APOLLO

Mathematics: applications and interpretation
SL Workbook

First assessment 2021
Whole questions arranged by syllabus topic.
Original question and scheme crops preserve source diagrams.

APOLLO

Number and algebra

52 questions

1. [Maximum mark: 4]

Katya approximates π, correct to four decimal places, by using the following expression.

$$3 + \frac{1}{6 + \frac{13}{16}}$$

(a) Calculate Katya's approximation of π, correct to four decimal places. [2]

(b) Calculate the percentage error in using Katya's four decimal place approximation of π, compared to the exact value of π in your calculator. [2]

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Question 1 - working space

8. [Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\,\text{m}$. On each subsequent day, Charlie ran $100\,\text{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme. [5]

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

(b) Find the value of n. [3]

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Question 8 - working space

10. [Maximum mark: 6]

Tommaso and Pietro have each been given 1500 euro to save for college.

Pietro invests his money in an account that pays a nominal annual interest rate of 2.75%, **compounded half-yearly**.

(a) Calculate the amount Pietro will have in his account after 5 years. Give your answer correct to 2 decimal places. [3]

Tommaso wants to invest his money in an account such that his investment will increase to 1.5 times the initial amount in 5 years. Assume the account pays a nominal annual interest of $r\%$ **compounded quarterly**.

(b) Determine the value of r. [3]

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Question 10 - working space

3. [Maximum mark: 19]

Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested $37\,000$ Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

(a) Calculate the value of Daisy's investment after 2 years. [3]

After m months, the amount of money in the fixed deposit account has appreciated to more than $50\,000$ AUD.

(b) Find the minimum value of m, where $m \in \mathbb{N}$. [4]

Daisy is saving to purchase a new apartment. The price of the apartment is $200\,000$ AUD.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

(c) Write down the amount of the loan. [1]

The loan is for 10 years, compounded monthly, with equal monthly payments of 1700 AUD made by Daisy at the end of each month.

(d) For this loan, find

(i) the amount of interest paid by Daisy.

(ii) the annual interest rate of the loan. [5]

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

(e) Find the amount of Daisy's final payment. [3]

(f) Find how much money Daisy saved by making one final payment after 5 years. [3]

Question 3 - working space

3. [Maximum mark: 16]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

(a) Find:

(i) the number of seats in the last row.

(ii) the total number of seats in the concert hall. [5]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

(b) Find the average number of visitors per concert in 2020. [2]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

(c) Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. [5]

It is assumed that the concert hall will host 50 concerts each year.

(d) Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025. [4]

Turn over

Question 3 - working space

10. [Maximum mark: 9]

In this question, give all answers correct to 2 decimal places.

Raul and Rosy want to buy a new house and they need a loan of 170 000 Australian dollars (AUD) from a bank. The loan is for 30 years and the annual interest rate for the loan is 3.8%, compounded monthly. They will pay the loan in fixed monthly instalments at the end of each month.

(a) Find the amount they will pay the bank each month. [3]

(b) (i) Find the amount Raul and Rosy will still owe the bank at the end of the first 10 years.

(ii) Using your answers to parts (a) and (b)(i), calculate how much interest they will have paid in total during the first 10 years. [6]

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Question 10 - working space

2. [Maximum mark: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	**Number of applications received in year n**
1	12 300
2	12 669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b) (i) Write down the common ratio of the sequence.

(ii) Find an expression for u_n.

(iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n. [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

Turn over

Question 2 - working space

2. [Maximum mark: 6]

The Osaka Tigers basketball team play in a multilevel stadium.



The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

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Question 2 - working space

1. [Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**.
No deposit required and repayments are made each quarter.

(a) (i) Find the repayment made each quarter.

(ii) Find the total amount paid for the car.

(iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of r% **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

(b) (i) Find the amount to be borrowed for this option.

(ii) Find the annual interest rate, r. [5]

(c) State which option Bryan should choose. Justify your answer. [2]

Bryan's car depreciates at an annual rate of 25% per year.

(d) Find the value of Bryan's car six years after it is purchased. [3]

Turn over

Question 1 - working space

5. [Maximum mark: 5]

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, $)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was $\$7816$.
- There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x, the number of child tickets sold be y, and the number of student tickets sold be z.

(a) Write down three equations that express the information given above. [3]

(b) Find the number of each type of ticket sold. [2]

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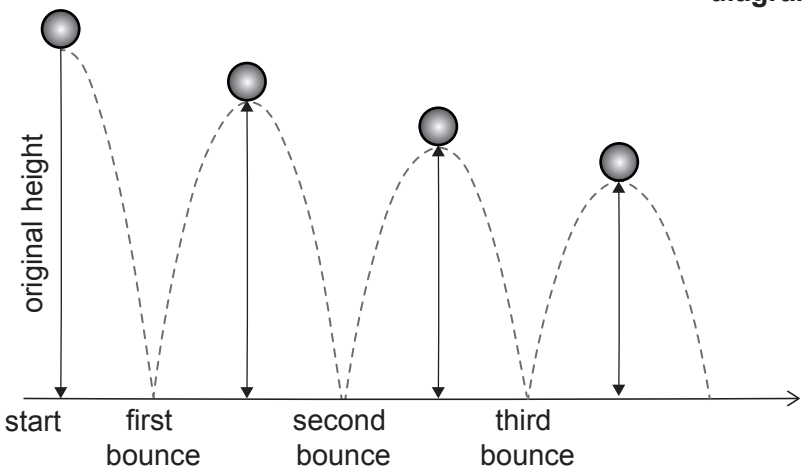


Question 5 - working space

13. [Maximum mark: 7]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.

diagram not to scale



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

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References:

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Question 13 - working space

13. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying $3.5\,\%$ interest, compounded annually. She expects the annual inflation rate to be $2\,\%$ per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $\$4000$, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $\$1000$ at the start of the 10-year period and then invest $\$x$ into the account at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3]

(b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. [3]

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References:

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Question 13 - working space

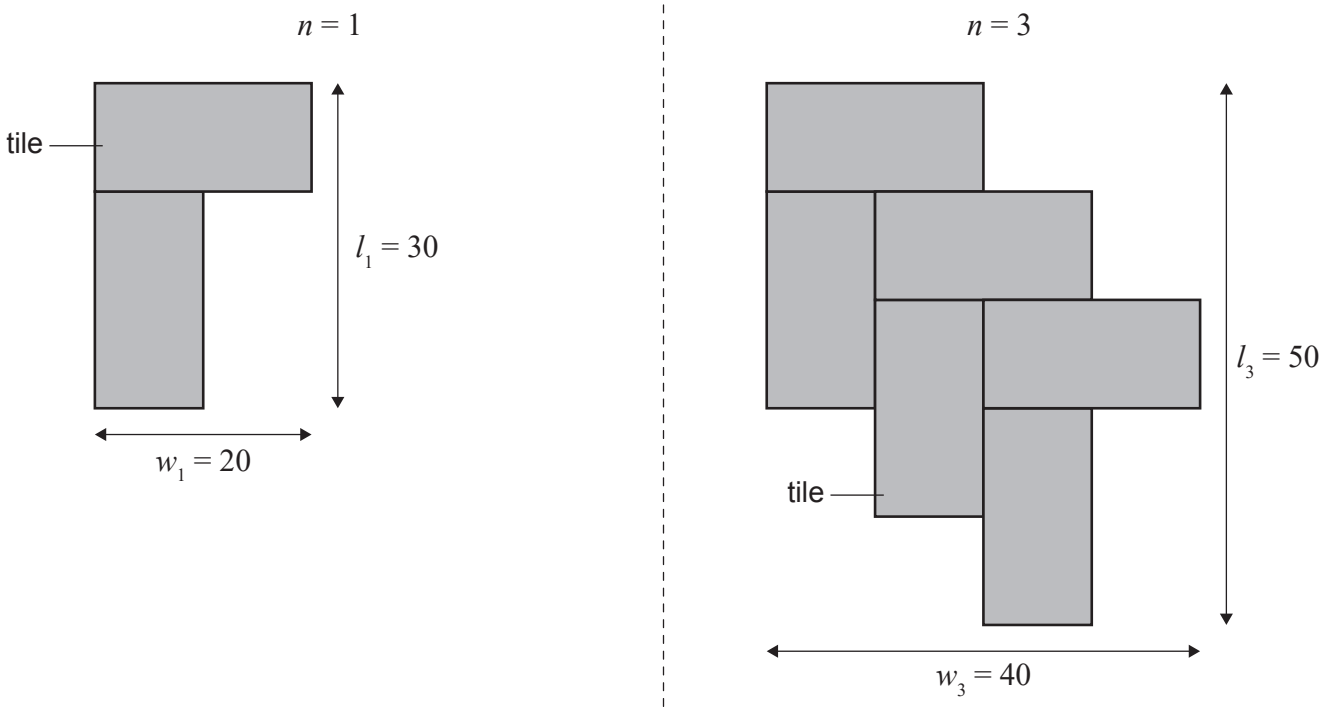
2. [Maximum mark: 19]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is $10\,\text{cm}$ wide and $20\,\text{cm}$ long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.

diagram not to scale



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

(This question continues on the following page)

(Question 2 continued)

(a) Find the value of

(i) a.

(ii) b. [2]

(b) Write down an expression in terms of n for

(i) w_n.

(ii) l_n. [3]

Eddie's lawn has a length $740\,\text{cm}$.

(c) (i) Show that Eddie needs 144 tiles.

(ii) Find the value of w_n for this path. [3]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles. [3]

To allow for breakages Eddie wants to have at least $8\,\%$ more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order. [3]

There is a fixed delivery cost of $\$35$.

(g) Find the total cost for Eddie's order. [2]

Turn over

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Question 2 - working space

2. [Maximum mark: 13]

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

(a) Find the number of cups of dog food

(i) fed to the dog per day;

(ii) remaining in the bag at the end of the first day. [4]

(b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

(c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]

(d) (i) Calculate the value of $\sum_{n=1}^{10}(625 \times 1.064^{(n-1)})$.

(ii) Describe what the value in part (d)(i) represents in this context. [3]

(e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

Question 2 - working space

2. [Maximum mark: 7]

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	Trees planted
1	85
2	115
3	145

(a) Find the number of trees to be planted in the 15th month. [3]

(b) Find the total number of trees to be planted in the first 15 months. [2]

(c) Find the mean number of trees planted per month during the first 15 months. [2]

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Question 2 - working space

7. [Maximum mark: 7]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional EUR 500 into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw EUR $10\,000$.

(a) Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. [4]

(b) For this value of k, find the interest earned in the savings account. Express your answer correct to the nearest EUR. [3]

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Question 7 - working space

10. [Maximum mark: 6]

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

(c) Find how many times brighter Acubens is compared to Ceres. [2]

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Question 10 - working space

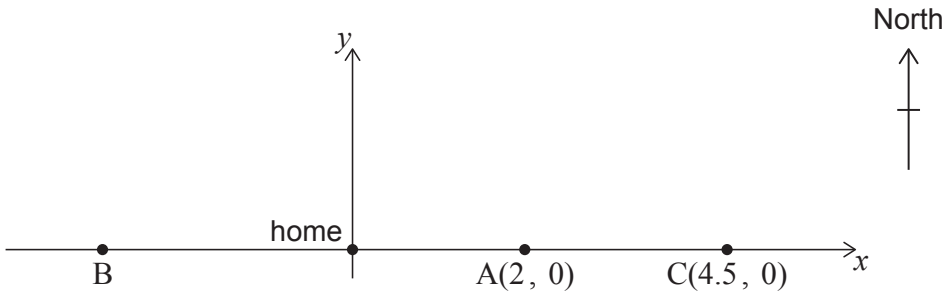
11. [Maximum mark: 7]

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0, 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $\mathrm{A}(2, 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $\mathrm{C}(4.5, 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a) Show that the common ratio, r, is -1.5. [2]

On the 6th day, Kristi runs to point F.

(b) Find the location of point F. [2]

(c) Find the total distance Kristi runs during the first 7 days of training. [3]

(This question continues on the following page)

(Question 11 continued)

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Question 11 - working space

3. [Maximum mark: 5]

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of 4%, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

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Question 3 - working space

8. [Maximum mark: 7]

"Password entropy" is a measure of the predictability of a computer password. The higher the entropy, the more difficult it is to guess the password.

The relationship between the password entropy, p, (measured in bits) and the number of guesses, G, required to decode the password is given by $0.301p = \log_{10} G$.

(a) Calculate the value of p for a password that takes 5000 guesses to decode. [2]

(b) Write down G as a function of p. [1]

(c) Find the number of guesses required to decode a password that has an entropy of 28 bits. Write your answer in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

There is a point on the graph of the function $G(p)$ with coordinates $(0, 1)$.

(d) Explain what these coordinate values mean in the context of computer passwords. [1]

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Question 8 - working space

1. [Maximum mark: 5]

Zaha is designing a bridge to cross a river. She believes that the weight of the steel needed for this bridge is approximately $53\,632\,000\,\text{kg}$.

The exact weight of the steel needed for the bridge is $55\,625\,000\,\text{kg}$.

(a) Find the percentage error in Zaha's approximation. [2]

Zaha's design is used to build five identical bridges.

(b) (i) Find the weight of the steel needed for these five bridges, **to three significant figures**.

(ii) Write down your answer to part (b)(i) in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

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Question 1 - working space

2. [Maximum mark: 6]

Angel has $\$520$ in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2% compounded quarterly.

(a) Calculate the amount of money Angel would have at the end of 5 years with the bank. Give your answer correct to two decimal places. [3]

Instead of investing the money, Angel decides to buy a phone that costs $\$520$. At the end of 5 years, the phone will have a value of $\$30$. It may be assumed that the depreciation rate per year is constant.

(b) Calculate the annual depreciation rate of the phone. [3]

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Question 2 - working space

2. [Maximum mark: 15]

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the 12th day. [3]

She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

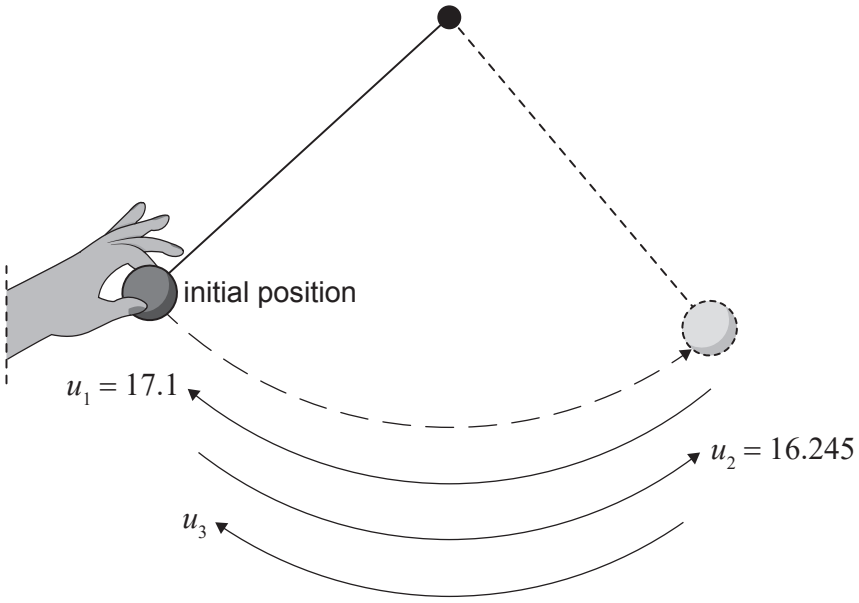
(b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.

diagram not to scale



(This question continues on the following page)

(Question 2 continued)

On each successive swing, the distance that the ball travelled was $95\,\%$ of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of $17.1\,\text{cm}$. During the second swing that she recorded, it travelled a distance of $16.245\,\text{cm}$.

(d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]

(e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]

(f) Calculate the distance that the ball travelled before Daina started recording. [2]

Turn over

Question 2 - working space

4. [Maximum mark: 16]

Ruben wants to buy a car for a price of $285\,000$ South African rand (ZAR). He goes to a bank to get a loan to buy the car. To be eligible for the loan, Ruben must make an initial down payment equal to $25\,\%$ of the price of the car.

The bank offers him a 5-year loan for the remaining balance, with a $4.5\,\%$ nominal interest rate per annum, compounded monthly. Ruben will pay the loan in fixed payments at the end of each month.

(a) (i) Find the original amount of the loan after the down payment is paid. Give the exact answer.

(ii) Calculate Ruben's monthly payment for this loan, to two decimal places. [5]

(b) Using your answer from part (a)(ii), calculate the total amount Ruben will pay over the life of the loan, to the nearest ZAR. Do **not** include the initial down payment. [2]

Ruben would like to repay the loan faster and increases his payments such that he pays $4600\,\text{ZAR}$ each month.

(c) Find the total number of monthly payments he will need to make to pay off the loan. [2]

This strategy will result in Ruben's final payment being less than $4600\,\text{ZAR}$.

(d) Determine the amount of Ruben's final payment, to two decimal places. [4]

(e) Hence, determine the total amount Ruben will save, to the nearest ZAR, by making the higher monthly payments. [3]

Question 4 - working space

4. [Maximum mark: 16]

Tiffany wants to buy a house for a price of $285\,000$ US Dollars (USD). She goes to a bank to get a loan to buy the house. To be eligible for the loan, Tiffany must make an initial down payment equal to $15\,\%$ of the price of the house.

The bank offers her a 30-year loan for the remaining balance, with a $4\,\%$ nominal interest rate per annum, compounded monthly. Tiffany will pay the loan in fixed payments at the end of each month.

(a) (i) Find the original amount of the loan after the down payment is paid. Give the exact answer.

(ii) Calculate Tiffany's monthly payment for this loan, to two decimal places. [5]

(b) Using your answer from part (a)(ii), calculate the total amount Tiffany will pay over the life of the loan, to the nearest dollar. Do **not** include the initial down payment. [2]

Tiffany would like to repay the loan faster and increases her payments such that she pays $1300\,\text{USD}$ each month.

(c) Find the total number of monthly payments she will need to make to pay off the loan. [2]

This strategy will result in Tiffany's final payment being less than $1300\,\text{USD}$.

(d) Determine the amount of Tiffany's final payment, to two decimal places. [4]

(e) Hence, determine the total amount Tiffany will save, to the nearest dollar, by making the higher monthly payments. [3]

Question 4 - working space

3. [Maximum mark: 7]

On 1 January 2025, the Faber Car Company will release a new car to global markets. The company expects to sell 40 cars in January 2025. The number of cars sold each month can be modelled by a geometric sequence where $r = 1.1$.

(a) Use this model to find the number of cars that will be sold in December 2025. [2]

(b) Use this model to find the total number of cars that will be sold in the year

(i) 2025.

(ii) 2026. [5]

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Question 3 - working space

5. [Maximum mark: 6]

Maan deposited $\$100\,000$ into a savings account with a nominal annual interest rate of $I\,\%$ **compounded monthly**. At the end of the eighth year, the amount in the account had increased to $\$150\,000$.

(a) Find the value of I. [3]

Maan withdraws the $\$150\,000$ and places it in an annuity, earning a nominal annual interest rate of $6.1\,\%$ **compounded monthly**. At the end of each month, Maan will receive a payment of $\$1000$.

(b) Find the amount of money remaining in the annuity at the end of 10 years. Express your answer to the nearest dollar. [3]

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Question 5 - working space

5. [Maximum mark: 9]

Imani invests $\$3000$ in a bank that pays a nominal annual interest rate of 1.25% compounded monthly.

(a) Calculate the amount of money Imani will have in the bank at the end of 6 years. Give your answer correct to two decimal places. [3]

(b) Calculate the number of months it takes until Imani has at least $\$3550$ in the bank. [2]

Imani uses the $\$3550$ as a partial payment for a used car costing $\$22\,000$. For the remainder she takes out a loan from a bank.

(c) Write down the amount of money that Imani takes out as a loan. [1]

The loan is for 8 years and the nominal annual interest rate is 12.6% compounded monthly. Imani will pay the loan in fixed monthly instalments at the end of each month.

(d) Calculate the amount, correct to the nearest dollar, that Imani will have to pay the bank each month. [3]

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Question 5 - working space

7. [Maximum mark: 7]

The pH scale is a measure of the acidity of a solution. Its value is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+],$$

where $[\text{H}^+]$ is the concentration of hydrogen ions in the solution (measured in moles per litre).

(a) Calculate the pH value if the concentration of hydrogen ions is 0.0003. [2]

The pH of milk is 6.6.

(b) Calculate the concentration of hydrogen ions in milk. [2]

The strength of an acid is measured by its concentration of hydrogen ions.

A lemon has a pH value of 2 and a tomato has a pH value of 4.5.

(c) Calculate how many times stronger the acid in a lemon is when compared to the acid in a tomato. [3]

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Question 7 - working space

9. [Maximum mark: 6]

A marathon is a race over a distance of $42.195\,\text{km}$. Two runners, Eefje and Shumay, are training to run a marathon.

The two runners train in different ways:

- Eefje runs $5\,\text{km}$ on the first day of training and then increases the distance she runs by $2\,\text{km}$ on each subsequent day.
- Shumay runs $5\,\text{km}$ on the first day of training and then increases the distance she runs by $13\,\%$ on each subsequent day.

Determine which runner will be the first to run the distance of a marathon on a particular day of their training, and state on which day of their training this will occur.

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Question 9 - working space

3. [Maximum mark: 5]

Tickets to enter a museum are priced at a dollars for adults and c dollars for children.

A school group of 7 adults and 60 children paid a total of $\$832$.

A family of 3 adults and 5 children paid a total of $\$108$.

(a) Write down **two** equations that represent this information. [2]

(b) Hence, find the price of

(i) an adult ticket

(ii) a child ticket. [2]

Rounded to the **nearest thousand**, there were $203\,000$ visitors at the museum last year.

(c) Write down the lower bound for the number of visitors last year. [1]

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Question 3 - working space

8. [Maximum mark: 5]

On 1 January in a particular year, Anton invests $\$18\,000$ in a new bank account. The account earns 4% simple interest, on the original $\$18\,000$, at the start of each subsequent year.

The amounts in the account at the start of each year form an arithmetic sequence.

(a) Find the common difference of this sequence. [2]

After k complete years, the amount in Anton's account will be greater than $\$32\,000$ for the first time.

(b) Find the value of k. [3]

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Question 8 - working space

10. [Maximum mark: 6]

When Humberto retires, he invests $\$300\,000$ in an annuity fund that earns interest at a nominal rate of 3.8% per year, compounded monthly.

Humberto then withdraws $\$2800$ at the end of every month to pay for his living expenses.

(a) Find how much is in the annuity fund after 8 years. [3]

(b) Calculate how many times Humberto is able to make these withdrawals. [3]

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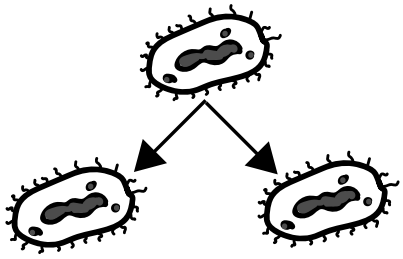
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Question 10 - working space

13. [Maximum mark: 7]

A type of bacteria reproduces by **dividing in two** every 10 minutes.



There were 1250 bacteria in a colony 10 minutes after the start of an experiment.

The following table is used to estimate the number of bacteria, u_n, for this colony.

The values of u_n form the terms of a sequence.

n	1	2	3		9		k
Time in minutes	10	20	30		90		$10k$
Number of bacteria, u_n	1250	2500	5000		…		…

(a) Complete the table by adding the two missing values. [3]

As the number of bacteria increases from 1250 to 2500, the total number of **bacterial divisions** is 1250.

(b) (i) Find the value of n when the number of bacteria is 1.28×10^6.

(ii) Hence or otherwise, find the total number of bacterial divisions as the number of bacteria increases from 1250 to 1.28×10^6. Give your answer correct to the nearest thousand bacterial divisions. [4]

(This question continues on the following page)

(Question 13 continued)

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Question 13 - working space

3. [Maximum mark: 5]

Tickets to enter a theme park are priced at a dollars for adults and c dollars for children.

A school group of 6 adults and 50 children paid a total of $\$1292$.

A family of 2 adults and 3 children paid a total of $\$130$.

(a) Write down **two** equations that represent this information. [2]

(b) Hence, find the price of

(i) an adult ticket

(ii) a child ticket. [2]

Rounded to the **nearest thousand**, there were $101\,000$ visitors at the theme park last year.

(c) Write down the lower bound for the number of visitors last year. [1]

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Question 3 - working space

8. [Maximum mark: 5]

On 1 January in a particular year, Eva invests $\$25\,000$ in a new bank account. The account earns 5% simple interest, on the original $\$25\,000$, at the start of each subsequent year.

The amounts in the account at the start of each year form an arithmetic sequence.

(a) Find the common difference of this sequence. [2]

After k complete years, the amount in Eva's account will be greater than $\$44\,000$ for the first time.

(b) Find the value of k. [3]

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Question 8 - working space

10. [Maximum mark: 6]

When Daniel retires, he invests $\$400\,000$ in an annuity fund that earns interest at a nominal rate of 4.5% per year, compounded monthly.

Daniel then withdraws $\$3600$ at the end of every month to pay for his living expenses.

(a) Find how much is in the annuity fund after 5 years. [3]

(b) Calculate how many times Daniel is able to make these withdrawals. [3]

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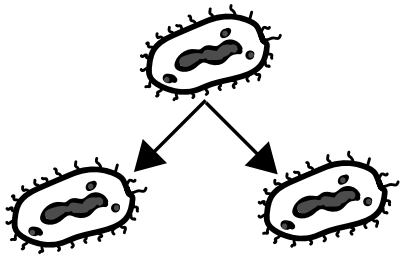
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Question 10 - working space

13. [Maximum mark: 7]

A type of bacteria reproduces by **dividing in two** every 10 minutes.



There were 1250 bacteria in a colony 10 minutes after the start of an experiment.

The following table is used to estimate the number of bacteria, u_n, for this colony.

The values of u_n form the terms of a sequence.

n	1	2	3		9		k
Time in minutes	10	20	30		90		$10k$
Number of bacteria, u_n	1250	2500	5000		…		…

(a) Complete the table by adding the two missing values. [3]

As the number of bacteria increases from 1250 to 2500, the total number of **bacterial divisions** is 1250.

(b) (i) Find the value of n when the number of bacteria is 1.28×10^6.

(ii) Hence or otherwise, find the total number of bacterial divisions as the number of bacteria increases from 1250 to 1.28×10^6. Give your answer correct to the nearest thousand bacterial divisions. [4]

(This question continues on the following page)

(Question 13 continued)

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Question 13 - working space

3. [Maximum mark: 5]

A bakery sells boxes of cupcakes at the following prices:

- a small box, containing 1 cupcake, costs $\$4.50$
- a medium box, containing 6 cupcakes, costs $\$23.50$
- a large box, containing 12 cupcakes, costs $\$44.75$.

Medium box



In a particular week, the bakery sold 88 boxes containing a total of 383 cupcakes. The bakery collected $\$1486$ from cupcake sales for the week.

The number of boxes of cupcakes sold during the week can be represented by

$$x + y + z = 88,$$

where x is the number of small boxes sold, y is the number of medium boxes sold, and z is the number of large boxes sold.

(a) Write down an equation in terms of x, y, and z to represent

(i) the number of cupcakes sold during the week.

(ii) the money collected from sales of cupcakes for the week. [3]

(b) Hence, use your graphic display calculator to determine the number of medium boxes that were sold during the week. [2]

(This question continues on the following page)

(Question 3 continued)

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Question 3 - working space

1. [Maximum mark: 7]

Give answers to this question correct to two decimal places.

Pierre invests 1500 euros (EUR) at the end of each month for 10 years into a savings plan that pays a nominal annual interest rate of 3.6% compounded monthly.

(a) Calculate the value of Pierre's savings plan at the end of the 10 years. [3]

At the end of the 10 years, Pierre withdraws $100\,000\,\text{EUR}$ from the savings plan to use as a deposit on a house.

Pierre invests the remainder into another account for 15 years at a nominal annual interest rate of 4.5% compounded quarterly.

(b) Calculate the amount in Pierre's account at the end of this time. [4]

(This question continues on the following page)

(Question 1 continued)

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Question 1 - working space

3. [Maximum mark: 6]

As part of an experiment, a colony of ants was created and observed. At the start of week one, 200 ants were observed in the colony. The number of ants in the colony increased by 15% each week.

(a) Calculate the number of ants in the colony at the start of week 11. [3]

The number of ants in the colony was first observed to exceed 3000 in week k.

(b) Find the value of k. [3]

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Question 3 - working space

6. [Maximum mark: 6]

A speed camera is used to determine whether a car is exceeding a speed limit of $8.3\,\text{m}\,\text{s}^{-1}$.

An exact distance of $10\,\text{m}$ is marked out.

The car travels this $10\,\text{m}$ distance in 1.2 seconds, measured to the nearest 0.1 second.

Determine whether it is certain that the car was exceeding the speed limit of $8.3\,\text{m}\,\text{s}^{-1}$.

Justify your answer.

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Question 6 - working space

7. [Maximum mark: 5]

The loudness of a sound, L, measured in decibels (dB) is determined by the intensity of the sound, I, measured in watts per square metre (Wm^{-2}). The relationship between loudness and intensity can be expressed using the logarithmic function

$$L = 10\log_{10}\left(\frac{I}{I_0}\right),\ I > 0$$

where I_0 is the reference intensity (the intensity of the least audible sound to the human ear).

The reference intensity I_0 is $10^{-12}\,\mathrm{Wm}^{-2}$.

The intensity of sound on a busy street is $10^{-5}\,\mathrm{Wm}^{-2}$.

(a) Calculate the loudness of the sound. [2]

The sound of a jet engine reaches a loudness of $185\,\mathrm{dB}$.

(b) Determine the intensity of its sound. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

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Question 7 - working space

11. [Maximum mark: 8]

Sofia takes out a loan for $50\,000$ euros. She pays a nominal interest rate of 4.2% per year, compounded monthly. She must pay back the loan in 25 years through regular monthly payments, made at the end of each month.

(a) Find the amount Sofia must pay back each month. Give your answer correct to two decimal places. [2]

At the end of eight years, Sofia wins the lottery and pays back the remainder of the loan early in one single payment.

(b) (i) Find how much Sofia pays back in this single payment. Give your answer correct to two decimal places.

(ii) Find the amount of money saved by Sofia by paying back the loan early. [6]

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Question 11 - working space

2. [Maximum mark: 15]

Give all answers in this question to the nearest dollar.

Anika invested $10\,000$ Canadian dollars (CAD) in an account which earns a nominal annual interest rate of 3.4%, compounded monthly.

(a) Calculate the value of Anika's investment after 6 years. [3]

Anika wants to purchase a house that costs $225\,000\,\text{CAD}$. Anika uses the value obtained in part (a) to make an initial payment on the house and takes out a loan to pay the rest.

(b) Determine the amount of Anika's loan. [1]

Anika amortizes her loan over 15 years. She arranges to make a loan payment at the end of each month. The nominal annual interest rate for the loan is 6.4%, compounded half-yearly.

(c) Determine Anika's monthly loan payment. [5]

After 2 complete years, Anika decides to increase her monthly loan payment to $2200\,\text{CAD}$. At the time, the remaining amount of the loan is $194\,572\,\text{CAD}$.

(d) (i) Calculate the amount by which the loan will decrease during the next 3 complete years.

(ii) Determine the total interest that Anika will pay over this 3-year period. [6]

Question 2 - working space

3. [Maximum mark: 17]

Felix is training for a competition consisting of running and swimming. He plans to train for 12 weeks to prepare for the competition.

The running part of the competition is a $5\,\text{km}$ race. During the 12 weeks, Felix runs on **3 days each week**. On the first day of training, Felix runs $2\,\text{km}$. Each subsequent day that he runs, he increases his distance by 5%.

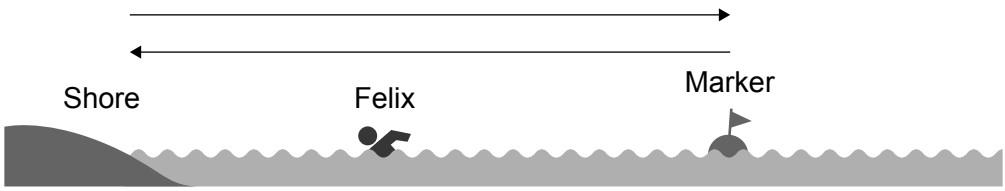
(a) Explain why the distances of each of Felix's runs form a geometric sequence and state the common ratio. [2]

(b) Find the total distance of Felix's first 15 runs. [2]

A published recommendation states that, on the last day of training, an athlete should run a distance that is at least twice as far as the race distance.

(c) Show that Felix's final run of his 12-week training plan meets this recommendation. [3]

For the swimming part of the competition, competitors must swim $375\,\text{m}$ out to a marker in the water and then $375\,\text{m}$ back to the shore.



Felix plans to swim **2 times per week** to train for the swimming portion of the competition. On each day he swims in the 1st week, Felix swims out to a marker that is $80\,\text{m}$ from the shore, and then swims back to shore.

At the start of each subsequent week, the marker is moved to increase the distance between the marker and the shore by a constant amount. Felix swims a total distance of $704\,\text{m}$ in the 4th week.

The distances that Felix swims each week form an arithmetic sequence, where each term, u_n, is the **total distance** he swims in the nth week.

(d) (i) Find u_1.

(ii) Determine the increase in the total distance Felix swims each week.

(iii) Hence calculate the distance between the marker and the shore in the 2nd week of training. [7]

(e) Calculate the distance of Felix's final swim in his 12th week of training. [3]

Turn over

Question 3 - working space

1. [Maximum mark: 16]

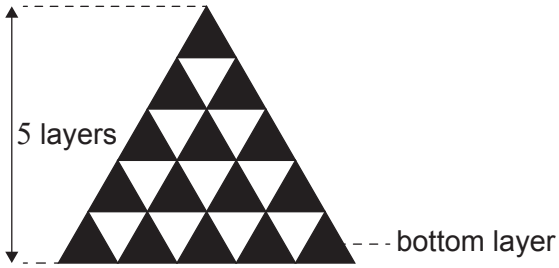
Thai cushions are designed with a triangular cross-section and are made from layers of smaller cushions. These cushions can be modelled as triangular prisms.

This is shown in the diagram.

diagram not to scale



Thai cushion with 4 layers



Cross-section of Thai cushion with 5 layers

(a) Write down the number of triangular prisms in the bottom layer of the cushion with

(i) 4 layers.

(ii) 5 layers. [2]

Mayumi notices that the number of triangular prisms in the bottom layer of the cushions forms an arithmetic sequence.

(b) (i) Write down the common difference of this sequence.

(ii) Find an expression for the number of triangular prisms in the bottom layer of a cushion with n layers. [3]

(This question continues on the following page)

(Question 1 continued)

Mayumi wants to extend this design to create a cushion with 9 layers.

(c) (i) Find the number of triangular prisms in the bottom layer of Mayumi's cushion.

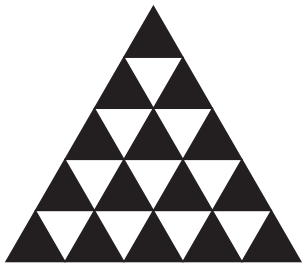
(ii) Calculate the **total** number of triangular prisms in Mayumi's cushion. [3]

(d) Find an expression for the **total** number of triangular prisms in a cushion with n layers, giving your answer in its simplest form. [2]

The cross-section of the cushion consists of black triangles and white triangles.



This cushion with 4 layers has a total of 6 white triangles.



This cushion with 5 layers has 4 white triangles in its bottom layer.

(e) Write down the total number of black triangles in a cushion with 4 layers. [1]

The number of black triangles in each layer forms an arithmetic sequence.

(f) Find and simplify an expression for the total number of black triangles in a cushion with n layers. [2]

The total number of white triangles in a cushion with n layers is $\frac{n(n-1)}{2}$.

(g) Using both the given expression and your answer to part (f), find and simplify an expression for the total number of black and white triangles in a cushion with n layers. [3]

Question 1 - working space

1. [Maximum mark: 18]

Cathie is a financial analyst studying the growth of two investment accounts, Account 1 and Account 2, for a new client.

Account 1 has an initial amount of 5000 US Dollars (USD). Interest is added to the amount in Account 1 at the end of each year in the following manner: $200\,\text{USD}$ at the end of the first year, $260\,\text{USD}$ at the end of the second year, $320\,\text{USD}$ at the end of the third year, $380\,\text{USD}$ at the end of the fourth year and $440\,\text{USD}$ at the end of the fifth year.

Assume the amount of interest continues to increase each year so that it follows an arithmetic sequence.

(a) Find

(i) the common difference.

(ii) the amount of interest, in USD, added at the end of the 10th year. [3]

(b) Show that the amount of money in Account 1 after n years may be expressed as

$$5000+\frac{n}{2}(340+60n).$$ [3]

(c) Hence or otherwise, find the amount of money in Account 1 at the end of 10 years. [2]

Account 2 has the same initial amount of $5000\,\text{USD}$. Account 2 pays 6.5% interest compounded annually. The interest is added to the amount in the account at the end of each year.

The amount in Account 2 after n years can be expressed as $5000\times B^n$ where $B\in\mathbb{R}$.

(d) (i) Write down the value of B.

(ii) Hence or otherwise, show that Account 1 will have more money than Account 2 at the end of 10 years. [4]

The client is interested in a longer-term investment. Cathie finds that it will take at least m complete years for the amount in Account 2 to exceed the amount in Account 1.

(e) Find the value of m. [3]

(f) Determine the total interest added to Account 2 at the end of m years. Give your answer correct to the nearest dollar. [3]

Question 1 - working space

1. [Maximum mark: 5]

The first three terms of a geometric sequence are 2, 6 and 18.

(a) Write down the common ratio, r. [1]

(b) Find the 8th term of the sequence. [2]

(c) Find the sum of the first 10 terms of the sequence. [2]

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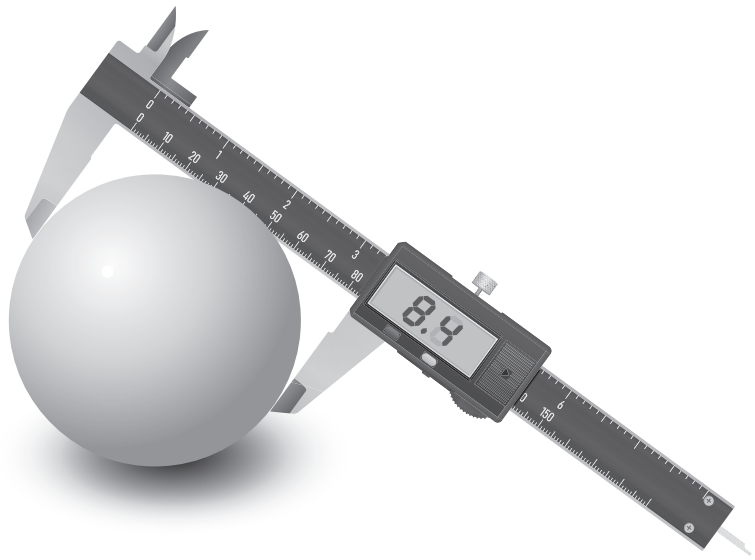
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Question 1 - working space

2. [Maximum mark: 6]

A digital caliper is used to measure the diameter of a spherical ball.

diagram not to scale



The digital caliper displays the diameter of the ball as $8.4\,\text{cm}$, and is accurate to one decimal place.

(a) (i) Write down the smallest possible diameter of the ball.

(ii) Hence, calculate the smallest possible volume of the ball. [4]

The exact diameter of the ball is $8.38\,\text{cm}$.

(b) Calculate the percentage error in the measurement of $8.4\,\text{cm}$, as shown on the digital caliper. [2]

(This question continues on the following page)

(Question 2 continued)

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Question 2 - working space

5. [Maximum mark: 6]

Mikko deposits $\$10\,000$ into a savings account with a fixed nominal annual interest rate of $r\%$ compounded monthly. At the end of each month, he deposits $\$800$ into the account.

After 10 years, the value in the savings account is $\$160\,000$.

(a) Find the value of r. [3]

(b) Find the amount of interest earned over the 10 years. [3]

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Question 5 - working space

1. [Maximum mark: 15]

A marathon is a race over a distance of $42.19\,\text{km}$.

(a) (i) Write down $42.19\,\text{km}$ correct to the nearest kilometre.

(ii) Find the percentage error if $42.19\,\text{km}$ is rounded to the nearest kilometre. [3]

Rolando and Davi are training to run a marathon. They both start their training in the same week.

Rolando runs $6\,\text{km}$ in the first week, $8\,\text{km}$ in the second week and $10\,\text{km}$ in the third week. In each subsequent week he runs $2\,\text{km}$ further than the previous week.

(b) Find the week during which Rolando runs $42\,\text{km}$. [3]

(c) Find the **total** distance that Rolando runs in the first 10 weeks. [2]

Davi runs $3\,\text{km}$ in the first week. In each subsequent week he runs $20\,\%$ further than the previous week. These distances form the terms of a geometric sequence.

(d) Find the first week during which Davi runs more than $42\,\text{km}$. [4]

(e) Find the first week that the **total** distance that Davi has run, since the start of training, exceeds $150\,\text{km}$. [3]

Turn over

Question 1 - working space

APOLLO

Functions

64 questions

4. [Maximum mark: 7]

The price of gas at Leon's gas station is $\$1.50$ per litre. If a customer buys a minimum of 10 litres, a discount of $\$5$ is applied.

This can be modelled by the following function, L, which gives the total cost when buying a minimum of 10 litres at Leon's gas station.

$$L(x) = 1.50x - 5,\ x \geq 10$$

where x is the number of litres of gas that a customer buys.

(a) Find the total cost of buying 40 litres of gas at Leon's gas station. [2]

(b) Find $L^{-1}(70)$. [2]

The price of gas at Erica's gas station is $\$1.30$ per litre. A customer must buy a minimum of 10 litres of gas. The total cost at Erica's gas station is cheaper than Leon's gas station when $x > k$.

(c) Find the minimum value of k. [3]

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Question 4 - working space

7. [Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\,\mathrm{e}^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten $50\,\%$ of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]

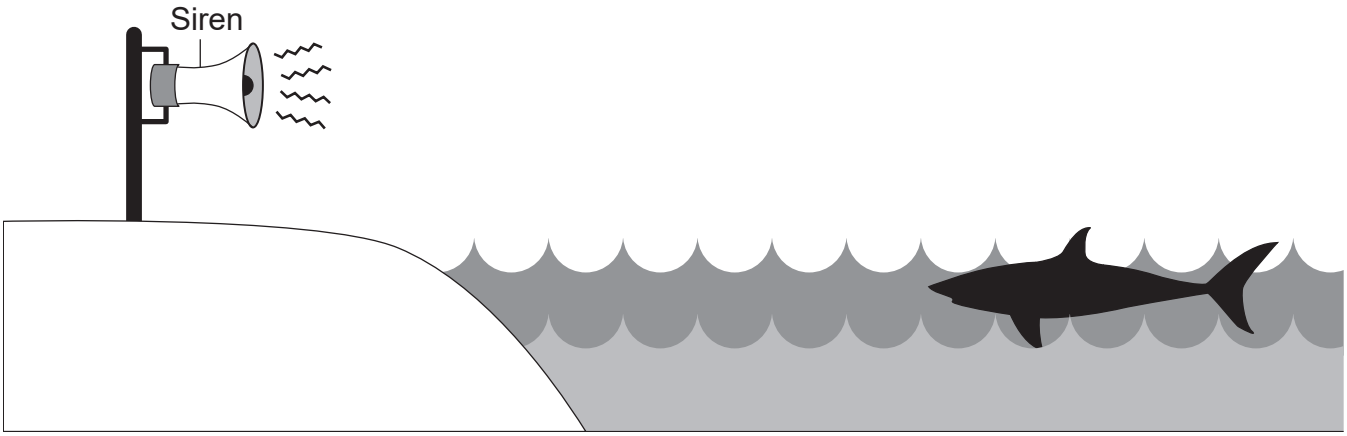
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Question 7 - working space

11. [Maximum mark: 6]

If a shark is spotted near to Brighton beach, a lifeguard will activate a siren to warn swimmers.

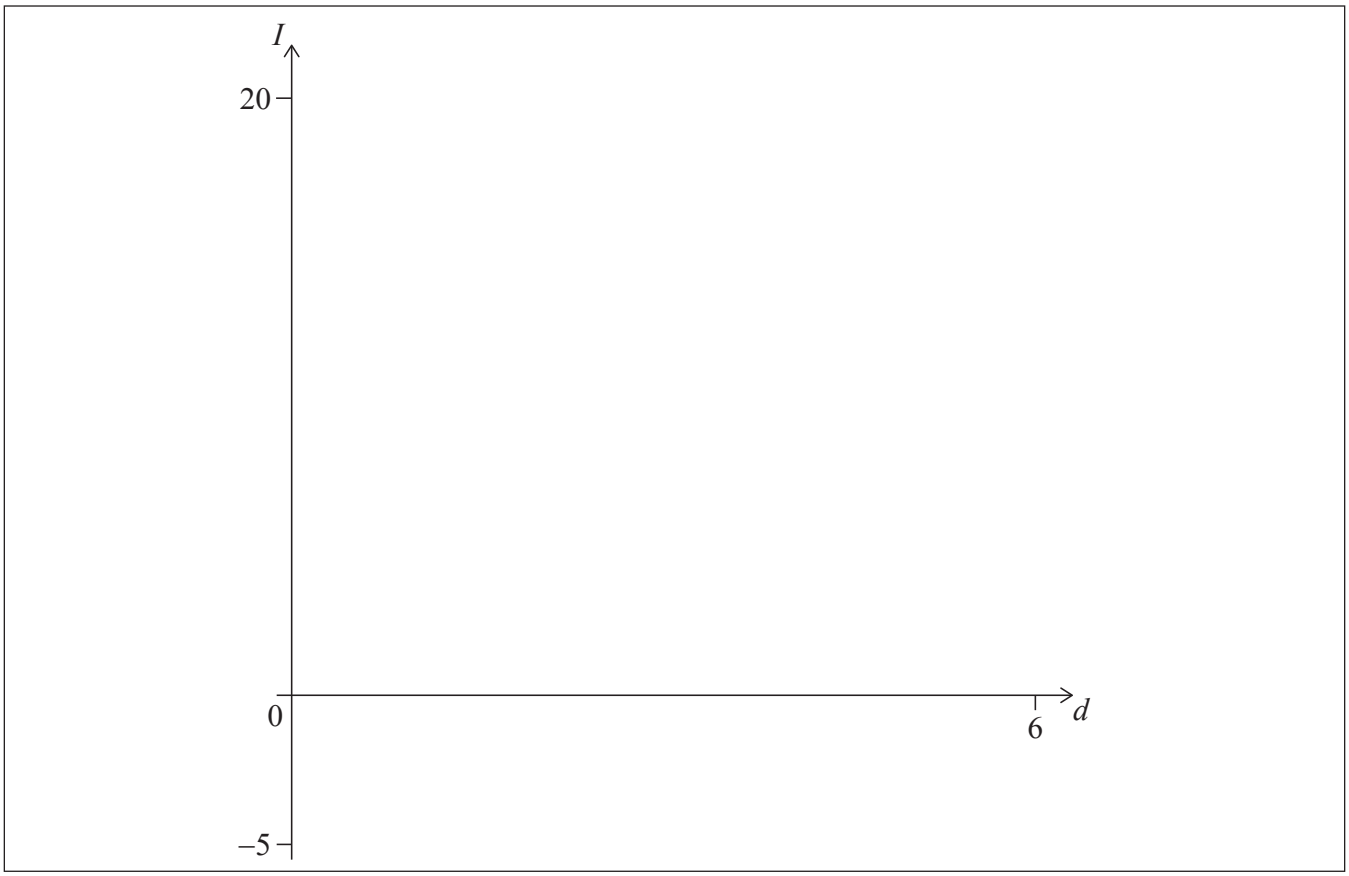


The sound intensity, I, of the siren varies inversely with the square of the distance, d, from the siren, where $d > 0$.

It is known that at a distance of 1.5 metres from the siren, the sound intensity is 4 watts per square metre $(\mathrm{W\,m^{-2}})$.

(a) Show that $I = \frac{9}{d^2}$. [2]

(b) Sketch the curve of I on the axes below showing clearly the point $(1.5, 4)$. [2]



(This question continues on the following page)

(Question 11 continued)

Whilst swimming, Scarlett can hear the siren only if the sound intensity at her location is greater than $1.5 \times 10^{-6}\,\text{Wm}^{-2}$.

(c) Find the values of d where Scarlett cannot hear the siren. [2]

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Question 11 - working space

1. [Maximum mark: 5]

The amount, in milligrams, of a medicinal drug in the body t hours after it was injected is given by $D(t) = 23(0.85)^t,\ t \geq 0$. Before this injection, the amount of the drug in the body was zero.

(a) Write down

(i) the initial dose of the drug.

(ii) the percentage of the drug that leaves the body each hour. [3]

(b) Calculate the amount of the drug remaining in the body 10 hours after the injection. [2]

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Question 1 - working space

7. [Maximum mark: 5]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \leq x \leq 7$, $x \neq -5$.

(a) Find the range of f. [3]

(b) Find the value of $f^{-1}(0)$. [2]

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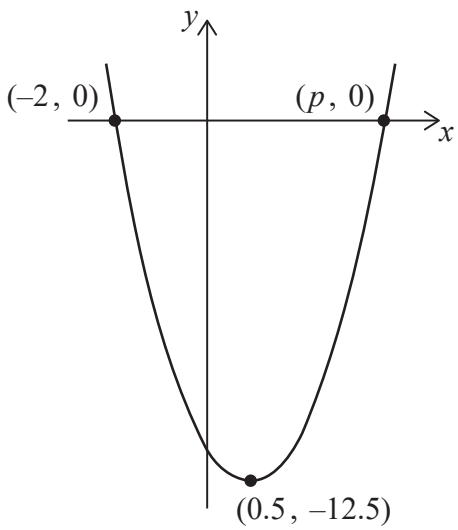
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Question 7 - working space

12. [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x-axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale



(a) Find the value of p. [1]

(b) Find the value of

(i) a.

(ii) b.

(iii) c. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

(This question continues on the following page)

(Question 12 continued)

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Question 12 - working space

3. [Maximum mark: 4]

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = A\mathrm{e}^{kt},$$

where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is $112\,\mathrm{mm}^2$ at the start of the experiment and $360\,\mathrm{mm}^2$ after 5 days.

(a) Write down the value of A. [1]

(b) Find the value of k. [3]

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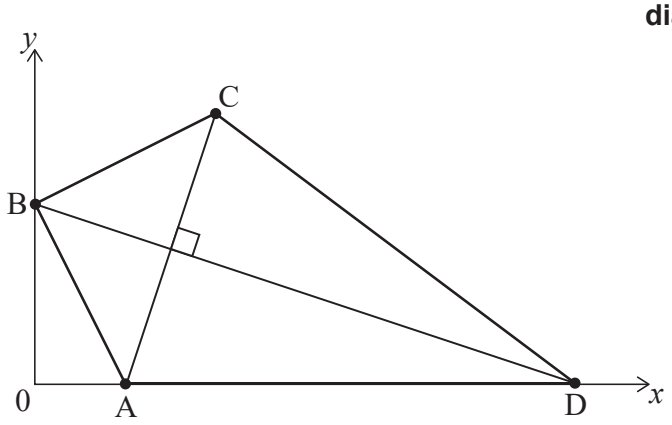
Question 3 - working space

4. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents $10\,\mathrm{cm}$.

The coordinates of A, B and C are $(2,\ 0)$, $(0,\ 4)$ and $(4,\ 6)$ respectively. Point D lies on the x-axis. $[\mathrm{AC}]$ is perpendicular to $[\mathrm{BD}]$. This information is shown in the following diagram.

diagram not to scale



(a) Find the gradient of the line through A and C. [2]

(b) Write down the gradient of the line through B and D. [1]

(c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

(d) Write down the x-coordinate of point D. [1]

(This question continues on the following page)

(Question 4 continued)

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Question 4 - working space

5. [Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter $x\,\text{cm}$.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \leq x \leq 14.$$

(a) Find the range of h. [3]

The function h^{-1} is the inverse function of h.

(b) (i) Find $h^{-1}(10)$.

(ii) In the context of the question, interpret your answer to part (b)(i).

(iii) Write down the range of h^{-1}. [4]

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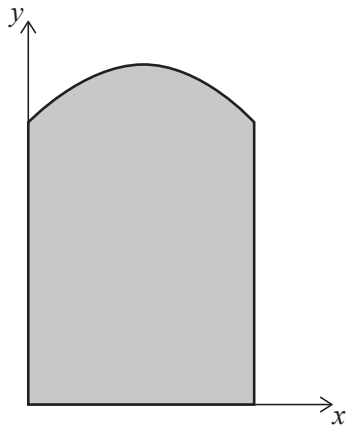
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Question 5 - working space

13. [Maximum mark: 8]

Irina uses a set of coordinate axes to draw her design of a window. The base of the window is on the x-axis, the upper part of the window is in the form of a quadratic curve and the sides are vertical lines, as shown on the diagram. The curve has end points $(0, 10)$ and $(8, 10)$ and its vertex is $(4, 12)$. Distances are measured in centimetres.



The quadratic curve can be expressed in the form $y = ax^2 + bx + c$ for $0 \leq x \leq 8$.

(a) (i) Write down the value of c.

(ii) Hence form two equations in terms of a and b.

(iii) Hence find the equation of the quadratic curve. [5]

(b) Find the area of the shaded region in Irina's design. [3]

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(This question continues on the following page)

(Question 13 continued)

References:

2. Bermuda Triangle map [online] Available at: https://commons.wikimedia.org/wiki/File:Bermuda_Triangle_map_(de).svg Thomas Römer. This file is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license. (CC BY-SA 3.0) https://creativecommons.org/licenses/by-sa/3.0/deed.en [Accessed 17 December 2020] Source adapted.

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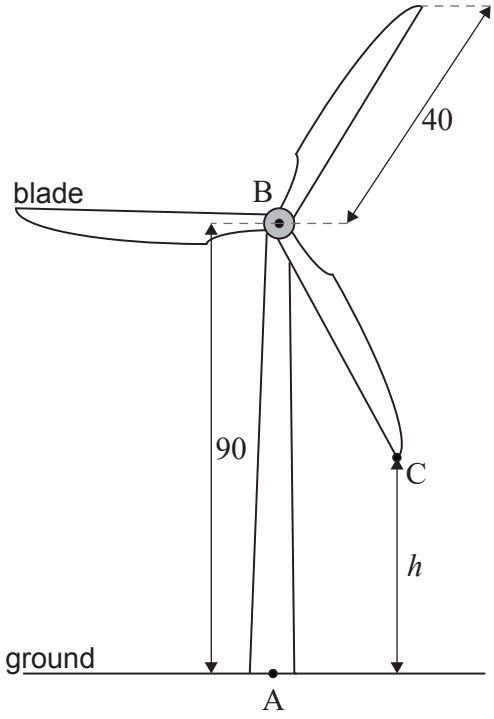
Question 13 - working space

3. [Maximum mark: 20]

A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is $90\,\mathrm{m}$. The blades of the turbine are centred at B and are each of length $40\,\mathrm{m}$. This is shown in the following diagram.

diagram not to scale



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

(a) Find the

(i) maximum value of h.

(ii) minimum value of h. [2]

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

(b) (i) Find the time, in seconds, it takes for the blade $[\mathrm{BC}]$ to make one complete rotation under these conditions.

(ii) Calculate the angle, in degrees, that the blade $[\mathrm{BC}]$ turns through in one second. [3]

(This question continues on the following page)

(Question 3 continued)

The height, h, of point C can be modelled by the following function. Time, t, is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40\cos(72t^\circ),\ t \geq 0$$

(c) (i) Write down the amplitude of the function.

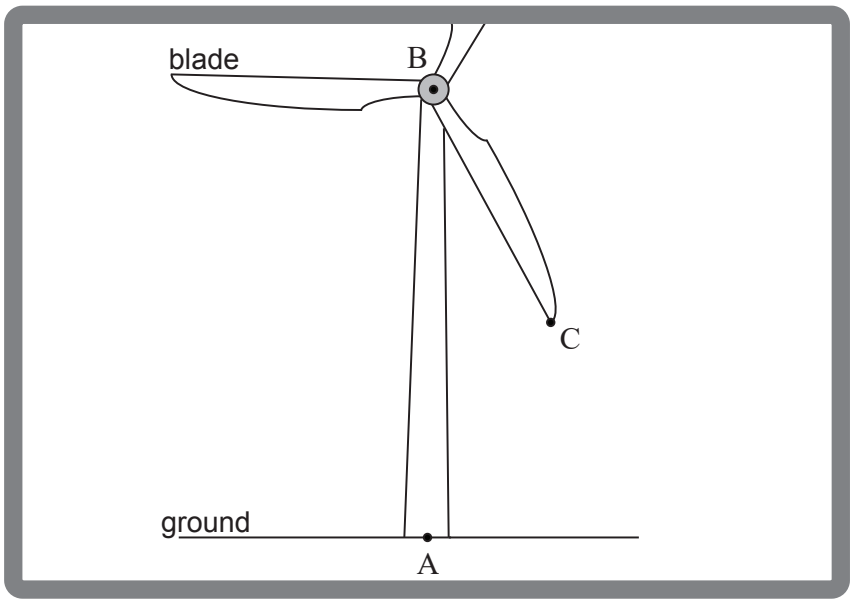
(ii) Find the period of the function. [2]

(d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points. [3]

(e) (i) Find the height of C above the ground when $t = 2$.

(ii) Find the time, in seconds, that point C is above a height of $100\,\text{m}$, during each complete rotation. [5]

Looking through his window, Tim has a partial view of the rotating wind turbine. The position of his window means that he cannot see any part of the wind turbine that is **more than $100\,\text{m}$** above the ground. This is illustrated in the following diagram.



(f) (i) At any given instant, find the probability that point C is visible from Tim's window.

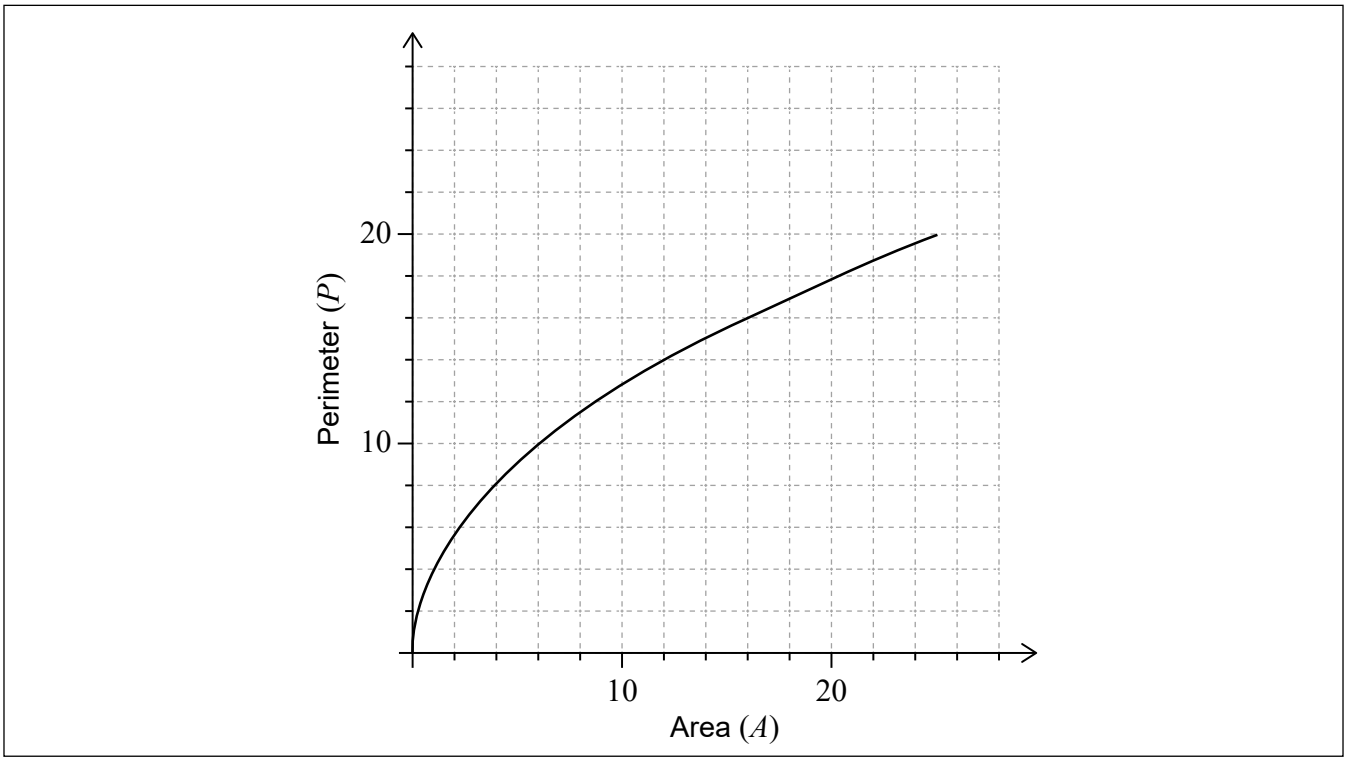
The wind speed increases. The blades rotate at twice the speed, but still at a constant rate.

(ii) At any given instant, find the probability that Tim can see point C from his window. Justify your answer. [5]

Question 3 - working space

4. [Maximum mark: 6]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

The range of $P(A)$ is $0 \leq P(A) \leq n$.

(b) Hence write down the value of n. [1]

(c) On the axes above, draw the graph of the inverse function, P^{-1}. [3]

(d) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

(This question continues on the following page)

(Question 4 continued)

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Question 4 - working space

5. [Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

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Question 5 - working space

8. [Maximum mark: 4]

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre ($\mathrm{W\,m^{-2}}$). The intensity level is given by the following formula.

$$L = 10\log_{10}(S \times 10^{12}),\ S \geq 0$$

(a) An orchestra has a sound intensity of $6.4 \times 10^{-3}\,\mathrm{W\,m^{-2}}$. Calculate the intensity level, L of the orchestra. [2]

(b) A rock concert has an intensity level of $112\,\mathrm{dB}$. Find the sound intensity, S. [2]

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Question 8 - working space

5. [Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s\,\text{m s}^{-1}$, and braking distance, $d\,\text{m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s\,\text{m s}^{-1}$	0	6	10
Braking distance, $d\,\text{m}$	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where p, $q \in \mathbb{Z}$

At a speed of $6\,\text{m s}^{-1}$, Model A can be represented by the equation $6p + q = 2$.

(a) (i) Write down a second equation to represent Model A, when the speed is $10\,\text{m s}^{-1}$.

(ii) Find the values of p and q [4]

(b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d(s) = 0.95s^2 - 3.92s$

(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\,\text{m s}^{-1}$. [2]

The actual braking distance at $20\,\text{m s}^{-1}$ is $320\,\text{m}$.

(f) Calculate the percentage error in the estimate in part (e). [2]

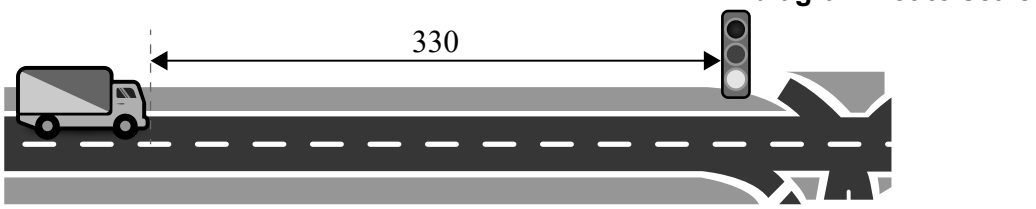
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(Question 5 continued)

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s\ \text{m s}^{-1}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330\,\text{m}$.

diagram not to scale



(g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection. [3]

Question 5 - working space

3. [Maximum mark: 5]

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

(b) Find the value of t when the ball hits the ground. [2]

(c) State an appropriate domain for t in this model. [2]

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Question 3 - working space

11. [Maximum mark: 8]

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$$\log_{10} N = a - M, \text{ for some } a \in \mathbb{R}.$$

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

(c) Given $0 < M < 8$, find the range for N. [2]

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(d) Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year. [2]

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Question 11 - working space

4. [Maximum mark: 5]

The pH of a solution measures its acidity and can be determined using the formula $\text{pH} = -\log_{10} C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

(a) Calculate the pH of the coffee. [2]

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

(b) Determine whether the unknown liquid is more or less acidic than the coffee. Justify your answer mathematically. [3]

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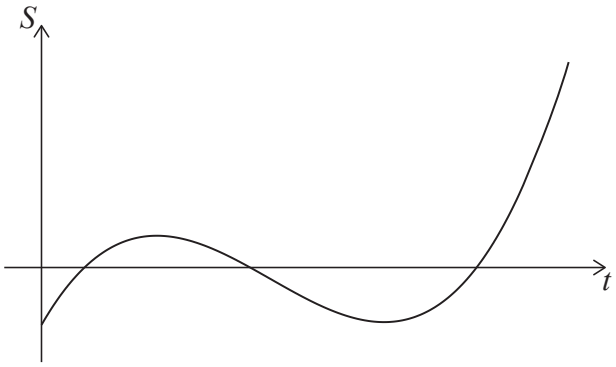
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Question 4 - working space

9. [Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t, the number of years after graduating from university.



(a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario. [1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a, b, c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	−5	3	−1	−5

(b) (i) Write down the value of d.

(ii) Write down three simultaneous equations for a, b and c.

(iii) Hence, or otherwise, find the values of a, b and c. [4]

A negative value of S indicates that a graduate is expected to be in debt.

(c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university. [3]

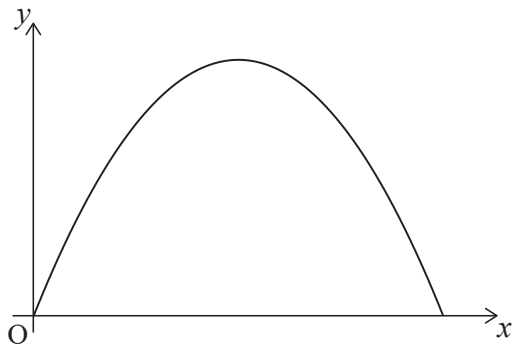
(This question continues on the following page)

(Question 9 continued)

Question 9 - working space

12. [Maximum mark: 5]

The cross-section of an arched entrance into the ballroom of a hotel is in the shape of a parabola. This cross-section can be modelled by part of the graph $y = -1.6x^2 + 4.48x$, where y is the height of the archway, in metres, at a horizontal distance, x metres, from the point O, in the bottom corner of the archway.



(a) Determine an equation for the axis of symmetry of the parabola that models the archway. [2]

To prepare for an event, a square-based crate that is $1.6\,\text{m}$ wide and $2.0\,\text{m}$ high is to be moved through the archway into the ballroom. The crate must remain upright while it is being moved.

(b) Determine whether the crate will fit through the archway. Justify your answer. [3]

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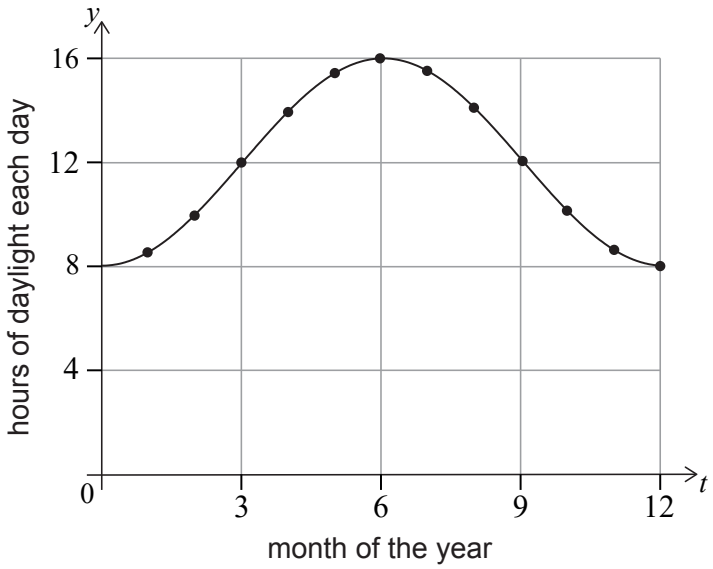
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Question 12 - working space

1. [Maximum mark: 15]

Boris recorded the number of daylight hours on the first day of each month in a northern hemisphere town.

