

APOLLO

Functions

SL

Mathematics: applications and interpretation

First assessment 2021

64 questions | mark schemes

4. (a) $L(40) = 1.50 \times 40 - 5$

(M1)

$= \$ 55$

A1

[2 marks]

(b) $70 = 1.50x - 5$

(M1)

$(x =) 50$ litres

A1

[2 marks]

(c) $1.30x$

(A1)

$1.30x < 1.50x - 5$

(M1)

Note: Award **M1** for a graph showing two intersecting linear functions, provided one function has a y -intercept of 0 and the other function has a negative y -intercept.

(minimum value of $k =$) 25

A1

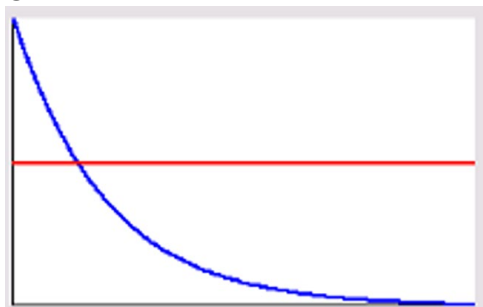
Note: Accept $x > 25$.

[3 marks]

Total [7 marks]

7. (a) **EITHER**

$$50 = 100e^{-1 \times p} \quad \text{OR} \quad 0.5 = e^{-1 \times p}$$

(M1)**OR****(M1)****THEN**

$$0.693 \quad (0.693147..., \ln 2)$$

A1**[2 marks]**

(b) $R(1.5) = 100e^{-0.693147... \times 1.5}$

(M1)

$$35.4(\%) \quad (35.3553...)$$

A1**[2 marks]**

(c) $R(t) > 0$ **OR** $R(t)$ has a horizontal asymptote

R1**[1 mark]**

(d) Award **A1** for **one** reasonable limitation of the domain:

A1

small values of t produce unrealistic results

$$R(0) = 100\%$$

large values of t are not possible

people do not live forever

model is not valid at small or large values of t

The reason should focus on the domain $t \geq 0$. Do not accept answers such as:

recollection varies for different people

memories are discrete not continuous

the nature of the information will change how easily it is recalled

emotional/physical stress can affect recollection/concentration

Note: Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

[1 mark]**Total [6 marks]**

11. (a) $I = \frac{k}{d^2}$ (M1)

$4 = \frac{k}{1.5^2}$ M1

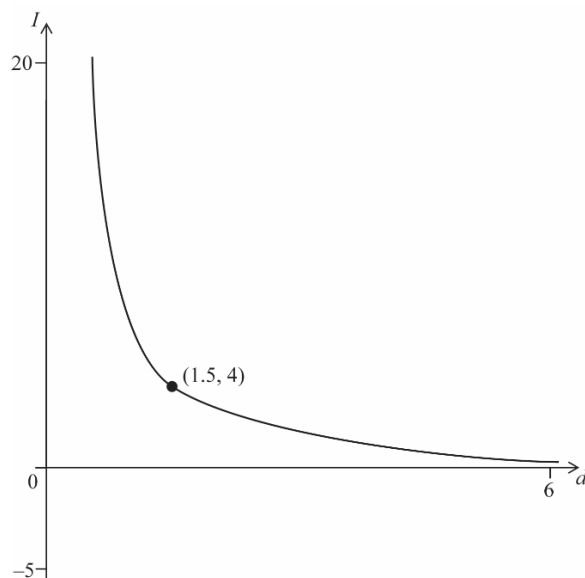
$I = \frac{9}{d^2}$ AG

Note: The **AG** line must be seen for the second **M1** to be awarded.

Award no marks for substituting 1.5 and 4 into $I = \frac{9}{d^2}$ (i.e., working backwards).

[2 marks]

(b)



A1A1

Note: Award **A1** for correct general shape (concave up) with no I -intercept, passing through the marked point (1.5, 4); the point must be labelled with either the coordinates or the values 1.5 and 4 on the x and y axes. Award **A1** for the curve showing asymptotic behavior (i.e. I tends to 0, as d tends to infinity), extending to at least $d = 6$; the curve must not cross nor veer away from the horizontal asymptote.

[2 marks]

(c) $1.5 \times 10^{-6} \geq \frac{9}{d^2}$ (M1)

Note: Award **(M1)** for a correct inequality.

$d \geq 2450$ (m) (2449.48...)

A1

Note: Award **A0** for $d = 2450$.

[2 marks]

Total [6 marks]

1. (a) (i) 23 mg

A1

(ii) $1 - 0.85$ **OR** $\frac{23 - 19.55}{23}$ **OR** 0.15
15 (%)

(M1)

A1

[3 marks]

(b) $23(0.85)^{10}$
4.53 mg (4.52811...)

(M1)

A1

[2 marks]

Total [5 marks]

7. (a) $(f(-7) =) 8$ and $(f(7) =) 1$ (A1)

range is $f(x) \leq 1$, $f(x) \geq 8$ A1A1

Note: Award at most **A1A1A0** if strict inequalities are used.

[3 marks]

- (b) **EITHER**
 sketch of f and $y = 0$ or sketch of f^{-1} and $x = 0$ (M1)

OR

finding the correct expression of $f^{-1}(x) = \frac{-2-5x}{x-2}$ (M1)

OR

$f^{-1}(0) = \frac{-2-5(0)}{0-2}$ (M1)

OR

$f(x) = 0$ (M1)

THEN

$f^{-1}(0) = 1$ A1

[2 marks]

Total [5 marks]

12. (a) 3

A1

Note: Accept (3, 0) seen.

[1 mark]

(b) METHOD 1

$$0 = 4a - 2b + c, \quad 0 = 9a + 3b + c, \quad -\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$$

(M1)(A1)

(i) 2

A1

(ii) -2

A1

(iii) -12

A1

Note: Award the (M1)(A1) if at least one correct value is seen.
Do not apply **FT** from part (a) if workings are not shown.

METHOD 2

$$-12.5 = a(0.5 + 2)(0.5 - 3)$$

(M1)

(i) $a = 2$

A1

$$0 = 2 \times (3)^2 + 3b + c$$

$$0 = 2 \times (-2)^2 + (-2)b + c$$

(M1)

(ii) $b = -2$

A1

(iii) $c = -12$

A1

[5 marks]

(c) $x = 0.5$

A1

Note: Do not **FT** from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

3. (a) $(A =) 112$

A1

[1 mark]

(b) $112e^{5k} = 360$

(M1)

Note: Award **(M1)** for their correct equation.

EITHER

graph of $y = 112e^{5x}$ and $y = 360$ with indication of point of intersection

(M1)

OR

$$(k =) \frac{1}{5} \ln\left(\frac{360}{112}\right)$$

(M1)

Note: Award **(M1)** for correct rearranging and use of log.

THEN

$(k =) 0.234 \text{ (0.233521...)}$

A1

Note: Award **(M1)(M1)(A0)** for 0.233.

[3 marks]

Total: [4 marks]

4. (a) $m = \frac{6-0}{4-2} = 3$

(M1)A1**[2 marks]**

(b) $(m =) -\frac{1}{3} (-0.333, -0.333333...)$

A1**[1 mark]**

- (c) an equation of line with a correct intercept and either of their gradients from (a) or (b)

(M1)

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

Note: Award **(M1)** for substituting either of their gradients from parts (a) or (b) and point B or (3, 3) into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple

A1**[2 marks]**

(d) $(x =) 12$

A1**[1 mark]****Total: [6 marks]**

5. (a) $h(4) = \frac{640}{4^2} + 0.5$ **OR** $h(14) = \frac{640}{14^2} + 0.5$ **(M1)**

Note: Award **(M1)** for substituting 4 or 14 into h . This can be implicit from seeing 3.77 (3.76530...) or 40.5.

$$3.77 \leq h(x) \leq 40.5 \quad (3.76530... \leq h(x) \leq 40.5) \quad \textbf{A1A1}$$

Note: Award **A1** for both correct endpoints seen, **A1** for the endpoints in a correct interval.

[3 marks]

(b) (i) $h(x) = 10$ **OR** $h^{-1}(x) = \sqrt{\frac{640}{x-0.5}}$ **OR** $h^{-1}(10) = \sqrt{\frac{640}{10-0.5}}$ **(M1)**

A1

$(x =) 8.21 \text{ cm } (8.20782...)$

(ii) a tin that is 10 cm high will have a diameter of 8.21 cm (8.20782...) **A1**

Note: Condone a correct answer expressed as the converse.

(iii) $4 \leq h^{-1} \leq 14$ **A1**

Note: Accept $4 \leq y \leq 14$. Do not **FT** in this part.

[4 marks]

Total: [7 marks]

13. (a) (i) $c = 10$ **A1**

(ii) $64a + 8b + 10 = 10$ **A1**

$16a + 4b + 10 = 12$ **A1**

Note: Award **A1** for each equivalent expression or **A1** for the use of the axis of symmetry formula to find $4 = \frac{-b}{2a}$ or from use of derivative.

Award **A0A1** for $64a + 8b + c = 10$ and $16a + 4b + c = 12$.

(iii) $y = -\frac{1}{8}x^2 + x + 10$ **A1A1**

Note: Award **A1A0** if one term is incorrect, **A0A0** if two or more terms are incorrect. Award at most **A1A0** if correct a , b and c values are seen but answer not expressed as an equation.

[5 marks]

(b) recognizing the need to integrate their expression **(M1)**

$\int_0^8 -\frac{1}{8}x^2 + x + 10 \, dx$ **(A1)**

Note: Award **(A1)** for correct integral, including limits. Condone absence of dx .

$90.7 \text{ cm}^2 \left(\frac{272}{3}, 90.6666\dots \right)$ **A1**

[3 marks]

Total: [8 marks]

3. (a) (i) maximum $h = 130$ metres
(ii) minimum $h = 50$ metres

A1

A1

[2 marks]

- (b) (i) $(60 \div 12 =) 5$ seconds

A1

- (ii) $360 \div 5$

(M1)

Note: Award (M1) for 360 divided by their time for one revolution.

$$= 72^\circ$$

A1

[3 marks]

- (c) (i) (amplitude $=$) 40

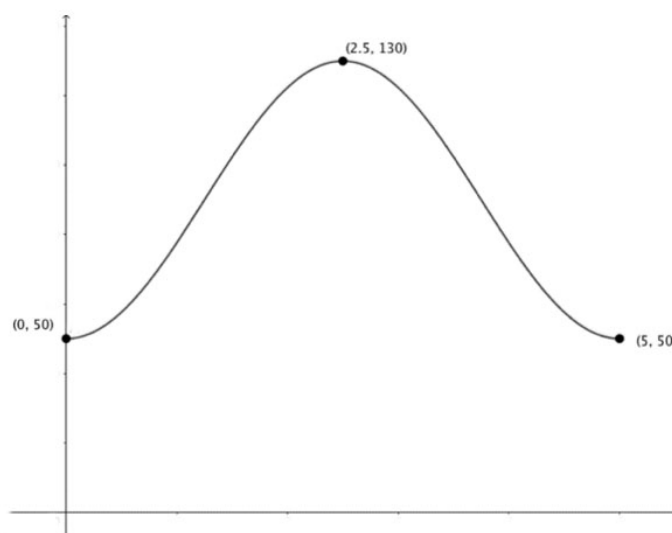
A1

- (ii) (period $= \frac{360}{72} =$) 5

A1

[2 marks]

- (d)



Maximum point labelled with correct coordinates.

