

APOLLO

Calculus

SL

Mathematics: analysis and approaches

First assessment 2021

83 questions | questions

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $y = \frac{\ln x}{x^4}$ for $x > 0$.

- (a) Show that $\frac{dy}{dx} = \frac{1 - 4 \ln x}{x^5}$. [3]

Consider the function defined by $f(x) = \frac{\ln x}{x^4}$ for $x > 0$ and its graph $y = f(x)$.

- (b) The graph of f has a horizontal tangent at point P. Find the coordinates of P. [5]
- (c) Given that $f''(x) = \frac{20 \ln x - 9}{x^6}$, show that P is a local maximum point. [3]
- (d) Solve $f(x) > 0$ for $x > 0$. [2]
- (e) Sketch the graph of f , showing clearly the value of the x -intercept and the approximate position of point P. [3]

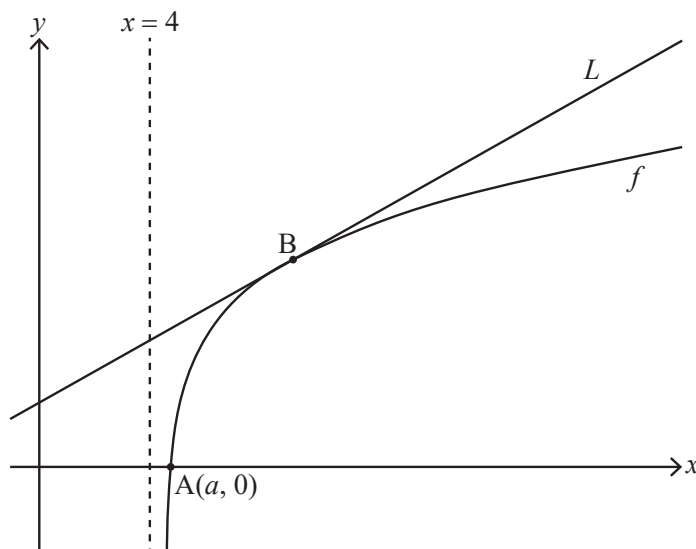


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5. [Maximum mark: 9]



- (a) Find the exact value of α . [3]
- (b) Given that the gradient of L is $\frac{1}{3}$, find the x -coordinate of B. [6]

[illegible]

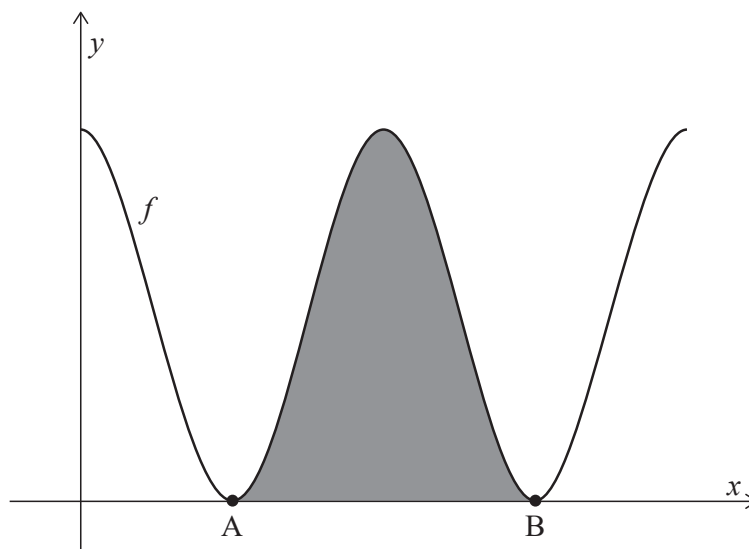
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the function f defined by $f(x) = 6 + 6\cos x$, for $0 \leq x \leq 4\pi$.

The following diagram shows the graph of $y = f(x)$.



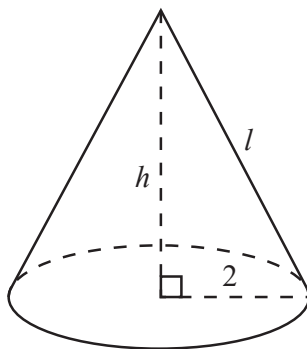
The graph of f touches the x -axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x -axis, between the points A and B.

- (a) Find the x -coordinates of A and B. [3]
- (b) Show that the area of the shaded region is 12π . [5]

The right cone in the following diagram has a total surface area of 12π , equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h , and slant height l .

diagram not to scale



- (c) Find the value of l . [3]
- (d) Hence, find the volume of the cone. [4]



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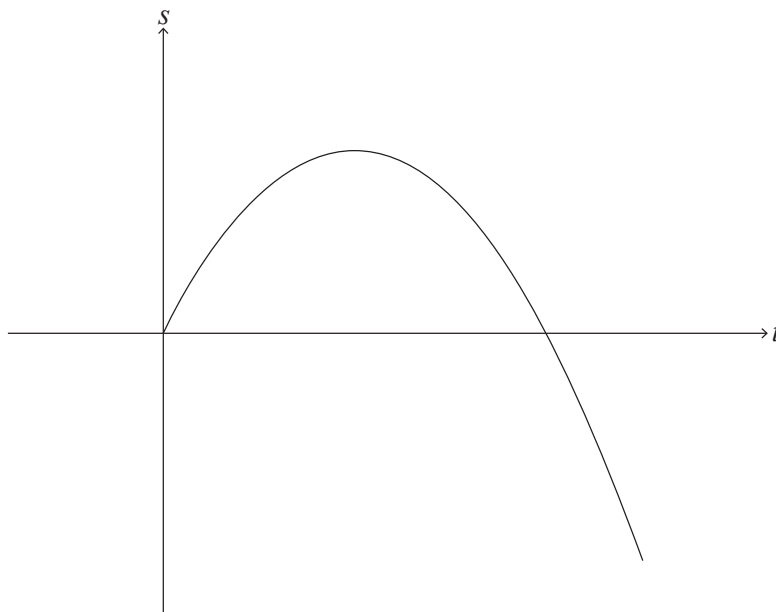
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

(a) Find the value of p . [2]

Particle A changes direction when $t = q$.

- (b) (i) Find the value of q .
 (ii) Find the displacement of particle A from the origin when $t = q$. [4]

(c) Find the distance of particle A from the origin when $t = 10$. [2]

The total distance travelled by particle A is given by d .

(d) Find the value of d . [2]

A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d .

(e) Find the value of k . [4]

References:

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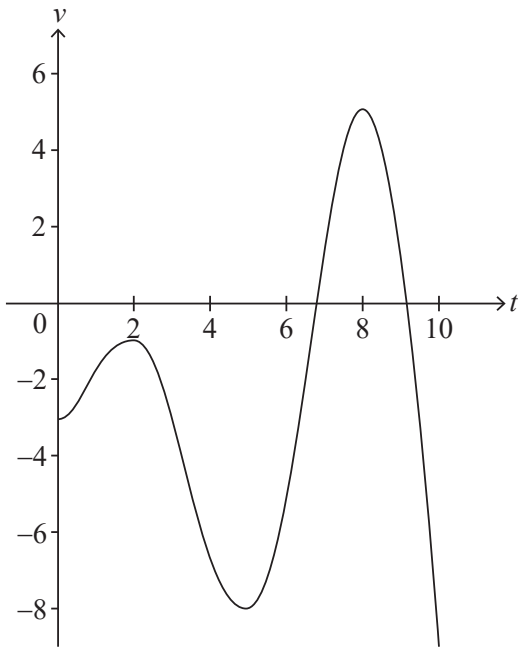
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\text{ ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t \sin t - 3$, for $0 \leq t \leq 10$.

The following diagram shows the graph of v .



- (a) Find the smallest value of t for which the particle is at rest. [2]
- (b) Find the total distance travelled by the particle. [2]
- (c) Find the acceleration of the particle when $t = 7$. [2]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider the function f defined by $f(x) = 90e^{-0.5x}$ for $x \in \mathbb{R}^+$.

The graph of f and the line $y = x$ intersect at point P.

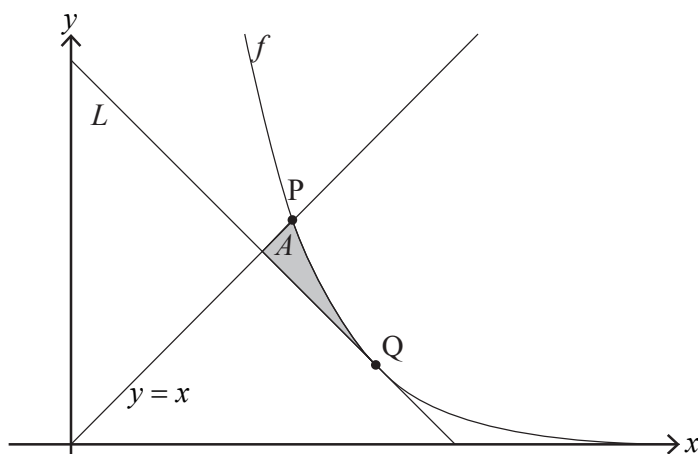
(a) Find the x -coordinate of P. [2]

The line L has a gradient of -1 and is a tangent to the graph of f at the point Q.

(b) Find the exact coordinates of Q. [4]

(c) Show that the equation of L is $y = -x + 2 \ln 45 + 2$. [2]

The shaded region A is enclosed by the graph of f and the lines $y = x$ and L .



(d) (i) Find the x -coordinate of the point where L intersects the line $y = x$.

