

APOLLO

Statistics and probability

SL

Mathematics: analysis and approaches

First assessment 2021

81 questions | mark schemes

4. (a) attempt to use definition of outlier (M1)
 $1.5 \times 20 + Q_3$ A1
 $1.5 \times 20 + U \geq 75$ ($\Rightarrow U \geq 45$, accept $U > 45$) OR $1.5 \times 20 + Q_3 = 75$ A1
 minimum value of $U = 45$ A1
 [3 marks]
- (b) attempt to use interquartile range (M1)
 $U - L = 20$ (may be seen in part (a)) OR $L \geq 25$ (accept $L > 25$) A1
 minimum value of $L = 25$
 [2 marks]
 Total [5 marks]

9. (a) recognising probabilities sum to 1 (M1)

$$p + p + p + \frac{1}{2}p = 1$$

$$p = \frac{2}{7}$$

A1

[2 marks]

- (b) valid attempt to find $E(X)$ (M1)

$$1 \times p + 2 \times p + 3 \times p + 4 \times \frac{1}{2}p (= 8p)$$

$$E(X) = \frac{16}{7}$$

A1

[2 marks]

- (c) (i) $0 \leq r \leq 1$ A1

- (ii) attempt to find a value of q (M1)

$$0 \leq 1 - 3q \leq 1 \quad \text{OR} \quad r = 0 \Rightarrow q = \frac{1}{3} \quad \text{OR} \quad r = 1 \Rightarrow q = 0$$

$$0 \leq q \leq \frac{1}{3}$$

A1

[3 marks]

- (d) $E(Y) = 1 \times q + 2 \times q + 3 \times q + 4 \times r (= 2 + 2r \text{ OR } 4 - 6q)$ (A1)

one correct boundary value

A1

$$1 \times \frac{1}{3} + 2 \times \frac{1}{3} + 3 \times \frac{1}{3} + 4 \times 0 (= 2) \text{ OR}$$

$$1 \times 0 + 2 \times 0 + 3 \times 0 + 4 \times 1 (= 4) \text{ OR}$$

$$2 + 2(0) (= 2) \text{ OR}$$

$$2 + 2(1) (= 4) \text{ OR}$$

$$4 - 6(0) (= 4) \text{ OR } 4 - 6\left(\frac{1}{3}\right) (= 2)$$

$$2 \leq E(Y) \leq 4$$

A1

[3 marks]

continued...

Question 9 continued(e) **METHOD 1**evidence of choosing at least four correct outcomes from
1&2, 1&3, 1&4, 2&3, 2&4, 3&4**(M1)**

$$\frac{6}{7}q + \frac{6}{7}r \text{ OR } 3pq + 3pr \text{ OR } pq + pq + p(1-3q) + pq + p(1-3q) + p(1-3q) \text{ (A1)}$$

solving for either q or r **M1**

$$\frac{6}{7}(q+1-3q) = \frac{1}{2} \text{ OR } \frac{6}{7}\left(\frac{1-r}{3} + r\right) = \frac{1}{2} \text{ OR } 3pq + 3p(1-3q) = \frac{1}{2}$$

$$\text{OR } 3p\left(\frac{1-r}{3}\right) + 3pr = \frac{1}{2}$$

EITHER two correct values

$$q = \frac{5}{24} \text{ and } r = \frac{3}{8}$$

A1A1**OR** one correct value

$$q = \frac{5}{24} \text{ OR } r = \frac{3}{8}$$

A1substituting their value for q or r **A1**

$$4 - 6\left(\frac{5}{24}\right) \text{ OR } 2 + 2\left(\frac{3}{8}\right)$$

THEN

$$E(Y) = \frac{11}{4}$$

A1**[6 marks]***continued...*

*Question 9 continued***METHOD 2** (solving for $E(Y)$)

evidence of choosing at least four correct outcomes from

1&2, 1&3, 1&4, 2&3, 2&4, 3&4

(M1)

$$\frac{6}{7}q + \frac{6}{7}r \text{ OR } 3pq + 3pr \text{ OR } pq + pq + p(1-3q) + pq + p(1-3q) + p(1-3q) \text{ (A1)}$$

rearranging to make q the subject**M1**

$$q = \frac{4 - E(Y)}{6}$$

$$3pq + 3p(1-3q) = \frac{1}{2}$$

M1

$$\frac{6}{7} \times \left(\frac{4 - E(Y)}{6} \right) + \frac{6}{7} \left(1 - 3 \left(\frac{4 - E(Y)}{6} \right) \right) = \frac{1}{2}$$

A1

$$\frac{2(E(Y) - 1)}{7} = \frac{1}{2}$$

$$E(Y) = \frac{11}{4}$$

A1**[6 marks]****Total [16 marks]**

7. (a) evidence of median position (M1)
 40 students
 median = 14 (hours) A1
[2 marks]
- (b) recognizing there are 8 students in the top 10% (M1)
 72 students spent less than k hours (A1)
 $k = 18$ (hours) A1
[3 marks]
- (c) 15 hours is 60 students OR $p = 60 - 4$ (M1)
 $p = 56$ A1
 21 hours is 76 students OR $q = 80 - 76$ OR $q = 80 - 4 - 56 - 16$ (A1)
 $q = 4$ A1
[4 marks]
- (d) 20 of the 80 students OR $\frac{1}{4}$ spend more than 15 hours doing homework (A1)
 $\frac{20}{80} = \frac{x}{320}$ OR $\frac{1}{4} \times 320$ OR 4×20 (A1)
 80 (students) A1
[3 marks]
- (e) (i) only year 12 students surveyed OR amount of homework might be different for different year levels R1
- (ii) stratified sampling OR survey students in all years R1
[2 marks]
- Total [14 marks]**

2. (a) $a = 0.433156\dots$, $b = 4.50265\dots$

$$a = 0.433, b = 4.50$$

A1A1

[2 marks]

- (b) attempt to substitute $x = 18$ into their equation

(M1)

$$y = 0.433 \times 18 + 4.50$$

$$= 12.2994\dots$$

$$= 12.3$$

A1

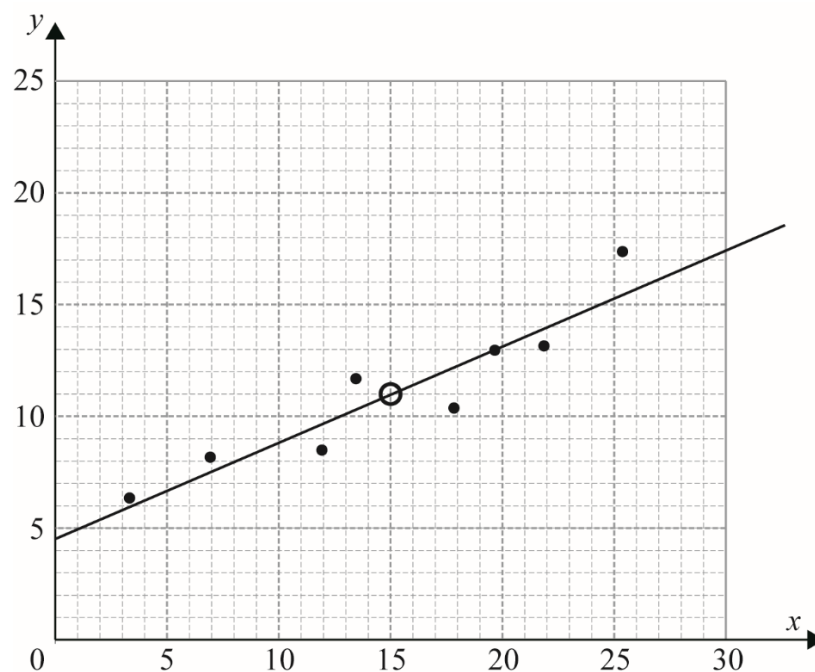
[2 marks]

- (c) $\bar{x} = 15$, $\bar{y} = 11$

A1

[1 mark]

- (d)



A1A1

Note: Award marks as follows:

A1 for a straight line going through (15,11)

A1 for intercepting the y-axis between their $b \pm 1.5$ (when their line is extended), which includes all the data for $3.3 \leq x \leq 25.3$.

If the candidate does not use a ruler, award **A0A1** where appropriate.

[2 marks]

Total [7 marks]

3.

Note: In this question, do not penalise incorrect use of strict inequality signs.

Let X = mass of a bag of sugar

(a) evidence of identifying the correct area **(M1)**

$$P(X < 995) = 0.0765637...$$

$$= 0.0766$$

A1**[2 marks]**

(b) 0.0766×100
 ≈ 8

A1**[1 mark]**

Note: Accept 7.66.

(c) recognition that $P(X > 1005 | X \geq 995)$ is required **(M1)**

$$\frac{P(X \geq 995 \cap X > 1005)}{P(X \geq 995)}$$

$$\frac{P(X > 1005)}{P(X \geq 995)}$$

(A1)

$$\frac{0.0765637...}{1 - 0.0765637...} \left(= \frac{0.0765637...}{0.923436...} \right)$$

$$= 0.0829$$

A1**[3 marks]****Total [6 marks]**

1. (a) (i) $a = 0.805084\dots$ and $b = 2.88135\dots$
 $a = 0.805$ and $b = 2.88$

A1A1

- (ii) $r = 0.97777\dots$
 $r = 0.978$

A1

[3 marks]

- (b) a represents the (average) increase in waiting time (0.805 mins) per additional customer (waiting to receive their coffee)

R1

[1 mark]

- (c) attempt to substitute $x = 7$ into their equation

(M1)

8.51693...

8.52 (mins)

A1

[2 marks]

Total [6 marks]

4. (a) **EITHER**

$$P(S) + P(T) + P(S' \cap T') - P(S \cap T) = 1 \text{ OR } P(S \cup T) = P((S' \cap T')') \quad (M1)$$

$$0.7 + 0.2 + 0.18 - P(S \cap T) = 1 \text{ OR } P(S \cup T) = 1 - 0.18$$

OR

a clearly labelled Venn diagram (M1)

THEN

$$P(S \cap T) = 0.08 \text{ (accept 8\%)} \quad A1$$

Note: To obtain the **M1** for the Venn diagram all labels must be correct and in the correct sections. For example, do not accept 0.7 in the area corresponding to $S \cap T'$.

[2 marks]

(b) **EITHER**

$$P(T \cap S') = P(T) - P(T \cap S) (= 0.2 - 0.08) \text{ OR}$$

$$P(T \cap S') = P(T \cup S) - P(S) (= 0.82 - 0.7)$$

(M1)

OR

a clearly labelled Venn diagram including $P(S)$, $P(T)$ and $P(S \cap T)$ (M1)

THEN

$$= 0.12 \text{ (accept 12\%)} \quad A1$$

[2 marks]

continued...

Question 4 continued

(c) $P(G \cap T) = P(T | G)P(G) \quad (0.25 \times 0.48)$ **(M1)**
 $= 0.12$ **A1**

[2 marks](d) **METHOD 1**

$$P(G) \times P(T) (= 0.48 \times 0.2) = 0.096$$
 A1

$$P(G) \times P(T) \neq P(G \cap T) \Rightarrow G \text{ and } T \text{ are not independent}$$
 R1

METHOD 2

$$P(T | G) = 0.25$$
 A1

$$P(T | G) \neq P(T) \Rightarrow G \text{ and } T \text{ are not independent}$$
 R1

Note: Do not award **A0R1**.
[2 marks]**Total [8 marks]**

8. (a) use of inverse normal to find z -score (M1)

$$z = 2.0537...$$

$$2.0537... = \frac{82 - 75}{\sigma} \quad (A1)$$

$$\sigma = 3.408401...$$

$$\sigma = 3.41$$

A1

[3 marks]

- (b) evidence of identifying the correct area under the normal curve (M1)

$$P(T > 80) = 0.071193...$$

$$P(T > 80) = 0.0712$$

A1

[2 marks]

- (c) recognition that $P(80 < T < 82)$ is required (M1)

$$P(T < 82 | T > 80) = \frac{P(80 < T < 82)}{P(T > 80)} = \left(\frac{0.051193...}{0.071193...} \right) \quad (M1)(A1)$$

$$= 0.719075...$$

$$= 0.719$$

A1

[4 marks]

continued...

Question 8 continued

(d) recognition of binomial probability **(M1)**

$$X \sim B(64, 0.071193\dots) \text{ or } E(X) = 64 \times 0.071193\dots \quad \textbf{(A1)}$$

$$E(X) = 4.556353\dots$$

$$E(X) = 4.56 \text{ (flights)}$$

A1**[3 marks]**

(e) $P(X > 6) = P(X \geq 7) = 1 - P(X \leq 6)$ **(M1)**

$$= 1 - 0.83088\dots$$

(A1)

$$= 0.1691196\dots$$

$$= 0.169$$

A1**[3 marks]****Total [15 marks]**

9. (a) considering that sum of probabilities is 1 (M1)
 $0.85 + c + 0.03 + 0.002 + 0.0001 = 1$
 0.1179 A1
[2 marks]

- (b) valid attempt to find $E(D)$ (M1)
 $E(D) = (0 \times 0.85) + (2 \times 0.1179) + (10 \times 0.03) + (50 \times 0.002) + (1000 \times 0.0001)$
 $E(D) = 0.7358$ A1
 No, not a fair game A1
 for a fair game, $E(D)$ would be \$2 OR players expected winnings
 are 1.264 R1
[4 marks]

- (c) recognition of GP with $r = 2$ (M1)
 $1000 \times 2^{n-1}$ OR $500(2^n)$ A1
[2 marks]

continued...