This data was plotted onto a scatter diagram. The points were then joined by a smooth curve, with minimum point $(0,\ 8)$ and maximum point $(6,\ 16)$ as shown in the following diagram.



Let the curve in the diagram be $y = f(t)$, where t is the time, measured in months, since Boris first recorded these values.

Boris thinks that $f(t)$ might be modelled by a quadratic function.

(a) Write down one reason why a quadratic function would not be a good model for the number of hours of daylight per day, across a number of years. [1]

(This question continues on the following page)

(Question 1 continued)

Paula thinks that a better model is $f(t) = a\cos(bt) + d$, $t \geq 0$, for specific values of a, b and d.

(b) For Paula's model, use the diagram to write down

(i) the amplitude.

(ii) the period.

(iii) the equation of the principal axis. [4]

(c) Hence or otherwise find the equation of this model in the form: [3]

$$f(t) = a\cos(bt) + d$$

(d) For the first year of the model, find the length of time when there are more than 10 hours and 30 minutes of daylight per day. [4]

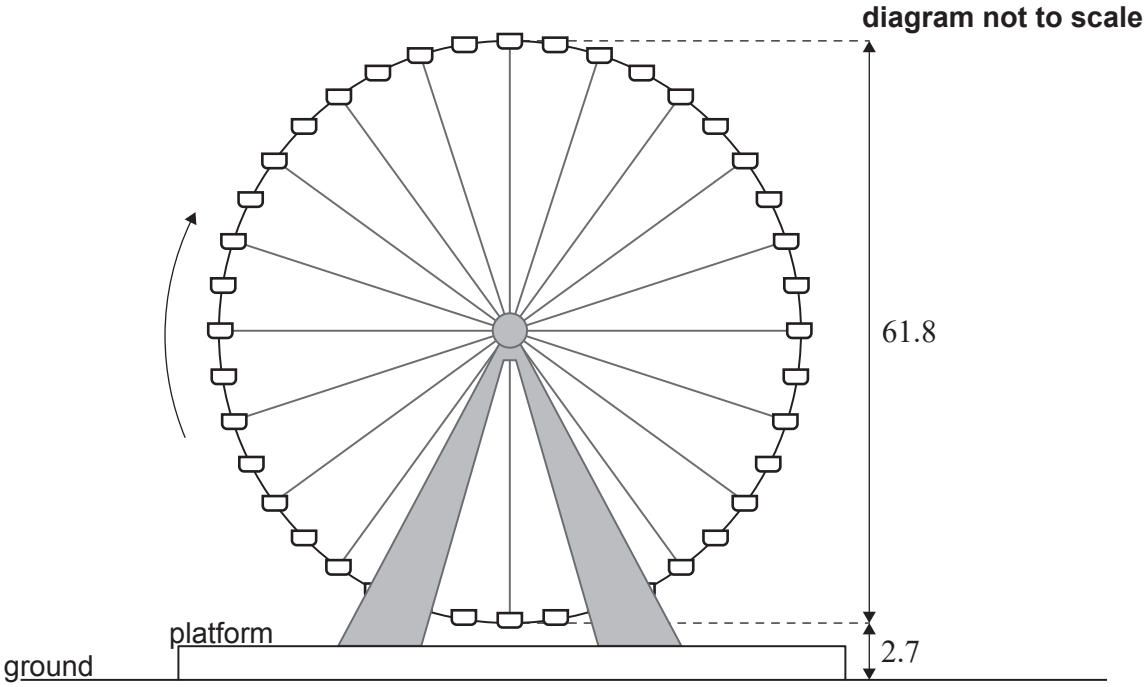
The true maximum number of daylight hours was 16 hours and 14 minutes.

(e) Calculate the percentage error in the maximum number of daylight hours Boris recorded in the diagram. [3]

Question 1 - working space

4. [Maximum mark: 17]

The Texas Star is a Ferris wheel at the state fair in Dallas. The Ferris wheel has a diameter of $61.8\,\text{m}$. To begin the ride, a passenger gets into a chair at the lowest point on the wheel, which is $2.7\,\text{m}$ above the ground, as shown in the following diagram. A ride consists of multiple revolutions, and the Ferris wheel makes 1.5 revolutions per minute.



The height of a chair above the ground, h, measured in metres, during a ride on the Ferris wheel can be modelled by the function $h(t) = -a\cos(bt) + d$, where t is the time, in seconds, since a passenger began their ride.

(a) Calculate the value of

(i) a;

(ii) b;

(iii) d. [6]

A ride on the Ferris wheel lasts for 12 minutes in total.

(b) Calculate the number of revolutions of the Ferris wheel per ride. [2]

(c) For exactly one ride on the Ferris wheel, suggest

(i) an appropriate domain for $h(t)$;

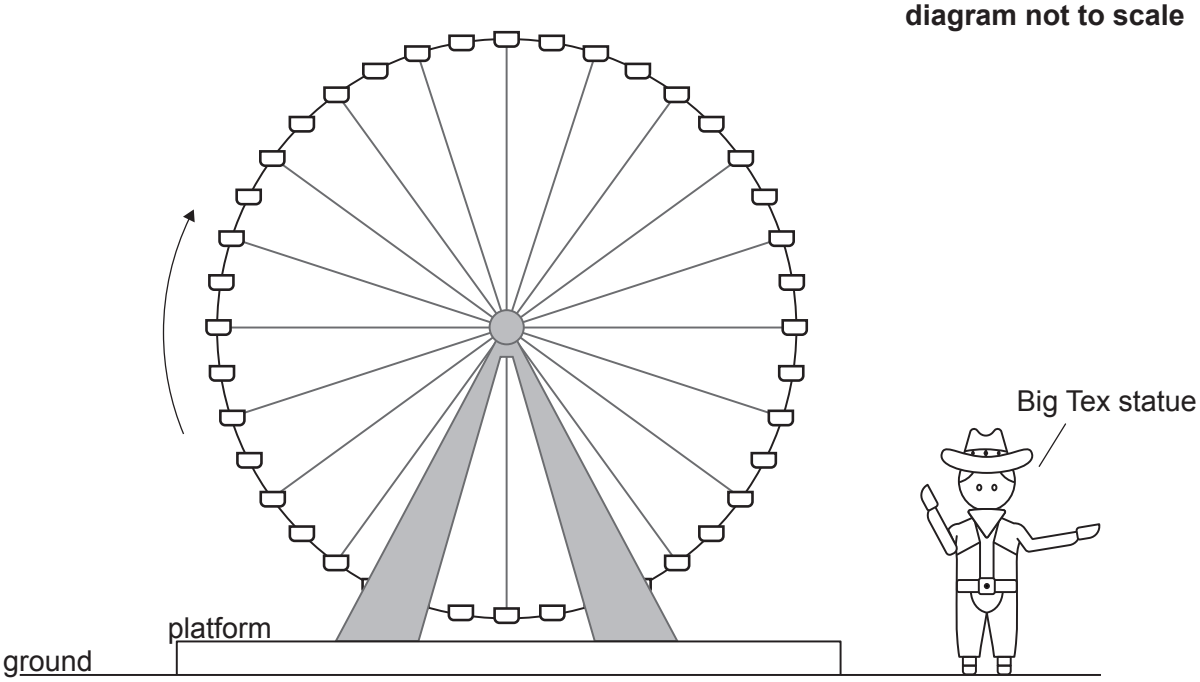
(ii) an appropriate range for $h(t)$. [3]

(This question continues on the following page)

(Question 4 continued)

Big Tex is a 16.7 metre-tall cowboy statue that stands on the horizontal ground next to the Ferris wheel.

diagram not to scale



(d) By considering the graph of $h(t)$, determine the length of time during one revolution of the Ferris wheel for which the chair is higher than the cowboy statue. [3]

There is a plan to relocate the Texas Star Ferris wheel onto a taller platform which will increase the maximum height of the Ferris wheel to $65.2\,\text{m}$. This will change the value of one parameter, a, b or d, found in part (a).

(e) (i) Identify which parameter will change.

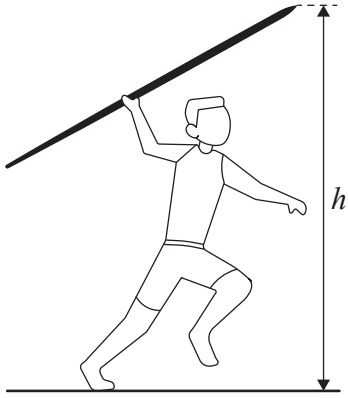
(ii) Find the new value of the parameter identified in part (e)(i). [3]

Turn over

Question 4 - working space

3. [Maximum mark: 5]

DeVaughn throws a javelin in a school track and field competition.



The height, h, of the front tip of the javelin above the ground, in metres, is modelled by the following quadratic function,

$$h(t) = -3.6t^2 + 10.8t + 1.8,\ t \geq 0$$

where t is the time in seconds after the javelin is thrown.

(a) Write down the height of the front tip of the javelin at the time it is thrown. [1]

(b) Find the value of t when the front tip of the javelin reaches its maximum height. [2]

(c) Find the value of t when the front tip of the javelin strikes the ground. [2]

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Question 3 - working space

5. [Maximum mark: 6]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^{\circ}\mathrm{C}$, could be modelled by the following function,

$$T(t) = 71\,\mathrm{e}^{-0.0514t} + 23\,,\ t \geq 0\,,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

The graph of T has a horizontal asymptote.

(b) Write down the equation of the horizontal asymptote. [1]

(c) Write down the room temperature. [1]

(d) Given that $T^{-1}(50) = k$, find the value of k. [2]

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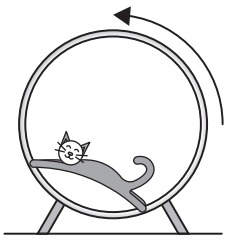
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Question 5 - working space

12. [Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and a, b, $c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i) a.

(ii) b. [3]

(c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

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Question 12 - working space

5. [Maximum mark: 5]

Line L_1 is tangent to the graph of a function $f(x)$ at the point $\mathrm{P}(3, -1)$. Line L_2 is given by the equation $y = -\frac{1}{2}x - \frac{5}{2}$ and is perpendicular to L_1.

(a) Write down the gradient of L_1. [1]

(b) Find the equation of L_1 in the form $y = mx + c$. [2]

(c) Show that L_2 is not the line that is normal to $f(x)$ at point P. [2]

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Question 5 - working space

6. [Maximum mark: 5]

When the brakes of a car are fully applied the car will continue to travel some distance before it completely stops. This stopping distance, d, in metres is directly proportional to the square of the speed of the car, v, in kilometres per hour ($\mathrm{km\,h^{-1}}$).

When a car is travelling at a speed of $50\,\mathrm{km\,h^{-1}}$ it will travel $12.3\,\mathrm{m}$ after the brakes are fully applied before it completely stops.

(a) Determine an equation for d in terms of v. [2]

The police can use this equation to estimate if cars are exceeding the speed limit.

A car is found to have travelled $33\,\mathrm{m}$, after fully applying its brakes, before it completely stopped.

(b) Use your equation from part (a) to estimate the speed at which this car was travelling before the brakes were applied. [2]

(c) After the brakes have been fully applied, identify one other variable besides speed that could affect stopping distance. [1]

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Question 6 - working space

10. [Maximum mark: 7]

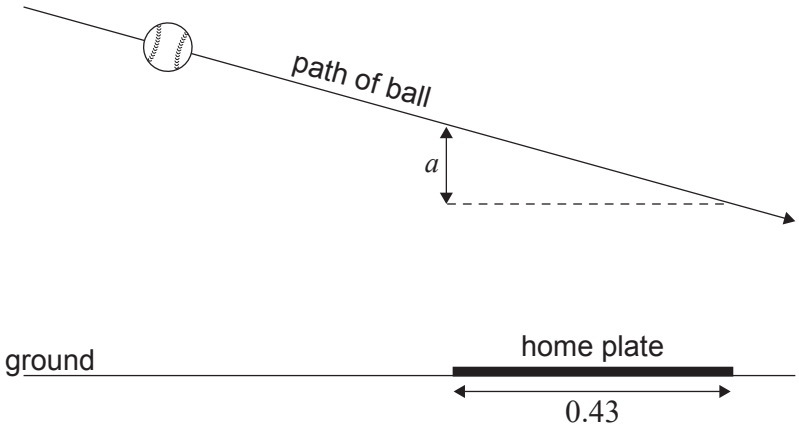
In a baseball game, Sakura is the batter standing beside home plate. The ball is thrown towards home plate along a path that can be modelled by the following function

$$y = -0.045x + 2.$$

In this model, x is the horizontal distance of the ball from the point the ball is thrown and y is the vertical height of the ball above the ground. Both measured in metres.

The outcome of the throw is called a strike if the height of the ball is between $0.53\,\text{m}$ and $1.24\,\text{m}$ at some point while it travels over home plate. The length of home plate is $0.43\,\text{m}$.

diagram not to scale



When the ball reaches the front of home plate, the height of the ball above the ground is $1.25\,\text{m}$. The height of the ball changes by a metres as the ball travels over the length of home plate.

(a) (i) Find the value of a.

(ii) Justify why this throw is a strike. [4]

On the next throw, Sakura hits the ball towards a wall that is 5 metres high. The horizontal distance of the wall from the point where the ball was hit is 96 metres. The path of the ball after it is hit can be modelled by the function $h(d)$.

$$h(d) = -0.01d^2 + 1.04d + 0.66, \text{ for } h,\ d > 0$$

In this model, h is the height of the ball above the ground and d is the horizontal distance of the ball from the point where it was hit. Both h and d are measured in metres.

(b) Determine whether the ball will go over the wall. Justify your answer. [3]

(This question continues on the following page)

(Question 10 continued)

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Question 10 - working space

10. [Maximum mark: 8]

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5,\ t \geq 0,$$

where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

(a) Find how long it takes for the basketball to reach its maximum height. [2]

(b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

Another player catches the basketball when it is at a height of 1.2 metres.

(c) Find the value of t when this player catches the basketball. [2]

(d) Write down two limitations of using $h(t)$ to model the height of the basketball. [2]

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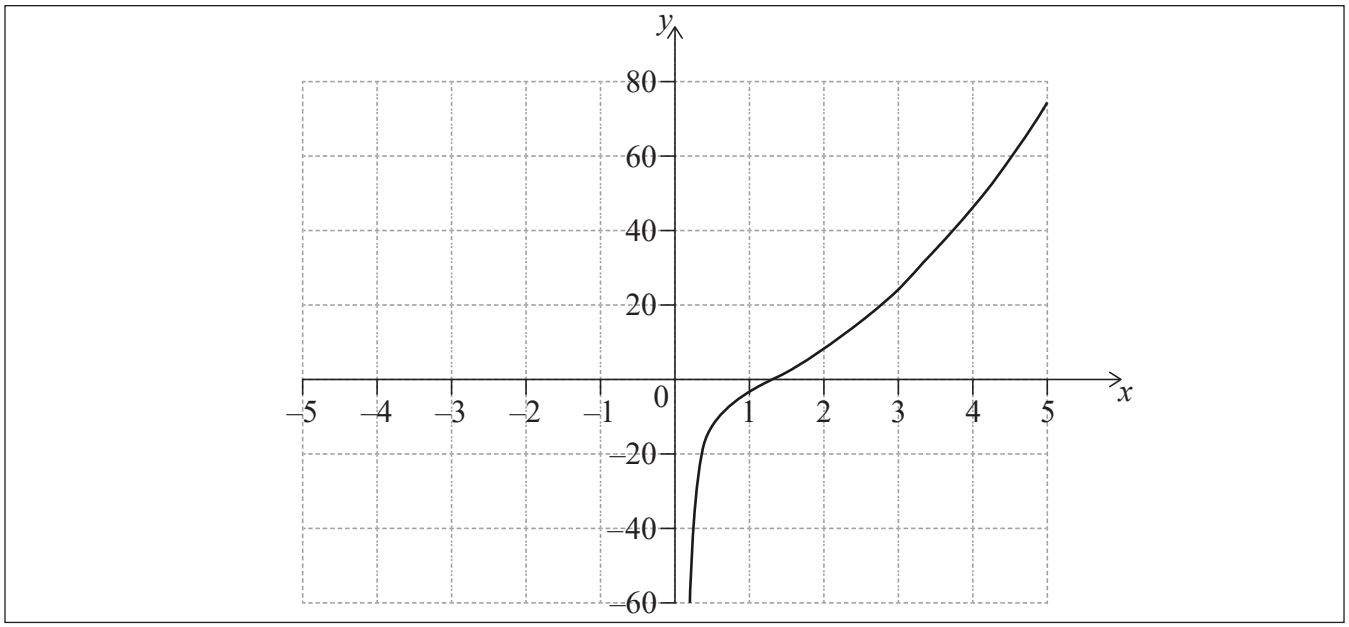
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Question 10 - working space

11. [Maximum mark: 7]

Consider $f(x) = 3x^2 - \frac{5}{x}$, $x \neq 0$. The graph of f for $0 < x \leq 5$ is shown on the following axes.



(a) (i) Sketch the graph of f, for $-5 \leq x < 0$, on the same axes.

(ii) Write down the x-coordinate of the local minimum point. [4]

(b) Use your graphic display calculator to find the solutions to the equation $f(x) = 20$. [2]

(c) Write down the equation of the vertical asymptote for the graph of f. [1]

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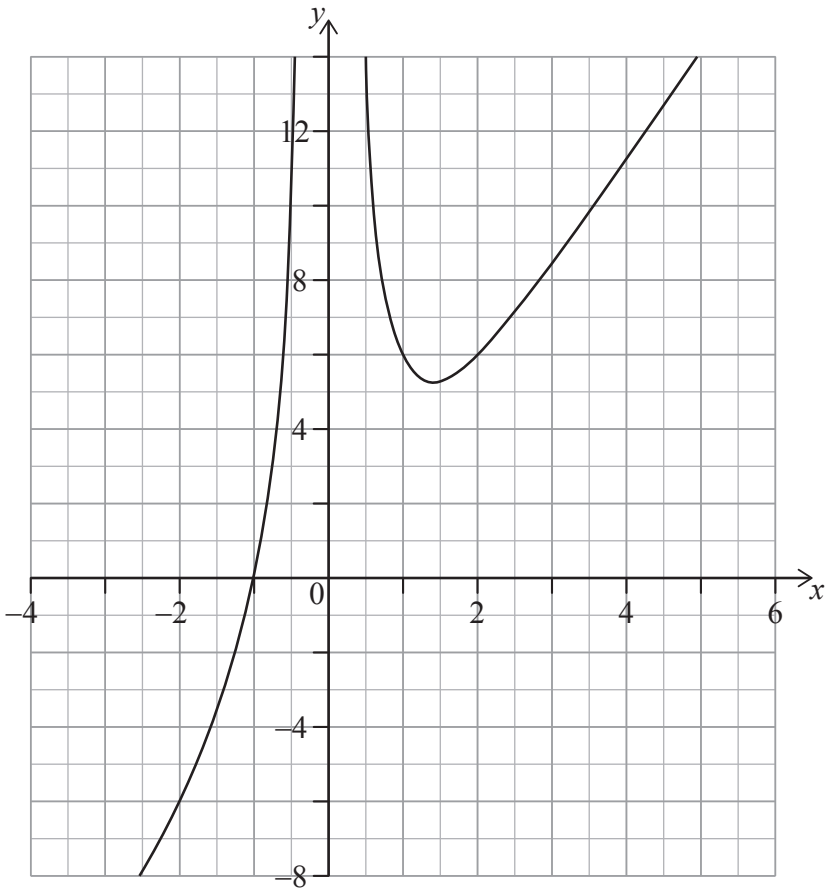
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Question 11 - working space

2. [Maximum mark: 16]

Consider the function $f(x) = 3x - 1 + 4x^{-2}$. Part of the graph of $y = f(x)$ is shown below.



The function is defined for all values of x except for $x = a$.

(a) Write down the value of a. [1]

(b) Use your graphic display calculator to find the coordinates of the local minimum. [2]

The equation $f(x) = w$, where $w \in \mathbb{R}$, has three solutions.

(c) Identify one possible value for w. [1]

The line $y = mx - \frac{1}{4}$ is tangent to $f(x)$ when $x = -4$.

(d) Write down whether the value of m is positive or negative. Justify your answer. [2]

(This question continues on the following page)

(Question 2 continued)

A second function is given by $g(x) = kp^x - 9$, where $p > 0$. The graph of $y = g(x)$ intersects the y-axis at point $\mathrm{A}(0, -5)$ and passes through point $\mathrm{B}(3, 4.5)$.

(e) Find the value of

(i) k;

(ii) p. [4]

(f) Write down the equation of the horizontal asymptote of $y = g(x)$. [2]

(g) Find the solution of $f(x) = g(x)$ when $x > 0$. [2]

Consider a third function, h, where $h(x) = f(x) + g(x)$. The point $\mathrm{C}(-1, q)$ lies on the graph of $g(x)$.

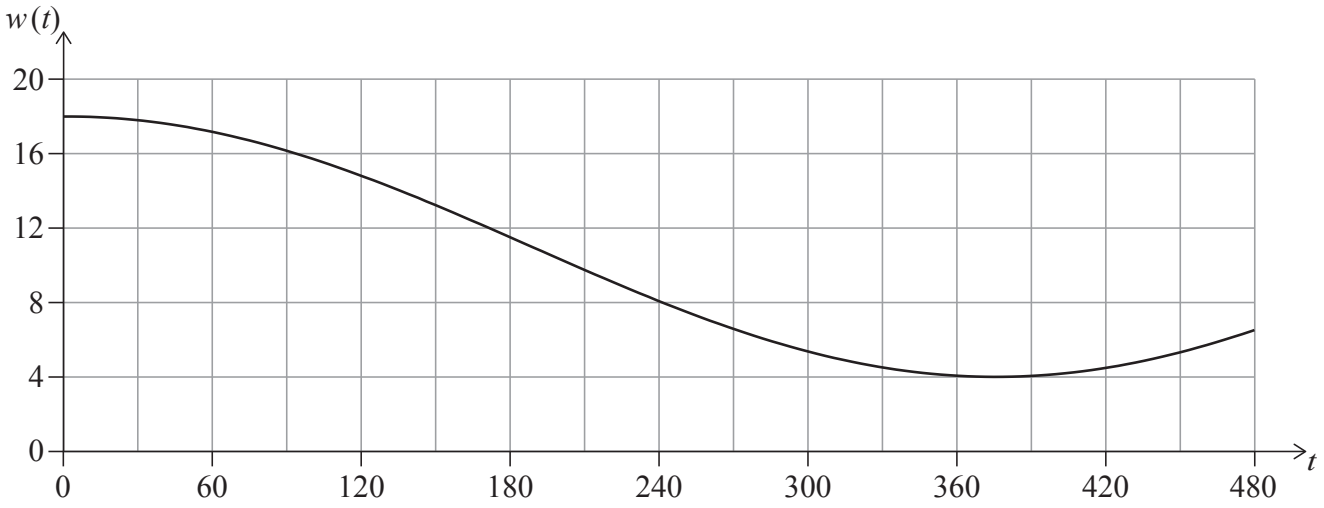
(h) State whether C also lies on the graph of $h(x)$. Justify your answer. [2]

Question 2 - working space

3. [Maximum mark: 15]

The depth of water, w metres, in a particular harbour can be modelled by the function $w(t) = a\cos(bt^\circ) + d$ where t is the length of time, in minutes, after $06{:}00$.

On 20 January, the first high tide occurs at $06{:}00$, at which time the depth of water is $18\,\text{m}$. The following low tide occurs at $12{:}15$ when the depth of water is $4\,\text{m}$. This is shown in the diagram.



(a) Find the value of a. [2]

(b) Find the value of d. [2]

(c) Find the period of the function in minutes. [3]

(d) Find the value of b. [2]

Naomi is sailing to the harbour on the morning of 20 January. Boats can enter or leave the harbour only when the depth of water is at least $6\,\text{m}$.

(e) Find the latest time before $12{:}00$, to the nearest minute, that Naomi can enter the harbour. [4]

(f) Find the length of time (in minutes) between $06{:}00$ and $15{:}00$ on 20 January during which Naomi **cannot** enter or leave the harbour. [2]

Question 3 - working space

3. [Maximum mark: 15]

A scientist is conducting an experiment on the growth of a certain species of bacteria.

The population of the bacteria, P, can be modelled by the function

$$P(t) = 1200 \times k^t,\ t \geq 0,$$

where t is the number of hours since the experiment began, and k is a positive constant.

(a) (i) Write down the value of $P(0)$.

(ii) Interpret what this value means in this context. [2]

3 hours after the experiment began, the population of the bacteria is $18\,750$.

(b) Find the value of k. [2]

(c) Find the population of the bacteria 1 hour and 30 minutes after the experiment began. [2]

The scientist conducts a second experiment with a different species of bacteria.
The population of this bacteria, S, can be modelled by the function

$$S(t) = 5000 \times 1.65^t,\ t \geq 0,$$

where t is the number of hours since both experiments began.

(d) Find the value of t when the two populations of bacteria are equal. [2]

It takes 2 hours and m minutes for the number of bacteria in the second experiment to reach $19\,000$.

(e) Find the value of m, giving your answer as an integer value. [4]

The bacteria in the second experiment are growing inside a container. The scientist models the volume of each bacterium in the second experiment to be $1 \times 10^{-18}\,\text{m}^3$, and the available volume inside the container is $2.1 \times 10^{-5}\,\text{m}^3$.

(f) Determine how long it would take for the bacteria to fill the container. [3]

Question 3 - working space

6. [Maximum mark: 4]

Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}, \text{ for } x > 1.$$

(a) Find $h^{-1}(1)$. [2]

(b) Find the domain of $h^{-1}(x)$. [2]

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Question 6 - working space

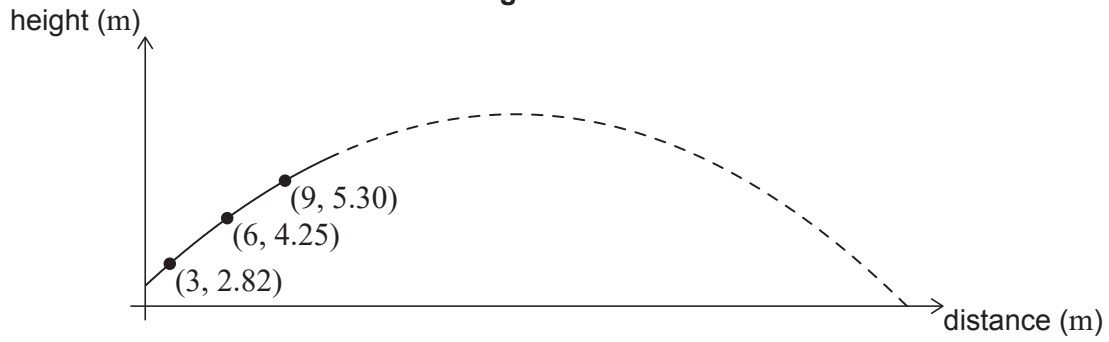
7. [Maximum mark: 9]

An athlete on a horizontal athletic field throws a discus. The height of the discus above the field, in metres, after it is thrown can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the discus has travelled from the athlete.

A specialized camera tracks the initial path of the discus after it is thrown by the athlete. The camera records that the discus travels through the three points $(3,\ 2.82)$, $(6,\ 4.25)$ and $(9,\ 5.30)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1



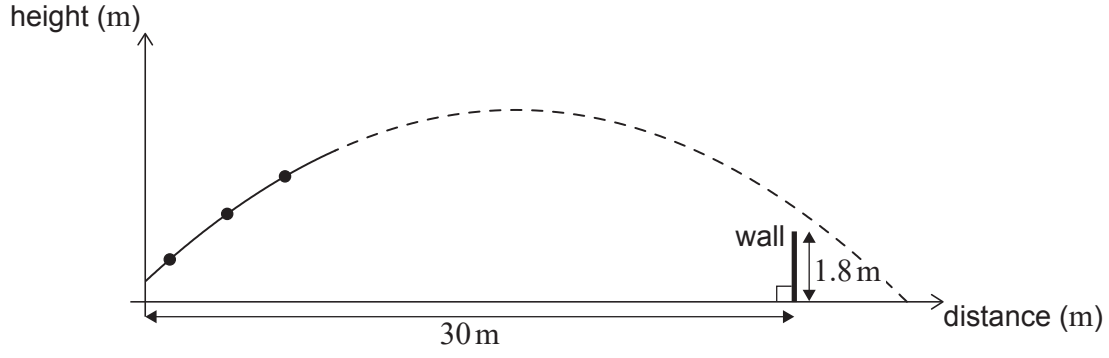
(a) Use the coordinates $(3,\ 2.82)$ to write down an equation in terms of a, b and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the discus. [3]

A 1.8-metre-high wall is 30 metres from where the athlete threw the discus, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the discus will go over the wall. [3]

(d) Find the horizontal distance that the discus will travel, from the athlete until it first hits the ground, according to this model. [2]

(This question continues on the following page)

(Question 7 continued)

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Question 7 - working space

6. [Maximum mark: 4]

Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}, \text{ for } x > 1.$$

(a) Find $h^{-1}(1)$. [2]

(b) Find the domain of $h^{-1}(x)$. [2]

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Question 6 - working space

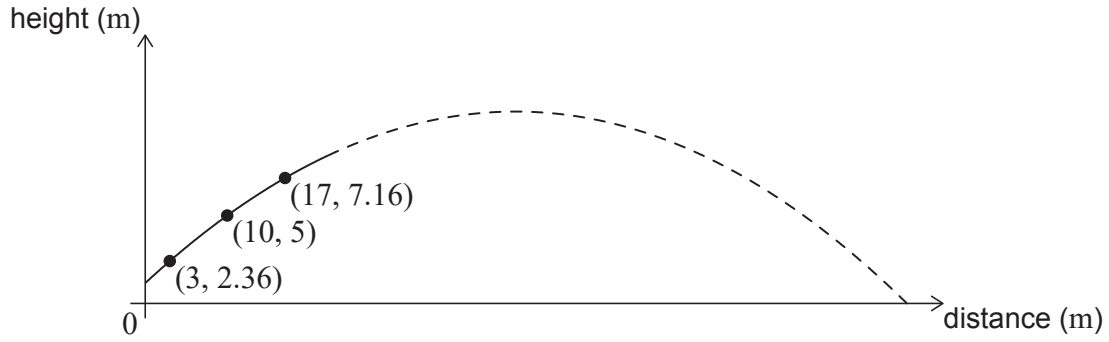
7. [Maximum mark: 9]

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3, 2.36)$, $(10, 5)$ and $(17, 7.16)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1



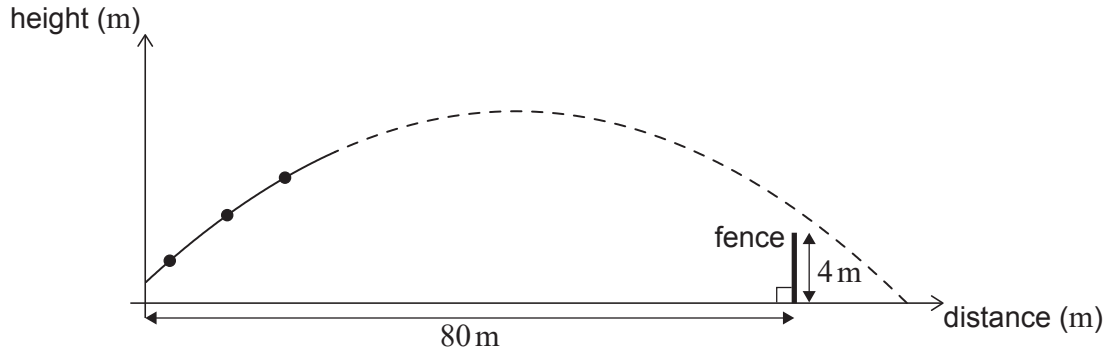
(a) Use the coordinates $(3, 2.36)$ to write down an equation in terms of a, b, and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the ball will go over the fence. [3]

(d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

(This question continues on the following page)

(Question 7 continued)

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Question 7 - working space

4. [Maximum mark: 7]

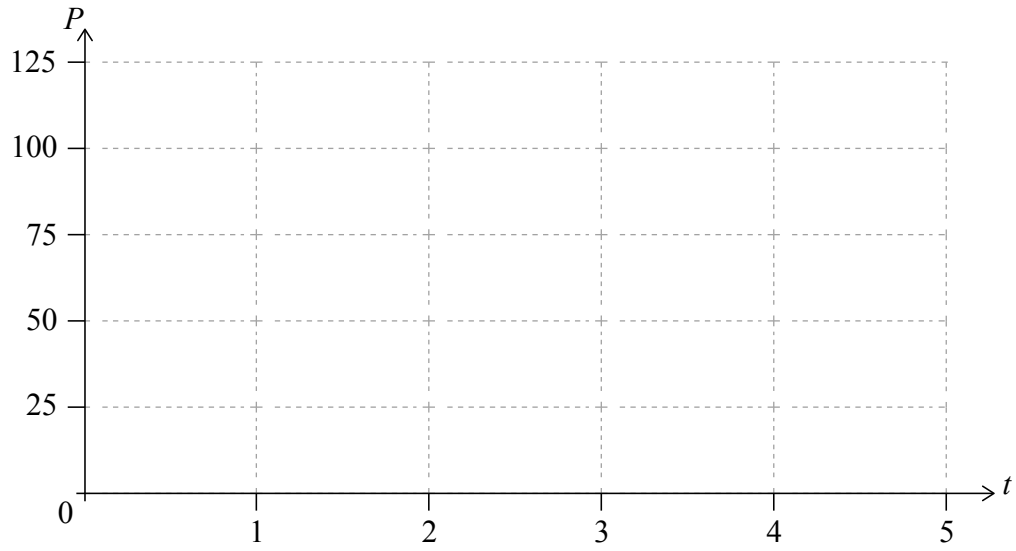
A cell phone starts charging at $07{:}00$. While being charged, the percentage of power, P, in the phone is modelled by the function $P = 100 - 60 \times a^{-t}$, where t is the number of hours after $07{:}00$.

(a) Find the percentage of power in the phone at $07{:}00$. [2]

The percentage of power in the phone reaches $75\,\%$ at $08{:}00$.

(b) Find the value of a. [2]

(c) Draw the graph of $P = 100 - 60 \times a^{-t}$ on the following set of axes. [2]



(d) State a mathematical reason why the model predicts the percentage of power in the phone will never reach $100\,\%$. [1]

(This question continues on the following page)

(Question 4 continued)

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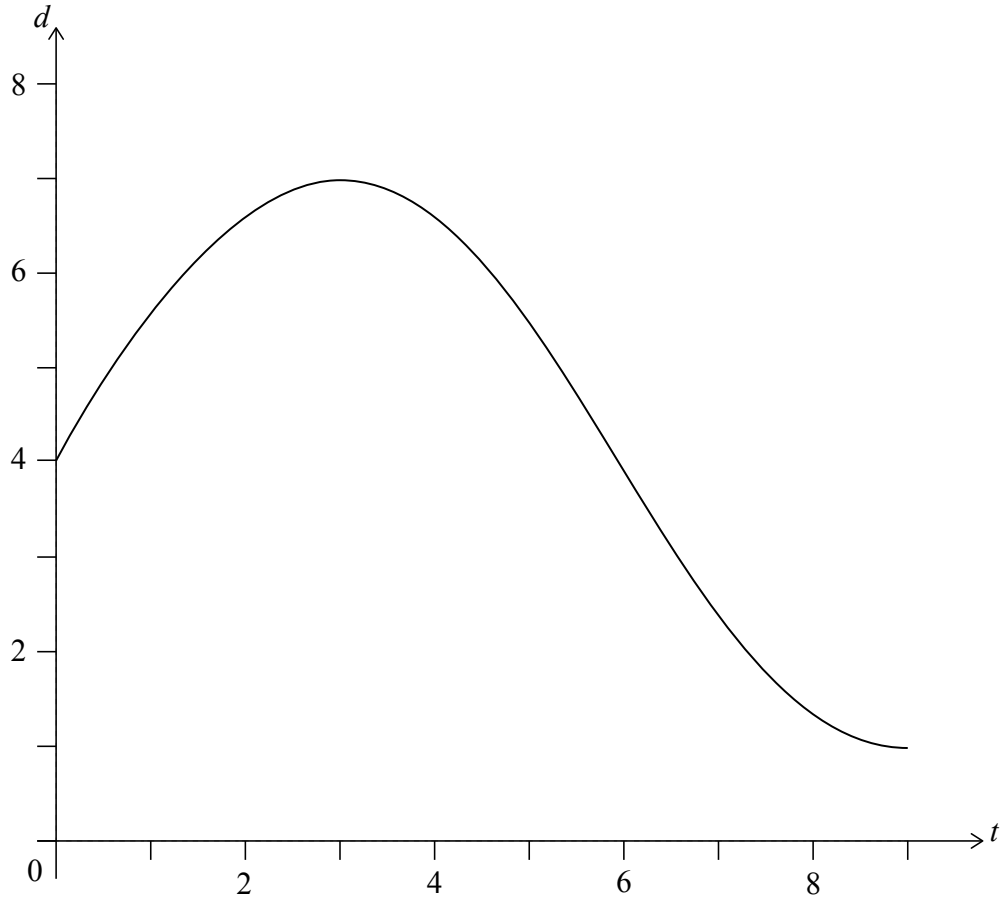
Question 4 - working space

7. [Maximum mark: 6]

The following graph shows the depth of water, d metres, in a river at t hours after $12{:}00$.

At $15{:}00$, the depth of water reaches $7\,\text{m}$, its highest level. At $21{:}00$, the depth of water drops to $1\,\text{m}$, its lowest level.

The depth can be modelled by the function $d(t) = a\sin(bt) + 4$.



(a) Find the value of a. [1]

(b) Find the value of b. [2]

(c) Find the first time after $12{:}00$ when the depth is equal to $3\,\text{m}$. Give your answer to the nearest minute. [3]

(This question continues on the following page)

(Question 7 continued)

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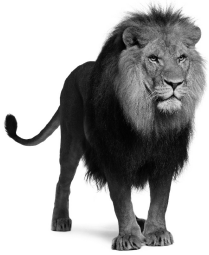
Question 7 - working space

10. [Maximum mark: 8]

When studying big cats, researchers use a model in which the mass (m kilograms) of an animal is directly proportional to the cube of its shoulder height (h metres).



Cheetah



Lion

A cheetah has a mass of $64\,\text{kg}$ and shoulder height of 0.8 metres.

(a) (i) Use the model to find an expression for m in terms of h.

(ii) Hence find the mass of a different cheetah, with a shoulder height of 0.75 metres. [4]

'Rubner's law' states that the energy needs of an animal (E) are directly proportional to the square of h.

The energy needs of a lion of mass $220\,\text{kg}$ are k times the energy needs of a cheetah of mass $64\,\text{kg}$.

(b) Find the value of k. [4]

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Question 10 - working space

4. [Maximum mark: 6]

At a particular building site, the number of square metres of bricks, n, that can be laid in one working day varies directly with the number of bricklayers, B.

Five bricklayers can lay an area of $60\,\mathrm{m}^2$ of bricks in one working day.

(a) Calculate the area that can be laid by 7 bricklayers in one working day. [3]

At another building site, the time, t hours, it takes to lay bricks varies inversely with the number of bricklayers, B.

Five bricklayers take 8 hours to lay an area of $60\,\mathrm{m}^2$.

(b) Calculate how long it takes 12 bricklayers to lay an area of $60\,\mathrm{m}^2$. [3]

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Question 4 - working space

5. [Maximum mark: 7]

The total cost, $C(d)$, in Canadian dollars (CAD), to hire a bicycle for d days from *Pedal Paradise* is given by the function

$$C(d) = 60d + 10,\ d \geq 3,\ d \in \mathbb{Z}.$$

The total cost includes a fixed charge to hire both a helmet and a repair kit.

(a) State, in context, what the values 60 and 10 represent. [2]

(b) Calculate the cost of hiring a bicycle for 5 days. [2]

Hema hires a bicycle from *Pedal Paradise*.

(c) Write down the minimum number of days she can hire the bicycle. [1]

(d) Given that $C^{-1}(1270) = k$, find the value of k. [2]

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Question 5 - working space

6. [Maximum mark: 6]

Radioactive carbon is a material that decays over time.

The mass, $m(t)$ (in nanograms), of radioactive carbon in a fossil of a plant, after t years, can be modelled by the function

$$m(t) = 120\mathrm{e}^{-0.000121t}$$

where t is the time since the plant died.

(a) Write down the initial mass of the radioactive carbon. [1]

(b) Find the mass of the radioactive carbon after $20\,000$ years. [2]

(c) Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]

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Question 6 - working space

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The total cost includes a fixed charge to hire both a helmet and a repair kit.

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(b) Find the mass of the radioactive carbon after $20\,000$ years. [2]

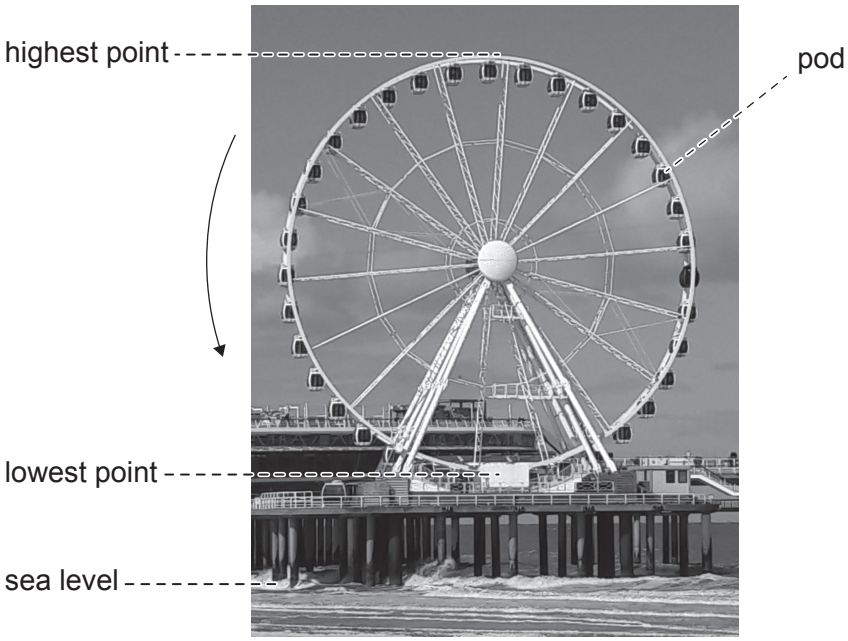
(c) Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]

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Question 6 - working space

2. [Maximum mark: 16]

The Scheveningen Ferris wheel's lowest point is $8\,\text{m}$ above sea level, and its highest point is $45\,\text{m}$ above sea level.



(a) (i) Show that the radius of the Ferris wheel is $18.5\,\text{m}$.

(ii) Calculate the circumference of the Ferris wheel. [2]

There are pods, equally spaced around the wheel, that carry passengers.

(b) When the wheel rotates 10°, find the distance that a pod travels along the circumference. [3]

(This question continues on the following page)

(Question 2 continued)

The height in metres, above sea level, of a particular pod is modelled by the function:

$$h(t) = a\sin(bt) + d, \text{ for } a, b > 0,$$

where t is the time, measured in minutes.

The wheel takes 15 minutes to complete 1 revolution.

(c) (i) Find the value of b.

(ii) Find the value of d.

(iii) Hence, write down the equation of the sinusoidal model. [5]

(d) Use the model to find the values of t when the height of this pod is $33\,\text{m}$ above sea level for $0 \leq t \leq 15$. [3]

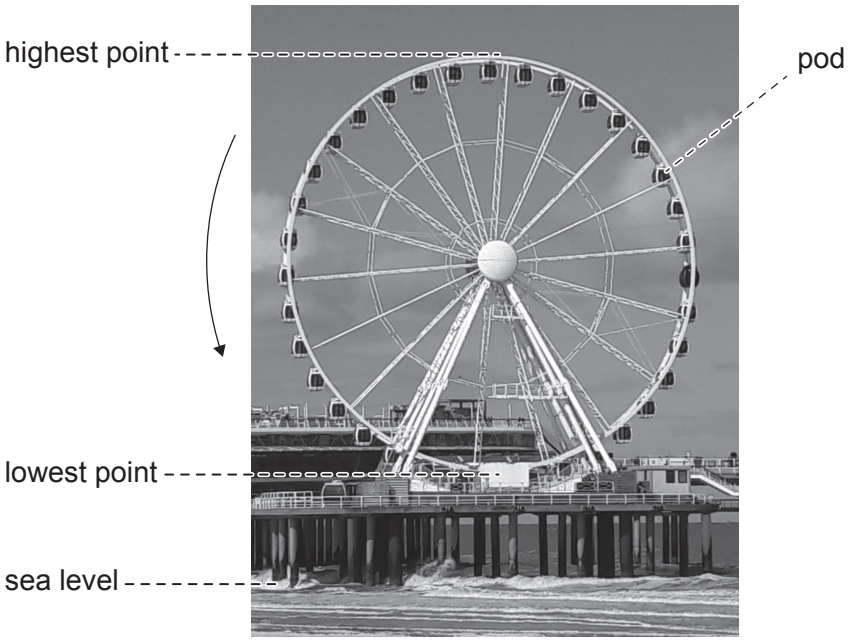
Since the Ferris wheel opened, it has been operating for 3000 days, and each day it rotates nonstop for 8 hours.

(e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

Question 2 - working space

2. [Maximum mark: 16]

The Scheveningen Ferris wheel's lowest point is $8\,\text{m}$ above sea level, and its highest point is $45\,\text{m}$ above sea level.



(a) (i) Show that the radius of the Ferris wheel is $18.5\,\text{m}$.

(ii) Calculate the circumference of the Ferris wheel. [2]

There are pods, equally spaced around the wheel, that carry passengers.

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(This question continues on the following page)

(Question 2 continued)

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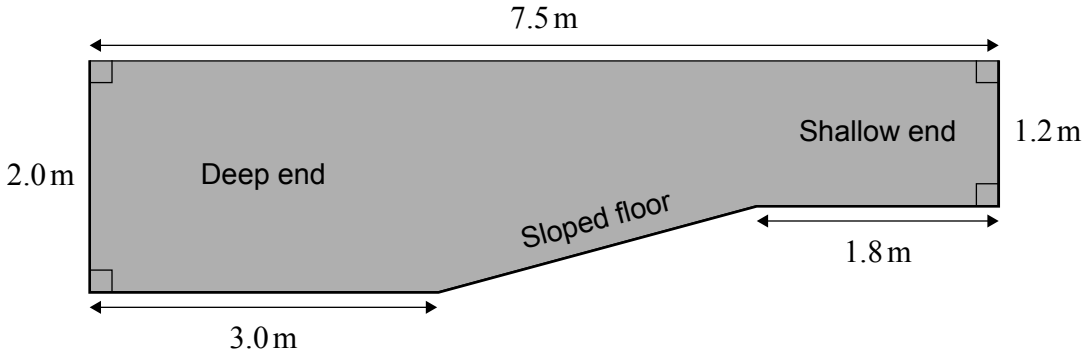
(e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer. [3]

Question 2 - working space

4. [Maximum mark: 5]

A swimming pool has a shallow end where the depth of the water is $1.2\,\text{m}$ and a deep end where the depth of the water is $2.0\,\text{m}$. A cross-section of the water in the swimming pool, with additional dimensions, is shown in the diagram.

diagram not to scale



A safety regulation states that the gradient of the sloped floor of the swimming pool, as shown in the cross-section, must not be greater than $\frac{1}{3}$.

(a) Show that the swimming pool satisfies the safety regulation. [2]

The time to fill the pool with water is inversely proportional to the water flow rate, in litres per minute, of the hose being used. A hose with a water flow rate of 300 litres per minute will completely fill this pool from empty in 4.5 hours if the hose is run continuously.

(b) Determine the time it takes to completely fill the pool from empty, using a hose run continuously with a water flow rate of 170 litres per minute. [3]

(This question continues on the following page)

(Question 4 continued)

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Question 4 - working space

5. [Maximum mark: 7]

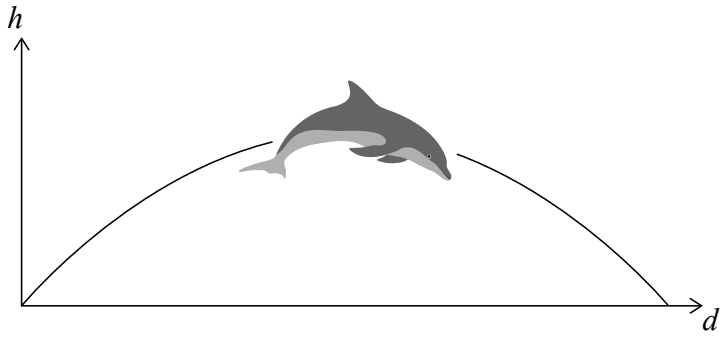
A dolphin jumps out of water. The path of its jump can be modelled by a quadratic function using the following variables:

d is the horizontal distance from the point where the dolphin leaves the water

h is the height of the dolphin above the surface of the water.

All distances are measured in metres, from the point where the dolphin leaves the water.

diagram not to scale



On a particular jump, the dolphin first reaches a height of $5.1\,\text{m}$ above the surface of the water when the horizontal distance is $3\,\text{m}$ and again when the horizontal distance is $8.5\,\text{m}$.

(a) Write down the equation of the axis of symmetry of the quadratic function. [2]

The quadratic function that models the dolphin's jump is $h(d) = -0.2d^2 + bd$, where $\text{d} > 0$ and b is a constant.

(b) Calculate the value of b. [2]

(c) Find the horizontal distance when the dolphin re-enters the water. [2]

(d) Explain what a negative h-value would mean in this context. [1]

(This question continues on the following page)

(Question 5 continued)

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Question 5 - working space

10. [Maximum mark: 7]

A manufacturer designs hats that businesses can order.

For an order of 15 hats or fewer, the price per hat is p euros (EUR). For larger orders, the price of **each** hat after the first 15 ordered is reduced by $5\,\text{EUR}$.

$C(n)$ represents the total cost of purchasing n hats from the manufacturer.

(a) Write down an expression in terms of p for the total cost of ordering 15 hats. [1]

(b) Write down an equation for $C(n)$, in terms of p and n, for the total cost of ordering n hats when $n > 15$. [3]

A company decides to order 100 hats. The mean price per hat for this order is $25.73\,\text{EUR}$.

(c) (i) Calculate the exact total cost of the order.

(ii) Hence, determine the value of p, correct to 2 decimal places. [3]

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Question 10 - working space

11. [Maximum mark: 5]

Mongolia is one of the coldest countries in the world, with the air temperature in winter as low as $-40°\text{C}$. It can feel even colder if a wind is blowing. This effect is called wind chill.

On a day when the air temperature is $-40°\text{C}$, the approximate wind chill index, W, can be calculated by the equation

$$W = -34.1 - 7.33\ln(v),$$

where v is the wind speed, in kilometres per hour (km h^{-1}).

(a) Find the approximate wind chill index on a day when the air temperature is $-40°\text{C}$ and the wind speed is measured as $13\,\text{km h}^{-1}$. [2]

Due to errors in the measuring device, the percentage error in the **approximate wind chill index** calculated in part (a) could be as high as 6%.

(b) Predict the maximum **and** the minimum wind chill index for this day. [3]

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Question 11 - working space

9. [Maximum mark: 5]

Consider the following function, $f(x)$, defined on the domain of integers from 0 to 4 inclusive.

x	0	1	2	3	4
$f(x)$	3	1	0	4	2

(a) Solve $f(x) = 4$. [1]

(b) Solve $f(x) = x$. [1]

(c) Complete the following table.

x	0	1	2	3	4
$f^{-1}(x)$					

[3]

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Question 9 - working space

3. [Maximum mark: 6]

The water level, h, in metres, in a water tank after t hours of irrigation is modelled by the following function.

$$h(t) = \frac{20}{2t+5},\ t \geq 0$$

(a) Find the value of $h(0.5)$. [2]

(b) (i) Find the value of $h^{-1}(2.5)$.

(ii) Interpret this value in context. [3]

(c) Write down the range of h^{-1}. [1]

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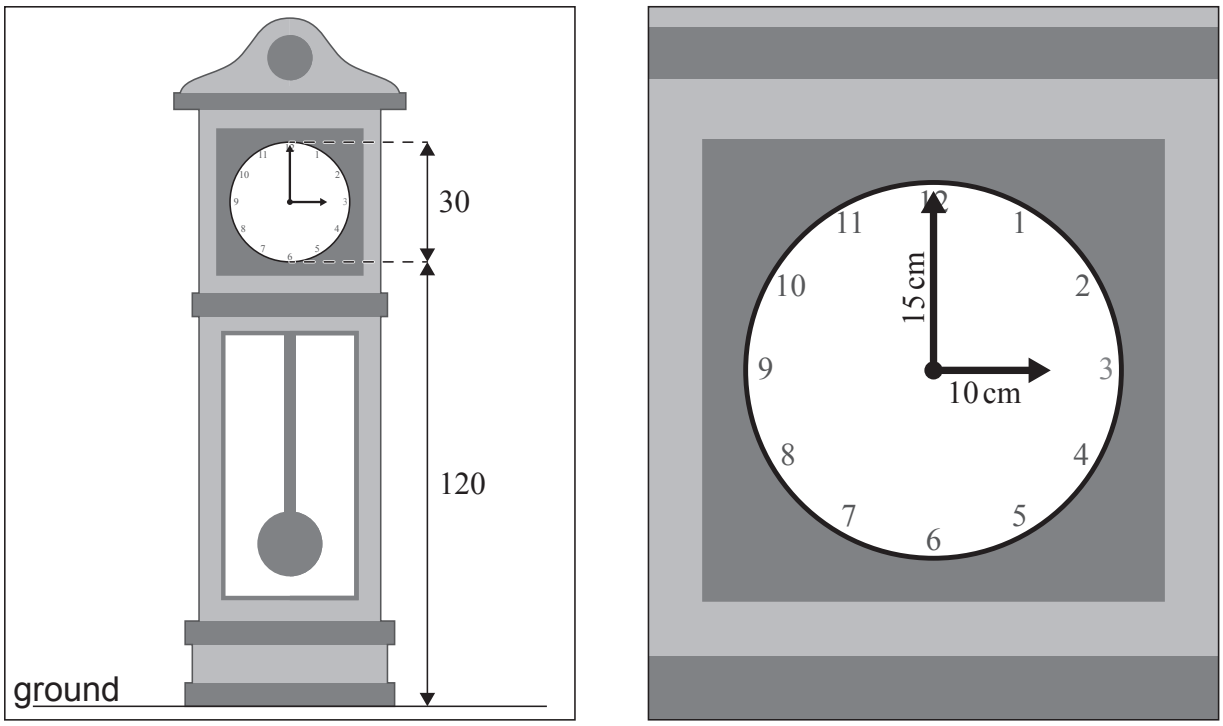
Question 3 - working space

4. [Maximum mark: 16]

The circular clock face of a grandfather clock is $30\,\text{cm}$ in diameter and its lowest point is $120\,\text{cm}$ above the ground. The length of the minute hand is $15\,\text{cm}$ and the length of the hour hand is $10\,\text{cm}$. This is shown in the diagram.

The clock is started at $3\,\text{pm}$ precisely.

diagram not to scale



(a) (i) Write down the size of the angle between the hour hand and the minute hand of the clock at $3\,\text{pm}$.

(ii) Calculate the distance between the tips of the two hands of the clock at $3\,\text{pm}$. [3]

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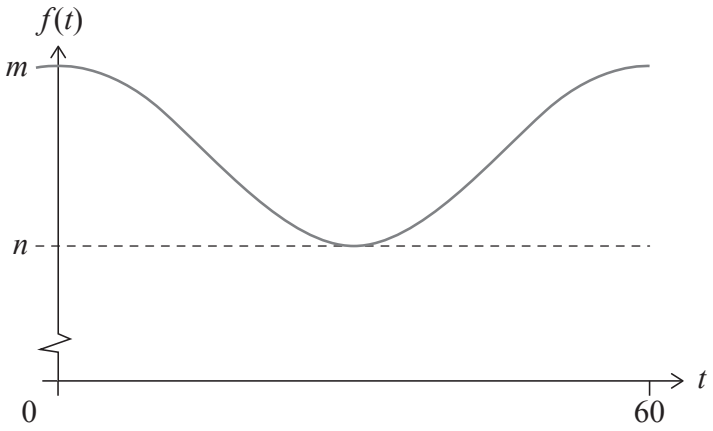
(Question 4 continued)

The height of the tip of the minute hand, $f(t)$, above the ground can be modelled by the cosine function

$$f(t) = a\cos(b\mathrm{t}) + d,\ a,\ b > 0$$

where the angle $b\mathrm{t}$ is measured in degrees and t is the number of minutes after 3 pm.

A sketch of the graph of $y = f(t)$ as time passes from 3 pm to 4 pm is shown.



(b) Write down the value of

(i) m.

(ii) n. [2]

(c) Hence find the value of

(i) a.

(ii) b.

(iii) d. [3]

(This question continues on page 9)

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(Question 4 continued)

The height of the tip of the hour hand, $g(t)$, above the ground can be modelled by the sine function

$$g(t) = p\sin(q\mathrm{t}) + d$$

where $q\mathrm{t}$ is measured in degrees and t is the number of minutes after 3 pm.

(d) (i) Show that $q = 0.5$.

(ii) Find the value of p. [3]

(e) Find the height of the tip of the hour hand above the ground at 4 pm. [2]

(f) Find the first time after 3 pm at which the tips of the two hands of the clock are at the same height above the ground.

Give your answer correct to the nearest minute. [3]

Question 4 - working space

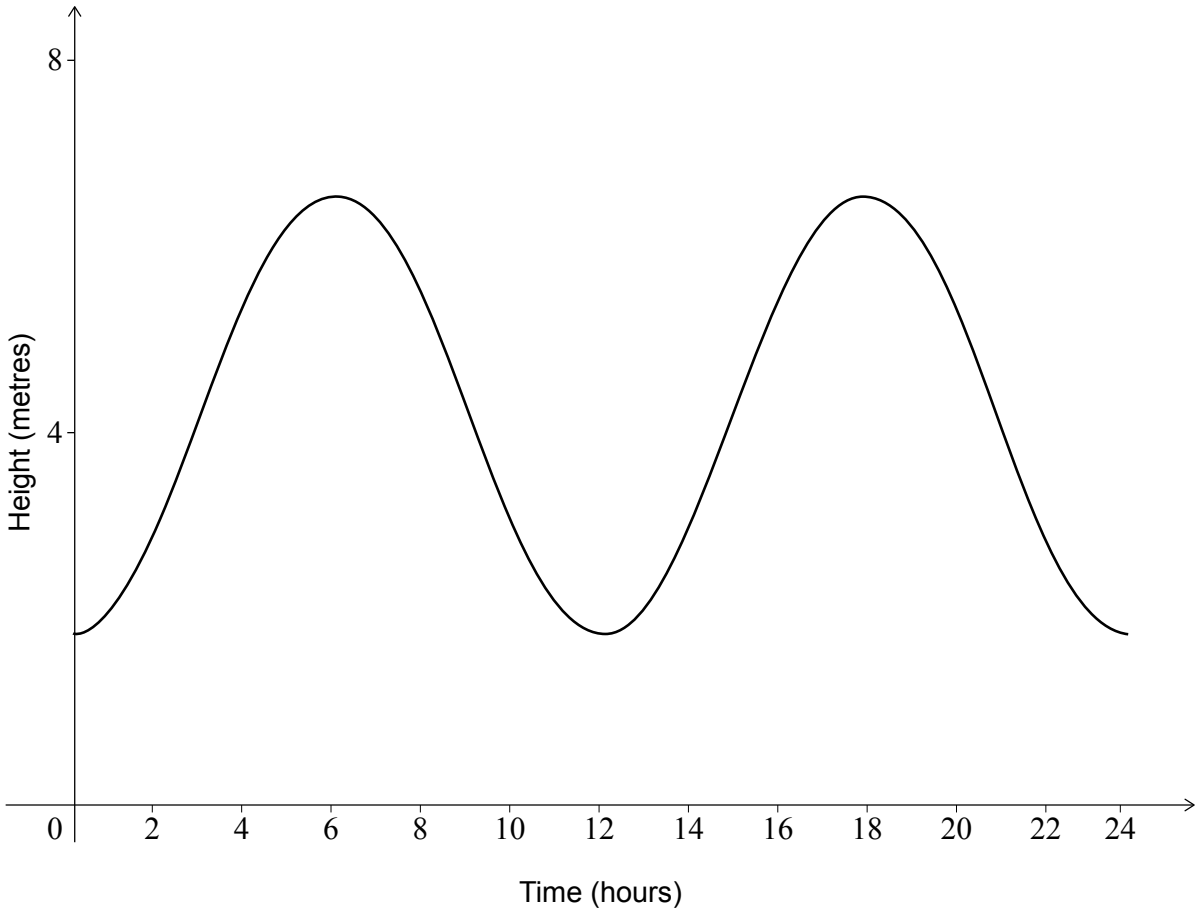
4. [Maximum mark: 13]

On a particular day the height of the tide, h, in metres, at Albion harbour can be modelled by the function

$$h(t) = -2.5\cos(bt°) + 4.5, \text{ where } b \in \mathbb{R}, 0 \leq t \leq 24$$

and t represents the number of hours after midnight.

The graph of h is shown in the following diagram.



(a) Show that the value of b is 30. [1]

(b) Find the height of the tide when $t = 5$. [2]

(c) Write down

(i) the amplitude of h.

(ii) the equation of the principal axis. [3]

(This question continues on the following page)

(Question 4 continued)

Boats can only leave or return to Albion harbour when $h(t) \geq 2.65$. Robin wants to leave the harbour to go fishing as soon as possible after the time is 12:00.

(d) Determine the earliest possible time that Robin could leave the harbour.
Give your answer to the nearest minute. [3]

The boat will take 15 minutes to travel from the harbour to the fishing site. Robin intends to return to the harbour on the same day.

(e) Determine the maximum length of time he could spend at the fishing site, in hours, and still be certain he will be able to enter the harbour on his return. [4]

Turn over

Question 4 - working space

3. [Maximum mark: 6]

TurboEats is a food delivery company that charges customers $\$0.80$ per kilometre plus a fixed amount of $\$1.50$. Their delivery charge, T in $\$$, is represented by the function

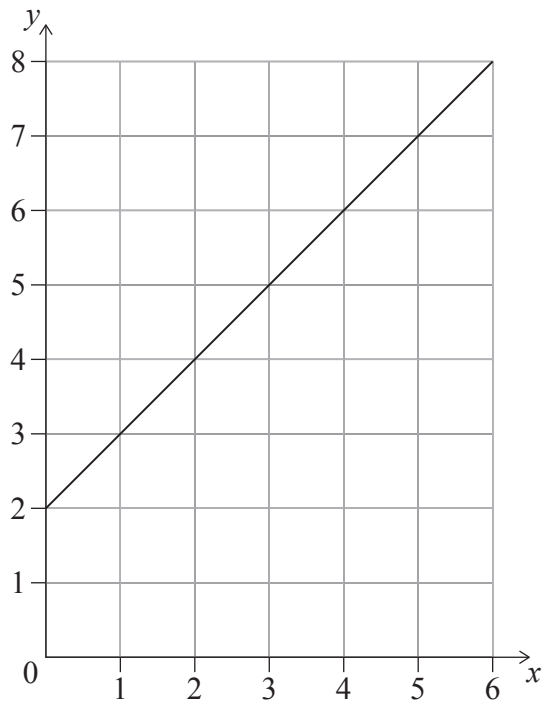
$$T(x) = 0.80x + 1.50, \text{ for } x \geq 0,$$

where x is the distance in kilometres from where the food is picked up to where it is delivered.

Lorena's home is $5.1\,\text{km}$ from a pizza café.

(a) Find the charge for TurboEats to deliver a pizza from the café to her home. [2]

A second food delivery company, GogoDart, charges customers according to the function $G(x) = ax + b$, for $x \geq 0$. Their charges for the first $6\,\text{km}$ are shown in the following graph of $y = G(x)$. The line passes through integer grid points.



(b) (i) Write down the value of a.

(ii) Write down the value of b. [2]

(c) Explain why TurboEats is **always** cheaper than GogoDart. [2]

(This question continues on the following page)

(Question 3 continued)

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Question 3 - working space

8. [Maximum mark: 8]

Sheena purchased a new mobile phone. The value of Sheena's mobile phone, V in Philippine Pesos (PHP), can be modelled by the function

$$V(t) = 2000 + 30\,000(\text{e}^{-kt}), \text{ for } t \geq 0,$$

where t is the time, in months, after the mobile phone was purchased.

(a) Calculate the value of the mobile phone when it was purchased. [2]

Ten months after it was purchased, the value of the mobile phone has decreased by 40%.

(b) Find the value of k. [4]

Sheena believes the value of the mobile phone will always be more than $\text{PHP}\,2000$.

(c) (i) State a mathematical property of the graph of $y = V(t)$ that supports Sheena's belief.

(ii) State one contextual reason why Sheena's belief might be incorrect. [2]

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Question 8 - working space

10. [Maximum mark: 7]

At 12:05 pm, Navam starts draining water from a small reservoir into an empty pond.

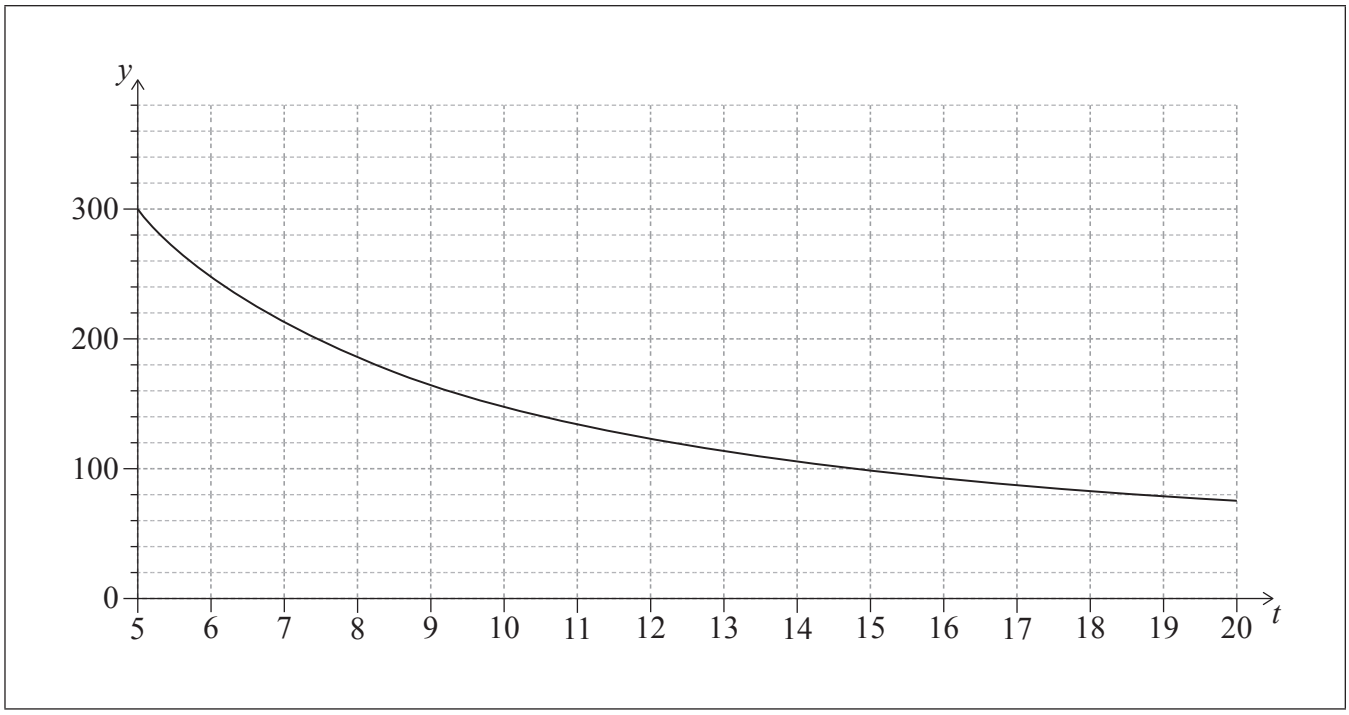
He controls the rate the water is drained so that the volume of water **remaining** in the reservoir, $V\,\text{m}^3$, varies inversely with the time, t, where t is the number of minutes after 12:00 pm.

At 12:05 pm, the volume of water in the reservoir is $300\,\text{m}^3$.

(a) Show that $V = \frac{1500}{t}$. [2]

(b) Find the value of t when the volume of water remaining in the reservoir equals the volume of water in the pond. [2]

The following diagram shows part of the graph of $y = V(t)$.