A1

At least one minimum point labelled. Coordinates seen for any minimum points must be correct.

A1

Correct shape with an attempt at symmetry and "concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain.

A1

[3 marks]

continued...

Question 3 continued

(e) (i) $h = 90 - 40 \cos(144^\circ)$ (M1)
 $(h =) 122 \text{ m } (122.3606\dots)$ A1

(ii) evidence of $h = 100$ on graph OR $100 = 90 - 40 \cos(72t)$ (M1)
 t coordinates 3.55 (3.54892...) OR 1.45 (1.45107...) or equivalent (A1)

Note: Award **A1** for either t -coordinate seen.

$= 2.10 \text{ seconds } (2.09784\dots)$ A1
[5 marks]

(f) (i) $5 - 2.09784\dots$ (M1)
 $\frac{(2.902153\dots)}{5}$ (M1)
 $0.580 \text{ } (0.580430\dots)$ A1

(ii) **METHOD 1**
 changing the frequency/dilation of the graph will not change the proportion of time that point C is visible. A1
 $0.580 \text{ } (0.580430\dots)$ A1

METHOD 2
 correct calculation of relevant found values
 $\frac{(2.902153\dots)}{5/2}$ A1
 $0.580 \text{ } (0.580430\dots)$ A1

Note: Award **A0A1** for an unsupported correct probability.

[5 marks]
Total: [20 marks]

4. (a) 20

A1
[1 mark]

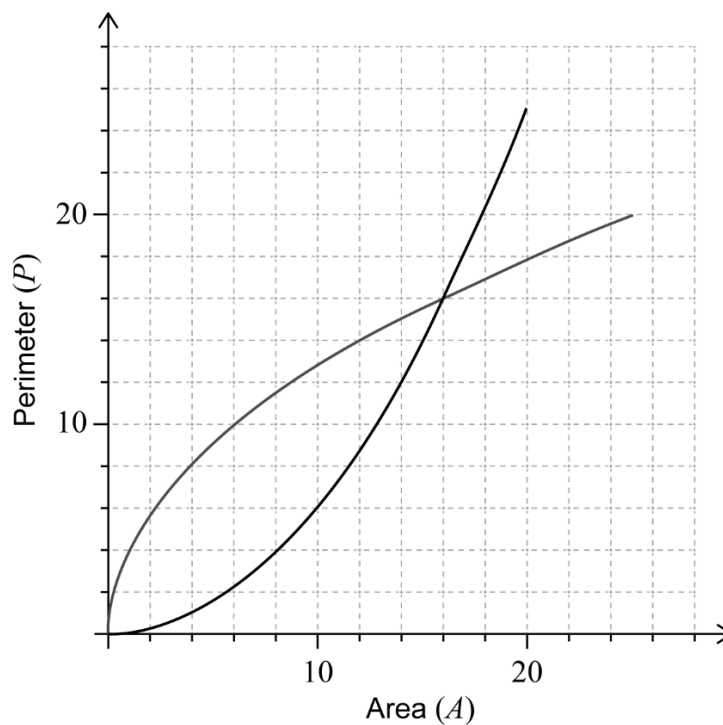
(b) $n = 20$

A1

Note: Follow through from part (a).

[1 mark]

(c)



(M1)A1A1

Note: Award **(M1)** for reflection in the line $P = A$, award **A1** for endpoint at (20, 25), award **A1** for passing through (16, 16).

[3 marks]

(d) when the perimeter is 8, the area is 4

A1
[1 mark]

Total [6 marks]

5. (a) (i) 1750

A1

(ii) $1350 + 400(1.25)^{-5}$

(M1)

$=1480$

A1

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400(1.25)^{-t}$

(M1)

9.32 (days (9.31885...) (days))

A1

[2 marks]

(c) 1350

A1

Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.

[1 mark]

Total [6 marks]

8. (a) $10\log_{10}(6.4 \times 10^{-3} \times 10^{12})$ (M1)
 $= 98.1(\text{dB})$ (98.06179...) A1
[2 marks]
- (b) $112 = 10\log_{10}(S \times 10^{12})$ (M1)
 $0.158(\text{W m}^{-2})(0.158489...(\text{W m}^{-2}))$ A1
[2 marks]
- Total [4 marks]**

5. (a) (i) $p(10)^2 + q(10) = 60$

M1

$$10p + q = 6 \quad (100p + 10q = 60)$$

A1

(ii) $p = 1, q = -4$

A1A1

Note: If p and q are both incorrect then award **M1A0** for an attempt to solve simultaneous equations.

[4 marks]

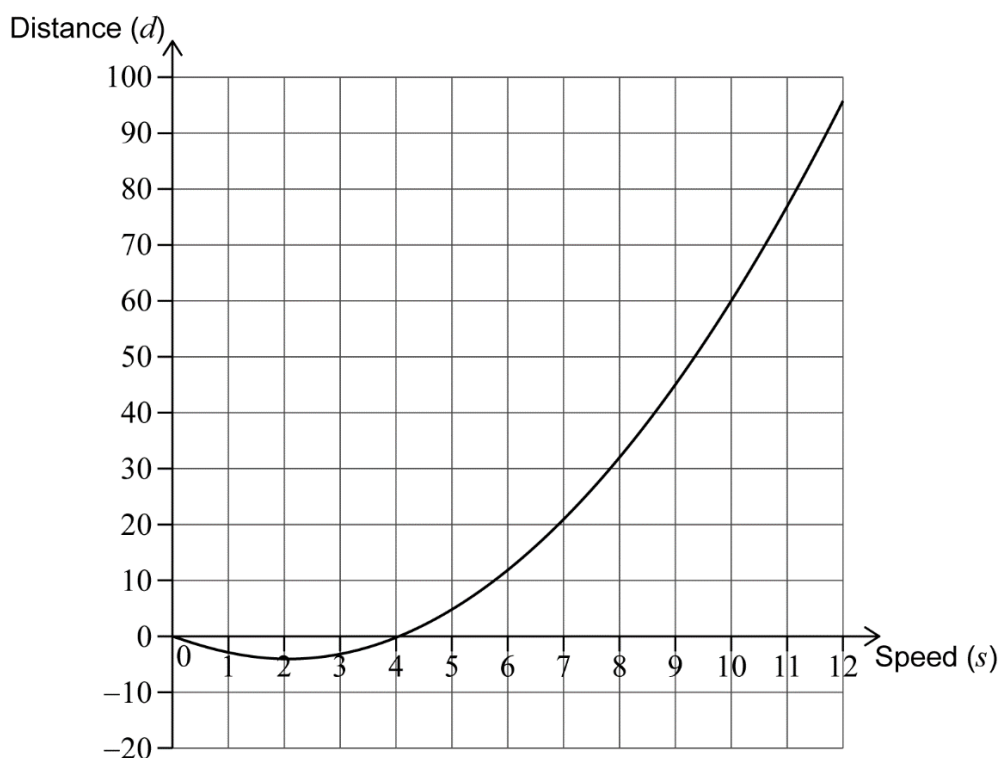
(b) $(2, -4)$

A1A1

Note: Award **A1** for each correct coordinate.
Award **A0A1** if parentheses are missing.

[2 marks]

(c)



A3

Note: Award **A1** for smooth quadratic curve on labelled axes and within correct window. Award **A1** for the curve passing through $(0, 0)$ and $(10, 60)$. Award **A1** for the curve passing through their vertex. Follow through from part (b).

[3 marks]

(d) the graph indicates there are negative stopping distances (for low speeds)

R1

Note: Award **R1** for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances...).

[1 mark]

continued...

Question 5 continued

$$(e) \quad 0.95 \times 20^2 - 3.92 \times 20$$

$$= 302(\text{m}) \quad (301.6\dots)$$

(M1)**A1****[2 marks]**

$$(f) \quad \left| \frac{301.6 - 320}{320} \right| \times 100$$

$$= 5.75(\%)$$

M1**A1****[2 marks]**

$$(g) \quad 330 = 1.6 \times s + 0.95 \times s^2 - 3.92 \times s$$

M1A1

Note: Award **M1** for an attempt to find an expression including stopping distance (model B) and reaction distance, equated to 330.
Award **A1** for a completely correct equation.

$$19.9(\text{ms}^{-1}) \quad (19.8988\dots)$$

A1**[3 marks]****Total [17 marks]**

3. (a) 1.2 metres **A1**

[1 mark]

(b) $-4.8t^2 + 21t + 1.2 = 0$
 $(t \Rightarrow) 4.43 \text{ s } (4.431415... \text{ s})$

(M1)

A1

Note: If both values for t are seen do not award the **A1** mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \leq t \leq 4.43$ **OR** $[0, 4.43]$

A1A1

Note: Award **A1** for correct endpoints and **A1** for expressing answer with correct notation. Award at most **A1A0** for use of x instead of t .

[2 marks]

[Total 5 marks]

11. (a) $\log_{10} 100 = a - 3$ (M1)
 $a = 5$

A1
[2 marks]

(b) **EITHER**
 $N = 10^{5-M}$ (M1)
 $= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$

OR
 $100 = \frac{b}{10^3}$ (M1)

THEN
 $b = 100000 (= 10^5)$ **A1**
[2 marks]

(c) $0.001 < N < 100000$ ($10^{-3} < N < 10^5$) **A1A1**

Note: Award A1 for correct endpoints and A1 for correct inequalities/interval notation.
--

[2 marks]

(d) $N = \frac{10^5}{10^{7.2}}$ ($= 0.0063095\dots$) (M1)

length of time = $\frac{1}{0.0063095\dots} = 10^{2.2}$
 $= 158$ years

A1
[2 marks]
[Total 8 marks]

4. (a) $(\text{pH} =) -\log_{10}(1.3 \times 10^{-5})$ (M1)
 4.89 (4.88605...) A1

[2 marks]

- (b) **EITHER**
 calculating pH
 $(\text{pH} =) -\log_{10}(10 \times 1.3 \times 10^{-5})$ (M1)
 3.89 (3.88605...) A1
 (3.89 < 4.89, therefore) the unknown liquid is more acidic (than coffee). A1

Note: Follow through within the part for the final **A1**. A correct conclusion must be supported by a mathematical justification linking the *C*-value to the pH level to earn the final **A1**; a comparison of *C*-values only earns **M0A0A0**.

