

APOLLO

Functions

SL

Mathematics: analysis and approaches

First assessment 2021

58 questions | mark schemes

1. (a) (i) $f(2) = 6$

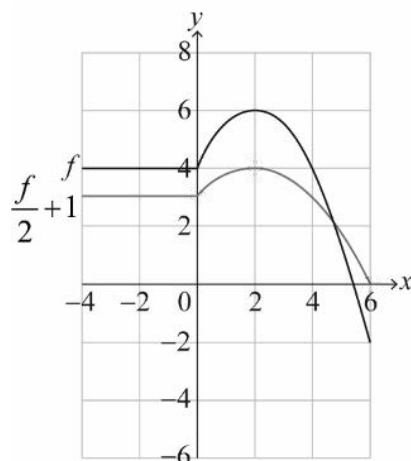
A1

(ii) $(f \circ f)(2) = -2$

A1

[2 marks]

(b)



M1A1A1

Note: Award **M1** for an attempt to apply any vertical stretch or vertical translation, **A1** for a correct horizontal line segment between -4 and 0 (located roughly at $y = 3$), **A1** for a correct concave down parabola including max point at $(2, 4)$ and for correct end points at $(0, 3)$ and $(6, 0)$ (within circles). Points do not need to be labelled.

[3 marks]

Total [5 marks]

7. (a) METHOD 1 (discriminant)

$$mx^2 - 2mx = mx - 9 \quad (M1)$$

$$mx^2 - 3mx + 9 = 0$$

recognizing $\Delta = 0$ (seen anywhere) **M1**

$$\Delta = (-3m)^2 - 4(m)(9) \quad (\text{do not accept only in quadratic formula for } x) \quad \mathbf{A1}$$

valid approach to solve quadratic for m **(M1)**

$$9m(m-4) = 0 \quad \text{OR} \quad m = \frac{36 \pm \sqrt{36^2 - 4 \times 9 \times 0}}{2 \times 9}$$

both solutions $m = 0, 4$ **A1**

$m \neq 0$ with a valid reason **R1**

the two graphs would not intersect OR $0 \neq -9$

$m = 4$ **AG**

METHOD 2 (equating slopes)

$$mx^2 - 2mx = mx - 9 \quad (\text{seen anywhere}) \quad (M1)$$

$$f'(x) = 2mx - 2m \quad \mathbf{A1}$$

equating slopes, $f'(x) = m$ (seen anywhere) **M1**

$$2mx - 2m = m$$

$$x = \frac{3}{2} \quad \mathbf{A1}$$

substituting their x value **(M1)**

$$\left(\frac{3}{2}\right)^2 m - 2m \times \frac{3}{2} = m \times \frac{3}{2} - 9$$

$$\frac{9}{4}m - \frac{12}{4}m = \frac{6}{4}m - 9 \quad \mathbf{A1}$$

$$\frac{-9m}{4} = -9$$

$$m = 4 \quad \mathbf{AG}$$

continued...

Question 7(a) continued

METHOD 3 (using $\frac{-b}{2a}$)

$$mx^2 - 2mx = mx - 9 \quad (M1)$$

$$mx^2 - 3mx + 9 = 0$$

attempt to find x -coord of vertex using $\frac{-b}{2a} \quad (M1)$

$$\frac{-(-3m)}{2m} \quad A1$$

$$x = \frac{3}{2} \quad A1$$

substituting their x value $(M1)$

$$\left(\frac{3}{2}\right)^2 m - 3m \times \frac{3}{2} + 9 = 0$$

$$\frac{9}{4}m - \frac{9}{2}m + 9 = 0 \quad A1$$

$$-9m = -36$$

$$m = 4 \quad AG$$

[6 marks]

(b) $4x(x-2) \quad (A1)$

$$p = 0 \text{ and } q = 2 \text{ OR } p = 2 \text{ and } q = 0 \quad A1$$

[2 marks]

(c) attempt to use valid approach $(M1)$

$$\frac{0+2}{2}, \frac{-(-8)}{2 \times 4}, f(1), 8x-8=0 \text{ OR } 4(x^2-2x+1-1) (= 4(x-1)^2-4)$$

$$h=1, k=-4 \quad A1A1$$

[3 marks]

(d) **EITHER**

recognition $x = h$ to 2 (may be seen on sketch) $(M1)$

OR

recognition that $f(x) < 0$ and $f'(x) > 0 \quad (M1)$

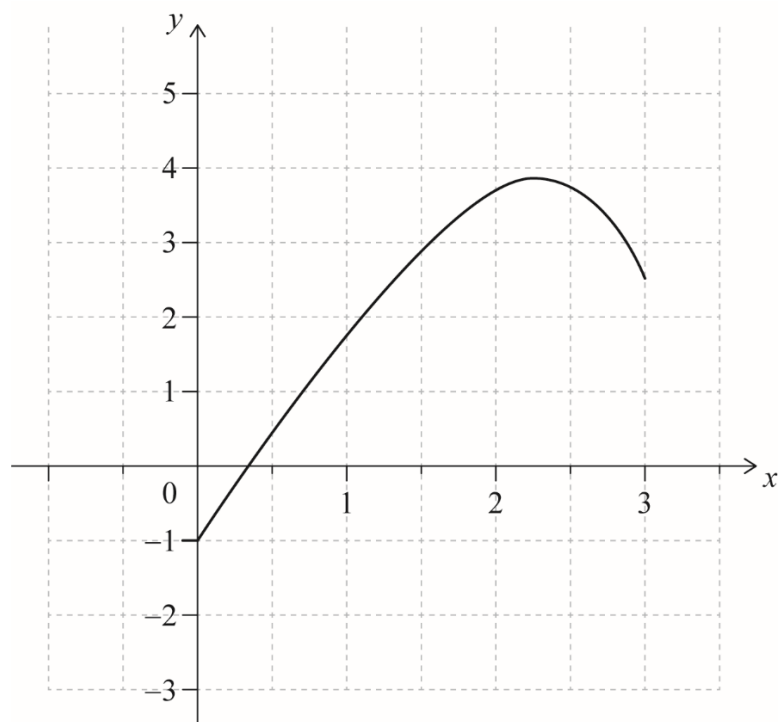
THEN

$$1 < x < 2 \quad A1A1$$

Note: Award A1 for two correct values, A1 for correct inequality signs.
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[3 marks]**Total [14 marks]**

2. (a)

**A1A1A1**

Note: Award **A1** for a smooth concave down curve with generally correct shape. If first mark is awarded, award **A1** for local maximum and x -intercept in approximately correct position, award **A1** for endpoints at $x = 0$ and $x = 3$ with approximately correct y -coordinates.

[3 marks](b) recognizing that $f'(x) = 0$ at local maximum**(M1)**

$$x = 2.33084\dots$$

$$x = 2.33$$

A1**[2 marks]****Total [5 marks]**

5. (a) attempting to find the vertex (M1)

$$x=1 \text{ OR } y=-5 \text{ OR } f(x)=6(x-1)^2-5$$

$$\text{range is } y \geq -5$$

(A1)

[2 marks]

- (b) **METHOD 1**

$$(g \circ f)(x) = -(6x^2 - 12x + 1) + c \quad \left(= -(6(x-1)^2 - 5) + c \right) \quad \text{(A1)}$$

EITHER

relating to the range of f OR attempting to find $g(-5)$ (M1)

$$5 + c \leq 0 \quad \text{(A1)}$$

OR

attempting to find the discriminant of $(g \circ f)(x)$ (M1)

$$144 + 24(c-1) \leq 0 \quad (120 + 24c \leq 0) \quad \text{(A1)}$$

THEN

$$c \leq -5 \quad \text{A1}$$

[4 marks]**METHOD 2**

vertical reflection followed by vertical shift (M1)

new vertex is $(1, 5 + c)$ (A1)

$$5 + c \leq 0 \quad \text{(A1)}$$

$$c \leq -5 \quad \text{A1}$$

[4 marks]**Total [6 marks]**

6. (a) $100 = A_0 e^0$ **A1**
 $A_0 = 100$ **AG**

[1 mark]

(b) correct substitution of values into exponential equation **(M1)**

$$50 = 100e^{-5730k} \quad \text{OR} \quad e^{-5730k} = \frac{1}{2}$$

EITHER

$$-5730k = \ln \frac{1}{2} \quad \text{A1}$$

$$\ln \frac{1}{2} = -\ln 2 \quad \text{OR} \quad -\ln \frac{1}{2} = \ln 2 \quad \text{A1}$$

OR

$$e^{5730k} = 2 \quad \text{A1}$$

$$5730k = \ln 2 \quad \text{A1}$$

THEN

$$k = \frac{\ln 2}{5730} \quad \text{AG}$$

Note: There are many different ways of showing that $k = \frac{\ln 2}{5730}$ which involve showing different steps. Award full marks for at least two correct algebraic steps seen.

[3 marks]

continued...

Question 6 continued

- (c) if 25 % of the carbon-14 has decayed, 75 % remains ie, 75 units remain **(A1)**

$$75 = 100e^{-\frac{\ln 2}{5730}t}$$

EITHER

using an appropriate graph to attempt to solve for t **(M1)**

OR

manipulating logs to attempt to solve for t **(M1)**

$$\ln 0.75 = -\frac{\ln 2}{5730}t$$

$$t = 2378.164\dots$$

THEN

$$t = 2380 \text{ (years) (correct to the nearest 10 years)}$$

A1

[3 marks]

Total [7 marks]

Section B

1. (a) (i) setting $f(x) = 0$ (M1)
 $x = 1$, $x = -3$ (accept $(1,0), (-3,0)$) A1

(ii) **METHOD 1**
 $x = -1$ A1
 substituting their x -coordinate into f (M1)
 $y = 8$ A1
 $(-1, 8)$

METHOD 2
 attempt to complete the square (M1)
 $-2((x+1)^2 - 4)$
 $x = -1$, $y = 8$ A1A1
 $(-1, 8)$

[5 marks]

(b) $h = -1$ A1
 $k = 8$ A1

[2 marks]

Total [7 marks]

- 3.** (a) (i) $x = 3$ **A1**
(ii) $y = -2$ **A1**

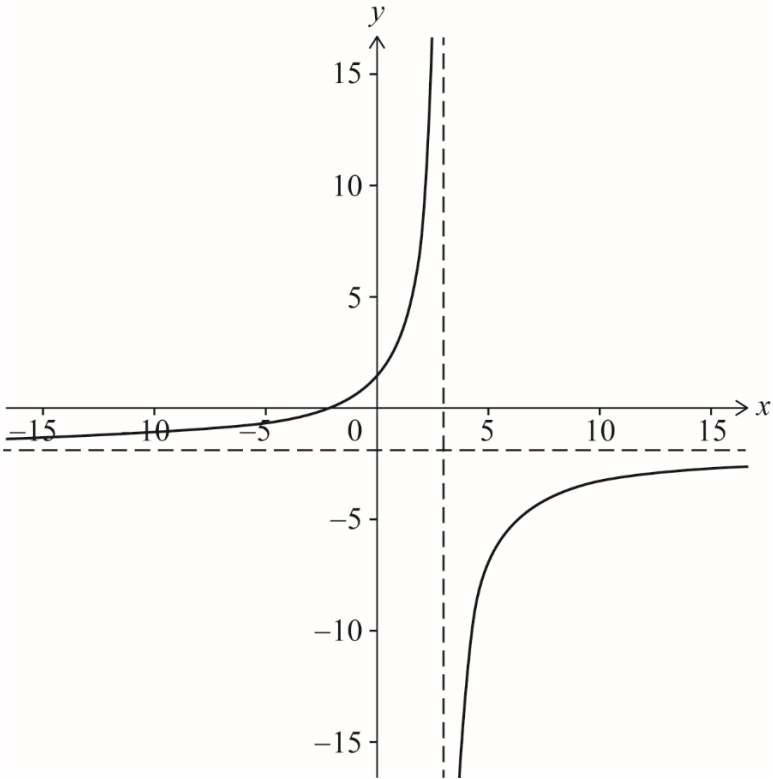
[2 marks]

- (b) (i) $(-2, 0)$ (accept $x = -2$) **A1**
(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$) **A1**

[2 marks]
continued...

Question 3 continued.

(c)



A1

Note: Award **A1** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

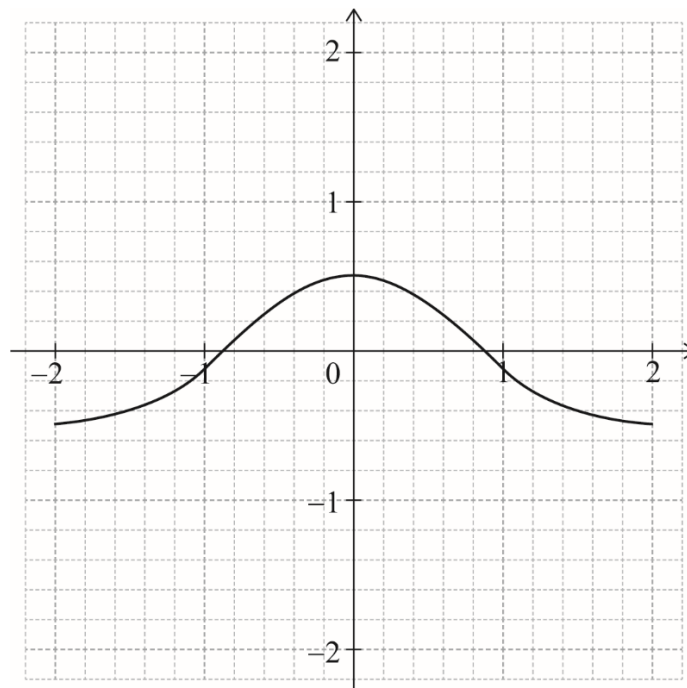
[1 mark]
Total [5 marks]

2. (a) $x = -0.832554\dots$, $x = 0.832554\dots$
 $x = -0.833$, $x = 0.833$

A1A1

[2 marks]

(b)



A1A1A1

Note: Award **A1** for approximately correct shape. Only if this mark is awarded, award **A1** for approximately correct roots and maximum point and **A1** for approximately correct endpoints.
 Allow $-1 < x \leq -0.8$, $0.8 \leq x < 1$ for roots, $x = 0$, $0.4 \leq y \leq 0.6$ for maximum and $x = \pm 2$, $-0.6 \leq y \leq -0.4$ for endpoints.