(c) On the same diagram, sketch the graph of $y = P(t)$, where P is the volume of water in the pond. [2]

(d) Write down an expression for $P(t)$. [1]

(This question continues on the following page)

(Question 10 continued)

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Question 10 - working space

2. [Maximum mark: 6]

The number of words that a child understands is modelled by the equation

$$y = -167 + 6264\log_{10}x, \text{ for } x \in \mathbb{R}^+,$$

where x is the number of years since the child was born, and y is the number of words the child understands.

(a) Use the model to predict the number of words that a child understands when $x = 10$. [2]

To be fluent in a language, a child must understand at least 5000 words.

(b) Use the model to predict how long it takes for a child to become fluent. Give your answer correct to four significant figures. [2]

(c) Determine if the model is realistic for a child on their first birthday. Give a reason to support your decision. [2]

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Question 2 - working space

6. [Maximum mark: 6]

A function is defined by $f(x) = \frac{4}{x} - 3$, for $-6 \leq x \leq 10, x \neq 0$.

(a) Find $f(10)$. [1]

(b) Find the range of f. [3]

(c) Find $f^{-1}(-2)$. [2]

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Question 6 - working space

8. [Maximum mark: 6]

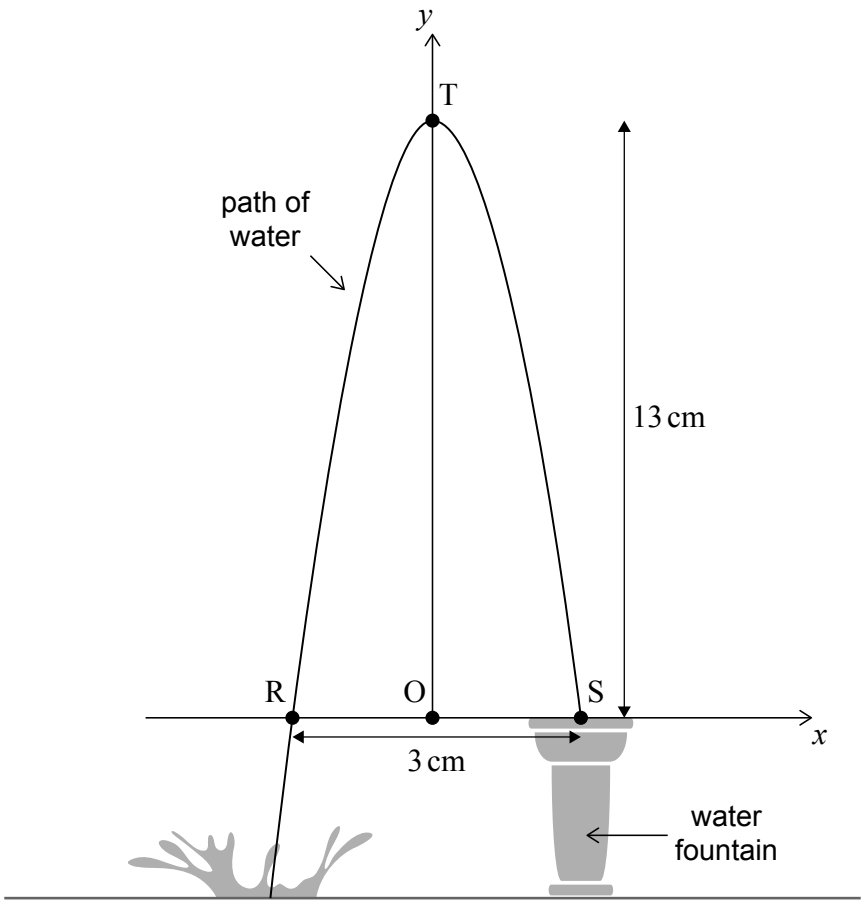
Eduardo takes a photograph of a water fountain. He notices the path of the water looks like a parabola, and he decides to model the curve using the equation

$$y = ax^2 + bx + c.$$

Eduardo takes measurements from the photograph. He finds the distance between the x-intercepts, R and S, to be $3\,\mathrm{cm}$ and the maximum height of the water to be $13\,\mathrm{cm}$ above the fountain.

These measurements are shown on the following diagram, where the point $(0, 0)$ is indicated as O and the vertex, T, is on the y-axis.

diagram not to scale



(a) Write down the value of c. [1]

(b) (i) Write down the coordinates of S.

(ii) Use the coordinates for S to write down an equation, in terms of only a and b. [2]

(c) Find the value of a and the value of b in Eduardo's model. [3]

(This question continues on the following page)

(Question 8 continued)

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Question 8 - working space

12. [Maximum mark: 6]

The amount of a radioactive material present in a sample, in micrograms, can be modelled by

$$f(t) = ab^t, \text{ for } t \geq 0,$$

where t represents the number of days since the sample was taken.

Paula takes two samples of this material, sample X and sample Y.

The initial amount of radioactive material in sample X is 100 micrograms, and after 138 days the amount of radioactive material present is 50 micrograms.

(a) For sample X,

(i) write down a.

(ii) find b, given that $b > 0$. [3]

After 30 days there were 4 micrograms of the radioactive material present in sample Y.

(b) Use your value of b found in part (a)(ii) to find the initial amount of radioactive material in sample Y. [3]

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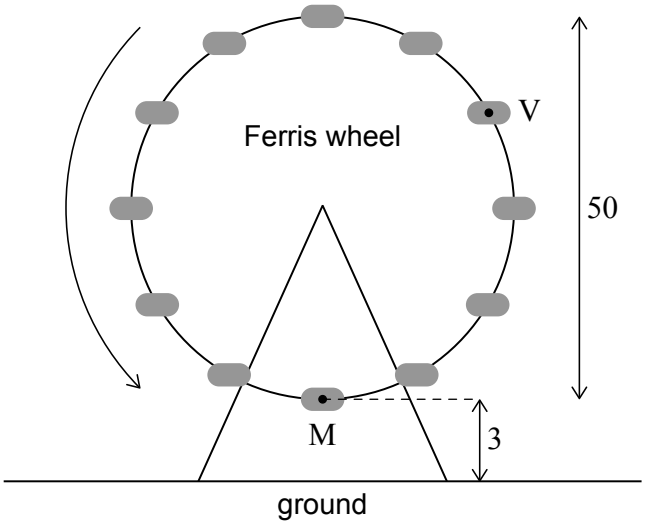
Question 12 - working space

5. [Maximum mark: 18]

The following diagram represents a Ferris wheel with 50-metre diameter and lowest point 3 metres above the ground. There are twelve evenly spaced chairs on the Ferris wheel. The Ferris wheel rotates in an anticlockwise (counterclockwise) direction.

Martz is seated in the chair labelled M, which **initially** is located at the lowest point of the Ferris wheel when it starts rotating. At the same time, Victoria is seated in the chair labelled V.

diagram not to scale



Let t be the length of time, in minutes, after the Ferris wheel started rotating.

(a) Find the height of V above the ground when $t = 0$. [3]

The wheel completes one revolution every 12 minutes.

(b) Find the height of M above the ground when $t = 9$. [2]

(c) Write down the values of t when M and V have the same height above the ground, for $0 \leq t \leq 12$. You may assume that Victoria stays on the ride during this interval. [3]

The height of M above the ground is given by the function

$$h(t) = -25\cos bt + d, \text{ for } t \geq 0.$$

(d) Write down the amplitude of h. [1]

(e) Find the value of

(i) b, given that $b > 0$;

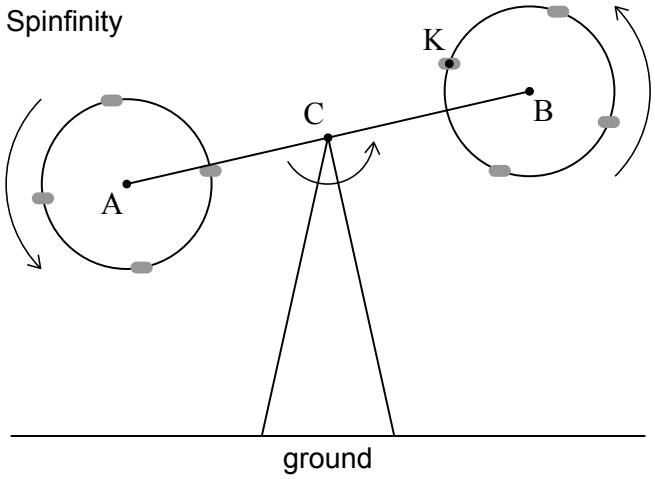
(ii) d. [3]

(This question continues on the following page)

(Question 5 continued)

Kofi rides “Spinfinity”, a modified Ferris wheel. The beam AB rotates in an anticlockwise (counterclockwise) direction about its midpoint C. The beam supports two circular wheels with centres A and B, each rotating in an anticlockwise (counterclockwise) direction. Kofi is in a chair labelled K. This information is illustrated in the following diagram.

diagram not to scale



The height in metres of B above the ground, T seconds after “Spinfinity” starts moving, is modelled by the function $b(T) = 20\sin(36T) + 40$.

The height in metres, of K above B, after T seconds, is modelled by the function $k(T) = 10\cos(72T)$.

(f) Find the furthest distance from Kofi to point C. [2]

(g) Find Kofi’s height above the ground after 18 seconds. [4]

Question 5 - working space

4. [Maximum mark: 18]

A particular island is only accessible by road at times when the water level is low enough. Otherwise, the road is underwater.

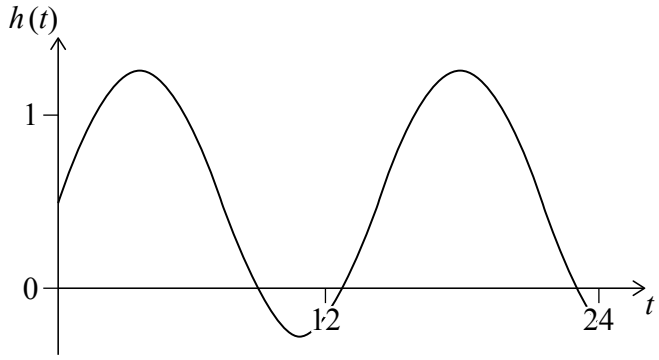
The height of the water above the road, h metres, can be modelled by

$$h(t) = a\sin(bt) + d,$$

where t is time measured in **hours** from $00{:}00$ (midnight) on Tuesday.

The first maximum (high tide) is at $03{:}36$ with a height of $1.26\,\text{m}$, and the first minimum (low tide) occurs at $10{:}48$ with a height of $-0.26\,\text{m}$.

This is shown in the graph.



(a) Find the value of a, given that $a > 0$. [2]

(b) Find the value of d. [2]

(c) Find the period of the function in hours. [3]

(d) Find the equation of the model. [3]

(e) Find the value of $h(t)$ at $02{:}30$. [2]

(f) State the meaning of $h(t) < 0$ in context. [1]

(g) Find the length of time on Tuesday when $h(t) < 0$. [5]

Turn over

Question 4 - working space

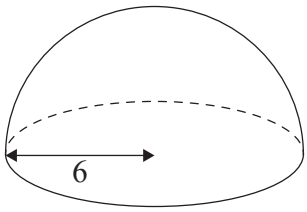
APOLLO

Geometry and trigonometry

70 questions

3. [Maximum mark: 6]

A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is 6 mm.



(a) Calculate the **total** surface area of one piece of candy. [4]

The total surface of the candy is coated in chocolate. It is known that 1 gram of the chocolate covers an area of $240\,\text{mm}^2$.

(b) Calculate the weight of chocolate required to coat one piece of candy. [2]

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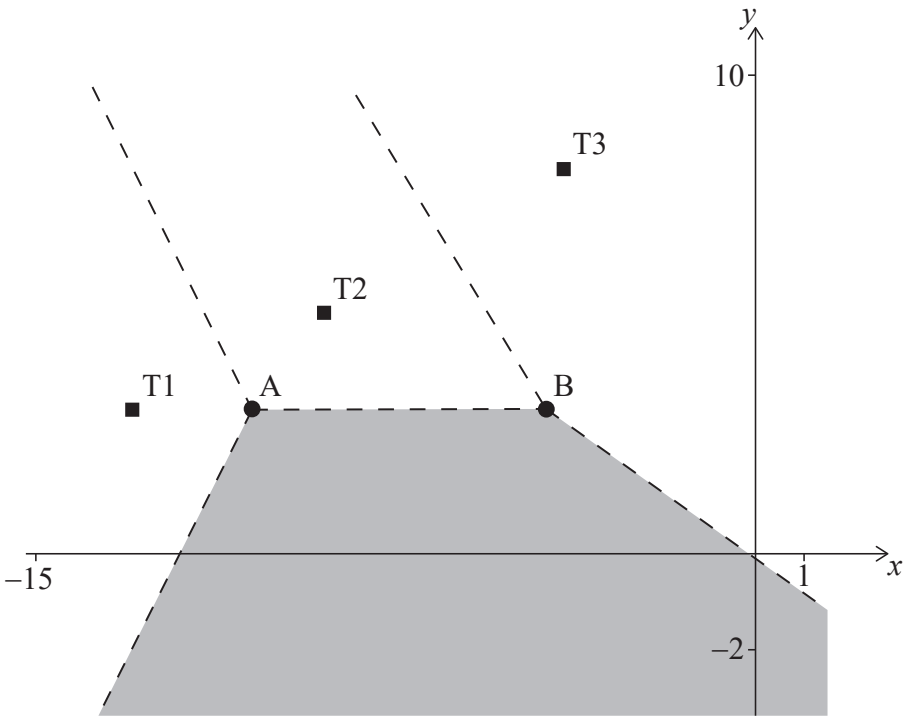
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Question 3 - working space

5. [Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.
Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]

(This question continues on the following page)

(Question 5 continued)

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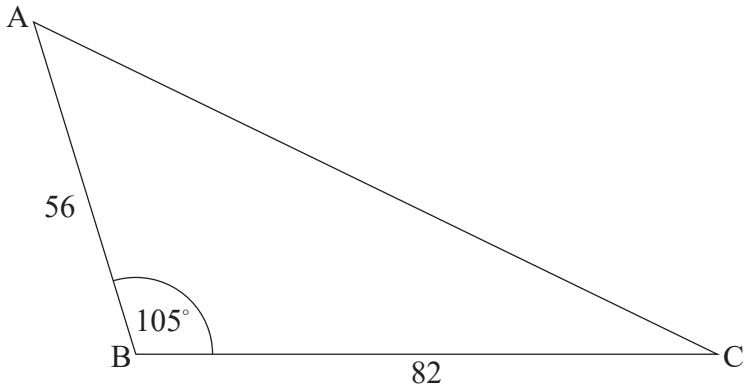


Question 5 - working space

9. [Maximum mark: 5]

A triangular field ABC is such that $AB = 56\,\text{m}$ and $BC = 82\,\text{m}$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

diagram not to scale



Calculate the maximum possible area of the field.

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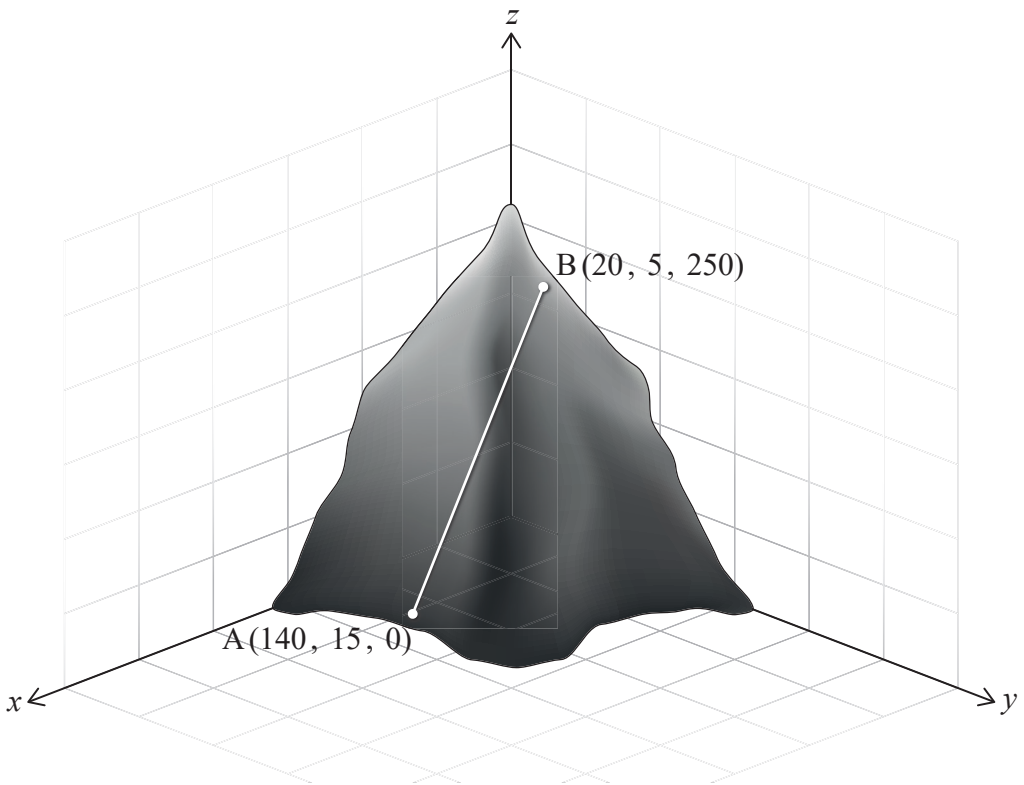
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Question 9 - working space

2. [Maximum mark: 5]

An inclined railway travels along a straight track on a steep hill, as shown in the diagram.

diagram not to scale



The locations of the stations on the railway can be described by coordinates in reference to x, y, and z-axes, where the x and y axes are in the horizontal plane and the z-axis is vertical.

The ground level station A has coordinates $(140, 15, 0)$ and station B, located near the top of the hill, has coordinates $(20, 5, 250)$. All coordinates are given in metres.

(a) Find the distance between stations A and B. [2]

Station M is to be built halfway between stations A and B.

(b) Find the coordinates of station M. [2]

(c) Write down the height of station M, in metres, above the ground. [1]

(This question continues on the following page)

(Question 2 continued)

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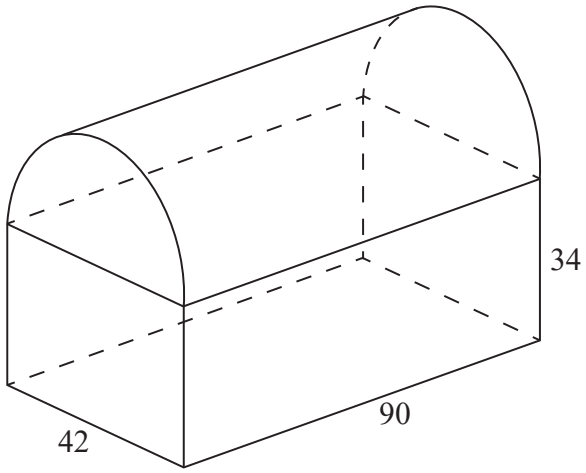
Question 2 - working space

3. [Maximum mark: 7]

A storage container consists of a box of length $90\,\text{cm}$, width $42\,\text{cm}$ and height $34\,\text{cm}$, and a lid in the shape of a half-cylinder, as shown in the diagram. The lid fits the top of the box exactly. The total exterior surface of the storage container is to be painted.

Find the area to be painted.

diagram not to scale



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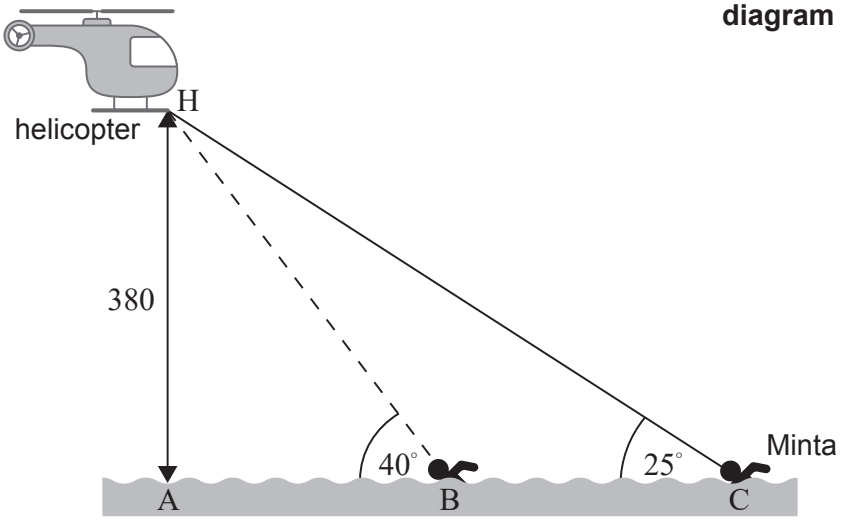
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Question 3 - working space

4. [Maximum mark: 7]

The diagram below shows a helicopter hovering at point H, $380\,\text{m}$ vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.

diagram not to scale



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Write down the size of the angle of depression from H to C. [1]

(b) Find the distance from A to C. [2]

(c) Find the distance from B to C. [3]

(d) Find Minta's speed, in metres per hour. [1]

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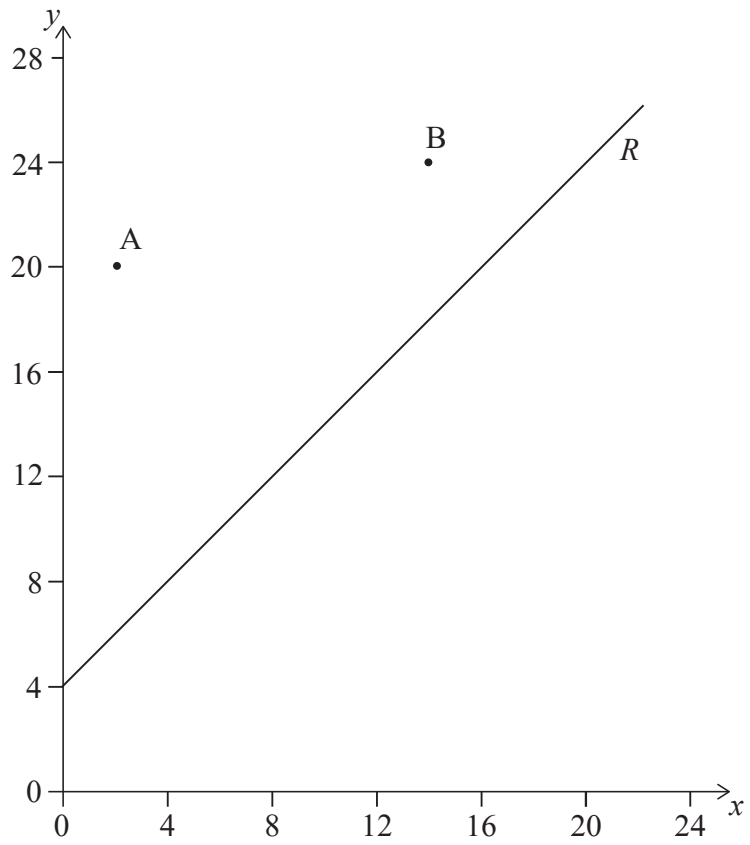
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Question 4 - working space

6. [Maximum mark: 7]

Two schools are represented by points $\mathrm{A}(2,\ 20)$ and $\mathrm{B}(14,\ 24)$ on the graph below. A road, represented by the line R with equation $-x + y = 4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of $[\mathrm{AB}]$. Give your equation in the form $y = mx + c$. [5]

(b) Determine the coordinates of the point on R where the bus stop should be located. [2]

(This question continues on the following page)

(Question 6 continued)

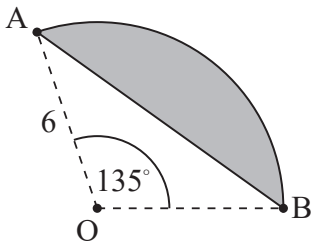
Question 6 - working space

9. [Maximum mark: 7]

A garden includes a small lawn. The lawn is enclosed by an arc AB of a circle with centre O and radius $6\,\text{m}$, such that $\text{A}\hat{\text{O}}\text{B} = 135^\circ$. The straight border of the lawn is defined by chord [AB].

The lawn is shown as the shaded region in the following diagram.

diagram not to scale



(a) A footpath is to be laid around the curved side of the lawn. Find the length of the footpath. [3]

(b) Find the area of the lawn. [4]

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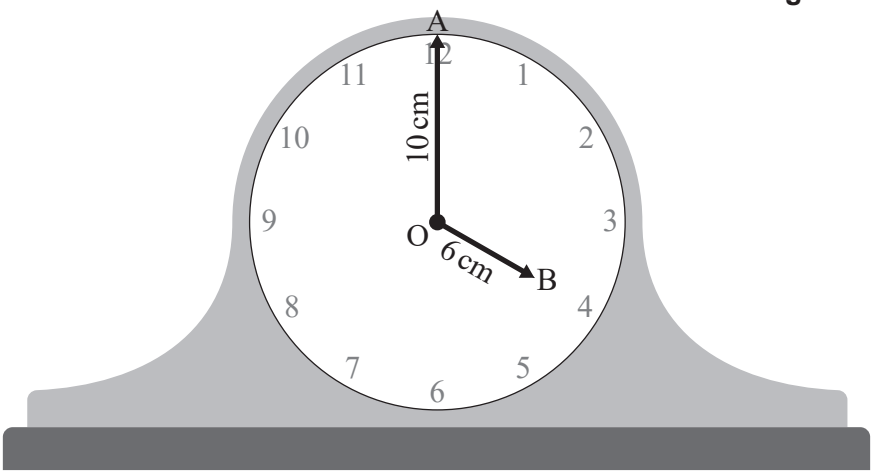
Question 9 - working space

2. [Maximum mark: 17]

The diagram below shows a circular clockface with centre O. The clock's minute hand has a length of $10\,\text{cm}$. The clock's hour hand has a length of $6\,\text{cm}$.

At $4{:}00$ pm the endpoint of the minute hand is at point A and the endpoint of the hour hand is at point B.

diagram not to scale

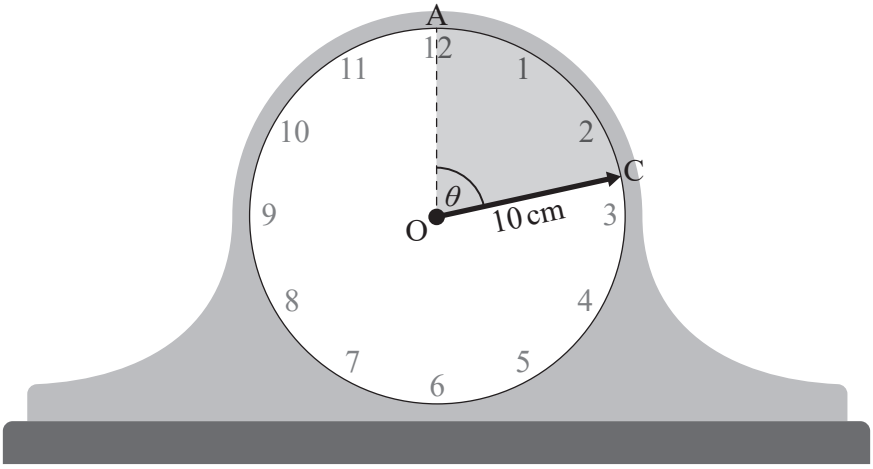


(a) Find the size of angle $\text{A}\hat{\text{O}}\text{B}$ in degrees. [2]

(b) Find the distance between points A and B. [3]

Between $4{:}00$ pm and $4{:}13$ pm, the endpoint of the **minute hand** rotates through an angle, θ, from point A to point C. This is illustrated in the diagram.

diagram not to scale



(c) Find the size of angle θ in degrees. [2]

(d) Calculate the length of arc AC. [2]

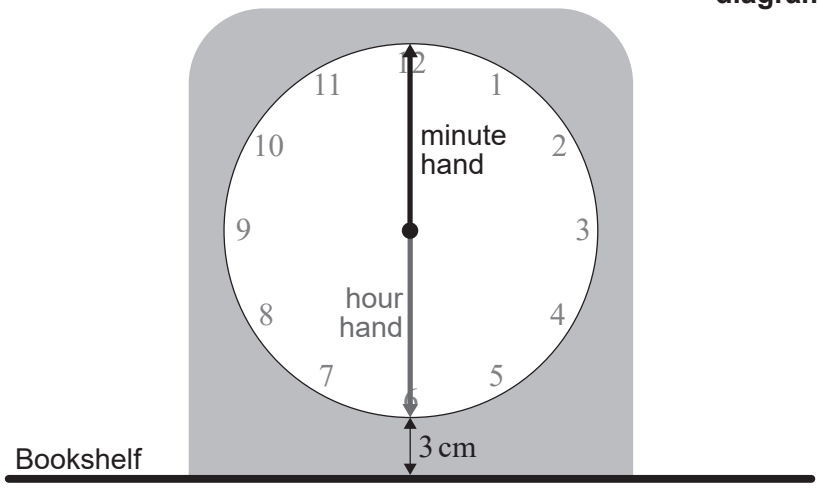
(e) Calculate the area of the shaded sector, AOC. [2]

(This question continues on the following page)

(Question 2 continued)

A **second** clock is illustrated in the diagram below. The clock face has radius $10\,\text{cm}$ with minute and hour hands both of length $10\,\text{cm}$. The time shown is $6{:}00\,\text{am}$. The bottom of the clock face is located $3\,\text{cm}$ above a horizontal bookshelf.

diagram not to scale



(f) Write down the height of the endpoint of the minute hand above the bookshelf at $6{:}00$ am. [1]

The height, h centimetres, of the endpoint of the minute hand above the bookshelf is modelled by the function

$$h(\theta) = 10\cos\theta + 13,\ \theta \geq 0,$$

where θ is the angle rotated by the minute hand from $6{:}00$ am.

(g) Find the value of h when $\theta = 160^\circ$. [2]

The height, g centimetres, of the endpoint of the **hour hand** above the bookshelf is modelled by the function

$$g(\theta) = -10\cos\left(\frac{\theta}{12}\right) + 13,\ \theta \geq 0,$$

where θ is the angle in degrees rotated by the minute hand from $6{:}00$ am.

(h) Write down the amplitude of $g(\theta)$. [1]

The endpoints of the minute hand and hour hand meet when $\theta = k$.

(i) Find the smallest possible value of k. [2]

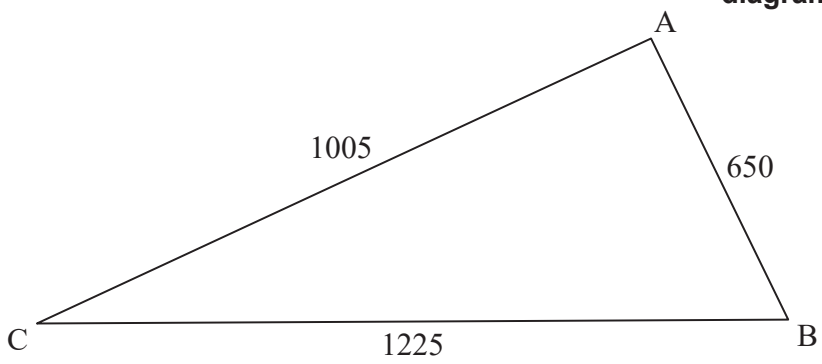
Turn over

Question 2 - working space

2. [Maximum mark: 15]

A farmer owns a field in the shape of a triangle ABC such that $AB = 650\,\text{m}$, $AC = 1005\,\text{m}$ and $BC = 1225\,\text{m}$.

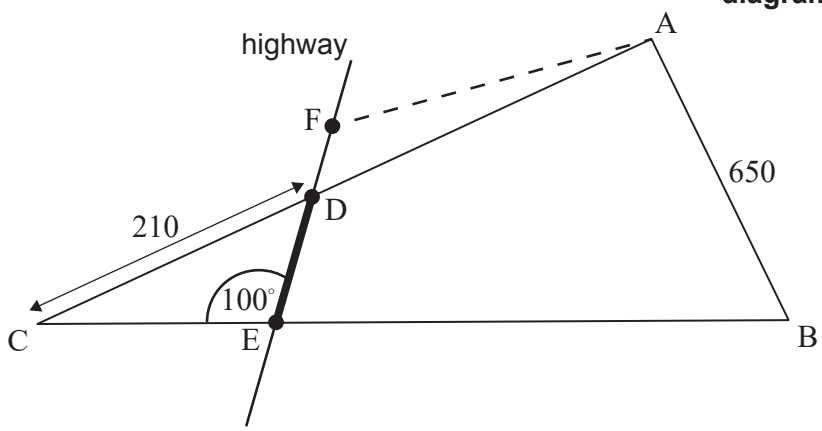
diagram not to scale



(a) Find the size of $A\hat{C}B$. [3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $DC = 210\,\text{m}$ and $C\hat{E}D = 100^\circ$, as shown in the diagram below.

diagram not to scale



(b) Find DE. [3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

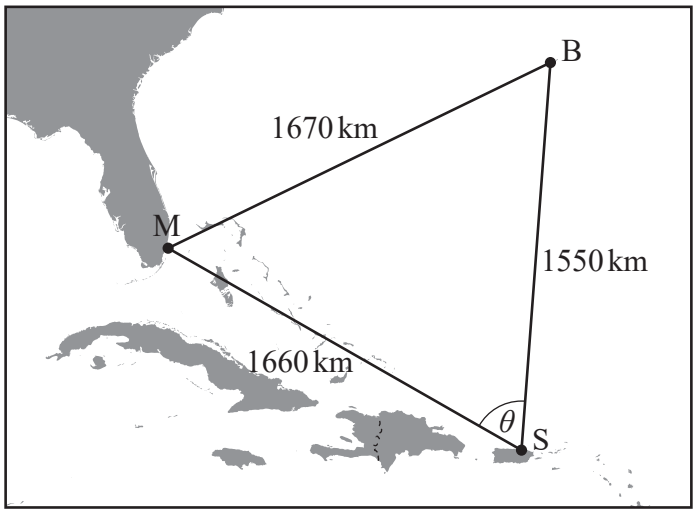
(d) Estimate DF. You may assume the highway has a width of zero. [4]

Question 2 - working space

2. [Maximum mark: 5]

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

(a) Calculate the value of θ, the measure of angle $\mathrm{M\hat{S}B}$. [3]

(b) Find the area of the Bermuda Triangle. [2]

(This question continues on the following page)

(Question 2 continued)

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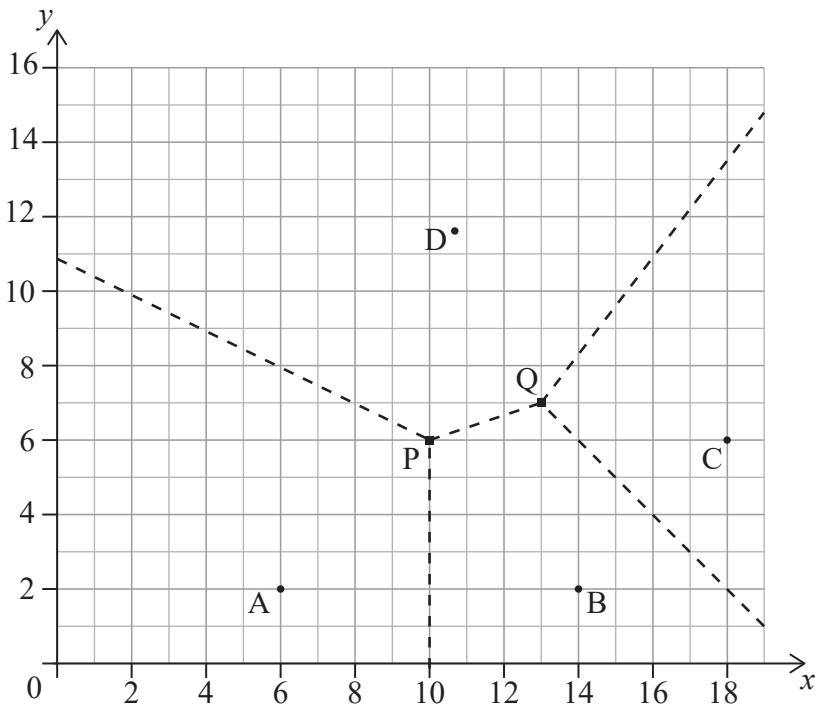
Question 2 - working space

7. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral $ABCD$. The new station will be located so that it is as far as possible from the nearest existing station.

(a) Show that the new station should be built at P. [3]

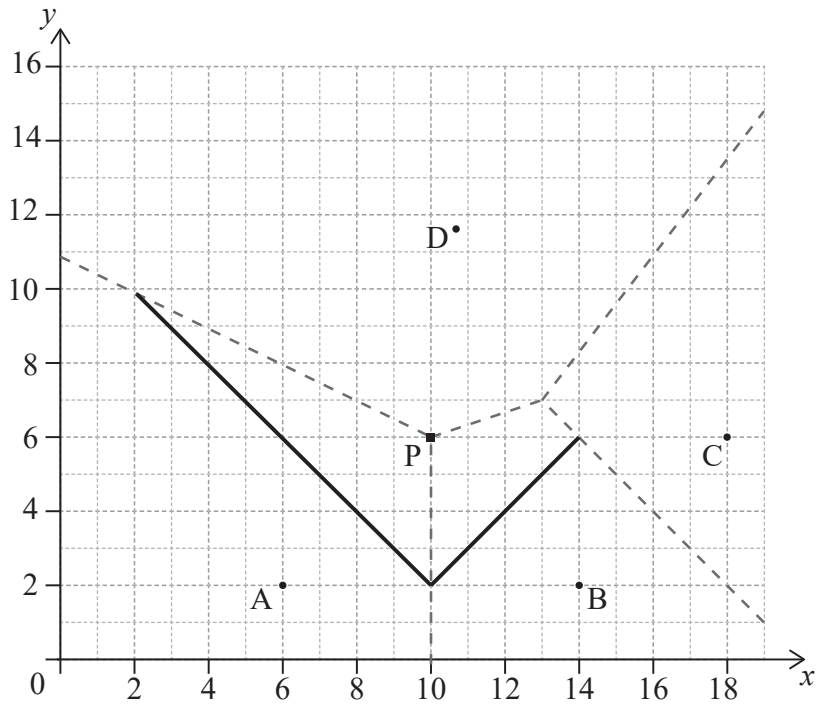
The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of $[AP]$ and $[BP]$ have been added to the following diagram.

(b) (i) Write down the equation of the perpendicular bisector of $[PC]$.

(ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

(This question continues on the following page)

(Question 7 continued)



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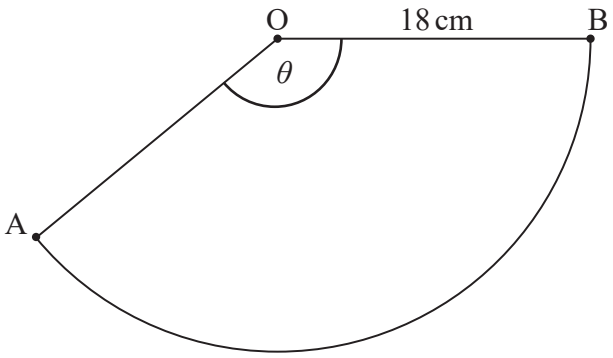
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Question 7 - working space

8. [Maximum mark: 5]

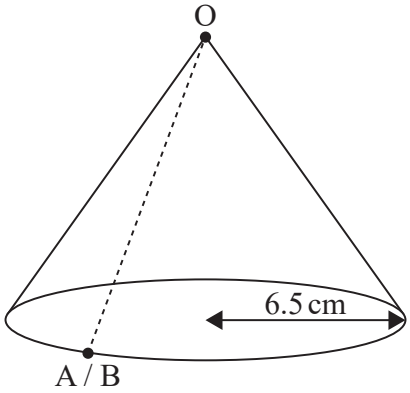
Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB, of a circular piece of paper with a radius of $18\,\mathrm{cm}$ and $\mathrm{A\hat{O}B} = \theta$ as shown in the diagram.

diagram not to scale



To make the hat, sides $[\mathrm{OA}]$ and $[\mathrm{OB}]$ are joined together. The hat has a base radius of $6.5\,\mathrm{cm}$.

diagram not to scale



(a) (i) Write down the perimeter of the base of the hat in terms of π.

(ii) Find the value of θ. [3]

(b) Find the surface area of the outside of the hat. [2]

(This question continues on the following page)

(Question 8 continued)

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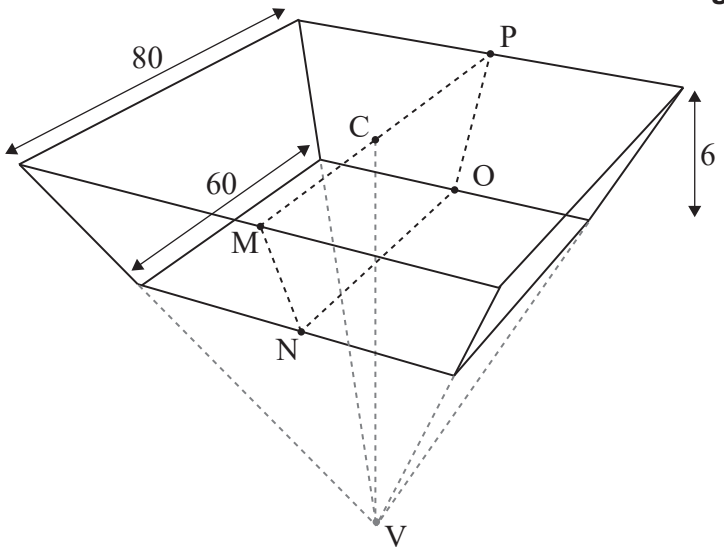


Question 8 - working space

4. [Maximum mark: 14]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.

diagram not to scale



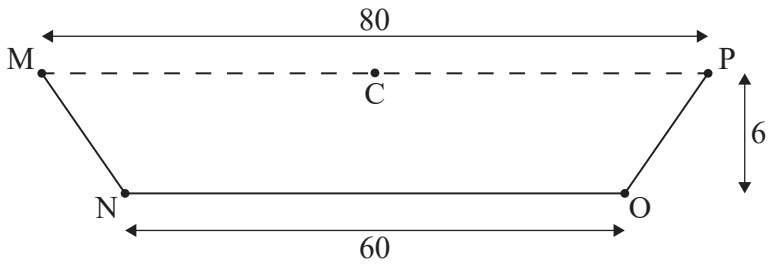
The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

(This question continues on the following page)

(Question 4 continued)

The second diagram shows a vertical cross section, MNOPC, of the reservoir.

diagram not to scale



(a) Find the angle of depression from M to N. [2]

(b) (i) Find CV.

(ii) Hence or otherwise, show that the volume of the reservoir is $29\,600\,\text{m}^3$. [5]

Every day $80\,\text{m}^3$ of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

(c) By finding an appropriate value, determine whether Joshua is correct. [2]

To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

(d) Find the area that was painted. [5]

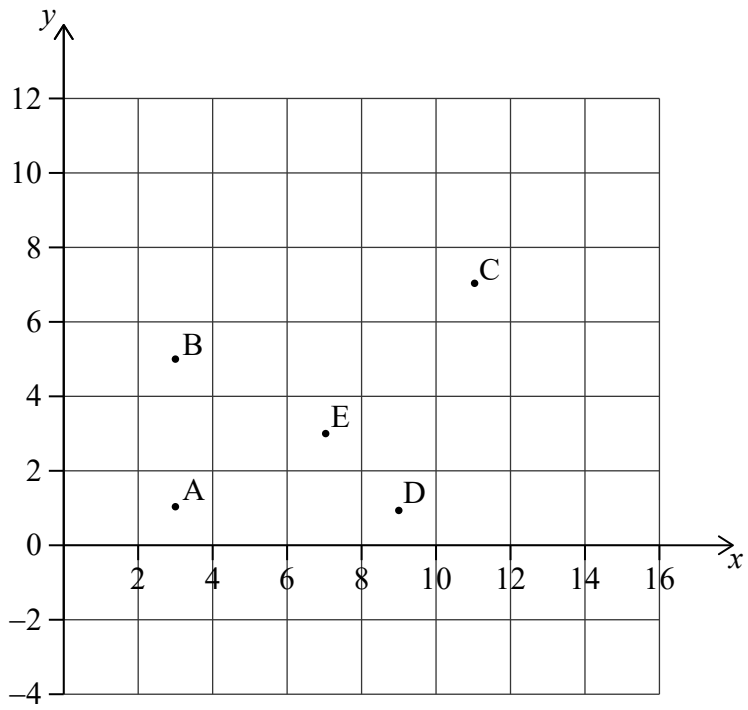
Question 4 - working space

7. [Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents $1\,\text{km}$.

Vertical scale: 1 unit represents $1\,\text{km}$.

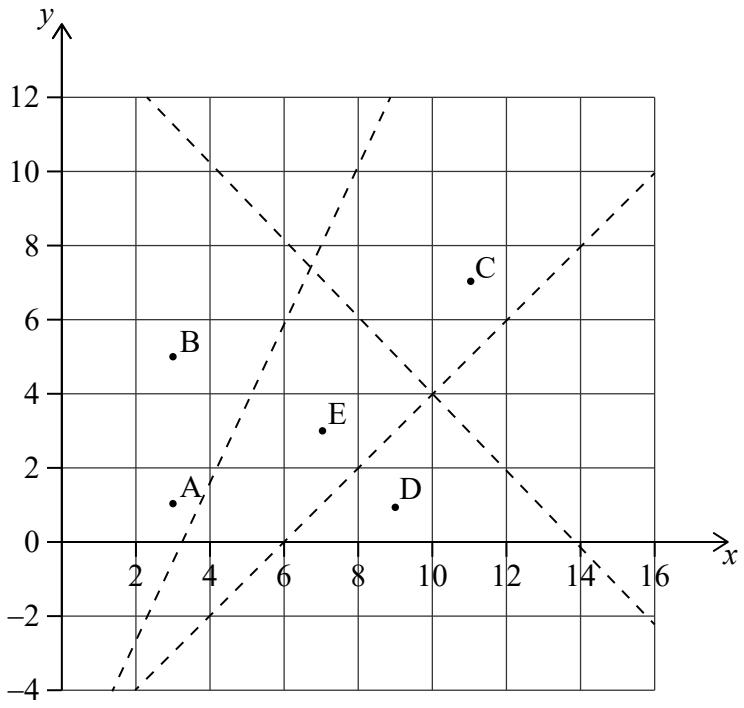


(a) Calculate the gradient of the line segment AE. [2]

(This question continues on the following page)

(Question 7 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



(b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

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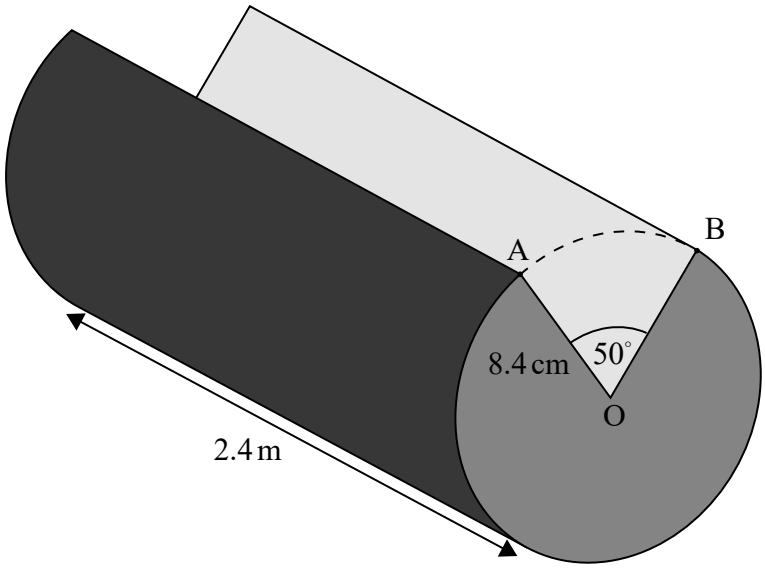
Turn over

Question 7 - working space

11. [Maximum mark: 4]

Helen is building a cabin using cylindrical logs of length $2.4\,\text{m}$ and radius $8.4\,\text{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



Find the volume of this log.

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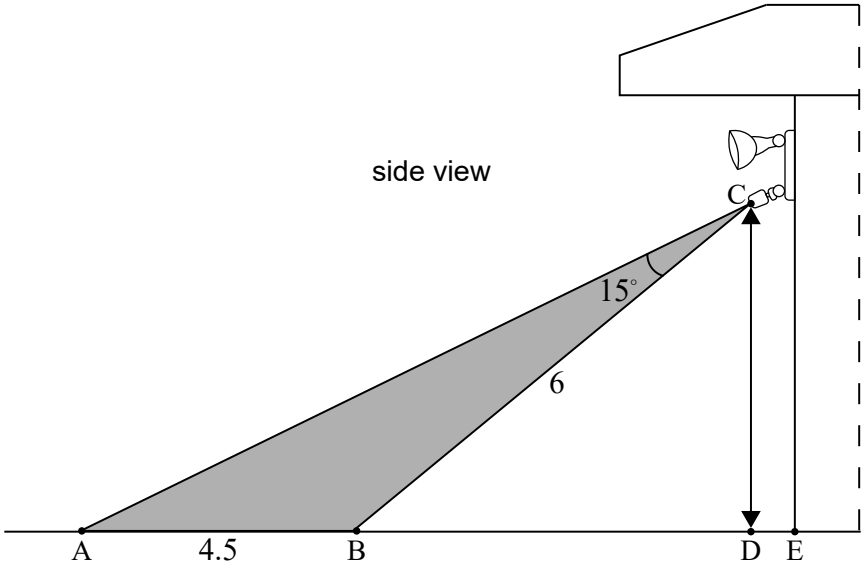
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Question 11 - working space

14. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is $4.5\,\text{m}$ and the distance from B to C is $6\,\text{m}$. Angle $A\hat{C}B$ is 15°.

diagram not to scale



(a) Find $C\hat{A}B$. [3]

Point B on the ground is $5\,\text{m}$ from point E at the entrance to Ollie's house. He is $1.8\,\text{m}$ tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

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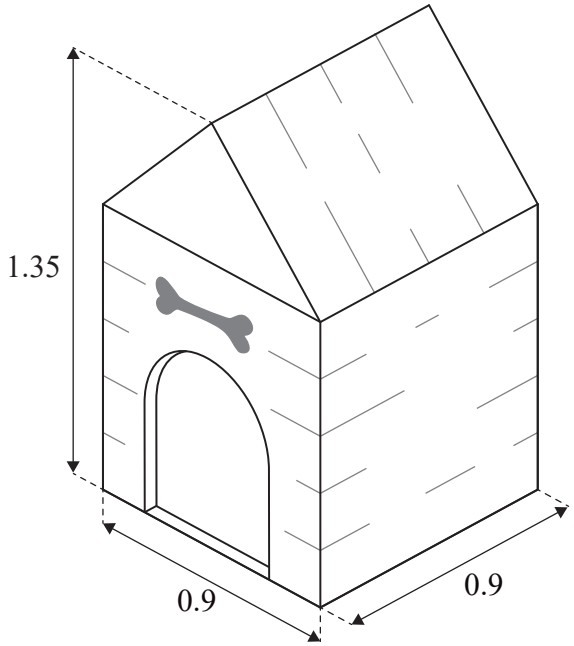
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will not be marked.

Question 14 - working space

1. [Maximum mark: 5]

The front view of a doghouse is made up of a square with an isosceles triangle on top. The doghouse is $1.35\,\text{m}$ high and $0.9\,\text{m}$ wide, and sits on a square base.

diagram not to scale



The top of the rectangular surfaces of the roof of the doghouse are to be painted.

Find the area to be painted.

(This question continues on the following page)

(Question 1 continued)

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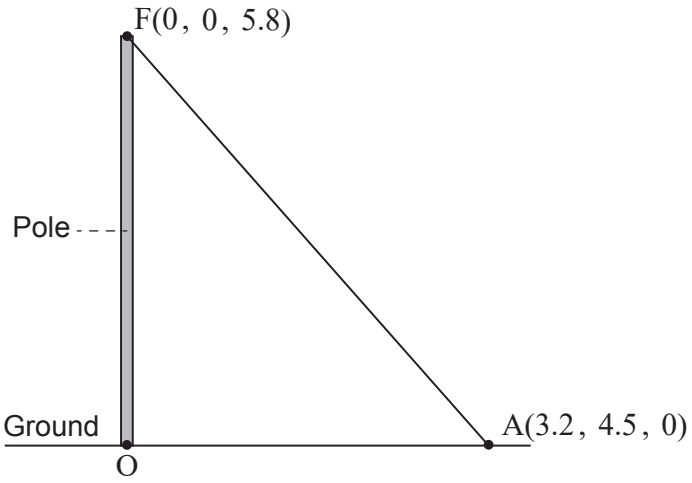
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Question 1 - working space

2. [Maximum mark: 4]

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0, 0, 5.8)$. All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F.

One of the ropes is attached to the ground at a point A with coordinates $(3.2, 4.5, 0)$. The rope forms a straight line from A to F.

(a) Find the length of the rope connecting A to F. [2]

(b) Find $\hat{FAO}$, the angle the rope makes with the ground. [2]

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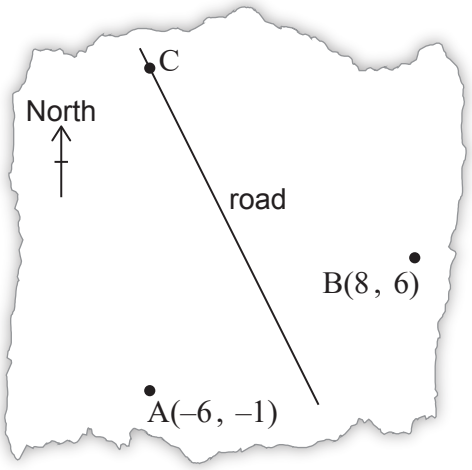
Question 2 - working space

4. [Maximum mark: 7]

Three towns, A, B and C are represented as coordinates on a map, where the x and y axes represent the distances east and north of an origin, respectively, measured in kilometres.

Town A is located at $(-6, -1)$ and town B is located at $(8, 6)$. A road runs along the perpendicular bisector of [AB]. This information is shown in the following diagram.

diagram not to scale



(a) Find the equation of the line that the road follows. [5]

Town C is due north of town A and the road passes through town C.

(b) Find the y-coordinate of town C. [2]

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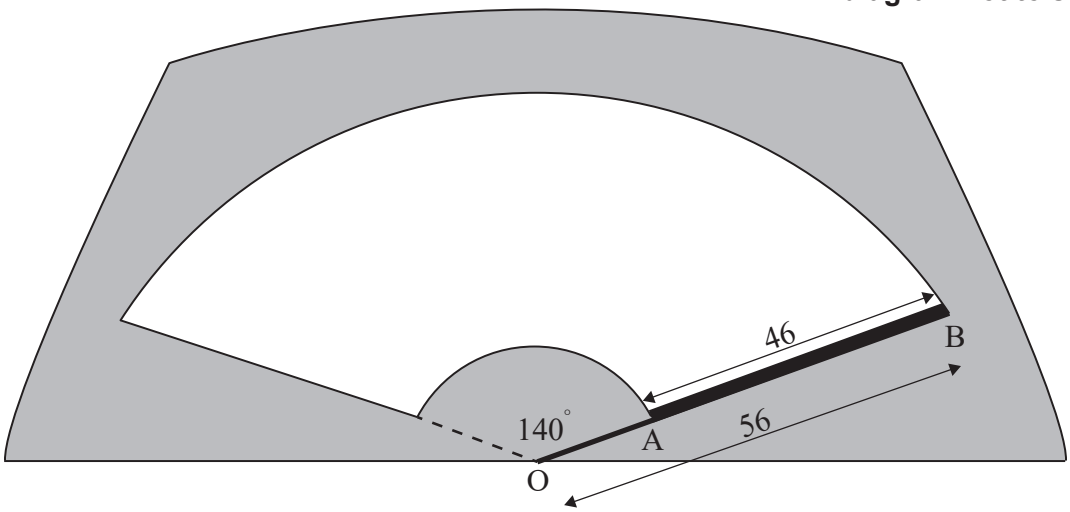
Question 4 - working space

1. [Maximum mark: 5]

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O, through an angle of $140°$. The windscreen is cleared by a rubber blade of length $46\,\text{cm}$ that is attached to the metal arm between points A and B. The total length of the metal arm, OB, is $56\,\text{cm}$.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.

diagram not to scale



(a) Calculate the length of the arc made by B, the end of the rubber blade. [2]

(b) Determine the area of the windscreen that is cleared by the rubber blade. [3]

(This question continues on the following page)

(Question 1 continued)

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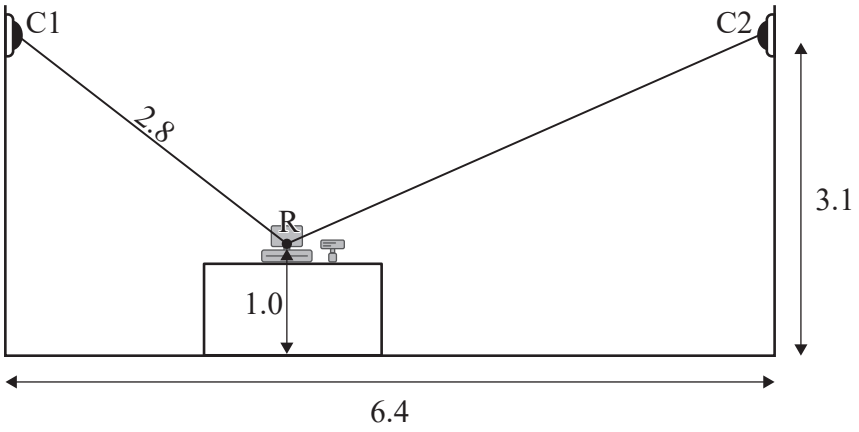
Question 1 - working space

3. [Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points $C1$ and $C2$. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.

diagram not to scale



The cameras are positioned at a height of $3.1\,\text{m}$, and the horizontal distance between the cameras is $6.4\,\text{m}$. The cash register is sitting on a counter so that its centre, R, is $1.0\,\text{m}$ above the floor.

The distance from Camera 1 to the centre of the cash register is $2.8\,\text{m}$.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

(This question continues on the following page)

(Question 3 continued)

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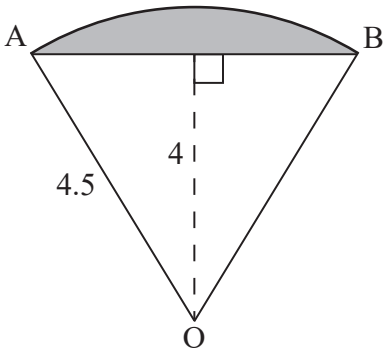
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Question 3 - working space

4. [Maximum mark: 15]

A sector of a circle, centre O and radius $4.5\,\text{m}$, is shown in the following diagram.

diagram not to scale

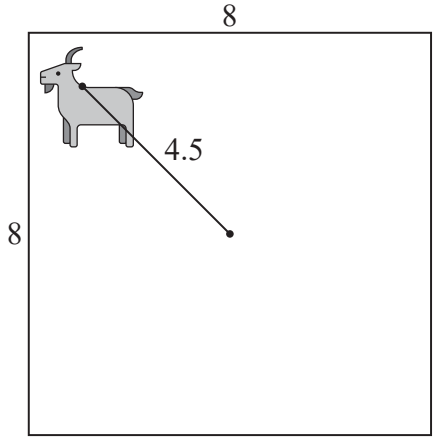


(a) (i) Find the angle $\text{A}\hat{\text{O}}\text{B}$.

(ii) Find the area of the shaded segment. [8]

A square field with side $8\,\text{m}$ has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to $4.5\,\text{m}$ from the post.

diagram not to scale



(b) (i) Find the area of a circle with radius $4.5\,\text{m}$.

(ii) Find the area of the field that can be reached by the goat. [5]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

The goat eats grass at the rate of $\frac{\text{d}V}{\text{d}t} = 0.3\,t\text{e}^{-t}$.

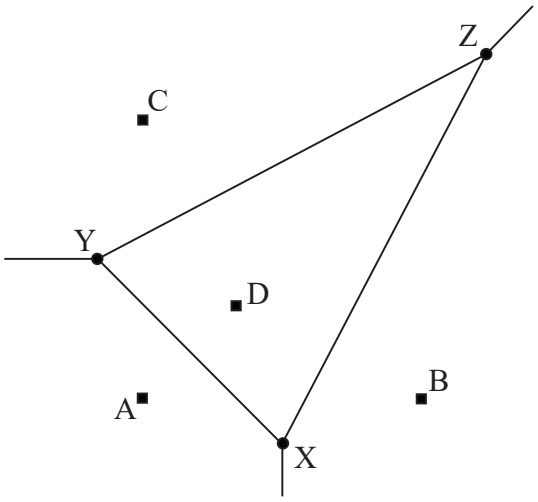
(c) Find the value of t at which the goat is eating grass at the greatest rate. [2]

Question 4 - working space

3. [Maximum mark: 18]

The Voronoi diagram below shows four supermarkets represented by points with coordinates $\mathrm{A}(0,\ 0)$, $\mathrm{B}(6,\ 0)$, $\mathrm{C}(0,\ 6)$ and $\mathrm{D}(2,\ 2)$. The vertices X, Y, Z are also shown. All distances are measured in kilometres.

diagram not to scale



(a) Find the midpoint of $[\mathrm{BD}]$. [2]

(b) Find the equation of (XZ). [4]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X. [3]

The coordinates of Y are $(-1,\ 3)$ and the coordinates of Z are $(7,\ 7)$.

(d) Determine the exact length of $[\mathrm{YZ}]$. [2]

(e) Given that the exact length of $[\mathrm{XY}]$ is $\sqrt{32}$, find the size of $\mathrm{X\hat{Y}Z}$ in degrees. [4]

(f) Hence find the area of triangle XYZ. [2]

A town planner believes that the larger the area of the Voronoi cell XYZ, the more people will shop at supermarket D.

(g) State one criticism of this interpretation. [1]

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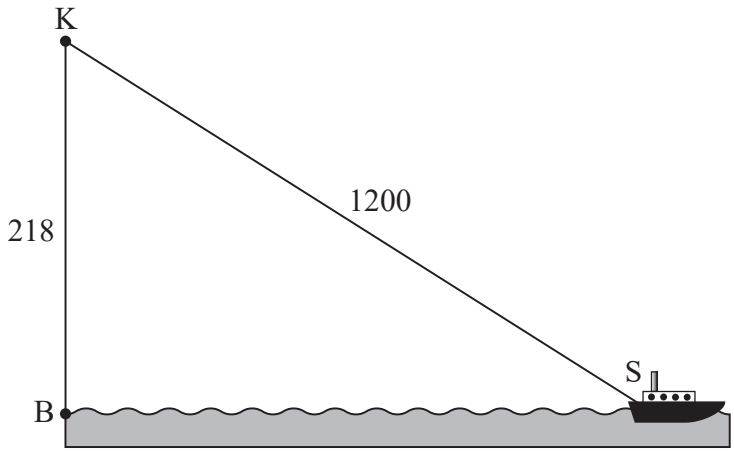
Question 3 - working space

1. [Maximum mark: 6]

Kacheena stands at point K, the top of a $218\,\mathrm{m}$ vertical cliff. The base of the cliff is located at point B. A ship is located at point S, $1200\,\mathrm{m}$ from Kacheena.

This information is shown in the following diagram.

diagram not to scale



(a) Find the angle of elevation from the ship to Kacheena. [2]

(b) Find the horizontal distance from the base of the cliff to the ship. [2]

(c) Write down your answer to part (b) in the form $a \times 10^k$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [2]

(This question continues on the following page)

(Question 1 continued)

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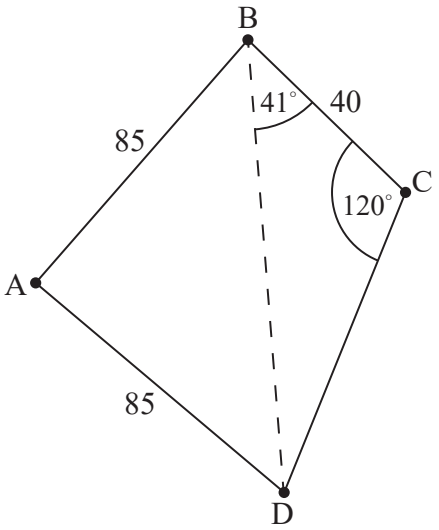
Question 1 - working space

2. [Maximum mark: 17]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral $ABCD$. A straight path crosses through the park from B to D.

$$AB = 85\,\text{m},\ AD = 85\,\text{m},\ BC = 40\,\text{m},\ C\hat{B}D = 41^\circ,\ B\hat{C}D = 120^\circ$$

diagram not to scale



(a) (i) Write down the value of angle BDC.

(ii) Hence use triangle BDC to find the length of path BD. [4]

(b) Calculate the size of angle $B\hat{A}D$, correct to five significant figures. [3]

The size of angle $B\hat{A}D$ rounds to 77°, correct to the nearest degree. Use $B\hat{A}D = 77^\circ$ for the rest of this question.

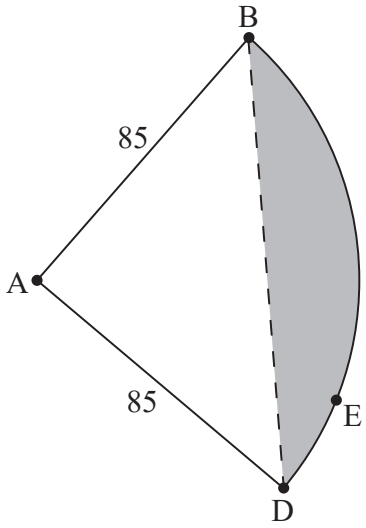
(c) Find the area bounded by the path BD, and fences AB and AD. [3]

(This question continues on the following page)

(Question 2 continued)

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

(f) Find the area of the shaded region in the proposed park. [3]

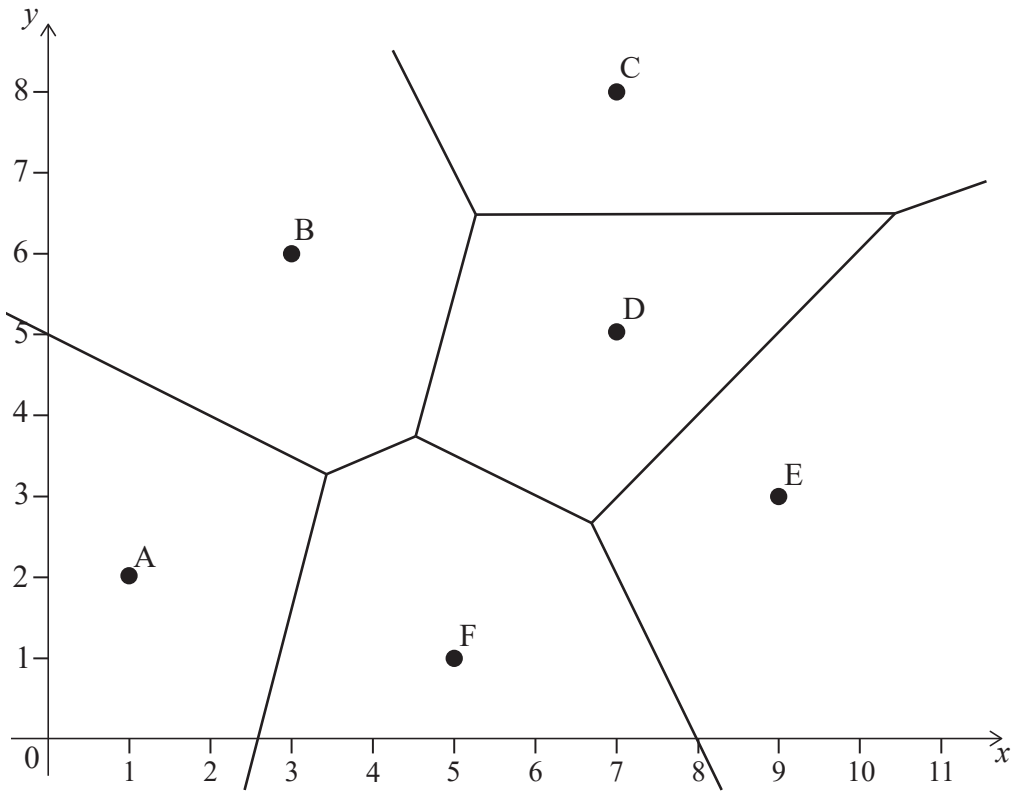
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Question 2 - working space

3. [Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



(a) Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(i) $(2, 7)$.

