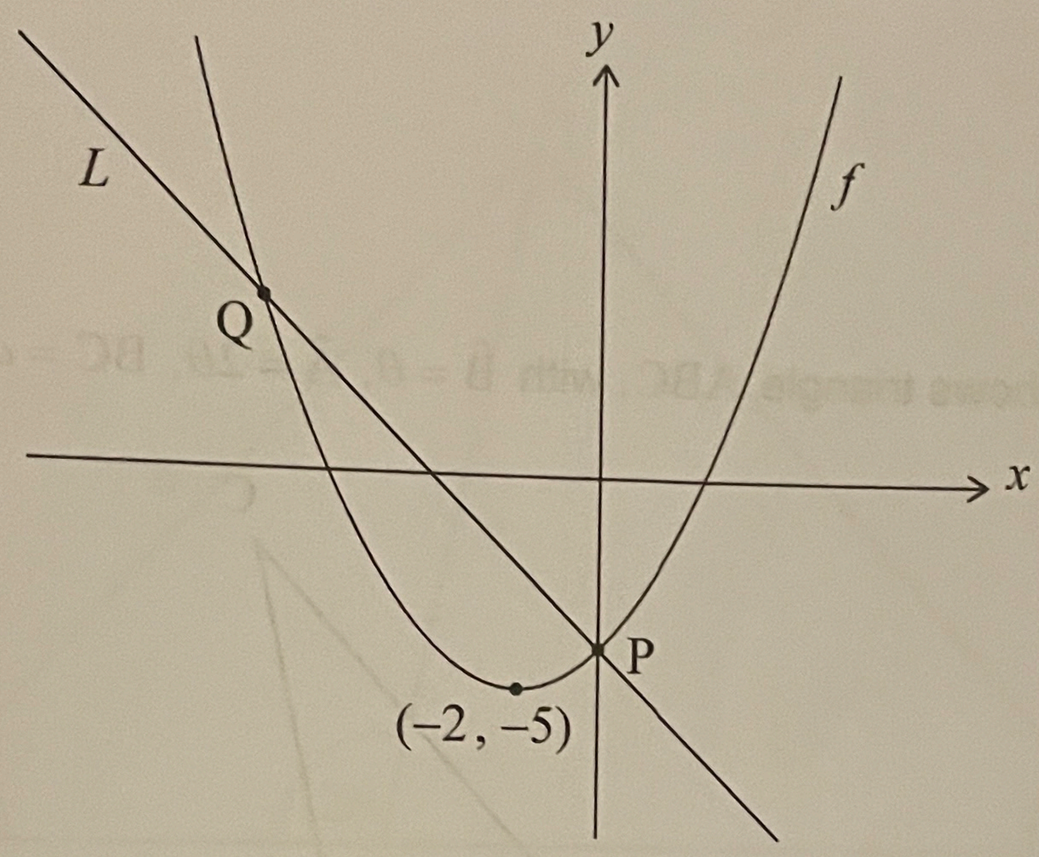
(c) Find the y-coordinate of P. [2]

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(Question 7 continued)

In the following diagram, the line L is normal to the graph of f at point P.



(d) Find the equation of the line L, in the form $y = ax + b$. [4]

The line L intersects the graph of f at a second point, Q, as shown above.

(e) Calculate the distance between P and Q. [8]

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8. [Maximum mark: 14]

