

APOLLO

Geometry and trigonometry

SL

Mathematics: analysis and approaches

First assessment 2021

72 questions | mark schemes

6. (a)

Note: Do not award the final **A1** for proofs which work from both sides to find a common expression other than $2 \sin x \cos x - 2 \sin^2 x$.

METHOD 1 (LHS to RHS)attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ **M1**LHS = $2 \sin x \cos x + \cos 2x - 1$ OR $\sin 2x + 1 - 2 \sin^2 x - 1$ OR $2 \sin x \cos x + 1 - 2 \sin^2 x - 1$ $= 2 \sin x \cos x - 2 \sin^2 x$ **A1** $\sin 2x + \cos 2x - 1 = 2 \sin x (\cos x - \sin x) = \text{RHS}$ **AG****METHOD 2 (RHS to LHS)**RHS = $2 \sin x \cos x - 2 \sin^2 x$ attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ **M1** $= \sin 2x + 1 - 2 \sin^2 x - 1$ **A1** $= \sin 2x + \cos 2x - 1 = \text{LHS}$ **AG****[2 marks]**

(b) attempt to factorise

M1 $(\cos x - \sin x)(2 \sin x + 1) = 0$ **A1**recognition of $\cos x = \sin x \Rightarrow \frac{\sin x}{\cos x} = \tan x = 1$ OR $\sin x = -\frac{1}{2}$ **(M1)**

one correct reference angle seen anywhere, accept degrees

(A1) $\frac{\pi}{4}$ OR $\frac{\pi}{6}$ (accept $-\frac{\pi}{6}, \frac{7\pi}{6}$)

Note: This **(M1)(A1)** is independent of the previous **M1A1**.

 $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{4}$ **A2**

Note: Award **A1** for any two correct (radian) answers.

Award **A1A0** if additional values given with the four correct (radian) answers.Award **A1A0** for four correct answers given in degrees.**[6 marks]****Total [8 marks]**

1. (a) minor arc AB has length r (A1)
 recognition that perimeter of shaded sector is $3r$ (A1)
 $3r = 12$
 $r = 4$ A1
[3 marks]

- (b) **EITHER**
 $\theta = 2\pi - \angle AOB (= 2\pi - 1)$ (M1)
 Area of non-shaded region $= \frac{1}{2}(2\pi - 1)(4^2)$ (A1)

OR

- area of circle - area of shaded sector (M1)
 $16\pi - \left(\frac{1}{2} \times 1 \times 4^2\right)$ (A1)

THEN

- area $= 16\pi - 8 (= 8(2\pi - 1))$ A1
[3 marks]
Total [6 marks]

3. (a) METHOD 1

correct substitution of $\cos^2 x = 1 - \sin^2 x$

A1

$$2(1 - \sin^2 x) + 5\sin x = 4$$

$$2\sin^2 x - 5\sin x + 2 = 0$$

AG

METHOD 2

correct substitution using double-angle identities

A1

$$(2\cos^2 x - 1) + 5\sin x = 3$$

$$1 - 2\sin^2 x + 5\sin x = 3$$

$$2\sin^2 x - 5\sin x + 2 = 0$$

AG

[1 mark]

continued...

Question 3 continued(b) **EITHER**

attempting to factorise

M1

$$(2\sin x - 1)(\sin x - 2)$$

A1**OR**

attempting to use the quadratic formula

M1

$$\sin x = \frac{5 \pm \sqrt{5^2 - 4 \times 2 \times 2}}{4} \left(= \frac{5 \pm 3}{4} \right)$$

A1**THEN**

$$\sin x = \frac{1}{2}$$

(A1)

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

A1A1**[5 marks]****Total [6 marks]**

6. METHOD 1attempt to use the cosine rule to find the value of x **(M1)**

$$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$$

A1

$$2x^2 = 100$$

$$x^2 = 50 \text{ OR } x = \sqrt{50} \left(= 5\sqrt{2}\right)$$

A1attempt to find $\sin \hat{C}$ (seen anywhere)**(M1)**

$$\sin^2 \hat{C} + \left(\frac{3}{4}\right)^2 = 1 \text{ OR } x^2 + 3^2 = 4^2 \text{ or right triangle with side 3 and hypotenuse 4}$$

$$\sin \hat{C} = \frac{\sqrt{7}}{4}$$

(A1)

Note: The marks for finding $\sin \hat{C}$ may be awarded independently of the first three marks for finding x .

correct substitution into the area formula using their value of x (or x^2) and their value of $\sin \hat{C}$ **(M1)**

$$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4} \text{ or } A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$$

$$A = \frac{25\sqrt{7}}{2}$$

A1*continued...*

*Question 6 continued***METHOD 2**attempt to find the height, h , of the triangle in terms of x **(M1)**

$$h^2 + \left(\frac{3}{4}x\right)^2 = x^2 \text{ OR } h^2 + \left(\frac{5}{4}x\right)^2 = 10^2 \text{ OR } h = \frac{\sqrt{7}}{4}x$$
A1

equating their expressions for either h^2 or h **(M1)**

$$x^2 - \left(\frac{3}{4}x\right)^2 = 10^2 - \left(\frac{5}{4}x\right)^2 \text{ OR } \sqrt{100 - \frac{25}{16}x^2} = \frac{\sqrt{7}}{4}x \text{ (or equivalent)}$$
A1

$$x^2 = 50 \text{ OR } x = \sqrt{50} \text{ } (=5\sqrt{2})$$
A1

correct substitution into the area formula using their value of x (or x^2) **(M1)**

$$A = \frac{1}{2} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}\sqrt{50} \text{ OR } A = \frac{1}{2}(2 \times 5\sqrt{2})\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$$

$$A = \frac{25\sqrt{7}}{2}$$
A1

Total [7 marks]

Section B

4 . amplitude is $\frac{110}{2} = 55$ **(A1)**

$a = -55$ **A1**

$c = 65$ **A1**

$\frac{2\pi}{b} = 20$ OR $-55\cos(20b) + 65 = 10$ **(M1)**

$b = \frac{\pi}{10} (= 0.314)$ **A1**

Total [5 marks]

8. (a) $r = \frac{28}{\theta}$

A1**[1 mark]**

(b) recognising sum of area of sector and area of triangle required

(M1)

$$\frac{1}{2}r^2\theta + \frac{1}{2}r \times r \times \sin(\pi - \theta) \left(= \frac{r^2}{2}(\theta + \sin(\pi - \theta)) \right)$$

A1

$$\sin(\pi - \theta) = \sin \theta \text{ (substitution seen anywhere)}$$

A1

$$\frac{1}{2}\left(\frac{28}{\theta}\right)^2 \theta + \frac{1}{2}\left(\frac{28}{\theta}\right)^2 \sin \theta \text{ OR } \frac{1}{2}\left(\frac{28}{\theta}\right)^2 (\theta + \sin \theta)$$

A1

$$\text{area} = \frac{392}{\theta^2}(\theta + \sin \theta)$$

AG**[4 marks]**

$$(c) \quad \frac{392}{\theta^2}(\theta + \sin \theta) = 460$$

(M1)

$$\theta = 1.43917\dots$$

$$\theta = 1.44$$

A1**[2 marks]**

$$(d) \quad \frac{\pi - (\pi - \theta)}{2} \text{ OR } \frac{\theta}{2}$$

(M1)

$$\hat{D}AE = 0.719588\dots$$

$$\hat{D}AE = 0.720$$

A1**[2 marks]**

$$(e) \quad (i) \quad \hat{A}BC = 195 - 180 + 90 \\ = 105^\circ$$

(A1)**A1**

(ii) choosing sine rule

(M1)

$$\frac{BC}{\sin \hat{D}AE} = \frac{800}{\sin 105} \text{ OR } \frac{BC}{\sin \hat{D}AE} = \frac{800}{\sin 1.83}$$

A1

$$BC = 546 \text{ (m)}$$

A1**[5 marks]****Total [14 marks]**

7. (a) $\tan 0.6 = \frac{h}{12}$ (M1)

8.20964... (A1)

8.21 (m) (A1)

[2 marks]

(b) $\tan B = \frac{8.2096...}{5}$ OR $\tan^{-1} 1.6419...$ (A1)

1.02375... (A1)

1.02 (radians) (accept 58.7°) (A1)

[2 marks]

(c) $x + 1.8 + 2.5 = 8.20964...$ (or equivalent) (A1)

3.90964... (A1)

3.91 (m) (A1)

[2 marks]

(d) **METHOD 1**

recognition that blade length = amplitude, $p = \frac{\max - \min}{2}$ (M1)

$p = 3.91$ (A1)

centre of windmill = vertical shift, $q = \frac{\max + \min}{2}$ (M1)

$q = 8.21$ (A1)

METHOD 2attempting to form two equations in terms of p and q (M1) (M1)

$$12.1192... = p \cos\left(\frac{3\pi}{10} \cdot 0\right) + q, \quad 4.3000... = p \cos\left(\frac{3\pi}{10} \cdot \frac{10}{3}\right) + q$$

$p = 3.91$ (A1)

$q = 8.21$ (A1)

[4 marks]

(e) appropriate working towards finding the period

(M1)

$$\text{period} = \frac{2\pi}{\frac{3\pi}{10}} (=6.6666\dots)$$

$$\text{rotations per minute} = \frac{60}{\text{their period}}$$

(M1)

$$n = 9 \text{ (must be an integer) (accept } n = 10, n = 18, n = 19)$$

A1

[3 marks]

Total [13 marks]

6. (a) METHOD 1

attempt to write all LHS terms with a common denominator of $x-1$ **(M1)**

$$2x-3-\frac{6}{x-1} = \frac{2x(x-1)-3(x-1)-6}{x-1} \quad \text{OR} \quad \frac{(2x-3)(x-1)}{x-1} - \frac{6}{x-1}$$

$$= \frac{2x^2-2x-3x+3-6}{x-1} \quad \text{OR} \quad \frac{2x^2-5x+3}{x-1} - \frac{6}{x-1}$$
A1

$$= \frac{2x^2-5x-3}{x-1}$$
AG

METHOD 2

attempt to use algebraic division on RHS **(M1)**

correctly obtains quotient of $2x-3$ and remainder -6 **A1**

$$= 2x-3-\frac{6}{x-1} \text{ as required.}$$
AG

[2 marks]
continued...

Question 6 continued.

(b) consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ (M1)

$$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$$

EITHER

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ (M1)

Note: Accept any variable in place of $\sin 2\theta$.

$$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$$

OR

attempt to substitute into quadratic formula (M1)

$$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$$

THEN

$$\sin 2\theta = -\frac{1}{2} \text{ or } \sin 2\theta = 3$$
 (A1)

Note: Award **A1** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) (A1)

$$\theta = \frac{7\pi}{12}, \frac{11\pi}{12} \text{ (must be in radians)}$$
 A1

Note: Award **A0** if additional answers given.

[5 marks]

Total [7 marks]

Section B

3. EITHER

attempt to use cosine rule

(M1)

$$12^2 + AB^2 - 2 \times 12 \times \cos 25^\circ \times AB = 7^2 \quad \text{OR} \quad AB^2 - 21.7513...AB + 95 = 0$$

(A1)at least one correct value for AB **(A1)**

$$AB = 6.05068... \quad \text{OR} \quad AB = 15.7007...$$

using their smaller value for AB to find minimum perimeter**(M1)**

$$12 + 7 + 6.05068...$$

OR

attempt to use sine rule

(M1)

$$\frac{\sin B}{12} = \frac{\sin 25^\circ}{7} \quad \text{OR} \quad \sin B = 0.724488... \quad \text{OR} \quad \hat{B} = 133.573...^\circ \quad \text{OR} \quad \hat{B} = 46.4263...^\circ$$

(A1)at least one correct value for C **(A1)**

$$\hat{C} = 21.4263...^\circ \quad \text{OR} \quad \hat{C} = 108.573...^\circ$$

using their acute value for $\hat{C}$ to find minimum perimeter**(M1)**

$$12 + 7 + \sqrt{12^2 + 7^2 - 2 \times 12 \times 7 \cos 21.4263...^\circ} \quad \text{OR} \quad 12 + 7 + \frac{7 \sin 21.4263...^\circ}{\sin 25^\circ}$$

THEN

$$25.0506...$$

$$\text{minimum perimeter} = 25.1.$$

A1**Total [5 marks]**

5. (a) attempt to find the area of either shaded region in terms of r and θ (M1)

Note: Do not award **M1** if they have only copied from the booklet and not applied to the shaded area.

$$\text{Area of segment} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta \quad \text{A1}$$

$$\text{Area of triangle} = \frac{1}{2}r^2\sin(\pi - \theta) \quad \text{A1}$$

correct equation in terms of θ only (A1)

$$\theta - \sin\theta = \sin(\pi - \theta)$$

$$\theta - \sin\theta = \sin\theta \quad \text{A1}$$

$$\theta = 2\sin\theta \quad \text{AG}$$

Note: Award a maximum of **M1A1A0A0A0** if a candidate uses degrees

(i.e., $\frac{1}{2}r^2\sin(180^\circ - \theta)$), even if later work is correct.

Note: If a candidate directly states that the area of the triangle is

$\frac{1}{2}r^2\sin\theta$, award a maximum of **M1A1A0A1A1**.

[5 marks]

(b) $\theta = 1.89549\dots$

$$\theta = 1.90 \quad \text{A1}$$

Note: Award **A0** if there is more than one solution. Award **A0** for an answer in degrees.

[1 mark]

Total [6 marks]

8. (a) $12 = \frac{2\pi}{b}$ OR $b = \frac{2\pi}{12}$

A1

$$b = \frac{\pi}{6}$$

AG

[1 mark]

(b) $a = \frac{6.8 - 2.2}{2}$ OR $a = \frac{\text{max} - \text{min}}{2}$

(M1)

$$= 2.3 \text{ (m)}$$

A1

[2 marks]

(c) $d = \frac{6.8 + 2.2}{2}$ OR $d = \frac{\text{max} + \text{min}}{2}$

(M1)

$$= 4.5 \text{ (m)}$$

A1

[2 marks]

continue...

Question 8 continued.

(d) **METHOD 1**

substituting $t = 4.5$ and $H = 6.8$ for example into their equation for H (A1)

$$6.8 = 2.3 \sin\left(\frac{\pi}{6}(4.5 - c)\right) + 4.5$$

attempt to solve their equation (M1)

$$c = 1.5 \quad \text{A1}$$

METHOD 2

using horizontal translation of $\frac{12}{4}$ (M1)

$$4.5 - c = 3 \quad \text{(A1)}$$

$$c = 1.5 \quad \text{A1}$$

METHOD 3

$$H'(t) = (2.3) \left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{6}(t - c)\right) \quad \text{(A1)}$$

attempts to solve their $H'(4.5) = 0$ for c (M1)

$$(2.3) \left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{6}(4.5 - c)\right) = 0$$

$$c = 1.5 \quad \text{A1}$$

[3 marks]

(e) attempt to find H when $t = 12$ or $t = 0$, graphically or algebraically (M1)

$$H = 2.87365\dots$$

$$H = 2.87(\text{m}) \quad \text{A1}$$

[2 marks]

continue...

Question 8 continued.

(f) attempt to solve $5 = 2.3 \sin\left(\frac{\pi}{6}(t - 1.5)\right) + 4.5$ **(M1)**

times are $t = 1.91852\dots$ and $t = 7.08147\dots$, ($t = 13.9185\dots, t = 19.0814\dots$) **(A1)**

total time is $2 \times (7.081\dots - 1.919\dots)$

10.3258...

= 10.3 (hours)

A1

Note: Accept 10.

[3 marks]

Total [13 marks]

/ - - - - \

1. (a) valid approach using Pythagorean identity

(M1)

$$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1 \text{ (or equivalent)}$$

(A1)

$$\sin A = \frac{\sqrt{11}}{6}$$

A1

[3 marks]

- (b) $\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent)

(A1)

$$\text{area} = 4\sqrt{11}$$

A1

[2 marks]

Total [5 marks]

6. (a) attempting to use the change of base rule

M1

$$\log_9(\cos 2x + 2) = \frac{\log_3(\cos 2x + 2)}{\log_3 9}$$

A1

$$= \frac{1}{2} \log_3(\cos 2x + 2)$$

A1

$$= \log_3 \sqrt{\cos 2x + 2}$$

AG**[3 marks]**

(b) $\log_3(2 \sin x) = \log_3 \sqrt{\cos 2x + 2}$

$$2 \sin x = \sqrt{\cos 2x + 2}$$

M1

$$4 \sin^2 x = \cos 2x + 2 \text{ (or equivalent)}$$

A1

$$\text{use of } \cos 2x = 1 - 2 \sin^2 x$$

(M1)

$$6 \sin^2 x = 3$$

$$\sin x = (\pm) \frac{1}{\sqrt{2}}$$

A1

$$x = \frac{\pi}{4}$$

A1

Note: Award **A0** if solutions other than $x = \frac{\pi}{4}$ are included.