(ii) Hence, find the area of A . [5]

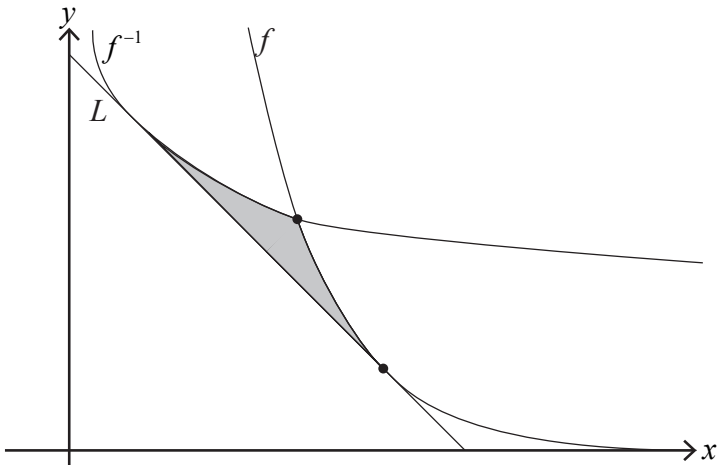
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(Question 9 continued)

The line L is tangent to the graphs of both f and the inverse function f^{-1} .



- (e) Find the shaded area enclosed by the graphs of f and f^{-1} and the line L . [2]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle P moves along the x -axis. The velocity of P is $v \text{ m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0$, P is at the origin O .

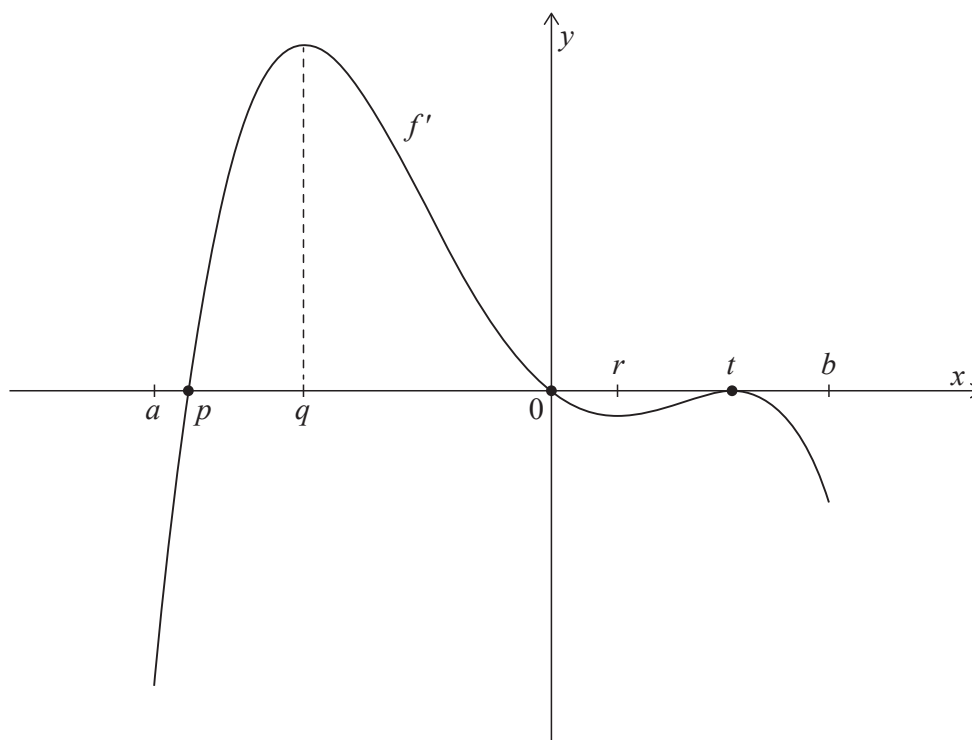
- (a) (i) Find the value of t when P reaches its maximum velocity. [7]
- (ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [4]
- (b) Sketch a graph of v against t , clearly showing any points of intersection with the axes. [5]
- (c) Find the total distance travelled by P .

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

Consider a function f with domain $a < x < b$. The following diagram shows the graph of f' , the derivative of f .



The graph of f' , the derivative of f , has x -intercepts at $x = p$, $x = 0$ and $x = t$. There are local maximum points at $x = q$ and $x = t$ and a local minimum point at $x = r$.

- (a) Find all the values of x where the graph of f is increasing. Justify your answer. [2]
- (b) Find the value of x where the graph of f has a local maximum. [1]
- (c)
 - (i) Find the value of x where the graph of f has a local minimum. Justify your answer.
 - (ii) Find the values of x where the graph of f has points of inflexion. Justify your answer. [5]
- (d) The total area of the region enclosed by the graph of f' , the derivative of f , and the x -axis is 20.

Given that $f(p) + f(t) = 4$, find the value of $f(0)$. [6]

References:

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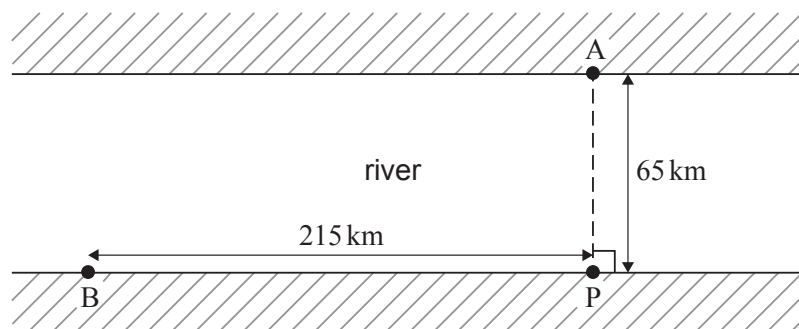
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Points A and P lie on opposite banks of a river, such that AP is the shortest distance across the river. Point B represents the centre of a city which is located on the riverbank. $PB = 215$ km, $AP = 65$ km and $\hat{APB} = 90^\circ$.

The following diagram shows this information.



A boat travels at an average speed of 42 km h^{-1} . A bus travels along the straight road between P and B at an average speed of 84 km h^{-1} .

(a) Find the travel time, in hours, from A to B given that

- (i) the boat is taken from A to P, and the bus from P to B;
- (ii) the boat travels directly to B.

[4]

There is a point D, which lies on the road from P to B, such that $BD = x$ km. The boat travels from A to D, and the bus travels from D to B.

- (b) (i) Find an expression, in terms of x for the travel time T , from A to B, passing through D.
- (ii) Find the value of x so that T is a minimum.
- (iii) Write down the minimum value of T .

[6]

(c) An excursion involves renting the boat and the bus. The cost to rent the boat is \$200 per hour, and the cost to rent the bus is \$150 per hour.

- (i) Find the new value of x so that the total cost C to travel from A to B via D is a minimum.
- (ii) Write down the minimum total cost for this journey.

[4]





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8. [Maximum mark: 16]

(a) Find $f'(x)$. [2]

(b) Find the value of a and the value of b . [3]

(ii) Hence explain why the graph of f has a local maximum point at $x = a$. [2]

(ii) Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$. [4]

(e) Find the value of p and the value of q . [5]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

Let $f(x) = \frac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

(a) Show that $f'(x) = \frac{1 - \ln 5x}{kx^2}$. [3]

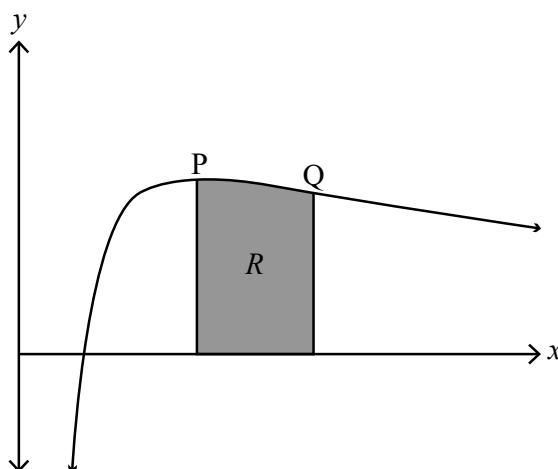
The graph of f has exactly one maximum point P.

(b) Find the x -coordinate of P. [3]

The second derivative of f is given by $f''(x) = \frac{2 \ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

(c) Show that the x -coordinate of Q is $\frac{1}{5}e^{\frac{3}{2}}$. [3]

The region R is enclosed by the graph of f , the x -axis, and the vertical lines through the maximum point P and the point of inflexion Q.



(d) Given that the area of R is 3, find the value of k . [7]



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2. [Maximum mark: 5]

- (a) The expression $\frac{3\sqrt{x}-5}{\sqrt{x}}$ can be written as $3 - 5x^p$. Write down the value of p . [1]
- (b) Hence, find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}} \right) dx$. [4]

[illegible]

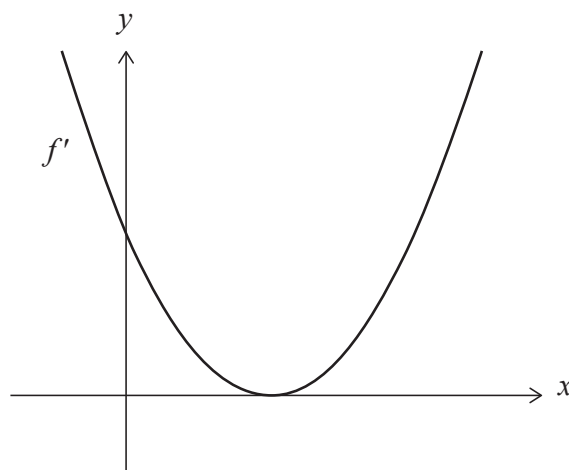
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A function, f , has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f' .