Question 9 continued

(d) recognizing $E(D) > 2$ (M1)

correct expression for w^{th} week (or n^{th} week) (A1)

$$(0 \times 0.85) + (2 \times 0.1179) + (10 \times 0.03) + (50 \times 0.002) + (1000 \times 2^{w-1} \times 0.0001)$$

correct inequality (accept equation) (A1)

$$0.6358 + (1000 \times 2^{w-1} \times 0.0001) > 2 \quad \text{OR} \quad 2^{n-1} > 13.642$$

EITHER

$$n-1 > 3.76998 \quad \text{OR} \quad w = 4.76998... \quad (A1)$$

OR

$$E(D) = 1.4358 \text{ in week 4 or } E(D) = 2.2358 \text{ in week 5} \quad (A1)$$

THEN

$$w = 5 \quad A1$$

expected profit per ticket = their $E(D) - 2$ (M1)

$$= 0.2358 \quad A1$$

[7 marks]

Total [15 marks]

4. (a) valid approach to find $P(R)$ (M1)

tree diagram (must include probability of picking box) with correct required probabilities

OR $P(R \cap B_1) + P(R \cap B_2)$ OR $P(R|B_1)P(B_1) + P(R|B_2)P(B_2)$

$$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2}$$

(A1)

$$P(R) = \frac{9}{14}$$

A1

[3 marks]

- (b) events A and R are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box

A2

Note: Both conclusion and reasoning are required. Do not split the A2.

[2 marks]

Total [5 marks]

1. (a) use of GDC to give (M1)

$$r = 0.883529...$$

$$r = 0.884$$

A1

Note: Award the **(M1)** for any correct value of r , a , b or $r^2 = 0.780624...$ seen in part (a) or part (b).

[2 marks]

- (b) $a = 1.36609...$, $b = 64.5171...$

$$a = 1.37, b = 64.5$$

A1

[1 mark]

- (c) attempt to find their difference (M1)

$$5 \times 1.36609... \text{ OR } 1.36609...(h+5) + 64.5171... - (1.36609...h + 64.5171...)$$

$$6.83045...$$

$$= 6.83 \text{ (6.85 from 1.37)}$$

the student could have expected her score to increase by 7 marks.

A1

Note: Accept an increase of 6, 6.83 or 6.85.

[2 marks]

Total [5 marks]

4. (a) recognize that the variable has a Binomial distribution (M1)

$$X \sim B(30, 0.05)$$

attempt to find $P(X \geq 1)$ (M1)

$$1 - P(X = 0) \text{ OR } 1 - 0.95^{30} \text{ OR } 1 - 0.214638... \text{ OR } 0.785361...$$

Note: The two **M** marks are independent of each other.

$$P(X \geq 1) = 0.785 \quad \text{A1}$$

[3 marks]

- (b) recognition of conditional probability (M1)

$$P(X \leq 2 | X \geq 1) \text{ OR } P(\text{at most 2 defective} | \text{at least 1 defective})$$

Note: Recognition must be shown in context either in words or symbols but not just $P(A | B)$.

$$\frac{P(1 \leq X \leq 2)}{P(X \geq 1)} \text{ OR } \frac{P(X = 1) + P(X = 2)}{P(X \geq 1)} \quad \text{(A1)}$$

$$\frac{0.597540...}{0.785361...} \text{ OR } \frac{0.812178... - 0.214638...}{0.785361...} \text{ OR } \frac{0.338903... + 0.258636...}{0.785361...} \quad \text{(A1)}$$

$$= 0.760847...$$

$$P(X \leq 2 | X \geq 1) = 0.761 \quad \text{A1}$$

[4 marks]

Total [7 marks]

$$9. \quad (a) \quad P\left(\frac{\mu - 1.5\sigma - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{\mu + 1.5\sigma - \mu}{\sigma}\right) \quad (M1)$$

$$P(-1.5 < Z < 1.5) \text{ OR } 1 - 2 \times P(Z < -1.5) \quad (A1)$$

$$P(-1.5 < Z < 1.5) = 0.866385\dots$$

$$P(\mu - 1.5\sigma < X < \mu + 1.5\sigma) = 0.866 \quad A1$$

Note: Do not award any marks for use of their answers from part (b).

[3 marks]

$$(b) \quad z_1 = -1.75068\dots \text{ and } z_2 = 1.30468\dots \text{ (seen anywhere)} \quad (A1)$$

correct equations (A1)(A1)

$$\frac{106.2 - \mu}{\sigma} = -1.75068\dots, \quad \mu + 1.30468\dots\sigma = 182.6$$

attempt to solve their equations involving z values (M1)

$$\mu = 149.976\dots, \sigma = 25.0051\dots$$

$$\mu = 150, \sigma = 25.0 \quad A1$$

[5 marks]

$$(c) \quad (i) \quad \text{new sample space is 96\% (may be seen in (ii) or (iii))} \quad (M1)$$

$$P(\text{medium}|\text{not small}) \text{ OR } \frac{0.576}{0.96}$$

$$P(\text{Medium}) = 0.6 \quad A1$$

$$(ii) \quad P(\text{Large}) = 0.3 \quad A1$$

$$(iii) \quad P(\text{Premium}) = 0.1 \quad A1$$

[4 marks]

continued...

Question 9 continued.

(d) attempt to express revenue from avocados (M1)

$$1.1 \times 0.6 + 1.29 \times 0.3 + 1.96 \times 0.1 \text{ OR } 1.243n$$

correct inequality or equation for net profit in terms of n (A1)

$$1.1 \times 0.6n + 1.29 \times 0.3n + 1.96 \times 0.1n - 200 \geq 438 \text{ OR } 1.243n - 200 = 438$$

attempt to solve the inequality (M1)

sketch OR $n = 513.274\dots$

$n = 514$ A1

Note: Only award follow through in part (d) for 3 probabilities which add up to 1. FT of probabilities from c) that do not add up to 1 should only be awarded **M** marks, where appropriate, in d).

[4 marks]

Total [16 marks]

2. attempt to substitute into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ **(M1)**

Note: Accept use of Venn diagram or other valid method.

$$0.6 = 0.5 + 0.4 - P(A \cap B) \quad \text{A1}$$

$$P(A \cap B) = 0.3 \text{ (seen anywhere)} \quad \text{A1}$$

$$\text{attempt to substitute into } P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \text{(M1)}$$

$$= \frac{0.3}{0.4}$$

$$P(A|B) = 0.75 \left(= \frac{3}{4} \right) \quad \text{A1}$$

Total [5 marks]

7. (a) evidence of median position (M1)
80th employee
40 minutes A1
[2 marks]
- (b) valid attempt to find interval (25–55) (M1)
18 (employees), 142 (employees) A1
124 A1
[3 marks]
- (c) recognising that there are 16 employees in the top 10% (M1)
144 employees travelled more than k minutes (A1)
 $k = 56$ A1
[3 marks]
- (d) $b = 70$ A1
[1 mark]
- (e) (i) recognizing a is first quartile value (M1)
40 employees
 $a = 33$ A1
- (ii) $47 - 33$ (M1)
IQR = 14 A1
[4 marks]
- (f) attempt to find $1.5 \times \text{their IQR}$ (M1)
 $33 - 21$
12 (A1)
[2 marks]
- [Total 15 marks]

4. (a) recognition of binomial **(M1)**
 $X \sim B(5, 0.7)$
 attempt to find $P(X \leq 3)$ **M1**
 $= 0.472 (= 0.47178)$ **A1**
[3 marks]
- (b) recognition of 2 sixes in 4 tosses **(M1)**
 $P(\text{3rd six on the 5th toss}) = \left[\binom{4}{2} \times (0.7)^2 \times (0.3)^2 \right] \times 0.7 (= 0.2646 \times 0.7)$ **A1**
 $= 0.185 (= 0.18522)$ **A1**
[3 marks]
Total [6 marks]

5. (a) $a = 1.29$ and $b = -10.4$

A1A1
[2 marks]

(b) recognising both lines pass through the mean point
 $p = 28.7, q = 30.3$

(M1)
A2
[3 marks]

Total [5 marks]

8. (a) attempt to use the symmetry of the normal curve (M1)
 eg diagram, $0.5 - 0.1446$
 $P(24.15 < X < 25) = 0.3554$ A1
 [2 marks]
- (b) (i) use of inverse normal to find z score (M1)
 $z = -1.0598$
 correct substitution $\frac{24.15 - 25}{\sigma} = -1.0598$ (A1)
 $\sigma = 0.802$ A1
- (ii) $P(X > 26) = 0.106$ (M1)A1
 [5 marks]
- (c) recognizing binomial probability (M1)
 $E(Y) = 10 \times 0.10621$ (A1)
 $= 1.06$ A1
 [3 marks]
- (d) $P(Y = 3)$ (M1)
 $= 0.0655$ A1
 [2 marks]
- (e) recognizing conditional probability (M1)
 correct substitution A1
 $\frac{0.3554}{1 - 0.10621}$
 $= 0.398$ A1
 [3 marks]
- Total [15 marks]**

- 3.** (a) $IQR = 10 - 6 (= 4)$ **(A1)**
- attempt to find $Q_3 + 1.5 \times IQR$ **(M1)**
- $10 + 6$
- 16 **A1**

[3 marks]

- (b) (i) choosing $c = \frac{1}{2}a - 9$ **(M1)**
- $\frac{1}{2} \times 42 - 9$
- $= 12$ (years old) **A1**

- (ii) attempt to solve system by substitution or elimination **(M1)**
- 34 (years old) **A1**

[4 marks]

Total [7 marks]

9. (a) uses $\sum P(X = x) = 1$ to form a linear equation in p and q (M1)
 correct equation in terms of p and q from summing to 1 A1
 $p + 0.3 + q + 0.1 = 1$ OR $p + q = 0.6$ (or equivalent)
 uses $E(X) = 2$ to form a linear equation in p and q (M1)
 correct equation in terms of p and q from $E(X) = 2$ A1
 $p + 0.6 + 3q + 0.4 = 2$ OR $p + 3q = 1$ (or equivalent)

Note: The marks for using $\sum P(X = x) = 1$ and the marks for using $E(X) = 2$ may be awarded independently of each other.

evidence of correctly solving these equations simultaneously A1
 for example, $2q = 0.4 \Rightarrow q = 0.2$ or $p + 3 \times (0.6 - p) = 1 \Rightarrow p = 0.4$
 so $p = 0.4$ and $q = 0.2$ AG

[5 marks]

- (b) valid approach (M1)
 $P(X > 2) = P(X = 3) + P(X = 4)$ OR $P(X > 2) = 1 - P(X = 1) - P(X = 2)$
 $= 0.3$ A1

[2 marks]

continued...

Question 9 continued

- (c) recognises at least one of the valid scores (6, 7, or 8) required to win the game **(M1)**

Note: Award **M0** if candidate also considers scores other than 6, 7, or 8 (such as 5).

let T represent the score on the last two rolls

a score of 6 is obtained by rolling $(2,4), (4,2)$ or $(3,3)$

$$P(T=6) = 2(0.3)(0.1) + (0.2)^2 \quad (= 0.1) \quad \mathbf{A1}$$

a score of 7 is obtained by rolling $(3,4)$ or $(4,3)$

$$P(T=7) = 2(0.2)(0.1) \quad (= 0.04) \quad \mathbf{A1}$$

a score of 8 is obtained by rolling $(4,4)$

$$P(T=8) = (0.1)^2 \quad (= 0.01) \quad \mathbf{A1}$$

Note: The above 3 **A1** marks are independent of each other.

$$P(\text{Nicky wins}) = 0.1 + 0.04 + 0.01$$

$$= 0.15 \quad \mathbf{A1}$$

[5 marks]

- (d) $3+b=8$ **(M1)**

$$b=5 \quad \mathbf{A1}$$

[2 marks]

continued...

Question 9 continued(e) **METHOD 1****EITHER**

$$P(S = 5) = \frac{4}{16}$$

$$P(S = a + 2) = \frac{4}{16}$$

$$\Rightarrow a + 2 = 5$$

A1**OR**

$$P(S = 6) = \frac{3}{16}$$

$$P(S = a + 3) = \frac{2}{16} \text{ and } P(S = 5 + 1) = \frac{1}{16}$$

$$\Rightarrow a + 3 = 6$$

A1**OR**

$$P(S = 4) = \frac{3}{16}$$

$$P(S = a + 1) = \frac{2}{16} \text{ and } P(S = 1 + 3) = \frac{1}{16}$$

$$\Rightarrow a + 1 = 4$$

A1**THEN**

$$\Rightarrow a = 3$$

A1

Note: Award **A0A0** for $a = 3$ obtained without working/reasoning/justification.

[2 marks]*continued...*

*Question 9 continued***METHOD 2****EITHER**

correctly lists a relevant part of the sample space

A1for example, $\{S = 4\} = \{(3,1), (1,a), (1,a)\}$ or $\{S = 5\} = \{(2,a), (2,a), (2,a), (2,a)\}$ or $\{S = 6\} = \{(3,a), (3,a), (1,5)\}$

$$a + 3 = 6$$

OReliminates possibilities (exhaustion) for $a < 5$ convincingly shows that $a \neq 2, 4$ **A1** $a \neq 4$, for example, $P(S = 7) = \frac{2}{16}$ from $(2,5), (2,5)$ and so

$$(3,a), (3,a) \Rightarrow a + 3 \neq 7$$

THEN

$$\Rightarrow a = 3$$

A1**[2 marks]****Total [16 marks]**

2. (a) EITHER

recognising that half the total frequency is 10 (may be seen in an ordered list or indicated on the frequency table) (A1)

OR

$$5 + 1 + 4 = 3 + x \quad \text{A1}$$

OR

$$\sum f = 20 \quad \text{A1}$$

THEN

$$x = 7 \quad \text{A1}$$

[2 marks]**(b) METHOD 1**

$$1.58429\dots$$

$$1.58 \quad \text{A2}$$

METHOD 2**EITHER**

$$\sigma^2 = \frac{5 \times (2 - 4.3)^2 + 1 \times (3 - 4.3)^2 + 4 \times (4 - 4.3)^2 + 3 \times (5 - 4.3)^2 + 7 \times (6 - 4.3)^2}{20} (= 2.51) \quad \text{A1}$$

OR

$$\sigma^2 = \frac{5 \times 2^2 + 1 \times 3^2 + 4 \times 4^2 + 3 \times 5^2 + 7 \times 6^2}{20} - 4.3^2 (= 2.51) \quad \text{A1}$$

THEN

$$\sigma = \sqrt{2.51} = 1.58429\dots$$

$$= 1.58 \quad \text{A1}$$

[2 marks]**Total [4 marks]**

4. (a) $0.41 + k - 0.28 + 0.46 + 0.29 - 2k^2 = 1$ OR $k - 2k^2 + 0.01 = 0.13$ (or equivalent) **A1**
 $2k^2 - k + 0.12 = 0$ **AG**

[1 mark]

- (b) one of 0.2 OR 0.3 **(M1)**

$k = 0.3$ **A1**

reasoning to reject $k = 0.2$ eg $P(1) = k - 0.28 \geq 0$ therefore $k \neq 0.2$ **R1**

[3 marks]

- (c) attempting to use the expected value formula **(M1)**

$E(X) = 0 \times 0.41 + 1 \times (0.3 - 0.28) + 2 \times 0.46 + 3 \times (0.29 - 2 \times 0.3^2)$

$= 1.27$ **A1**

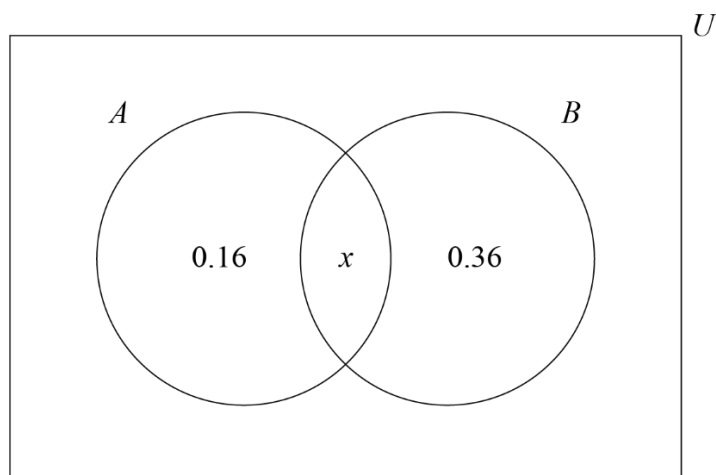
Note: Award **M1A0** if additional values are given.