[5 marks]**Total [8 marks]****Section B**

2. (a) METHOD 1

attempt to use the cosine rule

(M1)

$$\cos \theta = \frac{4^2 + 4^2 - 5^2}{2 \times 4 \times 4} \quad (\text{or equivalent})$$

A1

$$\theta = 1.35$$

A1**[3 marks]****METHOD 2**

attempt to split triangle AOB into two congruent right triangles

(M1)

$$\sin\left(\frac{\theta}{2}\right) = \frac{2.5}{4}$$

A1

$$\theta = 1.35$$

A1**[3 marks]****(b) attempt to find the area of the shaded region****(M1)**

$$\frac{1}{2} \times 4 \times 4 \times (2\pi - 1.35\dots)$$

A1

$$= 39.5 \text{ (cm}^2\text{)}$$

A1**[3 marks]****Total [6 marks]**

7. (a) $\frac{4.2}{60} \times 45$ **A1**
 $AB = 3.15 \text{ (km)}$ **A1**
[2 marks]
- (b) (i) 66° or $(180 - 114)$ **A1**
 $35 + 66$ **A1**
 $\hat{A}BC = 101^\circ$ **AG**
- (ii) attempt to use cosine rule **(M1)**
 $AC^2 = 3.15^2 + 4.6^2 - 2 \times 3.15 \times 4.6 \cos 101^\circ$ (or equivalent) **A1**
 $AC = 6.05 \text{ (km)}$ **A1**
[5 marks]
- (c) valid approach to find angle BCA **(M1)**
eg sine rule
correct substitution into sine rule **A1**
eg $\frac{\sin(\hat{B}CA)}{3.15} = \frac{\sin 101}{6.0507...}$
 $\hat{B}CA = 30.7^\circ$ **A1**
[3 marks]
- (d) $\hat{B}AC = 48.267$ (seen anywhere) **A1**
valid approach to find correct bearing **(M1)**
eg $48.267 + 35$
bearing = 83.3° (accept 083°) **A1**
[3 marks]
- (e) attempt to use time = $\frac{\text{distance}}{\text{speed}}$ **M1**
 $\frac{6.0507}{3.9}$ or 0.065768 km/min **(A1)**
 $t = 93 \text{ (minutes)}$ **A1**
[3 marks]

Total [16 marks]

9. (a) correct approach **A1**

eg $\frac{\pi}{6} = \frac{2\pi}{\text{period}}$ (or equivalent)

period = 12

A1

[2 marks]

(b) (i) valid approach **(M1)**

eg $\frac{\text{max} + \text{min}}{2}$ $b = \text{max} - \text{amplitude}$

$\frac{21.8 + 10.2}{2}$, or equivalent

$b = 16$

A1

continued...

Question 9 continued

- (ii) attempt to substitute into
- their**
- function

(M1)

$$5.8 \sin\left(\frac{\pi}{6}(6+1)\right) + 16$$

$$f(6) = 13.1$$

A1**[4 marks]**

- (c) valid attempt to set up a system of equations
-
- two correct equations

(M1)**A1**

$$p \sin\left(\frac{2\pi}{9}(3-3.75)\right) + q = 2.5, \quad p \sin\left(\frac{2\pi}{9}(6-3.75)\right) + q = 15.1$$

valid attempt to solve system

(M1)

$$p = 8.4; q = 6.7$$

A1A1**[5 marks]**

- (d) attempt to use
- $|f(x) - g(x)|$
- to find maximum difference
-
- $x = 1.64$

(M1)**A1****[2 marks]****Total [13 marks]**

4. (a) $(f \circ g)(x) = f(2x)$ **(A1)**

$$f(2x) = \sqrt{3} \sin 2x + \cos 2x$$
 A1

[2 marks]

(b) $\sqrt{3} \sin 2x + \cos 2x = 2 \cos 2x$

$$\sqrt{3} \sin 2x = \cos 2x$$

recognising to use $\tan$ or $\cot$

M1

$$\tan 2x = \frac{1}{\sqrt{3}} \text{ OR } \cot 2x = \sqrt{3} \text{ (values may be seen in right triangle)}$$
 (A1)

$$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) = \right) \frac{\pi}{6} \text{ (seen anywhere) (accept degrees)}$$
 (A1)

$$2x = \frac{\pi}{6}, \frac{7\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{7\pi}{12}$$

A1A1

Note: Do not award the final **A1** if any additional solutions are seen.
Award **A1A0** for correct answers in degrees.
Award **A0A0** for correct answers in degrees with additional values.

[5 marks]

Total [7 marks]

5. determines $\frac{\pi}{4}$ (or 45°) as the first quadrant (reference) angle **(A1)**

attempts to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}$ **(M1)**

Note: Award **M1** for attempting to solve $\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4}, \frac{7\pi}{4}, \dots$

$\frac{x}{2} + \frac{\pi}{3} = \frac{\pi}{4} \Rightarrow x < 0$ and so $\frac{\pi}{4}$ is rejected **(R1)**

$\frac{x}{2} + \frac{\pi}{3} = 2\pi - \frac{\pi}{4} \left(= \frac{7\pi}{4} \right)$ **A1**

$x = \frac{17\pi}{6}$ (must be in radians) **A1**

[5 marks]

3. (a) valid approach to find area of segment by finding area of sector – area of triangle (M1)

$$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$\frac{1}{2}(2)^2\theta - \frac{1}{2}(2)^2\sin\theta \quad (A1)$$

$$\text{area} = 2\theta - 2\sin\theta \quad A1$$

[3 marks]

- (b) EITHER

area of logo = area of rectangle – area of segments (M1)

$$5 \times 4 - 2 \times (2\theta - 2\sin\theta) = 13.4 \quad (A1)$$

OR

$$\text{area of one segment} = \frac{20 - 13.4}{2} (= 3.3) \quad (M1)$$

$$2\theta - 2\sin\theta = 3.3 \quad (A1)$$

THEN

$$\theta = 2.35672\dots$$

$$\theta = 2.36 \text{ (do not accept an answer in degrees)} \quad A1$$

Note: Award (M1)(A1)A0 if there is more than one solution.

Award (M1)(A1FT)A0 if the candidate works in degrees and obtains a final answer of 135.030...

[3 marks]

Total [6 marks]

7. (a) attempt to use the distance formula to find AV (M1)

$$\sqrt{(1 - (-1))^2 + (5 - 1)^2 + (0 - 6)^2}$$

$$= 7.48331\dots$$

$$= 7.48 \text{ (cm)} \left(= \sqrt{56} \text{ or } 2\sqrt{14} \right)$$

(A1)

[2 marks]

- (b) **METHOD 1**

attempt to apply cosine rule OR sine rule to find AB (M1)

$$(AB =) \sqrt{7.48\dots^2 + 7.48\dots^2 - 2 \times 7.48\dots \times 7.48\dots \cos(40^\circ)} \text{ OR } \frac{AB}{\sin 40^\circ} = \frac{\sqrt{56}}{\sin 70^\circ} \quad (\text{A1})$$

$$= 5.11888\dots$$

$$= 5.12 \text{ (cm)}$$

(A1)

METHOD 2

Let M be the midpoint of [AB]

attempt to apply right-angled trigonometry on triangle AVM (M1)

$$= 2 \times 7.48\dots \times \sin(20^\circ) \quad (\text{A1})$$

$$= 5.11888\dots$$

$$= 5.12 \text{ (cm)}$$

(A1)

[3 marks]*continued...*

*Question 7 continued***(c) METHOD 1**

equating volume of pyramid formula to 57.2

(M1)

$$\frac{1}{3} \times 5.11...^2 \times h = 57.2$$

(A1)

$$h = 6.54886...$$

$$h = 6.55 \text{ (cm)}$$

A1**METHOD 2**

Let M be the midpoint of [AB]

$$AV^2 = AM^2 + MX^2 + XV^2$$

(M1)

$$\Rightarrow XV = \sqrt{7.48...^2 - \left(\frac{5.11...}{2}\right)^2 - \left(\frac{5.11...}{2}\right)^2}$$

(A1)

$$h = 6.54886...$$

$$h = 6.55 \text{ (cm)}$$

A1**[3 marks]***continued...*

Question 7 continued

(d) $V = x \times 2x \times y = 57.2$ **(A1)**

$$S = 2(2x^2 + xy + 2xy)$$
 A1

Note: Condone use of A.

attempt to substitute $y = \frac{57.2}{2x^2}$ into their expression for surface area **(M1)**

$$(S(x) =) 4x^2 + 6x\left(\frac{57.2}{2x^2}\right)$$

EITHER

attempt to find minimum turning point on graph of area function **(M1)**

OR

$$\frac{dS}{dx} = 8x - 171.6x^{-2} = 0 \text{ OR } x = 2.77849... \quad \text{ **(M1)** }$$

THEN

92.6401...

minimum surface area = 92.6 (cm²) **A1**

[5 marks]

Total [13 marks]

1. (a) **EITHER**

uses the cosine rule

(M1)

$$AB^2 = 5^2 + 5^2 - 2 \times 5 \times 5 \times \cos 1.9$$

(A1)**OR**

uses right-angled trigonometry

(M1)

$$\frac{AB}{5} = \sin 0.95$$

(A1)**OR**

uses the sine rule

(M1)

$$\alpha = \frac{1}{2}(\pi - 1.9) (= 0.6207\dots)$$

$$\frac{AB}{\sin 1.9} = \frac{5}{\sin 0.6207\dots}$$

(A1)**THEN**

$$AB = 8.13415\dots$$

$$AB = 8.13 \text{ (m)}$$

A1**[3 marks]***continued...*

Question 1 continued

- (b) let the shaded area be
- A

METHOD 1

attempt at finding reflex angle

(M1)

$$\hat{AOB} = 2\pi - 1.9 \quad (= 4.3831\dots)$$

substitution into area formula

(A1)

$$A = \frac{1}{2} \times 5^2 \times 4.3831\dots \quad \text{OR} \quad \left(\frac{2\pi - 1.9}{2\pi} \right) \times \pi(5^2)$$

$$= 54.7898\dots$$

$$= 54.8 \text{ (m}^2\text{)}$$

A1**METHOD 2**let the area of the circle be A_c and the area of the unshaded sector be A_u

$$A = A_c - A_u$$

(M1)

$$A = \pi \times 5^2 - \frac{1}{2} \times 5^2 \times 1.9 \quad (= 78.5398\dots - 23.75)$$

(A1)

$$= 54.7898\dots$$

$$= 54.8 \text{ (m}^2\text{)}$$

A1**[3 marks]****Total [6 marks]**

7. (a) $\hat{A}BC = 27^\circ$ (A1)
 attempt to substitute into cosine rule (M1)
 $175^2 + 230^2 - 2(175)(230)\cos 27^\circ$ (A1)
 $108.62308\dots$
 $AC = 109 \text{ (m)}$ A1

[4 marks]

- (b) correct substitution into area formula (A1)
 $\frac{1}{2} \times 175 \times 230 \times \sin 27^\circ$
 $9136.55\dots$
 $\text{area} = 9140 \text{ (m}^2\text{)}$ A1

[2 marks]

- (c) attempt to substitute into sine rule or cosine rule (M1)
 $\frac{\sin 27^\circ}{108.623\dots} = \frac{\sin \hat{A}}{175}$ OR $\cos A = \frac{(108.623\dots)^2 + 230^2 - 175^2}{2 \times 108.623\dots \times 230}$ (A1)
 $47.0049\dots$
 $\hat{C}AB = 47.0^\circ$ A1

[3 marks]

continued...

Question 7 continued(d) **METHOD 1**recognizing that for areas to be equal, $AD=DC$ **(M1)**

$$AD = \frac{1}{2} AC = 54.3115\dots$$

A1

attempt to substitute into cosine rule to find BD

(M1)

correct substitution into cosine rule

(A1)

$$BD^2 = 230^2 + 54.3115^2 - 2(230)(54.3115)\cos 47.0049^\circ$$

$$BD = 197.009\dots$$

$$BD = 197(\text{m})$$

A1**[5 marks]***continued...*

*Question 7 continued***METHOD 2**

correct expressions for areas of triangle BDA and triangle BCD using BD

A1

$$\frac{1}{2} \times BD \times 230 \times \sin x^\circ \quad \text{and} \quad \frac{1}{2} \times BD \times 175 \times \sin(27 - x)^\circ \quad \text{OR}$$

$$\frac{1}{2} \times BD \times 230 \times \sin(27 - x)^\circ \quad \text{and} \quad \frac{1}{2} \times BD \times 175 \times \sin x^\circ$$

correct equation in terms of x **(A1)**

$$175 \sin(27 - x) = 230 \sin x \quad \text{or} \quad 175 \sin x = 230 \sin(27 - x)$$

$$x = 11.6326... \quad \text{or} \quad x = 15.3673...$$

(A1)substituting their value of x into equation to solve for BD**(M1)**

$$\frac{1}{2} \times BD \times 230 \times \sin 11.6326... = \frac{1}{2} \times BD \times 175 \times \sin 15.3673... \quad \text{or}$$

$$\frac{1}{2} \times BD \times 230 \times \sin 11.6326... = \frac{1}{2} \times 9136.55...$$

$$BD = 197(\text{m})$$

A1**[5 marks]****Total [14 marks]**

3. (a) (i) attempt to find midpoint of A and B (M1)

centre $(-1, 3, -2)$ (accept vector notation and/or missing brackets) A1

(ii) attempt to find AB or half of AB or distance between the centre and A (or B) (M1)

$$\frac{\sqrt{4^2 + 2^2 + 4^2}}{2} \text{ or } \sqrt{2^2 + 1^2 + 2^2}$$

$= 3$ A1

[4 marks]

(b) attempt to find the distance between their centre and V
(the perpendicular height of the cone) (M1)

$$\sqrt{0^2 + 4^2 + 2^2} \text{ OR } \sqrt{(\text{their slant height})^2 - (\text{their radius})^2}$$

$$= \sqrt{20} (= 2\sqrt{5}) \quad \text{(A1)}$$

$$\text{Volume} = \frac{1}{3} \pi 3^2 \sqrt{20}$$

$$= 3\pi\sqrt{20} (= 6\pi\sqrt{5}) \quad \text{A1}$$

[3 marks]

Total [7 marks]

9. (a) $\cos^2 x - 3\sin^2 x = 0$

valid attempt to reduce equation to one involving one trigonometric function (M1)

$$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3} \quad \text{OR} \quad 1 - \sin^2 x - 3\sin^2 x = 0 \quad \text{OR} \quad \cos^2 x - 3(1 - \cos^2 x) = 0$$

$$\text{OR} \quad \cos 2x - 1 + \cos 2x = 0$$

correct equation (A1)

$$\tan^2 x = \frac{1}{3} \quad \text{OR} \quad \cos^2 x = \frac{3}{4} \quad \text{OR} \quad \sin^2 x = \frac{1}{4} \quad \text{OR} \quad \cos 2x = \frac{1}{2}$$

$$\tan x = \pm \frac{1}{\sqrt{3}} \quad \text{OR} \quad \cos x = \pm \frac{\sqrt{3}}{2} \quad \text{OR} \quad \sin x = (\pm) \frac{1}{2} \quad \text{OR} \quad 2x = \frac{\pi}{3} \left(\frac{5\pi}{3} \right) \quad \text{(A1)}$$

$$x = \frac{\pi}{6}, x = \frac{5\pi}{6} \quad \text{A1A1}$$

Note: Award **M1A1A0A1A0** for candidates who omit the $\pm$ (for tan or cos) and give only $x = \frac{\pi}{6}$.

Award **M1A1A0A0A0** for candidates who omit the $\pm$ (for tan or cos) and give only $x = 30^\circ$.

Award **M1A1A1A1A0** for candidates who give both answers in degrees.

Award **M1A1A1A1A0** for candidates who give both correct answers in radians, but who include additional solutions outside the domain.

Award a maximum of **M1A0A0A1A1** for correct answers with no working.

[5 marks]

continued...

Question 9 continued

- (b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) **(M1)**

$$f'(x) = -2 \cos x \sin x - 6 \sin x \cos x (= -8 \sin x \cos x = -4 \sin 2x) \quad \textbf{A1}$$

- (ii) valid attempt to solve their $f'(x) = 0$ **(M1)**

At least 2 correct x -coordinates (may be seen in coordinates) **(A1)**

$$x = 0, \quad x = \frac{\pi}{2}, \quad x = \pi$$

Note: Accept additional correct solutions outside the domain.

