APOLLO

# Mathematics: applications and interpretation
# SL Answers

First assessment 2021
Whole questions arranged by syllabus topic.
Original question and scheme crops preserve source diagrams.

APOLLO

# Number and algebra

52 questions

**1.** (a) $\pi \approx 3 + \dfrac{1}{6 + \dfrac{13}{16}}$

$= 3.14678... \left( \dfrac{343}{109}, 3\frac{16}{109} \right)$ ***(A1)***

$= 3.1468$ ***A1***

**Note:** Award ***A1*** for correct rounding to 4 decimal places. Follow through within this part.

***[2 marks]***

(b) $\left| \dfrac{3.1468 - \pi}{\pi} \right| \times 100$ ***(M1)***

**Note:** Award ***M1*** for substitution of their final answer in part (a) into the percentage error formula. Candidates should use the exact value of $\pi$ from their GDC.

$= 0.166(\%)$ $(0.165754...)$ ***A1***

***[2 marks]***

***Total [4 marks]***

**8.** (a) (i) attempt to find $u_{20}$ using an arithmetic sequence ***(M1)***

e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700, \ldots$

(Charlie ran) $2400 \text{ m}$ ***A1***

(ii) $(r =)\ 1.02$ ***(A1)***

attempt to find $u_{20}$ using a geometric sequence ***(M1)***

e.g. identifying $u_1 = 500$ and a value for $r$ **OR** $500 \times r^{19}$ **OR** $500,\ 510,\ 520.2, \ldots$

(Daniella ran) $728 \text{ m}$ $(728.405...)$ ***A1***

***[5 marks]***

(b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***

$n > 184.215...$

$n = 185$ ***A1***

***[3 marks]***

***Total [8 marks]***

**10.** (a) **METHOD 1**

| | | |
|---|---|---|
| $N = 5$ | **OR** | $N = 10$ |
| $I\% = 2.75$ | | $I\% = 2.75$ |
| $PV = -1500$ | | $PV = -1500$ |
| $PMT = 0$ | | $PMT = 0$ |
| $P/Y = 1$ | | $P/Y = 2$ |
| $C/Y = 2$ | | $C/Y = 2$ |

***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, ***A1*** for all entries correct.

**METHOD 2**

$1500\left(1+\frac{2.75}{2\times 100}\right)^{2\times 5}$ ***(M1)(A1)***

$1719.49$ euro ***A1***

***[3 marks]***

(b) **METHOD 1**

| | | |
|---|---|---|
| $N = 5$ | **OR** | $N = 20$ |
| $PV = \pm 1500$ | | $PV = \pm 1500$ |
| $FV = \mp 2250$ | | $FV = \mp 2250$ |
| $PMT = 0$ | | $PMT = 0$ |
| $P/Y = 1$ | | $P/Y = 4$ |
| $C/Y = 4$ | | $C/Y = 4$ |

***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, ***A1*** for all entries correct. $PV$ and $FV$ must have opposite signs.

**METHOD 2**

$1500\left(1+\frac{r}{4\times 100}\right)^{4\times 5} = 2250$ **OR** $\left(1+\frac{r}{4\times 100}\right)^{4\times 5} = 1.5$ ***(M1)(A1)***

**Note:** Award ***M1*** for substitution in compound interest formula, ***A1*** for correct substitution and for equating to $2250$ (if using LHS equation) or to $1.5$ (if using RHS equation).

$r = 8.19$ $(8.19206...)$ ***A1***

**Note:** Accept $r = 8.19\%$.
Accept a trial and error method which leads to $r = 8.19$.

***[3 marks]***

***Total [6 marks]***

**3.** (a) **EITHER**

$N = 2$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

**OR**

$N = 8$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 4$
$C/Y = 4$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

**OR**

$$FV = 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4\times 2}$$ ***(M1)(A1)***

**Note:** Award ***M1*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

$= 42\,010 \text{ AUD}$ ***A1***

**Note:** Award ***(M1)(A1)A0*** for unsupported $42009.87$.

***[3 marks]***

(b) **EITHER**

$PV = -37\,000$
$FV = 50\,000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. The final mark can still be awarded for the correct number of months (multiple of $3$).

*continued…*

*Question 3 continued*

**OR**

$PV = -37\,000$
$FV = 50\,000$
$I\% = 6.4$
$P/Y = 4$
$C/Y = 4$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

**OR**

$50000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4 \times n}$ **OR** $50\,000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{n}$ ***(M1)(A1)***

**Note:** Award ***M1*** for the correct inequality, $50\,000$ and substituted compound interest formula. Allow an equation. Award ***A1*** for correct substitution.

**THEN**

$N = 4.74$ (years) $(4.74230...)$ **OR** $N = 18.9692...$ (quarters) ***(A1)***

$m = 57$ months ***A1***

**Note:** Award ***A1*** for rounding their $m$ to the correct number of months. The final answer must be a multiple of $3$. Follow through within this part.

***[4 marks]***

(c) $150\,000$ AUD ***A1***

***[1 mark]***

*continued…*

*Question 3 continued*

(d) (i) $120 \times 1700 - 150\,000$ ***(M1)***

$= 54\,000$ AUD ***A1***

(ii) $N = 120$
$PV = -150\,000$
$PMT = 1700$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula or $FV = 0$ seen. If a compound interest formula is equated to zero, award ***M1***, otherwise award ***M0*** for a substituted compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in annuity formula, but award ***A0*** for a substituted compound interest formula.
Follow through marks in part (d)(ii) are contingent on working seen.

$r = 6.46$ (%) $(6.45779...)$ ***A1***

***[5 marks]***

(e) $N = 60$
$I = 6.46$ $(6.45779...)$
$PV = -150\,000$
$PMT = 1700$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula. Award ***(M0)*** for a substituted compound interest formula. Award ***A1*** for all entries correct. Follow through marks in part (e) are contingent on working seen.

$FV = 86\,973$ AUD ***A1***

***[3 marks]***

(f) $204\,000 - (60 \times 1700 + 86\,973)$ **OR** $204\,000 - 188\,973$ ***(M1)(M1)***

**Note:** Award ***M1*** for $60 \times 1700$. Award ***M1*** for subtracting their $(60 \times 1700 + 86\,973)$ from their $(204\,000)$. Award at most ***M1M0*** for their $204\,000 - (60 \times 1700)$ or ***M0M0*** for their $204\,000 - (86\,973)$. Follow through from parts (d)(i) and (e).
Follow through marks in part (f) are contingent on working seen.

$15\,027$ AUD ***A1***

***[3 marks]***

***Total [19 marks]***

**3.** (a) (i) recognition of arithmetic sequence with common difference $2$ ***(M1)***

use of arithmetic sequence formula ***(M1)***

$14+2(20-1)$

$52$ ***A1***

(ii) use of arithmetic series formula ***(M1)***

$\frac{14+52}{2}\times 20$

$660$ ***A1***

***[5 marks]***

(b) $584+(584\times 0.012)$ **OR** $584\times(1.012)^1$ ***(M1)***

$591\ (591.008)$ ***A1***

**Note:** Award ***M0A0*** if incorrect $r$ used in part (b), and ***FT*** with their $r$ in parts (c) and (d).

***[2 marks]***

(c) recognition of geometric sequence ***(M1)***

equating their $n$th geometric sequence term to their $660$ ***(M1)***

**Note:** Accept inequality.

**METHOD 1**

**EITHER**

$660=584\times(1.012)^{x-1}$ ***A1***

$(x-1=)\ 10.3\ (10.2559\ldots)$

$x=11.3\ (11.2559\ldots)$ ***A1***

$2030$ ***A1***

**OR**

$660=584\times(1.012)^{x}$ ***A1***

$x=10.3\ (10.2559\ldots)$ ***A1***

$2030$ ***A1***

*continued…*

*Question 3 continued*

**METHOD 2**

$11^{th}$ term $658$ $(657.987...)$ ***(M1)A1***
$12^{th}$ term $666$ $(665.883...)$ ***(M1)A1***
$2030$ ***A1***

**Note:** The last mark can be awarded if both their $11^{th}$ and $12^{th}$ correct terms are seen.

***[5 marks]***

(d) $7$ seen ***(A1)***

**EITHER**

$584\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)***

multiplying their sum by $50$ ***(M1)***

**OR**

sum of the number of visitors for their $r$ and their seven years ***(M1)***
multiplying their sum by $50$ ***(M1)***

**OR**

$29\,200\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)(M1)***

**THEN**

$212000\ \ (211907.3\ldots)$ ***A1***

**Note:** Follow though from their $r$ from part (b).

***[4 marks]***

***Total [16 marks]***

**10.** (a) $N = 360$
$I\% = 3.8$
$PV = (\pm)170\,000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct. Accept a positive or negative value for $PV$.

$(PMT =)\ 792.13\ \ \text{AUD}$ ***A1***

**Note:** Accept an answer of $-792.13$. Do not award final ***A1*** if answer is not given correct to $2$ dp

***[3 marks]***

(b) (i) $N = 120$
$I\% = 3.8$
$PV = (\pm)170\,000$
$PMT = (\mp)792.13$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct. $PV$ and $PMT$ must have opposite signs.

$(FV =)\ 133019.94\ \ \text{AUD}$ ***A1***

**Note:** Do not award final ***A1*** if answer is not given correct to $2$ dp, unless already penalized in part (a). Accept $133020.30$ from use of exact value for PMT.

(ii) amount of money paid: $120 \times 792.13\ (= 95055.60)$ ***(M1)***
loan paid off: $170\,000 - 133019.94\ \ (= 36980.06)$ ***(M1)***
interest paid: $(95\,055.60 - 36\,980.06 =)\ 58\,075.54\ \ \text{AUD}$ ***A1***

**Note:** Allow $58075.60$ or $58075.90$ from use of some exact values from parts (a) and (b)(i). If their answer to part (b)(i) is greater than $170\,000$ then award at most ***(M1)(M1)(A0)*** for follow through in part (b)(ii).

***[6 marks]***
***Total: [9 marks]***

**2.** (a) $\frac{12669-12300}{12300}\times 100$ ***(M1)***

$3\%$ ***A1***

***[2 marks]***

(b) (i) $1.03$ ***A1***

**Note:** Follow through from part (a).

(ii) $(u_n =)\ 12300\times 1.03^{n-1}$ ***A1***

(iii) $(u_{11} =)\ 12300\times 1.03^{10}$ ***(M1)***

$16530$ ***A1***

**Note:** Answer must be to the nearest integer. Do not accept $16500$.

***[4 marks]***

(c) $(v_n =)\ 10380+600(n-1)$ **OR** $600n+9780$ ***M1A1***

**Note:** Award ***M1*** for substituting into arithmetic sequence formula, ***A1*** for correct substitution.

***[2 marks]***

(d) $80\times\frac{10}{2}\left(2(10380)+9(600)\right)$ ***(M1)(M1)***

**Note:** Award ***(M1)*** for multiplying by $80$ and ***(M1)*** for substitution into sum of arithmetic sequence formula.

$\$10\,500\,000$ $(\$10\,464\,000)$ ***A1***

***[3 marks]***

*continued…*

*Question 2 continued*

(e) $12300\times1.03^{n-1} < 10380 + 600(n-1)$ or equivalent ***(M1)***

**Note:** Award ***M1*** for equating their expressions from parts (b) and (c).

**EITHER**

graph showing $y = 12300\times1.03^{n-1}$ and $y = 10380 + 600(n-1)$ ***(M1)***

**OR**

graph showing $y = 12300\times1.03^{n-1} - \left(10380 + 600(n-1)\right)$ ***(M1)***

**OR**

list of values including, $(u_n =)\ 17537$ and $(v_n =)\ 17580$ ***(M1)***

**OR**

$12.4953...$ from graphical method or solving numerical equality ***(M1)***

**Note:** Award ***(M1)*** for a valid attempt to solve.

**THEN**

$(k =)\ 13$ ***A1***

***[3 marks]***

(f) this will not guarantee enough places. ***A1***

**EITHER**

A written statement that $u_n > v_n$, with range of $n$. ***R1***

*Example:* "when $n = 24$ (or greater), the number of applications will exceed the number of places again" ("$u_n > v_n$, $n \geq 24$").

**OR**

exponential growth will always exceed linear growth ***R1***

**Note:** Accept an equivalent sketch. Do not award ***A1R0***.

***[2 marks]***

***Total: [16 marks]***

**2.** (a) $(d =) -250$ ***A1***

***[1 mark]***

(b) $(u_{16} =) 6800 + (16 - 1)(-250)$ ***M1***

$(¥) 3050$ ***A1***

***[2 marks]***

(c) $(S_{16} =) \left(\frac{16}{2}\right)(2 \times 6800 + (16 - 1)(-250)) \times 2$ ***M1M1***

**Note:** Award ***M1*** for correct substitution into arithmetic series formula.
Award ***M1*** for multiplication by $2$ seen.

***OR***

$(S_{16} =) \left(\frac{16}{2}\right)(6800 + 3050) \times 2$ ***M1M1***

**Note:** Award ***M1*** for correct substitution into arithmetic series formula.
Award ***M1*** for multiplication by $2$ seen.

$(¥) 158\,000\ (157\,600)$ ***A1***

***[3 marks]***

***Total [6 marks]***

**1.** (a) (i) $N = 24$

$I\% = 14$

$PV = -14000$

$FV = 0$

$P/Y = 4$

$C/Y = 4$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $PV = 14000$.

$(€)871.82$ ***A1***

(ii) $4 \times 6 \times 871.82$ ***(M1)***

$(€)20923.68$ ***A1***

(iii) $20923.68 - 14000$ ***(M1)***

$(€)6923.68$ ***A1***

***[7 marks]***

(b) (i) $0.9 \times 14000$ $(= 14000 - 0.10 \times 14000)$ ***M1***

$(€)12600.00$ ***A1***

(ii) $N = 72$

$PV = 12600$

$PMT = -250$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $PV = -12600$ provided $PMT = 250$.

$12.56(\%)$ ***A1***

***[5 marks]***

*continued…*

*Question 1 continued*

(c) ***EITHER***

Bryan should choose Option A ***A1***
no deposit is required ***R1***

**Note:** Award ***R1*** for stating that no deposit is required. Award ***A1*** for the correct choice from that fact. Do not award ***R0A1.***

***OR***

Bryan should choose Option B ***A1***
cost of Option A $(6923.69) >$ cost of Option B $(72 \times 250 - 12\,600 = 5400)$ ***R1***

**Note:** Award ***R1*** for a correct comparison of costs. Award ***A1*** for the correct choice from that comparison. Do not award ***R0A1***.

***[2 marks]***

(d) $14\,000\left(1-\frac{25}{100}\right)^{6}$ ***(M1)(A1)***

**Note:** Award ***M1*** for substitution into compound interest formula. Award ***A1*** for correct substitutions.

$= 2491.70 \text{ (USD)}$ ***A1***

***OR***

$N = 6$

$I\% = -25$

$PV = \pm 14\,000$

$P/Y = 1$

$C/Y = 1$ ***(A1)(M1)***

**Note:** Award ***A1*** for $PV = \pm 14\,000$, ***M1*** for other entries correct.

$2491.70 \text{ (USD)}$ ***A1***

***[3 marks]***

***Total [17 marks]***

**5.** (a) $x + y + z = 600$ ***A1***

$15x + 10y + 12z = 7816$ ***A1***

$x = 2y$ ***A1***

**Note:** Condone other labelling if clear, e.g. $a$ (adult), $c$ (child) and $s$ (student). Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

***[3 marks]***

(b) $x = 308,\ y = 154,\ z = 138$ ***A1A1***

**Note:** Award ***A1*** for all three correct values seen, ***A1*** for correctly labelled as $x$, $y$ or $z$. Accept answers written in words: e.g. $308$ adult tickets.

***[2 marks]***

***[Total 5 marks]***

**13.** (a) use of geometric sequence with $r = 0.85$ ***M1***

**EITHER**

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68 \text{ m}$

$= 68 \text{ cm}$ ***AG***

**OR**

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68 \text{ cm}$ ***AG***

***[2 marks]***

*continued…*

*Question 13 continued*

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

**Note:** If $1.8$ m (or $180$ cm) is used then ***(M1)*** only awarded for use of $n$ in $(0.85)^n(1.8) > 0.1$.
If $1.53$ m (or $153$ cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

$17$ ***A1***

**OR**

$(0.85)^{17}(1.8) = 0.114$ m and $(0.85)^{18}(1.8) = 0.0966$ m ***(M1)***

$17$ ***A1***

**OR**

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

$17$ ***A1***

**Note:** Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

***[2 marks]***

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \dfrac{(1.8 \times 0.85)(1 - 0.85^3)}{1 - 0.85}$ $(= 3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(3.935925)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

**OR**

distance (in one direction) travelled between drop and fourth bounce

$= \dfrac{(1.8)(1 - 0.85^4)}{1 - 0.85}$ $(= 5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

**OR**

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8)$ $(= 3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

**Note:** Answers may be given in cm.

***[3 marks]***
***[Total 7 marks]***

**13.** (a) **METHOD 1** – *(with FV = 4000)*

**EITHER**

N= 10
I=1.5
FV= 4000
P/Y= 1
C/Y= 1 ***(A1)(M1)***

**Note:** Award ***A1*** for $(3.5-2=)\ 1.5$ seen and ***M1*** for all other entries correct.

**OR**

$4000 = A(1+0.015)^{10}$ ***(A1)(M1)***

**Note:** Award ***A1*** for $1.5$ or $0.015$ seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to $4000$.

**THEN**

(PV =) $3447 ***A1***

**Note:** Award ***A0*** if not rounded to a whole number or a negative sign given.

*continued…*

*Question 13 continued*

**METHOD 2** – *(With FV including inflation)*
calculate FV with inflation
$4000 \times 1.02^{10}$ ***(A1)***
(=4875.977…)

**EITHER**
$4000 \times 1.02^{10} = \text{PV} \times 1.035^{10}$ ***(A1)***
**OR**
N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

**Note:** Award ***M1*** for *their* FV and all other entries correct.

**THEN**
(PV =) $3457 ***A1***

**Note:** Award ***A0*** if not rounded to a whole number or a negative sign given.

**METHOD 3** – *(Using formula to calculate real rate of return)*
(real rate of return =) 1.47058…(%) ***(A1)***

**EITHER**
$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***
**OR**
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

**Note:** Award ***M1*** for all entries correct.

**THEN**
(PV =) $3457 ***A1***

***[3 marks]***

*continued…*

*Question 13 continued*

(b) **METHOD 1** – *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

**Note:** Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) \$4861.87 **OR** \$4875.97 ***(A1)***
so payment required (from TVM) will be \$294 **OR** \$295 ***A1***

**Note:** Award ***A0*** if a negative sign given, unless already penalized in part (a).

**METHOD 2** – *(Using FV)*
N= 10
I=3.5
PV= −1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

**Note:** Award ***A1*** for I=3.5 **and** FV= ±4875.977…, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) \$295 (295.393) ***A1***

**Note:** Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

***[3 marks]***
***Total [6 marks]***

**2.** (a) (i) $30$ ***A1***

(ii) $40$ ***A1***

***[2 marks]***

(b) arithmetic formula chosen ***(M1)***

(i) $w_n = 20 + (n-1)10 \quad (= 10 + 10n)$ ***A1***

(ii) $l_n = 30 + (n-1)10 \quad (= 20 + 10n)$ ***A1***

***[3 marks]***

(c) (i) $740 = 30 + (n-1)10$ **OR** $740 = 20 + 10n$ ***M1***

$n = 72$ ***A1***

144 tiles ***AG***

**Note:** The ***AG*** line must be stated for the final ***A1*** to be awarded.

(ii) $w_{72} = 730$ ***A1***

***[3 marks]***

(d) $(10 \times 20) \times 144$ ***(M1)***

$= 28800$ ***(A1)***

$2.88 \times 10^4 \quad \text{cm}^2$ ***A1***

**Note:** Follow through within the question for correctly converting *their* intermediate value into standard form (but only if the pre-conversion value is seen).

***[3 marks]***

*continued…*

*Question 2 continued*

(e) **EITHER**

$1$ square metre $= 100\,\text{cm} \times 100\,\text{cm}$ ***(M1)***

(so, $50$ tiles) and hence $10$ packs of tiles in a square metre ***(A1)***

(so each pack is $\frac{\$24.50}{10 \text{ packs}}$)

**OR**

area covered by one pack of tiles is $(0.2\,\text{m} \times 0.1\,\text{m} \times 5 =)\ 0.1\ \text{m}^2$ ***(A1)***

$24.5 \times 0.1$ ***(M1)***

**THEN**

$\$2.45$ per pack (of $5$ tiles) ***A1***

***[3 marks]***

(f) $\frac{1.08 \times 144}{5}\ (= 31.104)$ ***(M1)(M1)***

**Note:** Award ***M1*** for correct numerator, ***M1*** for correct denominator.

$32$ (packs of tiles) ***A1***

***[3 marks]***

(g) $35 + (32 \times 2.45)$ ***(M1)***

$\$113\ \ (113.4)$ ***A1***

***[2 marks]***

***[Total 19 marks]***

**2.** (a) (i) **EITHER**

$115.5 = u_1 + (3-1)\times d \quad (115.5 = u_1 + 2d)$

$108 = u_1 + (8-1)\times d \quad (108 = u_1 + 7d)$ ***(M1)(A1)***

**Note:** Award ***M1*** for attempting to use the arithmetic sequence term formula, ***A1*** for both equations correct. Working for ***M1*** and ***A1*** can be found in parts (i) or (ii).

$(d = -1.5)$

$1.5$ (cups/day) ***A1***

**Note:** Answer must be written as a positive value to award ***A1***.

**OR**

$(d =) \ \dfrac{115.5-108}{5}$ ***(M1)(A1)***

**Note:** Award ***M1*** for attempting a calculation using the difference between term $3$ and term $8$; ***A1*** for a correct substitution.

$(d =) \ 1.5$ (cups/day) ***A1***

(ii) $(u_1 =) \ 118.5$ (cups) ***A1***

***[4 marks]***

(b) attempting to substitute their values into the term formula for arithmetic sequence equated to zero ***(M1)***

$0 = 118.5 + (n-1)\times(-1.5)$

$(n =) \ 80$ days ***A1***

**Note:** Follow through from part (a) only if their answer is positive.

***[2 marks]***

(c) $(t_5 =) \ 625\times 1.064^{(5-1)}$ ***(M1)(A1)***

**Note:** Award ***M1*** for attempting to use the geometric sequence term formula; ***A1*** for a correct substitution

$\$ 801$ ***A1***

**Note:** The answer must be rounded to a whole number to award the final ***A1***.

***[3 marks]***

*continued…*

*Question 2 continued*

(d) (i) $(S_{10} =)$ (\$) 8390 (8394.39...) ***A1***

(ii) **EITHER**

the total cost (of dog food) ***R1***
for 10 years beginning in 2021 **OR** 10 years before 2031 ***R1***

**OR**

the total cost (of dog food) ***R1***
from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 ***R1***

***[3 marks]***

(e) **EITHER**
According to the model, the cost of dog food per year will eventually be too high to keep a dog.
**OR**
The model does not necessarily consider changes in inflation rate.
**OR**
The model is appropriate as long as inflation increases at a similar rate.
**OR**
The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.
**OR**
The model is appropriate since dog food bags can only be bought in discrete quantities. ***R1***

**Note:** Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either "model" is mentioned specifically, or other mathematical terms such as "increasing" or "discrete quantities" are seen. Do not accept a contextual argument in isolation, e.g. "The dog will eventually die".

***[1 mark]***
***Total [13 marks]***

**2.** (a) use of the $n^{\text{th}}$ term of an arithmetic sequence formula ***(M1)***

$u_{15} = 85 + (15-1)\times 30$ ***(A1)***

$505$ ***A1***

***[3 marks]***

(b) use of the sum of $n$ terms of an arithmetic sequence formula ***(M1)***

$S_{15} = \frac{15}{2}(85+505)$ **OR** $\frac{15}{2}\left(2\times 85+(15-1)\times 30\right)$

$4430\ \ (4425)$ ***A1***

***[2 marks]***

(c) $\frac{4425}{15}$ **OR** $85+(8-1)\times 30$ ***(M1)***

$295$ ***A1***

**Note:** Accept $295.333\ldots$ from use of 3sf value from part (b).

***[2 marks]***

***Total [7 marks]***

**7.** (a) $I\% = 7.5$
$PV = \mp 800$
$PMT = \mp 500$
$FV = \pm 10\,000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award ***A1*** for $\text{PMT} = -500$ or $\text{PMT} = 500$, with same sign to PV and opposite sign to FV.

$17.3070\ldots$ ***(A1)***
$(k =)\ 18$ ***A1***

**Note:** Award ***(M0)(A0)(A0)A0*** for a final answer of $17$ with no working. The final answer must be an integer.

***[4 marks]***

(b) $10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$ ***(A1)(M1)***

**Note:** Award ***(A1)*** for $10389$ $(10389.38\ldots)$ seen. Award ***(M1)*** for subtraction of their $(18 \times 500 + 800)$ from FV. ***FT*** from their value of $k$. Award ***A0M1A0*** for $10\,000 - (18 \times 500 + 800)$. Do not award the final ***A1FT*** if their answer is negative.

589 EUR ***A1***

**Note:** Final answer must be to the nearest euro.

***[3 marks]***
***Total [7 marks]***

**10.** (a) $m = 1 - 2.5\log_{10}(0.0525)$ ***(M1)***

$= 4.20 \ (4.19960\ldots)$ ***A1***

***[2 marks]***

(b) attempt to solve $7 = 1 - 2.5\log_{10}(b)$ ***(M1)***

**Note:** Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5\log_{10}(b)$.

$b = 0.00398 \ (0.00398107...)$ ***A1***

***[2 marks]***

(c) $\frac{0.0525}{0.00398107}$ ***(M1)***

$= 13.2 \ (13.1874\ldots)$ ***A1***

***[2 marks]***

***Total [6 marks]***

**11.** (a) $4.5 = 2(r)^{3-1}$ ***M1***

$r = \pm 1.5$, ***R1***

(Some $x$-values are negative or direction from house changes each day)

$r = -1.5$ ***AG***

**Note:** Award ***M0R0AG*** for a verification approach $4.5 = 2(-1.5)^{3-1}$.

***[2 marks]***

(b) $2(-1.5)^{6-1}$ ***(M1)***

**EITHER**

$(-15.2, 0)$ $(-15.1875..., 0)$ ***A1***

**OR**

$x = -15.2$ km ***A1***

**OR**

15.2 km west (of the origin) ***A1***

**Note:** Award ***(M1)A0*** for an answer of "$-15.2$ (km)" without indicating that it is the $x$-value.

***[2 marks]***

(c) choosing $r = 1.5$ ***(A1)***

$\frac{2\left((1.5)^7 - 1\right)}{1.5 - 1}$ ***(M1)***

**Note:** Award ***M1*** for an attempt at a substituted GP formula with $n = 7$. Award ***A0M1A0*** for substitution of $r = -1.5$, with $n = 7$ (this can be implied from a final answer of $14.4687...$).

64.3... km (64.3437...) ***A1***

***[3 marks]***

***Total [7 marks]***

**3.** (a) $N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).
Approaches that use the compound interest formula receive no marks.
Award ***A1*** for correct values of $PV$ and $PMT$ (signs must be the same) **and** a correct value of $N$.

$FV = (\$)3577.43$ ***A1***

**Note:** Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to $2$ dp.

***[3 marks]***

(b) $N = 36.5 \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

**Note:** Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that $PV$ and $FV$ have opposite signs and the resulting value of $N$ is positive.

***[2 marks]***

***[Total: 5 marks]***

**8.** (a) attempt to substitute $5000$ for $G$ ***(M1)***

$0.301p = \log_{10} 5000$

$(p =)\ 12.3$ (bits) $(12.2889\ldots)$ ***A1***

***[2 marks]***

(b) $(G =) 10^{0.301p}$ **OR** $2^p$ ***A1***

***[1 mark]***

(c) attempt to substitute $28$ for $p$ in given equation or $G(p)$ ***(M1)***

$0.301 \times 28 = \log_{10} G$ **OR** $(G =)\ 10^{0.301 \times 28}$

$(G =)\ 2.68 \times 10^8$ $(2.67916\ldots \times 10^8)$ ***A1A1***

**Note:** Award ***A1*** for $2.68$, ***A1*** for $10^8$. Award ***M1A1A0*** for a correct final answer not written in scientific notation or written incorrectly in scientific notation (e.g., $268\ 000\ 000$ or $26.8 \times 10^7$ or 2.68E08).

***[3 marks]***

(d) if a password has an **entropy of 0** (bits), then the password can be **guessed in one** try / then the **password is known** ***R1***

**Note:** Reference must be made to both entropy and number of guesses/password known for ***R1*** to be awarded.
Do not accept "no password" as this contradicts the context.

***[1 mark]***

***Total [7 marks]***

**1.** (a) attempt to substitute into percentage error formula ***(M1)***

$$\left|\frac{53632000-55625000}{55625000}\right|\times 100$$

$3.58\,(\%)$ $(3.58292\ldots(\%))$ ***A1***

**Note:** Award ***(M1)A0*** for a final answer of $-3.58(\%)$ or $0.0358$.

***[2 marks]***

(b) (i) $278\,000\,000$ ***A1***

(ii) $2.78\times10^{8}$ ***A1A1***

**Note:** Award ***A1*** for correct mantissa, consistent with their answer in part (b)(i).
Award ***A1*** for a correct exponent, consistent with their answer in part (b)(i).
Award ***A0A0*** for answers such as $27.8\times10^{7}$.

***[3 marks]***
***Total [5 marks]***

**2.** (a) **METHOD 1 (use of financial app in GDC)**

| | | |
|---|---|---|
| $N = 5$ | **OR** | $N = 20$ |
| $I\% = 1.2$ | | $I\% = 1.2$ |
| $PV = \pm 520$ | | $PV = \pm 520$ |
| $P/Y = 1$ | | $P/Y = 4$ |
| $C/Y = 4$ | | $C/Y = 4$ |

***(M1)(A1)***

**Note:** Award ***M1*** for evidence of using the financial app on the calculator, ***A1*** for all correct entries.

$(\$)\ 552.11$ ***A1***

**Note:** Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

**METHOD 2 (use of formula)**

attempt to substitute into compound interest formula ***(M1)***

$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4}$ ***(A1)***

$(\$)\ 552.11$ ***A1***

**Note:** Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

***[3 marks]***

(b) **EITHER**

$N = 5$

$I\% = 43.5\ \ (43.4772...(\%))$

$PV = \pm 520$

$FV = \mp 30$ ***(M1)(A1)A1***

**Note:** Award ***M1*** for evidence of using the finance app on the calculator, ***A1*** for all correct entries, ***A1*** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

**OR**

$30 = 520\left(1 - \frac{r}{100}\right)^{5}$ (or equivalent) ***(M1)(A1)***

$(r =)\ \ 43.5\%\ \ (43.477...\%)$ ***A1***

**Note:** Award ***M1*** for using the compound interest formula, ***A1*** for correct substitutions and for equating to $30$, ***A1*** for correct final answer. Accept $(r =)\ -43.5\%$.
Award ***M1A1A0*** for a final answer of $56.5\%$.

***[3 marks]***
***Total [6 marks]***

**2.** (a) recognizing arithmetic sequence (may be seen in part (b)) ***(M1)***

$(u_{12} =)\ 10+(12-1)\times 6$ ***(A1)***

$76$ ***A1***

***[3 marks]***

(b) correct substitution into either arithmetic series formula ***(A1)***

$(S_{15} =)\ \frac{15}{2}\left(2\times 10+(15-1)\times 6\right)$ **OR** $(S_{15} =)\ \frac{15}{2}(10+94)$

$780$ ***A1***

***[2 marks]***

(c) attempt to use either arithmetic series formula equated to $1000$ ***(M1)***

$\frac{15}{2}\left(2\times 10+(15-1)\times x\right)=1000$ **OR** $\frac{15}{2}(10+u_{15})=1000$

$x=8.09523...$ ***(A1)***

$x=9$ ***A1***

**Note:** Follow through within question part for final ***A1*** for candidates correctly rounding their value of $x$ up to the nearest integer. Award ***(M0)(A0)A0*** for a response of $x=8$ with no working shown.

***[3 marks]***

(d) recognizing geometric sequence (may be seen in part (e)) ***(M1)***

$17.1\times 0.95^{5-1}$ ***(A1)***

$13.9\ (\text{cm})(13.9280\ldots)$ ***A1***

***[3 marks]***

(e) correct substitution into geometric series formula ***(A1)***

$\frac{17.1(1-0.95^{16})}{1-0.95}$

$191\ (\text{cm})\ (191.476\ldots(\text{cm}))$ ***A1***

***[2 marks]***

(f) correct method to find $u_0$ ***(M1)***

$u_0=17.1\times(0.95)^{0-1}$ **OR** $17.1=0.95x$ **OR** $\frac{17.1}{0.95}$ (seen)

**Note:** Award ***(M0)A0*** for any attempt to find answer using $0.05$ or $1.05$.

$18\ (\text{cm})$ ***A1***

***[2 marks]***

***Total [15 marks]***

**4.** (a) (i) attempt to find $25\%$ or $75\%$ of $285000$ ***(M1)***

$285000 \times 0.75$

$213750$ (ZAR) ***A1***

**Note:** Do not award ***A1*** if answer is not given exact.

(ii) $N = 60$

$I\% = 4.5$

$PV = (\pm)213750$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct.

$(PMT =)\ 3984.95$ (ZAR) ***A1***

**Note:** Do not award final ***A1*** if answer is not given to $2$ dp.

***[5 marks]***

(b) $3984.95 \times 60$ ***(M1)***

$239097$ (ZAR) ***A1***

**Note:** Do not award ***A1*** if answer is not given to the nearest rand, unless already penalized in part (a)(ii).

***[2 marks]***

(c) $I\% = 4.5$

$PV = (\pm)213750$

$PMT = (\mp)4600$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(A1)***

**Note:** Award ***A1*** for $PMT = (\mp)4\,600$.

$(N =)\ 52$ ***A1***

***[2 marks]***

(d) **METHOD 1**

$N = 51$
$I\% = 4.5$
$PV = (\pm)\,213750$
$PMT = (\mp)\,4600$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

**Note:** Award ***A1*** for $N = 51$ seen.

$(FV =)\ 704.156...$ ***A1***

valid attempt to find interest in final month (e.g. $N = 1$ **OR** $PV = 704.156...$) ***(M1)***

$N = 1$
$I\% = 4.5$
$PV = 704.16\ \ (704.156...)$
$FV = 0$
$P/Y = 12$
$F/Y = 12$

$(PMT =)\ \ 706.80$ (ZAR) ***A1***

**Note:** Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

***[4 marks]***

**METHOD 2**

$N = 52$
$I\% = 4.5$
$PV = (\pm)\,213750$
$PMT = (\mp)\,4600$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

**Note:** Award ***A1*** for $N = 52$ seen.

$(FV =)\ 3893.20...$ ***A1***

$4600 - 3893.20...$ ***(A1)***
$(PMT =)\ \ 706.80$ (ZAR) ***A1***

**Note:** Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

***[4 marks]***

(e) $51 \times 4600 + 706.80$ ***(M1)***

$235306.80$

attempt to find difference between their value and their part (b) ***(M1)***
$(239097 - 235306.80)$

$3790$ (ZAR) ***A1***

**Note:** Do not penalize for not rounding to nearest rand if this has already been penalized in part (b).

***[3 marks]***
***[Total 16 marks]***

**4.** (a) (i) attempt to find $15\%$ or $85\%$ of $285000$ ***(M1)***

$285000 \times 0.85$

$242\,250$ (USD) ***A1***

**Note:** Do not award ***A1*** if answer is not given exact.

(ii) $N = 360$

$I\% = 4$

$PV = (\pm)\,242\,250$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct.

$(PMT =)\ 1156.54$ (USD) ***A1***

**Note:** Do not award final ***A1*** if answer is not given to $2$ dp.

***[5 marks]***

(b) $1156.54 \times 360$ ***(M1)***

$416\,354$ (USD) ***A1***

**Note:** Do not award ***A1*** if answer is not given to the nearest dollar, unless already penalized in part (a)(ii).

***[2 marks]***

(c) $I\% = 4$

$PV = (\pm)\,242\,250$

$PMT = (\mp)1300$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(A1)***

**Note:** Award ***A1*** for $PMT = (\mp)1300$

$(N =)\ 292$ ***A1***

***[2 marks]***

(d) **METHOD 1**

$N = 291$
$I\% = 4$
$PV = (\pm)\,242\,250$
$PMT = (\mp)1300$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

**Note:** Award ***A1*** for $N = 291$ seen.

$(FV =)\ 871.91\ (871.908\ldots)$ ***A1***

valid attempt to find interest in final month (e.g. $N = 1$ **OR** $PV = 871.91$) ***(M1)***

$N = 1$
$I\% = 4$
$PV = 871.91\ (871.908\ldots)$
$FV = 0$
$P/Y = 12$
$F/Y = 12$

$(PMT =)\ 874.82$ (USD) ***A1***

**Note:** Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

**METHOD 2**

$N = 292$
$I\% = 4$
$PV = (\pm)\,242\,250$
$PMT = (\mp)1300$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

**Note:** Award ***A1*** for $N = 292$ seen.

$(FV =)\ 425.185\ldots$ ***A1***

$1300 - 425.185...$ ***(A1)***
$(PMT =)\ 874.82$ (USD) ***A1***

**Note:** Accept $874.81$. Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

***[4 marks]***

(e) $291 \times 1300 + 874.82$ ***(M1)***

$379174.82$

attempt to find difference between their value and their part (b) ***(M1)***
$(416354 - 379174.82)$

$37179$ (USD) ***A1***

**Note:** Accept $37180$ (USD) from using the 2 dp. answer from part (b). Do not penalize for not rounding to nearest dollar if this has already been penalized in part (b).

***[3 marks]***
***[Total 16 marks]***

**3.** (a) attempt to substitute into geometric sequence formula for twelfth term **OR** at least three correct terms of the sequence ***(M1)***

$u_{12} = 40 \times 1.1^{12-1}$ **OR** $40, 44, 48.4 \ldots$

$114 \quad (114.124\ldots)$ ***A1***

***[2 marks]***

(b) (i) attempt to substitute into the geometric series formula **OR** a sum of at least the first three terms ***(M1)***

$S_{12} = \dfrac{40\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $\sum_{1}^{12}\left(40 \times 1.1^{n-1}\right)$ **OR** $40+44+48.4+\ldots$

**Note:** Award ***M1*** for $u_1 = 40$ and $r = 1.1$ seen as part of a geometric series formula, or ***M1*** for sigma notation and their $u_n$ formula (condone missing limits), or ***M1*** for the sum of at least the **correct** first three terms of the sequence.

$S_{12} = 855 \quad (855.371\ldots)$ ***A1***

(ii) finding $S_{24} = 3539.89...$ or attempt to find the sum between $u_{13}$ and $u_{24}$ ***(M1)***

**Note:** Award ***M1*** for $S_{24} = 3539.89\ldots$ **or** sigma notation that includes correct limits and their $u_n$ formula **or** a substituted geometric series formula that includes $125.537\ldots$ and $n = 12$ **or** a list of terms that includes at least the 13th term and the 24th term.

