



Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches

Standard level

Paper 1

15 May 2025

Zone A afternoon | Zone B afternoon | Zone C afternoon

1 hour 30 minutes

Candidate session number

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Write each of the following expressions in the form $\ln k$, where $k \in \mathbb{Z}^+$.

(a) $\ln 3 + \ln 4$ [1]

(b) $3 \ln 2$ [2]

(c) $-\ln \frac{1}{2}$ [2]

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2. [Maximum mark: 5]

Consider the function $f(x) = \frac{4x^3}{3} - 16x$, where $x \in \mathbb{R}$.

The graph of $y = f(x)$ has a local minimum point at (p, q) where $p > 0$.

Find the value of p and the value of q .

3. [Maximum mark: 7]

Bob invests 1000 dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1000(1 + k)^4$ where $k \in \mathbb{Q}$.

- (a) Write down the value of k . [1]
- (b) Expand and simplify $(1 + x)^4$. [2]
- (c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

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4. [Maximum mark: 4]

Find the area completely enclosed by the curves $y = e^x$, $y = -e^x$, and the lines $x = -1$ and $x = 1$.

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6. [Maximum mark: 8]

Consider a sequence of ten rectangular picture frames $F_1, F_2, \dots, F_9, F_{10}$.

Picture frame F_1 has width 4 cm and height 5 cm.

The width and height of picture frame F_n , are each increased by 50% to generate the width and height of the next picture frame F_{n+1} , for $n \in \mathbb{Z}^+$, $1 \leq n \leq 9$.

(a) (i) Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1} \text{ cm}^2$.

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^a - 1\right) \text{ cm}^2$, where $p \in \mathbb{Q}^+$, $a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^4 \text{ cm}^2$, where $q \in \mathbb{Q}^+$. [3]

Do **not** write solutions on this page

Section B

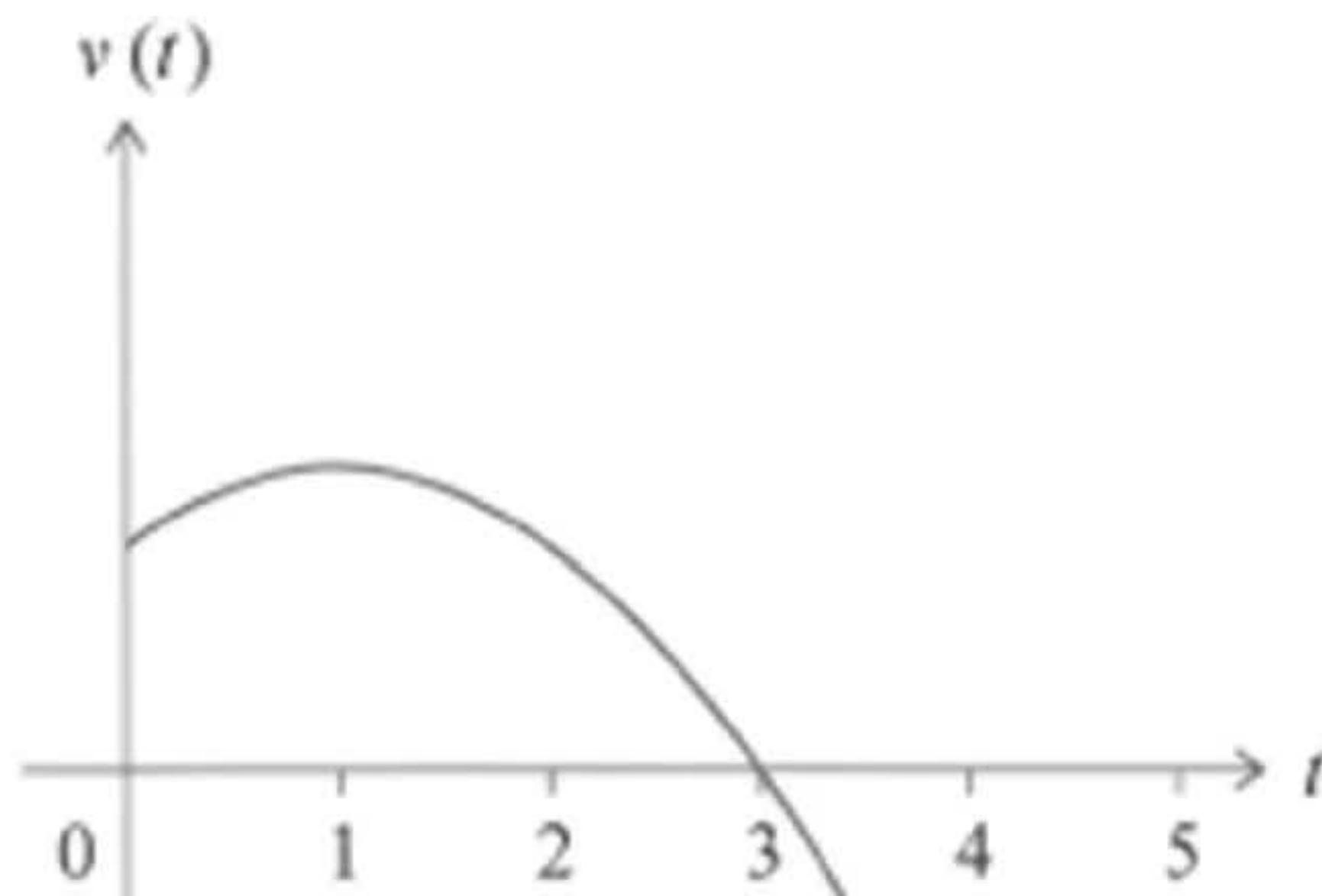
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