OR

referencing the graph

The graph of $y = -\log_{10}(x)$ shows that as the value of x increases, the value of y decreases.

M1

Since the *C*-value (x -value) of the unknown liquid is larger than that of the coffee, the pH level (y -value) is lower.

R1

The unknown liquid is more acidic (than coffee).

A1

Note: Follow through within the part for the final **A1**. A correct conclusion must be supported by a mathematical justification linking the *C*-value to the pH level to earn the final **A1**; a comparison of *C*-values only earns **M0R0A0**.

[3 marks]

Total [5 marks]

9. (a) *Accept any one of the following (or equivalent):*
 one minimum and one maximum point
 three x -intercepts or three roots (or zeroes)
 one point of inflexion

R1

Note: Do not accept “S shape” as a justification.

[1 mark]

(b) (i) $(d =) -5$

A1

(ii) $8 = a + b + c$
 $4 = 8a + 4b + 2c$
 $0 = 27a + 9b + 3c$

A2

Note: Award **A2** if all three equations are correct.
 Award **A1** if at least one is correct. Award **A1** for three correct equations that include the letter “ d ”.

(iii) $a = 2, b = -12, c = 18$

A1**[4 marks]**

- (c) equating found expression to zero

(M1)

$$0 = 2t^3 - 12t^2 + 18t - 5$$

$$t = 0.358216..., 1.83174..., 3.81003...$$

(A1)

(so total time in debt is $3.81003... - 1.83174... + 0.358216 \approx$)

2.34 (2.33650...) years

A1**[3 marks]****Total [8 marks]**

12. (a) $(x =) -\frac{4.48}{2(-1.6)}$ **OR** coordinates of maximum point (1.4, 3.136) **(M1)**

$x = 1.4$

A1

[2 marks]

(b) **METHOD 1**

the cart is centred in the archway when it is between

$x = 0.6$ and $x = 2.2$,

where $y \geq 2.112$ (m) (which is greater than 2)

A1

R1

the archway is tall enough for the crate

A1

Note: Do not award **R0A1**.

METHOD 2

the height of the archway is greater or equal to 2.0 between

$x = 0.557385\dots$ and $x = 2.24261\dots$

width of this section of archway =

$(2.24261\dots - 0.557385\dots) = 1.68522\dots$ (m) (which is greater than 1.6)

A1

R1

the archway is wide enough for the crate

A1

Note: Do not award **R0A1**.

[3 marks]

Total [5 marks]

1. (a) **EITHER**
annual cycle for daylight length **R1**

OR
there is a minimum length for daylight (cannot be negative) **R1**

OR
a quadratic could not have a maximum and a minimum or equivalent **R1**

Note: Do not accept “Paula's model is better”.

[1 mark]

(b) (i) 4 **A1**

(ii) 12 **A1**

(iii) $y = 12$ **A1A1**

Note: Award **A1** “ $y = (\text{a constant})$ ” and **A1** for that constant being 12.

[4 marks]

(c) $f(t) = -4 \cos(30t) + 12$ **OR** $f(t) = -4 \cos(-30t) + 12$ **A1A1A1**

Note: Award **A1** for $b = 30$ (or $b = -30$), **A1** for $a = -4$, and **A1** for $d = 12$. Award at most **A1A1A0** if extra terms are seen or form is incorrect. Award at most **A1A1A0** if x is used instead of t .

[3 marks]

(d) $10.5 = -4 \cos(30t) + 12$ **(M1)**

EITHER
 $t_1 = 2.26585\dots, t_2 = 9.73414\dots$ **(A1)(A1)**

OR
 $t_1 = \frac{1}{30} \cos^{-1} \frac{3}{8}$ **(A1)**

$t_2 = 12 - t_1$ **(A1)**

THEN
 $9.73414\dots - 2.26585\dots$
 7.47 (7.46828...) months (0.622356...years) **A1**

Note: Award **M1A1A1A0** for an unsupported answer of 7.46. If there is only one intersection point, award **M1A1A0A0**.

[4 marks]

continued...

Question 1 continued

$$(e) \left| \frac{16 - \left(16 + \frac{14}{60}\right)}{16 + \frac{14}{60}} \right| \times 100\% \quad (M1)(M1)$$

Note: Award **M1** for correct values and absolute value signs, **M1** for $\times 100$.

$$= 1.44\% \text{ (1.43737... \%)}$$

A1**[3 marks]****[Total 15 marks]**

4. (a) (i) an attempt to find the amplitude (M1)
 $\frac{61.8}{2}$ OR $\frac{64.5 - 2.7}{2}$
 (a =) 30.9 m A1

Note: Accept an answer of (a =) –30.9 m.

- (ii) (period = $\frac{60}{1.5}$ =) 40 (s) (A1)
 $((b =) \frac{360^\circ}{40})$
 (b =) 9 A1

Note: Accept an answer of (b =) –9.

- (iii) attempt to find d (M1)
 $(d =) 30.9 + 2.7$ OR $\frac{64.5 + 2.7}{2}$
 (d =) 33.6 m A1
[6 marks]

- (b) 12×1.5 OR $\frac{12 \times 60}{40}$ (M1)
 18 (revolutions per ride) A1
[2 marks]

- (c) (i) $0 \leq t \leq 720$ A1
 (ii) $2.7 \leq h \leq 64.5$ A1A1

Note: Award **A1** for correct endpoints of domain and **A1** for correct endpoints of range. Award **A1** for correct direction of both inequalities.

[3 marks]

- (d) graph of $h(t)$ and $y = 16.7$ OR $h(t) = 16.7$ (M1)
 $6.31596\dots$ and $33.6840\dots$ (A1)
 27.4 (s) ($27.3680\dots$) A1
[3 marks]

continued...

Question 4 continued

(e) (i) d **A1**

(ii) **EITHER**
 $d + 30.9 = 65.2$ **(A1)**

OR
 $65.2 - (61.8 + 2.7) = 0.7$ **(A1)**

OR
 3.4 (new platform height) **(A1)**

THEN
 $(d =) 34.3 \text{ m}$ **A1**

[3 marks]

Total [17 marks]

3. (a) 1.8 (m) **A1**
[1 mark]

(b) **EITHER**

$$\frac{-10.8}{2(-3.6)}$$
(M1)

OR
 $-7.2(t) + 10.8 = 0$
(M1)

OR
 sketch indicating maximum
 (M1)

THEN
 $(t =) 1.5$ seconds
 A1

Note: Award **(M1)A0** for (1.5, 9.9) seen.

[2 marks]

(c) **EITHER**
 $0 = -3.6t^2 + 10.8t + 1.8$
(M1)

OR
 sketch indicating a root
 (M1)

THEN
 $(t =) 3.16$ seconds (3.15831...)
 A1

Note: Award at most **M1A0** if -0.158 ($-0.158312\dots$) is part of the final answer unless clearly rejected.

[2 marks]

Total [5 marks]

5. (a) $71e^{-0.0514(16)} + 23$ (M1)

54.2 °C (54.1956...) A1

[2 marks]

(b) $T = 23$ A1

Note: Condone $y = 23$.

[1 mark]

(c) 23 °C A1

[1 mark]

(d) $50 = 71e^{-0.0514(k)} + 23$ (M1)

$k = 18.8 \left(\frac{-5000}{257} \ln \left(\frac{27}{71} \right), 18.8101... \right)$ A1

Note: Award **M1** for a sketch showing a point of intersection between the exponential function and $y = 50$.

[2 marks]

Total [6 marks]

12. (a) 78 **A1**
[1 mark]
- (b) (i) 65 **A1**
- (ii) **EITHER**
 (period =) 16 (could be seen on sketch) **(M1)**
 $b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$
 $(b =) 0.393 \left(0.392699..., \frac{\pi}{8} \right)$ **OR** $(b =) 22.5^\circ$ **A1**
- OR**
 $143 = 65 \sin(4b) + 78$ **(M1)**
 $(\sin(4b) = 1)$
 $(4b = \frac{\pi}{2} \text{ **OR** } 4b = 90^\circ)$
 $(b =) 0.393 \left(0.392699..., \frac{\pi}{8} \right)$ **OR** $(b =) 22.5^\circ$ **A1**
- [3 marks]**
- (c) 13 **A1**
- Note:** Apply follow through marking only if their final answer is positive.
- [1 mark]**
- (d) $(b =) 0.196 \left(0.196349..., \frac{\pi}{16} \right)$ **OR** $(b =) 11.3^\circ (11.25^\circ)$ **A1**
- [1 mark]**
Total [6 marks]

5. (a) 2

A1

[1 mark]

(b) attempt to substitute their part (a) and point $(3, -1)$ into the slope-intercept form or point-slope form of an equation **(M1)**

$$-1 = 2 \times 3 + c \quad \text{OR} \quad y + 1 = 2(x - 3)$$

$$y = 2x - 7$$

A1

Note: Equation must be in the form $y = mx + c$ for **A1** to be awarded.

[2 marks]

continued...

*Question 5 continued***(c) METHOD 1**

attempt to show that P does not lie on L_2 **(M1)**

e.g. $-\frac{1}{2}(3) - \frac{5}{2}$ **OR** graph showing L_2 and P in approximate correct locations

$-1 \neq -\frac{1}{2}(3) - \frac{5}{2}$ ($-1 \neq -4$) **OR** $(3, -1)$ does not lie on the graph of L_2 **R1**

hence L_2 is not the normal line to $f(x)$ at point P **AG**

METHOD 2

attempt to find the equation of the normal line at $(3, -1)$ **(M1)**

$(-1 = -\frac{1}{2}(3) + c$ **OR** $y + 1 = -\frac{1}{2}(x - 3))$

the normal line is $y = -\frac{1}{2}x + \frac{1}{2}$ **R1**

hence L_2 is not the normal line to $f(x)$ at point P **AG**

METHOD 3

attempt to find the intersection of L_1 and L_2 **(M1)**

Intersection of $y = 2x - 7$ and $y = -\frac{1}{2}x - \frac{5}{2}$ is $(1.8, -3.4)$

$x = 1.8 \neq 3$ **OR** $y = -3.4 \neq -1$ **R1**

hence L_2 is not the normal line to $f(x)$ at point P **AG**

Note: Accept equivalent written arguments provided values are seen.

Methods 1 and 2 are independent of the answers in (a) and (b) but FT marks can be given for Method 3.

