

APOLLO

Statistics and probability

HL

Mathematics: analysis and approaches

First assessment 2021

87 questions | mark schemes

3. (a) attempt to use definition of outlier

$$1.5 \times 20 + Q_3$$

(M1)

$$1.5 \times 20 + U \geq 75 \quad (\Rightarrow U \geq 45, \text{ accept } U > 45) \quad \text{OR } 1.5 \times 20 + Q_3 = 75$$

A1

$$\text{minimum value of } U = 45$$

A1

[3 marks]

- (b) attempt to use interquartile range

(M1)

$$U - L = 20 \quad (\text{may be seen in part (a)}) \quad \text{OR } L \geq 25 \quad (\text{accept } L > 25)$$

$$\text{minimum value of } L = 25$$

A1

[2 marks]

Total [5 marks]

10. (a) recognising probabilities sum to 1 (M1)

$$p + p + p + \frac{1}{2}p = 1$$

$$p = \frac{2}{7}$$

A1

[2 marks]

- (b) valid attempt to find $E(X)$ (M1)

$$1 \times p + 2 \times p + 3 \times p + 4 \times \frac{1}{2}p (= 8p)$$

$$E(X) = \frac{16}{7}$$

A1

[2 marks]

- (c) (i) $0 \leq r \leq 1$ A1

- (ii) Attempt to find a value of q (M1)

$$0 \leq 1 - 3q \leq 1 \quad \text{OR} \quad r = 0 \Rightarrow q = \frac{1}{3} \quad \text{OR} \quad r = 1 \Rightarrow q = 0$$

$$0 \leq q \leq \frac{1}{3}$$

A1

[3 marks]

- (d) $E(Y) = 1 \times q + 2 \times q + 3 \times q + 4 \times r (= 2 + 2r \text{ OR } 4 - 6q)$ (A1)

one correct boundary value

A1

$$1 \times \frac{1}{3} + 2 \times \frac{1}{3} + 3 \times \frac{1}{3} + 4 \times 0 (= 2) \text{ OR}$$

$$1 \times 0 + 2 \times 0 + 3 \times 0 + 4 \times 1 (= 4) \text{ OR}$$

$$2 + 2(0) (= 2) \text{ OR}$$

$$2 + 2(1) (= 4) \text{ OR}$$

$$4 - 6(0) (= 4) \text{ OR } 4 - 6\left(\frac{1}{3}\right) (= 2)$$

$$2 \leq E(Y) \leq 4$$

A1

[3 marks]

continued...

Question 10 continued

(e) **METHOD 1**

evidence of choosing at least four correct outcomes from
1&2, 1&3, 1&4, 2&3, 2&4, 3&4

(M1)

$$\frac{6}{7}q + \frac{6}{7}r \text{ OR } 3pq + 3pr \text{ OR } pq + pq + p(1-3q) + pq + p(1-3q) + p(1-3q) \text{ (A1)}$$

solving for either q or r

M1

$$\frac{6}{7}(q+1-3q) = \frac{1}{2} \text{ OR } \frac{6}{7}\left(\frac{1-r}{3} + r\right) = \frac{1}{2} \text{ OR } 3pq + 3p(1-3q) = \frac{1}{2}$$

$$\text{OR } 3p\left(\frac{1-r}{3}\right) + 3pr = \frac{1}{2}$$

EITHER two correct values

$$q = \frac{5}{24} \text{ and } r = \frac{3}{8}$$

A1A1

OR one correct value

$$q = \frac{5}{24} \text{ OR } r = \frac{3}{8}$$

A1

substituting their value for q or r

A1

$$4 - 6\left(\frac{5}{24}\right) \text{ OR } 2 + 2\left(\frac{3}{8}\right)$$

THEN

$$E(Y) = \frac{11}{4}$$

A1**[6 marks]**

continued...

*Question 10 continued***METHOD 2** (solving for $E(Y)$)evidence of choosing at least four correct outcomes from
1&2, 1&3, 1&4, 2&3, 2&4, 3&4**(M1)**

$$\frac{6}{7}q + \frac{6}{7}r \text{ OR } 3pq + 3pr \text{ OR } pq + pq + p(1-3q) + pq + p(1-3q) + p(1-3q) \text{ (A1)}$$

rearranging to make q the subject**M1**

$$q = \frac{4 - E(Y)}{6}$$

$$3pq + 3p(1-3q) = \frac{1}{2}$$

M1

$$\frac{6}{7} \times \left(\frac{4 - E(Y)}{6} \right) + \frac{6}{7} \left(1 - 3 \left(\frac{4 - E(Y)}{6} \right) \right) = \frac{1}{2}$$

A1

$$\frac{2(E(Y) - 1)}{7} = \frac{1}{2}$$

$$E(Y) = \frac{11}{4}$$

A1**[6 marks]****Total [16 marks]**

1. (a) $a = 0.433156\dots$, $b = 4.50265\dots$

$$a = 0.433, b = 4.50$$

A1A1

[2 marks]

- (b) attempt to substitute $x = 18$ into their equation

(M1)

$$y = 0.433 \times 18 + 4.50$$

$$= 12.2994\dots$$

$$= 12.3$$

A1

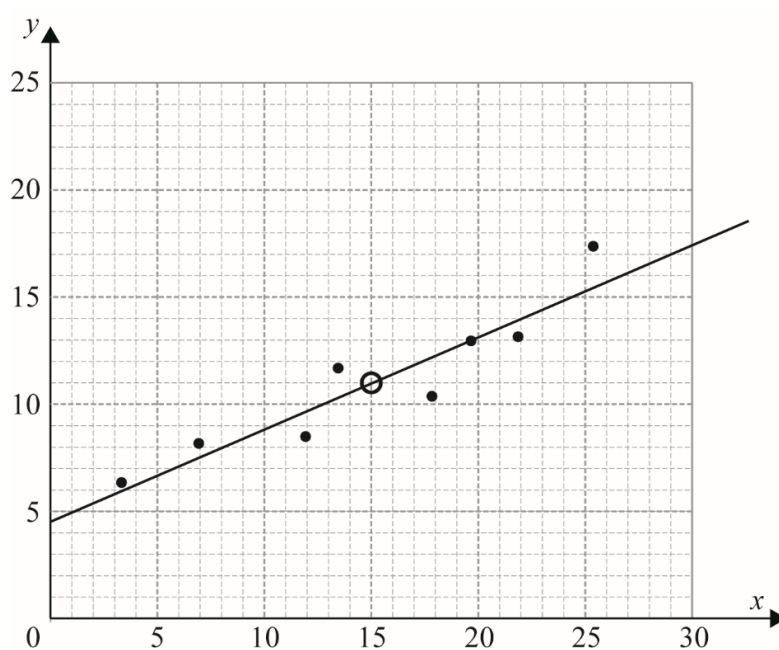
[2 marks]

- (c) $\bar{x} = 15$, $\bar{y} = 11$

A1

[1 mark]

- (d)



A1A1

Note: Award marks as follows:

A1 for a straight line going through (15, 11)

A1 for intercepting the y-axis between their $b \pm 1.5$ (when their line is extended), which includes all the data for $3.3 \leq x \leq 25.3$.

If the candidate does not use a ruler, award **A0A1** where appropriate.

[2 marks]

Total [7 marks]

2.

Note: In this question, do not penalise incorrect use of strict inequality signs.

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area **(M1)**

$$P(X < 995) = 0.0765637...$$

$$= 0.0766$$

A1**[2 marks]**

(b) 0.0766×100

$$\approx 8$$

A1**[1 mark]**

Note: Accept 7.66 .

(c) recognition that $P(X > 1005 | X \geq 995)$ is required **(M1)**

$$\frac{P(X \geq 995 \cap X > 1005)}{P(X \geq 995)}$$

$$\frac{P(X > 1005)}{P(X \geq 995)}$$

(A1)

$$\frac{0.07656...}{1 - 0.07656...} \left(= \frac{0.07656...}{0.9234...} \right)$$

$$= 0.0829$$

A1**[3 marks]****Total [6 marks]**

7. (a) recognition of the need to integrate $\frac{x}{\sqrt{(x^2 + k)^3}}$ **(M1)**

$$\int \frac{x}{\sqrt{(x^2 + k)^3}} dx (=1)$$

EITHER

$$u = x^2 + k \Rightarrow \frac{du}{dx} = 2x \text{ (or equivalent)} \quad \textbf{(A1)}$$

$$\int \frac{x}{\sqrt{(x^2 + k)^3}} dx = \frac{1}{2} \int u^{-\frac{3}{2}} du$$

$$= -u^{-\frac{1}{2}} (+c) \left(= -(x^2 + k)^{-\frac{1}{2}} (+c) \right) \quad \textbf{A1}$$

continued...

Question 7 continued

OR

$$\int \frac{x}{\sqrt{(x^2 + k)^3}} dx = \frac{1}{2} \int \frac{2x}{\sqrt{(x^2 + k)^3}} dx \quad (\text{A1})$$

$$= -(x^2 + k)^{-\frac{1}{2}} (+c) \quad \text{A1}$$

THENattempt to use correct limits for their integrand and set equal to 1 **M1**

$$\left[-u^{-\frac{1}{2}} \right]_k^{16+k} = 1 \text{ OR } \left[-(x^2 + k)^{-\frac{1}{2}} \right]_0^4 = 1$$

$$-(16+k)^{-\frac{1}{2}} + k^{-\frac{1}{2}} = 1 \left(\Rightarrow \frac{1}{\sqrt{k}} - \frac{1}{\sqrt{16+k}} = 1 \right) \quad \text{A1}$$

$$\sqrt{16+k} - \sqrt{k} = \sqrt{k}\sqrt{16+k} \quad \text{AG}$$

[5 marks]

(b) attempt to solve $\sqrt{16+k} - \sqrt{k} = \sqrt{k}\sqrt{16+k}$ **(M1)**

$$k = 0.645038...$$

$$= 0.645 \quad \text{A1}$$

[2 marks]**Total [7 marks]**

1. (a) (i) $a = 0.805084\dots$ and $b = 2.88135\dots$
 $a = 0.805$ and $b = 2.88$ **A1A1**
- (ii) $r = 0.97777\dots$
 $r = 0.978$ **A1**
- [3 marks]**
- (b) a represents the (average) increase in waiting time (0.805 mins) per additional customer (waiting to receive their coffee) **R1**
[1 mark]
- (c) attempt to substitute $x = 7$ into their equation **(M1)**
- 8.51693...
8.52 (mins) **A1**
- [2 marks]**
Total [6 marks]

3. (a) **EITHER**

$$P(S) + P(T) + P(S' \cap T') - P(S \cap T) = 1 \text{ OR } P(S \cup T) = P((S' \cap T')') \quad (M1)$$

$$0.7 + 0.2 + 0.18 - P(S \cap T) = 1 \text{ OR } P(S \cup T) = 1 - 0.18$$

OR

a clearly labelled Venn diagram

(M1)**THEN**

$$P(S \cap T) = 0.08 \text{ (accept 8\%)}$$

A1

Note: To obtain the **M1** for the Venn diagram all labels must be correct and in the correct sections.
For example, do not accept 0.7 in the area corresponding to $S \cap T'$.

[2 marks](b) **EITHER**

$$P(T \cap S') = P(T) - P(T \cap S) (= 0.2 - 0.08) \text{ OR}$$

$$P(T \cap S') = P(T \cup S) - P(S) (= 0.82 - 0.7) \quad (M1)$$

ORa clearly labelled Venn diagram including $P(S)$, $P(T)$ and $P(S \cap T)$ **(M1)****THEN**

$$= 0.12 \text{ (accept 12\%)}$$

A1**[2 marks]**

$$(c) \quad P(G \cap T) = P(T|G)P(G) \quad (0.25 \times 0.48)$$

(M1)

$$= 0.12$$

A1**[2 marks]***continued...*

Question 3 continued(d) **METHOD 1**

$$P(G) \times P(T) (= 0.48 \times 0.2) = 0.096$$

A1

$$P(G) \times P(T) \neq P(G \cap T) \Rightarrow G \text{ and } T \text{ are not independent}$$

R1**METHOD 2**

$$P(T | G) = 0.25$$

A1

$$P(T | G) \neq P(T) \Rightarrow G \text{ and } T \text{ are not independent}$$

R1
Note: Do not award **A0R1**.
[2 marks]**Total [8 marks]**

6. (a) $E(X) = (n+1) \int_0^1 x^{n+1} dx$ **M1**

$$= (n+1) \left[\frac{x^{n+2}}{n+2} \right]_0^1$$

A1

leading to $E(X) = \frac{n+1}{n+2}$ **AG**

[2 marks](b) **METHOD 1**

use of $\text{Var}(X) = E(X^2) - [E(X)]^2$ **M1**

$$\text{Var}(X) = (n+1) \int_0^1 x^{n+2} dx - \left(\frac{n+1}{n+2} \right)^2$$

$$= (n+1) \left[\frac{1}{n+3} x^{n+3} \right]_0^1 - \left(\frac{n+1}{n+2} \right)^2$$

$$= \frac{n+1}{n+3} - \left(\frac{n+1}{n+2} \right)^2$$

A1

$$= \frac{(n+1)(n+2)^2 - (n+1)^2(n+3)}{(n+2)^2(n+3)}$$

M1

EITHER

$$= \frac{(n+1)(n^2 + 4n + 4 - (n^2 + 4n + 3))}{(n+2)^2(n+3)}$$

A1

OR

$$= \frac{(n^3 + 5n^2 + 8n + 4) - (n^3 + 5n^2 + 7n + 3)}{(n+2)^2(n+3)}$$

A1

THEN

so $\text{Var}(X) = \frac{n+1}{(n+2)^2(n+3)}$ **AG**

continued...

*Question 6 continued***METHOD 2**

$$\text{use of } \text{Var}(X) = E(X - E(X))^2 \quad \mathbf{M1}$$

$$\text{Var}(X) = (n+1) \int_0^1 \left(x - \frac{n+1}{n+2}\right)^2 x^n dx$$

$$= (n+1) \left[\frac{1}{n+3} x^{n+3} - \frac{2(n+1)}{(n+2)^2} x^{n+2} + \frac{n+1}{(n+2)^2} x^{n+1} \right]_0^1$$

$$= \frac{n+1}{n+3} - \left(\frac{n+1}{n+2}\right)^2 \quad \mathbf{A1}$$

$$= \frac{(n+1)((n+2)^2 - (n+1)(n+3))}{(n+2)^2(n+3)} \quad \mathbf{M1}$$

EITHER

$$= \frac{(n+1)(n^2 + 4n + 4 - (n^2 + 4n + 3))}{(n+2)^2(n+3)} \quad \mathbf{A1}$$

OR

$$= \frac{(n^3 + 5n^2 + 8n + 4) - (n^3 + 5n^2 + 7n + 3)}{(n+2)^2(n+3)} \quad \mathbf{A1}$$

THEN

$$\text{so } \text{Var}(X) = \frac{n+1}{(n+2)^2(n+3)} \quad \mathbf{AG}$$

[4 marks]**Total [6 marks]**

10. (a) use of inverse normal to find z -score (M1)

$$z = 2.0537...$$

$$2.0537... = \frac{82 - 75}{\sigma} \quad \text{(A1)}$$

$$\sigma = 3.408401...$$

$$\sigma = 3.41 \quad \text{A1}$$

[3 marks]

- (b) evidence of identifying the correct area under the normal curve (M1)

$$P(T > 80) = 0.071193...$$

$$P(T > 80) = 0.0712 \quad \text{A1}$$

[2 marks]

- (c) recognition that $P(80 < T < 82)$ is required (M1)

$$P(T < 82 | T > 80) = \frac{P(80 < T < 82)}{P(T > 80)} = \left(\frac{0.051193...}{0.071193...} \right) \quad \text{(M1)(A1)}$$

$$= 0.719075...$$

$$= 0.719 \quad \text{A1}$$

[4 marks]

continued...

Question 10 continued

- (d) recognition of binomial probability

(M1)

$$X \sim B(64, 0.071193...) \text{ or } E(X) = 64 \times 0.071193...$$

(A1)

$$E(X) = 4.556353...$$

$$E(X) = 4.56 \text{ (flights)}$$

A1**[3 marks]**

- (e)
- $P(X > 6) = P(X \geq 7) = 1 - P(X \leq 6)$

(M1)

$$= 1 - 0.83088...$$

(A1)

$$= 0.1691196...$$

$$= 0.169$$

A1**[3 marks]****Total [15 marks]**

4

4. (a) valid approach to find $P(R)$ (M1)

tree diagram (must include probability of picking box) with correct required probabilities

OR $P(R \cap B_1) + P(R \cap B_2)$ OR $P(R|B_1)P(B_1) + P(R|B_2)P(B_2)$

$$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2} \quad \text{A1}$$

$$P(R) = \frac{9}{14} \quad \text{A1}$$

[3 marks]

- (b) events A and R are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box A2

Note: Both conclusion and reasoning are required. Do not split the A2.

[2 marks]

Total [5 marks]

1. (a) use of GDC to give (M1)
 $r = 0.883529...$
 $r = 0.884$ A1

Note: Award the **(M1)** for any correct value of r , a , b or $r^2 = 0.780624...$ seen in part (a) or part (b).

[2 marks]

- (b) $a = 1.36609...$, $b = 64.5171...$
 $a = 1.37$, $b = 64.5$ A1
[1 mark]

- (c) attempt to find their difference (M1)
 $5 \times 1.36609... \text{ OR } 1.36609...(h+5) + 64.5171... - (1.36609...h + 64.5171...)$
 $6.83045...$
 $= 6.83(6.85 \text{ from } 1.37)$
the student could have expected her score to increase by 7 marks. A1

Note: Accept an increase of 6, 6.83 or 6.85.

[2 marks]

- (d) Lucy is incorrect in suggesting there is a causal relationship.
This might be true, but the data can only indicate a correlation. R1

Note: Accept 'Lucy is incorrect as correlation does not imply causation' or equivalent.