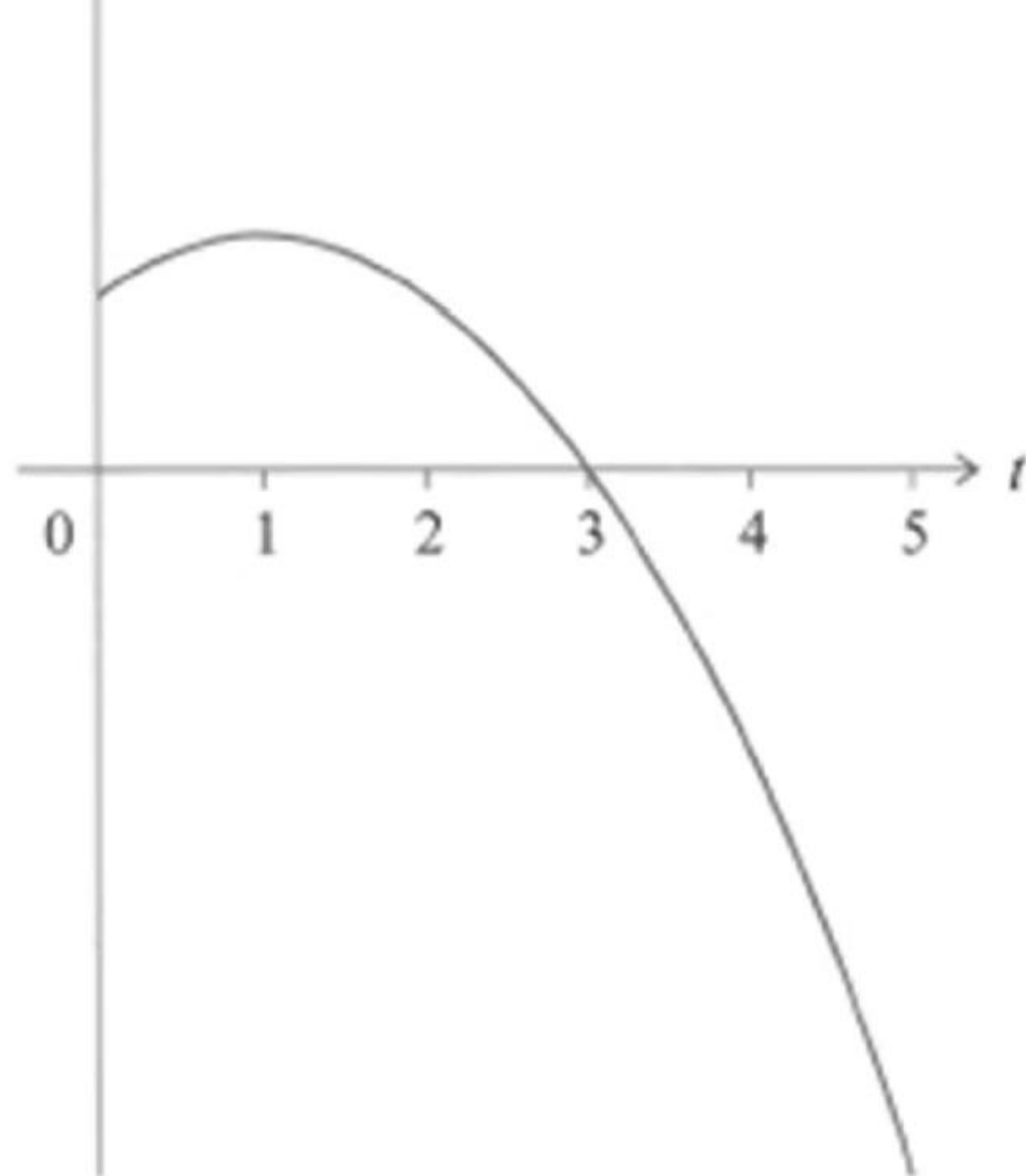
7. [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v \text{ ms}^{-1}$, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \leq t \leq 5$.