[3 marks]

Total [5 marks]

5. (a) attempt to form composition **M1**
 correct substitution $g\left(\frac{x+3}{4}\right) = 8\left(\frac{x+3}{4}\right) + 5$ **A1**
 $(g \circ f)(x) = 2x + 11$ **AG**
[2 marks]
- (b) attempt to substitute 4 (seen anywhere) **(M1)**
 correct equation $a = 2 \times 4 + 11$ **(A1)**
 $a = 19$ **A1**
[3 marks]
- Total [5 marks]**

1. (a) $m_{BC} = \frac{12-6}{-14-4} \left(= -\frac{1}{3} \right)$ (A1)

finding $m_L = \frac{-1}{m_{BC}}$ using their m_{BC} (M1)

$$m_L = 3$$

$$y - 20 = 3(x + 2), \quad y = 3x + 26 \quad \text{A1}$$

Note: Do not accept $L = 3x + 26$.

[3 marks]

(b) substituting $(k, 2)$ into their L (M1)

$$2 - 20 = 3(k + 2) \quad \text{OR} \quad 2 = 3k + 26$$

$$k = -8 \quad \text{A1}$$

[2 marks]

Total [5 marks]

6. (a) translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction **A1**
 translation (shift) by q upwards/positive vertical direction **A1**

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

(b) **METHOD 1**

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) **(M1)**

$$-4 + 2.5 + q \geq 7$$

$q \geq 8.5$ (accept $q = 8.5$) **A1**

substituting $x = 0$ and their q ($= 8.5$) to find r **(M1)**

$$(r =) 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + 8.5$$

$4 + 2.5 + 8.5$ **(A1)**

smallest value of r is 15 **A1**

continued...

*Question 6 continued***METHOD 2**

substituting $x = 0$ to find an expression (for r) in terms of q **(M1)**

$$(g(0) = r =) 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + q$$

$$(r =) 6.5 + q \quad \text{A1}$$

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 **(M1)**

$$-4 + 2.5 + q \geq 7$$

$$-4 + 2.5 + (r - 6.5) \geq 7 \quad (\text{accept } =) \quad \text{A1}$$

smallest value of r is 15 **A1**

METHOD 3

$$4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q = 4\cos x + 2.5 + q \quad \text{A1}$$

y -intercept of $4\cos x + 2.5 + q$ is a maximum **(M1)**

amplitude of $g(x)$ is 4 **(A1)**

attempt to find least maximum **(M1)**

$$r = 2 \times 4 + 7$$

smallest value of r is 15 **A1**

[5 marks]

Total [7 marks]

Section B

1. (a) $g(0) = -2$

A1

[1 mark]

(b) evidence of using composite function

(M1)

$f(g(0))$ OR $f(-2)$

$(f \circ g)(0) = 8$

A1

[2 marks]

(c) $x = 3$

A2

[2 marks]

Total [5 marks]

4.

(a)

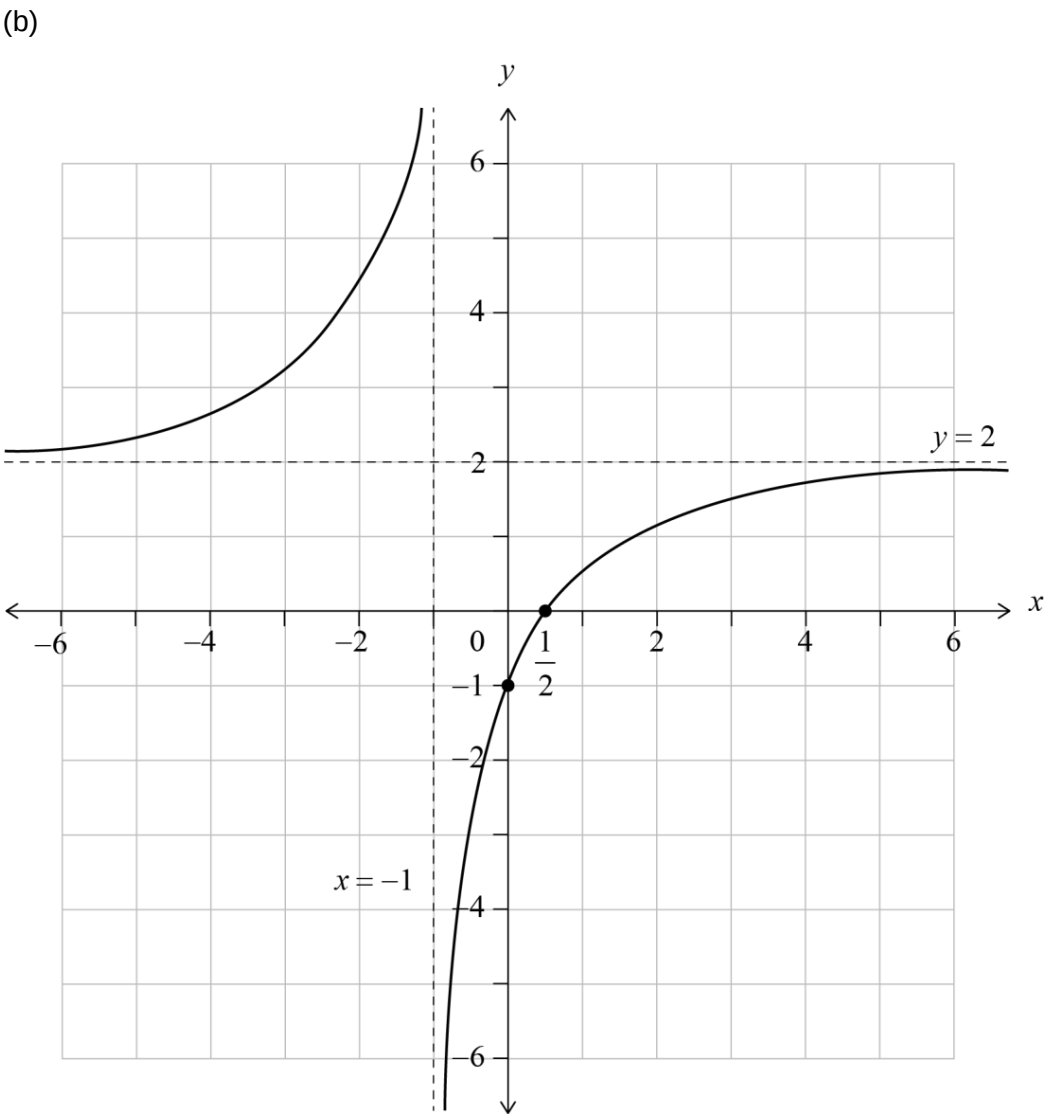
(i)

$x = -1$

A1
- (ii)

$y = 2$

A1
- [2 marks]



continued...

Question 4 continued

rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown

A1

Note: The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x = -1$ and $y = 2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x = \frac{1}{2}$ and $y = -1$

A1A1**[3 marks]**

(c) $x > \frac{1}{2}$

A1

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2}, \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]**Total [6 marks]**

8. (a) (i) $x = -4$

A1

(ii) attempt to substitute into $y = \frac{a}{c}$ OR table with large values of x OR sketch

of f showing asymptotic behaviour

(M1)

$y = 4$

A1

[3 marks]

(b) (i) $y = \frac{4x+1}{x+4}$

attempt to interchange x and y (seen anywhere)

M1

$xy + 4y = 4x + 1$ OR $xy + 4x = 4y + 1$

(A1)

$xy - 4x = 1 - 4y$ OR $xy - 4y = 1 - 4x$

(A1)

$f^{-1}(x) = \frac{1-4x}{x-4}$ (accept $y = \frac{1-4x}{x-4}$)

A1

continued...

Question 8 continued(ii) reflection in y -axis given by $f(-x)$ **(M1)**

$$f(-x) = \frac{-4x+1}{-x+4}$$
(A1)

reflection of their $f(-x)$ in x -axis given by $-f(-x)$ accept “now $-f(x)$ ” **M1**

$$(-f(-x)) = -\frac{-4x+1}{-x+4}$$

$$= \frac{-4x+1}{x-4} \text{ OR } \frac{4x-1}{-x+4}$$
A1

$$= \frac{1-4x}{x-4} (= f^{-1}(x))$$
AG

Note: If the candidate attempts to show the result using a particular coordinate on the graph of f rather than a general coordinate on the graph of f , where appropriate, award marks as follows:

M0A0 for eg $(2,3) \rightarrow (-2,3)$

M0A0 for $(-2,3) \rightarrow (-2,-3)$

[8 marks]*continued...*

Question 8 continued

(c) (i) attempt to solve $f(x) = f^{-1}(x)$ using graph or algebraically **(M1)**

$p = -1$ AND $q = 1$ **A1**

Note: Award **(M1)A0** if only one correct value seen.

(ii) attempt to set up an integral to find area between f and f^{-1} **(M1)**

$$\int_{-1}^1 \left(\frac{4x+1}{x+4} - \frac{1-4x}{x-4} \right) dx \quad \text{span style="float: right;">**(A1)**$$

$$= 0.675231\dots$$

$$= 0.675 \quad \text{span style="float: right;">**A1**$$

[5 marks]

Total [16 marks]

1. (a) gradient of g is -2 (may be seen in function, do not accept $-2x+3$) (A1)

$$g(x) = -2x \quad \text{A1}$$

[2 marks]

- (b) gradient is $\frac{1}{2}$ (may be seen in function) (A1)

attempt to substitute **their** gradient and $(-1, 2)$ into any form of equation for straight line (M1)

$$y - 2 = \frac{1}{2}(x + 1) \text{ OR } 2 = \frac{1}{2} \cdot (-1) + c$$

$$h(x) = \frac{1}{2}(x + 1) + 2 \quad \left(= \frac{1}{2}x + \frac{5}{2} \right) \quad \text{A1}$$

[3 marks]

- (c) $(g \circ h)(x) = -2\left(\frac{1}{2}x + \frac{5}{2}\right)$ OR $h(0) = \frac{5}{2}$ OR $g\left(\frac{5}{2}\right)$ (A1)

$$(g \circ h)(0) = -5 \quad \text{A1}$$

[2 marks]

Total [7 marks]

7. (a) correct substitution of $h = 3$ and $k = 2$ into $f(x)$ (A1)

$$f(x) = a(x-3)^2 + 2$$

correct substitution of $(5, 0)$ (A1)

$$0 = a(5-3)^2 + 2 \quad \left(a = -\frac{1}{2} \right)$$

Note: The first two A marks are independent.

$$f(x) = -\frac{1}{2}(x-3)^2 + 2$$

A1

[3 marks]

- (b) (i) **METHOD 1**

correct substitution of $(1, 4)$ (A1)

$$p + (t-1) - p = 4$$

$$t = 5$$

A1

substituting their value of t into $9p - 3(t-1) - p = 4$ (M1)

$$8p - 12 = 4$$

$$p = 2$$

A1

METHOD 2

correct substitution of ONE of the coordinates $(-3, 4)$ or $(1, 4)$ **(A1)**

$$9p - 3(t - 1) - p = 4 \quad \text{OR} \quad p + (t - 1) - p = 4$$

valid attempt to solve their two equations **(M1)**

$$p = 2, t = 5 \quad \textbf{A1A1}$$

$$(g(x) = 2x^2 + 4x - 2)$$

(ii) attempt to find the x -coordinate of the vertex **(M1)**

$$x = \frac{-3+1}{2} (= -1) \quad \text{OR} \quad \frac{-4}{2 \times 2} \quad \text{OR} \quad 4x + 4 = 0 \quad \text{OR} \quad 2(x+1)^2 - 4$$

y -coordinate of the vertex $= -4$ **(A1)**

correct range **A1**

$$[-4, +\infty[\quad \text{OR} \quad y \geq -4 \quad \text{OR} \quad g \geq -4 \quad \text{OR} \quad [-4, \infty)$$

[7 marks]

(c) equating the two functions or equations **(M1)**

$$g(x) = j(x) \text{ OR } px^2 + (t-1)x - p = -x + 3p$$

$$px^2 + tx - 4p = 0 \quad \text{A1}$$

attempt to find discriminant (do not accept only in quadratic formula) **(M1)**

$$\Delta = t^2 + 16p^2 \quad \text{A1}$$

$\Delta = t^2 + 16p^2 > 0$, because $t^2 \geq 0$ and $p^2 > 0$, therefore the sum will be positive **R1R1**

Note: Award **R1** for recognising that Δ is positive and **R1** for the reason.

There are two distinct points of intersection between the graphs of g and j . **AG**

[6 marks]

Total [16 marks]

8. (a) (i) valid approach to find the required logarithm **(M1)**

$$2^x = \frac{1}{16} \quad \text{OR} \quad 2^x = 2^{-4} \quad \text{OR} \quad \frac{1}{16} = 2^{-4} \quad \text{OR} \quad \log_2 1 - \log_2 16$$

$$\log_2 \frac{1}{16} = -4 \quad \text{A1}$$

- (ii) valid approach to find the required logarithm **(M1)**

$$9^x = 3 \quad \text{OR} \quad 3^{2x} = 3 \quad \text{OR} \quad 3 = 9^{\frac{1}{2}} \quad \text{OR} \quad \frac{\log_3 3}{\log_3 9}$$

$$\log_9 3 = \frac{1}{2} \quad \text{A1}$$

- (iii) $(\sqrt{3})^x = 81 \quad \text{OR} \quad \frac{\log_3 81}{\log_3 \sqrt{3}}$ **(A1)**

$$(3)^{\frac{x}{2}} = 3^4 \quad \text{OR} \quad \frac{x}{2} = 4 \quad \text{OR} \quad \frac{4}{\frac{1}{2}} \quad \text{A1}$$

$$x = 8 \quad \text{A1}$$

[7 marks]

continued...

Question 8 continued

(b) (i)

Note: There are many valid approaches to the question, and the steps may be seen in different ways. Some possible methods are given here, but candidates may use a combination of one or more of these methods.

In all methods, the final **A** mark is awarded for working which leads directly to the **AG**.