Award **A0** if any additional incorrect solutions are given.

correct coordinates (may be seen in graph for part (c))

A1A1A1

$$(0, 1), (\pi, 1), \left(\frac{\pi}{2}, -3\right)$$

Note: Award a maximum of **M1A1A1A1A0** if any additional solutions are given.

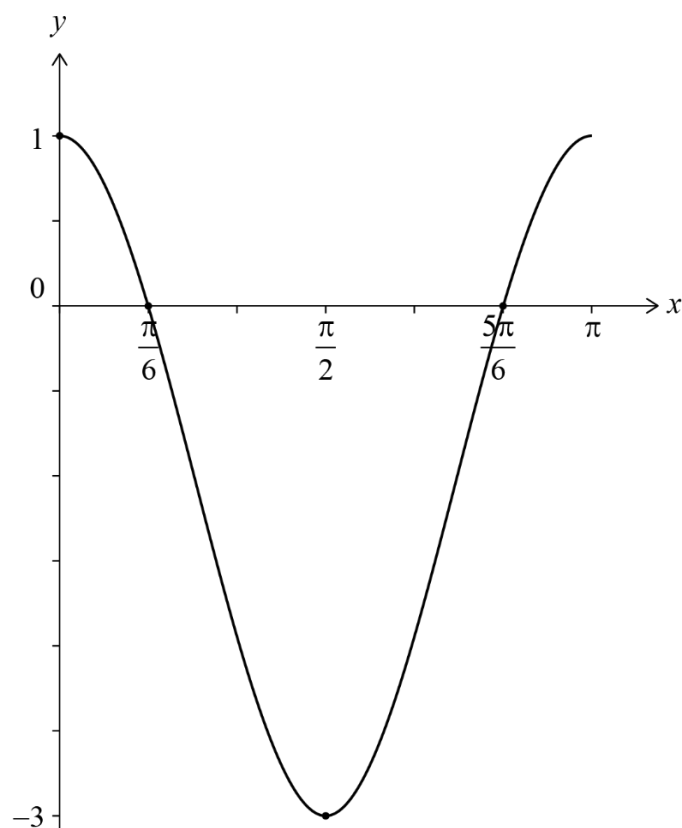
Note: If candidates do not find at least two correct x -coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as **M1A0A0A1A0**.

[7 marks]

continued...

Question 9 continued

(c)



Note: In this question do not award follow through from incorrect values found in earlier parts.

approximately correct smooth curve with minimum at $\left(\frac{\pi}{2}, -3\right)$

A1

Note: If candidates do not gain this mark then award no further marks.

endpoints at $(0,1)$, $(\pi,1)$, x -intercepts at $\frac{\pi}{6}$, $\frac{5\pi}{6}$

A1

correct concavity clearly shown at $(0,1)$ and $(\pi,1)$

A1

Note: The final two marks may be awarded independently of each other.

[3 marks]**Total [15 marks]**

2. (a) attempt to use sine rule (M1)

$$\frac{24}{\sin 113^\circ} = \frac{17}{\sin \hat{BAC}} \text{ OR } (\sin \hat{BAC} =) 0.652024... \quad (A1)$$

$$40.6943...$$

$$\hat{BAC} = 40.7^\circ \quad A1$$

[3 marks]

- (b) **METHOD 1** (cosine rule with $\hat{ABC}$ or $\hat{BAC}$)

attempt to use the cosine rule (M1)

$$24^2 = AB^2 + 17^2 - 2 \cdot 17 \cdot AB \cdot \cos 113^\circ \quad (AB^2 + 13.2848...AB - 287 = 0) \text{ OR}$$

$$17^2 = AB^2 + 24^2 - 2 \cdot 24 \cdot AB \cdot \cos 40.6943...^\circ \quad (AB^2 - 36.3935...AB + 287 = 0) \quad (A1)$$

$$11.5543...$$

$$AB = 11.6 \quad A1$$

METHOD 2 (cosine rule with $\hat{BCA}$)

attempt to use cosine rule (M1)

correct substitution (A1)

$$AB^2 = 17^2 + 24^2 - 2 \cdot 17 \cdot 24 \cdot \cos 26.3056...^\circ \text{ OR } AB^2 = 133.502...$$

$$11.5543...$$

$$AB = 11.6 \quad A1$$

continued...

*Question 2 continued***METHOD 3** (sine rule)

attempt to use sine rule

(M1)

correct substitution

(A1)

$$\frac{AB}{\sin 26.3056\dots^\circ} = \frac{24}{\sin 113^\circ} = \frac{17}{\sin 40.6943\dots^\circ} \quad \text{OR} \quad AB = \frac{24 \cdot \sin(180^\circ - 113^\circ - 40.6943\dots^\circ)}{\sin 113^\circ}$$

11.5543...

AB = 11.6

A1**[3 marks]****Total [6 marks]**

8.

Note: In parts (a) and (b) of this question, candidates may consider either triangle AOD or triangle AOE and work correctly to obtain the answer. Side AD is interchangeable with side AE in the following MS.

- (a) attempt to use right angled trigonometry or sine rule to find AE in terms of r and α

(M1)

$$\tan \alpha = \frac{r}{AE} \text{ OR } \frac{AE}{\sin\left(\frac{\pi}{2} - \alpha\right)} = \frac{r}{\sin \alpha}$$

$$AE = \frac{r}{\tan \alpha} \text{ OR } AE = \frac{r \sin\left(\frac{\pi}{2} - \alpha\right)}{\sin \alpha} \text{ OR } AE = \frac{r \cos \alpha}{\sin \alpha}$$

A1

valid approach to find the area of ADOE

(M1)

2x area of triangle AOE OR area of triangle AED + area of triangle OED OR
OE $\times$ AE

$$\text{Area ADOE} = 2\left(\frac{1}{2} \cdot \frac{r}{\tan \alpha} \cdot r\right) \text{ OR } r \times AE$$

A1

$$\text{Area ADOE} = \frac{r^2}{\tan \alpha}$$

AG**[4 marks]***continued...*

Question 8 continued

- (b) (i) recognizing that the sum of the angles of a kite is 2π (M1)

$$\hat{DOE} + \hat{OEA} + \hat{EAD} + \hat{ADO} = 2\pi \text{ OR } 2\alpha + 2 \cdot \frac{\pi}{2} + \hat{DOE} = 2\pi$$

$$\hat{DOE} = \pi - 2\alpha$$

A1

Note: Award **M1A0** if candidate uses degrees (i.e.

$$\hat{DOE} + \hat{OEA} + \hat{EAD} + \hat{ADO} = 360^\circ \text{ or } 2\alpha + 2 \cdot \frac{\pi}{2} + \hat{DOE} = 360^\circ) \text{ and obtains}$$

$$\hat{DOE} = 180^\circ - 2\alpha.$$

- (ii) valid approach to find the area of R (M1)

area of kite – area of sector OR 2(area of triangle AOE – 0.5 area of sector OED)

$$\text{Area of sector} = \frac{1}{2}r^2 \cdot \hat{DOE} \left(= \frac{1}{2}r^2(\pi - 2\alpha) \right) \text{ seen anywhere (A1)}$$

$$\text{Area of R} = \frac{r^2}{\tan \alpha} - \frac{1}{2}r^2(\pi - 2\alpha) \quad \text{A1}$$

Note: Accept $\frac{r^2}{\tan \alpha} - \frac{1}{2}r^2 \cdot \hat{DOE}.$

[5 marks]

continued...

Question 8 continued

- (c) equating their area formula to πr^2 **(M1)**

$$\frac{r^2}{\tan \alpha} - \frac{1}{2}r^2(\pi - 2\alpha) = \pi r^2$$

correct equation in terms of α

A1

$$\frac{1}{\tan \alpha} - \frac{1}{2}(\pi - 2\alpha) = \pi$$

valid approach to solve the equation

(M1)

$$\alpha = 0.218979\dots$$

$$\alpha = 0.219$$

A1

[4 marks]

Total [13 marks]

4. (a) $1 - 2\sin^2 x = \sin x$ **A1**
 $2\sin^2 x + \sin x - 1 = 0$ **AG**
[1 mark]

(b) valid attempt to solve quadratic **(M1)**

$$(2\sin x - 1)(\sin x + 1) \text{ OR } \frac{-1 \pm \sqrt{1 - 4(2)(-1)}}{2(2)}$$

recognition to solve for $\sin x$ **(M1)**

$$\sin x = \frac{1}{2} \text{ OR } \sin x = -1$$

any correct solution from $\sin x = -1$ **A1**

any correct solution from $\sin x = \frac{1}{2}$ **A1**

Note: The previous two marks may be awarded for degree or radian values, irrespective of domain.

$$x = -\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$$
A1

Note: If no working shown, award no marks for a final value(s).

Award **A0** for $-\frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ if additional values also given.

[5 marks]

Total [6 marks]

6. (a) recognition that period is $4m$ OR substitution of a point on f (except the origin) **(M1)**

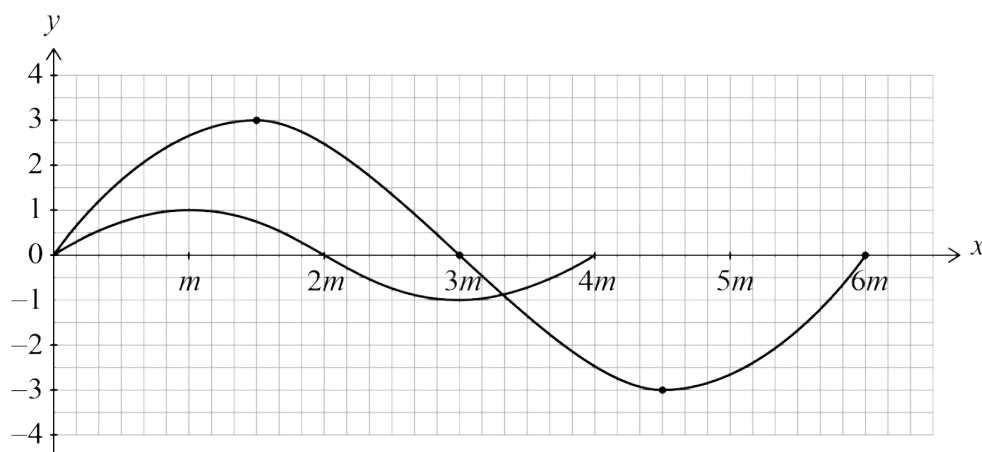
$$4m = \frac{2\pi}{q} \text{ OR } 1 = \sin qm$$

$$m = \frac{\pi}{2q}$$

A1**[2 marks]**

- (b) horizontal scale factor is $\frac{3}{2}$ (seen anywhere) **(A1)**

Note: This **(A1)** may be earned by seeing a period of $6m$, half period of $3m$ or the correct x -coordinate of the maximum/minimum point.

**A1A1A1**

Note: Curve must be an approximate sinusoidal shape (sine or cosine).
Only in this case, award the following:
A1 for correct amplitude.
A1 for correct domain.
A1 for correct max and min points **and** correct x -intercepts.

[4 marks]**Total [6 marks]**

Section B

1. (a) attempts to find perimeter (M1)
 $\text{arc} + 2 \times \text{radius}$ OR $10 + 4 + 4$
 $= 18 \text{ (cm)}$ A1

[2 marks]

- (b) $10 = 4\theta$ (A1)
 $\theta = \frac{10}{4} \left(= \frac{5}{2}, 2.5 \right)$ A1

[2 marks]

- (c) $\text{area} = \frac{1}{2} \left(\frac{10}{4} \right) (4^2)$ (A1)
 $= 20 \text{ (cm}^2\text{)}$ A1

[2 marks]

Total [6 marks]

8. (a) (i) attempt to use Pythagoras (M1)

$$\sin^2 \theta + \left(\frac{2}{3}\right)^2 = 1 \quad \text{OR} \quad x^2 + 2^2 = 3^2 \quad \text{OR} \quad \text{right triangle with side 2 and hypotenuse 3}$$

$$\sin \theta = \frac{\sqrt{5}}{3} \quad \text{A1}$$

- (ii) attempt to substitute into double-angle identity using their value of $\sin \theta$ (M1)

$$\sin 2\theta = 2 \left(\frac{\sqrt{5}}{3} \right) \left(\frac{2}{3} \right)$$

$$\sin 2\theta = \frac{4\sqrt{5}}{9} \quad \text{A1}$$

[4 marks]

- (b) **METHOD 1 (using values from part (a))**

$$\frac{b}{\sin \theta} = \frac{a}{\sin 2\theta}$$

attempt to use sine rule with their values from part (a) (M1)

$$\frac{b}{\left(\frac{\sqrt{5}}{3}\right)} = \frac{a}{\left(\frac{4\sqrt{5}}{9}\right)} \quad \text{OR} \quad \frac{\left(\frac{\sqrt{5}}{3}\right)}{b} = \frac{\left(\frac{4\sqrt{5}}{9}\right)}{a}$$

correct working that leads to **AG** A1

$$\frac{\sqrt{5}}{3}a = \frac{4\sqrt{5}}{9}b \quad \text{OR} \quad \frac{3b}{\sqrt{5}} = \frac{9a}{4\sqrt{5}} \quad \text{OR} \quad \frac{a}{3} = \frac{4b}{9} \quad (\text{or equivalent})$$

$$b = \frac{3a}{4} \quad \text{AG}$$

continued...

*Question 8 continued***METHOD 2 (double-angle identity)**

$$\frac{b}{\sin \theta} = \frac{a}{\sin 2\theta}$$

using double-angle identity

(A1)

$$\frac{b}{\sin \theta} = \frac{a}{2 \sin \theta \cos \theta} \quad \text{OR} \quad b = \frac{a \sin \theta}{2 \sin \theta \cos \theta} \quad \text{OR} \quad b = \frac{a}{2 \cos \theta}$$

correct working (involving substituting $\cos \theta = \frac{2}{3}$) that leads to **AG****A1**

$$b = \frac{a \sin \theta}{2 \sin \theta \left(\frac{2}{3}\right)} \quad \text{OR} \quad b = \frac{a \left(\frac{\sqrt{5}}{3}\right)}{2 \left(\frac{\sqrt{5}}{3}\right) \left(\frac{2}{3}\right)} \quad \text{OR} \quad b = \frac{a}{2 \left(\frac{2}{3}\right)} \quad (\text{or equivalent})$$

$$b = \frac{3a}{4}$$

AG**[2 marks]****(c) METHOD 1 (using supplementary angles)**recognizing $\hat{CAD}$ and $\hat{BAC}$ are supplementary**(M1)**

recognizing supplementary angles have the same sine value

(A1)

$$\sin \hat{CAD} = \sin 2\theta$$

$$\sin \hat{CAD} = \frac{4\sqrt{5}}{9}$$

A1*continued...*

Question 8 continued

METHOD 2 (using sine rule)recognizing $CD = a$ **(M1)**

$$\frac{a}{\sin \hat{CAD}} = \frac{b}{\sin \theta}$$

correct substitution of $\sin \theta = \frac{\sqrt{5}}{3}$ and $b = \frac{3a}{4}$ into sine rule**(A1)**

$$\frac{a}{\sin \hat{CAD}} = \frac{\left(\frac{3a}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)} \quad \text{OR} \quad \sin \hat{CAD} = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{3a}{4}\right)} \quad (\text{or equivalent})$$

$$\sin \hat{CAD} = \frac{4\sqrt{5}}{9}$$

A1**[3 marks]****(d) METHOD 1 (using $\hat{CAD}$ in area formula)**recognizing $\hat{DCA} = \theta$ **(A1)**recognizing $AD = b \left(= \frac{3a}{4} \right)$ **(A1)**correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \hat{CAD}$)**(M1)**

$$\text{area} = \frac{1}{2}(b)(b)\left(\frac{4\sqrt{5}}{9}\right) \quad \text{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin 2\theta \quad \text{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin \hat{CAD}$$

correct substitution in terms of a **(A1)**

$$\text{area} = \frac{1}{2}\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\left(\frac{4\sqrt{5}}{9}\right)$$

$$\text{area} = \frac{\sqrt{5}a^2}{8}$$

A1*continued...*

*Question 8 continued***METHOD 2 (using $\hat{A}CD$ or $\hat{A}DC$ in area formula)**recognizing $CD = a$ (A1)recognizing $AD = b \left(= \frac{3a}{4} \right)$ and/or $\hat{D}CA = \theta$ (A1)correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \hat{A}DC$ or $\sin \hat{A}CD$) (M1)

$$\text{area} = \frac{1}{2}(a)(b)\left(\frac{\sqrt{5}}{3}\right) \text{ OR } \text{area} = \frac{1}{2}(a)(b)\sin \theta \text{ OR } \text{area} = \frac{1}{2}(a)(b)\sin \hat{A}DC$$

$$\text{OR } \text{area} = \frac{1}{2}(a)(b)\sin \hat{A}CD$$

correct substitution in terms of a (A1)

$$\text{area} = \frac{1}{2}(a)\left(\frac{3a}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$$

$$\text{area} = \frac{\sqrt{5}a^2}{8} \quad \text{A1}$$

[5 marks]**Total [14 marks]**

1. (a) Let N be North

$$\hat{N}\hat{J}\hat{D} = 34^\circ \text{ OR } \hat{D}\hat{J}\hat{L} = 56^\circ \text{ (must be labelled or indicated in diagram):} \quad (\text{A1})$$

$$\hat{J}\hat{D}\hat{L} = 99^\circ \quad \text{A1}$$

Note: Accept $\frac{11\pi}{20}$, 1.73 (radians).