The graph of v is shown in the following diagram.





The graph of v has a local maximum point where $t = 1$ and intersects the t -axis at $t = 3$.

- (a) Determine the object's
- maximum velocity;



The graph of v has a local maximum point where $t = 1$ and intersects the t -axis at $t = 3$.

(a) Determine the object's

(i) maximum velocity;

(ii) maximum speed.

[4]

At $t = T$, the object changes direction.

(b) (i) Write down the value of T .

(ii) Find the distance travelled by the object in the first T seconds.

[5]

(c) Determine whether the object returns to its initial position during the time period $0 \leq t \leq 5$, justifying your answer.

[4]

Do **not** write solutions on this page

8. [Maximum mark: 15]

The function f is defined by $f(x) = 5(x + 1)(x + 3)$, where $x \in \mathbb{R}$.

- (a) Write $f(x)$ in the form $a(x - h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]
- (b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]
- (c) Solve the inequality $f(x) \leq 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}, x > 0$.

- (d)
 - (i) Write down an expression for $(f \circ g)(x)$.
 - (ii) Solve the inequality $(f \circ g)(x) \leq 40$. [3]

9. [Maximum mark: 17]

A solid cylinder has height h cm and base radius R cm.

The cylinder fits exactly inside a hollow sphere of radius r cm.