METHOD 1

$$(ab)^3 = a \quad (A1)$$

attempt to isolate b or a power of b **(M1)**

correct working **(A1)**

$$b = \frac{a}{a^3 b^2} \quad \text{OR} \quad b^3 = a^{-2} \quad \text{OR} \quad b^{-1} = (ab)^2 \quad \text{OR} \quad b^3 = \frac{1}{a^2}$$

$$b = \frac{1}{a^2 b^2} \quad \text{OR} \quad b = (ab)^{-2} \quad \text{OR} \quad 3 \log_{ab} b = -2 \log_{ab} a \quad \text{OR} \quad -\log_{ab} b = 2 \log_{ab} ab \quad \mathbf{A1}$$

$$\log_{ab} b = -2 \quad \mathbf{AG}$$

METHOD 2

$$(ab)^3 = a \quad (A1)$$

taking logarithm to base ab on both sides **(M1)**

$$\log_{ab} (ab)^3 = \log_{ab} a \quad \text{OR} \quad \log_{ab} a^3 b^3 = \log_{ab} a$$

correct application of log rules leading to equation in terms of $\log_{ab}$ **(A1)**

$$3 \log_{ab} a + 3 \log_{ab} b = \log_{ab} a \quad \text{OR} \quad 3 \log_{ab} b = -2 \log_{ab} a \quad \text{OR} \quad \log_{ab} b^3 = \log_{ab} a^{-2}$$

$$\log_{ab} b = \log_{ab} a^{-\frac{2}{3}} \quad \text{OR} \quad \log_{ab} b = -\frac{2}{3} \log_{ab} a \quad \text{OR} \quad \log_{ab} b = -\frac{2}{3}(3) \quad A1$$

$$\log_{ab} b = -2 \quad AG$$

Note: Candidates may substitute $\log_{ab} a = 3$ at any point in their working.

continued...

*Question 8 continued***METHOD 3**

$$\log_{ab} a = 3$$

writing in terms of base a **(M1)**

$$\frac{\log_a a}{\log_a ab} (= 3)$$

correct application of log rules

(A1)

$$\frac{\log_a a}{\log_a a + \log_a b} (= 3) \quad \text{OR} \quad \frac{1}{1 + \log_a b} (= 3) \quad \text{OR} \quad 3\log_a b = -2 \quad \text{OR}$$

$$\log_a b = -\frac{2}{3}$$

writing $\log_{ab} b$ in terms of base a **(A1)**

$$\log_{ab} b = \frac{\log_a b}{\log_a a + \log_a b}$$

correct working

A1

$$\log_{ab} b = \frac{-\frac{2}{3}}{1 - \frac{2}{3}} \quad \text{OR} \quad \frac{\left(-\frac{2}{3}\right)}{\left(\frac{1}{3}\right)}$$

$$\log_{ab} b = -2$$

AG*continued...*

*Question 8 continued***METHOD 4**

$$\log_{ab} ab = 1 \quad \text{A2}$$

$$\log_{ab} a + \log_{ab} b = 1 \quad \text{(A1)}$$

$$3 + \log_{ab} b = 1 \quad \text{A1}$$

$$\log_{ab} b = -2 \quad \text{AG}$$

(ii) applying the quotient rule or product rule for logs

$$\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} - \log_{ab} \sqrt{b} \quad \text{OR} \quad \log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} + \log_{ab} \frac{1}{\sqrt{b}} \quad \text{(A1)}$$

correct working (A1)

$$= \frac{1}{3} \log_{ab} a - \frac{1}{2} \log_{ab} b \quad \text{OR} \quad \log_{ab} ab - \log_{ab} \sqrt{b}$$

$$= \frac{1}{3} \cdot 3 - \frac{1}{2}(-2) \quad \text{(A1)}$$

$$= 2 \quad \text{A1}$$

Note: Award **A1A0A0A1** for a correct answer with no working.

[8 marks]

Total [15 marks]

5. recognition that initial population is 15000 (seen anywhere) (A1)

$$P(0) = 15000 \text{ OR } 0.11 \times 15000 \text{ OR } 0.89 \times 15000$$

population after 11% decrease is $15000 \times 0.89 (=13350)$ (A1)

recognizing that $t = 8$ on 1 January 2022 (seen anywhere) (A1)

substitution of their value of t for 1 January 2022 and their value of $P(8)$ into the model (M1)

$$15000 \times 0.89 = 15000e^{8k} \text{ OR } 13350 = 15000e^{8k}$$

$$k = \frac{\ln 0.89}{8} (-0.014566) \text{ (A1)}$$

substitution of $t = 2041 - 2014 (= 27)$ and their value for k into the model (M1)

$$P(27) = 15000e^{-0.0145... \times 27}$$

$$10122.3...$$

$$P(27) = 10100 \text{ (10122)} \text{ A1}$$

Total [7 marks]

1. (a) $M(6, -3)$

A1A1

[2 marks]

(b) gradient of $[PQ] = -\frac{5}{9}$

(A1)

gradient of $L = \frac{9}{5}$

A1

[2 marks]

(c) $y + 3 = \frac{9}{5}(x - 6)$ OR $y = \frac{9}{5}x - \frac{69}{5}$ (or equivalent)

A1

Note: Do not accept $L = \frac{9}{5}x - \frac{69}{5}$.

[1 mark]

Total [5 marks]

2. (a) recognizing $f(x) = 0$ (M1)
 $x = -1$ A1
[2 marks]

- (b) (i) $x = 2$ (must be an equation with x) A1
(ii) $y = \frac{7}{2}$ (must be an equation with y) A1
[2 marks]

- (c) **EITHER**
interchanging x and y (M1)
 $2xy - 4x = 7y + 7$
correct working with y terms on the same side: $2xy - 7y = 4x + 7$ (A1)

- OR**
 $2yx - 4y = 7x + 7$
correct working with x terms on the same side: $2yx - 7x = 4y + 7$ (A1)
interchanging x and y OR making x the subject $x = \frac{4y+7}{2y-7}$ (M1)

- THEN**
 $f^{-1}(x) = \frac{4x+7}{2x-7}$ (or equivalent) $\left(x \neq \frac{7}{2}\right)$ A1
[3 marks]

Total [7 marks]

5. recognition of quadratic in e^x **(M1)**
- $(e^x)^2 - 3e^x + \ln k (= 0)$ OR $A^2 - 3A + \ln k (= 0)$
- recognizing discriminant ≥ 0 (seen anywhere) **(M1)**
- $(-3)^2 - 4(1)(\ln k)$ OR $9 - 4 \ln k$ **(A1)**
- $\ln k \leq \frac{9}{4}$ **(A1)**
- $e^{9/4}$ (seen anywhere) **A1**
- $0 < k \leq e^{9/4}$ **A1**
- [6 marks]**

- 3.** (a) (i) $x = 2$
 (ii) $y = 1$

A1

A1

[2 marks]

- (b) (i) $\left(0, \frac{3}{2}\right)$
 (ii) $(3, 0)$

A1

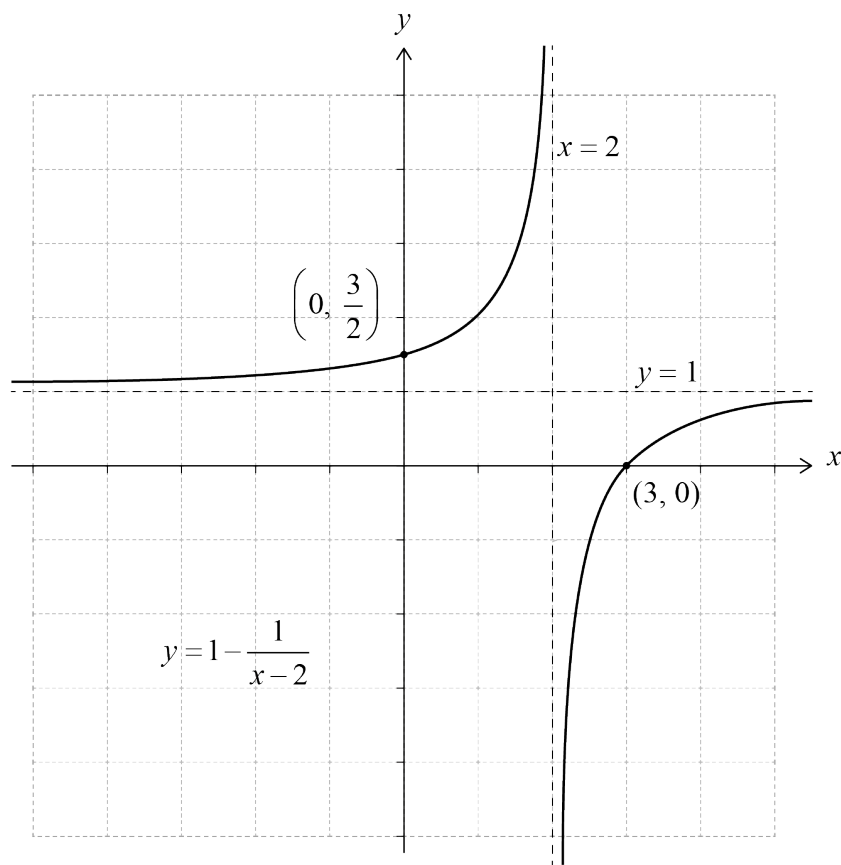
A1

[2 marks]

continued...

Question 3 continued

(c)



two correct branches with correct asymptotic behaviour and intercepts clearly shown

A1

[1 mark]

Total [5 marks]

6. attempts to form $(g \circ f)(x)$ **(M1)**

$$[f(x)]^2 + f(x) + 3 \quad \text{OR} \quad (ax + b)^2 + ax + b + 3$$

$$a^2x^2 + 2abx + b^2 + ax + b + 3 (= 4x^2 - 14x + 15) \quad \textbf{(A1)}$$

equates their corresponding terms to form at least one equation **(M1)**

$$a^2x^2 = 4x^2 \quad \text{OR} \quad a^2 = 4 \quad \text{OR} \quad 2abx + ax = -14x \quad \text{OR} \quad 2ab + a = -14 \quad \text{OR} \quad b^2 + b + 3 = 15$$

$$a = \pm 2 \text{ (seen anywhere)} \quad \textbf{A1}$$

attempt to use $2ab + a = -14$ to pair the correct values (seen anywhere) **(M1)**

$$f(x) = 2x - 4 \text{ (accept } a = 2 \text{ with } b = -4), f(x) = -2x + 3 \text{ (accept } a = -2 \text{ with } b = 3) \quad \textbf{A1A1}$$

[7 marks]

Section B

7. (a) $x = -2$ (must be an equation)

A1

[1 mark]

- (b) $h = -2, k = -5$

A1A1

[2 marks]

- (c) substituting $x = 0$ into $f(x)$

(M1)

$$y = \frac{1}{4}(0+2)^2 - 5$$

$$y = -4 \text{ (accept } P(0, -4))$$

A1

[2 marks]

- (d) $f'(x) = \frac{1}{2}(x+2)\left(\frac{1}{2}x+1\right)$

(A1)

substituting $x = 0$ into their derivative

(M1)

$$f'(0) = 1$$

gradient of normal is -1 (may be seen in their equation)

A1

$$y = -x - 4 \text{ (accept } a = -1, b = -4)$$

A1

Note: Award **A0** for $L = -x - 4$ (without the $y =$).

[4 marks]

continued...

Question 7 continued

(e) equating their $f(x)$ to their L **(M1)**

$$\frac{1}{4}(x+2)^2 - 5 = -x - 4$$

$$\frac{1}{4}x^2 + 2x = 0 \quad (\text{or equivalent}) \quad \text{A1}$$

valid attempt to solve their quadratic **(M1)**

$$\frac{1}{4}x(x+8) = 0 \quad \text{OR} \quad x(x+8) = 0$$

$$x = -8 \quad \text{A1}$$

Note: Accept both solutions $x = -8$ and $x = 0$ here, $x = -8$ may be seen in working to find coordinates of Q or distance.

substituting their value of x (not $x = 0$) into their $f(x)$ or their L **(M1)**

$$y = -(-8) - 4 \quad \text{or} \quad y = \frac{1}{4}(-8+2)^2 - 5$$

$$Q(-8, 4) \quad \text{A1}$$

correct substitution into distance formula **(A1)**

$$\sqrt{(-8-0)^2 + (4-(-4))^2}$$

$$\text{distance} = \sqrt{128} \quad (= 8\sqrt{2}) \quad \text{A1}$$

[8 marks]

Total [17 marks]

3. (a) attempt to substitute g into f
 $(f \circ g)(x) = 2 \tan x - \tan^3 x$

(M1)

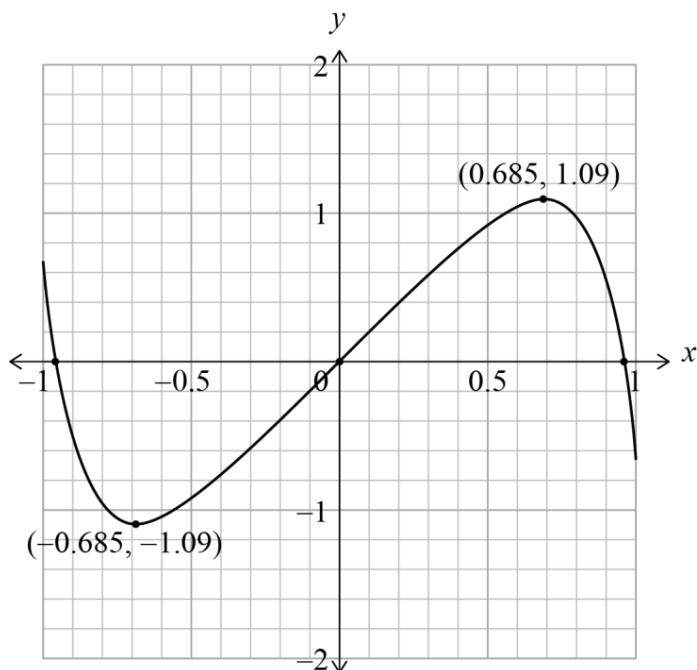
A1

[2 marks]

continued...