[2 marks]

[Total: 5 marks]

6. (a) attempt to set up a direct variation equation that includes a constant, k , or the calculation of a constant using 12.3 and 50 (M1)

e.g., $d = kv^2$ **OR** $12.3 = k \times 50^2$

$(k =) 0.00492 \left(\frac{1}{203.252...} \right)$

$d = 0.00492v^2$ **OR** $d = \frac{v^2}{203}$ A1

[2 marks]

- (b) substituting 33 for d in their part (a) (A1)

$33 = 0.00492 \times v^2$ **OR** $33 = \frac{v^2}{203.252...}$

$(v =) 81.9 \text{ (km h}^{-1}\text{)} \text{ (81.8982... (km h}^{-1}\text{))}$ A1

[2 marks]

- (c) Award **R1** for a reasonable variable that exists after the brakes are applied such as:

- | | |
|----------------------------|----------------------------|
| • road material | • gradient/incline of road |
| • weather conditions | • traction |
| • condition/type of brakes | • wind resistance |
| • weight/type of vehicle | • friction |

R1

Note: Do not accept a variable that refers to the timing of the brakes being applied such as:

- slow reaction time
- inexperienced driver

[1 mark]

[Total: 5 marks]

10. (a) (i) METHOD 1

attempt to find change in height of the ball using gradient

(M1)

$$\frac{a}{0.43} = (-)0.045$$

$$a = (-)0.045 \times 0.43$$

$$a = (-)0.0194(\text{m}) \quad (0.01935(\text{m}))$$

A1**METHOD 2**

attempt to find height at back of home plate

(M1)

horizontal distance to the front of the home plate = 16.6666... (m)

height at the back of the home plate = $-0.045(16.6666... + 0.43) + 2$

$$(\text{= } 1.23065(\text{m}))$$

Note: The **M1** can be awarded for $16.6666... + 0.43$ seen at some point.

$$(a = 1.25 - 1.23065...)$$

$$(a =) (-)0.0194(\text{m}) \quad (0.01935(\text{m}))$$

A1

$$(ii) \quad 1.25 - 0.01935 = 1.23065 \quad (\text{may be seen in part (a)(i)})$$

A1

$$0.53 < 1.23065 < 1.24$$

R1

therefore a strike

AG

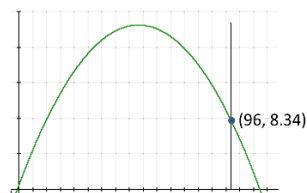
Note: Do not award **A0R1**.

[4 marks]*continued...*

Question 10 continued

(b) **METHOD 1**indication of $d = 96$ in the function $h(d)$ or its graph**(M1)****EITHER**

$$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$$

OR**THEN**

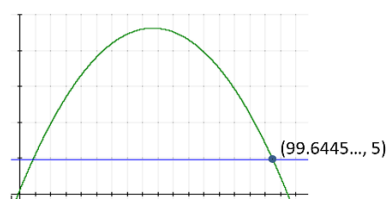
$$(h(96) =) 8.34 \text{ (m)}$$

A1

8.34 > 5 so the ball will go over the wall.

A1**METHOD 2**indication of $h = 5$ in the function $h(d)$ or its graph**(M1)****EITHER**

$$5 = -0.01d^2 + 1.04d + 0.66$$

OR**THEN**

$$d = 99.6 \text{ (m)} \quad (99.6445... \text{ (m)}) \quad (d = 4.35548... \text{ (m)} \text{ may also be seen})$$

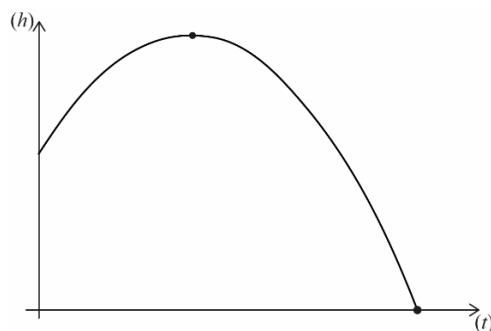
A1

96 < 99.6445... so the ball will go over the wall.

A1**[3 marks]****[Total: 7 marks]**

10. (a) METHOD 1

correct sketch with some indication of maximum point

(M1)

$$0.921 \text{ (seconds)} \left(0.921052..., \frac{35}{38} \right)$$

A1**METHOD 2**

correct substitution into equation for line of symmetry

(M1)

$$(t =) -\frac{8.75}{2 \times -4.75}$$

$$0.921 \text{ (seconds)} \left(0.921052..., \frac{35}{38} \right)$$

A1**METHOD 3**

equating the correct derivative to 0

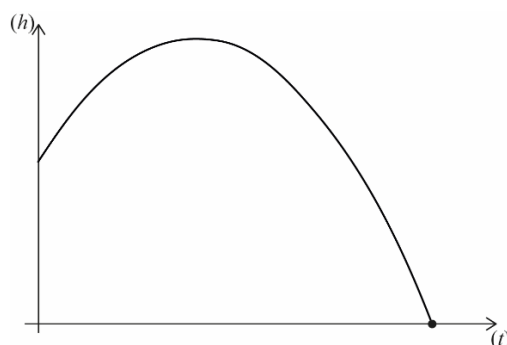
(M1)

$$-9.5t + 8.75 = 0$$

$$0.921 \text{ (seconds)} \left(0.921052..., \frac{35}{38} \right)$$

A1

Note: Award **M1A0** for a final answer of 0.92 seen with no working.

[2 marks]**(b) METHOD 1**correct sketch with some indication of x -intercept**(M1)**

Note: May be seen in part (a).

2 (seconds)

A1*continued...*

Question 10 continued

METHOD 2

setting the equation to zero

(M1)

$$-4.75t^2 + 8.75t + 1.5 = 0$$

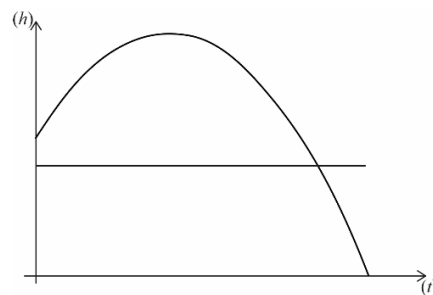
2 (seconds)

A1

Note: If both roots are given, with or without working, award **(M1)A0**.

[2 marks]**(c) METHOD 1**

correct sketch of quadratic function and a straight line in approximate correct position

(M1)

1.88 (seconds) (1.87577...(seconds))

A1**METHOD 2**

setting the equation equal to 1.2

(M1)

$$-4.75t^2 + 8.75t + 1.5 = 1.2$$

1.88 (seconds) (1.87577...(seconds))

A1

Note: Award **(M1)A0** if -0.0336702... seen as (part of) a final answer.
Award **M1A0** for answer of 1.9 seen without working.

[2 marks]**(d) Award R1 for each sensible reason, in the context of the question:****R1R1**

e.g.

The model ignores air resistance (or wind)

The model treats the ball as a point

The model assumes gravity is constant

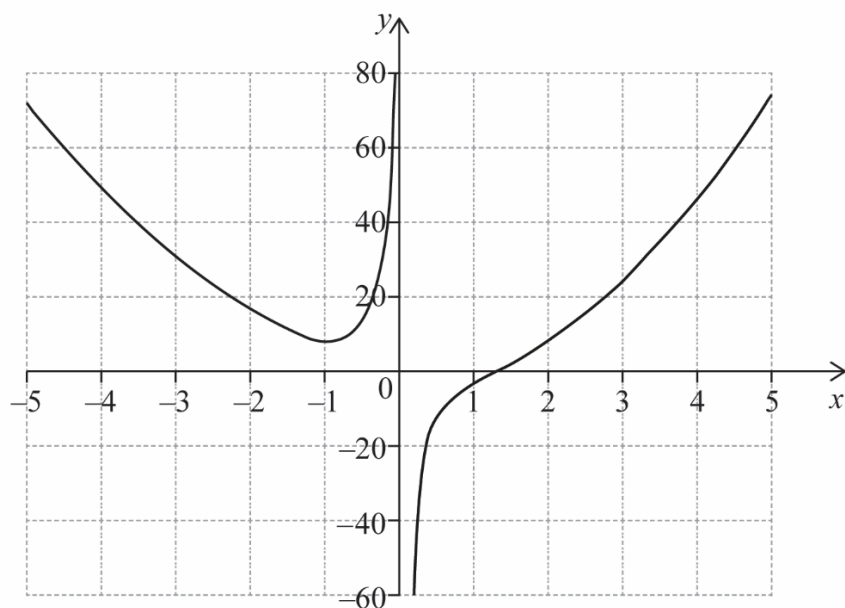
The model assumes that the ball continues to follow the trajectory even after hitting the ground

This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[2 mark]**Total [8 marks]**

11. (a) (i)

**A1A1A1**

Note: Award **A1** for correct shape (curve must be smooth) in the second quadrant only, **A1** for asymptotic behaviour, **A1** for minimum point in approximately correct position AND left-most point around $(-5, 76)$ (allow a tolerance of half a square for these two points).

(ii) -0.941 ($-0.941035\dots$)**A1****[4 marks]**(b) -2.45 ($-2.44651\dots$), -0.252 (-0.252412), 2.70 ($2.69892\dots$)**A2**

Note: Award **A2** for all three correct, **A1** for two correct and **A0** otherwise.
Award at most **A1** if additional solutions are seen.
Award **A1A0** for a final answer given as coordinates $(-2.45, 20)$, $(-0.252, 20)$, $(2.70, 20)$ or $(2.7, 20)$.
Award **A1A0** for three correct answers given in 2 sf.

[2 marks](c) $x = 0$ **A1****[1 mark]****Total [7 marks]**

2. (a) $(a =) 0$ **A1**

[1 mark]

(b) $(1.39, 5.24)$ $((1.38672\dots, 5.24025\dots))$ **A1A1**

Note: Award **A1** for each correct coordinate; do not accept 2 sf values. Award at most **A0A1** if parentheses are missing. Accept $x = 1.39$, $y = 5.24$.

[2 marks]

(c) any value greater than 5.24025... **A1**

Note: Accept a specific value **OR** a description of the correct interval for all values.

[1 mark]

continued...

Question 2 continued

(d) **METHOD 1**

attempt to relate gradient of function to graph of f **R1**

e.g. the function is increasing at $x = -4$

$m > 0$ **A1**

METHOD 2

attempt to find value of $f'(-4)$ **R1**

$$\left(= \frac{25}{8} = 3.125 \right)$$

$m > 0$ **A1**

Note: The **A1** can only be awarded if the correct value of $f'(-4)$ or m is found

METHOD 3

attempt to substitute $(-4, -12.75)$ into equation of tangent

$$-12.75 = m(-4) - \frac{1}{4} \text{ so } m = 3.125 \quad \text{R1}$$

$m > 0$ **A1**

Note: Do not award **R0A1**.

[2 marks]

continued...

Question 2 continued

- (e) (i) attempt to substitute $(0, -5)$ or $(3, 4.5)$ into $g(x)$ **(M1)**

$$-5 = k \times p^0 - 9$$

$$(k =) 4 \quad \text{A1}$$

- (ii) attempt to substitute $(3, 4.5)$ and their part (e)(i) into $g(x)$ **(M1)**

$$4.5 = 4 \times p^3 - 9$$

$$p^3 = 3.375$$

$$(p =) 1.5 \quad \text{A1}$$

[4 marks]

- (f) $y = -9$

A2

Note: Award **A1** for -9 seen, **A2** for a completely correct equation.