[1 mark]

- (e) no effect A1
[1 mark]
Total [7 marks]

3. (a) recognize that the variable has a Binomial distribution (M1)

$$X \sim B(30, 0.05)$$

attempt to find $P(X \geq 1)$ (M1)

$$1 - P(X = 0) \text{ OR } 1 - 0.95^{30} \text{ OR } 1 - 0.214638... \text{ OR } 0.785361...$$

Note: The two **M** marks are independent of each other.

$$P(X \geq 1) = 0.785 \quad \text{A1}$$

[3 marks]

- (b) recognition of conditional probability (M1)

$$P(X \leq 2 | X \geq 1) \text{ OR } P(\text{at most 2 defective} | \text{at least 1 defective})$$

Note: Recognition must be shown in context either in words or symbols but not just $P(A|B)$.

$$\frac{P(1 \leq X \leq 2)}{P(X \geq 1)} \text{ OR } \frac{P(X = 1) + P(X = 2)}{P(X \geq 1)} \quad \text{(A1)}$$

$$\frac{0.597540...}{0.785361...} \text{ OR } \frac{0.812178... - 0.214638...}{0.785361...} \text{ OR } \frac{0.338903... + 0.258636...}{0.785361...} \quad \text{(A1)}$$

$$= 0.760847...$$

$$P(X \leq 2 | X \geq 1) = 0.761 \quad \text{A1}$$

[4 marks]

Total [7 marks]

7. (a) recognises that $\int_0^m \arccos x \, dx = 0.5$ (M1)

$$m \arccos m - \sqrt{1-m^2} - (0 - \sqrt{1}) = 0.5$$

$$m = 0.360034...$$

$$m = 0.360$$

A1
[2 marks]

- (b) **METHOD 1**

attempts to find at least one endpoint (limit) both in terms of m (or their m) and a (M1)

$$P(m-a \leq X \leq m+a) = 0.3$$

$$\int_{0.360034...-a}^{0.360034...+a} \arccos x \, dx = 0.3$$

(A1)

Note: Award (A1) for $\int_{m-a}^{m+a} \arccos x \, dx = 0.3$.

$$\left[x \arccos x - \sqrt{1-x^2} \right]_{0.360034...-a}^{0.360034...+a}$$

attempts to solve their equation for a

(M1)

Note: The above (M1) is dependent on the first (M1).

$$a = 0.124861...$$

$$a = 0.125$$

A1

METHOD 2

$$\int_{-a}^a \arccos |x - 0.360034...| \, dx (= 0.3)$$

(M1)(A1)

Note: Only award (M1) if at least one limit has been translated correctly.

Note: Award (M1)(A1) for $\int_{-a}^a \arccos |x - m| \, dx (= 0.3)$.

attempts to solve their equation for a

(M1)

$$a = 0.124861...$$

$$a = 0.125$$

A1

METHOD 3**EITHER**

$$\int_{-a}^a \arccos(x + 0.360034...) \, dx \quad (= 0.3)$$

(M1)(A1)

Note: Only award **(M1)** if at least one limit has been translated correctly.

Note: Award **(M1)(A1)** for $\int_{-a}^a \arccos(x + m) \, dx \quad (= 0.3)$.

OR

$$\int_{2(0.360034...) - a}^{2(0.360034...) + a} \arccos(x - 0.360034...) \, dx \quad (= 0.3)$$

(M1)(A1)

Note: Only award **(M1)** if at least one limit has been translated correctly.

Note: Award **(M1)(A1)** for $\int_{2m-a}^{2m+a} \arccos(x - m) \, dx \quad (= 0.3)$.

THENattempts to solve their equation for a **(M1)**

Note: The above **(M1)** is dependent on the first **(M1)**.

$$a = 0.124861...$$

$$a = 0.125$$

A1**[4 marks]****Total [6 marks]**

1. attempt to substitute into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **(M1)**

Note: Accept use of Venn diagram or other valid method.

$$0.6 = 0.5 + 0.4 - P(A \cap B) \quad \text{span style="float: right;">**(A1)**$$

$$P(A \cap B) = 0.3 \text{ (seen anywhere)} \quad \text{span style="float: right;">**A1**$$

$$\text{attempt to substitute into } P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{span style="float: right;">**(M1)**$$

$$= \frac{0.3}{0.4}$$

$$P(A|B) = 0.75 \left(= \frac{3}{4} \right) \quad \text{span style="float: right;">**A1**$$

Total [5 marks]

7. attempting integration by parts, *eg*

$$u = \frac{\pi x}{36}, du = \frac{\pi}{36} dx, dv = \sin\left(\frac{\pi x}{6}\right) dx, v = -\frac{6}{\pi} \cos\left(\frac{\pi x}{6}\right) \quad (M1)$$

$$P(0 \leq X \leq 3) = \frac{\pi}{36} \left(\left[-\frac{6x}{\pi} \cos\left(\frac{\pi x}{6}\right) \right]_0^3 + \frac{6}{\pi} \int_0^3 \cos\left(\frac{\pi x}{6}\right) dx \right) \text{ (or equivalent)} \quad A1A1$$

Note: Award **A1** for a correct uv and **A1** for a correct $\int v du$.

attempting to substitute limits

M1

$$\frac{\pi}{36} \left[-\frac{6x}{\pi} \cos\left(\frac{\pi x}{6}\right) \right]_0^3 = 0 \quad (A1)$$

$$\text{so } P(0 \leq X \leq 3) = \frac{1}{\pi} \left[\sin\left(\frac{\pi x}{6}\right) \right]_0^3 \text{ (or equivalent)} \quad A1$$

$$= \frac{1}{\pi} \quad A1$$

Total [7 marks]

3. (a) recognition of binomial **(M1)**
 $X \sim B(5, 0.7)$
 attempt to find $P(X \leq 3)$ **M1**
 $= 0.472 (= 0.47178)$ **A1**
[3 marks]
- (b) recognition of 2 sixes in 4 tosses **(M1)**
 $P(\text{3rd six on the 5th toss}) = \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2 \right] \times 0.7 (= 0.2646 \times 0.7)$ **A1**
 $= 0.185 (= 0.18522)$ **A1**
[3 marks]
Total [6 marks]

4. (a) $a = 1.29$ and $b = -10.4$

A1A1

[2 marks]

(b) recognising both lines pass through the mean point
 $p = 28.7$, $q = 30.3$

(M1)

A2

[3 marks]

(c) substitution into **their** x on y equation
 $x = 1.29082(29) - 10.3793$
 $x = 27.1$

(M1)

A1

Note: Accept 27.

[2 marks]

Total [7 marks]

6. (a) **METHOD 1**

attempting to use the expected value formula (M1)

$$E(X) = (1 \times 0.60) + (2 \times 0.30) + (3 \times 0.03) + (4 \times 0.05) + (5 \times 0.02)$$

$$E(X) = 1.59(\$)$$
 (A1)

use of $E(1.20X + 2.40) = 1.20E(X) + 2.40$ (M1)

$$E(T) = 1.20(1.59) + 2.40$$

$$= 4.31(\$)$$
 A1

METHOD 2attempting to find the probability distribution for T (M1)

t	3.60	4.80	6.00	7.20	8.40
$P(T=t)$	0.60	0.30	0.03	0.05	0.02

(A1)

attempting to use the expected value formula (M1)

$$E(T) = (3.60 \times 0.60) + (4.80 \times 0.30) + (6.00 \times 0.03) + (7.20 \times 0.05) + (8.40 \times 0.02)$$

$$= 4.31(\$)$$
 A1

[4 marks]

(b) **METHOD 1**using $\text{Var}(1.20X + 2.40) = (1.20)^2 \text{Var}(X)$ with $\text{Var}(X) = 0.8419$ (M1)

$$\text{Var}(T) = 1.21$$
 A1

METHOD 2finding the standard deviation for **their** probability distribution found in part (a) (M1)

$$\text{Var}(T) = (1.101\dots)^2$$

$$= 1.21$$
 A1

Note: Award **M1A1** for $\text{Var}(T) = (1.093\dots)^2 = 1.20$.

[2 marks]

Total [6 marks]

10. (a) attempt to use the symmetry of the normal curve (M1)
 eg diagram, $0.5 - 0.1446$
 $P(24.15 < X < 25) = 0.3554$ A1
 [2 marks]
- (b) (i) use of inverse normal to find z score (M1)
 $z = -1.0598$
 correct substitution $\frac{24.15 - 25}{\sigma} = -1.0598$ (A1)
 $\sigma = 0.802$ A1
- (ii) $P(X > 26) = 0.106$ (M1)A1
 [5 marks]
- (c) recognizing binomial probability (M1)
 $E(Y) = 10 \times 0.10621$ (A1)
 $= 1.06$ A1
 [3 marks]
- (d) $P(Y = 3)$ (M1)
 $= 0.0655$ A1
 [2 marks]
- (e) recognizing conditional probability (M1)
 correct substitution A1
 $\frac{0.3554}{1 - 0.10621}$
 $= 0.398$ A1
 [3 marks]
- Total [15 marks]**

2. (a) $IQR = 10 - 6 (= 4)$ (A1)
- attempt to find $Q_3 + 1.5 \times IQR$ (M1)
- $10 + 6$
- 16 A1

[3 marks]

- (b) (i) choosing $c = \frac{1}{2}a - 9$ (M1)

$$\frac{1}{2} \times 42 - 9$$

$$= 12 \text{ (years old)}$$

A1

- (ii) attempt to solve system by substitution or elimination (M1)
- 34 (years old) A1

[4 marks]

Total [7 marks]

7. (a) attempt to integrate $\frac{k}{\sqrt{4-3x^2}}$ **(M1)**

$$= k \left[\frac{1}{\sqrt{3}} \arcsin \left(\frac{\sqrt{3}}{2} x \right) \right] \quad \text{A1}$$

Note: Award **(M1)A0** for $\arcsin \left(\frac{\sqrt{3}}{2} x \right)$.

Condone absence of k up to this stage.

equating their integrand to 1 **M1**

$$k \left[\frac{1}{\sqrt{3}} \arcsin \left(\frac{\sqrt{3}}{2} x \right) \right]_0^1 = 1$$

$$k = \frac{3\sqrt{3}}{\pi} \quad \text{A1}$$

[4 marks]

continued...

Question 7 continued

$$(b) \quad E(X) = \frac{3\sqrt{3}}{\pi} \int_0^1 \frac{x}{\sqrt{4-3x^2}} dx \quad \text{A1}$$

Note: Condone absence of limits if seen at a later stage.

EITHER

attempt to integrate by inspection

(M1)

$$= \frac{3\sqrt{3}}{\pi} \times -\frac{1}{6} \int -6x(4-3x^2)^{-\frac{1}{2}} dx$$

$$= \frac{3\sqrt{3}}{\pi} \left[-\frac{1}{3} \sqrt{4-3x^2} \right]_0^1$$

A1

Note: Condone the use of k up to this stage.

OR

$$\text{for example, } u = 4 - 3x^2 \Rightarrow \frac{du}{dx} = -6x$$

Note: Other substitutions may be used. For example, $u = -3x^2$.

$$= -\frac{\sqrt{3}}{2\pi} \int_4^1 u^{-\frac{1}{2}} du$$

M1

Note: Condone absence of limits up to this stage.

$$= -\frac{\sqrt{3}}{2\pi} \left[2\sqrt{u} \right]_4^1$$

A1

Note: Condone the use of k up to this stage.

THEN

$$= \frac{\sqrt{3}}{\pi}$$

A1

Note: Award **A0M1A1A0** for their $k \left[-\frac{1}{3} \sqrt{4-3x^2} \right]$ or $k \left[-2\sqrt{u} \right]$ for working with incorrect or no limits.

[4 marks]**Total [8 marks]**

8. let m be the median

EITHER

attempts to find the area of the required triangle

M1

base is $(m-a)$

(A1)

and height is $\frac{2}{(b-a)(c-a)}(m-a)$

$$\text{area} = \frac{1}{2}(m-a) \times \frac{2}{(b-a)(c-a)}(m-a) \left(= \frac{(m-a)^2}{(b-a)(c-a)} \right)$$

A1

OR

attempts to integrate the correct function

M1

$$\int_a^m \frac{2}{(b-a)(c-a)}(x-a) \, dx$$

$$= \frac{2}{(b-a)(c-a)} \left[\frac{1}{2}(x-a)^2 \right]_a^m \quad \text{OR} \quad \frac{2}{(b-a)(c-a)} \left[\frac{x^2}{2} - ax \right]_a^m$$

A1A1

Note: Award **A1** for correct integration and **A1** for correct limits.

THEN

$$\text{sets up (their) } \int_a^m \frac{2}{(b-a)(c-a)}(x-a) \, dx \text{ or area} = \frac{1}{2}$$

M1

Note: Award **M0A0A0M1A0A0** if candidates conclude that $m > c$ and set up their area or sum of integrals $= \frac{1}{2}$.

$$\frac{(m-a)^2}{(b-a)(c-a)} = \frac{1}{2}$$

$$m = a \pm \sqrt{\frac{(b-a)(c-a)}{2}}$$

(A1)

$$\text{as } m > a, \text{ rejects } m = a - \sqrt{\frac{(b-a)(c-a)}{2}}$$

$$\text{so } m = a + \sqrt{\frac{(b-a)(c-a)}{2}}$$

A1

[6 marks]

10. (a) uses $\sum P(X = x) = 1$ to form a linear equation in p and q (M1)
 correct equation in terms of p and q from summing to 1 A1
 $p + 0.3 + q + 0.1 = 1$ OR $p + q = 0.6$ (or equivalent)
 uses $E(X) = 2$ to form a linear equation in p and q (M1)
 correct equation in terms of p and q from $E(X) = 2$ A1
 $p + 0.6 + 3q + 0.4 = 2$ OR $p + 3q = 1$ (or equivalent)

Note: The marks for using $\sum P(X = x) = 1$ and the marks for using $E(X) = 2$ may be awarded independently of each other.

evidence of correctly solving these equations simultaneously A1
 for example, $2q = 0.4 \Rightarrow q = 0.2$ or $p + 3 \times (0.6 - p) = 1 \Rightarrow p = 0.4$
 so $p = 0.4$ and $q = 0.2$ AG

[5 marks]

- (b) valid approach (M1)
 $P(X > 2) = P(X = 3) + P(X = 4)$ OR $P(X > 2) = 1 - P(X = 1) - P(X = 2)$
 $= 0.3$ A1

[2 marks]

continued...

Question 10 continued

- (c) recognises at least one of the valid scores (6, 7, or 8) required to win the game **(M1)**

Note: Award **M0** if candidate also considers scores other than 6, 7, or 8 (such as 5).

let T represent the score on the last two rolls

a score of 6 is obtained by rolling $(2,4), (4,2)$ or $(3,3)$

$$P(T = 6) = 2(0.3)(0.1) + (0.2)^2 (= 0.1) \quad \mathbf{A1}$$

a score of 7 is obtained by rolling $(3,4)$ or $(4,3)$

$$P(T = 7) = 2(0.2)(0.1) (= 0.04) \quad \mathbf{A1}$$

a score of 8 is obtained by rolling $(4,4)$

$$P(T = 8) = (0.1)^2 (= 0.01) \quad \mathbf{A1}$$

Note: The above 3 **A1** marks are independent of each other.

$$P(\text{Nicky wins}) = 0.1 + 0.04 + 0.01$$

$$= 0.15 \quad \mathbf{A1}$$

[5 marks]

- (d) $3 + b = 8$ **(M1)**

$$b = 5 \quad \mathbf{A1}$$

[2 marks]

continued...

Question 10 continued(e) **METHOD 1****EITHER**

$$P(S = 5) = \frac{4}{16}$$

$$P(S = a + 2) = \frac{4}{16}$$

$$\Rightarrow a + 2 = 5$$

A1**OR**

$$P(S = 6) = \frac{3}{16}$$

$$P(S = a + 3) = \frac{2}{16} \text{ and } P(S = 5 + 1) = \frac{1}{16}$$

$$\Rightarrow a + 3 = 6$$

A1**OR**

$$P(S = 4) = \frac{3}{16}$$

$$P(S = a + 1) = \frac{2}{16} \text{ and } P(S = 1 + 3) = \frac{1}{16}$$

$$\Rightarrow a + 1 = 4$$

A1**THEN**

$$\Rightarrow a = 3$$

A1

Note: Award **A0A0** for $a = 3$ obtained without working/reasoning/justification.

[2 marks]*continued...*

*Question 10 continued***METHOD 2****EITHER**

correctly lists a relevant part of the sample space

A1for example, $\{S = 4\} = \{(3,1), (1,a), (1,a)\}$ or $\{S = 5\} = \{(2,a), (2,a), (2,a), (2,a)\}$ or $\{S = 6\} = \{(3,a), (3,a), (1,5)\}$

$$a + 3 = 6$$

OReliminates possibilities (exhaustion) for $a < 5$ convincingly shows that $a \neq 2, 4$ **A1** $a \neq 4$, for example, $P(S = 7) = \frac{2}{16}$ from $(2,5), (2,5)$ and so $(3,a), (3,a) \Rightarrow a + 3 \neq 7$ **THEN**

$$\Rightarrow a = 3$$

A1**[2 marks]****Total [16 marks]**

1. (a) **EITHER**

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table)

(A1)

OR

$$5 + 1 + 4 = 3 + x$$

(A1)

OR

$$\sum f = 20$$

(A1)

THEN

$$x = 7$$

A1

[2 marks]

continued...

Question 1 continued(b) **METHOD 1**

1.58429...

1.58

A2**METHOD 2****EITHER**

$$\sigma^2 = \frac{5 \times (2 - 4.3)^2 + 1 \times (3 - 4.3)^2 + 4 \times (4 - 4.3)^2 + 3 \times (5 - 4.3)^2 + 7 \times (6 - 4.3)^2}{20} (= 2.51) \quad \textbf{(A1)}$$

OR

$$\sigma^2 = \frac{5 \times 2^2 + 1 \times 3^2 + 4 \times 4^2 + 3 \times 5^2 + 7 \times 6^2}{20} - 4.3^2 (= 2.51) \quad \textbf{(A1)}$$

THEN

$$\sigma = \sqrt{2.51} = 1.58429...$$

= 1.58

A1**[2 marks]****Total [4 marks]**

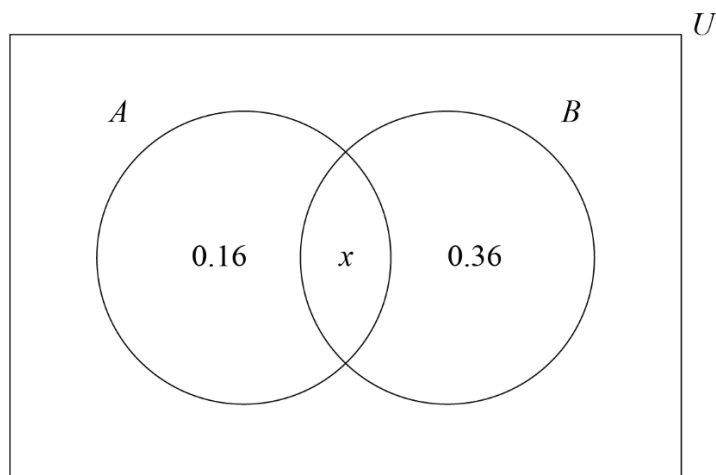
3. (a) $0.41 + k - 0.28 + 0.46 + 0.29 - 2k^2 = 1$ OR $k - 2k^2 + 0.01 = 0.13$ (or equivalent) **A1**
 $2k^2 - k + 0.12 = 0$ **AG**
[1 mark]

- (b) one of 0.2 OR 0.3 **(M1)**
 $k = 0.3$ **A1**
 reasoning to reject $k = 0.2$ eg $P(1) = k - 0.28 \geq 0$ therefore $k \neq 0.2$ **R1**
[3 marks]

- (c) attempting to use the expected value formula **(M1)**
 $E(X) = 0 \times 0.41 + 1 \times (0.3 - 0.28) + 2 \times 0.46 + 3 \times (0.29 - 2 \times 0.3^2)$
 $= 1.27$ **A1**

Note: Award M1A0 if additional values are given.
--

[2 marks]**Total [6 marks]**

5. (a) METHOD 1**EITHER**one of $P(A) = x + 0.16$ OR $P(B) = x + 0.36$ **A1****OR****A1****THEN**attempt to equate their $P(A \cap B)$ with their expression for $P(A) \times P(B)$ **M1**

$$P(A \cap B) = P(A) \times P(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$$

A1

$$x = 0.24$$

A1**METHOD 2**attempt to form at least one equation in $P(A)$ and $P(B)$ using independence**M1**

$$(P(A \cap B') = P(A) \times P(B') \Rightarrow) P(A) \times (1 - P(B)) = 0.16 \text{ OR}$$

$$(P(A' \cap B) = P(A') \times P(B) \Rightarrow) (1 - P(A)) \times P(B) = 0.36$$

$$P(A) = 0.4 \text{ AND } P(B) = 0.6$$

A1

$$P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.6$$

(A1)

$$x = 0.24$$

A1**[4 marks]***continued...*

Question 5 continued(b) **METHOD 1**recognising $P(A' | B') = P(A')$ **(M1)**

$$= 1 - 0.16 - 0.24$$

$$= 0.6$$
 A1

METHOD 2

$$P(B) = 0.36 + 0.24 (= 0.6)$$

$$P(A' | B') = \frac{P(A' \cap B')}{P(B')} \left(= \frac{0.24}{0.4} \right)$$
 (A1)

$$= 0.6$$
 A1

[2 marks]**Total [6 marks]**

11. (a) $P(C < 61)$ **(M1)**
 $= 0.365112...$
 $= 0.365$ **A1**
[2 marks]

(b) recognition of binomial eg $X \sim B(12, 0.365...)$ **(M1)**
 $P(X = 5) = 0.213666...$
 $= 0.214$ **A1**
[2 marks]

continued...

