Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

21 pages

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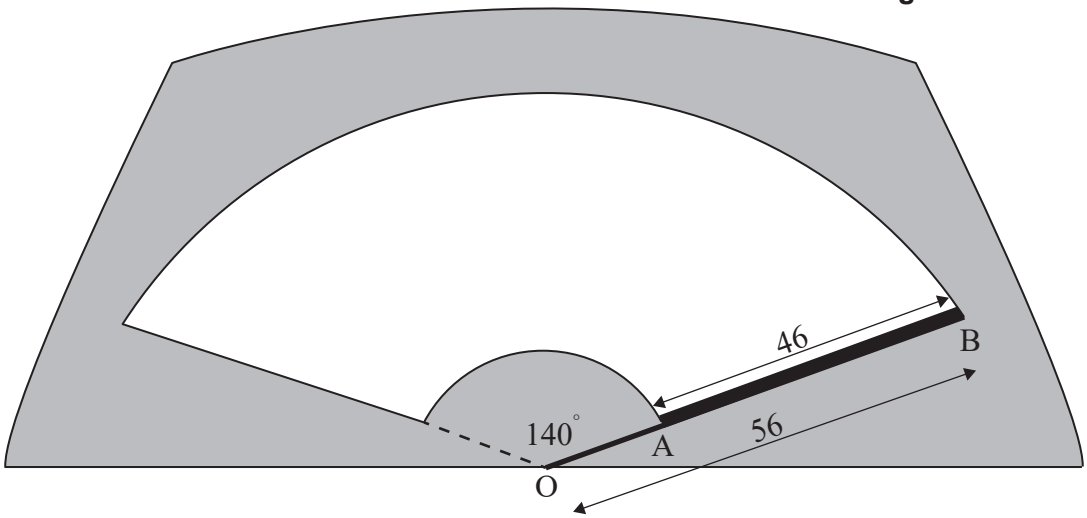
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O, through an angle of $140°$. The windscreen is cleared by a rubber blade of length $46\,\text{cm}$ that is attached to the metal arm between points A and B. The total length of the metal arm, OB, is $56\,\text{cm}$.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.

diagram not to scale



(a) Calculate the length of the arc made by B, the end of the rubber blade. [2]

(b) Determine the area of the windscreen that is cleared by the rubber blade. [3]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	**Rejected**
Arts programme	17	24
Sciences programme	25	64

(a) Find the probability that a randomly chosen applicant from this group was accepted by the university. [1]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

(b) Find the probability that the applicant applied for the Arts programme. [2]

Two different applicants are chosen at random from the original group.

(c) Find the probability that both applicants applied to the Arts programme. [3]

(This question continues on the following page)

(Question 2 continued)

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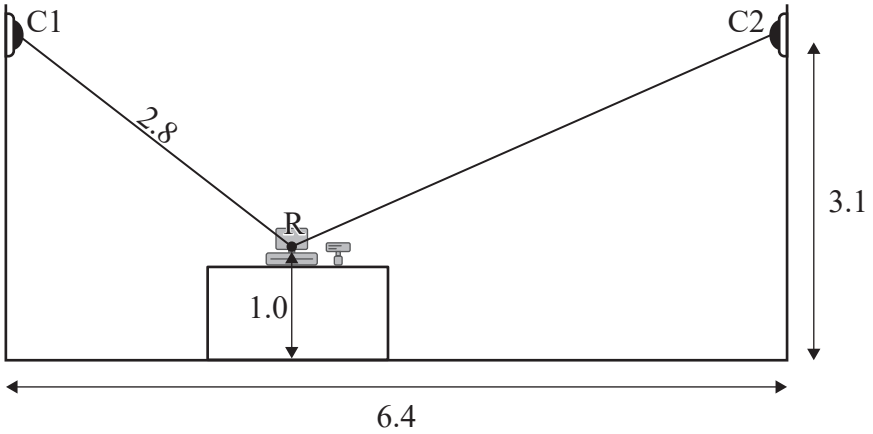
3. [Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points $C1$ and $C2$. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.

diagram not to scale



The cameras are positioned at a height of $3.1\,\text{m}$, and the horizontal distance between the cameras is $6.4\,\text{m}$. The cash register is sitting on a counter so that its centre, R, is $1.0\,\text{m}$ above the floor.