(ii) $(0, 1)$, when restaurant A is closed. [2]

Restaurant C is at $(7, 8)$ and restaurant D is at $(7, 5)$.

(b) Find the equation of the perpendicular bisector of CD. [2]

Restaurant B is at $(3, 6)$.

(c) Find the equation of the perpendicular bisector of BC. [5]

(d) Hence find

(i) the coordinates of the point which is of equal distance from B, C and D.

(ii) the distance of this point from D. [4]

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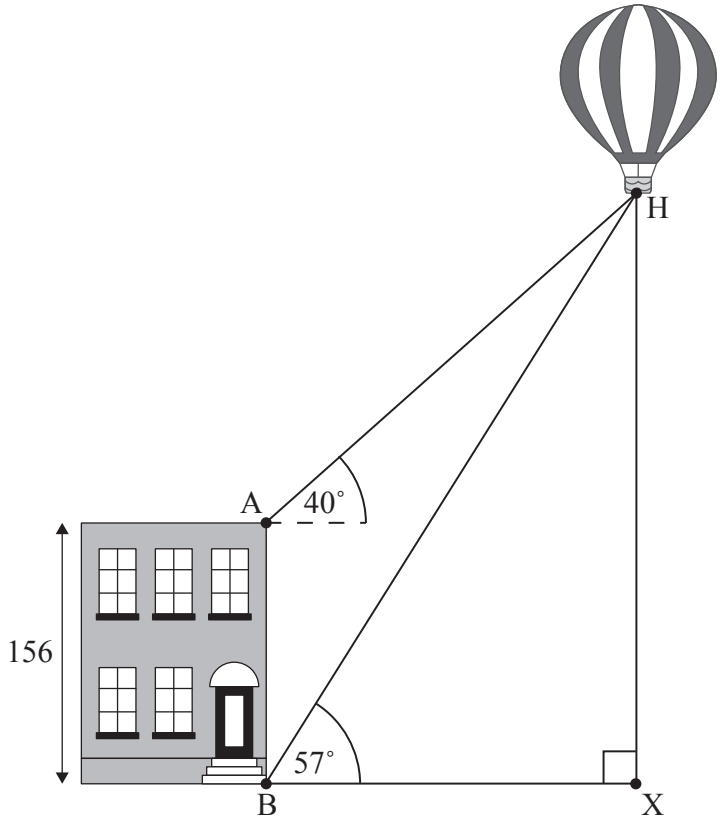
Question 3 - working space

2. [Maximum mark: 6]

Point H on a hot-air balloon is sighted at the same time by two observers. One observer is at the top of a vertical building that is 156 metres tall. The other observer is at the base of the building.

The angle of elevation from point A (at the top of the building) to H is $40°$, and the angle of elevation from point B (at the base of the building) to H is $57°$. Point X is the ground directly below point H. This information is shown in the diagram.

diagram not to scale



(a) Find the size of angle $A\hat{H}B$. [2]

(b) Calculate the distance from point B to point H. [3]

The hot-air balloon remains at a constant height as it moves further away from the building.

(c) Describe, in words, the change in the angle of depression from point H to point B as the horizontal distance between the balloon and the building increases. [1]

(This question continues on the following page)

(Question 2 continued)

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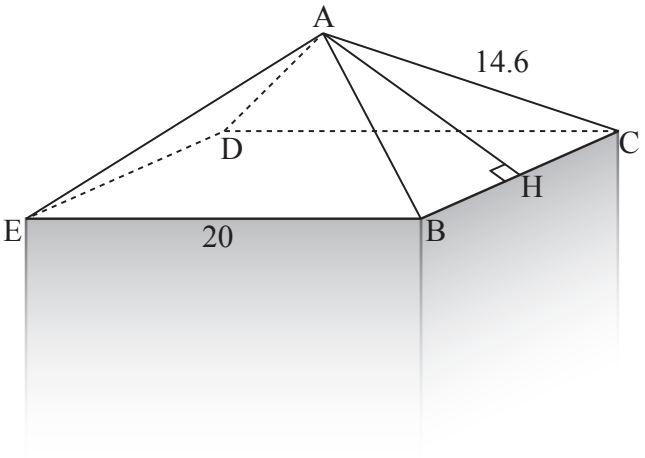
Question 2 - working space

11. [Maximum mark: 7]

Vertical posts are to be placed around the outer edge of a children's park. Each post is formed from a cuboid with a right square-based pyramid on top.

The cuboid part of the post is machine-made such that its width, and hence the width of the pyramid, is exactly $20\,\text{cm}$. The length from the apex of the pyramid, A, to any corner of the base of the pyramid is $14.6\,\text{cm}$, **but** this is only accurate to the nearest tenth of a centimetre. The post is shown in the diagram.

diagram not to scale



(a) Write down the upper bound and lower bound for the possible lengths of edge AC. [2]

Point H is the midpoint of BC.

(b) Determine the upper bound and lower bound for AH, the slant height of the pyramid. [3]

For the post to be safe for children, the angle between the slant height and the base of the pyramid must be less than $22°$.

(c) Show that this post is safe for children. Justify your answer. [2]

(This question continues on the following page)

(Question 11 continued)

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Question 11 - working space

13. [Maximum mark: 5]

A boat travels $8\,\text{km}$ on a bearing of $315°$ and then a further $6\,\text{km}$ on a bearing of $045°$. Find the bearing on which the boat should travel to return directly to the starting point.

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References:

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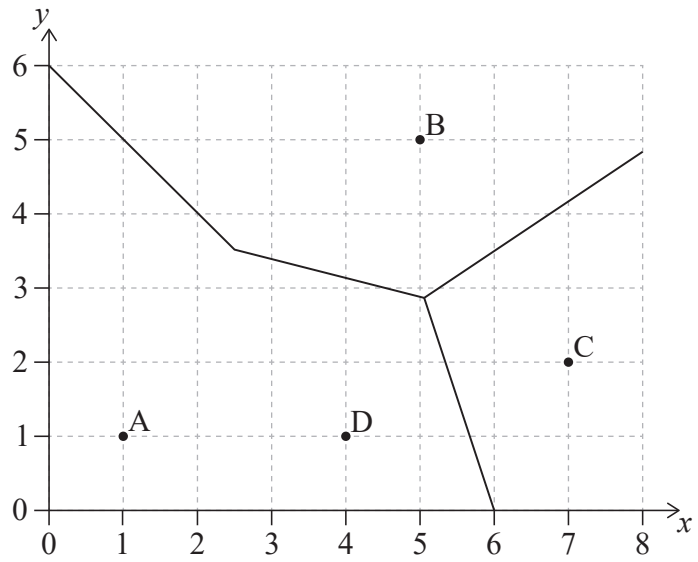
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Answers written on this page
will not be marked.

Question 13 - working space

7. [Maximum mark: 6]

Ani owns four cafes represented by points A, B, C and D. Ani wants to divide the area into delivery regions. This process has been started in the following incomplete Voronoi diagram, where 1 unit represents 1 kilometre.



The midpoint of CD is $(5.5, 1.5)$.

(a) Show that the equation of the perpendicular bisector of [CD] is $y = -3x + 18$. [3]

(b) Complete the Voronoi diagram shown above. [1]

Ani opens an office equidistant from three of the cafes, B, C and D. The equation of the perpendicular bisector of [BC] is $3y = 2x - 1.5$.

(c) Find the coordinates of the office. [2]

(This question continues on the following page)

(Question 7 continued)

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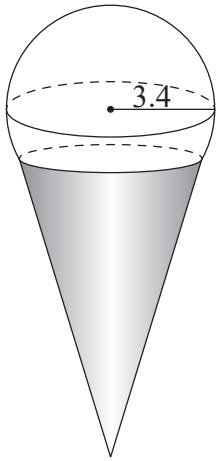
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Question 7 - working space

8. [Maximum mark: 5]

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of $3.4\,\text{cm}$. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

The cone has a slant height of $11\,\text{cm}$ and a radius of $3\,\text{cm}$.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2. [2]

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Question 8 - working space

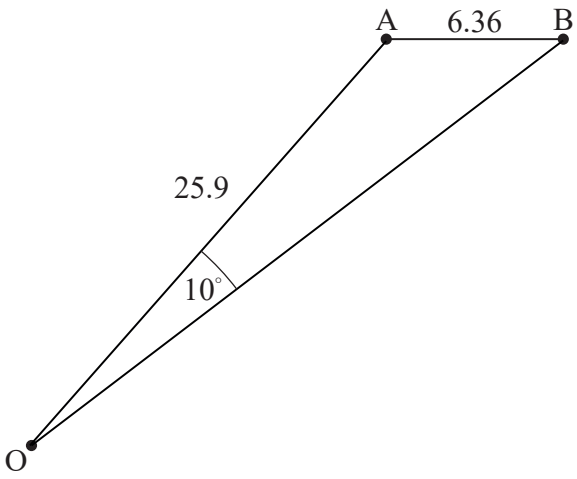
1. [Maximum mark: 17]

The diagram shows points in a park viewed from above, at a specific moment in time.

The distance between two trees, at points A and B, is $6.36\,\text{m}$.

Odette is playing football in the park and is standing at point O, such that $\text{A}\hat{\text{O}}\text{B} = 10^\circ$, $\text{OA} = 25.9\,\text{m}$ and $\text{O}\hat{\text{A}}\text{B}$ is obtuse.

diagram not to scale



(a) Calculate the size of $\text{A}\hat{\text{B}}\text{O}$. [3]

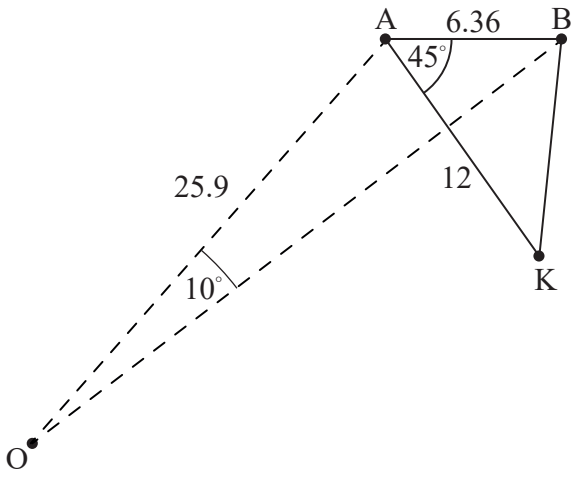
(b) Calculate the area of triangle AOB. [4]

(This question continues on the following page)

(Question 1 continued)

Odette's friend, Khemil, is standing at point K such that he is $12\,\text{m}$ from A and $\text{K}\hat{\text{A}}\text{B} = 45^\circ$.

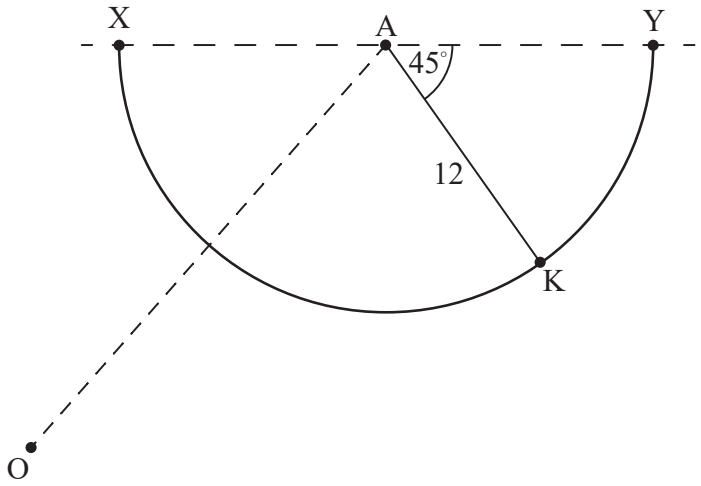
diagram not to scale



(c) Calculate Khemil's distance from B. [3]

XY is a semicircular path in the park with centre A, such that $\text{K}\hat{\text{A}}\text{Y} = 45^\circ$. Khemil is standing on the path and Odette's football is at point X. This is shown in the diagram below.

diagram not to scale



The length $\text{KX} = 22.2\,\text{m}$, $\text{K}\hat{\text{O}}\text{X} = 53.8^\circ$ and $\text{O}\hat{\text{K}}\text{X} = 51.1^\circ$.

(d) Find whether Odette or Khemil is closer to the football. [4]

Khemil runs along the semicircular path to pick up the football.

(e) Calculate the distance that Khemil runs. [3]

Turn over

Question 1 - working space

2. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).



[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

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Question 2 - working space

4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

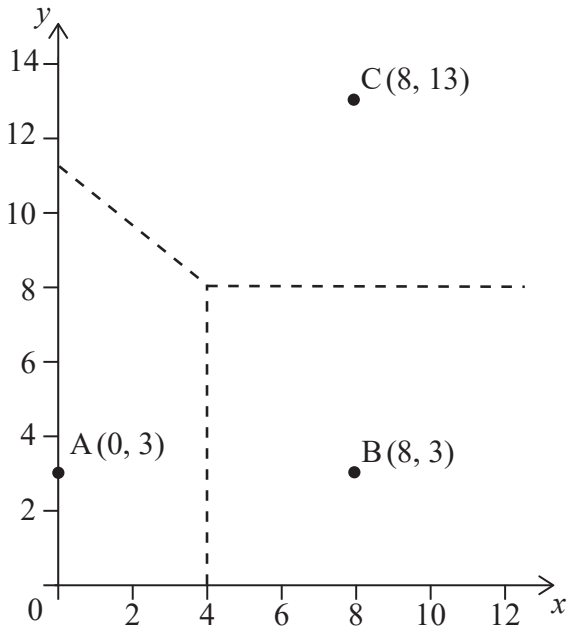
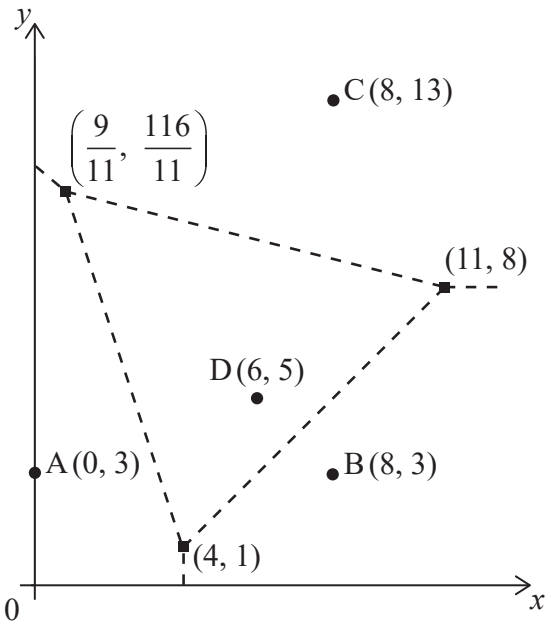


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

(This question continues on the following page)

(Question 4 continued)

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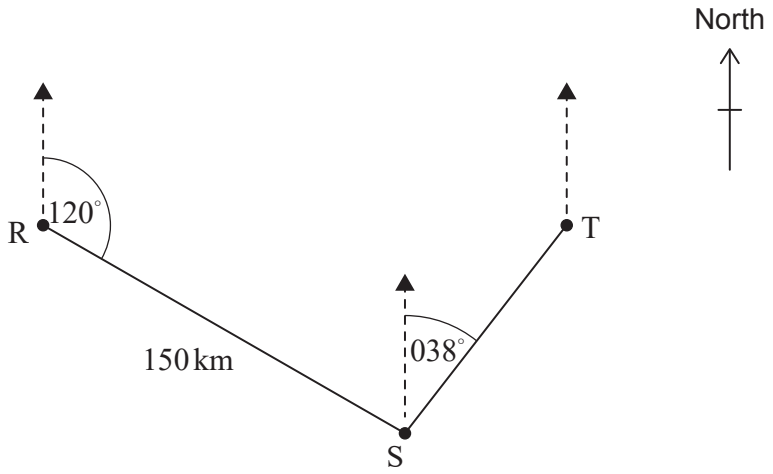


Question 4 - working space

5. [Maximum mark: 6]

Ron sails his boat from point R for a distance of $150\,\text{km}$, on a bearing of 120°, to arrive at point S. He then sails on a bearing of 038° to point T. Ron's journey is shown in the diagram.

diagram not to scale



(a) Find $\text{R}\hat{\text{S}}\text{T}$. [2]

Point T is directly east of point R.

(b) Calculate the distance that Ron sails to return directly from point T to point R. [4]

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Question 5 - working space

2. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).



[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

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Question 2 - working space

4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

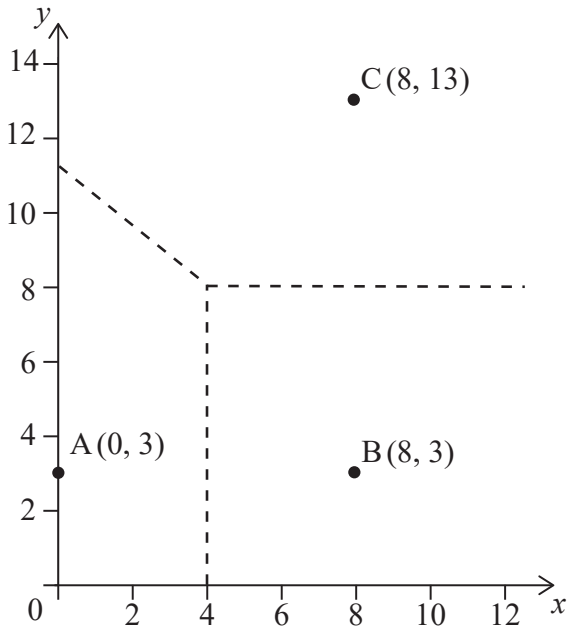
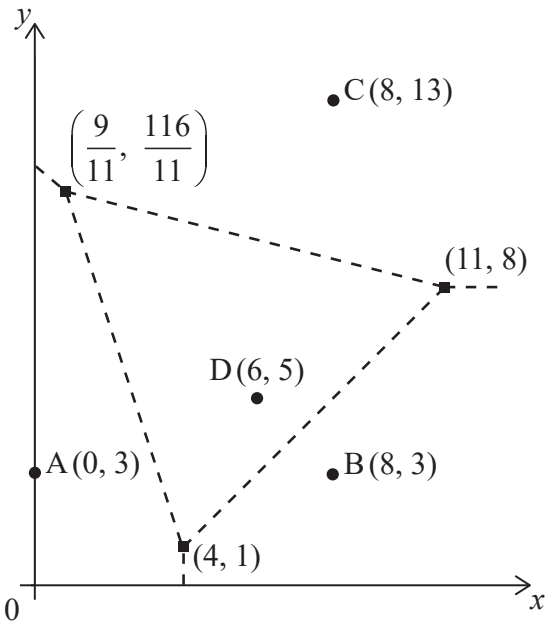


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

(This question continues on the following page)

(Question 4 continued)

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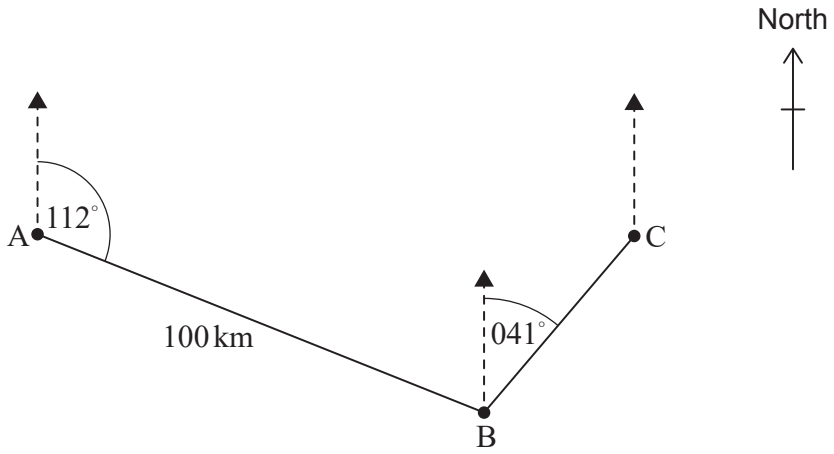


Question 4 - working space

5. [Maximum mark: 6]

Jason sails his boat from point A for a distance of $100\,\text{km}$, on a bearing of 112°, to arrive at point B. He then sails on a bearing of 041° to point C. Jason's journey is shown in the diagram.

diagram not to scale



(a) Find $A\hat{B}C$. [2]

Point C is directly east of point A.

(b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

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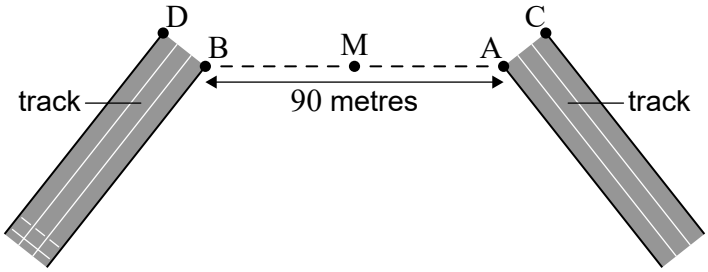
Question 5 - working space

2. [Maximum mark: 15]

Ansel is designing a racing track for a local bicycle club. The following diagram shows an incomplete portion of the track.

Ansel wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 90 metres.

diagram not to scale

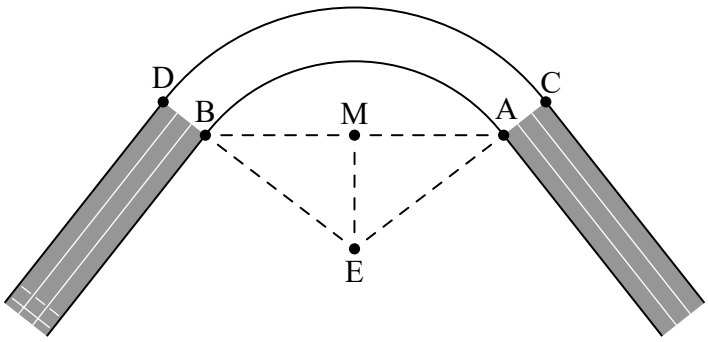


To create a smooth curve, Ansel first walks to M, the midpoint of $[AB]$.

(a) Write down the length of $[BM]$. [1]

Ansel then walks 32 metres in a direction perpendicular to $[AB]$ to get from point M to point E. Point E is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of $[BE]$.

(ii) Find $B\hat{E}M$. [4]

(c) Hence, find the length of arc AB. [3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre E, such that the track is 4 metres wide.

(d) Calculate the area of the curved portion of the track, $ABDC$. [4]

The base of the track will be made of concrete that is $15\,\text{cm}$ deep.

(e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

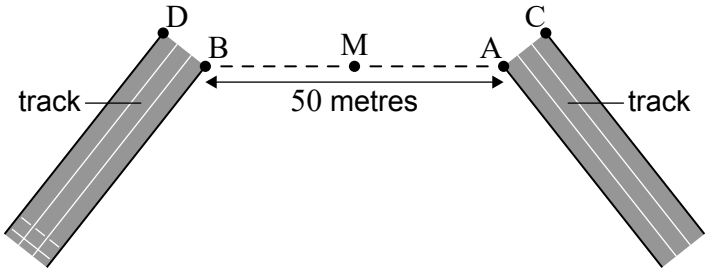
Question 2 - working space

2. [Maximum mark: 15]

Madhu is designing a jogging track for the campus of her school. The following diagram shows an incomplete portion of the track.

Madhu wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 50 metres.

diagram not to scale

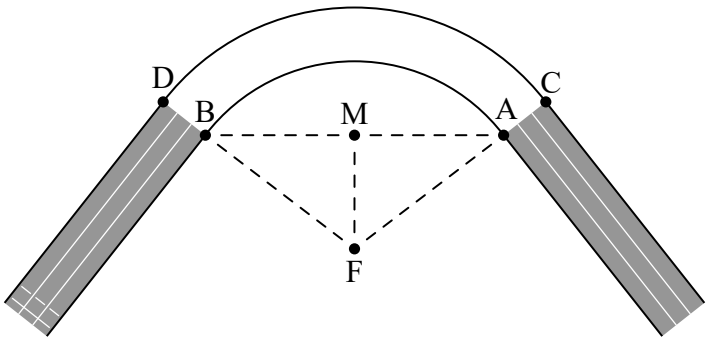


To create a smooth curve, Madhu first walks to M, the midpoint of $[AB]$.

(a) Write down the length of $[BM]$. [1]

Madhu then walks 20 metres in a direction perpendicular to $[AB]$ to get from point M to point F. Point F is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of $[BF]$.

(ii) Find $B\hat{F}M$. [4]

(c) Hence, find the length of arc AB. [3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre F, such that the track is 2 metres wide.

(d) Calculate the area of the curved portion of the track, ABDC. [4]

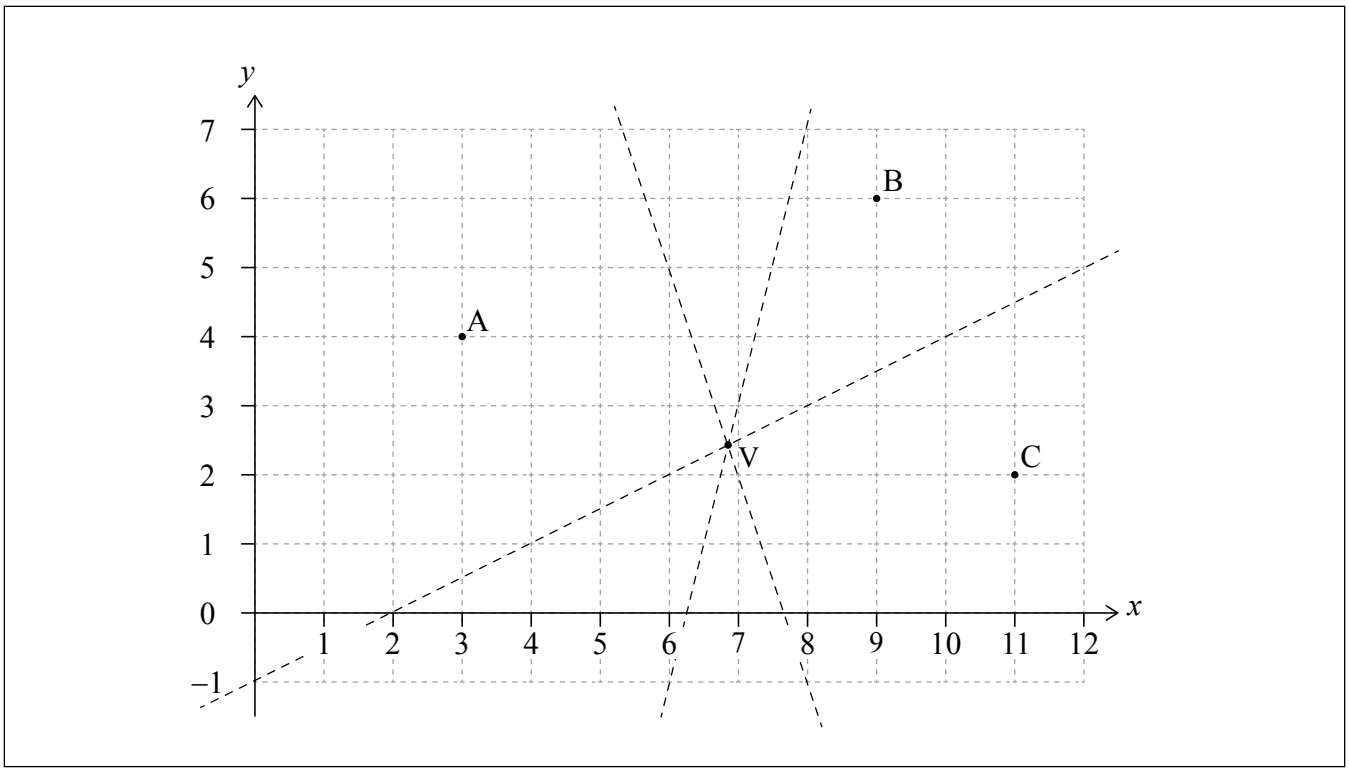
The base of the track will be made of concrete that is 12 cm deep.

(e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

Question 2 - working space

6. [Maximum mark: 6]

Points $\mathrm{A}(3,\ 4)$, $\mathrm{B}(9,\ 6)$ and $\mathrm{C}(11,\ 2)$ are shown on the following diagram, along with the perpendicular bisectors of [AB], [AC] and [BC].



The perpendicular bisector of [BC] intercepts the axes at coordinates $(0,\ -1)$ and $(2,\ 0)$.

(a) Write down the equation of the perpendicular bisector of [BC]. [2]

The equation of the perpendicular bisector of [AB] is $y = -3x + 23$.

(b) Find the coordinates of point V where the perpendicular bisectors meet. Give your answer to four significant figures. [2]

A Voronoi diagram is constructed with points A, B and C as the three sites.

(c) Draw, clearly, the edges of the Voronoi diagram on the given diagram. [2]

(This question continues on the following page)

(Question 6 continued)

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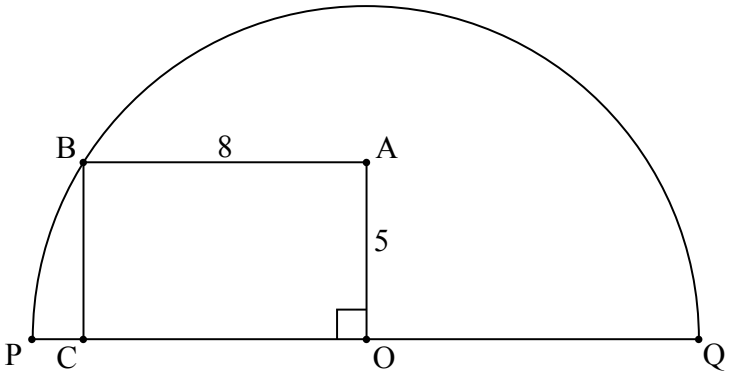


Question 6 - working space

11. [Maximum mark: 5]

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle $OABC$ is also shown, such that $AB = 8$ and $OA = 5$.

diagram not to scale



Find the length of the arc BQ.

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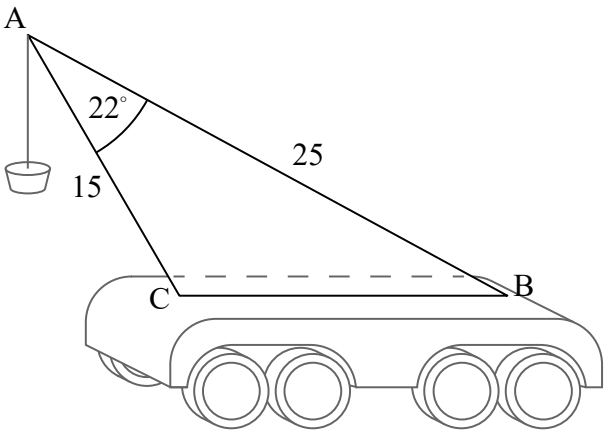
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Question 11 - working space

2. [Maximum mark: 6]

The diagram shows a toy crane.

diagram not to scale



$AB = 25\,\text{cm}$, $AC = 15\,\text{cm}$ and $B\hat{A}C = 22^\circ$.

(a) Calculate BC. [3]

(b) Given that $A\hat{B}C$ is acute, calculate $A\hat{B}C$. [3]

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Question 2 - working space

11. [Maximum mark: 6]

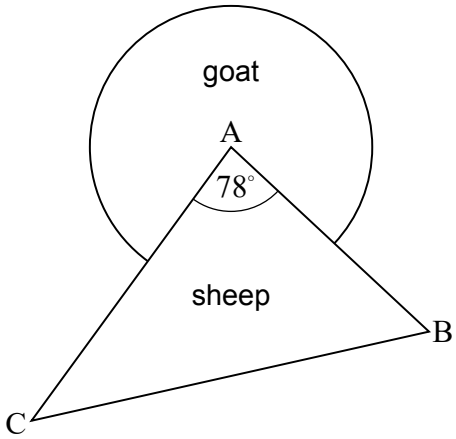
A sheep is in a field in the shape of a triangle, ABC.

$\mathrm{AC} = 21$ metres, $\mathrm{AB} = 15$ metres and $\mathrm{C\hat{A}B} = 78^\circ$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

The fields are shown in the diagram.

diagram not to scale



Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field.

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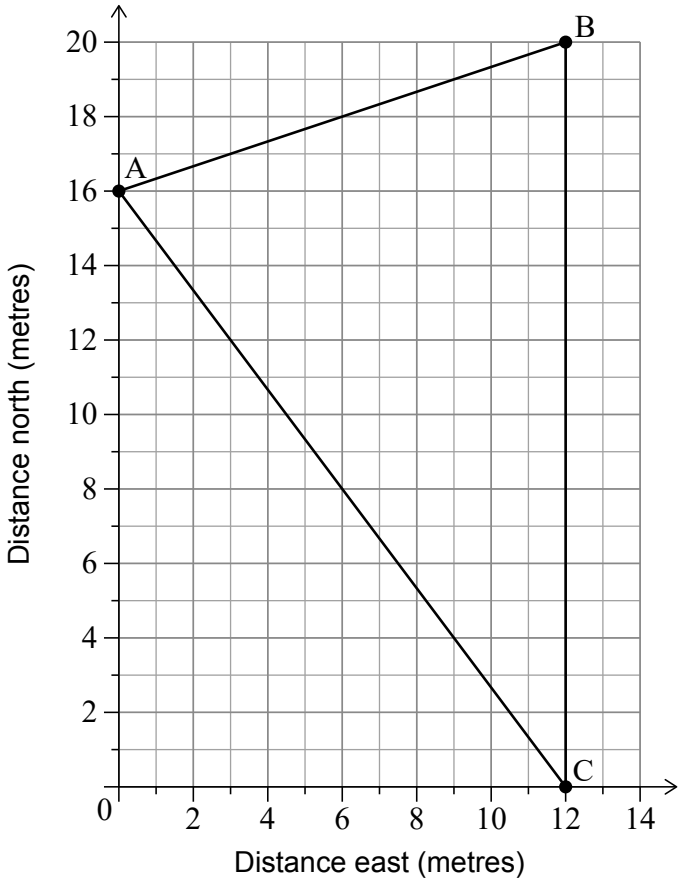
Turn over

Question 11 - working space

2. [Maximum mark: 14]

Mai is at an amusement park. A map of part of the amusement park is represented on the following coordinate axes.

Mai's favourite three attractions are positioned at $A(0, 16)$, $B(12, 20)$ and $C(12, 0)$.
All measurements are in metres.



(a) Write down the distance between B and C. [1]

(b) Calculate the distance between A and B. [2]

(This question continues on the following page)

(Question 2 continued)

Mai is standing at the attraction at B and wants to walk directly to the attraction at A.

(c) Calculate the bearing of A from B. [3]

A drinking fountain is to be installed at a point that is an equal distance from each of the attractions at A, B and C.

(d) (i) Write down the gradient of $[\mathrm{AC}]$.

(ii) Write down the mid-point of $[\mathrm{AC}]$.

(iii) Hence calculate the coordinates of the drinking fountain. [8]

Turn over

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Question 2 - working space

5. [Maximum mark: 16]

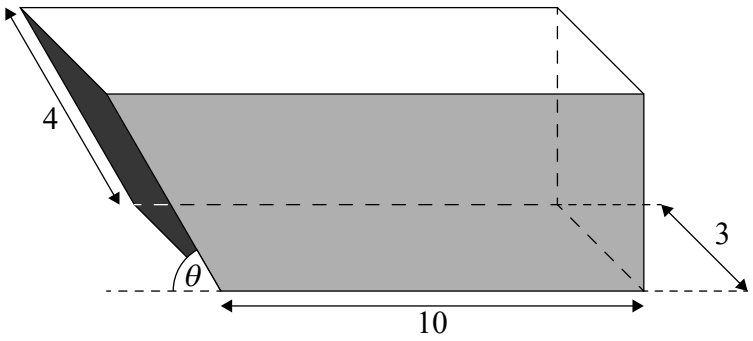
A skip is a container used to carry garbage away from a construction site. For safety reasons the garbage must not extend beyond the top of the skip. The maximum volume of garbage to be removed is therefore equal to the volume of the skip.



A particular design of skip can be modelled as a prism with a trapezoidal cross section. For the skip to be transported, it must have a rectangular base of length $10\,\text{m}$ and width $3\,\text{m}$. The length of the sloping edge is fixed at $4\,\text{m}$, and makes an angle of θ with the horizontal.

The following diagram shows such a skip.

diagram not to scale



(This question continues on the following page)

(Question 5 continued)

(a) Find the volume of this skip,

(i) if the length of the top edge of the skip is $11\,\text{m}$.

(ii) if the height of the skip is $3.2\,\text{m}$.

(iii) if θ is $60°$. [9]

(b) Show that the volume, $V\,\text{m}^3$, of the skip is given by

$$24\sin(\theta)(5+\cos(\theta)).$$ [2]

(c) Explain, in context, why $\theta \neq 0$. [1]

(d) (i) Sketch the graph of $V = 24\sin(\theta)(5+\cos(\theta))$, $0 < \theta < 90°$.

(ii) Find the maximum volume of the skip and the value of θ for which this maximum volume occurs. [4]

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Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

References:

5. Andyqwe, n.d. *Dumpster truck* [image online] Available at: https://www.gettyimages.co.uk/detail/photo/dumpster-truck-royalty-free-image/157611454 [Accessed 18 April 2023] Source adapted.

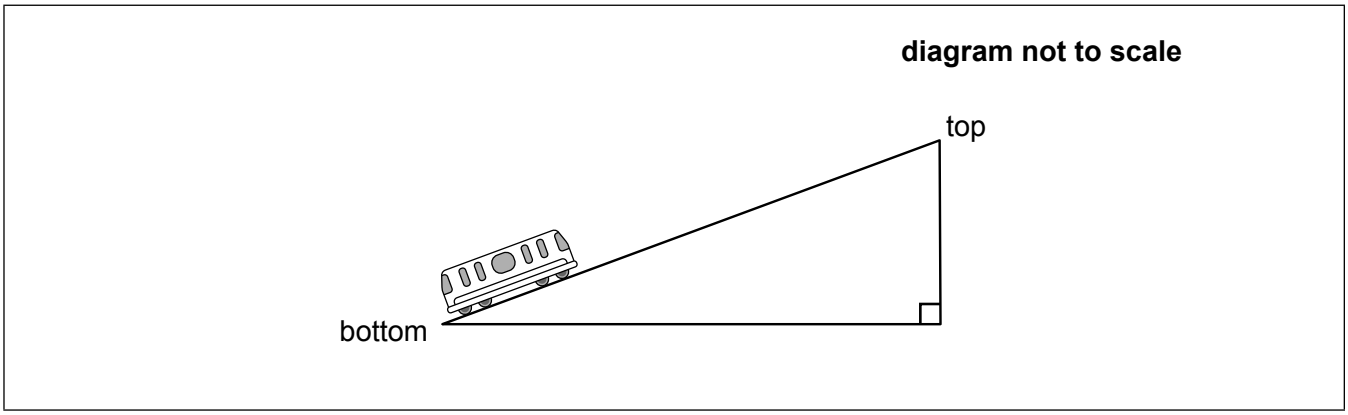
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Question 5 - working space

1. [Maximum mark: 4]

One of the steepest train tracks in the world is in Tennessee, USA.

This track is $1.52\,\text{km}$ long, and the angle of elevation from the bottom of the track to the top is 36.1°.



(a) Label the diagram with the given values for the track length and the angle of elevation. [2]

(b) Find the vertical change in height from the bottom of the track to the top. [2]

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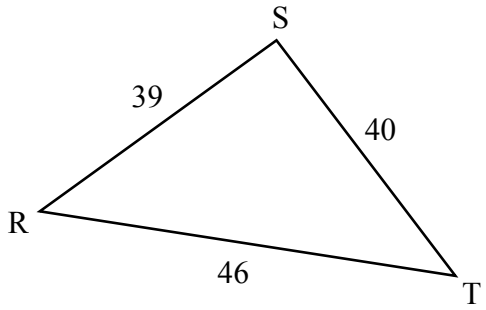
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Question 1 - working space

4. [Maximum mark: 5]

Consider the following triangle, RST, such that $\mathrm{RS} = 39\,\mathrm{cm}$, $\mathrm{ST} = 40\,\mathrm{cm}$, and $\mathrm{TR} = 46\,\mathrm{cm}$.

diagram not to scale



(a) Find the value of $\mathrm{T\hat{R}S}$. [3]

(b) Find the area of the triangle RST. [2]

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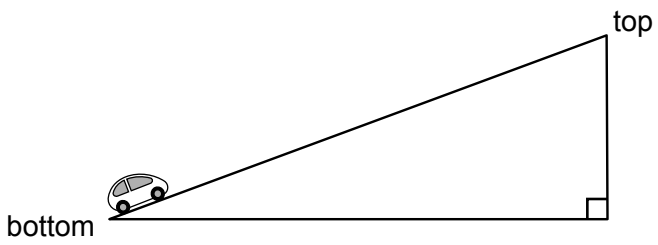
Question 4 - working space

1. [Maximum mark: 4]

One of the steepest straight roads in the world is in Dunedin, New Zealand.

This road is $161\,\text{m}$ long, and the angle of elevation from the bottom of the road to the top is $16.3°$.

diagram not to scale



(a) Label the diagram with the given values for the road length and the angle of elevation. [2]

(b) Find the vertical change in height from the bottom of the road to the top. [2]

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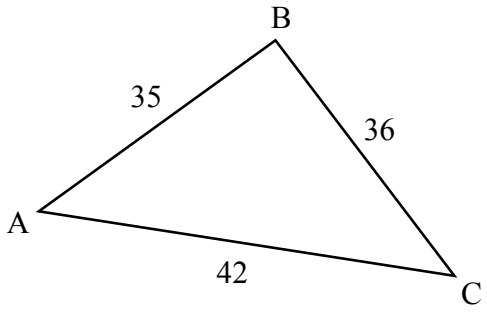
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Question 1 - working space

4. [Maximum mark: 5]

Consider the following triangle, ABC, such that $AB = 35\,\text{cm}$, $BC = 36\,\text{cm}$, and $CA = 42\,\text{cm}$.

diagram not to scale



(a) Find the value of $C\hat{A}B$. [3]

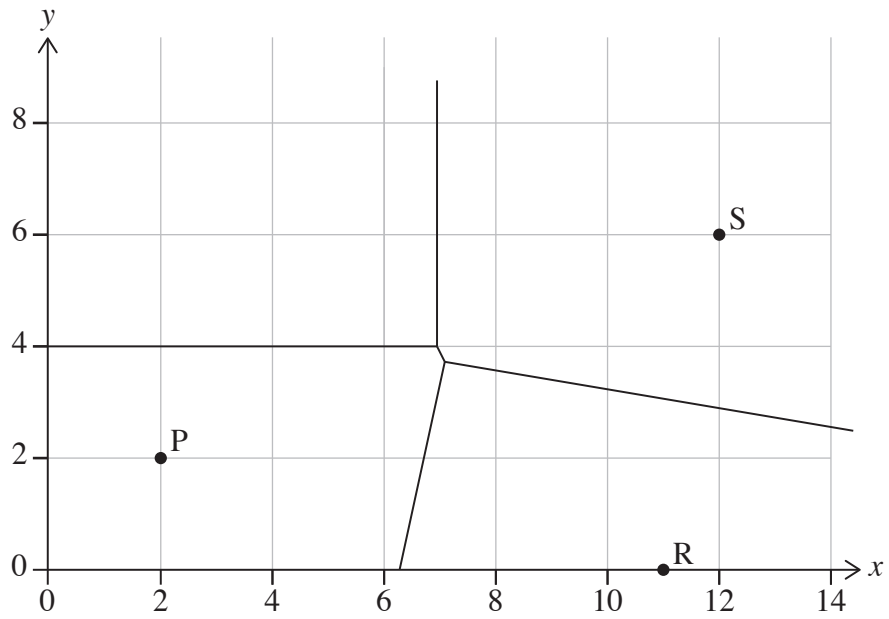
(b) Find the area of the triangle ABC. [2]

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Question 4 - working space

1. [Maximum mark: 13]

The following grid shows a restaurant's floorplan. There are four food stations centred at points P, Q, R and S. The Voronoi diagram for these four points is shown. Point Q is not shown.



One unit represents 1 metre.

Point P is located at $(2, 2)$.

The equation of the perpendicular bisector of $[\mathrm{PQ}]$ is $y = 4$.

(a) Write down the coordinates of Q. [1]

Points R and S are located at $(11, 0)$, and $(12, 6)$, respectively.

(b) Find

(i) the coordinates of the midpoint of $[\mathrm{RS}]$

(ii) the equation of the perpendicular bisector of $[\mathrm{RS}]$. Give your answer in the form $y = mx + c$. [6]

(This question continues on the following page)

(Question 1 continued)

Customers in the restaurant take their food from the nearest food station.

The following table shows the average waiting time, in minutes, for each food station.

Food station	P	Q	R	S
Average waiting time (minutes)	8	4	9	6

(c) Using nearest-neighbour interpolation, find the average waiting time for a customer at point $(6, 8)$. [1]

The restaurant owner wishes to determine whether customers spend more money during the weekend. She decides to use a two-sample t-test at a 5% level of significance.

For this test, the null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

where μ_1 is the mean amount of money spent by all customers on weekdays, and μ_2 is the mean amount of money spent by all customers on weekends.

(d) State the alternative hypothesis. [1]

The owner considers collecting her sample data from 2 weekdays and 1 weekend by recording the amount spent on the first 12 orders on each of these days.

(e) (i) State which one of the following methods best describes the owner's planned sampling:

convenience, systematic, or stratified.

(ii) State one disadvantage of this sampling technique. [2]

The restaurant owner instead collects the data using simple random sampling.

The p-value is 0.0897 for the sampled data.

(f) State the conclusion for the test. Justify your answer. [2]

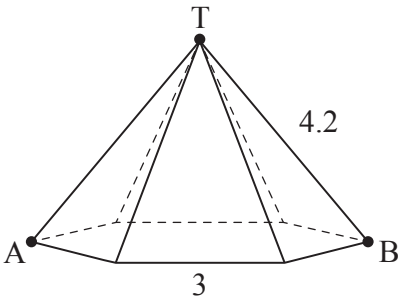
Question 1 - working space

5. [Maximum mark: 17]

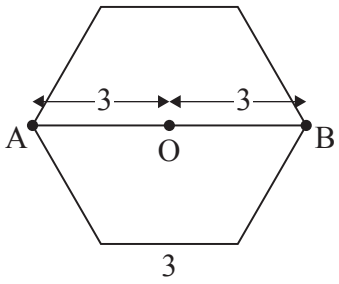
Jamali is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre O. The length of each side of the base is $3\,\text{m}$, the length of each sloping edge is $4.2\,\text{m}$, and the distance between each vertex on the base and O is $3\,\text{m}$, as shown in the diagrams.

diagrams not to scale

3D view of tent



2D view of base

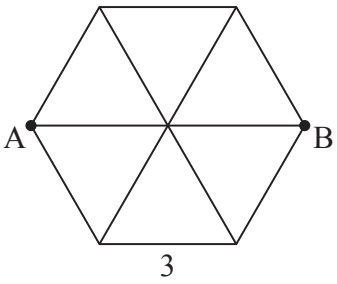


The top of the tent, T, will be supported by a vertical pole from O.

(a) Find the length of the pole, OT. [2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



(b) Find

(i) the area of the base

(ii) the volume of the tent. [5]

(c) Find the value of $\text{O}\hat{\text{A}}\text{T}$. [2]

(This question continues on the following page)

(Question 5 continued)

For extra support, Jamali decides to attach a rope, with length $3.5\,\text{m}$, to the tent at a point, P, on the edge AT.

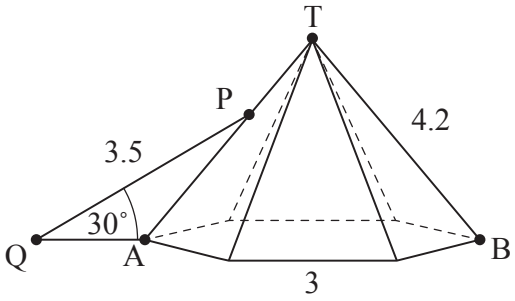
The rope will be fixed to the ground at point Q, such that:

- the rope, $[\text{PQ}]$, is straight
- points Q, A and B lie on a straight line
- $\text{A}\hat{\text{Q}}\text{P} = 30°$.

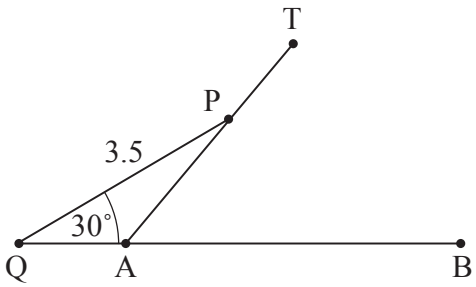
This is shown in the diagrams.

diagrams not to scale

3D view of tent and rope



2D view of triangle APQ



(d) Find AQ. [4]

For decoration at night, Jamali wants to fix a strip of lights from point A to a point, R, along the rope $[\text{PQ}]$.

The strip of lights, $[\text{AR}]$, is straight and has length $0.8\,\text{m}$.

(e) Find the two possible values of QR. [4]

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Content used in IB assessments is taken from authentic, third-party sources. The views expressed within them belong to their individual authors and/or publishers and do not necessarily reflect the views of the IB.

References:

3. InterNations. 2022. *Expat Insider 2022*. [online] Available at: https://www.internations.org/expat-insider/2022/. Source adapted.

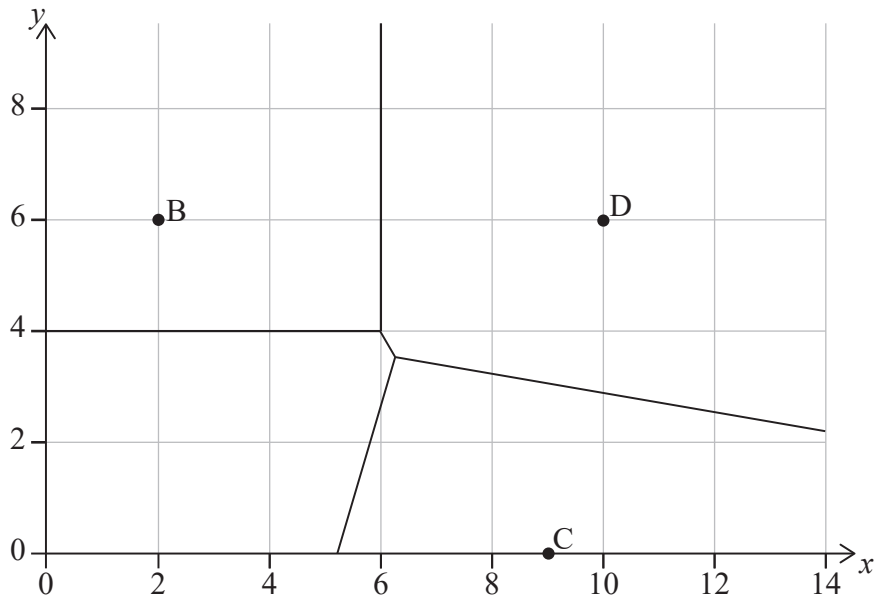
Heliwell, J. F., Huang, H., Wang, S. and Norton, M., 2022. Statistical Appendix for "Happiness, benevolence, and trust during COVID-19 and beyond," Chapter 2 of *World Happiness Report 2022*. [pdf online] Available at: https://worldhappiness.report/ed/2022/happiness-benevolence-and-trust-during-covid-19-and-beyond/ [Accessed 13 November 2023]. Source adapted.

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Question 5 - working space

1. [Maximum mark: 13]

The following grid shows a restaurant's floorplan. There are four food stations centred at points A, B, C and D. The Voronoi diagram for these four points is shown. Point A is not shown.



One unit represents 1 metre.

Point B is located at $(2, 6)$.

The equation of the perpendicular bisector of $[\mathrm{AB}]$ is $y = 4$.

(a) Write down the coordinates of A. [1]

Points C and D are located at $(9, 0)$, and $(10, 6)$, respectively.

(b) Find

(i) the coordinates of the midpoint of $[\mathrm{CD}]$

(ii) the equation of the perpendicular bisector of $[\mathrm{CD}]$. Give your answer in the form $y = mx + c$. [6]

(This question continues on the following page)

(Question 1 continued)

Customers in the restaurant take their food from the nearest food station.

The following table shows the average waiting time, in minutes, for each food station.

Food station	A	B	C	D
Average waiting time (minutes)	9	5	10	7

(c) Using nearest-neighbour interpolation, find the average waiting time for a customer at point $(4, 6)$. [1]

The restaurant owner wishes to determine whether customers spend more money during the weekend. She decides to use a two-sample t-test at a 5% level of significance.

For this test, the null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

where μ_1 is the mean amount of money spent by all customers on weekdays, and μ_2 is the mean amount of money spent by all customers on weekends.

(d) State the alternative hypothesis. [1]

The owner considers collecting her sample data from 2 weekdays and 1 weekend by recording the amount spent on the first 12 orders on each of these days.

(e) (i) State which one of the following methods best describes the owner's planned sampling:

convenience, systematic, or stratified.

(ii) State one disadvantage of this sampling technique. [2]

The restaurant owner instead collects the data using simple random sampling.

The p-value is 0.0897 for the sampled data.

(f) State the conclusion for the test. Justify your answer. [2]

Turn over

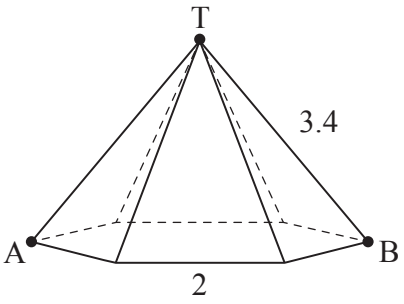
Question 1 - working space

5. [Maximum mark: 17]

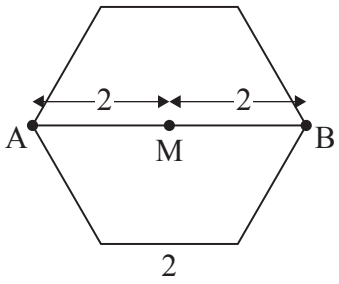
Gaurika is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre M. The length of each side of the base is $2\,\mathrm{m}$, the length of each sloping edge is $3.4\,\mathrm{m}$, and the distance between each vertex on the base and M is $2\,\mathrm{m}$, as shown in the diagrams.

diagrams not to scale

3D view of tent



2D view of base

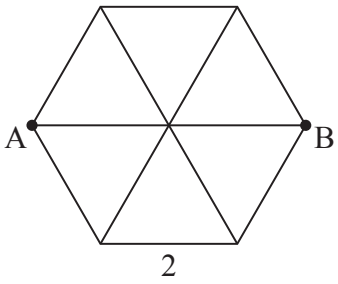


The top of the tent, T, will be supported by a vertical pole from M.

(a) Find the length of the pole, MT. [2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



(b) Find

(i) the area of the base

(ii) the volume of the tent. [5]

(c) Find the value of $\mathrm{M\hat{A}T}$. [2]

(This question continues on the following page)

(Question 5 continued)

For extra support, Gaurika decides to attach a rope, with length $2.6\,\text{m}$, to the tent at a point, X, on the edge AT.

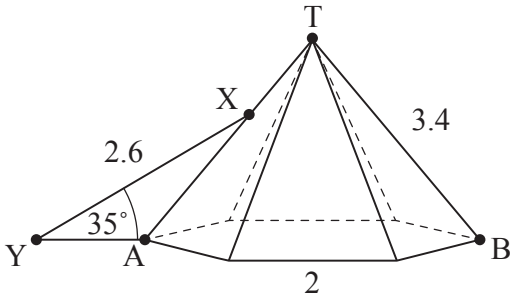
The rope will be fixed to the ground at point Y, such that:

- the rope, $[\text{XY}]$, is straight
- points Y, A and B lie on a straight line
- $\text{A}\hat{\text{Y}}\text{X} = 35°$.

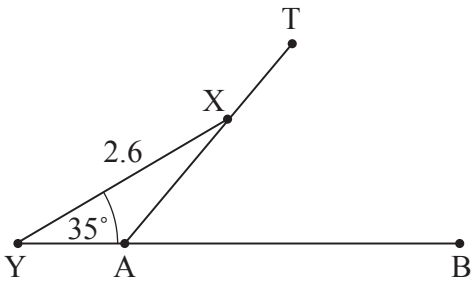
This is shown in the diagrams.

diagrams not to scale

3D view of tent and rope



2D view of triangle AXY



(d) Find AY. [4]

For decoration at night, Gaurika wants to fix a strip of lights from point A to a point, Z, along the rope $[\text{XY}]$.

The strip of lights, $[\text{AZ}]$, is straight and has length $0.9\,\text{m}$.

(e) Find the two possible values of YZ. [4]

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References:

3. InterNations. 2022. *Expat Insider 2022*. [online] Available at: https://www.internations.org/expat-insider/2022/. Source adapted.

Heliwell, J. F., Huang, H., Wang, S. and Norton, M., 2022. Statistical Appendix for "Happiness, benevolence, and trust during COVID-19 and beyond," Chapter 2 of *World Happiness Report 2022*. [pdf online] Available at: https://worldhappiness.report/ed/2022/happiness-benevolence-and-trust-during-covid-19-and-beyond/ [Accessed 13 November 2023]. Source adapted.

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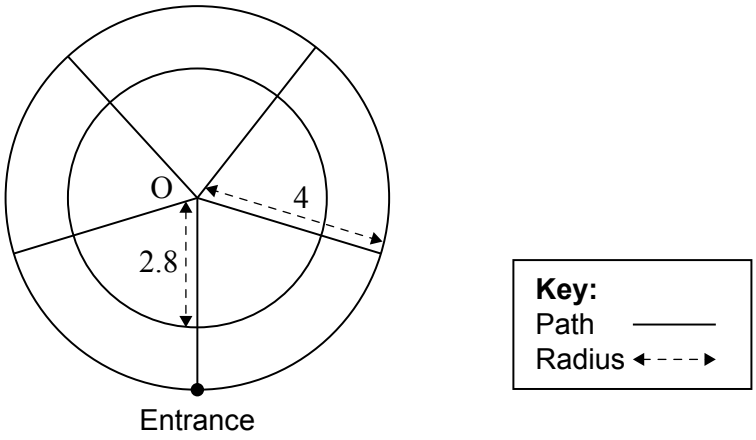
Question 5 - working space

6. [Maximum mark: 6]

A child's game is played by making paths in the snow. First, two circular paths are made using the same centre, O. The radius of the smaller circle is $2.8\,\text{m}$, and the radius of the larger circle is $4\,\text{m}$. Additional paths are then made from O to the outer edge of the larger circle, dividing each circle into 5 equal sectors, as shown in the following diagram.

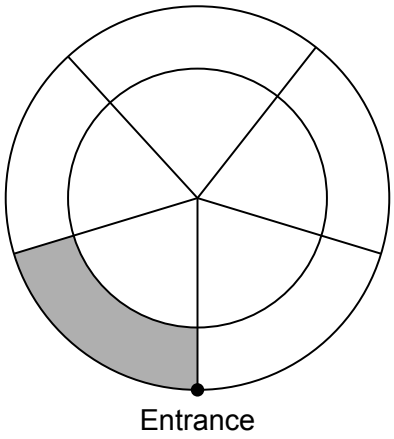
For your calculations, ignore the widths of the paths.

diagram not to scale



Each region **between** paths will be coloured with environmentally friendly dye. The shaded region below will be coloured orange.

diagram not to scale



Maureen has enough orange dye to cover an area of $6\,\text{m}^2$.

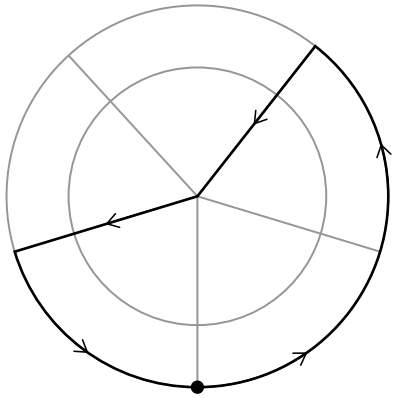
(a) Show that Maureen has enough orange dye to cover the shaded region. [3]

(This question continues on the following page)

(Question 6 continued)

During the game, the players start at the entrance and must travel only along the paths made in the snow. Maureen travels from the entrance along the path shown in the following diagram.

diagram not to scale



(b) Calculate the distance Maureen travels along this path, starting from the entrance and returning to the entrance. [3]

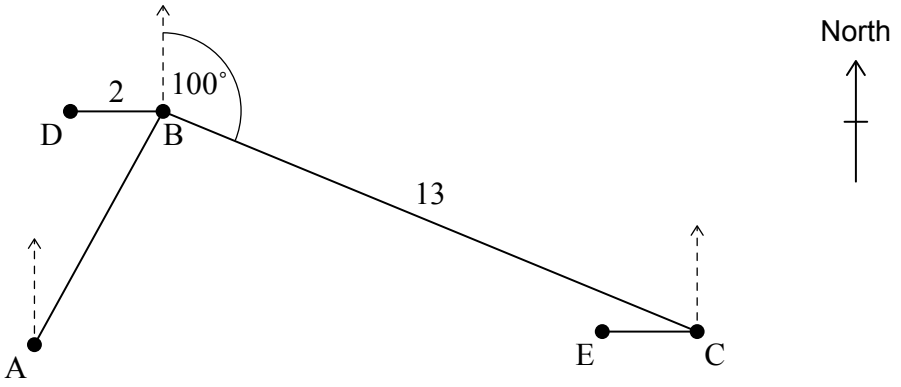
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Question 6 - working space

13. [Maximum mark: 9]

Ticky is sailing a boat in the ocean. For the first part of her trip, she plans to start at point A and travel to point B. She then plans to turn and travel $13\,\text{km}$ on a bearing of $100°$ to point C as shown in the following diagram.

diagram not to scale



Due to navigational error in the first part of the trip, Ticky arrives at point D instead of point B. Point D is $2\,\text{km}$ due west of point B.

(a) Determine

(i) the size of $\text{D}\hat{\text{B}}\text{C}$.

(ii) the distance from point D to point C.

(iii) the bearing Ticky must use to travel directly from point D to point C. [7]

Due to navigational error in the second part of the trip, Ticky arrives at point E instead of point C. Point E is $2\,\text{km}$ due west of point C.

(b) Write down the distance between point D and point E. Justify your answer. [2]

(This question continues on the following page)

(Question 13 continued)

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References:

3. ONYXprj, 2017. *Realistic cupcakes and cookies. Biscuits muffins packaging, creamy and chocolate bakery products in white box vector illustration* [image online] Available at: https://www.gettyimages.co.uk/detail/illustration/realistic-cupcakes-and-cookies-biscuits-royalty-free-illustration/1257757782 [Accessed 19 April 2024] Source adapted.

Question 13 - working space

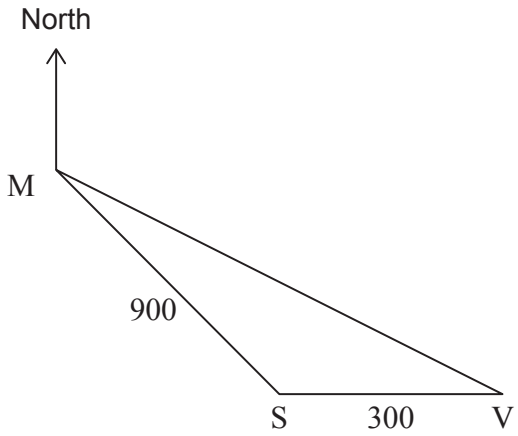
5. [Maximum mark: 8]

A ship sails $900\,\text{km}$ on a bearing of $135°$ (south-east) from Lomé, M, to São Tomé, S, where she unloads her cargo.

The ship then sails $300\,\text{km}$ due east from São Tomé to Libreville, V. From Libreville she then returns directly to Lomé.

These journeys are shown in the diagram.

diagram not to scale



(a) Find the size of $\text{V}\hat{\text{S}}\text{M}$. [1]

(b) Calculate MV. [3]

(c) Calculate the bearing of Lomé, M, from Libreville, V. [4]

(This question continues on the following page)

(Question 5 continued)

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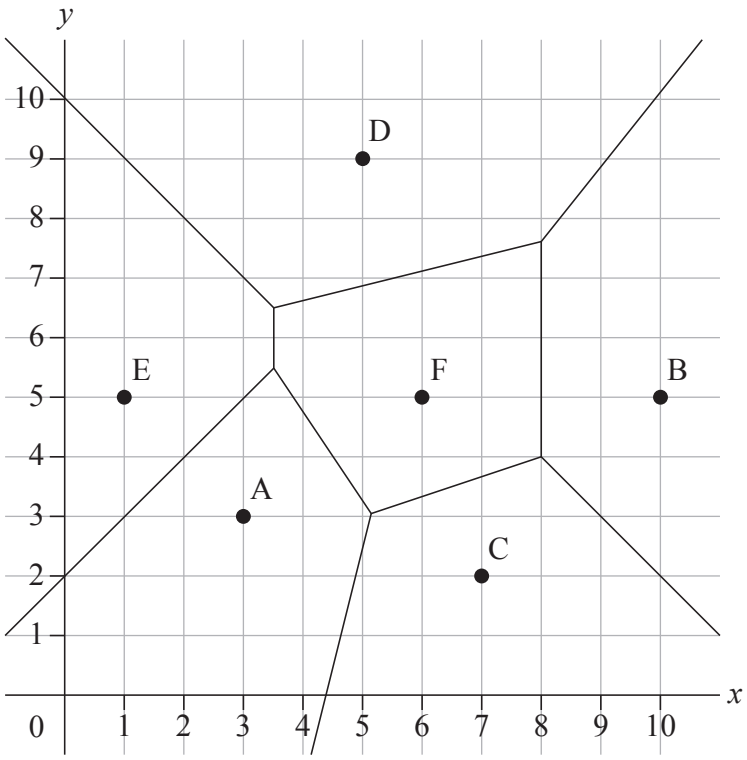
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Question 5 - working space

7. [Maximum mark: 9]

Consider the Voronoi diagram which shows the sites $\mathrm{A}(3, 3)$, $\mathrm{B}(10, 5)$, $\mathrm{C}(7, 2)$, $\mathrm{D}(5, 9)$, $\mathrm{E}(1, 5)$ and $\mathrm{F}(6, 5)$. The diagram also shows the cells formed by each site and their boundaries.



(a) Find the equation of the boundary between sites D and E. [2]

Vertex X is equidistant from sites B, C and F.

(b) (i) Write down the coordinates of X.

(ii) The exact value of BX is $\sqrt{n}$. Write down the value of n. [2]

Vertex $\mathrm{Y}(a, b)$ is equidistant from sites B, D and F.

(c) (i) Write down the value of a.

(ii) Find the exact value of b. [5]

(This question continues on the following page)

(Question 7 continued)

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Question 7 - working space

11. [Maximum mark: 9]

The point A has coordinates $(1, 2, 1)$ and the point B has coordinates $(3, 5, 2)$.

(a) Find AB. [2]

Triangle ABC is right-angled with its right angle at B. The point C has coordinates $(2, 8, k)$.

(b) Find the value of k. [4]

(c) Calculate the size of $\mathrm{B\hat{A}C}$. [3]

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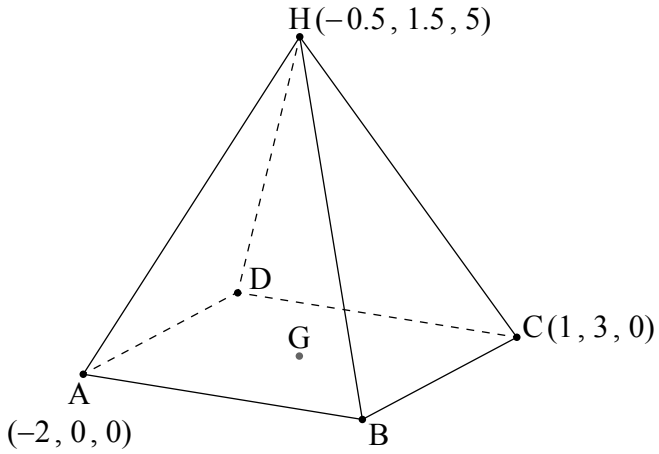
Question 11 - working space

1. [Maximum mark: 5]

A metal structure on a flat surface is in the form of a right-pyramid with rectangular base $ABCD$ and vertex $H(-0.5, 1.5, 5)$. Point A has coordinates $(-2, 0, 0)$ and point C has coordinates $(1, 3, 0)$. This is shown in the following diagram.