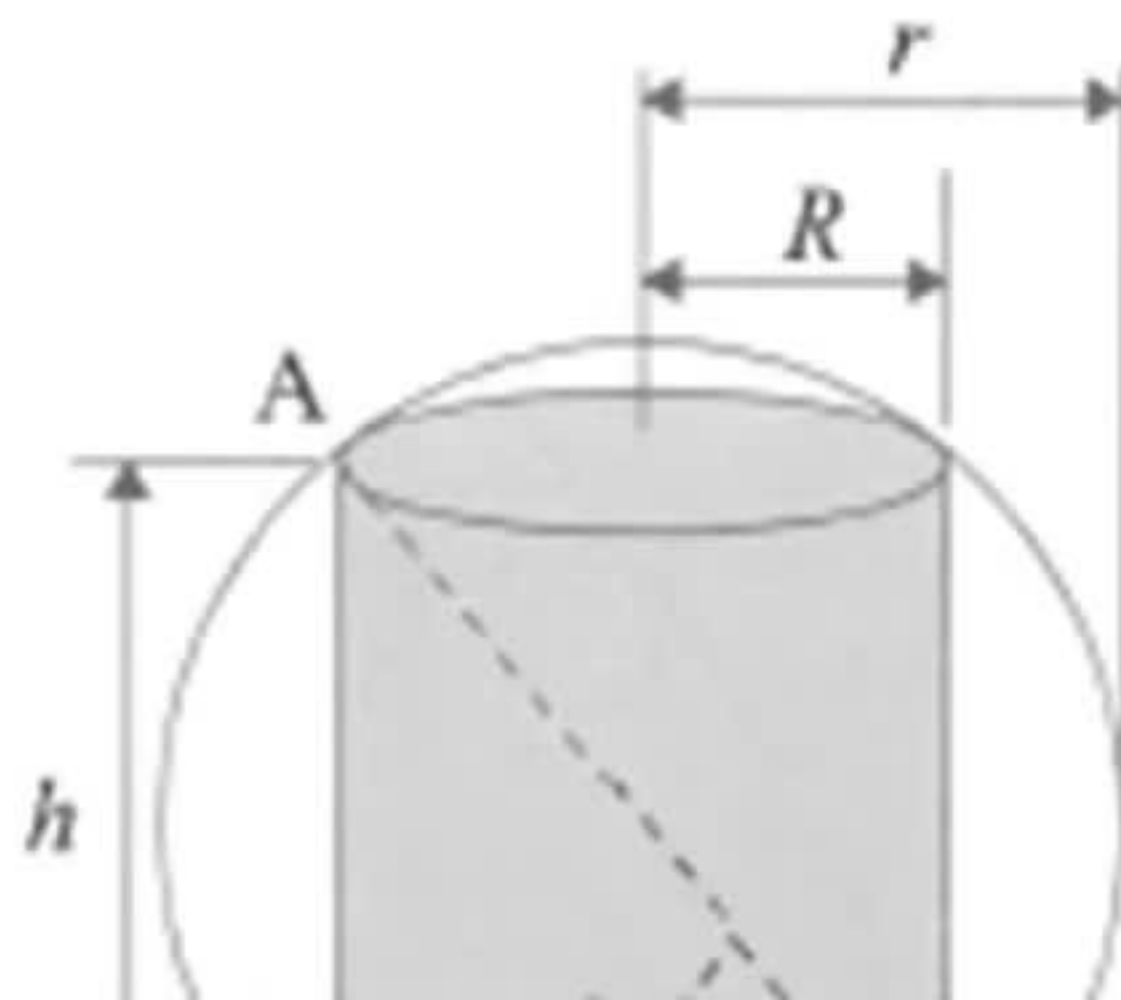
Points A, B and C are points where the surface of the cylinder touches the surface of the sphere.

The line segment [AB] is a diameter of the sphere.

The line segment [BC] is a diameter of the base of the cylinder and $\hat{ABC} = \theta$.

This information is shown on the following diagram.