[2 marks]

Total [6 marks]

6. (a) METHOD 1**EITHER**

one of $P(A) = x + 0.16$ OR $P(B) = x + 0.36$

A1**OR****A1****THEN**

attempt to equate their $P(A \cap B)$ with their expression for $P(A) \times P(B)$

M1

$$P(A \cap B) = P(A) \times P(B) \Rightarrow x = (x + 0.16) \times (x + 0.36)$$

A1

$$x = 0.24$$

A1**METHOD 2**

attempt to form at least one equation in $P(A)$ and $P(B)$ using independence

M1

$$(P(A \cap B') = P(A) \times P(B') \Rightarrow) P(A) \times (1 - P(B)) = 0.16 \text{ OR}$$

$$(P(A' \cap B) = P(A') \times P(B) \Rightarrow) (1 - P(A)) \times P(B) = 0.36$$

$$P(A) = 0.4 \text{ AND } P(B) = 0.6$$

A1

$$P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.6$$

(A1)

$$x = 0.24$$

A1**[4 marks]***continued...*

Question 6 continued(b) **METHOD 1**recognising $P(A' | B') = P(A')$ **(M1)**

$$= 1 - 0.16 - 0.24$$

$$= 0.6$$
 A1

METHOD 2

$$P(B) = 0.36 + 0.24 (= 0.6)$$

$$P(A' | B') = \frac{P(A' \cap B')}{P(B')} \left(= \frac{0.24}{0.4} \right)$$
 (A1)

$$= 0.6$$
 A1

[2 marks]**Total [6 marks]**

Section B

9. (a) $P(C < 61)$ (M1)

$$= 0.365112...$$

$$= 0.365$$

A1

[2 marks]

(b) recognition of binomial eg $X \sim B(12, 0.365...)$ (M1)

$$P(X = 5) = 0.213666...$$

$$= 0.214$$

A1

[2 marks]

(c) (i) Let CM represent 'chocolate muffin' and BM represent 'banana muffin'

$$P(B < 61) = 0.0197555... \quad (A1)$$

EITHER

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) \text{ (or equivalent in words)} \quad (M1)$$

ORtree diagram showing two ways to have a muffin weigh < 61 (M1)**THEN**

$$(0.6 \times 0.365...) + (0.4 \times 0.0197...) \quad (A1)$$

$$= 0.226969...$$

$$= 0.227 \quad A1$$

(ii) recognizing conditional probability (M1)

Note: Recognition must be shown in context either in words or symbols, not just $P(A|B)$.

$$\frac{0.6 \times 0.365112...}{0.226969...} \quad (A1)$$

$$= 0.965183...$$

$$= 0.965 \quad A1$$

[7 marks]

continued...

Question 9 continued

(d) **METHOD 1**

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) = 0.157 \quad (M1)$$

$$(0.6 \times P(C < 61)) + (0.4 \times 0.0197555...) = 0.157$$

$$P(C < 61) = 0.248496... \quad (A1)$$

attempt to solve for σ using GDC (M1)

Note: Award **(M1)** for a graph or table of values to show their $P(C < 61)$ with a variable standard deviation.

$$\sigma = 1.47225...$$

$$\sigma = 1.47 \text{ (g)} \quad A2$$

METHOD 2

$$P(CM) \times P(C < 61 | CM) + P(BM) \times P(B < 61 | BM) = 0.157 \quad (M1)$$

$$(0.6 \times P(C < 61)) + (0.4 \times 0.0197555...) = 0.157$$

$$P(C < 61) = 0.248496... \quad (A1)$$

use of inverse normal to find z score of their $P(C < 61)$ (M1)

$$z = -0.679229...$$

correct substitution (A1)

$$\frac{61 - 62}{\sigma} = -0.679229...$$

$$\sigma = 1.47225...$$

$$\sigma = 1.47 \text{ (g)} \quad A1$$

[5 marks]

Total [16 marks]

3. (a) METHOD 1

using geometric sequence with $r = 1.02$ **(M1)**

correct expression or listing terms correctly **(A1)**

$$45000 \times 1.02^{10} \text{ OR } 45000 \times 1.02^{11-1} \text{ OR listing terms}$$

Gemma's salary is \$54855 (must be to the nearest dollar) **A1**

[3 marks]

METHOD 2

$$N = 10$$

$$PV = \mp 45000$$

$$I\% = 2$$

$$P/Y = 1$$

$$C/Y = 1$$

$$FV = \pm 54854.7489... \quad \text{span style="float: right;">**(M1)(A1)**$$

Gemma's salary is \$54855 (must be to the nearest dollar) **A1**

[3 marks]

(b) finds $a = 1096.89...$ and $b = -2160753.8...$ (accept $b = -2.16 \times 10^6$) **(A1)(A1)**

Note: Award **(A1)(A1)** for $S = 1096.89...x + 33028.49...$, or
 $S = 1096.89...x + 43997.4...$, or $S = 1096.89...x + 45094.3...$

Kaia's salary in 2021 is \$56063.21 (accept \$56817.09 from $b = -2.16 \times 10^6$) **A1**

Kaia had a higher salary than Gemma in 2021 **AG**

[3 marks]

Total [6 marks]

4. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.68$

substitution of $P(A) \cdot P(B)$ for $P(A \cap B)$ in $P(A \cup B)$ (M1)

$$P(A) + P(B) - P(A)P(B) (= 0.68)$$

substitution of $3P(B)$ for $P(A)$ (M1)

$$3P(B) + P(B) - 3P(B)P(B) = 0.68 \text{ (or equivalent)} \quad \text{(A1)}$$

Note: The first two M marks are independent of each other.
--

attempts to solve their quadratic equation (M1)

$$P(B) = 0.2, 1.133... \left(\frac{1}{5}, \frac{17}{15} \right)$$

$$P(B) = 0.2 \left(= \frac{1}{5} \right) \quad \text{A2}$$

Note: Award A1 if both answers are given as final answers for $P(B)$.
--

[6 marks]

5. (a) 0.28 (s)

A1

[1 mark]

(b) $\text{IQR} = 0.35 - 0.27 (= 0.08)$ (s)

(A1)

substituting **their** IQR into correct expression for upper fence

(A1)

$0.35 + 1.5 \times 0.08 (= 0.47)$ (s)

$0.46 < 0.47$

R1

so 0.46 (s) is not an outlier

AG

[3 marks]

(c) **EITHER**

the median is closer to the lower quartile (positively skewed)

R1

OR

The distribution is positively skewed

R1

OR

the range of reaction times below the median is smaller than the range of reaction times above the median

R1

Note: These are sample answers from a range of acceptable correct answers.
Award **R1** for any correct statement that explains this.

Do not award **R1** if there is also an incorrect statement, even if another statement in the answer is correct. Accept a correctly and clearly labelled diagram.

[1 mark]

continued...

Question 5 continued(d) **EITHER**

the distribution for 'not sleeping well' is centred at a higher reaction time

R1**OR**

The median reaction time after not sleeping well is equal to the upper quartile reaction time after sleeping well

R1**OR**

75% of reaction times are <0.35 seconds after sleeping well, compared with 50% after not sleeping well

R1**OR**

the sample size of 9 is too small to draw any conclusions

R1

Note: These are sample answers from a range of acceptable correct answers. Accept any relevant correct statement **that relates to the median and/or quartiles shown in the box plots. Do not accept** a comparison of means. Do not award **R1** if there is also an incorrect statement, even if another statement in the answer is correct. Award **R0** to "correlation does not imply causation".

[1 mark]**Total [6 marks]**

9. (a) METHOD 1

$$T \sim N(35, \sigma^2)$$

$$P(T > 40) = 0.25 \text{ or } P(T < 40) = 0.75 \quad (M1)$$

attempt to solve for σ graphically or numerically using the GDC (M1)

graph of normal curve $T \sim N(35, \sigma^2)$ for $P(T > 40)$ and $y = 0.25$ OR $P(T < 40)$ and $y = 0.75$ OR table of values for $P(T < 40)$ or $P(T > 40)$

$$\sigma = 7.413011\dots$$

$$\sigma = 7.41 \text{ (min)} \quad A2$$

METHOD 2

$$T \sim N(35, \sigma^2)$$

$$P(T > 40) = 0.25 \text{ or } P(T < 40) = 0.75 \quad (M1)$$

$$z = 0.674489\dots \quad (A1)$$

valid equation using their z -score (clearly identified as z -score and not a probability) (M1)

$$\frac{40 - 35}{\sigma} = 0.674489\dots \text{ OR } 5 = 0.674489\dots\sigma$$

$$7.413011\dots$$

$$\sigma = 7.41 \text{ (min)} \quad A1$$

[4 marks]

(b) $P(T > 45) \quad (M1)$

$$= 0.0886718\dots$$

$$= 0.0887 \quad A1$$

[2 marks]

continued...

Question 9 continued

- (c) recognizing binomial probability **(M1)**

$$L \sim B(5, 0.0886718\dots)$$

$$P(L \geq 1) = 1 - P(L = 0) \text{ OR}$$

$$P(L \geq 1) = P(L = 1) + P(L = 2) + P(L = 3) + P(L = 4) + P(L = 5) \quad \textbf{(M1)}$$

$$0.371400\dots$$

$$P(L \geq 1) = 0.371 \quad \textbf{A1}$$

[3 marks]

- (d) recognizing conditional probability in context **(M1)**

finding $\{L < 3\} \cap \{L \geq 1\} = \{L = 1, L = 2\}$ (may be seen in conditional probability) **(A1)**

$$P(L = 1) + P(L = 2) = 0.36532\dots \text{ (may be seen in conditional probability)} \quad \textbf{(A1)}$$

$$P(L < 3 | L \geq 1) = \frac{0.36532\dots}{0.37140\dots} \quad \textbf{(A1)}$$

$$0.983636\dots$$

$$0.984 \quad \textbf{A1}$$

[5 marks]

continued...

*Question 9 continued***(e) METHOD 1**

recognizing that Suzi can be late no more than once (in the remaining six days) **(M1)**

$X \sim B(6, 0.0886718\dots)$, where X is the number of days late **(A1)**

$$P(X \leq 1) = P(X = 0) + P(X = 1) \quad \textbf{(M1)}$$

$$= 0.907294\dots$$

$$P(\text{Suzi gets a bonus}) = 0.907 \quad \textbf{A1}$$

Note: The first two marks may be awarded independently.

METHOD 2

recognizing that Suzi must be on time at least five times (of the remaining six days) **(M1)**

$X \sim B(6, 0.911328\dots)$, where X is the number of days on time **(A1)**

$$P(X \geq 5) = 1 - P(X \leq 4) \text{ OR } 1 - 0.0927052\dots \text{ OR } P(X = 5) + P(X = 6) \text{ OR } 0.334434\dots + 0.572860\dots \quad \textbf{(M1)}$$

$$= 0.907294\dots$$

$$P(\text{Suzi gets a bonus}) = 0.907 \quad \textbf{A1}$$

Note: The first two marks may be awarded independently.

[4 marks]

Total [18 marks]

6. (a) $P(A \cap B) = 0.24$

A1

[1 mark]

(b) $P(A \cup B) = 1.1 - P(A \cap B)$

(A1)

$(0 \leq) P(A \cup B) \leq 1$

(M1)

Note: This may be conveyed in a clearly labelled diagram or written explanation where $P(A \cup B) = 1$

the minimum value of $P(A \cap B)$ is 0.1

A1

[3 marks]

(c) A is a subset of B (so $P(A \cap B) = P(A)$).

R1

Note: This may be conveyed in a clearly labelled diagram where A is completely inside B, or in a written explanation indicating that $P(A \cap B) = P(A)$

so the maximum value of $P(A \cap B)$ is 0.3

A1

Note: Do not award **R0A1**.

[2 marks]

Total [6 marks]

Section B

1. (a) 1.01206..., 2.45230...

$$a = 1.01, b = 2.45 \text{ (} 1.01x + 2.45 \text{)}$$

A1A1

[2 marks]

- (b) 0.981464...

$$r = 0.981$$

A1

Note: A common error is to enter the data incorrectly into the GDC, and obtain the answers $a = 1.01700...$, $b = 2.09814...$ and $r = 0.980888...$. Some candidates may write the 3 sf answers, ie. $a = 1.02$, $b = 2.10$ and $r = 0.981$ or 2 sf answers, ie. $a = 1.0$, $b = 2.1$ and $r = 0.98$. In these cases award **A0A0** for part (a) and **A0** for part (b). Even though some values round to an accepted answer, they come from incorrect working.