Question 11 continued

- (c) (i) Let CM represent ‘chocolate muffin’ and BM represent ‘banana muffin’

$$P(B < 61) = 0.0197555... \quad (A1)$$

EITHER

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) \text{ (or equivalent in words)} \quad (M1)$$

OR

tree diagram showing two ways to have a muffin weigh < 61 (M1)

THEN

$$(0.6 \times 0.365...) + (0.4 \times 0.0197...) \quad (A1)$$

$$= 0.226969...$$

$$= 0.227 \quad A1$$

- (ii) recognizing conditional probability (M1)

Note: Recognition must be shown in context either in words or symbols, not just $P(A|B)$.

$$\frac{0.6 \times 0.365112...}{0.226969...} \quad (A1)$$

$$= 0.965183...$$

$$= 0.965 \quad A1$$

[7 marks]

continued...

Question 11 continued(d) **METHOD 1**

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) = 0.157 \quad \textbf{(M1)}$$

$$(0.6 \times P(C < 61)) + (0.4 \times 0.0197555...) = 0.157$$

$$P(C < 61) = 0.248496... \quad \textbf{(A1)}$$

attempt to solve for σ using GDC **(M1)**

Note: Award **(M1)** for a graph or table of values to show their $P(C < 61)$ with a variable standard deviation.

$$\sigma = 1.47225...$$

$$\sigma = 1.47 \text{ (g)}$$

A2*continued...*

*Question 11 continued***METHOD 2**

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) = 0.157 \quad (M1)$$

$$(0.6 \times P(C < 61)) + (0.4 \times 0.0197555...) = 0.157$$

$$P(C < 61) = 0.248496... \quad (A1)$$

use of inverse normal to find z score of their $P(C < 61)$ (M1)

$$z = -0.679229...$$

correct substitution (A1)

$$\frac{61 - 62}{\sigma} = -0.679229...$$

$$\sigma = 1.47225...$$

$$\sigma = 1.47 \text{ (g)} \quad A1$$

[5 marks]

Total [16 marks]

3. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.68$

substitution of $P(A) \cdot P(B)$ for $P(A \cap B)$ in $P(A \cup B)$ (M1)

$$P(A) + P(B) - P(A)P(B) (= 0.68)$$

substitution of $3P(B)$ for $P(A)$ (M1)

$$3P(B) + P(B) - 3P(B)P(B) = 0.68 \text{ (or equivalent)} \quad \text{(A1)}$$

Note: The first two marks are independent of each other.

attempts to solve their quadratic equation (M1)

$$P(B) = 0.2, 1.133... \left(\frac{1}{5}, \frac{17}{15} \right)$$

$$P(B) = 0.2 \left(= \frac{1}{5} \right) \quad \text{A2}$$

Note: Award A1 if both answers are given as final answers for $P(B)$.
--

[6 marks]

4. (a) 0.28 (s)

A1

[1 mark]

(b) $\text{IQR} = 0.35 - 0.27 (= 0.08)$ (s)

(A1)

substituting **their** IQR into correct expression for upper fence

(A1)

$0.35 + 1.5 \times 0.08 (= 0.47)$ (s)

$0.46 < 0.47$

R1

so 0.46 (s) is not an outlier

AG

[3 marks]

(c) **EITHER**

the median is closer to the lower quartile (positively skewed)

R1

OR

the distribution is positively skewed

R1

OR

the range of reaction times below the median is smaller than the range of reaction times above the median

R1

Note: These are sample answers from a range of acceptable correct answers.
Award **R1** for any correct statement that explains this.

Do not award **R1** if there is also an incorrect statement, even if another statement in the answer is correct. Accept a correctly and clearly labelled diagram.

[1 mark]

continued...

Question 4 continued(d) **EITHER**

the distribution for ‘not sleeping well’ is centred at a higher reaction time

R1**OR**

the median reaction time after not sleeping well is equal to the upper quartile reaction time after sleeping well

R1**OR**

75% of reaction times are <0.35 seconds after sleeping well, compared with 50% after not sleeping well

R1**OR**

the sample size of 9 is too small to draw any conclusions

R1

Note: These are sample answers from a range of acceptable correct answers. Accept any relevant correct statement **that relates to the median and/or quartiles shown in the box plots**. Do not accept a comparison of means.

Do not award **R1** if there is also an incorrect statement, even if another statement in the answer is correct

Award **R0** to “correlation does not imply causation”.

[1 mark]**Total [6 marks]**

8. Rachel: $R \sim N(56.5, 3^2)$

$$P(R \geq 60) = 0.1216... \quad (A1)$$

Sophia: $S \sim N(57.5, 1.8^2)$

$$P(S \geq 60) = 0.0824... \quad (A1)$$

recognises binomial distribution with $n = 5$ (M1)

let N_R represent the number of Rachel's throws that are longer than 60 metres

$$N_R \sim B(5, 0.1216...)$$

$$\text{either } P(N_R \geq 1) = 0.4772... \text{ or } P(N_R = 0) = 0.5227... \quad (A1)$$

let N_S represent the number of Sophia's throws that are longer than 60 metres

$$N_S \sim B(5, 0.0824...)$$

$$\text{either } P(N_S \geq 1) = 0.3495... \text{ or } P(N_S = 0) = 0.6504... \quad (A1)$$

EITHER

$$\text{uses } P(N_R \geq 1)P(N_S = 0) + P(N_S \geq 1)P(N_R = 0) \quad (M1)$$

$$P(\text{one of Rachel or Sophia qualify}) = (0.4772... \times 0.6504...) + (0.3495... \times 0.5227...)$$

OR

$$\text{uses } P(N_R \geq 1) + P(N_S \geq 1) - 2 \times P(N_R \geq 1) \times P(N_S \geq 1) \quad (M1)$$

$$P(\text{one of Rachel or Sophia qualify}) = 0.4772... + 0.3495... - 2 \times 0.4772... \times 0.3495...$$

THEN

$$= 0.4931...$$

$$= 0.493$$

A1

[7 marks]

Note: *M* marks are not dependent on the previous *A* marks.

6. (a) $P(A \cap B) = 0.24$

A1

[1 mark]

(b) $P(A \cup B) = 1.1 - P(A \cap B)$

(A1)

$(0 \leq) P(A \cup B) \leq 1$

(M1)

Note: This may be conveyed in a clearly labelled diagram or written explanation where $P(A \cup B) = 1$

the minimum value of $P(A \cap B)$ is 0.1

A1

[3 marks]

(c) A is a subset of B (so $P(A \cap B) = P(A)$).

R1

Note: This may be conveyed in a clearly labelled diagram where A is completely inside B, or in a written explanation indicating that $P(A \cap B) = P(A)$

so the maximum value of $P(A \cap B)$ is 0.3

A1

Note: Do not award **R0A1**.

[2 marks]

Total [6 marks]

1. (a) 1.01206..., 2.45230...

$$a = 1.01, b = 2.45 \text{ (} 1.01x + 2.45 \text{)}$$

A1A1

[2 marks]

- (b) 0.981464...

$$r = 0.981$$

A1

Note: A common error is to enter the data incorrectly into the GDC, and obtain the answers $a = 1.01700...$, $b = 2.09814...$ and $r = 0.980888...$. Some candidates may write the 3 sf answers, ie. $a = 1.02$, $b = 2.10$ and $r = 0.981$ or 2 sf answers, ie. $a = 1.0$, $b = 2.1$ and $r = 0.98$. In these cases award **A0A0** for part (a) and **A0** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

[1 mark]

- (c) correct substitution of 78 into **their** regression equation

(M1)

81.3930... , 81.23 from 3 sf answer

81

A1

[2 marks]

Total [5 marks]

6. (a) $\int_0^b axe^x dx = 1$ (seen anywhere) **M1**

attempt to use integration by parts (either way around) **(M1)**

$$\left[axe^x \right]_0^b - \int_0^b ae^x dx (=1) \quad \text{A1}$$

$$\left[axe^x \right]_0^b - \left[ae^x \right]_0^b (=1) \quad \text{A1}$$

Note: Condone incorrect or absent limits up to this point.

$$abe^b - ae^b + a = 1$$

$$a = \frac{1}{be^b - e^b + 1} \quad \text{A1}$$

[5 marks]

(b) $\int_0^m xe^x dx = \frac{1}{2}$ **(M1)**

$$\left[xe^x \right]_0^m - \left[e^x \right]_0^m = \frac{1}{2}$$

$$me^m - e^m + 1 = \frac{1}{2} \quad \text{A1}$$

$$m = 0.768039\dots$$

$$m = 0.768 \quad \text{A1}$$

[3 marks]

Total [8 marks]

10.

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for $P(T < 55 | T > 40)$ accept $P(T \leq 55 | T > 40)$, $P(T \leq 55 | T \geq 40)$, etc.

(a) recognising to find $P(T > 40)$ **(M1)**

$$P(T > 40) = 0.574136...$$

$$P(T > 40) = 0.574$$

A1**[2 marks]**(b) attempt to multiply four independent probabilities using their $P(T > 40)$ and

$$P(T < 40)$$

(M1)

$$(1-p)^3 \cdot p \text{ OR } (1-0.574136...)^3 \cdot 0.574136... \text{ OR } (0.425863...)^3 \cdot 0.574136...$$

(A1)

$$0.0443430...$$

$$0.0443, 0.0444 \text{ from 3 sf values}$$

A1**[3 marks]***continued...*

Question 10 continued

- (c) (i) recognizing conditional probability

(M1)

$$P(T < 55 | T > 40)$$

Note: Award (M1) for an expression or description in context. Accept $P(T > 40 | T < 55)$ but do not accept just $P(A | B)$.

$$\frac{P(40 < T < 55)}{P(T > 40)}$$

(A1)

$$\frac{0.461944...}{0.574136...}$$

(A1)

$$P(T < 55 | T > 40) = 0.804590...$$

$$= 0.805$$

A1

- (ii) recognizing binomial probability

(M1)

$$X \sim B(n, p)$$

$$n = 10 \text{ and } p = 0.804589...$$

(A1)

$$0.0242111..., 0.0240188... \text{ using } p = 0.805$$

$$P(X = 5) = 0.0242$$

A1

[7 marks]

continued...

Question 10 continued(d) Let $P(T < a) = x$

recognition that probabilities sum to 1 (seen anywhere)

(M1)**EITHER**

expressing the three regions in one variable

(M1)

$$x + 0.904 + 2x \text{ OR } P(T < a) + 0.904 + 2P(T < a) \text{ OR } \frac{1}{2}P(T > b) + 0.904 + P(T > b)$$

OR x and $2x$ correctly indicated on labelled bell diagram

$$P(T < a) + 0.904 + 2P(T < a) = 1 \text{ OR } \frac{1}{2}P(T > b) + 0.904 + P(T > b) = 1 \text{ (or$$

equivalent)

(A1)**OR**expressing either $P(T < a)$ or $P(T > b)$ only in terms of $P(a \leq T \leq b)$ **(M1)**

$$(P(T < a) =) \frac{1}{3}(1 - P(a \leq T \leq b)) \text{ OR } (P(T > b) =) \frac{2}{3}(1 - P(a \leq T \leq b))$$

$$x = \frac{1}{3}(1 - 0.904)(= 0.032) \text{ OR } P(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$$

(A1)**THEN**

$$P(T < a) = 0.032$$

$$a = 22.18167...$$

$$a = 22.2 \text{ (accept 22.1)}$$

A1**[4 marks]****Total [16 marks]**

2. (a) (i) summing frequencies of riders or finding complement (M1)

$$\text{probability} = \frac{34}{40} \quad \text{A1}$$

- (ii) attempt to find expected value (M1)

$$\frac{16}{40} + \left(2 \times \frac{13}{40}\right) + \left(3 \times \frac{2}{40}\right) + \left(4 \times \frac{3}{40}\right)$$

$$\frac{60}{40} (=1.5) \quad \text{A1}$$

[4 marks]

- (b) evidence of **their** rides/visitor $\times 1000$ or $\div 10$ (M1)

1500 OR 0.15

150 (times) A1

[2 marks]

Total [6 marks]

3. substitutes into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to form

$$0.55 = 0.4 + P(B) - P(A \cap B) \quad (\text{or equivalent}) \quad \textbf{(A1)}$$

$$\text{substitutes into } P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ to form } 0.25 = \frac{P(A \cap B)}{P(B)} \quad (\text{or equivalent}) \quad \textbf{(A1)}$$

attempts to combine their two probability equations to form an equation in $P(B)$ **(M1)**

Note: The above two **A** marks are awarded independently.

correct equation in $P(B)$ **A1**

$$0.55 = 0.4 + P(B) - 0.25P(B) \quad \text{OR} \quad \frac{P(B) - 0.15}{P(B)} = 0.25 \quad \text{OR} \quad P(B) - 0.15 = 0.25P(B)$$

(or equivalent)

$$P(B) = \frac{15}{75} \left(= \frac{1}{5} = 0.2 \right) \quad \textbf{A1}$$

Total [5 marks]

6. (a) $E(X) = 2a$ (by symmetry)

A1**[1 mark]**(b) **METHOD 1**

uses $\text{Var}(X) = E(X^2) - [E(X)]^2$

(M1)

$$\text{Var}(X) = \int_a^{3a} \frac{x^2}{2a} dx - (2a)^2$$

$$= \left[\frac{x^3}{6a} \right]_a^{3a} - (2a)^2$$

A1

$$= \frac{13a^2}{3} - (2a)^2$$

(A1)

$$= \frac{a^2}{3}$$

A1

Note: Award as above if $E(X^2)$ and $[E(X)]^2$ are calculated separately

leading to $\text{Var}(X) = \frac{a^2}{3}$. Award **(M1)A0A0A0** for $\text{Var}(X) = \frac{13a^2}{3}$.

continued...

*Question 6 continued***METHOD 2**

uses $\text{Var}(X) = E(X - E(X))^2$ **(M1)**

$$\text{Var}(X) = \int_a^{3a} \frac{(x-2a)^2}{2a} dx$$

$$= \left[\frac{(x-2a)^3}{6a} \right]_a^{3a}$$
A1

$$= \frac{a^3 - (-a^3)}{6a} \text{ or equivalent}$$
(A1)

$$= \frac{a^2}{3}$$
A1

[4 marks]**Total [5 marks]**

3. (a) recognising to find $y(25)$ (M1)

$$y(25) = -0.6 \times 25^2 + 23 \times 25 + 110$$

$$= 310 \text{ (children)}$$

(A1)

[2 marks]

- (b) recognizing x on y is required (M1)

$$0.0935114... \text{ and } 7.43053...$$

(A1)

$$x = 0.0935y + 7.43$$

(A1)

[3 marks]

- (c) attempt to substitute their answer to part (a) into their regression equation for either x or y (M1)

$$x = 0.0935114... \times 310 + 7.43053... (= 36.4190...)$$

$$36 \text{ (accept 37 or 36.4)}$$

(A1)

Note: Award **(M1)A1FT** for $x = 37$ found from using $y = 9.39x - 41.5$.

Award **(M1)A0FT** for a correct **FT** answer that lies outside $[15, 46]$.

[2 marks]**Total [7 marks]**

4. METHOD 1

$$Q_1 = 31.86 \text{ OR } Q_3 = 32.14 \quad (\text{A1})$$

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) (M1)

EITHER

equating an appropriate correct normal CDF function to its correct probability (0.25 or 0.5 or 0.75) (A2)

OR

$$z = -0.674489... \text{ OR } z = 0.674489... \text{ (seen anywhere)} \quad (\text{A1})$$

$$-0.674489... = \frac{31.86 - 32}{\sigma} \text{ OR } 0.674489... = \frac{32.14 - 32}{\sigma} \quad (\text{A1})$$

THEN

$$0.207564...$$

$$\sigma = 0.208 \text{ (mm)} \quad \text{A1}$$

METHOD 2

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) (M1)

$$z = -0.674489... \text{ OR } z = 0.674489... \quad (\text{A1})$$

$$(Q_1 =) 32 - 0.674489... \sigma \text{ OR } (Q_3 =) 32 + 0.674489... \sigma \quad (\text{A1})$$

$$(Q_3 - Q_1 =) 2 \times 0.674489... \sigma$$

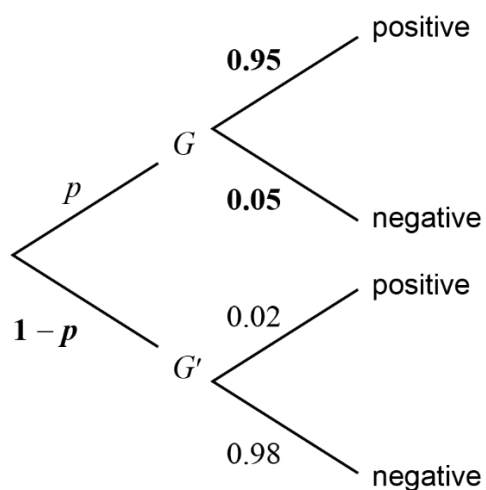
$$2 \times 0.674489... \sigma = 0.28 \quad (\text{A1})$$

$$0.207564...$$

$$\sigma = 0.208 \text{ (mm)} \quad \text{A1}$$

Total [5 marks]

7. (a)

**A1A1**

Note: award **A1** for branch correctly labelled $1 - p$
 award **A1** for branches correctly labelled 0.95 and 0.05
 award **A0** for G' branch labelled p'
 award **A0** for G' branch labelled q unless explicitly defined as $1 - p$

[2 marks]*continued...*

*Question 7 continued***(b) METHOD 1**

recognizing conditional probability

(M1)

$$P(G'|pos) \text{ OR } P(G|pos)$$

$$\frac{0.02(1-p)}{0.95p + 0.02(1-p)} \left(= \frac{18}{150} \right) \text{ OR } \frac{0.95p}{0.95p + 0.02(1-p)} \left(= \frac{132}{150} \right)$$

(A1)(A1)

Note: Award **A1** for a correct numerator and **A1** for a correct denominator.

$$p = 0.133738$$

$$p = 0.134$$

A1**METHOD 2**attempt to set up a system of equations (S = sample size)**(M1)**

$$p(0.95S) = 132 \text{ and } (1-p)(0.02S) = 18$$

(A1)attempt to solve for p or S **(M1)**

$$\frac{0.95p}{0.02(1-p)} = \frac{132}{18}$$

$$\text{OR } S = pS + (1-p)S = \frac{132}{0.95} + \frac{18}{0.02} = 138.947... + 900 = 1038.94...$$

$$p = 0.133738...$$

$$p = 0.134$$

A1

METHOD 3

attempt to find the number of parrots with the gene and the number without **(M1)**

number of parrots with the gene $\approx \frac{132}{0.95} = 138.947\dots$ AND

number of parrots without the gene $\approx \frac{18}{0.02} = 900$ **(A1)**

number of parrots in the sample $\approx 138.947\dots + 900 = 1038.94\dots$

attempt to find proportion of sample with the gene **(M1)**

$$p \approx \frac{138.947\dots}{1038.94\dots} = 0.133738\dots$$

$$p = 0.134 \quad \textbf{A1}$$

[4 marks]

Total [6 marks]

(3)

11. (a) $E(X) = \int_0^2 \frac{6x}{\pi\sqrt{16-x^2}} dx$ (A1)

Note: Condone the absence of dx .