Question 3 continued

(b)

**A1A1A1**

Note: **A1** for approximately correct odd function passing through the origin with a maximum above $y = 1$ and a minimum below $y = -1$.

A1 for endpoints at $x = \pm 1$ and y in the intervals $[0.6, 0.8]$ and $[-0.8, -0.6]$

A1 for maximum in approximately correct position and labelled $(0.685, 1.09)$ AND minimum in approximately correct position and labelled $(-0.685, -1.09)$. For approximate position, allow $-0.8 \leq x \leq -0.6$, $-1.2 \leq y \leq -1$ for minimum and $0.6 \leq x \leq 0.8$, $1 \leq y \leq 1.2$ for maximum. If the candidate gives the coordinates of extrema below their sketch, only award this mark if extrema are marked in the correct interval (eg by a dot).

[3 marks]**Total [5 marks]**

3. (a) $A(0) = 500 \text{ (mg)}$

A1

[1 mark]

(b) $280 = 500e^{-3k}$

(A1)

$k = 0.193272\dots$

$k = 0.193 \left(= -\frac{1}{3} \ln \left(\frac{280}{500} \right) \right)$

A1

[2 marks]

(c) $500e^{-0.193272\dots T} = 140$

(A1)

$T = 6.58636\dots$

$T = 6.59 \text{ (h)}$

A1

[2 marks]

Total [5 marks]

7. (a) (i) swapping x and y , or $h(h^{-1}(x)) = x$ (M1)

$$h^{-1}(x) = \frac{x^2 + 2}{4} \quad \text{A1}$$

recognizing range of h is domain of h^{-1} (M1)

Domain: $x \geq 0$ A1

- (ii) range of h^{-1} is $y \geq \frac{1}{2}$ A1

[5 marks]

(b) $\sqrt{4x-2} = \frac{x^2+2}{4}$ OR $\sqrt{4x-2} = x$ OR $\frac{x^2+2}{4} = x$ (M1)

$$x = 0.585786..., x = 3.414213... (= 2 + \sqrt{2})$$

$$x = 0.586, x = 3.41 \quad \text{A1A1}$$

[3 marks]

- (c) attempt to form integral of the difference between $h(x)$ and their h^{-1} , using their limits from part (b) (M1)

$$\int_{0.585786...}^{3.414213...} (h(x) - h^{-1}(x)) dx \quad \text{OR} \quad \int_{0.585786...}^{3.414213...} \left(\sqrt{4x-2} - \frac{x^2+2}{4} \right) dx \quad \text{OR}$$

$$6.5996632... - 4.7140452...$$

$$1.88561...$$

$$\text{area} = 1.89 \quad \text{A1}$$

[2 marks]

continued...

Question 7 continued

- (d) attempt to use chain rule or power rule

(M1)

$$h'(x) = 4 \cdot \frac{1}{2} (4x - 2)^{-\frac{1}{2}}$$

$$h'(x) = \frac{2}{\sqrt{4x - 2}}$$

A1**[2 marks]**

- (e)
- EITHER**

$$(h^{-1})'(x) = \frac{x}{2}$$

(A1)

equating their $h'(x)$ to the derivative of their $h^{-1}(x)$ and attempting to solve for x

(M1)

$$\frac{2}{\sqrt{4x - 2}} = \frac{x}{2}$$

OR

finding intersection of graphs of their derivatives

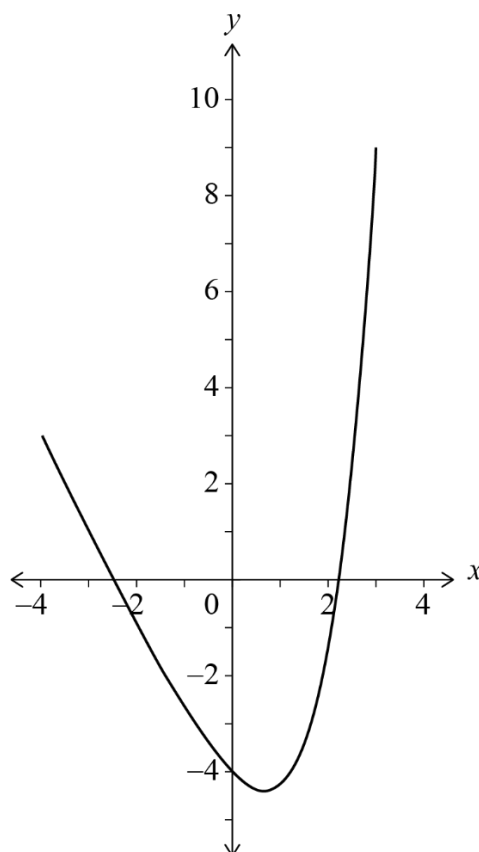
(M2)**THEN**

1.772776...

$x = 1.77$

A1**[3 marks]****Total [15 marks]**

3. (a)

**A1A1A1****Note:** Award marks as follows:**A1** for approximately correct roots, in the intervals $-3 < x < -2$ and $2 < x < 3$.**A1** for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-4.5 < y < -3.5$, and for local minimum $0.2 < x < 1.2$, $-5 < y < -4$.**A1** for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5$, $2.5 < y < 3.5$ and the right end in the intervals $2.5 < x < 3.5$, $8.5 < y < 9.5$.**[3 marks]**

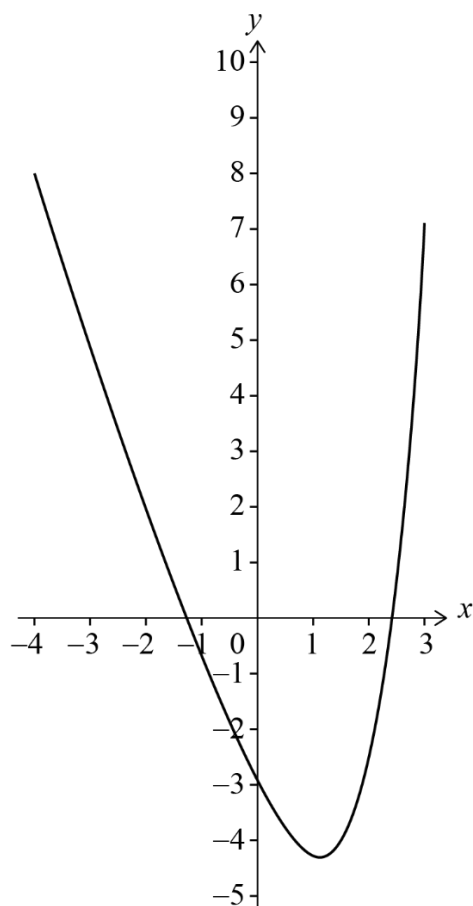
(b) $k = \frac{1}{3}$

A1

$c = -2$ (accept translate/shift 2 (units) down)

A1**[2 marks]****Total [5 marks]**

3. (a)

**A1A1A1****Note:** Award marks as follows:**A1** for approximately correct roots, in the intervals $-2 < x < -1$ and $2 < x < 3$.**A1** for y-intercept AND local minimum in approximately correct positions. Allow for y-intercept $-3.5 < y < -2.5$, and for local minimum $0.5 < x < 1.5$, $-5 < y < -4$.**A1** for approximately correct endpoints, with the left end in the intervals $-4.5 < x < -3.5$, $7.5 < y < 8.5$ and the right end in the intervals $2.5 < x < 3.5$, $6.5 < y < 7.5$.**[3 marks]**

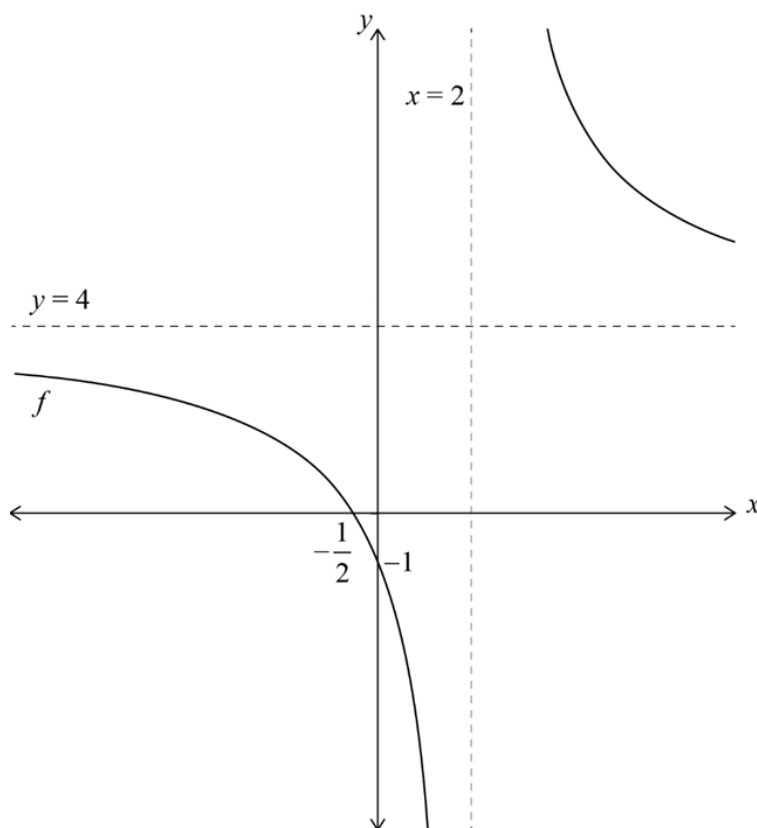
(b) $k = \frac{1}{2}$

A1

$c = -3$ (accept translate/shift 3 (units) down)

A1**[2 marks]****Total [5 marks]**

8. (a)

vertical asymptote $x = 2$ sketched and labelled with correct equation**A1**horizontal asymptote $y = 4$ sketched and labelled with correct equation**A1**

For an approximate rational function shape:

labelled intercepts $-\frac{1}{2}$ on x -axis, -1 on y -axis**A1A1**

two branches in correct opposite quadrants with correct asymptotic behaviour

A1**Note:** These marks may be awarded independently.**[5 marks]**(b) $y \neq 4$ (or equivalent)**A1****[1 mark]***continued...*

Question 8 continued

$$(c) \quad 2 + \frac{5}{2} \text{ OR } \left(-\frac{1}{2}\right) + 2 \times \frac{5}{2} \text{ OR } \frac{-\frac{1}{2} + p}{2} = 2 \text{ OR } -4 = -p + \frac{1}{2} \quad \mathbf{A1}$$

$$p = \frac{9}{2} \quad \mathbf{AG}$$

[1 mark]

(d) **METHOD 1**attempt to substitute both roots to form a quadratic **(M1)****EITHER**

$$\left(x + \frac{1}{2}\right)\left(x - \frac{9}{2}\right) \text{ OR } x^2 - \left(-\frac{1}{2} + \frac{9}{2}\right)x + \left(-\frac{1}{2} \times \frac{9}{2}\right)$$

$$= x^2 - 4x - \frac{9}{4} \quad \mathbf{A1A1}$$

$$\left(b = -4, c = -\frac{9}{4}\right)$$

Note: Award **A1** for each correct value. They may be embedded or stated explicitly.

OR

$$(2x + 1)(2x - 9) = 4\left(x^2 - 4x - \frac{9}{4}\right)$$

$$b = -4, c = -\frac{9}{4} \quad \mathbf{A1A1}$$

Note: Award **A1** for each correct value. They must be stated explicitly.

METHOD 2

$$-\frac{b}{2} = 2 \text{ OR } 4 + b = 0 \Rightarrow b = -4 \quad \mathbf{A1}$$

attempt to form a valid equation to find c using their b **(M1)**

$$\left(-\frac{1}{2}\right)^2 + -4\left(-\frac{1}{2}\right) + c = 0 \text{ OR } \left(\frac{9}{2}\right)^2 + -4\left(\frac{9}{2}\right) + c = 0$$

$$c = -\frac{9}{4} \quad \mathbf{A1}$$

continued...

Question 8 continued**METHOD 3**attempt to form two valid equations in b and c **(M1)**

$$\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + c = 0, \left(\frac{9}{2}\right)^2 + b\left(\frac{9}{2}\right) + c = 0$$

$$b = -4, c = -\frac{9}{4}$$

A1A1**METHOD 4**attempt to write $g(x)$ in the form $(x-h)^2 + k$ and substitute for x, h and $g(x)$ **(M1)**

$$\left(-\frac{1}{2} - 2\right)^2 + k = 0 \Rightarrow k = -\frac{25}{4}$$

$$(x-2)^2 - \frac{25}{4}$$

$$= x^2 - 4x - \frac{9}{4}$$

A1A1

$$\left(b = -4, c = -\frac{9}{4}\right)$$

Note: Award **A1** for each correct value. They may be embedded or stated explicitly.