$3539.89\ldots - 855.371\ldots$ **OR** $\sum_{13}^{24}\left(40 \times 1.1^{n-1}\right)$ **OR**

$\left(S_{13 \text{ to } 24} =\right)\dfrac{125.537\ldots\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $125.537 + \ldots + 358.172\ldots$ ***(A1)***

**Note:** Accept a calculation using $u_{13} = 125$ or $126$.

$2680 \quad \left(2684.52\ldots, 2685\right)$ ***A1***

**Note:** For $u_{13} = 125$, the sum is $2673.03\ldots$ and for $u_{13} = 126$, the sum is $2694.41\ldots$

***[5 marks]***

***[Total: 7 marks]***

**5.** (a) **EITHER**

$N = 96$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**OR**

$N = 8$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 1$
$C/Y = 12$ ***(M1)(A1)***

**OR**

$150000 = 100000\left(1 + \frac{I}{100 \times 12}\right)^{12 \times 8}$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use a compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in compound interest formula.

**THEN**

$I = 5.08\ (5.07903\ldots)$ ***A1***

***[3 marks]***

(b) $N = 120$
$I\% = 6.1$
$PV = \mp 150000$
$PMT = \pm 1000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award ***A1*** for all entries correct in financial app (condone missing -/+ sign if the correct final answer is seen).

$FV = (\$)\ 110867$ ***A1***

**Note:** Answer must be correct to nearest dollar to award the final ***A1***. Award ***(M1)(A1)A0*** for an unsupported final answer to a greater degree of accuracy eg. (\$) $110866.70\ldots$
Award ***M1A1A0*** for a truncated answer of $110866$ if no working is shown.

***[3 marks]***
***[Total: 6 marks]***

**5.** (a) **EITHER**

| | | |
|---|---|---|
| $N = 72$ | | $N = 6$ |
| $I = 1.25$ | | $I = 1.25$ |
| $PV = -3000$ | **OR** | $PV = -3000$ |
| $P/Y = 12$ | | $P/Y = 1$ |
| $C/Y = 12$ | | $C/Y = 12$ |

***(M1)(A1)***

**Note:** Award ***M1*** for attempt to use finance app on GDC, ***A1*** for all entries correct.

**OR**

$$3000\left(1+\frac{1.25}{1200}\right)^{72}$$ ***(M1)(A1)***

**Note:** Award ***M1*** for using CI formula, ***A1*** for correct entries.

**THEN**

$FV = (\$)\,3233.53$ ***A1***

**Note:** Answer must be correct to 2 dp for the final ***A1*** to be awarded.

***[3 marks]***

(b) **EITHER**

| | | |
|---|---|---|
| $I = 1.25$ | | $I = 1.25$ |
| $PV = -3000$ | | $PV = -3000$ |
| $FV = 3550$ | **OR** | $FV = 3550$ |
| $P/Y = 12$ | | $P/Y = 1$ |
| $C/Y = 12$ | | $C/Y = 12$ |

***(A1)***

**Note:** Award ***A1*** for all entries correct and opposite signs for $PV$ and $FV$ values.

**OR**

$$3550 = 3000\left(1+\frac{1.25}{1200}\right)^{12N}$$ ***(A1)***

**Note:** Award ***A1*** for all entries correct.

**THEN**

$N = 162\ (161.686\ldots)$ (months) ***A1***

***[2 marks]***

(c) $18450$ ***A1***

***[1 mark]***

(d)

| | | |
|---|---|---|
| $N = 96$ | | $N = 96$ |
| $I = 12.6$ | | $I = 1.05$ |
| $PV = -18450$ | **OR** | $PV = -18450$ |
| $FV = 0$ | | $FV = 0$ |
| $P/Y = 12$ | | $P/Y = 1$ |
| $C/Y = 12$ | | $C/Y = 1$ |

***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use finance app on GDC, ***A1*** for $N = 96$, $PV = -18450$ and $FV = 0$,

$PMT = (\$)\ 306$ ***A1***

**Note:** The answer must be correct to the nearest dollar for the final ***A1*** to be awarded.

***[3 marks]***
***[Total: 9 marks]***

**7.** (a) correct substitution of $0.0003$ into the formula ***(A1)***

$\text{pH} = -\log_{10}(0.0003)$

$= 3.52 \quad (3.52287\ldots)$ ***A1***

***[2 marks]***

(b) **EITHER**

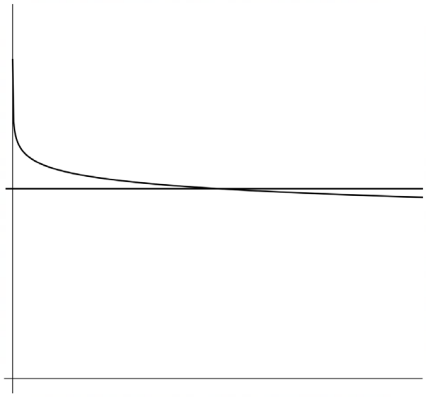
attempt to change to exponential form ***(M1)***

$\left[\text{H}^+\right] = 10^{-6.6}$

**OR**

attempt to solve $6.6 = -\log_{10}\left[\text{H}^+\right]$ ***(M1)***

(graphically or using numerical solver)



**THEN**

$\left[\text{H}^+\right] = 0.000000251$ (moles per litre) $\quad (0.000000251188..., 2.51\times10^{-7})$ ***A1***

**Note:** Award ***M1A0*** for an answer of $2.51$ $(2.51188...)$ seen. For an attempt to solve $6.6 = -\log_{10}\left[\text{H}^+\right]$, award ***M0A0*** if a substitution of $6.6$ into the formula is seen without an answer or some indication of using numerical solver.

***[2 marks]***

(c) $2 = -\log_{10}\left[\text{H}^+\right],\ 4.5 = -\log_{10}\left[\text{H}^+\right]$

$10^{-2} \quad (0.01)$ **OR** $10^{-4.5} \quad (0.0000316227...)$ ***(A1)***

substitution of their values into correct ratio ***(M1)***

$\dfrac{10^{-2}}{10^{-4.5}}$ **OR** $\dfrac{0.01}{0.0000316227...}$

$= 316.227... = 316$ ***A1***

**Note:** Some candidates may subtract logs and hence look to solve $\log_{10}[\text{H}^+] = 2.5$.

***[3 marks]***

***[Total: 7 marks]***

**9. EITHER**

recognizing that Eefje's is an AP **OR** that Shumay's is a GP ***(M1)***

$42.195 = 5 + (n-1)(2)$ ***(A1)***

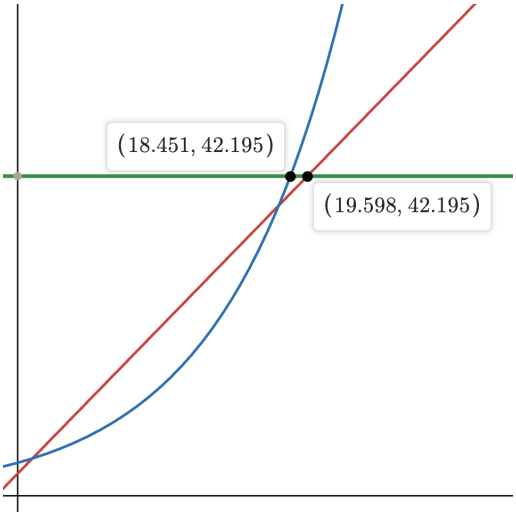
$n_E = 19.6 \ \ (19.5975\ldots)$ (accept 20) ***A1***

$42.195 = 5(1.13)^{n-1}$ ***(A1)***

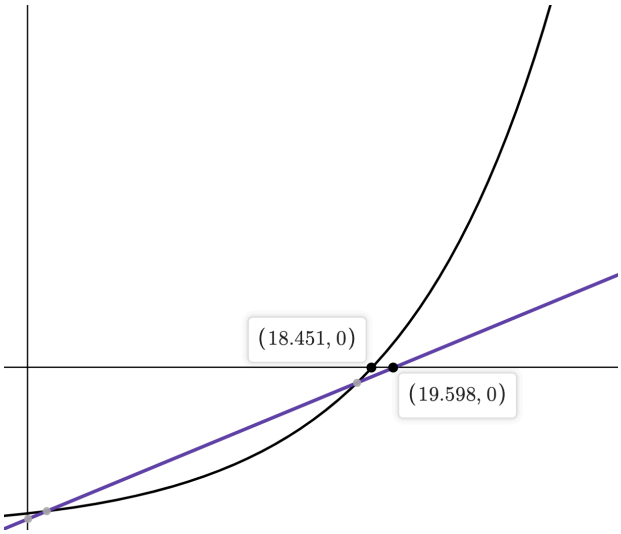
$n_S = 18.5 \ \ (18.4513\ldots)$ (accept 19) ***A1***

**OR (Graphical method)**

e.g




**OR**




Sketching a straight line (Eefje's) **OR** an exponential curve (Shumay's) ***(M1)***

**Note:** The horizontal line (marathon length) does not need to be seen to be awarded the ***M1***.

Horizontal line (marathon length) **and** Eefje's linear drawn ***(A1)***

$x$-coordinate of the point of intersection $19.6$ $(19.5975....)$ ***A1***

Horizontal line (marathon length) **and** Shumay's curved drawn ***(A1)***

$x$-coordinate of the point of intersection $18.5$ $(18.4513....)$ ***A1***

**Note:** A ruler does not need to be used for the lines.

**THEN**
so Shumay (will be the first to run the distance of a marathon) on day 19 ***A1***

**Note:** Award the final ***A*** mark for a correctly rounded-up integer value. **FT** from their $n_E$ and $n_s$ values. Award ***A0*** for a final result of "Shumay, day 19" if it is clear that Eefje/the AP has not been considered (either by finding the $19.5975$, or by substituting $n = 19$ into AP and showing distance is less than marathon).
Accept lists showing recognition that Eefje's is an AP **OR** that Shumay's is a GP
**If trial and error method is used:**
Award ***A1A1*** for finding the distance for Eefje on day $20 = 43$ and ***A1A1*** for finding the distance for Shumay on day $19 = 45.12$.

***[Total: 6 marks]***

**3.** (a) $7a + 60c = 832$ ***A1***

$3a + 5c = 108$ ***A1***

**Note:** Award ***A0A1FT*** for correct equations but different variables such as $x$ & $y$.

***[2 marks]***

(b) (i) $(a =)\ 16$ ***A1***

(ii) $(c =)\ 12$ ***A1***

***[2 marks]***

(c) $202\,500$ ***A1***

***[1 mark]***

***[Total 5 marks]***

**8.** (a) recognition that % increase gives the common difference ***(M1)***

$18\,000 \times 0.04 = 720$ ***A1***

***[2 marks]***

(b) $18000 + 720n > 32000$ **OR** $18000 + 720(n-1) > 32000$ (accept equality) ***(A1)***

attempt to solve their inequality **OR** equality (sketch **OR** cross-over values **OR** algebraic manipulation) ***(M1)***

$k = 20$ ***A1***

**Note:** Award ***(A1)(M1)A0*** for $19.4$, $20.4$ or $21$ seen.

***[3 marks]***

***[Total: 5 marks]***

**10.** (a) $N = 96,\ I = 3.8,\ PV = +300\,000,\ PMT = -2800,\ P/Y = 12,\ C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for use of Finance solver with any 2 entries correct, Award ***A1*** for all correct.

$FV = \$92\,800\ \ (\$92\,828.2...)$ ***A1***

***[3 marks]***

(b) $I = 3.8,\ PV = +300\,000,\ PMT = -2800,\ FV = 0,\ P/Y = 12,\ C/Y = 12$ ***(A1)(A1)***

**Note:** Award ***A1*** for $FV = 0$, award ***A1*** for all correct.

$N = 131$ (months) ***A1***

***[3 marks]***
***[Total 6 marks]***

**13.** (a) recognizing sequence is geometric (e.g. $r = 2$ seen anywhere) ***(M1)***

$u_9 = 1250 \times 2^{9-1}$

$320000$ ***A1***

$1250 \times 2^{k-1}$ ***A1***

| $n$ | 1 | 2 | 3 | 9 | $k$ |
|---|---|---|---|---|---|
| Time in minutes | 10 | 20 | 30 | 90 | $10k$ |
| Number of bacteria $u_n$ | 1250 | 2500 | 5000 | **320000** | $1250 \times 2^{k-1}$ |

***[3 marks]***

(b) (i) $(n =)\ 11$ ***A1***

(ii) **METHOD 1**

recognition of summation of sequence ***(M1)***

$(S_{10} =)\ \dfrac{1250(2^{10}-1)}{2-1} \quad (= 1278750)$ ***(A1)***

$1279000$ ***A1***

**METHOD 2**

all of the bacteria must come from divisions, apart from initial ***(M1)***

$1.28 \times 10^6 - 1250$ ***(A1)***

$1279000$ ***A1***

**Note:** Final answer must be to the nearest thousand to earn the final ***A1***.

Award ***(M1)(A1)A0*** for a final answer of $1280000$ without working.

***[4 marks]***
***[Total 7 marks]***

**3.** (a) $6a + 50c = 1292$ ***A1***

$2a + 3c = 130$ ***A1***

**Note:** Award ***A0A1FT*** for correct equations but different variables such as $x$ & $y$.

***[2 marks]***

(b) (i) $(a =)\ 32$ ***A1***

(ii) $(c =)\ 22$ ***A1***

***[2 marks]***

(c) $100\,500$ ***A1***

***[1 mark]***

***[Total 5 marks]***

**8.** (a) recognition that % increase gives the common difference ***(M1)***

$25000 \times 0.05 = 1250$ ***A1***

***[2 marks]***

(b) $25000 + 1250n > 44000$ **OR** $25000 + 1250(n-1) > 44000$ (accept equality) ***(A1)***

attempt to solve their inequality **OR** equality (sketch **OR** cross-over values **OR** algebraic manipulation) ***(M1)***

$k = 16$ ***A1***

**Note:** Award ***(A1)(M1)A0*** for $15.2$, $16.2$ or $17$ seen.

***[3 marks]***

***[Total: 5 marks]***

**10.** (a) $N = 60,\ I = 4.5,\ PV = +400\,000,\ PMT = -3600,\ P/Y = 12,\ C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for use of Finance solver with any 2 entries correct.

$FV = \$259\,000 (\$258\,994)$ ***A1***

***[3 marks]***

(b) $I = 4.5,\ PV = +400\,000,\ PMT = -3600,\ FV = 0,\ P/Y = 12,\ C/Y = 12$ ***(A1)(A1)***

**Note:** Award ***A1*** for $FV = 0$, award ***A1*** for all correct.

$N = 144$ ***A1***

***[3 marks]***

***[Total 6 marks]***

**13.** (a) recognizing sequence is geometric (e.g. $r = 2$ seen anywhere) ***(M1)***

$u_9 = 1250 \times 2^{9-1}$

$320000$ ***A1***

$1250 \times 2^{k-1}$ ***A1***

| $n$ | 1 | 2 | 3 | 9 | $k$ |
|---|---|---|---|---|---|
| Time in minutes | 10 | 20 | 30 | 90 | $10k$ |
| Number of bacteria $u_n$ | 1250 | 2500 | 5000 | **320000** | $1250 \times 2^{k-1}$ |

***[3 marks]***

(b) (i) $(n =)\ 11$ ***A1***

(ii) **METHOD 1**

recognition of summation of sequence ***(M1)***

$(S_{10} =)\ \dfrac{1250(2^{10}-1)}{2-1} \quad (= 1278750)$ ***(A1)***

$1279000$ ***A1***

**METHOD 2**

all of the bacteria must come from divisions, apart from initial ***(M1)***

$1.28 \times 10^6 - 1250$ ***(A1)***

$1279000$ ***A1***

**Note:** Final answer must be to the nearest thousand to earn the final ***A1***.

Award ***(M1)(A1)A0*** for a final answer of $1280000$ without working.

***[4 marks]***
***[Total 7 marks]***

**3.** (a) (i) $x+6y+12z=383$ ***M1A1***

**Note:** Award ***M1*** for at least one correct term other than $x$ **and** equating to $383$, ***A1*** for correct equation.

(ii) $4.50x+23.50y+44.75z=1486$ ***A1***

**Note:** If no marks were awarded for part (i), award ***M1*** for at least one correct term **and** equating to $1486$, ***A1*** for correct equation.

***[3 marks]***

(b) **EITHER**
attempt to solve the simultaneous equations ***(M1)***
e.g., $(x=)\ 53$ or $(y=)\ 15$ or $(z=)\ 20$

**OR**
elimination of a variable using algebraic method ***(M1)***

**THEN**
$15$ (boxes) ***A1***

**Note:** The answer $15$ must be identified as the final answer to award ***A1***. Award ***M1A0*** if all $3$ values are shown as the final answer.

***[2 marks]***

***[Total: 5 marks]***

**1.** (a) after 10 years

*N:* 120
*I%:* 3.6
*PV:* 0
*PMT:* (-)1500
*P/Y*: 12
*C/Y:* 12 ***(M1)(A1)***

**Note:** Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct except PV.

*FV:* 216 278.58 (euros) ***A1***

**Note:** Award ***A0*** for a negative answer or answer not given to 2 decimal places.

***[3 marks]***

(b) **METHOD 1 (C.I. formula)**

invests: 116 278.58 ***(M1)***

after 15 years: $116278.58\times(1+\frac{4.5}{400})^{60}$ ***(M1)(A1)***

*FV:* 227 515.92 (euros) ***A1***

**Note:** Award the first ***M1*** for subtracting *their* FV from part (a) even if the value of the investment is negative.
Award the second ***M1*** for use of the C.I. formula.
Award first ***A1*** for all entries correct except PV.
Allow full follow through from part (a).
Award final ***A0*** for FV not given to 2 decimal places unless this has already been penalized in (a).
Accept 227 515.93 from use of unrounded value.

**METHOD 2 (financial app)**

invests: 116 278.58 ***(M1)***

**THEN**

| **EITHER** | | **OR** | | **OR** | |
|---|---|---|---|---|---|
| *N:* | 60 | *N:* | 15 | *N:* | 180 |
| *I%:* | 4.5 | *I%:* | 4.5 | *I%:* | 4.5 |
| *PV:* | 116 278.58 | *PV:* | 116 278.58 | *PV:* | 116 278.58 |
| *PMT*: | 0 | *PMT:* | 0 | *PMT:* | 0 |
| *P/Y:* | 4 | *P/Y:* | 1 | *P/Y:* | 12 |
| *C/Y:* | 4 | *C/Y:* | 4 | *C/Y:* | 4 |

***(M1)(A1)***

*continued…*

*Question 1 continued*

**Note:** Award the first ***M1*** for subtracting *their* $FV$ from part (a) even if the value of the investment is negative.
Award ***(M1)*** for an attempt to use a financial app in their technology with a least two entries seen, award ***A1*** for all entries correct except $PV$.

**THEN**

$FV$: 227 515.92 (euros) ***A1***

**Note:** Award ***A0*** for a negative answer or answer not given to 2 decimal places unless already penalized in part (a).
Accept 227 515.93 from use of unrounded value.

***[4 marks]***
***Total [7 marks]***

**3.** (a) attempt to use geometric sequence formula ***(M1)***

$200 \times 1.15^{10}$ ***(A1)***

$809.111$

$809$ (accept $810$) ***A1***

***[3 marks]***

**Note:** Award ***M1A0A1*** for $703$ or $704$ seen.
If list used award ***M1*** for list of at least 5 geometric terms; award ***A1*** for at least 5 correct terms.

(b) $200 \times 1.15^{n-1} > 3000$ (accept as equation) ***(A1)***

**Note:** Accept $3001$ instead of $3000$.
The $200 \times 1.15^{n-1} > 3000$ may be seen as a graph sketch.

$n = 20.3761...$ ***(A1)***

$20^{\text{th}}$ week (of observation) ***A1***

**OR**

use of table

$200 \times 1.15^{n-1} > 3000$ (accept as equation) ***(A1)***

$200 \times 1.15^{19} = 2846.35...$ **AND** $200 \times 1.15^{20} = 3273.30...$ ***(A1)***

$20^{\text{th}}$ week (of observation) ***A1***

***[3 marks]***

***Total [6 marks]***

**6.** use of $\text{speed} = \frac{\text{distance}}{\text{time}}$ **OR** $\text{speed} \times \text{time} = \text{distance}$ **OR** $\text{time} = \frac{\text{distance}}{\text{speed}}$

seen anywhere ***(M1)***

attempt to find a bounding value of $t$ (**OR** use appropriate values of $t$ that support and contradict the statement that the car is speeding) ***(M1)***

$(t =) 1.25\,(\text{s})$ OR $(t =) 1.15\,(\text{s})$ ***(A1)***

**THEN**

**EITHER**

use of their upper bound for $t$ to find minimum speed (**OR** of any value of $t$ where $1.20481... < t \leq 1.25$ to find speed)

(speed =) $\frac{10}{1.25}$ (or $\frac{10}{\text{their time}}$) ***A1***

(speed =) $8\,(\text{m s}^{-1})$ (or a value where $8 \leq \text{speed} < 8.3$) seen

hence speed $< 8.3\,(\text{ms}^{-1})$ ***A1***

**OR**

use of their upper bound for $t$ to find maximum distance (**OR** of any value of $t$ where $1.20481... < t \leq 1.25$ to find distance)

(distance=) $8.3 \times 1.25$ (or $8.3 \times \text{their time}$) ***A1***

(distance =) $10.375\ \text{m}$ (or a value where $10 < \text{distance} \leq 10.375$) seen

hence distance $> 10\,(\text{m})$ ***A1***

**OR**

time taken at speed limit $(\text{time} =) \frac{10}{8.3}$ ***A1***

$(\text{time} =) 1.20481...(\text{s})$

hence time of $1.2(\text{s})$ to the nearest $0.1(\text{s})$ could be greater than or less than this ***A1***

**THEN**

conclusion: it is not certain that the car was exceeding the speed limit. ***A1***

**Note:** The conclusion must have been justified by further showing that for some appropriate value of $t$ the car is speeding

Award at most ***M1M1A0A1A0A1*** for the use of a value of $t$ where $t > 1.25$

Award at most ***M1M0A0A0A0A1*** for $t = 1.2$ used with a corresponding justified conclusion.

***Total [6 marks]***

**Q7.**

*(a)* attempt to substitute the given values in the function ***(M1)***

$\left(L=\right)10\log_{10}\left(\frac{10^{-5}}{10^{-12}}\right)$

$=70\left(\text{dB}\right)$ ***A1***

***[2 marks]***

*(b)* $185=10\log_{10}\left(\frac{I}{10^{-12}}\right)$ ***(A1)***

attempt to solve their logarithmic equation using graph, solver or analytically ***(M1)***

$I=3160000\left(\text{Wm}^{-2}\right)\left(3162277.66\right)$

$I=3.16\times10^{6}\ \ \left(3.16227...\times10^{6}\right)$ ***A1***

| |
|---|
| **Note***:* Award at most ***A1M1A0*** or ***A0M1A1FT*** for an incorrect answer written correctly in the required form. |

***[3 marks]***

***Total [5 marks]***

**Q11**.

(a) evidence of correct method ***(M1)***

$N = 300, I\% = 4.2, PV = \mp 50000, FV = 0, P/Y = 12, C/Y = 12$ OR $269.4711$ seen

**Note**: Award ***M1*** for using the financial app on their GDC with at least 2 correct entries.

$269.47$( euros) ***A1***

**Note**: Award at most ***(M1)A0*** for $-269.47$ . Accept $269.48$.

***[2 marks]***

(b) (i) evidence of correct method $N = 96$ ***(M1)***

$N = 96,\ I\% = 4.2,\ PV = \mp 50000,\ PMT = \pm 269.47,\ P/Y = 12,\ C/Y = 12$

**OR**

$N = 96,\ I\% = 0.35,\ PV = \mp 50000,\ PMT = \pm 269.47,\ P/Y = 1,\ C/Y = 1$

**Note**: Award ***M1*** for using the financial app on their GDC with at least 2 correct entries.

$39,\ 243.42$ ***A1***

**Note**: Accept $39{,}243.29$ which results from the full calculator value for the payment in part (a). Accept $39{,}243.28$ which results from the use of $269.48$.

**Note**: If two decimal place accuracy was penalised in part (a) do not penalise again.

(ii) **METHOD 1**

full payment $12 \times 25 \times 269.47 = 80{,}841.00$ ***(A1)***
actual payment $12 \times 8 \times 269.47 + 39243.42$ ***(M1)***

= $65,\ 112.54$ ***(A1)***

money saved = $15{,}700$ $(15{,}728.46)$ ( euros) ***A1***

**Note**: Accept $15{,}728.59$ from using $269.47$ from part (a) and $39{,}243.29$ from part (b)(i).
Accept $15{,}730.64$ from using $269.48$ from part (a) and $39{,}243.28$ from part b(i).

*continued*

*Question 11 continued*

**METHOD 2**

if Sophie did not pay back the loan early her subsequent payments would be $269.47 \times 12 \times 17$ ***(M1)***

**Note**: Award ***M1*** for attempting to find the remaining amount to pay if loan not paid back.

$= 54971.88$ (euros) ***(A1)***

money saved is therefore
$54971.88 - 39243.42$ ***(M1)***

$= 15,700 (15,728.46)$ ( euros) ***A1***

***[6 marks]***
***Total [8 marks]***

**2.** (a) **EITHER**

$N = 72$ ***(M1)(A1)***

$I\% = 3.4$

$PV = \pm 10\,000$

$P/Y = 12$

$C/Y = 12$

**Note:** Award ***M1*** for an attempt to use a financial app in their technology (i.e., at least three entries seen, but not necessarily correct), ***A1*** for all entries correct.

**OR**

$N = 72$ ***(M1)(A1)***

$I\% = 3.4/12$

$PV = \pm 10\,000$

$P/Y = 1$

$C/Y = 1$

**Note:** Award ***M1*** for an attempt to use a financial app in their technology (i.e., at least three entries seen, but not necessarily correct), ***A1*** for all entries correct.

**OR**

attempt to substitute into the compound interest formula ***(M1)***

$FV = 10\,000 \times \left(1 + \frac{3.4}{100 \times 12}\right)^{6 \times 12}$ ***(A1)***

**THEN**

($FV$ =) 12 259 (CAD) ***A1***

***[3 marks]***

(b) 212 741 (CAD) ***A1***

***[1 mark]***

(c) $N = 180$ ***(M1)(A1)(A1)(A1)***
$FV = 0$
$I\% = 6.4$
$PV = \pm 212\,741$
$P/Y = 12$
$C/Y = 2$

**Note:** Award ***M1*** for attempt to use a financial app with at least three entries seen, but not necessarily correct, ***A1*** for $N$ **and** $I$, ***A1*** for $P/Y$ **and** $C/Y$, ***A1*** for their $PV$ from (b) **and** $FV = 0$. Approaches that use the compound interest formula receive no marks.

$(PMT =)\ 1832$ (CAD) ***A1***

**Note:** Accept $-1832$ (CAD).

***[5 marks]***

(d) (i) $N = 36$ ***(M1)(A1)***
$PV = \pm 194\,572$
$PMT = 2200$

**Note:** Award ***M1*** for adjusting at least one of $N$, $PV$, or $PMT$; ***A1*** for correct entries ($PV$ and $PMT$ must have opposite signs). Allow ***FT*** from incorrect entries for $P/Y$ and/or $C/Y$ from part (c).

$(FV =) 148\,099$ (CAD) ***(A1)***
$(194\,572 - 148\,099)$

46 473 (CAD) ***A1***

(ii) $2200 \times 12 \times 3\ (= 79\,200)$ ***(A1)***
$(79\,200 - 46\,473 =)$

32 727 (CAD) ***A1***

***[6 marks]***

***Total [15 marks]***

**3.** (a) **EITHER**
The ratio of consecutive terms is constant. ***R1***

**OR**
Each term of the sequence is found by multiplying the previous term by a constant. ***R1***

**Note:** Do not accept "it has a common ratio" unless it is stated or illustrated that the ratio is between consecutive terms. Do not accept "because $r = 1.05$" or " because each subsequent day that he runs he increases his distance by 5%" for the explanation. The explanation must refer to a constant ratio or repeated multiplication by a constant.

**THEN**
$r = 1.05$ ***A1***

**Note:** Award ***R0A1*** for $r = 1.05$ seen with incorrect or no explanation.

***[2 marks]***

(b) attempt to substitute into the sum formula for geometric sequence **OR**
attempt to write the sum in sigma notation using the term formula **OR** a sum of geometric sequence terms ***(M1)***

$(S_{15} =) \dfrac{2\left(1.05^{15} - 1\right)}{1.05 - 1}$ **OR** $\sum_{1}^{15}(2 \times 1.05^{n-1})$ **OR** $2 + 2 \times 1.05 + 2 \times 1.05^2 + \ldots$

$43.2\ (\text{km})$ $(43.1571\ldots\ (\text{km}))$ ***A1***

***[2 marks]***

(c) choosing $n = 36$ ***(A1)***

attempt to substitute into term formula for geometric sequence ***(M1)***
$(u_{36} =)\ 2 \times 1.05^{36-1}$

$11.0\ (11.0320\ldots) > 5 \times 2$ **OR** $11.0$ verbally compared to $5 \times 2$ ***A1***

**Note:** Award at most ***A1M1A0*** if a final distance $(11.0$ or $11)$ is not given or if the comparison to $10$ $(5 \times 2)$ is not shown. Accept $11$ for $u_{36}$ (correct 2sf answer).

***[3 marks]***

(d) (i) recognition of swimming out and back **AND/OR** swimming two times per week ***(M1)***

$(u_1 =)\ 80 \times 2 \times 2$

$(u_1 =)\ 320\,(\text{m})$ ***A1***

(ii) attempt to substitute into term formula of arithmetic sequence ***(M1)***

$704 = 320 + (4 - 1)\,d$ **OR** $\frac{704-320}{3}$ ***(A1)***

**Note:** Equating to $704$ must be seen to award the ***(A1)***.

$(d =)\ 128\ (\text{m})$ ***A1***

(iii) **METHOD 1: Using increased swimming distance**

$(128 \div 2 \div 2 =)\ 32\ (\text{m})$ (further each week) ***(A1)***

$(80 + 32 =)\ 112\ (\text{m})$ (from the shore in week 2) ***A1***

**METHOD 2: Using total swimming distance for week 2**

$(320 + 128 =)\ 448\ (\text{m})$ (swam in week 2) ***(A1)***

$(448 \div 2 \div 2 =)\ 112\ (\text{m})$ (from the shore in week 2) ***A1***

***[7 marks]***

(e) **METHOD 1: Using sequence for distance swam per week (i.e. 320, 448, 576…)**

attempt to substitute *their* answers from (d)(i) and (d)(ii) into term formula ***(M1)***

$(u_{12} =)\ 320 + (12 - 1) \times 128$

$1728\ (\text{m})$ (for the week) ***(A1)***

$864\ (\text{m})$ (on a day in week $12$) ***A1***

**METHOD 2: Using a sequence $(t_n)$ for distance swam per day each week (i.e. 160, 224, 288…)**

recognizing $t_1 = 160$ and $d = 64$ ***(A1)***

attempt to substitute $160$ and $64$ into term formula ***(M1)***

$(t_{12} =)\ 160 + (12 - 1) \times 64$

$864\ (\text{m})$ (on a day in week $12$) ***A1***

**METHOD 3**: **Using a sequence $(v_n)$ for distance of marker from shore each week (i.e. 80, 112, 144…)**

attempt to substitute $80$ and $32$ into term formula ***(M1)***

$(v_{12} =)\ 80 + (12 - 1) \times 32$

(marker is) $432\ (\text{m})$ (from shore) ***(A1)***

$(432 \times 2 =)\ 864\ (\text{m})$ (on a day in week $12$) ***A1***

***[3 marks]***

***[Total: 17 marks]***

**1.** (a) (i) $7$ ***A1***

(ii) $9$ ***A1***

***[2 marks]***

(b) (i) $2$ ***A1***

(ii) $1+2(n-1)\left(=2n-1\right)$ ***(M1)A1***

**Note:** ***(M1)*** for substituting their (b)(i) into the arithmetic sequence formula.

***[3 marks]***

(c) (i) $17$ ***A1***

(ii) $\frac{9}{2}(1+17)$ OR $\frac{9}{2}(2\times1+(9-1)\times2)$ OR $1+3+\ldots+15+17$ ***(M1)***

$=81$ ***A1***

***[3 marks]***

(d) $\frac{n}{2}(1+(2n-1))$ OR $\frac{n}{2}(2\times1+(n-1)\times2)$ ***(M1)***

$=n^2$ ***A1***

***[2 marks]***

(e) $10$ ***A1***

***[1 mark]***

(f) $1+2+3+\ldots+n$ ***(M1)***

$=\frac{n}{2}(1+n)$ ***A1***

**Note:** Award ***(M1)*** for attempt for use of sum to $n$ terms of an arithmetic sequence.

***[2 marks]***

(g) $\frac{n(1+n)}{2}+\frac{n(n-1)}{2}$ ***M1***

**Note:** Award ***M1*** for addition of answers to parts (f) and given expression.

$=\frac{n}{2}\left[\left(1+n\right)+\left(n-1\right)\right]$ OR $\frac{n+n^2+n^2-n}{2}$ ***M1***

**Note:** Award ***M1*** for valid non-trivial attempt at further simplification of their sum.

$=\frac{n}{2}(2n)$ OR $\frac{2n^2}{2}$

$=n^2$ ***A1***

**Note:** Award a maximum of ***M1M1A0*** for **ft** from an incorrect part (f).

***[3 marks]***

***Total [16 marks]***

**Q1.**

(a) (i) 60 (USD) ***A1***

(ii) attempt to substitute into the arithmetic sequence formula ***(M1)***

$(u_{10} =)(200 + 9 \times 60)$

740 (USD) ***A1***

***[3 marks]***

(b) attempt to substitute into the sum of an arithmetic sequence formula ***M1***

$\frac{n}{2}(2 \times 200 + (n-1) \times 60)$ ***A1***

$5000 + \frac{n}{2}(400 + 60n - 60)$ ***A1***

$5000 + \frac{n}{2}(340 + 60n)$ ***AG***

**Note**: Award ***A1*** for adding $5000$ and a correct step in simplifying the expression which leads to the given answer. The final line need not be seen. Award ***M0A0A0*** if the student works backwards, i.e. $5000 + \frac{10}{2}(340 + 60 \times 10)$.

***[3 marks]***

(c) attempt to substitute $n = 10$ into $5000 + \frac{n}{2}(340 + 60n)$ ***(M1)***

$5000 + \frac{10}{2}(340 + 60 \times 10)$

$= 9700$ (USD) ***A1***

***[2 marks]***

(d) (i) $B = 1.065$ ***A1***

(ii) **EITHER**

use of their $B$ from part (i), in given formula ***(M1)***

$(u_{11} =) 5000 \times 1.065^{10}$ ***(A1)***

**OR**

$N = 10, I\% = 6.5, PV = \pm 5000, P/Y = 1, C/Y = 1$ ***(M1)(A1)***

**Note**: Award ***M1*** for using the finance app on the GDC, ***A1*** for all correct entries.

*continued…*

*Question 1 continued.*

**THEN**

$9390\ (\text{USD})\ (9385.68\ldots)$ ***A1***

hence Account 1> Account 2 (after 10 years) OR $9700 > 9390$ ***AG***

**Note:** The given answer (Account 1> Account 2) or numerical comparison must be seen to award final ***A1***. Rounding of $B$ to $1.07$ (7% in TVM) will lead to Account 2> Account 1. Award at most ***M1A1A0***. If $B = 6.5$, the final answer is $6.73\times10^{11}$. If $B = 0.065$, the final answer is $6.73\times10^{-9}$. These answers are unrealistic, award at most ***M1A1A0***.

***[4 marks]***

(e) attempt at comparing the accounts after $m$ years (e.g. graph, table, solver, etc..) ***(M1)***

$5000\times1.065^{m} > 5000+\frac{m}{2}(340+60\times m)$ ***(A1)***

$(m=)38.0462\ldots$

$(m=)39$ (years) ***A1***

**Note**: Follow through in (e) for their equation (account 2), if explicitly seen in d(ii).

***[3 marks]***

(f) $FV = 58\ 291.42\ldots$ after their $39$ years ***(A1)***

attempt to find the interest by subtracting $5000$ from their $FV$ after $39$ years ***(M1)***

$58\ 291.42\ldots - 5000$

$53\ 291$ (USD) ***A1***

**Note:** Award at most ***(A1)(M1)A0*** if the final answer is not rounded to the nearest dollar. Award at most ***A0M1A0*** for a non-integer exponent, even if it follows through from part (e).

***[3 marks]***
***Total [18 marks]***

**1.** (a) $(r =) 3$ ***A1***

***[1 mark]***

(b) $(u_8 =) 2 \times 3^7$ ***(A1)***

$4374$ (accept $4370$) ***A1***

***[2 marks]***

(c) $(S_{10} =) \frac{2(3^{10}-1)}{3-1}$ ***(A1)***

$59\,048$ (accept $59000$) ***A1***

***[2 marks]***

***Total [5 marks]***

**2.** (a) (i) $8.35\ (\text{cm})$ ***A1***

(ii) $\frac{4\pi(4.175)^3}{3}$ ***(A1)(M1)***

**Note:** Award ***A1*** for their correct radius or a correct calculation of the radius (i.e., $\frac{8.35}{2}$) seen.
Award ***M1*** for an attempt to substitute into the volume of sphere formula.

$305\ (\text{cm}^3)\ (304.830\ldots, 97.0304\ldots\pi)$ ***A1***

***[4 marks]***

(b) $\left|\frac{8.4-8.38}{8.38}\right| \times 100$ ***(M1)***

$0.239\ (\%)(0.238663\ldots)$ ***A1***

**Note:** Award ***M1*** for an attempt to substitute both $8.4$ and $8.38$ into the percentage error formula.

***[2 marks]***

***Total [6 marks]***

**5.** (a) $N = 120$
$PV = \mp 10\,000$
$PMT = \mp 800$
$FV = \pm 160\,000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app. Award ***A1*** for all entries correct in financial app. Approaches that use the compound interest formula receive no marks.

$(r =) 7.15\ (\%)\quad (7.14842\ldots)$ ***A1***
***[3 marks]***

(b) $160\,000 - 10000 - 10 \times 12 \times 800$ ***(A1)(M1)***

**Note:** Award ***A1*** for $10 \times 12 \times 800\ (= 96\,000)$ seen. Award ***M1*** for subtracting $10\,000$ **and** their total payments from $160\,000$.