The graph of f' has an axis of symmetry $x = q$.

(a) Find the value of q . [2]

The vertex of the graph of f' lies on the x -axis.

(b) (i) Write down the value of the discriminant of f' .
(ii) Hence or otherwise, find the value of p . [4]

(c) Find the value of the gradient of the graph of f' at $x = 0$. [3]

(d) Sketch the graph of f'' , the second derivative of f . Indicate clearly the x -intercept and the y -intercept. [2]

The graph of f has a point of inflexion at $x = a$.

(e) (i) Write down the value of a .
(ii) Find the values of x for which the graph of f is concave-down. Justify your answer. [3]





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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

(a) (i) Expand and simplify $(1 - a)^3$ in ascending powers of a .

(ii) By using a suitable substitution for a , show that
 $1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x = 8 \sin^6 x$.

[6]

Consider $f(x) = 4 \cos x (1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x)$.

(b) (i) Show that $\int_0^m f(x) \, dx = \frac{32}{7} \sin^7 m$, where m is a positive real constant.

(ii) It is given that $\int_m^{\frac{\pi}{2}} f(x) \, dx = \frac{127}{28}$, where $0 \leq m \leq \frac{\pi}{2}$. Find the value of m .

[9]

References:

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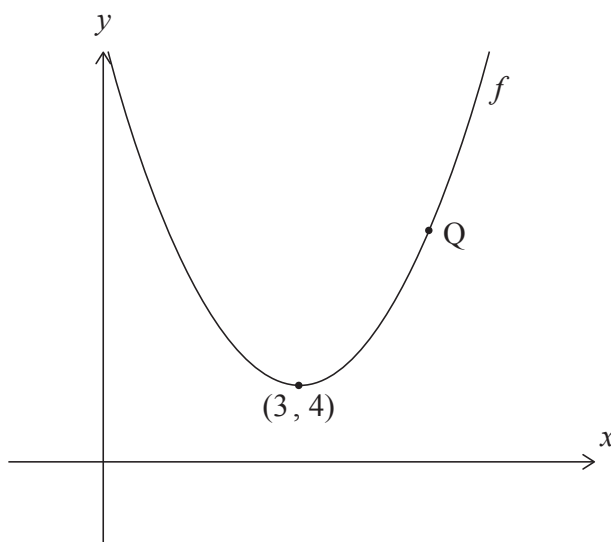
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

The following diagram shows part of the graph of a quadratic function f .

The graph of f has its vertex at $(3, 4)$, and it passes through point Q as shown.



- (a) Write down the equation of the axis of symmetry. [1]
- (b) The function can be written in the form $f(x) = a(x - h)^2 + k$.
 - (i) Write down the values of h and k .
 - (ii) Point Q has coordinates $(5, 12)$. Find the value of a . [4]

The line L is tangent to the graph of f at Q.

- (c) Find the equation of L . [4]

Now consider another function $y = g(x)$. The derivative of g is given by $g'(x) = f(x) - d$, where $d \in \mathbb{R}$.

- (d) Find the values of d for which g is an increasing function. [3]
- (e) Find the values of x for which the graph of g is concave-up. [3]



Turn over

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4444-7117

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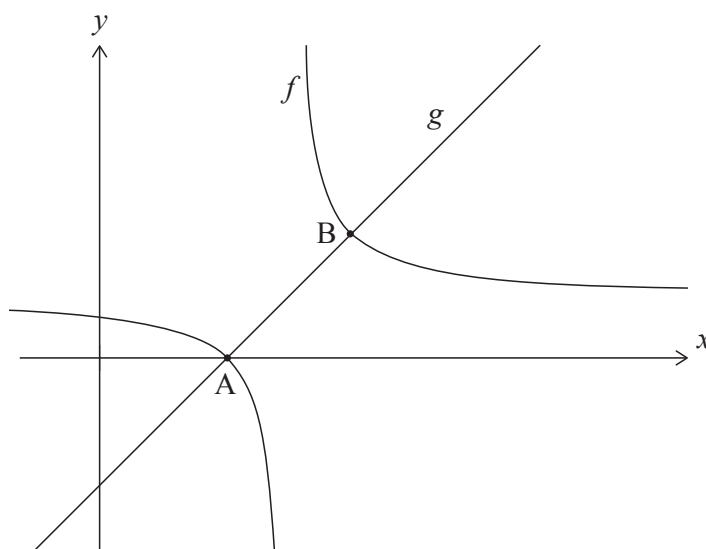
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the functions $f(x) = \frac{1}{x-4} + 1$, for $x \neq 4$, and $g(x) = x - 3$ for $x \in \mathbb{R}$.

The following diagram shows the graphs of f and g .



The graphs of f and g intersect at points A and B. The coordinates of A are $(3, 0)$.

(a) Find the coordinates of B.

[5]

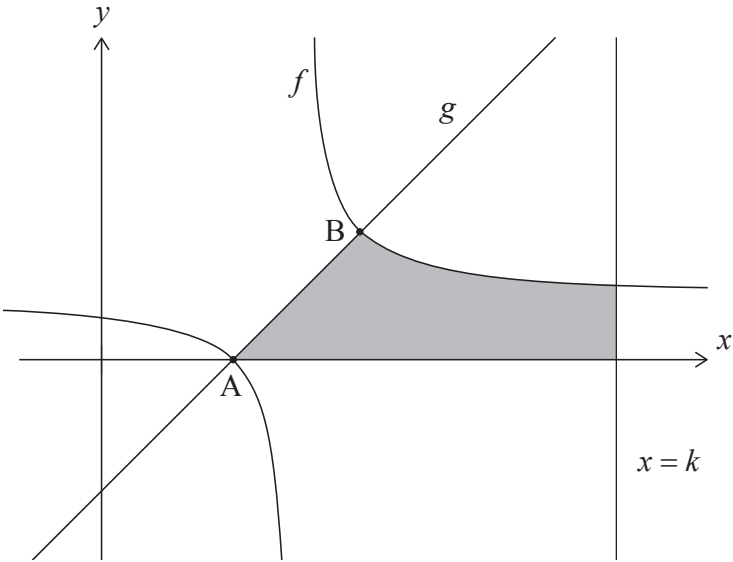
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(Question 8 continued)

In the following diagram, the shaded region is enclosed by the graph of f , the graph of g , the x -axis, and the line $x = k$, where $k \in \mathbb{Z}$.



The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

- (b) Find the value of k and the value of p .
- [10]



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4444-1117

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5. [Maximum mark: 7]

(a) Find the value of t when the particle is at rest. [2]

(b) Find the acceleration of the particle when it changes direction. [3]

(c) Find the total distance travelled by the particle. [2]

[illegible]

The derivative of a function g is given by $g'(x) = 3x^2 + 5e^x$, where $x \in \mathbb{R}$. The graph of g passes through the point $(0, 4)$. Find $g(x)$.

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4444-7113

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 12]

A scientist conducted a nine-week experiment on two plants, A and B , of the same species. He wanted to determine the effect of using a new plant fertilizer. Plant A was given fertilizer regularly, while Plant B was not.

The scientist found that the height of Plant A , h_A cm, at time t weeks can be modelled by the function $h_A(t) = \sin(2t + 6) + 9t + 27$, where $0 \leq t \leq 9$.

The scientist found that the height of Plant B , h_B cm, at time t weeks can be modelled by the function $h_B(t) = 8t + 32$, where $0 \leq t \leq 9$.

(a) Use the scientist's models to find the initial height of

(i) Plant B ;

(ii) Plant A correct to three significant figures.

[3]

(b) Find the values of t when $h_A(t) = h_B(t)$.

[3]

(c) For $0 \leq t \leq 9$, find the total amount of time when the rate of growth of Plant B was greater than the rate of growth of Plant A .

[6]



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3. [Maximum mark: 6]

(c) Find the total area enclosed by the graph of f , the x -axis and the line $x = 2$. [3]

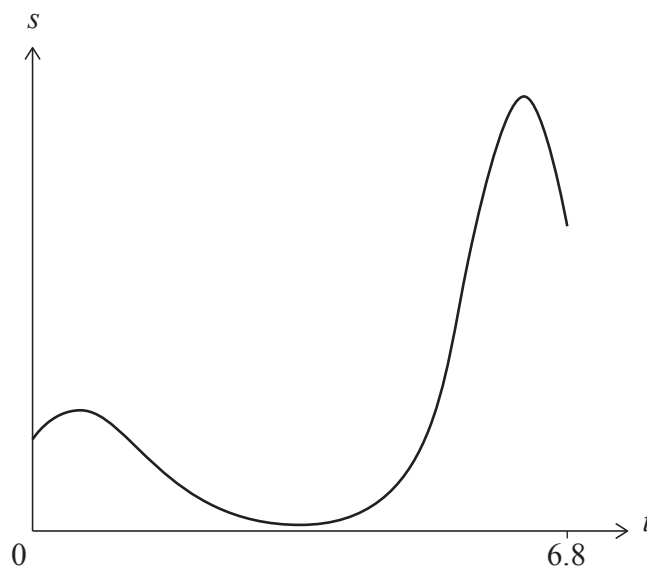
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s(t) = 3(t + 2)^{\cos t}$, for $0 \leq t \leq 6.8$, as shown in the following graph.