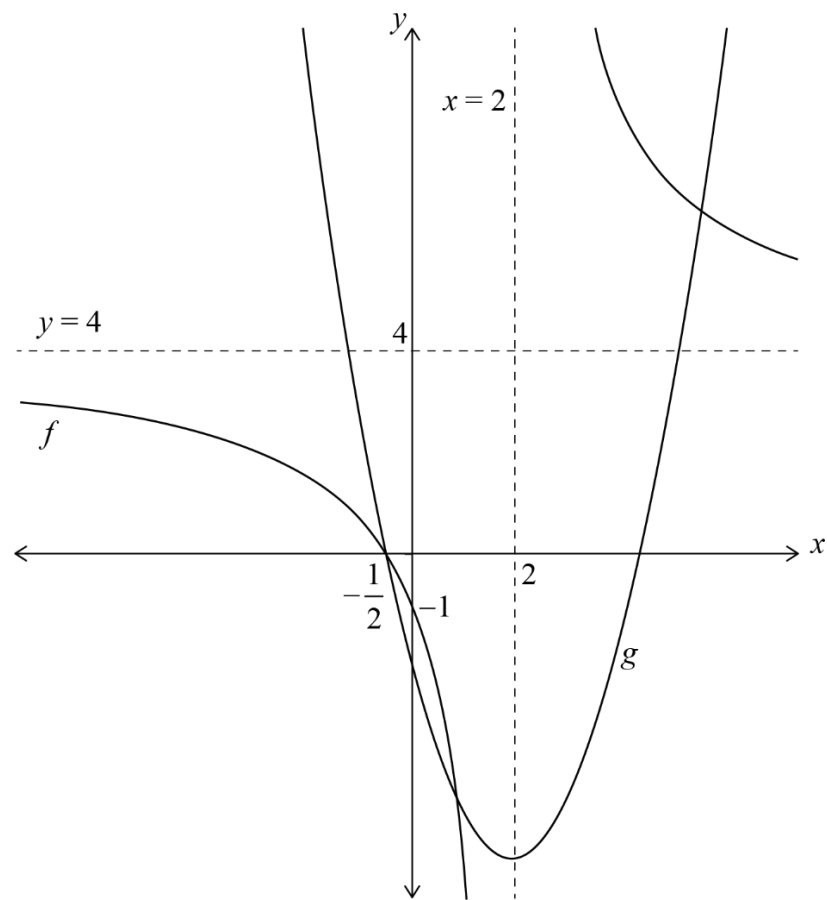
[3 marks](e) attempt to substitute $x = 2$ into their $g(x)$ ORcomplete the square on their $g(x)$ (may be seen in part (d))**(M1)**

$$y = -\frac{25}{4}$$

A1**[2 marks]***continued...*

Question 8 continued

(f)



both graphs sketched on same axes and identifying points of intersection **(M1)**

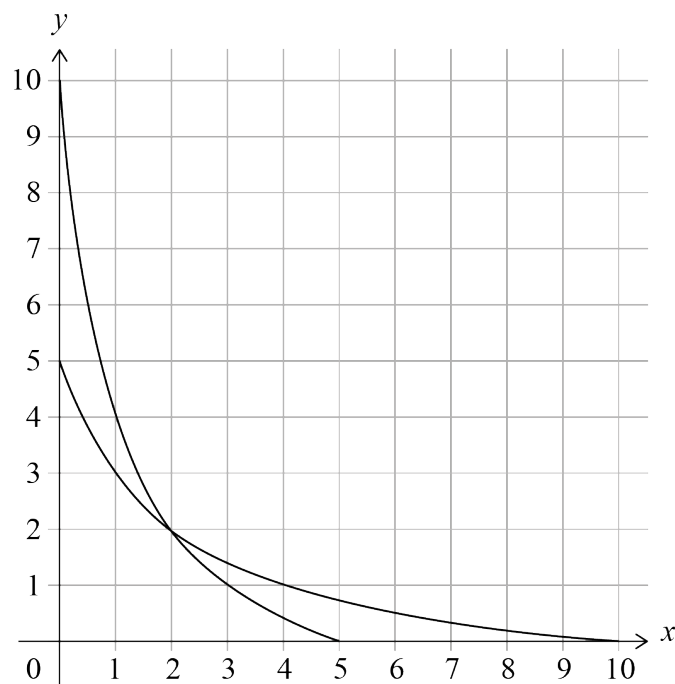
3 solutions **A1**

Note: Exception to **FT**: If the candidate's graph in part (a) is incorrect, the **M1** may be awarded for a sketch of their graph from part (a) and $g(x)$. Do not award the final **A1** in this case.

[2 marks]
Total [14 marks]

1. (a) (i) $f(4)=1$ **A1**
- (ii) $f \circ f(4)=3$ **A1**
- (iii) $f^{-1}(3)=1$ **A1**
- [3 marks]**

(b)



concave up curve with y intercept at $(0,10)$ and x intercept at $(5,0)$ **A1**

curve passes through $(2,2)$ OR through $(1, 4)$ and $(3, 1)$ **A1**

Note: Do not award the second mark unless the first mark has been awarded.(Do not award **A0A1**).

[2 marks]

Total [5 marks]

3. (a) valid attempt to solve a quadratic equation (factorising, use of formula, completing square, or otherwise) **(M1)**

$$(3m-1)(m+2)=0 \text{ OR } m = \frac{-5 \pm \sqrt{25+24}}{6} \text{ (or equivalent)} \quad \textbf{(A1)}$$

$$m = \frac{1}{3}, m = -2 \quad \textbf{A1}$$

[3 marks]

- (b) setting their m -value(s) $= 3^x$ OR recognising a quadratic in 3^x **(M1)**

$$3^x = \frac{1}{3} \text{ (or } 3^x = -2 \text{) OR } 3 \times (3^x)^2 + 5 \times 3^x - 2 = 0$$

$$x = -1 \quad \textbf{A1}$$

Note: Award the final **A1** if candidate's answer includes $x = -1$ and $x = \log_3(-2)$. Award **A0** if other incorrect answers are given.

[2 marks]

Total [5 marks]

✓ ✓ ✓

4. (a) (i) $\left(\frac{9}{2}, \frac{3\sqrt{3}}{2}\right)$ (accept $x = \frac{9}{2}$ and $y = \frac{3\sqrt{3}}{2}$) **A1**

(ii) using $m = \frac{\text{change in } y}{\text{change in } x}$ with their midpoint OR gradient perpendicular to AC

OR $m = \tan 30^\circ$ **(M1)**

$m = \frac{\sqrt{3}}{3}$ **(A1)**

$y = \frac{\sqrt{3}}{3}x$ OR $y - \frac{3\sqrt{3}}{2} = \frac{\sqrt{3}}{3}\left(x - \frac{9}{2}\right)$ (must be written as an equation) **A1**

[4 marks]

(b) substituting $x = 6$ into their equation **(M1)**

so at B $y = 2\sqrt{3}$ **(A1)**

area of triangle OAB $= \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$

area of quadrilateral OABC $= 12\sqrt{3}$ **A1**

[3 marks]

Total [7 marks]

6. (a) $y = \frac{2}{3}$ (must be written as equation with $y =$,)

A1**[1 mark]**

(b) (i) 2

A1

(ii) **EITHER**

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation

(M1)

$$2x + 6 = 3mx^2 + 6mx + 3x + 6$$

$$3mx^2 + (6m+1)x = 0 \quad \text{OR} \quad 3mx^2 + 6mx + x = 0$$

A1

recognition that discriminant $\Delta = 0$ for one solution

(M1)

$$(6m+1)^2 = 0$$

continued...

Question 6 continued.

OR

$$\frac{2(x+3)}{3(x+2)} = mx + 1$$

attempt to expand to obtain a quadratic equation **(M1)**

$$2x + 6 = 3mx^2 + 6mx + 3x + 6$$

$$3mx^2 + (6m + 1)x = 0 \quad \text{OR} \quad 3mx^2 + 6mx + x = 0 \quad \textbf{A1}$$

attempt to solve their quadratic for x and equating their solutions **(M1)**

$$x(3mx + 6m + 1) = 0$$

$$x = 0 \quad \text{OR} \quad x = -\frac{6m+1}{3m} (= 0)$$

$$-\frac{6m+1}{3m} = 0$$

OR

attempt to find $f'(x)$ using the quotient rule **(M1)**

$$f'(x) = \frac{2}{3} \left(\frac{(x+2) - (x+3)}{(x+2)^2} \right) = \left(\frac{-2}{3(x+2)^2} \right) \quad \text{OR} \quad \frac{2(3x+6) - 3(2x+6)}{(3x+6)^2} \quad \text{or}$$

equivalent **A1**

recognition that m is the derivative of $f(x)$ at $x = 0$ **(M1)**

THEN

$$\Rightarrow m = -\frac{1}{6} \quad \textbf{A1}$$

continued...

Question 6 continued.

(iii)

Note: In this part, FT may be awarded only for values of m between -1 and 0 .

$$-\frac{1}{6} < m < 0$$

A2

Note: Award **A1** for only $m > -\frac{1}{6}$. Award **A1** for only $m < 0$.

[7 marks]

Total [8 marks]

SECTION B

7. (a) METHOD 1attempt to interchange x and y **M1****Note:** This **M1** may be awarded at any stage in the working.

attempt to rearrange using definition of natural log or take the natural log of both sides

M1

$$\frac{2x}{3} = e^{y-2} \Rightarrow \ln\left(\frac{2x}{3}\right) = y-2 \quad \text{OR} \quad x = \frac{3}{2}e^{y-2} \Rightarrow \ln(x) = \ln\left(\frac{3}{2}\right) + y-2$$

A1

$$y = 2 + \ln\left(\frac{2x}{3}\right)$$

$$\text{so } f^{-1}(x) = 2 + \ln\left(\frac{2x}{3}\right)$$

AG**[3 marks]****METHOD 2**attempt to verify that $(f \circ f^{-1})(x) = x$ **M1**

$$(f \circ f^{-1})(x) = \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)+2-2} \left(= \frac{3}{2}e^{\ln\left(\frac{2x}{3}\right)} \right)$$

attempt to use definition of natural log

M1

$$(f \circ f^{-1})(x) = \frac{3}{2} \times \frac{2x}{3}$$

A1

$$(f \circ f^{-1})(x) = x$$

AG**[3 marks]***continued...*

Question 7 continued

- (b) $(0.264456..., 0.264456...)$ AND $(2.51799..., 2.51799...)$ (A1)

Note: Award **A1** for 0.264456... and 2.51799... seen.

attempt to put their values in distance formula or use of the isosceles right-angled triangle (M1)

$$\begin{aligned} & \sqrt{(2.51799... - 0.264456...)^2 + (2.51799... - 0.264456...)^2} \quad \text{OR} \\ & \sqrt{2} \times (2.51799... - 0.264456...) \\ & = 3.18689... \\ & = 3.19 \end{aligned}$$

A1

[3 marks]

- (c) (i) $g(x) = -\frac{3}{2}e^{x-2} + 5$ OR $g(x) = -f(x) + 5$ A1A1

Note: Award **A1** for each correct term.

- (ii) $0 \leq x \leq 4$ A1
[3 marks]

- (d) $\frac{3}{2}e^{x-2} = -\frac{3}{2}e^{x-2} + 5$ OR $f(x) = -f(x) + 5$

attempt to collect together terms in e^{x-2} or $f(x)$ (M1)

$$3e^{x-2} = 5 \quad \text{OR} \quad 2f(x) = 5$$

$$e^{x-2} = \frac{5}{3} \quad \text{OR} \quad x = f^{-1}\left(\frac{5}{2}\right) \quad (A1)$$

$$x = 2 + \ln\left(\frac{5}{3}\right) \quad A1$$

$$\left(a = 2, b = \frac{5}{3}\right)$$

Note: Award **A1** for each correct term given in exact form.

[3 marks]

Total [12 marks]

4. (a) $I = 2 \times 10^{-6} \left(= \frac{1}{500000} \right)$ (units)

A1**[1 mark]**

(b) substitutes their doubled I -value from part (a) into L

(M1)

$$L = 10 \log_{10} (2 \times 10^{-6} \times 10^{12}) (= 63.0102\dots)$$

$$= 63.0 \text{ (decibels)}$$

A1

Note: Accept $60 + 10 \log_{10} 2$ (decibels) as a final answer.
Do not award the final **A1** for $L = 0$ (from $I = 10^{-12}$).

[2 marks]

(c) $115 = 10 \log_{10} (I \times 10^{12})$

(A1)

attempts to solve for I

(M1)

$$I = \frac{10^{11.5}}{10^{12}} \text{ (or equivalent) } (= 0.316227\dots)$$

$$I = 0.316 \text{ (units)}$$

A1

Note: Accept exact final answers such as $10^{-0.5}$ and $\frac{1}{\sqrt{10}}$.

[3 marks]**Total [6 marks]**

8. (a) (i) substitution of $x=2$ in part (i) or $x=\frac{1}{8}$ in part (ii) **(M1)**

$$\log_2(8 \times 2) \text{ OR } \log_2(16)$$

$$\log_2(16) = 4 \quad \text{A1}$$

$$(ii) \quad f\left(\frac{1}{8}\right) = 0 \quad \text{A1}$$

[3 marks]

- (b) swap x and y **(M1)**

$$x = \log_2(8y) \text{ OR } x = 3 + \log_2 y$$

attempt to write as exponential **(M1)**

$$2^x = 8y \quad \text{A1}$$

$$\left(f^{-1}(x) = \frac{2^x}{8} \quad (= 2^{x-3}) \quad \left(\text{accept } y = \frac{2^x}{8} \text{ or } y = 2^{x-3}\right)\right) \quad \text{A1}$$

[4 marks]

- (c) $\frac{1}{8}$ **A1**

[1 mark]

continued...

Question 8 continued**(d) METHOD 1**

$$f(4x^2) = \log_2(8 \times 4x^2)$$

attempt to use addition rule for logs

(M1)

$$\log_2 8 + \log_2 4 + \log_2 x^2 \text{ OR } \log_2 32 + \log_2 x^2 \text{ (or equivalent)}$$

(A1)

attempt to use exponent property for logarithms

(M1)

$$f(4x^2) = 5 + 2 \log_2 x \text{ (or equivalent)}$$

A1

the graph of g must be vertically stretched (dilated) by a scale factor of 2 and then vertically translated (shifted) 5 units upwards.

A2**METHOD 2**

$$f(4x^2) = \log_2(8 \times 4x^2)$$

attempt to write argument as a power

(M1)

$$\log_2(32x^2) = \log_2\left(\left(\sqrt{32}x\right)^2\right) \text{ (or equivalent)}$$

(A1)

attempt to use exponent property for logarithms

(M1)

$$f(4x^2) = 2 \log_2(\sqrt{32}x) \text{ (or equivalent)}$$

A1**EITHER**

the graph of g must be vertically stretched (dilated) by a scale factor of 2 and stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$.

A1**OR**

the graph of g must be stretched (dilated) horizontally by a scale factor of $\frac{1}{\sqrt{32}}$ and

vertically stretched (dilated) by a scale factor of 2.

A1

Note: In this method, the final mark is **A1**, as the question specifically asks for a translation and a stretch.

[6 marks]**[Total: 14 marks]**

8. (a) (i) substitution of $x = 8$ in part (i) or $x = \frac{1}{4}$ in part (ii) (M1)

$$\log_2(4 \times 8) \text{ OR } \log_2(32)$$

$$\log_2(32) = 5 \quad \text{A1}$$

(ii) $f\left(\frac{1}{4}\right) = 0$ A1

[3 marks]

- (b) swap x and y (M1)

$$x = \log_2(4y) \text{ OR } x = 2 + \log_2 y$$

attempt to write as exponential (M1)

$$2^x = 4y \quad \text{(A1)}$$

$$(f^{-1}(x) =) \frac{2^x}{4} (= 2^{x-2}) \quad \text{A1}$$

[4 marks]

- (c) $\frac{1}{4}$ A1

[1 mark]

continued...