Condone $g(x) = -9$

[2 marks]

- (g) $(x =) 4$ **(M1)A1**

Note: Award **M1** for equating $f(x)$ to **their** $g(x)$

Award **(M1)A0** for the y -coordinate (11.25) as the answer or part of the answer (e.g. coordinates).

Award **(M1)A0** for an answer of $(x =) -2.39421\dots$

[2 marks]

continued...

Question 2 continued

(h) **METHOD 1:**

$$f(-1) = 0 \Rightarrow h(-1) = g(-1) \quad (\text{This justification could be graphical.}) \quad \mathbf{R1}$$

yes **A1**

Note: Award **R1** for any reasoning based on the fact that -1 is the x -intercept of f .
Accept equivalent arguments in words. Do not award **R0A1**.

METHOD 2:

$$g(-1) = -\frac{19}{3} \quad (-6.33333\dots)$$

$$h(x) = 3x - 1 + 4x^{-2} + 4 \times 1.5^x - 9$$

$$h(-1) = 3(-1) - 1 + 4(-1)^{-2} + 4 \times 1.5^{-1} - 9$$

$$h(-1) = -\frac{19}{3} \quad (-6.33333\dots) \quad \mathbf{R1}$$

yes **A1**

Note: Award **R1** for showing that $g(-1) = -\frac{19}{3}$ and $h(-1) = -\frac{19}{3}$. Do not award **R0A1**.

[2 marks]**[Total: 16 marks]**

3. (a) $\frac{18-4}{2}$ (M1)

(a) = 7 A1

[2 marks]

(b) $\frac{18+4}{2}$ OR $18-7$ OR $4+7$ (M1)

(d) = 11 A1

[2 marks]

(c) (time between high and low tide is) 6h15m OR 375 minutes (A1)

multiplying by 2 (M1)

750 minutes A1

[3 marks]

(d) EITHER

$\frac{360^\circ}{b} = 750$ (A1)

OR

$7\cos(b \times 375) + 11 = 4$ (A1)

THEN

(b =) 0.48 A1

Note: Award **A1A0** for an answer of $\frac{2\pi}{750} \left(= \frac{\pi}{375} = 0.00837758... \right)$

[2 marks]

(e) equating their cos function to 6 or graphing their cos function and 6

(M1)

$7\cos(0.48t) + 11 = 6$

$\Rightarrow t = 282.468... \text{ (minutes)}$ (A1)

$= 4.70780... \text{ (hr)}$ OR 4hr 42 mins (4hr 42.4681... mins) (A1)

so the time is 10:42 A1

[4 marks]

continued...

Question 3 continued

(f) next solution is $t = 467.531\dots$ (A1)

$$467.531\dots - 282.468\dots$$

185 (mins) (185.063\dots) A1

Note: Accept an (unsupported) answer of 186 (from correct 3 sf values for t)

[2 marks]

[Total: 15 marks]

3. (a) (i) 1200 A1
- (ii) the initial population of the bacteria A1
[2 marks]
- (b) $1200 \times k^3 = 18750$ (A1)
- $(k =) 2.5$ A1
[2 marks]
- (c) $1200 \times 2.5^{1.5}$ (A1)
- 4740 (4743.41...) A1
- Note:** Do not penalize if final answer is not given as an integer. Award **(A1)A0** for an answer of 3950 (3949.14...) from use of 1.3 in the exponent, but only if working is shown.
- [2 marks]
- (d) equating $P(t)$ and $S(t)$ **OR** equating each function to a common variable (M1)
- $1200 \times 2.5^t = 5000 \times 1.65^t$; $1200 \times 2.5^t = x$ and $5000 \times 1.65^t = x$
- $t = 3.43$ (hours) (3.43456...) A1
[2 marks]

continued...

Question 3 continued

(e) **METHOD 1**

$$5000 \times 1.65^t = 19000 \quad (M1)$$

$$(t =) 2.66586... \quad \text{OR} \quad (t - 2 =) 0.66586... \text{ (seen)} \quad (A1)$$

multiplying by 60 seen to convert to minutes (M1)
 $(m = 39.9521...)$

$(m =) 40$ (minutes) **OR** 2 hours and 40 minutes A1

METHOD 2

equating an expression for $S(t)$ to 19000 (M1)

expressing t as $2 + \frac{m}{60}$ (A1)

$$5000 \times 1.65^{2 + \frac{m}{60}} = 19000$$

$$2 + \frac{m}{60} = 2.66586... \quad A1$$

$(m =) 40$ (minutes) **OR** 2 hours and 40 minutes A1

Note: Award **(M1)(A1)(M1)A0** for an answer of 39.9521... or 39 with or without working.

[4 marks]

(f) **EITHER (find volume of all bacteria)**

multiplying total population of bacteria by the volume of bacterium (M1)

$$1 \times 10^{-18} \times (5000 \times 1.65^t)$$

setting expression equal to 2.1×10^{-5} and attempt to solve (e.g. sketch) (M1)

OR (find total number of bacteria to fill container)

attempting to find the total number of bacteria to fill container (M1)

$$2.1 \times 10^{-5} = n \times 1 \times 10^{-18} \quad \text{OR} \quad \frac{2.1 \times 10^{-5}}{1 \times 10^{-18}} \quad \text{OR} \quad 2.1 \times 10^{13}$$

setting value equal to $S(t)$ and attempt to solve (e.g. sketch) (M1)

Note: If $\frac{2.1 \times 10^{-5}}{1 \times 10^{-18}}$ is seen but candidate has an incorrect total number of bacteria, the second **(M1)** can still be awarded for setting their incorrect value equal to $S(t)$ and attempting to solve.

THEN

$$t = 44.2 \text{ (hours)} \text{ (44.2480...)} \quad A1$$

[3 marks]

Total [15 marks]

6. (a) setting $h(x) = 1$
 $(h^{-1}(1) =)17$

(M1)

A1

[2 marks]

(b) $x > \frac{1}{2}$

A1A1

Note: Award **A1** for $\frac{1}{2}$ seen, **A1** for completely correct answer.

[2 marks]

[Total 4 marks]

7. (a) $2.82 = a(3)^2 + b(3) + c$ **OR** $2.82 = 9a + 3b + c$

A1**[1 mark]**

(b) finding other equations to solve simultaneously

(M1)

$$4.25 = a(6)^2 + b(6) + c \text{ **AND** } 5.30 = a(9)^2 + b(9) + c$$

$$\text{OR } 4.25 = 36a + 6b + c \text{ **AND** } 5.30 = 81a + 9b + c$$

any one coefficient in equation correct

(A1)

$$f(x) = -0.0211x^2 + 0.667x + 1.01$$

A1

$$(f(x) = -0.02111111\dots x^2 + 0.666666\dots x + 1.01)$$

$$\left(f(x) = -\frac{19}{900}x^2 + \frac{2}{3}x + \frac{101}{100} \right)$$

Note: Award at most **(M1)(A1)A0** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 30 into their equation

(M1)

$$(f(30) =) 2.01$$

A1

$$2.01 > 1.8 \text{ **OR** therefore the discus will go over the wall}$$

R1

Note: Do not award **A0R1**; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero **OR** graph with the zero indicated

(M1)

$$0 = -0.02111111\dots x^2 + 0.666666\dots x + 1.01\dots \text{ **OR** } f(x) = 0$$

$$33.0 \text{ (33.0275\dots) (m)}$$

A1**[2 marks]****[Total 9 marks]**

6. (a) setting $h(x) = 1$
 $(h^{-1}(1) =) 17$

(M1)

A1

[2 marks]

- (b) $x > \frac{1}{2}$

A1A1

Note: Award **A1** for $\frac{1}{2}$ seen, **A1** for completely correct answer.

[2 marks]

[Total 4 marks]

7. (a) $2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$

A1**[1 mark]**

(b) finding other equations to solve simultaneously

(M1)

$$5 = a(10)^2 + b(10) + c \text{ **AND** } 7.16 = a(17)^2 + b(17) + c$$

$$\text{OR } 5 = 100a + 10b + c \text{ **AND** } 7.16 = 289a + 17b + c$$

any one coefficient in equation correct

(A1)

$$f(x) = -0.00490x^2 + 0.441x + 1.08$$

A1

Note: Condone the 2 sf answer for the coefficient of x^2 i.e. -0.0049 .

$$(f(x) = -0.00489795\dots x^2 + 0.440816\dots x + 1.08163\dots)$$

$$\left(f(x) = -\frac{6}{1225}x^2 + \frac{108}{245}x + \frac{53}{49} \right)$$

Note: Award at most **(M1)(A1)A0** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 80 into their equation

(M1)

$$(f(80) =) 5$$

A1

$5 > 4$ **OR** therefore the ball will go over the fence

R1

Note: Do not award **A0R1**; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero, graph

(M1)

$$0 = -0.00489795\dots x^2 + 0.440816\dots x + 1.08163\dots \text{ **OR** } f(x) = 0$$

$$92.4 \text{ (92.3902\dots) (m)}$$

A1**[2 marks]****[Total 9 marks]**

4. (a) correct substitution of $t = 0$ (A1)

$$100 - 60 \times a^0$$

$$= 40 (\%)$$

A1

[2 marks]

- (b) correct substitution of $P = 75$ and $t = 1$ (A1)

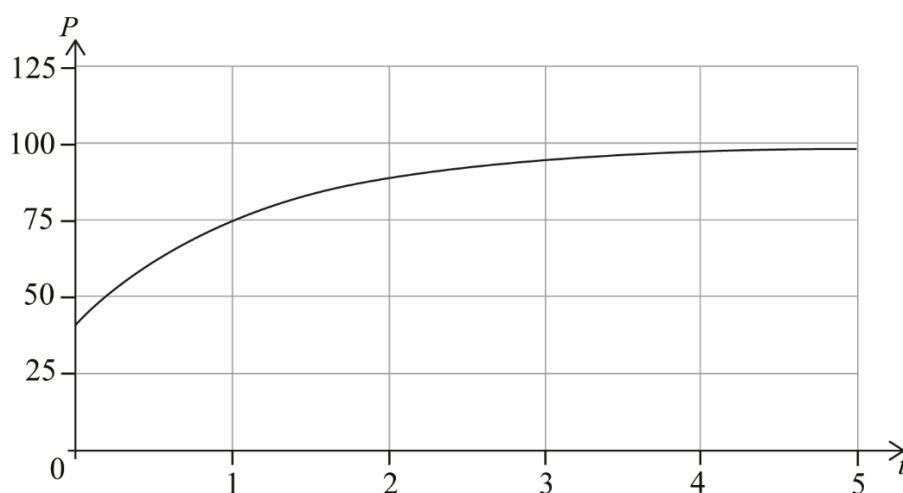
$$75 = 100 - 60 \times a^{-1}$$

$$a = 2.4$$

A1

[2 marks]

- (c)



A1A1

Note: Award **A1** for correct general shape (increasing, concave down) passing through (1, 75) and continuing on, **A1** for a curve starting from (0, 40), finishing around (5, 100).