(\$) $54\,000$ ***A1***
***[3 marks]***
***Total [6 marks]***

**1.** (a) (i) $42$ (km) ***A1***

(ii) $(\varepsilon =)\left|\frac{42-42.19}{42.19}\right|\times 100\%$ ***(M1)***

$0.450\ (\%)\ \ (0.450343\ldots(\%))$ ***A1***

**Note:** Award ***(M1)*** for correct substitution of their part (a)(i) into percentage error formula.

***[3 marks]***

(b) **METHOD 1**

attempt to substitute into arithmetic sequence formula ***(M1)***

$42=6+(n-1)\times 2$ **OR** $42=2n+4$ ***(A1)***

$(n=)\ 19$ ***A1***

**METHOD 2**

list of any consecutive terms ending on/above $42$ ***(M1)***

at least two correct terms (not including $6$, $8$ and $10$) ***(A1)***

$(n=)\ 19$ ***A1***

***[3 marks]***

(c) **METHOD 1**

$(S_{10}=)\frac{10}{2}(2\times 6+(10-1)2)$ ***(A1)***

$150(\text{km})$ ***A1***

**METHOD 2**

summing (some) correct terms up to $24$ **OR** list of correct cumulative values ***(A1)***

$6+8+10+\ldots+24$ **OR** $6, 14, 24, 36\ldots.$

$150\ (\text{km})$ ***A1***

***[2 marks]***

(d) **METHOD 1**

$3(1.2)^{n-1}$ ***(A1)***

**Note:** Award ***(A1)*** for correct substitution into geometric sequence formula.

comparing their geometric expression to $42$ ***(M1)***

*e.g.* $3(1.2)^{n-1} > 42$ **OR** $3(1.2)^{n-1} = 42$

$(n =)\ 15.4747\ldots$ seen **OR** $38.5175\ldots$ and $46.2210...$ seen (from table) ***(A1)***

$16$ (weeks) ***A1***

**METHOD 2**

List of values see. At least 3, 3.6, 4.32 etc ***(M1)(A1)***

38.5175… and 46.2210… seen ***(A1)***

$16$ (weeks) ***A1***

***[4 marks]***

(e) **METHOD 1**

attempt to substitute into geometric series formula ***(M1)***

**Note:** ***FT*** their $r$ from (d).

$150 < \frac{3\left(1-1.2^n\right)}{1-1.2}$ **OR** $150 = \frac{3\left(1-1.2^n\right)}{1-1.2}$ **OR** $13.1520\ldots$ ***(A1)***

$14$ (weeks) ***A1***

**METHOD 2**

evidence using tables in GDC or adding the first 14 terms ***(M1)***

both crossover values seen, e.g $145.589\ldots$ and $177.587\ldots$ ***(A1)***

$14$ (weeks) ***A1***

***[3 marks]***

***Total [15 marks]***

APOLLO

# Functions

64 questions

**4.** (a) $L(40) = 1.50 \times 40 - 5$ ***(M1)***

$= \$\ 55$ ***A1***

***[2 marks]***

(b) $70 = 1.50x - 5$ ***(M1)***

$(x =)\ 50$ litres ***A1***

***[2 marks]***

(c) $1.30x$ ***(A1)***

$1.30x < 1.50x - 5$ ***(M1)***

**Note:** Award ***M1*** for a graph showing two intersecting linear functions, provided one function has a $y$-intercept of $0$ and the other function has a negative $y$-intercept.

(minimum value of $k =$) $25$ ***A1***

**Note:** Accept $x > 25$.

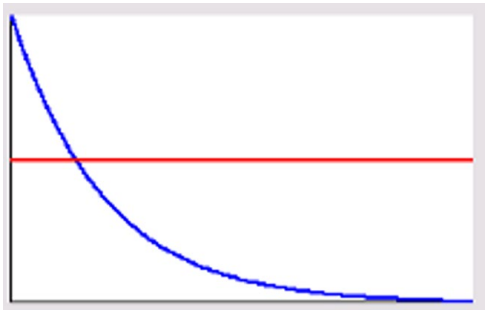
***[3 marks]***

***Total [7 marks]***

**7.** (a) **EITHER**

$50 = 100\,e^{-1\times p}$ **OR** $0.5 = e^{-1\times p}$ ***(M1)***

**OR**



***(M1)***

**THEN**

$0.693$ $(0.693147..., \ln 2)$ ***A1***

***[2 marks]***

(b) $R(1.5) = 100\,e^{-0.693147...\times 1.5}$ ***(M1)***

$35.4\,(\%)$ $(35.3553...)$ ***A1***

***[2 marks]***

(c) $R(t) > 0$ **OR** $R(t)$ has a horizontal asymptote ***R1***

***[1 mark]***

(d) *Award **A1** for **one** reasonable limitation of the domain:* ***A1***

small values of $t$ produce unrealistic results
$R(0) = 100\%$
large values of $t$ are not possible
people do not live forever
model is not valid at small or large values of $t$

*The reason should focus on the domain $t \geq 0$. Do not accept answers such as:*

recollection varies for different people
memories are discrete not continuous
the nature of the information will change how easily it is recalled
emotional/physical stress can affect recollection/concentration

**Note:** Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

***[1 mark]***

***Total [6 marks]***

**11.** (a) $I = \frac{k}{d^2}$ ***(M1)***

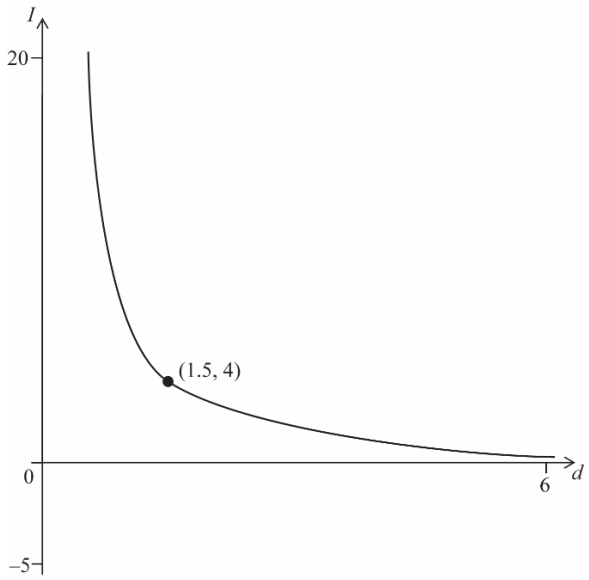
$4 = \frac{k}{1.5^2}$ ***M1***

$I = \frac{9}{d^2}$ ***AG***

**Note:** The ***AG*** line must be seen for the second ***M1*** to be awarded.
Award no marks for substituting $1.5$ and $4$ into $I = \frac{9}{d^2}$ (i.e., working backwards).

***[2 marks]***

(b)




***A1A1***

**Note:** Award ***A1*** for correct general shape (concave up) with no $I$-intercept, passing through the marked point $(1.5, 4)$; the point must be labelled with either the coordinates or the values $1.5$ and $4$ on the $x$ and $y$ axes. Award ***A1*** for the curve showing asymptotic behavior (i.e. $I$ tends to $0$, as $d$ tends to infinity), extending to at least $d = 6$; the curve must not cross nor veer away from the horizontal asymptote.

***[2 marks]***

(c) $1.5 \times 10^{-6} \geq \frac{9}{d^2}$ ***(M1)***

**Note:** Award ***(M1)*** for a correct inequality.

$d \geq 2450$ (m) $(2449.48...)$ ***A1***

**Note:** Award ***A0*** for $d = 2450$.

***[2 marks]***

***Total [6 marks]***

**1.** (a) (i) $23$ mg ***A1***

(ii) $1-0.85$ **OR** $\frac{23-19.55}{23}$ **OR** $0.15$ ***(M1)***

15 (%) ***A1***

***[3 marks]***

(b) $23(0.85)^{10}$ ***(M1)***

4.53 mg (4.52811…) ***A1***

***[2 marks]***

***Total [5 marks]***

**7.** (a) $(f(-7) =)\ 8$ and $(f(7) =)\ 1$ ***(A1)***

range is $f(x) \leq 1,\ f(x) \geq 8$ ***A1A1***

**Note:** Award at most ***A1A1A0*** if strict inequalities are used.

***[3 marks]***

(b) **EITHER**

sketch of $f$ and $y = 0$ or sketch of $f^{-1}$ and $x = 0$ ***(M1)***

**OR**

finding the correct expression of $f^{-1}(x) = \frac{-2-5x}{x-2}$ ***(M1)***

**OR**

$f^{-1}(0) = \frac{-2-5(0)}{0-2}$ ***(M1)***

**OR**

$f(x) = 0$ ***(M1)***

**THEN**

$f^{-1}(0) = 1$ ***A1***

***[2 marks]***

***Total [5 marks]***

**12.** (a) $3$ ***A1***

**Note:** Accept $(3, 0)$ seen.

***[1 mark]***

(b) **METHOD 1**

$0 = 4a - 2b + c$, $0 = 9a + 3b + c$, $-\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$ ***(M1)(A1)***

(i) $2$ ***A1***

(ii) $-2$ ***A1***

(iii) $-12$ ***A1***

**Note:** Award the ***(M1)(A1)*** if at least one correct value is seen.
Do not apply ***FT*** form part (a) if workings are not shown.

**METHOD 2**

$-12.5 = a(0.5 + 2)(0.5 - 3)$ ***(M1)***

(i) $a = 2$ ***A1***

$0 = 2 \times (3)^2 + 3b + c$
$0 = 2 \times (-2)^2 + (-2)b + c$ ***(M1)***

(ii) $b = -2$ ***A1***

(iii) $c = -12$ ***A1***

***[5 marks]***

(c) $x = 0.5$ ***A1***

**Note:** Do not ***FT*** from their part (b), this is a contradiction with the diagram.

***[1 mark]***

***Total [7 marks]***

**3.** (a) $(A =)\ 112$ ***A1***

***[1 mark]***

(b) $112\,\mathrm{e}^{5k} = 360$ ***(M1)***

**Note:** Award ***(M1)*** for their correct equation.

**EITHER**

graph of $y = 112\,\mathrm{e}^{5x}$ and $y = 360$ with indication of point of intersection ***(M1)***

**OR**

$(k =)\ \frac{1}{5}\ln\left(\frac{360}{112}\right)$ ***(M1)***

**Note:** Award ***(M1)*** for correct rearranging and use of $\log$.

**THEN**

$(k =)\ 0.234\ (0.233521...)$ ***A1***

**Note:** Award ***(M1)(M1)(A0)*** for $0.233$.

***[3 marks]***
***Total: [4 marks]***

**4.** (a) $m = \frac{6-0}{4-2} = 3$ ***(M1)A1***

***[2 marks]***

(b) $(m =)\ -\frac{1}{3}\ (-0.333, -0.333333...)$ ***A1***

***[1 mark]***

(c) an equation of line with a correct intercept and either of their gradients from (a) or (b) ***(M1)***

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

**Note:** Award ***(M1)*** for substituting either of their gradients from parts (a) or (b) and point B or $(3, 3)$ into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple ***A1***

***[2 marks]***

(d) $(x =)\ 12$ ***A1***

***[1 mark]***

***Total: [6 marks]***

**5.** (a) $h(4) = \frac{640}{4^2} + 0.5$ **OR** $h(14) = \frac{640}{14^2} + 0.5$ ***(M1)***

**Note:** Award ***(M1)*** for substituting $4$ or $14$ into $h$. This can be implicit from seeing $3.77\ (3.76530\ldots)$ or $40.5$.

$3.77 \leq h(x) \leq 40.5\ \ (3.76530... \leq h(x) \leq 40.5)$ ***A1A1***

**Note:** Award ***A1*** for both correct endpoints seen, ***A1*** for the endpoints in a correct interval.

***[3 marks]***

(b) (i) $h(x) = 10$ **OR** $h^{-1}(x) = \sqrt{\frac{640}{x - 0.5}}$ **OR** $h^{-1}(10) = \sqrt{\frac{640}{10 - 0.5}}$ ***(M1)***

$(x =)\ 8.21\ \text{cm}\ (8.20782...)$ ***A1***

(ii) a tin that is $10\ \text{cm}$ high will have a diameter of $8.21\ \text{cm}\ (8.20782...)$ ***A1***

**Note:** Condone a correct answer expressed as the converse.

(iii) $4 \leq h^{-1} \leq 14$ ***A1***

**Note:** Accept $4 \leq y \leq 14$. Do not ***FT*** in this part.

***[4 marks]***
***Total: [7 marks]***

**13.** (a) (i) $c = 10$ ***A1***

(ii) $64a + 8b + 10 = 10$ ***A1***

$16a + 4b + 10 = 12$ ***A1***

**Note:** Award ***A1*** for each equivalent expression or ***A1*** for the use of the axis of symmetry formula to find $4 = \frac{-b}{2a}$ or from use of derivative.
Award ***A0A1*** for $64a + 8b + c = 10$ and $16a + 4b + c = 12$.

(iii) $y = -\frac{1}{8}x^2 + x + 10$ ***A1A1***

**Note:** Award ***A1A0*** if one term is incorrect, ***A0A0*** if two or more terms are incorrect. Award at most ***A1A0*** if correct $a$, $b$ and $c$ values are seen but answer not expressed as an equation.

***[5 marks]***

(b) recognizing the need to integrate their expression ***(M1)***

$\int_0^8 -\frac{1}{8}x^2 + x + 10 \, \mathrm{d}x$ ***(A1)***

**Note:** Award ***(A1)*** for correct integral, including limits. Condone absence of $\mathrm{d}x$.

$90.7 \text{ cm}^2 \left(\frac{272}{3}, 90.6666\ldots\right)$ ***A1***

***[3 marks]***
***Total: [8 marks]***

**3.** (a) (i) maximum $h = 130$ metres ***A1***

(ii) minimum $h = 50$ metres ***A1***

***[2 marks]***

(b) (i) $(60 \div 12 =)\ 5$ seconds ***A1***

(ii) $360 \div 5$ ***(M1)***

**Note:** Award ***(M1)*** for $360$ divided by their time for one revolution.

$= 72^\circ$ ***A1***

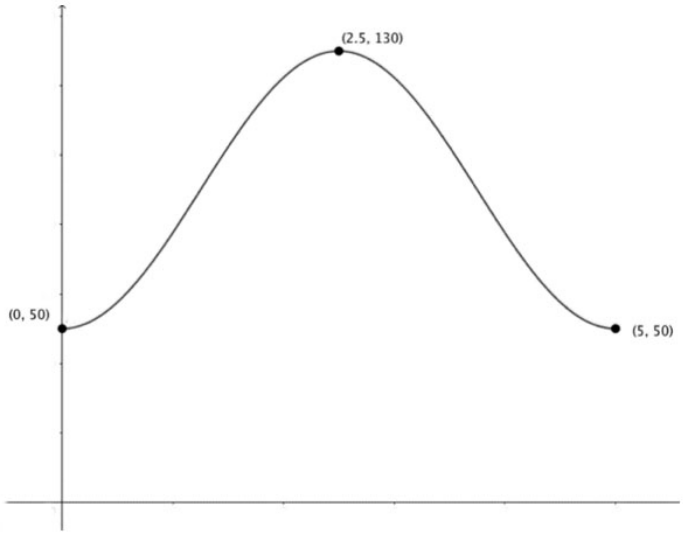
***[3 marks]***

(c) (i) (amplitude $=)\ 40$ ***A1***

(ii) (period $= \frac{360}{72} =\ )\ 5$ ***A1***

***[2 marks]***

(d)




Maximum point labelled with correct coordinates. ***A1***

At least one minimum point labelled. Coordinates seen for any minimum points must be correct. ***A1***

Correct shape with an attempt at symmetry and “concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain. ***A1***

***[3 marks]***

*continued…*

*Question 3 continued*

(e) (i) $h = 90 - 40\cos(144^\circ)$ ***(M1)***

$(h =)\ 122\,\text{m}\ \ (122.3606.....)$ ***A1***

(ii) evidence of $h = 100$ on graph **OR** $100 = 90 - 40\cos(72t)$ ***(M1)***

$t$ coordinates $3.55\ (3.54892...)$ **OR** $1.45\ (1.45107...)$ or equivalent ***(A1)***

**Note:** Award ***A1*** for either $t$-coordinate seen.

$= 2.10$ seconds $(2.09784\ldots)$ ***A1***

***[5 marks]***

(f) (i) $5 - 2.09784\ldots$ ***(M1)***

$\dfrac{(2.902153...)}{5}$ ***(M1)***

$0.580\ \ (0.580430...)$ ***A1***

(ii) **METHOD 1**

changing the frequency/dilation of the graph will not change the proportion of time that point C is visible. ***A1***

$0.580\ \ (0.580430...)$ ***A1***

**METHOD 2**

correct calculation of relevant found values

$\dfrac{(2.902153...)/2}{5/2}$ ***A1***

$0.580\ \ (0.580430...)$ ***A1***

**Note:** Award ***A0A1*** for an unsupported correct probability.

***[5 marks]***

***Total: [20 marks]***

**4.** (a) $20$ ***A1***

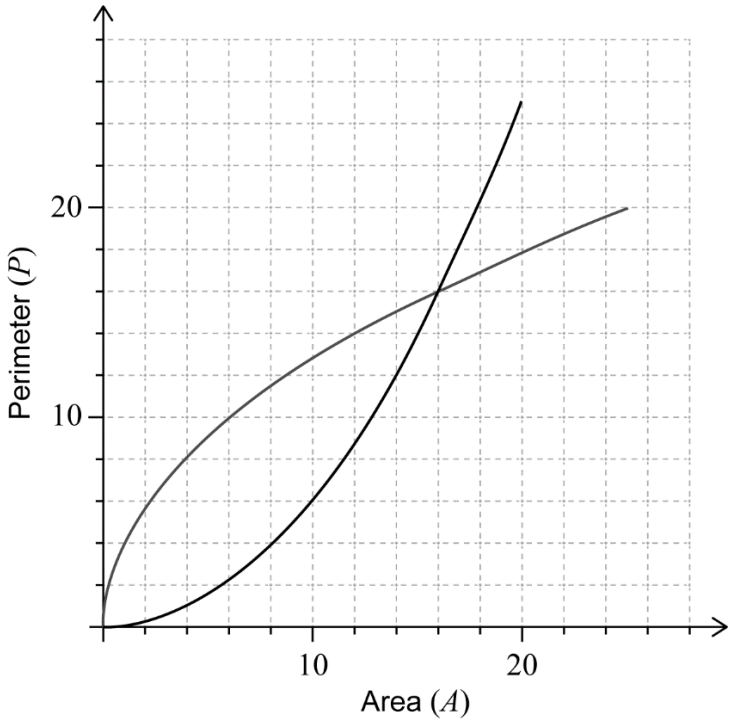
***[1 mark]***

(b) $n = 20$ ***A1***

**Note:** Follow through from part (a).

***[1 mark]***

(c)




***(M1)A1A1***

**Note:** Award ***(M1)*** for reflection in the line $P = A$, award ***A1*** for endpoint at $(20, 25)$, award ***A1*** for passing through $(16, 16)$.

***[3 marks]***

(d) when the perimeter is $8$, the area is $4$ ***A1***

***[1 mark]***

***Total [6 marks]***

**5.** (a) (i) $1750$ ***A1***

(ii) $1350 + 400\,(1.25)^{-5}$ ***(M1)***

$= 1480$ ***A1***

**Note:** Accept $1481$.

***[3 marks]***

(b) $1400 = 1350 + 400(1.25)^{-t}$ ***(M1)***

$9.32$ (days ($9.31885\ldots$) (days)) ***A1***

***[2 marks]***

(c) $1350$ ***A1***

**Note:** Accept $1351$ as a valid interpretation of the model as $P = 1350$ is an asymptote.

***[1 mark]***

***Total [6 marks]***

**8.** (a) $10\log_{10}\left(6.4\times10^{-3}\times10^{12}\right)$ ***(M1)***

$=98.1(\text{dB})\ (98.06179\ldots)$ ***A1***

***[2 marks]***

(b) $112=10\log_{10}\left(S\times10^{12}\right)$ ***(M1)***

$0.158\left(\text{W m}^{-2}\right)\left(0.158489\ldots\left(\text{W m}^{-2}\right)\right)$ ***A1***

***[2 marks]***

***Total [4 marks]***

**5.** (a) (i) $p(10)^2 + q(10) = 60$ ***M1***

$10p + q = 6 \ (100p + 10q = 60)$ ***A1***

(ii) $p = 1, \ q = -4$ ***A1A1***

**Note:** If $p$ and $q$ are both incorrect then award ***M1A0*** for an attempt to solve simultaneous equations.

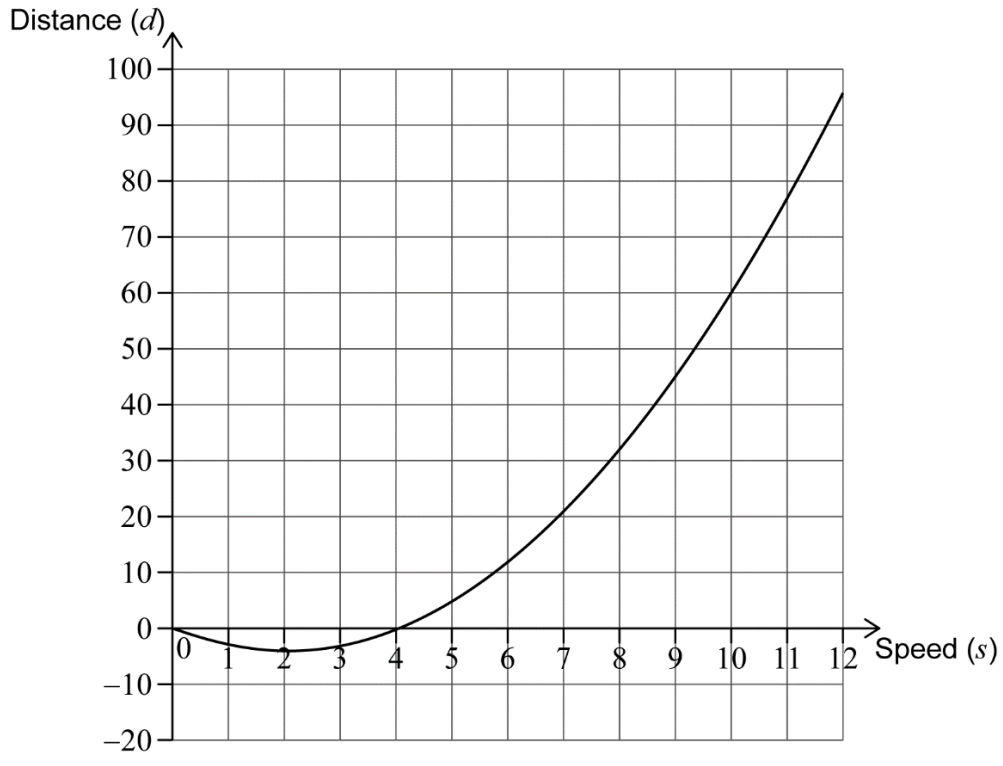
***[4 marks]***

(b) $(2, -4)$ ***A1A1***

**Note:** Award ***A1*** for each correct coordinate. Award ***A0A1*** if parentheses are missing.

***[2 marks]***

(c)




***A3***

**Note:** Award ***A1*** for smooth quadratic curve on labelled axes and within correct window. Award ***A1*** for the curve passing through $(0, 0)$ and $(10, 60)$. Award ***A1*** for the curve passing through their vertex. Follow through from part (b).

***[3 marks]***

(d) the graph indicates there are negative stopping distances (for low speeds) ***R1***

**Note:** Award ***R1*** for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances…).

***[1 mark]***

*continued…*

*Question 5 continued*

(e) $0.95\times 20^2 - 3.92\times 20$ ***(M1)***

$= 302\,(\text{m})$ $(301.6\ldots)$ ***A1***

***[2 marks]***

(f) $\left|\frac{301.6-320}{320}\right|\times 100$ ***M1***

$= 5.75\,(\%)$ ***A1***

***[2 marks]***

(g) $330 = 1.6\times s + 0.95\times s^2 - 3.92\times s$ ***M1A1***

**Note:** Award ***M1*** for an attempt to find an expression including stopping distance (model B) and reaction distance, equated to 330.
Award ***A1*** for a completely correct equation.

$19.9\,(\text{m s}^{-1})$ $(19.8988\ldots)$ ***A1***

***[3 marks]***

***Total [17 marks]***

**3.** (a) $1.2$ metres ***A1***

***[1 mark]***

(b) $-4.8t^2 + 21t + 1.2 = 0$ ***(M1)***

$(t =)\ 4.43\ \text{s}\ \ (4.431415...\ \text{s})$ ***A1***

**Note:** If both values for $t$ are seen do not award the ***A1*** mark unless the negative is explicitly excluded.

***[2 marks]***

(c) $0 \le t \le 4.43$ **OR** $[0,\ 4.43]$ ***A1A1***

**Note:** Award ***A1*** for correct endpoints and ***A1*** for expressing answer with correct notation. Award at most ***A1A0*** for use of $x$ instead of $t$.

***[2 marks]***
***[Total 5 marks]***

**11.** (a) $\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

***[2 marks]***

(b) **EITHER**

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left( = \frac{100000}{10^M} \right)$

**OR**

$100 = \frac{b}{10^3}$ ***(M1)***

**THEN**

$b = 100000 \ (= 10^5)$ ***A1***

***[2 marks]***

(c) $0.001 < N < 100000 \ (10^{-3} < N < 10^5)$ ***A1A1***

**Note:** Award ***A1*** for correct endpoints and ***A1*** for correct inequalities/interval notation.

***[2 marks]***

(d) $N = \frac{10^5}{10^{7.2}} \ (= 0.0063095...)$ ***(M1)***

length of time = $\frac{1}{0.0063095...} = 10^{2.2}$

= $158$ years ***A1***

***[2 marks]***

***[Total 8 marks]***

**4.** (a) $(\text{pH} =) \ -\log_{10}(1.3\times10^{-5})$ ***(M1)***

$4.89 \quad (4.88605\ldots)$ ***A1***

***[2 marks]***

(b) **EITHER**

calculating pH

$(\text{pH} =) \ -\log_{10}(10\times1.3\times10^{-5})$ ***(M1)***

$3.89 \quad (3.88605\ldots)$ ***A1***

$(3.89 < 4.89$, therefore) the unknown liquid is more acidic (than coffee). ***A1***

**Note:** Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the $C$-value to the pH level to earn the final ***A1***; a comparison of $C$-values only earns ***M0A0A0***.

**OR**

referencing the graph

The graph of $y = -\log_{10}(x)$ shows that as the value of $x$ increases, the value of $y$ decreases. ***M1***

Since the $C$-value ($x$-value) of the unknown liquid is larger than that of the coffee, the pH level ($y$-value) is lower. ***R1***

The unknown liquid is more acidic (than coffee). ***A1***

**Note:** Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the $C$-value to the pH level to earn the final ***A1***; a comparison of $C$-values only earns ***M0R0A0***.

***[3 marks]***

***Total [5 marks]***

**9.** (a) *Accept any one of the following (or equivalent):*
one minimum and one maximum point
three $x$-intercepts or three roots (or zeroes)
one point of inflexion ***R1***

**Note:** Do not accept "S shape" as a justification.

***[1 mark]***

(b) (i) $(d =) -5$ ***A1***

(ii) $8 = a + b + c$
$4 = 8a + 4b + 2c$
$0 = 27a + 9b + 3c$ ***A2***

**Note:** Award ***A2*** if all three equations are correct.
Award ***A1*** if at least one is correct. Award ***A1*** for three correct equations that include the letter "$d$".

(iii) $a = 2,\ b = -12,\ c = 18$ ***A1***

***[4 marks]***

(c) equating found expression to zero ***(M1)***
$0 = 2t^3 - 12t^2 + 18t - 5$
$t = 0.358216\ldots,\ 1.83174\ldots,\ 3.81003\ldots$ ***(A1)***

(so total time in debt is $3.81003\ldots - 1.83174\ldots + 0.358216 \approx$)
$2.34\ \ (2.33650\ldots)$ years ***A1***

***[3 marks]***
***Total [8 marks]***

**12.** (a) $(x=)\ -\frac{4.48}{2(-1.6)}$ **OR** coordinates of maximum point $(1.4,\ 3.136)$ ***(M1)***

$x = 1.4$ ***A1***

***[2 marks]***

(b) **METHOD 1**

the cart is centred in the archway when it is between
$x = 0.6$ and $x = 2.2$, ***A1***
where $y \geq 2.112\ (\text{m})$ (which is greater than $2$) ***R1***

the archway is tall enough for the crate ***A1***

**Note:** Do not award ***R0A1***.

**METHOD 2**

the height of the archway is greater or equal to $2.0$ between
$x = 0.557385\ldots$ and $x = 2.24261\ldots$ ***A1***
width of this section of archway =
$(2.24261\ldots - 0.557385\ldots =)\ 1.68522\ldots\ (\text{m})$ (which is greater than $1.6$) ***R1***

the archway is wide enough for the crate ***A1***

**Note:** Do not award ***R0A1***.

***[3 marks]***
***Total [5 marks]***

**1.** (a) **EITHER**
annual cycle for daylight length ***R1***

**OR**
there is a minimum length for daylight (cannot be negative) ***R1***

**OR**
a quadratic could not have a maximum and a minimum or equivalent ***R1***

**Note:** Do not accept "Paula's model is better".

***[1 mark]***

(b) (i) $4$ ***A1***

(ii) $12$ ***A1***

(iii) $y = 12$ ***A1A1***

**Note:** Award ***A1*** "$y = (\text{a constant})$" and ***A1*** for that constant being $12$.

***[4 marks]***

(c) $f(t) = -4\cos(30t) + 12$ **OR** $f(t) = -4\cos(-30t) + 12$ ***A1A1A1***

**Note:** Award ***A1*** for $b = 30$ (or $b = -30$), ***A1*** for $a = -4$, and ***A1*** for $d = 12$. Award at most ***A1A1A0*** if extra terms are seen or form is incorrect. Award at most ***A1A1A0*** if $x$ is used instead of $t$.

***[3 marks]***

(d) $10.5 = -4\cos(30t) + 12$ ***(M1)***

**EITHER**
$t_1 = 2.26585\ldots,\ t_2 = 9.73414\ldots$ ***(A1)(A1)***

**OR**
$t_1 = \frac{1}{30}\cos^{-1}\frac{3}{8}$ ***(A1)***
$t_2 = 12 - t_1$ ***(A1)***

**THEN**
$9.73414\ldots - 2.26585\ldots$
$7.47 \quad (7.46828\ldots)$ months $(0.622356\ldots \text{years})$ ***A1***

**Note:** Award ***M1A1A1A0*** for an unsupported answer of $7.46$. If there is only one intersection point, award ***M1A1A0A0***.

***[4 marks]***

*continued…*

*Question 1 continued*

(e) $\left|\dfrac{16-\left(16+\dfrac{14}{60}\right)}{16+\dfrac{14}{60}}\right|\times 100\,\%$ ***(M1)(M1)***

**Note:** Award ***M1*** for correct values and absolute value signs, ***M1*** for $\times 100$.

$=1.44\,\%\ \ (1.43737...\%)$ ***A1***

***[3 marks]***

***[Total 15 marks]***

**4.** (a) (i) an attempt to find the amplitude ***(M1)***

$\frac{61.8}{2}$ **OR** $\frac{64.5-2.7}{2}$

$(a=)\ 30.9\ \text{m}$ ***A1***

**Note:** Accept an answer of $(a=)\ -30.9\ \text{m}$.

(ii) $(\text{period} = \frac{60}{1.5} =)\ \ 40\ (\text{s})$ ***(A1)***

$((b=)\ \frac{360^\circ}{40})$

$(b=)\ 9$ ***A1***

**Note:** Accept an answer of $(b=)\ -9$.

(iii) attempt to find $d$ ***(M1)***

$(d=)\ 30.9+2.7$ **OR** $\frac{64.5+2.7}{2}$

$(d=)\ 33.6\ \text{m}$ ***A1***

***[6 marks]***

(b) $12\times1.5$ **OR** $\frac{12\times60}{40}$ ***(M1)***

$18$ (revolutions per ride) ***A1***

***[2 marks]***

(c) (i) $0\le t\le 720$ ***A1***

(ii) $2.7\le h\le 64.5$ ***A1A1***

**Note:** Award ***A1*** for correct endpoints of domain and ***A1*** for correct endpoints of range. Award ***A1*** for correct direction of both inequalities.

***[3 marks]***

(d) graph of $h(t)$ and $y=16.7$ **OR** $h(t)=16.7$ ***(M1)***

$6.31596\ldots$ and $33.6840\ldots$ ***(A1)***

$27.4\ (\text{s})\ \ (27.3680\ldots)$ ***A1***

***[3 marks]***

*continued…*

*Question 4 continued*

(e) (i) $d$ ***A1***

(ii) **EITHER**
$d + 30.9 = 65.2$ ***(A1)***

**OR**
$65.2 - (61.8 + 2.7) = 0.7$ ***(A1)***

**OR**
3.4 (new platform height) ***(A1)***

**THEN**
$(d =)$ 34.3 m ***A1***

***[3 marks]***
***Total [17 marks]***

**3.** (a) $1.8\ (\text{m})$ ***A1***

***[1 mark]***

(b) **EITHER**

$\dfrac{-10.8}{2(-3.6)}$ ***(M1)***

**OR**

$-7.2(t)+10.8=0$ ***(M1)***

**OR**

sketch indicating maximum ***(M1)***

**THEN**

$(t=)\ 1.5$ seconds ***A1***

**Note:** Award ***(M1)A0*** for $(1.5, 9.9)$ seen.

***[2 marks]***

(c) **EITHER**

$0=-3.6t^2+10.8t+1.8$ ***(M1)***

**OR**

sketch indicating a root ***(M1)***

**THEN**

$(t=)\ 3.16$ seconds $(3.15831...)$ ***A1***

**Note:** Award at most ***M1A0*** if $-0.158\ (-0.158312..)$ is part of the final answer unless clearly rejected.

***[2 marks]***

***Total [5 marks]***

**5**. (a) $71e^{-0.0514(16)} + 23$ ***(M1)***

$54.2\ °C\ \ (54.1956...)$ ***A1***

***[2 marks]***

(b) $T = 23$ ***A1***

**Note:** Condone $y = 23$.

***[1 mark]***

(c) $23\ °C$ ***A1***

***[1 mark]***

(d) $50 = 71e^{-0.0514(k)} + 23$ ***(M1)***

$k = 18.8\ \ \left(\frac{-5000}{257}\ln\left(\frac{27}{71}\right), 18.8101...\right)$ ***A1***

**Note:** Award ***M1*** for a sketch showing a point of intersection between the exponential function and $y = 50$.

***[2 marks]***
***Total [6 marks]***

**12.** (a) 78 ***A1***

***[1 mark]***

(b) (i) 65 ***A1***

(ii) **EITHER**

(period $=$) 16 (could be seen on sketch) ***(M1)***

$b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

**OR**

$143 = 65\sin(4b) + 78$ ***(M1)***

$(\sin(4b) = 1)$

$(4b = \frac{\pi}{2}$ **OR** $4b = 90^\circ)$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

***[3 marks]***

(c) 13 ***A1***

**Note:** Apply follow through marking only if their final answer is positive.

***[1 mark]***

(d) $(b =)\ 0.196\ \left(0.196349..., \frac{\pi}{16}\right)$ **OR** $(b =) 11.3^\circ\ (11.25^\circ)$ ***A1***

***[1 mark]***

***Total [6 marks]***

**5.** (a) $2$ ***A1***

***[1 mark]***

(b) attempt to substitute their part (a) and point $(3, -1)$ into the slope-intercept form or point-slope form of an equation ***(M1)***

$-1 = 2 \times 3 + c$ **OR** $y + 1 = 2(x - 3)$

$y = 2x - 7$ ***A1***

**Note:** Equation must be in the form $y = mx + c$ for ***A1*** to be awarded.

***[2 marks]***

*continued…*

*Question 5 continued*

(c) **METHOD 1**

attempt to show that P does not lie on $L_2$ ***(M1)***

e.g. $-\frac{1}{2}(3)-\frac{5}{2}$ **OR** graph showing $L_2$ and P in approximate correct locations

$-1 \neq -\frac{1}{2}(3)-\frac{5}{2}$ $(-1 \neq -4)$ **OR** $(3, -1)$ does not lie on the graph of $L_2$ ***R1***

hence $L_2$ is not the normal line to $f(x)$ at point P ***AG***

**METHOD 2**

attempt to find the equation of the normal line at $(3, -1)$ ***(M1)***

$(-1 = -\frac{1}{2}(3)+c$ **OR** $y+1 = -\frac{1}{2}(x-3))$

the normal line is $y = -\frac{1}{2}x+\frac{1}{2}$ ***R1***

hence $L_2$ is not the normal line to $f(x)$ at point P ***AG***

**METHOD 3**

attempt to find the intersection of $L_1$ and $L_2$ ***(M1)***

Intersection of $y = 2x-7$ and $y = -\frac{1}{2}x-\frac{5}{2}$ is $(1.8, -3.4)$

$x = 1.8 \neq 3$ **OR** $y = -3.4 \neq -1$ ***R1***

hence $L_2$ is not the normal line to $f(x)$ at point P ***AG***

**Note:** Accept equivalent written arguments provided values are seen.
Methods 1 and 2 are independent of the answers in (a) and (b) but FT marks can be given for Method 3.