[1 mark]

- (c) correct substitution of 78 into **their** regression equation

(M1)

$$81.3930... \text{ } 81.23 \text{ from 3 sf answer}$$

$$81$$

A1

[2 marks]

Total [5 marks]

9.

Note: Do not penalize for inclusion or non-inclusion of endpoints for probabilities using a normal distribution. For example, for $P(T < 55 | T > 40)$ accept $P(T \leq 55 | T > 40)$, $P(T \leq 55 | T \geq 40)$, etc.

(a) recognising to find $P(T > 40)$ **(M1)**

$$P(T > 40) = 0.574136...$$

$$P(T > 40) = 0.574$$

A1**[2 marks]**(b) attempt to multiply four independent probabilities using their $P(T > 40)$ and

$$P(T < 40)$$

(M1)

$$(1-p)^3 \cdot p \text{ OR } (1-0.574136...)^3 \cdot 0.574136... \text{ OR } (0.425863...)^3 \cdot 0.574136...$$

(A1)

$$0.0443430...$$

$$0.0443, 0.0444 \text{ from 3 sf values}$$

A1**[3 marks]***continued...*

Question 9 continued

- (c) (i) recognizing conditional probability

(M1)

$$P(T < 55 | T > 40)$$

Note: Award **(M1)** for an expression or description in context. Accept

$P(T > 40 | T < 55)$ but do not accept just $P(A | B)$.

$$\frac{P(40 < T < 55)}{P(T > 40)}$$

(A1)

$$\frac{0.461944...}{0.574136...}$$

(A1)

$$P(T < 55 | T > 40) = 0.804590...$$

$$= 0.805$$

A1

- (ii) recognizing binomial probability

(M1)

$$X \sim B(n, p)$$

$$n = 10 \text{ and } p = 0.804589...$$

(A1)

$$0.0242111..., 0.0240188... \text{ using } p = 0.805$$

$$P(X = 5) = 0.0242$$

A1**[7 marks]***continued...*

Question 9 continued(d) Let $P(T < a) = x$

recognition that probabilities sum to 1 (seen anywhere)

(M1)**EITHER**

expressing the three regions in one variable

(M1)

$$x + 0.904 + 2x \text{ OR } P(T < a) + 0.904 + 2P(T < a) \text{ OR } \frac{1}{2}P(T > b) + 0.904 + P(T > b)$$

OR x and $2x$ correctly indicated on labelled bell diagram

$$P(T < a) + 0.904 + 2P(T < a) = 1 \text{ OR } \frac{1}{2}P(T > b) + 0.904 + P(T > b) = 1 \text{ (or equivalent)}$$

(A1)**OR**expressing either $P(T < a)$ or $P(T > b)$ only in terms of $P(a \leq T \leq b)$ **(M1)**

$$(P(T < a) = \frac{1}{3}(1 - P(a \leq T \leq b)) \text{ OR } (P(T > b) = \frac{2}{3} \cdot (1 - P(a \leq T \leq b)))$$

$$x = \frac{1}{3}(1 - 0.904)(= 0.032) \text{ OR } P(T > b) = \frac{2}{3}(1 - 0.904)(= 0.064)$$

(A1)**THEN**

$$P(T < a) = 0.032$$

$$a = 22.18167...$$

$$a = 22.2 \text{ accept } 22.1$$

A1**[4 marks]****Total [16 marks]**

3. (a) (i) summing frequencies of riders or finding complement **(M1)**
 probability = $\frac{34}{40}$ **A1**

- (ii) attempt to find expected value **(M1)**

$$\frac{16}{40} + \left(2 \times \frac{13}{40}\right) + \left(3 \times \frac{2}{40}\right) + \left(4 \times \frac{3}{40}\right)$$

$$\frac{60}{40} (= 1.5)$$
 A1

[4 marks]

- (b) evidence of **their** rides/visitor $\times 1000$ or $\div 10$ **(M1)**
 1500 OR 0.15
 150 (times) **A1**

[2 marks]

Total [6 marks]

2. (a) $u_1 + 3d = u_4$ (M1)

$$0.6 + 3d = 0.15$$

$$d = -0.15$$

(A1)

[2 marks](b) **METHOD 1**

$$u_2 = 0.45 \text{ or } u_3 = 0.3 \text{ (may be seen in their equation)} \quad (A1)$$

summing their probabilities to 1 (seen anywhere) (M1)

$$\frac{0.6}{k} + \frac{u_2}{k} + \frac{u_3}{k} + \frac{0.15}{k} = 1$$

$$\frac{0.6}{k} + \frac{0.45}{k} + \frac{0.3}{k} + \frac{0.15}{k} = 1 \text{ (or equivalent)} \quad (A1)$$

$$\frac{1.5}{k} = 1$$

$$k = 1.5$$

(A1)

[4 marks]**METHOD 2** (using S_n formula)

$$S_4 = \frac{4}{2}(2(0.6) + (4-1)(-0.15)) \text{ OR } S_4 = \frac{4}{2}\left(2\left(\frac{0.6}{k}\right) + (4-1)\left(\frac{-0.15}{k}\right)\right)$$

$$\text{OR } S_4 = \frac{4}{2}(0.6 + 0.15) \text{ OR } S_4 = \frac{4}{2}\left(\frac{0.6}{k} + \frac{0.15}{k}\right) \text{ (or equivalent)} \quad (A1)$$

summing their probabilities to 1 (seen anywhere) (M1)

$$\frac{u_1}{k} + \frac{u_2}{k} + \frac{u_3}{k} + \frac{u_4}{k} = 1 \text{ OR } u_1 + u_2 + u_3 + u_4 = k \text{ OR } S_4 = 1 \text{ OR } S_4 = k$$

$$\frac{4}{2}(2(0.6) + (4-1)(-0.15)) = k \text{ (or equivalent)} \quad (A1)$$

$$k = 1.5$$

(A1)

[4 marks]**Total [6 marks]**

4. substitutes into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to form

$$0.55 = 0.4 + P(B) - P(A \cap B) \quad (\text{or equivalent}) \quad \textbf{(A1)}$$

$$\text{substitutes into } P(A|B) = \frac{P(A \cap B)}{P(B)} \text{ to form } 0.25 = \frac{P(A \cap B)}{P(B)} \quad (\text{or equivalent}) \quad \textbf{(A1)}$$

attempts to combine their two probability equations to form an equation in $P(B)$ **(M1)**

Note: The above two **A** marks are awarded independently.

correct equation in $P(B)$ **A1**

$$0.55 = 0.4 + P(B) - 0.25P(B) \quad \text{OR} \quad \frac{P(B) - 0.15}{P(B)} = 0.25 \quad \text{OR} \quad P(B) - 0.15 = 0.25P(B)$$

(or equivalent)

$$P(B) = \frac{15}{75} \left(= \frac{1}{5} = 0.2 \right) \quad \textbf{A1}$$

Total [5 marks]

4. (a) recognising to find $y(25)$ **(M1)**
- $$y(25) = -0.6 \times 25^2 + 23 \times 25 + 110$$
- $$= 310 \text{ (children)}$$
- A1**
[2 marks]

- (b) recognizing x on y is required **(M1)**
- 0.0935114... and 7.43053... **(A1)**
- $$x = 0.0935y + 7.43$$
- A1**

[3 marks]
continued...

Question 4 continued

- (c) attempt to substitute their answer to part (a) into their regression equation for either x or y

(M1)

$$x = 0.0935114... \times 310 + 7.43053... (= 36.4190...)$$

36 (accept 37 or 36.4)

A1

Note: Award **(M1)A1FT** for $x = 37$ found from using $y = 9.39x - 41.5$.

Award **(M1)A0FT** for a correct **FT** answer that lies outside $[15, 46]$.

[2 marks]**Total [7 marks]**

5. METHOD 1

$$Q_1 = 31.86 \text{ OR } Q_3 = 32.14 \quad (\text{A1})$$

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) (M1)

EITHER

equating an appropriate correct normal CDF function to its correct probability (0.25 or 0.5 or 0.75) (A2)

OR

$$z = -0.674489... \text{ OR } z = 0.674489... \text{ (seen anywhere)} \quad (\text{A1})$$

$$-0.674489... = \frac{31.86 - 32}{\sigma} \text{ OR } 0.674489... = \frac{32.14 - 32}{\sigma} \quad (\text{A1})$$

THEN

$$0.207564...$$

$$\sigma = 0.208 \text{ (mm)} \quad \text{A1}$$

METHOD 2

recognition that the area under the normal curve below Q_1 or above Q_3 is 0.25 OR the area between Q_1 and Q_3 is 0.5 (seen anywhere including on a diagram) (M1)

$$z = -0.674489... \text{ OR } z = 0.674489... \quad (\text{A1})$$

$$(Q_1 =) 32 - 0.674489... \sigma \text{ OR } (Q_3 =) 32 + 0.674489... \sigma \quad (\text{A1})$$

$$(Q_3 - Q_1 =) 2 \times 0.674489... \sigma$$

$$2 \times 0.674489... \sigma = 0.28 \quad (\text{A1})$$

$$0.207564...$$

$$\sigma = 0.208 \text{ (mm)} \quad \text{A1}$$

Total [5 marks]

8. (a) (i) $p = 12$ **A1**
 (ii) $q = 100$ **A1**

[2 marks]

- (b) $P(\text{Adult}) = \frac{100}{160} (= 0.625)$ (seen anywhere) **(A1)**

Note: Award **A1** for $(X \sim) B(3, 0.625)$ or $\left(\frac{100}{160}\right)^3$ but no further marks.

recognition that choice of adults is without replacement (may be seen in tree diagram) **(M1)**

$$\frac{100}{160} \times \frac{99}{159} \times \frac{98}{158} \quad \text{span style="float: right;">**(A1)**$$

0.241372...

0.241 **A1**

[4 marks]

- (c) (i) $\frac{x}{48+x} \left(= \frac{1}{3} \right)$ OR $\frac{\frac{x}{160}}{\frac{48+x}{160}}$ **(A1)(A1)**

Note: Award **A1** for correct numerator and **A1** for correct denominator.

$$x = 24 \quad \text{span style="float: right;">**A1**$$

- (ii) $P(A \cap M) = \frac{24}{160} \left(= \frac{3}{20} \right)$ **A1**

[4 marks]*continued...*

Question 8 continued(d) **METHOD 1 (using $P(A|B) = P(A)$)**recognition that A and M are independent if $P(A|M) = P(A)$ **(M1)**

$$\frac{1}{3} \neq \frac{100}{160} \quad \textbf{R1}$$

so they are not independent **A1****METHOD 2 (using $P(A) \cdot P(B) = P(A \cap B)$)**attempt to find the product $P(A) \times P(M)$ OR $P(A) \times P(D)$ **(M1)**

$$\frac{100}{160} \times \frac{72}{160} \neq \frac{24}{160} \left(\frac{9}{32} \neq \frac{3}{20} \right) \quad \text{OR} \quad \frac{100}{160} \times \frac{88}{160} \neq \frac{76}{160} \left(\frac{11}{32} \neq \frac{19}{40} \right) \quad \textbf{R1}$$

so they are not independent **A1****Note:** Do not award **R0A1**.**[3 marks]**(e) $P(\text{dark chocolate}) = \frac{88}{160}$ (= 0.55) (maybe seen in part (d)) **(A1)**recognize that the variable has a Binomial distribution **(M1)**

$$X \sim B(10, 0.55)$$

recognition that $P(X \geq 5)$ or $1 - P(X \leq 4)$ is required **(M1)****Note:** These two M marks are independent of each other.

0.738437...

0.738 **A1****[4 marks]****Total [17 marks]**

1. (a) $a = 1.93258\dots$, $b = 7.21662\dots$
 $a = 1.93$, $b = 7.22$

A1A1
[2 marks]

- (b) $r = 0.991087\dots$
 $r = 0.991$

A1
[1 mark]

- (c) attempt to substitute $d = 20$ into their equation
height = 45.8683...
height = 45.9 (cm)

(M1)
A1
[2 marks]
Total [5 marks]

4. (a) evidence of attempting to find correct area under normal curve (M1)
- $P(W > 210)$ OR sketch
- $P(W > 210) = 0.115069\dots$
- $P(W > 210) = 0.115$ A1

[2 marks]

- (b) recognizing $P(W < w) = 1 - P(w < W < 210) - P(W > 210)$ (M1)
- $P(W < w) = 1 - 0.8 - 0.115069\dots$
- $P(W < w) = 0.084930\dots$
- $P(W < w) = 0.0849$ A1

[2 marks]

- (c) evidence of attempting to use inverse normal function (M1)
- $w = 197.136\dots$
- $w = 197$ (grams) A1

[2 marks]

- (d) recognition of binomial distribution (M1)
- $X \sim B(10, 0.0849302\dots)$
- $P(X = 1) = 0.382076\dots$
- $P(X = 1) = 0.382$ A1

[2 marks]

Total [8 marks]

9. (a) (i) $\frac{10}{n}$ **A1**
- (ii) multiplying probabilities for GG **(M1)**

$$P(GG) = \frac{10}{n} \times \frac{9}{n-1}$$

$$P(GG) = \frac{90}{n^2 - n} \left(\text{accept } \frac{90}{n(n-1)} \right) \quad \textbf{A1}$$

[3 marks]

- (b) $P(\text{First red}) = \frac{15}{25}$ and $P(\text{Second red}) = \frac{14}{24}$ (seen anywhere) **(A1)**

$$P(RR) = \frac{15}{25} \times \frac{14}{24} \text{ (or equivalent)} \quad \textbf{A1}$$

$$= 0.35 \quad \textbf{AG}$$

[2 marks]

- (c) $\frac{15}{25} \times \frac{14}{24} \times \frac{13}{23}$ OR $0.35 \times \frac{13}{23}$ **(A1)**

$$0.197826...$$

$$P(\text{three red}) = 0.198 \text{ (exact answer is } \frac{91}{460} \text{)} \quad \textbf{A1}$$

[2 marks]*continued...*

Question 9 continued

- (d)
- $P(\text{at least one green}) = 1 - P(\text{three red})$
- OR

$$P(\text{at least one G}) = P(\text{one G}) + P(\text{two G}) + P(\text{three G}) \quad (\text{M1})$$

$$1 - \left(\frac{15}{25} \times \frac{14}{24} \times \frac{13}{23} \right) \text{ OR } 3 \left(\frac{10}{25} \times \frac{15}{24} \times \frac{14}{23} \right) + 3 \left(\frac{10}{25} \times \frac{9}{24} \times \frac{15}{23} \right) + \left(\frac{10}{25} \times \frac{9}{24} \times \frac{8}{23} \right)$$

$$0.802173\dots$$

$$P(\text{at least one green}) = 0.802 \text{ (exact answer is } \frac{369}{460} \text{)} \quad \text{A1}$$

[2 marks]

$$(e) \quad P(\text{first green on third draw}) = \frac{15}{25} \times \frac{14}{24} \times \frac{10}{23} \times \frac{22}{22} \left(= \frac{7}{46} = 0.152173\dots \right) \quad (\text{A1})$$

$$P(\text{first green on fourth draw}) = \frac{15}{25} \times \frac{14}{24} \times \frac{13}{23} \times \frac{10}{22} \left(= \frac{91}{1012} = 0.0899209\dots \right) \quad (\text{A1})$$

Note: The first two (A1) are independent.

attempt to substitute their probabilities into expected value formula (M1)

$$\text{expected points per game} = 10 \times \frac{7}{46} + 50 \times \frac{91}{1012} \left(= \frac{3045}{506} = 6.01778\dots \right) \quad (\text{A1})$$

setting up inequality or equation in k (M1)

$$\frac{3045}{506}k > 100$$

$$k > 16.6174\dots \left(= \frac{10120}{609} \right)$$

Millie must play at least 17 times.