[2 marks]

- (b) attempt to apply the sine rule (M1)

$$\frac{DL}{\sin 56^\circ} = \frac{500}{\sin 99^\circ} \text{ OR } \frac{DL}{\sin 0.977384\dots} = \frac{500}{\sin 1.72787\dots} \quad (\text{A1})$$

$$419.685\dots$$

$$DL = 420 \text{ (km)} \quad \text{A1}$$

Note: Award **M1A1A0** for 261 (km) from use of degrees with GDC set in radians (with or without working).

[3 marks]

Total [5 marks]

9. (a) recognition that $45 = 10 + 10 + \text{arc length}$ (M1)
arc length = 25 (cm) (A1)
 $25 = 12\theta$ A1
 $\theta = 2.08$ correct to 3 significant figures AG

[3 marks]

continued...

Question 9 continued

(b)

Note: There are many different ways to dissect the cross-section to determine its area. In all approaches, candidates will need to find w or $\frac{w}{2}$. Award the first three marks for work seen anywhere.

EITHER

evidence of using the cosine rule OR sine rule

(M1)

$$w^2 = 12^2 + 12^2 - 2 \cdot 12 \cdot 12 \cos(2.08) \quad \text{OR} \quad \frac{w}{\sin(2.08)} = \frac{12}{\sin(0.530796\dots)}$$

(A1)

$$w = 20.6977\dots \quad \text{or} \quad \frac{w}{2} = 10.3488\dots$$

(A1)**OR**using trig ratios in a right triangle with angle $\frac{2.08}{2}$ and side length $\frac{w}{2}$ **(M1)**

$$\sin\left(\frac{2.08}{2}\right) = \frac{\frac{w}{2}}{12}$$

(A1)

$$w = 20.6977\dots \quad \text{or} \quad \frac{w}{2} = 10.3488\dots$$

(A1)

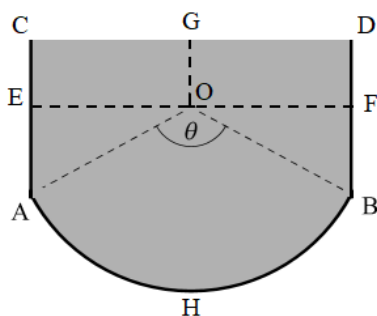
Note: Accept $w = 20.7179\dots$ from use of $\frac{\theta}{2} = \frac{25}{24}$.

continued...

Question 9 continued

THEN

Let the points A, B, C, D, E, F, G, H lie on the figure as follows:



EITHER

(segment AHB =) sector OAB – triangle OAB

(M1)

$$= \frac{1}{2} \times 12^2 \times 2.08 - \frac{1}{2} \times 12^2 \times \sin 2.08 (= 149.76 - 62.8655... = 86.8944...)$$

(A1)

valid approach to find total cross-sectional area (seen anywhere)

(M1)

sector OAB – triangle OAB + rectangle CDBA

$$= 86.8944... + 10w (= 86.8944... + 206.977...)$$

Note: Use of $\theta = \frac{25}{12}$ throughout leads to segment OAB = 87.2517... and cross-sectional area = 87.2517... + 207.179...

continued...

*Question 9 continued***OR**trapezium CGOA (= rectangle CGOE + triangle EOA) **(M1)**

$$= \frac{1}{2} \times (10 + (10 - 12 \cos(1.04))) \times \frac{20.6977...}{2} (= 72.0557) \quad \textbf{(A1)}$$

valid approach to find total cross-sectional area (seen anywhere) **(M1)**

2 × trapezium CGOA + sector OAB

$$= 2(72.0557...) + \frac{1}{2} \times 12^2 \times 2.08 (= 144.111... + 149.76)$$

Note: Use of $\theta = \frac{25}{12}$ leads to area of trapezium CGOA = 72.2154... and cross-sectional area = 144.430... + 150.

OR2 x area of trapezium CGOA (= area of rectangle CDFE + 2 x triangle EOA) **(M1)**

$$20.6977... \times (10 - 12 \cos(1.04)) + 2 \times \frac{1}{2} \times 12 \cos(1.04) \times 12 \sin(1.04) \quad \textbf{(A1)}$$

$$(= 81.2458... + 62.8655...)$$

valid approach to find total cross-sectional area (seen anywhere) **(M1)**

2 x trapezium CGOA + sector OAB

$$= 144.111... + \frac{1}{2} \times 12^2 \times 2.08 (= 144.111... + 149.76)$$

Note: Use of $\theta = \frac{25}{12}$ leads to 2 x area of trapezium CGOA = 144.430... and cross-sectional area = 144.430... + 150.

continued...

Question 9 continued

THEN

area of cross-section = 293.871... (294.430... from exact answer)

= 294 (cm²)

A1

[7 marks]

continued...

*Question 9 continued***(c) METHOD 1**

volume of gutter = 176323 OR 176658 (OR $600 \times$ their area) (seen anywhere) **A1**

recognising rainfall can be represented by an integral **(M1)**

$$\int_0^{60} R'(t) dt \left(= \frac{250}{2\pi} \sin\left(\frac{2\pi \times 60}{5}\right) + 3000 \times 60 \right) \quad \textbf{(A1)}$$

Note: Accept any 60 second interval or any interval which is a multiple of 5 seconds (one period) scaled up to 60 seconds e.g. $12 \int_0^5 R'(t) dt$.

rainfall over 60 seconds = 180000 (cm^3) **A1**

the gutter will overflow because the rainfall > gutter volume **A1**

METHOD 2

volume of gutter = 176323 OR 176658 (OR $600 \times$ their area) (seen anywhere) **A1**

recognition that cosine has a minimum value of -1 **(M1)**

$$R'(t) \geq -1 \times 50 + 3000 (\text{cm}^3 \text{s}^{-1}) \quad \textbf{(A1)}$$

rainfall over 60 seconds ≥ 177000 **(A1)**

the gutter will overflow because the rainfall > gutter volume **A1**

continued...

*Question 9 continued***METHOD 3**

volume of gutter = 176323 OR 176658 (OR $600 \times$ their area) (seen anywhere) **A1**

recognising rainfall can be represented by an integral **(M1)**

attempt to solve $60 > 58.8$ OR $\int_0^T R'(t) dt = 176658$ **(M1)**

time to reach overflow point = 58.7875... OR 58.8990... **A1**

the gutter will overflow because $60 > 58.8$ OR $60 > 58.9$ **A1**

[5 marks]

Total [15 marks]

2. (a) attempt to substitute into cosine rule (M1)

$$154^2 = 150^2 + 90^2 - 2(150)(90)\cos \hat{A}PB \quad \text{OR} \quad \cos \hat{A}PB = \frac{150^2 + 90^2 - 154^2}{2(150)(90)} \quad (\text{A1})$$

$$\hat{A}PB = 75.2286...^\circ \quad \text{OR} \quad 1.31298... \text{ radians}$$

$$\hat{A}PB = 75.2^\circ \quad \text{OR} \quad 1.31 \text{ radians}$$

A1

[3 marks]

- (b) valid approach to find θ (M1)

$$\theta = \frac{180^\circ - \hat{A}PB}{2} \quad \text{OR} \quad \theta = \frac{180^\circ - 75.2286...^\circ}{2} \quad (= 52.3856...) \quad \text{OR}$$

$$\theta = \frac{\pi - 1.31298...}{2} \quad (= 0.914302...)$$

valid approach to express h in terms of θ

(M1)

$$\sin \theta = \frac{h}{150} \quad \text{OR} \quad h = 150 \sin 52.3856...^\circ$$

$$h = 118.820...$$

$$h = 119 \text{ (m)}$$

A1

[3 marks]

Total [6 marks]

8. (a) $7.8 = \frac{2\pi}{\text{period}}$ **(M1)**

$$\frac{2\pi}{7.8} = 0.805536\dots$$

$$\text{period} = 0.806 \left(= \frac{10\pi}{39} \right)$$

A1**[2 marks]****(b) METHOD 1**

(i) $\text{amplitude} = \frac{\text{max} - \text{min}}{2}$ **(M1)**

$$\frac{1.8 - 1}{2}$$

$$a = -0.4$$

A1

(ii) $b = 1.4$

A1**METHOD 2**attempt to form two simultaneous equations in a and b **(M1)**

$$H(0) = 1 \Rightarrow a + b = 1, \quad H\left(\frac{\pi}{7.8}\right) = 1.8 \Rightarrow -a + b = 1.8$$

$$a = -0.4, b = 1.4$$

A1A1**[3 marks]***continued...*

Question 8 continued(c) **EITHER**

$$\frac{5}{\text{period}} = 6.207... < 6\frac{1}{2} \quad (\text{A1})$$

OR

consideration of number of maximums on graph in first 5 seconds (A1)

OR

maximums when $t = 0.403, 1.21, 2.01, 2.82, 3.62, 4.43$ (A1)

THEN

6 times

A1
[2 marks]

(d) recognizing that $H(t) = 1.5$ **(M1)**

$$-0.4 \cos(7.8t) + 1.4 = 1.5$$

$$0.233779...$$

$$t = 0.234 \text{ (seconds)}$$

A1
[2 marks]

continued...

Question 8 continued

(e) finding second time height is 1.5 metres **(M1)**

$$t = 0.571757...$$

in each period, height is greater than 1.5 metres for 0.337978... seconds **(A1)**

Note: Award **(M1)(A1)** for total time 2.02787... seen.

multiplying their value by 6 and dividing by 5 **(M1)**

$$\frac{0.337978... \times 6}{5} \text{ OR } \frac{2.02787...}{5}$$

$$= 0.405574...$$

$P(\text{height is greater than 1.5 m}) = 0.406$ **A1**

[4 marks]

Total [13 marks]

2. (a) $BV = \sqrt{(8-4)^2 + (6-3)^2 + (0-10)^2}$ (A1)
 $= 11.1803...$
 $= 11.2 (= \sqrt{125} = 5\sqrt{5})$ A1

[2 marks]

(b) **METHOD 1**

$$BV = VC \text{ AND } BC = 6 \text{ (seen anywhere)} \quad (A1)$$

attempt to use the cosine rule on triangle BVC for any angle (M1)

Note: Recognition must be shown in context either in terms of labelled sides or side lengths.

$$\cos \hat{BVC} = \frac{11.1...^2 + 11.1...^2 - 6^2}{2 \times 11.1... \times 11.1...} \text{ OR}$$

$$6^2 = 11.1...^2 + 11.1...^2 - 2 \times 11.1... \times 11.1... \cos \hat{BVC} \quad (A1)$$

$$\hat{BVC} = 0.543314...$$

$$\hat{BVC} = 0.543 \text{ (0.542 from 3 sf) (accept } 31.1^\circ) \quad A1$$

continued...

*Question 2 continued***METHOD 2**

let M be the midpoint of BC

$$BM = 3 \text{ (seen anywhere)} \quad \textbf{(A1)}$$

attempt to use sine or cosine in triangle BMV or CMV **(M1)**

$$\arcsin \frac{3}{\sqrt{125}} \text{ OR } \frac{\pi}{2} - \arccos \frac{3}{\sqrt{125}} \text{ OR } 0.271657... \quad \textbf{(A1)}$$

$$\hat{BVC} = 0.543314...$$

$$\hat{BVC} = 0.543 \text{ (0.542 from 3 sf) (accept } 31.1^\circ) \quad \textbf{A1}$$

[4 marks]**Total [6 marks]**

4. (a) use of sector area formula to find area of at least one sector (M1)

$$\frac{1}{2} \times 4.8 \times 100 - \frac{1}{2} \times 4.8 \times r^2 \quad \text{OR} \quad 10^2 \pi - \pi r^2 - \left(\frac{1}{2} 10^2 \times (2\pi - 4.8) - \frac{1}{2} \times (2\pi - 4.8) r^2 \right) \quad \text{A1}$$

$$(\text{area}) = 240 - 2.4r^2 \quad \text{AG}$$

[2 marks]

- (b) (i) $240 - 2.4r^2 = 176$ (A1)

$$r = 5.16397\dots$$

$$= 5.16 \text{ (cm)} \left(\frac{4\sqrt{15}}{3} \text{ exact} \right) \quad \text{A1}$$

- (ii) 10×4.8 OR $5.16\dots \times 4.8$ (A1)

substituting their value of r into $10 \times 4.8 + r \times 4.8 + 2(10 - r)$ (or equivalent) (M1)

$$\text{Perimeter} = 10 \times 4.8 + 5.16\dots \times 4.8 + 2(10 - 5.16\dots) (= 48 + 24.7870\dots + 9.67204\dots)$$

$$= 82.4591\dots$$

$$= 82.5 \text{ (cm)} \text{ (82.4 from 3 sf)} \quad \text{A1}$$

[5 marks]

Total [7 marks]

2. (a) $BV = \sqrt{(6-3)^2 + (8-4)^2 + (0-9)^2}$ (A1)
 $= 10.2956...$
 $= 10.3 (= \sqrt{106})$ A1

[2 marks]

(b) **METHOD 1**

$BV = VC$ AND $BC = 8$ (seen anywhere) (A1)

attempt to use the cosine rule on triangle BVC for any angle (M1)

Note: Recognition must be shown in context either in terms of labelled sides or in side lengths.

$$\cos \hat{BVC} = \frac{10.2...^2 + 10.2...^2 - 8^2}{2 \times 10.2... \times 10.2...} \text{ OR}$$

$$8^2 = 10.2...^2 + 10.2...^2 - 2 \times 10.2... \times 10.2... \cos \hat{BVC} \quad (A1)$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ) \quad A1$$

METHOD 2

let M be the midpoint of BC

$BM = 4$ (seen anywhere) (A1)

attempt to use sine or cosine in triangle BMV or CMV (M1)

$$\arcsin \frac{4}{\sqrt{106}} \text{ OR } \frac{\pi}{2} - \arccos \frac{4}{\sqrt{106}} \text{ OR } 0.399018 \quad (A1)$$

$$\hat{BVC} = 0.798037...$$

$$\hat{BVC} = 0.798 \text{ (accept } 45.7^\circ) \quad A1$$

[4 marks]

Total [6 marks]

4. (a) use of sector area formula to find area of at least one sector (M1)

$$\frac{1}{2} \times 5.2 \times 100 - \frac{1}{2} \times 5.2 \times r^2 \quad \text{OR} \quad 10^2 \pi - \frac{1}{2} 10^2 \times (2\pi - 5.2) - \left(\pi r^2 - \frac{1}{2} \times (2\pi - 5.2) \times r^2 \right) \quad \text{A1}$$

$$(\text{area}) = 260 - 2.6r^2 \quad \text{AG}$$

Note: There are many different ways to find the area of the “C”. In all methods, the **A** mark is awarded for working which leads directly to the **AG**.