***[2 marks]***

***[Total: 5 marks]***

**6.** (a) attempt to set up a direct variation equation that includes a constant, $k$, or the calculation of a constant using $12.3$ and $50$ ***(M1)***

e.g., $d = kv^2$ **OR** $12.3 = k \times 50^2$

$(k =)\ 0.00492 \quad \left(\frac{1}{203.252...}\right)$

$d = 0.00492v^2$ **OR** $d = \frac{v^2}{203}$ ***A1***

***[2 marks]***

(b) substituting $33$ for $d$ in their part (a) ***(A1)***

$33 = 0.00492 \times v^2$ **OR** $33 = \frac{v^2}{203.252...}$

$(v =)\ 81.9\ (\text{km h}^{-1}) \quad (81.8982\ldots\ (\text{km h}^{-1}))$ ***A1***

***[2 marks]***

(c) *Award **R1** for a reasonable variable that exists after the brakes are applied such as:*

- road material
- weather conditions
- condition/type of brakes
- weight/type of vehicle
- gradient/incline of road
- traction
- wind resistance
- friction

***R1***

**Note:** Do not accept a variable that refers to the timing of the brakes being applied such as:

- slow reaction time
- inexperienced driver

***[1 mark]***

***[Total: 5 marks]***

**10.** (a) (i) **METHOD 1**

attempt to find change in height of the ball using gradient ***(M1)***

$\frac{a}{0.43} = (-)0.045$

$a = (-)0.045 \times 0.43$

$a = (-)0.0194\,(\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

**METHOD 2**

attempt to find height at back of home plate ***(M1)***

horizontal distance to the front of the home plate $= 16.6666\ldots\ (\text{m})$

height at the back of the home plate $= -0.045(16.6666\ldots + 0.43) + 2$

$(= 1.23065\ (\text{m}))$

**Note:** The ***M1*** can be awarded for $16.6666\ldots + 0.43$ seen at some point.

$(a = 1.25 - 1.23065\ldots)$

$(a =)\ (-)0.0194\ (\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

(ii) $1.25 - 0.01935 = 1.23065$ (may be seen in part (a)(i)) ***A1***

$0.53 <\ 1.23065\ < 1.24$ ***R1***

therefore a strike ***AG***

**Note:** Do not award ***A0R1***.

***[4 marks]***

*continued…*

*Question 10 continued*

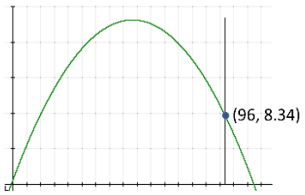
(b) **METHOD 1**

indication of $d = 96$ in the function $h(d)$ or its graph ***(M1)***

**EITHER**

$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$

**OR**




**THEN**

$(h(96) =)\ 8.34\ (\text{m})$ ***A1***

$8.34 > 5$ so the ball will go over the wall. ***A1***

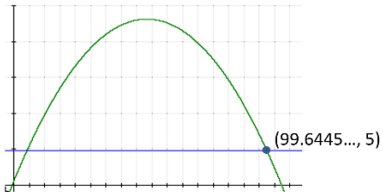
**METHOD 2**

indication of $h = 5$ in the function $h(d)$ or its graph ***(M1)***

**EITHER**

$5 = -0.01d^2 + 1.04d + 0.66$

**OR**




**THEN**

$d = 99.6\ (\text{m})\ (99.6445\ldots\ (\text{m}))\ (d = 4.35548\ldots\ (\text{m})$ may also be seen) ***A1***

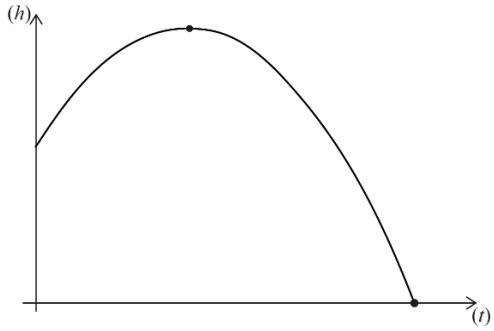
$96 < 99.6445\ldots$ so the ball will go over the wall. ***A1***

***[3 marks]***

***[Total: 7 marks]***

**10.** (a) **METHOD 1**

correct sketch with some indication of maximum point ***(M1)***




$0.921$ (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

**METHOD 2**

correct substitution into equation for line of symmetry ***(M1)***

$(t =) \ -\frac{8.75}{2 \times -4.75}$

$0.921$ (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

**METHOD 3**

equating the correct derivative to $0$ ***(M1)***

$-9.5t + 8.75 = 0$

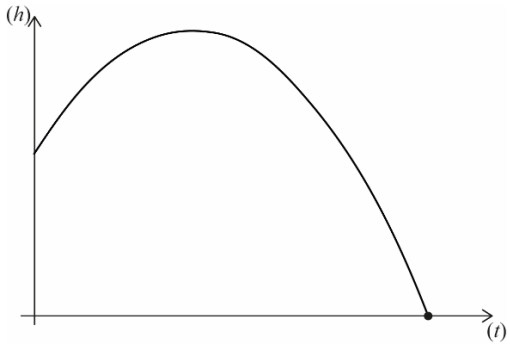
$0.921$ (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

**Note:** Award ***M1A0*** for a final answer of $0.92$ seen with no working.

***[2 marks]***

(b) **METHOD 1**

correct sketch with some indication of $x$-intercept




***(M1)***

**Note:** May be seen in part (a).

$2$ (seconds) ***A1***

*continued…*

*Question 10 continued*

**METHOD 2**
setting the equation to zero ***(M1)***

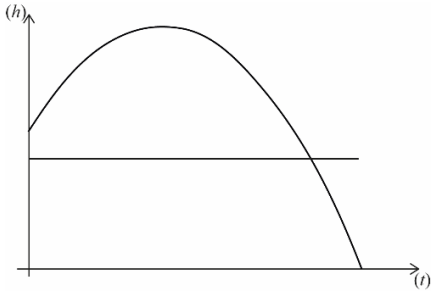
$-4.75t^2 + 8.75t + 1.5 = 0$

$2$ (seconds) ***A1***

**Note:** If both roots are given, with or without working, award ***(M1)A0***.

***[2 marks]***

(c) **METHOD 1**
correct sketch of quadratic function and a straight line in approximate correct position ***(M1)***




$1.88$ (seconds) $(1.87577\ldots$(seconds)) ***A1***

**METHOD 2**
setting the equation equal to $1.2$ ***(M1)***
$-4.75t^2 + 8.75t + 1.5 = 1.2$
$1.88$ (seconds) $(1.87577\ldots$(seconds)) ***A1***

**Note:** Award ***(M1)A0*** if $-0.0336702\ldots$ seen as (part of) a final answer.
Award ***M1A0*** for answer of $1.9$ seen without working.

***[2 marks]***

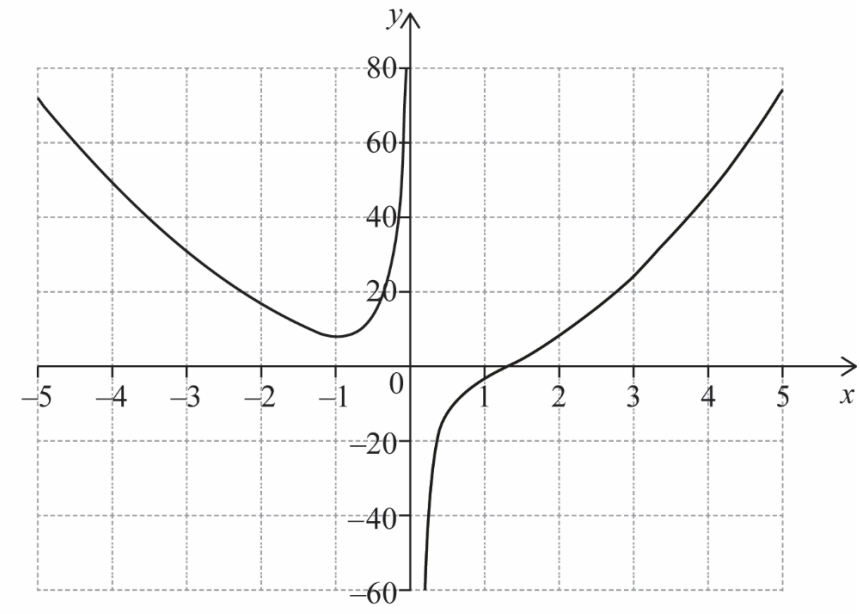
(d) *Award **R1** for each sensible reason, in the context of the question:* ***R1R1***
*e.g.*
The model ignores air resistance (or wind)
The model treats the ball as a point
The model assumes gravity is constant
The model assumes that the ball continues to follow the trajectory even after hitting the ground
This model ignores the bouncing back of the ball after hitting the ground

**Note:** Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

***[2 mark]***

***Total [8 marks]***

**11.** (a) (i)




***A1A1A1***

**Note:** Award ***A1*** for correct shape (curve must be smooth) in the second quadrant only, ***A1*** for asymptotic behaviour, ***A1*** for minimum point in approximately correct position AND left-most point around $(-5, 76)$ (allow a tolerance of half a square for these two points).

(ii) $-0.941\ (-0.941035...)$ ***A1***

***[4 marks]***

(b) $-2.45\ (-2.44651\ldots),\ -0.252\ (-0.252412),\ 2.70\ (2.69892\ldots)$ ***A2***

**Note:** Award ***A2*** for all three correct, ***A1*** for two correct and ***A0*** otherwise.
Award at most ***A1*** if additional solutions are seen.
Award ***A1A0*** for a final answer given as coordinates $(-2.45, 20)$, $(-0.252, 20)$, $(2.70, 20)$ or $(2.7, 20)$.
Award ***A1A0*** for three correct answers given in $2$ sf.

***[2 marks]***

(c) $x = 0$ ***A1***

***[1 mark]***

***Total [7 marks]***

**2.** (a) $(a =) 0$ ***A1***

***[1 mark]***

(b) $(1.39, 5.24)$ $((1.38672\ldots, 5.24025\ldots)$ ***A1A1***

**Note:** Award ***A1*** for each correct coordinate; do not accept 2 sf values. Award at most ***A0A1*** if parentheses are missing. Accept $x = 1.39$, $y = 5.24$.

***[2 marks]***

(c) any value greater than $5.24025\ldots$ ***A1***

**Note:** Accept a specific value **OR** a description of the correct interval for all values.

***[1 mark]***

*continued…*

*Question 2 continued*

(d) **METHOD 1**

attempt to relate gradient of function to graph of $f$ ***R1***

e.g. the function is increasing at $x = -4$

$m > 0$ ***A1***

**METHOD 2**

attempt to find value of $f'(-4)$ ***R1***

$\left( = \frac{25}{8} = 3.125 \right)$

$m > 0$ ***A1***

**Note:** The ***A1*** can only be awarded if the correct value of $f'(-4)$ or $m$ is found

**METHOD 3**

attempt to substitute $(-4, -12.75)$ into equation of tangent

$-12.75 = m(-4) - \frac{1}{4}$ so $m = 3.125$ ***R1***

$m > 0$ ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***

*continued…*

*Question 2 continued*

(e) (i) attempt to substitute $(0, -5)$ or $(3, 4.5)$ into $g(x)$ ***(M1)***

$-5 = k \times p^0 - 9$

$(k =)\ 4$ ***A1***

(ii) attempt to substitute $(3, 4.5)$ and their part (e)(i) into $g(x)$ ***(M1)***

$4.5 = 4 \times p^3 - 9$

$p^3 = 3.375$

$(p =)\ 1.5$ ***A1***

***[4 marks]***

(f) $y = -9$

***A2***

**Note:** Award ***A1*** for $-9$ seen, ***A2*** for a completely correct equation.
Condone $g(x) = -9$

***[2 marks]***

(g) $(x =)\ 4$ ***(M1)A1***

**Note:** Award ***M1*** for equating $f(x)$ to **their** $g(x)$
Award ***(M1)A0*** for the $y$-coordinate $(11.25)$ as the answer or part of the answer (e.g. coordinates).
Award ***(M1)A0*** for an answer of $(x =) -2.39421\ldots$

***[2 marks]***

*continued…*

*Question 2 continued*

(h) **METHOD 1:**

$f(-1) = 0 \Rightarrow h(-1) = g(-1)$ (This justification could be graphical.) ***R1***

yes ***A1***

**Note:** Award ***R1*** for any reasoning based on the fact that $-1$ is the $x$-intercept of $f$. Accept equivalent arguments in words. Do not award ***R0A1***.

**METHOD 2:**

$g(-1) = -\frac{19}{3}$ $(-6.33333\ldots)$

$h(x) = 3x - 1 + 4x^{-2} + 4 \times 1.5^{x} - 9$

$h(-1) = 3(-1) - 1 + 4(-1)^{-2} + 4 \times 1.5^{-1} - 9$

$h(-1) = -\frac{19}{3}$ $(-6.33333\ldots)$ ***R1***

yes ***A1***

**Note:** Award ***R1*** for showing that $g(-1) = -\frac{19}{3}$ and $h(-1) = -\frac{19}{3}$. Do not award ***R0A1***.

***[2 marks]***

***[Total: 16 marks]***

**3.** (a) $\frac{18-4}{2}$ ***(M1)***

$(a) = 7$ ***A1***

***[2 marks]***

(b) $\frac{18+4}{2}$ **OR** $18-7$ **OR** $4+7$ ***(M1)***

(d) $=11$ ***A1***

***[2 marks]***

(c) (time between high and low tide is) $6\text{h}15\text{m}$ **OR** $375$ minutes ***(A1)***

multiplying by $2$ ***(M1)***

$750$ minutes ***A1***

***[3 marks]***

(d) **EITHER**

$\frac{360^\circ}{b} = 750$ ***(A1)***

**OR**

$7\cos(b \times 375) + 11 = 4$ ***(A1)***

**THEN**

$(b =)\ 0.48$ ***A1***

**Note:** Award ***A1A0*** for an answer of $\frac{2\pi}{750}\left(=\frac{\pi}{375}=0.00837758...\right)$

***[2 marks]***

(e) equating their cos function to $6$ or graphing their cos function and $6$ ***(M1)***

$7\cos(0.48t)+11=6$

$\Rightarrow t = 282.468...$ (minutes) ***(A1)***

$=4.70780\ldots$ (hr) **OR** 4hr 42 mins (4hr $42.4681\ldots$ mins) ***(A1)***

so the time is $10{:}42$ ***A1***

***[4 marks]***

*continued…*

*Question 3 continued*

(f) next solution is $t = 467.531...$ ***(A1)***

$467.531... - 282.468...$

$185\ (\text{mins})\ (185.063...)$ ***A1***

**Note:** Accept an (unsupported) answer of 186 (from correct 3 sf values for ***t)***

***[2 marks]***

***[Total: 15 marks]***

**3.** (a) (i) $1200$ ***A1***

(ii) the initial population of the bacteria ***A1***

***[2 marks]***

(b) $1200 \times k^3 = 18750$ ***(A1)***

$(k =)\ 2.5$ ***A1***

***[2 marks]***

(c) $1200 \times 2.5^{1.5}$ ***(A1)***

$4740\ (4743.41\ldots)$ ***A1***

**Note:** Do not penalize if final answer is not given as an integer. Award ***(A1)A0*** for an answer of $3950\ (3949.14\ldots)$ from use of $1.3$ in the exponent, but only if working is shown.

***[2 marks]***

(d) equating $P(t)$ and $S(t)$ **OR** equating each function to a common variable ***(M1)***

$1200 \times 2.5^t = 5000 \times 1.65^t$; $1200 \times 2.5^t = x$ and $5000 \times 1.65^t = x$

$t = 3.43$ (hours) $(3.43456...)$ ***A1***

***[2 marks]***

*continued…*

*Question 3 continued*

(e) **METHOD 1**

$5000\times1.65^{t}=19000$ ***(M1)***

$(t=)\ 2.66586...$ **OR** $(t-2=)\ 0.66586...$ (seen) ***(A1)***

multiplying by $60$ seen to convert to minutes ***(M1)***
$(m=39.9521\ldots)$

$(m=)\ 40$ (minutes) **OR** $2$ hours and $40$ minutes ***A1***

**METHOD 2**

equating an expression for $S(t)$ to $19000$ ***(M1)***

expressing $t$ as $2+\frac{m}{60}$ ***(A1)***

$5000\times1.65^{2+\frac{m}{60}}=19000$

$2+\frac{m}{60}=2.66586\ldots$ ***A1***

$(m=)\ 40$ (minutes) **OR** $2$ hours and $40$ minutes ***A1***

**Note:** Award ***(M1)(A1)(M1)A0*** for an answer of $39.9521...$ or $39$ with or without working.

***[4 marks]***

(f) **EITHER (find volume of all bacteria)**

multiplying total population of bacteria by the volume of bacterium ***(M1)***
$1\times10^{-18}\times\left(5000\times1.65^{t}\right)$

setting expression equal to $2.1\times10^{-5}$ and attempt to solve (e.g. sketch) ***(M1)***

**OR (find total number of bacteria to fill container)**

attempting to find the total number of bacteria to fill container ***(M1)***

$2.1\times10^{-5}=n\times1\times10^{-18}$ **OR** $\frac{2.1\times10^{-5}}{1\times10^{-18}}$ **OR** $2.1\times10^{13}$

setting value equal to $S(t)$ and attempt to solve (e.g. sketch) ***(M1)***

**Note:** If $\frac{2.1\times10^{-5}}{1\times10^{-18}}$ is seen but candidate has an incorrect total number of bacteria, the second ***(M1)*** can still be awarded for setting their incorrect value equal to $S(t)$ and attempting to solve.

**THEN**

$t=44.2$ (hours) $(44.2480...)$ ***A1***

***[3 marks]***
***Total [15 marks]***

**6.** (a) setting $h(x) = 1$ ***(M1)***

$\left(h^{-1}(1) =\right) 17$ ***A1***

***[2 marks]***

(b) $x > \frac{1}{2}$ ***A1A1***

**Note:** Award ***A1*** for $\frac{1}{2}$ seen, ***A1*** for completely correct answer.

***[2 marks]***
***[Total 4 marks]***

**7.** (a) $2.82 = a(3)^2 + b(3) + c$ **OR** $2.82 = 9a + 3b + c$ ***A1***

***[1 mark]***

(b) finding other equations to solve simultaneously ***(M1)***

$4.25 = a(6)^2 + b(6) + c$ **AND** $5.30 = a(9)^2 + b(9) + c$
**OR** $4.25 = 36a + 6b + c$ **AND** $5.30 = 81a + 9b + c$

any one coefficient in equation correct ***(A1)***

$f(x) = -0.0211x^2 + 0.667x + 1.01$ ***A1***
$\left(f(x) = -0.0211111\ldots x^2 + 0.666666\ldots x + 1.01\right)$
$\left(f(x) = -\frac{19}{900}x^2 + \frac{2}{3}x + \frac{101}{100}\right)$

**Note:** Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

***[3 marks]***

(c) attempt to substitute $30$ into their equation ***(M1)***
$\left(f(30) =\right) 2.01$ ***A1***

$2.01 > 1.8$ **OR** therefore the discus will go over the wall ***R1***

**Note:** Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

***[3 marks]***

(d) setting their equation equal to zero **OR** graph with the zero indicated ***(M1)***
$0 = -0.0211111\ldots x^2 + 0.666666\ldots x + 1.01\ldots$ **OR** $f(x) = 0$

$33.0$ $(33.0275\ldots)$ (m) ***A1***

***[2 marks]***
***[Total 9 marks]***

**6.** (a) setting $h(x) = 1$ ***(M1)***

$\left(h^{-1}(1) =\right)\ 17$ ***A1***

***[2 marks]***

(b) $x > \frac{1}{2}$ ***A1A1***

**Note:** Award ***A1*** for $\frac{1}{2}$ seen, ***A1*** for completely correct answer.

***[2 marks]***
***[Total 4 marks]***

**7.** (a) $2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$ ***A1***

***[1 mark]***

(b) finding other equations to solve simultaneously ***(M1)***

$5 = a(10)^2 + b(10) + c$ **AND** $7.16 = a(17)^2 + b(17) + c$
**OR** $5 = 100a + 10b + c$ **AND** $7.16 = 289a + 17b + c$

any one coefficient in equation correct ***(A1)***
$f(x) = -0.00490x^2 + 0.441x + 1.08$ ***A1***

**Note:** Condone the 2 sf answer for the coefficient of $x^2$ i.e. $-0.0049$.

$\left(f(x) = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots\right)$
$\left(f(x) = -\frac{6}{1225}x^2 + \frac{108}{245}x + \frac{53}{49}\right)$

**Note:** Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

***[3 marks]***

(c) attempt to substitute $80$ into their equation ***(M1)***
$\left(f(80) =\right) 5$ ***A1***

$5 > 4$ **OR** therefore the ball will go over the fence ***R1***

**Note:** Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

***[3 marks]***

(d) setting their equation equal to zero, graph ***(M1)***
$0 = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots$ **OR** $f(x) = 0$

$92.4$ $(92.3902\ldots)$ (m) ***A1***

***[2 marks]***
***[Total 9 marks]***

**4.** (a) correct substitution of $t = 0$ ***(A1)***

$100 - 60 \times a^0$

$= 40\ (\%)$ ***A1***

***[2 marks]***

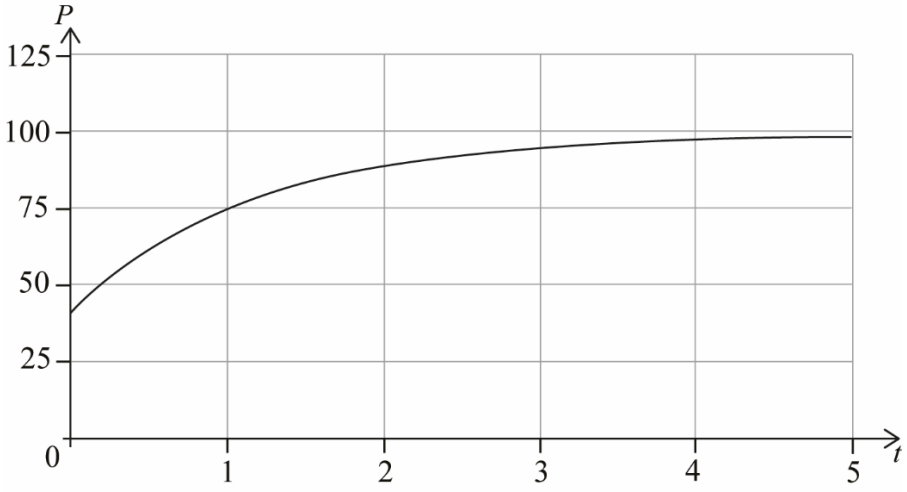
(b) correct substitution of $P = 75$ and $t = 1$ ***(A1)***

$75 = 100 - 60 \times a^{-1}$

$a = 2.4$ ***A1***

***[2 marks]***

(c)




***A1A1***

**Note:** Award ***A1*** for correct general shape (increasing, concave down) passing through $(1, 75)$ and continuing on, ***A1*** for a curve starting from $(0, 40)$, finishing around $(5, 100)$.

***[2 marks]***

(d) **EITHER**

has a horizontal asymptote at $P = 100$ ***A1***

**Note:** Accept a description of the asymptote in words.

**OR**

equation is $100$ minus a positive value and hence cannot equal $100$ ***A1***

**Note:** Award ***A0*** for a contextualized answer with no mathematical justification, e.g. “battery may degrade”.

***[1 mark]***

***[Total: 7 marks]***

**7.** (a) $a = 3$ ***A1***

***[1 mark]***

(b) period = $12$ ***(A1)***

$\left(\frac{360}{b} = 12 \quad \textbf{OR} \quad \frac{2\pi}{b} = 12\right)$

$b = 30 \qquad b = \frac{\pi}{6}$ ***A1***

***[2 marks]***

(c) equating their expression to $3$ ***(M1)***

$3 = 3\sin(30t) + 4 \quad \textbf{OR} \quad 3 = 3\sin\left(\frac{\pi}{6}t\right) + 4$

$t = 6.64904\ldots$ ***(A1)***

6:39 (pm) (18:39) ***A1***

**Note:** Follow through within the part for the final ***A1***; this mark is awarded for expressing **their** intermediate answer (seen) as a time correct to the nearest minute.

***[3 marks]***

***[Total: 6 marks]***

**10.** (a) (i) $m = ah^3$ ***(M1)***

$64 = a \times 0.8^3 \ \ (\Rightarrow a = 125)$ ***(A1)***

$m = 125h^3$ ***A1***

**Note:** The final answer must be written as $m$ in terms of $h$ to award the final ***A1***.

(ii) $m = (125(0.75)^3 =)\ 52.7\ (\text{kg})\ \ (52.7343...)$ ***A1***

***[4 marks]***

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$

$h = \sqrt[3]{1.76}\ \ (1.20736...)$ ***(A1)***

$E = k_1 h^2$ (seen anywhere) ***(A1)***

$k = \dfrac{\left(\sqrt[3]{1.76}\right)^2}{0.8^2}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

**OR (finding a formula for E in terms of m)**

$\left(m = 125h^3\right)$

$E = k_1 h^2$ ***(A1)***

$E = k_2 m^{\frac{2}{3}}$ ***(A1)***

$k = \dfrac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

***[4 marks]***

***[Total 8 marks]***

**4.** (a) expressing one variable in terms of the other ***(M1)***

$n = kB$ **OR** $n \propto B$

$60 = 5k$

$k = 12$ ***(A1)***

$n = 12(7)$

$= 84$ ***A1***

***[3 marks]***

(b) expressing $t$ terms of reciprocal of $B$ ***(M1)***

$t = \frac{c}{B}$ **OR** $t \propto \frac{1}{B}$

$8 = \frac{c}{5}$

$c = 40$ ***(A1)***

$(t =) \frac{40}{12}$ $\left(\frac{10}{3},\ 3.33,\ 3.33333\ldots\right)$ (hours) **OR** 3 hours 20 minutes ***A1***

**Note:** Award at most ***M1A1A0*** for the use of $t = cB$ in part (b) if $n = \frac{k}{B}$ was seen and penalized in part (a).

***[3 marks]***

***[Total: 6 marks]***

**5.** (a) $60$ is the daily rate of hire ***A1***
$10$ is the fixed charge for a helmet and repair kit ***A1***
***[2 marks]***

(b) $(C(5) =)\ 60 \times 5 + 10$ ***(A1)***
$310 \quad (\text{CAD})$ ***A1***
***[2 marks]***

(c) $3$ ***A1***
***[1 mark]***

(d) recognizing that $1270$ is the total cost ***(M1)***
$1270 = 60k + 10$
$k = 21$ ***A1***
***[2 marks]***
***[Total 7 marks]***

**6.** (a) $120$ (ng) ***A1***

**Note:** Only accept an exact answer of $120$.

***[1 mark]***

(b) $\left(m(20\,000) = \right) 120\,\mathrm{e}^{-0.000121 \times 20\,000}$ ***(A1)***

$10.7$ $(10.6705\ldots..)$ (ng) ***A1***

***[2 marks]***

(c) attempt to write or solve an inequality (or equality) with a value of $60$ seen ***(M1)***

$\left(m(t) = \right) 120\,\mathrm{e}^{-0.000121t} \leq 60$ **OR** sketch **OR** table of values

$(t >)$ $5728.489\ldots.$ ***(A1)***

$5729$ (years) ***A1***

**Note:** Do not accept $5728$ or $5730$ for the final ***A1***.

***[3 marks]***
***[Total: 6 marks]***

**5.** (a) $60$ is the daily rate of hire ***A1***
$10$ is the fixed charge for a helmet and repair kit ***A1***
***[2 marks]***

(b) $(C(5) =)\ 60 \times 5 + 10$ ***(A1)***
$310 \quad (\text{CAD})$ ***A1***
***[2 marks]***

(c) $3$ ***A1***
***[1 mark]***

(d) recognizing that $1270$ is the total cost ***(M1)***
$1270 = 60k + 10$
$k = 21$ ***A1***
***[2 marks]***
***[Total 7 marks]***

**6.** (a) $120$ (ng) ***A1***

**Note:** Only accept an exact answer of $120$.

***[1 mark]***

(b) $(m(20\,000) =)\ 120\,\mathrm{e}^{-0.000121\times 20\,000}$ ***(A1)***

$10.7\ (10.6705\ldots..)$ (ng) ***A1***

***[2 marks]***

(c) attempt to write or solve an inequality (or equality) with a value of $60$ seen ***(M1)***

$(m(t) =)120\,\mathrm{e}^{-0.000121t} \leq 60$ **OR** sketch **OR** table of values

$(t >)\ 5728.489\ldots.$ ***(A1)***

$5729$ (years) ***A1***

**Note:** Do not accept $5728$ or $5730$ for the final ***A1***.

***[3 marks]***
***[Total: 6 marks]***

**2.** (a) (i) $\frac{45-8}{2}$ ***A1***

$18.5\ (\text{m})$ ***AG***

**Note:** Use of $18.5$ in the calculation is reverse engineering, and falls short of the demand of the command term "Show that'. In such cases, award ***A0***.

(ii) $(2\pi \times 18.5 =)\ 116\ \ (116.2389\ldots,\ 37\pi)\ (\text{m})$. ***A1***

***[2 marks]***

(b) recognition the distance is the arc length ***(M1)***

$\frac{10°}{360°} \times 2\pi \times 18.5$ **OR** $\frac{116.2389\ldots}{36}$ ***(A1)***

$= 3.23\ \left(3.228859\ldots,\ \frac{37\pi}{36}\right)(\text{m})$ ***A1***

***[3 marks]***

(c) (i) $\left(b = \frac{360}{15} =\right)\ 24$ ***A1***

(ii) $\frac{\max + \min}{2}$ ***(A1)***

$(d =)\ \frac{45+8}{2}$ **OR** $45 - 18.5$ **OR** $8 + 18.5$

$= 26.5$ ***A1***

(iii) $h(t) = 18.5\sin(24t) + 26.5$ ***A1A1***

**Note:** Award ***A1*** for $18.5$ seen as parameter $a$, ***A1*** for a completely correct equation (including LHS).

***[5 marks]***

(d) attempt to equate $33$ with $h(t)$ **OR** sketch graph of curve and line ***(M1)***

$h(t)(= 18.5\sin(24t) + 26.5) = 33$

$t = 0.857\ \ (0.857083\ldots)$ and $t = 6.64\ \ (6.64291\ldots)$ (minutes) ***A1A1***

***[3 marks]***

(e) $4$ (revolutions per hour) seen **OR** $32$ (revolutions per day) seen ***(A1)***

$(4 \times 8 \times 3000 =)\ 96000$ ***(A1)***

$9.6 \times 10^4$ ***A1***

***[3 marks]***

***[Total 16 marks]***

**2.** (a) (i) $\frac{45-8}{2}$ ***A1***

$18.5\ (\text{m})$ ***AG***

**Note:** Use of $18.5$ in the calculation is reverse engineering, and falls short of the demand of the command term "Show that'. In such cases, award ***A0***.

(ii) $(2\pi \times 18.5 =)\ 116\ \ (116.2389\ldots,\ 37\pi)\ (\text{m})$ ***A1***

***[2 marks]***

(b) recognition the distance is the arc length ***(M1)***

$\frac{10°}{360°} \times 2\pi \times 18.5$ **OR** $\frac{116.2389\ldots}{36}$ ***(A1)***

$= 3.23\ \left(3.228859\ldots,\ \frac{37\pi}{36}\right)(\text{m})$ ***A1***

***[3 marks]***

(c) (i) $\left(b = \frac{360}{15} =\right)\ 24$ ***A1***

(ii) $\frac{\max + \min}{2}$ ***(A1)***

$(d =)\ \frac{45+8}{2}$ **OR** $45 - 18.5$ **OR** $8 + 18.5$

$= 26.5$ ***A1***

(iii) $h(t) = 18.5\sin(24t) + 26.5$ ***A1A1***

**Note:** Award ***A1*** for $18.5$ seen as parameter $a$, ***A1*** for a completely correct equation (including LHS).

***[5 marks]***

(d) attempt to equate $33$ with $h(t)$ **OR** sketch graph of curve and line ***(M1)***

$h(t)(= 18.5\sin(24t) + 26.5) = 33$

$t = 0.857\ \ (0.857083\ldots)$ and $t = 6.64\ \ (6.64291\ldots)$ (minutes) ***A1A1***

***[3 marks]***

(e) $4$ (revolutions per hour) seen **OR** $32$ (revolutions per day) seen ***(A1)***

$(4 \times 8 \times 3000 =)\ 96000$ ***(A1)***

$9.6 \times 10^4$ ***A1***

***[3 marks]***

***[Total 16 marks]***

**4.** (a) (gradient =) $\frac{0.8}{2.7}$ $\left(0.296,\ 0.296296\ldots,\ \frac{8}{27}\right)$ ***A1***

$0.296 < \frac{1}{3}$ $(0.333333\ldots)$ **OR** $\frac{8}{27} < \frac{9}{27}$ ***R1***

the gradient of the pool is less than the gradient in the regulation / the swimming pool satisfies the safety regulation ***AG***

***[2 marks]***

(b) expressing inverse proportionality using notation or using multiplication to calculate $k$ ***(M1)***

$w \propto \frac{1}{t}$ **OR** $w = \frac{k}{t}$ **OR** $(k =)\ 300 \times 4.5$

$300 \times 4.5 = 170 \times t$ or equivalent ***(A1)***

**Note:** Accept a calculation in minutes (i.e., $300 \times 270 = 170 \times t$ ).

$(t =)\ 7.94\ (7.94117\ldots)$ (h) ***A1***

**Note:** Award full marks for an answer given in minutes (i.e. $476$ $(476.470\ldots)$ minutes or $7$ hours and $56$ minutes). "Minutes" must be shown as the unit.

***[3 marks]***

***[Total: 5 marks]***

**5**. (a) $d = 5.75 \left( d = \frac{23}{4} \right)$ ***A1A1***

**Note:** Award ***A1*** for $5.75$ seen, ***A1*** for completely correct equation.

***[2 marks]***

(b) attempt to substitute into the axis of symmetry formula using *their* $5.75$ **OR**
attempt to substitute the coordinates $(3, 5.1)$ or $(8.5, 5.1)$ into the function ***(M1)***

$5.75 = \frac{-b}{2\times(-0.2)}$ **OR** $5.1 = -0.2(3)^2 + 3b$ **OR** $5.1 = -0.2(8.5)^2 + 8.5b$

$(b =)\ 2.3$ ***A1***

***[2 marks]***

(c) indication of finding zero or attempt to use symmetry ***(M1)***
$h(d) = 0$ **OR** $0 = -0.2d^2 + 2.3d$ **OR** $2 \times 5.75$ **OR** $3 - 0 = d - 8.5$ **OR**
resketch of graph with zero indicated

$(d =)\ 11.5\ (\text{m})$ ***A1***

**Note:** Award ***A0*** for a FT answer that is less than $8.5$ (contradicts the given information).

***[2 marks]***

(d) **EITHER**
the dolphin is below the surface of the water / the dolphin is underwater / the dolphin is in the water ***R1***

**OR**

the value of $h$ cannot be negative since the definition of the model only applies to the jump ***R1***

***[1 mark]***

***[Total: 7 marks]***

**10.** (a) $15p$ ***A1***

***[1 mark]***

(b) $(p-5)$ or $(n-15)$ seen ***A1***

adding a cost expression (multiplying a $p$ and an $n$) to their $15p$ ***(M1)***

$C(n)=15p+(p-5)(n-15)$ **OR** $C(n)=np-5n+75$ (or equivalent) ***A1***

***[3 marks]***

(c) (i) $(25.73\times100=)\ 2573$ (EUR) ***A1***

**Note:** Answer must be exact.

(ii) correctly substituting *their* total cost and $n=100$ into *their* cost function ***(A1)***
$2573=15p+(p-5)(100-15)$

$(p=)\ 29.98$ (EUR) ***A1***

**Note:** Answer must be written to 2 dp.

***[3 marks]***

***[Total: 7 marks]***

**11.** (a) $(W =) -34.1 - 7.33\ln(13)$ ***(A1)***

$-52.9$ $(-52.9010\ldots)$ ***A1***

***[2 marks]***

(b) attempt to work back from the approximate to the exact value ***(M1)***

$\frac{-52.9010\ldots - v_E}{v_E} = (\pm)0.06$ **OR** $(v_E =) \frac{-52.9010\ldots}{1 \pm 0.06}$ **OR** $v_E(1 \pm 0.06) = -52.9010\ldots$

$-56.3,\ -49.9$ $(-56.2777\ldots,\ -49.9066\ldots)$ ***A1A1***

**Note:** Award ***A1*** for each correct value.

***[3 marks]***

***[Total: 5 marks]***

**9.** (a) 3

***A1***

***[1 mark]***

(b) 1

***A1***

***[1 mark]***

(c)

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $f^{-1}(x)$ | 2 | 1 | 4 | 0 | 3 |

award ***A1*** for 1 correct, ***A2*** for 3 correct, ***A3*** for all 5 correct ***A1A1A1***

***[3 marks]***
***Total [5 marks]***

**Q3.**

(a) attempt to substitute $0.5$ into function ***(M1)***

$h(0.5) = \frac{20}{2 \times 0.5 + 5}$

$3.33\ (\text{m})\ (3.33333\ldots, \frac{10}{3})$ ***A1***

***[2 marks]***

(b) (i) **EITHER**

attempt to equate $h(t)$ to $2.5$ ***(M1)***

$\frac{20}{2t+5} = 2.5$

**OR**

attempt to substitute $2.5$ in $h^{-1}$ ***(M1)***

$\frac{20 - 5 \times 2.5}{2 \times 2.5}$

**OR**

graph of $h$, with an indication of the point $(t, 2.5)$ ***(M1)***

$t = 1.5$ (hours) ***A1***

(ii) It takes $1.5$ hours for the water level to reach 2.5 metres. ***R1***

***[3 marks]***

(c) $t \geq 0$ ***A1***

**Note**: Accept $h^{-1} \geq 0$.

***[1 mark]***

***Total [6 marks]***

4. (a) (i) $90^\circ$ ***A1***

(ii) $\sqrt{10^2+15^2}$ ***(M1)***

$18(.0\ )(\text{cm}\ )\quad (18.0277...)\quad \left(\sqrt{325}\right)$ ***A1***

***[3 marks]***

(b) (i) 150 ***A1***

(ii) 120 ***A1***

**Note:** Award ***A0A1*FT** for (0, 150) and (0, 120)

***[2 marks]***

(c) (i) 15 ***A1***

(ii) $\left(\frac{360}{60}=\right)6$ ***A1***

(iii) 135 ***A1***

***[3 marks]***

(d) (i) $(q=)\frac{360}{(60\times12)}=0.5$ or in 60 minutes, 30 degrees travelled so $(q=)\frac{30}{60}=\frac{1}{2}$ ***A1AG***

(ii) -10 ***A2***

**Note:** Award ***A1*** for 10.

***[3 marks]***

(e) substitution of $t=60$ into their $g(t)$ ***(M1)***

130 (cm) ***A1***

***[2 marks]***

(f) $15\cos(6t)+135=-10\sin(0.5t)+135$ ***(M1)***

$(t=)15.8806...$ ***(A1)***

(time =) 3:16 (pm) (15:16) ***A1***

**Note:** If the answer to part (d)(ii) is 10, then $t=14.2115$ and the time is 3:14 (pm) (15:14)

***[3 marks]***

***Total [16 marks]***

**Q4.**

(a) $\frac{360}{12}$ ***M1***

$=30$ ***AG***

**Note:** Award ***M0*** for use of given value, $30$.

***[1 mark]***

(b) substitution of $t=5$ into model $\left(-2.5\cos(30\times5)+4.5\right)$ ***(M1)***

$h(5)=6.67\left(6.66506...\right)$ ***A1***

**Note:** Award ***M1A0*** for $2.75187...$ (radian answer).

***[2 marks]***

(c) (i) $2.5$ (m) ***A1***

**Note:** Award ***A0*** for $-2.5$ (m)

(ii) $h(t)=4.5$ ***A1A1***

**Note**: Award ***A1*** for an equation of a horizontal line $h=$ constant ( accept $y=$ constant), ***A1*** for that constant being 4.5. Second ***A1*** is dependent on first ***A1***. Do not award ***A0A1***.

***[3 marks]***

(d) attempt at finding point of intersection between $h(t)=2.65$ and the curve. ***(M1)***

13.4 (13.4089…) ***(A1)***
13:25 (1:25 p.m.) ***A1***
***[3 marks]***

**Note**: Award ***M1A1A1FT*** for consistent use of radians 12.1720… 12:10, 0:10 p.m.
Award at most ***M1A0A0*** if the student works in the first cycle $\left(t=1.40895…\right)$

(e) $22.5910…-13.4089….-\ 0.5$ ***(A1)(M1)(M1)***

**Note**: Award ***A1*** for $22.5910…$seen, ***M1*** for subtracting their two times for harbour, ***M1*** for subtracting $0.5$ from their answer at any stage of the work.