Question 8 continued.

(d) **METHOD 1**

$$f(16x^3) = \log_2(4 \times 16x^3)$$

attempt to use addition rule for logs (M1)

$$\log_2 4 + \log_2 16 + \log_2 x^3 \text{ OR } \log_2 64 + \log_2 x^3 \text{ (or equivalent) (A1)}$$

attempt to use exponent property for logarithms (M1)

$$f(16x^3) = 6 + 3 \log_2 x \text{ (or equivalent) A1}$$

the graph of g must be vertically stretched (dilated) by a scale factor of 3 and then vertically translated (shifted) 6 units upwards. A2

METHOD 2 (5 marks maximum)

$$f(16x^3) = \log_2(4 \times 16x^3)$$

attempt to write argument as a power (M1)

$$\log_2(64x^3) = \log_2((4x)^3) \text{ (or equivalent) (A1)}$$

attempt to use exponent property for logarithms (M1)

$$f(16x^3) = 3 \log_2(4x) \text{ (or equivalent) A1}$$

EITHER

the graph of g must be vertically stretched (dilated) by a scale factor of 3 and stretched (dilated) horizontally by a scale factor of $\frac{1}{4}$. A1

OR

the graph of g must be stretched (dilated) horizontally by a scale factor of $\frac{1}{4}$ and vertically stretched (dilated) by a scale factor of 3. A1

Note: In this method, the final mark is **A1**, as the question specifically asks for a translation and a stretch.

[6 marks]

[Total: 14 marks]

1. (a) (i) $f(0) = -7$ **A1**

(ii) $-12.7782\dots$

$$f(60) = -12.8 \left(= 7\sqrt{60} - 67 = 14\sqrt{15} - 67 \right) \quad \textbf{A1}$$

[2 marks]

(b) attempt to find at least one root **(M1)**

$$x = 1.46098\dots \text{ and } x = 33.5390\dots$$

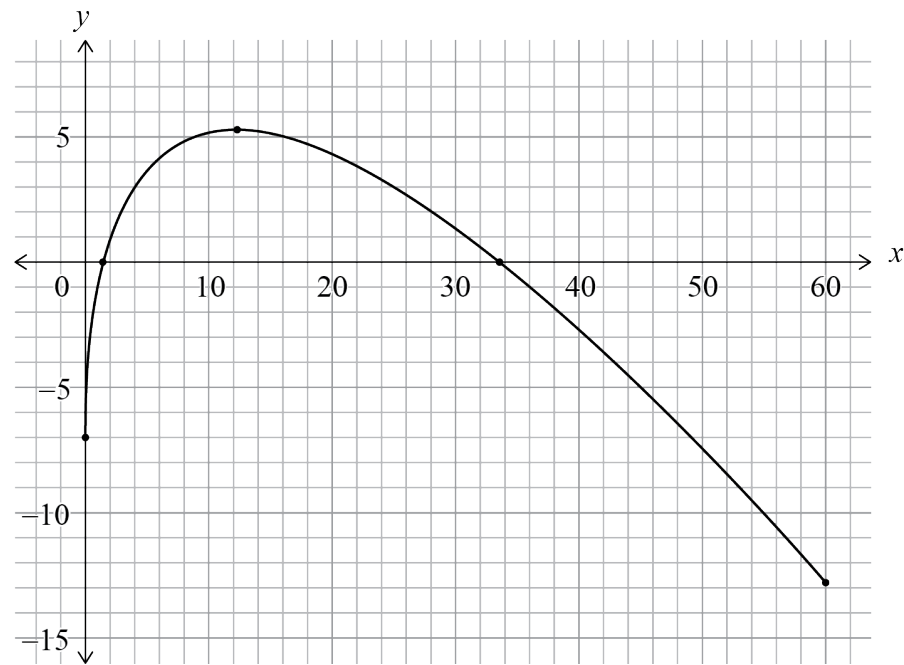
$$x = 1.46 \text{ and } x = 33.5 \quad \textbf{A1}$$

[2 marks]

continued...

Question 1 continued

(c)



A1A1A1

Note: Follow through from their part (a).

Award **A1** for endpoints at approximately $(0, -7)$ and $(60, -12.8)$. Allow for y -intercept $-8 < y < -6$ and for the right endpoint in the interval $x = 60, -13 < y < -12$.

The following two **A** marks may only be awarded if the approximate shape is correct.

Award **A1** for x -intercepts at approximately $x = 1.5$ and $x = 33.5$ and award **A1** for maximum at approximately $(12.2, 5.3)$. Allow for x -intercepts in the intervals $0 < x < 2, 32 < x < 34$, and maximum in the intervals $10 < x < 14, 4.5 < y < 6$.

[3 marks]

Total [7 marks]

6. METHOD 1

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) **R1**

(discriminant =) $(-r)^2 - 4(3)(r - 2)$ $(= r^2 - 12r + 24)$ (seen anywhere) **(A1)**

$2.53589... (= 6 - 2\sqrt{3})$ AND $9.46410... (= 6 + 2\sqrt{3})$ (seen anywhere) **(A1)**

recognition that discriminant of $3x^2 - rx + r - 2$ is less than zero **(M1)**

$2.54 < r < 9.46$ $(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3})$ **A1**

Note: Accept $2.54 \leq r \leq 9.46$.

METHOD 2

recognition that $3x^2 - rx + r - 2$ must be greater than zero (seen anywhere) **R1**

EITHER

minimum when $x = \frac{r}{6} \Rightarrow (y =) 3\left(\frac{r}{6}\right)^2 - r\left(\frac{r}{6}\right) + r - 2$ (> 0) **(A1)**

attempt to solve their inequality for y (must be in terms of r and r^2) **(M1)**

OR

$x < 1 \Rightarrow r > \frac{3x^2 - 2}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{3x^2 - 2}{x - 1}$ **(A1)**

attempt to find local minimum AND local maximum of $r = \frac{3x^2 - 2}{x - 1}$ **(M1)**

THEN

$(r >) 2.53589... (= 6 - 2\sqrt{3})$ AND $(r <) 9.46410... (= 6 + 2\sqrt{3})$ (seen anywhere) **(A1)**

$2.54 < r < 9.46$ $(6 - 2\sqrt{3} < r < 6 + 2\sqrt{3})$ **A1**

Note: Accept $2.54 \leq r \leq 9.46$.

[5 marks]

Section B

1. (a) (i) $f(0) = -11$

A1

(ii) $-1.80650\dots$

$$f(20) = -1.81 \left(= 11\sqrt{20} - 51 = 22\sqrt{5} - 51 \right)$$

A1

[2 marks]

(b) attempt to find at least one root

(M1)

$$x = 1.72622\dots \text{ and } x = 17.5237\dots$$

$$x = 1.73 \text{ and } x = 17.5$$

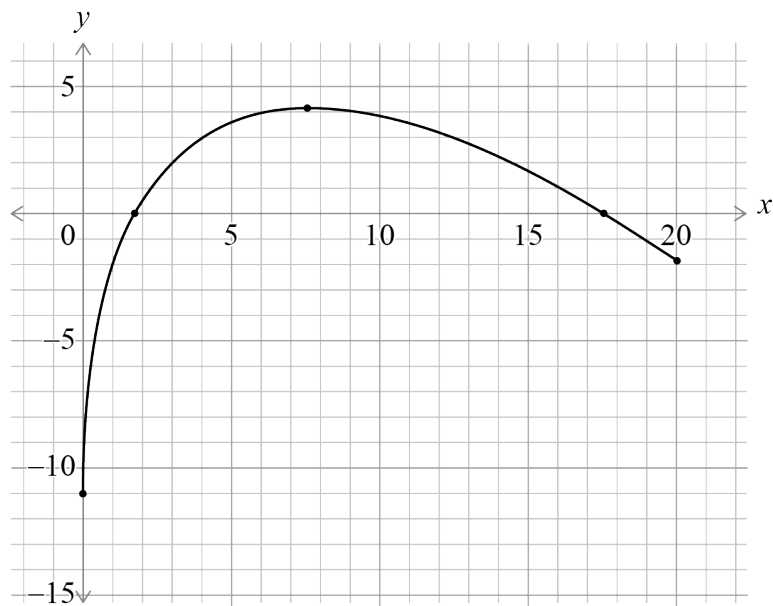
A1

[2 marks]

continued...

Question 1 continued.

(c)



A1A1A1

Note: Follow through from their part (a).

Award **A1** for endpoints at approximately $(0, -11)$ and $(20, -1.8)$. Allow for y -intercept $-12 < y < -10$ and for the right endpoint in the interval $x = 20, -2 < y < -1$.

The following two **A** marks may only be awarded if the approximate shape is correct.

Award **A1** for x -intercepts at approximately $x = 1.7$ and $x = 17.5$ and award **A1** for maximum at approximately $(7.6, 4.1)$. Allow for x -intercepts in the intervals $1 < x < 2$, $17 < x < 18$, and maximum in the intervals $6.5 < x < 8.5$, $3.5 < y < 5$.

[3 marks]

Total [7 marks]

6. METHOD 1

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) **R1**

(discriminant =) $(-r)^2 - 4(4)(r-1)$ $(= r^2 - 16r + 16)$ (seen anywhere) **(A1)**

$1.07179... (= 8 - 4\sqrt{3})$ AND $14.9282... (= 8 + 4\sqrt{3})$ (seen anywhere) **(A1)**

recognition that discriminant of $4x^2 - rx + r - 1$ is less than zero **(M1)**

$1.07 < r < 14.9$ $(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3})$ **A1**

Note: Accept $1.08 \leq r \leq 14.9$.

METHOD 2

recognition that $4x^2 - rx + r - 1$ must be greater than zero (seen anywhere) **R1**

EITHER

minimum when $x = \frac{r}{8} \Rightarrow (y =) 4\left(\frac{r}{8}\right)^2 - r\left(\frac{r}{8}\right) + r - 1$ (> 0) **(A1)**

attempt to solve their inequality for y (must be in terms of r and r^2) **(M1)**

OR

$x < 1 \Rightarrow r > \frac{4x^2 - 1}{x - 1}$ OR $x > 1 \Rightarrow r < \frac{4x^2 - 1}{x - 1}$ **(A1)**

attempt to find local minimum AND local maximum of $r = \frac{4x^2 - 1}{x - 1}$ **(M1)**

THEN

$(r >) 1.07179... (= 8 - 4\sqrt{3})$ AND $(r <) 14.9282... (= 8 + 4\sqrt{3})$ (seen anywhere) **(A1)**

$1.07 < r < 14.9$ $(8 - 4\sqrt{3} < r < 8 + 4\sqrt{3})$ **A1**

Note: Accept $1.08 \leq r \leq 14.9$.

[5 marks]

Section B

1. (a) $\ln 12$

A1
[1 mark]

(b) $\ln 2^3$
 $= \ln 8$

(A1)
A1
[2 marks]

(c) $\ln\left(\frac{1}{2}\right)^{-1}$ OR $-(\ln 1 - \ln 2)$ OR $-\ln 2^{-1}$
 $= \ln 2$

(A1)
A1
[2 marks]
Total [5 marks]

8. (a) METHOD 1

$$a = 5$$

(A1)attempt to use roots and symmetry to find h **(M1)**

$$h = \frac{(-1) + (-3)}{2} \text{ OR half the distance between the roots } \frac{(-1) - (-3)}{2} = 1 \text{ (may be seen on a diagram)}$$

$$h = -2 \text{ (accept } x = -2)$$

(A1)

$$f(x) = 5(x - (-2))^2 - 5 (= 5(x + 2)^2 - 5)$$

A1

$$(a = 5, h = -2, k = -5)$$

METHOD 2

$$a = 5$$

(A1)

attempt to expand

$$(x + 1)(x + 3) = x^2 + 4x + 3 \text{ OR } 5(x + 1)(x + 3) = 5x^2 + 20x + 15$$

EITHER

uses their expansion to attempt to complete the square to the form

(M1)

$$p(x + q)^2 + r, \text{ where } q \text{ is half the coefficient of their } x \text{ term}$$

$$= (x + 2)^2 - 2^2 + 3 (= (x + 2)^2 - 1) \text{ OR } 5[(x + 2)^2 - 2^2 + 3] (= 5(x + 2)^2 - 5)$$

(A1)**OR**

uses their expansion to attempt to differentiate and sets equal to zero

(M1)

$$\frac{dy}{dx} = 2x + 4 = 0 \text{ OR } \frac{dy}{dx} = 10x + 20 = 0$$

$$h = -2 \text{ (accept } x = -2)$$

(A1)**OR**uses their expansion to attempt to find axis of symmetry using $h = \frac{-b}{2a}$ **(M1)**

$$h = \frac{-4}{2} \text{ OR } h = \frac{-20}{10}$$

$$h = -2 \text{ (accept } x = -2)$$

(A1)**THEN**

$$f(x) = 5(x - (-2))^2 - 5 (= 5(x + 2)^2 - 5)$$

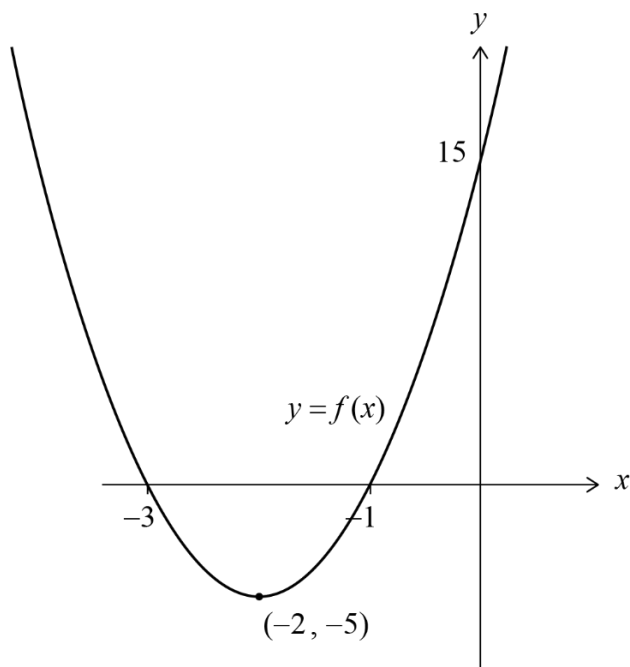
A1

$$(a = 5, h = -2, k = -5)$$

[4 marks]*continued...*

Question 8 continued.