[2 marks]

- (b) (i) $260 - 2.6r^2 = 64$ (A1)

$$r = 8.68243\dots$$

$$= 8.68(\text{cm}) \left(\frac{14\sqrt{65}}{13} \text{ exact} \right) \quad \text{A1}$$

- (ii) 10×5.2 OR $8.68\dots \times 5.2$ (A1)

substituting their value of r into $10 \times 5.2 + r \times 5.2 + 2(10 - r)$ (or equivalent) (M1)

$$\text{Perimeter} = 10 \times 5.2 + 8.68\dots \times 5.2 + 2(10 - 8.68\dots) \quad (= 52 + 45.1486\dots + 2.63513\dots)$$

$$= 99.7837\dots$$

$$= 99.8 \text{ (cm)} \quad \text{A1}$$

[5 marks]

Total [7 marks]

4. (a) $2r + r\theta = 10$ **A1**

$$\frac{1}{2}r^2\theta = 6.25$$
 A1

attempt to eliminate θ to obtain an equation in r **M1**

correct intermediate equation in r **A1**

$$10 - 2r = \frac{25}{2r} \quad \text{OR} \quad \frac{10}{r} - 2 = \frac{25}{2r^2} \quad \text{OR} \quad \frac{1}{2}r^2\left(\frac{10}{r} - 2\right) = 6.25 \quad \text{OR} \quad 12.5 + 2r^2 = 10r$$

$$4r^2 - 20r + 25 = 0$$
 AG

[4 marks]

(b) attempt to solve quadratic by factorizing or use of formula or completing the square **(M1)**

$$(2r - 5)^2 = 0 \quad \text{OR} \quad r = \frac{20 \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)} \left(= \frac{20 \pm \sqrt{400 - 400}}{8} \right)$$

$$r = \frac{5}{2}$$
 A1

attempt to substitute their value of r into their perimeter or area equation **(M1)**

$$\theta = \frac{10 - 2\left(\frac{5}{2}\right)}{\left(\frac{5}{2}\right)} \quad \text{or} \quad \theta = \frac{25}{2\left(\frac{5}{2}\right)^2}$$

$$\theta = 2$$
 A1

[4 marks]

Total [8 marks]

2. $\tan^{-1} 1 = 45^\circ$ or equivalent (A1)

attempt to equate $2x - 5^\circ$ to their reference angle (M1)

Note: Do not accept $2x - 5^\circ = 1$.

$$2x - 5^\circ = 45^\circ, (225^\circ)$$

$$x = 25^\circ, 115^\circ \quad \text{A1A1}$$

Note: Do not award the final **A1** if any additional solutions are seen.

[4 marks]

7. (a) $12b = 2\pi$ OR $(b =) \frac{2\pi}{12}$ OR $12 = \frac{2\pi}{b}$

A1

$$b = \frac{\pi}{6}$$

AG

[1 mark]

(b) (i) $a = 5$

A1

(ii) $c = 15$

A1

[2 marks]

continued...

Question 7 continued.

- (c) (i) attempt to substitute $x = 5$ into $g(x)$ **(M1)**

$$g(5) = 3.5 \sin \frac{5\pi}{6} + 11$$

$$\sin \frac{5\pi}{6} = \frac{1}{2}$$
 (A1)

$$g(5) = 3.5 \times \frac{1}{2} + 11$$

$$g(5) = 12.75 \left(= \frac{51}{4} \right)$$
 A1

- (ii) **METHOD 1 (finding maximum temperature)**

considering the maximum value of $\sin \frac{\pi}{6} x (=1)$ OR $g'(x) = 0$ at maximum

OR maximum = vertical shift + amplitude (may be seen on a graph) **(M1)**

$$g_{\max} = 3.5 + 11 \quad \text{OR} \quad \frac{\pi}{6} \cdot 3.5 \cos \left(\frac{\pi}{6} x \right) = 0 \quad \text{OR} \quad x = 3$$

$$g_{\max} = 14.5$$
 A1

$14.5 < 15$ (hence the midday water temperature is never warm enough for Alex to swim)

R1

Note: Do not award the R mark unless the previous marks been awarded
(Do not award **M1A0R1** or **M0A0R1**).

Worded conclusions are acceptable for the **R1**, as long as the reasoning is clear that the water does not reach 15° , so not warm enough for Alex.

continued...

Question 7 continued.

METHOD 2 (working with inequality)

$$3.5\sin\left(\frac{\pi}{6}x\right) + 11 \geq 15 \quad (\mathbf{M1})$$

$$\sin\left(\frac{\pi}{6}x\right) \geq \frac{8}{7} \quad \mathbf{A1}$$

sine values can never be greater than 1 (hence the midday water temperature is never warm enough for Alex to swim) **R1**

Note: Do not award the R mark unless the previous marks been awarded (Do not award **M1A0R1** or **M0A0R1**).

If candidate works with an equation throughout, the **M1** and **A1** may be awarded, if appropriate. A correct inequality is required for the **R1** to be awarded.

[6 marks]

continued...

Question 7 continued.

(d) **EITHER**

attempt to find $0.7 f(x)$ OR $0.7 f(x) + q$ **(M1)**

$$0.7 f(x) = 3.5 \sin \frac{\pi}{6} x + 10.5 \quad \text{OR} \quad 0.7 f(x) + q = 3.5 \sin \frac{\pi}{6} x + 10.5 + q \quad \text{OR}$$

$$10.5 + q = 11 \quad \text{A1}$$

OR

attempt to find $0.7 f(x)$ for a particular value of x **(M1)**

$$\text{eg maximum } 20 \times 0.7 = 14 \quad \text{A1}$$

THEN

$$q = 0.5 \quad \text{A1}$$

[3 marks]

Total [12 marks]

3. (a) attempt to use trigonometry to find the radius of the cone OR Oliver's distance from centre $(r + 5)$ (M1)

$$\tan 58^\circ = \frac{18.2}{r+5} \quad \text{OR} \quad \frac{r+5}{\sin 32^\circ} = \frac{18.2}{\sin 58^\circ} \quad \text{OR} \quad (r+5) = 11.3726... \quad \text{(A1)}$$

$$r = 6.37262... \text{ (m)}$$

$$(r =) 6.37 \text{ (m)} \quad \text{A1}$$

[3 marks]

- (b) attempt to substitute $h = 20$ and their radius into the correct volume of cone formula (M1)

$$V = \frac{\pi(6.37262...)^2(20)}{3}$$

$$= 850.540...$$

$$= 851 \text{ (m}^3\text{)} \quad \text{A1}$$

Note: Accept 849.840... (850) obtained from using $r = 6.37$.

[2 marks]

Total [5 marks]

8. (a) EITHERattempt to find value of t for the first low tide OR the first high tide **(M1)**

$$11.2619... - 5.13801...$$

$$= 6.12396... \quad \textbf{(A1)}$$

ORattempt to find half of the period **(M1)**

$$\frac{1}{2} \times \frac{2\pi}{0.513}$$

$$= 6.12396... \quad \textbf{(A1)}$$

THEN

$$m = (6.12396... - 6) \times 60 = 7.43773...$$

$$m = 7 \quad \textbf{A1}$$

[3 marks]**(b)** attempt to solve $H(t) = 1$ **(M1)**

$$3.56919... \text{ OR } 6.70684... \text{ OR } 15.8171... \text{ OR } 18.9547...$$

$$(6.70684... - 3.56919...) = 3.13764...$$

$$= 3.14 \text{ (hours)} \quad \textbf{A1}$$

[2 marks]**(c)** recognition that $H'(13)$ is required **(M1)**

$$= -0.650622...$$

$$= -0.651 \text{ (m/h)} \quad \textbf{A1}$$

[2 marks]*continued...*

Question 8 continued

(d)

Note: In part (d), award the marks for a , b , c and d independent of each other.

METHOD 1

$$a = 1.17$$

A1

$$d = 1.57$$

A1

attempt to find time between low and high tide in hours

(M1)

6 hours and 21 minutes = 6.35 hours

(period =) 12.7

(A1)

$$b = \frac{2\pi}{12.7} = 0.494739\dots$$

$$b = 0.495 \left(= \frac{60\pi}{381} \right)$$

A1

attempt to find mean of low and high tide times OR substitute values of a known point

(M1)

$$c = \frac{1}{2} \left(2 \frac{41}{60} + 9 \frac{2}{60} \right) \text{ OR eg } 0.40 = 1.17 \sin(0.495(2.68333\dots - c)) + 1.57$$

$$c = 5.85833\dots$$

$$c = 5.86$$

A1

Note: Award **(M1)A1** for $c = 18.6$.
Award **(M1)A0** for $c = -6.84$.

[7 marks]

continued...

Question 8 continued**METHOD 2**

$$a = 1.17 \quad \text{A1}$$

$$d = 1.57 \quad \text{A1}$$

substituting at least one point into $h(t)$ (M1)

$$1.17 \sin\left(b\left(2\frac{41}{60} - c\right)\right) + 1.57 = 0.4 \quad \text{OR} \quad 1.17 \sin\left(b\left(9\frac{2}{60} - c\right)\right) + 1.57 = 2.74$$

$$b\left(2\frac{41}{60} - c\right) = -\frac{\pi}{2} (= -1.57) \quad \text{AND} \quad b\left(9\frac{2}{60} - c\right) = \frac{\pi}{2} (= 1.57) \quad \text{(A1)}$$

Note: accept any angles of the form $-\frac{\pi}{2} + 2\pi k$ and $\frac{\pi}{2} + 2\pi k$.

EITHER

use of graph or table to find their intersection (M1)

OR

attempt to solve their equations simultaneously (M1)

$$\frac{2\frac{41}{60} - c}{9\frac{2}{60} - c} = -1$$

THEN

$$c = 5.85833\dots$$

$$c = 5.86 \quad \text{A1}$$

$$b = 0.494739\dots$$

$$b = 0.495 \quad \text{A1}$$

[7 marks]

(e) attempt to find point of intersection of two graphs (M1)

$$T = 4.16292\dots \quad \text{OR} \quad T = 4.16417\dots \text{ (using 3 sf)}$$

$$T = 4.16 \quad \text{A1}$$

[2 marks]

Total [16 marks]

9. (a) (i) METHOD 1

attempt to use the sine rule to find $\hat{A}BO$ **(M1)**

$$\frac{25}{\sin 28^\circ} = \frac{50}{\sin \hat{A}BO}$$

$$(\hat{A}BO =) 69.8748\dots^\circ \text{ or } (\hat{A}BO =) 110.125\dots^\circ \quad \textbf{(A1)(A1)}$$

Note: Award **A1** for each value.

attempt to find at least one possible angle for $\hat{O}AB$ **(M1)**

$$(\hat{O}AB =) 180^\circ - 28^\circ - 69.8748\dots^\circ \text{ OR } (\hat{O}AB =) 180^\circ - 28^\circ - 110.125\dots^\circ$$

$$\hat{O}AB = 82.1251\dots^\circ, 41.8748\dots^\circ$$

$$\hat{O}AB = 82.1^\circ, 41.9^\circ \quad \textbf{A1}$$

METHOD 2

attempt to use the cosine rule to find OB **(M1)**

$$25^2 = 50^2 + OB^2 - 2(50)(OB)\cos 28^\circ$$

$$OB = 52.7491\dots \text{ or } 35.5455\dots \quad \textbf{(A1) (A1)}$$

attempt to use the sine rule to find $\hat{O}AB$ **(M1)**

$$\frac{25}{\sin 28^\circ} = \frac{OB}{\sin \hat{O}AB}$$

$$\hat{O}AB = 82.1251\dots^\circ, 41.8748\dots^\circ$$

$$\hat{O}AB = 82.1^\circ, 41.9^\circ \quad \textbf{A1}$$

continued...

Question 9 continued

- (ii) attempt to substitute two sides of triangle OAB and one of their angles into the

area formula $\frac{1}{2}ab \sin C$ **(M1)**

$$\frac{1}{2}(OA)(AB)\sin 82.1251\dots^\circ \text{ OR } \frac{1}{2}(50)(25)\sin 41.8748\dots^\circ \text{ OR}$$

$$\frac{1}{2}(50)(52.7491\dots)\sin 28^\circ \text{ OR } \frac{1}{2}(50)(35.5455)\sin 28^\circ$$

$$\text{Area} = 619.106\dots \text{ OR } = 417.190\dots$$

$$= 619 \text{ (m}^2\text{)} \text{ OR } = 417 \text{ (m}^2\text{)}$$

A1A1**[8 marks]**

- (b) attempt to use the cosine rule in triangle OCD
- (M1)**

$$10^2 = x^2 + y^2 - 2xy \cos 28^\circ$$

A1

- attempt to use the area formula in triangle OCD
- (M1)**

$$\frac{1}{2}xy \sin 28^\circ = 60$$

A1

Note: Award **(M1)A1** for use of the area formula independently of the **(M1)A1** for use of the cosine rule.

$$xy = \frac{120}{\sin 28^\circ}$$

$$100 = x^2 + y^2 - \frac{240 \cos 28^\circ}{\sin 28^\circ}$$

A1

$$x^2 + y^2 = 100 + \frac{240}{\tan 28^\circ}$$

AG**[5 marks]***continued...*

Question 9 continued(c) **EITHER**attempt to eliminate y or x **(M1)**

$$100 = x^2 + \left(\frac{120}{x \sin 28^\circ} \right)^2 - \frac{240}{\tan 28^\circ}$$

OR

attempt to find the intersection of the graph of their cosine rule equation and the graph of their area formula

(M1)

Note: Award **(M1)** only if their graphs are of functions with the same subject e.g. both " $y = \dots$ " or both " $y^2 = \dots$ ".

THEN

$$x = 13.1300\dots \text{ or } x = 19.4673\dots$$

$$x = 13.1 \text{ or } x = 19.5$$

A1A1

$$OC = 13.1(\text{m}) \text{ or } 19.5(\text{m})$$

[3 marks]**Total [16 marks]**

9. (a) METHOD 1

let M be the midpoint of $[AB]$ and so $AB = 2AM$

attempts to use Pythagoras' theorem to find AM^2 OR AM **(M1)**

$$AM^2 = 20^2 - 14^2 (= 204) \text{ OR } AM = \sqrt{20^2 - 14^2} (= 14.2828... = \sqrt{204} = 2\sqrt{51})$$

recognizes that $AB = 2AM$ **(A1)**

$$AB = 2 \times 14.2828... (= 28.5657...) (= 2\sqrt{204} = 4\sqrt{51}) \quad \textbf{A1}$$

$$AB = 28.5657...$$

$$AB = 28.57 \text{ (m)} \quad \textbf{AG}$$

METHOD 2

let M be the midpoint of $[AB]$ and so $AB = 2AM$

let $\theta = \hat{ASM}$

$$\theta = 0.795398... \left(= \cos^{-1} \frac{14}{20} \right) \quad \textbf{(A1)}$$

attempts to use a valid trigonometric ratio **(M1)**

EITHER

$$AM = 14 \tan(0.795398...) \left(= 14.2828... = 14 \tan \left(\cos^{-1} \frac{14}{20} \right) \right) \quad \textbf{A1}$$

OR

$$AM = 20 \sin(0.795398...) \left(= 14.2828... = 20 \sin \left(\cos^{-1} \frac{14}{20} \right) \right) \quad \textbf{A1}$$

THEN

$$AB = 28.5657...$$

$$AB = 28.57 \text{ (m)} \quad \textbf{AG}$$

[3 marks]

continued...

Question 9 continued.

(b) **EITHER**

the sprinkler rotates through (an angle of) 2π (radians) every 16 seconds and

hence rotates through $\frac{2\pi}{16}$ (radians) in 1 second

A1

OR

$$\left(\frac{2\pi}{n} = 16 \Rightarrow n = \right) \frac{2\pi}{16} \left(= \frac{\pi}{8} \right)$$

A1

THEN

sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second

AG

[1 mark]

continued...

Question 9 continued.

(c)

Note: For candidates that used Method 2 in part (a) apply full FT from their value of θ .

attempts to find 2θ where $\theta = \hat{A}\hat{S}M$ **(M1)**

$$= 2(0.795398...) \left(1.59079... = 2\cos^{-1}\frac{14}{20} \right)$$

uses $\frac{\theta}{t}$ (rad/s) or similar to form an equation involving T **(M1)**

$$\frac{2\pi}{16} = \frac{1.59079...}{T} \left(\frac{2\pi}{16} = \frac{2\cos^{-1}\frac{14}{20}}{T} \right) \quad \text{A1}$$

$$T = 4.05093... \left(= \frac{1.59079...}{\frac{2\pi}{16}} \right) \left(= \frac{2\cos^{-1}\frac{14}{20}}{\frac{2\pi}{16}} \right)$$

$$T = 4.05 \text{ (s)} \quad \text{A1}$$

[4 marks]

continued...

Question 9 continued.

(d) $\alpha = \frac{\pi t}{8}$

A1**[1 mark]**

(e) applies sine rule in $\triangle ASD$

A1

$$\frac{d}{\sin \alpha} = \frac{20}{\sin \hat{ADS}}$$

attempts to find $\hat{ADS}$ in terms of α

M1

$$\hat{ADS} = \pi - \beta - \alpha \quad (= \pi - 0.7754 - \alpha) \quad (= 2.366... - \alpha) \quad (= 2.37 - \alpha)$$

$$d = \frac{20 \sin \alpha}{\sin(2.366... - \alpha)} \left(= \frac{20 \sin \alpha}{\sin(2.37 - \alpha)} \right) \text{ (accept } d = \frac{20 \sin \alpha}{\sin(\pi - \beta - \alpha)} \text{)}$$

A1

$$d = \frac{20 \sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}$$

AG**[3 marks]**

(f) 18 (m)

A1**[1 mark]***continued...*

Question 9 continued.