All units are in centimetres.

diagram not to scale



The centre of the base, G, is the midpoint of AC.

(a) Find the coordinates of G. [2]

(b) Write down the vertical height HG. [1]

(c) Find the distance between C and H. [2]

(This question continues on the following page)

(Question 1 continued)

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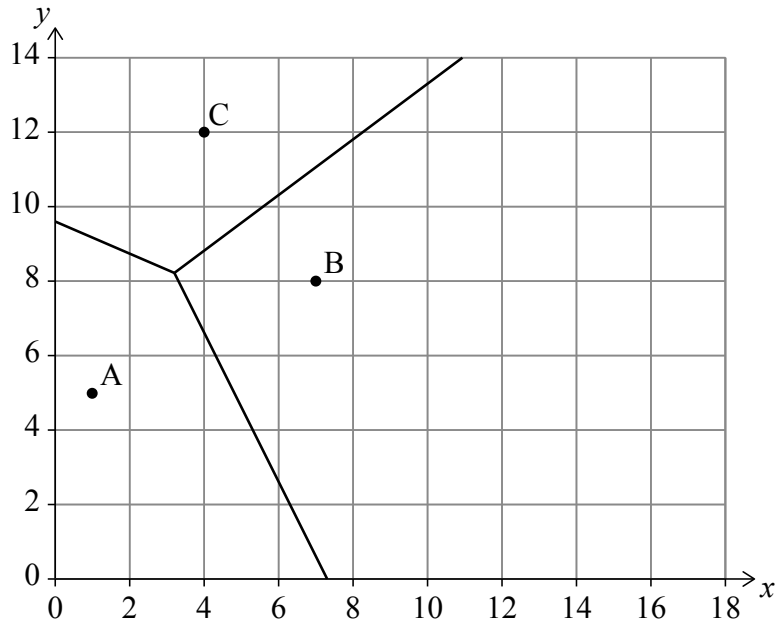
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Question 1 - working space

5. [Maximum mark: 9]

A telecommunications company has identical cell towers in a rural area. They are located at the points $A(1, 5)$, $B(7, 8)$ and $C(4, 12)$. The coverage areas are divided as shown in the Voronoi diagram. All distances are in kilometres.



(a) Find the equation of the perpendicular bisector of [AB]. [4]

(This question continues on the following page)

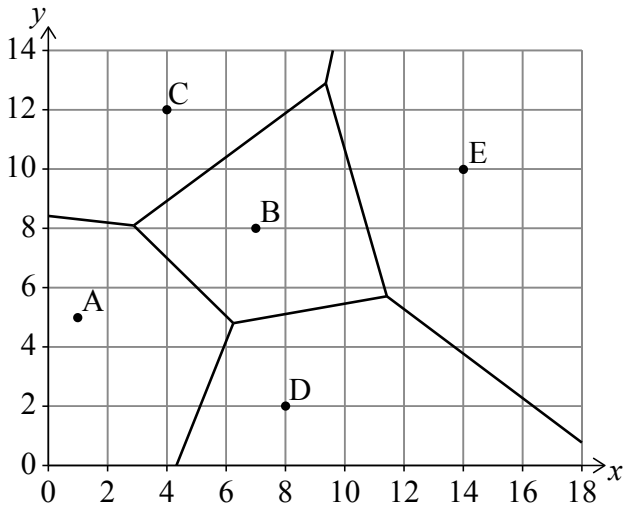
(Question 5 continued)

The company is planning to improve the coverage of its cellular network in the area by adding two new towers. It identifies potential locations at the points $D(8, 2)$ and $E(14, 10)$.

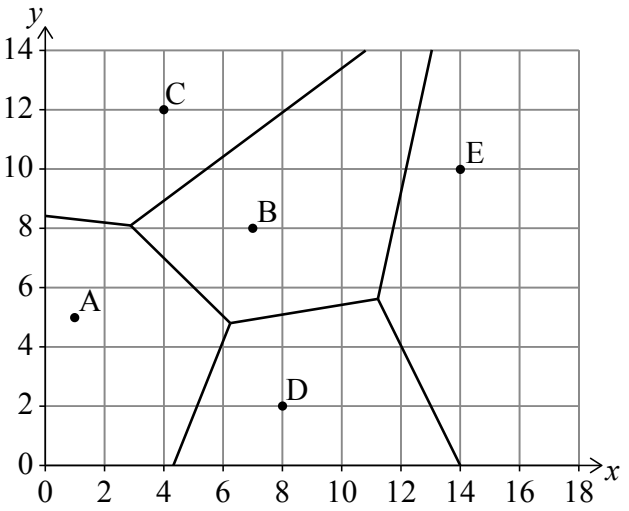
The company reviews the coverage areas and draws a new Voronoi diagram.

(b) Identify the correct Voronoi diagram from the options shown in the following diagrams. [2]

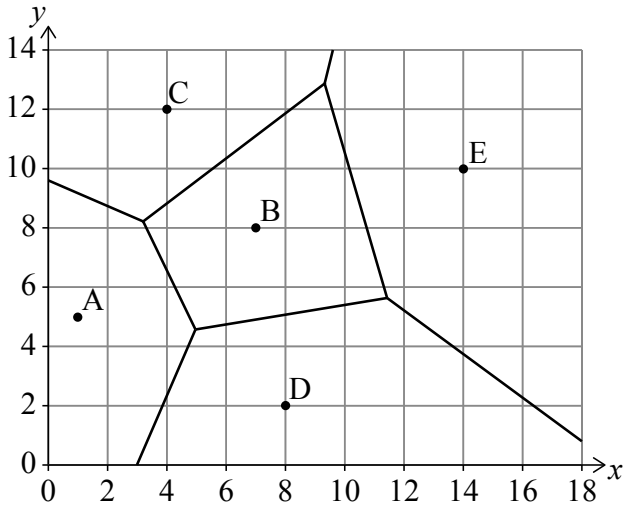
Option 1



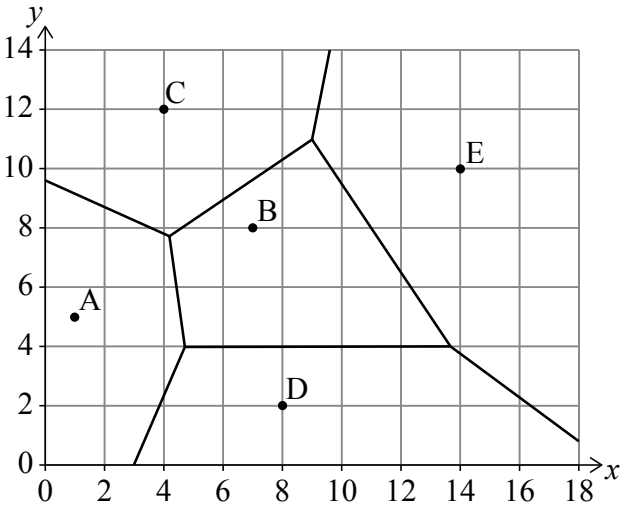
Option 2



Option 3



Option 4



(This question continues on the following page)

(Question 5 continued)

Each tower provides guaranteed excellent coverage within a radius of $3\,\text{km}$.

Pooja is at a beauty parlour located at the point $(6, 4)$.

(c) State whether excellent coverage is guaranteed for Pooja at the beauty parlour. Justify your answer. [3]

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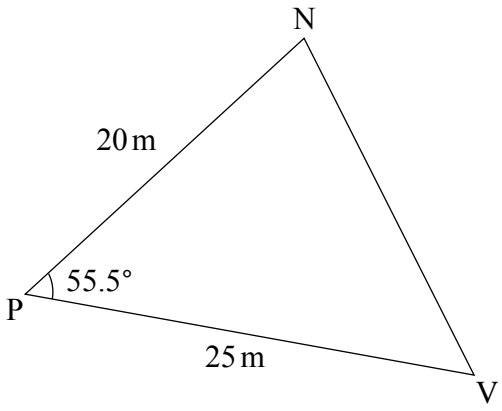
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Question 5 - working space

6. [Maximum mark: 9]

Three points N, P, and V are shown on the following diagram. NP is 20 metres, PV is 25 metres and $V\hat{P}N$ is $55.5°$.

diagram not to scale



(a) Find NV. [3]

(b) Find $P\hat{N}V$. [3]

(c) Hence or otherwise, find the shortest distance between P and [NV]. [3]

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(Question 6 continued)

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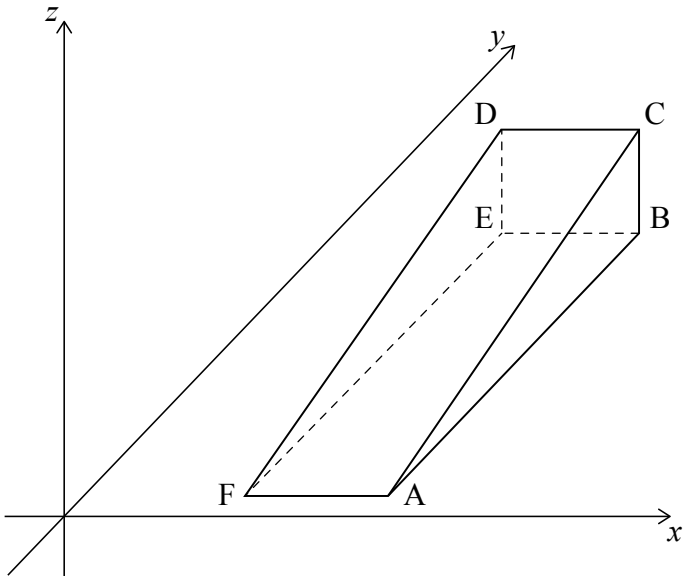
Question 6 - working space

4. [Maximum mark: 14]

AirFlow Industries is designing a doorstop in the shape of a triangular prism, ABCDEF.

The points $\mathrm{A}(13, 1, 0)$, $\mathrm{B}(13, 25, 0)$ and $\mathrm{C}(13, 25, 7)$ are the coordinates of the vertices of one of the triangular faces of the prism. Point $\mathrm{D}(4, 25, 7)$ is another vertex as shown in the following diagram. All the measurements are in centimetres.

diagram not to scale



(a) Write down the coordinates of F. [2]

(b) Find AC. [2]

(c) Show that triangle ABC is a right-angled triangle. [2]

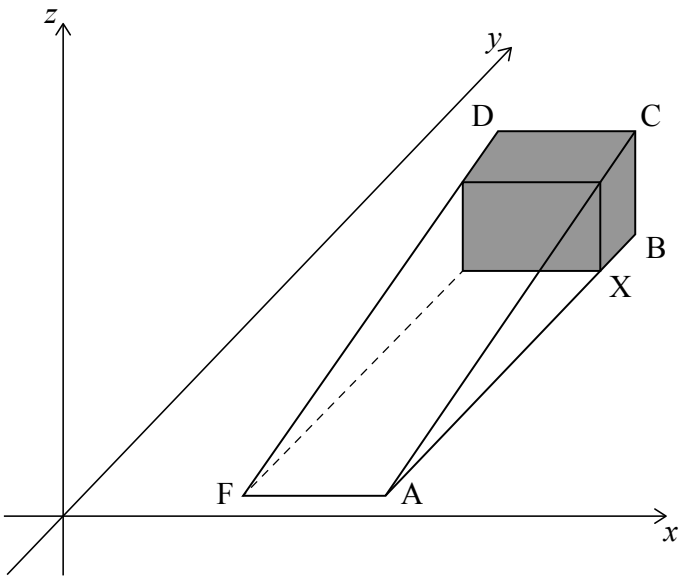
(d) Find the volume of material needed to make the doorstop. [3]

(This question continues on the following page)

(Question 4 continued)

To lower the cost of the doorstop, AirFlow decides to reduce its volume to $625\,\text{cm}^3$ by removing the shaded section shown in the following diagram. The remaining triangular faces are still right-angled triangles.

diagram not to scale



The length of the smaller doorstop, AX, can be calculated by

$$\text{AX} = \text{AB} \times \sqrt{\frac{V_S}{V_O}}$$

where V_S is the volume of the smaller doorstop, and V_O is the volume of the original doorstop, both in cubic centimetres.

(e) Find AX. [2]

The material needed to make the doorstop costs 0.025 US dollars (USD) per cubic centimetre. The production of each doorstop requires $10\,\%$ more material than its final volume, due to wastage in the production process.

To make a profit, the company will sell the doorstop for $20\,\%$ more than the cost of the material.

(f) Determine the price, to two decimal places, the company will charge for the **new** doorstop. [3]

Turn over

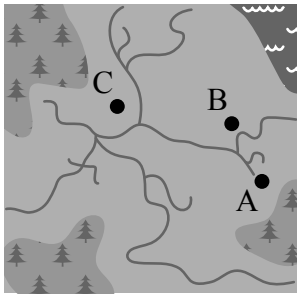
Question 4 - working space

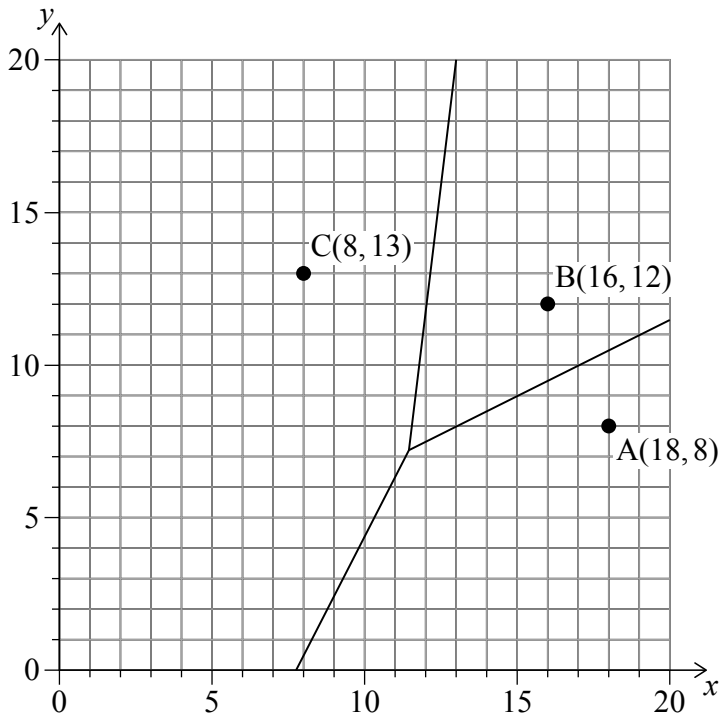
5. [Maximum mark: 19]

The locations of three fire stations within a 50 by 50 kilometre rural area of Japan are shown on the following map.

A Voronoi diagram can be used to determine the part of the rural area for which each fire station is responsible. The Voronoi diagram for the three fire stations is shown on the coordinate grid beside the map, where 1 unit represents $2.5\,\text{km}$.

Map





(a) (i) Find the midpoint of $[\text{BC}]$.

(ii) Find the gradient of the perpendicular bisector of $[\text{BC}]$.

(iii) Hence find the equation of the edge between sites B and C. [6]

(b) (i) Identify the fire station that is expected to respond, based on the Voronoi diagram, if a fire is reported at a location with coordinates $(14, 10)$. Justify your response.

(ii) Suggest a reason why a different fire station might respond to this fire. [2]

(This question continues on the following page)

(Question 5 continued)

A fire is reported at a location D with coordinates $\left(11\frac{1}{3}, 7\frac{1}{6}\right)$. The distance of D from A on the Voronoi diagram, to six significant figures, is 6.71855 units ($16.7964\,\mathrm{km}$ on the map).

(c) (i) Show that the distance of D from B, to six significant figures, is also 6.71855 units.

(ii) Show that any of the three fire stations would be expected to respond to the fire reported at D. [4]

The equation of the edge between fire stations A and B is $y = 0.5x + 1.5$. The edge between fire stations A and C has an x-intercept of 7.75.

(d) (i) Determine the area, to the nearest square unit, of the region within the given Voronoi diagram for which fire station A is responsible.

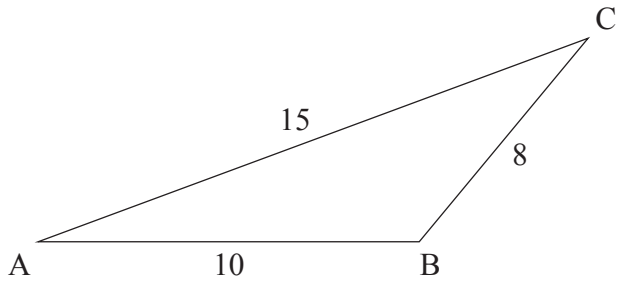
(ii) Hence, calculate the actual area, in square kilometres, of this region. [7]

Question 5 - working space

6. [Maximum mark: 8]

Triangle ABC has sides of length $AB = 10\,\text{cm}$, $BC = 8\,\text{cm}$ and $AC = 15\,\text{cm}$, as shown in the diagram.

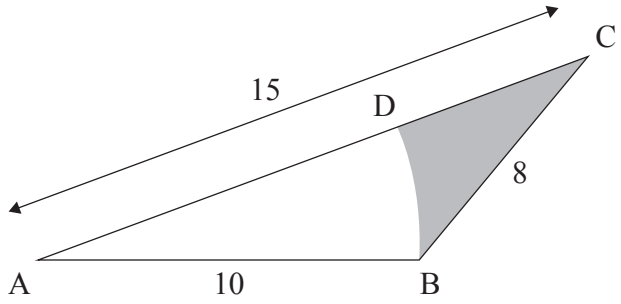
diagram not to scale



(a) Find the size of $B\hat{A}C$. [3]

The circular arc BD, with centre A, is drawn inside the triangle, such that D lies on AC.

diagram not to scale



(b) Find the area of the shaded region. [5]

(This question continues on the following page)

(Question 6 continued)

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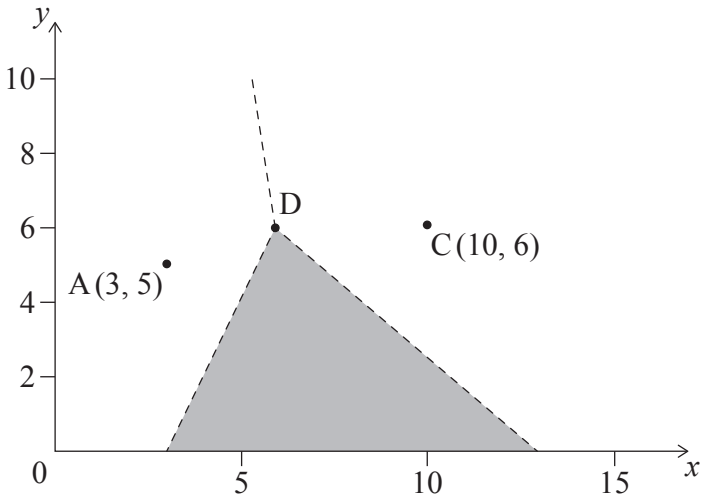
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Question 6 - working space

12. [Maximum mark: 7]

The city of Benoulli has three police stations, located at $\mathrm{A}(3, 5)$, $\mathrm{C}(10, 6)$ and B. In the following Voronoi diagram, A and C represent sites. The dashed lines represent the edges and the cell for site B is shaded.



(a) State the geometrical significance of

(i) the shaded region;

(ii) point D. [2]

The equation of the edge between A and B is $y = 2x - 6$. The equation of the edge between C and B is $y = -x + 13$.

(b) Calculate the coordinates of B. [5]

(This question continues on the following page)

(Question 12 continued)

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References:

2. Pepifoto, 2021. *Digital caliper measuring screw*. [image online] Available at: https://www.gettyimages.co.uk/detail/photo/digital-caliper-measuring-screw-royalty-free-image/1302481493?phrase=digital+caliper+hand&adppopup=true Images [Accessed 20 November 2024]. Source adapted.

Lesikvit, 2024. *White round sphere or 3d ball*. [image online] Available at: https://www.gettyimages.co.uk/detail/illustration/white-round-sphere-or-3d-ball-royalty-free-illustration/2157157426?phrase=grey+sphere&adppopup=true Images [Accessed 20 November 2024]. Source adapted.

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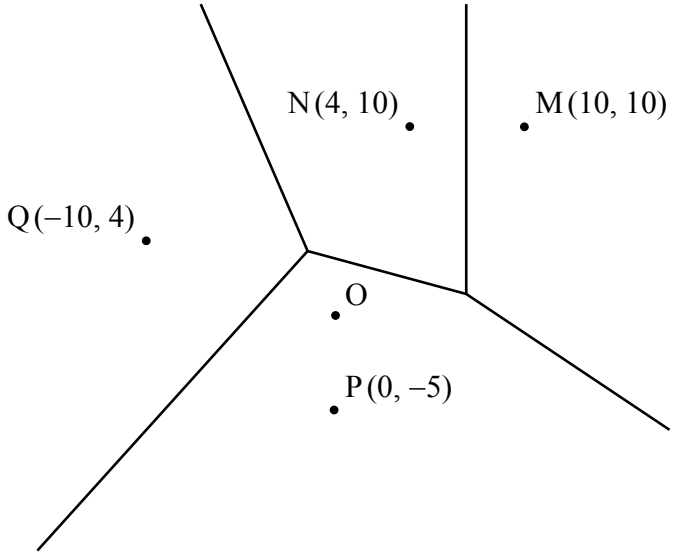
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Question 12 - working space

5. [Maximum mark: 9]

The following Voronoi diagram represents a map of a town. It shows the location of four bakeries, at points $\mathrm{M}(10, 10)$, $\mathrm{N}(4, 10)$, $\mathrm{P}(0, -5)$ and $\mathrm{Q}(-10, 4)$. The town centre is at location O with coordinates $(0, 0)$.



Olga is at the town centre and wants to go to the nearest bakery.

(a) Write down the nearest bakery to Olga. [1]

Before going to the nearest bakery, Olga checks on her phone and finds that it is closed.

(b) Find which bakery or bakeries Olga should go to instead. Justify your answer. [3]

(c) Write down the equation of the line containing all the points that are of equal distance from bakery M and bakery N. [1]

(d) Find the equation of the perpendicular bisector of $[\mathrm{MP}]$. [4]

(This question continues on the following page)

(Question 5 continued)

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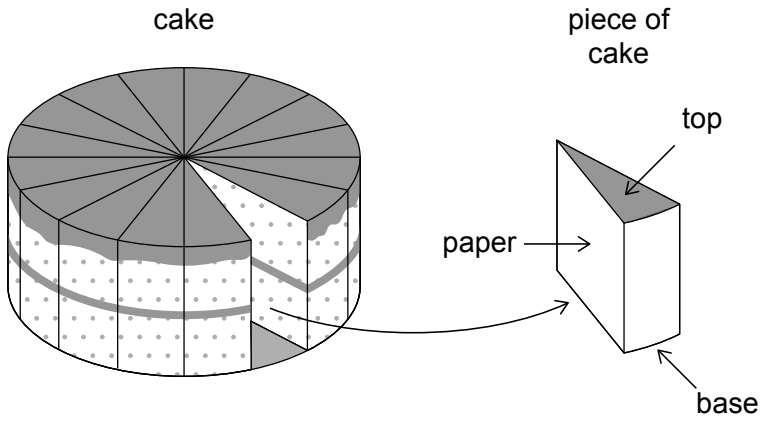
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Question 5 - working space

11. [Maximum mark: 6]

A cylindrical-shaped cake has diameter $22\,\text{cm}$ and height $10\,\text{cm}$. Coco cuts the cake into 16 pieces of equal size, as shown in the diagram.



(a) Find the volume of one piece of cake. [3]

Paper is wrapped around all three **vertical** sides of a piece of cake.

(b) Find the area of the paper used to wrap around one piece of cake, given that the paper fits exactly. [3]

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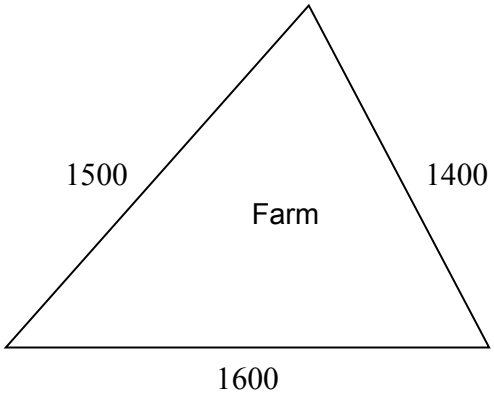
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Question 11 - working space

3. [Maximum mark: 15]

Inola has a triangular-shaped farm with boundaries of length $1600\,\text{m}$, $1500\,\text{m}$ and $1400\,\text{m}$.



(a) Find the area of the farm. Give your answer in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$. [6]

Inola's farm has a barn and a farmhouse.

Inola is constructing a $2\,\text{m}$-wide straight path on the level ground between the barn and the farmhouse. The barn is located $300\,\text{m}$ east and $200\,\text{m}$ north of the farmhouse.

(b) Find the distance from the barn to the farmhouse. [2]

The path costs $\$120$ per square metre.

(c) Find the cost, c, of the path, to the nearest dollar. [2]

Inola takes out a loan to pay for supplies to make the path. Inola borrows c dollars for five years, compounded monthly. To repay the loan, she makes a payment of $\$1750$ at the end of each month.

(d) (i) Find the total amount of interest paid.

(ii) Find the nominal annual interest rate of the loan. [5]

Question 3 - working space

APOLLO

Statistics and probability

112 questions

2. [Maximum mark: 4]

Deb used a thermometer to record the maximum daily temperature over ten consecutive days. Her results, in degrees Celsius ($^{\circ}C$), are shown below.

$$14, 15, 14, 11, 10, 9, 14, 15, 16, 13$$

For this data set, find the value of

(a) the mode. [1]

(b) the mean. [2]

(c) the standard deviation. [1]

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Question 2 - working space

6. [Maximum mark: 5]

Arriane has geese on her farm. She claims the mean weight of eggs from her black geese is less than the mean weight of eggs from her white geese.

She recorded the weights of eggs, in grams, from a random selection of geese. The data is shown in the table.

Weights of eggs from black geese	136	134	142	141	128	126
Weights of eggs from white geese	135	138	141	140	136	134

In order to test her claim, Arriane performs a t-test at a 10% level of significance. It is assumed that the weights of eggs are normally distributed and the samples have equal variances.

(a) State, in words, the null hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) State whether the result of the test supports Arriane's claim. Justify your reasoning. [2]

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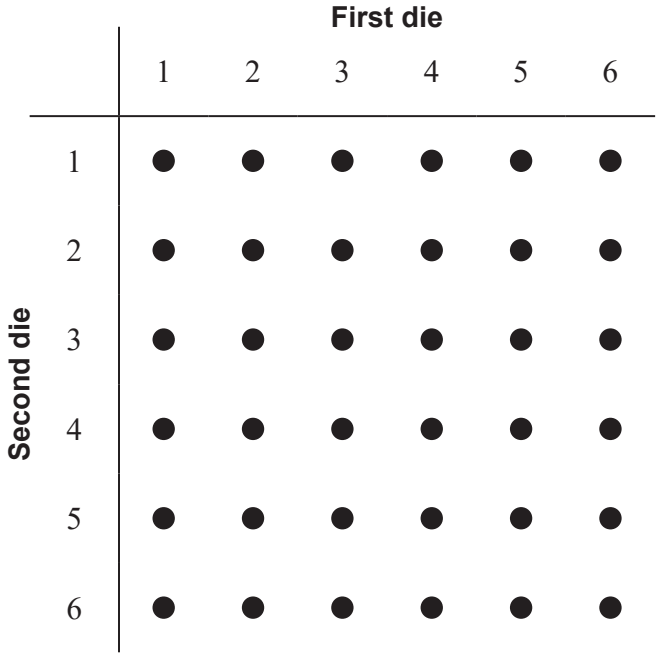
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Question 6 - working space

10. [Maximum mark: 7]

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable "the score in a game".

(a) Complete the table to show the probability distribution of T. [2]

t	1	2	3	4	5	6
$\mathbf{P}(T = t)$						

(b) Find the probability that

(i) a player scores at least 3 in a game.

(ii) a player scores 6, given that they scored at least 3. [3]

(c) Find the expected score of a game. [2]

(This question continues on the following page).

(Question 10 continued)

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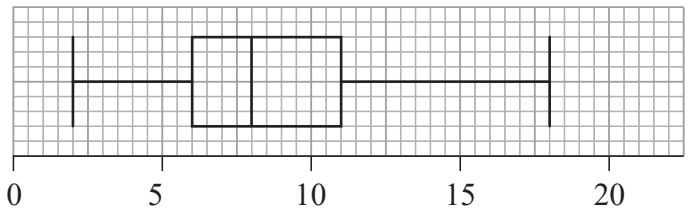
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Question 10 - working space

5. [Maximum mark: 5]

The number of sick days taken by each employee in a company during a year was recorded. The data was organized in a box and whisker diagram as shown below:



(a) For this data, write down

(i) the minimum number of sick days taken during the year.

(ii) the lower quartile.

(iii) the median. [3]

Paul claims that this box and whisker diagram can be used to infer that the percentage of employees who took fewer than six sick days is smaller than the percentage of employees who took more than eleven sick days.

(b) State whether Paul is correct. Justify your answer. [2]

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Question 5 - working space

8. [Maximum mark: 6]

At Springfield University, the weights, in kg, of 10 chinchilla rabbits and 10 sable rabbits were recorded. The aim was to find out whether chinchilla rabbits are generally heavier than sable rabbits. The results obtained are summarized in the following table.

Weight of chinchilla rabbits, kg	4.9	4.2	4.1	4.4	4.3	4.6	4.0	4.7	4.5	4.4
Weight of sable rabbits, kg	4.2	4.1	4.1	4.2	4.5	4.4	4.5	3.9	4.2	4.0

A t-test is to be performed at the 5% significance level.

(a) Write down the null and alternative hypotheses. [2]

(b) Find the p-value for this test. [2]

(c) Write down the conclusion to the test. Give a reason for your answer. [2]

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Question 8 - working space

11. [Maximum mark: 6]

A newspaper vendor in Singapore is trying to predict how many copies of *The Straits Times* they will sell. The vendor forms a model to predict the number of copies sold each weekday. According to this model, they expect the same number of copies will be sold each day.

To test the model, they record the number of copies sold each weekday during a particular week. This data is shown in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of copies sold	74	97	91	86	112

A goodness of fit test at the 5% significance level is used on this data to determine whether the vendor's model is suitable.

The critical value for the test is 9.49 and the hypotheses are

H_0 : The data satisfies the model.
H_1 : The data does not satisfy the model.

(a) Find an estimate for how many copies the vendor expects to sell each day. [1]

(b) (i) Write down the degrees of freedom for this test.

(ii) Write down the conclusion to the test. Give a reason for your answer. [5]

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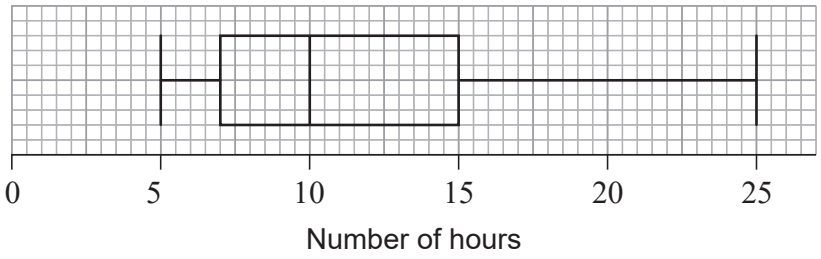
Question 11 - working space

1. [Maximum mark: 18]

As part of his mathematics exploration about classic books, Jason investigated the time taken by students in his school to read the book *The Old Man and the Sea*. He collected his data by stopping and asking students in the school corridor, until he reached his target of 10 students from **each** of the literature classes in his school.

(a) State which of the two sampling methods, systematic or quota, Jason has used. [1]

Jason constructed the following box and whisker diagram to show the number of hours students in the sample took to read this book.



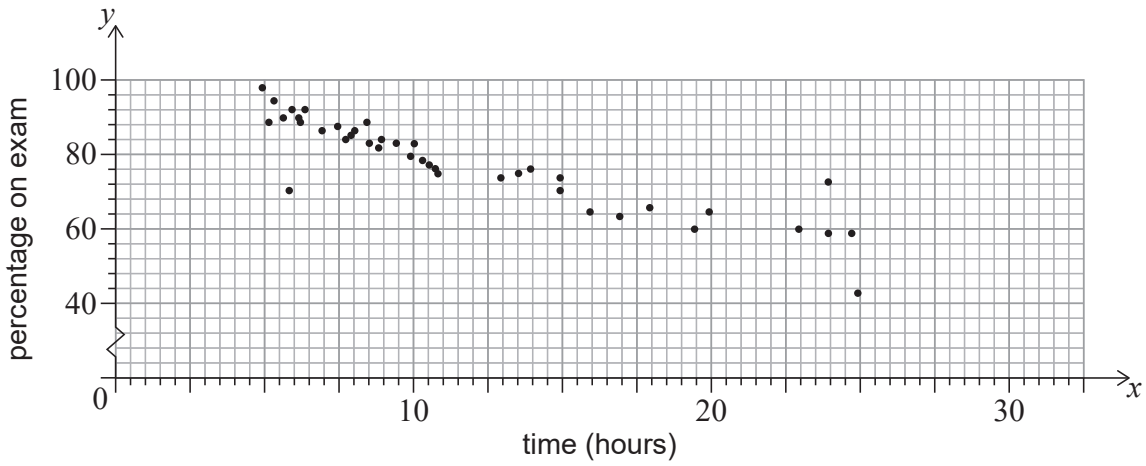
(b) Write down the median time to read the book. [1]

(c) Calculate the interquartile range. [2]

Mackenzie, a member of the sample, took 25 hours to read the novel. Jason believes Mackenzie's time is not an outlier.

(d) Determine whether Jason is correct. Support your reasoning. [4]

For each student interviewed, Jason recorded the time taken to read *The Old Man and the Sea* (x), measured in hours, and paired this with their percentage score on the final exam (y). These data are represented on the scatter diagram.



(e) Describe the correlation. [1]

(This question continues on the following page)

(Question 1 continued)

Jason correctly calculates the equation of the regression line y on x for these students to be

$$y = -1.54x + 98.8 .$$

He uses the equation to estimate the percentage score on the final exam for a student who read the book in 1.5 hours.

(f) Find the percentage score calculated by Jason. [2]

(g) State whether it is valid to use the regression line y on x for Jason's estimate. Give a reason for your answer. [2]

Jason found a website that rated the 'top 50' classic books. He randomly chose eight of these classic books and recorded the number of pages. For example, Book H is rated 44th and has 281 pages. These data are shown in the table.

Book	A	B	C	D	E	F	G	H
Number of pages (n)	4215	863	585	1225	366	209	624	281
Top 50 rating (t)	1	2	5	7	13	22	40	44

Jason intends to analyse the data using Spearman's rank correlation coefficient, r_s.

(h) Copy and complete the information in the following table. [2]

Book	A	B	C	D	E	F	G	H
Rank – Number of pages	1							
Rank – Top 50 Rating	1							

(i) (i) Calculate the value of r_s.

(ii) Interpret your result. [3]

Question 1 - working space

4. [Maximum mark: 13]

The stopping distances for bicycles travelling at $20\,\text{km}\,\text{h}^{-1}$ are assumed to follow a normal distribution with mean $6.76\,\text{m}$ and standard deviation $0.12\,\text{m}$.

(a) Under this assumption, find, correct to four decimal places, the probability that a bicycle chosen at random travelling at $20\,\text{km}\,\text{h}^{-1}$ manages to stop

(i) in less than $6.5\,\text{m}$.

(ii) in more than $7\,\text{m}$. [3]

1000 randomly selected bicycles are tested and their stopping distances when travelling at $20\,\text{km}\,\text{h}^{-1}$ are measured.

(b) Find, correct to four significant figures, the expected number of bicycles tested that stop between

(i) $6.5\,\text{m}$ and $6.75\,\text{m}$.

(ii) $6.75\,\text{m}$ and $7\,\text{m}$. [3]

The measured stopping distances of the 1000 bicycles are given in the table.

Measured stopping distance	**Number of bicycles**
Less than $6.5\,\text{m}$	12
Between $6.5\,\text{m}$ and $6.75\,\text{m}$	428
Between $6.75\,\text{m}$ and $7\,\text{m}$	527
More than $7\,\text{m}$	33

It is decided to perform a χ^2 goodness of fit test at the 5% level of significance to decide whether the stopping distances of bicycles travelling at $20\,\text{km}\,\text{h}^{-1}$ can be modelled by a normal distribution with mean $6.76\,\text{m}$ and standard deviation $0.12\,\text{m}$.

(c) State the null and alternative hypotheses. [2]

(d) Find the p-value for the test. [3]

(e) State the conclusion of the test. Give a reason for your answer. [2]

Turn over

Question 4 - working space

1. [Maximum mark: 19]

A medical centre is testing patients for a certain disease. This disease occurs in 5% of the population.

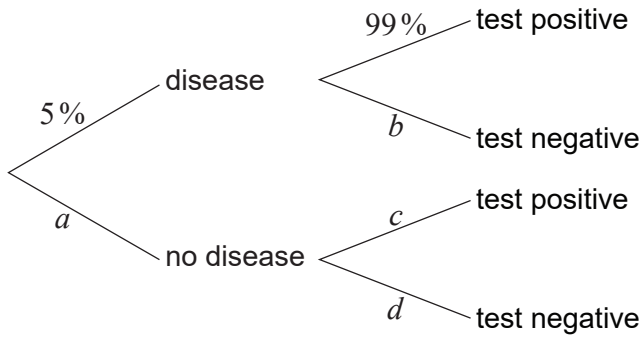
They test every patient who comes to the centre on a particular day.

(a) State the sampling method being used. [1]

It is intended that if a patient has the disease, they test "positive", and if a patient does not have the disease, they test "negative".

However, the tests are not perfect, and only 99% of people who have the disease test positive. Also, 2% of people who **do not** have the disease test positive.

The tree diagram shows some of this information.



(b) Write down the value of

(i) a.

(ii) b.

(iii) c.

(iv) d. [4]

(This question continues on the following page)

(Question 1 continued)

(c) Use the tree diagram to find the probability that a patient selected at random

(i) will not have the disease and will test positive.

(ii) will test negative.

(iii) has the disease given that they tested negative. [8]

(d) The medical centre finds the actual number of positive results in their sample is different than predicted by the tree diagram. Explain why this might be the case. [1]

The staff at the medical centre looked at the care received by all visiting patients on a randomly chosen day. All the patients received at least one of these services: they had medical tests (M), were seen by a nurse (N), or were seen by a doctor (D). It was found that:

- 78 had medical tests,
- 45 were seen by a nurse;
- 30 were seen by a doctor;
- 9 had medical tests and were seen by a doctor and a nurse;
- 18 had medical tests and were seen by a doctor but were not seen by a nurse;
- 11 patients were seen by a nurse and had medical tests but were not seen by a doctor;
- 2 patients were seen by a doctor without being seen by nurse and without having medical tests.

(e) Draw a Venn diagram to illustrate this information, placing all relevant information on the diagram. [3]

(f) Find the total number of patients who visited the centre during this day. [2]

Question 1 - working space

4. [Maximum mark: 14]

It is known that the weights of male Persian cats are normally distributed with mean $6.1\,\text{kg}$ and variance $0.5^2\,\text{kg}^2$.

(a) Sketch a diagram showing the above information. [2]

(b) Find the proportion of male Persian cats weighing between $5.5\,\text{kg}$ and $6.5\,\text{kg}$. [2]

A group of 80 male Persian cats are drawn from this population.

(c) Determine the expected number of cats in this group that have a weight of less than $5.3\,\text{kg}$. [3]

(d) It is found that 12 of the cats weigh more than $x\,\text{kg}$. Estimate the value of x. [3]

(e) Ten of the cats are chosen at random. Find the probability that exactly one of them weighs over $6.25\,\text{kg}$. [4]

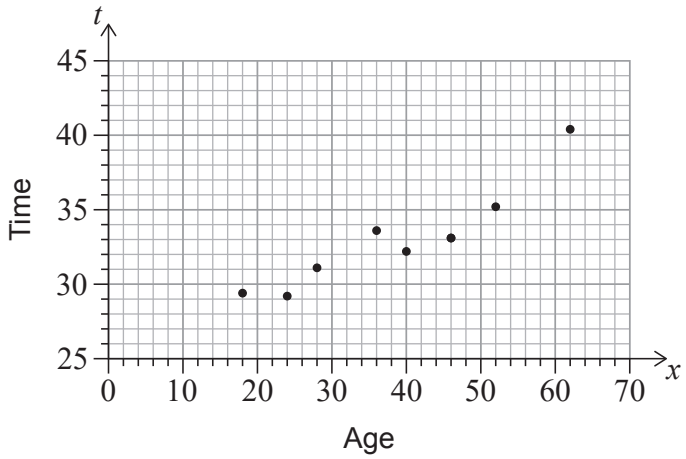
Question 4 - working space

1. [Maximum mark: 6]

Eduardo believes that there is a linear relationship between the age of a male runner and the time it takes them to run 5000 metres.

To test this, he recorded the age, x years, and the time, t minutes, for eight males in a single $5000\,\text{m}$ race. His results are presented in the following table and scatter diagram.

***x*, years**	18	24	28	36	40	46	52	62
***t*, minutes**	29.4	29.2	31.1	33.6	32.2	33.1	35.2	40.4



(a) For this data, find the value of the Pearson's product-moment correlation coefficient, r. [2]

Eduardo looked in a sports science text book. He found that the following information about r was appropriate for athletic performance.

Value of $\lvert r \rvert$	**Description of the correlation**
$0 \leq \lvert r \rvert < 0.4$	weak
$0.4 \leq \lvert r \rvert < 0.8$	moderate
$0.8 \leq \lvert r \rvert \leq 1$	strong

(b) Comment on your answer to part (a), using the information that Eduardo found. [1]

(c) Write down the equation of the regression line of t on x, in the form $t = ax + b$. [1]

(This question continues on the following page)

(Question 1 continued)

A 57-year-old male also ran in the 5000 m race.

(d) Use the equation of the regression line to estimate the time he took to complete the 5000 m race. [2]

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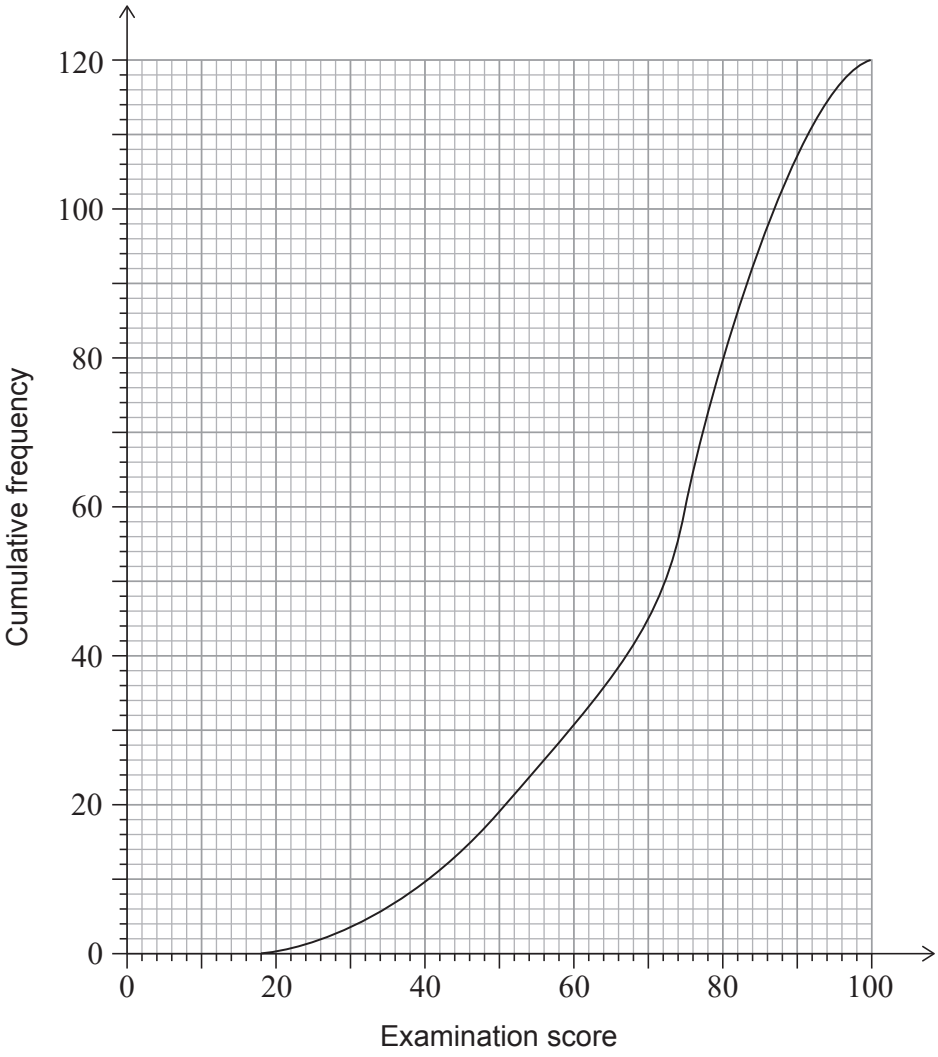
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Question 1 - working space

9. [Maximum mark: 8]

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained. [1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

Grade	1	2	3	4	5
Number of students	6	13	26	a	b

(b) Find an expression for a in terms of b. [2]

(This question continues on the following page)

(Question 9 continued)

(c) The mean grade for these students is 3.65.

(i) Find the number of students who obtained a grade 5.

(ii) Find the minimum score needed to obtain a grade 5. [5]

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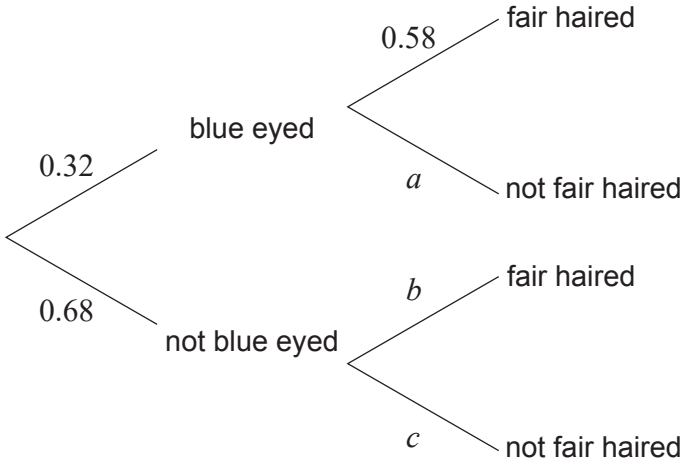
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Question 9 - working space

11. [Maximum mark: 5]

In a city, 32% of people have blue eyes. If someone has blue eyes, the probability that they also have fair hair is 58%. This information is represented in the following tree diagram.



(a) Write down the value of a. [1]

(b) Find an expression, in terms of b, for the probability of a person not having blue eyes **and** having fair hair. [1]

It is known that 41% of people in this city have fair hair.

(c) Calculate the value of

(i) b.

(ii) c. [3]

(This question continues on the following page)

(Question 11 continued)

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Question 11 - working space

1. [Maximum mark: 16]

A group of 1280 students were asked which electronic device they preferred. The results per age group are given in the following table.

	Age			
Preferred device	**11–13**	**14–16**	**17–18**	**Total**
Laptop	143	160	153	456
Tablet	205	224	131	560
Mobile phone	72	128	64	264
Total	420	512	348	1280

(a) A student from the group is chosen at random. Calculate the probability that the student

(i) prefers a tablet.

(ii) is $11–13$ years old and prefers a mobile phone.

(iii) prefers a laptop **given that** they are $17–18$ years old.

(iv) prefers a tablet or is $14–16$ years old. [9]

A χ^2 test for independence was performed on the collected data at the $1\,\%$ significance level. The critical value for the test is 13.277.

(b) State the null and alternative hypotheses. [1]

(c) Write down the number of degrees of freedom. [1]

(d) (i) Write down the χ^2 test statistic.

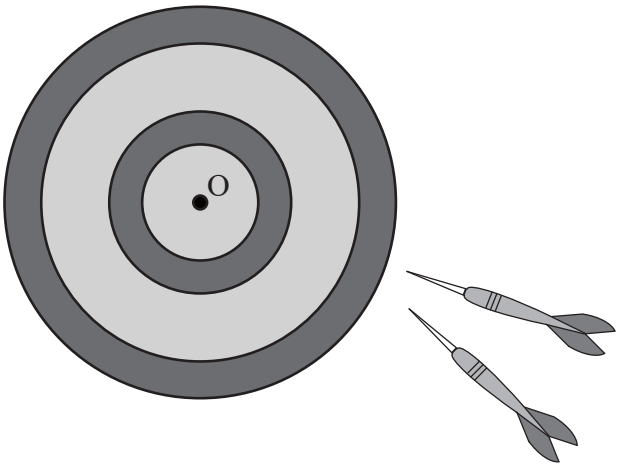
(ii) Write down the p-value.

(iii) State the conclusion for the test in context. Give a reason for your answer. [5]

Question 1 - working space

5. [Maximum mark: 14]

Arianne plays a game of darts.



The distance that her darts land from the centre, O, of the board can be modelled by a normal distribution with mean $10\,\text{cm}$ and standard deviation $3\,\text{cm}$.

(a) Find the probability that

(i) a dart lands less than $13\,\text{cm}$ from O.

(ii) a dart lands more than $15\,\text{cm}$ from O. [3]

Each of Arianne's throws is independent of her previous throws.

(b) Find the probability that Arianne throws two consecutive darts that land more than $15\,\text{cm}$ from O. [2]

In a competition a player has three darts to throw on each turn. A point is scored if a player throws **all** three darts to land within a central area around O. When Arianne throws a dart the probability that it lands within this area is 0.8143.

(c) Find the probability that Arianne does **not** score a point on a turn of three darts. [2]

In the competition Arianne has ten turns, each with three darts.

(d) (i) Find the probability that Arianne scores at least 5 points in the competition.

(ii) Find the probability that Arianne scores at least 5 points and less than 8 points.

(iii) Given that Arianne scores at least 5 points, find the probability that Arianne scores less than 8 points. [7]

References:

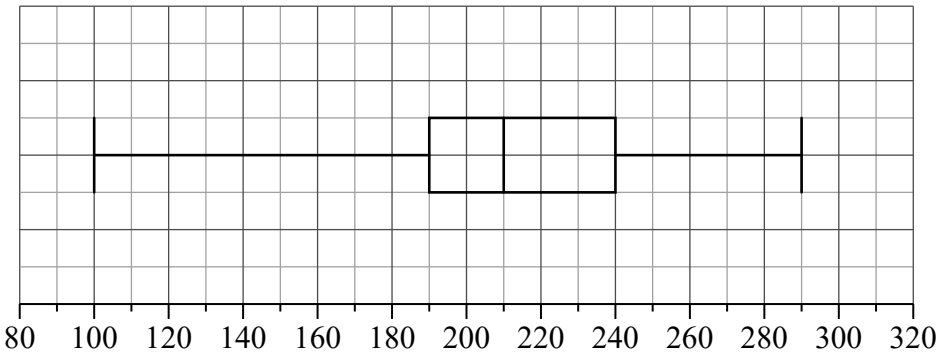
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Question 5 - working space

1. [Maximum mark: 6]

Palvinder breeds Labrador puppies at his farm. Over many years he recorded the weight (g) of the puppies.

The data is illustrated in the following box and whisker diagram.



Weight of Labrador puppies (g)

(a) Write down the median weight of the puppies. [1]

(b) Write down the upper quartile. [1]

(c) Find the interquartile range. [2]

The weights of these Labrador puppies are normally distributed.

(d) Find the weight of the heaviest possible puppy that is not an outlier. [2]

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(Question 1 continued)

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Question 1 - working space

3. [Maximum mark: 6]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

(a) State whether the data is discrete or continuous. [1]

The mean number of classes in which a student used the internet is 2.

(b) Find the value of k. [4]

It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

(c) Identify the sampling technique used in the survey. [1]

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Question 3 - working space

6. [Maximum mark: 5]

As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

	Salad meals per week			
	0	**1 – 2**	**3 – 4**	**> 4**
Students	45	26	18	6
Professors	15	8	5	12
Staff and Administration	16	13	10	6

Jing conducted a χ^2 test for independence at a $5\,\%$ level of significance.

(a) State the null hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether the null hypothesis should be accepted. [2]

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Question 6 - working space

9. [Maximum mark: 6]

Ms Calhoun measures the heights of students in her mathematics class. She is interested to see if the mean height of male students, μ_1, is the same as the mean height of female students, μ_2. The information is recorded in the table.

Male height (cm)	150	148	143	152	151	149	147	
Female height (cm)	148	152	154	147	146	153	152	150

At the 10% level of significance, a t-test was used to compare the means of the two groups. The data is assumed to be normally distributed and the standard deviations are equal between the two groups.

(a) (i) State the null hypothesis.

(ii) State the alternative hypothesis. [2]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether Ms Calhoun should accept the null hypothesis. [2]

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Question 9 - working space

12. [Maximum mark: 6]

Jae Hee plays a game involving a biased six-sided die.
The faces of the die are labelled -3, -1, 0, 1, 2 and 5.
The score for the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathbf{P}(X = x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Jae Hee plays the game twice and adds the two scores together.

(c) Find the probability Jae Hee has a **total** score of -3. [3]

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Question 12 - working space

13. [Maximum mark: 6]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1]

In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times. [2]

(c) Find the probability he will choose a male student at most 9 times. [3]

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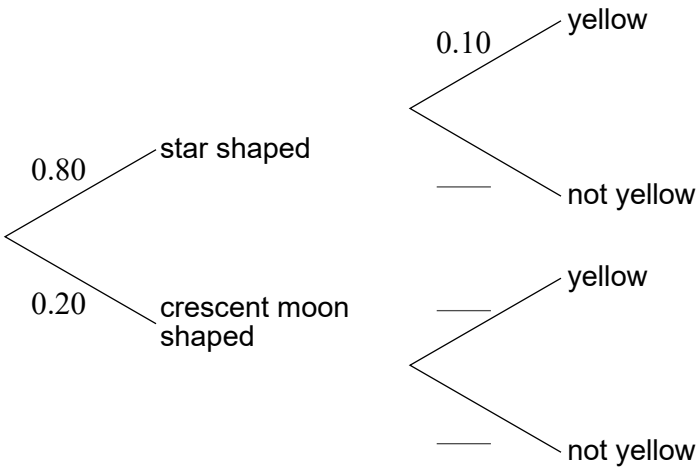
Question 13 - working space

2. [Maximum mark: 14]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a) Using the given information, **copy** and complete the following tree diagram. [2]



(b) A sweet is selected at random.

(i) Find the probability that the sweet is yellow.

(ii) Given that the sweet is yellow, find the probability it is star shaped. [4]

(This question continues on the following page)

(Question 2 continued)

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

(c) Write down the null hypothesis for this test. [1]

(d) **Copy** and complete the following table in your answer booklet. [2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

(e) Write down the number of degrees of freedom. [1]

(f) Find the p-value for the test. [2]

(g) State the conclusion of the test. Give a reason for your answer. [2]

Question 2 - working space

3. [Maximum mark: 17]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	B	C	D	E	F	G	H
Stan's score (x)	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score (y)	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

(a) (i) Write down the value of the Pearson's product–moment correlation coefficient, r.

(ii) Using the value of r, interpret the relationship between Stan's score and Minsun's score. [4]

(b) Write down the equation of the regression line y on x. [2]

(c) (i) Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10.

(ii) State whether this estimate is reliable. Justify your answer. [4]

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

(d) **Copy** and complete the information in the following table. [2]

Competitors	A	B	C	D	E	F	G	H
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

(e) (i) Find the value of the Spearman's rank correlation coefficient, r_s.

(ii) Comment on the result obtained for r_s. [4]

The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1.

(f) Explain why the value of the Spearman's rank correlation coefficient r_s does not change. [1]

Question 3 - working space

7. [Maximum mark: 7]

Leo is investigating whether a six-sided die is fair. He rolls the die 60 times and records the observed frequencies in the following table:

Number on die	1	2	3	4	5	6
Observed frequency	8	7	6	15	12	12

Leo carries out a χ^2 goodness of fit test at a 5% significance level.

(a) Write down the null and alternative hypotheses. [1]

(b) Write down the degrees of freedom. [1]

(c) Write down the expected frequency of rolling a 1. [1]

(d) Find the p-value for the test. [2]

(e) State the conclusion of the test. Give a reason for your answer. [2]

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Question 7 - working space

8. [Maximum mark: 6]

A factory produces bags of sugar with a labelled weight of $500\,\text{g}$. The weights of the bags are normally distributed with a mean of $500\,\text{g}$ and a standard deviation of $3\,\text{g}$.

(a) Write down the percentage of bags that weigh more than $500\,\text{g}$. [1]

A bag that weighs less than $495\,\text{g}$ is rejected by the factory for being underweight.

(b) Find the probability that a randomly chosen bag is rejected for being underweight. [2]

A bag that weighs more than k grams is rejected by the factory for being overweight.
The factory rejects $2\,\%$ of bags for being overweight.

(c) Find the value of k. [3]

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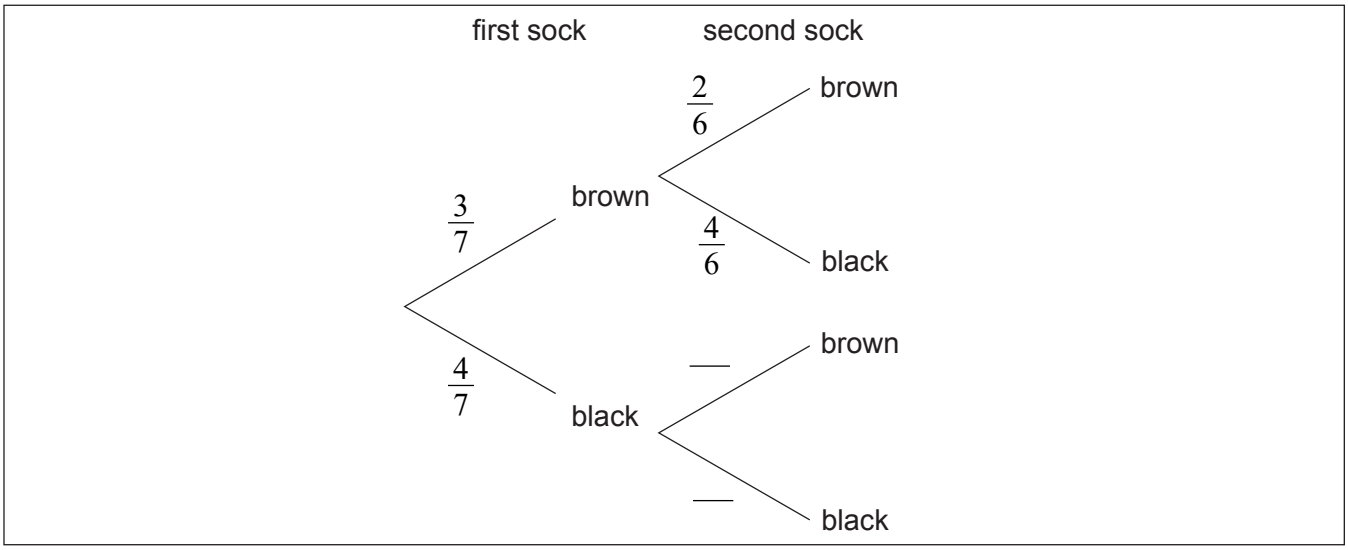
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Question 8 - working space

10. [Maximum mark: 6]

Karl has three brown socks and four black socks in his drawer. He takes two socks at random from the drawer.

(a) Complete the tree diagram. [1]



(b) Find the probability that Karl takes two socks of the same colour. [2]

(c) Given that Karl has two socks of the same colour find the probability that he has two brown socks. [3]

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Question 10 - working space

2. [Maximum mark: 6]

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	**Rejected**
Arts programme	17	24
Sciences programme	25	64

(a) Find the probability that a randomly chosen applicant from this group was accepted by the university. [1]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

(b) Find the probability that the applicant applied for the Arts programme. [2]

Two different applicants are chosen at random from the original group.

(c) Find the probability that both applicants applied to the Arts programme. [3]

(This question continues on the following page)

(Question 2 continued)

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Question 2 - working space

5. [Maximum mark: 7]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

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Question 5 - working space

7. [Maximum mark: 5]

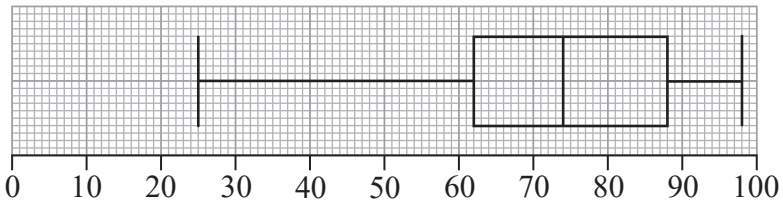
A college runs a mathematics course in the morning. Scores for a test from this class are shown below.

25 33 51 62 63 63 70 74 79 79 81 88 90 90 98

For these data, the lower quartile is 62 and the upper quartile is 88.

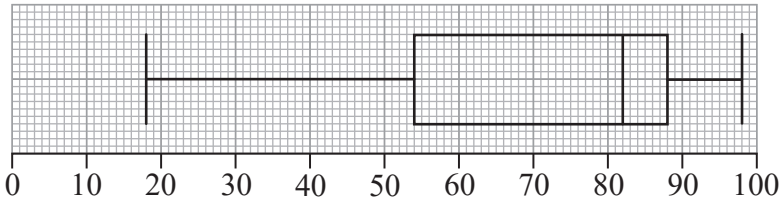
(a) Show that the test score of 25 would not be considered an outlier. [3]

The box and whisker diagram showing these scores is given below.



Test scores

Another mathematics class is run by the college during the evening. A box and whisker diagram showing the scores from this class for the same test is given below.



Test scores

A researcher reviews the box and whisker diagrams and believes that the evening class performed better than the morning class.

(b) With reference to the box and whisker diagrams, state one aspect that may support the researcher's opinion and one aspect that may counter it. [2]

(This question continues on the following page)

(Question 7 continued)

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Question 7 - working space

8. [Maximum mark: 6]

A study was conducted to investigate whether the mean reaction time of drivers who are talking on mobile phones is the same as the mean reaction time of drivers who are talking to passengers in the vehicle. Two independent groups were randomly selected for the study.

To gather data, each driver was put in a car simulator and asked to either talk on a mobile phone or talk to a passenger. Each driver was instructed to apply the brakes as soon as they saw a red light appear in front of the car. The reaction times of the drivers, in seconds, were recorded, as shown in the following table.

Talking on mobile phone	Talking to passenger
0.69	0.67
0.87	0.86
0.98	0.60
1.04	0.81
0.79	0.76
0.87	0.71
0.71	0.74

At the 10% level of significance, a t-test was used to compare the mean reaction times of the two groups. Each data set is assumed to be normally distributed, and the population variances are assumed to be the same.

Let μ_1 and μ_2 be the population means for the two groups. The null hypothesis for this test is $\mathrm{H}_0 : \mu_1 - \mu_2 = 0$.

(a) State the alternative hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) (i) State the conclusion of the test. Justify your answer.

(ii) State what your conclusion means in context. [3]

(This question continues on the following page)

(Question 8 continued)

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Question 8 - working space

10. [Maximum mark: 5]

The masses of Fuji apples are normally distributed with a mean of $163\,\text{g}$ and a standard deviation of $6.83\,\text{g}$.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between $172\,\text{g}$ and $183\,\text{g}$.

(a) Determine the probability that a Fuji apple selected at random will be a large apple. [2]

Approximately $68\,\%$ of Fuji apples have a mass within the medium-sized category, which is between k and $172\,\text{g}$.

(b) Find the value of k. [3]

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Question 10 - working space

3. [Maximum mark: 16]

The scores of the eight highest scoring countries in the 2019 Eurovision song contest are shown in the following table.

	Eurovision score
Netherlands	498
Italy	472
Russia	370
Switzerland	364
Sweden	334
Norway	331
North Macedonia	305
Azerbaijan	302

(a) For this data, find

(i) the upper quartile.

(ii) the interquartile range. [4]

(b) Determine if the Netherlands' score is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

Chester is investigating the relationship between the highest-scoring countries' Eurovision score and their population size to determine whether population size can reasonably be used to predict a country's score.

The populations of the countries, to the nearest million, are shown in the table.

	Population (x) (millions)	**Eurovision score (y)**
Netherlands	17	498
Italy	60	472
Russia	145	370
Switzerland	9	364
Sweden	10	334
Norway	5	331
North Macedonia	2	305
Azerbaijan	10	302

Chester finds that, for this data, the Pearson's product moment correlation coefficient is $r = 0.249$.

(c) State whether it would be appropriate for Chester to use the equation of a regression line for y on x to predict a country's Eurovision score. Justify your answer. [2]

Chester then decides to find the Spearman's rank correlation coefficient for this data, and creates a table of ranks.

	Population rank (to the nearest million)	**Eurovision score rank**
Netherlands	3	1
Italy	2	2
Russia	1	3
Switzerland	a	4
Sweden	b	5
Norway	7	6
North Macedonia	8	7
Azerbaijan	c	8

(This question continues on the following page)

(Question 3 continued)

(d) Write down the value of:

(i) a,

(ii) b,

(iii) c. [3]

(e) (i) Find the value of the Spearman's rank correlation coefficient r_s.

(ii) Interpret the value obtained for r_s. [3]

(f) When calculating the ranks, Chester incorrectly read the Netherlands' score as 478. Explain why the value of the Spearman's rank correlation r_s does not change despite this error. [1]

Question 3 - working space

5. [Maximum mark: 15]

The aircraft for a particular flight has 72 seats. The airline's records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

(a) The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. [3]

(b) (i) Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. [2]

(ii) Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. [2]

Each passenger pays $\$150$ for a ticket. If too many passengers arrive, then the airline will give $\$300$ in compensation to each passenger that cannot board.

(c) Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. [8]

References:

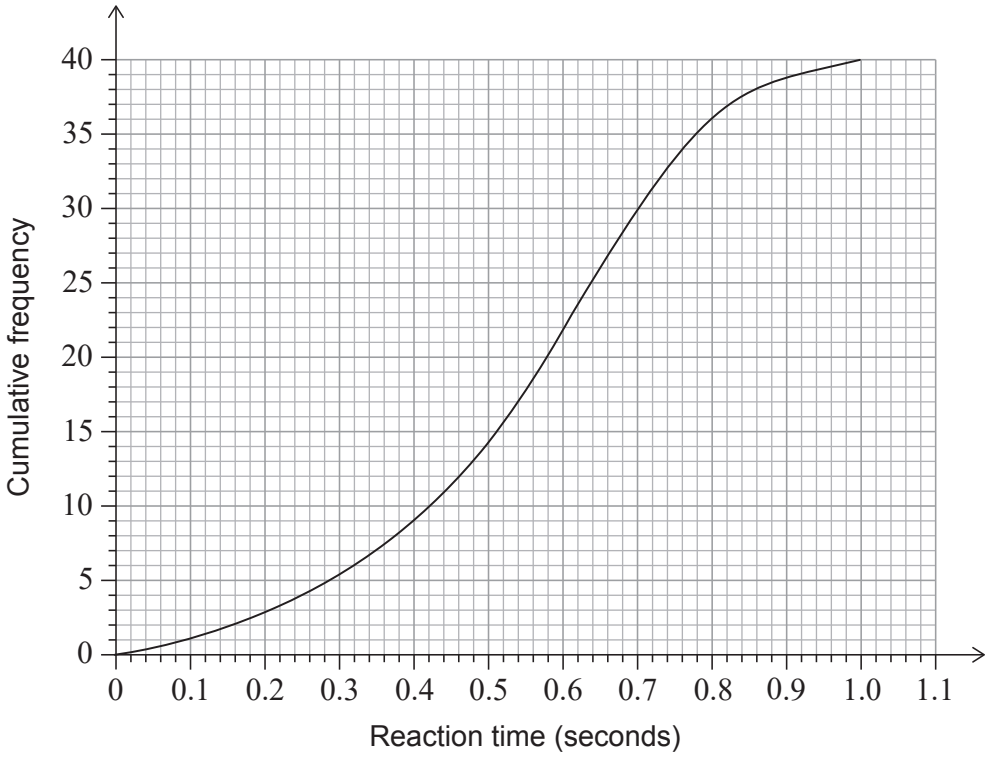
2. mynamepong, n.d. Goat [image online] Available at: https://thenounproject.com/term/goat/1761571/ This file is licensed under the Creative Commons Attribution-ShareAlike 3.0 Unported (CC BY-SA 3.0) https://creativecommons.org/licenses/by-sa/3.0/deed.en [Accessed 22 April 2010] Source adapted.