Accept $\int_0^2 xf(x) dx$

attempt to integrate $\frac{6x}{\pi\sqrt{16-x^2}}$ using inspection/substitution (M1)

$$-\frac{6}{2\pi} \int -2x(16-x^2)^{-\frac{1}{2}} dx \text{ or let } u = 16-x^2$$

$$-\frac{6}{2\pi} \left[2(16-x^2)^{\frac{1}{2}} \right]_0^2 \text{ OR } \frac{6}{\pi} \left[-u^{\frac{1}{2}} \right]_{16}^{12} \quad \text{A1}$$

Note: For this **A1** condone absent or incorrect limits.

attempt to substitute their limits and evaluate (M1)

$$\frac{24}{\pi} - \frac{6}{\pi} \sqrt{12} \left(= \frac{12}{\pi} (2 - \sqrt{3}) \right) \quad \text{A1}$$

Note: The substitution $\sin \theta = \frac{x}{4}$ may also be used, leading to

$$\frac{24}{\pi} \int_0^{\frac{\pi}{6}} \sin \theta d\theta = \frac{24}{\pi} [-\cos \theta]_0^{\frac{\pi}{6}} = \frac{24}{\pi} \left(1 - \cos \frac{\pi}{6} \right). \text{ Award marks as}$$

appropriate and accept $\frac{24}{\pi} \left(1 - \cos \frac{\pi}{6} \right)$ for the final **A1**.

[5 marks]

(b) $\int_0^{0.5} f(x) dx \left(= \int_0^{0.5} \frac{6}{\pi\sqrt{16-x^2}} dx \right)$ (M1)

$$P(X < 0.5) = 0.239358...$$

$$= 0.239$$

A1
[2 marks]

continued...

Question 11 continued(c) **EITHER**recognition $P(\text{at least one success after } n \text{ trials}) = 1 - P(\text{no successes after } n \text{ trials})$ (M1)

$$1 - (1 - 0.239\dots)^n \geq 0.99 \quad (\text{A1})$$

$$n = 16.8321\dots$$

Note: Use of 0.239 results in $n = 16.8612\dots$

ORrecognition that $Y \sim B(n, 0.239\dots)$ (M1)If $n = 16$ $P(\text{at least one success after } n \text{ trials}) = 0.987443\dots$ and if $n = 17$ $P(\text{at least one success after } n \text{ trials}) = 0.990448\dots$ (A1)

Note: Use of 0.239 results in the values 0.987348... and 0.990371...

THEN

17 trials

A1**[3 marks]**(d) recognition that $Y \sim B(10, \text{their part b})$ (M1)

$$B(10, 0.239\dots)$$

$$P(X = 3) = 0.242430\dots$$

$$= 0.242$$

A1**[2 marks]***continued...*

Question 11 continued

(e) 8

A1**[1 mark]**(f) (i) $n - 2$ **A1**(ii) ${}^nC_3$ (ways of 3 successes in n trials)**(A1)**

$$\frac{n-2}{{}^nC_3}$$

(A1)

Attempt to solve their $\frac{n-2}{{}^nC_3} > 0.05$ OR $\frac{6}{n(n-1)} > 0.05$ (or equivalent)

(M1)

Note: Accept an equation.

$$n = 11.4658... \text{ OR}$$

table values $n = 11, \frac{n-2}{{}^nC_3} = 0.0545454... \text{ and } n = 12, \frac{n-2}{{}^nC_3} = 0.0454545...$

(A1)

$$n = 11$$

A1**[6 marks]****Total [19 marks]**

1. (a) $a = 1.93258\dots$, $b = 7.21662\dots$
 $a = 1.93$, $b = 7.22$

A1A1

[2 marks]

- (b) attempt to substitute $d = 20$ into their equation
height = 45.8683...
height = 45.9 (cm)

(M1)

A1

[2 marks]

Total [4 marks]

3. (a) evidence of attempting to find correct area under normal curve (M1)
- $P(W > 210)$ OR sketch
- $P(W > 210) = 0.115069\dots$
- $P(W > 210) = 0.115$ A1

[2 marks]

- (b) recognizing $P(W < w) = 1 - P(w < W < 210) - P(W > 210)$ (M1)
- $P(W < w) = 1 - 0.8 - 0.115069\dots$
- $P(W < w) = 0.084930\dots$
- $P(W < w) = 0.0849$ A1

[2 marks]

- (c) evidence of attempting to use inverse normal function (M1)
- $w = 197.136\dots$
- $w = 197$ (grams) A1

[2 marks]

- (d) recognition of binomial distribution (M1)
- $X \sim B(10, 0.0849302\dots)$
- $P(X = 1) = 0.382076\dots$
- $P(X = 1) = 0.382$ A1

[2 marks]

Total [8 marks]

11. (a) attempts to form a numerator involving a product of two terms involving y
and a denominator involving a product of two terms involving $r + y$

(M1)

$$\frac{y(y-1)}{(r+y)(r+y-1)} = \frac{1}{3}$$

A1

attempts to remove the fractions and expand the brackets

(M1)

$$3y^2 - 3y = y^2 + 2ry - y + r^2 - r$$

A1

$$2y^2 - 2ry - 2y + r - r^2 = 0$$

$$2y^2 - 2(r+1)y + r - r^2 = 0$$

AG**[4 marks]**

- (b) attempts to solve for y

(M1)

$$y = \frac{2(r+1) \pm \sqrt{4(r+1)^2 - 8(r-r^2)}}{4}$$

A1

$$y = \frac{2(r+1) \pm \sqrt{12r^2 + 4}}{4}$$

A1

$$y = \frac{(r+1) \pm \sqrt{3r^2 + 1}}{2}$$

(since $r, y \in \mathbb{Z}^+$) and $\frac{(r+1) - \sqrt{3r^2 + 1}}{2} < 0$ for $r > 1$

R1

Note: Award the **R1** for stating that number of balls cannot be negative, or similar.

Note: Accept $y > 0$

$$\text{so } y = \frac{(r+1) + \sqrt{3r^2 + 1}}{2}$$

AG**[4 marks]***continued...*

Question 11 continued

- (c) attempts to find a pair of positive integer values
- eg*
- by using a table

(M1)

Note: Award **M0** if numbers are not positive integers.1 red ball and 2 yellow balls ($r = 1$ and $y = 2$)**A1**4 red balls and 6 yellow balls ($r = 4$ and $y = 6$)**A2****Note:** Award **A1** for one solution and **A2** for another.15 red balls and 21 yellow balls ($r = 15$ and $y = 21$) is the next solution.**[4 marks]**

- (d) attempts to form a numerator involving a product of three terms involving
- y
-
- and a denominator involving a product of three terms that includes a
- $(y+10)$
-
- term

(M1)

$$P(YYY) = \frac{y(y-1)(y-2)}{(y+10)(y-1+10)(y-2+10)} \left(= \frac{y(y-1)(y-2)}{(y+10)(y+9)(y+8)} \right)$$

A1A1**Note:** Award **A1** for a correct numerator and **A1** for a correct denominator.**[3 marks]**

$$(e) \quad P(\text{new } YYY) = \frac{(y+1)(y)(y-1)}{(y+1+10)(y+10)(y-1+10)} \left(= \frac{(y+1)(y)(y-1)}{(y+11)(y+10)(y+9)} \right)$$

(A1)equates their answer for $P(\text{new } YYY)$ to $2 \times$ their answer for part (d)**M1**

$$\frac{2y(y-1)(y-2)}{(y+10)(y+9)(y+8)} = \frac{(y+1)(y)(y-1)}{(y+11)(y+10)(y+9)}$$

attempts to solve for y

(M1)

Note: Award **(M1)** for attempting to write the above expression as

$$\frac{2(y-2)}{y+8} = \frac{y+1}{y+11}.$$

 $y = 4$ **A2****[5 marks]****Total [20 marks]**

2. (a) $(P(A \cup B) =) 0.65 + 0.75 - 0.6$ OR $0.05 + 0.6 + 0.15$ (A1)
 $= 0.8$ A1

[2 marks]

(b) recognition that $A' \cap B' = (A \cup B)'$ OR $A' \cap B' = 1 - A \cup B$ (M1)
 (region/value may be seen in a correctly shaded/labeled Venn diagram)
 $(= 1 - 0.8)$
 $= 0.2$ A1

Note: For the final mark, 0.2 must be stated as the candidate's answer, or labeled as $P(A' \cap B')$ in their Venn diagram. Just seeing an unlabeled 0.2 in the correct region of their diagram earns **M1A0**.

[2 marks]

Total [4 marks]

2. (a) $(P(A \cup B) =) 0.65 + 0.75 - 0.6$ OR $0.05 + 0.6 + 0.15$ (A1)
 $= 0.8$ A1

[2 marks]

(b) recognition that $A' \cap B' = (A \cup B)'$ OR $A' \cap B' = 1 - A \cup B$ (M1)
 (region/value may be seen in a correctly shaded/labeled Venn diagram)
 $(= 1 - 0.8)$
 $= 0.2$ A1

Note: For the final mark, 0.2 must be stated as the candidate's answer, or labeled as $P(A' \cap B')$ in their Venn diagram. Just seeing an unlabeled 0.2 in the correct region of their diagram earns **M1A0**.

[2 marks]

Total [4 marks]

$$5. \quad E(X) = k + 2k^2 + 3a + 4k^3 = 2.3 \quad (\text{A1})$$

$$k + k^2 + a + k^3 = 1 \quad (\text{A1})$$

Note: The first two **A** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations (M1)

$$a = 1 - k - k^2 - k^3 \text{ and } a = \frac{1}{3}(2.3 - k - 2k^2 - 4k^3)$$

use of graph or table to attempt to find intersection (M1)

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k (M1)

$$k^3 - k^2 - 2k + 0.7 = 0 \text{ OR } 3(1 - k - k^2 - k^3) = 2.3 - k - 2k^2 - 4k^3 \text{ (or equivalent)}$$

attempt to solve their cubic in k (M1)

THEN

$$a = 0.552839... \text{ OR } k = 0.315870... \text{ (other solutions to cubic are } k = -1.18538..., 1.86951... \text{)}$$

$$a = 0.553 \quad \text{A1}$$

Note: If no working shown, award **(A1)(A1)(M1)(M1)A0** for $a = 2.44587...$ OR $a = -10.8987...$ and award **(A0)(A0)(M1)(M1)A0** for $a = 0.55$ (2sf) .

Total [5 marks]

6. (a) $5.75 = 25p(1-p)$ (A1)

$$p = 0.641421..., 0.358578...$$

$$p = 0.641, 0.359 \left(= \frac{5 \pm \sqrt{2}}{10} \right) \quad \text{A1A1}$$

[3 marks]

(b) $\text{Var}(Y) = (-2)^2 \text{Var}(X) (= 4\text{Var}(X))$ (A1)

$$= 23 \quad \text{A1}$$

[2 marks]

Total [5 marks]

10. (a) recognizing probabilities sum to 1 (M1)

$$0.288 + P(94.6 < X < 98.1) + 0.434 = 1$$

$$P(94.6 < X < 98.1) = 0.278 \quad \text{A1}$$

Note: If no working shown, award **(M1)A0** for $P(94.6 < X < 98.1) = 0.28$ (2sf).

[2 marks]

- (b) **METHOD 1**

recognizing the need to use inverse normal with 0.288, $(1 - 0.434)$ or 0.434 (M1)

Note: Accept use of calculator notation eg $\text{invNorm}(0.288)$ (= -0.559236...).

$$\mu + \text{invNorm}(0.288)\sigma = 94.6, \mu + \text{invNorm}(1 - 0.434)\sigma = 98.1 \text{ (or equivalent)} \quad \text{(A1)(A1)}$$

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) (M1)

$$\mu = 97.2981..., \sigma = 4.82468...$$

$$\mu = 97.3, \sigma = 4.82 \quad \text{A1}$$

Note: Condone use of different variables throughout, but do not award the final **A1** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $P(Z < z) = 0.288$ or

$$P(Z < z) = 1 - 0.434 \quad \text{(M1)}$$

$$z_1 = -0.559236... \text{ OR } z_2 = 0.166199...$$

$$\frac{94.6 - \mu}{\sigma} = -0.559236..., \frac{98.1 - \mu}{\sigma} = 0.166199... \text{ (or equivalent)} \quad \text{(A1)(A1)}$$

attempt to solve their equations (that involve z-values rather than probabilities) (M1)

$$\mu = 97.2981..., \sigma = 4.82468...$$

$$\mu = 97.3, \sigma = 4.82 \quad \text{A1}$$

Note: Award marks as appropriate for work seen in part (a).

Note: If no working shown, award **(M1)(A0)(A0)(M1)A0** for $\mu = 97, \sigma = 4.8$ (2sf).

[5 marks]

Question 10 continued

- (c) (i) recognition of Binomial distribution (M1)

$$X \sim B(100, 0.434)$$

$$P(X = 34) = 0.0133198\dots$$

$$= 0.0133$$

A1

Note: If no working shown, award **(M1)A0** for $P(X = 34) = 0.013$ (2sf).

- (ii) $P(X < 49) = 0.848218\dots$ (seen anywhere) (A1)

recognition of conditional probability (M1)

Note: recognition must be shown in context, either in symbols eg $P(X = 34 | X < 49)$, or in words eg $P(34 \text{ plants} | \text{less than } 49 \text{ plants})$, not only as $P(A | B)$.

$$(P(X = 34 | X < 49) =) \frac{P(X = 34)}{P(X < 49)} \text{ OR } \frac{P(X = 34)}{P(X \leq 48)} \left(= \frac{0.0133198\dots}{0.848218\dots} \right) \quad \text{(A1)}$$

$$= 0.0157033\dots$$

$$P(X = 34 | X < 49) = 0.0157 \quad \text{A1}$$

Note: Exception to **FT**: If the candidate finds $P(X \leq 49) (= 0.890474\dots)$ and uses that to calculate $P(X = 34 | X \leq 49) = 0.0149581\dots$ award **(A0)(M1)(A1)A0**.

Note: If no working shown, award **(A0)(M1)(A0)A0** for $P(X = 34 | X < 49) = 0.016$ (2sf).

[6 marks]*continued...*

Question 10 continued

- (d) $Q_1 = 96.19$ OR $Q_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) **(A1)**

$P(96.19 < F < 101.01) = 0.5$ OR $P(F < 96.19) = 0.25$ OR $P(F < 101.01) = 0.75$
(or equivalent)

EITHER

attempt to find d using graph or table **(M1)**

OR

$$1 - 2P\left(Z < -\frac{2.41}{d}\right) = 0.5 \text{ OR } P(Z < -\frac{2.41}{d}) = 0.25 \text{ OR } P(Z < \frac{2.41}{d}) = 0.75$$

$$\text{OR } P(-\frac{2.41}{d} < Z < \frac{2.41}{d}) = 0.5 \text{ (or equivalent)} \quad \textbf{(M1)}$$

$$-\frac{2.41}{d} = -0.674489... \text{ OR } \frac{2.41}{d} = 0.674489...$$

THEN

$$3.57307...$$

$$d = 3.57 \quad \textbf{A1}$$

Note: Accept 3.56 using 96.2 or 101.

Note: If no working shown, award **(A0)(M1)A0** for $d = 3.6$ (2sf).

[3 marks]

Total [16 marks]

$$5. \quad E(X) = k + 2k^2 + 3a + 4k^3 = 2.3 \quad (\text{A1})$$

$$k + k^2 + a + k^3 = 1 \quad (\text{A1})$$

Note: The first two **A** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations (M1)

$$a = 1 - k - k^2 - k^3 \text{ and } a = \frac{1}{3}(2.3 - k - 2k^2 - 4k^3)$$

use of graph or table to attempt to find intersection (M1)

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k (M1)

$$k^3 - k^2 - 2k + 0.7 = 0 \text{ OR } 3(1 - k - k^2 - k^3) = 2.3 - k - 2k^2 - 4k^3 \text{ (or equivalent)}$$

attempt to solve their cubic in k (M1)

THEN

$$a = 0.552839... \text{ OR } k = 0.315870... \text{ (other solutions to cubic are } k = -1.18538..., 1.86951... \text{)}$$

$$a = 0.553 \quad \text{A1}$$

Note: If no working shown, award **(A1)(A1)(M1)(M1)A0** for $a = 2.44587...$ OR $a = -10.8987...$ and award **(A0)(A0)(M1)(M1)A0** for $a = 0.55$ (2sf) .

Total [5 marks]

6. (a) $5.75 = 25p(1-p)$ (A1)

$$p = 0.641421..., 0.358578...$$

$$p = 0.641, 0.359 \left(= \frac{5 \pm \sqrt{2}}{10} \right) \quad \text{A1A1}$$

[3 marks]

(b) $\text{Var}(Y) = (-2)^2 \text{Var}(X) (= 4\text{Var}(X))$ (A1)

$$= 23 \quad \text{A1}$$

[2 marks]

Total [5 marks]

10. (a) recognizing probabilities sum to 1 (M1)

$$0.288 + P(94.6 < X < 98.1) + 0.434 = 1$$

$$P(94.6 < X < 98.1) = 0.278 \quad \text{A1}$$

Note: If no working shown, award **(M1)A0** for $P(94.6 < X < 98.1) = 0.28$ (2sf).