(b)

**M1A1A1A1**award **M1** for a roughly symmetric curve which is concave upaward **A1** for x intercepts at -3 and -1 award **A1** for y intercept at 15 award **A1** for vertex at $(-2, -5)$ **[4 marks]**(c) $5(x+2)^2 - 5 \leq 40$ OR $5(x+1)(x+3) \leq 40$ OR $(x+1)(x+3) \leq 8$ leading to $(x+2)^2 \leq 9$ OR $5x^2 + 20x - 25 \leq 0$ OR $x^2 + 4x - 5 \leq 0$ **(A1)**valid attempt to find the critical values for their quadratic inequality **(M1)** $x+2 = \pm 3$ OR $(x+5)(x-1) = 0$ $x = -5, x = 1$ **(A1)** $-5 \leq x \leq 1$ **A1****Note:** Accept $(x \in)[-5, 1]$ or equivalent.**[4 marks]**

continued...

Question 8 continued.

(d) (i) $(f \circ g)(x) = 5(\ln x + 1)(\ln x + 3)$ OR $5(\ln x + 2)^2 - 5$ OR
 $5(\ln x)^2 + 20\ln x + 15$

A1

(ii) attempt to replace x with $\ln x$ using their solution to part (c)

(M1)

$$-5 \leq \ln x \leq 1$$

$$e^{-5} \leq x \leq e$$

A1

Note: Accept $(x \in) [e^{-5}, e]$ or equivalent.

[3 marks]

Total [15 marks]

3.

(a) -2 (accept $(0, -2)$)

A1

[1 mark]

(b) $y = \frac{3}{2}$ (must be an equation)

A1

Note: Do not accept $\neq$ sign.

[1 mark]

(c) attempt to reflect the graph of f in the x -axis OR one correct value seen

(M1)

both correct values $-\frac{3}{2}$ and 2 (seen anywhere)

A1

$$-\frac{3}{2} < y \leq 2$$

A1

[3 marks]

Total [5 marks]

5.(a) recognising $\Delta > 0$ (seen anywhere) **(M1)**

$$\Delta = k^2 - 4(15 - k) \quad (= k^2 + 4k - 60) \quad \mathbf{A1}$$

valid attempt to solve quadratic (in)equality **(M1)**

$$(k - 6)(k + 10) \quad \text{OR} \quad k = \frac{-4 \pm \sqrt{4^2 - 4(-60)}}{2}$$

two correct values -10 and 6 (seen anywhere) **A1**

$$k < -10, \quad k > 6 \quad \mathbf{A1}$$

[5 marks](b) recognising constant or product of roots is positive **(M1)**

$$15 - k > 0 \Rightarrow k < 15$$

$$k < -10, \quad 6 < k < 15 \quad \mathbf{A1}$$

[2 marks]**Total [7 marks]**

9.

(a) $4^x = 8$ OR $f^{-1}(x) = \log_4 x$ OR $f^{-1}(8) = \log_4 8$ (A1)

attempt to use indices with same base OR change of base of logs (M1)

$$2^{2x} = 2^3, 4^x = 4^{\frac{3}{2}} \text{ OR } f^{-1}(8) = \log_4 4^{\frac{3}{2}} \text{ OR } f^{-1}(8) = \frac{\log_2 8}{\log_2 4}$$

$$f^{-1}(8) = \frac{3}{2} \quad \text{A1}$$

[3 marks]

(b) (i) interchanging x and y (seen anywhere) (M1)

$$x = 1 + \log_2 y \text{ OR } y - 1 = \log_2 x$$

$$x - 1 = \log_2 y \text{ OR } 2^{y-1} = x$$

$$g^{-1}(x) = 2^{x-1} \text{ (or equivalent)} \quad \text{A1}$$

(ii) **METHOD 1**

a horizontal translation/shift by 1 unit to the left (do not accept 'move')

followed by a horizontal stretch/dilation with scale factor $\frac{1}{2}$ (accept horizontal compression by a factor of 2)

one correct transformation (A1)

two correct transformations in the correct order (A1)

METHOD 2

horizontal stretch/dilation with scale factor $\frac{1}{2}$ (accept horizontal compression (A1)

by a factor of 2)

vertical stretch/dilation with scale factor 2 (A1)

Note: A1s can be awarded independently here as order is not important.
--

[4 marks]

continued...

Question 9 continued.

- (c) attempt to find composite function (in any order)

M1

$$f(1 + \log_2 x) (= 4^{1 + \log_2 x})$$

$$4 \times 4^{\log_2 x} \text{ OR } 4 \times 2^{2\log_2 x} \text{ OR } 4^{\log_2 2x} \text{ OR } 2^{(2+2\log_2 x)} \text{ OR } 4^{(\log_4 4 + 2\log_4 x)}$$

(A1)

$$= 4 \times 2^{\log_2 x^2} \text{ OR } 2^{\log_2 (2x)^2} \text{ OR } 4^{\log_4 (4x^2)}$$

A1

$$= 4x^2$$

AG**[3 marks]**

$$(d) \quad (i) \quad 2x - 1 + \frac{1}{2x+1} = \frac{(2x-1)(2x+1)+1}{2x+1}$$

(A1)

$$= \frac{4x^2 - 1 + 1}{2x+1}$$

A1

$$2x - 1 + \frac{1}{2x+1} = \frac{4x^2}{2x+1}$$

AG

Note: Accept working from RHS to LHS.

- (ii)
- METHOD 1**

$$\text{enclosed area is } \int_1^3 \left(2x - 1 + \frac{1}{2x+1} \right) dx$$

attempt to integrate (at least one correct term or $\ln(2x+1)$ seen)**(M1)**

$$= x^2 - x + \frac{1}{2} \ln|2x+1| (+c)$$

A1A1

Note: Award **A1** for $x^2 - x$ and **A1** for $\frac{1}{2} \ln|2x+1|$.

Accept $\frac{1}{2} \ln(2x+1)$.

substitute correct limits into their integrated expression and subtract

(M1)

$$= \left(9 - 3 + \frac{1}{2} \ln 7 \right) - \left(1 - 1 + \frac{1}{2} \ln 3 \right)$$

$$A = 6 + \frac{1}{2} \ln \frac{7}{3}$$

A1*continued...*

Question 9 continued.

METHOD 2

attempt to use integration by substitution

(M1)

$$\text{let } u = 2x + 1 \text{ OR } u = 2x - 1 \Rightarrow \frac{du}{dx} = 2$$

$$= \frac{1}{2} \int \left(u - 2 + \frac{1}{u} \right) du \text{ OR } \frac{1}{2} \int \frac{(u-1)^2}{u} du \text{ OR } \frac{1}{2} \int \left(u + \frac{1}{u+2} \right) du$$

A1

correct integration

$$= \frac{1}{4} u^2 - u + \frac{1}{2} \ln|u| (+c) \text{ OR } \frac{1}{4} u^2 + \frac{1}{2} \ln|u+2| (+c)$$

A1

substitution of their limits into their integrated expression and subtract

(M1)

$$= \left(\frac{49}{4} - 7 + \frac{1}{2} \ln 7 \right) - \left(\frac{9}{4} - 3 + \frac{1}{2} \ln 3 \right) \text{ OR } \left(\frac{25}{4} + \frac{1}{2} \ln 7 \right) - \left(\frac{1}{4} + \frac{1}{2} \ln 3 \right)$$

$$A = 6 + \frac{1}{2} \ln \frac{7}{3} \text{ (or equivalent)}$$

A1

[7 marks]

Total [17 marks]

2. (a) correct application of $\log_a xy = \log_a x + \log_a y$ or $\log_a x^m = m \log_a x$ (M1)

correct expression in terms of $\log_{10} 2$ AND $\log_{10} 3$ (arguments must be 2 and 3) (A1)

$$3\log_{10} 2 + \log_{10} 3 \quad \text{OR} \quad \log_{10} 2 + \log_{10} 2 + \log_{10} 2 + \log_{10} 3$$

$$3p + q \quad \text{A1}$$

[3 marks]

(b) $(\log_3 8 =) \frac{\log_{10} 8}{\log_{10} 3} \left(= \frac{3\log_{10} 2}{\log_{10} 3} \right)$ (A1)

$$= \frac{3p}{q} \quad \text{A1}$$

[2 marks]

Total [5 marks]

3. (a) $f(-3) = -1$

A1

[1 mark]

(b) $-3 \leq x \leq 5$

A1

Note: Award **A1** for answers using interval notation $[-3, 5]$.

[1 mark]

(c) $\left(f^{-1}(2x-7) = -3 \Rightarrow\right) 2x-7 = f(-3)$ **OR** $f^{-1}(-1) = -3$

(M1)

$2x-7 = -1$

(A1)

$x = 3$

A1

[3 marks]

Total [5 marks]

5. (a) equating $y = mx - 3$ and $y = x^2 - x - 1$ **M1**
- $$x^2 - x - 1 - (mx - 3) = 0 \quad \text{OR} \quad x^2 - mx - x + 2 = 0 \quad \text{A1}$$
- $$x^2 - (m+1)x + 2 = 0 \quad \text{AG}$$

[2 marks](b) **METHOD 1 (discriminant)**correct substitution into discriminant (do not award if seen only in quadratic formula) **(A1)**

$$(-(m+1))^2 - 4(1)(2) \quad \text{OR} \quad (m+1)^2 - 4(1)(2)$$

discriminant equals 0 (seen anywhere) **A1**

$$(m+1)^2 = 8 \quad \text{OR} \quad m^2 + 2m - 7 = 0 \quad \text{A1}$$

$$(m \Rightarrow) -1 - 2\sqrt{2}, -1 + 2\sqrt{2} \quad \text{A1A1}$$

METHOD 2 (derivative)

$$\frac{dy}{dx} = 2x - 1 \quad \text{A1}$$

substituting $m = 2x - 1$ or $x = \frac{m+1}{2}$ into **AG** from part (a) **(A1)**

$$x^2 - (2x - 1 + 1)x + 2 = 0 \quad \text{OR} \quad x^2 - 2x^2 + 2 = 0 \quad \text{OR} \quad \left(\frac{m+1}{2}\right)^2 - (m+1)\frac{m+1}{2} + 2 = 0$$

$$x = \pm\sqrt{2} \quad \text{OR} \quad -m^2 - 2m + 7 = 0 \quad \text{OR} \quad m^2 + 2m - 7 = 0 \quad \text{A1}$$

$$(m \Rightarrow) 2\sqrt{2} - 1, -2\sqrt{2} - 1 \quad \text{A1A1}$$

[5 marks]**Total [7 marks]**

9. (a) (i) recognizing need to factorise, complete the square or substitute into quadratic formula (M1)

$$(5+x)(1-x) \text{ OR } 9-(x+2)^2 \text{ OR } \frac{4 \pm \sqrt{(-4)^2 - 4 \times (-1) \times 5}}{2 \times (-1)}$$

$$x = -5, x = 1 \quad \text{A1}$$

- (ii) sign diagram or sketch of $y = 5 - 4x - x^2$ with -5 and 1 indicated (M1)

$$-5 < x < 1 \quad \text{A1}$$

Note: Award **A1** for answers using interval notation $(-5, 1)$.

[4 marks]

- (b) (i) $(a =) -5$ A1

- (ii) $(b =) 1$ A1

[2 marks]

- (c) $(\log_k(5-4x-x^2) = 0 \Rightarrow) 5-4x-x^2 = 1$ (A1)

$$x^2 + 4x - 4 = 0 \text{ OR } 4 - 4x - x^2 = 0$$

attempting to solve their quadratic equation set to 0 (M1)

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times (-4)}}{2 \times 1} \text{ OR } (x+2)^2 - 8 (=0) \quad \text{A1}$$

$$x = -2 \pm 2\sqrt{2} \quad \text{A1}$$

[4 marks]

continued...

Question 9 continued

(d) **EITHER**recognizing that maximum of f is when $5 - 4x - x^2$ is a maximum or when $f'(x) = 0$ **(M1)**substituting into $x = -\frac{b}{2a}$ **OR** $-4 - 2x = 0$ **OR** $\frac{-4 - 2x}{(5 - 4x - x^2)\ln k} = 0$ **OR**use of symmetry to find x -coordinate of maximum**(M1)** $\frac{a+b}{2}$ **OR** $x = \frac{\text{sum of their answers to (c)}}{2}$ **THEN** $x = -2$ (seen anywhere)**A1**maximum of quadratic is 9 **OR** argument of logarithm is 9**(A1)** $\log_k(9) = 2$ **OR** $\log_k(9) = \log_k(k^2)$ **OR** $k^2 = 5 - 4(-2) - (-2)^2$ (or equivalent)**A1**

valid approach to solve

(M1) $k^2 = 9$ **OR** $k = \pm 3$ $k = 3$ **A1****[6 marks]****Total [16 marks]**

9. (a) METHOD 1

attempt to rearrange and swap x and y (at any stage) **(M1)**

$$x = \frac{2-2y}{y+2} \Rightarrow xy + 2x = -2y + 2$$

$$xy + 2y = 2 - 2x \text{ OR } y(x+2) = 2 - 2x \quad \textbf{A1}$$

$$f^{-1}(x) = \frac{2-2x}{x+2} \text{ OR } y = \frac{2-2x}{x+2} \quad \textbf{A1}$$

$$\text{So } f^{-1}(x) = f(x) \quad \textbf{AG}$$

METHOD 2

attempt to find $f(f(x))$ **(M1)**

$$f(f(x)) = \frac{2 - 2\left(\frac{2-2x}{x+2}\right)}{\left(\frac{2-2x}{x+2}\right) + 2}$$

$$= \frac{2(x+2) - 2(2-2x)}{(2-2x) + 2(x+2)} \left(= \frac{6x}{6} \right) \quad \textbf{A1}$$

$$= x \quad \textbf{A1}$$

$$\text{so } f^{-1}(x) = f(x) \quad \textbf{AG}$$

[3 marks]

continued...