The distance from Camera 1 to the centre of the cash register is $2.8\,\text{m}$.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 5]

The pH of a solution measures its acidity and can be determined using the formula $\text{pH} = -\log_{10} C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

(a) Calculate the pH of the coffee. [2]

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

(b) Determine whether the unknown liquid is more or less acidic than the coffee. Justify your answer mathematically. [3]

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5. [Maximum mark: 7]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

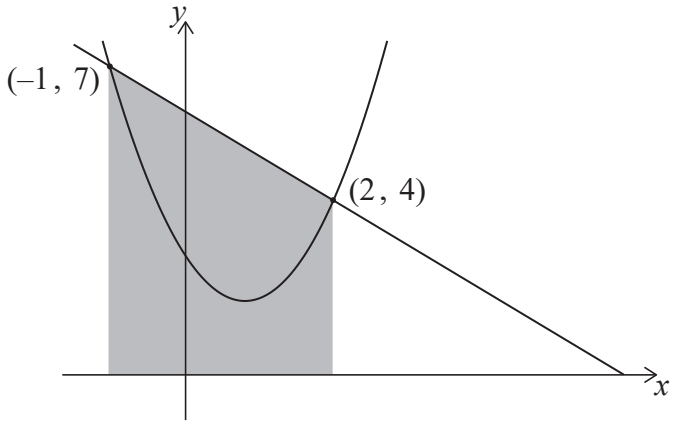
6. [Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2, 4)$ and $(-1, 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 1

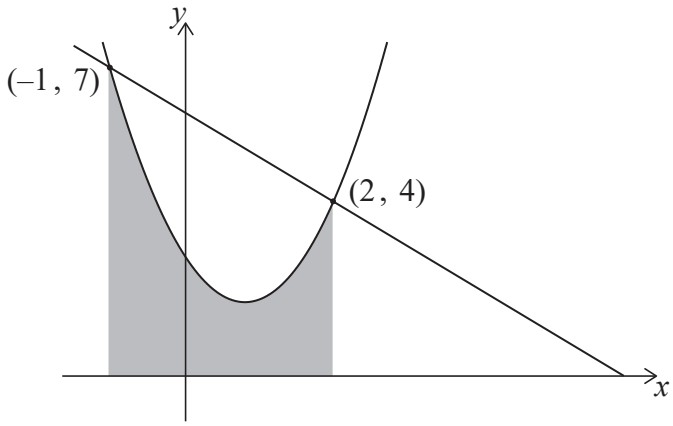


(a) Calculate the area of the shaded region in **diagram 1**. [2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 2



(b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

(ii) Calculate the area of this region. [3]

(c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$. [2]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 5]

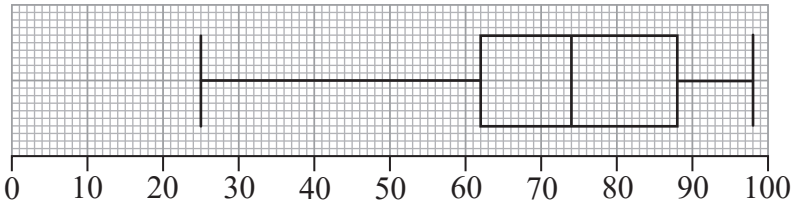
A college runs a mathematics course in the morning. Scores for a test from this class are shown below.

25 33 51 62 63 63 70 74 79 79 81 88 90 90 98

For these data, the lower quartile is 62 and the upper quartile is 88.

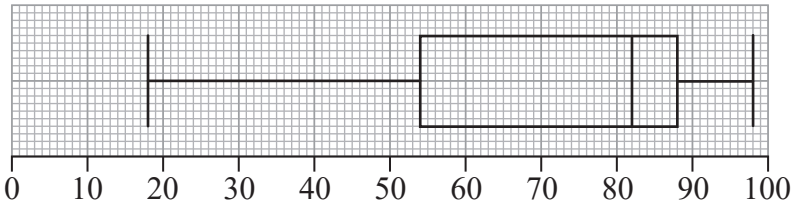
(a) Show that the test score of 25 would not be considered an outlier. [3]

The box and whisker diagram showing these scores is given below.



Test scores

Another mathematics class is run by the college during the evening. A box and whisker diagram showing the scores from this class for the same test is given below.



Test scores

A researcher reviews the box and whisker diagrams and believes that the evening class performed better than the morning class.

(b) With reference to the box and whisker diagrams, state one aspect that may support the researcher's opinion and one aspect that may counter it. [2]

(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 6]

A study was conducted to investigate whether the mean reaction time of drivers who are talking on mobile phones is the same as the mean reaction time of drivers who are talking to passengers in the vehicle. Two independent groups were randomly selected for the study.