- (a) Find the particle's initial displacement from the point P. [2]
- (b) Find the particle's velocity when $t = 2$. [2]
- (c) Determine the intervals of time when the particle is moving away from the point P. [5]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

- (d) Find the value of b and the value of c . [4]
- (e) Find the total distance travelled by the particle for $b \leq t \leq c$. [3]



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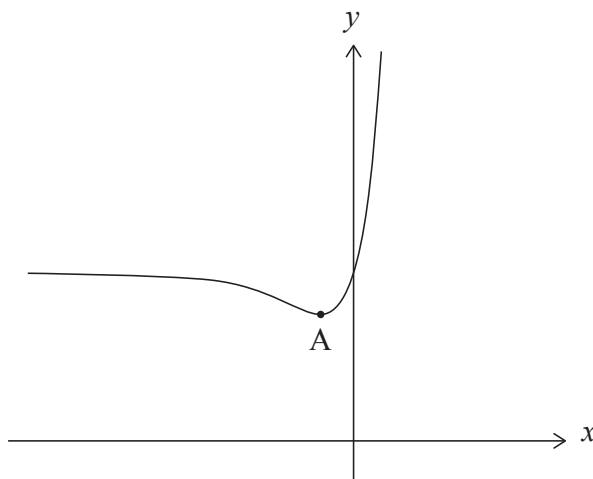
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The function h is defined by $h(x) = 2xe^x + 3$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of h , which has a local minimum at point A.



- (a) Find the value of the y -intercept. [2]
- (b) Find $h'(x)$. [2]
- (c) Hence, find the coordinates of A. [5]
- (d) (i) Show that $h''(x) = (2x + 4)e^x$.
- (ii) Find the values of x for which the graph of h is concave-up. [4]





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Section B

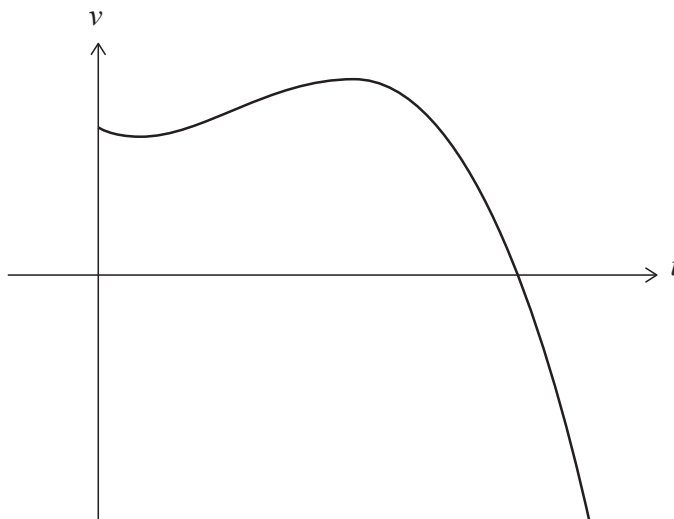
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by

$$v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6, \text{ for } 0 \leq t \leq 4. \text{ The object first comes to rest at } t = k.$$

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

- | | |
|--|-----|
| (a) Find the displacement of the object from the origin at $t = 1$. | [5] |
| (b) Find an expression for the acceleration of the object. | [2] |
| (c) Hence, find the greatest speed reached by the object before it comes to rest. | [5] |
| (d) Find the greatest speed reached by the object for $0 \leq t \leq 4$. | [2] |
| (e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. | [3] |

References:

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5. [Maximum mark: 6]

A graph in the xy -plane showing a shaded region R . The region is bounded by the x -axis, a curve that starts at the origin, rises to a peak, and then decreases, and a vertical line segment at $x=c$. The curve continues to the right of $x=c$, but the region R is only shaded up to $x=c$.

Find the value of c .

[illegible]

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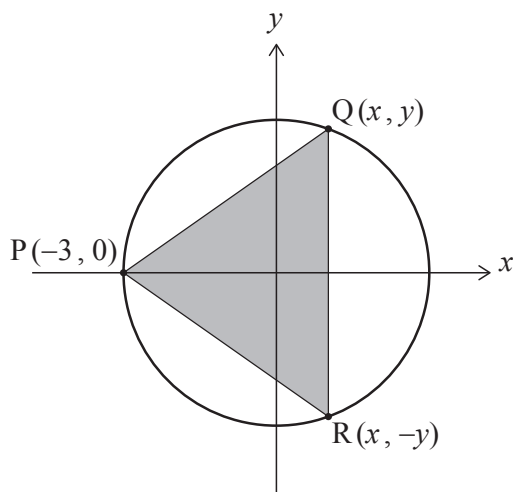
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $P(-3, 0)$, $Q(x, y)$ and $R(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



- (a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]
- (b) Hence, find an expression for A , the area of triangle PQR, in terms of x . [3]
- (c) Show that $\frac{dA}{dx} = \frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]
- (d) Hence or otherwise, find the y -coordinate of R such that A is a maximum. [6]

References:

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Section B

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7. [Maximum mark: 13]

The temperature of a cup of tea, t minutes after it is poured, can be modelled by $H(t) = 21 + 75e^{-0.08t}$, $t \geq 0$. The temperature is measured in degrees Celsius ($^{\circ}\text{C}$).

- (a) (i) Find the initial temperature of the tea.
- (ii) Find the temperature of the tea three minutes after it is poured. [2]
- (b) Write down the value of $H'(3)$. [2]
- (c) Interpret the meaning of your answer to part (b) in the given context. [2]
- (d) After k minutes, the tea will be below 67°C and cool enough to drink.

Find the least possible value of k , where $k \in \mathbb{Z}^+$. [3]

As the tea cools, $H(t)$ approaches the temperature of the room, which is constant.

- (e) Find the temperature of the room. [2]
- (f) Find the limit of $H'(t)$ as t approaches infinity. [2]



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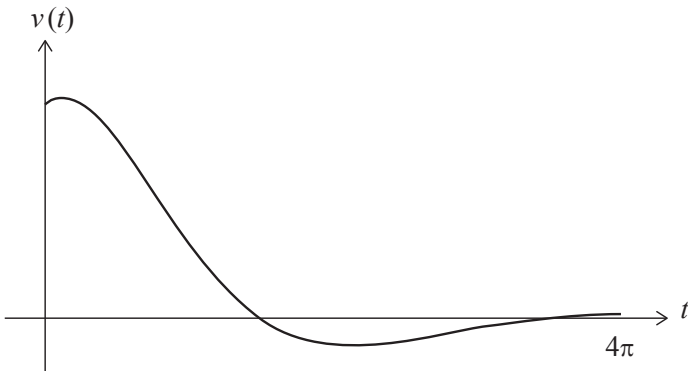
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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by $v(t) = 4e^{-\frac{t}{3}} \cos\left(\frac{t}{2} - \frac{\pi}{4}\right)$, for $0 \leq t \leq 4\pi$. The graph of v is shown in the following diagram.



Let t_1 be the first time when the particle's **acceleration** is zero.

(a) Find the value of t_1 . [2]

Let t_2 be the **second** time when the particle is instantaneously at rest.

(b) Find the value of t_2 . [2]

(c) Find the distance travelled by the particle between $t = t_1$ and $t = t_2$. [2]

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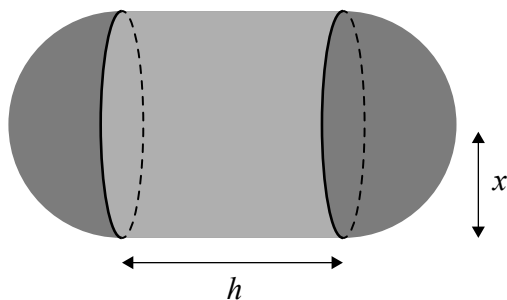
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height h cm and radius x cm. The hemispheres fit exactly onto either end of the cylinder.



The volume of the cylinder is 41 cm^3 .

(a) Show that the total surface area, $S \text{ cm}^2$, of the solid is given by $S = \frac{82}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (i) Find an expression for $\frac{dS}{dx}$.
 (ii) Hence, find the **exact** value of a . [5]

(c) (i) Find an expression for $\frac{d^2S}{dx^2}$.
 (ii) Use the second derivative of S to justify that S is a minimum when $x = a$.
 (iii) Find the minimum surface area of the solid. [6]



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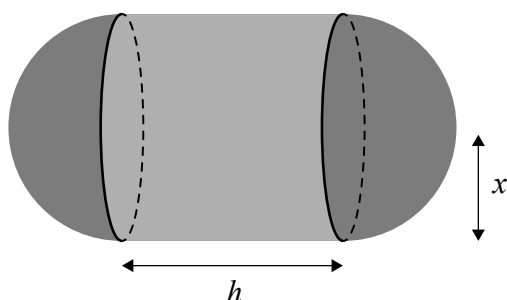
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height h cm and radius x cm. The hemispheres fit exactly onto either end of the cylinder.



The volume of the cylinder is 45 cm^3 .