A1**[6 marks]****Total [15 marks]**

6. $E(X) = k + 2k^2 + 3a + 4k^3 = 2.6$ (A1)

$k + k^2 + a + k^3 = 1$ (A1)

Note: The first two **A** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations (M1)

$$a = 1 - k - k^2 - k^3 \text{ and } a = \frac{1}{3}(2.6 - k - 2k^2 - 4k^3)$$

use of graph or table to attempt to find intersection (M1)

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k (M1)

$$k^3 - k^2 - 2k + 0.4 = 0 \text{ OR } 3(1 - k - k^2 - k^3) = 2.6 - k - 2k^2 - 4k^3 \text{ (or equivalent)}$$

attempt to solve their cubic in k (M1)

THEN

$$a = 0.773073... \text{ OR } k = 0.185928...$$

(Other solutions to the cubic are: $k = 1.92921...$, $k = -1.11514...$)

$a = 0.773$ A1

Total [5 marks]

Section B

9. (a) recognizing probabilities sum to 1 (M1)
- $$0.213 + P(82.4 < X < 87.3) + 0.409 = 1$$
- $$P(82.4 < X < 87.3) = 0.378 \quad \text{A1}$$

[2 marks]

(b) **METHOD 1**

recognizing the need to use inverse normal with 0.213, (1 - 0.409) or 0.409 (M1)

$$\mu \pm \text{invNorm}(0.213)\sigma = 82.4, \mu \pm \text{invNorm}(1 - 0.409)\sigma = 87.3 \text{ (or equivalent)} \quad \text{(A1)(A1)}$$

attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) (M1)

$$\mu = 86.2011..., \sigma = 4.77502...$$

$$\mu = 86.2, \sigma = 4.78 \quad \text{A1}$$

Note: Condone use of different variables throughout, but do not award the final **A1** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2use of inverse normal to find at least one z-score for $P(Z < z) = 0.213$, or

$$P(Z < z) = 1 - 0.409 \quad \text{(M1)}$$

$$z_1 = -0.796055... \text{ OR } z_2 = 0.230118...$$

$$\frac{82.4 - \mu}{\sigma} = -0.796055..., \frac{87.3 - \mu}{\sigma} = 0.230118... \text{ (or equivalent)} \quad \text{(A1)(A1)}$$

attempt to solve their equations (that involve z-values rather than probabilities) (M1)

$$\mu = 86.2011..., \sigma = 4.77502...$$

$$\mu = 86.2, \sigma = 4.78 \quad \text{A1}$$

[5 marks]

continued...

Question 9 continued

- (c) (i) recognition of Binomial distribution **(M1)**

$$X \sim B(100, 0.409)$$

$$P(X = 32) = 0.0157931\dots$$

$$= 0.0158$$

A1

- (ii) $P(X < 44) = 0.702975\dots$ (seen anywhere) **(A1)**

recognition of conditional probability **(M1)**

Note: recognition must be shown in context, either in symbols eg $P(X = 32 | X < 44)$, or in words eg $P(32 \text{ plants} | \text{less than } 44 \text{ plants})$, not only as $P(A | B)$.

$$(P(X = 32 | X < 44) =) \frac{P(X = 32)}{P(X < 44)} \text{ OR } \frac{P(X = 32)}{P(X \leq 43)} \left(= \frac{0.0157931\dots}{0.702975\dots} \right) \quad \textbf{(A1)}$$

$$= 0.0224661\dots$$

$$P(X = 32 | X < 44) = 0.0225$$

A1**[6 marks]***continued...*

Question 9 continued

- (d) $Q_1 = 90.54$ OR $Q_3 = 95.06$ (may be seen on a labelled diagram with areas indicated) **(A1)**

$P(90.54 < F < 95.06) = 0.5$ OR $P(F < 90.54) = 0.25$ OR $P(F < 95.06) = 0.75$ (or equivalent)

EITHER

attempt to find d using graph or table **(M1)**

OR

$$1 - 2P\left(Z < -\frac{2.26}{d}\right) = 0.5 \text{ OR } P\left(Z < -\frac{2.26}{d}\right) = 0.25 \text{ OR } P\left(Z < \frac{2.26}{d}\right) = 0.75$$

OR $P\left(-\frac{2.26}{d} < Z < \frac{2.26}{d}\right) = 0.5$ (or equivalent) **(M1)**

$$-\frac{2.26}{d} = -0.674489... \text{ OR } \frac{2.26}{d} = 0.674489...$$

THEN

$$3.35068...$$

$$d = 3.35 \quad \text{A1}$$

[3 marks]

Total [16 marks]

$$6. \quad E(X) = k + 2k^2 + 3a + 4k^3 = 2.3 \quad (A1)$$

$$k + k^2 + a + k^3 = 1 \quad (A1)$$

Note: The first two **A** marks are independent of each other.

EITHER (finding intersections of functions)

attempt to make a the subject in both of their equations **(M1)**

$$a = 1 - k - k^2 - k^3 \text{ and } a = \frac{1}{3}(2.3 - k - 2k^2 - 4k^3)$$

use of graph or table to attempt to find intersection **(M1)**

OR (solving algebraically)

attempt to solve their equations algebraically to find a cubic in k **(M1)**

$$k^3 - k^2 - 2k + 0.7 = 0 \text{ OR } 3(1 - k - k^2 - k^3) = 2.3 - k - 2k^2 - 4k^3 \text{ (or equivalent)}$$

attempt to solve their cubic in k **(M1)**

THEN

$$a = 0.552839... \text{ OR } k = 0.315870... \text{ (other solutions to cubic are } k = -1.18538..., 1.86951... \text{)}$$

$$a = 0.553 \quad \textbf{A1}$$

Total [5 marks]

Section B

9. (a) recognizing probabilities sum to 1 (M1)
- $$0.288 + P(94.6 < X < 98.1) + 0.434 = 1$$
- $$P(94.6 < X < 98.1) = 0.278 \quad \text{A1}$$
- [2 marks]

(b) **METHOD 1**

- recognizing the need to use inverse normal with 0.288, $(1 - 0.434)$ or 0.434 (M1)
- $$\mu + \text{invNorm}(0.288)\sigma = 94.6, \mu + \text{invNorm}(1 - 0.434)\sigma = 98.1 \text{ (or equivalent)} \quad \text{(A1)(A1)}$$
- attempt to solve their equations in two variables using the GDC (that involve either z-values or 'invNorm' rather than probabilities) (M1)
- $$\mu = 97.2981..., \sigma = 4.82468...$$
- $$\mu = 97.3, \sigma = 4.82 \quad \text{A1}$$

Note: Condone use of different variables throughout, but do not award the final **A1** if they do not clearly identify which variable is their mean and standard deviation.

METHOD 2

- use of inverse normal to find at least one z-score for $P(Z < z) = 0.288$ or $P(Z < z) = 1 - 0.434$ (M1)
- $$z_1 = -0.559236... \text{ OR } z_2 = 0.166199...$$
- $$\frac{94.6 - \mu}{\sigma} = -0.559236..., \frac{98.1 - \mu}{\sigma} = 0.166199... \text{ (or equivalent)} \quad \text{(A1)(A1)}$$
- attempt to solve their equations (that involve z-values rather than probabilities) (M1)
- $$\mu = 97.2981..., \sigma = 4.82468...$$
- $$\mu = 97.3, \sigma = 4.82 \quad \text{A1}$$
- [5 marks]

Question 9 continued

(c) (i) recognition of Binomial distribution **(M1)**

$$X \sim B(100, 0.434)$$

$$P(X = 34) = 0.0133198\dots$$

$$= 0.0133$$

A1

(ii) $P(X < 49) = 0.848218\dots$ (seen anywhere) **(A1)**

recognition of conditional probability **(M1)**

Note: recognition must be shown in context, either in symbols eg $P(X = 34 | X < 49)$, or in words eg $P(34 \text{ plants} | \text{less than 49 plants})$, not only as $P(A | B)$.

$$(P(X = 34 | X < 49) =) \frac{P(X = 34)}{P(X < 49)} \text{ OR } \frac{P(X = 34)}{P(X \leq 48)} \left(= \frac{0.0133198\dots}{0.848218\dots} \right) \quad \textbf{(A1)}$$

$$= 0.0157033\dots$$

$$P(X = 34 | X < 49) = 0.0157 \quad \textbf{A1}$$

[6 marks]*continued...*

Question 9 continued

- (d) $Q_1 = 96.19$ OR $Q_3 = 101.01$ (may be seen on a labelled diagram with areas indicated) **(A1)**

$P(96.19 < F < 101.01) = 0.5$ OR $P(F < 96.19) = 0.25$ OR $P(F < 101.01) = 0.75$
(or equivalent)

EITHER

attempt to find d using graph or table **(M1)**

OR

$$1 - 2P\left(Z < -\frac{2.41}{d}\right) = 0.5 \text{ OR } P\left(Z < -\frac{2.41}{d}\right) = 0.25 \text{ OR } P\left(Z < \frac{2.41}{d}\right) = 0.75$$

OR $P\left(-\frac{2.41}{d} < Z < \frac{2.41}{d}\right) = 0.5$ (or equivalent) **(M1)**

$$-\frac{2.41}{d} = -0.674489... \text{ OR } \frac{2.41}{d} = 0.674489...$$

THEN

$$3.57307...$$

$d = 3.57$ **A1**

[3 marks]

Total [16 marks]

2. (a) attempt to form equation for the sum of frequencies=16 or mean=3 (M1)

$$p + q + 4 + 2 + 3 = 16 (\Rightarrow p + q = 7) \quad \text{A1}$$

$$\frac{p + 2q + 12 + 8 + 18}{16} = 3 (\Rightarrow p + 2q = 10) \quad \text{OR} \quad \frac{p + 2q + 12 + 8 + 18}{9 + p + q} = 3 (\Rightarrow 2p + q = 11) \quad \text{A1}$$

attempt to eliminate one variable from their equations (M1)

$$p + 2(7 - p) + 38 = 48 \quad \text{OR} \quad 2(7 - q) + q = 11$$

$$p = 4 \quad \text{and} \quad q = 3 \quad \text{A1}$$

Note: Award **M1A0A0M0A1** for $p = 4, q = 3$ with no working.

[5 marks]

- (b) mean final score = 30 A1

[1 mark]

Total [6 marks]

9. (a) evidence of understanding that there are now 3R and 2B (M1)

$$p = \frac{3}{5}, q = \frac{2}{5} \quad \text{A1}$$

[2 marks]

- (b) attempt to add two products (M1)

$$P(\text{same}) = P(\text{RR or BB}) = \frac{3}{4} \times \frac{4}{5} + \frac{1}{4} \times \frac{2}{5} \quad \text{A1}$$

$$= \frac{14}{20}$$

$$= \frac{7}{10} \quad \text{AG}$$

[2 marks]

- (c) attempt to use conditional probability formula in context (M1)

$$P(\text{RR}|\text{same}) = \frac{P(\text{RR})}{P(\text{same})}$$

Note: Award **M0** if candidate only writes $P(A|B)$ formula and nothing else.

$$= \frac{\binom{12}{20}}{\binom{14}{20}} \quad \text{(A1)}$$

$$= \frac{12}{14} \left(= \frac{6}{7} \right) \quad \text{A1}$$

[3 marks]

continued...

Question 9 continued

$$(d) \quad a = \frac{6}{20} \left(= \frac{3}{10} \right), \quad b = \frac{12}{20} \left(= \frac{6}{10} \right)$$

A1A1**[2 marks]**(e) attempt to use the formula for $E(X)$ **(M1)**

$$E(X) = 0 \times \frac{1}{10} + 1 \times \frac{6}{20} + 2 \times \frac{12}{20}$$

$$= \frac{30}{20} \left(= \frac{3}{2} \right)$$

A1**[2 marks]**

$$(f) \quad \frac{1}{6}$$

A1**[1 mark]**(g) **METHOD 1**

$$P(n-1 \text{ reds}) = \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{n+1}{n+2} \left(= \frac{3}{n+2} \right)$$

(A1)

$$P(\text{next one blue}) = \frac{1}{n+3}$$

(A1)

$$P(n-1 \text{ reds then 1 blue}) = P(n-1 \text{ reds}) \times P(\text{next one blue})$$

(M1)

$$\frac{3}{n+2} \times \frac{1}{n+3} = \frac{3}{56}$$

(A1)

$$(n+2)(n+3) = 56$$

$$n = 5$$

A1

Note: If no working shown, award **M1A0A0A0A1** for $n = 5$.

continued...