(g) (i) $w = \left| 0.05t^2 + 1.1t + 18 - \frac{20 \sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)} \right|$ **A1**

(ii) attempts to solve $w = 0$ for t **(M1)**

$t = 3.34880\dots(12.7765\dots)$

$t = 3.35$ (s) **A1**

$22.2444\dots$

22.2 (m) (south of A) **A1**

[4 marks]

Total [17 marks]

2. (a) $\frac{1}{2}r^2(\theta) = 6$ OR $\frac{1}{2}r^2(0.75) = 6$ (A1)

attempt to solve their equation to find r or r^2 (M1)

Note: To award the **M1**, candidate's equation must include r^2 and $\theta = 1.5$, and they must isolate r^2 or r .

$$r^2 = 16$$

$$r = 4 \text{ (cm)}$$

A1

[3 marks]

(b) evidence of summing the two radii and the arc length

(M1)

$$\text{perimeter} = 2r + r\theta$$

$$= 8 + 4(0.75)$$

$$= 11 \text{ (cm)}$$

A1

[2 marks]

Total [5 marks]

5. attempt to substitute into cosine rule

(M1)

$$(\cos 2\theta =) \frac{3^2 + 10^2 - 8^2}{2 \times 3 \times 10} \text{ OR } 8^2 = 3^2 + 10^2 - 2 \times 3 \times 10 \times \cos 2\theta$$

$$(\cos 2\theta =) \frac{45}{60} \left(= \frac{3}{4} \right)$$

(A1)

attempt to use $\cos 2\theta = 2\cos^2 \theta - 1$

(M1)

$$\cos^2 \theta = \frac{1 + \frac{45}{60}}{2} \left(= \frac{1 + \frac{3}{4}}{2} \right)$$

$$\cos^2 \theta = \frac{105}{120} \left(= \frac{7}{8} \right)$$

A1

$$\cos \theta = (\pm) \sqrt{\frac{105}{120}} \left(= \sqrt{\frac{7}{8}} \right)$$

(A1)

$$= \frac{\sqrt{7}}{2\sqrt{2}}$$

$$= \frac{\sqrt{14}}{4} \quad (p=14, q=4)$$

A1

Note: The final answer must be positive.

[6 marks]

2. (a) $\frac{1}{2}r^2\theta = 48$ OR $\frac{1}{2}r^2(1.5) = 48$ (A1)

attempt to solve their equation to find r or r^2 (M1)

Note: To award the **M1**, candidate's equation must include r^2 and $\theta = 1.5$, and they must attempt to isolate r^2 or r .

$$r^2 = 64$$

$$r = 8 \text{ (cm)}$$

A1

[3 marks]

(b) evidence of summing the two radii and the arc length (M1)

$$\text{perimeter} = 2r + r\theta$$

$$= 16 + 8(1.5)$$

$$= 28 \text{ (cm)}$$

A1

[2 marks]

Total [5 marks]

5. attempt to substitute into cosine rule (M1)

$$(\cos 2\theta =) \frac{4^2 + 6^2 - 5^2}{2 \times 4 \times 6} \text{ OR } 5^2 = 4^2 + 6^2 - 2 \times 4 \times 6 \times \cos 2\theta$$

$$(\cos 2\theta =) \frac{27}{48} \left(= \frac{9}{16} \right) \quad \text{(A1)}$$

attempt to use $\cos 2\theta = 2\cos^2 \theta - 1$ (M1)

$$\cos^2 \theta = \frac{1 + \frac{27}{48}}{2} \left(= \frac{1 + \frac{9}{16}}{2} \right)$$

$$\cos^2 \theta = \frac{75}{96} \left(= \frac{25}{32} \right) \quad \text{A1}$$

$$\cos \theta = (\pm) \sqrt{\frac{75}{96}} \left(= \sqrt{\frac{25}{32}} = \frac{5}{\sqrt{32}} \right) \quad \text{(A1)}$$

$$= \frac{5}{4\sqrt{2}}$$

$$= \frac{5\sqrt{2}}{8} \quad (p=5, q=8) \quad \text{A1}$$

Note: The final answer must be positive.

[6 marks]

5. (a) METHOD 1attempt to use right triangle trigonometry **(M1)**

$$\tan \hat{BAE} = \frac{15}{4} \text{ OR } \tan(90^\circ - \hat{BAE}) = \frac{4}{15} \quad \textbf{(A1)}$$

75.0685...

$$\hat{BAE} = 75.1^\circ \quad \textbf{A1}$$

Note: Award **(M1)(A1)A0** for the equivalent radian value of 1.31.
METHOD 2attempt to find $\hat{BAE}$ using sine rule OR cosine rule **(M1)**

$$\frac{\sin \hat{BAE}}{15} = \frac{\sin 90}{\sqrt{15^2 + 4^2}} \text{ OR } 15^2 = 4^2 + 241 - 2 \times 4 \times \sqrt{15^2 + 4^2} \times \cos \hat{BAE} \quad \textbf{(A1)}$$

75.0685...

$$\hat{BAE} = 75.1^\circ \quad \textbf{A1}$$

Note: Award **(M1)(A1)A0** for the equivalent radian value of 1.31.
[3 marks]**(b) (i) METHOD 1**attempt to find DE using right angle trigonometry **(M1)**

$$\sin 75.0685...^\circ = \frac{350}{DE} \text{ OR equivalent} \quad \textbf{(A1)}$$

$$DE = 362.230...$$

$$CE = 362.230... + 40$$

$$= 402.230...$$

$$= 402 \text{ (cm)} \quad \textbf{A1}$$

METHOD 2

$$\text{Let } DE = EF = x$$

attempt to find DE using their $\hat{DEF}$ and the sine rule OR cosine rule **(M1)**

$$\frac{700}{\sin(150.137...)} = \frac{DE}{\sin(14.9314...)} \text{ OR } x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 14.9314... \quad \textbf{(A1)}$$

$$DE = 362.230...$$

$$CE = 362.230... + 40$$

$$= 402.230...$$

$$= 402 \text{ (cm)} \quad \textbf{A1}$$

continued...

*Question 5 continued***METHOD 3**

Let G be the midpoint of DF

$$EG = \frac{4}{15} \times 350 \left(= \frac{280}{3} = 93.3333... \right) \quad (\text{A1})$$

use of Pythagoras' with their EG to find DE (M1)

$$(DE =) \sqrt{93.3333...^2 + 350^2} (= 362.230...)$$

$$CE = 362.230... + 40$$

$$= 402.230...$$

$$= 402 \text{ (cm)} \quad (\text{A1})$$

$$(ii) \quad \tan(75.0685...^\circ) = \frac{20}{x} \text{ OR } \frac{15}{4} = \frac{20}{x} \quad (\text{A1})$$

$$x = 5.33$$

$$BA = 402.230... + 5.33333...$$

$$= 407.564...$$

$$= 408 \text{ (cm) (accept 407 from previous 3sf answer)} \quad (\text{A1})$$

[5 marks]

Total [8 marks]

9. (a) (i) 3 A1
- (ii) attempt to find period (M1)
- $$\frac{2\pi}{b} \text{ or } \frac{2\pi}{4\pi}$$
- $$\text{period} = \frac{1}{2} \quad \text{A1}$$
- [3 marks]**
- (b) (i) evidence of considering the graph of $3\sin(4\pi x) - 4\cos(4\pi x)$ (seen in i, ii, or iii) (M1)
- $$a = 5 \quad \text{A1}$$
- (ii) $b = 4\pi$ A1
- (iii) 0.198792... A1
- $$c = 0.199 \quad \text{A1}$$
- [4 marks]**
- (c) 19 (seconds) A1
- [1 mark]**
- (d) 5 (speed bumps) A1
- [1 mark]**
- (e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{14}$) (M1)
- $$(3.5)\left(\frac{\pi}{14}\right)\sin\left(\frac{\pi}{14}(t-5)\right) \text{ OR } 0.785398\dots$$
- $$v'(t) = 0.785\sin\left(\frac{\pi}{14}(t-5)\right) \left(= \frac{\pi}{4}\sin\left(\frac{\pi}{14}(t-5)\right) \right) \quad \text{A1}$$
- (ii) recognition that $v' = a$ (M1)
- $$0.785(\text{ms}^{-2}) \left(= \frac{\pi}{4} \right) \quad \text{A1}$$

[4 marks]*continued...*

Question 9 continued

- (f) recognition that a definite integral of the velocity function is needed **(M1)**
using a correct set of limits (any limits which differ by 28 seconds) **(A1)**

$$\int_5^{33} v(t)dt \left(= \int_k^{k+28} |v(t)|dt \right)$$

252(m) (exact) **A1**

[3 marks]

Total [16 marks]

5. (a) METHOD 1

attempt to use right triangle trigonometry (M1)

$$\tan \hat{BAE} = \frac{12}{7} \text{ OR } \tan(90^\circ - \hat{BAE}) = \frac{7}{12} \quad (\text{A1})$$

59.7435...

$$\hat{BAE} = 59.7^\circ \quad \text{A1}$$

Note: Award (M1)(A1)A0 for the equivalent radian value of 1.04.
METHOD 2attempt to find $\hat{BAE}$ using sine rule OR cosine rule (M1)

$$\frac{\sin \hat{BAE}}{12} = \frac{\sin 90}{\sqrt{12^2 + 7^2}} \text{ OR } 12^2 = 7^2 + 193 - 2 \times 7 \times \sqrt{12^2 + 7^2} \times \cos \hat{BAE} \quad (\text{A1})$$

 $\hat{BAE} = 59.7435...$

$$\hat{BAE} = 59.7^\circ \quad \text{A1}$$

Note: Award (M1)(A1)A0 for the equivalent radian value of 1.04.
[3 marks]**(b) (i) METHOD 1**

attempt to find DE using right angle trigonometry (M1)

$$\sin 59.7435...^\circ = \frac{350}{DE} \text{ OR equivalent} \quad (\text{A1})$$

$$DE = 405.196...$$

$$CE = 405.196... + 50$$

$$= 455.196...$$

$$= 455 \text{ (cm)} \quad \text{A1}$$

METHOD 2

$$\text{Let } DE = EF = x$$

attempt to find DE using their $\hat{DEF}$ and the sine rule OR cosine rule (M1)

$$\frac{700}{\sin(119.487...)} = \frac{DE}{\sin(30.2564...)} \text{ OR } x^2 = 700^2 + x^2 - 2 \times 700 \times x \times \cos 30.2564... \quad (\text{A1})$$

$$DE = 405.196...$$

$$CE = 405.196... + 50$$

$$= 455.196...$$

$$= 455 \text{ (cm)} \quad \text{A1}$$

continued...

Question 5 continued.

METHOD 3

Let G be the midpoint of DF

$$EG = \frac{7}{12} \times 350 \left(= \frac{1225}{6} = 204.166... \right) \quad (\text{A1})$$

use of Pythagoras' with their EG to find DE (M1)

$$DE = \sqrt{204.166...^2 + 350^2} \quad (= 405.196...)$$

$$CE = 405.196... + 50$$

$$= 455.196...$$

$$= 455 \text{ (cm)} \quad \text{A1}$$

$$(ii) \quad \tan(59.7435...^\circ) = \frac{30}{x} \text{ OR } \frac{12}{7} = \frac{30}{x} \quad (\text{A1})$$

$$x = 17.5$$

$$BA = 455.196... + 17.5$$

$$= 472.696...$$

$$= 473 \text{ (cm)} \quad \text{A1}$$

[5 marks]

Total [8 marks]

9. (a) (i) 6 **A1**
 (ii) attempt to find period **(M1)**
 $\frac{2\pi}{b}$ or $\frac{2\pi}{\pi}$
 period = 2 **A1**

[3 marks]

- (b) (i) evidence of considering the graph of $6\cos(\pi x) - 8\sin(\pi x)$ (seen in i, ii, or iii) **(M1)**
 $a = 10$ **A1**

- (ii) $b = \pi$ **A1**

- (iii) 1.70483...
 $c = 1.70$ **A1**

[4 marks]

- (c) 23 (seconds) **A1**
[1 mark]

- (d) 4 (speedbumps) **A1**
[1 mark]

continued...

Question 9 continued.

- (e) (i) attempt to use chain rule (multiplication by $\frac{\pi}{19}$) **(M1)**

$$(4.5)\left(\frac{\pi}{19}\right)\sin\left(\frac{\pi}{19}(t-4)\right) \text{ OR } 0.744061\dots$$

$$v'(t) = 0.744\sin\left(\frac{\pi}{19}(t-4)\right)\left(=\left(\frac{9\pi}{38}\right)\sin\left(\frac{\pi}{19}(t-4)\right)\right) \quad \textbf{A1}$$

- (ii) recognition that $v' = a$ **(M1)**

$$0.744(\text{ms}^{-2}) \quad \textbf{A1}$$

[4 marks]

- (f) recognition that a definite integral of the velocity function is needed **(M1)**

using a correct set of limits (any limits which differ by 38 seconds) **(A1)**

$$\int_4^{42} v(t)dt \left(= \int_k^{k+38} |v(t)|dt \right)$$

$$399(\text{m}) (\text{exact}) \quad \textbf{A1}$$

[3 marks]**Total [16 marks]**

9. (a) (i) recognition that $AB = 2r$ and $BC = 2R$ (seen anywhere) (M1)

$$\cos \theta = \frac{2R}{2r} \text{ OR } \frac{2r}{\sin 90^\circ} = \frac{2R}{\sin(90^\circ - \theta)} \left(= \frac{2R}{\cos \theta} \right) \quad \text{A1}$$

$$R = r \cos \theta \quad \text{AG}$$

- (ii) attempt to use Pythagoras, sine or the sine rule or cosine rule in triangle $\hat{A}BC$ or similar (M1)

$$\sin \theta = \frac{h}{2r} \text{ OR } h^2 + (2R)^2 = (2r)^2 \text{ OR } \frac{2r}{\sin 90^\circ} = \frac{h}{\sin \theta} \text{ OR}$$

$$h^2 = (2R)^2 + (2r)^2 - 2 \times 2R \times 2r \cos(\theta)$$

$$h = 2r \sin \theta \text{ OR } h = \sqrt{4r^2 - 4r^2 \cos^2 \theta} \quad \text{A1}$$

[4 marks]

- (b) area of one circle $= \pi R^2$

$$= \pi (r \cos \theta)^2 \quad \text{A1}$$

$$\text{curved surface area} = 2\pi R h$$

$$= 2\pi (r \cos \theta) (2r \sin \theta) \text{ OR } = 2\pi (r \cos \theta) \sqrt{4r^2 - 4r^2 \cos^2 \theta} \quad \text{A1}$$

Note: these **A1** marks can be awarded independently.

recognition that total surface area = area of two circles + curved surface area

$$= 2\pi R^2 + 2\pi R h \text{ (seen anywhere)} \quad \text{M1}$$

$$= 2\pi r^2 \cos^2 \theta + 4\pi r^2 \sin \theta \cos \theta \text{ OR } = 2\pi r^2 \cos^2 \theta + 2\pi r \cos \theta \sqrt{4r^2 - 4r^2 \cos^2 \theta}$$

$$= 2\pi r^2 (1 - \sin^2 \theta) + 4\pi r^2 \sin \theta \cos \theta \quad \text{A1}$$

$$\text{total surface area of the cylinder} = 2\pi r^2 (1 + 2 \sin \theta \cos \theta - \sin^2 \theta) \quad \text{AG}$$

[4 marks]

- (c) attempt to equate external surface area of the sphere to $2S$

$$4\pi r^2 = 4\pi r^2 (1 + 2 \sin \theta \cos \theta - \sin^2 \theta) \text{ or equivalent} \quad \text{M1}$$

$$2 \sin \theta \cos \theta - \sin^2 \theta = 0 \quad \text{A1}$$

$$\sin \theta \neq 0 \quad \text{R1}$$

$$2 \cos \theta - \sin \theta = 0 \quad \text{A1}$$

$$\tan \theta = 2 \quad \text{AG}$$

[4 marks]

continued...

Question 9 continued.