(a) Show that the total surface area, $S \text{ cm}^2$, of the solid is given by $S = \frac{90}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (i) Find an expression for $\frac{dS}{dx}$.
 (ii) Hence, find the **exact** value of a . [5]

(c) (i) Find an expression for $\frac{d^2S}{dx^2}$.
 (ii) Use the second derivative of S to justify that S is a minimum when $x = a$.
 (iii) Find the minimum surface area of the solid. [6]



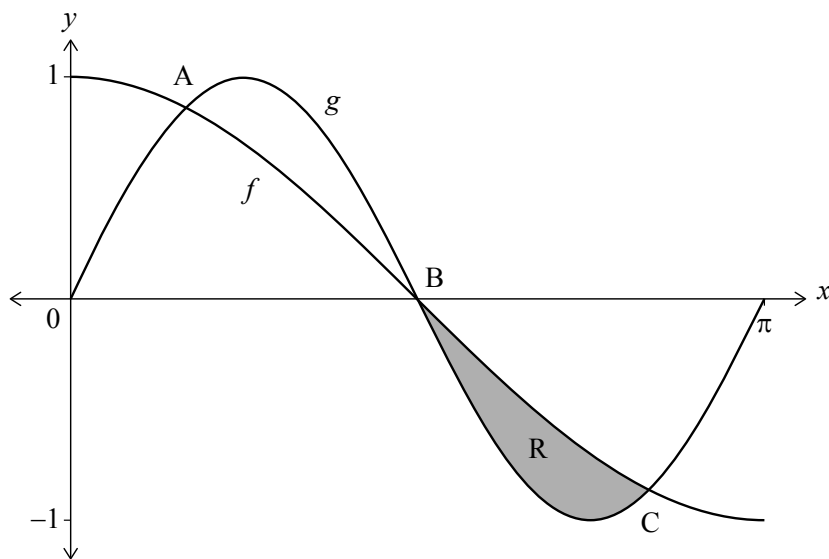
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0020-7110

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5. [Maximum mark: 7]

The graph of f intersects the graph of g at the point A, the point B $\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



- The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

- (b) Find the area of R . [4]

[illegible]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Consider the curve with equation $y = x^3 - x^2 - x + 1$.

(a) Find

(i) $\frac{dy}{dx}$;

(ii) $\frac{d^2y}{dx^2}$. [3]

The curve has a local maximum at A.

(b) Find the coordinates of A, using your answer to part (a)(ii) to justify your answer. [6]

The curve has a point of inflexion at B.

(c) Find the x -coordinate of B. [2]

The line L is the normal to the curve at the point $(0, 1)$.

(d) Find the equation of L . [3]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 17]

The derivative of a function f is given by $f'(x) = \frac{2x+2}{x^2+2x+2}$, for $x \in \mathbb{R}$.

- (a) (i) Show that $x^2 + 2x + 2 > 0$ for all values of x .
 (ii) Hence, find the values of x for which f is increasing. [3]
- (b) (i) Write down the value of x for which $f'(x) = 0$.
 (ii) Show that $f''(x) = \frac{-2x^2 - 4x}{(x^2 + 2x + 2)^2}$.
 (iii) Hence, justify that the value of x found in part (b)(i) corresponds to a local minimum point on the graph of f . [7]

It is given that $f(2) = 3 + \ln 10$.

- (c) Find an expression for $f(x)$. [4]
 (d) Find the equation of the normal to the graph of f at $(2, 3 + \ln 10)$. [3]



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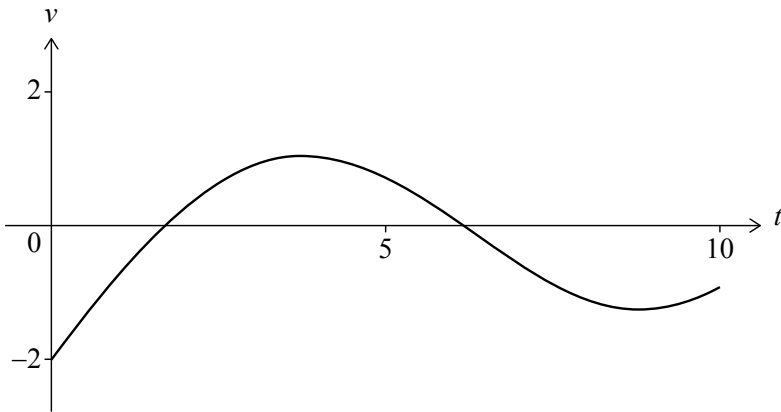
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

A particle moves in a straight line such that it passes through a fixed point O at time $t = 0$, where t represents time measured in seconds after passing O . For $0 \leq t \leq 10$ its velocity, v metres per second, is given by $v = 2 \sin(0.5t) + 0.3t - 2$.

The graph of v is shown in the following diagram.



(a) Find the smallest value of t when the particle changes direction. [2]

The displacement of the particle is measured in metres from O .

(b) Find the range of values of t for which the displacement of the particle is increasing. [2]

(c) Find the displacement of the particle relative to O when $t = 10$. [2]

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2. [Maximum mark: 6]

$$f(x) = 1 - x^2$$

$$g(x) = e^{2x}.$$

(a) Find the value of a and the value of b . [3]

(b) Find the area of the region enclosed by the graphs of f and g . [3]

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5. [Maximum mark: 6]

(a) Find the velocity of the particle at $t = 2$. [1]

(b) Find the maximum velocity of the particle. [2]

(c) Find the acceleration of the particle at the instant it changes direction. [3]

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Section B

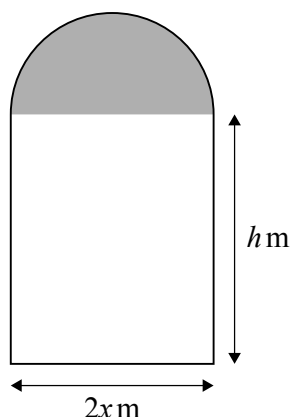
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A window is designed in the shape of a semicircle attached to a rectangle.

The rectangular section of the window has dimensions $2x$ metres by h metres.

The window consisting of its two sections is shown in the following diagram.



Let the area of the window be A square metres.

(a) Write down an expression for A in terms of x and h . [2]

Let the perimeter of the window be P metres.

(b) Given that $P = 10$, show that $h = \frac{1}{2}(10 - 2x - \pi x)$. [2]

The window is designed to let in the maximum amount of light.

The rectangular section of the window consists of clear glass and lets in three units of light per square metre.

The semicircular section of the window consists of tinted glass and lets in one unit of light per square metre.

(c) Show that the amount of light, L units, let in by the window is given by

$$L = 30x - 6x^2 - \frac{5}{2}\pi x^2. \quad [4]$$

(d) (i) Find an expression for $\frac{dL}{dx}$.

(ii) Find the value of x so that the window lets in the maximum amount of light. Justify that this value of x gives a maximum.

(iii) Find the value of h so that the window lets in the maximum amount of light. [7]



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Section B

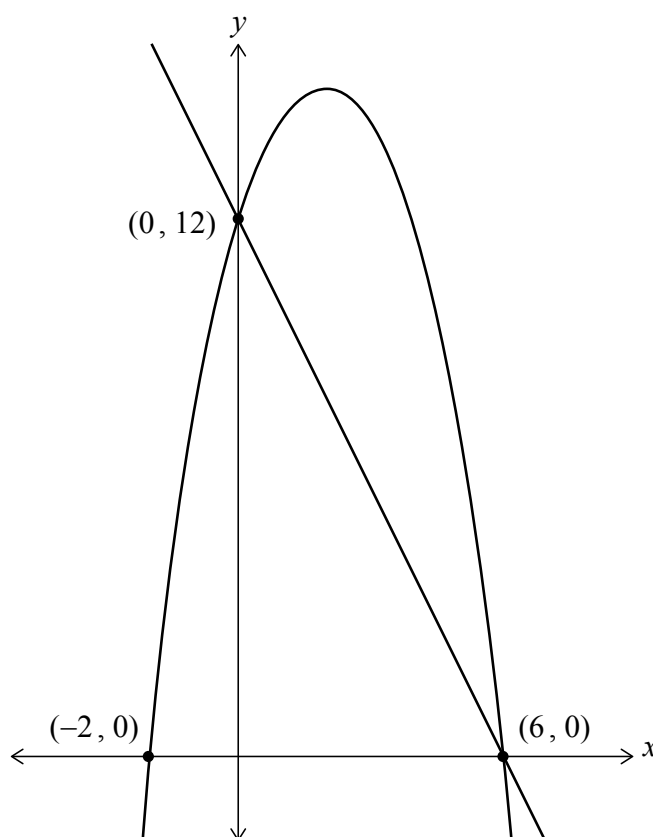
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g .

The graph of f is linear, has an x -intercept at $(6, 0)$ and a y -intercept at $(0, 12)$.

The graph of g has x -intercepts at $(-2, 0)$ and $(6, 0)$ and a y -intercept at $(0, 12)$.



(a) Write down the equation for f in the form $f(x) = mx + c$.

[2]

(This question continues on the following page)



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(Question 7 continued)

The function g is given by $g(x) = -x^2 + bx + 12$, where b is a real constant.

(b) Find the value of b . [3]

(c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^6 (-x^2 + 6x)dx$. [2]

(d) Hence, find the area of the region enclosed by the graph of f and the graph of g . [4]

Point P is on the graph of g . The tangent to the graph of g at P is parallel to the graph of f .

(e) Find the coordinates of P. [5]



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Section B

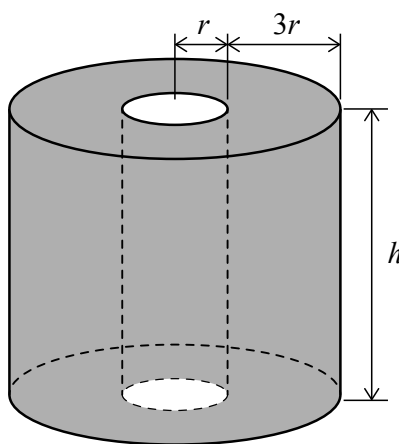
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h . A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2 , is given by S .

The volume of the hollow cylinder, in cm^3 , is given by V .