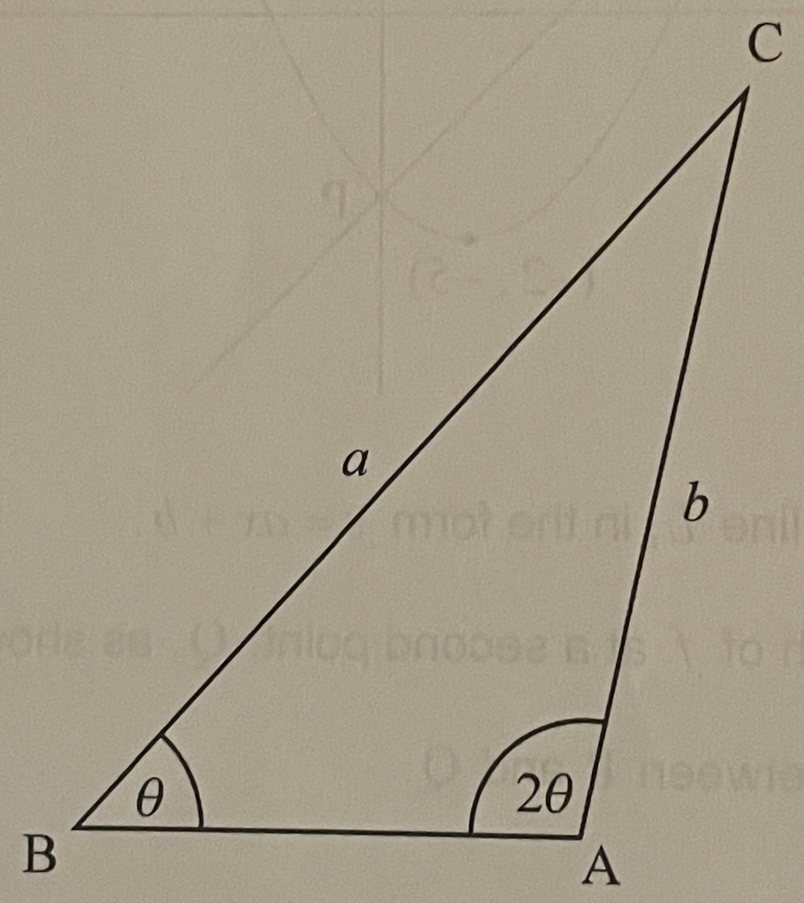
Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$.

(a) Find the value of

(i) $\sin\theta$;

(ii) $\sin 2\theta$. [4]

The following diagram shows triangle ABC, with $\hat{B} = \theta$, $\hat{A} = 2\theta$, $BC = a$ and $AC = b$.



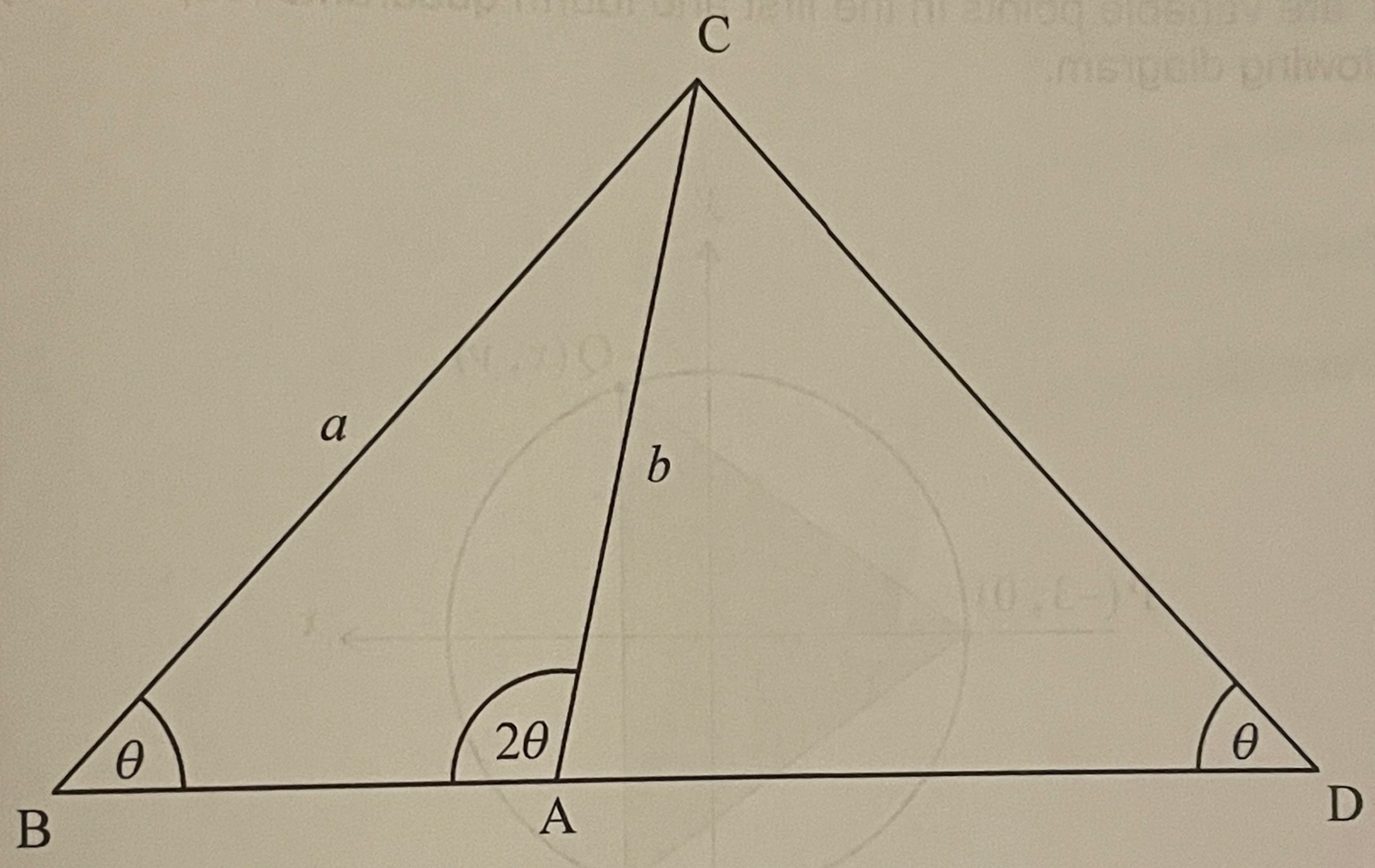
(b) Show that $b = \frac{3a}{4}$. [2]