- (d) attempt to find volume of the cylinder in terms of r and θ (M1)

$$\text{Volume of cylinder} = \pi R^2 h = \pi (r \cos \theta)^2 (2r \sin \theta) \quad \text{OR} \quad \pi r^2 \cos^2 \theta \sqrt{4r^2 - 4r^2 \cos^2 \theta}$$

$$\text{OR} \quad \pi (r \cos \theta)^2 \times 4r \cos \theta \quad \text{A1}$$

attempt to use right angled triangle to find $\cos \theta$ and $\sin \theta$ (M1)

$$\sqrt{1^2 + 2^2} = \sqrt{5}, \quad \cos \theta = \frac{1}{\sqrt{5}}, \quad \sin \theta = \frac{2}{\sqrt{5}}$$

$$\text{volume} = 2\pi r^3 \left(\frac{1}{\sqrt{5}} \right)^2 \left(\frac{2}{\sqrt{5}} \right) \quad \text{OR} \quad 4\pi r^3 \left(\frac{1}{\sqrt{5}} \right)^3 \quad \text{A1}$$

$$= \frac{4}{25} \pi r^3 \sqrt{5} \quad \text{A1}$$

$$p = \frac{4}{25}$$

[5 marks]

Total [17 marks]

4.

(a) correct substitution in sine rule

(A1)

$$\frac{\sin \theta}{5} = \frac{\sin 2\theta}{6\sqrt{2}} \text{ (or equivalent)}$$

attempt to use double angle rule for $\sin 2\theta$

(M1)

$$\frac{\sin \theta}{5} = \frac{2\sin \theta \cos \theta}{6\sqrt{2}}$$

$$6\sqrt{2} \sin \theta = 10 \sin \theta \cos \theta \quad \text{OR} \quad \frac{1}{5} = \frac{2\cos \theta}{6\sqrt{2}} \quad \text{OR equivalent}$$

A1

$$\cos \theta = \frac{3\sqrt{2}}{5}$$

AG

[3 marks](b) valid attempt to find $\sin \theta$

(M1)

$$\sin^2 \theta + \left(\frac{3\sqrt{2}}{5} \right)^2 = 1 \quad \text{OR right triangle with adjacent side and hypotenuse labelled}$$

$$\sin \theta = \frac{\sqrt{7}}{5}$$

A1

[2 marks]

$$(c) \quad \frac{1}{2} \times 6\sqrt{2} \times DC \times \frac{\sqrt{7}}{5} = 2\sqrt{14}$$

(A1)

$$DC = \frac{10}{3}$$

A1

[2 marks]**Total [7 marks]**

8. (a) (i) amplitude is $\frac{72}{2} = 36$ OR $8 = a \cos(b \times 0) + 44$ (A1)

$a = -36$ A1

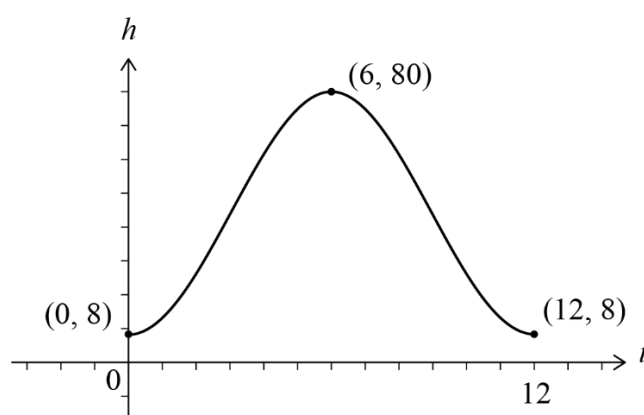
Note: If no working shown, award **A1A0** for $a = 36$.

(ii) $\frac{2\pi}{b} = 12$ (accept $\frac{360}{b} = 12$) OR $80 = -36 \cos(6b) + 44$ (M1)

$b = \frac{\pi}{6}$ (do not accept 30) A1

[4 marks]

(b)



A1A1

Note: Award **A1** for maximum at $(6, 80)$.

For an approximately correct sinusoidal shape,
award **A1** for correct end points (accept y -intercept labelled as 8).

Accept coordinate values clearly aligned and labelled on the axes.

[2 marks]

continued...

Question 8 continued.

(c) equating their $-36 \cos\left(\frac{\pi}{6}t\right) + 44 \geq 26$ (M1)

Note: Accept working with = or > sign.

$$\cos\left(\frac{\pi}{6}t\right) \leq \frac{1}{2}$$

$$\frac{\pi}{6}t = \frac{\pi}{3} \text{ OR } \frac{\pi}{6}t = \frac{5\pi}{3} \quad (\text{accept } 30t = 60 \text{ or } 30t = 300)$$
 (A1)

$$t = 2 \text{ OR } t = 10$$
 A1

valid attempt to find interval (M1)

$$12 - 2 \times 2 \text{ OR } 10 - 2$$

$$T = 8 \text{ (minutes)}$$
 A1

Note: If their values from part (a) result in a single solution for t only, award **M1A1FTA1FTM0A0** as appropriate.

[5 marks]

(d) (i) attempt to differentiate using chain rule (M1)

$$h'(t) = -36 \times -\sin\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6} \left(= 6\pi \sin\left(\frac{\pi}{6}t\right) \right)$$
 A1

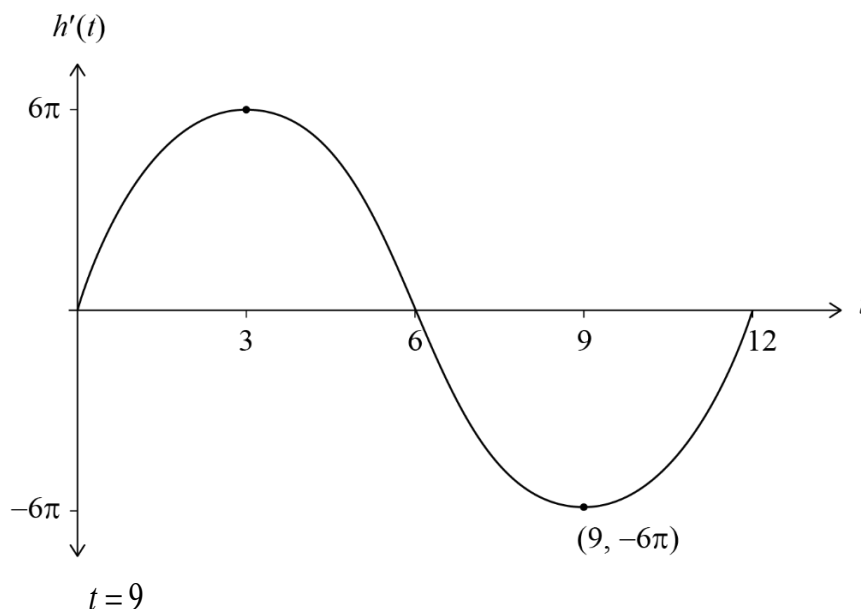
continued...

Question 8 continued.

(ii) **METHOD 1**

sketch of their h' (with their minimum point labelled)

(M1)



A1

METHOD 2

attempt to find and solve their $h''(t) = 0$

(M1)

$$h''(t) = -36 \times -\cos\left(\frac{\pi}{6}t\right) \times \frac{\pi}{6} \times \frac{\pi}{6} \left(= \pi^2 \cos\left(\frac{\pi}{6}t\right) \right) = 0$$

$$t = 3, t = 9$$

$$t = 9$$

A1

METHOD 3

recognising (minimum value of sine -1) occurs at $\frac{3}{4}$ of revolution

(M1)

$$\frac{\pi}{6}t = \frac{3\pi}{2} \quad (\text{accept } 30t = 270)$$

$$t = 9$$

A1

[4 marks]

Total [15 marks]

2. (a) EITHER

$$210^\circ = \frac{210\pi}{180} \left(= \frac{7\pi}{6} = 3.66519... \right) \text{ radians} \quad (\text{A1})$$

attempt to use radian formula for area of sector (M1)

$$\text{area} = \frac{1}{2}(19.5)^2 \left(\frac{7\pi}{6} \right)$$

OR

attempt to use degree formula for area of sector (M1)

$$\text{area} = \frac{210}{360} \pi (19.5)^2 \quad (\text{A1})$$

THEN

$$\text{area} = \frac{3549\pi}{16} = 696.844...$$

$$= 697 \left(= \frac{3549\pi}{16} \right) (\text{cm}^2) \quad \text{A1}$$

[3 marks]**(b) EITHER**

$$\text{arc length} = 19.5 \left(\frac{7\pi}{6} \right) \text{ OR } = \frac{210}{360} (2\pi(19.5)) \left(= \frac{91\pi}{4} = 71.4712... \right) \quad (\text{A1})$$

attempt to set $2\pi r$ equal to arc length (M1)

$$2\pi r = 71.4712...$$

OR

attempt to set $\pi r l$ equal to their area from (a) (M1)

$$19.5\pi r = 696.844... \quad (\text{A1})$$

THEN

$$r = 11.4 \left(= \frac{91}{8} = 11.375 \right) (\text{cm}) \quad \text{A1}$$

[3 marks]**Total[6 marks]**

3. (a) period is $\frac{\pi}{2} (=1.57079\dots = 1.57)$ **A1**

[1 mark]

(b) attempt to substitute $x = \frac{\pi}{12}, f(x) = 5$ and $x = \frac{\pi}{3}, f(x) = 7$ to obtain two equations **(M1)**

Note: accept work where x values have been converted into degrees
--

$$a \tan\left(\frac{\pi}{6}\right) + b = 5 \text{ and } a \tan\left(\frac{2\pi}{3}\right) + b = 7 \left(\Rightarrow \frac{a}{\sqrt{3}} + b = 5 \text{ and } -a\sqrt{3} + b = 7 \right)$$

$$a = -\frac{\sqrt{3}}{2} (= -0.866025\dots = -0.866)$$
 A1

$$b = \frac{11}{2} (= 5.5)$$
 A1

Note: These A1 marks may be awarded independently.
--

[3 marks]

Total [4 marks]

2.

- (a) attempt to find distance between two points **(M1)**

$$\sqrt{(-3-1)^2 + (4-7)^2 + (2-0)^2} (= 5.38516...)$$

$$VX = 5.39 (= \sqrt{29})$$

A1**[2 marks]**

- (b) attempt to use Pythagoras' theorem **(M1)**

$$\sqrt{5^2 + 5^2} (= 7.07106...)$$

$$AC = 7.07 (= 5\sqrt{2})$$

A1**[2 marks]**

- (c) valid attempt to use trig ratio in triangle VXC **(M1)**

$$\tan \theta = \frac{\frac{\sqrt{29}}{5\sqrt{2}}}{\frac{2}{3.53553...}} \left(= \frac{5.38516...}{3.53553...} \right)$$

$$56.7138...^\circ \text{ OR } 0.989842... \text{ rad}$$

$$\theta = 56.7^\circ \text{ OR } \theta = 0.990 \text{ rad (accept } \theta = 0.99)$$

A1**[2 marks]****Total [6 marks]**

6.

METHOD 1

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) **(M1)**

$\text{area}_{\text{triangle}} = 21.3333\ldots$ OR $\text{area}_{\text{sector}} = 34.1333\ldots$ **A1**

correct equation in r and θ **(A1)**

$$\frac{1}{2}r^2\theta = 34.1333\ldots \text{ OR } \frac{1}{2}r^2 \sin \theta = 21.3333\ldots \text{ OR } \frac{1}{2}r^2(\theta - \sin \theta) = 12.8 \text{ (seen anywhere)}$$

correct equation in one variable **A1**

$$\frac{1}{2}r^2 \left(\frac{68.2666\ldots}{r^2} - \sin \left(\frac{68.2666\ldots}{r^2} \right) \right) = 12.8 \text{ OR } \frac{1}{2} \left(\frac{68.2666\ldots}{\theta} \right) \sin \theta = 21.3333\ldots \text{ OR}$$

$$\frac{1}{2} \left(\frac{68.2666\ldots}{\theta} \right) (\theta - \sin \theta) = 12.8$$

attempt to solve their equation or use of graph **(M1)**

$$\theta = 1.59934\ldots$$

$$6.53330\ldots$$

$$r = 6.53 \quad \textbf{A1}$$

METHOD 2

recognising $\frac{\text{area}_{\text{segment}}}{\text{area}_{\text{triangle}}} = \frac{3}{5}$ (or equivalent) (seen anywhere) **(M1)**

$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin \theta)$ (seen anywhere) **(A1)**

$$\frac{\frac{1}{2}r^2(\theta - \sin \theta)}{\frac{1}{2}r^2 \sin \theta} = \frac{3}{5}$$

correct equation without r **A1**

$$\frac{(\theta - \sin \theta)}{\sin \theta} = \frac{3}{5}$$

$$\theta = 1.59934\ldots \quad \textbf{(A1)}$$

continued...

Question 6 continuedattempt to solve for r using their θ **(M1)**

$$\frac{1}{2}r^2(1.59... - \sin 1.59...) = 12.8$$

$$6.53330...$$

$$r = 6.53$$

A1**METHOD 3**recognising $\text{area}_{\text{segment}} = \frac{3}{8} \times \text{area}_{\text{sector}}$ (seen anywhere)**(M1)**

$$\text{area}_{\text{segment}} = \frac{1}{2}r^2(\theta - \sin \theta) \quad (\text{seen anywhere})$$

(A1)

$$\frac{1}{2}r^2(\theta - \sin \theta) = \frac{3}{8} \times \frac{1}{2}r^2\theta$$

correct equation without r **A1**

$$\frac{1}{2}(\theta - \sin \theta) = \frac{3}{8} \times \frac{1}{2}\theta$$

$$\theta = 1.59934...$$

(A1)attempt to solve for r using their θ **(M1)**

$$\frac{1}{2}r^2(1.59... - \sin 1.59...) = 12.8$$

$$6.53330...$$

$$r = 6.53$$

A1**[6 marks]**

SECTION B

7. (a) $\hat{JCL} = 28^\circ$ (A1)
 attempt to substitute into cosine rule (M1)
 $JL^2 = (5.5)^2 + 8^2 - 2(5.5)(8)\cos 28^\circ$ (A1)
 4.068244...
 $JL = 4.07 \text{ (km)}$ A1

Note: Award (A1)(M1)(A1)A0 for an answer of 13.4(13.3775...) (km) for working in radians.

[4 marks]

- (b) valid attempt to solve (M1)
 use a trigonometric ratio or sine rule in triangle BLX
 $\tan 0.94^\circ = \frac{60}{BL}$ OR $\tan 89.06^\circ = \frac{BL}{60}$ OR $\frac{BL}{\sin 89.06^\circ} = \frac{60}{\sin 0.94^\circ}$ (A1)
 3656.849288... OR 3.656849...
 $BL = 3660 \text{ (m)}$ OR $BL = 3.66 \text{ (km)}$ A1

[3 marks]

- (c) valid approach (M1)
 clear diagram OR $180 + 121$ OR $360 - (180 - 121)$ OR $270 + (121 - 90)$
 301° (accept N59°W) A1

[2 marks]

- (d) $\frac{t}{3.656...} = \frac{60}{48}$ OR $t = \frac{3.65684...}{48} (= 0.0761843...)$ (A1)
 4.57106...
 $t = 4.57 \text{ (min)}$ A1

[2 marks]

continued...

Question 7 continued

(e) attempt to use cosine rule to find distance BC **(M1)**

$$(3.656\dots)^2 + 8^2 - 2(3.656\dots)(8)\cos 31^\circ \quad (= 27.2200\dots)$$

$$BC = 5.21728\dots \quad \textbf{(A1)}$$

$$\frac{t}{5.21728\dots} = \frac{60}{55} \quad \text{OR} \quad t = \frac{5.21728\dots}{60} (= 0.094859\dots)$$

time for coast guard rescue boat = 5.69158...(min) OR 0.094859...(hour) **A1**

5.69158... > 4.57106....(min) OR 0.094859... > 0.076184...(hour)

the lighthouse rescue boat will reach the jet-ski first. **A1**

[4 marks]

Total [15 marks]

1. (a) substituting into distance between two points formula in 3D (M1)

$$\sqrt{(6-4)^2 + (-1-(-5))^2 + (-3-(-9))^2}$$

$$= 7.48331...$$

$$(\text{radius}) = 7.48 \left(= \sqrt{56} \text{ or } 2\sqrt{14} \text{ (exact)} \right) \quad \text{A1}$$

[2 marks]

- (b) valid approach to find surface area of solid (M1)

$$\frac{1}{2} \times 4\pi r^2 + \pi r^2$$

$$\frac{1}{2} \times 4\pi \times (7.48331...)^2 + \pi \times (7.48331...)^2 \quad \text{(A1)}$$

$$= 527.787...$$

$$(\text{surface area}) = 528 \quad (= 168\pi) \quad (\text{Accept } 527 \text{ from 3sf answer to (a)}) \quad \text{A1}$$

Note: Award (M1)A0A0 for $351.858... (= 112\pi)$ from only considering curved surface.