(a) Show that $S = 30\pi r^2 + 10\pi r h$. [3]

(b) The total surface area of the hollow cylinder is $240\pi \text{ cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{dV}{dr}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p . [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]



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Section B

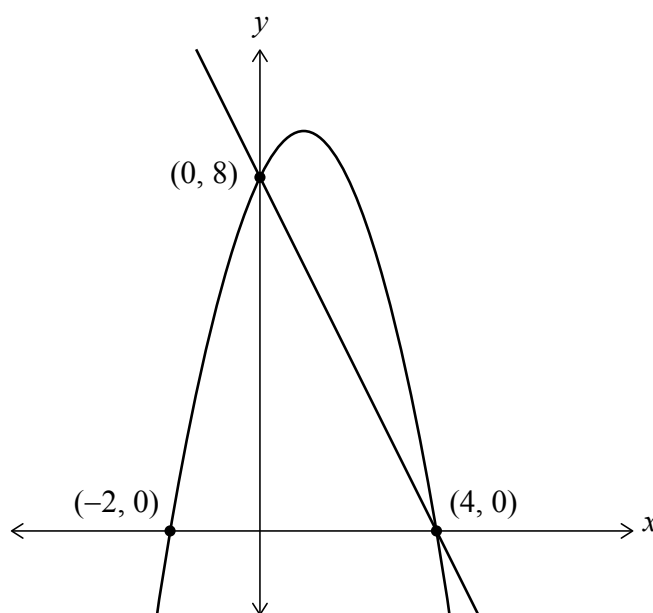
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g .

The graph of f is linear, has an x -intercept at $(4, 0)$ and a y -intercept at $(0, 8)$.

The graph of g has x -intercepts at $(-2, 0)$ and $(4, 0)$ and a y -intercept at $(0, 8)$.



- (a) Write down the equation for f in the form $f(x) = mx + c$. [2]

The function g is given by $g(x) = -x^2 + bx + 8$, where b is a real constant.

- (b) Find the value of b . [3]

- (c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^4 (-x^2 + 4x) dx$. [2]

- (d) Hence, find the area of the region enclosed by the graph of f and the graph of g . [4]

Point P is on the graph of g . The tangent to the graph of g at P is parallel to the graph of f .

- (e) Find the coordinates of P . [5]





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Section B

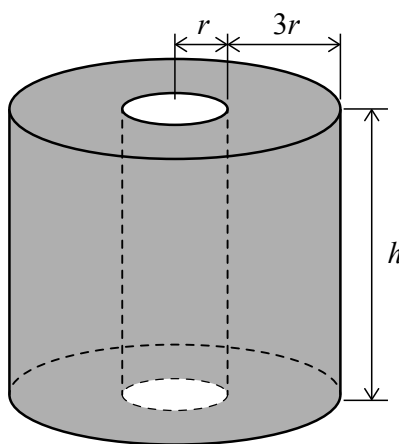
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h . A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2 , is given by S .

The volume of the hollow cylinder, in cm^3 , is given by V .

(a) Show that $S = 30\pi r^2 + 10\pi r h$. [3]

(b) The total surface area of the hollow cylinder is $240\pi \text{ cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{dV}{dr}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p . [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.6	32.3	34.1	35.6	38.0

A student uses linear regression to model the population of Canada using these data. The student model is $p = at + b$.

- (a) (i) Write down the value of a and the value of b .
- (ii) Interpret, in context, the value of a . [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.4 million people.

- (b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 30.6(1.007)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (c) (i) Use Benoit's model to predict the population of Canada in the year 2100.
- (ii) Interpret, in context, the value 1.007 in Benoit's model. [3]

(This question continues on the following page)



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(Question 8 continued)

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{61}{1 + e^{-0.03t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]
- (e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]
- (f) Find the value of
 - (i) $B'(40)$;
 - (ii) $C'(40)$. [2]
- (g) Compare and interpret, in context, the values of $B'(40)$ and $C'(40)$. [2]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The following table shows the population of Canada t years after the year 2000.

t (years after 2000)	0	5	10	15	20
p (population in millions)	30.7	32.2	34.0	35.7	37.9

A student uses linear regression to model the population of Canada using these data.
The student model is $p = at + b$.

- (a) (i) Write down the value of a and the value of b .
- (ii) Interpret, in context, the value of a . [3]

The student uses this model to predict the population of Canada in the year 2030, where $t = 30$, and calculates a population of approximately 41.3 million people.

- (b) Comment on the reliability of the student's prediction. [1]

A data scientist, Benoit, uses additional information to develop an exponential model for Canada's future population.

In this model, $B(t) = 33.5(1.005)^t$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (c) (i) Use Benoit's model to predict the population of Canada in the year 2100.
- (ii) Interpret, in context, the value 1.005 in Benoit's model. [3]

(This question continues on the following page)



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(Question 8 continued)

Another data scientist, Cecilia, develops a third model for the Canadian population.

In this model, $C(t) = \frac{62}{1 + e^{-0.02t}}$ represents the millions of people in Canada t years after the year 2000, where $25 \leq t \leq 100$.

- (d) Use Cecilia's model to predict the population of Canada in the year 2100. [1]
- (e) Determine the year in which the difference between the predictions from Benoit's model and Cecilia's model is greatest. [3]
- (f) Find the value of
 - (i) $B'(75)$;
 - (ii) $C'(75)$. [2]
- (g) Compare and interpret, in context, the values of $B'(75)$ and $C'(75)$. [2]



Turn over

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2. [Maximum mark: 5]

The graph of $y = f(x)$ has a local minimum point at (p, q) where $p > 0$.

Find the value of p and the value of q .

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Find the area completely enclosed by the curves $y = e^x$, $y = -e^x$, and the lines $x = -1$ and $x = 1$.

[illegible]

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Section B

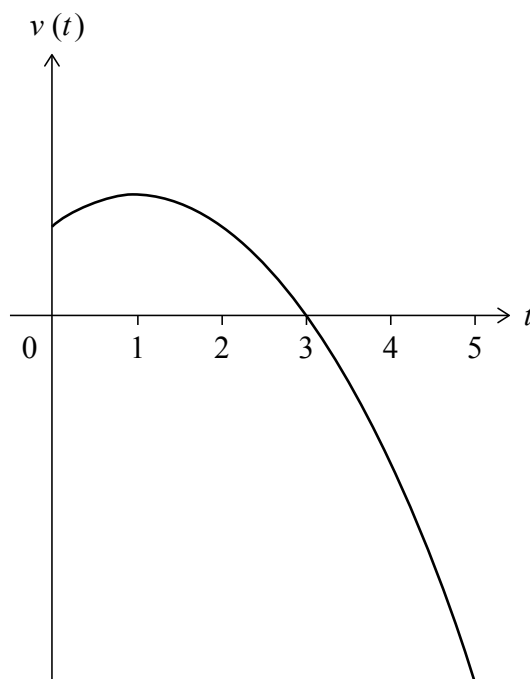
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v \text{ m s}^{-1}$, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \leq t \leq 5$.

The graph of v is shown in the following diagram.



The graph of v has a local maximum point where $t = 1$ and intersects the t -axis at $t = 3$.

(a) Determine the object's

(i) maximum velocity;

(ii) maximum speed.

[4]

At $t = T$, the object changes direction.

(b) (i) Write down the value of T .

(ii) Find the distance travelled by the object in the first T seconds.

[5]

(c) Determine whether the object returns to its initial position during the time period $0 \leq t \leq 5$, justifying your answer.

[4]



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1. [Maximum mark: 4]

(a) Find $f'(1)$. [2]

(b) Find the equation of the tangent to the graph of f at $x = 1$. [2]

[illegible]

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

6. [Maximum mark: 7]

Consider the function $f(x) = \sqrt{x^2 \ln x + 4 - x^2}$, where $x \in \mathbb{R}$, $x > 0$.

- (a) Show that the distance, l , between the origin and any point on the graph of f is given by $l = \sqrt{x^2 \ln x + 4}$. [1]
- (b) Hence, find the x -coordinate of the point on the graph of f which is closest to the origin. [6]



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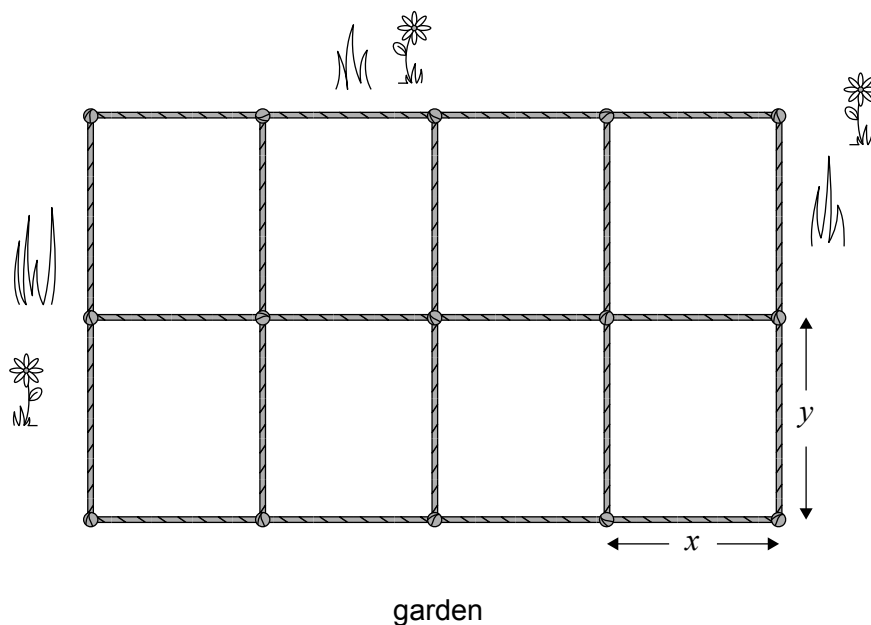
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

A gardener plans to enclose part of their garden with rope. The total area being enclosed is 60m^2 . This will be further divided by rope to make eight identical rectangular areas, each measuring x metres by y metres, where $x, y > 0$. This is shown in the following diagram.

diagram not to scale



(a) Find an expression for y in terms of x . [2]

(b) Show that the total length, T metres, of rope required is given by

$$T = 12x + \frac{75}{x}. \quad [2]$$

(c) Find an expression for $\frac{dT}{dx}$. [2]

(This question continues on the following page)



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(Question 7 continued)

When $x = k$, $\frac{dT}{dx} = 0$.