Question 9 continued.

(b) (i) **EITHER**

$$OP = \sqrt{k^2 + \left(\frac{2-2k}{k+2}\right)^2} \quad (\text{A1})$$

attempt to minimise distance (M1)

OR

recognition that P lies on $y = x$ due to symmetry (M1)

$$k = -4.44948... \text{ or } k = 0.449489... \quad (\text{A1})$$

THEN

minimum when $k = 0.449489...$

$$k = 0.449 \quad \text{A1}$$

Note: If no working seen, award **(M0)(A0)A0** for 0.45.

(ii) coordinates of P (0.449, 0.449) A1

Note: Award **A1FT** for $f(k)$, using their k .

[4 marks]

(c) (i) $x = -\frac{d}{c}$ A1

(ii) $y = -\frac{3}{c}$ A1

Note: Answers in (i) and (ii) must be correct equations.

[2 marks]

(d) $g^{-1}(x) = \frac{2-dx}{cx+3}$ OR $-\frac{d}{c} = -\frac{3}{c}$ (A1)

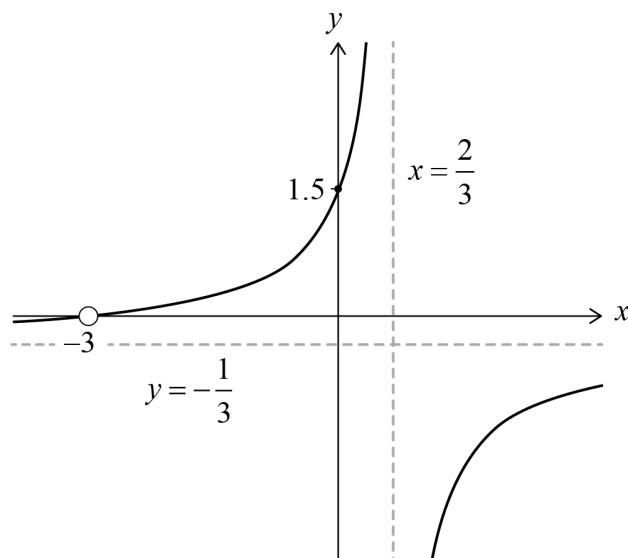
$$d = 3 \quad \text{A1}$$

[2 marks]

continued...

Question 9 continued.

(e)



Only if shape is approximately correct for this rational function (i.e. two branches in opposite corners and displaying appropriate asymptotic behavior), award the following marks independently.

asymptotes $x = \frac{2}{3}$ and $y = -\frac{1}{3}$ drawn with correct labels

A1A1

labelled point discontinuity (or x -intercept) at $(-3, 0)$, labelled y -intercept at $(0, \frac{3}{2})$

A1A1

Note: Award **A1FTA1FT**, as appropriate, for their value of d (i.e. x -intercept $= -d$, y -intercept $\frac{d}{2}$).

[4 marks]**Total [15 marks]**

1.

Function	Letter
$y = f(x + 1)$	B
$y = f(x) - 1$	A
$y = f(-x)$	F
$y = f(2x)$	D

A1A1A1A1
Total [4 marks]

5. (a) attempt to factorize at least one expression (M1)

$$\frac{x}{(x-7)(x-1)} \times \frac{(x-1)(x+1)}{x+1} \quad \text{OR} \quad \frac{x(x^2-1)}{(x-7)(x^2-1)} \quad \text{OR} \quad \frac{x(x^2-1)}{(x-7)(x-1)(x+1)} \quad \text{A1}$$

$$\frac{x}{x-7} \quad \text{AG}$$

[2 marks]

- (b) **METHOD 1** (combines given log terms)

attempt to use the subtraction/quotient property of logs (M1)

$$\log_2 \left(\frac{x(x^2-1)}{(x^2-8x+7)(x+1)} \right)$$

valid attempt to rewrite equation with (or without) logs on both sides (seen anywhere) (M1)

$$2 = \frac{x}{x-7} \quad \text{OR} \quad \log_2 \left(\frac{x}{x-7} \right) = \log_2 2 \quad \text{OR} \quad 2^{\log_2 \left(\frac{x}{x-7} \right)} = 2^1 \quad \text{(A1)}$$

$$x = 14 \quad \text{A1}$$

METHOD 2 (uses log laws with LHS)

attempt to use $1 = \log_2 2$ (seen anywhere) (M1)

attempt to use the subtraction/quotient property of logs (M1)

$$\log_2 \left(\frac{x^3-x}{2} \right) = \log_2 \left((x^2-8x+7)(x+1) \right) \quad \text{OR} \quad \frac{x(x^2-1)}{2} = (x-1)(x-7)(x+1) \quad \text{(A1)}$$

$$\frac{x}{2} = x-7$$

$$x = 14 \quad \text{A1}$$

[4 marks]

Total [6 marks]

6. (a) METHOD 1 x -coordinate of the vertex at A is 2 (A1)attempt to substitute their x -coordinate into equation of line (M1)

$$2(2) - 1 (= 3)$$

$$3 = -(2)^2 + 4(2) + p \quad \text{A1}$$

$$p = -1 \quad \text{AG}$$

METHOD 2 x -coordinate of the vertex at A is 2 (A1)attempt to substitute their x -coordinate into $f(x)$ (M1)

$$f(2) = p + 4$$

$$p + 4 = 2(2) - 1 \quad \text{A1}$$

$$p = -1 \quad \text{AG}$$

METHOD 3attempt to complete the square on $f(x)$ (M1)

$$-(x-2)^2 + p + 4$$

$$x_A = 2 \quad \text{OR} \quad y_A = p + 4 \quad \text{(A1)}$$

$$p + 4 = 2(2) - 1 \quad \text{A1}$$

$$p = -1 \quad \text{AG}$$

Note: Award no marks if $x = 0$ is substituted into $-x^2 + 4x + p = 2x - 1$ to show $p = -1$.**[3 marks]***continued...*

Question 6 continued.

(b) **METHOD 1** (equating $g(x)$ and $2x - 1$)

x -coordinate is $\frac{-q}{2}$ (seen anywhere) (A1)

attempt to find intersection points, equating $g(x)$ and $2x - 1$ (M1)

$$x^2 + qx - 1 = 2x - 1$$

attempt to substitute their x -coordinate (M1)

$$\left(\frac{-q}{2}\right)^2 + q\left(\frac{-q}{2}\right) - 1 = 2\left(\frac{-q}{2}\right) - 1$$

$$q^2 - 4q = 0$$
 (A1)

$$q = 4 \text{ (since } q \neq 0 \text{)}$$
 A1

METHOD 2 (find coordinates of B in terms of q)

x -coordinate is $\frac{-q}{2}$ (A1)

EITHER

attempt to substitute their x -coordinate into $y = 2x - 1$ (M1)

$$2\left(\frac{-q}{2}\right) - 1 \Rightarrow B\left(\frac{-q}{2}, -q - 1\right)$$

attempt to substitute their coordinates of B into $g(x)$ (M1)

$$-q - 1 = \left(\frac{-q}{2}\right)^2 + q\left(\frac{-q}{2}\right) - 1$$

OR

attempt to substitute their x -coordinate into $g(x)$ (M1)

$$\left(\frac{-q}{2}\right)^2 + q\left(\frac{-q}{2}\right) - 1 \Rightarrow B\left(\frac{-q}{2}, -\frac{q^2}{4} - 1\right)$$

attempt to substitute their coordinates of B into the line (M1)

$$-\frac{q^2}{4} - 1 = 2\left(\frac{-q}{2}\right) - 1$$

continued...

Question 6 continued.

THEN

$$q^2 - 4q = 0 \quad (\text{A1})$$

$$q = 4 \text{ (since } q \neq 0 \text{)} \quad \text{A1}$$

METHOD 3 (completing the square)

attempt to complete the square on $g(x)$ (M1)

$$\left(x + \frac{q}{2}\right)^2 - \left(\frac{q}{2}\right)^2 - 1$$

$$x_B = \frac{-q}{2} \quad \text{OR} \quad y_B = -\frac{q^2}{4} - 1 \quad (\text{A1})$$

attempt to substitute their coordinates of B into equation of line (M1)

$$-\frac{q^2}{4} - 1 = 2\left(\frac{-q}{2}\right) - 1$$

$$q^2 - 4q = 0 \quad (\text{A1})$$

$$q = 4 \text{ (since } q \neq 0 \text{)} \quad \text{A1}$$

[5 marks]

Total [8 marks]

Section B

7. (a) (i) 7 A1

(ii) attempt to interchange x and y (seen anywhere) (M1)

$$f^{-1}(x) = \frac{x-3}{2} \text{ (accept } (y =) \frac{x-3}{2} \text{)}$$
A1

[3 marks]

(b) $h(x) = \frac{1}{2}x - 1$ (or equivalent) A1A1

Note: Award **A1** for $\frac{1}{2}x$ and **A1** for -1 .

[2 marks]

Special note: The following parts (c), (d) and (e) may use their expression for f^{-1} and/or their expression for h . Please ensure to apply **FT** to all these parts, as appropriate.

(c) **METHOD 1**

recognition that $h^{-1}(x) = -2 \Rightarrow h(-2) = x$ (M1)

$$x = -2$$
A1

METHOD 2

attempt to find inverse of h (M1)

$$h^{-1}(x) = 2(x+1)$$

$$2(x+1) = -2$$

$$x = -2$$
A1

[2 marks]

continued...

Question 7 continued.

(d) **METHOD 1**

attempt to use properties of inverse functions

(M1)

$$f(h(x)) \text{ OR } f(f^{-1}(g(x))) \text{ OR } f\left(\frac{1}{2}x-1\right) = g(x)$$

$$2\left(\frac{1}{2}x-1\right)+3 = g(x) \text{ OR } 2\left(\frac{1}{2}x-1\right)+3 = mx+c \text{ (or equivalent)}$$

(A1)

attempt to solve for $g(x)$ OR equate terms (one of $m = 2 \times \frac{1}{2}$ or $c = -2 + 3$)

OR substituting $x = 0$ into their equation to solve for c

(M1)

$m = 1, c = 1$ (accept $g(x) = x + 1$)

A1

METHOD 2

attempt to compose f^{-1} and g (in any order)

(M1)

$$\frac{g(x)-3}{2} \text{ OR } \frac{1}{2}(mx+c) - \frac{3}{2}$$

$$\frac{g(x)-3}{2} = \frac{1}{2}x-1 \text{ OR } \frac{1}{2}(mx+c) - \frac{3}{2} = \frac{1}{2}x-1 \text{ (or equivalent)}$$

(A1)

attempt to solve for $g(x)$ OR equate terms (one of $\frac{1}{2}m = \frac{1}{2}$ or $\frac{1}{2}c - \frac{3}{2} = -1$)

OR substituting $x = 0$ into their equation to solve for c

(M1)

$m = 1, c = 1$ (accept $g(x) = x + 1$)

A1

Note: If $g \circ f^{-1}\left(\frac{mx}{2} - \frac{3m}{2} + c\right)$ is seen, then **M1A0M1A0** may be awarded for attempting to equate their terms.

[4 marks]

continued...

Question 7 continued.

- (e) attempt to compose functions OR recognition that $k(x) = h^{-1}(x)$ **(M1)**

$$\frac{1}{2}(k(x)) - 1 = x$$

$$k(x) = 2x + 2$$

A1**[2 marks]**

- (f) reflection in the line $y = x$

A1**[1 mark]****Total [14 marks]**

5. (a) substitute $P(-10,1)$ or $Q(3,-12)$ into $y = \frac{Ax+B}{x-4}$ (M1)

$$-14 = -10A + B \text{ (or equivalent)}$$

$$12 = 3A + B \text{ (or equivalent)}$$

attempt to solve their equations simultaneously (M1)

$$A = 2, B = 6$$

A1
[3 marks]

(b) EITHER

attempt to express y in the form $P + \frac{Q}{x-4}$ (M1)

OR

attempt to find one or both asymptotes using $x-4=0$ or $y = \frac{a}{c}$ (dividing coefficients) (M1)

THEN

recognizing vertical asymptote at $x = 4$ (may be seen anywhere, including labeled on graph or in part (a)) OR horizontal shift 4 units to the right A1

recognizing horizontal asymptote at $y = 2$ (may be seen anywhere, including labeled on graph or in part (a)) OR vertical shift 2 units up A1

$$\frac{2x+6}{x-4} = 2 + \frac{14}{x-4} \quad (A1)$$

EITHER

horizontal translation 4 units to the right;

vertical stretch scale factor 14 followed by vertical translation 2 up. A1

Note: Horizontal translation may be seen anywhere; y -axis transformations must be seen in the order above.

OR

vertical stretch scale factor 14; followed by translation (through/by) $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ A1

Note: In this question, the transformations may be described using terms such as *translate* or *shift*, and *dilate* or *stretch*. Do not accept “move” to describe a transformation.

[5 marks]
Total [8 marks]

6.

Note: Candidates may approach this problem in different ways and may do their steps in many different orders.
The **M** marks are independent and may be awarded in any order. Use the description of each mark to determine when the mark may be awarded.

$$3\log_8 10x - \log_4 x = 1$$

attempt to apply power rule, quotient rule, or product rule (seen anywhere) **(M1)**

attempt to apply change of base rule (seen anywhere) **(M1)**

correct equation with same base in all logarithms **A1**

$$\log_2 10x - \log_2 x^{\frac{1}{2}} = 1 \quad \text{or} \quad \log_2 10 + \log_2 x - \frac{1}{2}\log_2 x = 1 \quad \text{or} \quad \frac{\log_4 (10x)^3}{\log_4 8} - \log_4 x = 1 \quad (\text{or equivalent})$$

correct equation without logarithms **A1**

$$10\sqrt{x} = 2 \quad \text{or} \quad \sqrt{x} = \frac{2}{10} \left(= \frac{1}{5} \right) \quad (\text{or equivalent})$$

$$x = \frac{1}{25} \quad \text{A1}$$

[5 marks]

Section B