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Question 5 - working space

1. [Maximum mark: 17]

Mackenzie conducted an experiment on the reaction times of teenagers. The results of the experiment are displayed in the following cumulative frequency graph.



(a) Use the graph to estimate the

(i) median reaction time;

(ii) interquartile range of the reaction times. [4]

(b) Find the estimated number of teenagers who have a reaction time greater than 0.4 seconds. [2]

(c) Determine the 90th percentile of the reaction times from the cumulative frequency graph. [2]

(This question continues on the following page)

(Question 1 continued)

Mackenzie created the cumulative frequency graph using the following grouped frequency table.

Reaction time, t (s)	Frequency
$0 < t \leq 0.2$	3
$0.2 < t \leq 0.4$	a
$0.4 < t \leq 0.6$	13
$0.6 < t \leq 0.8$	14
$0.8 < t \leq 1.0$	b

(d) Write down the value of

(i) a;

(ii) b. [2]

(e) Write down the modal class from the table. [1]

(f) Use your graphic display calculator to find an estimate of the mean reaction time. [2]

Upon completion of the experiment, Mackenzie realized that some values were grouped incorrectly in the frequency table. Some reaction times recorded in the interval $0 < t \leq 0.2$ should have been recorded in the interval $0.2 < t \leq 0.4$.

(g) Suggest how, if at all, the estimated mean and estimated median reaction times will change if the errors are corrected. Justify your response. [4]

Question 1 - working space

4. [Maximum mark: 5]

Sergio is interested in whether an adult's favourite breakfast berry depends on their income level. He obtains the following data for 341 adults and decides to carry out a χ^2 test for independence, at the 10% significance level.

		Income level		
		Low	**Medium**	**High**
Favourite berry	**Strawberry**	21	39	30
	Blueberry	39	67	42
	Other berry	32	45	26

(a) Write down the null hypothesis. [1]

(b) Find the value of the χ^2 statistic. [2]

The critical value of this χ^2 test is 7.78.

(c) Write down Sergio's conclusion to the test in context. Justify your answer. [2]

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Question 4 - working space

6. [Maximum mark: 5]

Manny and Annabelle, mathematics teachers at Burnham High School, give their students the same examination. A random sample of the examination scores were collected from each of their classes.

Examination scores from Manny's class	76	77	82	84	88	90	91	98
Examination scores from Annabelle's class	68	79	81	89	91	92	92	95

Annabelle uses these scores to conduct a two-tailed t-test to compare the means of the two classes, at the 5% level of significance. It is assumed the examination scores for both classes have the same variance and are normally distributed.

The null hypothesis is $\mu_1 = \mu_2$, where μ_1 is the mean examination score from Manny's class and μ_2 is the mean examination score from Annabelle's class.

(a) Write down the alternative hypothesis. [1]

(b) Find the p-value for this test. Give your answer correct to five decimal places. [2]

Annabelle concludes there is insufficient evidence to reject the null hypothesis.

(c) State whether Annabelle's conclusion is correct. Give a reason for your answer. [2]

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Question 6 - working space

8. [Maximum mark: 5]

Roy is a member of a motorsport club and regularly drives around the Port Campbell racetrack.

The times he takes to complete a lap are normally distributed with mean 59 seconds and standard deviation 3 seconds.

(a) Find the probability that Roy completes a lap in less than 55 seconds. [2]

Roy will complete a 20 lap race. It is expected that 8.6 of the laps will take more than t seconds.

(b) Find the value of t. [3]

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Question 8 - working space

9. [Maximum mark: 7]

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

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Question 9 - working space

1. [Maximum mark: 17]

Elsie, a librarian, wants to investigate the length of time, T minutes, that people spent in her library on a particular day.

(a) State whether the variable T is discrete or continuous. [1]

Elsie's data for 160 people who visited the library on that particular day is shown in the following table.

***T* (minutes)**	$0 \leq T < 20$	$20 \leq T < 40$	$40 \leq T < 60$	$60 \leq T < 80$	$80 \leq T < 100$
Frequency	50	62	k	14	8

(b) Find the value of k. [2]

(c) (i) Write down the modal class.

(ii) Write down the mid-interval value for this class. [2]

(d) Use Elsie's data to calculate an estimate of the mean time that people spent in the library. [2]

(e) Using the table, write down the maximum possible number of people who spent 35 minutes or less in the library on that day. [1]

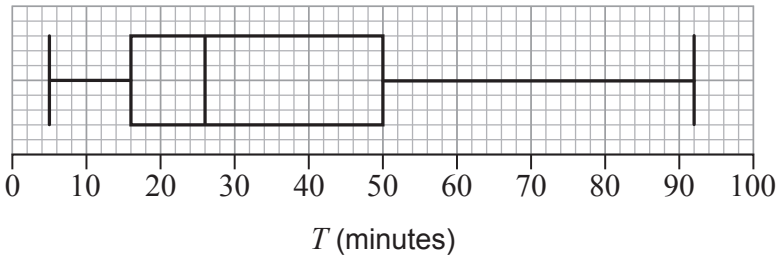
Elsie assumes her data to be representative of future visitors to the library.

(f) Find the probability a visitor spends at least 60 minutes in the library. [2]

(This question continues on the following page)

(Question 1 continued)

The following box and whisker diagram shows the times, in minutes, that the 160 visitors spent in the library.



(g) Write down the median time spent in the library. [1]

(h) Find the interquartile range. [2]

(i) Hence show that the longest time that a person spent in the library is not an outlier. [3]

Elsie believes the box and whisker diagram indicates that the times spent in the library are not normally distributed.

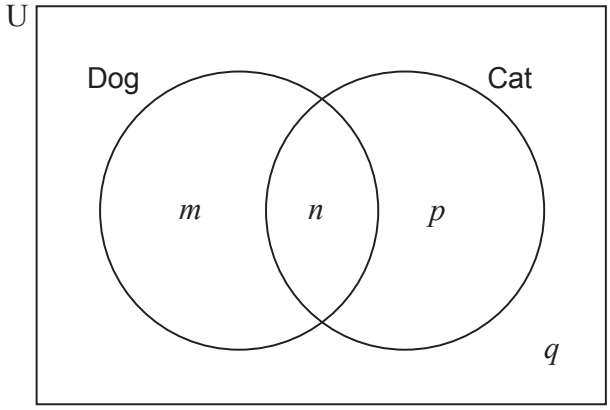
(j) Identify one feature of the box and whisker diagram which might support Elsie's belief. [1]

Question 1 - working space

4. [Maximum mark: 16]

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m, n, p and q represent the percentage of students within each region.



(a) Find the value of

(i) m.

(ii) n.

(iii) p.

(iv) q. [4]

(b) Find the percentage of students who have a dog or a cat or both. [1]

(c) Find the probability that a randomly chosen student

(i) has a dog but does not have a cat.

(ii) has a dog given that they do not have a cat. [3]

(This question continues on the following page)

(Question 4 continued)

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events “being a school captain” and “having a dog” are independent.

Use Tim’s model to find the probability that in the next 10 years

(d) (i) 5 school captains have a dog.

(ii) more than 3 school captains have a dog.

(iii) exactly 9 school captains in succession have a dog. [7]

John randomly chooses 10 students from the survey.

(e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. [1]

Question 4 - working space

1. [Maximum mark: 6]

The decathlon is a competition where athletes compete in ten events. Two of those events are long jump and high jump. In both events, a greater distance means a better ranking.

The table shows results for these two events at the World Championships.

	Event		Rank	
Athlete's Country	**Long Jump (m)**	**High Jump (m)**	**Long Jump Rank**	**High Jump Rank**
Germany	7.64	2.11	1	
France	7.52	2.08	2	
Estonia	7.49	1.84	3	
Canada	7.44	2.02	4	
Netherlands	7.33	2.05	5	
Ukraine	7.28	2.02	6	
Algeria	7.22	1.90	7	
Austria	7.11	1.87	8	
Grenada	6.98	1.99	9	
Japan	6.64	1.96	10	

The Spearman's rank correlation coefficient is used to determine if there is a linear correlation between an athlete's ranking in long jump and their ranking in high jump.

(a) Complete the table to show the athletes' rankings in high jump. [2]

(b) Find the value of the Spearman's rank correlation coefficient r_s. [2]

(This question continues on the following page)

(Question 1 continued)

The following guide is used by the coach to determine the strength of the correlation between the ranks for long jump and high jump.

$\lvert r_s \rvert$	**Strength**
0.000 to 0.199	Very weak
0.200 to 0.399	Weak
0.400 to 0.599	Moderate
0.600 to 0.799	Strong
0.800 to 1.000	Very strong

(c) State the strength of the correlation between the rankings as indicated by the table and interpret this in the context of the question. [2]

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Question 1 - working space

4. [Maximum mark: 6]

Carys believes that, on a memory retention test, the mean score of bilingual people (μ_b) will be higher than the mean score of monolingual people (μ_m). Carys gave a memory retention test to a random sample of students in her class. The results are shown in the two tables.

	Scores									
Bilingual	100	94	100	90	100	94	98	98	98	98

	Scores							
Monolingual	97	92	88	98	88	94	100	100

Carys performs a one-tailed t-test at a 5% level of significance. It is assumed that the scores are normally distributed and the samples have equal variances.

(a) State the null and alternative hypotheses. [2]

(b) Calculate the p-value for this test. [2]

(c) State the conclusion of the test in the context of the question. Justify your answer. [2]

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Question 4 - working space

12. [Maximum mark: 7]

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\,\text{km h}^{-1}$.

A speed of $75.7\,\text{km h}^{-1}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

Speeding tickets are issued to all drivers travelling at a speed greater than $72\,\text{km h}^{-1}$.

(b) Find the probability that a randomly selected driver who passes the speed camera receives a speeding ticket. [2]

It is found that 82% of cars on this road travel at speeds between $p\,\text{km h}^{-1}$ and $q\,\text{km h}^{-1}$, where $p < q$. This interval includes cars travelling at a speed of $74\,\text{km h}^{-1}$.

(c) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean. [3]

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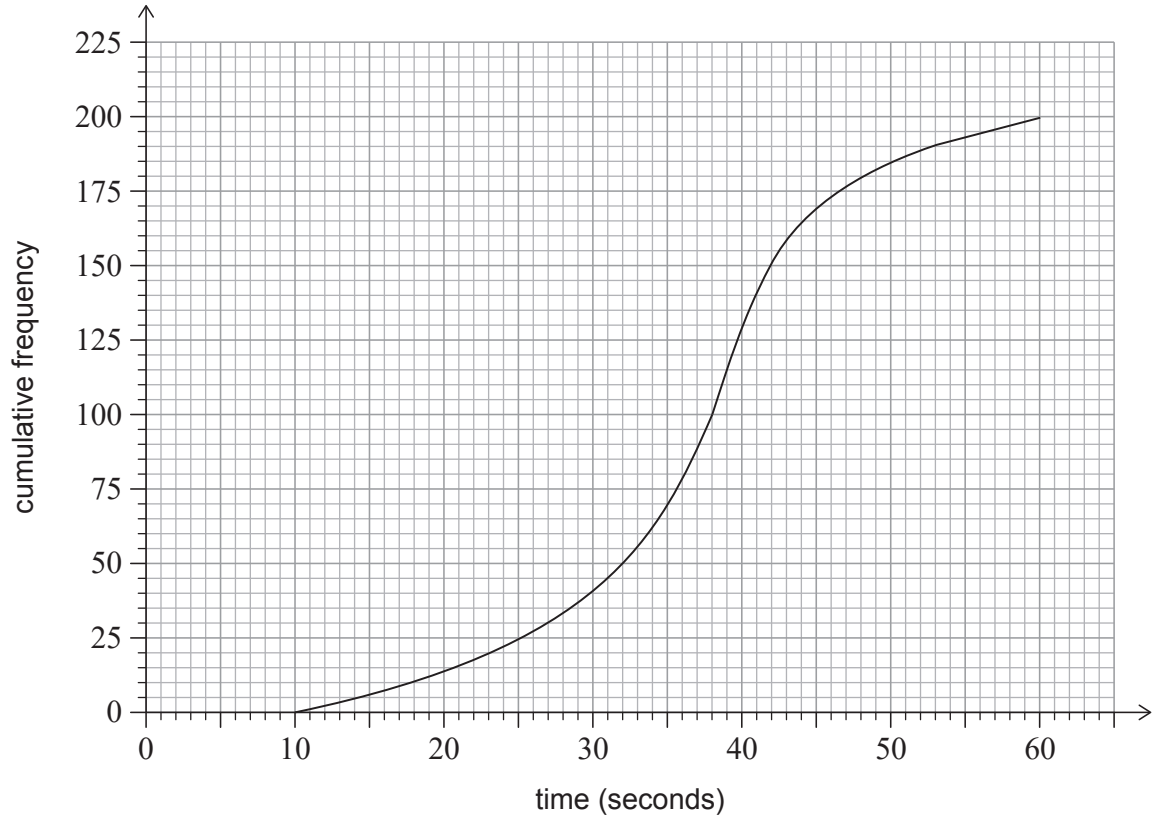
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Question 12 - working space

3. [Maximum mark: 7]

In a school, 200 students solved a problem in a mathematics competition. Their times to solve the problem were recorded and the following cumulative frequency graph was produced.



(a) Use the graph to find

(i) the median time;

(ii) the lower quartile;

(iii) the upper quartile;

(iv) the interquartile range. [4]

Cedric took 14 seconds to solve the problem.

(b) Determine whether Cedric's time is an outlier. [3]

(This question continues on the following page)

(Question 3 continued)

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Question 3 - working space

4. [Maximum mark: 6]

At a running club, Sung-Jin conducts a test to determine if there is any association between an athlete's age and their best time taken to run $100\,\text{m}$. Eight athletes are chosen at random, and their details are shown below.

Athlete	A	B	C	D	E	F	G	H
Age (years)	13	17	22	18	19	25	11	36
Time (seconds)	13.4	14.6	13.4	12.9	12.0	11.8	17.0	13.1

Sung-Jin decides to calculate the Spearman's rank correlation coefficient for his set of data.

(a) Complete the table of ranks. [2]

Athlete	A	B	C	D	E	F	G	H
Age rank			3					
Time rank							1	

(b) Calculate the Spearman's rank correlation coefficient, r_s. [2]

(c) Interpret this value of r_s in the context of the question. [1]

(d) Suggest a mathematical reason why Sung-Jin may have decided not to use Pearson's product-moment correlation coefficient with his data from the original table. [1]

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Question 4 - working space

5. [Maximum mark: 4]

The following frequency distribution table shows the test grades for a group of students.

Grade	1	2	3	4	5	6	7
Frequency	1	4	7	9	p	9	4

For this distribution, the mean grade is 4.5.

(a) Write down the total number of students in terms of p. [1]

(b) Calculate the value of p. [3]

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Question 5 - working space

6. [Maximum mark: 6]

A company that owns many restaurants wants to determine if there are differences in the quality of the food cooked for three different meals: breakfast, lunch and dinner.

Their quality assurance team randomly selects 500 items of food to inspect. The quality of this food is classified as perfect, satisfactory, or poor. The data is summarized in the following table.

		Quality			
		Perfect	**Satisfactory**	**Poor**	**Total**
Meal	**Breakfast**	101	124	7	**232**
	Lunch	68	81	5	**154**
	Dinner	35	69	10	**114**
	Total	**204**	**274**	**22**	**500**

An item of food is chosen at random from these 500.

(a) Find the probability that its quality is not perfect, given that it is from breakfast. [2]

A χ^2 test at the 5% significance level is carried out to determine if there is significant evidence of a difference in the quality of the food cooked for the three meals.

The critical value for this test is 9.488.

The hypotheses for this test are:
H_0: The quality of the food and the type of meal are independent.
H_1: The quality of the food and the type of meal are not independent.

(b) Find the χ^2 statistic. [2]

(c) State, with justification, the conclusion for this test. [2]

(This question continues on the following page)

(Question 6 continued)

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Question 6 - working space

9. [Maximum mark: 6]

The lengths of the seeds from a particular mango tree are approximated by a normal distribution with a mean of $4\,\text{cm}$ and a standard deviation of $0.25\,\text{cm}$.

A seed from this mango tree is chosen at random.

(a) Calculate the probability that the length of the seed is less than $3.7\,\text{cm}$. [2]

It is known that $30\,\%$ of the seeds have a length greater than $k\,\text{cm}$.

(b) Find the value of k. [2]

For a seed of length $d\,\text{cm}$, chosen at random, $\text{P}(4 - m < d < 4 + m) = 0.6$.

(c) Find the value of m. [2]

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Question 9 - working space

12. [Maximum mark: 5]

In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$\mathbf{P}(X = x)$	0.15	0.2	k	0.16	$2k$	0.25

(a) Find the value of k. [2]

The player has a chance to win money based on how many times they hit the target.

The gain for the player, in \$, is shown in the following table, where a negative gain means that the player loses money.

x	0	1	2	3	4	5
Player's gain (\$)	–4	–3	–1	0	1	4

(b) Determine whether this game is fair. Justify your answer. [3]

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Question 12 - working space

1. [Maximum mark: 15]

The mean annual temperatures for Earth, recorded at fifty-year intervals, are shown in the table.

Year (x)	1708	1758	1808	1858	1908	1958	2008
Temperature °C (y)	8.73	9.22	9.10	9.12	9.13	9.45	9.76

Tami creates a linear model for this data by finding the equation of the straight line passing through the points with coordinates $(1708, 8.73)$ and $(1958, 9.45)$.

(a) Calculate the gradient of the straight line that passes through these two points. [2]

(b) (i) Interpret the meaning of the gradient in the context of the question.

(ii) State appropriate units for the gradient. [2]

(c) Find the equation of this line giving your answer in the form $y = mx + c$. [2]

(d) Use Tami's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses linear regression to obtain a model for the data.

(e) (i) Find the equation of the regression line y on x.

(ii) Find the value of r, the Pearson's product-moment correlation coefficient. [3]

(f) Use Thandizo's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses his regression line to predict the year when the mean annual temperature will first exceed $15\,°\text{C}$.

(g) State two reasons why Thandizo's prediction may not be valid. [2]

Turn over

Question 1 - working space

4. [Maximum mark: 17]

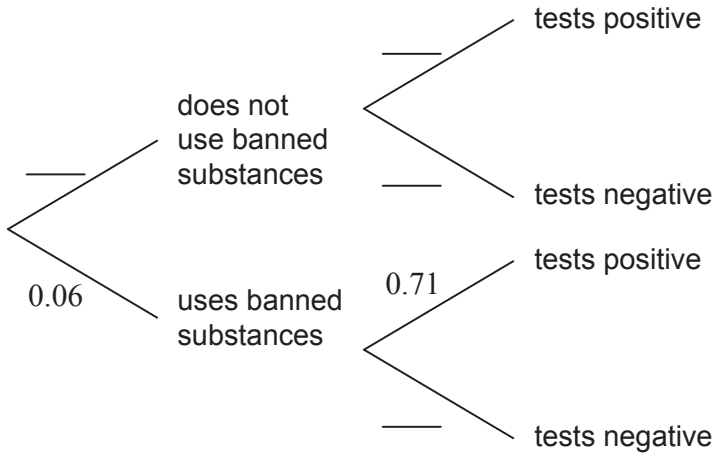
A large international sports tournament tests their athletes for banned substances. They interpret a positive test result as meaning that the athlete uses banned substances. A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06.

If an athlete **uses** banned substances, the probability that they will test positive is 0.71.

If an athlete does **not use** banned substances, the probability that they will test negative is 0.98.

(a) Using the information given, **copy** (into your answer booklet) and complete the following tree diagram. [2]



(b) (i) Determine the probability that a randomly selected athlete does not use banned substances and tests negative.

(ii) If two athletes are selected at random, calculate the probability that both athletes do not use banned substances and both test negative. [4]

(c) (i) Calculate the probability that a randomly selected athlete will receive an **incorrect** test result.

(ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result. [5]

Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

(d) Calculate the probability that none of the athletes in Team X will test positive. [4]

(e) Determine the probability that more than 2 athletes in Team X will test positive. [2]

Turn over

Question 4 - working space

4. [Maximum mark: 17]

It is claimed that a new remedy cures 82% of the patients with a particular medical problem.

This remedy is to be used by 115 patients, and it is assumed that the 82% claim is true.

(a) Find the probability that exactly 90 of these patients will be cured. [3]

(b) Find the probability that at least 95 of these patients will be cured. [2]

(c) Find the variance in the possible number of patients that will be cured. [2]

The probability that at least n patients will be cured is less than 30%.

(d) Find the least value of n. [3]

A clinic is interested to see if the mean recovery time of their patients who tried the new remedy is less than that of their patients who continued with an older remedy. The clinic randomly selects some of their patients and records their recovery time in days. The results are shown in the table below.

	Recovery time (days)								
Group N (new remedy)	12	13	9	13	14	15	17		

	Recovery time (days)								
Group O (old remedy)	17	11	10	18	20	22	14	15	18

The data is assumed to follow a normal distribution and the population variance is the same for the two groups. A t-test is used to compare the means of the two groups at the 10% significance level.

(e) State the appropriate null and alternative hypotheses for this t-test. [2]

(f) Find the p-value for this test. [2]

(g) State the conclusion for this test. Give a reason for your answer. [2]

(h) Explain what the p-value represents. [1]

Turn over

Question 4 - working space

1. [Maximum mark: 7]

Billy is a keen walker who keeps a record of his performance. The following table shows the time, in minutes, it takes him to walk one kilometre up hills with different gradients. The gradient of each hill is constant.

Gradient G (%)	0	4	10	15	20
Time T (min.)	6.85	8.42	11.20	14.49	17.88

(a) (i) Find the equation of the regression line of T on G.

(ii) Describe the correlation between T and G with reference to the value of r, the Pearson's product-moment correlation coefficient. [4]

On Sunday, Billy intends to walk up a hill with a gradient of $13\,\%$.

(b) Estimate the time it will take Billy to walk one kilometre up the hill. [2]

This morning, Billy walked one kilometre up a hill, and it took 22 minutes.

(c) Explain why it would be inappropriate to use the equation found in part (a) to estimate the gradient of this hill. [1]

(This question continues on the following page)

(Question 1 continued)

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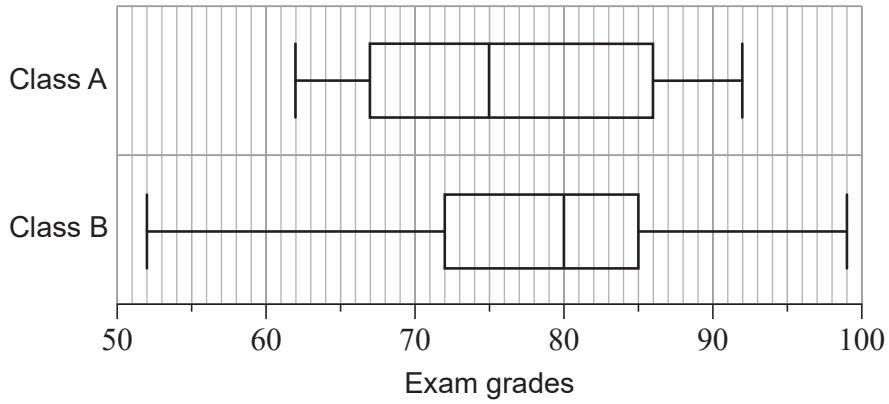
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Question 1 - working space

9. [Maximum mark: 8]

Mrs Whitehouse is a chemistry teacher. After grading her final exams, she creates the following box and whisker diagram to compare the grades of her two classes.



(a) Identify which **two** of the following statements **must** be true according to the box and whisker diagram. Indicate your choices by placing tick marks in the second column of the following table. [2]

Statement	True (✓)
The data for Class A is normally distributed.	
A higher percentage of students in Class A received a grade less than 70 on the exam, than in Class B.	
More students in Class B received a grade greater than 90 on the exam than in Class A.	
The interquartile range for Class B is less than the interquartile range for Class A.	

At the end of the year, Mrs Whitehouse surveyed a random sample of students from each of her two large classes to determine how satisfied they were with her teaching.

Each student independently selected a value from 1 to 10, with 1 meaning that they were not satisfied at all and 10 meaning that they were very satisfied.

Her collected data from the student surveys is shown.

Class A	7	5	3	4	3	8	6	5		
Class B	6	9	8	10	1	9	10	9	8	3

(This question continues on the following page)

(Question 9 continued)

Mrs Whitehouse believes that there was no difference in the general satisfaction between the two classes. She assumes that the data is drawn from a population that can be modelled by a normal distribution and proposes to conduct a t-test at the 5% significance level.

(b) Write down the null and alternative hypotheses for her test. [2]

(c) Find the p-value for her test. [2]

(d) Write down the conclusion to the test. Give a reason for your answer. [2]

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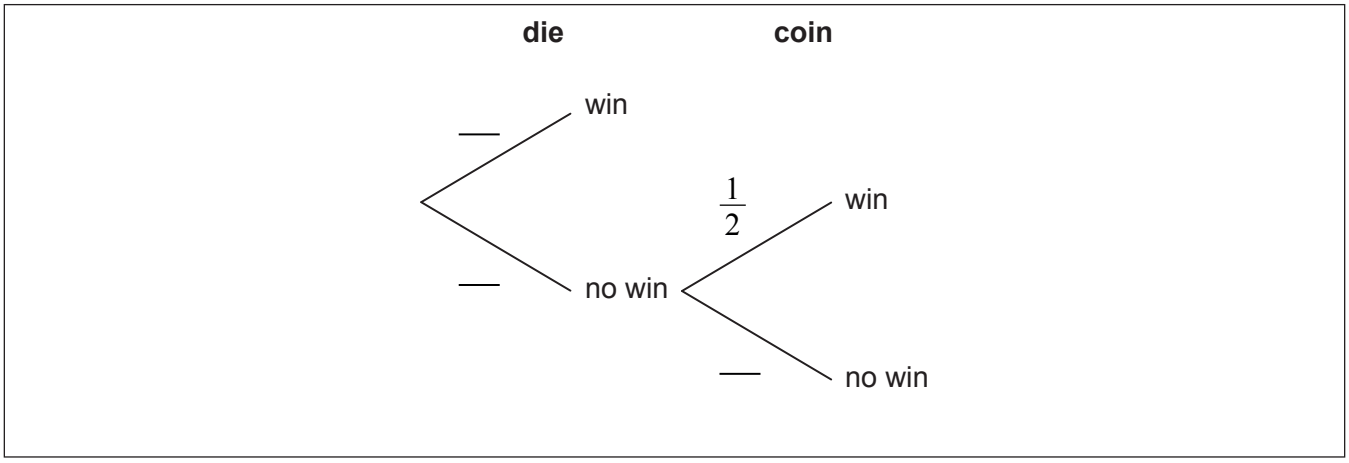
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Question 9 - working space

10. [Maximum mark: 7]

Rita is playing a game. In the game, she must roll a fair six-sided die. If she gets a five or six then she wins a prize. If not, then she has another chance but this time she must flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then Rita wins a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]



(b) Find the probability that Rita does **not** win a prize. [2]

(c) Given that Rita won a prize, find the probability that she got a five or six when she rolled the die. [3]

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Question 10 - working space

11. [Maximum mark: 6]

Nicole works at a local school 5 days each week. She drives an old car to work that has a 72% probability of starting on any given morning. The probability of the car starting on a given morning is independent of it starting on any other morning.

(a) Find the probability that Nicole's car starts on exactly three mornings in a particular 5 day workweek. [2]

Nicole walks to work on mornings when her car does not start and it is **not** raining.
Nicole takes the bus to work on mornings when her car does not start and it is raining.

Where Nicole lives, there is a 42% probability of rain on any given morning, independent of any other morning. The probability of Nicole's car starting is independent of the weather.

(b) Find the probability that Nicole will **not** have to take the bus in a particular workweek. [4]

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Question 11 - working space

12. [Maximum mark: 6]

Thurston believes that more popular musical artists sell more albums.

He begins to investigate this belief by randomly selecting eight musical artists and collecting data on the number of followers each of the artists has on a particular social media platform. He then collects data on the number of albums each artist sold in the first week after releasing an album. His data is shown in **Table 1**.

Table 1

	Artist 1	**Artist 2**	**Artist 3**	**Artist 4**	**Artist 5**	**Artist 6**	**Artist 7**	**Artist 8**
Number of social media followers (in thousands)	11 500	12 400	1300	2300	674	49 500	315	94 400
Number of albums sold in first week (in thousands)	123	62.4	17.4	94.9	52.5	27	21.6	595.5

Thurston decides to calculate the Spearman's rank correlation coefficient.

(a) Complete the table of ranks shown in **Table 2**. [1]

Table 2

	Artist 1	**Artist 2**	**Artist 3**	**Artist 4**	**Artist 5**	**Artist 6**	**Artist 7**	**Artist 8**
Rank – social media followers	4	3	6	5	7	2	8	1
Rank – albums sold in first week								1

(This question continues on the following page)

(Question 12 continued)

(b) Calculate the value of r_s, Spearman's rank correlation coefficient. [2]

Thurston believes that artists with a higher number of social media followers sell more albums in the first week. He carries out a hypothesis test using a 10% significance level with the following null hypothesis:

H_0: In the population, there is no monotonic relationship between the number of social media followers and the number of albums sold in the first week.

(c) Write down Thurston's alternative hypothesis. [1]

The critical value of r_s for this test is 0.643.

(d) State the conclusion of the hypothesis test, giving a reason. [2]

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References:

2. Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.

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Question 12 - working space

1. [Maximum mark: 7]

Joel is a keen cyclist who keeps a record of his performance. The following table shows the time, in minutes, it takes him to ride one kilometre on hills with different gradients. The gradient of each hill is constant.

Gradient G (%)	0	4	10	15	20
Time T (min.)	2.11	5.39	10.56	13.20	18.58

(a) (i) Find the equation of the regression line of T on G.

(ii) Describe the correlation between T and G with reference to the value of r, the Pearson's product-moment correlation coefficient. [4]

On Saturday, Joel intends to ride a hill with a gradient of 17%.

(b) Estimate the time it will take Joel to ride one kilometre up the hill. [2]

This morning, Joel rode one kilometre up a hill, and it took 22 minutes.

(c) Explain why it would be inappropriate to use the equation found in part (a) to estimate the gradient of this hill. [1]

(This question continues on the following page)

(Question 1 continued)

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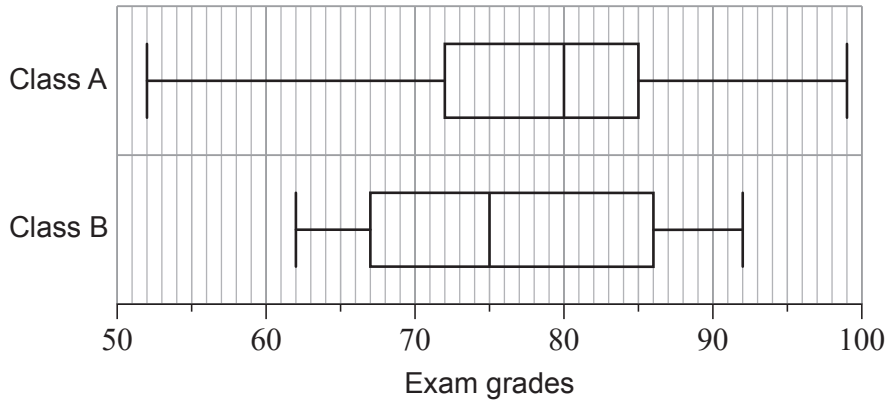
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Question 1 - working space

9. [Maximum mark: 8]

Mr Kelly is a history teacher. After grading his final exams, he creates the following box and whisker diagram to compare the grades of his two classes.



(a) Identify which **two** of the following statements **must** be true according to the box and whisker diagram. Indicate your choices by placing tick marks in the second column of the following table. [2]

Statement	True (✓)
A higher percentage of students in Class B received a grade less than 70 on the exam, than in Class A.	
The data for Class B is normally distributed.	
More students in Class A received a grade greater than 90 on the exam than in Class B.	
The interquartile range for Class A is less than the interquartile range for Class B.	

At the end of the year, Mr Kelly surveyed a random sample of students from each of his two large classes to determine how satisfied they were with his teaching.

Each student independently selected a value from 1 to 10, with 1 meaning that they were not satisfied at all and 10 meaning that they were very satisfied.

His collected data from the student surveys is shown.

Class A	6	9	8	10	1	9	10	9	8	4

Class B	7	5	3	4	3	8	6	7

(This question continues on the following page)

(Question 9 continued)

Mr Kelly believes that there was no difference in the general satisfaction between the two classes. He assumes that the data is drawn from a population that can be modelled by a normal distribution and proposes to conduct a t-test at the 5% significance level.

(b) Write down the null and alternative hypotheses for his test. [2]

(c) Find the p-value for his test. [2]

(d) Write down the conclusion to the test. Give a reason for your answer. [2]

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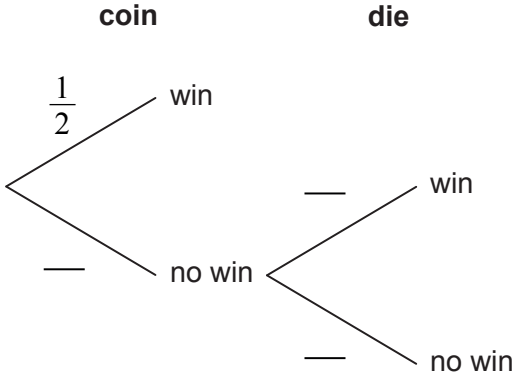
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Question 9 - working space

10. [Maximum mark: 7]

Michèle is playing a game. In the game, she must first flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then she wins a prize. If it lands on tails, then she has another chance but this time she must roll a fair six-sided die and get a five or six in order to win a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]



(b) Find the probability that Michèle does **not** win a prize. [2]

(c) Given that Michèle won a prize, find the probability that the coin landed on heads. [3]

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Question 10 - working space

11. [Maximum mark: 6]

Toktam works at a local bakery 5 days each week. She drives an old car to work that has a 65% probability of starting on any given morning. The probability of the car starting on a given morning is independent of it starting on any other morning.

(a) Find the probability that Toktam's car starts on exactly two mornings in a particular 5 day workweek. [2]

Toktam walks to work on mornings when her car does not start and it is **not** raining.
Toktam takes a taxi to work on mornings when her car does not start and it is raining.

Where Toktam lives, there is a 45% probability of rain on any given morning, independent of any other morning. The probability of Toktam's car starting is independent of the weather.

(b) Find the probability that Toktam will **not** have to take a taxi in a particular workweek. [4]

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Question 11 - working space

12. [Maximum mark: 6]

Thurston believes that more popular musical artists sell more albums.

He begins to investigate this belief by randomly selecting eight musical artists and collecting data on the number of followers each of the artists has on a particular social media platform. He then collects data on the number of albums each artist sold in the first week after releasing an album. His data is shown in **Table 1**.

Table 1

	Artist 1	**Artist 2**	**Artist 3**	**Artist 4**	**Artist 5**	**Artist 6**	**Artist 7**	**Artist 8**
Number of social media followers (in thousands)	11 500	12 400	1300	2300	674	49 500	315	94 400
Number of albums sold in first week (in thousands)	123	62.4	17.4	94.9	52.5	27	21.6	595.5

Thurston decides to calculate the Spearman's rank correlation coefficient.

(a) Complete the table of ranks shown in **Table 2**. [1]

Table 2

	Artist 1	**Artist 2**	**Artist 3**	**Artist 4**	**Artist 5**	**Artist 6**	**Artist 7**	**Artist 8**
Rank – social media followers	4	3	6	5	7	2	8	1
Rank – albums sold in first week								1

(This question continues on the following page)

(Question 12 continued)

(b) Calculate the value of r_s, Spearman's rank correlation coefficient. [2]

Thurston believes that artists with a higher number of social media followers sell more albums in the first week. He carries out a hypothesis test using a 10% significance level with the following null hypothesis:

H_0: In the population, there is no monotonic relationship between the number of social media followers and the number of albums sold in the first week.

(c) Write down Thurston's alternative hypothesis. [1]

The critical value of r_s for this test is 0.643.

(d) State the conclusion of the hypothesis test, giving a reason. [2]

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References:

2. Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.

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Question 12 - working space

3. [Maximum mark: 18]

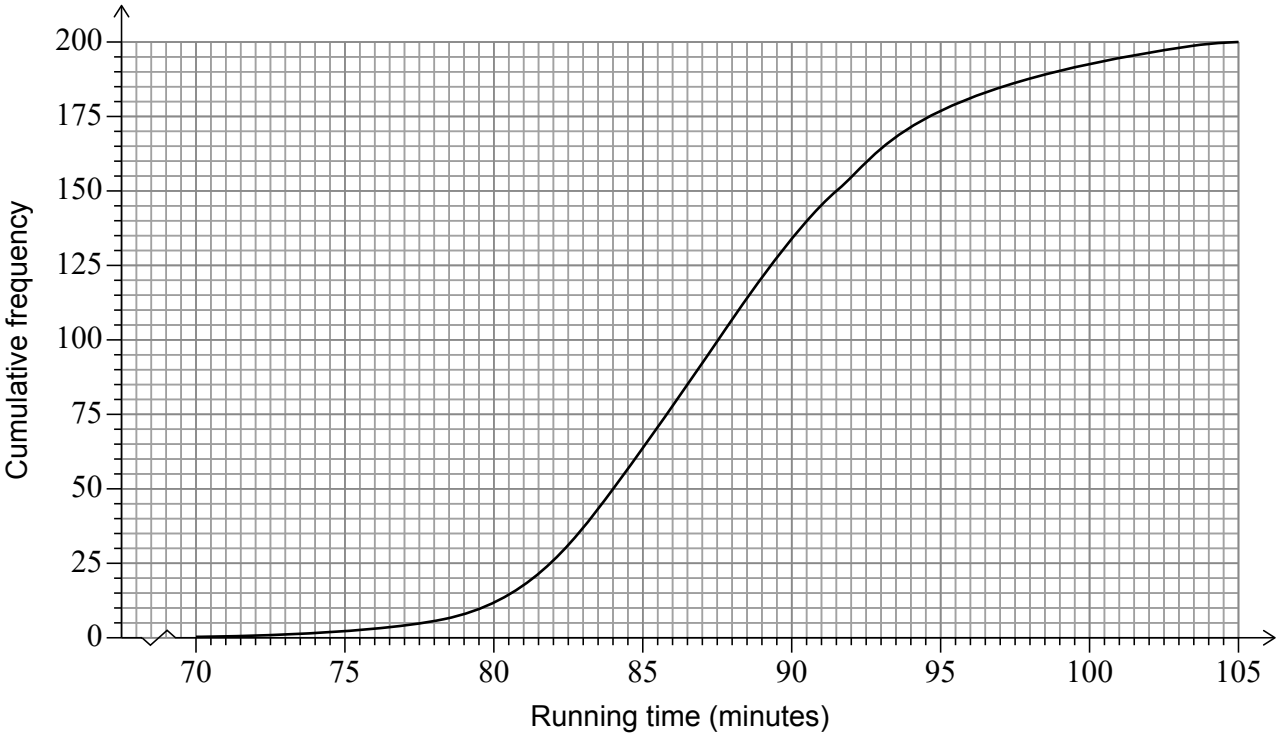
The running time, t (minutes), of 200 family movies are recorded in the following table.

Running time (minutes)	Frequency
$70 \le t < 80$	11
$80 \le t < 85$	51
$85 \le t < 90$	68
$90 \le t < 95$	47
$95 \le t < 105$	23

(a) (i) Write down the mid-interval value of $70 \le t < 80$.

(ii) Calculate an estimate of the mean running time of the 200 movies. [3]

This table is used to create the following cumulative frequency graph.



(b) Use the cumulative frequency curve to estimate the interquartile range. [2]

"Star Feud" is a movie in the data set and its running time is 100 minutes.

(c) Use your answer to part (b) to estimate whether "Star Feud's" running time is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

It is believed that the running times of family movies follow a normal distribution with mean 88 minutes and standard deviation 6.75 minutes.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 movies could have plausibly been drawn from an underlying distribution $\mathrm{N}(88,\ 6.75^2)$.

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Movie running time (minutes)	Observed frequency	Expected frequency
$t < 80$	11	23.6
$80 \leq t < 85$	51	42.1
$85 \leq t < 90$	68	a
$90 \leq t < 95$	47	46.7
$95 \leq t$	23	b

(e) (i) Find the value of a and the value of b.

(ii) Hence, perform the test to a 5% significance level, clearly stating the conclusion in context. [8]

Question 3 - working space

3. [Maximum mark: 18]

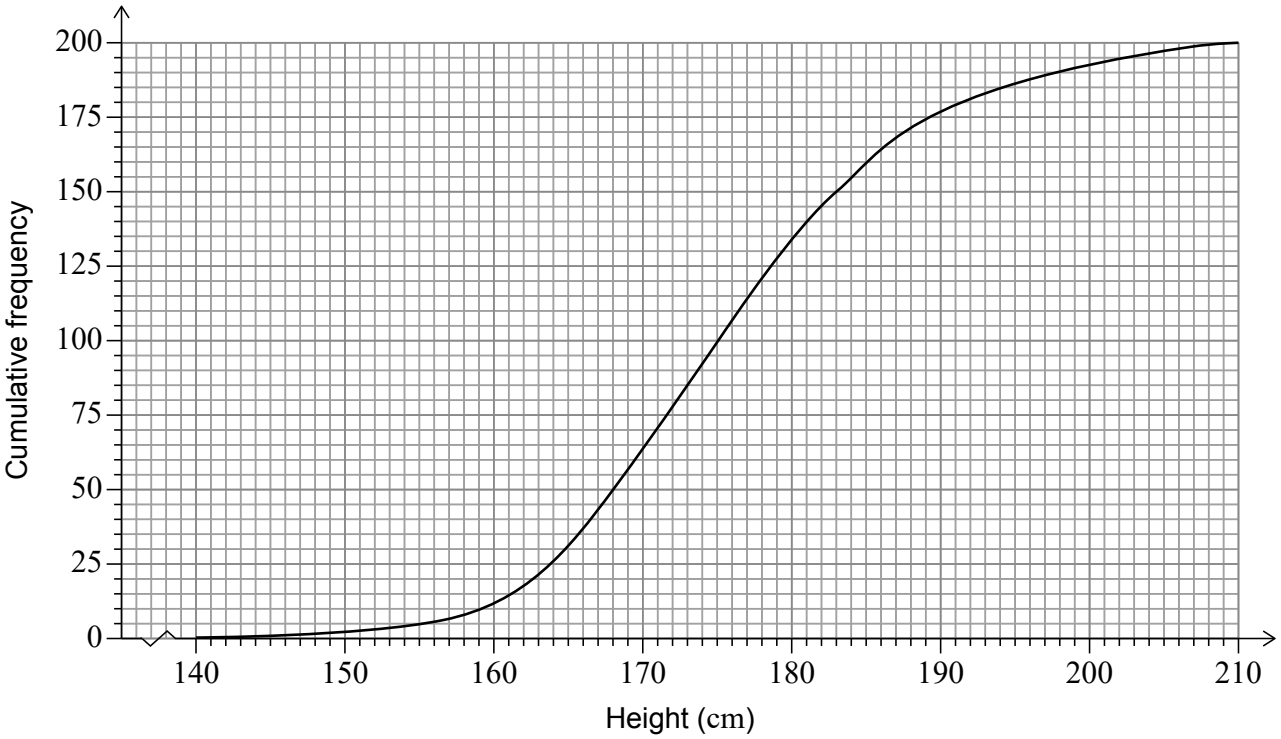
The heights, h, of 200 university students are recorded in the following table.

Height (cm)	Frequency
$140 \leq h < 160$	11
$160 \leq h < 170$	51
$170 \leq h < 180$	68
$180 \leq h < 190$	47
$190 \leq h < 210$	23

(a) (i) Write down the mid-interval value of $140 \leq h < 160$.

(ii) Calculate an estimate of the mean height of the 200 students. [3]

This table is used to create the following cumulative frequency graph.



(b) Use the cumulative frequency curve to estimate the interquartile range. [2]

Laszlo is a student in the data set and his height is $204\,\text{cm}$.

(c) Use your answer to part (b) to estimate whether Laszlo's height is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

It is believed that the heights of university students follow a normal distribution with mean $176\,\text{cm}$ and standard deviation $13.5\,\text{cm}$.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 students could have plausibly been drawn from an underlying distribution $\text{N}(176, 13.5^2)$.

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Height of student (cm)	Observed frequency	Expected frequency
$h < 160$	11	23.6
$160 \leq h < 170$	51	42.1
$170 \leq h < 180$	68	a
$180 \leq h < 190$	47	46.7
$190 \leq h$	23	b

(e) (i) Find the value of a and the value of b.

(ii) Hence, perform the test to a $5\,\%$ significance level, clearly stating the conclusion in context. [8]

Question 3 - working space

1. [Maximum mark: 7]

The following data show the heights, in metres, of six players in a basketball team.

1.67	1.60	1.68	2.31	2.31	2.19

(a) For these six players, find

(i) the mean height.

(ii) the median height.

(iii) the modal height.

(iv) the range of the heights. [6]

A new player, Gheorghe, joins the team. Their height is measured as 1.98 metres to the nearest centimetre.

(b) Write down the shortest possible height of Gheorghe. [1]

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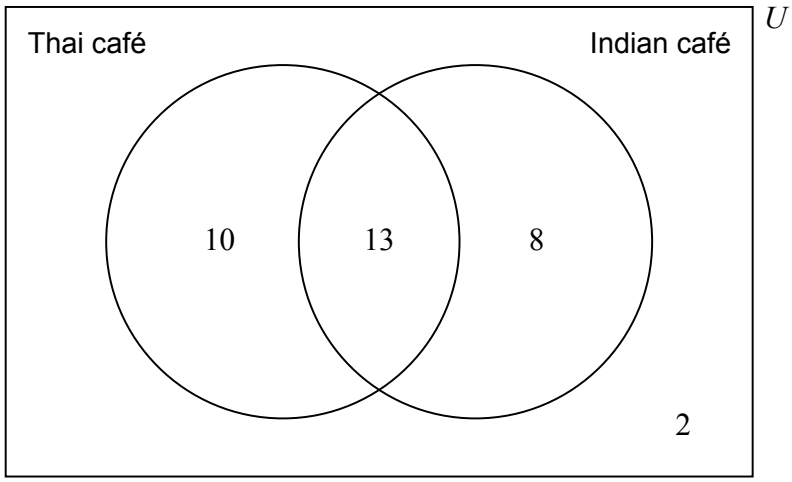
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Question 1 - working space

2. [Maximum mark: 6]

A teacher surveys their students to find out if they have eaten at the local Thai and Indian cafés. The results of the survey are shown in the following Venn diagram.



(a) Write down the number of students surveyed. [1]

(b) Write down the number of students who have not eaten at the Indian café. [1]

A student is chosen at random from those surveyed.

(c) Find the probability this student has eaten at both the Thai café and the Indian café. [1]

Let T be the event: a student has eaten at the Thai café.
Let I be the event: a student has eaten at the Indian café.

(d) Find $\mathrm{P}(T \cup I)$. [1]

(e) State whether the events T and I are mutually exclusive. Justify your answer. [2]

(This question continues on the following page)

(Question 2 continued)

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Question 2 - working space

8. [Maximum mark: 6]

The formula $F = 1.8C + 32$ is used to convert a temperature in degrees Celsius, C, to degrees Fahrenheit, F.

(a) (i) Find a formula for converting a temperature in degrees Fahrenheit to degrees Celsius.

(ii) Find the temperature in degrees Celsius that is recorded as 77 degrees Fahrenheit. [3]

Over one year, the mean daily temperature in Mexico City was calculated to be 17 degrees Celsius with a standard deviation of 9 degrees Celsius.

(b) For the same year, find in degrees Fahrenheit

(i) the mean daily temperature in Mexico City.

(ii) the standard deviation of the daily temperature in Mexico City. [3]

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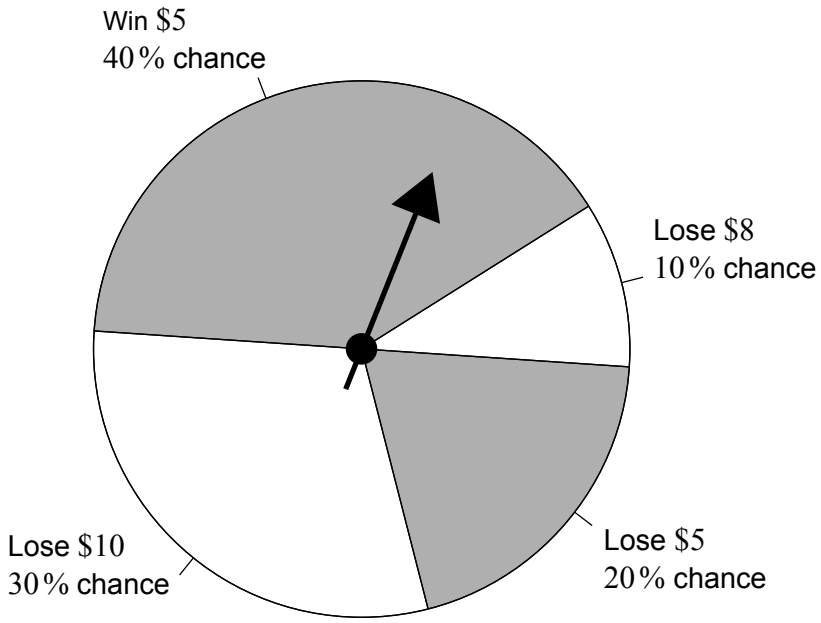
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Question 8 - working space

12. [Maximum mark: 8]

Zac raises funds for a library by running a game where players spin a needle. The final position of the needle results in an outcome where a player wins or loses money. The outcomes, with associated probabilities, are shown in the following diagram.

diagram not to scale



Let X represent the amount that a player of this game wins.

(a) (i) Find the expected value of X.

(ii) Interpret your answer to part (a)(i). [3]

To encourage a person to keep playing this game, Zac increases the winning prize for the second game they play from $\$5$ to $\$6$. For each successive game they play, the winning prize continues to increase by $\$1$.

Emily plays k games. The kth game is fair.

(b) (i) Find the value of k.

(ii) Explain why Zac expects to raise money from the games Emily plays. [5]

(This question continues on the following page)

(Question 12 continued)

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10. Saddako, n.d. *Cheetah (Acinonyx jubatus) Running - stock photo.* [image online] Available at: https://www.gettyimages.co.uk/detail/photo/cheetah-running-royalty-free-image/523244194?phrase=cheetah+speed&adppopup=true [Accessed 2 May 2023]. Source adapted.

GlobalP, n.d. *Lion, Panthera leo, 8 years old, standing - stock photo.* [image online] Available at: https://www.gettyimages.co.uk/detail/photo/lion-panthera-leo-8-years-old-standing-royalty-free-image/134976936?phrase=Lion+standing&adppopup=true [Accessed 2 May 2023]. Source adapted.

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Question 12 - working space

1. [Maximum mark: 7]

The prices, in dollars, of 10 different garden chairs are:

$$79 \quad 139 \quad 255 \quad 99 \quad 50 \quad 209 \quad 229 \quad 193 \quad 69 \quad 49$$

(a) Find the range of the prices of the 10 chairs. [2]

(b) Use your graphic display calculator to find

(i) the mean price of the chairs.

(ii) the standard deviation of the price of the chairs. [3]

In a sale, the price of each of the 10 garden chairs is reduced by $\$20$.

(c) Write down

(i) the new mean.

(ii) the new standard deviation. [2]

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Question 1 - working space

3. [Maximum mark: 5]

Sunita sorts 300 peppers into sizes of small, medium or large. Some peppers are red, some are green, and some are yellow.

The following table shows her results.

	Small	**Medium**	**Large**
Red	18	31	46
Green	26	32	21
Yellow	34	66	26

Sunita wants to test, at the 5% significance level, whether the size of the peppers is independent of the colour.

(a) State the null and alternative hypotheses for this test. [1]

The critical value for this test is 9.49.

(b) (i) Calculate χ^2_{calc}.

(ii) State a conclusion to the test. Give a reason for your answer. [4]

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Question 3 - working space

6. [Maximum mark: 6]

Jerry makes handcrafted chocolates. On average, 1 in 25 of the chocolates that Jerry makes is flawed. Whether or not a chocolate is flawed is independent of all other chocolates.

(a) In a batch of 20 chocolates, chosen at random, find the probability that

(i) two are flawed.

(ii) more than two are flawed. [4]

Jerry sells the perfect chocolates for 50 pesos each and the flawed ones for 15 pesos each.

(b) Calculate the expected number of pesos Jerry makes from selling a batch of 20 randomly selected chocolates. [2]

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Question 6 - working space

8. [Maximum mark: 7]

Gustav plays a game in which he first tosses an unbiased coin and then rolls an unbiased six-sided die.

If the coin shows tails, the score on the die is Gustav's final number of points.

If the coin shows heads, one is added to the score on the die for Gustav's final number of points.

(a) Find the probability that Gustav's final number of points is 7. [2]

(b) Complete the following table. [3]

Final number of points	1	2	3	4	5	6	7
Probability							

(c) Calculate the expected value of Gustav's final number of points. [2]

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Question 8 - working space

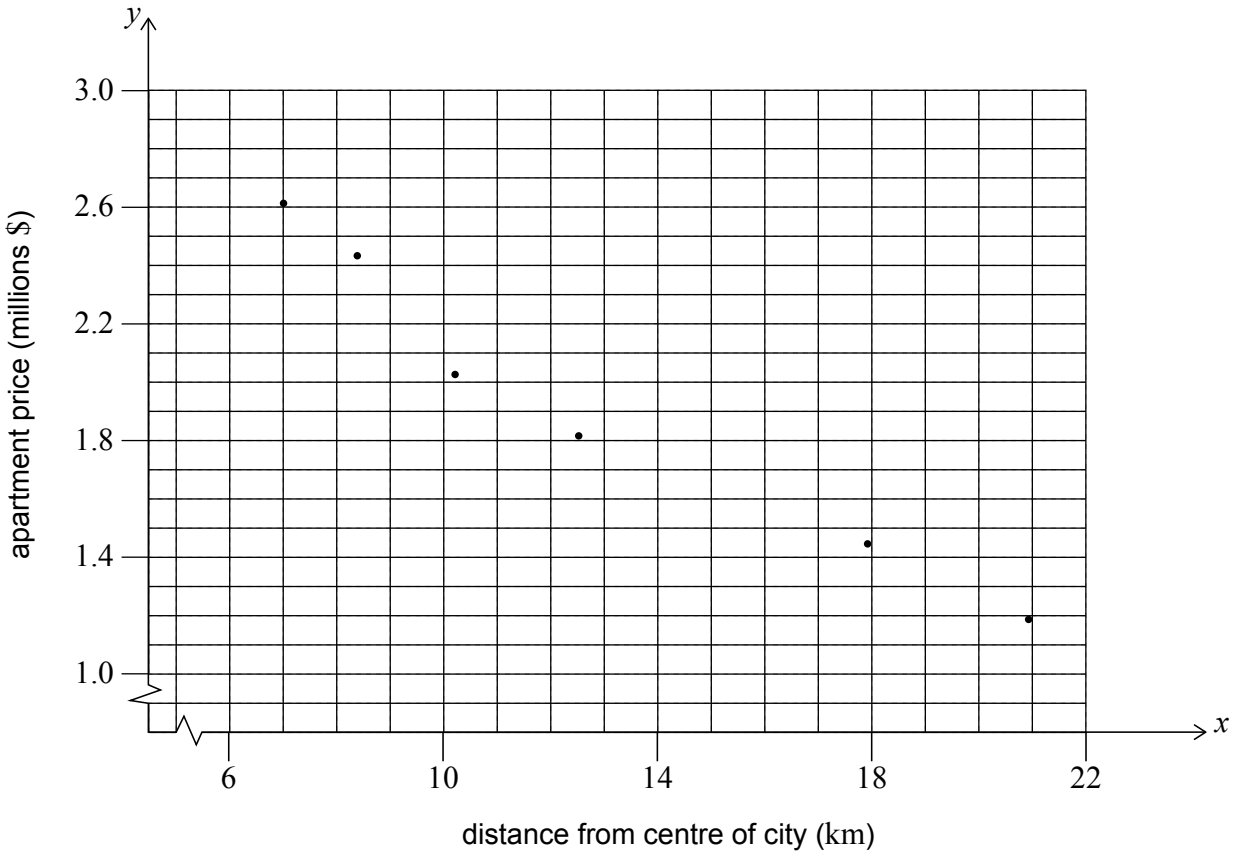
1. [Maximum mark: 19]

Xavie conducted a study to see if there is a relationship between the price of an apartment, y, and its distance, x, from the city centre of Melbourne.

They took a random sample of six typical apartments along a train line in the city. Xavie obtained the data shown in the following table.

x (kilometres)	7.0	8.4	10.3	12.5	17.8	20.9
y (millions of dollars)	2.61	2.44	2.03	1.81	1.45	1.18

A plot of these data is seen in the following graph.



(This question continues on the following page)

(Question 1 continued)

(a) Write down the value of the Spearman's rank correlation coefficient, r_s. [1]

(b) (i) Find the Pearson's product-moment correlation coefficient, r.

(ii) Use your value of r to state which **two** of the following would best describe the correlation between the variables.

Positive	Negative	Strong	Weak	No correlation

[4]

The relationship between the variables can be modelled by the regression equation $y = ax + b$.

(c) (i) Write down the value of a.

(ii) Write down the value of b.

(iii) According to this model, state in context what the value of b represents. [3]

(d) Xavie uses the regression equation to estimate the price of a typical apartment located $19.6\,\text{km}$ from the city centre.

(i) Find this estimated price.

(ii) State two reasons that Xavie might use to justify the validity of this estimate. [5]

(This question continues on page 5)

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(Question 1 continued)

To verify whether this relationship applies in a different direction from the city centre, Xavie considers two locations, A and B, both an equal distance from the city centre. They take a random sample of seven apartments from each location and record the prices (in millions of dollars) in the following tables.

Apartment price in location A						
1.21	1.25	1.31	1.32	1.58	1.95	2.13

Apartment price in location B						
1.51	1.58	1.69	2.61	2.72	2.81	2.95

Xavie conducts a t-test, at the 5 % level of significance, to see if the mean apartment price in location A is different to the mean apartment price in location B. They assume the population variances are the same.

For this test, Xavie takes the null hypothesis to be $\mu_A = \mu_B$.

(e) Write down the alternative hypothesis. [1]

(f) Find the p-value for this test. [2]

(g) State the conclusion of the test. Justify your answer. [2]

(h) State one additional assumption Xavie has made about the distributions to conduct this test. [1]

Question 1 - working space

2. [Maximum mark: 15]

The company Fred Express delivers packages. From past experience, the time taken, T, to deliver a package follows a normal distribution with mean 64 hours and standard deviation 12 hours.

(a) State $\mathrm{P}(T < 64)$. [1]

(b) Find $\mathrm{P}(44 < T < 64)$. [2]

$30\,\%$ of packages are delivered in less than k hours.

(c) (i) Sketch a diagram of this normal distribution, shading the region that represents $\mathrm{P}(T < k)$.

(ii) Find the value of k. [4]

For quality control, the manager randomly selects five outgoing packages. These selections are independent.

(d) Find the probability that exactly two of these packages are delivered in less than k hours. [3]

Fred Express charges a fixed amount of $\$4.50$ for any package weighing $1\,\mathrm{kg}$ or less. Heavier packages are charged an additional fee of $\$2.00$ per kg. This fee is applied for any weight **in excess of** $1\,\mathrm{kg}$. For example, a $1.5\,\mathrm{kg}$ package is charged an additional $\$1.00$.

(e) Write down an expression for the amount charged to deliver a package of weight $x\,\mathrm{kg}$, where $x > 1$. [2]

(f) Find the amount Fred Express charges for a $5.3\,\mathrm{kg}$ package. [1]

Meiling is charged $\$7.20$ for the delivery of a package.

(g) Find the weight of Meiling's package. [2]

Question 2 - working space

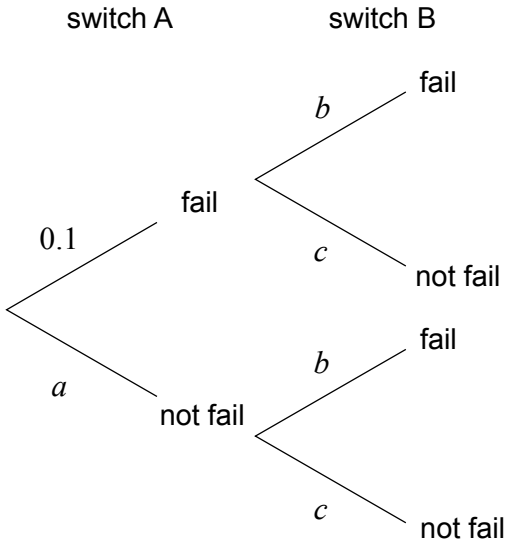
4. [Maximum mark: 12]

A type of generator will only function if a particular switch is working. The generator has a main switch, A, and a 'back up' switch, B.

The manufacturer claims the probability of switch A failing within one month of being fitted is 0.1 and the probability of the cheaper switch B failing within one month is 0.3. Whether or not a switch fails is independent of the state of the other switch.

If both switches fail, the generator needs to shut down to replace the switches. Both switches are replaced after a month of use (whether they have failed or not) or whenever the generator needs to be shut down.

The following tree diagram shows the probabilities of a switch failing within one month of them both being replaced, assuming the manufacturer's claim is correct.



(a) Write down the values of

(i) a.

(ii) b.

(iii) c. [2]

(b) Hence find the probability that the generator needs to shut down within one month of the switches being replaced. [1]

(This question continues on the following page)

(Question 4 continued)

The owner of the generator is suspicious of the switch manufacturer's claims, so they look back through the past 200 occasions when the switches were replaced. The records show whether no switches, one switch or two switches had failed.

The data the owner collected are shown in the following table.

No switch fails	One switch fails	Two switches fail
118	72	10

(c) Show that the expected value of no switches failing in the generator, during the last 200 occasions when the switches were replaced, is 126. [2]

(d) Perform a χ^2 goodness of fit test at the 5% significance level to test whether the manufacturer's claims are correct using the following hypotheses.

H_0: The manufacturer's claims are correct.
H_1: The manufacturer's claims are not both correct. [7]

Question 4 - working space

1. [Maximum mark: 16]

In a given week, the number of students in a particular primary school that were absent due to headlice (H), influenza (I) and/or chickenpox (C) were recorded as follows.

The primary school has 500 students.

35 students had headlice only
20 students had influenza only
5 students had chickenpox only
4 students had headlice and influenza but not chickenpox
2 students had headlice and chickenpox but not influenza
3 students had influenza and chickenpox but not headlice
1 student had headlice, influenza and chickenpox

(a) Draw a Venn diagram to represent this information. [3]

(b) Calculate the number of students who did not have headlice or influenza or chickenpox. [2]

A student is chosen at random from all the students in the school.

(c) Find the probability that this student has

(i) headlice.

(ii) influenza given that the student has headlice. [4]

(This question continues on the following page)

(Question 1 continued)

Diego is a teacher in the school. He believes that the number of students, n, who have had influenza during the first t days of the school year, can be modelled by the function

$$n(t) = 250 - 240(2)^{kt},\ k \in \mathbb{R}.$$

(d) Use Diego's model to calculate the number of students who started the school year with influenza. [2]

It is known that 130 students have had influenza during the first 10 days of the school year.

(e) Find the value of k. [2]

(f) Using this model, calculate how many days it will take for 200 students to have had influenza since the start of the school year. [2]

By the last day of the school year, it is known that 300 students have had influenza.

(g) Comment on the appropriateness of Diego's model. [1]

Question 1 - working space

4. [Maximum mark: 19]

A recent study found that the heights of Dutch women can be modelled by a normal distribution with mean $170.7\,\text{cm}$ and standard deviation $6.3\,\text{cm}$.

A Dutch woman is chosen at random.

(a) Calculate the probability that her height is

(i) less than $160\,\text{cm}$.

(ii) between $160\,\text{cm}$ and $170\,\text{cm}$. [4]

$27\,\%$ of Dutch women have a height of more than h metres.

(b) Calculate the value of h. [2]

Janneke selects a random sample of 200 Dutch women from Amsterdam and measures their heights. She wants to determine whether this sample could have been chosen from a normally distributed population with mean of $170.7\,\text{cm}$ and standard deviation of $6.3\,\text{cm}$.

She performs a χ^2 goodness of fit test at the $5\,\%$ significance level. She begins by creating the following frequency table.

Height, h	Observed frequency	Expected frequency
$h \leq 160$	6	8.943
$160 < h \leq 170$	85	a
$170 < h \leq 180$	92	b
$h > 180$	17	13.99

(c) Calculate, correct to four significant figures, the value of

(i) a.

(ii) b. [2]

(This question continues on the following page)

(Question 4 continued)

The hypotheses for Janneke's test are

H_0: the heights are drawn from a normally distributed population with mean $170.7\,\text{cm}$ and standard deviation $6.3\,\text{cm}$

H_1: the heights are not drawn from a normally distributed population with mean $170.7\,\text{cm}$ and standard deviation $6.3\,\text{cm}$

(d) Write down the degrees of freedom for this test. [1]

The critical value for this test is 7.815.

(e) Perform the χ^2 goodness of fit test and state your conclusion, justifying your reasoning. [4]

Gundega claims that, on average, Latvian women are taller than Dutch women.

Random samples of 10 Latvian women and 10 Dutch women are chosen, and their heights are measured.

Heights of Latvian women (cm)									
171	163	180	159	169	182	166	168	171	170

Heights of Dutch women (cm)									
173	182	181	166	175	161	169	165	172	169

Gundega performs a t-test at the $5\,\%$ significance level. It is assumed that the populations are normally distributed and have equal variances.

(f) Write down the null and alternative hypotheses for this test. [2]

(g) Perform the t-test and state the conclusion, justifying your reasoning. [4]

Question 4 - working space

2. [Maximum mark: 7]

The scores on a test, out of 7 points, for 240 students are shown in the following table.

Score	1	2	3	4	5	6	7
Frequency	11	29	31	34	65	47	23

(a) Find the mean **and** standard deviation of these test scores. Give your answers correct to four significant figures. [3]

(b) The scores are multiplied by ten. Write down the new mean and standard deviation. [2]

(c) After the scores have been multiplied by ten, 30 points are added to each of them. Write down the new mean and standard deviation. [2]

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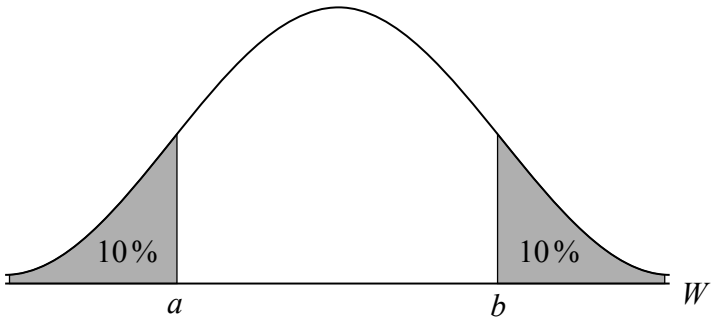
Question 2 - working space

7. [Maximum mark: 7]

The mass, W, of Scottish Terriers (a breed of dog) is normally distributed with a mean of $8.9\,\text{kg}$ and a standard deviation of $1.2\,\text{kg}$.

(a) A Scottish Terrier is selected at random. Calculate the probability this dog's mass is more than $7.2\,\text{kg}$. [2]

The following curve represents this distribution. It is known that $\text{P}(W < a) = 0.1$ and $\text{P}(W > b) = 0.1$.



(b) Find the value of

(i) a

(ii) b. [3]

(c) Two Scottish Terriers are selected at random from a large population. Find the probability that both dogs have a mass less than $7.2\,\text{kg}$. [2]

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Question 7 - working space

9. [Maximum mark: 7]

The records at a driving school show that 60% of students pass their driving test on the first attempt.

A group of 30 students take their driving test for the first time.

As part of its quality control, the driving school uses the model $X \sim \mathrm{B}(30, 0.60)$, where X is the number of students who pass the driving test.

(a) Calculate the

(i) mean of X

(ii) variance of X. [2]

(b) Find the probability that

(i) exactly 21 students pass the test

(ii) fewer than 12 students pass the test. [4]

(c) State one assumption that the driving school makes in using this model. [1]

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Question 9 - working space

11. [Maximum mark: 7]

A fair game is played where points are scored as follows:

- A win scores w points.
- A draw scores 0 points.
- A loss scores -7 points.