[2 marks]

- (d) **EITHER**
has a horizontal asymptote at $P = 100$ A1

Note: Accept a description of the asymptote in words.

OR

equation is 100 minus a positive value and hence cannot equal 100 A1

Note: Award **A0** for a contextualized answer with no mathematical justification, e.g. "battery may degrade".

[1 mark]

[Total: 7 marks]

7. (a) $a = 3$

A1

[1 mark]

(b) period = 12

(A1)

$$\left(\frac{360}{b} = 12 \quad \text{OR} \quad \frac{2\pi}{b} = 12 \right)$$

$$b = 30 \qquad b = \frac{\pi}{6}$$

A1

[2 marks]

(c) equating their expression to 3

(M1)

$$3 = 3 \sin(30t) + 4 \quad \text{OR} \quad 3 = 3 \sin\left(\frac{\pi}{6}t\right) + 4$$

$$t = 6.64904\dots$$

(A1)

$$6:39 \text{ (pm)} \quad (18:39)$$

A1

Note: Follow through within the part for the final **A1**; this mark is awarded for expressing **their** intermediate answer (seen) as a time correct to the nearest minute.

[3 marks]

[Total: 6 marks]

10. (a) (i) $m = ah^3$ (M1)
 $64 = a \times 0.8^3 \Rightarrow a = 125$ (A1)

$m = 125h^3$ A1

Note: The final answer must be written as m in terms of h to award the final A1.

(ii) $m = (125(0.75)^3) = 52.7 \text{ (kg)} \text{ (52.7343...)} \quad \text{A1}$

[4 marks]

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$
 $h = \sqrt[3]{1.76} \text{ (1.20736...)} \quad \text{(A1)}$

$E = k_1 h^2 \text{ (seen anywhere)} \quad \text{(A1)}$

$k = \frac{(\sqrt[3]{1.76})^2}{0.8^2} \quad \text{(M1)}$
 $= 2.28 \text{ (2.27769...)} \quad \text{A1}$

OR (finding a formula for E in terms of m)

$(m = 125h^3)$
 $E = k_1 h^2 \quad \text{(A1)}$

$E = k_2 m^{\frac{2}{3}} \quad \text{(A1)}$

$k = \frac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}} \quad \text{(M1)}$

$= 2.28 \text{ (2.27769...)} \quad \text{A1}$

[4 marks]

[Total 8 marks]

4. (a) expressing one variable in terms of the other (M1)

$$n = kB \quad \text{OR} \quad n \propto B$$

$$60 = 5k$$

$$k = 12$$

$$n = 12(7)$$

$$= 84$$

(A1)

A1

[3 marks]

- (b) expressing t terms of reciprocal of B (M1)

$$t = \frac{c}{B} \quad \text{OR} \quad t \propto \frac{1}{B}$$

$$8 = \frac{c}{5}$$

$$c = 40$$

(A1)

$$(t =) \frac{40}{12} \left(\frac{10}{3}, 3.33, 3.33333... \right) (\text{hours}) \quad \text{OR} \quad 3 \text{ hours } 20 \text{ minutes}$$

A1

Note: Award at most **M1A1A0** for the use of $t = cB$ in part (b) if $n = \frac{k}{B}$ was seen and penalized in part (a).

[3 marks]

[Total: 6 marks]

5. (a) 60 is the daily rate of hire **A1**
 10 is the fixed charge for a helmet and repair kit **A1**
[2 marks]
- (b) $(C(5) =) 60 \times 5 + 10$ **(A1)**
 310 (CAD) **A1**
[2 marks]
- (c) 3 **A1**
[1 mark]
- (d) recognizing that 1270 is the total cost **(M1)**
 $1270 = 60k + 10$
 $k = 21$ **A1**
[2 marks]
[Total 7 marks]

6. (a) 120 (ng)

A1

Note: Only accept an exact answer of 120.

[1 mark]

(b) $(m(20000) =) 120e^{-0.000121 \times 20\,000}$
 10.7 (10.6705.....) (ng)

(A1)**A1****[2 marks]**

(c) attempt to write or solve an inequality (or equality) with a value of 60 seen
 $(m(t) =) 120e^{-0.000121t} \leq 60$ **OR** sketch **OR** table of values
 $(t >) 5728.489....$
 5729 (years)

(M1)**(A1)****A1**

Note: Do not accept 5728 or 5730 for the final **A1**.

[3 marks]**[Total: 6 marks]**

5. (a) 60 is the daily rate of hire **A1**
 10 is the fixed charge for a helmet and repair kit
[2 marks]
- (b) $(C(5) =) 60 \times 5 + 10$ **(A1)**
 310 (CAD) **A1**
[2 marks]
- (c) 3 **A1**
[1 mark]
- (d) recognizing that 1270 is the total cost **(M1)**
 $1270 = 60k + 10$
 $k = 21$ **A1**
[2 marks]
[Total 7 marks]

6. (a) 120 (ng)

A1**Note:** Only accept an exact answer of 120.**[1 mark]**

(b) $(m(20000) =) 120e^{-0.000121 \times 20\,000}$
 10.7 (10.6705.....) (ng)

(A1)**A1****[2 marks]**

(c) attempt to write or solve an inequality (or equality) with a value of 60 seen
 $(m(t) =) 120e^{-0.000121t} \leq 60$ **OR** sketch **OR** table of values
 $(t >) 5728.489....$
 5729 (years)

(M1)**(A1)****A1****Note:** Do not accept 5728 or 5730 for the final **A1**.**[3 marks]****[Total: 6 marks]**

2. (a) (i) $\frac{45-8}{2}$ **A1**
 18.5 (m) **AG**

Note: Use of 18.5 in the calculation is reverse engineering, and falls short of the demand of the command term “Show that”. In such cases, award **A0**.

(ii) $(2\pi \times 18.5 =) 116 \text{ (} 116.2389\dots, 37\pi \text{) (m).}$ **A1**
[2 marks]

(b) recognition the distance is the arc length **(M1)**
 $\frac{10^\circ}{360^\circ} \times 2\pi \times 18.5$ **OR** $\frac{116.2389\dots}{36}$ **(A1)**
 $= 3.23 \left(3.228859\dots, \frac{37\pi}{36} \right) \text{ (m)}$ **A1**
[3 marks]

(c) (i) $\left(b = \frac{360}{15} = \right) 24$ **A1**

(ii) $\frac{\text{max} + \text{min}}{2}$ **(A1)**
 $(d =) \frac{45+8}{2}$ **OR** $45 - 18.5$ **OR** $8 + 18.5$
 $= 26.5$ **A1**

(iii) $h(t) = 18.5 \sin(24t) + 26.5$ **A1A1**

Note: Award **A1** for 18.5 seen as parameter a , **A1** for a completely correct equation (including LHS).

[5 marks]

(d) attempt to equate 33 with $h(t)$ **OR** sketch graph of curve and line **(M1)**

$$h(t) (= 18.5 \sin(24t) + 26.5) = 33$$

$t = 0.857 \text{ (} 0.857083\dots \text{) and } t = 6.64 \text{ (} 6.64291\dots \text{) (minutes)}$ **A1A1**
[3 marks]

(e) 4 (revolutions per hour) seen **OR** 32 (revolutions per day) seen **(A1)**
 $(4 \times 8 \times 3000 =) 96000$ **(A1)**
 9.6×10^4 **A1**

[3 marks]

[Total 16 marks]

2. (a) (i) $\frac{45-8}{2}$ **A1**
 18.5 (m) **AG**

Note: Use of 18.5 in the calculation is reverse engineering, and falls short of the demand of the command term “Show that”. In such cases, award **A0**.

- (ii) $(2\pi \times 18.5 =) 116 \text{ (} 116.2389\dots, 37\pi \text{) (m)}$ **A1**
[2 marks]

- (b) recognition the distance is the arc length **(M1)**
 $\frac{10^\circ}{360^\circ} \times 2\pi \times 18.5$ **OR** $\frac{116.2389\dots}{36}$ **(A1)**
 $= 3.23 \left(3.228859\dots, \frac{37\pi}{36} \right) \text{ (m)}$ **A1**
[3 marks]

- (c) (i) $\left(b = \frac{360}{15} = \right) 24$ **A1**

- (ii) $\frac{\text{max}+\text{min}}{2}$ **(A1)**
 $(d =) \frac{45+8}{2}$ **OR** $45 - 18.5$ **OR** $8 + 18.5$
 $= 26.5$ **A1**

- (iii) $h(t) = 18.5 \sin(24t) + 26.5$ **A1A1**

Note: Award **A1** for 18.5 seen as parameter a , **A1** for a completely correct equation (including LHS).

[5 marks]

- (d) attempt to equate 33 with $h(t)$ **OR** sketch graph of curve and line **(M1)**

$$h(t) (= 18.5 \sin(24t) + 26.5) = 33$$

- $t = 0.857 \text{ (} 0.857083\dots \text{) and } t = 6.64 \text{ (} 6.64291\dots \text{) (minutes)}$ **A1A1**
[3 marks]

- (e) 4 (revolutions per hour) seen **OR** 32 (revolutions per day) seen **(A1)**
 $(4 \times 8 \times 3000 =) 96000$ **(A1)**
 9.6×10^4 **A1**

[3 marks]
[Total 16 marks]

4. (a) (gradient =) $\frac{0.8}{2.7} \left(0.296, 0.296296\dots, \frac{8}{27} \right)$ **A1**

$0.296 < \frac{1}{3} (0.333333\dots)$ **OR** $\frac{8}{27} < \frac{9}{27}$ **R1**

the gradient of the pool is less than the gradient in the regulation / the swimming pool satisfies the safety regulation

AG
[2 marks]

- (b) expressing inverse proportionality using notation or using multiplication to calculate k

(M1)

$w \propto \frac{1}{t}$ **OR** $w = \frac{k}{t}$ **OR** $(k =) 300 \times 4.5$

$300 \times 4.5 = 170 \times t$ or equivalent

(A1)

Note: Accept a calculation in minutes (i.e., $300 \times 270 = 170 \times t$).

$(t =) 7.94 (7.94117\dots) \text{ (h)}$

A1

Note: Award full marks for an answer given in minutes (i.e. 476 (476.470...) minutes or 7 hours and 56 minutes). "Minutes" must be shown as the unit.

[3 marks]

[Total: 5 marks]

✓ ✓

5. (a) $d = 5.75 \left(d = \frac{23}{4} \right)$ **A1A1**

Note: Award **A1** for 5.75 seen, **A1** for completely correct equation.