Question 9 continued**METHOD 2**

Let X be the number of selections in total made when first blue picked

attempt to establish pattern for $X = 1, 2, 3, \dots$ with at least 3 cases **(M1)**

$$P(X = 1) = \frac{1}{4} \text{ and } P(\text{second pick}) = \frac{3}{4} \times \frac{1}{5} \quad \textbf{(A1)}$$

$$P(X = 3) \left(= \frac{3}{4} \times \frac{4}{5} \times \frac{1}{6} \right) = \frac{3}{5} \times \frac{1}{6} \quad \textbf{(A1)}$$

$$P(X = 5) = \frac{3}{7} \times \frac{1}{8} \left(= \frac{3}{56} \right) \quad \textbf{(A1)}$$

so $n = 5$ **A1**

METHOD 3

$$P(\text{next one blue}) = \frac{1}{n+3} \quad \textbf{(A1)}$$

recognising $P(n-1 \text{ R then 1B}) = P(n-1 \text{ R}) \times P(\text{next one B})$ OR $\frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{1}{n+3}$ **(M1)**

$$\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{1}{8} \left(= \frac{3}{56} \right) \quad \textbf{(A1)(A1)}$$

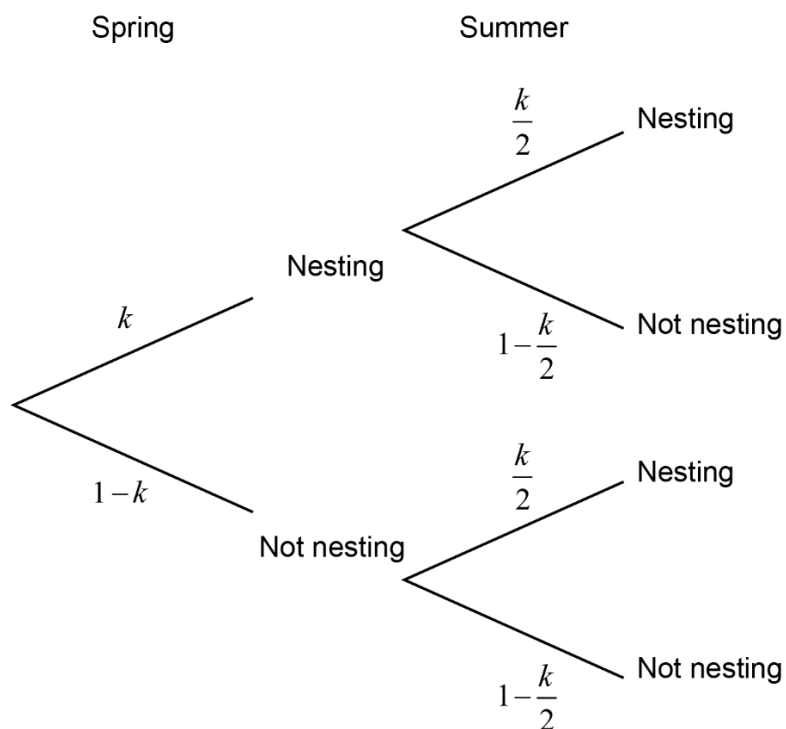
Note: Award **A1** for $\frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7}$ (seen anywhere) and **A1** for $\times \frac{1}{8}$.

so $n = 5$ **A1**

[5 marks]

Total [17 marks]

5. (a)



$1-k$ for Spring

A1

$1-\frac{k}{2}$ for both Summers

A1

[2 marks]

continued...

Question 5 continued.

- (b) (i) multiplying the two correct branches **(A1)**

$$(1-k)\left(1-\frac{k}{2}\right)$$

attempt to expand and equate to $\frac{5}{9}$ **(M1)**

$$1-k-\frac{k}{2}+\frac{k^2}{2}=\frac{5}{9}$$

$$18-18k-9k+9k^2=10 \quad \text{OR} \quad \frac{k^2}{2}-\frac{3k}{2}+\frac{4}{9}=0 \quad \text{OR} \quad \frac{9k^2}{2}-\frac{27k}{2}+4=0 \quad \text{A1}$$

$$9k^2-27k+8=0 \quad \text{AG}$$

- (ii) $\left(k=\frac{1}{3} \text{ is the only valid solution as } \frac{8}{3}>1\right)$ **R1**

Note: Accept any valid reasoning indicating that any probability cannot be greater than 1 and/or probability cannot be less than 0.

[4 marks]

Total [6 marks]

1. (a) median = 75 **A1**
 (upper quartile =) 84 OR (lower quartile =) 68 **(A1)**
 Interquartile range = 16 **A1**
[3 marks]

(b)

Note: In this part, their reasoning and answer must be consistent with their values in part (a).

In both part (i) and part (ii), award **R0A1** for a correct answer with no reasoning.

- (i) $80 > 75$ OR Australia has a higher median OR Spain's median is lower **R1**

Note: Award **R1** for correct reasoning based on a comparison of their medians.

in general (on average), rabbits in Australia have longer ears (than rabbits in Spain)

OR

in the top 50% of each distribution, some rabbits in Spain have smaller ears than those in Australia

OR

in the bottom 50% of each distribution, some rabbits in Australia have longer ears than those in Spain **A1**

- (ii) $16 > 11$ OR higher IQR in Spain OR lower IQR in Australia **R1**

Note: Award **R1** for correct reasoning based on a comparison of their IQR's.

(the IQR's suggest that) there is more variation/spread in (the middle 50% of) rabbit ears in Spain (than those in Australia) (or equivalent) **A1**

Note: Award **A1** for any correct answer which uses the IQR's to compare each distribution.

[4 marks]

Total [7 marks]

4. (a) recognition of $X > 13$ OR $Z > 1.5$ (could be seen in a diagram) **(M1)**

$$P(X > 13) = 0.0668072...$$

$$= 0.0668$$

A1

[2 marks]

- (b) **EITHER**

equating an appropriate correct normal CDF function to 0.1 or 0.9 **(M1)**

$$P(X > 10 + 2k) = 0.1 \text{ OR } P(Z < k) = 0.9 \text{ OR } P(X < 10 - 2k) = 0.1 \text{ OR } P(Z < -k) = 0.1$$

OR

recognising need to use inverse normal with 0.1 or 0.9 **(M1)**

THEN

$$1.28155...$$

$$k = 1.28$$

A1

Total [4 marks]

6. (a) $r = 0.901017\dots$

$r = 0.901$

A2

[2 marks]

(b) Student 11 Test B: should not extrapolate

R1

[1 mark]

(c) (i) Student 12 Test A: should not use line of y on x to predict x from y (or equivalent)

R1

(ii) attempt to find the equation of the regression line of x on y

(M1)

$(x =) 0.987124\dots y - 3.21970\dots \quad ((x =) 0.987y - 3.22)$

A1

$(x =) 0.987124\dots(90) - 3.21970\dots \quad (= 85.6214\dots)$

A1

$= 86$ to nearest integer.

AG

Note: Condone notation for x and y switched if values are correct.

[4 marks]

Total [7 marks]

Section B

1. (a) (i) 4.45 (hours) **A1**

(ii) one correct quartile either $Q_1 = 1.9$ or $Q_3 = 5.7$ **(A1)**

$$\text{IQR} = 5.7 - 1.9$$

$$= 3.8 \text{ (hours)}$$

A1

[3 marks]

(b) attempts to find the upper fence value **(M1)**

$$\text{upper fence} = 11.4$$

(A1)

$$11.7 > 11.4$$

R1

$\Rightarrow 11.7$ is an outlier

AG

[3 marks]

Total [6 marks]

3. (a) **EITHER**

$$\bar{y} = 2.1875 \times 7 + 0.6875$$

A1**OR**

$$\bar{y} = 15.3125 + 0.6875$$

A1**THEN**

$$\bar{y} = 16$$

AG**[1 mark]**

(b) attempts to use $16 = \frac{\sum y}{n}$ to form a linear equation in p and q **(M1)**

$$16 = \frac{9+13+p+q+21}{5} \quad (80 = p+q+43 \Rightarrow p+q=37) \quad \textbf{(A1)}$$

attempts to solve two linear equations simultaneously for p and q (one of which is $q = p + 3$) **(M1)**

$$16 = \frac{9+13+p+p+3+21}{5} \quad (80 = 2p+46)$$

$$p = 17 \text{ and } q = 20$$

A1**[4 marks]****Total [5 marks]**

6.

METHOD 1correct inequality or equation involving $P(X = 0)$ (A1)

$$1 - P(X = 0) > 0.99 \text{ OR } P(X = 0) < 0.01 \text{ OR } 1 - P(X = 0) = 0.99 \text{ OR } P(X = 0) = 0.01$$

attempts to solve their inequality (equality) involving 0.75^n for n (M1)

$$1 - 0.75^n > 0.99 \text{ OR } 0.75^n < 0.01 \text{ OR } 0.75^n = 0.01 \text{ OR } 1 - 0.75^n = 0.99$$

Note: Valid solving attempts include graphical, use of logarithms, tabular or trial and error.

EITHER

$$n > 16.0078... \text{ OR } n = 16.0078... \quad (A2)$$

the least value of n is 17 A1**OR**

$$P(X = 0) = 0.010022... (> 0.01) \text{ (corresponding to } n = 16) \quad (A1)$$

$$P(X = 0) = 0.0075169... (< 0.01) \quad (A1)$$

corresponding to $n = 17$ (which is the least value of n) A1*continued...*

Question 6 continued.

METHOD 2 (TABLE ONLY APPROACH)

attempts to use binomial cdf to calculate a correct value of $P(X \geq 1)$ for one value of n **(M1)**

calculates correct values of $P(X \geq 1)$ for at least one value of n **(A1)**

$P(X \geq 1) = 0.989977\dots$ (< 0.99) (corresponding to $n = 16$) **(A1)**

$P(X \geq 1) = 0.992483\dots$ (> 0.99) **(A1)**

corresponding to $n = 17$ (which is the least value of n) **A1**

[5 marks]

SECTION B

7. (a) recognition to add μ and σ (M1)

49.8 (cm) A1

[2 marks]

(b) $P(L > 48)$ (M1)

Note: Award (M1) for a clearly labelled diagram.

$= 0.283854\dots$

$= 0.284$ A1

[2 marks]

(c) $P(L > x) = 0.99$ OR $P(L < x) = 0.01$ (M1)

Note: Award (M1) for a clearly labelled diagram or the use of inverse normal.

$x = 35.8293\dots$

$x = 35.8$ (cm) A1

[2 marks]

continued...

Question 7 continued.

(d) (i) $P(40 < L < 56) = 0.902149\dots$ (may be seen in part (ii)) **(A1)**

attempts to find $100 \times P(40 < L < 56)$ with their probability **(M1)**

$$= 90.2149\dots$$

$$= 90.2 \quad \mathbf{A1}$$

Note: Accept 90 or 91.

(ii) recognizes binomial distribution **(M1)**

$$X \sim B(100, 0.902149\dots)$$

$$P(X = 95) = 0.038105\dots$$

$$= 0.0381 \quad \mathbf{A1}$$

[5 marks]

continued...

Question 7 continued.

(e) $P(45.55 \leq L < 45.65)$

(M1)(A1)

Note: Award **(M1)** for any reasonable interval centred on 45.6 , no wider than $P(45.5 \leq L < 45.7)$.

Accept either of $P(45.55 \leq L \leq 45.65)$ or $P(45.55 < L < 45.65)$.

= 0.009498...

= 0.00950

A1

[3 marks]

Total [14 marks]

1.

(a)

(i)

27

A1

(ii)

30

A1

(iii)

35

A1
- [3 marks]

- (b)

$Q_1 = 31$ and $Q_3 = 46$

(A1)

attempt to subtract their upper and lower quartiles

(M1)

Note: Award **M1** only for correct values, or for values clearly indicated as candidate's Q_1 and Q_3 .

46 – 31

IQR = 15

A1

[3 marks]

[Total: 6 marks]

3. (a) use of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

$$P(A \cap B) = 0.45 + 0.65 - 0.8 \text{ (or equivalent)} \quad (\text{A1})$$

$$= 0.3 \quad \text{A1}$$

[3 marks]

(b) $P(A' \cap B') = 0.2$ (may be seen in Venn diagram) (A1)

attempt to substitute their values into $P(A' | B') = \frac{P(A' \cap B')}{P(B')}$ (M1)

$$P(A' | B') = \frac{0.2}{0.35}$$

$$= \frac{20}{35} \left(= \frac{4}{7} \right) \quad \text{A1}$$

[3 marks]

Total [6 marks]

1. (a) (i) 22 **A1**
 (ii) 20 **A1**
 (iii) 30 **A1**
[3 marks]

- (b) $Q_1 = 26$ and $Q_3 = 38$ **(A1)**
 attempt to subtract their upper and lower quartiles **(M1)**

Note: Award **M1** only for correct values, or for values clearly indicated as candidate's Q_1 and Q_3 .

$38 - 26$
 $IQR = 12$ **A1**
[3 marks]
[Total: 6 marks]

3. (a) use of $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)

$$P(A \cap B) = 0.65 + 0.45 - 0.85 \quad (\text{or equivalent}) \quad (A1)$$

$$= 0.25$$

A1
[3 marks]

(b) $P(A' \cap B') = 0.15$ (may be seen in Venn diagram) (A1)

attempt to substitute their values into $P(A' | B') = \frac{P(A' \cap B')}{P(B')}$ (M1)

$$= \frac{15}{55} \left(= \frac{3}{11} \right)$$

A1

[3 marks]

Total [6 marks]

4. (a) recognition of sum of probabilities equals 1 (M1)

$$\frac{2k}{15} + \frac{4k}{15} + \frac{7k}{15} + \frac{10k}{15} = 1$$

$$k = 0.652173\dots$$

$$k = 0.652 \left(= \frac{15}{23} \right)$$

A1**[2 marks]**

- (b) correct probabilities: $\frac{2}{23}, \frac{4}{23}, \frac{7}{23}, \frac{10}{23}$ OR 0.0870, 0.174, 0.304, 0.435 (A1)

substitution of their probabilities into formula for expected value (M1)

$$2 \times \frac{2}{23} + 4 \times \frac{4}{23} + 7 \times \frac{7}{23} + 10 \times \frac{10}{23} \quad \text{OR} \quad \frac{169k}{15}$$

$$= 7.34782\dots$$

$$E(X) = 7.35 \left(= \frac{169}{23} \right) \quad (\text{same 3sf from previous 3sf answer})$$

A1**[3 marks]****Total [5 marks]**

7. (a) 16.7037...