[2 marks]

- (b) **METHOD 1**

recognizing the need to use inverse normal with 0.288, $(1 - 0.434)$ or 0.434 (M1)

Note: Accept use of calculator notation eg $\text{invNorm}(0.288)$ (= -0.559236...).

$$\mu + \text{invNorm}(0.288)\sigma = 94.6, \mu + \text{invNorm}(1 - 0.434)\sigma = 98.1 \text{ (or equivalent)} \quad \text{(A1)(A1)}$$

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) (M1)

$$\mu = 97.2981..., \sigma = 4.82468...$$

$$\mu = 97.3, \sigma = 4.82 \quad \text{A1}$$

Note: Condone use of different variables throughout, but do not award the final **A1** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

use of inverse normal to find at least one z-score for $P(Z < z) = 0.288$ or

$$P(Z < z) = 1 - 0.434 \quad \text{(M1)}$$

$$z_1 = -0.559236... \text{ OR } z_2 = 0.166199...$$

$$\frac{94.6 - \mu}{\sigma} = -0.559236..., \frac{98.1 - \mu}{\sigma} = 0.166199... \text{ (or equivalent)} \quad \text{(A1)(A1)}$$

attempt to solve their equations (that involve z-values rather than probabilities) (M1)

$$\mu = 97.2981..., \sigma = 4.82468...$$

$$\mu = 97.3, \sigma = 4.82 \quad \text{A1}$$

Note: Award marks as appropriate for work seen in part (a).

Note: If no working shown, award **(M1)(A0)(A0)(M1)A0** for $\mu = 97, \sigma = 4.8$ (2sf).

[5 marks]

Question 10 continued

- (c) (i) recognition of Binomial distribution (M1)

$$X \sim B(100, 0.434)$$

$$P(X = 34) = 0.0133198\dots$$

$$= 0.0133$$

A1

Note: If no working shown, award (M1)A0 for $P(X = 34) = 0.013$ (2sf).

- (ii)
- $P(X < 49) = 0.848218\dots$
- (seen anywhere) (A1)

recognition of conditional probability (M1)

Note: recognition must be shown in context, either in symbols eg $P(X = 34 | X < 49)$, or in words eg $P(34 \text{ plants} | \text{less than } 49 \text{ plants})$, not only as $P(A | B)$.

$$(P(X = 34 | X < 49) =) \frac{P(X = 34)}{P(X < 49)} \text{ OR } \frac{P(X = 34)}{P(X \leq 48)} \left(= \frac{0.0133198\dots}{0.848218\dots} \right) \quad (A1)$$

$$= 0.0157033\dots$$

$$P(X = 34 | X < 49) = 0.0157 \quad A1$$

Note: Exception to **FT**: If the candidate finds $P(X \leq 49) (= 0.890474\dots)$ and uses that to calculate $P(X = 34 | X \leq 49) = 0.0149581\dots$ award (A0)(M1)(A1)A0.

Note: If no working shown, award (A0)(M1)(A0)A0 for $P(X = 34 | X < 49) = 0.016$ (2sf).

[6 marks]

continued...

Question 10 continued

- (d) $Q_1 = 96.19$ OR $Q_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) **(A1)**

$P(96.19 < F < 101.01) = 0.5$ OR $P(F < 96.19) = 0.25$ OR $P(F < 101.01) = 0.75$
(or equivalent)

EITHER

attempt to find d using graph or table **(M1)**

OR

$$1 - 2P\left(Z < -\frac{2.41}{d}\right) = 0.5 \text{ OR } P(Z < -\frac{2.41}{d}) = 0.25 \text{ OR } P(Z < \frac{2.41}{d}) = 0.75$$

OR $P(-\frac{2.41}{d} < Z < \frac{2.41}{d}) = 0.5$ (or equivalent) **(M1)**

$$-\frac{2.41}{d} = -0.674489... \text{ OR } \frac{2.41}{d} = 0.674489...$$

THEN

$$3.57307...$$

$$d = 3.57 \quad \text{A1}$$

Note: Accept 3.56 using 96.2 or 101.

Note: If no working shown, award **(A0)(M1)A0** for $d = 3.6$ (2sf).

[3 marks]

Total [16 marks]

1. (a) attempt to form equation for the sum of frequencies=16 or mean=3 (M1)

$$p + q + 4 + 2 + 3 = 16 (\Rightarrow p + q = 7) \quad \text{A1}$$

$$\frac{p + 2q + 12 + 8 + 18}{16} = 3 (\Rightarrow p + 2q = 10) \quad \text{OR} \quad \frac{p + 2q + 12 + 8 + 18}{9 + p + q} = 3 (\Rightarrow 2p + q = 11) \quad \text{A1}$$

attempt to eliminate one variable from their equations (M1)

$$p + 2(7 - p) + 38 = 48 \quad \text{OR} \quad 2(7 - q) + q = 11$$

$$p = 4 \quad \text{and} \quad q = 3 \quad \text{A1}$$

Note: Award **M1A0A0M0A1** for $p = 4, q = 3$ with no working.

[5 marks]

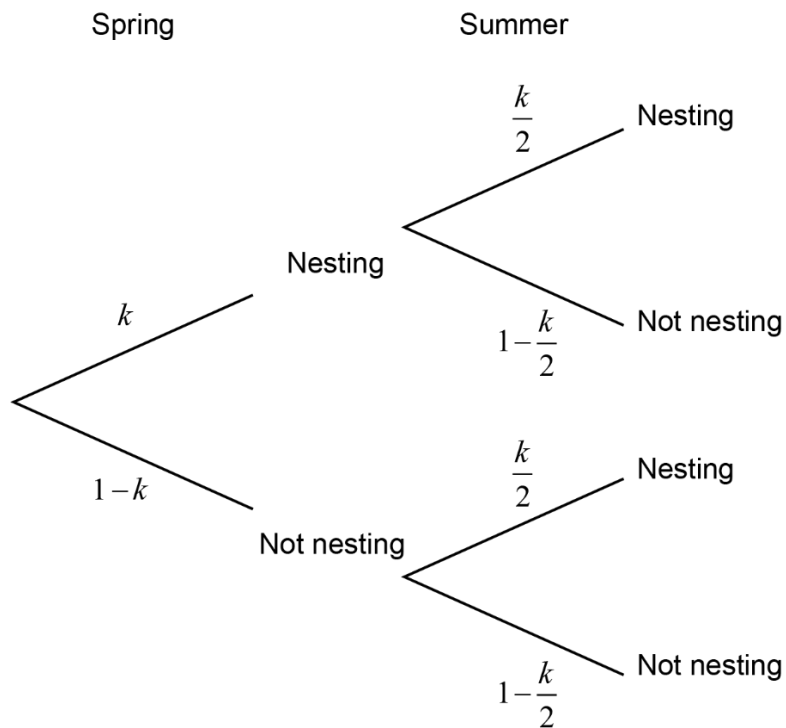
- (b) mean final score = 30 A1

[1 mark]

Total [6 marks]

1

4. (a)



$1-k$ for Spring

A1

$1-\frac{k}{2}$ for both Summers

A1

[2 marks]

continued...

Question 4 continued.

- (b) (i) multiplying the two correct branches **(A1)**

$$(1-k)\left(1-\frac{k}{2}\right)$$

attempt to expand and equate to $\frac{5}{9}$ **(M1)**

$$1-k-\frac{k}{2}+\frac{k^2}{2}=\frac{5}{9}$$

$$18-18k-9k+9k^2=10 \quad \text{OR} \quad \frac{k^2}{2}-\frac{3k}{2}+\frac{4}{9}=0 \quad \text{OR} \quad \frac{9k^2}{2}-\frac{27k}{2}+4=0 \quad \text{A1}$$

$$9k^2-27k+8=0 \quad \text{AG}$$

- (ii) $\left(k=\frac{1}{3} \text{ is the only valid solution as } \frac{8}{3}>1\right)$ **R1**

Note: Accept any valid reasoning indicating that any probability cannot be greater than 1 and/or probability cannot be less than 0.

[4 marks]

Total [6 marks]

6. (a) 2.5(%)

A1**[1 mark]**(b) $P(\text{weight of cooking apples} > 140) = 0.5 (50\%)$ (seen anywhere)**(A1)**

recognition of conditional probability in context

(M1)

$$P(\text{eating apple} | \text{weight} > 140) = \frac{P(\text{eating apple and weight of eating apple} > 140)}{P(\text{weight of apple} > 140)}$$

OR

$$\frac{P(\text{eating apple} \cap \text{weight of eating apple} > 140)}{P(\text{eating apple} \cap \text{weight of eating apple} > 140) + P(\text{cooking apple} \cap \text{weight of cooking apple} > 140)}$$

OR

$$\frac{P(\text{eating apple})P(\text{weight of eating apple} > 140)}{P(\text{eating apple})P(\text{weight of eating apple} > 140) + P(\text{cooking apple})P(\text{weight of cooking apple} > 140)}$$

$$= \frac{0.8 \times 0.025}{0.8 \times 0.025 + 0.2 \times 0.5} \left(= \frac{80 \times 2.5}{80 \times 2.5 + 20 \times 50} \right) \quad \textbf{(A1)}$$

$$= \frac{200}{1200} \left(= \frac{1}{6} \right) \quad \textbf{A1}$$

Note: Accept any equivalent exact answer written as a fraction.

[4 marks]**Total [5 marks]**

3. (a) recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) (M1)

$$P(X > 13) = 0.0668072\dots$$

$$= 0.0668$$

A1

[2 marks]

- (b) **EITHER**

equating an appropriate correct normal CDF function to 0.1 or 0.9 (M1)

$$P(X > 10 + 2k) = 0.1 \text{ OR } P(Z < k) = 0.9 \text{ OR } P(X < 10 - 2k) = 0.1 \text{ OR } P(Z < -k) = 0.1$$

OR

recognising need to use inverse normal with 0.1 or 0.9 (M1)

THEN

$$1.28155\dots$$

$$k = 1.28$$

A1

[2 marks]

Total [4 marks]

5. (a) $r = 0.901017\dots$

$r = 0.901$

A2

[2 marks]

(b) Student 11 Test B: should not extrapolate

R1

[1 mark]

(c) (i) Student 12 Test A: should not use line of y on x to predict x from y (or equivalent)

R1

(ii) attempt to find the equation of the regression line of x on y

(M1)

$(x =) 0.987124\dots y - 3.21970\dots \quad ((x =) 0.987y - 3.22)$

A1

$(x =) 0.987124\dots(90) - 3.21970\dots \quad (= 85.6214\dots)$

A1

$= 86$ to nearest integer.

AG

Note: Condone notation for x and y switched if values are correct.

[4 marks]

Total [7 marks]

6. let X be the number of days of rain in May

(a) recognition of binomial distribution

(M1)

$$X \sim B(31, 0.2) \text{ or } {}^{31}C_{10} (0.2)^{10} (0.8)^{21} \text{ or } X \sim B(n, p) \text{ or } {}^nC_r p^r (1-p)^{n-r}$$

$$P(X=10) = 0.0418894\dots$$

$$= 0.0419$$

A1

Note: If no working shown, award (M1)A0 for 0.042 (2 sf)

[2 marks]

(b) recognition of need to find $P(X \geq 10) (= 1 - P(X \leq 9))$

(M1)

$$= 0.0745998\dots (= 1 - 0.925400\dots)$$

$$= 0.0746$$

A1

Note: If no working shown, award (M1)A0 for 0.075 (2 sf)

[2 marks]

(c) recognition of 9 days with no rain followed by a day of rain

(M1)

$$0.8^9 \times 0.2 = 0.0268435\dots$$

$$= 0.0268$$

A1

Note: If no working shown, award (M1)A0 for 0.027 (2 sf)

[2 marks]

Total [6 marks]

8. (a) $\int_0^k kx \, dx + \int_k^{2k} (2kx - x^2) \, dx = 1$ **A1**

$$\left[\frac{kx^2}{2} \right]_0^k + \left[kx^2 - \frac{x^3}{3} \right]_k^{2k} = 1$$

$$\frac{k^3}{2} + \left(4k^3 - \frac{8k^3}{3} \right) - \left(k^3 - \frac{k^3}{3} \right) = 1$$
 A1

$$7k^3 = 6$$
 AG

[2 marks]

(b) recognition that the median m is the value such that $\int_0^m f(x) \, dx = 0.5$ or

$$\int_m^{2k} f(x) \, dx = 0.5$$
 (M1)

$$(k = 0.949914... \left(= \sqrt[3]{\frac{6}{7}} \right) \text{ so } \int_0^k kx \, dx = 0.428571... \left(= \frac{3}{7} \right) \text{ seen anywhere} \quad \textbf{(A1)}$$

Note: This **A1** is independent of **M1**.

EITHER

$$(m > k \text{ so } \int_0^k kx \, dx + \int_k^m (2kx - x^2) \, dx = 0.5 \text{ OR } \int_k^m (2kx - x^2) \, dx = 0.0714285... \left(= \frac{1}{14} \right) \quad \textbf{(A1)}$$

$$\left[kx^2 - \frac{x^3}{3} \right]_k^m = 0.0714285...$$

$$m = 1.02925...$$

continued...

*Question 8 continued***OR**

$$(m > k \text{ so}) \int_m^{2k} (2kx - x^2) dx = 0.5 \quad (\mathbf{A1})$$

$$\left[kx^2 - \frac{x^3}{3} \right]_m^{2k} = 0.5$$

$$m = 1.02925\dots$$

THEN

$$m = 1.03$$

A1

Note: The correct 3sf answer can be found by solving $\int_0^m kx dx = 0.5$. This method is not valid since

$m > k$. In this case award **M1A0M0A0**.

[4 marks]**Total [6 marks]**

2. (a) **EITHER**

$$\bar{y} = 2.1875 \times 7 + 0.6875$$

A1**OR**

$$\bar{y} = 15.3125 + 0.6875$$

A1**THEN**

$$\bar{y} = 16$$

AG**[1 mark]**

(b) attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in p and q **(M1)**

$$16 = \frac{9+13+p+q+21}{5} \quad (80 = p+q+43 \Rightarrow p+q=37) \quad \textbf{(A1)}$$

attempts to solve two linear equations simultaneously for p and q (one of which is $q = p + 3$) **(M1)**

$$16 = \frac{9+13+p+p+3+21}{5} \quad (80 = 2p+46)$$

$$p = 17 \text{ and } q = 20$$

A1**[4 marks]****Total [5 marks]**

5.

METHOD 1correct inequality or equation involving $P(X = 0)$ (A1)

$$1 - P(X = 0) > 0.99 \text{ OR } P(X = 0) < 0.01 \text{ OR } 1 - P(X = 0) = 0.99 \text{ OR } P(X = 0) = 0.01$$

attempts to solve their inequality (equality) involving 0.75^n for n (M1)

$$1 - 0.75^n > 0.99 \text{ OR } 0.75^n < 0.01 \text{ OR } 0.75^n = 0.01 \text{ OR } 1 - 0.75^n = 0.99$$

Note: Valid solving attempts include graphical, use of logarithms, tabular or trial and error.

EITHER

$$n > 16.0078... \text{ OR } n = 16.0078... \quad (A2)$$

the least value of n is 17 A1**OR**

$$P(X = 0) = 0.010022... (> 0.01) \text{ (corresponding to } n = 16) \quad (A1)$$

$$P(X = 0) = 0.0075169... (< 0.01) \quad (A1)$$

corresponding to $n = 17$ (which is the least value of n) A1*continued...*

Question 5 continued.

METHOD 2 (TABLE ONLY APPROACH)

attempts to use binomial cdf to calculate a correct value of $P(X \geq 1)$ for one value of n **(M1)**

calculates correct values of $P(X \geq 1)$ for at least one value of n **(A1)**

$P(X \geq 1) = 0.989977\dots$ (< 0.99) (corresponding to $n = 16$) **(A1)**

$P(X \geq 1) = 0.992483\dots$ (> 0.99) **(A1)**

corresponding to $n = 17$ (which is the least value of n) **A1**

[5 marks]

10. (a) recognizes that the mode is a value of x at which f has a maximum value (M1)

a clearly labelled graph of f OR states $f'(x) = 0$ OR considers the axis of symmetry

mode is 1.5 (kg) A1

Note: Award **M1A0** for (1.5, 0.441) or 0.441 stated as the final answer.

[2 marks]

- (b) attempts to find $\int_1^2 f(x) dx$ (M1)

= 0.435294...

= 0.435 $\left(= \frac{37}{85} \right)$ A1

[2 marks]

continued...

Question 10 continued.

(c) **METHOD 1**

recognizes that $\int_{0.5}^m f(x) dx = 0.5$ **(M1)**

$$m = 1.68701\dots$$

$$m = 1.69 \text{ (kg)} \quad \textbf{A2}$$

METHOD 2

recognizes that $\int_{0.5}^m f(x) dx = 0.5$ **(M1)**

$$\frac{6}{85} \left(4m + \frac{3}{2}m^2 - \frac{1}{3}m^3 \right) - \frac{6}{85} \left(2 + \frac{3}{8} - \frac{1}{24} \right) = 0.5 \quad \textbf{A1}$$

$$m = 1.68701\dots$$

$$m = 1.69 \text{ (kg)} \quad \textbf{A1}$$

[3 marks]

(d) $0.5 \leq x \leq 2$ (can be seen in a definite integral) **(A1)**

attempts to evaluate their definite integral **(M1)**

$$\int_{0.5}^2 f(x) dx = 0.635294\dots$$

$$= 0.635 \quad \textbf{A1}$$

[3 marks]

continued...

Question 10 continued.

(e) an attempt at forming an expected value integral $\int_{x_1}^{x_2} x f(x) dx$ (M1)

$$\int_{0.5}^{0.75} x f(x) dx (= 0.060592...) \text{ OR } \int_{0.5}^{0.75} 25x f(x) dx (= 1.51482...) \quad (\text{A1})$$

$$\int_{0.75}^3 x f(x) dx (= 1.64345...) \text{ OR } \int_{0.75}^3 24x f(x) dx (= 39.4428...) \quad (\text{A1})$$

sums their two definite integrals (M1)

$$(\text{expected amount spent per customer is}) = \int_{0.5}^{0.75} 25x f(x) dx + \int_{0.75}^3 24x f(x) dx$$

$$= 40.9576...$$

(expected amount spent per customer is) \$40.96 A1

[5 marks]

Total [15 marks]

1. (a) (i) $0.9 \times 0.9 \times 0.1$ (M1)

Note: Award **M1** for 0.9 seen as part of a multiplication.

$$= 0.081$$

A1

[2 marks]

- (ii) let A = number of boosts in the first 6 actions

METHOD 1

Recognition of binomial distribution

(M1)

with parameters $n = 6$ and $p = 0.1$

(A1)

$$(P(A \geq 1) =) 0.469 \quad (0.468558\dots)$$

A1

Note: Award **(M1)(A1)A0** for an answer of 0.114 (from finding $P(A > 1)$).

METHOD 2

$$(P(A \geq 1) =) 1 - P(\text{no boost in first 6})$$

(M1)

$$= 1 - 0.9^6$$

(A1)

$$= 0.469 \quad (0.468558\dots)$$

A1

METHOD 3

$$0.1 + 0.9 \times 0.1 + 0.9^2 \times 0.1 + 0.9^3 \times 0.1 + 0.9^4 \times 0.1 + 0.9^5 \times 0.1$$

(M1)(A1)

Note: Award **(M1)** for at least 3 correct terms in a sum.

$$= 0.469 \quad (0.468558\dots)$$

A1

[3 marks]

continued...