[2 marks]

- (b) attempt to substitute into the axis of symmetry formula using *their* 5.75 **OR**
attempt to substitute the coordinates (3, 5.1) or (8.5, 5.1) into the function **(M1)**

$$5.75 = \frac{-b}{2 \times (-0.2)} \quad \text{OR} \quad 5.1 = -0.2(3)^2 + 3b \quad \text{OR} \quad 5.1 = -0.2(8.5)^2 + 8.5b$$

$$(b =) 2.3$$

A1
[2 marks]

- (c) indication of finding zero or attempt to use symmetry **(M1)**
 $h(d) = 0$ **OR** $0 = -0.2d^2 + 2.3d$ **OR** 2×5.75 **OR** $3 - 0 = d - 8.5$ **OR**
resketch of graph with zero indicated

$$(d =) 11.5 \text{ (m)}$$

A1

Note: Award **A0** for a FT answer that is less than 8.5 (contradicts the given information).

[2 marks]

- (d) **EITHER**
the dolphin is below the surface of the water / the dolphin is underwater / the dolphin
is in the water **R1**

OR

the value of h cannot be negative since the definition of the model only applies
to the jump

R1
[1 mark]

[Total: 7 marks]

10. (a) $15p$ A1

[1 mark]

(b) $(p-5)$ or $(n-15)$ seen A1

adding a cost expression (multiplying a p and an n) to their $15p$ (M1)

$C(n) = 15p + (p-5)(n-15)$ **OR** $C(n) = np - 5n + 75$ (or equivalent) A1
[3 marks]

(c) (i) $(25.73 \times 100 =) 2573$ (EUR) A1

Note: Answer must be exact.

(ii) correctly substituting *their* total cost and $n = 100$ into *their* cost function (A1)
 $2573 = 15p + (p-5)(100-15)$

$(p =) 29.98$ (EUR) A1

Note: Answer must be written to 2 dp.

[3 marks]

[Total: 7 marks]

11. (a) $(W \Rightarrow) -34.1 - 7.33 \ln(13)$ (A1)

-52.9 ($-52.9010\dots$) A1

[2 marks]

(b) attempt to work back from the approximate to the exact value (M1)

$$\frac{-52.9010\dots - v_E}{v_E} = (\pm)0.06 \quad \text{OR} \quad (v_E \Rightarrow) \frac{-52.9010\dots}{1 \pm 0.06} \quad \text{OR} \quad v_E(1 \pm 0.06) = -52.9010\dots$$

$-56.3, -49.9$ ($-56.2777\dots, -49.9066\dots$) A1A1

Note: Award A1 for each correct value.

[3 marks]

[Total: 5 marks]

9. (a) 3

A1
[1 mark]

(b) 1

A1
[1 mark]

(c)

x	0	1	2	3	4
$f^{-1}(x)$	2	1	4	0	3

award **A1** for 1 correct, **A2** for 3 correct, **A3** for all 5 correct

A1A1A1
[3 marks]
Total [5 marks]

Q3.

- (a) attempt to substitute 0.5 into function **(M1)**

$$h(0.5) = \frac{20}{2 \times 0.5 + 5}$$

$$3.33 \text{ (m)} \left(3.33333\ldots, \frac{10}{3} \right)$$

A1**[2 marks]**

- (b) (i) **EITHER**

attempt to equate $h(t)$ to 2.5

(M1)

$$\frac{20}{2t + 5} = 2.5$$

OR

attempt to substitute 2.5 in h^{-1}

(M1)

$$\frac{20 - 5 \times 2.5}{2 \times 2.5}$$

OR

graph of h , with an indication of the point $(t, 2.5)$

(M1)

$$t = 1.5 \text{ (hours)}$$

A1

- (ii) It takes 1.5 hours for the water level to reach 2.5 metres.

R1**[3 marks]**

- (c) $t \geq 0$

A1

Note: Accept $h^{-1} \geq 0$.

[1 mark]**Total [6 marks]**

4. (a) (i) 90° **A1**
 (ii) $\sqrt{10^2 + 15^2}$ **(M1)**
 $18(.0)(\text{cm})$ $(18.0277\dots)$ $(\sqrt{325})$ **A1**

[3 marks]

- (b) (i) 150 **A1**
 (ii) 120 **A1**

Note: Award **A0A1FT** for (0, 150) and (0, 120)

[2 marks]

- (c) (i) 15 **A1**
 (ii) $\left(\frac{360}{60} = \right) 6$ **A1**
 (iii) 135 **A1**

[3 marks]

- (d) (i) $(q =) \frac{360}{(60 \times 12)} = 0.5$ or in 60 minutes, 30 degrees travelled so $(q =) \frac{30}{60} = \frac{1}{2}$ **A1AG**
 (ii) -10 **A2**

Note: Award **A1** for 10.

[3 marks]

- (e) substitution of $t = 60$ into their $g(t)$ **(M1)**
 130 (cm) **A1**

[2 marks]

- (f) $15 \cos(6t) + 135 = -10 \sin(0.5t) + 135$ **(M1)**
 $(t =) 15.8806\dots$ **(A1)**
 (time =) 3:16 (pm) (15:16) **A1**

Note: If the answer to part (d)(ii) is 10, then $t = 14.2115$ and the time is 3:14 (pm) (15:14)

[3 marks]**Total [16 marks]**

Q4.

(a) $\frac{360}{12}$ **M1**
 $= 30$ **AG**

Note: Award **M0** for use of given value, 30.

(b) substitution of $t = 5$ into model $(-2.5 \cos(30 \times 5) + 4.5)$ **[1 mark]**
(M1)
 $h(5) = 6.67(6.66506...)$ **A1**

Note: Award **M1A0** for 2.75187... (radian answer).

(c) (i) 2.5 (m) **[2 marks]**
A1

Note: Award **A0** for -2.5 (m)

(ii) $h(t) = 4.5$ **A1A1**

Note: Award **A1** for an equation of a horizontal line $h = \text{constant}$ (accept $y = \text{constant}$), **A1** for that constant being 4.5. Second **A1** is dependent on first **A1**. Do not award **A0A1**.

(d) attempt at finding point of intersection between $h(t) = 2.65$ and the curve. **[3 marks]**
(M1)
13.4 (13.4089...) **(A1)**
13:25 (1:25 p.m.) **A1**
[3 marks]

Note: Award **M1A1A1FT** for consistent use of radians 12.1720... 12:10, 0:10 p.m.
Award at most **M1A0A0** if the student works in the first cycle ($t = 1.40895...$)

(e) $22.5910... - 13.4089... - 0.5$ **(A1)(M1)(M1)**

Note: Award **A1** for 22.5910... seen, **M1** for subtracting their two times for harbour, **M1** for subtracting 0.5 from their answer at any stage of the work.

8.68 (hours)(8.68209...) (hours) **A1**
[4 marks]

Note: Award at most **A0M0M1A0** for consistent use of radians. There is no 15 minute window for the ship to reach nor return from the fishing site.

Total [13 marks]

		Total [6 marks]
3.	(a) $0.80(5.1) + 1.50$	(A1)
	(\$) 5.58	A1
		[2 marks]
(b)	(i) $(a =)1$	A1
	(ii) $(b =)2$	A1
		[2 marks]
(c)	the gradient of TurboEats (0.8) is less than the gradient of Gogodart (1)	R1
	the y -intercept of TurboEats (1.50) is less than the y -intercept of Gogodart (2)	R1

Note: Accept contextual language, e.g. “cost per km” for gradient, and “fixed amount/constant” for y -intercept, in their explanations.
Award **R1R1** only for explicit consideration of both the gradient and y -intercept.

[2 marks]
Total [6 marks]

8. (a) $2000 + 30\,000(e^{-k \times 0})$ (A1)
 32 000 (PHP) A1
 [2 marks]

- (b) $32\,000 \times 0.60$ OR $32\,000 - 32\,000 \times 0.40$ OR $32\,000 \times 0.40$ (M1)
 19 200 (A1)
 $19\,200 = 2000 + 30\,000(e^{-k \times 10})$ (M1)

Note: Award **M1** for a correctly substituted equation including 10 and *their* 19 200 (or equivalent LHS expression).

$$k = 0.0556 \text{ (0.0556287...)} \quad \text{A1}$$

[4 marks]

- (c) (i) (the curve of $y = V(t)$ has a horizontal asymptote at $y = 2000$ OR $V = 2000$ A1

- (ii) **EITHER**
 newer models/updated technology will render her cell phone obsolete. R1

- OR**
 the cell phone will eventually stop working R1

[2 mark]

Total [8 marks]

10. (a) recognizing the form of inverse variation (M1)

$$V = \frac{k}{t} \quad \text{OR} \quad V \propto \frac{1}{t}$$

$$300 = \frac{k}{5} \quad \text{OR} \quad k = 5 \times 300 \text{ (the } k \text{ must be shown)} \quad \text{A1}$$

$$V = \frac{1500}{t} \quad \text{AG}$$

Note: Award **MOA0** for only verifying that 300 and 5 satisfy given equation, e.g. $300 = \frac{1500}{5}$ seen.

[2 marks]

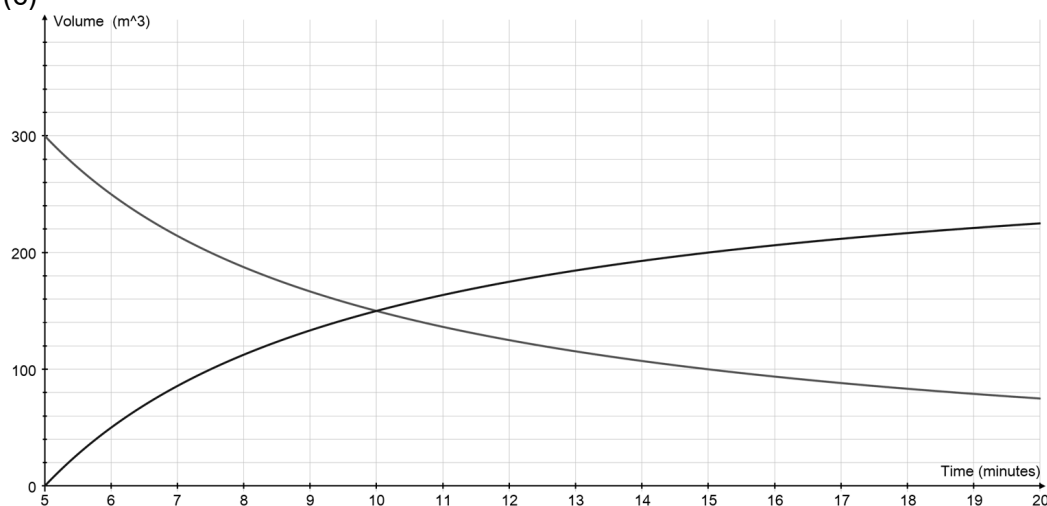
- (b) 150 seen (A1)

$$150 = \frac{1500}{t}$$

$$t = 10 \text{ (minutes)}$$

A1
[2 marks]

- (c)



A1A1

Note: Award **A1** for an **increasing curve** from point (5, 0). Award **A1** for a smooth curve, with approximately correct shape, eg concave down, approximately passing through (10,150). Accept any endpoint in the range [150, 300].