- (d) (i) Find the value of k .
- (ii) Hence, calculate the value of T when $x = k$.
- (iii) Find the value of y when $x = k$. [7]
- (e) (i) Find an expression for $\frac{d^2T}{dx^2}$.
- (ii) Hence, justify whether T has a local minimum or a local maximum when $x = k$. [2]



Turn over

Do **not** write solutions on this page.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the function $f(x) = \frac{(x-1)^2}{x}$, where $x \in \mathbb{R}$, $x \neq 0$.

(a) Show that $\frac{(x-1)^2}{x} = x - 2 + \frac{1}{x}$. [2]

(b) Hence, find $\int f(x) \, dx$. [3]

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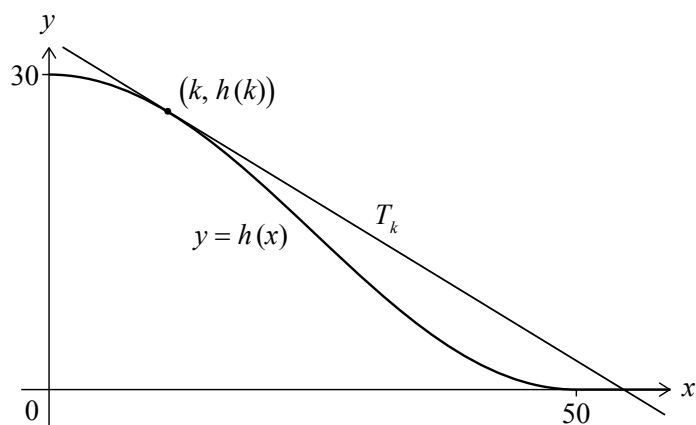
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6. [Maximum mark: 6]

The tangent, T_k , to the curve $y = h(x)$ at the point $(k, h(k))$ is shown on the following diagram.



- (a) Find the gradient of T_k in terms of k . [3]

Consider the case where the angle between T_k and the x -axis is $\frac{\pi}{8}$ radians.

- (b) Find the possible values of k . [3]

[illegible]

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3. [Maximum mark: 6]

(a) Find the values of x for which f is decreasing. [3]

(b) Find the values of x for which the graph of f is concave-up. [3]

[illegible]

6. [Maximum marks: 7]

(a) Find an expression for $f'(x)$, in terms of a . [3]

(b) Find the two possible values of a . [4]

[illegible]

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4440-1120

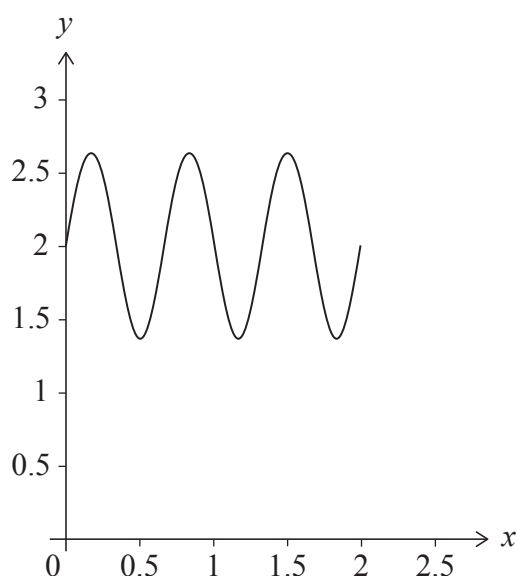
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Consider the function $f(x) = \frac{2}{\pi} \sin(3\pi x) + 2$, where $0 \leq x \leq 2$. The following diagram shows the graph of f .



- (a) (i) Write down the amplitude of f .
- (ii) Find the period of f . [3]
- (b) The point P has coordinates $(1.63, 2.16)$. State whether P lies above, below or on the graph of f . Justify your answer. [3]
- The line L_1 has equation $x - 6y + 11 = 0$.
- (c) Write down the gradient of the line L_1 . [1]

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(Question 7 continued)

The line L_1 is normal to the graph of f at point $A(1, 2)$.

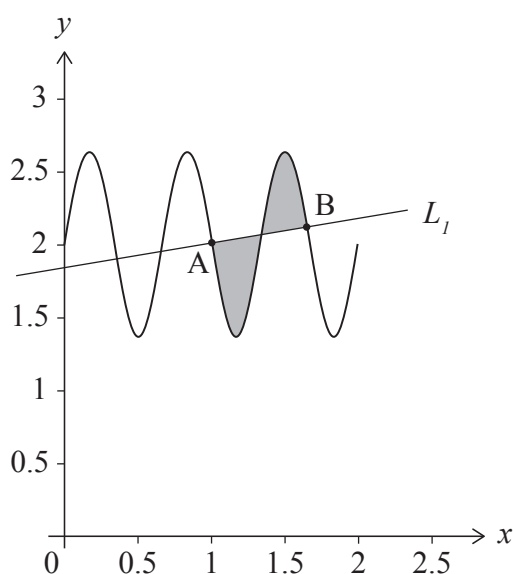
The line L_2 is tangent to the graph of f at A .

(d) (i) Find the gradient of L_2 .

(ii) Hence, or otherwise, find the equation of L_2 .

[3]

The line L_1 intersects the graph of f at another point B , where the x -coordinate of B is greater than 1.5. This is shown in the following diagram.



(e) Find the coordinates of B .

[2]

The shaded region is enclosed by the graph of f and the line L_1 between A and B .

(f) Find the area of the shaded region.

[3]



Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

Two athletes, Fiona and Lucy, compete in a 200 metres race along a straight track.

Fiona's velocity, in m s^{-1} , during the race can be modelled by $v(t) = \frac{8.14t}{\sqrt{t^2 + 0.2}}$, where $t \geq 0$. Time, t , is measured in seconds from when the race starts.

- (a) (i) Write down the value of $v(1)$.
- (ii) Find the time when Fiona's velocity is 5 m s^{-1} . [3]
- (b) Find the time when Fiona's acceleration is 4 m s^{-2} . [2]
- (c) (i) Write down the limit of $v(t)$ as t approaches infinity.
- (ii) State a reason why the value in part (c)(i) is not valid in the context of this question. [3]

Lucy's velocity, in m s^{-1} , during the race can be modelled by $w(t) = \frac{8t}{\sqrt{t^2 + 0.3}}$, where $t \geq 0$.

Fiona completes the race and crosses the finishing line in front of Lucy.

- (d) Find the distance Lucy is from the finishing line when Fiona completes the 200 metres. [6]



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Section A

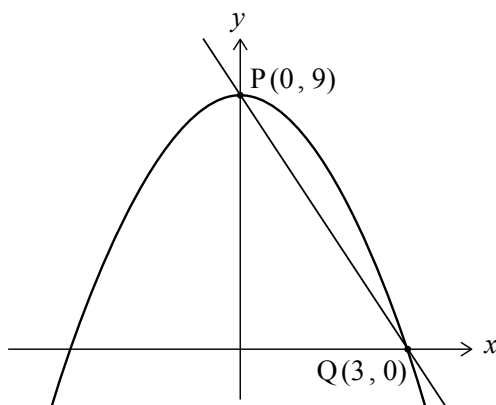
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

A line with equation $y = -3x + 9$ intersects the axes at the points $P(0, 9)$ and $Q(3, 0)$.

A parabola of the form $y = ax^2 + c$, where $a, c \in \mathbb{Z}$, also passes through the points P and Q .

This is shown in the following diagram.

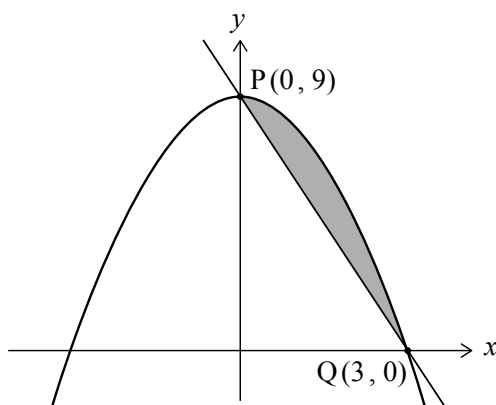


(a) (i) Write down the value of c .

(ii) Find the value of a .

[3]

The region enclosed by the line and the parabola is shaded in the following diagram.



(b) Find the area of the shaded region.

[4]

(This question continues on the following page)



[illegible]

Turn over

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

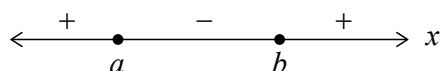
8. [Maximum mark: 14]

The function f has a derivative given by $f'(x) = 3x^2 + 12x - 15$.

The graph of $y = f(x)$ has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(a) Find the value of a and the value of b . [3]

The following diagram shows three intervals along the x -axis defined by a and b . The sign of the **first derivative** of f is shown in each interval.

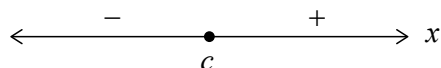


(b) State, with a reason, whether there is a local maximum point or a local minimum point on the graph of $y = f(x)$ at $x = a$. [2]

The second derivative $f''(x)$ is zero at $x = c$.

(c) Find the value of c . [3]

The following diagram shows two intervals along the x -axis defined by c . The sign of the **second derivative** is shown in each interval.