$8.68$ (hours)$(8.68209…)$ (hours) ***A1***
***[4 marks]***

**Note**: Award at most ***A0M0M1A0*** for consistent use of radians. There is no 15 minute window for the ship to reach nor return from the fishing site.

***Total [13 marks]***

***Total [6 marks]***

**3.** (a) $0.80(5.1) + 1.50$ ***(A1)***
$(\$)5.58$ ***A1***
***[2 marks]***

(b) (i) $(a =)1$ ***A1***
(ii) $(b =)2$ ***A1***
***[2 marks]***

(c) the gradient of TurboEats $(0.8)$ is less than the gradient of Gogodart $(1)$ ***R1***
the $y$-intercept of TurboEats $(1.50)$ is less than the $y$-intercept of Gogodart $(2)$ ***R1***

**Note:** Accept contextual language, e.g. “cost per km” for gradient, and “fixed amount/constant” for y-intercept, in their explanations.
Award ***R1R1*** only for explicit consideration of both the gradient and y-intercept.

***[2 marks]***
***Total [6 marks]***

**8.** (a) $2000 + 30\,000(e^{-k\times 0})$ ***(A1)***

$32\,000$ (PHP) ***A1***

***[2 marks]***

(b) $32\,000 \times 0.60$ **OR** $32\,000 - 32\,000 \times 0.40$ **OR** $32\,000 \times 0.40$ ***(M1)***

$19\,200$ ***(A1)***

$19\,200 = 2000 + 30\,000(e^{-k\times 10})$ ***(M1)***

**Note:** Award ***M1*** for a correctly substituted equation including $10$ and *their* $19\,200$ (or equivalent LHS expression).

$k = 0.0556\ (0.0556287\ldots)$ ***A1***

***[4 marks]***

(c) (i) (the curve of $y = V(t)$ has a) horizontal asymptote at $y = 2000$ **OR** $V = 2000$ ***A1***

(ii) **EITHER**

newer models/updated technology will render her cell phone obsolete. ***R1***

**OR**

the cell phone will eventually stop working ***R1***

***[2 mark]***

***Total [8 marks]***

**10.** (a) recognizing the form of inverse variation ***(M1)***

$V = \frac{k}{t}$ **OR** $V \propto \frac{1}{t}$

$300 = \frac{k}{5}$ **OR** $k = 5 \times 300$ (the $k$ must be shown) ***A1***

$V = \frac{1500}{t}$ ***AG***

**Note:** Award ***M0A0*** for only verifying that $300$ and $5$ satisfy given equation, e.g $300 = \frac{1500}{5}$ seen.

***[2 marks]***

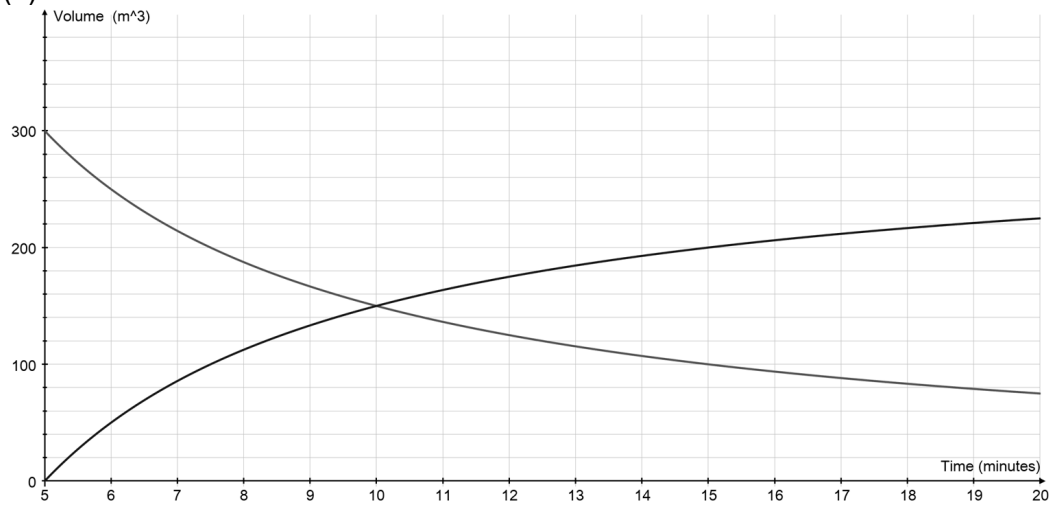
(b) $150$ seen ***(A1)***

$150 = \frac{1500}{t}$

$t = 10$ (minutes) ***A1***

***[2 marks]***

(c)




***A1A1***

**Note:** Award ***A1*** for an **increasing curve** from point $(5, 0)$. Award ***A1*** for a smooth curve, with approximately correct shape, eg concave down, approximately passing through $(10,150)$. Accept any endpoint in the range $[150, 300]$.

***[2 marks]***

(d) $(P(t) =) 300 - \frac{1500}{t}$ **OR** $300 - V(t)$ ***A1***

***[1 mark]***

***Total [7 marks]***

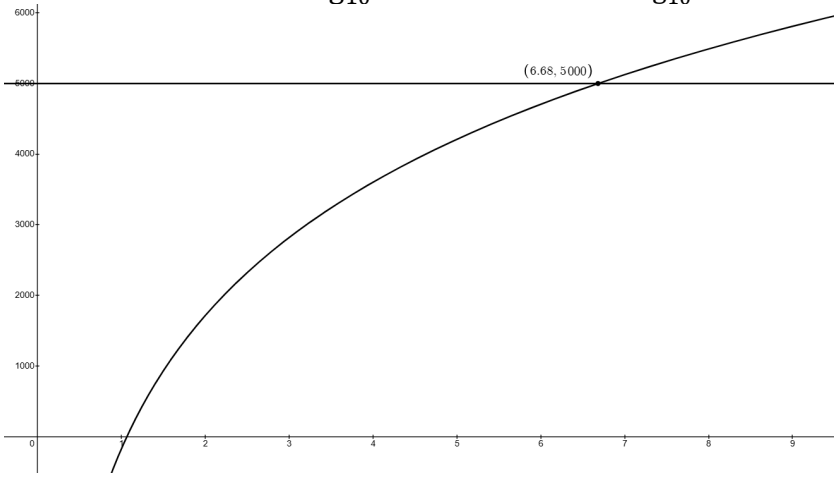
**2.** (a) $-167 + 6264\log_{10} 10$ ***(A1)***

$6100\ (6097)$ ***A1***

**Note:** Award ***A1A0***, as a special case if $14256.3\ldots$ seen as an unsupported answer, from use of $\ln$ in place of $\log$.

***[2 marks]***

(b) $5000 = -167 + 6264\log_{10} x$ **OR** $-167 + 6264\log_{10} x \geq 5000$ **OR** graph drawn ***(A1)***




$(x =) 6.681$ (years) ***A1***

**Note:** Award at most ***(A1)A0*** if the final answer is not given to 4 sf.

***[2 marks]***

(c) unrealistic ***A1***

$(-167 + 6264\log_{10} 1 = -167$ **OR** $-167 + 6264\log_{10} 1 < 0)$ **OR** graph drawn:

you cannot know a negative number of words ***R1***

**Note:** Do not award ***A1R0***. Award ***R1*** for $-167$ seen even if the explanation is not correct e.g. "Given that when $x = 1$, $y = -167$ the model is not realistic because it would mean that the child would be less than 1". Mention of "negative number of words", is sufficient for the ***R1***; the calculations can be implied.

***[2 marks]***

***Total [6 marks]***

**6.** (a) $-2.6$ $\left(\frac{-13}{5}\right)$ ***A1***

***[1 mark]***

(b) **EITHER**

$(-\infty, -3.67] \cup [-2.6, \infty)$ ***A1A1A1***

**OR**

$y \leq -3.67$ or $y \geq -2.6$ ***A1A1A1***

**Note:** Award ***A1*** for $-3.67$ $\left(-3.66666\ldots, -\frac{11}{3}\right)$ seen

***A1*** for their $-2.6\left(-\frac{13}{5}\right)$ seen and **both** sets of inequalities/brackets correct;

***A1*** if completely correct, including union symbol or "or".

It is possible to award ***A0A1A0*** for correctly presenting *their* $-3.67$ (and their $-2.6$).

The $-3.67$ maybe seen as part of a sketch.

Award ***A1A0A0*** for $-3.67 \leq y \leq -2.6$ or $[-3.67, -2.6]$.

***[3 marks]***

(c) setting $f(x)$ equal to $-2$ ***(M1)***

$f(x) = -2$ **OR** $-2 = \frac{4}{x} - 3$ **OR** sketch of graph with line labelled $-2$

4 ***A1***

***[2 marks]***

***Total [6 marks]***

**8.** (a) $(c =)13$ ***A1***

***[1 mark]***

(b) (i) $(1.5,0)$ ***A1***

(ii) $0 = 2.25a + 1.5b + 13$ ***A1***

**Note:** Accept $0 = (1.5)^2a + 1.5b + 13$.

***[2 marks]***

(c) **METHOD 1**

attempting to find another equation (e.g. in term of $R$, or setting $\frac{dy}{dx}$ to zero) ***(M1)***

$0 = 2.25a - 1.5b + 13$ **OR** $-\frac{b}{2a} = 0$ ***(A1)***

$a = -5.78\ \left(-\frac{52}{9},\ -5.77777\ ...\right),\ b = 0$ ***A1***

**METHOD 2**

deciding to use factorized form ***(M1)***

$y = a(x + 1.5)(x - 1.5)$

correct substitution of $(0,13)$ ***(A1)***

$13 = a(1.5)(-1.5)$

$a = -5.78\ \left(-\frac{52}{9},\ -5.77777\ ...\right),\ b = 0$ ***A1***

***[3 marks]***

***Total [6 marks]***

**12.** (a) (i) $(a =)100$ ***A1***

(ii) attempt to substitute values into given equation (at least two correct) ***(M1)***
$50 = ab^{138}$ **OR** $50 = 100b^{138}$ **OR** $0.5 = b^{138}$

**Note:** Award ***M1*** for a sketch of $y = 100x^{138}$ and $y = 50$.

$(b =)0.995(0.994989 \ldots)$ ***A1***
***[3 marks]***

(b) attempt to substitute (must include two of: $30$, their $b$, equated to $4$) ***(M1)***
$4 = ab^{30}$ **OR** $f(30) = a \times (0.994989 \ldots)^{30}$ **OR** $4 = a \times (0.994989 \ldots)^{t}$
$4 = a \times (0.994989 \ldots)^{30}$ ***(A1)***
$a = 4.65(4.65051 \ldots.)$ (micrograms) ***A1***
***[3 marks]***
***Total [6 marks]***

**5.** (a) $28 + 25\sin 30$ ***(A1)(M1)***

**Note:** Award ***A1*** for $30^\circ$ seen. Award ***M1*** for expressing the height of V as "28 + another value".

**OR**

$\left(\frac{3}{4}\right)(50)+3$ **OR** $3\times 12.5+3$ ***(A1)(M1)***

**THEN**

$40.5$ (m) ***A1***

***[3 marks]***

(b) $\frac{9}{12}$ **OR** $0.75$ seen ***(A1)***

$28$ (m) ***A1***

**Note:** Award ***(A1)A1FT*** for an answer of $25$ seen as the height of their midpoint in part (a).

***[2 marks]***

(c) $\frac{1}{3}$ seen **OR** sketch diagram with M and V at same (approx.) height/location

**OR** $\mathbf{28 - 25\cos(30t) = 28 + 25\sin(30(t+1))}$ ***(M1)***

$t = 4$ ***A1***

$t = 10$ ***A1***

***[3 marks]***

(d) $25$ ***A1***

***[1 mark]***

(e) (i) $(b =) \frac{360}{12}$ **OR** an attempt to form an equation from a known point ***(M1)***

$(b =)\ 30$ ***A1***

**Note:** Accept an answer of $\frac{2\pi}{12}, \frac{\pi}{6}, 0.523598...$, from use of radians (as degrees not explicit in question).

(ii) $(d =)\ 28$ ***A1***

**Note:** Accept an answer of $25$, if consistent with their working and answers in parts (a) and (b).

***[3 marks]***

(f) $10$ and $20$ seen ***(A1)***

$30$ (metres) ***A1***

***[2 marks]***

(g) $\left(b(18) =\right)\ 20\sin(36\times18)+40$ **OR** $21.0\ (20.9788...)$ ***(A1)***

$(k(18) =)\ 10\cos(72\times18)$ **OR** $-8.09\ (-8.09016...)$ ***(A1)***

$10\cos(72\times18)+20\sin(36\times18)+40$ ***(M1)***

**Note:** Award ***M1*** for summing their expressions or values.

$12.9\ (\text{m})\ (12.8886...)$ ***A1***

**Note:** Accept an answer of $(54.7844... - 0.92896... =)\ 53.8554...$, from use of radians (as degrees not explicit in question).

***[4 marks]***
***Total [18 marks]***

**4.** (a) $\frac{1.26-(-0.26)}{2}$ **OR** $\frac{1.52}{2}$ ***(A1)***

$0.76$ ***A1***

***[2 marks]***

(b) $\frac{1.26+(-0.26)}{2}$ ***(A1)***

$0.5$ ***A1***

***[2 marks]***

(c) **METHOD 1**

attempt to subtract the times at the minimum and maximum ***(M1)***

$10:48-03:36 \ (=7:12)$

$(7:12\times 2=)$ 14:24 ***(A1)***

$14.4$ ***A1***

**METHOD 2**

correct conversion of at least one time in hours, e.g $10.8$ or $3.6$ seen ***(A1)***

attempt to subtract their times ***(M1)***

$10.8-3.6 \ (=7.2)$

$(7.2\times 2 \text{ or } 3.6\times 4=) \ 14.4$ ***A1***

***[3 marks]***

(d) $\frac{360}{14\cdot 4}$ **OR** $\frac{2\pi}{14.4}$ ***(A1)***

**Note:** Award full marks for (d), (e) and (g) if radians used. ***FT*** from their (c).

$(b=) \ 25$ (seen, including in an incorrect equation) **OR** $0.436 \ (0.436332\ldots)$ ***(A1)***

$h(t)=0.76\sin(25t)+0.5$ **OR** $h(t)=0.76\sin(0.436t)+0.5$ ***A1***

**Note:** Do not accept $y$ and $x$ in place of $h(t)$ and $t$.

***[3 marks]***

(e) attempt to substitute $2.5$ into their equation ***(M1)***

**Note:** If they lost the mark in (c) because they did not convert to real number, **AND** substitute 2.3 in the formula, then award full ***FT*** marks.

$(h(2.5)=) \ 0.76\sin(25\times 2.5)+0.5$ **OR** $0.76\sin(0.436\times 2.5)+0.5$

$1.17 \ (1.17412\ldots)(\text{m})$ ***A1***

***[2 marks]***

(f) the road is above the water-level **OR** the island is accessible **OR** cars can drive on the road ***R1***

**Note:** Accept equivalent statements.

***[1 mark]***

(g) setting their equation to zero ***(M1)***

$0 = 0.76\sin(25t) + 0.5$ **OR** $0 = 0.76\sin(0.436332\ldots t) + 0.5$

$t = 8.84558\ldots,\ t = 12.7544\ldots,\ t = 23.2455\ldots$ ***(A1)(A1)***

**Note:** Award ***A1A0*** if only one or two correct $t$-values are seen.

finding differences between $t$-values ***(M1)***

$12.7544\ldots - 8.84558\ldots$ **OR** $24 - 23.2455\ldots$

4.66 (4.66325…) (hours) ***A1***

**Note:** Answer does not need to be in minutes, but accept "4 hours 40 minutes" or "280 minutes" as a correct final answer.

Award ***(M1)(A1)A0(M1)A0*** for an answer of "3.91 hours", "3 hours and 55 minutes", or "235 minutes" (i.e., only considering first minimum).

***[5 marks]***

***Total [18 marks]***

APOLLO

# Geometry and trigonometry

70 questions

**3**. (a) $\frac{1}{2}\times 4\times \pi \times 6^2 + \pi \times 6^2$ **OR** $3\times \pi \times 6^2$ ***(M1)(A1)(M1)***

**Note:** Award ***M1*** for use of surface area of a sphere formula (or curved surface area of a hemisphere), ***A1*** for substituting correct values into hemisphere formula, ***M1*** for adding the area of the circle.

$= 339 \text{ mm}^2 \ (108\pi, 339.292...)$ ***A1***

***[4 marks]***

(b) $\frac{339.292...}{240}$ ***(M1)***

$= 1.41 \text{ (g)} \left(\frac{9\pi}{20}, 0.45\pi, 1.41371...\right)$ ***A1***

***[2 marks]***

***Total [6 marks]***

**5.** (a) every point in the shaded region is closer to tower T4 ***R1***

**Note:** Specific reference must be made to the closeness of tower T4.

***[1 mark]***

(b) $(-9, 1)$ ***A1A1***

**Note:** Award ***A1*** for each correct coordinate. Award at most ***A0A1*** if parentheses are missing.

***[2 marks]***

(c) correct use of gradient formula ***(M1)***

*e.g.* $(m =) \frac{5-3}{-9--13} \left(= \frac{1}{2}\right)$

taking negative reciprocal of **their** $m$ (at any point) ***(M1)***

edge gradient $= -2$ ***A1***

***[3 marks]***

***Total [6 marks]***

**9.** attempt to find any relevant maximum value ***(M1)***
largest sides are $56.5$ and $82.5$ ***(A1)***
smallest possible angle is $102.5$ ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***

$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$

$= 2280 (\text{m}^2)$ $(2275.37...)$ ***A1***

***Total [5 marks]***

**2.** (a) attempt at substitution into 3D distance formula ***(M1)***

$\text{AB} = \sqrt{(140-20)^2 + (15-5)^2 + 250^2} \quad \left(= \sqrt{77\,000}\right)$

$= 277 \text{ m} \quad (10\sqrt{770},\ 277.488...)$ ***A1***

***[2 marks]***

(b) attempt at substitution in the midpoint formula ***(M1)***

$\left(\frac{140+20}{2}, \frac{15+5}{2}, \frac{0+250}{2}\right)$

$(80, 10, 125)$ ***A1***

***[2 marks]***

(c) 125 m ***A1***

***[1 mark]***

***Total [5 marks]***

**3.** $2\times 90\times 34\ (=6120)$ **AND** $2\times 42\times 34\ (=2856)$ ***(A1)***

$90\times 42\ (=3780)$ ***(A1)***

$r=21$ ***(A1)***

$\pi\times 21^2\ (=441\pi, 1385.44...)$ ***(M1)***

use of curved surface area formula ***(M1)***

$21\pi\times 90\ (=1890\pi, 5937.61...)$ ***(A1)***

$20100\ \text{cm}^2\ (20079.0\ldots)$ ***A1***

***Total [7 marks]***

**4.** (a) $25^\circ$ ***A1***

***[1 mark]***

(b) $\text{AC} = \dfrac{380}{\tan 25^\circ}$ **OR** $\text{AC} = \sqrt{\left(\dfrac{380}{\sin 25^\circ}\right)^2 - 380^2}$ **OR** $\dfrac{380}{\sin 25^\circ} = \dfrac{\text{AC}}{\sin 65^\circ}$ ***(M1)***

$\text{AC} = 815 \text{ m } (814.912\ldots)$ ***A1***

***[2 marks]***

(c) **METHOD 1**

attempt to find AB ***(M1)***

$\text{AB} = \dfrac{380}{\tan 40^\circ}$

$= 453 \text{ m } (452.866\ldots)$ ***(A1)***

$\text{BC} = 814.912\ldots - 452.866\ldots$

$= 362 \text{ m } (362.046\ldots)$ ***A1***

**METHOD 2**

attempt to find HB ***(M1)***

$\text{HB} = \dfrac{380}{\sin 40^\circ}$

$591 \text{ m } (= 591.175...)$ ***(A1)***

$\text{BC} = \dfrac{591.175... \times \sin 15^\circ}{\sin 25^\circ}$

$= 362 \text{ m } (362.046...)$ ***A1***

***[3 marks]***

(d) $362.046\ldots \times 4$

$= 1450 \text{ m h}^{-1} \ (1448.18\ldots)$ ***A1***

***[1 mark]***

***Total [7 marks]***

**6.** (a) gradient $\mathrm{AB} = \frac{4}{12}\left(\frac{1}{3}\right)$ ***(A1)***

midpoint $\mathrm{AB}: (8, 22)$ ***(A1)***

gradient of bisector $= -\frac{1}{\text{gradient AB}} = -3$ ***(M1)***

perpendicular bisector: $22 = -3 \times 8 + b$ **OR** $(y - 22) = -3(x - 8)$ ***(M1)***

perpendicular bisector: $y = -3x + 46$ ***A1***

***[5 marks]***

(b) attempt to solve simultaneous equations ***(M1)***

$x + 4 = -3x + 46$

$(10.5, 14.5)$ ***A1***

***[2 marks]***

***Total [7 marks]***

**9.** (a) $135^\circ \times \frac{12\pi}{360^\circ}$ ***(M1)(A1)***

$14.1\ (\text{m})\ \ (14.1371\ldots)$ ***A1***

***[3 marks]***

(b) evidence of splitting region into two areas ***(M1)***

$135^\circ \times \frac{\pi 6^2}{360^\circ} - \frac{6 \times 6 \times \sin 135^\circ}{2}$ ***(M1)(M1)***

**Note:** Award ***M1*** for correctly-substituting into area of sector formula, ***M1*** for evidence of substituting into area of triangle formula.

$42.4115\ldots - 12.7279\ldots$

$29.7\ \text{m}^2\ \ (29.6835\ldots)$ ***A1***

***[4 marks]***

***Total [7 marks]***

**2.** (a) $4 \times \frac{360^\circ}{12}$ **OR** $4 \times 30^\circ$ ***(M1)***

$120^\circ$ ***A1***

***[2 marks]***

(b) substitution in cosine rule ***(M1)***

$AB^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos(120^\circ)$ ***(A1)***

$AB = 14 \text{ cm}$ ***A1***

**Note:** Follow through marks in part (b) are contingent on working seen.

***[3 marks]***

(c) $\theta = 13 \times 6$ ***(M1)***

$= 78^\circ$ ***A1***

***[2 marks]***

(d) substitution into the formula for arc length ***(M1)***

$l = \frac{78}{360} \times 2 \times \pi \times 10$ **OR** $l = \frac{13\pi}{30} \times 10$

$= 13.6 \text{ cm} \left( 13.6135..., 4.33\pi, \frac{13\pi}{3} \right)$ ***A1***

***[2 marks]***

(e) substitution into the area of a sector ***(M1)***

$A = \frac{78}{360} \times \pi \times 10^2$ **OR** $l = \frac{1}{2} \times \frac{13\pi}{30} \times 10^2$

$= 68.1 \text{ cm}^2 \left( 68.0678..., 21.7\pi, \frac{65\pi}{3} \right)$ ***A1***

***[2 marks]***

(f) 23 ***A1***

***[1 mark]***

*continued…*

*Question 2 continued*

(g) correct substitution ***(M1)***

$h = 10\cos(160^\circ) + 13$

$= 3.60 \text{ cm} \quad (3.60307...)$ ***A1***

***[2 marks]***

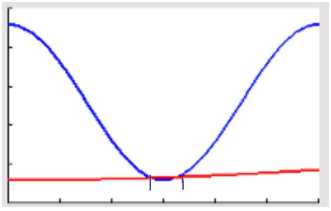
(h) $10$ ***A1***

***[1 mark]***

(i) **EITHER**

$10 \times \cos(\theta) + 13 = -10 \times \cos\left(\frac{\theta}{12}\right) + 13$ ***(M1)***

**OR**



***(M1)***

**Note:** Award ***M1*** for equating the functions. Accept a sketch of $h(\theta)$ and $g(\theta)$ with point(s) of intersection marked.

**THEN**

$k = 196^\circ \quad (196.363...)$ ***A1***

**Note:** The answer $166.153\ldots$ is incorrect but the correct method is implicit. Award ***(M1)A0***.

***[2 marks]***

***Total [17 marks]***

**2.** (a) use of cosine rule ***(M1)***

$$\hat{ACB} = \cos^{-1}\left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225}\right) \quad \textit{(A1)}$$

$= 32.0^\circ \ (31.9980...)$ ***A1***

***[3 marks]***

(b) use of sine rule ***(M1)***

$$\frac{\text{DE}}{\sin 31.9980...^\circ} = \frac{210}{\sin 100^\circ} \quad \textit{(A1)}$$

$(\text{DE} =)\ 113\ \text{m}\ \ (112.9937...)$ ***A1***

***[3 marks]***

(c) **METHOD 1**

$180^\circ - (100^\circ + \text{ their part } (a))$ ***(M1)***

$= 48.0019\ldots^\circ$ **OR** $0.837791...$ ***(A1)***

substituted area of triangle formula ***(M1)***

$$\frac{1}{2} \times 112.9937... \times 210 \times \sin 48.002^\circ \quad \textit{(A1)}$$

$8820\ \text{m}^2\ \ (8817.18\ldots)$ ***A1***

**METHOD 2**

$$\frac{\text{CE}}{\sin(180 - 100 - \text{their part } (a))} = \frac{210}{\sin 100} \quad \textit{(M1)}$$

$(\text{CE} =)\ 158.472...$ ***(A1)***

substituted area of triangle formula ***(M1)***

**EITHER**

$$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100 \quad \textit{(A1)}$$

**OR**

$$\frac{1}{2} \times 210... \times 158.472... \times \sin(\text{their part } (a)) \quad \textit{(A1)}$$

**THEN**

$8820\ \text{m}^2\ \ (8817.18\ldots)$ ***A1***

*continued…*

*Question 2 continued*

**METHOD 3**

$\text{CE}^2 = 210^2 + 112.993...^2 - \left(2 \times 210 \times 112.993... \times \cos(180 - 100 - \text{their part } (a))\right)$ ***(M1)***

$(\text{CE} =)\ 158.472\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$ ***(A1)***

$8820\ \text{m}^2\ \ (8817.18\ldots)$ ***A1***

***[5 marks]***

(d) $1005 - 210$ **OR** $795$ ***(A1)***

equating answer to part (c) to area of a triangle formula ***(M1)***

$8817.18\ldots = \frac{1}{2} \times \text{DF} \times (1005 - 210) \times \sin 48.002...^\circ$ ***(A1)***

$(\text{DF} =)\ 29.8\ \text{m}\ \ (29.8473...)$ ***A1***

***[4 marks]***

***Total [15 marks]***

**2.** (a) attempt at substituting the cosine rule formula ***(M1)***

$$\cos\theta = \frac{1660^2 + 1550^2 - 1670^2}{2(1660)(1550)}$$ ***(A1)***

$(\theta =)\ 62.6^\circ\ (62.5873\ldots)$ (accept 1.09 rad (1.09235…)) ***A1***

***[3 marks]***

(b) correctly substituted area of triangle formula ***(M1)***

$$A = \frac{1}{2}(1660)(1550)\sin(62.5873\ldots)$$

$(A =)\ 1140000\ (1.14\times10^6, 1142043.327\ldots)\ \text{km}^2$ ***A1***

**Note:** Accept $1150000\ (1.15\times10^6, 1146279.893\ldots)\ \text{km}^2$ from use of $63^\circ$. Other angles and their corresponding sides may be used.

***[2 marks]***

***Total: [5 marks]***

**7.** (a) (the best placement is either point P or point Q)
attempt at using the distance formula ***(M1)***

$\text{AP} = \sqrt{(10-6)^2+(6-2)^2}$ **OR**

$\text{BP} = \sqrt{(10-14)^2+(6-2)^2}$ **OR**

$\text{DP} = \sqrt{(10-10.8)^2+(6-11.6)^2}$ **OR**

$\text{BQ} = \sqrt{(13-14)^2+(7-2)^2}$ **OR**

$\text{CQ} = \sqrt{(13-18)^2+(7-6)^2}$ **OR**

$\text{DQ} = \sqrt{(13-10.8)^2+(7-11.6)^2}$

$(\text{AP or BP or DP} =)\ \sqrt{32} = 5.66\ \ (5.65685...)$ **AND**
$(\text{BQ or CQ or DQ} =)\ \sqrt{26} = 5.10\ \ (5.09901...)$ ***A1***

$\sqrt{32} > \sqrt{26}$ **OR** AP (or BP or DP) is greater than BQ (or CQ or DQ) ***A1***

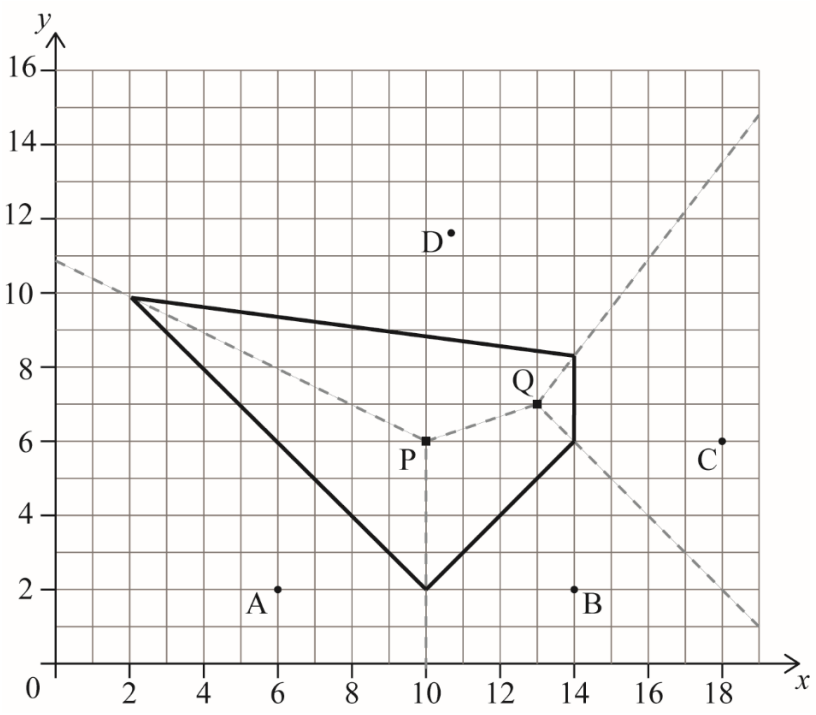
point P is the furthest away ***AG***

**Note:** Follow through from their values provided their AP (or BP or DP) is greater than their BQ (or CQ or DQ).

***[3 marks]***

(b) (i) $x = 14$ ***A1***

(ii)




***A1A1***

**Note:** Award ***A1*** for each correct straight line. Do not ***FT*** from their part (b)(i).

***[3 marks]***
***Total: [6 marks]***

**8.** (a) (i) $13\pi$ cm ***A1***

**Note:** Answer must be in terms of $\pi$.

(ii) **METHOD 1**

$\frac{\theta}{360} \times 2\pi(18) = 13\pi$ **OR** $\frac{\theta}{360} \times 2\pi(18) = 40.8407...$ ***(M1)***

**Note:** Award ***(M1)*** for correct substitution into length of an arc formula.

$(\theta =)\ 130^\circ$ ***A1***

**METHOD 2**

$\frac{\theta}{360} \times \pi \times 18^2 = \pi \times 6.5 \times 18$ ***(M1)***

$(\theta =)\ 130^\circ$ ***A1***

***[3 marks]***

(b) **EITHER**

$\frac{130}{360} \times \pi(18)^2$ ***(M1)***

**Note:** Award ***(M1)*** for correct substitution into area of a sector formula.

**OR**

$\pi(6.5)(18)$ ***(M1)***

**Note:** Award ***(M1)*** for correct substitution into curved area of a cone formula.

**THEN**

$(\text{Area} =)\ 368\ \text{cm}^2\ \ (367.566...,117\pi)$ ***A1***

**Note:** Allow ***FT*** from their part (a)(ii) even if their angle is not obtuse.

***[2 marks]***

***Total: [5 marks]***

**4.** (a) $\tan(\theta) = \frac{6}{10}$ ***(M1)***

$(\theta =)\ 31.0^\circ\ (30.9637...^\circ)$ **OR** $0.540\ (0.540419\ldots)$ ***A1***

***[2 marks]***

(b) (i) $(\text{CV} =)\ 40\tan(\theta)$ **OR** $(\text{CV} =)\ 4\times 6$ ***(M1)***

**Note:** Award ***(M1)*** for an attempt at trigonometry or similar triangles (e.g. ratios).

$(\text{CV} =)\ 24\ \text{m}$ ***A1***

(ii) $(V =)\ \frac{1}{3}80^2\times 24 - \frac{1}{3}60^2\times 18$ ***M1A1A1***

**Note:** Award ***M1*** for finding the difference between the volumes of two pyramids, ***A1*** for each correct volume expression. The final ***A1*** is contingent on correct working leading to the given answer.
If the correct final answer is not seen, award at most ***M1A1A0***. Award ***M0A0A0*** for any height derived from $V = 29\,600$, including $18.875$ or $13.875$.

$(V =)\ 29\,600\ \text{m}^3$ ***AG***

***[5 marks]***

(c) **METHOD 1**

$\left(\frac{29\,600}{80} =\right)\ 370$ (days) ***A1***

$(370 > 366)$ Joshua is correct ***A1***

**Note:** Award ***A0A0*** for unsupported answer of "Joshua is correct". Accept $1.01... > 1$ for the first ***A1*** mark.

**METHOD 2**

$80\times 366 = 29280\ \text{m}^3$ **OR** $80\times 365 = 29200\ \text{m}^3$ ***A1***

$(29280 < 29600)$ Joshua is correct ***A1***

**Note:** The second ***A1*** can be awarded for an answer consistent with their result.

***[2 marks]***

*continued…*

*Question 4 continued*

(d) height of trapezium is $\sqrt{10^2+6^2}$ $(=11.6619...)$ ***(M1)***

area of trapezium is $\frac{80+60}{2}\times\sqrt{10^2+6^2}$ $(=816.333...)$ ***(M1)(A1)***

$(SA=)$ $4\times\left(\frac{80+60}{2}\times\sqrt{10^2+6^2}\right)+60^2$ ***(M1)***

**Note:** Award ***M1*** for adding 4 times their (MNOP) trapezium area to the area of the $(60\times60)$ base.

$(SA=)$ $6870\ \text{m}^2$ $(6865.33\ \text{m}^2)$ ***A1***

**Note:** No marks are awarded if the correct shape is not identified.

***[5 marks]***
***Total: [14 marks]***

**7.** (a) $\frac{3-1}{7-3}$ ***(M1)***

$= 0.5$ ***A1***

***[2 marks]***

(b) $y-2=-2(x-5)$ ***(A1)(M1)***

**Note:** Award ***(A1)*** for their $-2$ seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

***[3 marks]***

(c) every point in the cell is closer to $\mathrm{E}$ than any other snow shelter ***A1***

***[1 mark]***

***Total [6 marks]***

**11.** volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

**Note:** Award ***M1*** $240 \times$ area , award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of their area of the sector from area of circle.

$= 45800\ (= 45811.96071)$ ***A1***

***Total [4 marks]***

**14.** (a) $\frac{\sin \text{CAB}}{6} = \frac{\sin 15^\circ}{4.5}$ ***(M1)(A1)***

$\text{C}\hat{\text{A}}\text{B} = 20.2^\circ \ (20.187415\ldots)$ ***A1***

**Note:** Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

***[3 marks]***

(b) $\text{C}\hat{\text{B}}\text{D} = 20.2 + 15 = 35.2^\circ$ ***A1***

*(let $X$ be the point on $BD$ where Ollie activates the sensor)*

$\tan 35.18741\ldots^\circ = \frac{1.8}{\text{BX}}$ ***(M1)***

**Note:** Award ***A1*** for their correct angle $\text{C}\hat{\text{B}}\text{D}$. Award ***M1*** for correctly substituted trigonometric formula.

$\text{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\text{m})\ (2.44714\ldots)$ ***A1***

***[5 marks]***

***Total [8 marks]***

**1.** height of triangle at roof $= 1.35 - 0.9 = 0.45$ ***(A1)***

**Note:** Award ***A1*** for $0.45$ (height of triangle) seen on the diagram.

slant height $= \sqrt{0.45^2 + 0.45^2}$ **OR** $\sin(45^\circ) = \dfrac{0.45}{\text{slant height}}$ ***(M1)***

$= \sqrt{0.405}$ $(0.636396..., 0.45\sqrt{2})$ ***A1***

**Note:** If using $\sin(45^\circ) = \dfrac{0.45}{\text{slant height}}$ then ***(A1)*** for angle of $45^\circ$, ***(M1)*** for a correct trig statement.

area of one rectangle on roof $= \sqrt{0.405} \times 0.9$ $(= 0.572756...)$ ***M1***

area painted $= (2 \times \sqrt{0.405} \times 0.9 = 2 \times 0.572756...)$

$1.15 \text{ m}^2$ $(1.14551... \text{ m}^2, 0.81\sqrt{2} \text{ m}^2)$ ***A1***

***[Total 5 marks]***

**2.** (a) $\sqrt{3.2^2 + 4.5^2 + 5.8^2}$ ***(M1)***

$= 8.01 \ (8.00812...)$ m ***A1***

***[2 marks]***

(b) $\text{F}\hat{\text{A}}\text{O} = \sin^{-1}\left(\frac{5.8}{8.00812...}\right)$ **OR** $\cos^{-1}\left(\frac{5.52177...}{8.00812...}\right)$ **OR** $\tan^{-1}\left(\frac{5.8}{5.52177...}\right)$ ***(M1)***

$46.4^\circ \ (46.4077...^\circ)$ ***A1***

***[2 marks]***

***[Total 4 marks]***

**4.** (a) midpoint $(1, 2.5)$ ***A1***

$m_{AB} = \frac{6-(-1)}{8-(-6)} = \frac{1}{2}$ ***(M1)A1***

**Note:** Accept equivalent gradient statements including using midpoint.

$m_{\perp} = -2$ ***M1***

**Note:** Award ***M1*** for finding the negative reciprocal of their gradient.

$y - 2.5 = -2(x-1)$ **OR** $y = -2x + \frac{9}{2}$ **OR** $4x + 2y - 9 = 0$ ***A1***

***[5 marks]***

(b) substituting $x = -6$ into their equation from part (a) ***(M1)***

$y = -2(-6) + \frac{9}{2}$

$y = 16.5$ ***A1***

**Note:** Award ***M1A0*** for $(-6, 16.5)$ as their final answer.

***[2 marks]***
***[Total 7 marks]***

**1.** (a) attempt to substitute into length of arc formula ***(M1)***

$\frac{140°}{360°} \times 2\pi \times 56$

$137 \text{ cm} \quad \left(136.833\ldots, \frac{392\pi}{9} \text{ cm}\right)$ ***A1***

***[2 marks]***

(b) subtracting two substituted area of sectors formulae ***(M1)***

$\left(\frac{140°}{360°} \times \pi \times 56^2\right) - \left(\frac{140°}{360°} \times \pi \times 10^2\right)$ **OR** $\frac{140°}{360°} \times \pi \times \left(56^2 - 10^2\right)$ ***(A1)***

$3710 \text{ cm}^2 \quad (3709.17\ldots \text{ cm}^2)$ ***A1***

***[3 marks]***

***Total [5 marks]***

**3.** (a) $\sin\theta = \frac{2.1}{2.8}$ **OR** $\tan\theta = \frac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)\ 48.6\ °\ \ (48.5903\ldots°)$ ***A1***

***[2 marks]***

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\frac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

**Note:** Award ***M1*** for attempt to use Pythagorean Theorem with $2.1$ seen or for attempt to use cosine or tangent ratio.