16.7(%)

A1

[1 mark]

(b) recognition of binomial

(M1)

$$X \sim B(1000, 0.167\dots)$$

attempt to find $P(X < 150)$ OR $P(X \leq 149)$

(M1)

0.0669378...

$$P(X < 150) = 0.0669 \text{ (accept 0.0673 from previous 3sf)}$$

A1

[3 marks]

(c) recognition of conditional probability in context

(M1)

$$P(\text{age} \geq 75 \mid \text{age} \geq 55) \text{ OR } \frac{P(\text{age} \geq 55 \cap \text{age} \geq 75)}{P(\text{age} \geq 55)}$$

Note: Recognition must be shown in context either in words or symbols, not just $P(A \mid B)$

$$\frac{P(\text{age} \geq 75)}{P(\text{age} \geq 55)}$$

(A1)

$$\frac{2.1}{(2.1+12.9)} \text{ OR } \frac{0.0233853\dots}{0.167}$$

$$\frac{2.1}{(2.1+12.9)} \text{ OR } \frac{0.0233853\dots}{0.167}$$

(A1)

$$0.14 \text{ (exact)} \left(= \frac{7}{50} \right)$$

A1

Note: Condone use of "a" or "X" or any letter for age.

[4 marks]

(d) 15, 26.5, 45, 65, 85

A1

[1 mark]

continued...

Question 7 continued

- (e)

(i)

38.0740...

mean = 38.1

A1

(ii)

variance is square of standard deviation

16.3639...²

267.778...

variance = 268

A1

[3 marks]

- (f)

(i)

Graph D

A1

Note: Allow **FT** from incorrect values for mean and variance in (e).

- (ii)

SmallTalk

A1

Note: Do not award **A0A1**.

[2 marks]

Total [14 marks]

4. (a) recognition of sum of probabilities equals 1

(M1)

$$\frac{3k}{20} + \frac{5k}{20} + \frac{8k}{20} + \frac{11k}{20} = 1$$

$$k = 0.740740$$

$$k = 0.741 \left(= \frac{20}{27} \right)$$

A1

[2 marks]

- (b) correct probabilities: $\frac{3}{27}, \frac{5}{27}, \frac{8}{27}, \frac{11}{27}$ OR 0.111, 0.185, 0.296, 0.407

(A1)

substitution of their probabilities into formula for expected value

(M1)

$$3 \times \frac{3}{27} + 5 \times \frac{5}{27} + 8 \times \frac{8}{27} + 11 \times \frac{11}{27} \text{ OR } \frac{219k}{20}$$

$$= 8.11111\dots$$

$$E(X) = 8.11 \left(= \frac{219}{27} = \frac{73}{9} \right) \text{ (same 3sf from previous 3sf answer)}$$

A1

[3 marks]

Total [5 marks]

7. (a) 22.4358...

22.4(%)

A1**[1 mark]**

(b) recognition of binomial

(M1)

$$X \sim B(1000, 0.224\dots)$$

attempt to find $P(X < 200)$ OR $P(X \leq 199)$ **(M1)**

0.0284945...

$$P(X < 200) = 0.0285 \text{ (accept 0.0303 from previous 3sf)}$$

A1**[3 marks]**

(c) recognition of conditional probability in context

(M1)

$$P(\text{age} \geq 65 \mid \text{age} \geq 45) \text{ OR } \frac{P(\text{age} \geq 45 \cap \text{age} \geq 65)}{P(\text{age} \geq 45)}$$

Note: Recognition must be shown in context either in words or symbols, not just $P(A \mid B)$

$$\frac{P(\text{age} \geq 65)}{P(\text{age} \geq 45)}$$

(A1)

$$\frac{3.7}{(3.7 + 17.3)} \text{ OR } \frac{0.0395}{0.224}$$

(A1)

0.176190...

$$0.176 \left(= \frac{37}{210} \right)$$

A1

Note: Condone use of “a” or “X” or any letter for age.

[4 marks]*continued...*

Question 7 continued.

(d) 15.5, 21.5, 35, 55, 75

A1
[1 mark]

(e) (i) 35.7280...
mean = 35.7

A1

(ii) variance is square of standard deviation
14.3159...²
204.945...
variance = 205

(M1)

A1
[3 marks]

(f)
(i) Graph C

A1

Note: Allow **FT** from incorrect values for mean and variance in (e).

(ii) MyLife

A1

Note: Do not award **A0A1**.

[2 marks]
Total [14 marks]

5. (a) $P(A) = \frac{1}{4}$ (A1)

attempt to use $P(B|A) = \frac{P(B \cap A)}{P(A)}$ (M1)

$$\frac{2}{3} = \frac{P(B \cap A)}{\left(\frac{1}{4}\right)}$$

$$P(A \cap B) = \frac{2}{3} \left(\frac{1}{4} \right)$$

$$= \frac{2}{12} \left(= \frac{1}{6} \right)$$

A1

[3 marks]

- (b) attempt to use $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ OR a Venn diagram, with their values of $P(A)$ and $P(B \cap A)$

M1

$$\frac{3}{4} = \frac{1}{4} + P(B) - \frac{1}{6}$$

$$P(B) = \frac{1}{2} + \frac{1}{6}$$

$$= \frac{4}{6} \left(= \frac{2}{3} \right)$$

A1

$P(B|A) = P(B)$ OR $P(A)P(B) = \frac{1}{6}$ so $P(A \cap B) = P(A)P(B)$ (hence A and B are independent)

R1

Note: The **R1** is dependent on all previous marks

[3 marks]

Total [6 marks]

7. (a) recognise sum of probabilities is 1 (seen anywhere) (M1)

$$k + 3k^2 + 2k^2 + k^2 = 1 \text{ OR } \sum P(X = x) = 1$$

$$6k^2 + k - 1 = 0 \text{ OR } 6k^2 + k = 1 \quad \text{A1}$$

valid attempt to solve their quadratic = 0 (M1)

$$(3k-1)(2k+1) \text{ OR } k = \frac{-1 \pm \sqrt{1-4(6)(-1)}}{2(6)}$$

$$k = \frac{1}{3} \text{ and } k = -\frac{1}{2} \quad \text{A1}$$

$$0 < k < 1 \text{ (so, disregard } k = -\frac{1}{2} \text{)} \quad \text{R1}$$

Note: accept $k > 0$ or $k \neq 0$ or k is a probability.

Do not award **R1** unless the previous mark has been awarded.

$$k = \frac{1}{3} \quad \text{AG}$$

Note: If a candidate uses $k = \frac{1}{3}$ to verify that the probabilities sum to 1, award
M1A0M0A0R0.

[5 marks]

- (b) $1 - P(X = 3a) \text{ OR } P(X = 0) + P(X = a) + P(X = 2a)$ (M1)

$$1 - k^2 \left(= 1 - \frac{1}{9} \right) \text{ OR } 5k^2 + k \left(= \frac{5}{9} + \frac{1}{3} \right) \text{ OR } \frac{1}{3} + \frac{3}{9} + \frac{2}{9}$$

$$P(X < 3a) = \frac{8}{9} \quad \text{A1}$$

[2 marks]

continued...

Question 7 continued.

- (c) valid attempt to identify the correct required outcomes (M1)

$$\frac{P(X \geq a \cap X < 3a)}{P(X < 3a)} \text{ OR } P(a \leq X < 3a) \text{ OR } P(X = a) + P(X = 2a) \text{ OR}$$

$$X = a \text{ and } 2a \text{ OR } P(X < 3a) - P(X < a)$$

correct numerator $3k^2 + 2k^2 (= 5k^2)$ OR $\frac{3}{9} + \frac{2}{9}$ OR $\frac{8}{9} - \frac{1}{9} (= \frac{7}{9})$ (A1)

$$\frac{\frac{5}{9}}{\frac{8}{9}}$$

$$P(X \geq a | X < 3a) = \frac{5}{8} \quad \text{A1}$$

[3 marks]

- (d) attempt to use the expected value formula (M1)

$$0 \times \frac{1}{3} + a \times \frac{1}{3} + 2a \times \frac{2}{9} + 3a \times \frac{1}{9} (= 20)$$

$$\frac{10a}{9} = 20 \text{ OR } 3a + 4a + 3a = 180 \text{ (or equivalent)} \quad \text{A1}$$

$$a = 18 \quad \text{A1}$$

[3 marks]

Total [13 marks]

1. (a) 68 **OR** 74 (A1)
74 – 68
= 6 A1
[2 marks]

(b) recognizing 25% of golfers scored between 70 and 74 (M1)
25% × 80 **OR** 60 – 40
= 20 A1
[2 marks]

Total [4 marks]

6. (a) recognizing constant change from Y to X and/or comparing the two distributions

$$b - 7 = 22 - 19 \quad \text{OR} \quad b = 7 + (22 - 19) \quad \text{OR} \quad \frac{b - 7}{a} = \frac{22 - 19}{a} \quad (M1)$$

$$b = 10 \quad A1$$

[2 marks]

- (b) $(P(7 - a < X < 7 + a) =) 0.68 \quad A1$

[1 mark]

- (c) **EITHER**

recognizing that 22 is one standard deviation above the mean (M1)

OR

recognizing symmetry of the normal curve and total area = 1 (M1)

THEN

0.16 or 0.34 (or equivalent) seen in correct sketch or probability statement (A1)

$$(P(Y < 22) =) 1 - \frac{0.32}{2} \quad \text{OR} \quad 0.5 + 0.34 \quad \text{OR} \quad 0.68 + 0.16$$

$$0.84 \quad A1$$

[3 marks]

Total [6 marks]

Section B

–

5. (a) attempt to substitute $F = 19.8$ into the regression line for A on F (M1)

$$A = 2.89(19.8) + 99.3$$

$$= 156.522 \text{ (cm)}$$

$$\text{arm span} = 157 \text{ (cm)}$$

A1

Note: Award **MOA0** for choosing the wrong regression line to get
 $A = 156.417...$ so $A = 156$.

[2 marks]

- (b) recognition that the lines intersect at the mean point (may be seen on a sketch) (M1)

$$2.89F + 99.3 = \frac{F + 32.6}{0.335} \text{ OR } 0.335A - 32.6 = \frac{A - 99.3}{2.89}$$

$$159.686... \text{ or } 20.8948...$$

$$\text{the mean arm span} = 160 \text{ (cm)}, \text{ the mean foot length} = 20.9 \text{ (cm)}$$

A1A1

[3 marks]

- (c) **METHOD 1**

recognition of symmetry of interval around mean (may be seen on a sketch) (M1)

$$P(H < 153) = 0.06 \text{ OR } P(H < 173) = 0.94 \text{ OR equivalent}$$

$$\frac{153 - 163}{\sigma} = -1.55477... \text{ OR } \frac{173 - 163}{\sigma} = 1.55477... \quad (\text{A1})$$

$$\sigma = 6.43181...$$

$$\sigma = 6.43 \text{ (cm)}$$

A1

METHOD 2

attempt to find σ by equating an appropriate correct normal CDF function to 0.88
 (or e.g. 0.06 or 0.94) (M1)

$$\sigma = 6.43181...$$

$$\sigma = 6.43 \text{ (cm)}$$

A2

Note: Accept use of calculator notation e.g.
 $\text{normcdf}(153, 173, 163, \sigma) = 0.88$

[3 marks]

Total [8 marks]

7. (a) $P(\text{scores 5 points}) = \frac{10}{36}$

A1A1

Note: Award **A1** for numerator of 10 and **A1** for denominator of 36.

$$= \frac{5}{18}$$

AG**[2 marks]**(b) **METHOD 1**

attempts to list or count all outcomes with a difference of more than one

(M1)

$$4 + 3 + 3 + 3 + 3 + 4 (= 20)$$

$$P(\text{scores 0 points}) = \frac{20}{36} \left(= \frac{5}{9} = 0.555555... = 0.556 \right)$$

A1**METHOD 2**attempts to subtract $P(\text{scores 5 points})$ and $P(\text{scores 10 points})$ from 1**(M1)**

$$P(\text{scores 0 points}) = 1 - \frac{10}{36} - \frac{6}{36}$$

$$= \frac{20}{36} \left(= \frac{5}{9} = 0.555555... = 0.556 \right)$$

A1**[2 marks]***continued...*

Question 7 continued.

(c) $P(\text{scores 10 points}) = \frac{6}{36} \left(= \frac{1}{6} \right)$ (may be seen in a table) **(A1)**

x	0	5	10
$P(X = x)$	$\frac{20}{36}$	$\frac{10}{36}$	$\frac{6}{36}$

attempt to use formula for expected value **(M1)**

$$E(X) = \left(0 \times \frac{20}{36} + \right) 5 \times \frac{10}{36} + 10 \times \frac{6}{36} \quad \textbf{(A1)}$$

$$= \frac{110}{36} \left(= \frac{55}{18} = 3.05555... = 3.06 \right) \quad \textbf{A1}$$

[4 marks]

(d) recognition that their part c) must be multiplied by 90 **(M1)**

$$\frac{110}{36} \times 90$$

$$= 275 \quad \textbf{A1}$$

[2 marks]*continued...*

Question 7 continued.

- (e) recognise that round scores must be five 10s or four 10s and a 5 **(M1)**

$$P(\text{five 10s}) = \left(\frac{1}{6}\right)^5, \quad P(\text{four 10s and a 5}) = 5\left(\frac{1}{6}\right)^4 \left(\frac{5}{18}\right) \quad \textbf{(A1) (A1)}$$

$$P(\text{Lynn wins a prize}) = 0.000128601\dots + 0.00107167\dots$$

$$= 0.00120027\dots \left(= 1.20027\dots \times 10^{-3}\right)$$

$$= 0.00120 \left(= 1.20 \times 10^{-3} = \frac{7}{5832}\right) \text{ (accept 0.0012) } \quad \textbf{A1}$$

[4 marks]**Total [14 marks]**

1.