Question 1 continued(b) (i) **EITHER**there will be $x - 1$ actions not boosted, followed by one boosted action **A1****OR**The x^{th} action is boosted and all previous actions are not boosted **A1****OR**

$$\underbrace{(1-p) \times (1-p) \times \dots \times (1-p)}_{(x-1) \text{ times}} \times p$$
A1
Note: For the **OR** method, the order of the operations must be clear (accept also a tree diagram etc.).
THEN

$$p(1-p)^{x-1}$$
AG
[1 mark]

(ii) $E(X) = \sum_{x=1}^{\infty} xp(1-p)^{x-1}$ **A1**
[1 mark]*continued...*

Question 1 continued

(c) (i) LHS:

$$a + 2ar + 3ar^2 + \dots$$

A1

Note: Award A1 for writing LHS = $\sum ar^n$ and differentiating to get = $\sum anr^{n-1}$.

RHS:

attempt to differentiate $a(1-r)^{-1}$ or use quotient rule**(M1)**

$$\frac{a}{(1-r)^2}$$

A1

$$\sum_{n=1}^{\infty} nar^{n-1} = \frac{a}{(1-r)^2}$$

A1

Note: To award the final **A1** “ $\sum_{n=1}^{\infty} nar^{n-1}$ ” must be seen either as part of the LHS working or as part of the final answer and all previous marks must be awarded.

[4 marks](ii) recognition that $a = p$ and $r = 1 - p$ (and $x = n$)**(M1)**

$$E(X) = \frac{p}{(1-(1-p))^2}$$

A1

Note: Award **A0 FT** if their answer does not lead to the **AG**.

$$= \frac{1}{p}$$

AG**[2 marks]**(d) $E(X) = 10$, $\text{Var}(X) = 90$ **A1A1****[2 marks]**

continued...

Question 1 continued

(e) $0.8 \times 0.6 \times 0.6$

(M1)A1

Note: Award **M1** for evidence of P(“not boosted”) changing, e.g. a labelled tree diagram and/or 0.8 AND 0.6 seen.

$= 0.288$

AG**[2 marks]**

- (f) after four actions that are not boosted, the probability that the next action is boosted is 1 (and $p = 1$ is a certainty)

R1

Note: Accept “when $Y = 5$, the probability of a boost is 1”.

Do not accept “on the 5th action the probability of a boost is 1”, unless there is reference to the previous actions being not boosted.

[1 mark]

(g) (i) $m = 0.32, n = 0.1536$ (0.154)

A1A1**[2 marks]**

- (ii) attempt to multiply outcomes by probabilities and find the sum
 $0.2 + 2 \times 0.32 + 3 \times 0.288 + 4 \times 0.1536 + 5 \times 0.0384$

(M1)**A1**

Note: The **A1** can only be awarded if m and n are correct and the exact value of n is used.

$= 2.5104$

AG**[2 marks]**

(iii) $0.2 + 2^2 m + 3^2 \times 0.288 + 4^2 \times n + 5^2 \times 0.0384 - 2.5104^2$
 $= 1.19$ (1.18749...)

(A1)**A1**

Note: Award **A1A1** for a correct answer found using the GDC.

[2 marks]*continued...*

Question 1 continued

(h) (i) $\left(p = \frac{1}{2.5104}\right) = 0.398 \text{ (0.398342...)}$ **A1**

[1 mark]

(ii) $\text{Var}(X) = 3.79 \text{ (3.79170...)}$ **A1**

Note: Award **A1** for an answer of 3.80 (3.80040...) (from using their 3sf answer from part (i).)
Condone 3.8.

[1 mark]

(iii) since $\text{Var}(X) > \text{Var}(Y)$, 2nd model provides more consistent experience **R1**

Note: Only award **R1 FT** if both their variances are positive.

[1 mark]**Total [27 marks]**

2. (a) 6^3 OR $6 \times 6 \times 6$ **A1**

Note: Accept a labelled diagram that clearly illustrates correct application of the multiplication principle leading to 216.

$$= 216$$

AG**[1 mark]**(b) **EITHER**attempts to find Δ **(M1)**

$$\Delta \left(= \left(4^2 - 4(1)(4) \right) \right) = 0$$

A1**OR**attempts to solve $x^2 + 4x + 4 = 0$ **(M1)**

$$\left((x+2)^2 = 0 \Rightarrow \right) x = -2$$

A1**OR**attempts to express $x^2 + 4x + 4 (= 0)$ as a perfect square**(M1)** $(x+2)^2 (= 0)$ is a perfect square**A1****OR**a graph of $y = x^2 + 4x + 4$ with the vertex touching the x -axis at $x = -2$ **A2****THEN**graph of f has only one x -intercept**AG****[2 marks]***continued...*

Question 2 continued.

Note: In parts (c) – (f), $(a, b, c) = (1, 2, 1)$, for example, represents an ordered 3-tuple $a = 1, b = 2$ and $c = 1$.

(c) recognizes that $b^2 - 4ac = 0$ (or equivalent) (M1)

EITHER

attempts to use $\frac{b^2}{ac} = 4 \left(\frac{b^2}{4} = ac \right)$ (M1)

determines one value of b from $b = 2, 4$ or 6 only (seen anywhere) OR one value of ac from $ac = 1, 4$ or 9 only (seen anywhere) (A1)

OR

attempts to find a possible value of b (M1)

determines one value of b from $b = 2, 4$ or 6 only (seen anywhere) (A1)

OR

recognizes that b^2 must be a multiple of 4 OR b must be a multiple of 2 (M1)

determines one value of b from $b = 2, 4$ or 6 only (seen anywhere) (A1)

OR

attempts to find a possible value of ac (M1)

determines one value of ac from $ac = 1, 4$ or 9 only (seen anywhere) (A1)

THEN

$b = 2$ and $ac = 1$:

$(a, b, c) = (1, 2, 1)$ OR 1 possible way OR $\frac{1}{216}$ A1

$b = 4$ and $ac = 4$:

$(a, b, c) = (1, 4, 4), (4, 4, 1), (2, 4, 2)$ OR 3 possible ways OR $\frac{3}{216}$ A1

$b = 6$ and $ac = 9$:

$(a, b, c) = (3, 6, 3)$ OR 1 possible way OR $\frac{1}{216}$ A1

therefore the required probability is $\frac{1}{216} + \frac{3}{216} + \frac{1}{216}$

$= \frac{5}{216}$ AG

[6 marks]

continued...

Question 2 continued.

(d) recognizes that $b^2 - 4ac > 0$ (or equivalent eg. $\frac{b^2}{4} > ac$) (M1)

maximum value of b^2 is 36 OR maximum value of ac is 8 (A1)

Note: The above (A1) is independent of the (M1).

$$ac = 1, 2, 3, 4, 5, 6, 8$$

A1

[3 marks]

(e) (i) $ac = 1$ ($b^2 > 4$)

$b = 3, 4, 5, 6$ OR 1×4 (quadratics) OR $6 - 2$ (quadratics)

A1

there are four quadratic functions

AG

[1 mark]

(ii)

$$ac = 2$$
 ($b^2 > 8$)

$$b = 3, 4, 5, 6$$

(A1)

Note: Award (A1) for referencing their result shown in part (e) (i).

EITHER

$$(a, b, c) = (1, 3, 2), (1, 4, 2), (1, 5, 2), (1, 6, 2), (2, 3, 1), (2, 4, 1), (2, 5, 1), (2, 6, 1)$$

A1

Note: Award A1 for listing the eight quadratic expressions.

OR

2×4 (quadratics)

A1

THEN

there are eight quadratics functions

AG

[2 marks]

continued...

Question 2 continued.

(f) **METHOD 1**

varies ac ($ac \neq 1, 2$) and determines possible values of b such that $\Delta > 0$ **(M1)**

correctly determines one of the following five cases **(A1)**

correctly determines a further two of the following five cases **(A1)**

correctly determines the remaining two cases **(A1)**

case 1: $ac = 3$ ($b^2 > 12 \Rightarrow b = 4, 5, 6$)

$(a, b, c) = (1, 4, 3), (1, 5, 3), (1, 6, 3), (3, 4, 1), (3, 5, 1), (3, 6, 1)$ OR

6 possible ways OR $\frac{6}{216}$

case 2: $ac = 4$ ($b^2 > 16 \Rightarrow b = 5, 6$)

$(a, b, c) = (1, 5, 4), (1, 6, 4), (2, 5, 2), (2, 6, 2), (4, 5, 1), (4, 6, 1)$ OR

6 possible ways OR $\frac{6}{216}$

case 3: $ac = 5$ ($b^2 > 20 \Rightarrow b = 5, 6$)

$(a, b, c) = (1, 5, 5), (1, 6, 5), (5, 5, 1), (5, 6, 1)$ OR 4 possible ways OR $\frac{4}{216}$

case 4: $ac = 6$ ($b^2 > 24 \Rightarrow b = 5, 6$)

$(a, b, c) = (1, 5, 6), (2, 5, 3), (3, 5, 2), (6, 5, 1), (1, 6, 6), (2, 6, 3), (3, 6, 2), (6, 6, 1)$

OR 8 possible ways OR $\frac{8}{216}$

case 5: $ac = 8$ ($b^2 > 32 \Rightarrow b = 6$)

$(a, b, c) = (2, 6, 4), (4, 6, 2)$ OR 2 possible ways OR $\frac{2}{216}$

adds their probabilities **(M1)**

Note: Award **(M1)** for adding at least 3 of their probabilities (denominator 216).

$$(p) = \frac{4}{216} + \frac{8}{216} + \frac{6}{216} + \frac{6}{216} + \frac{4}{216} + \frac{8}{216} + \frac{2}{216}$$

$$(= 0.0185... + 0.0370... + 0.0277... + 0.0277... + 0.0185... + 0.0370... + 0.0092...)$$

$$= \frac{38}{216} \quad (= \frac{19}{108}, = 0.176)$$

A1

continued...

Question 2 continued.

METHOD 2

varies $b^2 (\neq 1, 4)$ OR $b (\neq 1, 2)$ and determines possible values of ac such that $\Delta > 0$

(M1)

correctly determines one of the following four cases

(A1)

correctly determines another case from the following four cases

(A1)

correctly determines the remaining two cases

(A1)

case 1: $b^2 = 9$ ($b = 3$) ($ac = 1, 2$)

$(a, b, c) = (1, 3, 1), (1, 3, 2), (2, 3, 1)$ OR 3 possible ways OR $\frac{3}{216}$

case 2: $b^2 = 16$ ($b = 4$) ($ac = 1, 2, 3$)

$(a, b, c) = (1, 4, 1), (1, 4, 2), (2, 4, 1), (1, 4, 3), (3, 4, 1)$ OR 5 possible ways OR $\frac{5}{216}$

case 3: $b^2 = 25$ ($b = 5$) ($ac = 1, 2, 3, 4, 5, 6$)

$(a, b, c) = (1, 5, 1), (1, 5, 2), (2, 5, 1), (1, 5, 3), (3, 5, 1), (1, 5, 4), (2, 5, 2)$
 $(4, 5, 1), (1, 5, 5), (5, 5, 1), (1, 5, 6), (2, 5, 3), (3, 5, 2), (6, 5, 1)$

OR 14 possible ways OR $\frac{14}{216}$

case 4: $b^2 = 36$ ($b = 6$) ($ac = 1, 2, 3, 4, 5, 6, 8$)

$(a, b, c) = (1, 6, 1), (1, 6, 2), (2, 6, 1), (1, 6, 3), (3, 6, 1), (1, 6, 4), (2, 6, 2), (4, 6, 1)$
 $(1, 6, 5), (5, 6, 1), (1, 6, 6), (2, 6, 3), (3, 6, 2), (6, 6, 1), (2, 6, 4), (4, 6, 2)$

OR 16 possible ways OR $\frac{16}{216}$

adds their probabilities

(M1)

Note: Award (M1) for adding at least 3 of their probabilities (denominator 216).

$$(p =) \frac{3}{216} + \frac{5}{216} + \frac{14}{216} + \frac{16}{216}$$

$$(= 0.013889... + 0.023148... + 0.064815... + 0.074074...)$$

$$= \frac{38}{216} (= \frac{19}{108}, = 0.176)$$

A1

continued...

Question 2 continued.

METHOD 3

varies b^2 OR b and determines possible values of ac such that $\Delta < 0$ **(M1)**

correctly determines two of the following six cases **(A1)**

correctly determines a further two of the following six cases **(A1)**

correctly determines the remaining two cases **(A1)**

case 1: $b^2 = 1$ ($b = 1$) 36 possible ways OR $\frac{36}{216}$

case 2: $b^2 = 4$ ($b = 2$) 35 possible ways OR $\frac{35}{216}$

case 3: $b^2 = 9$ ($b = 3$) 33 possible ways OR $\frac{33}{216}$

case 4: $b^2 = 16$ ($b = 4$) 28 possible ways OR $\frac{28}{216}$

case 5: $b^2 = 25$ ($b = 5$) 22 possible ways OR $\frac{22}{216}$

case 6: $b^2 = 36$ ($b = 6$) 19 possible ways OR $\frac{19}{216}$

$(p =) 1 - \left(\frac{36}{216} + \frac{35}{216} + \frac{33}{216} + \frac{28}{216} + \frac{22}{216} + \frac{19}{216} + \frac{5}{216} \right)$ **(M1)**

$\left(= 1 - \left(0.16666... + 0.16203... + 0.15277... + 0.12962... + 0.10185... + 0.087962... + 0.023148... \right) \right)$

Note: Award **(M1)** for adding at least 3 of their probabilities inside the above bracket (denominator 216).

$= \frac{38}{216} \left(= \frac{19}{108}, = 0.176 \right)$ **A1**

[6 marks]

continued...

Question 2 continued.

(g) recognizes that $4Z^2 - 4 > 0$ ($Z^2 > 1$) (M1)

probability of two x -intercepts is

EITHER

$P(|Z| > 1)$ (A1)

OR

$P(Z < -1)$ or $P(Z > 1)$ (can be shown on a labelled diagram) (A1)

$= 0.158655... + 0.158655...$

OR

$1 - P(-1 \leq Z \leq 1)$ (can be shown on a labelled diagram) (A1)

$= 1 - 0.682689...$

THEN

$= 0.317310...$

$= 0.317$

A1

[3 marks]

(h) attempts to solve $X_1 > 0.5$ for Z (M1)

$-1.25 < Z \leq -1$ (A1)(A1)

Note: Award (M1)(A1) for obtaining $Z = -1.25$ from solving $X_1 = 0.5$ and award (A1) for stating the correct inequality.

Award (M1)(A1)(A1) for $-1.25 < Z < -1$.

Award (M1)(A1)(A0) for $-1.25 < Z$.

Award (M1) for rearranging to form $-\sqrt{Z^2 - 1} = Z + 0.5$ and then

attempting to square both sides $Z^2 - 1 = (Z + 0.5)^2$ ($= Z^2 + Z + 0.25$).

attempts to calculate their $P(X_1, X_2 \text{ both } > 0.5)$ (M1)

$P(-1.25 < Z \leq -1) = 0.053005...$ (A1)

attempts to calculate their $P(X_1, X_2 \text{ both } > 0.5 | x\text{-intercepts})$ (M1)

$= \frac{0.053005...}{0.317310...}$

$= 0.167$

A1

[7 marks]

Total [31 marks]

2. (a) use of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **(M1)**

$$P(A \cap B) = 0.65 + 0.45 - 0.85 \quad (\text{or equivalent}) \quad \textbf{(A1)}$$

$$= 0.25 \quad \textbf{A1}$$

[3 marks]

(b) $P(A' \cap B') = 0.15$ (may be seen in Venn diagram) **(A1)**

attempt to substitute their values into $P(A' | B') = \frac{P(A' \cap B')}{P(B')}$ **(M1)**

$$P(A' | B') = \frac{0.15}{0.55}$$

$$= \frac{15}{55} \left(= \frac{3}{11} \right)$$

A1

[3 marks]

Total [6 marks]

3. (a) recognition of sum of probabilities equals 1

(M1)

$$\frac{3k}{20} + \frac{5k}{20} + \frac{8k}{20} + \frac{11k}{20} = 1$$

$$k = 0.740740$$

$$k = 0.741 \left(= \frac{20}{27} \right)$$

A1

[2 marks]

- (b) correct probabilities: $\frac{3}{27}, \frac{5}{27}, \frac{8}{27}, \frac{11}{27}$ OR 0.111, 0.185, 0.296, 0.407

(A1)

substitution of their probabilities into formula for expected value

(M1)

$$3 \times \frac{3}{27} + 5 \times \frac{5}{27} + 8 \times \frac{8}{27} + 11 \times \frac{11}{27} \text{ OR } \frac{219k}{20}$$

$$= 8.11111\dots$$

$$E(X) = 8.11 \left(= \frac{219}{27} = \frac{73}{9} \right) \text{ (same 3sf from previous 3sf answer)}$$

A1

[3 marks]**Total [5 marks]**

6. (a) $E(X) = \int_0^2 \frac{x}{5} dx + \int_2^8 \left(-\frac{x^2}{30} + \frac{4x}{15} \right) dx$ (A1)(A1)

Note: Award (A1) $\int_0^2 \frac{x}{5} (dx)$ and (A1) for $\int_2^8 \left(-\frac{x^2}{30} + \frac{4x}{15} \right) (dx)$

$$= \frac{2}{5} + \frac{12}{5}$$

$$= \frac{14}{5} (= 2.8)$$

A1

[3 marks]

(b) attempt to use the expectation formula $E(aX + b) = aE(X) + b$ (M1)

$$E(c - 2X) = c - 2E(X) (= 0)$$

$$c = 2E(X)$$

$$= \frac{28}{5} (= 5.6)$$

A1

[2 marks]

(c) recognition that median m lies between 2 and 8 e.g. using a diagram or integral (M1)

$$\int_0^2 \frac{1}{5} dx + \int_2^m \left(-\frac{x}{30} + \frac{4}{15} \right) dx = \frac{1}{2} \text{ OR } \int_m^8 \left(-\frac{x}{30} + \frac{4}{15} \right) dx = \frac{1}{2} \text{ OR } \int_2^m \left(-\frac{x}{30} + \frac{4}{15} \right) dx = \frac{1}{10}$$
 (A1)

$$m = 2.52277...$$

$$m = 2.52$$

A1

[3 marks]

Total [8 marks]

4. (a) $P(A) = \frac{1}{4}$ (A1)

attempt to use $P(B|A) = \frac{P(B \cap A)}{P(A)}$ (M1)

$$\frac{2}{3} = \frac{P(B \cap A)}{\left(\frac{1}{4}\right)}$$

$$P(A \cap B) = \frac{2}{3} \left(\frac{1}{4} \right)$$

$$= \frac{2}{12} \left(= \frac{1}{6} \right)$$

A1

[3 marks]

- (b) attempt to use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ OR a Venn diagram, with their values of $P(A)$ and $P(B \cap A)$

M1

$$\frac{3}{4} = \frac{1}{4} + P(B) - \frac{1}{6}$$

$$P(B) = \frac{1}{2} + \frac{1}{6}$$

$$= \frac{4}{6} \left(= \frac{2}{3} \right)$$

A1

$P(B|A) = P(B)$ OR $P(A)P(B) = \frac{1}{6}$ so $P(A \cap B) = P(A)P(B)$ (hence A and B are independent)

R1

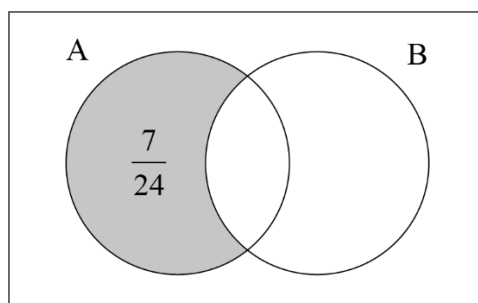
Note: The **R1** is dependent on all previous marks

[3 marks]

Total [6 marks]

4. (a) attempt to use a Venn diagram OR law of addition

(M1)



$$P(A \cup B) = P(A \cap B') + P(B)$$

$$\frac{5}{8} = \frac{7}{24} + P(B)$$

(A1)

$$P(B) = \frac{8}{24} \left(= \frac{1}{3} \right)$$

A1

[3 marks]

- (b) **METHOD 1:** finding $P(A)$

attempt to find $P(A)$

M1

$$P(A \cap B') = P(A) \times P(B') \quad \text{OR} \quad P(A \cup B) = P(A) + P(B) - P(A) \times P(B)$$

$$P(A) \times \frac{2}{3} = \frac{7}{24} \quad \text{OR} \quad \frac{5}{8} = \frac{2}{3} P(A) + \frac{1}{3}$$

$$P(A) = \frac{7}{16}$$

(A1)

EITHER

$$P(A' | B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(A') \times P(B)}{P(B)} = P(A') \quad (\text{seen anywhere}) \quad \text{OR} \quad 1 - \frac{7}{16}$$

(A1)

OR

$$P(A' | B) \left(= \frac{P(A' \cap B)}{P(B)} \right) = \frac{\frac{9}{16}}{\frac{1}{3}} = \frac{27}{16}$$

(A1)

THEN

$$P(A' | B) = \frac{9}{16}$$

A1

continued...