Let X be the number of points scored during a game. The probability distribution is shown.

x	w	0	-7
$\mathrm{P}(X = x)$	0.35	0.4	p

(a) Find the value of p. [2]

The game is played 60 times.

(b) Find the expected number of losses. [2]

(c) Calculate the value of w, given that the game is fair. [3]

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Question 11 - working space

2. [Maximum mark: 7]

The scores on a test, out of 7 points, for 220 students are shown in the following table.

Score	1	2	3	4	5	6	7
Frequency	11	24	26	35	61	43	20

(a) Find the mean **and** standard deviation of these test scores. Give your answers correct to four significant figures. [3]

(b) The scores are multiplied by ten. Write down the new mean and standard deviation. [2]

(c) After the scores have been multiplied by ten, 30 points are added to each of them. Write down the new mean and standard deviation. [2]

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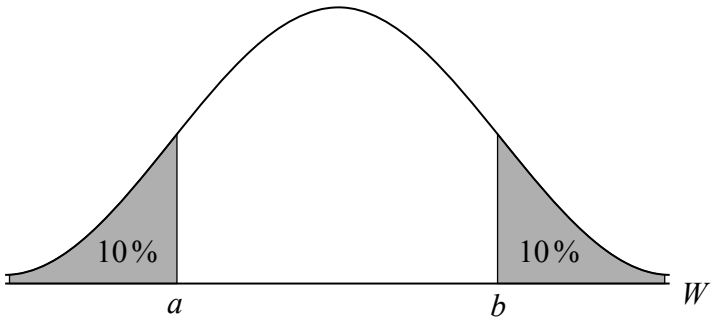
Question 2 - working space

7. [Maximum mark: 7]

The mass, W, of Manx cats is normally distributed with a mean of $4.5\,\text{kg}$ and a standard deviation of $0.4\,\text{kg}$.

(a) A Manx cat is selected at random. Calculate the probability this cat's mass is more than $3.5\,\text{kg}$. [2]

The following curve represents this distribution. It is known that $\text{P}(W < a) = 0.1$ and $\text{P}(W > b) = 0.1$.



(b) Find the value of

(i) a

(ii) b. [3]

(c) Two Manx cats are selected at random from a large population. Find the probability that they both have a mass less than $3.5\,\text{kg}$. [2]

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Question 7 - working space

9. [Maximum mark: 7]

The records at a driving school show that 55% of students pass their driving test on the first attempt.

A group of 20 students take their driving test for the first time.

As part of its quality control, the driving school uses the model $X \sim \mathrm{B}(20, 0.55)$, where X is the number of students who pass the driving test.

(a) Calculate the

(i) mean of X

(ii) variance of X. [2]

(b) Find the probability that

(i) exactly 14 students pass the test

(ii) fewer than 5 students pass the test. [4]

(c) State one assumption that the driving school makes in using this model. [1]

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Question 9 - working space

11. [Maximum mark: 7]

A fair game is played where points are scored as follows:

- A win scores w points.
- A draw scores 0 points.
- A loss scores -5 points.

Let X be the number of points scored during a game. The probability distribution is shown.

x	w	0	-5
$\mathrm{P}(X = x)$	0.25	0.4	p

(a) Find the value of p. [2]

The game is played 60 times.

(b) Find the expected number of losses. [2]

(c) Calculate the value of w, given that the game is fair. [3]

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Question 11 - working space

3. [Maximum mark: 14]

A survey was answered by $20\,000$ expatriates (people living in a country that is not their own). The data ranked countries in order of the country they felt was best for expatriates. The highest-ranked country was Switzerland.

These results were compared to happiness scores taken from *The World Happiness Report 2022*. The following table shows this data for the top 10 expatriate countries.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Expatriate country rank	1	2	3	4	5	6	7	8	9	10
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4

(a) For the **happiness score**, find

(i) the upper quartile

(ii) the interquartile range. [4]

(b) Show that Switzerland's happiness score is not an outlier for this data. [3]

(This question continues on the following page)

(Question 3 continued)

The happiness scores were ranked to calculate Spearman's rank correlation coefficient, r_s. These ranks are shown in the following table.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4
Happiness rank	1	a	b	c	9	10	5	7	6	2

(c) Write down the value of

(i) a

(ii) b

(iii) c. [3]

(d) (i) Find r_s.

(ii) If France's happiness score is upgraded to 6.9, explain why the value of r_s does not change. [3]

Jose concludes from this data that countries with high happiness scores are likely to be favourite expatriate countries.

(e) State, with a reason, whether Jose's conclusion is appropriate. [1]

Question 3 - working space

3. [Maximum mark: 14]

A survey was answered by $20\,000$ expatriates (people living in a country that is not their own). The data ranked countries in order of the country they felt was best for expatriates. The highest-ranked country was Switzerland.

These results were compared to happiness scores taken from *The World Happiness Report 2022*. The following table shows this data for the top 10 expatriate countries.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Expatriate country rank	1	2	3	4	5	6	7	8	9	10
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4

(a) For the **happiness score**, find

(i) the upper quartile

(ii) the interquartile range. [4]

(b) Show that Switzerland's happiness score is not an outlier for this data. [3]

(This question continues on the following page)

(Question 3 continued)

The happiness scores were ranked to calculate Spearman's rank correlation coefficient, r_s. These ranks are shown in the following table.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4
Happiness rank	1	a	b	c	9	10	5	7	6	2

(c) Write down the value of

(i) a

(ii) b

(iii) c. [3]

(d) (i) Find r_s.

(ii) If France's happiness score is upgraded to 6.9, explain why the value of r_s does not change. [3]

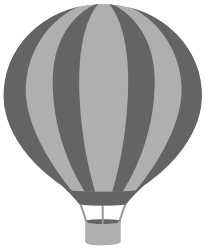
Jose concludes from this data that countries with high happiness scores are likely to be favourite expatriate countries.

(e) State, with a reason, whether Jose's conclusion is appropriate. [1]

Question 3 - working space

1. [Maximum mark: 6]

Hot air balloons are available in many sizes for different numbers of passengers.



The following table shows the recommended minimum volume of a hot air balloon, in cubic metres, for a specific number of passengers.

Number of Passengers (x)	1	2	4	5	7	15
Recommended Minimum Volume (y)	1000	1350	2225	2500	3150	5800

(a) Use your graphic display calculator to find the Pearson's product-moment correlation coefficient, r, for these values. [2]

(b) (i) Find the equation of the regression line y on x for this data in the form $y = ax + b$.

(ii) State what the value of a means in the context of the question. [2]

(c) Use your regression equation from part (b)(i) to find the recommended minimum volume of a balloon for 10 passengers. [2]

(This question continues on the following page)

(Question 1 continued)

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Question 1 - working space

2. [Maximum mark: 5]

A car manufacturer collected data about preferred car colour from drivers in various age groups. The results are presented in the following table.

	Preferred car colour				
	White	**Black**	**Silver**	**Red**	**Total**
18 ≤ Age < 25	12	7	4	17	40
25 ≤ Age < 45	15	b	10	12	58
Age ≥ 45	12	18	16	6	52
Total	39	46	30	35	150

(a) Write down the value of b. [1]

The car manufacturer performs a χ^2 test for independence, at the 1% significance level, to determine if there is significant evidence that different age groups have different car colour preferences.

The null hypothesis and alternative hypothesis are defined as:

H_0: age and car colour preference are independent.

H_1: age and car colour preference are not independent.

The χ^2 critical value for this test is 16.81.

(b) Find the χ^2 test statistic. [2]

(c) Write down the conclusion to the test, in context. Give a reason for your answer. [2]

(This question continues on the following page)

(Question 2 continued)

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Question 2 - working space

7. [Maximum mark: 5]

Children inherit genes from their biological parents that determine their eye colour.

Maria and Alex plan to have 5 children. The probability that a child born to Maria and Alex will have brown eyes is 0.75. It is assumed that a child's eye colour is independent of any other child's eye colour.

Maria and Alex wish to predict how many of their 5 children will have brown eyes.

(a) State one criterion, in addition to independence, that would support them using a binomial distribution to find probabilities for their prediction. [1]

(b) Calculate the probability that

(i) exactly 3 of the 5 children will have brown eyes.

(ii) at least 4 of the 5 children will have brown eyes. [4]

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Question 7 - working space

2. [Maximum mark: 8]

Two unbiased dice, each with faces numbered from 1 to 6 inclusive, are rolled. The numbers on the uppermost faces of the dice are noted.

Let the random variable X be the difference between the numbers on the dice.

(a) (i) Find $P(X = 0)$.

(ii) Find $P(X = 5)$. [2]

(b) Complete the table to show the probability distribution of X. [2]

The probability that $X = 3$ is shown.

x	0	1	2	3	4	5
$P(X = x)$				$\frac{6}{36}$		

(c) Calculate $\mathrm{E}(X)$. [2]

(d) Given that the difference between the numbers on the dice is an odd number, find the probability that $X = 5$. [2]

(This question continues on the following page)

(Question 2 continued)

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Question 2 - working space

4. [Maximum mark: 8]

Jane tested three different flavours of pet food on cats and dogs to determine whether the preferred flavour was independent of the type of animal.

The results are given in the table.

	Preferred flavour		
	Beef	Chicken	Fish
Cat	8	22	20
Dog	16	25	9

Perform a χ^2 test at both the 5% and 1% levels to investigate the results.

(a) State the null and alternative hypotheses. [1]

(b) Write down the number of degrees of freedom. [1]

(c) Calculate the p-value for the test. [2]

(d) State the conclusion to the test, giving your reason

(i) at the 1% level.

(ii) at the 5% level. [3]

Jane's conclusion to her test was that the preferred flavour was not independent of the type of animal.

(e) State which of the two significance levels Jane used in her test. [1]

(This question continues on the following page)

(Question 4 continued)

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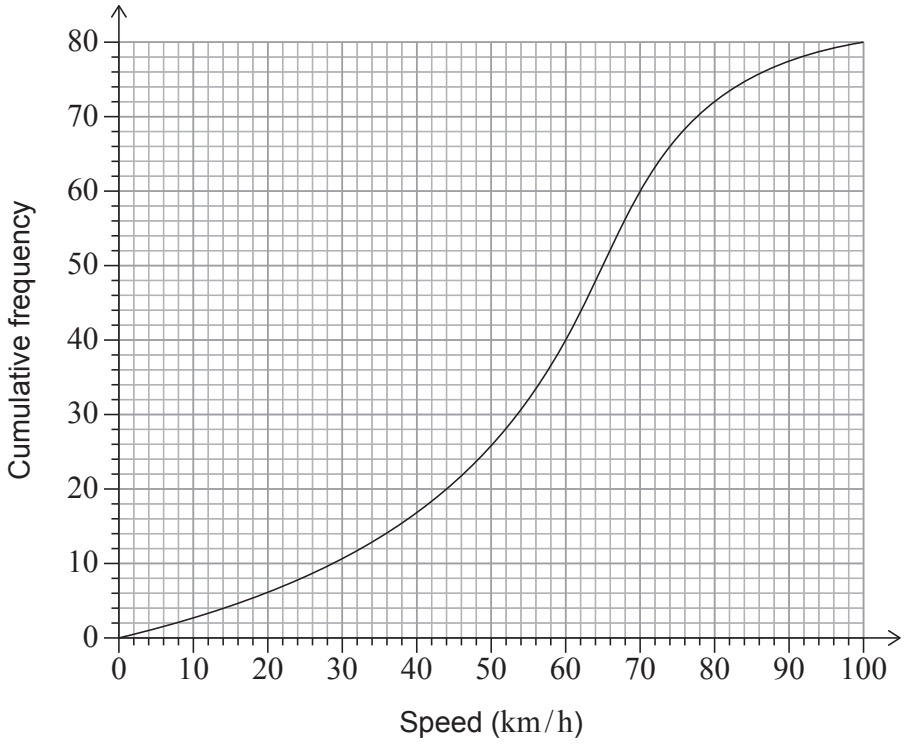
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Question 4 - working space

8. [Maximum mark: 8]

The cumulative frequency curve shows the speeds in kilometres per hour of 80 cars along a road.



(a) Calculate the percentage of these cars that exceed the speed limit of $80\,\text{km/h}$. [2]

Consider the grouped frequency table constructed using the cumulative frequency curve.

Speed $(x)\,(\text{km/h})$	$0 < x \leq p$	$p < x \leq m$	$m < x \leq q$	$q < x \leq 100$
Number of cars	20	20	20	20

(b) Find the value of

(i) p.

(ii) m.

(iii) q. [3]

(c) Hence calculate an estimate of the mean speed of these cars. [3]

(This question continues on the following page)

(Question 8 continued)

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Question 8 - working space

10. [Maximum mark: 6]

Two judges, Brett and Clarence, rank the skill levels of eight sheepdogs in a competition. The sheepdogs are labelled A to H and the judges rank the dogs as shown in the table.

Rank	1	2	3	4	5	6	7	8
Brett	A	C	D	B	E	F	G	H
Clarence	A	B	D	C	E	G	F	H

(a) Write down the rank that Brett awards sheepdog B. [1]

(b) Calculate Spearman's rank correlation coefficient for these data. [4]

(c) Comment on your answer to part (b) in terms of the ranks awarded by Brett and Clarence. [1]

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Question 10 - working space

2. [Maximum mark: 5]

The monthly energy consumption, x, in kilowatt hours (kWh) of 150 households in Helvetia is shown in the following table.

Energy consumption (x kWh)	Number of households
$500 \leq x < 700$	38
$700 \leq x < 900$	45
$900 \leq x < 1100$	25
$1100 \leq x < 1300$	18
$1300 \leq x < 1500$	16
$1500 \leq x < 1700$	8

(a) For this data set, use the mid-point interval values to find estimates for

(i) the mean.

(ii) the standard deviation. [4]

The standard deviation of the monthly energy consumption in another nearby residential area, Eureka, is found to be $95\,\text{kWh}$.

(b) Interpret the meaning of the value of the standard deviation in Eureka in comparison with the standard deviation in Helvetia. [1]

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Question 2 - working space

8. [Maximum mark: 7]

Prakash is the leader of a customer service team and is interested in determining whether there is a relationship between a customer's satisfaction level and the type of service interaction they have experienced.

He collects data from a random sample of 250 customers and tracks their satisfaction level after three types of service interactions: in-person, online chat bots and website contact forms.

He categorizes the satisfaction levels as satisfied, neutral and dissatisfied.

He records the data in the following table.

		Satisfaction level		
		Satisfied	Neutral	Dissatisfied
Type of service interaction	In-person	35	30	23
	Online chat bots	31	39	23
	Website contact forms	19	28	22

Prakash performs a χ^2 test for independence at the 5% significance level.

The critical value is 9.488.

The null hypothesis, H_0, is the satisfaction level and the type of service interaction are independent.

(a) State the alternative hypothesis for this test. [1]

(b) Find the degrees of freedom for this test. [1]

(c) Find χ^2_{calc}, the chi-squared test statistic. [2]

Prakash concludes that there is sufficient evidence to reject the null hypothesis.

(d) (i) State whether Prakash is correct. Justify your answer.

(ii) Write down the conclusion for this test in context. [3]

(This question continues on the following page)

(Question 8 continued)

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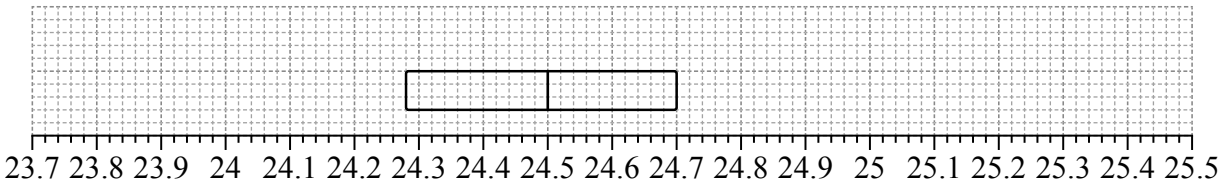
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Question 8 - working space

10. [Maximum mark: 7]

The times, in seconds, for the fastest 16 women in a $50\,\text{m}$ freestyle swimming championship event were recorded. All swimmers recorded different times.

Part of a box and whisker diagram for these times is shown in the following diagram.



Swimming times in seconds

(a) Write down the number of swimmers who took more than 24.70 seconds to complete the race. [1]

(b) Find the interquartile range (IQR) for the data. [2]

An outlier is defined as a value that satisfies one of the following:

- more than $1.5 \times \text{IQR}$ below the lower quartile
- more than $1.5 \times \text{IQR}$ above the upper quartile.

Of the 16 women, the two fastest swimmers took 23.96 and 24.12 seconds and the two slowest women took 25.12 and 25.40 seconds to complete the race.

(c) (i) Show that only one of these times is an outlier.

(ii) Complete the box and whisker diagram above. [4]

(This question continues on the following page)

(Question 10 continued)

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Question 10 - working space

12. [Maximum mark: 7]

A team of researchers is using a model to predict the relative happiness of different countries. To do this, a value x is calculated based on easily measured parameters, for example, life expectancy, or available social support. It is assumed that higher values of x indicate greater happiness.

To test the model a survey is conducted in six countries, A, B, C, D, E and F. In these countries the level of happiness is assessed directly using questionnaires and given a score y, out of 10, with higher scores indicating greater happiness.

To select the countries for the survey, all countries are divided into three equal groups based on wealth and two countries are chosen randomly from each group.

(a) Write down the name of this type of sampling. [1]

The results of the survey, along with the value obtained from the model, are given in the following table.

Country	A	B	C	D	E	F
Value from the model (x)	12.3	15.2	14.1	18.5	20.1	19.2
Happiness score (y)	5.2	7.3	6.2	6.9	8.0	7.2

The researchers will accept the model is a valid predictor of happiness score if the Pearson's product-moment correlation coefficient, r, is greater than 0.8.

(b) (i) Find the value of r.

(ii) Hence state whether the model can be regarded as a valid predictor of happiness score. [3]

(c) Find the equation of the regression line y on x. [1]

For a particular country $x = 17.2$.

(d) Use the regression line to predict the happiness score for this country. [2]

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(Question 12 continued)

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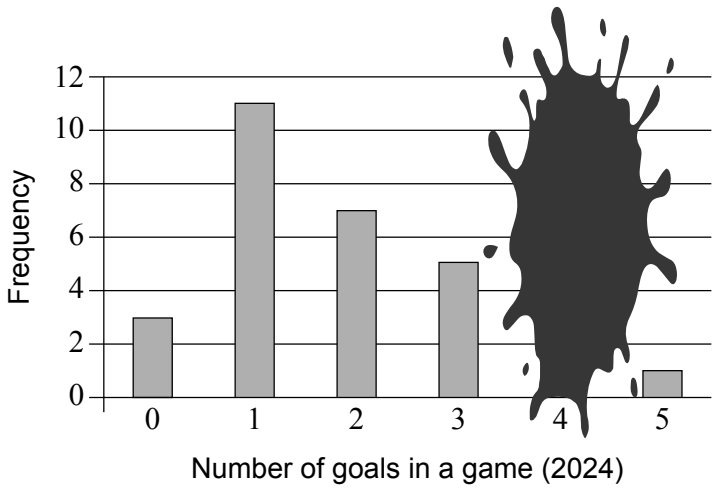
Question 12 - working space

1. [Maximum mark: 15]

Paul has a bar graph for the total number of goals scored in each game of a soccer tournament in 2024. The bar graph is shown below, however the frequency of 4 goals in a game is unreadable.

Paul uses this bar graph to create a frequency table.

Bar graph



Frequency table

Number of goals in a game (2024)	Frequency
0	3
1	11
2	7
3	k
4	p
5	1

(a) Write down the value of k. [1]

Paul knows that the mean number of goals per game scored during the tournament was 2.2.

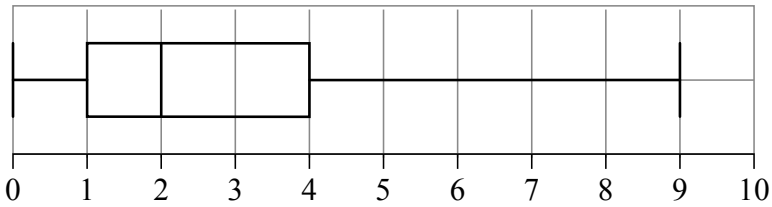
(b) (i) Write down an equation for the mean in terms of p.

(ii) Determine the value of p. [3]

(This question continues on the following page)

(Question 1 continued)

Data for the number of goals per game in the **2025 soccer tournament** are shown in the following box and whisker diagram.



Number of goals in a game (2025)

After comparing the box and whisker diagram from the 2025 tournament with the frequency table from the 2024 tournament, Paul concludes that the distribution of goals is consistent between the two tournaments.

(c) State two observations that support Paul's conclusion using values from the data to compare any **two** of:

range, symmetry, median, and interquartile range. [3]

Paul plans to watch all the games from the **2024** tournament in a random order.
He will watch each game once.

For the first game he watches, he defines event F as:

"scoring either 0 goals or exactly 1 goal".

(d) Write down the event(s) from the table that are equivalent to F'. There may be more than one correct event. [2]

Event	Description
A	Scoring exactly 2 goals in this game
B	Scoring more than 1 goal in this game
C	Scoring at least 2 goals in this game
D	Scoring either 0 goals or exactly 1 goal in all games except this game
E	Not scoring 0 goals or exactly 1 goal in any game

(e) If exactly 1 goal was scored in the first game Paul watches, write down the probability that exactly 1 goal was scored in the second game he watches. Give your answer as a fraction. [2]

(f) Calculate the probability that 5 goals were scored in the first game that Paul watches **and** 0 goals were scored in the second game he watches. [4]

Question 1 - working space

3. [Maximum mark: 20]

According to the timetable, the evening train from Cardiff arrives in London at $6{:}00$ pm. Records show that the arrival times for this train are normally distributed with a mean arrival time of $6{:}20$ pm and a standard deviation of 10 minutes.

Let X be the random variable "the number of minutes late the train arrives".

(a) Write down

(i) the mean of X.

(ii) the variance of X. [2]

(b) Find the probability that the evening train arrives in London between 15 and 25 minutes late. [2]

If the train arrives more than 30 minutes late, the train company must pay a fine.

(c) Calculate the expected number of days, in a 365-day year, that the train company pays a fine. [3]

Janine arrives in London on the evening train from Cardiff. She then has to catch a connecting train.

She allows 10 minutes to change platforms at the station.

She requires the minimum probability of arriving at the platform to catch the connecting train to be 99%.

(d) Find the earliest time, correct to the nearest minute, that the connecting train could depart to meet Janine's requirements. [4]

David, an employee of the train company, is concerned about the number of trains being late.

He spends 7 days in London noting the arrivals of trains.

It can be assumed that train arrivals are independent.

(e) Find the probability that the Cardiff to London evening train arrives late

(i) on the first day that David is in London.

(ii) on 5 of the 7 days that David is in London.

(iii) on at least 5 consecutive days that David is in London. [9]

Turn over

Question 3 - working space

2. [Maximum mark: 17]

A company produces electronic components on a large scale. They carry out quality control tests to determine whether the components meet the company's standards.

Zaakir, the owner of the company, wants the quality control team to analyse the distribution of the weights of the components.

Based on historical data, the quality control team knows that the weights of the components follow a normal distribution with a mean of 2.5 grams and a standard deviation of 0.15 grams.

(a) Find the probability that the weight of a component selected at random is greater than 2.8 grams. [2]

The probability that the weight of a component selected at random is greater than w grams is 0.8.

(b) (i) Sketch a diagram of a normal curve to show the area represented by this probability.

(ii) Find the value of w. [4]

To pass Test 1, the weight of a component must be between 2.3 grams and 2.7 grams.

(c) (i) Find the probability that a randomly selected component passes Test 1.

(ii) Find the expected number of components in a box of 200 that will pass Test 1. [3]

Zaakir asks the quality control team to conduct a more in-depth analysis by performing a new test, Test 2. The probability of a component passing Test 2 is 0.95. The team randomly selects one box and tests each of the 200 components.

(d) Find the probability that exactly 190 components pass Test 2. [2]

(e) Find the probability that at least 188 components pass Test 2. [2]

Instead of testing all 200 components, Zaakir now decides to test a random sample of 12 components from a box of 200 components. He decides that the box will only be dispatched if at least 10 of the 12 components pass both Test 1 and Test 2. The results of Test 1 and Test 2 are independent.

(f) Find the probability that the box is dispatched. [4]

Turn over

Question 2 - working space

4. [Maximum mark: 7]

Under controlled driving conditions, Jacob investigated the fuel efficiency of his car when using premium fuel compared to standard fuel.

Jacob recorded the distance travelled per litre (km L^{-1}) using standard fuel for six days and then using premium fuel for seven days. This information is shown in the following tables.

	Standard fuel						
Distance travelled per litre (km L^{-1})	8.1	8.2	7.9	8.2	7.9	8.0	

	Premium fuel						
Distance travelled per litre (km L^{-1})	8.3	8.4	8.1	8.3	8.2	8.0	8.2

At the 5% significance level, Jacob performs a t-test to determine whether there is sufficient evidence that his car travels further using premium fuel compared to standard fuel.

(a) State one mathematical assumption made for this test to be valid. [1]

(b) Write down the

(i) null hypothesis;

(ii) alternative hypothesis. [2]

(c) (i) Find the p-value.

(ii) State your conclusion to the test in context. Justify your answer. [4]

(This question continues on the following page)

(Question 4 continued)

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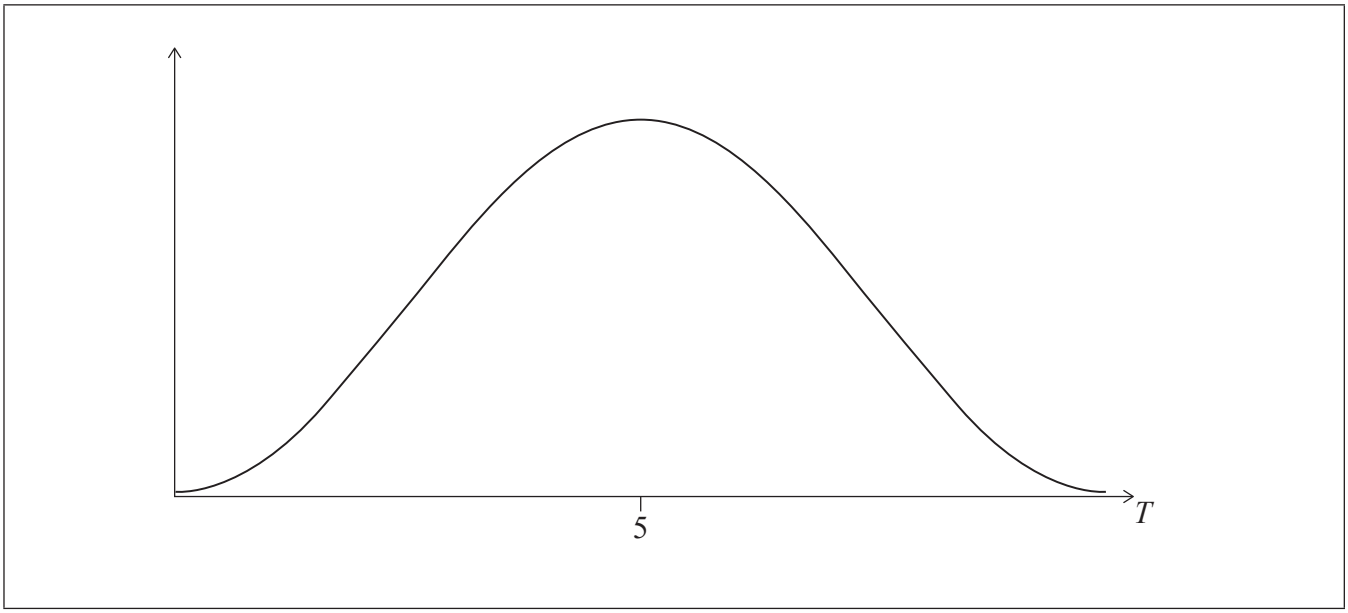
Question 4 - working space

7. [Maximum mark: 5]

In a reaction test, the time, T seconds, taken to complete the test can be modelled by a normal distribution with a mean of 5 seconds and a standard deviation of 1 second.

(a) Find the probability that a randomly selected person completes the reaction test in less than 4 seconds. [2]

The graph of the distribution of T is shown in the following diagram.



The slowest 15% of participants will be given training to reduce their time to complete the test.

(b) (i) Shade the region on the given diagram that corresponds to these participants.

(ii) Find the least time taken to complete the test by a person who will be given training. [3]

(This question continues on the following page)

(Question 7 continued)

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Question 7 - working space

9. [Maximum mark: 7]

On a television quiz show, a contestant is asked **ten questions**. Each question is multiple choice with four possible options, only one of which is correct.

Xiaoyi is a contestant and randomly guesses each answer. Her answers to the questions are independent of each other.

(a) Find the probability that Xiaoyi

(i) answers exactly five questions correctly;

(ii) answers at least three questions correctly. [4]

Xiaoyi answers all ten questions correctly and is offered the choice of two prizes:

- **Option A:** $\$50\,000$ in cash
- **Option B:** Toss a biased two-sided coin where the probability of showing heads is 0.3 and the probability of showing tails is 0.7. If the outcome is heads, Xiaoyi will win $\$500\,000$. If the outcome is tails, Xiaoyi will receive no money.

(b) Show that the expected value of Xiaoyi's winnings if she chooses **Option B** is $\$150\,000$. [2]

(c) State, giving a reason, whether Xiaoyi should choose **Option A** or **Option B**. [1]

(This question continues on the following page)

(Question 9 continued)

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Question 9 - working space

1. [Maximum mark: 5]

Consider the following list of 10 data items.

$$16 \quad 19 \quad 21 \quad 19 \quad 18 \quad 12 \quad 16 \quad 21 \quad 21 \quad 32$$

This data has a mean of 19.5.

Find the value of the

(a) mode. [1]

(b) median. [1]

(c) standard deviation. [1]

(d) range. [2]

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Question 1 - working space

3. [Maximum mark: 6]

Abigail is planning an end-of-school celebration. She asks all her classmates to choose both a main course and an ice cream flavour.

She organizes the replies from her classmates into the following table.

		Main course			
		Burgers	**Chicken skewers**	**Salad**	**Pasta**
Ice cream flavour	**Chocolate**	15	10	12	16
	Strawberry	11	14	13	12
	Vanilla	10	21	11	5

(a) Find the total number of classmates who replied. [1]

Abigail performs a χ^2 test for independence at the 5% significance level to see if the main course choice is independent of the ice cream flavour choice.

(b) Write down the null hypothesis. [1]

(c) Find the p-value of Abigail's test. [2]

(d) Write down the conclusion of the test in context. Justify your answer. [2]

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Question 3 - working space

4. [Maximum mark: 5]

The heights of pine trees in a particular forest are normally distributed, with a mean of 35 metres and a standard deviation of 5.5 metres.

(a) Write down the probability that a randomly chosen pine tree has a height of less than 35 metres. [1]

(b) Find the probability that a randomly chosen pine tree has a height of greater than 25 metres. [2]

(c) In the forest, 8% of pine trees have a height of less than k metres. Find the value of k. [2]

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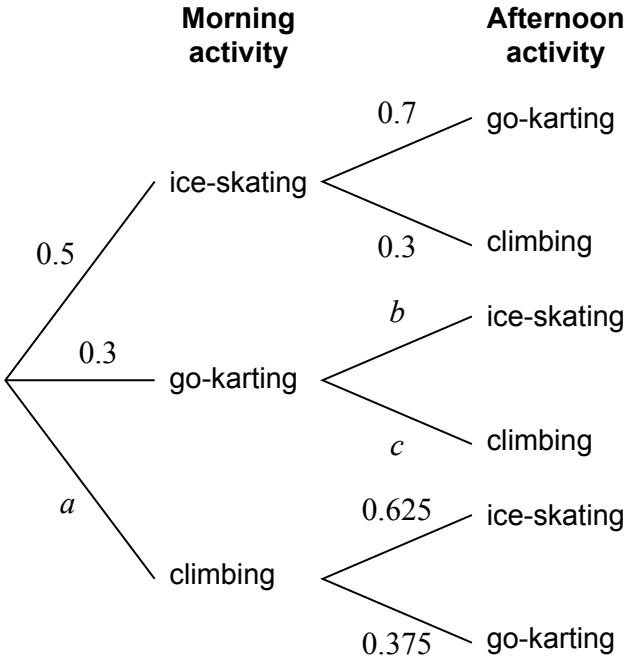
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Question 4 - working space

7. [Maximum mark: 7]

At a sports centre, students choose one activity in the morning and a second, different activity in the afternoon. The tree diagram shows the probability of students choosing each morning and afternoon activity.



(a) Write down the value of a. [1]

(b) Find the probability of a student choosing ice-skating in the morning and climbing in the afternoon. [2]

(c) Find an expression, in terms of b, for the probability of a student choosing go-karting in the morning and ice-skating in the afternoon. [1]

(d) At the end of the day, 87.5% of the students chose ice-skating. Calculate the value of

(i) b.

(ii) c. [3]

(This question continues on the following page)

(Question 7 continued)

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Question 7 - working space

10. [Maximum mark: 7]

In a job assessment there are 15 multiple choice questions, each with 5 answers. Only one answer is correct for each question.

Adil completes the assessment by randomly choosing every answer. All answers are equally likely to be chosen. Each of Adil's choices are independent.

(a) Write down the expected number of correct answers. [2]

(b) Find the probability that Adil correctly answers exactly 3 questions. [3]

If Adil answers 7 or more questions correctly, he will be offered a job.

(c) Find the probability that Adil is offered a job. [2]

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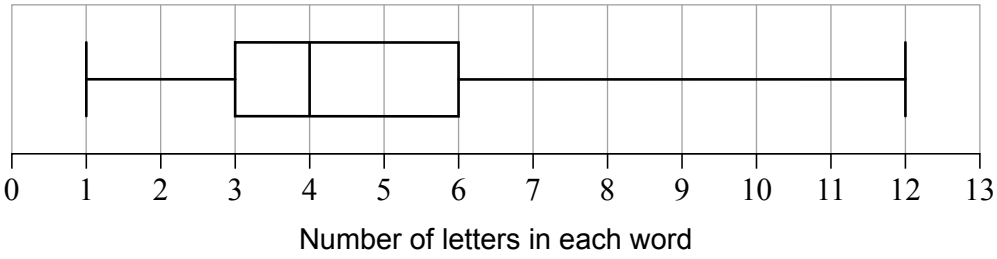
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Question 10 - working space

1. [Maximum mark: 15]

Dr Petrillo wrote a short scientific essay. He analysed the readability of his essay by counting the number of letters in each word.

Dr Petrillo constructs a box and whisker diagram for his data.



Number of letters in each word

(a) Write down

(i) the median;

(ii) the upper quartile, Q_3;

(iii) the interquartile range, IQR. [3]

Dr Petrillo now wants to modify his diagram to show any outliers. He considers the longer words in his data and uses the following formula:

$$\text{outliers} > (1.5 \times \mathrm{IQR}) + \mathrm{Q}_3.$$

Words with at least k letters are considered outliers.

(b) Find the value of k. [2]

(This question continues on the following page)

(Question 1 continued)

Dr Petrillo further considers the outliers and sees no reason to exclude them from his analysis.

The length of each word in the essay, n, and its associated frequency are given in the following table.

n	$1 \leq n \leq 2$	$3 \leq n \leq 4$	$5 \leq n \leq 6$	$7 \leq n \leq 8$	$9 \leq n \leq 10$	$11 \leq n \leq 12$	$n \geq 13$
Frequency	289	480	220	136	90	45	0

(c) Use the mid-interval values to calculate an estimate of the mean number of letters in each word. [3]

Dr Petrillo conducts a χ^2 goodness of fit test at the 1% significance level, to test the following null hypothesis:

H_0: The frequency of the number of letters in each word in his essay is consistent with the English language.

(d) Write down the alternative hypothesis for this test. [1]

The observed and expected frequencies of the number of letters in each word in his essay are listed in the following table.

n	$1 \leq n \leq 2$	$3 \leq n \leq 4$	$5 \leq n \leq 6$	$7 \leq n \leq 8$	$9 \leq n \leq 10$	$11 \leq n \leq 12$	$n \geq 13$
Observed frequency	289	480	220	136	90	45	0
Expected frequency	252.0	491.4	239.4	151.2	88.2	31.8	6.0

(e) (i) Write down the number of degrees of freedom.

(ii) Find the χ^2 statistic for this test.

(iii) Find the p-value for this test. [4]

The critical value for this test, at the 1% significance level, is 16.812.

(f) State whether Dr Petrillo should reject the null hypothesis. Justify your answer. [2]

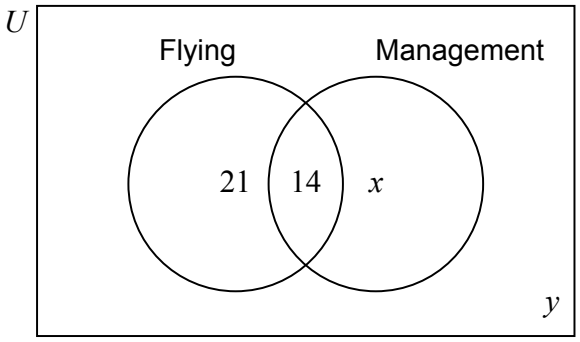
Question 1 - working space

3. [Maximum mark: 15]

Hawksburn Aviation School offers three courses, flying, management and engineering. The following information was collected about the 50 students enrolled in courses at the school.

- 21 students are enrolled in the flying course only.
- 14 students are enrolled in both the flying course and the management course.
- Students enrolled in the engineering course are not enrolled in any other courses.

The information about all 50 students is presented in the following Venn diagram.



A student is selected randomly from this group.

(a) Find the probability that

(i) the student is enrolled in the flying course;

(ii) the student is enrolled in the management course, given that they are enrolled in the flying course. [4]

Let F be the event that the student selected is enrolled in the flying course and M be the event that the student is enrolled in the management course.

(b) State whether F and M are mutually exclusive. Justify your answer. [2]

It is known that $\mathrm{P}(F|M) = 0.7$.

(c) (i) Find the value of x.

(ii) Find the value of y.

(iii) Write down the number of students enrolled in only one course. [6]

Three students are randomly selected from the group of 50 to give a presentation.

(d) Find the probability that all three students are enrolled in the flying course. [3]

Turn over

Question 3 - working space

2. [Maximum mark: 14]

Aiden works for WoodCraft Wonders, a company that makes wooden letters. He wants to predict the demand for different letters. He thinks that letters earlier in the alphabet are more popular.

Aiden finds that, in the year he was born, his name was the most popular in the United States. From the 10 most popular boys' names, his name was first alphabetically. This data is shown in the following table.

Name	**Popularity**	**Position in the alphabet**
Aiden	1	1
Ethan	2	3
Jacob	3	6
Jayden	4	7
Caden	5	2
Noah	6	9
Jackson	7	5
Jack	8	4
Logan	9	8
Ryan	10	10

Aiden calculates the Spearman's rank correlation coefficient, r_s, for this data.

(a) Find r_s, and use the following table to interpret its value. [4]

$\lvert r_s \rvert$	**Strength**
0.000 to 0.199	Very weak
0.200 to 0.399	Weak
0.400 to 0.599	Moderate
0.600 to 0.799	Strong
0.800 to 1.000	Very strong

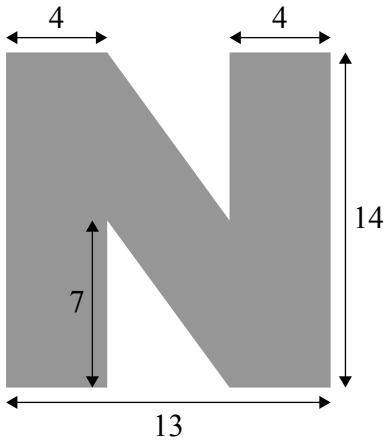
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(Question 2 continued)

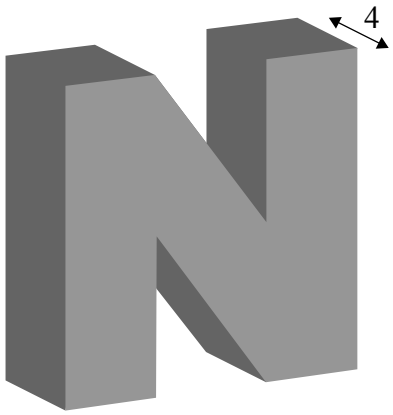
WoodCraft Wonders makes the letter N from a wooden rectangular prism with a depth of $4\,\text{cm}$. The front surface is a rectangle with a base length of $13\,\text{cm}$ and a height of $14\,\text{cm}$. Two identical right-angled triangular prisms are then cut out of the rectangular prism to make the letter N, as shown in the diagram.

diagrams not to scale

Front surface of letter N



Finished letter N



(b) Find the **area** of the front surface of the letter N. [3]

(c) Calculate the **volume** of the finished letter N. [1]

WoodCraft Wonders has two machines that make the wooden letters: Machine A and Machine B. Aiden wants to test if the machines are producing letters of different weights. A sample of wooden letters in the shape of an N are taken from both machines and weighed.

Machine A sample (grams)						
355	342	361	343	328	376	364

Machine B sample (grams)						
338	343	332	358	349	332	324

Aiden assumes the samples came from normally distributed populations with equal variances, and he performs a t-test at the 5% significance level.

(d) (i) Write down the null hypothesis and the alternative hypothesis.

(ii) Find the p-value for this test.

(iii) Write down the conclusion of the test. Give a reason for your answer. [6]

Turn over

Question 2 - working space

APOLLO

Calculus

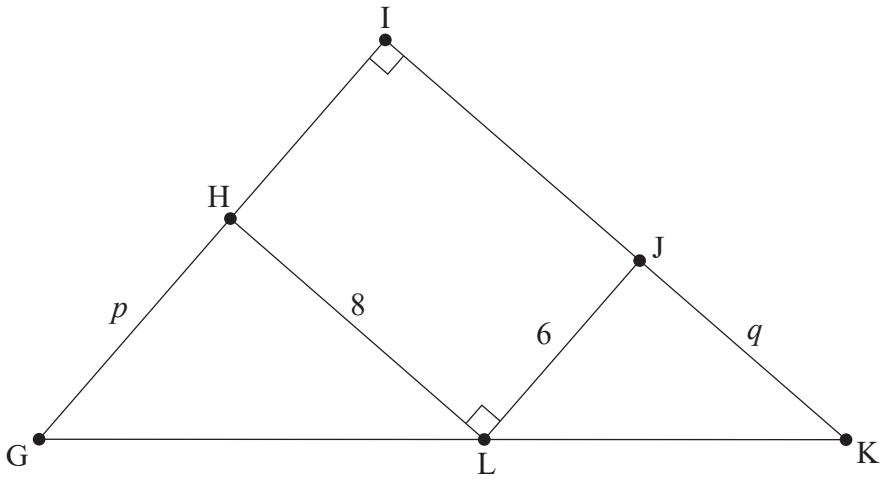
54 questions

12. [Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is A cm^2.

(a) (i) Find A in terms of p and q.

(ii) Show that $A = \frac{192}{q} + 3q + 48$. [4]

(b) Find $\frac{\mathrm{d}A}{\mathrm{d}q}$. [2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q.

(ii) Hence, or otherwise, find this value of q. [2]

(This question continues on the following page)

(Question 12 continued)

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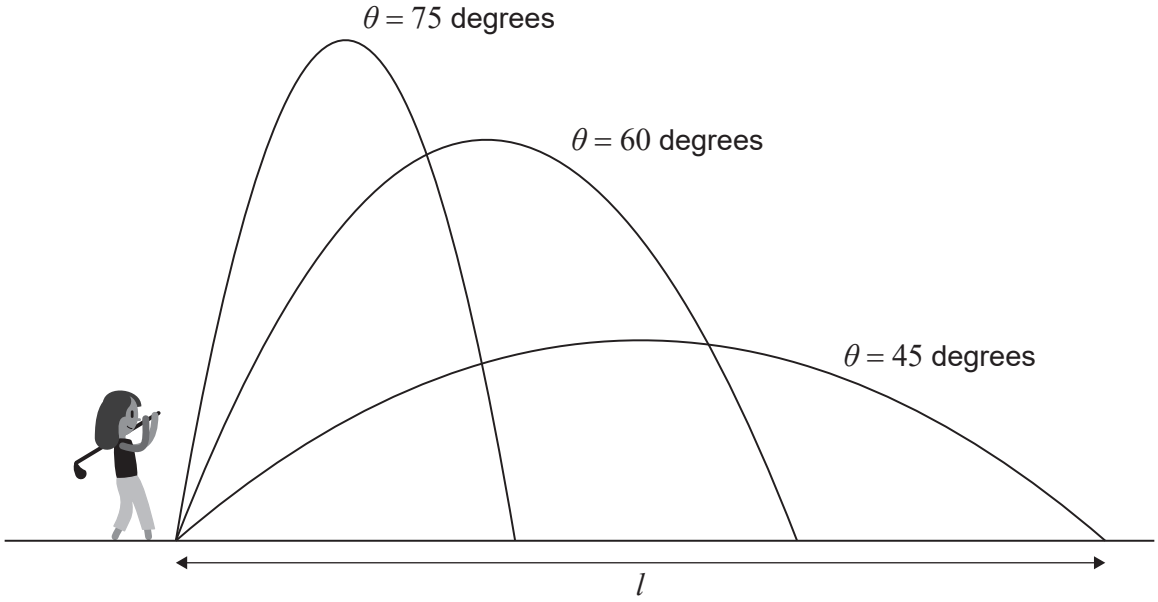
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Question 12 - working space

13. [Maximum mark: 8]

Sieun hits golf balls into the air. Each time she hits a ball she records θ, the angle at which the ball is launched into the air, and l, the horizontal distance, in metres, which the ball travels from the point of contact to the first time it lands. The diagram below represents this information.



Sieun analyses her results and concludes:

$$\frac{\mathrm{d}l}{\mathrm{d}\theta} = -0.2\theta + 9\,,\ 35^\circ \leq \theta \leq 75^\circ\,.$$

(a) Determine whether the graph of l against θ is increasing or decreasing at $\theta = 50^\circ$. [3]

Sieun observes that when the angle is 40°, the ball will travel a horizontal distance of $205.5\,\mathrm{m}$.

(b) Find an expression for the function $l(\theta)$. [5]

(This question continues on the following page)

(Question 13 continued)

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Question 13 - working space

13. [Maximum mark: 7]

A company produces and sells electric cars. The company's profit, P, in thousands of dollars, changes based on the number of cars, x, they produce per month.

The rate of change of their profit from producing x electric cars is modelled by

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -1.6x + 48,\ x \geq 0.$$

The company makes a profit of 260 (thousand dollars) when they produce 15 electric cars.

(a) Find an expression for P in terms of x. [5]

The company regularly increases the number of cars it produces.

(b) Describe how their profit changes if they increase production to over 30 cars per month and up to 50 cars per month. Justify your answer. [2]

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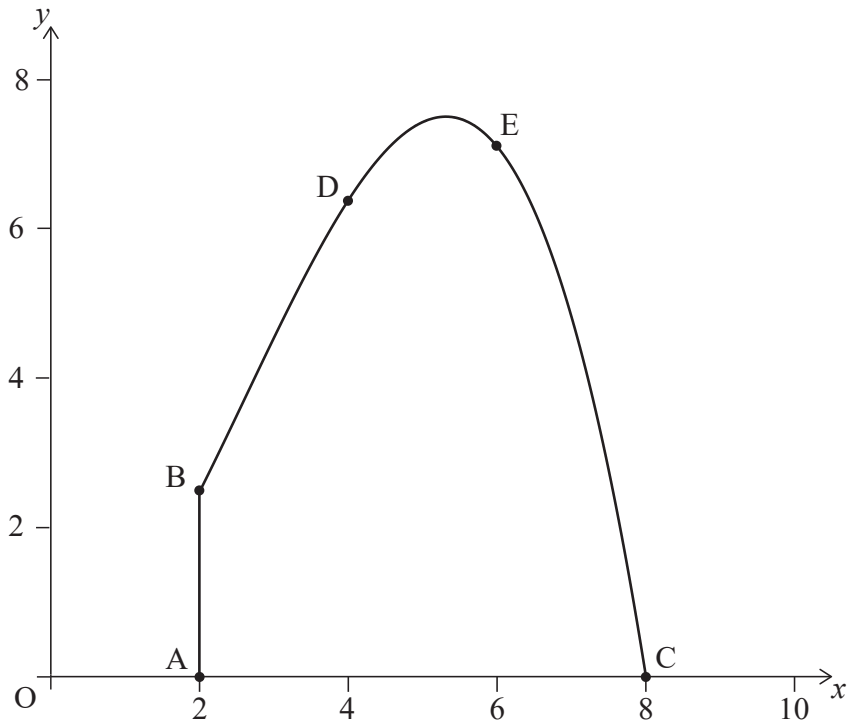
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Question 13 - working space

5. [Maximum mark: 13]

The cross-sectional view of a tunnel is shown on the axes below. The line $[AB]$ represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by $y = -0.1x^3 + 0.8x^2,\ 2 \le x \le 8$, relative to an origin O.



Point A has coordinates $(2, 0)$, point B has coordinates $(2, 2.4)$, and point C has coordinates $(8, 0)$.

(a) (i) Find $\frac{dy}{dx}$.

(ii) Hence find the maximum height of the tunnel. [6]

When $x = 4$ the height of the tunnel is $6.4\,\text{m}$ and when $x = 6$ the height of the tunnel is $7.2\,\text{m}$. These points are shown as D and E on the diagram, respectively.

(b) Use the trapezoidal rule, with three intervals, to estimate the cross-sectional area of the tunnel. [3]

(c) (i) Write down the integral which can be used to find the cross-sectional area of the tunnel.

(ii) Hence find the cross-sectional area of the tunnel. [4]

References:

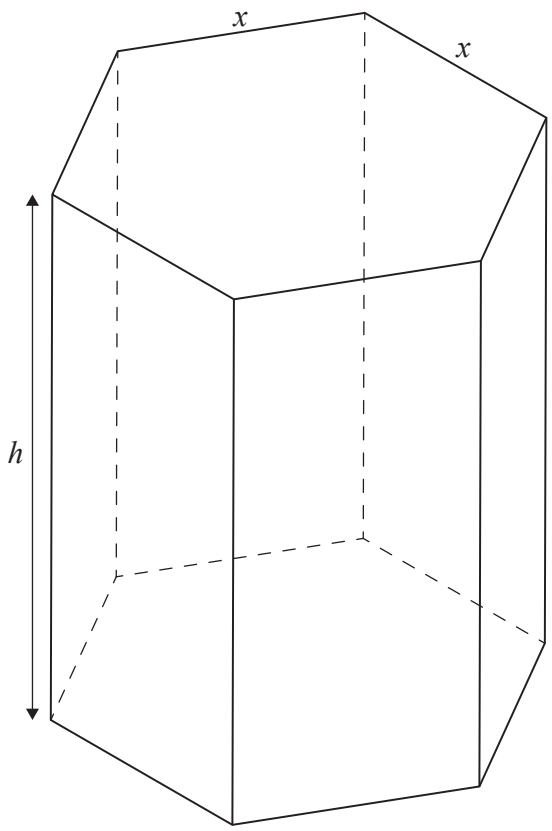
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Question 5 - working space

5. [Maximum mark: 16]

A hollow chocolate box is manufactured in the form of a right prism with a regular hexagonal base. The height of the prism is $h\,\text{cm}$, and the top and base of the prism have sides of length $x\,\text{cm}$.

diagram not to scale



(a) Given that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, show that the area of the base of the box is equal to $\frac{3\sqrt{3}x^2}{2}$. [2]

(b) Given that the total external surface area of the box is $1200\,\text{cm}^2$, show that the volume of the box may be expressed as $V = 300\sqrt{3}\,x - \frac{9}{4}x^3$. [5]

(c) Sketch the graph of $V = 300\sqrt{3}\,x - \frac{9}{4}x^3$, for $0 \le x \le 16$. [2]

(d) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

(e) Find the value of x which maximizes the volume of the box. [2]

(f) Hence, or otherwise, find the maximum possible volume of the box. [2]

The box will contain spherical chocolates. The production manager assumes that they can calculate the exact number of chocolates in each box by dividing the volume of the box by the volume of a single chocolate and then rounding down to the nearest integer.

(g) Explain why the production manager is incorrect. [1]

References:

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Question 5 - working space

6. [Maximum mark: 5]

Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t, the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t-axis.

The rate, R, is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

(a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours. [3]

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

(b) Find the percentage error of the estimate found in part (a). [2]

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Question 6 - working space

12. [Maximum mark: 6]

The surface area of an open box with a volume of $32\,\text{cm}^3$ and a square base with sides of length $x\,\text{cm}$ is given by $S(x) = x^2 + \frac{128}{x}$ where $x > 0$.

(a) Find $S'(x)$. [3]

(b) (i) Solve $S'(x) = 0$.

(ii) Interpret your answer to (b)(i) in context. [3]

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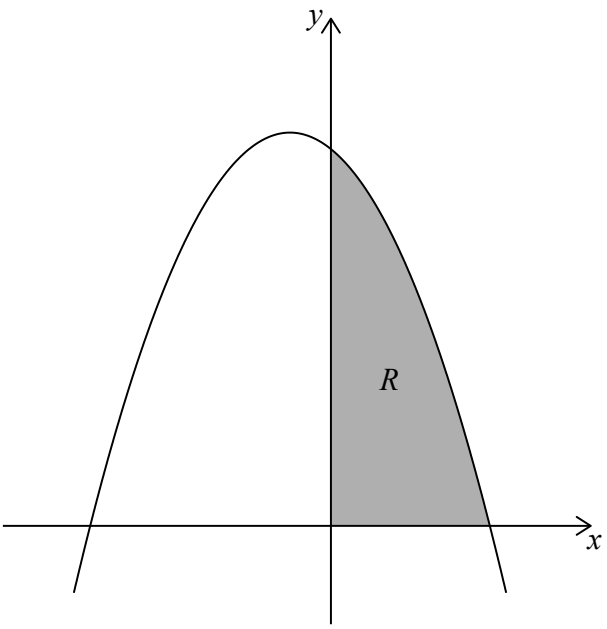
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Question 12 - working space

10. [Maximum mark: 5]

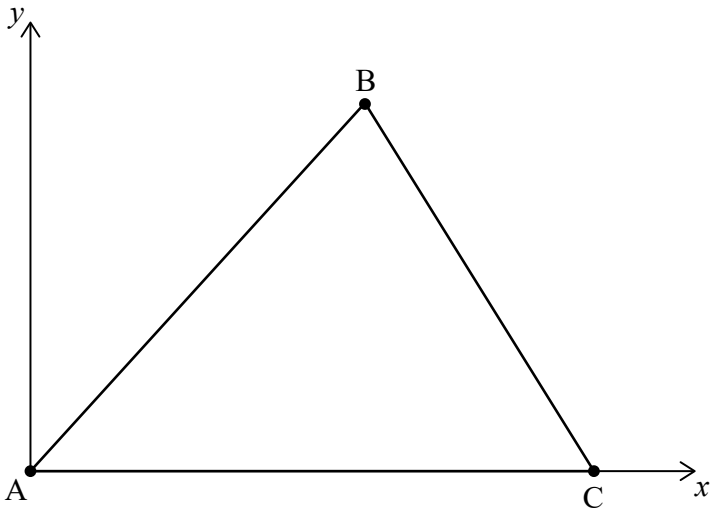
The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, y-axis and the graph of f.



(a) Write down an integral for the area of region R. [2]

(b) Find the area of region R. [1]

The three points $\mathrm{A}(0, 0)$, $\mathrm{B}(3, 10)$ and $\mathrm{C}(a, 0)$ define the vertices of a triangle.



(c) Find the value of a, the x-coordinate of C, such that the area of the triangle is equal to the area of region R. [2]

(This question continues on the following page)

(Question 10 continued)

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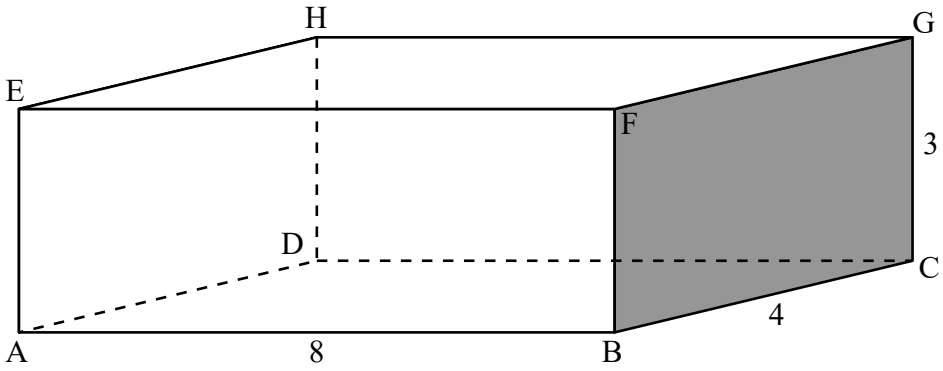
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Question 10 - working space

4. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length $8\,\text{cm}$, width $4\,\text{cm}$ and height $3\,\text{cm}$. The information is shown in the diagram.



(a) Calculate the surface area of the box in cm^2. [2]

(b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{\text{d}P}{\text{d}x} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of $20\,000$ boxes is $\$1700$.

(d) Find $P(x)$. [5]

(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

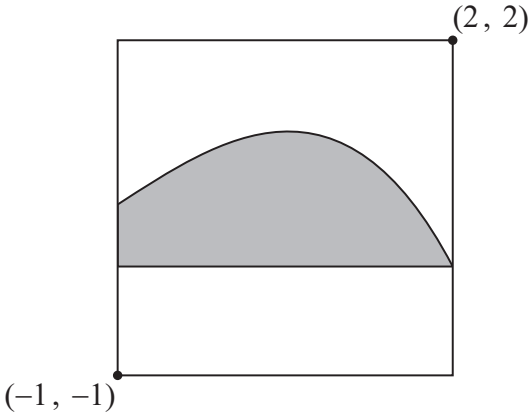
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Question 4 - working space

6. [Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1, -1)$ and the top right corner has coordinates $(2, 2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x-axis. Distances are measured in metres.

(a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region. [3]

x	-1	0	1	2
y	0.6	1.2	1.2	0

The artist used the equation $y = \frac{-x^3 - 3x^2 + 4x + 12}{10}$ to draw the curve.

(b) Find the exact area of the shaded region in the painting. [2]

(c) Find the area of the unshaded region in the painting. [2]

(This question continues on the following page)

(Question 6 continued)

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Question 6 - working space

9. [Maximum mark: 7]

The function f is defined by $f(x) = \frac{2}{x} + 3x^2 - 3$, $x \neq 0$.

(a) Find $f'(x)$. [3]

(b) Find the equation of the normal to the curve $y = f(x)$ at $(1, 2)$ in the form $ax + by + d = 0$, where $a, b, d \in \mathbb{Z}$. [4]

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Question 9 - working space

12. [Maximum mark: 6]

A company's profit per year was found to be changing at a rate of

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 3t^2 - 8t$$

where P is the company's profit in thousands of dollars and t is the time since the company was founded, measured in years.

(a) Determine whether the profit is increasing or decreasing when $t = 2$. [2]

One year after the company was founded, the profit was 4 thousand dollars.

(b) Find an expression for $P(t)$, when $t \geq 0$. [4]

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Question 12 - working space

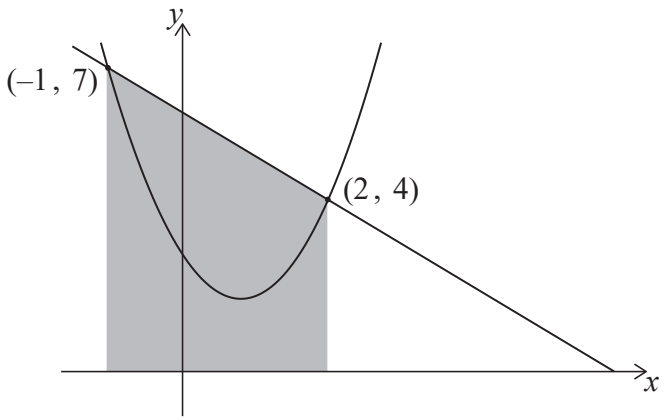
6. [Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2, 4)$ and $(-1, 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 1

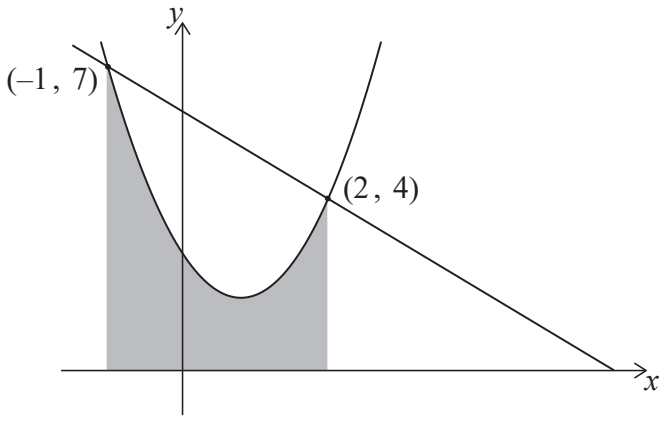


(a) Calculate the area of the shaded region in **diagram 1**. [2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x-axis has been shaded.

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Diagram 2



(b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

(ii) Calculate the area of this region. [3]

(c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$. [2]

(This question continues on the following page)

(Question 6 continued)

Question 6 - working space

11. [Maximum mark: 7]

Consider the function $f(x) = x^2 - \frac{3}{x},\ x \neq 0$.

(a) Find $f'(x)$. [2]

Line L is a tangent to $f(x)$ at the point $(1,\ -2)$.

(b) Use your answer to part (a) to find the gradient of L. [2]

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer. [3]

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Question 11 - working space

5. [Maximum mark: 15]

A cafe makes x litres of coffee each morning. The cafe's profit each morning, C, measured in dollars, is modelled by the following equation

$$C = \frac{x}{10}\left(k^2 - \frac{3}{100}x^2\right)$$

where k is a positive constant.

(a) Find an expression for $\frac{\mathrm{d}C}{\mathrm{d}x}$ in terms of k and x. [3]

(b) Hence find the maximum value of C in terms of k. Give your answer in the form pk^3, where p is a constant. [4]

The cafe's manager knows that the cafe makes a profit of $\$426$ when 20 litres of coffee are made in a morning.

(c) (i) Find the value of k.

(ii) Use the model to find how much coffee the cafe should make each morning to maximize its profit. [3]

(d) Sketch the graph of C against x, labelling the maximum point and the x-intercepts with their coordinates. [3]

The manager of the cafe wishes to serve as many customers as possible.

(e) Determine the maximum amount of coffee the cafe can make that will not result in a loss of money for the morning. [2]

References:

4. Aline Escobar., n.d. Cowboy. [image online] Available at: https://thenounproject.com/search/?q=cowboy&i=1080130 This file is licensed under the Creative Commons Attribution-ShareAlike 3.0 Unported (CC BY-SA 3.0) https://creativecommons.org/licenses/by-sa/3.0/deed.en [Accessed 13/05/2021]. Source adapted.

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Question 5 - working space

13. [Maximum mark: 8]

Giles charges a customer per hour to hire his boat. It is known that

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 20 - \frac{980}{t^2},\ 0 < t \leq 12$$

where P is the cost per hour, in Norwegian krone (NOK), that the customer is charged and t is the time, in hours, spent on the boat.

The cost per hour has a local minimum when the boat is hired for h hours.

(a) Find the value of h. [2]

Sandra hired Giles' boat for 5 hours and was charged $\mathrm{NOK}\ 328$ per hour. Yvonne hires Giles' boat for 7 hours.

(b) Show that the cost per hour for Yvonne is $\mathrm{NOK}\ 312$. [6]

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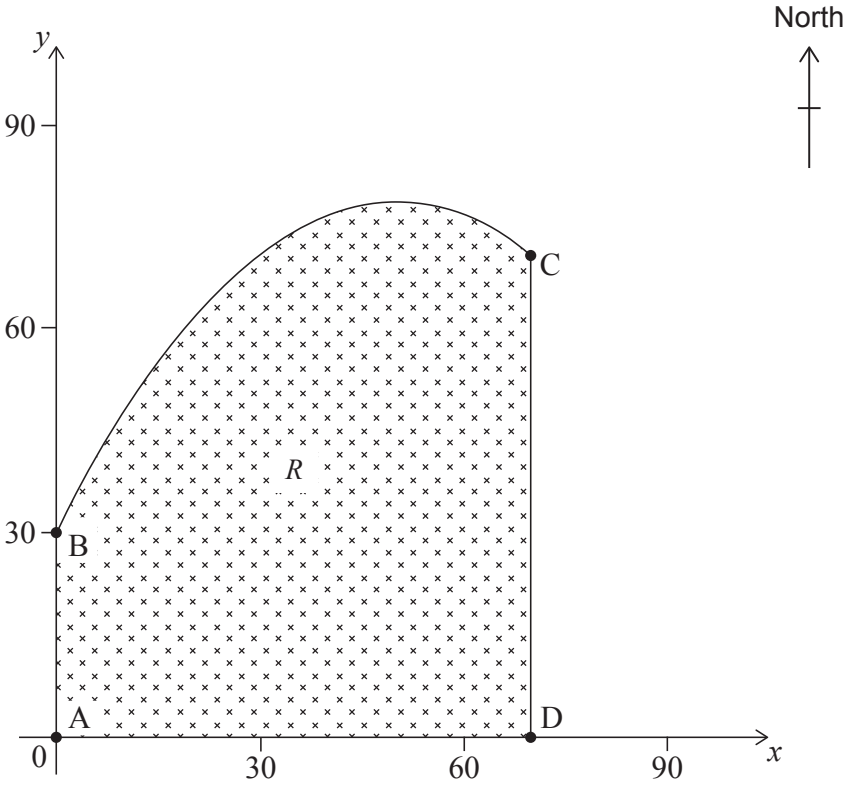
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Question 13 - working space

5. [Maximum mark: 17]

Linda owns a field, represented by the shaded region R. The plan view of the field is shown in the following diagram, where both axes represent distance and are measured in metres.



The segments [AB], [CD] and [AD] respectively represent the western, eastern and southern boundaries of the field. The function, $f(x)$, models the northern boundary of the field between points B and C and is given by

$$f(x) = \frac{-x^2}{50} + 2x + 30, \text{ for } 0 \leq x \leq 70.$$

(a) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the point on the field that is furthest north. [5]

Point A has coordinates $(0, 0)$, point B has coordinates $(0, 30)$, point C has coordinates $(70, 72)$ and point D has coordinates $(70, 0)$.

(b) (i) Write down the integral which can be used to find the area of the shaded region R.

(ii) Find the area of Linda's field. [4]

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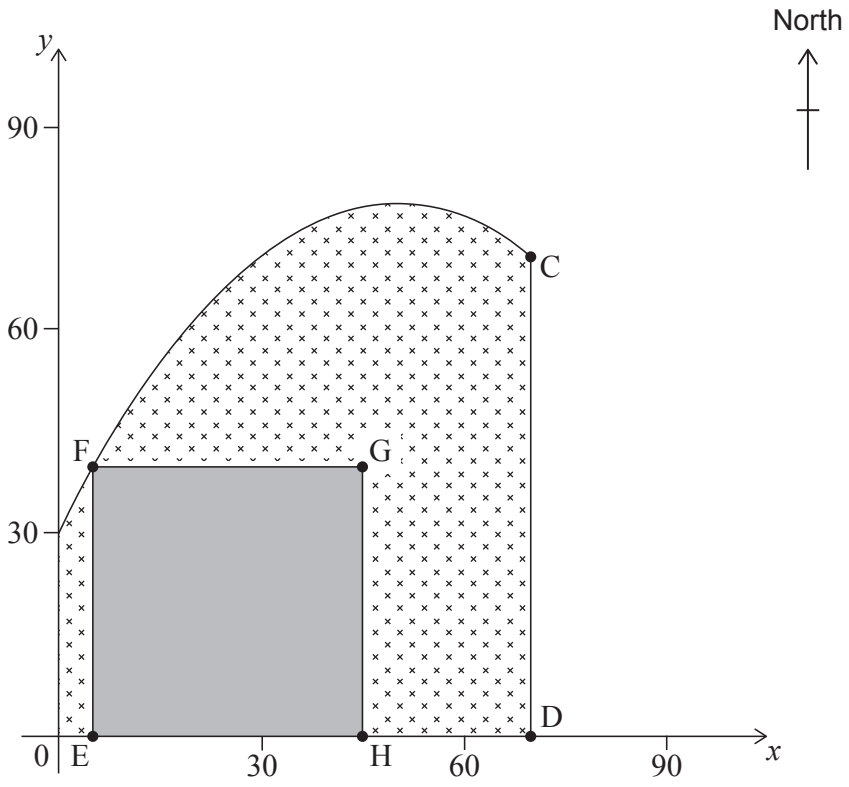
(Question 5 continued)

Linda used the trapezoidal rule with ten intervals to estimate the area. This calculation underestimated the area by $11.4\,\text{m}^2$.