(d) State, with a reason, whether there is a point of inflexion on the graph of $y = f(x)$ at $x = c$. [2]

(e) Given that $f(-2) = 36$, find $f(x)$. [4]



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The gradient of a curve is given by $\frac{dy}{dx} = 4 \cos x - \sin x$. The curve passes through the point $(\pi, 3)$. Find the equation of the curve.

[illegible]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

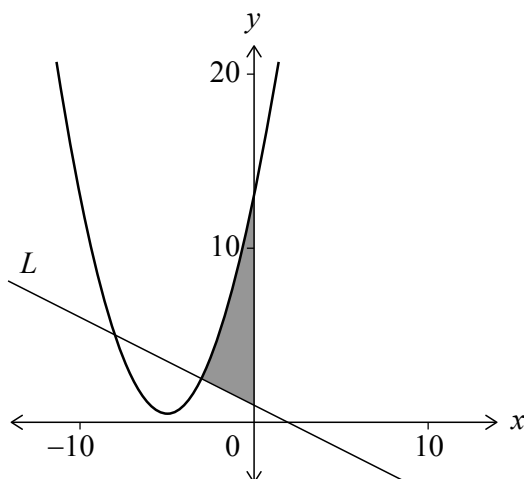
Consider the function defined by $f(x) = \frac{1}{2}x^2 + kx + 13$, where $x \in \mathbb{R}$ and $k \in \mathbb{Z}^+$.

- (a) Given that the equation $f(x) = 0$ has no real roots, show that the greatest possible value of k is 5. [2]

For the remainder of this question, consider the case $k = 5$.

- (b) (i) Write down the equation of the axis of symmetry of the graph of f .
 (ii) Hence, or otherwise, determine the coordinates of the minimum point on the graph of f . [3]

The following diagram shows the graph of f and a line L which is normal to the curve at $x = -3$. The shaded area shown is bounded by the curve, the line L and the y -axis.



- (c) Show that the equation of L is given by $y = -\frac{1}{2}x + 1$. [5]
 (d) Hence, find the shaded area. [5]



Turn over

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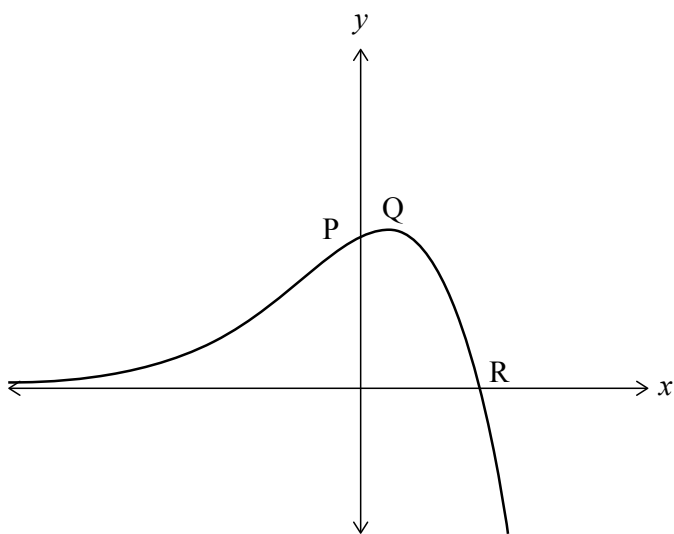
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

The function f is defined by $f(x) = \frac{5e^x - e^{2x}}{e^x + 1}$. The following diagram shows part of the graph of f . The graph intersects the y -axis at point P and intersects the x -axis at point R.

Point Q is a local maximum point with coordinates $(q, 7 - 2\sqrt{6})$.



- (a) (i) Find the coordinates of point P. [6]
- (ii) Find the coordinates of point R. [6]
- (b) State the range of f . [1]
- (c) Show that $f'(x) = \frac{-e^{3x} - 2e^{2x} + 5e^x}{(e^x + 1)^2}$. [3]
- (d) Hence, show that $q = \ln(\sqrt{6} - 1)$. [6]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

Consider the function $f(x) = \frac{3x+1}{2x-4}$ where $x \neq 2$.

- (a) Write down the equation of
- (i) the vertical asymptote of the graph of f ;
 - (ii) the horizontal asymptote of the graph of f . [2]
- (b) Find $f'(x)$. [4]
- (c) Hence, explain briefly why f is never increasing. [2]

The point $P(x, f(x))$ is on the graph of f such that $x < 2$.

Let d be the distance between the points P and $Q(3, 5)$.

- (d) Find an expression for d in terms of x . [2]
- (e) Hence, find the coordinates of P such that d is a minimum. [3]



Turn over

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

A population of frogs, F , in a swamp after t months, can be modelled by the function

$$F(t) = 1850 \times 1.105^t \text{ where } t \geq 0.$$

(a) Find the population of frogs after one year. [2]

(b) After x complete months, the population will be at least 35 000 frogs. Find the value of x . [3]

The function F can be written in the form $F(t) = 1850e^{kt}$.

(c) Find the exact value of k . [2]

(d) Find the rate at which the population of frogs is growing after 15 months. [2]

A more realistic model describing the population of frogs, G , after t months is given by

$$G(t) = \frac{35000}{1 + Ae^{-0.0998t}} \text{ where } t \geq 0.$$

(e) After 15 months, this model predicts a population of 6995 frogs. Find the value of A . [2]

(f) Find the value of t when the rate of population growth is the greatest. [2]

(g) By considering the graph of G or otherwise, state one reason why $G(t)$ is a more appropriate long-term model than $F(t)$. [1]



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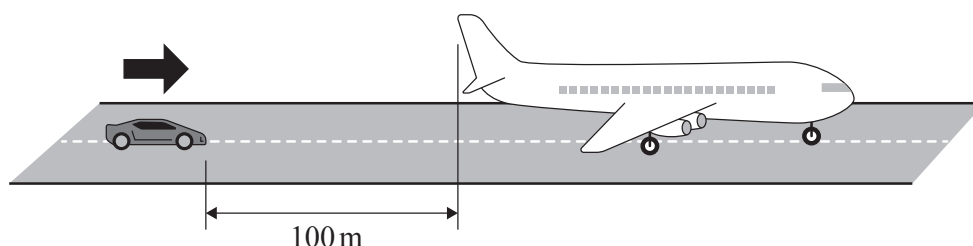
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

An airplane lands on a runway 100 metres in front of a stationary car. At the instant the airplane lands, the car begins to travel in the same direction towards the airplane.

diagram not to scale



Let t represent the number of seconds after the airplane lands. For $t \geq 0$, the velocities of the airplane and the car in m s^{-1} , can be modelled by the following equations:

$$v_{\text{air}} = 60e^{-0.1t}$$

$$v_{\text{car}} = 5t$$

- (a) When the airplane lands, write down the speed of
 - (i) the airplane;
 - (ii) the car. [2]
 - (b) Find
 - (i) the value of t when the airplane and the car have the same speed;
 - (ii) the speed at this time. [3]
- Let $d(t)$ represent the distance, in metres, between the car and the back of the airplane after t seconds.
- (c) If $d(0) = 100$, find $d(t)$. [7]
 - (d) Hence, find how long it takes for the car to reach the back of the airplane. [2]
 - (e) Find the distance travelled by the car when it reaches the back of the airplane. [3]



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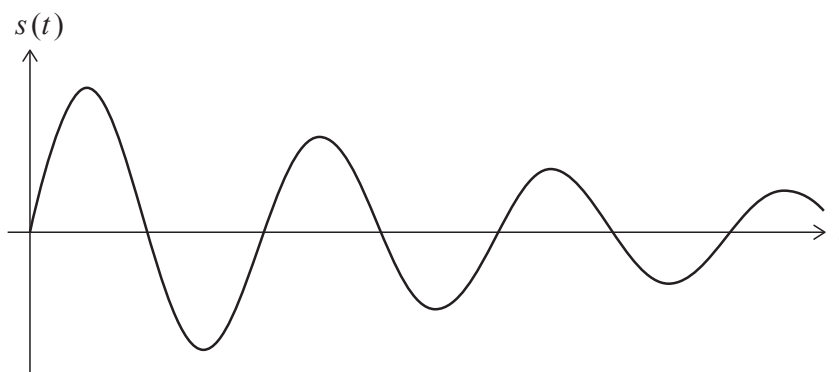
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

A particle P moves in a straight line so that its displacement, s cm, from a fixed point O at time t seconds is given by $s(t) = 2^{\left(1 - \frac{t}{5}\right)} \sin\left(\frac{2\pi t}{3}\right)$, where $t \geq 0$.

The following diagram shows part of the graph of $y = s(t)$.



- (a) Find
- (i) the maximum displacement of P from O ;
 - (ii) the maximum velocity of P . [5]
- (b) Find
- (i) the minimum value of the displacement function $s(t)$;
 - (ii) the displacement of P from O when $t = 3.5$. [3]
- (c) Hence, determine the **total distance** travelled by P in the first 3.5 seconds. [3]
- The first time that P returns and passes through O is when $t = T$.
- (d) Write down the value of T . [1]

(This question continues on the following page)



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(Question 9 continued)

The particle passes through O every T seconds.

A sequence $u_1, u_2, u_3 \dots$ is formed where $u_1, u_2, u_3 \dots$ are the largest **distances** from O in each of the intervals $0 < t < T$, $T < t < 2T$, $2T < t < 3T \dots$ respectively.

It is known that $u_1, u_2, u_3 \dots$ form a geometric sequence.

- (e) (i) Determine the value of the common ratio r of this geometric sequence.
- (ii) Calculate the **total distance** travelled by the particle if it were to continue to move in this way indefinitely.

[5]



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