[3 marks]

Total [5 marks]

2. (a) substituting into cosine rule (M1)

$$(BC^2 =) 12^2 + 7^2 - 2 \times 12 \times 7 \times \cos(116^\circ) \quad (A1)$$

$$\sqrt{266.646...} = 16.3293...$$

$$(BC =) 16.3 \quad A1$$

[3 marks]

- (b) substituting into sine rule or cosine rule for angle $\hat{ACB}$ or angle $\hat{ABC}$ (M1)

$$\frac{\sin(\hat{ACB})}{7} = \frac{\sin(116^\circ)}{16.3293...} \quad \text{OR} \quad 7^2 = 12^2 + 16.3293...^2 - 2 \times 12 \times 16.3293... \times \cos(\hat{ACB})$$

$$\text{OR } 180^\circ - 116^\circ - 41.3^\circ \text{ from finding angle } \hat{ABC} \quad (A1)$$

$$22.6618...$$

$$(\hat{ACB} =) 22.7 (^\circ) \text{ (accept } 22.8(^\circ) \text{ if using previous 3sf AND the cosine rule)} \quad A1$$

[3 marks]

Total [6 marks]

5. Attempts to use the area formula for triangle AOB **AND** equate it to 26 **(M1)**

$$\frac{1}{2}a \times b \times \sin(2.51) = 26 \text{ OR } \frac{1}{2}OA \times OB \times \sin AOB = 26$$

$$\frac{1}{2} \times r^2 \times \sin(2.51) = 26 \text{ or equivalent} \quad \textbf{(A1)}$$

$$r = 9.38462... \quad \textbf{(A1)}$$

use of arc length formula to find the major arc ACB **(M1)**

$$9.38462... \times (2\pi - 2.51) \text{ OR } 2\pi \times 9.38462... - 9.38462... \times 2.51 \text{ OR } 9.38462... \times 3.773185...$$

$$35.4099...$$

$$(\text{arc length} =) 35.4 \text{ (cm)} \quad \textbf{A1}$$

[5 marks]

9. (a) attempt to substitute $\frac{\sin \theta}{\cos \theta}$ for $\tan \theta$ (in numerator or denominator) **(M1)**

uses $\cos^2 \theta + \sin^2 \theta = 1$ or $2 \sin \theta \cos \theta = \sin 2\theta$ in their expression (seen anywhere) **(M1)**

correct working leading to given answer **A1**

$\sin 2\theta$ **AG**

Note: Candidates perform substitutions in different orders. Use the above as a guide.

Award **(M1)(M1)A0AG** if a candidate substitutes $\frac{\sin \theta}{\cos \theta}$ and with no other interim working writes $2 \sin \theta \cos \theta$ leading to the **AG**.

[3 marks]

- (b) $\sin 2\theta = \frac{1}{2}$ **(A1)**

$(2\theta =) 30, 150$ (accept $\frac{\pi}{6}, \frac{5\pi}{6}$ and condone extra values) **(A1)(A1)**

attempt to halve their angle(s) **(M1)**

$\theta = 15^\circ, 75^\circ$ **A1**

Note: Award final **A0** for extra or missing values of θ .

Award **(A1)(A1)(A1)(M1)A0** for $\theta = \frac{\pi}{12}, \frac{5\pi}{12}$.

Award **(A1)(A1)A0(M1)A0** for only one of $\theta = 15^\circ$ OR $\theta = 75^\circ$ with no working.

[5 marks]

continued...

Question 9 continued.

$$(c) \frac{2 \tan\left(22\frac{1}{2}^\circ\right)}{1 + \tan^2\left(22\frac{1}{2}^\circ\right)} = \sin 45^\circ \quad (\text{A1})$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \text{ (seen anywhere)} \quad (\text{A1})$$

attempt to form an equation in terms of x or $\tan(22.5)$ **M1**

$$\frac{2x}{1+x^2} = \frac{1}{\sqrt{2}} \quad \left(= \frac{\sqrt{2}}{2} \right) \quad \text{OR} \quad 2\sqrt{2} \tan(22.5) = 1 + (\tan(22.5))^2$$

Note: Accept working in radians.

$$2\sqrt{2}x = 1 + x^2 \quad \text{OR} \quad \sqrt{2}x^2 - 4x + \sqrt{2} = 0 \quad \text{OR} \quad \tan^2(22.5) - 2\sqrt{2} \tan(22.5) + 1 = 0 \quad (\text{A1})$$

$$x^2 - 2\sqrt{2}x + 1 = 0 \quad (\text{AG})$$

Note: Substituting $\tan(22.5)$ directly into the **AG** is working backwards; award no marks.

Exception: If a candidate rearranges the **AG** to $\frac{\sqrt{2} \times 2x}{1+x^2} = 1$ and then substitutes $\sin 45^\circ$ for $\frac{2x}{1+x^2}$, award the first **A1**; then a maximum of **A1M0A0** may be awarded after that.

[4 marks]

(d) attempt to use the quadratic formula **(M1)**

$$x = \frac{2\sqrt{2} \pm \sqrt{8-4}}{2}$$

$$x = \sqrt{2} \pm 1 \quad (\text{A1})$$

$$\tan\left(22\frac{1}{2}^\circ\right) = \sqrt{2} - 1, \text{ (since } \tan\left(22\frac{1}{2}^\circ\right) < 1) \quad (\text{A1})$$

[3 marks]

Total [15 marks]

7. (a) **METHOD 1** (cosine rule)

attempt to use cosine rule

(M1)

$$PQ^2 = 2^2 + 3^2 - 2(2)(3)\cos 60^\circ \quad \text{OR} \quad \cos 60^\circ = \frac{2^2 + 3^2 - PQ^2}{2(2)(3)} \quad \textbf{(A1)}$$

$$PQ^2 = 7$$

$$PQ = \sqrt{7}$$

A1**METHOD 2** (drawing altitude from Q perpendicular to PR to create two triangles)attempt to find length of altitude segment ($=\sqrt{3}$)**(M1)**

$$PQ^2 = 2^2 + (\sqrt{3})^2 \quad \textbf{(A1)}$$

$$= 7$$

$$PQ = \sqrt{7}$$

A1**[3 marks]**

(b) attempt to use cosine rule

(M1)

$$AB^2 = (x+2)^2 + (3x-1)^2 - 2(x+2)(3x-1)\cos 60^\circ \quad \text{OR} \quad \cos 60^\circ = \frac{(x+2)^2 + (3x-1)^2 - AB^2}{2(x+2)(3x-1)} \quad \textbf{(A1)}$$

$$(AB^2 =) (x^2 + 4x + 4) + (9x^2 - 6x + 1) - (3x^2 + 5x - 2) \quad \textbf{A1A1}$$

Note: Correct expansions may be seen anywhere in the candidate's working.Award **A1** for $(x^2 + 4x + 4)$ AND $(9x^2 - 6x + 1)$ Award **A1** for $-2(x+2)(3x-1)\left(\frac{1}{2}\right)$ OR $-(3x^2 + 5x - 2)$.

$$(AB^2 =) 7x^2 - 7x + 7$$

A1

$$(AB =) \sqrt{7x^2 - 7x + 7}$$

AG**[5 marks]***continued...*

Question 7 continued

(c) (i) $\sqrt{7x^2 - 7x + 7} = 7$

$7x^2 - 7x + 7 = 49$ OR $x^2 - x - 6 = 0$ (or equivalent) **(A1)**

valid attempt to solve their quadratic **(M1)**

$x = 3$ **A1**

Note: Award **A0** if $x = -2$ is included in the final answer.

(ii) attempt to use $A = \frac{1}{2}(x+2)(3x-1)\sin 60$ with their value of x **(M1)**

$\frac{1}{2} \cdot 5 \cdot 8 \cdot \sin 60$ OR $\frac{1}{2} \cdot 5 \cdot 8 \cdot \frac{\sqrt{3}}{2}$ (or equivalent) **(A1)**

$= 10\sqrt{3}$ **A1**

Note: Do not award FT for $x \leq \frac{1}{3}$.**[6 marks]****Total [14 marks]**

1. (a) (i) 1.25663...

$$\theta = 1.26 \left(= \frac{2\pi}{5} \right) \quad \text{A1}$$

$$(ii) \quad \frac{2\pi}{5} \times r = 12 \text{ or equivalent} \quad \text{(A1)}$$

9.54929...

$$r = 9.55 \text{ (cm)} \left(= \frac{60}{2\pi} = \frac{30}{\pi} \right) \quad \text{A1}$$

[3 marks](b) **METHOD 1**attempt to find area of at least one shaded segment (M1)

$$\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta \quad \text{OR} \quad \frac{1}{2}r^2(\theta - \sin\theta)$$

$$\frac{1}{2}(9.54929\dots)^2 \times 1.25663\dots - \frac{1}{2}(9.54929\dots)^2 \sin 1.25663\dots \text{ (or equivalent)} \quad \text{(A1)}$$

$$\text{shaded area} = 5 \times (57.2957\dots - 43.3629\dots) (= 5 \times 13.9328\dots)$$

69.6640...

$$= 69.7 \text{ (cm}^2\text{)} \quad \text{A1}$$

[3 marks]**METHOD 2**attempt to find area of one triangle (M1)

$$\frac{1}{2} \times (9.54929\dots)^2 \times \sin 1.25663\dots$$

attempt to subtract five times their triangle area from their circle area (M1)

$$\pi \times (9.54929\dots)^2 - 5 \times \frac{1}{2} \times (9.54929\dots)^2 \times \sin 1.25663\dots$$

69.6640...

$$= 69.7 \text{ (cm}^2\text{)} \quad \text{A1}$$

[3 marks]**Total [6 marks]**

5. (a) (i) $a = -5$ **A1**

(ii) **EITHER**
period = 32 **(A1)**

OR
correct substitution of a point on the graph of g and their a **(A1)**

$$2 = -5 \cos(b \times 16) - 3 \quad \text{OR} \quad -3 = -5 \cos(b \times 8) - 3$$

Note: Award **A0** if candidate substitutes $x = 0, y = -8$.

THEN

$$0.196349\dots$$

$$b = 0.196 \left(= \frac{2\pi}{32} = \frac{\pi}{16} \right) \quad \text{A1}$$

[3 marks]

(b) $k = 32$ **A1**

[1 mark]

(c) (i) **EITHER**
attempt to use a horizontal translation **(M1)**

$$24 - q = 16 \quad \text{OR} \quad 16 - q = 8$$

OR

substituting a point on the graph of g into $f(x) = p \sin(b(x - q)) - 3$ **(M1)**

$$-8 = p \sin\left(\frac{\pi}{16}(0 - q)\right) - 3 \quad \text{OR} \quad -3 = p \sin\left(\frac{\pi}{16}(8 - q)\right) - 3 \quad \text{OR} \quad 2 = p \sin\left(\frac{\pi}{16}(16 - q)\right) - 3$$

THEN

$$q = 8 \quad \text{A1}$$

(ii) $p = 5$ **A1**

[3 marks]

Total [7 marks]

2. (a) 34°

A1
[1 mark]

(b) $\hat{LBP} = 20^\circ$ (seen anywhere)

(A1)

attempt to use the sine rule

(M1)

$$\frac{LB}{\sin 34^\circ} = \frac{5}{\sin 20^\circ}$$

(A1)

$$LB = 8.17485\dots$$

$$LB = 8.17 \text{ (km)}$$

A1
[4 marks]

Total [5 marks]

3. (a) attempt to use double angle formula for $\cos 2x$ (seen anywhere)

M1

Note: Only award the **M1** if a double angle formula for $\cos 2x$ is used, and not only for stating it.

$$3(1 - 2\sin^2 x) = 3 - 6\sin^2 x \text{ (seen anywhere)}$$

A1

$$3\cos 2x + 11\sin x = 3 + 11\sin x - 6\sin^2 x$$

AG**[2 marks]**

- (b) **METHOD 1**

attempt to solve a quadratic equation in $\sin x$

(M1)

$$\sin x = 0.333333... \left(= \frac{1}{3} \right)$$

$$x = 19.4712..., 160.528...$$

$$x = 19.5, 161$$

A1A1**METHOD 2**

attempt to solve $3\cos 2x + 11\sin x - 6 = 0$ (may be seen in a sketch)

(M1)

$$x = 19.4712..., 160.528...$$

$$x = 19.5, 161$$

A1A1**[3 marks]****Total [5 marks]**

5. (a) perimeter of sector is $1.5 \times$ circumference of circle (seen anywhere) **A1**
 $1.5 \times 16\pi$ OR 24π OR 75.3982...

perimeter of sector $r\theta + 2r$ **(M1)**

$$24\pi = r\theta + 2r \quad \mathbf{A1}$$

$$r = \frac{24\pi}{2 + \theta} \quad \mathbf{AG}$$

[3 marks]

- (b) correct substitution of $r = \frac{24\pi}{2 + \theta}$ into $\frac{1}{2}r^2\theta$ **(A1)**

$$64\pi = \frac{1}{2} \times \theta \times \left(\frac{24\pi}{2 + \theta} \right)^2$$

attempt to solve their equation (must be quadratic) **(M1)**

$$\theta = 0.411273\dots$$

$$\theta = 0.411 \quad \mathbf{A1}$$

Note: Award **(A1)(M1)A0** if additional answers are given.

[3 marks]

Total [6 marks]

8. (a) (i) attempt to use volume of cone or sphere or hemisphere (M1)

$$\frac{1}{3}\pi \times 3^2 \times 8 = 75.3982... (24\pi) \text{ OR } \frac{4}{3}\pi \times 3^3 = 113.097... (36\pi) \text{ OR}$$

$$\frac{2}{3}\pi \times 3^3 = 56.5486... (18\pi)$$

recognize that total volume is half the volume of a sphere plus the volume of a cone (M1)

$$\frac{2}{3}\pi \times r^3 + \frac{1}{3}\pi \times r^2 h \text{ OR } V = 56.5486... + 75.3982... (18\pi + 24\pi)$$

$$V = 131.946... \text{ OR } V = 42\pi \quad \text{A1}$$

$$V = 132 \text{ (cm}^3\text{)} \quad \text{AG}$$

- (ii) EITHER

attempt to use Pythagoras to find the slant height. (M1)

$$l^2 = 3^2 + 8^2 \text{ OR } l = \sqrt{3^2 + 8^2}$$

OR

attempt to use trigonometry to find the angle **and** the slant height (M1)

$$\arctan \frac{8}{3} = 1.21202... \text{ rad or } 69.4439...^\circ \text{ and } l = \frac{8}{\sin 69.4439...} \text{ or } \frac{3}{\cos 69.4439...}$$

THEN

$$l = 8.54400... (\sqrt{73}) \quad \text{(A1)}$$

attempt to use formula for surface area with their l (M1)

$$S = \pi \times 3 \times 8.54400...$$

Note: Do not award the (M1) if they use $l = 8$.

$$S = 80.5 (80.5253... = 3\pi\sqrt{73}) (\text{cm}^2) \quad \text{A1}$$

[7 marks]

continued...

Question 8 continued

- (b) recognize that volume of cone + volume of hemisphere = 120 **(M1)**

$$\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = 120 \quad \text{A1}$$

$$h = \left(120 - \frac{2}{3}\pi r^3\right) \div \left(\frac{1}{3}\pi r^2\right) \quad \text{OR} \quad h = (360 - 2\pi r^3) \div (\pi r^2) \quad \text{OR} \quad \pi r^2 h = 360 - 2\pi r^3 \quad \text{A1}$$

$$h = \frac{360 - 2\pi r^3}{\pi r^2} \quad \text{AG}$$

[3 marks]

- (c) attempt to use Pythagoras to find l in terms of r and h **(M1)**

$$l = \sqrt{r^2 + h^2}$$

$$l = \sqrt{r^2 + \left(\frac{360 - 2\pi r^3}{\pi r^2}\right)^2} \quad \text{A1}$$

$$S = \pi r \sqrt{r^2 + \left(\frac{360 - 2\pi r^3}{\pi r^2}\right)^2} \quad \text{AG}$$

[2 marks]

- (d) attempt to find the minimum value of S **(M1)**

$$S = 44.4045\dots, \quad r = 3.67084\dots, \quad \text{sketch}$$

$$S = 44.4 \text{ (cm}^2\text{)}, \quad r = 3.67 \text{ (cm)} \quad \text{A1A1}$$

[3 marks]**Total [15 marks]**