(c) (i) Calculate the percentage error in Linda's estimate.

(ii) Suggest how Linda might be able to reduce the error whilst still using the trapezoidal rule. [3]

Linda would like to construct a building on her field. The **square** foundation of the building, EFGH, will be located such that [EH] is on the southern boundary and point F is on the northern boundary of the property. A possible location of the foundation of the building is shown in the following diagram.



The area of the square foundation will be largest when [GH] lies on [CD].

(d) (i) Find the x-coordinate of point E for the largest area of the square foundation of building EFGH.

(ii) Find the largest area of the foundation. [5]

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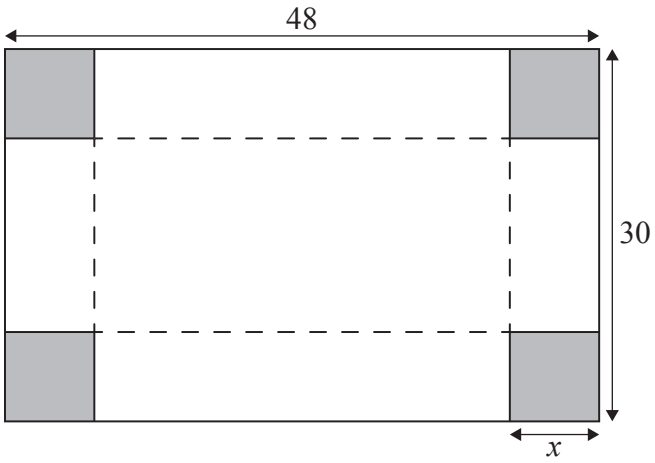
Question 5 - working space

7. [Maximum mark: 6]

A rectangular box, with an open top, is to be constructed from a piece of cardboard that measures $48\,\text{cm}$ by $30\,\text{cm}$.

Squares of equal size will be cut from the corners of the cardboard, as indicated by the shading in the diagram. The sides will then be folded along the dotted lines to form the box.

diagram not to scale



The volume of the box, in cubic centimetres, can be modelled by the function $V(x) = (48 - 2x)(30 - 2x)(x)$, for $0 < x < k$, where x is the length of the sides of the squares removed in centimetres.

(a) Write down the maximum possible value of k in this context. [1]

(b) Find the value of x that maximizes the volume of the box. [2]

A second piece of $48\,\text{cm}$ by $30\,\text{cm}$ cardboard is damaged and a strip $2\,\text{cm}$ wide must be removed from all four sides. A box will then be constructed in a similar manner from the remaining cardboard.

(c) Calculate the maximum possible volume of the box made from the second piece of cardboard. [3]

(This question continues on the following page)

(Question 7 continued)

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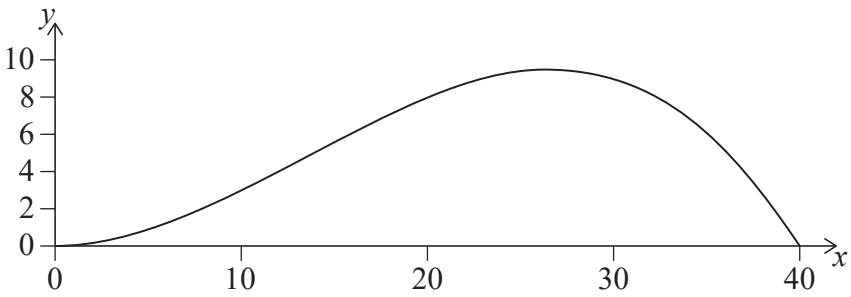
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Question 7 - working space

9. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3,\ 0 \le x \le 40$.

(b) (i) Write down an integral to find the exact cross-sectional area.

(ii) Calculate the value of the cross-sectional area to two decimal places. [4]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

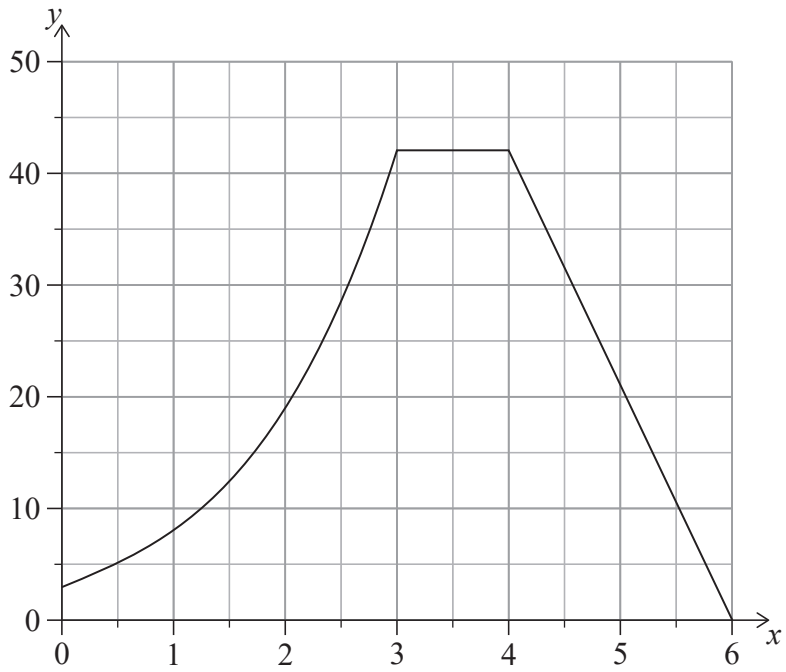
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Question 9 - working space

13. [Maximum mark: 9]

An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.



The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \le x \le 3$.

x	0	0.5	1	1.5	2	2.5	3
$y = f(x)$	3	5.13	8	12.4	19	28.6	42

(a) Calculate an estimate for the area in the interval $0 \le x \le 3$ by using the trapezoidal rule with three equal intervals. [2]

It is known that $f'(x) = 3x^2 + 4$ in the domain $0 < x < 3$.

(b) Find an expression for $f(x)$, in the domain $0 < x < 3$. [4]

(c) **Hence** find the actual area of the **entire** cross-section. [3]

(This question continues on the following page)

(Question 13 continued)

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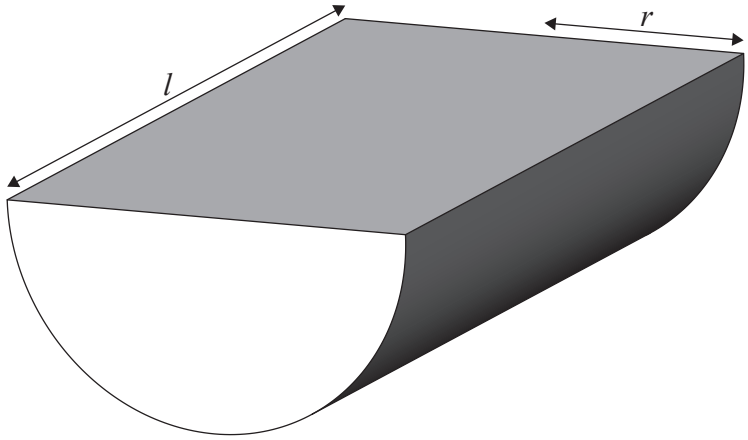
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Question 13 - working space

5. [Maximum mark: 17]

A large closed container, in the shape of a half cylinder with a rectangular lid, is to be constructed with a volume of $0.8\,\text{m}^3$. The container has a length of l metres and a radius of r metres.

diagram not to scale



(a) Find an exact expression for l in terms of r and π. [2]

The container will be constructed using two different materials. The material for both the curved surface and the rectangular lid of the container costs $\$4.40$ per square metre. The material for the semicircular ends of the container costs $\$p$ per square metre.

The cost, C, of the materials to construct the container can be written in terms of r and p (where $p > 0$ and $r > 0$).

(b) Show that $C = 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$. [4]

(c) Find $\frac{\text{d}C}{\text{d}r}$. [3]

The cost of materials to construct the container is minimized when the radius of the container, r, is $0.7\,\text{m}$.

(d) Find the value of p. [3]

In total, 350 containers will be constructed at this minimum cost.

(e) Calculate the cost of materials, to the nearest dollar, to construct all 350 containers. [3]

(This question continues on the following page)

(Question 5 continued)

The materials for constructing the containers can be purchased at a discount according to the information in the table.

Cost of materials ($\$ C$) before discount	**Discount applied to entire order**
$1000 \leq C < 2500$	1%
$2500 \leq C < 5000$	4%
$5000 \leq C < 10\,000$	8%
$C \geq 10\,000$	10%

(f) Determine the cost of materials for 350 containers after the discount is applied. [2]

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Question 5 - working space

5. [Maximum mark: 16]

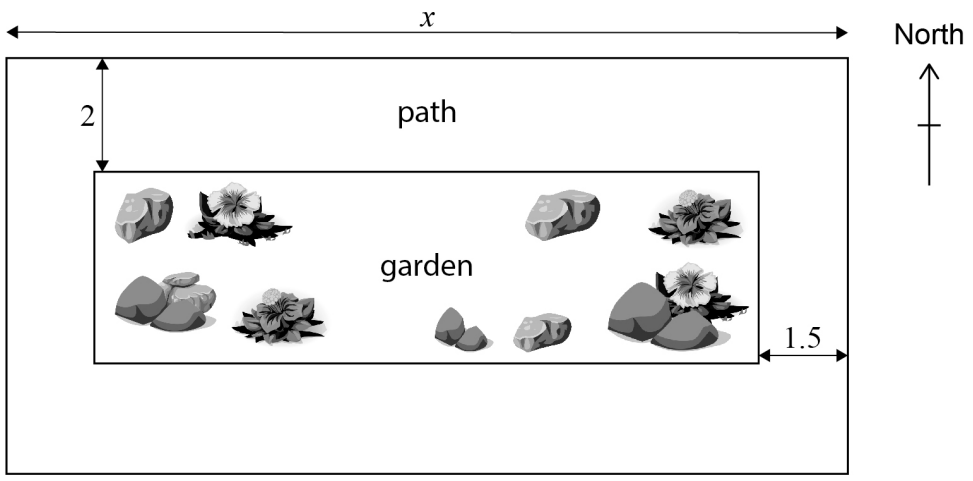
A particular park consists of a rectangular garden, of area $A\,\text{m}^2$, and a concrete path surrounding it. The park has a total area of $1200\,\text{m}^2$.

The width of the path at the north and south side of the park is $2\,\text{m}$.

The width of the path at the west and east side of the park is $1.5\,\text{m}$.

The length of the park (along the north and south sides) is x metres, $3 < x < 300$.

diagram not to scale



(a) (i) Write down the length of the garden in terms of x.

(ii) Find an expression for the width of the garden in terms of x.

(iii) Hence show that $A = 1212 - 4x - \frac{3600}{x}$. [5]

(b) Find the possible dimensions of the park if the area of the garden is $800\,\text{m}^2$. [4]

(c) Find an expression for $\frac{\text{d}A}{\text{d}x}$. [3]

(d) Use your answer from part (c) to find the value of x that will maximize the area of the garden. [2]

(e) Find the maximum possible area of the garden. [2]

References:

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Question 5 - working space

3. [Maximum mark: 6]

Consider the graph of the following function:

$$f(x) = \frac{16}{x} + \frac{x^2}{8}, \text{ for } x \neq 0.$$

(a) Write down the equation of the vertical asymptote of $f(x)$. [1]

(b) Find $f'(x)$. [3]

(c) Write down the interval in which $f(x)$ is increasing. [2]

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Question 3 - working space

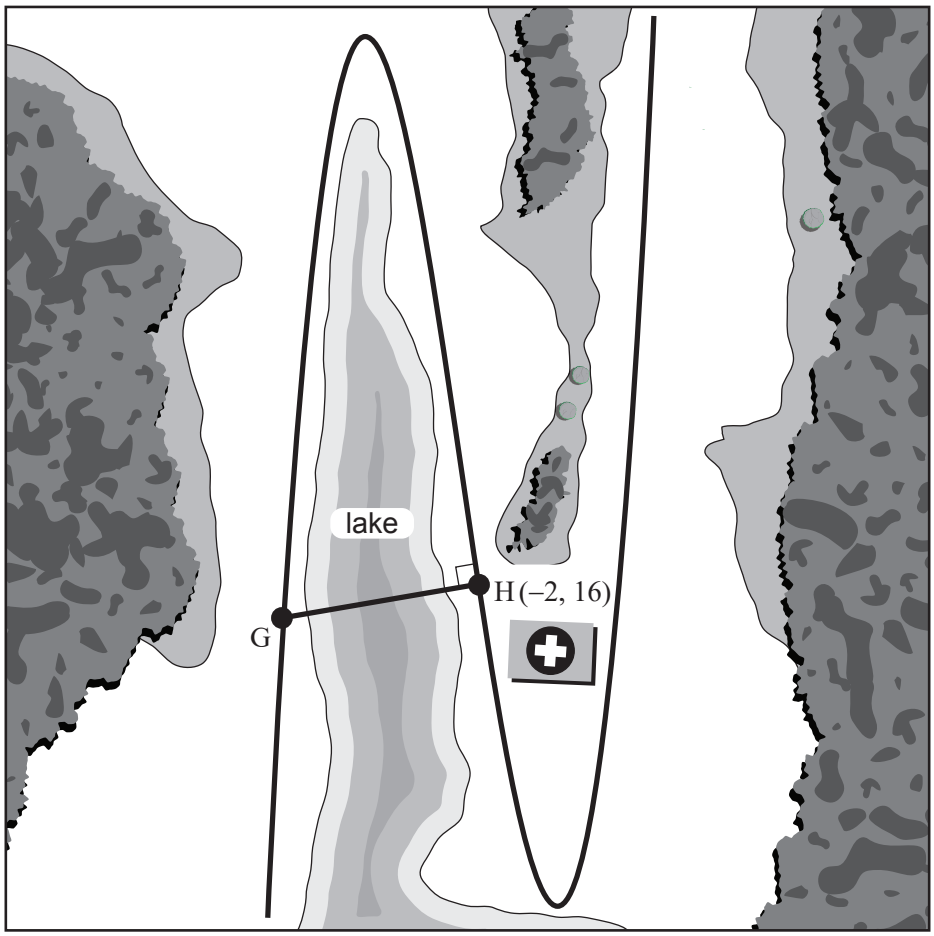
8. [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x+2)^3 + 3x^2 - 2x$. All distances in the map are in kilometres.

The local hospital is located at point H, which has coordinates $(-2, 16)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point H and will connect the hospital to point G.

diagram not to scale



(a) Using your graphic display calculator, find the value of $r'(-2)$. [2]

(b) Find the equation of the line normal to $r(x)$ at point H, which can be used to model the new bridge. [2]

(c) Hence, determine the length of the new bridge. [3]

(This question continues on the following page)

(Question 8 continued)

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Question 8 - working space

3. [Maximum mark: 6]

Consider the graph of the following function:

$$g(x) = \frac{8}{x} + \frac{x^2}{2}, \text{ for } x \neq 0.$$

(a) Write down the equation of the vertical asymptote of $g(x)$. [1]

(b) Find $g'(x)$. [3]

(c) Write down the interval in which $g(x)$ is increasing. [2]

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Question 3 - working space

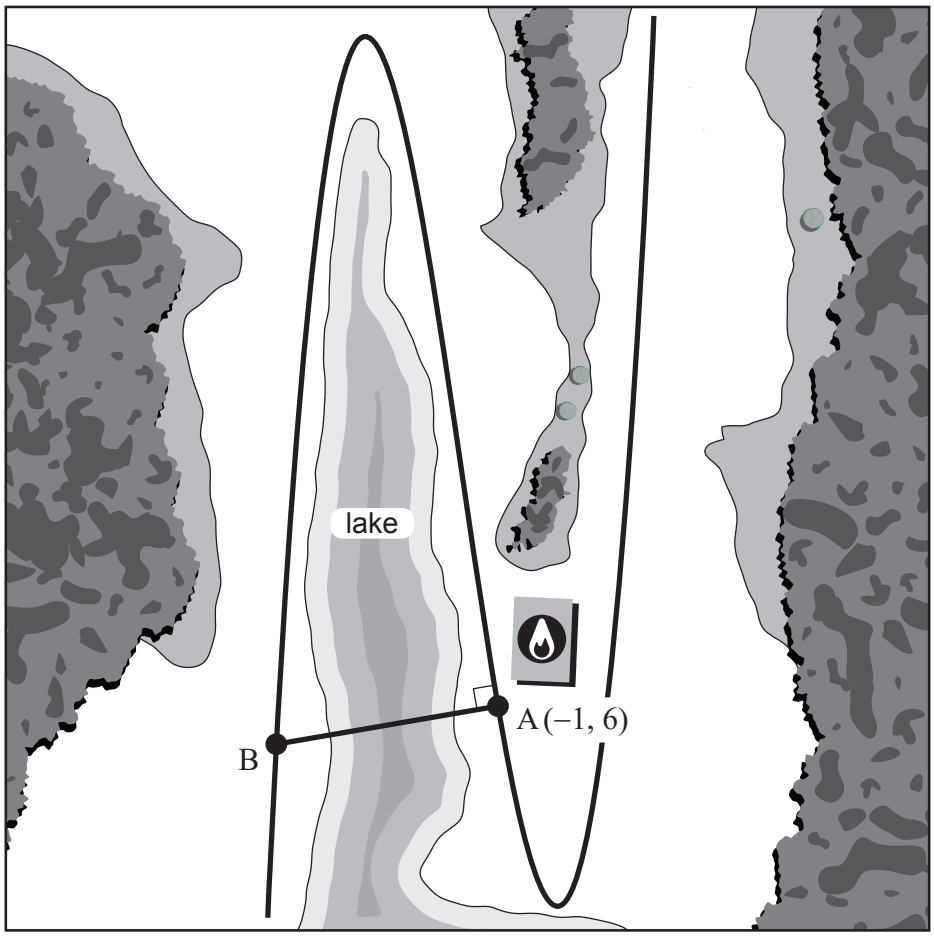
8. [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x+1)^3 + 2x^2 - 4x$. All distances in the map are in kilometres.

The local fire station is located at point A, which has coordinates $(-1, 6)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point A and will connect the fire station to point B.

diagram not to scale



(a) Using your graphic display calculator, find the value of $r'(-1)$. [2]

(b) Find the equation of the line normal to $r(x)$ at point A, which can be used to model the new bridge. [2]

(c) Hence, determine the length of the new bridge. [3]

(This question continues on the following page)

(Question 8 continued)

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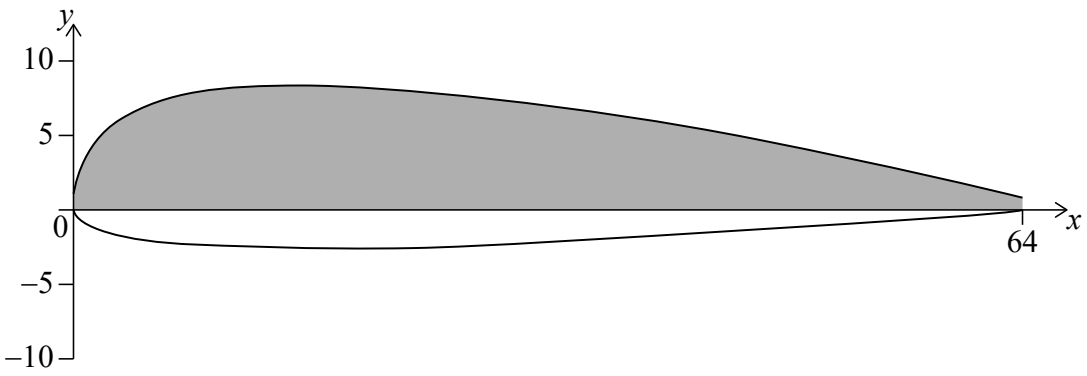
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Question 8 - working space

1. [Maximum mark: 12]

Jan is investigating the shape of model helicopter propeller blades. A cross-section of one of the blades is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x-axis and the curve with equation

$$y = 4\sqrt{x} - \frac{x}{2} + 1, \text{ for } 0 \leq x \leq 64$$

where x is the distance, in mm, from the edge of the blade and y is the height, in mm, above the horizontal axis through the blade, as shown in the diagram.

(a) Find the values of a, b and c, shown in the table. [3]

x (mm)	0	16	32	48	64
y (mm)	1	a	b	c	1

Jan uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the blade.

(b) Find Jan's estimate of the shaded area of the cross-section. [3]

(c) (i) Write down the integral that Jan can use to find the exact area of the shaded part of the cross-section.

(ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]

(d) Calculate the percentage error of Jan's estimate in part (b). [2]

Turn over

Question 1 - working space

5. [Maximum mark: 19]

Mosaic Industries is an international company that sells cell phone cases. Their market research predicts that the average number of cases they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each case.

A salesperson suggests that Mosaic Industries should sell the cases for $16\,\text{EUR}$ each.

(a) Find the average number of cases that this model predicts Mosaic Industries will sell at this price. [2]

(b) Calculate Mosaic Industries' average monthly income, before any expenses, at this selling price. [2]

(c) Hence, write down the function $R(x)$ that can be used to predict Mosaic Industries' average monthly income, before expenses, at any selling price, x. [1]

Mosaic Industries has $10\,000\,\text{EUR}$ of fixed monthly operational costs. Additionally, Mosaic Industries must pay their phone case supplier $10\,\text{EUR}$ for each case.

(d) Calculate Mosaic Industries' average monthly profit if they sell each case at a price of $16\,\text{EUR}$. [3]

(e) Show that the average monthly profit for any selling price, x, can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]

(f) (i) Find $P'(x)$.

(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Mosaic Industries negotiates a new deal with their phone case supplier. Under the new deal, the supplier agrees to discount the cost of each case based on the number of cases purchased by Mosaic Industries. The cost charged by the supplier for each case can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of cases sold by Mosaic Industries.

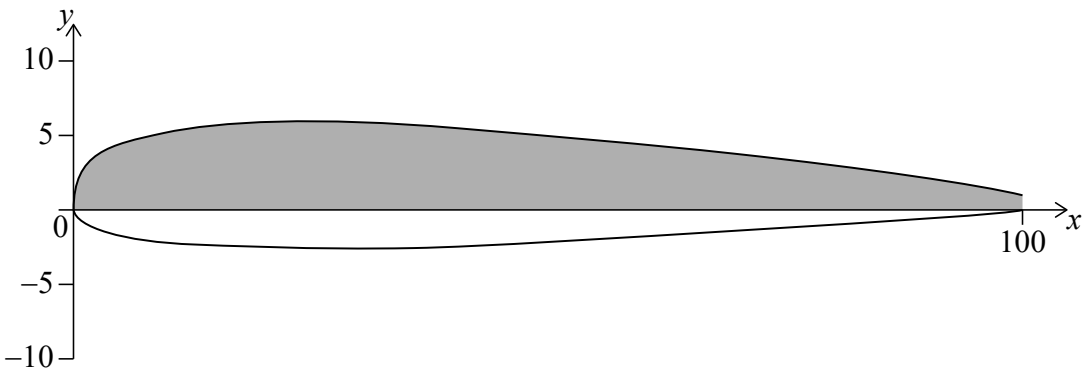
(g) Find the function that can be used to find Mosaic Industries' average monthly profit using the new deal from the supplier. [3]

(h) Hence, find the selling price, per case, that Mosaic Industries should choose in order to maximize their average monthly profit under the new deal. [2]

Question 5 - working space

1. [Maximum mark: 12]

Gabriel is investigating the shape of model airplane wings. A cross-section of one of the wings is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x-axis and the curve with equation

$$y = 2\sqrt{x} - \frac{x}{5} + 1, \text{ for } 0 \leq x \leq 100$$

where x is the distance, in cm, from the front of the wing and y is the height, in cm, above the horizontal axis through the wing, as shown in the diagram.

(a) Find the values of a, b and c, shown in the table. [3]

x (cm)	0	25	50	75	100
y (cm)	1	a	b	c	1

Gabriel uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the wing.

(b) Find Gabriel's estimate of the shaded area of the cross-section. [3]

(c) (i) Write down the integral that Gabriel can use to find the exact area of the shaded part of the cross-section.

(ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]

(d) Calculate the percentage error of Gabriel's estimate in part (b). [2]

Turn over

Question 1 - working space

5. [Maximum mark: 19]

Howell Industries is an international company that sells travel coffee mugs. Their market research predicts that the average number of mugs they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each mug.

A salesperson suggests that Howell Industries should sell the mugs for $16\,\text{EUR}$ each.

(a) Find the average number of mugs that this model predicts Howell Industries will sell at this price. [2]

(b) Calculate Howell Industries' average monthly income, before any expenses, at this selling price. [2]

(c) Hence, write down the function $R(x)$ that can be used to predict Howell Industries' average monthly income, before expenses, at any selling price, x. [1]

Howell Industries has $10\,000\,\text{EUR}$ of fixed monthly operational costs. Additionally, Howell Industries must pay their mug supplier $10\,\text{EUR}$ for each coffee mug.

(d) Calculate Howell Industries' average monthly profit if they sell each mug at a price of $16\,\text{EUR}$. [3]

(e) Show that the average monthly profit for any selling price, x, can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]

(f) (i) Find $P'(x)$.

(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Howell Industries negotiates a new deal with their mug supplier. Under the new deal, the supplier agrees to discount the cost of each mug based on the number of mugs purchased by Howell Industries. The cost charged by the supplier for each mug can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of mugs sold by Howell Industries.

(g) Find the function that can be used to find Howell Industries' average monthly profit using the new deal from the supplier. [3]

(h) Hence, find the selling price, per mug, that Howell Industries should choose in order to maximize their average monthly profit under the new deal. [2]

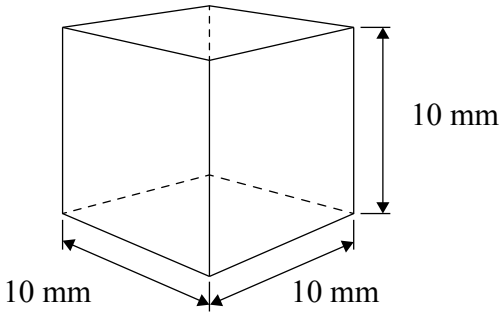
Question 5 - working space

9. [Maximum mark: 8]

Kyungyoon investigates the rate at which a cubical block of sugar dissolves in hot coffee. Initially, the cube has side lengths of $10\,\text{mm}$. This information is illustrated in the following diagrams.

diagram not to scale





Kyungyoon predicts that, as the block of sugar dissolves, each side length will decrease at a constant rate of $0.2\,\text{mm}$ per second.

(a) According to this model, find

(i) the length of one side of a block of sugar, 20 seconds after it is placed in hot coffee.

(ii) the volume of a block of sugar, 20 seconds after it is placed in hot coffee. [3]

Let the function $V(t)$ represent the volume of the block of sugar, mm^3, t seconds after it is placed in hot coffee. $V(t)$ is given by

$$V(t) = 1000 - 60t + 1.2t^2 - 0.008t^3, \text{ for } 0 \leq t \leq 50.$$

(b) Find $V'(t)$. [2]

(c) Find the rate of change of the volume of the block of sugar at $t = 20$. [2]

(d) State one reason why the side length of the cube may not always decrease at a constant rate. [1]

(This question continues on the following page)

(Question 9 continued)

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Question 9 - working space

10. [Maximum mark: 7]

The gradient of the **normal** to the curve $y = ax^2 + bx - 10$ at the point $\mathrm{T}(2, 4)$ is $-\frac{1}{3}$.

Calculate the value of a and the value of b.

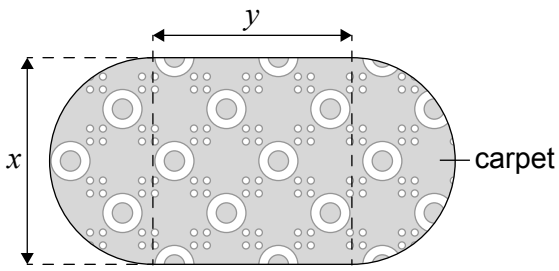
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Question 10 - working space

12. [Maximum mark: 8]

A company is designing a new carpet. The intended design of the carpet is in the shape of a rectangle with a semi-circle at each end.

The width of the rectangle is y metres and the diameter of each semi-circle is x metres, with $x > 0$ and $y \geq 0$.



The company has decided that the perimeter of the carpet will be 20 metres and would like to maximize its area.

(a) Find an expression for the perimeter in terms of x and y. [1]

(b) Show that the area, $A\,\text{m}^2$, of the carpet can be expressed as $A = 10x - \frac{\pi x^2}{4}$. [3]

(c) Find $\frac{\text{d}A}{\text{d}x}$. [2]

(d) Hence find the **exact** value of x for which the area is a maximum. [2]

(This question continues on the following page)

(Question 12 continued)

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will not be marked.

Question 12 - working space

3. [Maximum mark: 13]

A shop uses the following model to estimate n, the number of smoothies sold per day, in terms of x, the price of a single smoothie in pesos.

$$n = \frac{40\,000}{x^2}$$

The maximum number of smoothies the shop can make in a day is 400.

(a) Find the maximum price they could charge per smoothie for the shop to sell 400 in one day. [2]

(b) On a day when the shop sells smoothies at 50 pesos each, use the model to find

(i) the number of smoothies sold.

(ii) the total income from the smoothies sold. [2]

The cost of making each smoothie is 20 pesos. The profit per day (P) is the total income from the sale of smoothies that day minus the cost of making them.

(c) (i) Show that, according to the model, $P = \frac{40\,000}{x} - \frac{800\,000}{x^2}$.

(ii) Find $\frac{\mathrm{d}P}{\mathrm{d}x}$.

(iii) Find the value of x for which $\frac{\mathrm{d}P}{\mathrm{d}x} = 0$.

(iv) Find the number of smoothies sold when the profit is maximized. [9]

Turn over

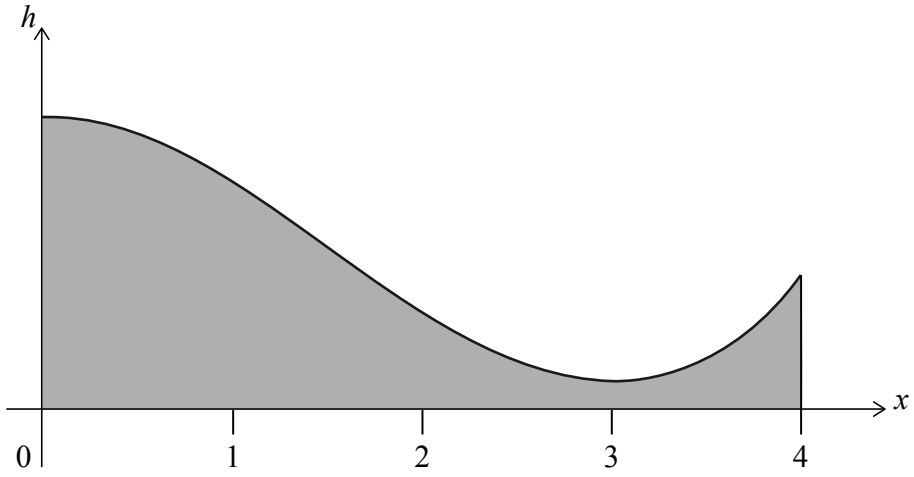
Question 3 - working space

5. [Maximum mark: 21]

Andrew designs a fibreglass water slide for a water park.

Let x be the horizontal distance, in metres, from the start of the water slide.
Let h be the height, in metres, of the water slide above the horizontal ground.

The following diagram shows a cross section of the water slide. The water slide will be supported by a concrete structure, represented by the shaded region in the diagram.



It is known that $h'(x) = 0.6x^2 - 1.8x$.

(a) Find the gradient of the water slide when $x = 2$. [2]

At the exit point of the water slide, $x = 4$ and $h = 1.4$.

(b) Find an expression for $h(x)$. [4]

(c) (i) Write down an integral that can be used to find the area of the shaded region.

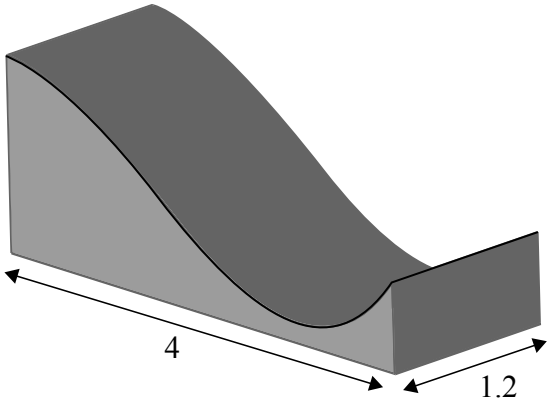
(ii) Find the cross-sectional area of the concrete structure. [4]

(This question continues on the following page)

(Question 5 continued)

The water slide and concrete structure have a uniform cross section and a width of 1.2 metres, as shown in the following diagram.

diagram not to scale



(d) Find the volume of the concrete structure. [1]

To comply with safety regulations, Andrew updates this design. The start of the design will stay the same. Then, over the domain $2 \leq x \leq 4$, the updated design must satisfy the following conditions.

$$h(x) = ax^2 + bx + c \qquad h(4) = 1 \qquad h'(2) = -1.2 \qquad h'(3) = 0$$

(e) Write down **three** equations, in terms of a, b and c, using the given conditions. [6]

(f) Hence find the values of a, b and c. [2]

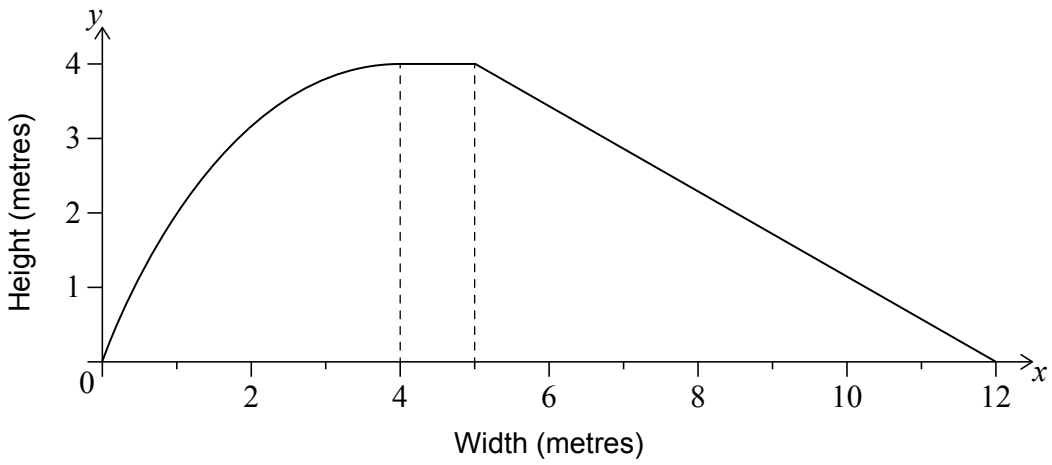
Andrew claims the updated design will still provide a smooth path for a person using the slide.

(g) Explain why Andrew's claim is correct at the point where $x = 2$. [2]

Question 5 - working space

3. [Maximum mark: 15]

The following diagram shows a model of the side view of a water slide. All lengths are measured in metres.



The curved edge of the slide is modelled by

$$f(x) = -\frac{1}{4}x^2 + 2x \text{ for } 0 \le x \le 4.$$

The remainder of the slide is modelled by

$$g(x) = \begin{cases} 4, \text{ for } 4 \le x \le 5 \\ \frac{48}{7} - \frac{4x}{7}, \text{ for } 5 \le x \le 12 \end{cases}$$

(a) Use the trapezoidal rule with an interval width of 1 to calculate the approximate area under the model of the slide in the interval $0 \le x \le 4$. [5]

(b) Find $\int\left(-\frac{1}{4}x^2 + 2x\right)\mathrm{d}x$. [3]

(c) Calculate the exact area under the entire model of the slide, for $0 \le x \le 12$. [4]

(d) Find the percentage error in the **total** area under the entire model of the slide when using the approximate value from part (a). [3]

Turn over

Question 3 - working space

12. [Maximum mark: 7]

Consider the curve $y = 5x^2 - \frac{3}{x^3}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Write down the gradient of the curve at $x = 1$. [1]

(c) Hence, find the equation of the normal to the curve at $x = 1$. [3]

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Question 12 - working space

12. [Maximum mark: 7]

Consider the curve $y = 4x^3 - \frac{2}{x^2}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Write down the gradient of the curve at $x = 1$. [1]

(c) Hence, find the equation of the normal to the curve at $x = 1$. [3]

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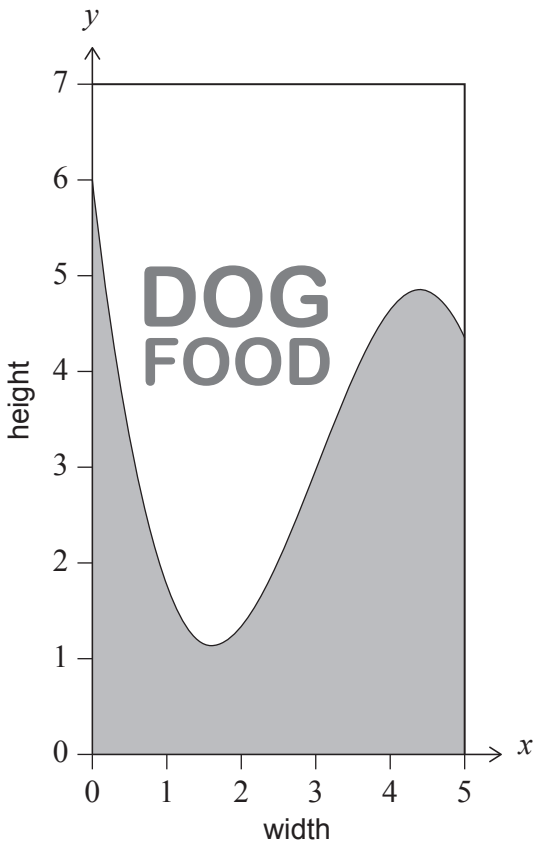
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Question 12 - working space

4. [Maximum mark: 20]

A type of dog food is sold in cylindrical containers. A new label for the container is being considered. The label will be a rectangle that is $5\,\text{cm}$ wide and $7\,\text{cm}$ high.

The design on the label is a curve, as shown on the following axes, where one unit represents $1\,\text{cm}$ for both axes.



The values in the table approximate points on the curve, correct to one decimal place.

Width, x	0	1	2	3	4	5
Height, y	6	1.7	1.3	3.0	4.7	4.3

(a) Use the trapezoidal rule with five intervals, and the values given in the table, to estimate the shaded area below the curve. [3]

The curve used in the label design can be modelled by:

$$y = -\frac{x^3}{3} + 3x^2 - 7x + 6, \text{ for } 0 \le x \le 5.$$

(b) (i) Write down the integral that represents the area of the shaded region.

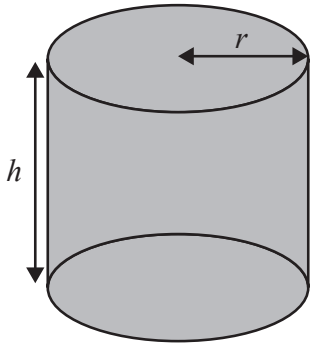
(ii) Hence, find the area of the shaded region. [4]

(This question continues on the following page)

(Question 4 continued)

The dog food is sold in closed cylindrical containers, with radius r and height h.

diagram not to scale



The whole container is made from one type of material, and it is assumed that the thickness of the material is negligible.

Each container has a volume of $400\,\text{cm}^3$.

(c) Write down an equation, in terms of r and h, that shows this information. [1]

The amount of material used for each container can be modelled by the external surface area of the container.

The external surface area, A, of the container can be expressed as

$$A = 2\pi r^2 + \frac{k}{r}, \text{ where } r > 0.$$

(d) Find the value of k. [4]

(e) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}r}$.

(ii) Given that A has a minimum value, find the value of r that will minimize the material used. [5]

The containers are made so that the surface area is minimized. The $5\,\text{cm}$ by $7\,\text{cm}$ rectangular label is to be glued to the curved surface of the container.

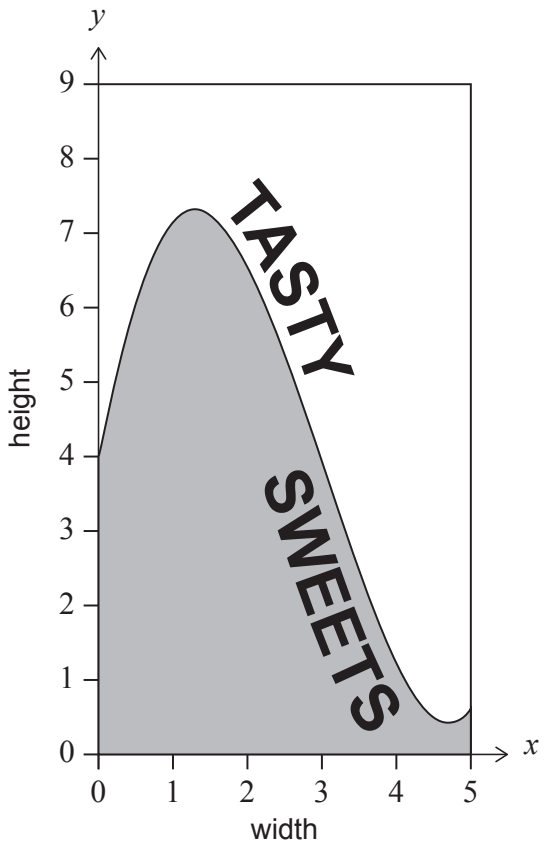
(f) Show that the label will fit on the container. [3]

Question 4 - working space

4. [Maximum mark: 20]

Sweets are sold in cylindrical containers. A new label for the container is being considered. The label will be a rectangle that is $5\,\text{cm}$ wide and $9\,\text{cm}$ high.

The design on the label is a curve, as shown on the following axes, where one unit represents $1\,\text{cm}$ for both axes.



The values in the table approximate points on the curve, correct to one decimal place.

Width, x	0	1	2	3	4	5
Height, y	4	7.3	6.7	4.0	1.3	0.7

(a) Use the trapezoidal rule with five intervals, and the values given in the table, to estimate the shaded area below the curve. [3]

The curve used in the label design can be modelled by:

$$y = \frac{x^3}{3} - 3x^2 + 6x + 4 \text{, for } 0 \le x \le 5 .$$

(b) (i) Write down the integral that represents the area of the shaded region.

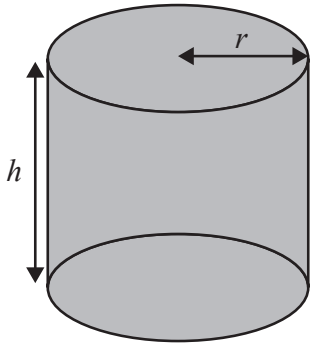
(ii) Hence, find the area of the shaded region. [4]

(This question continues on the following page)

(Question 4 continued)

The sweets are sold in closed cylindrical containers, with radius r and height h.

diagram not to scale



The whole container is made from one type of material, and it is assumed that the thickness of the material is negligible.

Each container has a volume of $600\,\text{cm}^3$.

(c) Write down an equation, in terms of r and h, that shows this information. [1]

The amount of material used for each container can be modelled by the external surface area of the container.

The external surface area, A, of the container can be expressed as

$$A = 2\pi r^2 + \frac{k}{r}, \text{ where } r > 0.$$

(d) Find the value of k. [4]

(e) (i) Find $\frac{\mathrm{d}A}{\mathrm{d}r}$.

(ii) Given that A has a minimum value, find the value of r that will minimize the material used. [5]

The containers are made so that the surface area is minimized. The $5\,\text{cm}$ by $9\,\text{cm}$ rectangular label is to be glued to the curved surface of the container.

(f) Show that the label will fit on the container. [3]

Question 4 - working space

8. [Maximum mark: 7]

Consider a function of the form $f(x) = 2x^{-2} + bx^{-1} + c$, where b and c are constants.
The graph of $y = f(x)$ has a gradient of 0.208 at $x = 5$.

(a) Determine the value of b. [5]

(b) Show that $f(x)$ is increasing at $x = 3.5$ [2]

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Question 8 - working space

9. [Maximum mark: 4]

Consider the function $f(x) = x^3 - 9x^2 + 23x - 15$ for $0 \leq x \leq 5$.

The graph of $f(x)$ crosses the x-axis when $x = 1$, $x = 5$ and $x = a$.

(a) Find the value of a. [2]

(b) Use your graphic display calculator to find the area enclosed by the curve $y = f(x)$ and the x-axis, when $1 \leq x \leq a$. [2]

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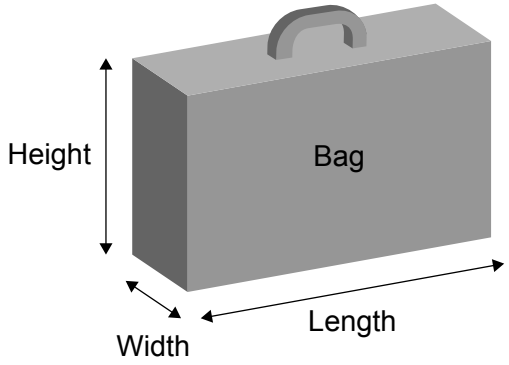
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Question 9 - working space

12. [Maximum mark: 9]

An airline restricts the size of bags by stating that the sum of a bag's width, length, and height (ignoring the handle) must not exceed a fixed value, M.

diagram not to scale



$$\text{Width} + \text{Length} + \text{Height} \leq M$$

A company is designing a cuboid-shaped bag so that the sum of the bag's width, length, and height **equals** M. Their design will have a length equal to three times its width.

As the width of the design varies, its volume changes according to the equation

$$\frac{\mathrm{d}V}{\mathrm{d}w} = 690w - 36w^2,$$

where w is the width in centimetres and V is the volume in cubic centimetres.

(a) Use your graphic display calculator to find the value of w that will produce the maximum volume. [2]

(b) Show that the maximum volume of the bag to three significant figures is $42\,200\,\mathrm{cm}^3$. [4]

(c) Hence find the value of M. [3]

(This question continues on the following page)

(Question 12 continued)

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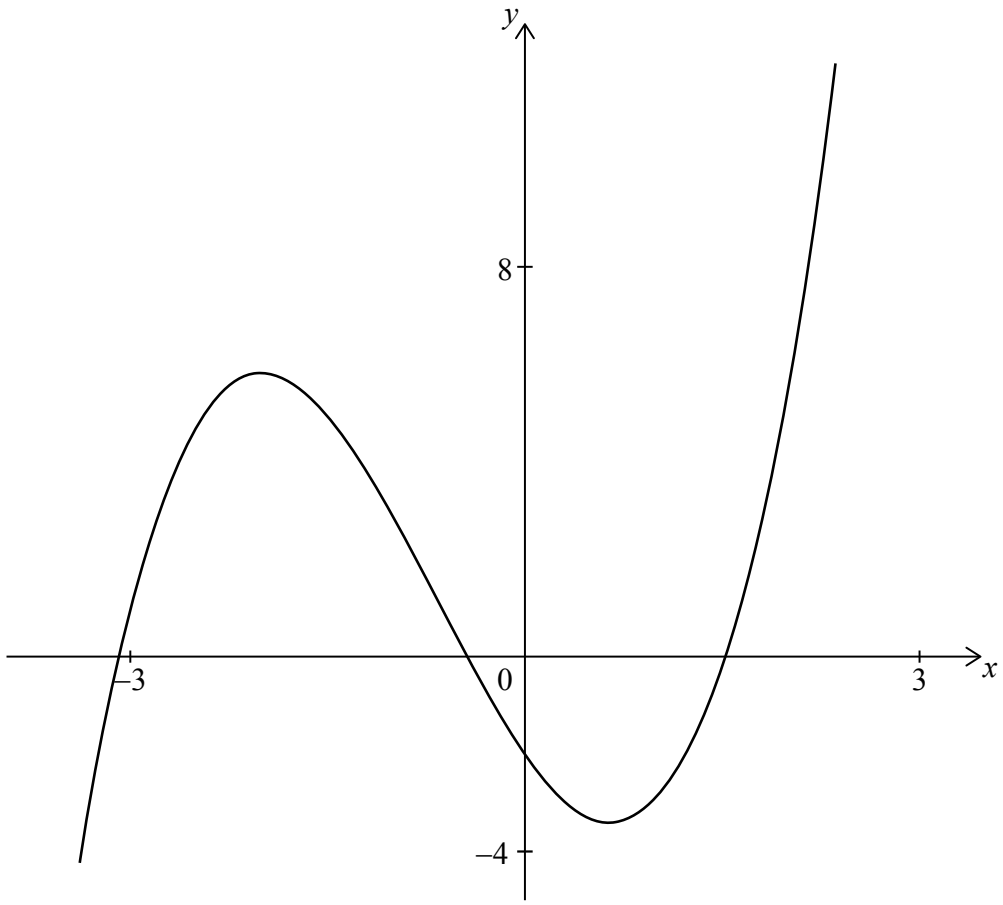
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Question 12 - working space

4. [Maximum mark: 5]

Consider the graph of the cubic function $f(x) = x^3 + 2x^2 - 4x - 2$. Part of the graph of $y = f(x)$ is shown in the following diagram.



(a) Write down the x-coordinate of

(i) the local maximum.

(ii) the local minimum. [2]

(b) Hence, write down the interval where the function is decreasing. [1]

The tangent to the curve at $(1, -3)$ is parallel to the straight line $y = 3x + 5$.

(c) Write down

(i) the gradient of the tangent.

(ii) the equation of the tangent. [2]

(This question continues on the following page)

(Question 4 continued)

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Question 4 - working space

9. [Maximum mark: 7]

Pascale owns a company that produces and sells curry powder. The rate of change of the company's profit, P, in Mauritian rupees (MUR) from producing x kilograms (kg) of curry powder is modelled by

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -10x + 460,\ x \geq 0.$$

She makes a profit of $3300\,\mathrm{MUR}$ when producing $10\,\mathrm{kg}$ of curry powder.

(a) Find an expression for the company's profit, P, in terms of x. [5]

Pascale decides to increase the production of curry powder from $25\,\mathrm{kg}$ to $50\,\mathrm{kg}$.

(b) Find the increase in profit. [2]

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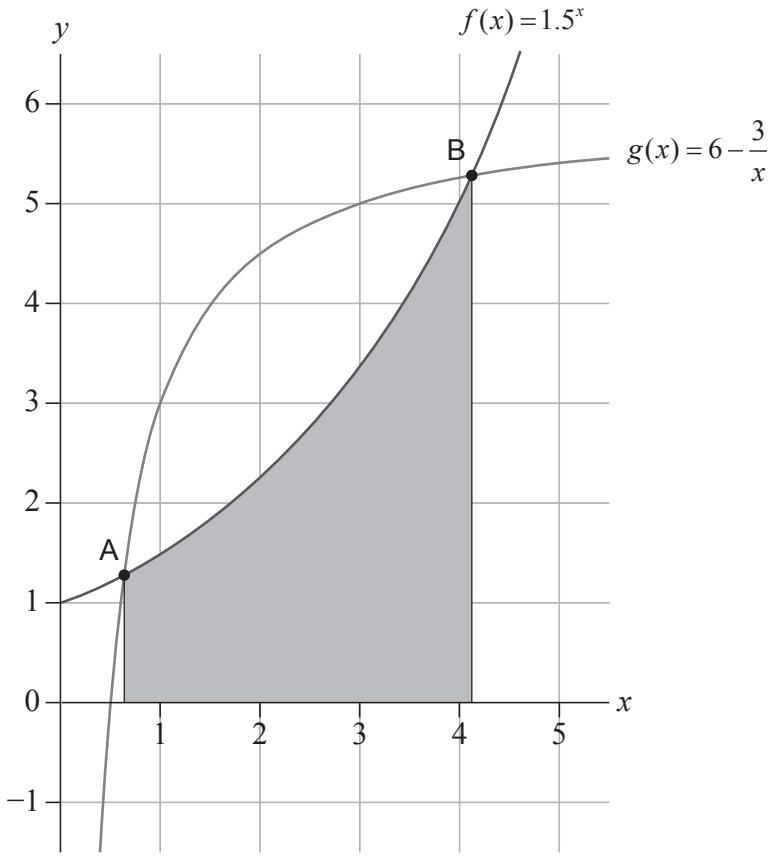
Question 9 - working space

2. [Maximum mark: 12]

The diagram shows part of the graphs of the functions

$$f(x) = 1.5^x \qquad x \geq 0$$

$$g(x) = 6 - \frac{3}{x} \qquad x > 0\,.$$



(a) Solve $f(x) = g(x)$. [3]

(b) (i) Write down the integral that represents the area of the shaded region.

(ii) Calculate the area of this shaded region.

(iii) Hence, or otherwise, calculate the area of the region enclosed between the curves $y = f(x)$ and $y = g(x)$. [6]

The tangent to the graph of $y = f(x)$ is parallel to the tangent to the graph of $y = g(x)$ at $x = k$.

(c) Find the value of k. [3]

Question 2 - working space

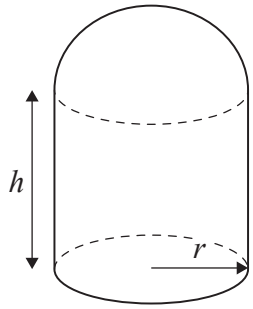
5. [Maximum mark: 16]

Ju Shen designs a plastic cover, in the shape of a cylinder combined with a hemisphere on top, as shown in the diagram.

The plastic used to make the cover forms the curved surface of both the hemisphere and the cylinder; there is no bottom to the cover, however it stands on a flat horizontal surface.

Let the height of the cylinder be $h\,\text{cm}$ and the internal radius of its base be $r\,\text{cm}$.

diagram not to scale



(a) Find an expression for the total internal surface area, $A\,\text{cm}^2$, of the cover in terms of r and h. [2]

(b) Show that the total volume, $V\,\text{cm}^3$, enclosed by the cover and the horizontal surface is given by the expression

$$V = \frac{2\pi r^3 + 3\pi r^2 h}{3}.$$ [2]

The total volume enclosed by the cover is $10\,000\,\text{cm}^3$.

(c) Hence show that $h = \frac{30\,000 - 2\pi r^3}{3\pi r^2}$. [2]

Ju Shen uses the total internal surface area to model the amount of plastic used to construct the cover.

(d) Show that A is given by the expression

$$A = \frac{2\pi r^2}{3} + \frac{20\,000}{r}.$$ [2]

(This question continues on the following page)

(Question 5 continued)

Ju Shen wants to use the minimum amount of plastic in the construction of the cover.

(e) Find an expression for $\frac{dA}{dr}$. [3]

(f) Find the value of r and the value of h that minimizes the use of plastic. [4]

(g) By interpreting your answer to part (f), suggest the best shape for Ju Shen's plastic cover. [1]

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References:

1. naisupakit, 2016. *Triangle Pillow tradition native Thai style pillow*. [image online] Available at: https://www.gettyimages.co.uk/detail/photo/triangle-pillow-tradition-native-thai-style-pillow-royalty-free-image/623127206 [Accessed 9 April 2024]. Source adapted.

4. bortonia, 2020. *Antique Grandfather Clock Icon on Transparent Background*. [image online] Available at: https://www.gettyimages.co.uk/detail/illustration/antique-grandfather-clock-icon-on-royalty-free-illustration/1284116855 [Accessed 9 April 2024]. Source adapted.

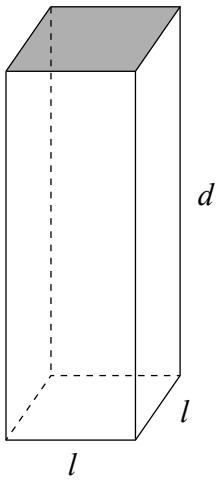
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Question 5 - working space

3. [Maximum mark: 20]

Kailash manufactures drink containers in the shape of a cuboid. The container has a square top and a square base of length, $l\,\text{cm}$. Its height, $d\,\text{cm}$, is three times the length of the base.

diagram not to scale



(a) Write down an expression for d in terms of l. [1]

The container can hold $375\,\text{cm}^3$ of drink.

(b) Find the value of l and d. [3]

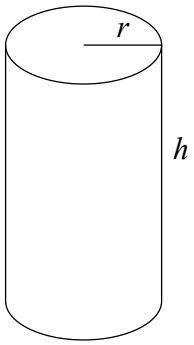
(c) Calculate the total external surface area of the container. [3]

(This question continues on the following page)

(Question 3 continued)

To reduce environmental impact, Kailash is trying to minimize the amount of material needed for the production of the $375\,\text{cm}^3$ container.

He is willing to change the shape to a cylinder with radius $r\,\text{cm}$, and height $h\,\text{cm}$, as shown below.



The cylindrical container of drink must also hold $375\,\text{cm}^3$.

(d) Find an expression for the height, h, of the container in terms of r. [2]

Let the total external surface area be $A\,\text{cm}^2$.

(e) Show that $A = 2\pi r^2 + \frac{750}{r}$. [2]

(f) Find $\frac{\text{d}A}{\text{d}r}$. [3]

(g) Hence or otherwise

(i) find the value of r that will minimize A.

(ii) find the minimum value of A needed for the cylinder. [3]

To produce the containers, additional material is required:

- $10\,\%$ additional surface area for the cuboid
- $25\,\%$ additional surface area for the cylinder.

Kailash will choose the container that requires the least total amount of material.

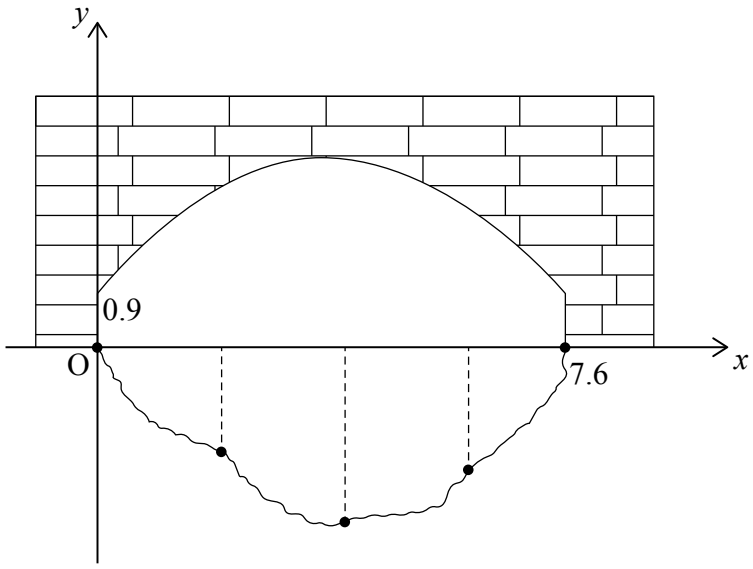
(h) Determine which container Kailash should choose. Justify your answer. [3]

Question 3 - working space

5. [Maximum mark: 12]

The diagram shows the cross-section of a bridge and a river. A coordinate system has been added with the origin, O, at the point where the bridge meets the water on one side. All units are in metres.

diagram not to scale



A researcher wants to calculate the volume of water that flows under the bridge. To do this he takes measurements of the depth every $1.9\,\mathrm{m}$ from O. The depths are shown in the following table.

Horizontal distance from O in metres	0	1.9	3.8	5.7	7.6
Vertical depth of water in metres	0	1.68	2.81	2.32	0

(a) Use the trapezoidal rule to find the cross-sectional area of the river as it passes under the bridge. [3]

The water flows under the bridge at a rate of $0.3\,\mathrm{m\,s^{-1}}$.

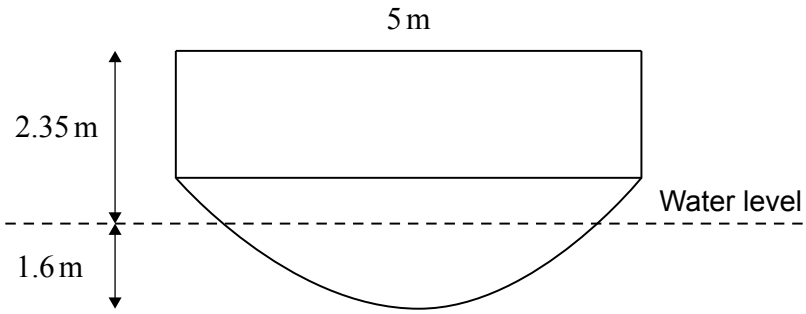
(b) Find the volume of water that passes under the bridge each second. [2]

(This question continues on the following page)

(Question 5 continued)

A boat is travelling along the river. The cross-section of the boat and the water level is shown in the following diagram.

The top of the boat is parallel to the water level and has a width of $5\,\text{m}$. The height of the boat is $2.35\,\text{m}$ above the water level and the lowest part of the boat is $1.6\,\text{m}$ below the water level.



The boat is travelling down the centre of the river.

(c) Find the vertical distance between the lowest part of the boat and the bottom of the river as it passes under the bridge. [1]

The curved arch of the bridge can be modelled by the equation

$$y = -0.15x^2 + 1.14x + 0.9,\ 0 \le x \le 7.6.$$

(d) Find the maximum height of the curved arch above the water level. [2]

(e) Determine whether the top of the boat will be able to pass under the bridge. [4]

Question 5 - working space

11. [Maximum mark: 8]

Consider the function $B(x) = 20 + \frac{32}{x} + 2x$, where $x \geq 1$, $x \in \mathbb{R}$.

(a) (i) Find $B'(x)$.

(ii) Use your answer to part (a)(i) to find the minimum value of $B(x)$. You may assume that $B(x)$ has no local maximum point over the given domain. [6]

Kim goes to a restaurant that offers a large family banquet. The time, T, in minutes, to prepare the family banquet depends on the number of chefs, n.

To predict the value of T, Kim uses the model $T(n) = 20 + \frac{32}{n} + 2n$, where $n \in \mathbb{Z}^+$.

The restaurant informs Kim that the time taken to prepare the family banquet is less than 40 minutes. There are k chefs preparing the family banquet.

(b) Find the possible values of k. [2]

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Question 11 - working space

9. [Maximum mark: 6]

Consider the function $f(x) = x^3 - 2x^2 + 5x - 7$.

(a) Find $f(1)$. [2]

(b) Find $f'(x)$. [2]

(c) Find the equation of the line that is tangent to the curve $y = f(x)$ at $x = 1$. [2]

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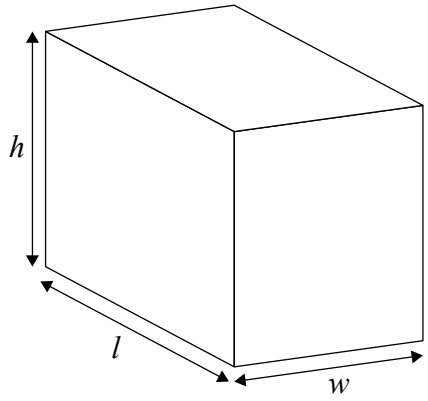
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Question 9 - working space

13. [Maximum mark: 5]

Soraya is designing a gift box in the shape of a rectangular prism, as shown in the diagram.

diagram not to scale



The length, l, of the box is 1.5 times the width, w, of the box. This can be expressed as $l = 1.5w$. The height, h, of the box is also labelled on the diagram.

The box has a volume of $10\,000\,\text{cm}^3$. Soraya wants to minimize the surface area of the box. Find the width of the box.

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(Question 13 continued)

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Question 13 - working space

2. [Maximum mark: 16]

The height of a plant, $h\,\text{cm}$, depends on the time, t days, after it is planted.

For the first 15 days, the rate of change of the height of the plant is modelled as

$$\frac{\mathrm{d}h}{\mathrm{d}t} = 1.2 - 0.1t \text{, where } 0 \leq t \leq 15.$$

(a) (i) Find the value of $\frac{\mathrm{d}h}{\mathrm{d}t}$ at $t = 3$.

(ii) Interpret the meaning of your answer to part (a)(i) in context. [2]

(b) Use $\frac{\mathrm{d}h}{\mathrm{d}t}$ to find the value of t when the plant reaches its maximum height. [2]

Two days after it is planted, the height of the plant is $6.5\,\text{cm}$.

(c) Find an expression for h in terms of t, for $0 \leq t \leq 15$. [5]

(d) Hence, find the maximum height of the plant. [2]

A second plant was planted at the same time as the first plant. Its height, $H\,\text{cm}$, is modelled by $H = 2(1.2)^t$, where $0 \leq t \leq 15$.

(e) Write down the initial height of the second plant. [1]

Each day, the height of the second plant increases by $p\%$.

(f) Find the value of p. [2]

(g) Find the value of t when the two plants have the same height. [2]

Question 2 - working space

5. [Maximum mark: 18]

Zheng models the height of a section of a rollercoaster.

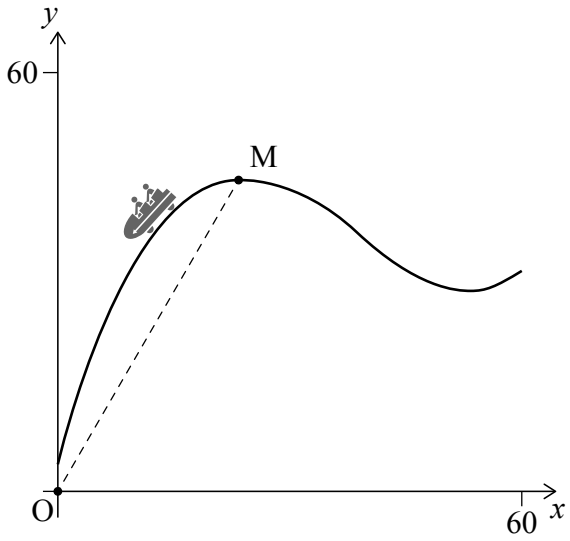
She uses the function

$$f(x) = 0.001x^3 - 0.12x^2 + 4.05x + 3, \text{ for } 0 \le x \le 60,$$

where x is the horizontal distance from the left-hand end of the section and $f(x)$ is the vertical height. Distances are measured in metres.

Zheng's model is shown on the following graph of $y = f(x)$.

diagram not to scale



Point O has coordinates $(0, 0)$.

(a) (i) Write down the definite integral for the exact area between the curve and the x-axis.

(ii) Calculate this area. [4]

A light beam from O passes through the local maximum at point M.

(b) (i) Find the coordinates of M.

(ii) Find the angle of elevation from O to M. [4]

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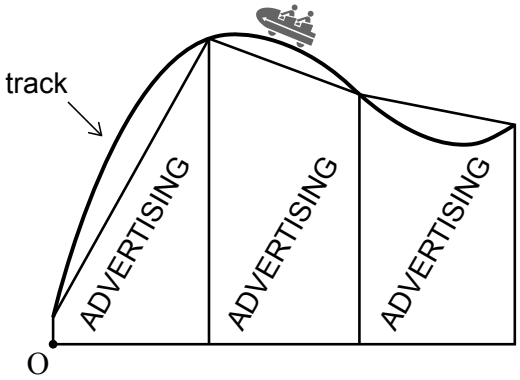
(Question 5 continued)

Zheng wants to place advertising boards in front of this section of the rollercoaster.
At four positions along the rollercoaster track, she records the horizontal distance from O and the vertical height of the track.

Her values are shown in the table and represented on the graph.

x **(horizontal distance)**	0	20	40	60
y **(vertical distance)**	3	44	37	30

diagram not to scale



(c) Find the total area of the advertising boards. [3]

The cost of space on the advertising boards per square metre varies according to the amount purchased.

The **rate of change** in the cost, in dollars ($\$$), for space on the advertising boards can be modelled by the function

$$C'(a) = -0.00292a + 4.71, \text{ for } a > 60,$$

where a is the total square metres purchased.

A company, E-Byte, wishes to advertise its new gaming device. Zheng tells E-Byte it will cost $\$1750$ for $500\,\mathrm{m}^2$ of space on the advertising boards.

(d) Find the cost function $C(a)$. [5]

E-Byte instead wants to purchase the entire space of all three boards.

(e) Using the cost function from part (d), estimate how much this would cost E-Byte. [2]

Question 5 - working space