To gather data, each driver was put in a car simulator and asked to either talk on a mobile phone or talk to a passenger. Each driver was instructed to apply the brakes as soon as they saw a red light appear in front of the car. The reaction times of the drivers, in seconds, were recorded, as shown in the following table.

Talking on mobile phone	**Talking to passenger**
0.69	0.67
0.87	0.86
0.98	0.60
1.04	0.81
0.79	0.76
0.87	0.71
0.71	0.74

At the 10% level of significance, a t-test was used to compare the mean reaction times of the two groups. Each data set is assumed to be normally distributed, and the population variances are assumed to be the same.

Let μ_1 and μ_2 be the population means for the two groups. The null hypothesis for this test is $\mathrm{H}_0 : \mu_1 - \mu_2 = 0$.

(a) State the alternative hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) (i) State the conclusion of the test. Justify your answer.

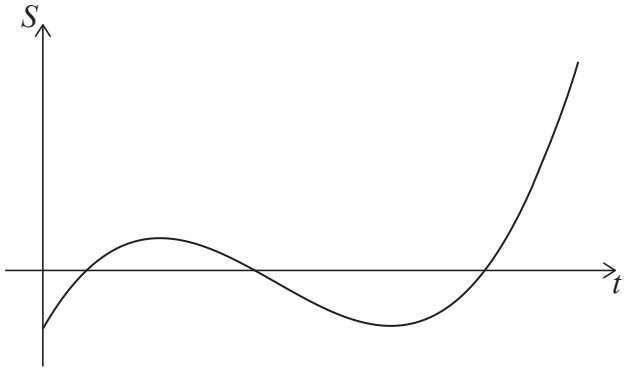
(ii) State what your conclusion means in context. [3]

(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t, the number of years after graduating from university.



(a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario. [1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a, b, c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	–5	3	–1	–5

(b) (i) Write down the value of d.

(ii) Write down three simultaneous equations for a, b and c.

(iii) Hence, or otherwise, find the values of a, b and c. [4]

A negative value of S indicates that a graduate is expected to be in debt.

(c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university. [3]

(This question continues on the following page)

(Question 9 continued)

10. [Maximum mark: 5]

The masses of Fuji apples are normally distributed with a mean of $163\,\text{g}$ and a standard deviation of $6.83\,\text{g}$.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between $172\,\text{g}$ and $183\,\text{g}$.

(a) Determine the probability that a Fuji apple selected at random will be a large apple. [2]

Approximately $68\,\%$ of Fuji apples have a mass within the medium-sized category, which is between k and $172\,\text{g}$.

(b) Find the value of k. [3]

11. [Maximum mark: 7]

Consider the function $f(x) = x^2 - \frac{3}{x}$, $x \neq 0$.

(a) Find $f'(x)$. [2]

Line L is a tangent to $f(x)$ at the point $(1, -2)$.

(b) Use your answer to part (a) to find the gradient of L. [2]

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer. [3]

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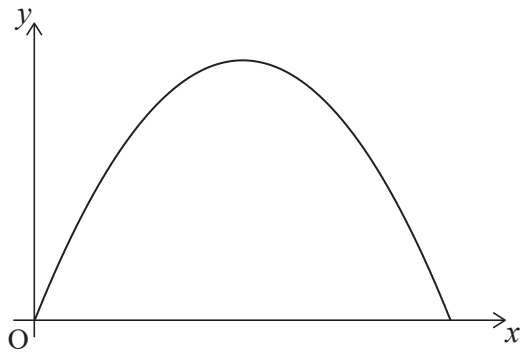
12. [Maximum mark: 5]

The cross-section of an arched entrance into the ballroom of a hotel is in the shape of a parabola. This cross-section can be modelled by part of the graph $y = -1.6x^2 + 4.48x$, where y is the height of the archway, in metres, at a horizontal distance, x metres, from the point O, in the bottom corner of the archway.



(a) Determine an equation for the axis of symmetry of the parabola that models the archway. [2]

To prepare for an event, a square-based crate that is $1.6\,\text{m}$ wide and $2.0\,\text{m}$ high is to be moved through the archway into the ballroom. The crate must remain upright while it is being moved.

(b) Determine whether the crate will fit through the archway. Justify your answer. [3]

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13. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying $3.5\,\%$ interest, compounded annually. She expects the annual inflation rate to be $2\,\%$ per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $\$4000$, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $\$1000$ at the start of the 10-year period and then invest $\$x$ into the account at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3]

(b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. [3]

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References:

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