$1.85\ (\text{m})\ \ (1.85202\ldots)$ ***(A1)***

**Note:** Award the ***M1A1*** if $1.85$ is seen in part (a).

$(6.4 - 1.85202\ldots)$
$4.55\ \text{m}\ \ (4.54797\ldots)$ ***(A1)***

**Note:** Award ***A1*** for $4.55$ or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$
$5.01\ \text{m}\ \ (5.00939\ldots\text{m})$ ***A1***

**METHOD 2**
attempt to use cosine rule ***(M1)***
$(c^2 =)\ 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

**Note:** Award ***A1*** for $48.5903...°$ substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01\ \text{m}\ \ (5.00939\ldots\text{m})$ ***A1***

***[4 marks]***

(c) camera $1$ is closer to the cash register (than camera $2$ and both cameras are at the same height on the wall) ***R1***
the larger angle of depression is from camera $1$ ***A1***

**Note:** Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept "$1.85<4.55$" or "$2.8<5.01$" as evidence for the ***R1***.

***[2 marks]***
***Total [8 marks]***

**4.** (a) (i) $\left(\frac{1}{2}\text{A}\hat{\text{O}}\text{B} =\right) \arccos\left(\frac{4}{4.5}\right) = 27.266...$ ***(M1)(A1)***

$\text{A}\hat{\text{O}}\text{B} = 54.532... \approx 54.5^\circ$ $(0.951764... \approx 0.952$ radians) ***A1***

**Note:** Other methods may be seen; award ***(M1)(A1)*** for use of a correct trigonometric method to find an appropriate angle and then ***A1*** for the correct answer.

(ii) finding area of triangle

**EITHER**

area of triangle = $\frac{1}{2} \times 4.5^2 \times \sin(54.532...)$ ***(M1)***

**Note:** Award ***M1*** for correct substitution into formula.

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

**OR**

$\text{AB} = 2 \times \sqrt{4.5^2 - 4^2} = 4.1231\ldots$

area triangle $= \frac{4.1231\ldots \times 4}{2}$ ***(M1)***

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

finding area of sector

**EITHER**

area of sector = $\frac{54.532...}{360} \times \pi \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

**OR**

area of sector = $\frac{1}{2} \times 0.9517641.. \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

**THEN**

area of segment = $9.63661... - 8.24621...$

$= 1.39 \text{ m}^2$ $(1.39040...)$ ***A1***

***[8 marks]***

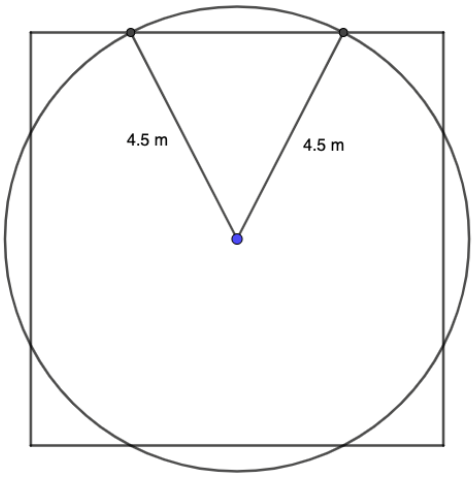
*continued…*

*Question 4 continued*

(b) (i) $\pi \times 4.5^2$ ***(M1)***

$63.6\ \text{m}^2\ \ (63.6172\ldots\ \text{m}^2)$ ***A1***

(ii) **METHOD 1**




$4 \times 1.39040...\ \ (5.56160)$ ***(A1)***

subtraction of four segments from area of circle ***(M1)***

$= 58.1\ \text{m}^2\ \ (58.055...)$ ***A1***

**METHOD 2**

$4(0.5 \times 4.5^2 \times \sin 54.532\ldots) + 4\left(\frac{35.4679}{360} \times \pi \times 4.5^2\right)$ ***(M1)***

$= 32.9845\ldots + 25.0707$ ***(A1)***

$= 58.1\ \text{m}^2\ \ (58.055\ldots)$ ***A1***

***[5 marks]***

(c) sketch of $\frac{\text{d}V}{\text{d}t}$ **OR** $\frac{\text{d}V}{\text{d}t} = 0.110363...$ **OR** attempt to find where $\frac{\text{d}^2V}{\text{d}t^2} = 0$ ***(M1)***

$t = 1$ hour ***A1***

***[2 marks]***

***[Total 15 marks]***

**3.** (a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ ***(M1)***

$(4, 1)$ ***A1***

**Note:** Award ***A0*** if parentheses are omitted in the final answer.

***[2 marks]***

(b) attempt to substitute values into gradient formula ***(M1)***

$\left(\frac{0-2}{6-2} = \right) -\frac{1}{2}$ ***(A1)***

therefore the gradient of perpendicular bisector is $2$ ***(M1)***

so $y-1=2(x-4)$ $(y=2x-7)$ ***A1***

***[4 marks]***

(c) identifying the correct equations to use: ***(M1)***

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or of finding intersection point graphically ***(M1)***

$(3, -1)$ ***A1***

**Note:** Accept an answer expressed as "$x=3$, $y=-1$".

***[3 marks]***

(d) attempt to use distance formula ***(M1)***

$\mathrm{YZ} = \sqrt{(7-(-1))^2 + (7-3)^2}$

$= \sqrt{80}$ $\left(4\sqrt{5}\right)$ ***A1***

***[2 marks]***

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

**Note:** Accept $8.94$ and $8.9$.

attempt to substitute into cosine rule ***(M1)***

$\cos \mathrm{X\hat{Y}Z} = \frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}$ $(=0.316227\ldots)$ ***(A1)***

**Note:** Award ***A1*** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} = \right) 71.6°$ $(71.5650\ldots°)$ ***A1***

**Note:** Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

*continued…*

*Question 3 continued*

**METHOD 2 (splitting isosceles triangle in half)**

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

**Note:** Accept $8.94$ and $8.9$.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ ***(M1)(A1)***

**Note:** Award ***A1*** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} =\right)\ 71.6°\ \ (71.5650°)$ ***A1***

**Note:** Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

***[4 marks]***

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\ldots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ ***(M1)***

$= 24\ \text{km}^2$ ***A1***

***[2 marks]***

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc. ***R1***

***[1 mark]***

***Total [18 marks]***

**1.** (a) $\sin(\hat{BSK}) = \frac{218}{1200}$ **OR** $\frac{\sin(BSK)}{218} = \frac{\sin(90^\circ)}{1200}$ ***(M1)***

**Note:** Award ***M1*** for a correct trig formula. Accept other variables representing $\hat{BSK}$.

$(\hat{BSK} =)\ 10.5^\circ\ (10.4668...)$ ***A1***

**Note:** Award ***A1*** for the radian answer, $0.182681...$. Award ***M1A0*** if the candidate finds the correct angle of elevation but then uses it to find a complementary angle as their final answer.

***[2 marks]***

(b) $SB^2 + 218^2 = 1200^2$ **OR** $\cos(10.4668...) = \frac{SB}{1200}$ **OR** $\tan(10.4668...) = \frac{218}{SB}$ **OR**

$\frac{BS}{\sin(79.5331...^\circ)} = \frac{1200}{\sin(90^\circ)}$ ***(M1)***

$1180\ (\text{m})\ \left(\sqrt{1392476}, 1180.03...\right)$ ***A1***

***[2 marks]***

(c) $1.18\times10^3$ ***A1A1***

**Note:** Award ***A1*** for $1.18$
Award ***A1*** for $10^3$
Accept their rounded answer to part (b).
Award ***A0A0*** for answers of the type: $11.8\times10^2$.

***[2 marks]***
***Total [6 marks]***

**2.** (a) (i) $19^\circ$ ***A1***

(ii) $\frac{\text{BD}}{\sin 120^\circ} = \frac{40}{\sin 19^\circ}$ ***(M1)(A1)***

**Note:** Award ***M1*** for substituted sine rule for BCD, ***A1*** for their correct substitution.

$(\text{BD} =) \ 106 \text{ m} \quad (106.401...)$ ***A1***

***[4 marks]***

(b) **METHOD 1 (cosine rule)**

$\cos \text{BAD} = \frac{85^2 + 85^2 - 106.401...^2}{2 \times 85 \times 85}$ ***(M1)(A1)***

**Note:** Award ***M1*** for substituted cosine rule, ***A1*** for their correct substitution.

$77.495$ ***A1***

**Note:** Accept an answer of $77.149$ from use of $3$ sf answer from part (a). The final answer must be correct to five significant figures.

**METHOD 2 (right angled trig/isosceles triangles)**

$\sin\left(\frac{\text{BAD}}{2}\right) = \frac{53.2008...}{85}$ ***(A1)(M1)***

**Note:** Award ***A1*** for $53.2008...$ seen. Award ***M1*** for correctly substituted trig ratio. Follow through from part (a).

$77.495...$ ***A1***

**Note:** Use of $3$ sf answer from part (a), results in $77.149$.

***[3 marks]***

(c) **EITHER**

$(\text{Area} =) \ \frac{1}{2} \times 85 \times 85 \times \sin(77^\circ)$ ***(M1)(A1)***

**Note:** Award ***M1*** for substituted area formula, ***A1*** for correct substitution. Award at most ***(M1)(A1)A0*** if an angle other than $77^\circ$ is used.

**OR**

$(\text{Area} =) \ \frac{1}{2} \times \left(2 \times 85 \times \sin(38.5^\circ)\right) \times \left(85 \times \cos(38.5^\circ)\right)$ ***(M1)(A1)***

**Note:** Award ***M1*** for substituted area formula $A = \frac{1}{2}bh$, ***A1*** for correct substitution.

$3520 \text{ m}^2 \ (3519.91...)$ ***A1***

***[3 marks]***

*continued…*

*Question 2 continued*

(d) $85\text{ m}$ ***A1***

***[1 mark]***

(e) $85+85+\frac{77}{360}\times 2\pi\times 85$ ***(M1)(M1)***

**Note:** Award ***M1*** for correctly substituted into $\frac{\theta}{360}\times 2\pi\times r$, ***M1*** for addition of AB and AD.

$284\text{ m}\ (284.231...)$ ***A1***

***[3 marks]***

(f) $\frac{77}{360}\times\pi\times(85)^2-3519.91...$ ***(M1)(M1)***

**Note:** Award ***M1*** for correctly substituted area of sector formula, ***M1*** for subtraction of their area from part (c).

$1330\text{ m}^2\ \ (1334.93...)$ ***A1***

***[3 marks]***

***Total [17 marks]***

**3.** (a) (i) B ***A1***

(ii) F ***A1***

***[2 marks]***

(b) correct substitution into the midpoint formula ***(M1)***

$\frac{8+5}{2}$

$y = 6.5$ ***A1***

**Note:** Answer must be an equation for the ***A1*** to be awarded.

***[2 marks]***

(c) midpoint $= (5,\ 7)$ ***(A1)***

correct use of gradient formula ***(M1)***

$\frac{8-6}{7-3}$

gradient of $\text{BC} = 0.5$ ***(A1)***

negative reciprocal of gradient ***(M1)***

perpendicular gradient $= -2$

$y - 7 = -2(x-5)$ (or $y = -2x+17$) ***A1***

***[5 marks]***

(d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) ***(M1)***

**Note:** This may be seen graphically or algebraically.

$6.5 - 7 = -2(x-5)$ **OR** $6.5 = -2x+17$

**Note:** Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x-5)$ **OR** $y = 4x - 14.5$

$x = 5.25,\ y = 6.5$ **OR** $(5.25,\ 6.5)$ ***A1***

**Note:** The $x$-coordinate must be exact or expressed to at least 3 sf.

(ii) their correct substitution into distance formula ***(M1)***

$\sqrt{(5.25-7)^2 + (6.5-5)^2}$

$= 2.30 \text{ km} \left(2.30488..., \frac{\sqrt{85}}{4}\right)$ ***A1***

***[4 marks]***

***Total [13 marks]***

**2.** (a) attempt to calculate $\text{A}\hat{\text{H}}\text{B}$ using $33$ **OR** use of alternate angles ***(M1)***

e.g., $180-(33+130)$ **OR** $90-(33+40)$ **OR** $57-40$

$17\ (°)$ ***A1***

***[2 marks]***

(b) attempt to use sine rule ***(M1)***

$$\frac{\text{BH}}{\sin(130^\circ)}=\frac{156}{\sin(17^\circ)}$$ ***(A1)***

$(\text{BH}=)\ 409\ (\text{m})\ (408.736\ldots)$ ***A1***

**Note:** If radians are used, answer is $151\ (150.922\ldots)$; award at most ***(M1)(A1)A0***.

***[3 marks]***

(c) (the angle of depression from the hot air balloon) gets smaller ***A1***
(as the horizontal distance increases)

***[1 mark]***

***[Total: 6 marks]***

**11.** (a) $14.55$ (cm) to $14.65$ (cm) ***A1A1***

**Note:** Award ***A1*** for each value. Accept $14.55 \leq \text{AC} < 14.65$.

***[2 marks]***

(b) attempt to use Pythagorean theorem **OR** trig ratio to find slant height ***(M1)***

a correct expression for either the **upper** or **lower** bound ***(A1)***

$\sqrt{14.55^2 - 10^2}$ **OR** $\sqrt{14.65^2 - 10^2}$ **OR**

$\sin(46.5844...^\circ) = \frac{\text{AH}}{14.55}$ **OR** $\sin(46.9533...^\circ) = \frac{\text{AH}}{14.65}$

(lower bound =) $10.6$ (cm) $(10.5689\ldots)$ **AND**

(upper bound =) $10.7$ (cm) $(10.7061\ldots)$ ***A1***

***[3 marks]***

*continued…*

*Question 11 continued*

(c) **METHOD 1**

attempt to find the maximum angle measure of the post using trigonometry ***(M1)***

e.g. $\cos\theta = \frac{10}{10.7061...}$ **OR** $\frac{\sin\theta}{3.82393\ldots} = \frac{\sin(90^\circ)}{10.7061\ldots}$

**Note:** Accept an inequality.

$(\theta =)\ 20.9\ (^\circ)\quad (20.9265\ldots(^\circ))$ ***A1***

and hence the post is safe ***AG***

**Note:** Use of radians gives an answer of $0.365\ \ (0.365237\ldots)$; award at most ***(M1)A0*** since this value cannot be directly compared to $22^\circ$.

Award at most ***(M1)A0*** for an angle calculated using their lower bound from part (b).

**METHOD 2**

attempt to find the longest slant height for angle to be a maximum of $22^\circ$ ***(M1)***

e.g. $\cos(22^\circ) = \frac{10}{x}$

$(x = 10.7853\ldots)$

$10.7061\ldots < 10.7853\ldots$ ***A1***

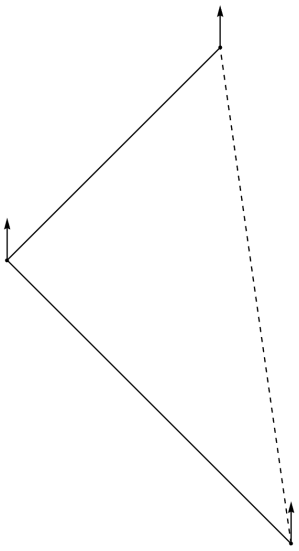
and hence the post is safe ***AG***

**Note:** A comparison to their upper bound from part (b) is required for ***A1*** to be awarded. Use of radians gives an unreasonable answer of $-10.0003...$ ; award at most ***(M1)A0***.

***[2 marks]***

***Total [7 marks]***

**13.** diagram showing (approximately) correct directions (and order) for the $315^\circ$ and $045^\circ$ ***(A1)***



**Note:** Values do not need to be seen on the diagram to award the ***A1***.

recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

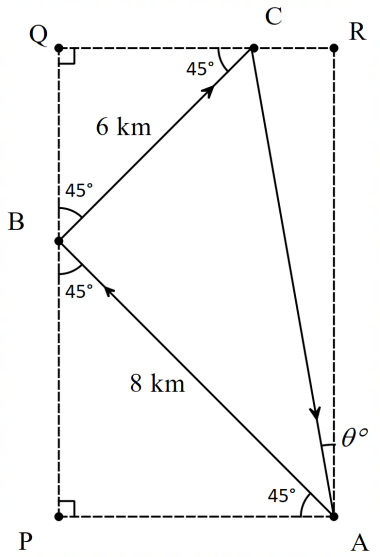
$=172^\circ$ $(171.869...^\circ)$ ***A1***

*continued…*

*Question 13 continued*

**METHOD 2**

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$ (these may be shown in reverse as the return journey) ***(A1)***




finding the lengths marked AP, BP, CQ and BQ in the diagram ***(M1)***

$$\text{AP} = \text{BP} = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$\text{CQ} = \text{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426...$$

**Note:** This may be done using a vector approach.

using $\tan\theta° = \frac{\text{AP} - \text{CQ}}{\text{PB} + \text{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$= 172° \quad (171.869...°)$ ***A1***

***[Total: 5 marks]***

**7.** (a) attempt to find gradient of $\mathrm{CD}$ ***(M1)***

gradient of $\mathrm{CD} = \frac{1}{3}$ therefore perpendicular gradient $\mathrm{CD} = -3$ ***A1***

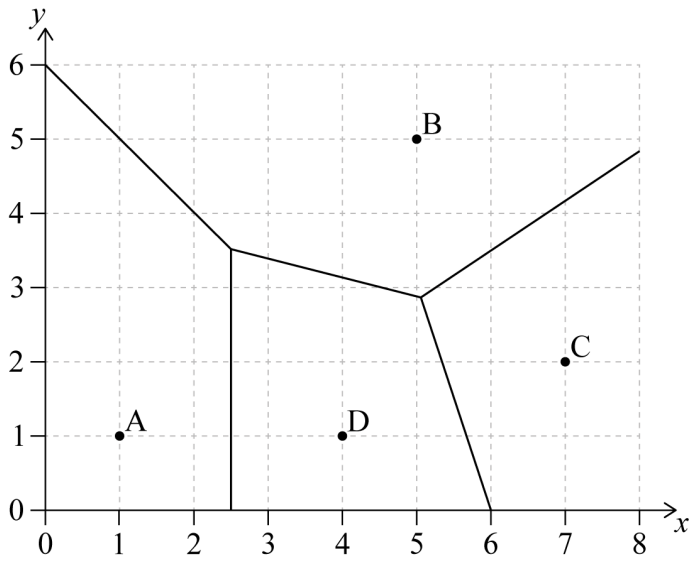
$y - 1.5 = -3(x - 5.5)$ **OR** $1.5 = -3(5.5) + c$ ***M1***

**Note:** Award ***M1*** for substituting the gradient and midpoint into equation of line, provided further work is seen leading to a correct answer.

$y = -3x + 18$ ***AG***

***[3 marks]***

(b)




perpendicular bisector AD: a vertical line with $x$ intercept $2.5$ ***A1***

**Note:** The perpendicular bisector should not go beyond the intersection point (should not enter site B).

***[1 mark]***

(c) attempt to solve simultaneous equations: $3y = 2x - 1.5$ and $y = -3x + 18$ ***(M1)***

$(5.05, 2.86)$ $((5.04545\ldots, 2.86363\ldots))$ ***A1***

**Note:** Accept $x = 5.05$ $(5.04545...)$, $y = 2.86$ $(2.86363...)$ in place of coordinates.
Accept $(5.05, 2.87)$ and $(5.05, 2.85)$ for using their $3$ sf or $4$ sf $x$-value to find $y$ from any of the two equations.

***[2 marks]***
***Total [6 marks]***

**8.** (a) **EITHER**

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

multiplying their volume by $\frac{4}{5}$ ***(M1)***

**OR**

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Subtracting $\frac{1}{5}$ of their volume ***(M1)***

$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5}\times\frac{4}{3}\pi(3.4)^3\right)$

**Note:** The ***M1*** can be awarded for a final answer of $32.9272\ldots$ seen without working.

**THEN**

$132\ \text{cm}^3\ \ (131.708\ldots\ \text{cm}^3)$ ***A1***

***[3 marks]***

(b) $\pi\times 3\times 11$ ***(A1)***

$103.672\ldots\ (\text{cm}^2)$ **OR** $33\pi(\text{cm}^2)$

$104\ (\text{cm}^2)$ ***A1***

***[2 marks]***

***Total [5 marks]***

**1.** (a) attempt to use sine rule ***(M1)***

$$\frac{\sin \hat{ABO}}{25.9} = \frac{\sin 10^\circ}{6.36} \quad \textit{(A1)}$$

$45.0^\circ \ (45.0036...^\circ)$ ***A1***

**Note:** Accept an answer of $45^\circ$ for full marks.

***[3 marks]***

(b) $(\hat{OAB} =) \ 124.996...^\circ$ ***(A1)***

attempt to use area of triangle formula ***(M1)***

$$\frac{1}{2} \times 25.9 \times 6.36 \times \sin(124.996...^\circ) \quad \textit{(A1)}$$

$67.5 \text{ m}^2 \ (67.4700... \text{ m}^2)$ ***A1***

**Note:** Units are required. The final ***A1*** is only awarded if the correct units are seen in their answer; hence award ***(A1)(M1)(A1)A0*** for an unsupported answer of $67.5$. Accept $67.4670\ldots \text{m}^2$ from use of 3 sf values.

Full follow through marks can be awarded for this part even if their $\hat{OAB}$ is not obtuse, provided that all working is shown.

***[4 marks]***

(c) attempt to use cosine rule ***(M1)***

$(BK =) \sqrt{12^2 + 6.36^2 - 2 \times 12 \times 6.36 \times \cos 45^\circ}$ ***(A1)***

$8.75 \text{ (m)} \ (8.74738\ldots \text{(m)})$ ***A1***

**Note:** Award ***(M1)(A1)(A0)*** for radian answer of $10.2 \text{ (m)} \ (10.2109\ldots \text{(m)})$ with or without working shown.

***[3 marks]***

*continued…*

*Question 1 continued*

(d) **METHOD 1**

attempt to use sine rule with measurements from triangle $OKX$ ***(M1)***

$\frac{OX}{\sin 51.1^\circ} = \frac{22.2}{\sin 53.8^\circ}$ ***(A1)***

$(OX =)\ 21.4\ (m)\ (21.4099\ldots)(m)$ ***A1***

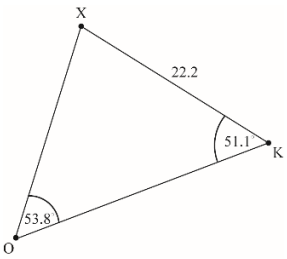
$(21.4\ (m) < 22.2\ (m))$

Odette is closer to the football / Khemil is further from the football ***A1***

**Note:** For the final ***A1*** to be given, $21.4\ (21.4099\ldots)$ must be seen. Follow through within question part for final ***A1*** for a consistent comparison with their $OX$.

**METHOD 2**

sketch of triangle $OXK$ with vertices, angles and lengths ***(A1)***




$51.1^o$ is smallest angle in triangle $OXK$ ***R1***

opposite side $(OX)$ is smallest length ***R1***

therefore Odette is closest ***A1***

***[4 marks]***

(e) attempt to use length of arc formula ***(M1)***

$\frac{135}{360} \times 2\pi \times 12$ ***(A1)***

$28.3(m)\ (9\pi,\ 28.2743...)\ (m)$ ***A1***

***[3 marks]***

***Total [17 marks]***

**2.** (a) (upper bound =) $0.525\ (\text{m})$ ***A1***
(lower bound =) $0.515\ (\text{m})$ ***A1***

**Note:** Accept an answer in interval notation or written as an inequality.

***[2 marks]***

(b) **METHOD 1 Convert REC to linear metres**
attempt to convert REC to metres using their lower bound ***(M1)***
$440\times0.515\ (=226.6)$ **OR** $280\times0.515\ (=144.2)$ seen

attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440\times0.515)^2(280\times0.515)$ ***(A1)***

$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

**METHOD 2 Convert REC to cubic metres**
attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440)^2(280)\ \ (=18\,069\,333.33...)$
attempt to convert 1 cubic REC to cubic metres using their lower bound ***(M1)***
(1 cubic REC = ) $0.515^3$

$(V=)\ \frac{1}{3}(440)^2(280)\times(0.515)^3$ ***(A1)***
$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

***[4 marks]***
***[Total 6 marks]***

**4.** (a) $(4, 8)$ ***A1***

***[1 mark]***

(b) attempt to find the gradient of AC ***(M1)***

$\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$

attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line ***(M1)***

$y-8=-\frac{4}{5}(x-4)$ **OR** $8=-\frac{4}{5}(4)+c$ **OR** $c=11.2$

$y-8=-\frac{4}{5}(x-4) \quad (y=-0.8x+11.2,\ 4x+5y-56=0)$ ***A1***

***[3 marks]***

(c) (i) attempt to find one distance from a farm to any closest vertex ***M1***

finding a correct distance from at least two distinct vertices ***A1***

$7.58968\ldots, 4.472135\ldots \left(\sqrt{20}\right), 5.830951\ldots \left(\sqrt{34}\right)$

$\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) ***A1***

(ii) $7.59$ (km) $(= 7.58968...)$ ***A1***

***[4 marks]***

***[Total 8 marks]***

**5.** (a) recognizing supplementary angles or acute angles in right-triangles ***(M1)***

$(\text{R}\hat{\text{S}}\text{T} =)\ 38^\circ + (180^\circ - 120^\circ),\ 38^\circ + (90^\circ - 30^\circ)$

$\text{R}\hat{\text{S}}\text{T} = 98^\circ$ ***A1***

***[2 marks]***

(b) $\text{R}\hat{\text{T}}\text{S} = 52^\circ$ (may be seen in part (a)) ***(A1)***

attempt to substitute into the sine rule (or equivalent) ***(M1)***

$\dfrac{\text{RT}}{\sin 98^\circ} = \dfrac{150}{\sin 52^\circ}$ ***(A1)***

$\text{RT} = 189\ (\text{km})\ \ (= 188.500...)$ ***A1***

***[4 marks]***

***[Total 6 marks]***

**2.** (a) (upper bound =) $0.525\ (\text{m})$ ***A1***
(lower bound =) $0.515\ (\text{m})$ ***A1***

**Note:** Accept an answer in interval notation or written as an inequality.

***[2 marks]***

(b) **METHOD 1 Convert REC to linear metres**
attempt to convert REC to metres using their lower bound ***(M1)***
$440\times0.515\ (=226.6)$ **OR** $280\times0.515\ (=144.2)$ seen

attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440\times0.515)^2(280\times0.515)$ ***(A1)***

$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

**METHOD 2 Convert REC to cubic metres**
attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440)^2(280)\ \ (=18\,069\,333.33...)$
attempt to convert 1 cubic REC to cubic metres using their lower bound ***(M1)***
(1 cubic REC = ) $0.515^3$

$(V=)\ \frac{1}{3}(440)^2(280)\times(0.515)^3$ ***(A1)***
$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

***[4 marks]***
***[Total 6 marks]***

**4.** (a) $(4, 8)$ ***A1***

***[1 mark]***

(b) attempt to find the gradient of $\mathrm{AC}$ ***(M1)***

$\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$

attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line ***(M1)***

$y-8=-\frac{4}{5}(x-4)$ **OR** $8=-\frac{4}{5}(4)+c$ **OR** $c=11.2$

$y-8=-\frac{4}{5}(x-4) \quad (y=-0.8x+11.2,\ 4x+5y-56=0)$ ***A1***

***[3 marks]***

(c) (i) attempt to find one distance from a farm to any closest vertex ***M1***

finding a correct distance from at least two distinct vertices ***A1***

$7.58968\ldots, 4.472135\ldots\ \left(\sqrt{20}\right), 5.830951\ldots\ \left(\sqrt{34}\right)$

$\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) ***A1***

(ii) $7.59\ (\mathrm{km})\ (=7.58968...)$ ***A1***

***[4 marks]***

***[Total 8 marks]***

**5.** (a) recognizing supplementary angles or acute angles in right-triangles ***(M1)***

$(\text{A}\hat{\text{B}}\text{C} =)\ 41^\circ + (180^\circ - 112^\circ)$, $41^\circ + (90^\circ - 22^\circ)$

$\text{A}\hat{\text{B}}\text{C} = 109^\circ$ ***A1***

***[2 marks]***

(b) $\text{A}\hat{\text{C}}\text{B} = 49^\circ$ (may be seen in part (a)) ***(A1)***

attempt to substitute into the sine rule (or equivalent) ***(M1)***

$\frac{\text{AC}}{\sin 109^\circ} = \frac{100}{\sin 49^\circ}$ ***(A1)***

$\text{AC} = 125\ (\text{km})\ \ (= 125.282...)$ ***A1***

***[4 marks]***

***[Total 6 marks]***

**2.** (a) $45\ (\text{m})$ ***A1***

***[1 mark]***

(b) (i) recognition of need to use Pythagoras theorem ***(M1)***

$\text{BE}^2 = 32^2 + 45^2$

$(\text{BE} =)\ 55.2\ (55.2177\ldots, \sqrt{3049})\ (\text{m})$ ***A1***

(ii) correct use of trig ratio for $\text{B}\hat{\text{E}}\text{M}$ ***(M1)***

$(\text{B}\hat{\text{E}}\text{M} =)\ \tan^{-1}\left(\frac{45}{32}\right)$ or equivalent

$(\text{B}\hat{\text{E}}\text{M} =)\ 54.6\ (54.5829...)$ ***A1***

***[4 marks]***

(c) attempt to use arc length formula ***(M1)***

(arc length =) $\frac{2\times 54.5829...}{360}\times 2\pi(55.2177\ldots)$ ***(A1)***

(arc length =) $105\ (105.206\ldots)\ (\text{m})$ ***A1***

***[3 marks]***

(d) $59.2177\ldots$ (seen anywhere) ***(A1)***

use of area of sector formula ***(M1)***

recognition of subtracting areas of two sectors ***(M1)***

(area =) $\frac{109.165\ldots}{360}\times\pi\left((59.2177\ldots)^2-(55.2177\ldots)^2\right)$

(area =) $436\ (\text{m}^2)\ (436.068\ldots)$ ***A1***

***[4 marks]***

(e) multiplying their area from part (d) by 0.15 or 15 ***(M1)***

$0.15$ (m) seen **OR** $4360688$ ($cm^2$) seen ***(A1)***

$436.068\ldots \times 0.15$ **OR** $4360688 \times 15$

$65.4$ $(65.4103\ldots)$ $m^3$ **OR** $65400000$ $(65410332)$ $cm^3$ ***A1***

***[3 marks]***

***[Total 15 marks]***

**2.** (a) $25\ (\text{m})$ ***A1***

***[1 mark]***

(b) (i) recognition of need to use Pythagoras theorem ***(M1)***

$\text{BF}^2 = 20^2 + 25^2$

$(\text{BF} =)\ 32.0\ \ (32.0156\ldots,\ \sqrt{1025},\ 5\sqrt{41})\ (\text{m})$ ***A1***

(ii) correct use of trig ratio for $\text{B}\hat{\text{F}}\text{M}$ ***(M1)***

$(\text{B}\hat{\text{F}}\text{M} =)\ \tan^{-1}\left(\frac{25}{20}\right)$ or equivalent

$(\text{B}\hat{\text{F}}\text{M} =)\ 51.3\ (51.3401...)$ ***A1***

**Note:** Accept an answer of $51.4$ from use of 3sf answer to part (b)(i) and then either cosine rule or inverse sine.

***[4 marks]***

(c) attempt to use arc length formula ***(M1)***

(arc length =) $\frac{2\times 51.3401...}{360}\times 2\pi(32.0156\ldots)$ ***(A1)***

(arc length =) $57.4\ \ (57.3755\ldots)\ (\text{m})$ ***A1***

**Note:** Accept $57.3$ from use of 3 sf. values of their answers from parts (b)(i) and (b)(ii).

***[3 marks]***

(d) $34.0156\ldots$ (seen anywhere) ***(A1)***

use of area of sector formula ***(M1)***

recognition of subtracting areas of two sectors ***(M1)***

(area =) $\frac{102.680\ldots}{360}\times\pi\left((34.0156\ldots)^2-(32.0156\ldots)^2\right)$

(area =) $118\ (\text{m}^2)\ \ (118.335\ldots)$ ***A1***

***[4 marks]***

(e) multiplying their area from part (d) by $0.12$ or $12$ ***(M1)***

$0.12$ (m) seen **OR** $1183350$ ($cm^2$) seen ***(A1)***

$118.335\ldots \times 0.12$ **OR** $1183350 \times 12$

$14.2$ $(14.2002\ldots)$ $m^3$ **OR** $14200000$ $(14200236)$ $cm^3$ ***A1***

***[3 marks]***

***[Total 15 marks]***

**6.** (a) $y = 0.5x - 1$ ***A1A1***

**Note:** Award ***A1*** for $0.5x$ and ***A1*** for $-1$ (or equivalent equation). Award at most ***A1A0*** if answer is not presented as an equation.

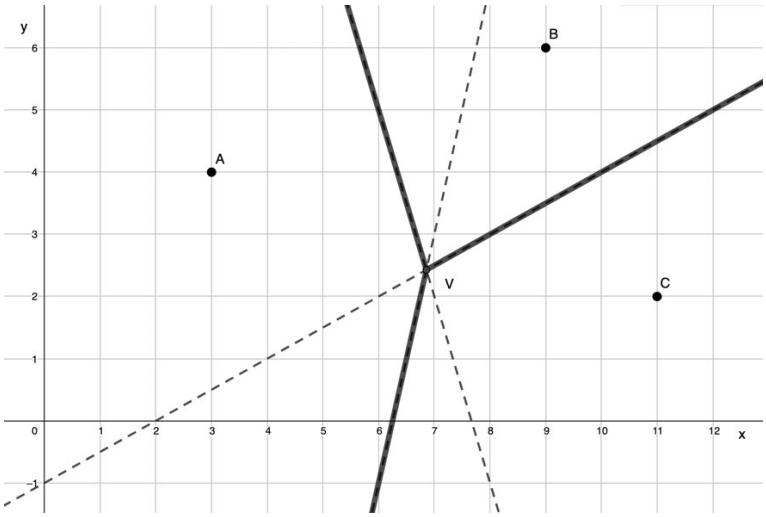
***[2 marks]***

(b) $(6.857,\ 2.429)$ ***A1A1***

**Note:** If both answers are not correct to 4 sig figs, award at most ***A1A0***.
Accept $x = 6.857$, $y = 2.429$.
Award ***A1A0*** for $\left(\frac{48}{7}, \frac{17}{7}\right)$. Award ***A0A1*** for $(2.429, 6.857)$.

***[2 marks]***

(c)




***A2***

**Note:** Award marks as shown in the table below. Condone edges that do not extend to the sides of the graph or beyond the $x$-axis.

| Correct edges | Incorrect edges | Marks |
|---|---|---|
| 3 | 0 | ***A2*** |
| 3 | 1 | ***A1A0*** |
| 3 | 2 or more | ***A0A0*** |
| 2 | 0 | ***A1A0*** |
| 2 | 1 | ***A1A0*** |
| 2 | 2 or more | ***A0A0*** |
| 1 | 0 | ***A1A0*** |
| 1 | 1 or more | ***A0A0*** |

***[2 marks]***
***[Total 6 marks]***

**11.** (recognition that $\mathrm{OB}$ is a radius)

(radius =) $\sqrt{5^2+8^2}$ $(=\sqrt{89})$ ***(A1)***

**EITHER (finding angle BOQ)**

correct calculation for finding $\mathrm{B\hat{O}A}$ ***(A1)***

$\mathrm{B\hat{O}A} = \arctan\left(\frac{8}{5}\right)$ **OR** $\tan \mathrm{B\hat{O}A} = \frac{8}{5}$

expressing $\mathrm{B\hat{O}Q}$ as $90 + \mathrm{B\hat{O}A}$ ***(M1)***

$\mathrm{B\hat{O}Q} = 90 + \arctan\left(\frac{8}{5}\right)$ **OR** $\mathrm{B\hat{O}Q} = \frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)$

$\left(\mathrm{B\hat{O}Q} =\right) 147.994°\ldots$ **OR** $2.58299\ldots$

substituting *their* radius and angle $\mathrm{B\hat{O}Q}$ correctly into arc length formula ***(M1)***

(arc BQ =) $\frac{90 + \arctan\left(\frac{8}{5}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$ **OR** $\left(\frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)\right) \times \left(\sqrt{5^2+8^2}\right)$

24.4 (m) (24.3679…) ***A1***

**OR (finding angle BOP)**

correct calculation for finding angle $\mathrm{B\hat{O}P}$ ***(A1)***

$\mathrm{B\hat{O}P} = \arctan\left(\frac{5}{8}\right)$ **OR** $\tan \mathrm{B\hat{O}P} = \frac{5}{8}$

substituting *their* radius and $\mathrm{B\hat{O}P}$ correctly into arc length formula ***(M1)***

(arc BP =) $\frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

subtracting *their* arc BP from arc PQ ***(M1)***

(arc BQ =) $\pi\sqrt{5^2+8^2} - \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

24.4 (m) (24.3679…) ***A1***

***[Total: 5 marks]***

**2.** (a) attempt to substitute into cosine rule formula ***(M1)***

$(BC^2 =)\ 15^2 + 25^2 - 2\times 15\times 25\times \cos(22)$ ***(A1)***

$(BC =)\ 12.4\ (\text{cm})\quad (12.4343\ldots)$ ***A1***

***[3 marks]***

(b) selecting sine rule formula **OR** cosine rule formula ***(M1)***

$\frac{12.4343\ldots}{\sin 22} = \frac{15}{\sin A\hat{B}C}$ **OR** $\left(\cos A\hat{B}C =\right)\frac{25^2 + 12.4343\ldots^2 - 15^2}{2\times 25\times 12.4343\ldots}$ ***(A1)***

**Note:** Award ***M1A1*** for correct cosine rule formula to find $A\hat{B}C$.

$(A\hat{B}C =)\ 26.9^\circ\quad (26.8658\ldots^\circ)$ ***A1***

**Note:** Accept $26.9461\ldots$ from use of $12.4$ in the sine rule formula and $26.7267\ldots$ in the cosine rule formula.