Question 4 continued.

METHOD 2: attempt to find $P(A \cap B)$

attempt to use $P(A \cap B) = P(A) \times P(B)$

M1

$$x = \left(x + \frac{7}{24} \right) \times \frac{1}{3}$$

$$x = \frac{7}{48}$$

A1

$$P(A' | B) = P(B) - P(A \cap B) + 1 - P(A \cup B) \quad \text{OR} \quad = \frac{1}{3} - \frac{7}{48} + 1 - \frac{5}{8}$$

(A1)

$$P(A' | B) = \frac{27}{48} \left(= \frac{9}{16} \right)$$

A1

[4 marks]

Total [7 marks]

5. (a) recognizing constant change from Y to X and/or comparing the two distributions

$$b - 7 = 22 - 19 \quad \text{OR} \quad b = 7 + (22 - 19) \quad \text{OR} \quad \frac{b - 7}{a} = \frac{22 - 19}{a} \quad (M1)$$

$$b = 10$$

A1

[2 marks]

- (b) $(P(7 - a < X < 7 + a) =) 0.68$

A1

[1 mark]

- (c) **EITHER**

recognizing that 22 is one standard deviation above the mean

(M1)

OR

recognizing symmetry of the normal curve and total area = 1

(M1)

THEN

0.16 or 0.34 (or equivalent) seen in correct sketch or probability statement

(A1)

$$(P(Y < 22) =) 1 - \frac{0.32}{2} \quad \text{OR} \quad 0.5 + 0.34 \quad \text{OR} \quad 0.68 + 0.16$$

$$0.84$$

A1

[3 marks]

Total [6 marks]

4. (a) attempt to substitute $F = 19.8$ into the regression line for A on F (M1)

$$A = 2.89(19.8) + 99.3$$

$$= 156.522 \text{ (cm)}$$

$$\text{arm span} = 157 \text{ (cm)}$$

A1

Note: Award **M0A0** for choosing the wrong regression line to get $A = 156.417...$ so $A = 156$.

[2 marks]

- (b) recognition that the lines intersect at the mean point (may be seen on a sketch) (M1)

$$2.89F + 99.3 = \frac{F + 32.6}{0.335} \text{ OR } 0.335A - 32.6 = \frac{A - 99.3}{2.89}$$

$$159.686... \text{ or } 20.8948...$$

$$\text{the mean arm span} = 160 \text{ (cm)}, \text{ the mean foot length} = 20.9 \text{ (cm)}$$

A1A1

[3 marks]

- (c) **METHOD 1**

recognition of symmetry of interval around mean (may be seen on a sketch) (M1)

$$P(H < 153) = 0.06 \text{ OR } P(H < 173) = 0.94 \text{ OR equivalent}$$

$$\frac{153 - 163}{\sigma} = -1.55477... \text{ OR } \frac{173 - 163}{\sigma} = 1.55477... \quad (\text{A1})$$

$$\sigma = 6.43181...$$

$$\sigma = 6.43 \text{ (cm)}$$

A1

METHOD 2

attempt to find σ by equating an appropriate correct normal CDF function to 0.88 (or e.g. 0.06 or 0.94) (M1)

$$\sigma = 6.43181...$$

$$\sigma = 6.43 \text{ (cm)}$$

A2

Note: Accept use of calculator notation eg $\text{normcdf}(153, 173, 163, \sigma) = 0.88$

[3 marks]

Total [8 marks]

$$11. \quad (a) \quad (i) \quad \int_{2.25}^{4.5} \frac{4}{21} \left(1 - \cos \left(\frac{4\pi}{9} (t - 2.25) \right) \right) dt$$

$$= \frac{3}{7} (= 0.428571... = 0.429) \quad \text{A1}$$

(ii) mode of $T = 4.5$ A1

(iii) the median is greater A1

$$P(T < 4.5) < 0.5 \quad \text{OR} \quad P(T > 4.5) > 0.5 \quad \text{OR} \quad P(T < 4.5) < P(T > 4.5) \quad \text{OR} \quad \text{median} = 4.69 \quad \text{R1}$$

Note: Accept reference to areas rather than probabilities.

[4 marks]

(b) recognition of the need to integrate f (M1)

$$\int_{2.25}^{3.5} f(t) dt$$

$$= 0.103749...$$

$$= 0.104$$

A1

[2 marks]

(c) attempt to use formula for conditional probability in context (M1)

$$\frac{P(T \leq 3)}{P(\text{fast})} \quad \text{OR} \quad \frac{P(T \leq 3)}{P(T \leq 3.5)} \quad \text{OR} \quad \frac{P(\text{very fast})}{P(\text{fast})} \quad (\text{accept strict inequality signs})$$

$$= \frac{0.0247152...}{0.103749...} \quad \text{(A1)}$$

$$= 0.238220...$$

$$= 0.238$$

A1

[3 marks]

continued...

Question 11 continued.

- (d) recognition that the lower quartile q is the value such that $\int_{2.25}^q f(t)dt = 0.25$ (M1)

$$\int_{2.25}^q \frac{4}{21} \left(1 - \cos \left(\frac{4\pi}{9}(t - 2.25) \right) \right) dt = 0.25 \quad (\text{A1})$$

Note: Condone the absence of dt for this **A1**.

$$\left[\frac{4}{21} \left(t - \frac{9}{4\pi} \sin \left(\frac{4\pi}{9}(t - 2.25) \right) \right) \right]_{2.25}^q = 0.25$$

$$q = 4.01290\dots$$

$$q = 4.01$$

A1

[3 marks]

- (e) attempt to find the expected value for a transformed linear variable (M1)

$$E(P) = E(a - bT) = a - bE(T)$$

$$a - 4.723b = 100$$

(A1)

recognition that max score is achieved with fastest time $t = 2.25$ (M1)

$$\text{maximum score } a - 2.25b = 150$$

(A1)

$$a = 195.491\dots, \quad b = 20.2183\dots$$

$$a = 195, \quad b = 20$$

A1

Note: these values must be given to the nearest integer for the **A1** to be awarded.

[5 marks]

- (f) attempt to find variance for a transformed linear variable (M1)

$$\text{Var}(P) = \text{Var}(a - bT) = b^2 \text{Var}(T)$$

$$0.906(20.2183\dots)^2 = 370.356\dots$$

$$\text{Var}(P) = 370$$

A1

Note: accept any answer which rounds to any value between 362 and 370 inclusive based on use of less accurate values of b .

[2 marks]

Total [19 marks]

1.

(a) 1.97651..., 40.2286...

$$a = 1.98, b = 40.2 \quad ((y =) 1.98x + 40.2)$$

A1A1

[2 marks]

(b) attempt to substitute $x = 20$ into their regression equation

(M1)

$$y = 1.97651... \times 20 + 40.2286...$$

$$79.7589...$$

$$79.8 \text{ (hours)}$$

A1

[2 marks]

Total [4 marks]

8. an attempt to set up an expression for mean or variance that includes total frequency and both p and q (M1)

$$\bar{x} = \frac{20 \times 12 + 35 \times q + p \times 8}{12 + q + 8} \left(= \frac{240 + 35q + 8p}{20 + q} \right)$$

$$\frac{240 + 35q + 8p}{20 + q} = 31 \text{ OR } 4q + 8p = 380 \text{ } (q + 2p = 95) \quad \text{(A1)}$$

$$\sigma^2 = \frac{12 \times 20^2 + q \times 35^2 + 8 \times p^2}{12 + q + 8} - 31^2 = 124 \left(\frac{4800 + 1225q + 8p^2}{20 + q} - 31^2 = 124 \right)$$

OR

$$\sigma^2 = \frac{12 \times (20 - 31)^2 + q \times (35 - 31)^2 + 8 \times (p - 31)^2}{12 + q + 8} = 124 \left(\frac{1452 + 16q + 8(p - 31)^2}{20 + q} = 124 \right) \quad \text{(A1)}$$

EITHER

$$q = 95 - 2p \quad \text{(A1)}$$

substitute their q into their expression for variance and solve for p (M1)

$$\frac{4800 + 1225(95 - 2p) + 8p^2}{20 + (95 - 2p)} - 31^2 = 124 \text{ OR } \frac{1452 + 16(95 - 2p) + 8(p - 31)^2}{20 + (95 - 2p)} = 124$$

$$p = 45 \quad \text{A1}$$

$$q = 5 \quad \text{A1}$$

OR

$$p = \frac{95 - q}{2} \quad \text{(A1)}$$

substitute their p into their expression for variance and solve for q M1

$$\frac{4800 + 1225q + 8\left(\frac{95 - q}{2}\right)^2}{20 + q} - 31^2 = 124 \text{ OR } \frac{1452 + 16q + 8\left(\frac{95 - q}{2} - 31\right)^2}{20 + q} = 124$$

$$q = 5 \quad \text{A1}$$

$$p = 45 \quad \text{A1}$$

Note: Award **A1A0** for $p = 45, q = 5$ and $p = -10, q = 115$.

Note: Award full marks for $p = 45, q = 5$ regardless of working.

[7 marks]

10.

- (a) recognising to find $P(W < 170)$ **(M1)**

$$P(W < 170) = 0.265985\dots$$

$$P(W < 170) = 0.266 \text{ (accept 26.6\%)}$$

A1**[2 marks]**

- (b) recognising $P(W > w) = 0.2$ OR labelled sketch (or equivalent) **(M1)**

$$w = 181.732\dots$$

$$w = 181.7 \text{ (grams) (must be 4 sf)}$$

A1**[2 marks]**

- (c) recognizing the need to find $P(170 < W < 185)$ **(M1)**

$$\text{diagram, } P(170 < W < 185) = 0.628(364\dots)$$

$$62.8\%$$

A1**[2 marks]**

- (d) recognising binomial probability **(M1)**

$$X \sim B(40, 0.628364\dots) \text{ OR } n = 40 \text{ and } p = 0.628364\dots$$

(A1)

$$P(X \geq 30) = 0.073861\dots$$

$$P(X \geq 30) = 0.0739 \text{ (accept 7.39\%)}$$

A1**[3 marks]***continued...*

Question 10 continued.

(e) $Y \sim B(10, 0.073861\dots)$ (A1)

$$P(Y = 4) = 0.003944\dots$$

$$P(Y = 4) = 0.00394 \text{ (accept 0.394\%)} \quad \text{A1}$$

[2 marks]

(f) $P(W < 170) = 0.12$ OR $P(W > 185) = 0.06$ (A1)

use of inverse normal to find z score for their probability (M1)

$$z = -1.17(4986\dots), z = 1.55(4773\dots) \quad \text{(A1)(A1)}$$

attempt to solve their two equations (M1)

$$-1.174986\dots = \frac{170 - \mu}{\sigma}$$

$$1.554773\dots = \frac{185 - \mu}{\sigma}$$

$$\mu = 176.456\dots$$

$$\mu = 176 \text{ (grams)} \quad \text{A1}$$

[6 marks]

Total [17 marks]

3. (a) (i) summing probabilities and equating to 1 **(M1)**
 $1.5a + 2a + 0.281 + a + 0.026 = 1$
 $(a =) 0.154$ **A1**
(ii) 2 (days) **A1**

[3 marks]

- (b) using expected value formula **(M1)**
(mean =) 2.44 (2.436 (exact)) **A1**

[2 marks]

- (c) convenience (sampling) **A1**

[1 mark]

Total [6 marks]

11. (a) (i) recognizing binomial distribution (M1)

$E \sim B(50, 0.08)$, where E is the number of inaccurate surveys

$$P(E \leq 6) = 0.898128...$$

$$(P(E \leq 6) =) 0.898 \text{ (accept 89.8\%)} \quad \text{A1}$$

- (ii) recognition of conditional probability (M1)

Note: Recognition must be shown in context either in words or symbols, not just $P(A|B)$.

$$P(E = 4 | E \leq 6) = \frac{P(E = 4)}{P(E \leq 6)}$$

$$= \frac{0.203654...}{0.898128...} \quad \text{(A1)}$$

$$= 0.227 \text{ (accept 22.7\%)} \quad \text{A1}$$

[5 marks]

- (b) use of tables (M1)

Note: Award (M1) for at least one correct value of $P(E \leq 6)$ for a value of n , where $n \neq 50$.

$$(n = 94 \Rightarrow) P(E \leq 6) = 0.366746...$$

$$n = 94 \quad \text{A2}$$

[3 marks]

- (c) (i) $P(I) = P(I \cap A) + P(I \cap B) + P(I \cap C)$ (M1)

$$= 0.55 \times 0.08 + 0.25 \times 0.06 + 0.2 \times 0.11 \quad \text{(A1)}$$

$$= 0.081 \text{ (accept 8.1\%)} \quad \text{A1}$$

- (ii) $P(A|I) = \frac{P(A \cap I)}{P(I)}$ (or use of Bayes' formula) (M1)

$$P(A|I) = \frac{0.55 \times 0.08}{0.081} \quad \text{(A1)}$$

$$0.543209...$$

$$= 0.543 \text{ (accept 54.3\%)} \quad \text{A1}$$

[6 marks]

continued...

Question 11 continued(d) **METHOD 1**

$$P(I | C) = \frac{x}{100} \text{ (seen anywhere)} \quad (M1)$$

$$P(I) = 0.55 \times 0.08 + 0.25 \times 0.06 + 0.2 \times \frac{x}{100} \quad (A1)$$

$$\text{equating their expression for } P(I) \text{ to } \frac{x}{100} \quad (M1)$$

$$\frac{x}{100} = 0.55 \times 0.08 + 0.25 \times 0.06 + 0.2 \times \frac{x}{100}$$

$$x = 7.375$$

$$x = 7.38 \text{ (accept 7.38\%)} \quad A1$$

METHOD 2

$$P(I) = 0.55 \times 0.08 + 0.25 \times 0.06 + 0.2 \times y \quad (A1)$$

$$\text{equating their expression for } P(I) \text{ to } y \quad (M1)$$

$$y = 0.55 \times 0.08 + 0.25 \times 0.06 + 0.2 \times y$$

$$y = 0.07375$$

$$\text{recognizing to multiply their } y \text{ value by 100} \quad (M1)$$

$$x = 7.38 \text{ (accept 7.38\%)} \quad A1$$

[4 marks]**Total [18 marks]**

1. (a) attempts to use $E(M) = \sum mP(M = m)$ (M1)

$$= 1 \times \frac{1}{16} + 2 \times \frac{3}{16} + 3 \times \frac{5}{16} + 4 \times \frac{7}{16} \text{ (or equivalent)}$$

$$= \frac{25}{8} \left(= 3.125, = \frac{50}{16}, = 3.13 \right) \quad \text{A1}$$

[2 marks]

- (b) $G(1) = 1$ A1

[1 mark]

- (c) (i) $G'(t) = \frac{1}{16} + \frac{3}{8}t + \frac{15}{16}t^2 + \frac{7}{4}t^3 \left(= \frac{1}{16} + \frac{6}{16}t + \frac{15}{16}t^2 + \frac{28}{16}t^3 \right)$ A2

Note: Award **A1A0** for three correct terms.

[2 marks]

- (ii) substitutes $t = 1$ into their $G'(t)$ (M1)

$$G'(1) = \frac{1}{16} + \frac{3}{8} + \frac{15}{16} + \frac{7}{4} \left(= \frac{1}{16} + \frac{6}{16} + \frac{15}{16} + \frac{28}{16} \right) \quad \text{A1}$$

$$= \frac{25}{8} \left(= 3.125, = \frac{50}{16} \right) \quad \text{A1}$$

$$= E(M) \quad \text{AG}$$

Note: Award a maximum of **M1A1A0FT** for an incorrect $E(M)$ obtained with an incorrect $G'(t)$.

Award a maximum of **M1A1A0FT** for a correct $E(M)$ obtained with an incorrect $G'(t)$.

[3 marks]

continued...

Question 1 continued.(d) attempts to find at least one required probability **(M1)**

$$P(X=0) = \frac{3}{5} \times \frac{2}{4} \text{ OR } P(X=0) = \frac{{}^2C_0 \times {}^3C_2}{{}^5C_2} \left(= \frac{{}^3C_2}{{}^5C_2} \right) \quad \textbf{A1}$$

$$= \frac{3}{10}$$

$$P(X=1) = \frac{2}{5} \times \frac{3}{4} + \frac{3}{5} \times \frac{2}{4} \left(= 2 \times \frac{6}{20} \right) \quad \textbf{A1A1}$$

Note: Award **A1** for $\frac{2}{5} \times \frac{3}{4}$ OR $\frac{6}{20}$ and **A1** for $\frac{3}{5} \times \frac{2}{4}$ OR $\frac{6}{20}$.