diagram not to scale



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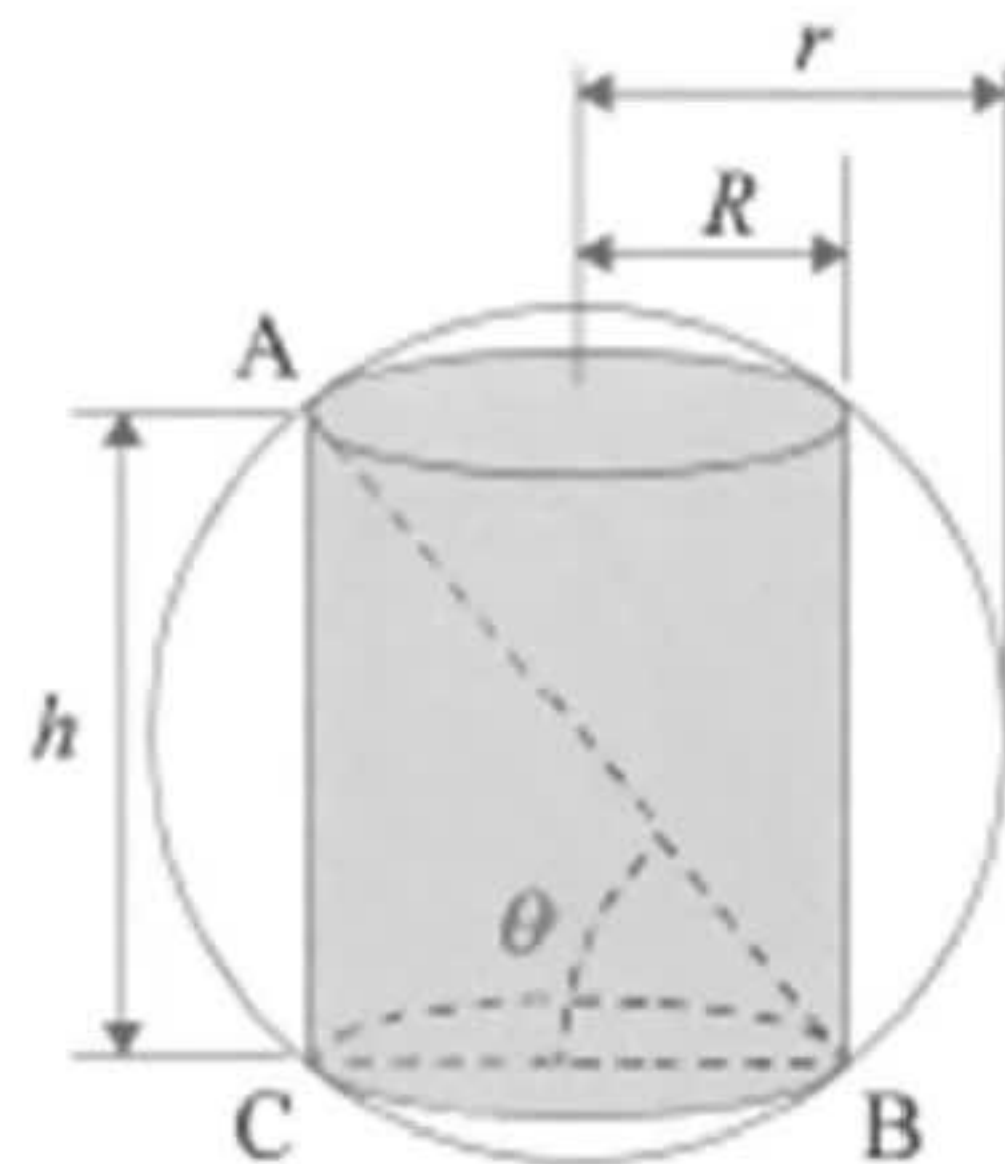
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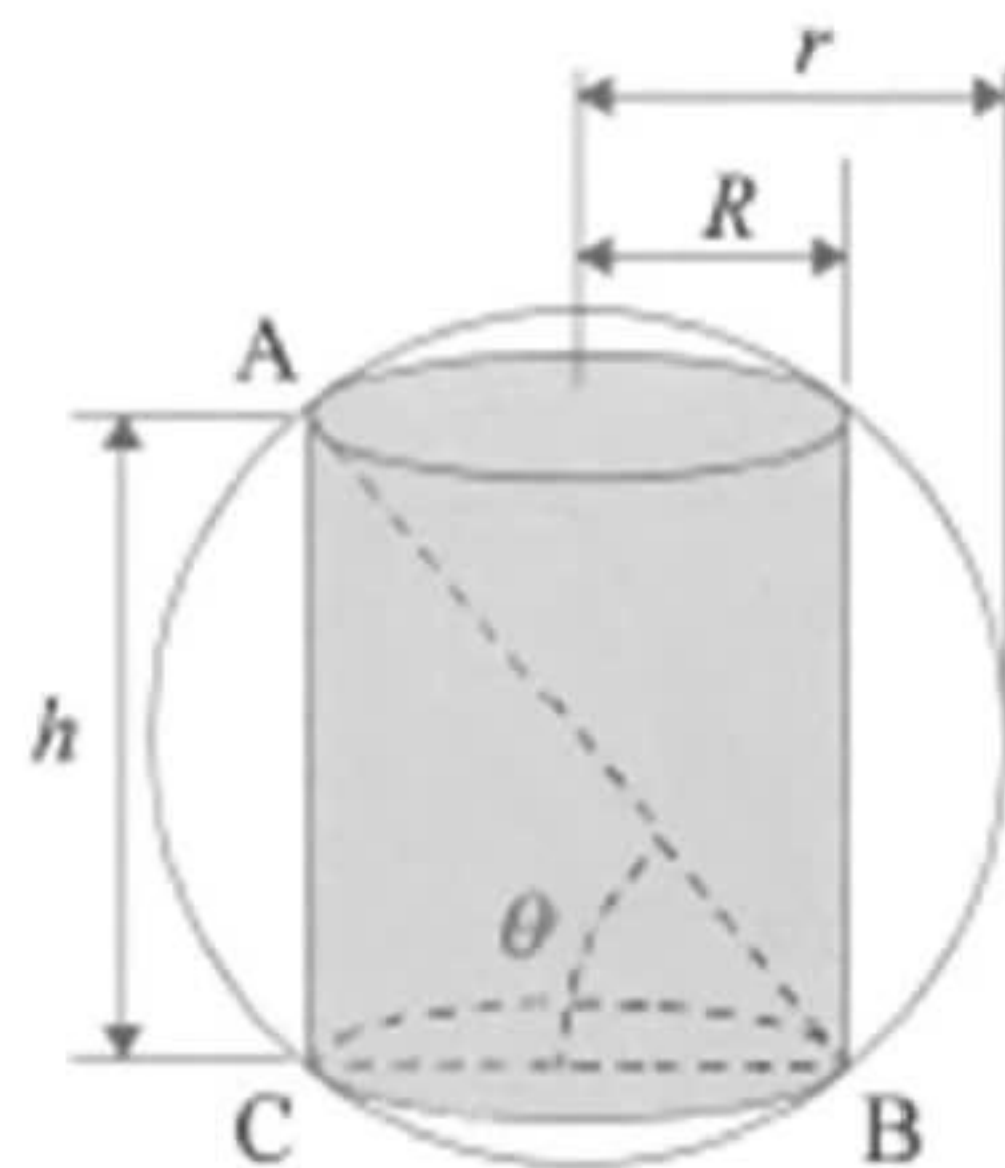
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The line segment $[BC]$ is a diameter of the base of the cylinder and $\angle ABC = \theta$.

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diagram not to scale





- (a) (i) By considering triangle ABC , show that $R = r \cos \theta$.
- (ii) Find an expression for h in terms of r and θ . [4]
- (b) Hence or otherwise, show that the total surface area, $S \text{ cm}^2$, of the cylinder is given by $S = 2\pi r^2 (1 + 2 \sin \theta \cos \theta - \sin^2 \theta)$. [4]

The external surface area of the sphere is $2S$.

- (c) Show that $\tan \theta = 2$. [4]

The volume of the cylinder is $V \text{ cm}^3$.

- (d) Find V , giving your answer in the form $p\pi r^3 \sqrt{5}$, where $p \in \mathbb{Q}^+$. [5]