***[3 marks]***

***[Total: 6 marks]***



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**11.** attempt to substitute into area of triangle formula ***(M1)***

(sheep's field area =) $0.5\times15\times21\times\sin(78°)$

$=154.058\ldots\ (\text{m}^2)$ ***A1***

**EITHER**

(goat's field area =) $\frac{282}{360}\times\pi\times8^2$ ***(A1)(A1)***

**Note:** Award ***A1*** for $282$, ***A1*** for correct entries in formula (including their $282$).

**OR**

$\pi\times8^2-\frac{78}{360}\times\pi\times8^2$ ***(M1)(A1)***

**Note:** Award ***A1*** for minor sector area, ***M1*** for subtracting their sector area from circle area.

**THEN**

$=157.498\ldots\left(\frac{752\pi}{15}\right)\ (\text{m}^2)$ ***A1***

the goat has most area by $3.44\ (\text{m}^2)\ (3.44026\ldots)$ ***A1***

**Note:** Accept $154$ and $157$ for the intermediate ***A1*** marks, but do NOT follow through within the question; a final answer of $3\ \text{m}^2$ is awarded ***A0***.

***[Total: 6 marks]***

**2.** (a) $BC = 20\ (\text{m})$ ***A1***

***[1 mark]***

(b) use of Pythagoras ***(M1)***

$AB = \sqrt{12^2 + 4^2}$

$= 12.6\ (\text{m})\quad (12.6491\ldots, \sqrt{160})$ ***A1***

***[2 marks]***

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $A\hat{B}C$ (or finding the bearing of B from A) ***(A1)***

e.g. $\tan\left(A\hat{B}C\right) = \frac{12}{4}$, $\cos A\hat{B}C = \frac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649}$, $\cos A\hat{B}C = \frac{6.3245}{20}$

$A\hat{B}C = 71.6\ (71.5650\ldots)$ ***(A1)***

**Note:** Angle $A\hat{B}C$ can be $71.5$ or $72.2$ depending on their working out. Bearings should be given in degrees.

$180 + 71.5650\ldots = 252°\ (251.565\ldots)$ ***A1***

**Note:** The final ***A1*** can be awarded for $180$ plus their $71.6$. If radians used, award ***A1A1*** for $1.24904\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

**METHOD 2 – finding angle that AB makes with the horizontal (angle *H*)**

correct use of a trig ratio to find $H$, the angle $AB$ makes with horizontal ***(A1)***

e.g. $\tan \hat{H} = \frac{4}{12}$, $\cos \hat{H} = \frac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$

$\hat{H} = 18.4\ (18.4349...)$ ***(A1)***

**Note:** Accept $18.5\ (18.5078\ldots)$ from use of 3sf answer from part (b). Bearings should be given in degrees.

$270 - 18.4348... = 252°\ (251.565\ldots)$ ***A1***

**Note:** The final ***A1*** can be awarded for $270$ minus their $18.4$. If radians used, award ***A1A1*** for $0.321750\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

***[3 marks]***

(d) (i) $-\frac{4}{3}$ $\left(-\frac{16}{12}\right)$ ***A1***

(ii) $(6, 8)$ ***A1A1***

**Note:** Award ***A1A0*** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ ***(M1)***

equation of perpendicular bisector of $\mathrm{AC}$ ***(A1)***

e.g. $(y-8)=\frac{3}{4}(x-6)$ **OR** $y=\frac{3}{4}x+3.5$

**EITHER**

equation of perpendicular bisector of $\mathrm{BC}$ is $y=10$ ***(A1)***

**OR**

equation of perpendicular bisector of $\mathrm{AB}$ is $y=-3x+36$ ***(A1)***

**Note:** The ***A1*** is for either equation of perpendicular bisector of $\mathrm{BC}$ or $\mathrm{AB}$.

point of intersection $\left(8\frac{2}{3}, 10\right)$ **OR** $(8.67, 10)$ $\left[(8.666\ldots, 10)\right]$ ***(M1)A1***

**Note:** Award ***M1*** for an attempt to equate their perpendicular bisectors
Award the final ***A1*** for the correct coordinate pair – parentheses omitted or not.

***[8 marks]***

***[Total: 14 marks]***

**5.** (a) (i) correct approach to find missing length ***(A1)***

$\sqrt{4^2-1^2}\quad(=\sqrt{15})$

attempt to find cross-section ***(M1)***

e.g. use of area of trapezoid formula or rectangle+triangle or rectangle – triangle

use of volume of prism formula ***(M1)***

(their cross-section multiplied by 3)

$3\left[\frac{1}{2}(10+11)\left(\sqrt{4^2-1^2}\right)\right]$

$=122\,(\text{m}^3)\ \ (121.998\ldots)$ ***A1***

(ii) correct approach to find missing height ***(A1)***

$\sqrt{4^2-3.2^2}\quad(=2.4)$

attempt to find volume ***(M1)***

(multiplication by $3.2$ and $3$ seen)

$3\left[\frac{1}{2}\left(10+10+\sqrt{4^2-3.2^2}\right)(3.2)\right]$

$=108\,(\text{m}^3)\ \ (107.52...)$ ***A1***

(iii) correct approach to find missing lengths ***(A1)***

$\sin(60^\circ)$ and $\cos(60^\circ)$ **OR** $\sin(60^\circ)$ and Pythagoras etc seen in work

$3\left[\frac{1}{2}(10+10+4\cos(60^\circ))4\sin(60^\circ)\right]$

$=114\,(\text{m}^3)\ \ (114.315\ldots)$ ***A1***

***[9 marks]***

(b) $V=3\left[\frac{1}{2}(10+10+4\cos(\theta))4\sin(\theta)\right]$ ***A1***

all correct intermediate working leading to given answer ***A1***

e.g. $V=6\sin(\theta)(20+4\cos(\theta))$

$V=24\sin(\theta)(5+\cos(\theta))$ ***AG***

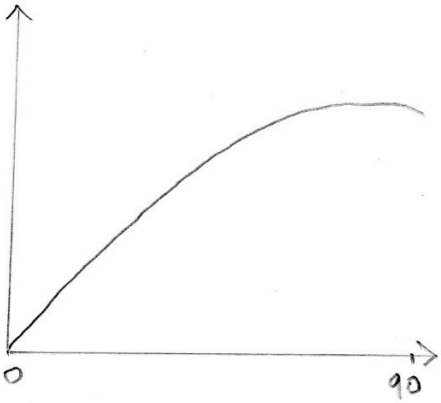
**Note:** The ***AG*** line must be seen for the final ***A1*** to be awarded.

***[2 marks]***

(c) *accept any reasoning along the lines:* "skip would have zero volume" or "if the angle is zero, then the contents would fall out" ***R1***

***[1 mark]***

(d) (i)




***A1A1***

**Note:** Award ***A1*** for the correct shape and ***A1*** for the graph on the correct, labelled, domain. Condone omission of $\theta / V$ labels (or $x/y$).

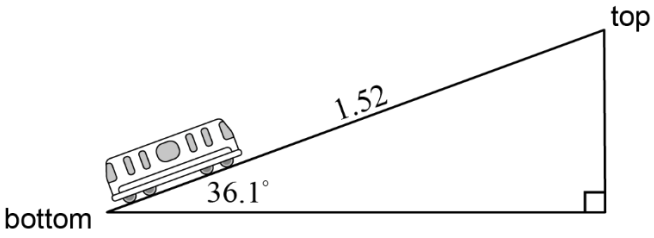
(ii) $\theta = 79.3^\circ$ $(79.2723...^\circ)$ $(1.38$ $(1.38356...))$, $V_{\max} = 122$ $(122.292...)$ ***A1A1***

**Note:** Award ***A0A1*** if values are reversed and ***A0A0*** for a coordinate pair.

***[4 marks]***

***[Total: 16 marks]***

**1.** (a)




***A1A1***

***A1*** for correct angle of elevation labelled as $36.1$
***A1*** for hypotenuse labelled as $1.52$ (km)

***[2 marks]***

(b) $1.52 \times \sin 36.1°$ ***(A1)***

$0.896$ (km) $(0.895578…\text{(km)}, 896\text{ m})$ ***A1***

***[2 marks]***
***[Total 4 marks]***

**4.** (a) attempt to use the cosine rule to find $\mathrm{T\hat{R}S}$ ***(M1)***

$(\mathrm{T\hat{R}S} =) \cos^{-1}\left(\frac{46^2 + 39^2 - 40^2}{2 \times 46 \times 39}\right)$ ***(A1)***

$(\mathrm{T\hat{R}S} =)\ 55.4^\circ\ (55.4082\ldots^\circ)$ **OR** $0.967\ (0.967055\ldots)$ ***A1***

***[3 marks]***

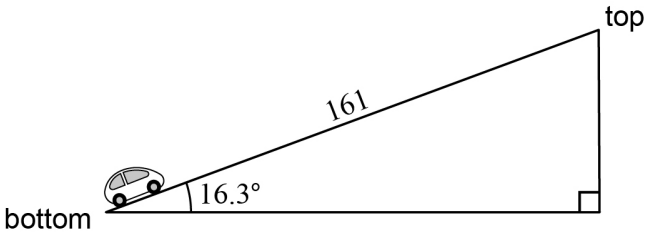
(b) $(\text{Area} =)\ \frac{1}{2} \times 46 \times 39 \times \sin 55.4082\ldots^\circ$ ***(A1)***

$(\text{Area} =)\ 738\ (738.426...)\ \text{cm}^2$ ***A1***

**Note:** Correct units must be seen for the final ***A1*** to be awarded.

***[2marks]***
***[Total 5 marks]***

**1.** (a)




***A1A1***

***A1*** for correct angle of elevation labelled as $16.3$
***A1*** for hypotenuse labelled as $161$ (m)

***[2 marks]***

(b) $161 \times \sin 16.3°$ ***(A1)***
$45.2$ $(45.1873\ldots.)$ (m) ***A1***

***[2 marks]***
***[Total 4 marks]***

**4.** (a) attempt to use the cosine rule to find $\mathrm{C\hat{A}B}$ ***(M1)***

$(\mathrm{C\hat{A}B} =)\ \cos^{-1}\left(\dfrac{42^2 + 35^2 - 36^2}{2 \times 42 \times 35}\right)$ ***(A1)***

$(\mathrm{C\hat{A}B} =)\ \ 54.8^\circ\ \ (54.8407\ldots^\circ)$ **OR** $0.957\ \ (0.957152\ldots)$ ***A1***

***[3 marks]***

(b) $(\text{Area} =)\ \dfrac{1}{2} \times 42 \times 35 \times \sin 54.8407\ldots^\circ$ ***(A1)***

$(\text{Area} =)\ 601\ \ (600.903)\ \text{cm}^2$ ***A1***

**Note:** Correct units must be seen for the final ***A1*** to be awarded.

***[2 marks]***
***[Total 5 marks]***

**1.** (a) $(2, 6)$ ***A1***

**Note**: Award ***A0*** if parentheses are omitted.

***[1 mark]***

(b) (i) attempt to use midpoint formula (at least one correct) ***(M1)***

$\left(\frac{11+12}{2}, \frac{0+6}{2}\right)$

$(11.5, 3)$ ***A1***

(ii) $\left(m = \frac{6-0}{12-11} =\right) 6$ ***(A1)***

finding negative reciprocal of their gradient ***(M1)***

$m_{\perp} = -\frac{1}{6}$

attempt to substitute their midpoint and their gradient into equation of straight line ***(M1)***

eg. $y-3 = -\frac{1}{6}\left(x-\frac{23}{2}\right)$ **OR** $3 = -\frac{1}{6}(11.5)+c$

$y = -0.167x + 4.92 \left(y = -0.166666\ldots x + 4.91666\ldots, y = -\frac{1}{6}x + \frac{59}{12}\right)$ ***A1***

**Note**: Substituting $m = 6$ (no negative reciprocal) and their point into the equation of straight line would receive at most ***A1M0M1A0***.

***[6 marks]***

(c) (closest to Q so) 4 ***A1***

***[1 mark]***

(d) **EITHER**

$.(\mathrm{H}_1 :)\ \mu_1 < \mu_2 .$ ***A1***

**OR**

**mean** amount of money spent by all customers on weekdays is less than the **mean** amount of money spent by all customers on weekends ***A1***

**Note**: Award ***A0*** for 'sample mean' or if 'mean" is omitted. For example, 'money spent on weekdays is less than the money spent on weekends'.

***[1 mark]***

*continue…*

*Question 1 continued.*

(e) (i) convenience ***A1***

(ii) not representative of the whole population ***A1***

*Do not accept the following reasons*
- disadvantages of other sampling methods
- there are five days during the week, only two on a weekend
- anything based on speculation/information not presented in the question
- people spend different amounts of money
- this is a sample (doesn't collect every data point) more data should have been collected

***[2 marks]***

(f) $p > 0.05 \ (\alpha)$ **OR** $0.0897\ldots. > 0.05$ ***R1***

There is insufficient evidence to reject $\mathrm{H}_0$ ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***
***[Total 13 marks]***

**5.** (a) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{4.2^2 - 3^2}$

$= 2.94\ (2.93938\ldots)$ (m) ***A1***

***[2 marks]***

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$60°$ ***(A1)***

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$ ***(M1)***

$\frac{1}{2} \times 3 \times 3 \times \sin(60°)$

$\left(6 \times \frac{1}{2} \times 3 \times 3 \times \sin(60^\circ)\right) = 23.4\ \ (23.3826..., \frac{27\sqrt{3}}{2})\ (\text{m}^2)$ ***A1***

**Note:** Award ***A0M0A0*** for $\frac{1}{2} \times 3 \times 3$ or equivalent.

**METHOD 2** (Use of altitude)

(altitude is) $\frac{3}{2}\sqrt{3}\ (2.59807\ldots)$ ***(A1)***

attempt to find area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude ***(M1)***

$\frac{1}{2} \times 3 \times \frac{3}{2}\sqrt{3}$

$\left(6 \times \frac{1}{2} \times 3 \times \frac{3}{2}\sqrt{3}\right) = 23.4 \left(23.3826\ldots, \frac{27\sqrt{3}}{2}\right)$ ($\text{m}^2$) ***A1***

**Note:** Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

**METHOD 3** (Finding the area of a trapezoid)

(altitude of one trapezoid is) $\frac{3}{2}\sqrt{3}\ (2.59807\ldots)$ ***(A1)***

attempt to find area of one trapezoid using $\frac{1}{2} \times (a+b)\,h$ ***(M1)***

$\frac{1}{2}(3+6)\left(\frac{3}{2}\sqrt{3}\right)$ **OR** $\frac{27}{4}\sqrt{3}$

$\left(2 \times \frac{1}{2}(3+6)\left(\frac{3}{2}\sqrt{3}\right)\right) = 23.4 \left(23.3826\ldots, \frac{27\sqrt{3}}{2}\right)$ ($\text{m}^2$) ***A1***

*continue…*

*Question 5 continued.*

(ii) $\frac{1}{3} \times 23.3826... \times 2.93938...$ ***(A1)***

$= 22.9 \text{ m}^3 \ (22.9102\ldots)$ ***A1***

**Note:** Units must be seen.

***[5 marks]***

(c) $\cos(\hat{OAT}) = \frac{3}{4.2}$ or correct equivalent ***(A1)***

$(\hat{OAT} =)\ 44.4^\circ$ (44.4153…, 0.775193… radians) ***A1***

***[2 marks]***

(d) Angle QAP = 180 – 44.4... = 135.584…° ***(A1)***

Angle QPA = 180 – 30 – 135.584... = 14.4153…° ***(A1)***

**Note:** These angles may be seen in the sine rule.

Attempt to substitute into sine rule ***(M1)***

$$\frac{AQ}{\sin(14.4153...)} = \frac{3.5}{\sin(135.584\ldots)}$$

$AQ = 1.25 \ \ (1.24500\ldots)$ (m) ***A1***

**Note:** Accept 1.24, from use of a 3sf value for $\hat{QAP}$ and $\hat{QPA}$.

***[4 marks]***

(e) attempt to substitute into cosine rule to form a quadratic for QR ***(M1)***

$0.8^2 = QR^2 + 1.24500...^2 - 2 \times 1.24500... \times QR \times \cos(30)$ ***(A1)***

$QR = 0.576 \ (0.575717\ldots)$ (m) , $1.58 \ (1.58068\ldots)$ (m) ***A1A1***

***[4 marks]***

***[Total: 17 marks]***

**1.** (a) $(2, 2)$ ***A1***

**Note:** Award ***A0*** if paretheses are omitted.

***[1 mark]***

(b) (i) attempt to use midpoint formula (at least one correct) ***(M1)***

$\left(\frac{9+10}{2}, \frac{0+6}{2}\right)$

$(9.5, 3)$ ***A1***

(ii) $\left(m = \frac{6-0}{10-9} = \right)6$ ***(A1)***

finding negative reciprocal of their gradient ***(M1)***

$m_{\perp} = -\frac{1}{6}$

attempt to substitute their midpoint and their gradient into equation of straight line ***(M1)***

eg. $y-3 = -\frac{1}{6}\left(x-\frac{19}{2}\right)$ **OR** $3 = -\frac{1}{6}(9.5)+c$

$y = -\frac{1}{6}x+\frac{55}{12}$ **OR** $y = -0.167x+4.58 \quad \left(y = -0.166666...x+4.58333...\right)$ ***A1***

**Note:** Substituting $m = 6$ (no negative reciprocal) and their point into the equation of straight line would receive at most ***A1M0M1A0***.

***[6 marks]***

(c) (closest to B so) $5$ ***A1***

***[1 mark]***

(d) **EITHER**

$(\mathrm{H}_1:)\ \mu_1 < \mu_2$ ***A1***

**OR**

**mean** amount of money spent by all customers on weekdays, is less than the **mean** amount of money spent by all customers on weekends ***A1***

**Note:** Award ***A0*** for "sample mean" or if 'mean' is omitted. Eg "money spent on weekdays, is less than the money spent on weekends".

***[1 mark]***

*continue…*

*Question 1 continued.*

(e) (i) convenience ***A1***

(ii) not representative of the whole population ***A1***

*Do not accept the following reasons*
- disadvantages of other sampling methods
- there are five days during the week, only two on a weekend
- anything based on speculation/information not presented in the question
- people spend different amounts of money
- this is a sample (doesn't collect every data point) more data should have been collected

***[2 marks]***

(f) $p > 0.05\ (\alpha)$ **OR** $0.0897\ldots. > 0.05$ ***R1***

There is insufficient evidence to reject $\mathrm{H}_0$. ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***
***[Total 13 marks]***

**5.** (a) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{3.4^2 - 2^2}$

$= 2.75\ (2.74954\ldots)$ (m) ***A1***

***[2 marks]***

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$60°$ ***(A1)***

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$ ***(M1)***

$\frac{1}{2} \times 2 \times 2 \times \sin(60°)$

$\left(6 \times \frac{1}{2} \times 2 \times 2 \times \sin(60^\circ)\right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

**Note:** Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

**METHOD 2** (Use of altitude)

(altitude is) $\sqrt{3}$ ***(A1)***

attempt to find the area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude. ***(M1)***

$\frac{1}{2} \times 2 \times \sqrt{3}$

$\left(6 \times \frac{1}{2} \times 2 \times \sqrt{3}\right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

**Note:** Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

**METHOD 3** (Finding the area of a trapezoid)

(altitude of one trapezoid is) $\sqrt{3}$ ***(A1)***

attempt to find area of one trapezoid using $\frac{1}{2} \times (a + b)\, h$ ***(M1)***

$\frac{1}{2} \times (2 + 4)\sqrt{3} \quad \left(3\sqrt{3}\right)$

$\left(2 \times \frac{1}{2} \times (2 + 4)\sqrt{3}\ \right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

*continue…*

*Question 5 continued.*

(ii) $\frac{1}{3} \times 10.3923... \times 2.74954...$ ***(A1)***

$= 9.52 \text{ m}^3 \ (9.52470\ldots)$ ***A1***

**Note:** Units must be seen.

***[5 marks]***

(c) $\cos(\text{M}\hat{\text{A}}\text{T}) = \frac{2}{3.4}$ or correct equivalent ***(A1)***

$(\text{M}\hat{\text{A}}\text{T} =) \ 54.0^\circ \quad (53.9681\ldots)$ ***A1***

***[2 marks]***

(d) Angle $\text{YAX} = 180 - 53.9681... = 126.031\ldots^\circ$ ***(A1)***

Angle $\text{YXA} = 180 - 35 - 126.031... = 18.9681\ldots^\circ$ ***(A1)***

**Note:** These angles may be seen in the sine rule.

Attempt to substitute into sine rule ***(M1)***

$$\frac{\text{AY}}{\sin(18.9681...)} = \frac{2.6}{\sin(126.031)}$$

$\text{AY} = 1.05 \quad (1.04503\ldots) \ (\text{m})$ ***A1***

***[4 marks]***

(e) attempt to substitute into cosine rule to form a quadratic for $\text{YZ}$ ***(M1)***

$0.9^2 = \text{YZ}^2 + 1.04503...^2 - 2 \times 1.04503... \times \text{YZ} \times \cos(35)$ ***(A1)***

$\text{YZ} = 0.185 \ (0.184692\ldots) \ (\text{m}) \ , \ 1.53 \ (1.52739\ldots) \ (\text{m})$ ***A1A1***

***[4 marks]***

***[Total: 17 marks]***

**6.** (a) attempt to calculate area of a circle using 4 or 2.8 ***M1***

recognition of multiplication by $\frac{1}{5}$ or $\frac{72}{360}$ ***M1***

$A = \frac{1}{5}\left(\pi(4)^2 - \pi(2.8)^2\right)$

$5.13\ (\text{m}^2)$ $(5.12707\ldots\ (\text{m}^2),\ \frac{204\pi}{125}\ (\text{m}^2),\ 1.632\pi\ (\text{m}^2))$ ***A1***

$5.12707\ldots < 6$ **OR** Maureen has enough ***AG***

**Note:** Do not award the final ***A1*** for the area unless a comparison to 6 is shown OR a statement of having enough is seen.

***[3 marks]***

(b) $(C =)\ 2\pi(4)\quad (25.1327\ldots)$ ***(A1)***

recognition to multiply by $\frac{3}{5}$ or $\frac{216}{360}$ ***(M1)***

$\frac{3}{5} \times 2\pi(4) + 2 \times 4$

$23.1\ (\text{m})\quad (23.0796\ldots(\text{m}),\ \frac{24\pi}{5} + 8)$ ***A1***

***[3 marks]***

***[Total: 6 marks]***

**13.** (a) (i) $170°$ ***(A1)***

(ii) attempt to substitute into the cosine rule formula ***(M1)***

$(DC^2) = 2^2 + 13^2 - 2(2)(13)\cos(170°)$ ***(A1)***

$(DC =)\ 15.0\ (14.9736\ldots)\ (\text{km})$ ***A1***

**Note:** Do not accept a 2sf answer of 15. Award ***M0A0A0*** for a 2sf answer of 15 with no working shown OR $2+13=15$.

(iii) correct substitution into sine rule or cosine rule ***(A1)***

$\frac{\sin B\hat{D}C}{13} = \frac{\sin 170°}{14.9736\ldots}$ **OR** $\cos(B\hat{D}C) = \frac{14.9736...^2 + 2^2 - 13^2}{2 \times 14.9736... \times 2}$

$(B\hat{D}C =)\ 8.67°\ (8.67097\ldots°)$ ***(A1)***

(bearing $=$) $098.7°\ (098.6709\ldots°)$ ***A1***

***[7 marks]***

(b) (distance between points D and E $=$) $13\ (\text{km})$ ***A1***

points B, C, D, and E make a parallelogram **OR** DB and EC are parallel **and** equal in length ***R1***

**Note:** It is possible to award ***A1R0*** for this question part.

***[2 marks]***
***[Total 9 marks]***

**5.** (a) $\hat{VSM} = 135°$ ***A1***

**Note:** Maybe seen on the diagram.

***[1 mark]***

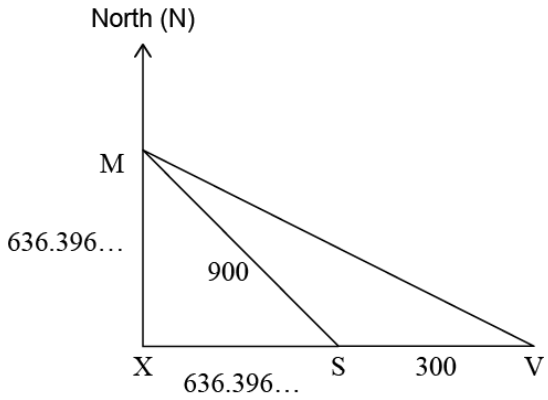
(b) **METHOD 1**

attempt to use cosine rule ***(M1)***

$MV^2 = 900^2 + 300^2 - 2(900)(300)\cos(135)$ ***(A1)***

$MV = 1130(\text{km})$ $(1132.18...)$ ***A1***

**METHOD 2**




attempt to use Pythagoras ***(M1)***

$MV^2 = (636.396...)^2 + (636.396... + 300)^2$ ***(A1)***

$MV = 1130(\text{km})$ ***A1***

***[3 marks]***

(c) **METHOD 1**

attempt to use sine rule ***(M1)***

$\frac{\sin V}{900} = \frac{\sin 135}{\text{their MV}}$ ***(A1)***

angle $V = 34.2°$ $(34.2009...°)$ ***(A1)***

**Note:** Award ***(M1)(A1)*** if $S\hat{M}V = 10.8°$ is seen

bearing $= 304°$ $(304.200...°)$ ***A1***

**Note:** Award ***(M1)*** for attempt to use trig, ***(A1)*** for correct substitution, ***(A1)*** for finding angle correctly and ***A1*** for correct bearing.

*continued…*

*Question 5 continued*

**METHOD 2**

attempt to use cosine rule ***(M1)***

$\cos^{-1}\left(\frac{300^2 + \text{their MV}^2 - 900^2}{2 \times 300 \times \text{their MV}}\right)$ ***(A1)***

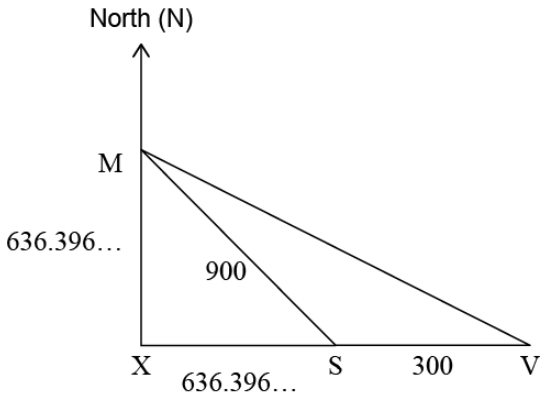
angle $V = 34.2^\circ \quad (34.2009...^\circ)$ ***(A1)***

**Note:** Award ***(M1)(A1)*** if $S\hat{M}V = 10.8^\circ$ is seen

bearing $= 304^\circ \quad (304.200...^\circ)$ ***A1***

**Note:** Award ***(M1)*** for attempt to use trig, ***(A1)*** for correct substitution, ***(A1)*** for finding angle correctly and ***A1*** for correct bearing.

**METHOD 3**




attempt to use tan. ***(M1)***

$\tan X\hat{M}V = \frac{636.396... + 300}{636.396...}$ ***(A1)***

$X\hat{M}V = 55.7990...^\circ$ ***)*** or $S\hat{V}M = 34.2...^\circ$ or $N\hat{M}V = 124.2...^\circ$ ***(A1)***

bearing $= 304^\circ \quad (304.200...^\circ)$ ***A1***

**Note:** Award ***(M1)*** for attempt to use trig, ***(A1)*** for correct substitution, ***(A1)*** for finding angle correctly and ***A1*** for correct bearing.

***[4 marks]***
***Total [8 marks]***

**7.** (a) $y = 10 - x$ OE ***A1A1***

***[2 marks]***

**Note:** Award **A*1*** for correct intercept seen, ***A1*** for fully correct equation.
Accept $y = -1x + 10$.
OE means for example $y - 7 = -1(x - 3)$ is ***A1A1***

(b) (i) $(8,\ 4)$ ***A1***

**Note:** Do not condone the omission of parenthesis.

(ii) $5$ ***A1***

***[2 marks]***

(c) (i) $8$ ***A1***

(ii) **EITHER**
attempt to use Pythagoras theorem **(*M1*)**

$4 + (b-5)^2 = 9 + (9-b)^2$ **(*A1*)**

$(b =) \frac{61}{8} = 7.625 \quad \left(\frac{61}{8}, 7\frac{5}{8}\right)$ ***A2***

**Note:** Award **A*1A0*** for 7.63

**OR**

$m(BD) = -\frac{4}{5}$ **(*A1*)**

midpoint $[BD] = (7.5,\ 7)$ **(*A1*)**

**Note:** Gradient and midpoint may be implied by further working. Condone as interim coordinate $7.5, 7$ instead of $(7.5, 7)$.

$(y - 7) = \frac{5}{4}(x - 7.5)$

substitute $x = 8$ in their perpendicular bisector **(*M1*)**

$b - 7 = \frac{5}{4}(8 - 7.5)$

$(b =) 7.625 \quad \left(\frac{61}{8}, 7\frac{5}{8}\right)$ ***A1***

**Note:** Award **A*1A1M1A0*** final answer of 7.63

**OR**

$m(\mathrm{DF}) = -4$ **(*A1*)**

midpoint $[\mathrm{DF}] = (5.5\ , 7)$ **(*A1*)**

*continued…*

*Question 7 continued*

**Note:** Gradient and midpoint may be implied by further working. Condone as interim coordinate $5.5, 7$ instead of $(5.5, 7)$

$(y-7)=\frac{1}{4}(x-5.5)$

substitute $x=8$ in their perpendicular bisector **(*M1*)**

$(8-7)=\frac{1}{4}(b-5.5)$

$(b=)7.625\left(\frac{61}{8}, 7\frac{5}{8}\right)$ ***A1***

**Note:** Award **A*1A1M1A0*** final answer of 7.63

***[5 marks]***
***Total [9 marks]***

**11.** (a) use of Pythagoras in 3 dimensions ***(M1)***

$AB^2 = 2^2 + 3^2 + 1^2$

$AB = 3.74 \quad (= 3.74165... = \sqrt{14})$ ***A1***

***[2 marks]***

(b) $1^2 + 6^2 + (k-1)^2 = 14 + 1^2 + 3^2 + (k-2)^2$ ***(M1)A1***

**Note:** Award ***M1*** for use of Pythagoras in three dimensions, ***A1*** for correct use.

$38 - 2k = 28 - 4k$ ***(M1)***

**Note:** Award ***M1*** for reducing equation to linear form.

$k = -5$ ***A1***

***[4 marks]***

(c) **EITHER**

$\cos(B\hat{A}C) = \frac{\sqrt{14}}{\sqrt{73}}$ ***(M1)(A1)***

**OR**

$\sin(B\hat{A}C) = \frac{\sqrt{59}}{\sqrt{73}}$ ***(M1)(A1)***

**OR**

$\tan(B\hat{A}C) = \frac{\sqrt{59}}{\sqrt{14}}$ ***(M1)(A1)***

**Note:** Award ***M1*** for use of trig ratio, ***A1*** for correct use.

**THEN**

$B\hat{A}C = 64.0° \quad (64.0282...^\circ) \quad (1.12\,(1.11750...))$ ***A1***

***[3 marks]***

***Total [9 marks]***

**Q1.**

(a) attempt to substitute coordinates A and C in the formula for midpoint in 3D ***(M1)***

$\left(\frac{-2+1}{2}, \frac{0+3}{2}, \frac{0+0}{2}\right)$

$(-0.5, 1.5, 0)$ OR $\left(-\frac{1}{2}, \frac{3}{2}, 0\right)$ ***A1***

**Note*:*** Award at most ***M1A0*** for answers not given as coordinates. Accept $x = -0.5, y = 1.5, z = 0$.

***[2 marks]***

(b) 5 (cm) ***A1***

***[1 mark]***

(c) attempt to substitute coordinates $\mathrm{C}$ and $\mathrm{H}$ in the formula for distance between two points in 3D ***(M1)***

$\sqrt{(1-(-0.5))^2 + (3-(1.5))^2 + (0-(5))^2}$

5.43 (5.43139…) (cm) ***A1***

**Note**: If use of 2D has been penalized in part (a), do not penalize in part (c)

***[2 marks]***

***Total [5 marks]***

**Q5.**

(a) midpoint of $[AB]$: $(4, 6.5)$ ***(A1)***

gradient of $[AB]$: $\frac{1}{2}$ ***(A1)***

gradient of perpendicular line: $-2$ ***(M1)***

**Note**: Award ***M1*** for gradient of perpendicular line. Allow follow through from their gradient of $[AB]$.

$y - 6.5 = -2(x - 4)$ OR $y = -2x + 14.5$ ***A1***

***[4 marks]***

(b) Option 3 ***A2***

***[2 marks]***

Note: Award ***A1A0*** if 2 options are selected and 1 of them is Option 3. Award ***A0A0*** in all other cases.

(c) attempt to find distance $(6, 4)$ to D or to draw a circle with radius 3 on the diagram ***M1***

$\sqrt{2^2 + 2^2} = \sqrt{8}, 2.82842...$

Yes (she is experiencing excellent coverage) ***A1***

**EITHER**

$2.83$ (km) < $3$ km ***R1***

**OR**

$(6, 4)$ is within a radius of $3$ km from site D ***R1***

**Note**: Allow follow through for the ***R1*** from their distance. Award at most ***M0R1FTA0*** for comparing their distance with 3 km. Condone truncated 2 or 3 sf values for the ***R*** mark.
Do not award ***A1R0.***

*continued*

*Question 5 continued.*

**Note**: Allow the use of point $B$ for candidates who assume the new towers have not yet been built.

attempt to find distance $(6, 4)$ to $B$ or to draw a circle with radius 3 on the diagram ***M1***

$\sqrt{4^2 + 1^2} = \sqrt{17} = 4.12310...$

No (she is not experiencing excellent coverage) ***A1***

**EITHER**

$4.12$ (km) > $3$ km ***R1***

**OR**

$(6,4)$ is not within a radius of $3$ km from site B ***R1***

***[3 marks]***

***Total [9 marks]***

**Q6.**

(a) attempt to use the cosine rule formula ***(M1)***

$(\text{NV} =)\sqrt{20^2 + 25^2 - 2\times 20\times 25\cos 55.5^\circ}$ ***(A1)***

$= 21.4\,(\text{m})\,(21.4148...)$ ***A1***

***[3 marks]***

(b) **EITHER**

attempt to use the sine rule formula ***(M1)***

$\frac{\sin \hat{\text{PNV}}}{25} = \frac{\sin 55.5^\circ}{21.4148...}$ ***(A1)***

**OR**

attempt to use the cosine rule formula ***(M1)***

$\cos \hat{\text{PNV}} = \frac{20^2 + 21.4148...^2 - 25^2}{2\times 20\times 21.4148...}$ ***(A1)***

**THEN**

$\hat{\text{PNV}} = 74.2^\circ\,(74.1749...^\circ)$ ***A1***

***[3 marks]***

(c) recognition that shortest distance is perpendicular to NV ***(M1)***

(may be seen on diagram)

use of right-angled triangle or other valid method ***(M1)***

$\sin 74.1749...\times 20$ OR $\frac{1}{2}\times 20\times 25\times \sin 55.5^\circ \times \frac{2}{21.4148...}$

$= 19.2\,(\text{m})\,(19.2419...)$ ***A1***

**Note**: Accept 19.3 (m) from using their 3 sf value in $\frac{1}{2}\times 20\times 25\times \sin 55.5^\circ \times \frac{2}{21.4148...}$

***[3 marks]***

***Total [9 marks]***

**4.** (a) $(4,1,0)$ ***A1A1***

**Note:** Award ***A1*** for $4$ seen as the $x$-coordinate or $1$ seen as the $y$- coordinate, ***A1*** for all coordinates correct and in the correct order. Accept $x=4, y=1, z=0$.

***[2 marks]***

(b) correct substitution into distance formula ***(A1)***

$\sqrt{(13-13)^2+(25-1)^2+(7-0)^2}$ **OR** $\sqrt{0^2+24^2+7^2}$

**Note:** Accept omission of $(13-13)^2$ or $0^2$.

$(\text{AC}=)\ 25\ (\text{cm})$ ***A1***

***[2 marks]***

(c) **Method 1**

$24$ and $7$ seen ***A1***

$24^2+7^2=25^2$ **OR** $625=625$ **OR** (angle $\text{B}=)\ \cos^{-1}\left(\frac{7^2+24^2-25^2}{2(7)(24)}\right)=90^\circ$ ***R1***

**Note:** Accept $\text{AB}^2+\text{BC}^2=625$ as long as the length of $\text{AB}$ and of $\text{BC}$ are defined.

therefore $\Delta\text{ABC}$ must be a right-angled triangle ***AG***

**Method 2**

A relevant statement describing $\vec{\text{AB}}$ and $\vec{\text{BC}}$, such as:
$\text{AB} \parallel y$-axis **AND** $\text{BC} \parallel z$-axis
**OR**
$x$-coordinates are the same for $\text{A}$ and $\text{B}$ **AND** $y$-coordinates are the same for $\text{B}$ and $\text{C}$ ***A1***
$\text{AB}$ perpendicular to $\text{BC}$ since $x$-axis is perpendicular to $z$-axis ***R1***
therefore $\Delta\text{ABC}$ must be a right-angled triangle ***AG***

***[2 marks]***

(d) width of $9$ or correct cross-section area $(84)$ seen ***(A1)***

correct substitution into volume formula ***(A1)***

$V=\frac{1}{2}(24)(7)(9)$

$(\text{Volume}=)756\ \text{cm}^3$ ***A1***

**Note:** Do not award final ***A1*** if correct units are not included.

***[3 marks]***

(e) correct substitution using *their* AB from part (c) or (d) and *their* original volume from part (d) ***(A1)***

$(\text{AX} =)\ 24 \times \sqrt{\frac{625}{756}}$

$21.8\ (\text{cm})\quad (21.8217\ldots\ (\text{cm}))$ ***A1***

***[2 marks]***

(f) multiplying $625$ by $1.1$ or $1.2$ or $0.025$ ***(M1)***

$625 \times 1.1 \times 0.025 \times 1.2$ ***(A1)***

(price $=$) $20.63\ \ (20.625)$ (USD) ***A1***

***[3 marks]***

***[Total 14 marks]***

**5.** (a) (i) attempt to substitute any given coordinates into midpoint formula ***(M1)***

$(12, 12.5)$ or $(12, \frac{25}{2})$ ***A1***

**Note:** Award ***A0*** if parentheses are omitted.

(ii) attempt to find gradient of $\mathrm{BC}$ using coordinates of $\mathrm{B}$ and $\mathrm{C}$ ***(M1)***

(gradient of $\mathrm{BC}=$ ) $\frac{12-13}{16-8}\left(=-\frac{1}{8}=-0.125\right)$ ***(A1)***

(gradient of $\perp=$ ) $8$ ***A1***

**Note:** Follow through within question part for final ***A1*** from their gradient of $\mathrm{BC}$ only if the gradient is seen.