Award **A2** for $P(X=1) = \frac{{}^2C_1 \times {}^3C_1}{{}^5C_2} \left(= \frac{2 \times {}^3C_1}{{}^5C_2} \right)$.

$$= \frac{3}{5}$$

EITHER

$$P(X=2) = \frac{2}{5} \times \frac{1}{4} \text{ OR } P(X=2) = \frac{{}^2C_2 \times {}^3C_0}{{}^5C_2} \left(= \frac{{}^2C_2}{{}^5C_2}, = \frac{{}^3C_0}{{}^5C_2} \right) \quad \textbf{A1}$$

OR

$$P(X=2) = 1 - \left(\frac{3}{10} + \frac{3}{5} \right) \quad \textbf{A1}$$

THEN

$$= \frac{1}{10}$$

leading to $G_X(t) = \frac{3}{10} + \frac{3}{5}t + \frac{1}{10}t^2 \quad \textbf{AG}$

Note: Award **A** marks as appropriate for clearly labelled correct tree diagrams.

Do not award **A** marks for $P(X=0) = \frac{3}{10}$, $P(X=1) = \frac{3}{5}$ and

$P(X=2) = \frac{1}{10}$ stated without any working shown.

[5 marks]*continued...*

Question 1 continued.

(e) (i) **EITHER**

$$P(Y = 2) = \frac{1}{2}p \quad (\text{A1})$$

OR

$$\frac{1}{2}p = \frac{1}{3} \quad (\text{A1})$$

THEN

$$p = \frac{2}{3} (= 0.667) \quad \text{A1}$$

[2 marks]

(ii) uses their $G_Y(t) = P(Y = 0)t^0 + P(Y = 1)t^1 + \frac{1}{3}t^2$ with their value of p **(M1)**

$$P(Y = 0) = \frac{1}{6} \quad (\text{A1})$$

$$P(Y = 1) = \frac{1}{2} \quad (\text{A1})$$

$$G_Y(t) = \frac{1}{6} + \frac{1}{2}t + \frac{1}{3}t^2 \quad \text{A1}$$

Note: Award a maximum **(M1)(A1)(A1)A0FT** from an incorrect value of p .

Award **(M1)(A1)(A1)A0FT** from $p = \frac{1}{3}$ leading to $G_Y(t) = \frac{1}{6} + \frac{1}{2}t + \frac{1}{3}t^2$.

[4 marks]

continued...

Question 1 continued.

(f) **METHOD 1**

$$G_Z(t) = \left(\frac{3}{10} + \frac{3}{5}t + \frac{1}{10}t^2 \right) \left(\frac{1}{6} + \frac{1}{2}t + \frac{1}{3}t^2 \right) \left(G_Z(t) = \frac{1}{20} + \frac{1}{4}t + \frac{5}{12}t^2 + \frac{1}{4}t^3 + \frac{1}{30}t^4 \right) \quad (\text{A1})$$

attempts to evaluate their $G_Z'(1)$ (M1)

$$G_Z'(1) = 1.96666\dots$$

$$E(Z) = 1.97 \left(= \frac{59}{30} \right) \quad \text{A2}$$

Note: Award a maximum of **(A1)FT(M1)(M1)A0** for use of an incorrect quadratic $G_Y(t)$ from part (e) (ii).

Do not award **(A1)FT** if $G_Y(t)$ is not a quadratic of the required form.

METHOD 2

$$G_Z(t) = \left(\frac{3}{10} + \frac{3}{5}t + \frac{1}{10}t^2 \right) \left(\frac{1}{6} + \frac{1}{2}t + \frac{1}{3}t^2 \right) \left(G_Z(t) = \frac{1}{20} + \frac{1}{4}t + \frac{5}{12}t^2 + \frac{1}{4}t^3 + \frac{1}{30}t^4 \right) \quad (\text{A1})$$

attempts to find $G_Z'(t)$ by differentiation M1

$$G_Z'(t) = \frac{1}{4} + \frac{5}{6}t + \frac{3}{4}t^2 + \frac{2}{15}t^3 \text{ OR}$$

$$G_Z'(t) = \left(\frac{3}{5} + \frac{1}{5}t \right) \left(\frac{1}{6} + \frac{1}{2}t + \frac{1}{3}t^2 \right) + \left(\frac{1}{2} + \frac{2}{3}t \right) \left(\frac{3}{10} + \frac{3}{5}t + \frac{1}{10}t^2 \right)$$

substitutes $t=1$ into their $G_Z'(t)$ (M1)

$$G_Z'(1) = 1.96666\dots$$

$$E(Z) = 1.97 \left(= \frac{59}{30} \right) \quad \text{A1}$$

Note: Award a maximum of **(A1)FT(M1)(M1)A0** for use of an incorrect quadratic $G_Y(t)$ from part (e) (ii).

Do not award **(A1)FT** if $G_Y(t)$ is not a quadratic of the required form.

continued...

Question 1 continued.

METHOD 3

$$G_Z(t) = \left(\frac{3}{10} + \frac{3}{5}t + \frac{1}{10}t^2 \right) \left(\frac{1}{6} + \frac{1}{2}t + \frac{1}{3}t^2 \right) \left(G_Z(t) = \frac{1}{20} + \frac{1}{4}t + \frac{5}{12}t^2 + \frac{1}{4}t^3 + \frac{1}{30}t^4 \right) \quad (\mathbf{A1})$$

$$\text{recognizes that } G_Z'(1) = G_X'(1) + G_Y'(1) \Rightarrow E(Z) = E(X) + E(Y) \quad (\mathbf{M1})$$

$$\text{attempts to evaluate their } G_X'(1) \text{ and } G_Y'(1) \text{ (or their } E(X) \text{ and } E(Y)) \quad (\mathbf{M1})$$

$$G_X'(1) = \frac{3}{5} + \frac{1}{5} \left(= \frac{4}{5} \right) \text{ and } G_Y'(1) = \frac{1}{2} + \frac{2}{3} \left(= \frac{7}{6} \right)$$

$$E(Z) = 1.97 \left(= \frac{59}{30} \right) \quad \mathbf{A1}$$

Note: Award a maximum of **(A1)FT(M1)(M1)A0** for use of an incorrect quadratic $G_Y(t)$ from part (e) (ii).

Do not award **(A1)FT** if $G_Y(t)$ is not a quadratic of the required form.

[4 marks]

Total [23 marks]

2. (a) attempt to use $P(A \cap B) = P(A|B)P(B) \left(= \frac{1}{4} \times \frac{1}{3} \right)$ (M1)

$$= \frac{1}{12}$$

A1

[2 marks]

(b) $P(B') = \frac{2}{3}$ or $P(A \cap B') = \frac{5}{12}$ (A1)

attempt to use $P(A|B') = \frac{P(A \cap B')}{P(B')}$ (M1)

$$= \frac{\left(\frac{5}{12} \right)}{\left(\frac{2}{3} \right)}$$

(A1)

$$= \frac{5}{8} \left(= \frac{15}{24} \right)$$

A1

[4 marks]

Total [6 marks]

7. (a) considers at least two of $0.6 - 2a \geq 0$, $3a \geq 0$ and $0.4 - a \geq 0$ (M1)
 $\Rightarrow 0 \leq a \leq 0.3$ A1
[2 marks]

(b) **METHOD 1**

recognition that $\text{Var}(2 - X) = \text{Var}(X)$ (may be seen anywhere) R1

$a = 0.2 \Rightarrow E(X) = 2$ (by symmetry) A1

attempt to use $E(X^2) = \sum x^2 P(X = x)$ (M1)

$$E(X^2) = 0.2 + 2.4 + 1.8$$

$= 4.4$ (A1)

attempt to use $\text{Var}(X) = E(X^2) - (E(X))^2$ OR $\text{Var}(X) = E[(X - \mu)^2]$ (M1)

$$4.4 - 4 \text{ OR } (-1)^2 \times 0.2 + (1)^2 \times 0.2$$

$\text{Var}(X) = 0.4$ A1

METHOD 2

attempt to find distribution for $Y = 2 - X$ M1

$a = 0.2 \Rightarrow E(Y) = 0$ (by symmetry) A1

attempt to use $E(Y^2) = \sum y^2 P(Y = y)$ (M1)

$$E(Y^2) = 0.2 + 0 + 0.2$$

$= 0.4$ (A1)

attempt to use $\text{Var}(Y) = E(Y^2) - (E(Y))^2$ OR $\text{Var}(Y) = E[(Y - \mu)^2]$ (M1)

$$0.4 - 0 \text{ OR } (1)^2 \times 0.2 + (-1)^2 \times 0.2$$

$\text{Var}(Y) = 0.4$ A1

[6 marks]

Total [8 marks]

3. (a) attempt to find mean of x -values (M1)
 $\bar{x} = 6.45$ A1
 [2 marks]

- (b) (i) attempt to substitute their $\bar{x}$ into regression equation (M1)
 $\bar{y} = 0.014375 + 2.1625 \times 6.45$
 $\bar{y} = 14.0$ (13.9625 exact) A1

- (ii) attempt to use formula for the mean (M1)

$$\sum_{i=1}^8 y_i = 8 \times 13.9625 \dots (\Rightarrow p + 93.2 = 111.7)$$

 $p = 18.5$ A1

Note: Accept $p = 18.8$ from the use of 3 sf in part (b)(i).

Some candidates are attempting part (b)(ii) before (b)(i), and substitute 8.9 into the given regression equation to find p . They then use their value of p to find $\bar{y}$. In this case, do not award any marks in this part. Allow **FT** from their incorrect p in part (c).

[4 marks]

- (c) 0.953958...
 $r = 0.954$ A1

[1 mark]

Total [7 marks]

- 5.** (a) (i) 27.5 **A1**
- (ii) 6.26374...
6.26 **A1**
- [2 marks]**
- (b) 75th percentile = 31.7248... **(A1)**
25th percentile = 23.2751... **(A1)**
8.44965...
IQR = 8.4 **A1**
- [3 marks]**
- Total [5 marks]**

7. (a) recognition of binomial distribution

(M1)

$$\binom{10}{2}(p)^2(1-p)^8, B(10, p)$$

Note: The **(M1)** may be awarded for just stating 'binomial', parameters are not required, accept incorrect parameters. Do not award the **(M1)** if candidates only state np . Accept the use of calculator notation, such as binompdf.

attempt to consider both machines using $p = 0.1$ AND $p = 0.05$ **(M1)**

$$0.5 \times (0.193710...) + 0.5 \times (0.0746347...)$$

(A1)

0.134172...

0.134

A1**[4 marks]***continued...*

Question 7 continued.

(b) recognition of conditional probability

(M1)

$$P(A|2 \text{ defective}) \text{ OR } \frac{P(A \cap 2 \text{ defective})}{P(2 \text{ defective})}$$

Note: recognition must be shown in context, either in words or symbols, not just $P(A|B)$.

$$\frac{0.0968...}{0.134...}$$

(A1)

$$0.72187...$$

$$0.722$$

A1

Note: Accept 0.723 or 0.724 from using 3 sf values from part (a).

[3 marks]

Total [7 marks]

11. (a) METHOD 1

attempt to use integration by parts

M1

$$u = \arccos x, \quad \frac{dv}{dx} = 1, \quad \frac{du}{dx} = -\frac{1}{\sqrt{1-x^2}}, \quad v = x$$

$$x \arccos x + \int \frac{x}{\sqrt{1-x^2}} dx$$

(A1)attempt to integrate $\frac{x}{\sqrt{1-x^2}}$ by substitution or inspection**(M1)**

$$-\frac{1}{2} \times 2u^{\frac{1}{2}}, \quad -\sqrt{1-x^2}$$

$$x \arccos x - \sqrt{1-x^2} + C$$

A1
Note: Do not award the final **A1** if $+C$ is missing.
METHOD 2

attempt to use substitution

(M1)

$$\cos \theta = x$$

$$\theta = \arccos x$$

$$\frac{d\theta}{dx} = -\frac{1}{\sqrt{1-x^2}}, \quad \sin \theta = \sqrt{1-x^2}$$

$$-\int \theta \sin \theta d\theta$$

(A1)

attempt to use integration by parts

M1

$$u = \theta, \quad \frac{du}{d\theta} = 1, \quad \frac{dv}{d\theta} = \sin \theta, \quad v = -\cos \theta$$

$$\theta \cos \theta - \sin \theta$$

$$x \arccos x - \sqrt{1-x^2} + C$$

A1
Note: Do not award the final **A1** if $+C$ is missing.
[4 marks]*continued...*

Question 11 continued.

$$(b) \quad (i) \quad \int_0^k 3x \arccos(x^2) dx = 1 \quad (\text{seen anywhere}) \quad \mathbf{M1}$$

attempt to integrate **(M1)**

$$\frac{3}{2} \int \arccos u \, du$$

$$\frac{3}{2} \left[u \arccos u - \sqrt{1-u^2} \right]_0^{k^2} (=1) , \quad \frac{3}{2} \left[x^2 \arccos x^2 - \sqrt{1-x^4} \right]_0^k (=1) \quad \mathbf{A1}$$

Note: Condone incorrect or absent limits up to this point.

$$\frac{3}{2} \left(k^2 \arccos(k^2) - \sqrt{1-k^4} - (0-1) \right) = 1 \quad \mathbf{OR} \quad k^2 \arccos(k^2) - \sqrt{1-k^4} - (0-1) = \frac{2}{3} \quad \mathbf{A1}$$

$$k^2 \arccos(k^2) - \sqrt{1-k^4} + \frac{1}{3} = 0 \quad \mathbf{AG}$$

$$(ii) \quad k = 0.713250 \quad \mathbf{A2}$$

[6 marks]

continued...

Question 11 continued.

- (c) (i) attempt to substitute their value of k into the formula for $E(X)$ (M1)

$$\int_0^k 3x^2 \arccos(x^2) dx$$

$$0.456309\dots$$

$$E(X) = 0.456$$

A1

- (ii) **METHOD 1**

$$E(X^2) = 0.237 \quad (0.237198)$$

(A1)

attempt to use formula for variance (M1)

$$E(X^2) - [E(X)]^2$$

$$0.0289805\dots$$

$$\text{Var}(X) = 0.0290$$

A1

[5 marks]

METHOD 2

$$\text{Var}(X) = \int_0^{0.713250} (x - 0.456309\dots)^2 f(x) dx$$

(M1)(A1)

$$\text{Var}(X) = 0.0290$$

(A1)

[5 marks]

continued...

Question 11 continued.

(d) $\sigma = 0.170236\dots$ (seen anywhere) (A1)

$\int_{0.28607\dots}^{0.626549\dots} f(x) \, dx$ (A1)

0.620012...

0.620 A1

[3 marks]

Total [18 marks]

2.

- (a) the median is the middle value and as Aiden's score is below 10.5 and Brett's score is above 10.5, hence the middle value stays in the same place **R1**

Note: Award the **R1** for any valid reasoning referring to the scores being on either side of the median.

[1 mark]

- (b) attempt to use that the sum of scores is $n \times \text{mean}$ **(M1)**
 10.5×28 OR 10.6×30

sum of Aiden and Brett's scores = 24 ($24 = B + A$) (seen anywhere) **(A1)**

difference between Aiden and Brett's scores = 14 ($14 = B - A$) (seen anywhere) **(A1)**

attempt to solve their two simultaneous equations or by trial and improvement **(M1)**

(Aiden's score =) 5, (Brett's score =) 19 **A1**

Note: The first two **A** marks are independent of each other.

[5 marks]**Total [6 marks]**

4. (a) recognition that $P(\text{height of large sunflowers} > 180) = 0.25$. (M1)

$$\frac{180 - \mu}{\sigma} = 0.674489... \text{ OR } \mu + 0.674\sigma = 180 \quad (\text{A1})$$

$$A = 0.674489...$$

$$A = 0.674 \quad \text{A1}$$

[3 marks]

- (b) correct parameters for the height of giant sunflowers (A1)

$$\mu + 35 \text{ and } 2\sigma \text{ OR } \mu + 35 \text{ and } (2\sigma)^2$$

$$\frac{180 - \mu - 35}{2\sigma} = -2.05374... \quad (\text{A1})$$

attempt to solve their two simultaneous equations in two unknowns, μ and σ . (M1)

$$\mu = 175.063..., \sigma = 7.31913....$$

$$\mu = 175, \sigma = 7.32 \quad \text{A1}$$

[4 marks]

Total [7 marks]

11. (a) attempt to use $\int f(x) dx = 1$

M1

Note: The = 1 can be seen at any stage before **AG** last line. If the = 1 is not seen before **AG** last line award a maximum of **M0M1A1A0**.

recognition for the need to integrate $a(x-3)^3 + b(x-3)^2$ with limits of 3 and 9

M1

$$\int_3^9 a(x-3)^3 + b(x-3)^2 dx (=1)$$

$$\left[\frac{a}{4}(x-3)^4 + \frac{b}{3}(x-3)^3 \right]_3^9 (=1) \text{ OR } \left[a \left(\frac{x^4}{4} - 3x^3 + \frac{27x^2}{2} - 27x \right) + b \left(\frac{x^3}{3} - 3x^2 + 9x \right) \right]_3^9 (=1) \quad \textbf{(A1)}$$

$$\frac{a}{4} \times 6^4 + \frac{b}{3} \times 6^3 (=1) \text{ OR } \left(\frac{1215a}{4} + 81b \right) - \left(\frac{-81a}{4} + 9b \right) (=1)$$

A1

$$324a + 72b = 1$$

AG

[4 marks]

(b) (i) attempt to substitute $x = 9$ into $a(x-3)^3 + b(x-3)^2$

(M1)

$$a(9-3)^3 + b(9-3)^2 (=0)$$

$$a(9-3)^3 + b(9-3)^2 = 0 \text{ OR } a \times 6^3 + b \times 6^2 = 0 \text{ OR } 216a + 36b = 0$$

A1

$$6a + b = 0$$

AG

(ii) attempt to solve two linear equations in a and b

(M1)

$$a = -\frac{1}{108}, b = \frac{1}{18} \text{ OR}$$

$$a = -0.00926(-0.00925925...), b = 0.0556(0.0555555...)$$

A1

[4 marks]

continued...

Question 11 continued

- (c) attempt to find the maximum point on the graph of
- f
- (M1)

(7, 0.296296...)

mode = 7

A1

EITHERattempt to find the area below f between 3 and 7 (M1)

area = 0.592592...

A1

area > 0.5 or area > 0.407407... (area below f between 7 and 9) R1**OR**attempt to find the median by solving $\int_3^m f(x) = 0.5$ OR $\int_m^9 f(x) = 0.5$ (M1)

$$-\frac{1}{432}(m-3)^4 + \frac{1}{54}(m-3)^3 = 0.5 \text{ OR } -\frac{6^4}{432} + \frac{6^3}{54} - \left(-\frac{1}{432}(m-3)^4 + \frac{1}{54}(m-3)^3 \right) = 0.5 \quad (\text{A1})$$

median = 6.68563...

A1

Note: Award (M1)(A1)A0 for 6.69 seen but not recognized as the median.

THEN

median < mode

AG

[5 marks]

- (d) recognising conditional probability in context (M1)

$$P(X > 8.52 \mid X > 8)$$

recognition that $P(X > 8.52 \cap X > 8) = P(X > 8.52)$ (M1)

$$\frac{\int_{8.52}^9 f(x) dx}{\int_8^9 f(x) dx} \left(= \frac{0.03442688...}{0.13194444...} \right) \quad (\text{A1})$$

0.260919....

0.261 (0.263 from using 3 sf values for a and b)

A1

[4 marks]

Total [17 marks]