(a) 1.97651..., 40.2286...

$$a = 1.98, b = 40.2 \quad ((y =) 1.98x + 40.2)$$

A1A1

[2 marks]

(b) attempt to substitute $x = 20$ into their regression equation

(M1)

$$y = 1.97651... \times 20 + 40.2286...$$

79.7589...

79.8 (hours)

A1

[2 marks]

Total [4 marks]

8.

- (a) recognising to find $P(W < 170)$ (M1)

$$P(W < 170) = 0.265985\dots$$

$$P(W < 170) = 0.266 \text{ (accept 26.6\%)} \quad \text{A1}$$

[2 marks]

- (b) recognising $P(W > w) = 0.2$ OR labelled sketch (or equivalent) (M1)

$$w = 181.732\dots$$

$$w = 181.7 \text{ (grams) (must be 4 sf)} \quad \text{A1}$$

[2 marks]

- (c) recognizing the need to find $P(170 < W < 185)$ (M1)

$$\text{diagram, } P(170 < W < 185) = 0.628(364\dots)$$

$$62.8\% \quad \text{A1}$$

[2 marks]

- (d) recognising binomial probability (M1)

$$X \sim B(40, 0.628364\dots) \text{ OR } n = 40 \text{ and } p = 0.628364\dots \quad \text{(A1)}$$

$$P(X \geq 30) = 0.073861\dots$$

$$P(X \geq 30) = 0.0739 \text{ (accept 7.39\%)} \quad \text{A1}$$

[3 marks]

continued...

Question 8 continued.

(e) $Y \sim B(10, 0.073861\dots)$ (A1)

$$P(Y = 4) = 0.003944\dots$$

$$P(Y = 4) = 0.00394 \text{ (accept 0.394\%)} \quad \text{A1}$$

[2 marks]

(f) $P(W < 170) = 0.12$ OR $P(W > 185) = 0.06$ (A1)

use of inverse normal to find z score for their probability (M1)

$$z = -1.17(4986\dots), z = 1.55(4773\dots) \quad \text{(A1)(A1)}$$

attempt to solve their two equations (M1)

$$-1.174986\dots = \frac{170 - \mu}{\sigma}$$

$$1.554773\dots = \frac{185 - \mu}{\sigma}$$

$$\mu = 176.456\dots$$

$$\mu = 176 \text{ (grams)} \quad \text{A1}$$

[6 marks]

Total [17 marks]

- 4.** (a) (i) summing probabilities and equating to 1 **(M1)**
 $1.5a + 2a + 0.281 + a + 0.026 = 1$
 $(a =) 0.154$ **A1**
(ii) 2 (days) **A1**
[3 marks]
- (b) using expected value formula **(M1)**
(mean =) 2.44 (2.436 (exact)) **A1**
[2 marks]
- (c) convenience (sampling) **A1**
[1 mark]
Total [6 marks]

8. (a) fixed number of trials

each trial has two possible outcomes (p , q with $P(p) + P(q) = 1$)

outcome of each trial is independent of all the others OR probability of success constant.

A1A1

Note: Award **A1** for each correct condition.

[2 marks]

- (b) 1900×0.37

(A1)

703 (people)

A1

[2 marks]

- (c) (i) Let D be the number of people who will ride *Daifong*
recognition of binomial distribution

(M1)

$$P(D = 712) = 0.0172556\dots$$

$$= 0.0173$$

A1

- (ii) recognizing the need to find $P(684 \leq D \leq 712)$

(M1)

$$P(D \leq 712) = 0.674739\dots$$

$$P(D \leq 683) = 0.177146\dots$$

$$P(684 \leq D \leq 712) = 0.497593\dots$$

$$= 0.498$$

A1

Note: If the normal approximation to the binomial distribution is used, award marks as appropriate.

[4 marks]

continued...

Question 8 continued

(d) recognizing conditional probability

(M1)

Note: Recognition must be shown in context either in words or symbols, not just $P(A|B)$.

$$P(D \leq 692 | 684 \leq D \leq 712) \left(= \frac{P(684 \leq D \leq 692)}{P(684 \leq D \leq 712)} \right) \text{ or equivalent in words}$$

$$P(684 \leq D \leq 692) = 0.132318... \text{ (seen anywhere)}$$

(A1)

$$\left(\frac{P(684 \leq D \leq 692)}{P(684 \leq D \leq 712)} = \right) \frac{0.132318...}{0.497593...}$$

(A1)

$$= 0.265917...$$

$$= 0.266$$

A1

[4 marks]

(e) substituting into formula for independent events

(M1)

$$= 0.37 \times 0.61$$

$$= 0.226 \text{ (= 0.2257 (exact))}$$

A1

[2 marks]

(f) (let T be the number of people who went on *Torbellino*)

use of tables, guess-and-check or inverse binomial

(M1)

Note: Award (M1) for at least one correct value of $P(T \leq 500)$ for a value of n .

Award (M1) for an attempt to set up the normal distribution with $z = 0.504$.

$$n = 809 \text{ (accept } n = 808)$$

A2

[3 marks]

Total [17 marks]

3. (a) $0.5 + 0.6 - 0.4$ (A1)
 $P(A \cup B) = 0.7$ A1

[2 marks]

- (b) $(P(A) \times P(B) =) 0.3 \neq 0.4 (= P(A \cap B))$ A1
 hence A and B are not independent AG

Note: Accept $0.5 \times 0.6 \neq 0.4$ or $0.3 \neq P(A \cap B)$.

[1 mark]

- (c) $P(A \cap B') = 0.1$ (seen anywhere, including Venn diagram) (A1)
 attempt to use conditional probability with A and B' (seen anywhere) (M1)

$$\frac{P(A \cap B')}{P(B')}, \frac{0.1}{0.4}$$

Note: Accept using their $P(A \cap B')$ and $P(B')$ values, which may be seen in their Venn diagram.

$$P(A | B') = \frac{1}{4} \quad \text{A1}$$

[3 marks]

Total [6 marks]

1. (a) recognition that sum of probabilities = 1 (M1)
 $0.2 + 0.3 + k + 0.1 = 1$
 $k = 0.4$ A1

[2 marks]

- (b) recognition that $P(X > 1.5) = P(X = 2) + P(X = 3) + P(X = 4)$ OR $1 - P(X = 1)$ (M1)
 $0.3 + 0.4 + 0.1$ OR $1 - 0.2$
 $= 0.8$ A1

Note: Award (M1)A1FT only if $0 < k < 1$ and their '0.8' is < 1 .

[2 marks]

- (c) attempt to substitute into the formula for $E(X)$ (M1)
 $= (1 \times 0.2) + (2 \times 0.3) + 3k + (4 \times 0.1)$
 $= 2.4$ A1

Note: Award (M1)A1FT only if $0 < k < 1$.

[2 marks]

Total [6 marks]

4. (a) attempt to use $P(A \cap B) = P(A|B)P(B) \left(= \frac{1}{4} \times \frac{1}{3} \right)$ (M1)

$$= \frac{1}{12}$$

A1

[2 marks]

(b) $P(B') = \frac{2}{3}$ or $P(A \cap B') = \frac{5}{12}$ (A1)

attempt to use $P(A|B') = \frac{P(A \cap B')}{P(B')}$ (M1)

$$= \frac{\left(\frac{5}{12} \right)}{\left(\frac{2}{3} \right)}$$

(A1)

$$= \frac{5}{8} \left(= \frac{15}{24} \right)$$

A1

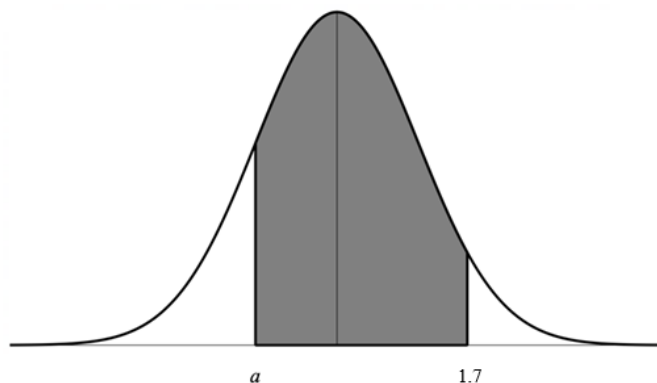
[4 marks]

Total [6 marks]

2. (a) 0.955434...
0.955

A1
[1 mark]

(b)



A1

Note: Award **A1** only if a is indicated to the left of the mean AND the correct area is indicated either by shading, or by labelling with 0.7865.

[1 mark]

- (c) attempt to solve $P(a < Z < 1.7) = 0.7865$ for $P(Z < a)$ or a **(M1)**
their $P(Z < 1.7) - P(Z < a) = 0.7865$ ($\Rightarrow P(Z < a) = 0.955434... - 0.7865 = 0.168934...$)

Note: Accept calculator notation such as $\text{normcdf}(a, 1.7, 0, 1) = 0.7865$.

$$a = -0.958384...$$

$$a = -0.958 \text{ (accept } -0.960 \text{ or } -0.96 \text{ from previous 3 sf value)}$$

A1
[2 marks]

Total [4 marks]

3. (a) attempt to find mean of x -values (M1)
 $\bar{x} = 6.45$ A1
 [2 marks]

- (b) (i) attempt to substitute their $\bar{x}$ into regression equation (M1)
 $\bar{y} = 0.014375 + 2.1625 \times 6.45$
 $\bar{y} = 14.0$ (13.9625 exact) A1

- (ii) attempt to use formula for the mean (M1)

$$\sum_{i=1}^8 y_i = 8 \times 13.9625 \dots (\Rightarrow p + 93.2 = 111.7)$$

 $p = 18.5$ A1

Note: Accept $p = 18.8$ from the use of 3 sf in part (b)(i).

Some candidates are attempting part (b)(ii) before (b)(i), and substitute 8.9 into the given regression equation to find p . They then use their value of p to find $\bar{y}$. In this case, do not award any marks in this part. Allow **FT** from their incorrect p in part (c).

[4 marks]

- (c) 0.953958...
 $r = 0.954$ A1

[1 mark]

Total [7 marks]

6. (a) recognition of binomial distribution

(M1)

$$\binom{10}{2}(p)^2(1-p)^8, B(10, p)$$

Note: The **(M1)** may be awarded for just stating 'binomial', parameters are not required, accept incorrect parameters. Do not award the **(M1)** if candidates only state np . Accept the use of calculator notation, such as binompdf.

attempt to consider both machines using $p = 0.1$ AND $p = 0.05$

(M1)

$$0.5 \times (0.193710...) + 0.5 \times (0.0746347...)$$

(A1)

0.134172...

0.134

A1

[4 marks]

continued...

Question 6 continued.

(b) recognition of conditional probability

(M1)

$$P(A|2 \text{ defective}) \text{ OR } \frac{P(A \cap 2 \text{ defective})}{P(2 \text{ defective})}$$

Note: recognition must be shown in context, either in words or symbols, not just $P(A|B)$.

$$\frac{0.0968...}{0.134...}$$

(A1)

$$0.72187...$$

$$0.722$$

A1

Note: Accept 0.723 or 0.724 from using 3 sf values from part (a).

[3 marks]

Total [7 marks]

Section B

1. (a) (i) $\bar{x} = 235.714...$ and $\bar{y} = 33.4285...$

$$\bar{x} = 236 \text{ and } \bar{y} = 33.4$$

A1

(ii) recognition that the gradient is $\frac{\bar{y}}{\bar{x}}$

(M1)

$$m = 0.141818... \left(\frac{33.4285...}{235.714...} \right)$$

$$m = 0.142$$

$$y = 0.142x$$

A1**[3 marks]**

(b) valid attempt to solve

(M1)

$$a = 0.125515.... \text{ OR } b = 3.84278...$$

$$a = 0.126, b = 3.84$$

A1**[2 marks]**

(c) attempt to substitute 350 into the regression equation from (b)

(M1)

$$y = 0.125515... \times 350 + 3.84278...$$

$$y = 47.7731...$$

$$y = 47.8.$$

A1**[2 marks]****Total [7 marks]**

4.

- (a) the median is the middle value and as Aiden's score is below 10.5 and Brett's score is above 10.5, hence the middle value stays in the same place **R1**

Note: Award the **R1** for any valid reasoning referring to the scores being on either side of the median.

[1 mark]

- (b) attempt to use that the sum of scores is $n \times \text{mean}$ **(M1)**
 10.5×28 OR 10.6×30

sum of Aiden and Brett's scores = 24 ($24 = B + A$) (seen anywhere) **(A1)**

difference between Aiden and Brett's scores = 14 ($14 = B - A$) (seen anywhere) **(A1)**

attempt to solve their two simultaneous equations or by trial and improvement **(M1)**

(Aiden's score =) 5, (Brett's score =) 19 **A1**

Note: The first two **A** marks are independent of each other.

[5 marks]**Total [6 marks]**

6. (a) recognition that $P(\text{height of large sunflowers} > 180) = 0.25$. (M1)

$$\frac{180 - \mu}{\sigma} = 0.674489... \text{ OR } \mu + 0.674\sigma = 180 \quad (\text{A1})$$

$$A = 0.674489...$$

$$A = 0.674 \quad \text{A1}$$

[3 marks]

- (b) correct parameters for the height of giant sunflowers (A1)

$$\mu + 35 \text{ and } 2\sigma \text{ OR } \mu + 35 \text{ and } (2\sigma)^2$$

$$\frac{180 - \mu - 35}{2\sigma} = -2.05374... \quad (\text{A1})$$

attempt to solve their two simultaneous equations in two unknowns, μ and σ . (M1)

$$\mu = 175.063..., \sigma = 7.31913....$$

$$\mu = 175, \sigma = 7.32 \quad \text{A1}$$

[4 marks]

Total [7 marks]

Section B