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(Question 8 continued)

[BA] is extended to form an isosceles triangle DAC, with $\hat{D} = \theta$, as shown in the following diagram.



(c) Find the value of $\sin C\hat{A}D$. [3]

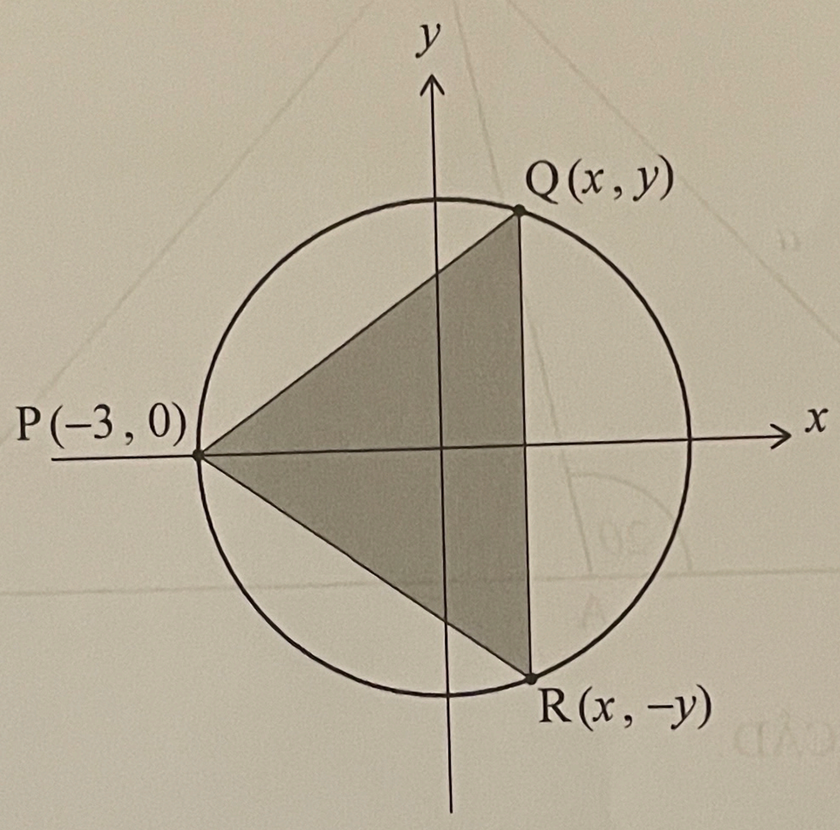
(d) Find the area of triangle DAC, in terms of a. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $\mathrm{P}(-3, 0)$, $\mathrm{Q}(x, y)$ and $\mathrm{R}(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



(a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]

(b) Hence, find an expression for A, the area of triangle PQR, in terms of x. [3]

(c) Show that $\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]

(d) Hence or otherwise, find the y-coordinate of R such that A is a maximum. [6]