[2 marks]

- (d) $(P(t) =) 300 - \frac{1500}{t}$ OR $300 - V(t)$

A1

[1 mark]

Total [7 marks]

2. (a) $-167 + 6264 \log_{10} 10$
6100 (6097)

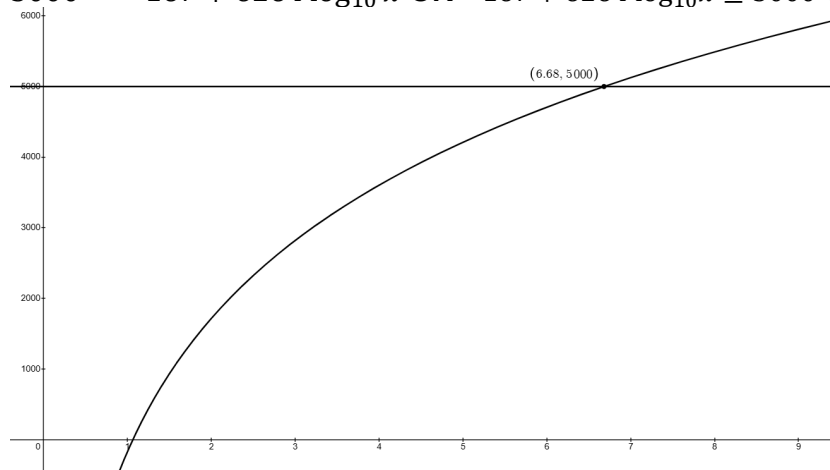
(A1)
A1

Note: Award **A1A0**, as a special case if 14256.3... seen as an unsupported answer, from use of $\ln$ in place of $\log$.

[2 marks]

- (b) $5000 = -167 + 6264 \log_{10} x$ **OR** $-167 + 6264 \log_{10} x \geq 5000$ **OR** graph drawn

(A1)



($x =$)6.681 (years)

A1

Note: Award at most (A1)A0 if the final answer is not given to 4 sf.

[2 marks]

- (c) unrealistic
($-167 + 6264 \log_{10} 1 = -167$ **OR** $-167 + 6264 \log_{10} 1 < 0$) **OR** graph drawn:
you cannot know a negative number of words

A1

R1

Note: Do not award **A1R0**. Award **R1** for -167 seen even if the explanation is not correct e.g. "Given that when $x = 1$, $y = -167$ the model is not realistic because it would mean that the child would be less than 1". Mention of "negative number of words", is sufficient for the **R1**; the calculations can be implied.

[2 marks]

Total [6 marks]

6. (a) $-2.6 \left(\frac{-13}{5}\right)$

A1
[1 mark]

(b) **EITHER**
 $(-\infty, -3.67] \cup [-2.6, \infty)$
OR
 $y \leq -3.67$ or $y \geq -2.6$

A1A1A1

A1A1A1

Note: Award **A1** for $-3.67 \left(-3.66666 \dots, -\frac{11}{3}\right)$ seen

A1 for their $-2.6 \left(-\frac{13}{5}\right)$ seen and **both** sets of inequalities/brackets correct;

A1 if completely correct, including union symbol or “or”.

It is possible to award **A0A1A0** for correctly presenting *their* -3.67 (and their -2.6).

The -3.67 maybe seen as part of a sketch.

Award **A1A0A0** for $-3.67 \leq y \leq -2.6$ or $[-3.67, -2.6]$.

[3 marks]

(c) setting $f(x)$ equal to -2
 $f(x) = -2$ **OR** $-2 = \frac{4}{x} - 3$ **OR** sketch of graph with line labelled -2

(M1)

4

A1
[2 marks]
Total [6 marks]

8. (a) $(c =)13$

A1
[1 mark]

(b) (i) $(1.5, 0)$

A1

(ii) $0 = 2.25a + 1.5b + 13$

A1**Note:** Accept $0 = (1.5)^2a + 1.5b + 13$.**[2 marks]**(c) **METHOD 1**attempting to find another equation (e.g. in term of R , or setting $\frac{dy}{dx}$ to zero)**(M1)**

$0 = 2.25a - 1.5b + 13 \text{ OR } -\frac{b}{2a} = 0$

(A1)

$a = -5.78 \left(-\frac{52}{9}, -5.77777 \dots\right), b = 0$

A1**METHOD 2**

deciding to use factorized form

(M1)

$y = a(x + 1.5)(x - 1.5)$

correct substitution of $(0, 13)$ **(A1)**

$13 = a(1.5)(-1.5)$

$a = -5.78 \left(-\frac{52}{9}, -5.77777 \dots\right), b = 0$

A1**[3 marks]****Total [6 marks]**

12. (a) (i) $(a =)100$ **A1**
- (ii) attempt to substitute values into given equation (at least two correct) **(M1)**
 $50 = ab^{138}$ **OR** $50 = 100b^{138}$ **OR** $0.5 = b^{138}$

Note: Award **M1** for a sketch of $y = 100x^{138}$ and $y = 50$.

- $(b =)0.995(0.994989 \dots)$ **A1**
[3 marks]
- (b) attempt to substitute (must include two of: 30, their b , equated to 4) **(M1)**
 $4 = ab^{30}$ **OR** $f(30) = a \times (0.994989 \dots)^{30}$ **OR** $4 = a \times (0.994989 \dots)^t$
 $4 = a \times (0.994989 \dots)^{30}$ **(A1)**
 $a = 4.65(4.65051 \dots)$ (micrograms) **A1**
[3 marks]
Total [6 marks]

5. (a) $28 + 25 \sin 30$ **(A1)(M1)**

Note: Award **A1** for 30° seen. Award **M1** for expressing the height of V as “ $28 +$ another value”.

OR

$$\left(\frac{3}{4}\right)(50) + 3 \quad \text{OR} \quad 3 \times 12.5 + 3 \quad \text{A1(M1)}$$

THEN

40.5 (m) **A1**
[3 marks]

- (b) $\frac{9}{12}$ **OR** 0.75 seen **(A1)**
28 (m) **A1**

Note: Award **(A1)A1FT** for an answer of 25 seen as the height of their midpoint in part (a).

[2 marks]

- (c) $\frac{1}{3}$ seen **OR** sketch diagram with M and V at same (approx.) height/location **(M1)**
OR $28 - 25 \cos(30t) = 28 + 25 \sin(30(t + 1))$ **A1**
 $t = 4$ **A1**
 $t = 10$ **[3 marks]**

- (d) 25 **A1**
[1 mark]

– 12 –

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- (e) (i) $(b =) \frac{360}{12}$ **OR** an attempt to form an equation from a known point (M1)
 $(b =) 30$ A1

Note: Accept an answer of $\frac{2\pi}{12}$, $\frac{\pi}{6}$, 0.523598..., from use of radians (as degrees not explicit in question).

- (ii) $(d =) 28$ A1

Note: Accept an answer of 25, if consistent with their working and answers in parts (a) and (b).

[3 marks]

- (f) 10 and 20 seen (A1)
 30 (metres) A1

[2 marks]

- (g) $(b(18) =) 20\sin(36 \times 18) + 40$ **OR** 21.0 (20.9788...) (A1)
 $(k(18) =) 10\cos(72 \times 18)$ **OR** –8.09 (–8.09016...) (A1)
 $10\cos(72 \times 18) + 20\sin(36 \times 18) + 40$ (M1)

Note: Award **M1** for summing their expressions or values.

- 12.9 (m) (12.8886...) A1

Note: Accept an answer of $(54.7844... - 0.92896... =) 53.8554...$, from use of radians (as degrees not explicit in question).

[4 marks]
Total [18 marks]

4. (a) $\frac{1.26 - (-0.26)}{2}$ OR $\frac{1.52}{2}$ (A1)
0.76 A1
[2 marks]

(b) $\frac{1.26 + (-0.26)}{2}$ (A1)
0.5 A1
[2 marks]

(c) **METHOD 1**
attempt to subtract the times at the minimum and maximum (M1)
 $10:48 - 03:36 (= 7:12)$
 $(7:12 \times 2 =) 14:24$ (A1)
14.4 A1

METHOD 2
correct conversion of at least one time in hours, e.g 10.8 or 3.6 seen (A1)
attempt to subtract their times (M1)
 $10.8 - 3.6 (= 7.2)$
 $(7.2 \times 2 \text{ or } 3.6 \times 4 =) 14.4$ A1
[3 marks]

(d) $\frac{360}{14.4}$ OR $\frac{2\pi}{14.4}$ (A1)

Note: Award full marks for (d), (e) and (g) if radians used. **FT** from their (c).

$(b =) 25$ (seen, including in an incorrect equation) OR 0.436 (0.436332...) (A1)

$h(t) = 0.76 \sin(25t) + 0.5$ OR $h(t) = 0.76 \sin(0.436t) + 0.5$ A1

Note: Do not accept y and x in place of $h(t)$ and t .

[3 marks]

(e) attempt to substitute 2.5 into their equation (M1)

Note: If they lost the mark in (c) because they did not convert to real number, **AND** substitute 2.3 in the formula, then award full **FT** marks.

$(h(2.5) =) 0.76 \sin(25 \times 2.5) + 0.5$ OR $0.76 \sin(0.436 \times 2.5) + 0.5$
1.17 (1.17412...)(m) A1
[2 marks]

- (f) the road is above the water-level **OR** the island is accessible **OR** cars can drive on the road

R1

Note: Accept equivalent statements.

[1 mark]

- (g) setting their equation to zero
 $0 = 0.76 \sin(25t) + 0.5$ **OR** $0 = 0.76 \sin(0.436332 \dots t) + 0.5$
 $t = 8.84558 \dots, t = 12.7544 \dots, t = 23.2455 \dots$

(M1)**(A1)(A1)**

Note: Award **A1A0** if only one or two correct t -values are seen.

finding differences between t -values
 $12.7544 \dots - 8.84558 \dots$ **OR** $24 - 23.2455 \dots$
 4.66 ($4.66325 \dots$) (hours)

(M1)**A1**

Note: Answer does not need to be in minutes, but accept “4 hours 40 minutes” or “280 minutes” as a correct final answer.

Award **(M1)(A1)A0(M1)A0** for an answer of “3.91 hours”, “3 hours and 55 minutes”, or “235 minutes” (i.e., only considering first minimum).

[5 marks]**Total [18 marks]**