(iii) **EITHER**

$8(x-12)=y-12.5 \quad \left(8\left(x-12\right)=y-12\frac{1}{2}\right)$ ***A1***

**OR**

$y=8x-83.5 \quad \left(y=8x-83\frac{1}{2}\right)$ ***A1***

***[6 marks]***

(b) (i) **EITHER**

$\mathrm{B}$ is expected to respond since $(14, 10)$ lies in the same cell (of the Voronoi diagram) as fire station $\mathrm{B}$ ***R1***

**OR**

$\mathrm{B}$ is expected to respond since $(14, 10)$ is closer to $\mathrm{B}$ than to any other fire station ***R1***

**OR**

$\mathrm{B}$ is expected to respond since it's the closest (nearest responder) to $(14, 10)$ ***R1***

**OR**

Fire station $\mathrm{B}$ will respond as the fire is within $\mathrm{B}$'s boundaries/area/region. ***R1***

**Note:** Do not award ***R1*** unless the statement refers to both the distance (closer) to $\mathrm{B}$ (or located in the $\mathrm{B}$ cell) **and** the responder (point $\mathrm{B}$ or fire station $\mathrm{B}$ ) For example, award ***R0*** for "Point $\mathrm{B}$ is closer to $(14, 10)$" as there is no reference to a responder.

(ii) *Award **R1** for any viable reason such as:*

- fire station $\mathrm{B}$ might have already responded to a different fire
- a different fire station might have more manpower or equipment available to fight the fire
- a different fire station might respond to assist fire station $\mathrm{B}$ if the fire is large
- the Voronoi diagram only considers straight-line distances, so a different fire station might have a shorter driving route to the fire ***R1***

***[2 marks]***

(c) (i) attempt to calculate the length $\mathrm{DB}$ ***(M1)***

a correct expression for $\mathrm{DB}$ ***A1***

$(\mathrm{DB}=)\ \sqrt{\left(11\frac{1}{3}-16\right)^2+\left(7\frac{1}{6}-12\right)^2}$

$\mathrm{DB}=6.71855$ ***AG***

(ii) **METHOD 1**

$(\mathrm{DC}=)\ \sqrt{\left(11\frac{1}{3}-8\right)^2+\left(7\frac{1}{6}-13\right)^2}$ ***A1***

$\mathrm{DC}=6.71855$ therefore, the fire is equidistant from all 3 fire stations ***R1***

so, any of the fire stations would be expected to respond ***AG***

**Note:** Do not award ***A0R1***. Do not award ***R1*** if ***AG*** statement is not seen.

**METHOD 2**

The intersection of bisector $\mathrm{AB}$ and bisector $\mathrm{BC}$:

$y=8x-83.5$ and $y=0.5x+1.5$ have an intersection at $\mathrm{D}\left(11\frac{1}{3},\ 7\frac{1}{6}\right)$ ***A1***

**Note:** Calculator solve is acceptable but $\mathrm{D}$ must be clearly identified from a calculation of intersecting lines.

Since $\mathrm{D}$ is the intersection of the bisector lines, the fire at $\mathrm{D}$ is equidistant from all $3$ fire stations ***R1***

so, any of the fire stations would be expected to respond ***AG***

**Note:** Do not award ***A0R1.*** Do not award ***R1*** if ***AG*** statement is not seen.

***[4 marks]***

(d) (i) **METHOD 1: Dividing into two triangles**

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

$(20-7.75=)\ 12.25$ or $(20-11.3333\ldots=)\ 8.66666\ldots$ seen ***(A1)***

substitution into triangle area formula for either triangle ***(M1)***

$(A=)\ 0.5\times(20-7.75)\times 7.16666\ldots+0.5\times 11.5\times(20-11.3333\ldots)$ ***(A1)***

$93.7291\ldots$ (sq. units)

$94$ (sq. units) ***A1***

**Note:** Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(A1)(M1)(A1)A0*** for an answer of $93.7$ without working.

**METHOD 2: Dividing vertically into triangle and trapezoid**

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

attempt to substitute into area of trapezoid formula ***(M1)***
$(A=)\ 0.5\times(7.16666\ldots+11.5)\times(20-11.3333\ldots)$

**Note:** Lengths of parallel sides must be seen to award the ***M1***.

correct calculation of right triangle area ***(A1)***
$(A=)\ 0.5\times(11.3333\ldots-7.75)\times 7.16666\ldots$

$(A=)\ 0.5\times(11.3333\ldots-7.75)\times 7.16666\ldots+0.5\times(7.16666\ldots+11.5)\times(20-11.3333\ldots)$ ***(A1)***

$93.7291\ldots$ (sq. units)

$94$ (sq. units) ***A1***

**Note:** Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(M1)(A1)(A1)A0*** for an answer of $93.7$ without working.

**METHOD 3: Dividing horizontally into triangle and trapezoid**

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

attempt to substitute into area of trapezoid formula ***(M1)***

$(A=)\ 0.5\times\left[(20-7.75)+(20-11.3333\ldots)\right]\times 7.16666\ldots$

**Note:** Lengths of parallel sides must be seen to award the ***M1***.

correct calculation of right triangle area ***(A1)***

$(A=)\ 0.5\times(20-11.3333\ldots)\times(11.5-7.16666\ldots)$

$(A=)\ 0.5\times(20-11.3333\ldots)\times(11.5-7.16666\ldots)+0.5\times[(20-7.75)+(20-11.3333\ldots)]\times 7.16666\ldots$ ***(A1)***

$93.7291\ldots$ (sq. units)

$94$ (sq. units) ***A1***

**Note:** Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(M1)(A1)(A1)A0*** for an answer of $93.7$ without working.

**METHOD 4: Dividing into two triangles and rectangle**

$(0.5\times 20+1.5=)11.5$ seen in an area calculation ***(A1)***

$(20-11.3333...)$ **OR** $(11.3333...-7.75)$ **OR** $(11.5-7.16666...)$ seen ***(A1)***

substitution into triangle area formula. ***(M1)***

$0.5\times(11.3333...-7.75)\times 7.16666...+(20-11.3333...)\times 7.16666...+$
$+0.5\times(11.5-7.16666...)\times(20-11.3333...)$

***(A1)***

$93.7291\ldots$ (sq. units))

$94$ (sq. units) ***A1***

**Note:** Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(A1)(M1)(A1)A0*** for an answer of $93.7$ without working.

**METHOD 5: Calculus**

A and C mid point $(13, 10.5)$

(gradient of AC= ) $\frac{13-8}{8-18} = -\frac{5}{10}$

(gradient of $\perp$=) $2$

AC bisector $2(x-13) = y-10.5$ **OR** $y = 2x-15.5$ ***A1***

attempt to integrate their equation and given equation ***(M1)***

$\text{area} = \int_{7.75}^{11.3} 2x-15.5\,\mathrm{d}x + \int_{11.3}^{20} 0.5x+1.5\,\mathrm{d}x$ ***(A1)(A1)***

**Note:** Award ***A1*** for each correct expression with correct limits.

93.7291… (sq. units))

94 (sq. units) ***A1***

**Note:** Answer must be given as a whole number to award final ***A1***.
Award ***A1(M1)(A1)(A1)A0*** for an answer of 93.7 without working.

(ii) (1 sq unit = $2.5\text{ km} \times 2.5\text{ km}$ =) $6.25\ (\text{km}^2)$ ***(A1)***

**Note:** Award ***A1*** for 6.25 seen.

$(94 \times 6.25\,\text{km}^2)$

588 (587.5 ) $(\text{km}^2)$ ***A1***

**Note:** Accept: 586 (585.807…) $(\text{km}^2)$ from $(93.7291\ldots \times 6.25\ \text{km}^2)$.

***[7 marks]***
***[Total: 19 marks]***

**6.** (a) attempt to use cosine rule ***(M1)***

$\cos\theta = \frac{15^2+10^2-8^2}{2\times15\times10}$ ***(A1)***

$29.5(°)$ **OR** $0.516$ $(29.5413\ldots, 0.515594\ldots)$ ***A1***

***[3 marks]***

(b) attempt to find the area of triangle ABC (substitution into the formula must be attempted) ***(M1)***

$\frac{1}{2}\times 15\times 10\times \sin(29.5413\ldots°)$ ***(A1)***

$(= 36.9788\ldots)$

attempt to find the area of sector ABD (substitution into the formula must be attempted) ***(M1)***

$\frac{29.5413\ldots}{360}\times\pi\times 10^2$ **OR** $\frac{1}{2}\times 10^2\times 0.515594\ldots$ ***(A1)***

$(= 25.7797\ldots)$

$(36.9788\ldots - 25.7797\ldots)$

$11.2\ \text{cm}^2$ $(11.1991\ldots)$ ***A1***

**Note:** Correct units are required to award the final ***A1***.

***[5 marks]***

***Total [8 marks]***

**12.** (a) (i) every point in the shaded region is closer to B than any other police station ***A1***

(ii) point D is equidistant from A, C **and** B **OR** equidistant from the three sites ***A1***
***[2 marks]***

(b) $m_{AB} = -0.5$ **OR** $m_{BC} = 1$ ***A1***

**Note:** Award ***A1*** for either perpendicular gradient seen. The gradient may be seen in the equation of the perpendicular bisector.

$y - 5 = -0.5(x - 3)$ **or** $y = -0.5x + 6.5$
**OR**
$y - 6 = 1(x - 10)$ **or** $y = x - 4$ ***M1***

**Note:** Award ***M1*** for an attempt to find the equation of AB or BC. One equation consistent with their gradient is sufficient to award this mark. If the gradient is incorrect, it must be explicitly identified as the gradient of $m_{AB}$ or $m_{BC}$.

**EITHER** (equating AB and BC)
$y - 5 = -0.5(x - 3)$ or $y = -0.5x + 6.5$ **AND** $y - 6 = 1(x - 10)$ or $y = x - 4$ ***(A1)***
$-0.5x + 6.5 = x - 4$ **OR** sketch of AB and BC with intersection marked ***(M1)***

**Note:** Award ***M1*** for an attempt to find the intersection of their lines AB and BC.

**OR** (equating AB or BC to their respective edges, and using midpoint)
$-0.5x + 6.5 = 2x - 6$ **OR** $x - 4 = -x + 13$ ***(M1)***
(midpoint of AB) $(5,4)$ **OR** (midpoint BC) $(8.5,4.5)$ ***(A1)***

**Note:** Award ***M1*** for an attempt to equate a line (AB or BC) and its associated perpendicular edge. This may be implied through the midpoint of AB $(5,4)$ or BC $(8.5,4.5)$ seen.

**THEN**
$(7,3)$ ***A1***
***[5 marks]***
***Total [7 marks]***

**5.** (a) P ***A1***

***[1 mark]***

(b) attempt to find one distance ***(M1)***

$\sqrt{116}(10.8, 10.7703\ldots, 2\sqrt{29})$ **OR** $\sqrt{200}(14.1, 14.1421\ldots, 10\sqrt{2})$ (seen) ***A1***

**Note:** Award ***(M1)*** for attempt to find one distance (i.e. OQ or ON or OM or PQ or PN or PM).
Award ***(M1)A1*** for finding correctly $\mathrm{OQ} = 10.8$ or $\mathrm{ON} = 10.8$ or $\mathrm{OM} = 14.1$.
Award ***(M0)A1A0*** for an unsupported final answer of "N & Q".
To be awarded the final ***A1*** must consider point M.
These values might be seen in working for part (a) and can be credited in part (b).

N and Q ***A1***

***[3 marks]***

(c) $x = 7$ ***A1***

**Note:** Answer must be an equation for the ***A1*** to be awarded.

***[1 mark]***

(d) the gradient of MP is $\frac{10-(-5)}{10-0} = \frac{15}{10}\ \left(or\ \frac{3}{2}\right)$ ***(A1)***

negative reciprocal of their gradient ***(M1)***

gradient of perpendicular bisector $= -\frac{2}{3}$

$\text{midpoint}_{\text{MP}}$ is $\left(\frac{10+0}{2},\ \frac{10+(-5)}{2}\right) = \left(5, \frac{5}{2}\right)$ ***(A1)***

$y - \frac{5}{2} = -\frac{2}{3}(x-5)$ **OR** $y = -\frac{2}{3}x + \frac{35}{6}$ **OR** $4x + 6y - 35 = 0$ ***A1***

**Note:** Allow follow-through, within the question, for the final ***A1*** from their midpoint and gradient of perpendicular bisector (seen), if the ***M1*** has been awarded.

***[4 marks]***

***Total [9 marks]***

**11.** (a) attempt to use volume of cylinder formula or area of sector formula ***(M1)***

$\pi \times 11^2 \times 10$ **OR** $\frac{22.5}{360} \times \pi \times 11^2$ **OR** $\frac{\pi \times 11^2}{16}$

$(V =) \frac{\pi \times 11^2}{16} \times 10$ **OR** $(V =) \frac{22.5}{360} \times \pi \times 11^2 \times 10$ ***(A1)***

$= 238 \text{ cm}^3 \left(237.582 \ldots, \frac{605}{8}\pi, 75.625\pi\right)$ ***A1***

**Note:** Do not award the final ***A1*** if units are missing or incorrect. Award ***(M1)(A1)A0*** for "238" seen without working. Accept $75.6\pi \text{ cm}^3$ as a correct final answer.

***[3 marks]***

(b) attempt find arc length **OR** area of one rectangle ***(M1)***

$\frac{2 \times 11 \times \pi}{16}$ **OR** $10 \times 11$ **OR** $\frac{22.5}{360} \times 2\pi \times 11$

$\frac{2 \times 11 \times \pi}{16} \times 10 + 2 \times 10 \times 11$ ***(A1)***

$= 263 \text{cm}^2 (263.196 \ldots)$ ***A1***

**Note:** Do not award the final ***A1*** if units are missing or incorrect. Award ***(M1)(A1)A0*** for "263" seen without working. If units were penalized in part (a) then do not penalize here so it is possible to be awarded ***(M1)(A1)A1ft***.

***[3 marks]***

***Total [6 marks]***

**3.** (a) **Cosine rule and area of triangle formula**

attempt to use cosine rule to find an angle ***(M1)***

**EITHER**

$(\theta =)\ \cos^{-1}\left(\dfrac{1600^2+1500^2-1400^2}{2\times 1600\times 1500}\right)\ \left(=53.5764\ldots^{(\circ)}\right)$ ***(A1)***

attempt to use area of triangle formula for **their angle**/expression ***(M1)***

$(A=)\ \dfrac{1}{2}1600\times 1500\times\sin\left(53.57642636\ldots^{(\circ)}\right)$ ***(A1)***

**OR**

$(\theta =)\ \cos^{-1}\left(\dfrac{1400^2+1500^2-1600^2}{2\times 1400\times 1500}\right)\ \left(=66.8676\ldots^{(\circ)}\right)$ ***(A1)***

attempt to use area of triangle formula for **their angle**/expression ***(M1)***

$(A=)\ \dfrac{1}{2}1400\times 1500\times\sin\left(66.8676036\ldots^{(\circ)}\right)$ ***(A1)***

**OR**

$(\theta =)\ \cos^{-1}\left(\dfrac{1600^2+1400^2-1500^2}{2\times 1600\times 1400}\right)\ \left(=59.5559\ldots^{(\circ)}\right)$ ***(A1)***

attempt to use area of triangle formula for their angle/expression ***(M1)***

$(A=)\ \dfrac{1}{2}1400\times 1600\times\sin\left(59.5559\ldots^{(\circ)}\right)$ ***(A1)***

**THEN**

$966\,000\quad (965\,579\ldots)(\text{m}^2)$ ***(A1)***

$9.66\times 10^5\quad (9.65579\ldots\times 10^5)\ (\text{m}^2)$ ***A1***

**Note:** Follow through within the question for correctly converting their intermediate value into standard form (but only if their pre-conversion value is seen).

***[6 marks]***

(b) $\sqrt{300^2+200^2}$ **OR** $300^2+200^2=d^2$ ***(A1)***

$361\ (360.555\ldots)\ (\text{m})$ ***A1***

***[2 marks]***

(c) $360.555\ldots\times 2\times(\$)120$ ***(A1)***

$(\$)86533.2\ldots$

$(\$)86533$ ***A1***

**Note:** Accept $86640$, $86544$, and $86534$, (from use of 3 sf, 4sf and 5sf answer to part(b), respectively).

***[2 marks]***

(d) (i) $1750 \times 60 - 86533$ ***(A1)***

$(\$)18467$ ***A1***

**Note:** Accept $18360$, $18456$, and $18466$, from 3sf, 4sf and 5sf answers to part (b).

(ii) $N = 60$

$PV = (\mp)86533$

$FV = 0$

$PMT = (\pm)1750$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use an annuity formula.
Award ***A1*** for all entries correct in financial app or correct substitution in an annuity formula.
Award ***M0*** for a substituted compound interest formula only.

$7.89(\%)$ $(7.88938\ldots)$ ***A1***

**Note:** Accept $7.84(\%)$ and $7.88(\%)$, from 3sf and 4sf answers to part (b).

***[5 marks]***
***Total [15 marks]***

APOLLO

# Statistics and probability

112 questions

**2.** (a) 14 ***A1***

***[1 mark]***

(b) $\frac{14+15+...}{10}$ ***(M1)***

$=13.1$ ***A1***

***[2 marks]***

(c) 2.21 (2.21133...) ***A1***

***[1 mark]***

***Total [4 marks]***

**6.** (a) **EITHER**

$H_0$: The population mean weight of eggs from (her/the) black geese is equal to/the same as the population mean weight of eggs from (her/the) white geese.

**OR**

$H_0$: The population mean weight of eggs from (her/the) black geese is not less than the population mean weight of eggs from (her/the) white geese. ***A1***

**Note:** Reference to the "population mean weight" must be explicit for the ***A1*** to be awarded. The term “population” can be implied by use of “all” or “on average” or “generally” when relating to the weight of eggs e.g. “the mean weight of eggs for all (her/the) black geese”.
Award ***A0*** if reference is made to the mean weights from the sample or the table.
Award ***A0*** for a null hypothesis written in symbolic form.

***[1 mark]***

(b) $p\text{-value} = 0.177 \ \ (0.176953...)$ ***A2***

**Note:** Award ***A1*** for an answer of $0.18221\ldots$, from “unpooled” settings on GDC.

***[2 marks]***

(c) $0.177 > 0.1$ ***R1***

(insufficient evidence to reject $H_0$)

Arriane’s claim is not supported by the evidence ***A1***

**Note:** Accept $p > 0.1$ or $p >$ *significance level* provided $p$ is explicitly seen in part (b).
Award ***A1*** only if reference is specifically made to Arriane's claim.
Do not award ***R0A1***.

***[2 marks]***

***Total [5 marks]***

**10.** (a)

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $\mathrm{P}(T=t)$ | $\frac{1}{36}$ $(0.027777\ldots)$ | $\frac{3}{36}$ $(0.083333\ldots)$ | $\frac{5}{36}$ $(0.138888\ldots)$ | $\frac{7}{36}$ $(0.194444\ldots)$ | $\frac{9}{36}$ $(0.25)$ | $\frac{11}{36}$ $(0.305555\ldots)$ |

***A2***

**Note:** Award ***A1*** if three to five probabilities are correct.

***[2 marks]***

(b) (i) $\frac{32}{36}$ $\left(\frac{8}{9}, 0.888888..., 88.9\%\right)$ ***(A1)***

(ii) use of conditional probability ***(M1)***
e.g. denominator of $32$ **OR** denominator of $0.888888\ldots$, etc.
$\frac{11}{32}$ $(0.34375, 34.4\%)$ ***A1***

***[3 marks]***

(c) $\frac{1\times1+3\times2+5\times3+...+11\times6}{36}$ ***(M1)***

$=\frac{161}{36}$ $\left(4\frac{17}{36}, 4.47, 4.47222...\right)$ ***A1***

***[2 marks]***

***Total [7 marks]***

**5.** (a) (i) $2$ ***A1***

(ii) $6$ ***A1***

(iii) $8$ ***A1***

***[3 marks]***

(b) **EITHER**
Each of these percentages represent approximately $25\%$ of the employees. ***R1***

**OR**
The diagram is not explicit enough to show what is happening at the quartiles regarding $6$ and $11$ / we do not have the data points ***R1***

**OR**
Discrete data not clear how to interpret "fewer". ***R1***

**THEN**
Hence, Paul is not correct (**OR** no such inference can be made). ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***

***Total [5 marks]***

**8.** (a) (let $\mu_c =$ population mean for chinchilla rabbits, $\mu_s =$ population mean for sable rabbits)

$H_0 : \mu_c = \mu_s$ ***A1***

$H_1 : \mu_c > \mu_s$ ***A1***

**Note:** Accept an equivalent statement in words, must include mean and reference to "**population** mean" / "mean for **all** chinchilla rabbits" for the first ***A1*** to be awarded.
Do not accept an imprecise *"the means are equal".*

***[2 marks]***

(b) $p$-value $= 0.0408$ $(0.0408065...)$ ***A2***

**Note:** Award ***A1*** for an answer of $0.041565\ldots$, from "unpooled" settings on GDC.

***[2 marks]***

(c) $0.0408 < 0.05$ . ***R1***

(there is sufficient evidence to) reject (or not accept) $H_0$ ***A1***

(there is sufficient evidence to suggest that chinchilla rabbits are heavier than sable rabbits)

**Note:** Do not award ***R0A1***. Accept 'accept $H_1$'.

***[2 marks]***
***Total [6 marks]***

**11.** (a) $\left(\frac{74+97+91+86+112}{5}\right)=92$ ***A1***

***[1 mark]***

(b) (i) $4$ ***A1***

(ii) $\chi^2_{\text{calc}}=8.54\ (8.54347...)$ **OR** $p\text{-value}=0.0736\ (0.0735802...)$ ***A2***

$8.54<9.49$ **OR** $0.0736>0.05$ ***R1***

therefore there is insufficient evidence to reject $\mathrm{H}_0$ ***A1***

(i.e. the data satisfies the model)

**Note:** Do not award ***R0A1***. Accept "accept" or "do not reject" in place of "insufficient evidence to reject".
Award the ***R1*** for comparing their $p$-value with $0.05$ or their $\chi^2$ value with $9.49$ and then ***FT*** their final conclusion.

***[5 marks]***

***Total [6 marks]***

**1.** (a) Quota sampling **A1**

***[1 mark]***

(b) $10$ (hours) **A1**

***[1 mark]***

(c) $15-7$ **(M1)**

**Note:** Award ***M1*** for $15$ and $7$ seen.

$8$ **A1**

***[2 marks]***

(d) indication of a valid attempt to find the upper fence **(M1)**

$15+1.5\times 8$

$27$ **A1**

$25<27$ (accept equivalent answer in words) **R1**

Jason is correct **A1**

**Note:** Do not award ***R0A1***. Follow through **within** this part from *their* $27$, but only if their value is supported by a valid attempt **or** clearly and correctly explains what their value represents.

***[4 marks]***

(e) "negative" seen **A1**

**Note:** Strength cannot be inferred visually; ignore "strong" or "weak".

***[1 mark]***

(f) correct substitution **(M1)**

$y=-1.54\times 1.5+98.8$

$96.5$ (%) $(96.49)$ **A1**

***[2 marks]***

(g) not reliable **A1**

extrapolation **OR** outside the given range of the data **R1**

**Note:** Do not award ***A1R0*.** Only accept reasoning that includes reference to the range of the data. Do not accept a contextual reason such as $1.5$ hours is too short to read the book.

***[2 marks]***

*continued…*

*Question 1 continued*

(h)

| | Book | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **A** | **B** | **C** | **D** | **E** | **F** | **G** | **H** |
| **Rank – Number of pages** | **1** | 3 | 5 | 2 | 6 | 8 | 4 | 7 |
| **Rank – Top 50 Rating** | **1** | 2 | 3 | 4 | 5 | 6 | 7 | 8 |

***A1A1***

**Note:** Award ***A1*** for correct ranks for 'number of pages'. Award ***A1*** for correct ranks for 'top 50 rating'.

***[2 marks]***

(i) (i) $0.714$ $(0.714285...)$ ***A2***

**Note:** ***FT*** from their table.

(ii) **EITHER**
there is a (strong/moderate) positive association between the number of pages and the top $50$ rating. ***A1***

**OR**
there is a (strong/moderate) agreement between the rank order of number of pages and the rank order top $50$ rating. ***A1***

**OR**
there is a (strong/moderate) positive (linear) correlation between the rank order of number of pages and the rank order top $50$ rating. ***A1***

**Note:** Follow through from their value of $r_s$ .

***[3 marks]***

***Total [18 marks]***

**4.** (a) (i) evidence of correct probability ***(M1)***
e.g sketch **OR** correct probability statement, $\mathrm{P}(X < 6.5)$

$0.0151$ ***A1***

(ii) $0.0228$ ***A1***

**Note:** Answers should be given to $4$ decimal place.

***[3 marks]***

(b) (i) multiplying **their** probability by $1000$ ***(M1)***
$451.7$ ***A1***

(ii) $510.5$ ***A1***

***[3 marks]***

**Note:** Answers should be given to $4$ sf.

(c) $\mathrm{H}_0$ : stopping distances can be modelled by $\mathrm{N}(6.76, 0.12^2)$
$\mathrm{H}_1$ : stopping distances cannot be modelled by $\mathrm{N}(6.76, 0.12^2)$ ***A1A1***

**Note:** Award ***A1*** for correct $\mathrm{H}_0$, including reference to the mean and standard deviation.
Award ***A1*** for the negation of their $\mathrm{H}_0$.

***[2 marks]***

(d) $15.1$ or $22.8$ seen ***(M1)***

$0.0727$ $(0.0726542\ldots, 7.27\%)$ ***A2***

***[3 marks]***

(e) $0.05 < 0.0727$ ***R1***
there is insufficient evidence to reject $\mathrm{H}_0$ (or "accept $\mathrm{H}_0$") ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***
***Total [13 marks]***

**1.** (a) convenience sampling ***A1***

***[1 mark]***

(b) (i) $95\%$ ***A1***

(ii) $1\%$ ***A1***

(iii) $2\%$ ***A1***

(iv) $98\%$ ***A1***

***[4 marks]***

(c) (i) $0.95 \times 0.02$ ***(M1)***

$0.019$ ***A1***

(ii) $0.05 \times 0.01 + 0.95 \times 0.98$ ***(M1)(M1)***

**Note:** Award ***M1*** for summing two products and ***M1*** for correct products seen.

$0.932 \quad (0.9315)$ ***A1***

(iii) recognition of conditional probability ***(M1)***

$$\frac{0.05 \times 0.01}{0.05 \times 0.01 + 0.95 \times 0.98}$$ ***A1***

$0.000537 \quad (0.000536768\ldots)$ ***A1***

**Note:** Accept $0.000536$ if $0.932$ used.

***[8 marks]***

(d) **EITHER**

sample may not be representative of population ***A1***

**OR**

sample is not randomly selected ***A1***

**OR**

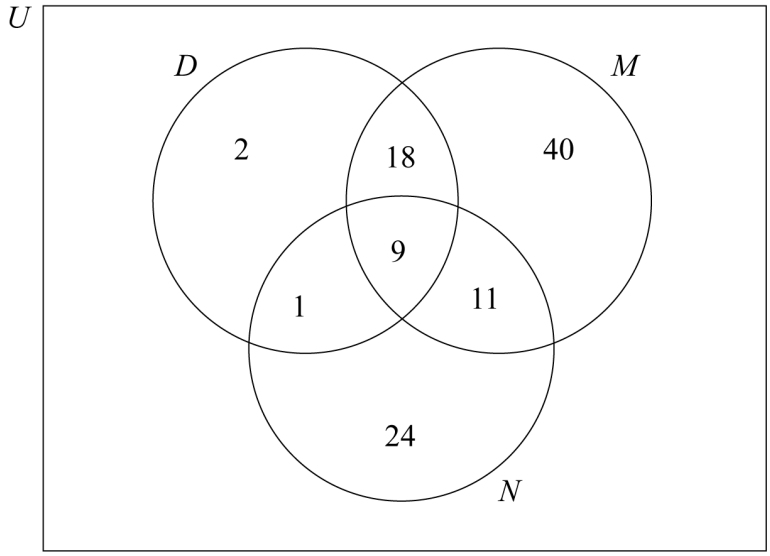
unrealistic to think expected and observed values will be exactly equal ***A1***

***[1 mark]***

*continued…*

*Question 1 continued*

(e)




***A1A1A1***

**Note:** Award ***A1*** for rectangle and 3 labelled circles and 9 in centre region; ***A1*** for 2, 40, 24; ***A1*** for 18, 1, and 11.

***[3 marks]***

(f) $18+9+1+11+2+40+24$ ***(M1)***

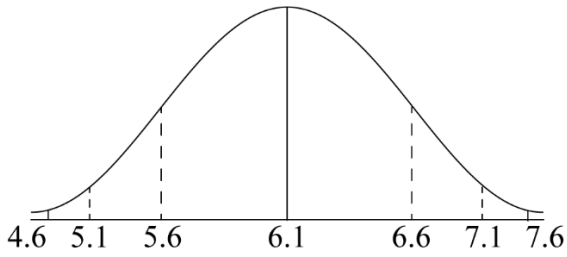
$105$ ***A1***

**Note:** Follow through from the entries on their Venn diagram in part (e). Working required for ***FT***.

***[2 marks]***

***Total [19 marks]***

**4.** (a)




***A1A1***

**Note:** Award ***A1*** for a normal curve with mean labelled $6.1$ or $\mu$, ***A1*** for indication of SD $(0.5)$: marks on horizontal axis at $5.6$ and/or $6.6$ **OR** $\mu - 0.5$ and/or $\mu + 0.5$ on the correct side and approximately correct position.

***[2 marks]***

(b) $X \sim \mathrm{N}(6.1,\ 0.5^2)$

$\mathrm{P}(5.5 < X < 6.5)$ **OR** labelled sketch of region ***(M1)***

$= 0.673\ \ (0.673074...)$ ***A1***

***[2 marks]***

(c) $(\mathrm{P}(X < 5.3) =)\ \ 0.0547992...$ ***(A1)***

$0.0547992... \times 80$ ***(M1)***

$= 4.38\ \ (4.38393...)$ ***A1***

***[3 marks]***

(d) $0.15$ **OR** $0.85$ ***(A1)***

$\mathrm{P}(X > x) = 0.15$ **OR** $\mathrm{P}(X < x) = 0.85$ **OR** labelled sketch of region ***(M1)***

$6.62\ \ (6.61821\ldots)$ ***A1***

***[3 marks]***

(e) $(\mathrm{P}(X > 6.25) =)\ \ 0.382088...$ ***(A1)***

recognition of binomial ***(M1)***

e.g. $\mathrm{B}(10,\ 0.382088...)$

$0.0502\ \ (0.0501768...)$ ***A2***

***[4 marks]***

***Total [14 marks]***

**1.** (a) $r = 0.933 \ (0.933419...)$ ***A2***

***[2 marks]***

(b) strong ***A1***

**Note:** Answer may include "positive", however this is not necessary for the mark.

***[1 mark]***

(c) $t = 0.228x + 24.3 \ (t = 0.227703...x + 24.3153...)$ ***A1***

**Note:** Condone $y$ in place of $t$. Answer must be an equation.

***[1 mark]***

(d) $(t =) \ 0.227703... \times 57 + 24.3153...$ ***(M1)***

**Note:** Award ***(M1)*** for correct substitution into their regression line.

$(t =) \ 37.3 \text{ minutes} \ (37.2944)$ ***A1***

**Note:** Accept $37.1$ and $37.4$ from use of 2sf and/or 3sf values.

***[2 marks]***
***Total: [6 marks]***

**9.** (a) $75$ ***A1***

***[1 mark]***

(b) recognition that all entries add up to $120$ ***(M1)***

$a = 120 - 6 - 13 - 26 - b$ **OR** $a = 75 - b$ ***A1***

***[2 marks]***

(c) (i) $\dfrac{6\times1+13\times2+26\times3+(75-b)\times4+b\times5}{120} = 3.65$ ***(M1)(A1)***

**Note:** Award ***(M1)*** for attempt to substitute into mean formula, LHS expression is sufficient for the ***M*** mark. Award ***(A1)*** for correct substitutions in one variable **OR** in two variables, followed by evidence of solving simultaneously with $a + b = 75$.

$(b =)\ 28$ ***A1***

(ii) $120 -$ their part (c)(i) seen (e.g. $92$ indicated on graph) ***(M1)***

$84$ ***A1***

***[5 marks]***

***Total: [8 marks]***

**11.** (a) $a = 0.42$ ***A1***

***[1 mark]***

(b) $(\mathrm{P}(B' \cap F) =)\ b \times 0.68$ ***A1***

***[1 mark]***

(c) (i) $0.32 \times 0.58 + 0.68b = 0.41$ ***(M1)***

**Note:** Award ***(M1)*** for setting up equation for fair-haired or equivalent.

$b = 0.33$ ***A1***

(ii) $c = 0.67$ ***A1***

***[3 marks]***

***Total: [5 marks]***

**1.** (a) (i) $\frac{560}{1280}\left(\frac{7}{16}, 0.4375\right)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator, ***A1*** for correct denominator.

(ii) $\frac{72}{1280}\left(\frac{9}{160}, 0.05625\right)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator, ***A1*** for correct denominator.

(iii) $\frac{153}{348}\left(\frac{51}{116}, 0.439655...\right)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator, ***A1*** for correct denominator.

(iv) $160+224+128+205+131$ **OR** $560+512-224$ ***(M1)***

$\frac{848}{1280}\left(\frac{53}{80}, 0.6625\right)$ ***A1A1***

**Note:** Award ***A1*** for correct denominator $(1280)$ seen, ***(M1)*** for correct calculation of the numerator, ***A1*** for the correct answer.

***[9 marks]***

(b) $H_0$ : the variables are independent

$H_1$ : the variables are dependent ***A1***

**Note:** Award ***A1*** for both hypotheses correct. Do not accept "not correlated" or "not related" in place of "independent".

***[1 mark]***

(c) 4 ***A1***

***[1 mark]***

(d) (i) $(\chi^2 =)\ 23.3\ (23.3258...)$ ***A2***

(ii) $0.000109\ (0.000108991...)$ **OR** $1.09\times10^{-4}$ ***A1***

(iii) **EITHER**

$23.3 > 13.277$ ***R1***

**OR**

$0.000109 < 0.01$ ***R1***

**THEN**

(there is sufficient evidence to accept $H_1$ that) preferred device and age group are not independent ***A1***

**Note:** For the final ***A1*** the answer must be in context. Do not award ***A1R0***.

***[5 marks]***

***Total: [16 marks]***

**5.** (a) (i) Let $X$ be the random variable "distance from O".

$X \sim \mathrm{N}(10, 3^2)$

$\mathrm{P}(X < 13) = 0.841 \ (0.841344...)$ ***(M1)A1***

(ii) $\left(\mathrm{P}(X > 15) =\right) \ 0.0478 \ (0.0477903)$ ***A1***

***[3 marks]***

(b) $\mathrm{P}(X > 15) \times \mathrm{P}(X > 15)$ ***(M1)***

$= 0.00228 \ (0.00228391...)$ ***A1***

***[2 marks]***

(c) $1 - (0.8143)^3$ ***(M1)***

$0.460 \ (0.460050...)$ ***A1***

***[2 marks]***

(d) (i) **METHOD 1**

let $Y$ be the random variable "number of points scored"

evidence of use of binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10, 0.539949...)$ ***(A1)***

$\left(\mathrm{P}(Y \geq 5) =\right) \ 0.717 \ (0.716650...)$. ***A1***

**METHOD 2**

let $Q$ be the random variable "number of times a point is not scored"

evidence of use of binomial distribution ***(M1)***

$Q \sim \mathrm{B}(10, 0.460050...)$ ***(A1)***

$\left(\mathrm{P}(Q \leq 5) =\right) \ 0.717 \ (0.716650...)$ ***A1***

(ii) $\mathrm{P}(5 \leq Y < 8)$ ***(M1)***

$0.628 \ (0.627788...)$ ***A1***

**Note:** Award ***M1*** for a correct probability statement or indication of correct lower and upper bounds, 5 and 7.

(iii) $\dfrac{\mathrm{P}(5 \leq Y < 8)}{\mathrm{P}(Y \geq 5)} \left(= \dfrac{0.627788...}{0.716650...}\right)$ ***(M1)***

$0.876 \ (0.876003...)$ ***A1***

***[7 marks]***

***Total: [14 marks]***

**1.** (a) $210\,\text{g}$ ***A1***

***[1 mark]***

(b) $240\,\text{g}$ ***A1***

***[1 mark]***

(c) $240 - 190$ ***(M1)***

$= 50\,\text{g}$ ***A1***

***[2 marks]***

(d) $240 + 1.5 \times (50)$ ***M1***

$= 315\,\text{g}$ ***A1***

***[2 marks]***

***Total [6 marks]***

**3.** (a) discrete ***A1***

***[1 mark]***

(b) $\frac{24 + 60 + 3k + 40 + 15 + 6}{88 + k} = 2$ ***M1A1***

**Note:** Award ***M1*** for substitution into the formula for the mean, award ***A1*** for a correct equation.

attempt to solve their equation ***(M1)***

$k = 31$ ***A1***

***[4 marks]***

(c) systematic ***A1***

***[1 mark]***

***Total [6 marks]***

**6.** (a) number of salad meals per week is independent of a person's position in the university ***A1***

**Note:** Accept "not associated" instead of independent.

***[1 mark]***

(b) $0.0201\ (0.0201118\ldots)$ ***A2***

***[2 marks]***

(c) $0.0201 < 0.05$ ***R1***

the null hypothesis is rejected ***A1***

***[2 marks]***

**Note:** Award ***(R1)*** for a correct comparison of their $p$-value to the test level, award ***(A1)*** for the correct interpretation from that comparison. Do not award ***(R0)(A1)***.

***Total [5 marks]***

**9.** (a) (i) $\mu_1 - \mu_2 = 0$ ***A1***

(ii) $\mu_1 - \mu_2 \neq 0$ ***A1***

**Note:** Accept equivalent statements in words.

***[2 marks]***

(b) $0.296\ (0.295739\ldots)$ ***A2***

***[2 marks]***

(c) $0.296 > 0.1$ ***R1***

fail to reject the null hypothesis, there is no difference between the mean height of male and female students ***A1***

**Note:** Award ***(R1)*** for a correct comparison of their $p$-value to the test level, award ***(A1)*** for the correct interpretation from that comparison. Do not award ***R0A1***.

***[2 marks]***

***Total [6 marks]***

**12.** (a) $\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

***[1 mark]***

(b) $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \ldots + 5 \times \frac{7}{18}$ ***(M1)***

**Note:** Award ***(M1)*** for their correct substitution into the formula for expected value.

$= 1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

***[2 marks]***

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$ ***(M1)(M1)***

**Note:** Award ***(M1)*** for $\frac{1}{18} \times \frac{3}{18}$, award ***(M1)*** for multiplying their product by $2$.

$= \frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

***[3 marks]***

***Total [6 marks]***



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**13.** (a) $\frac{6}{15}\left(0.4, \frac{2}{5}\right)$ ***A1***

***[1 mark]***

(b) $\mathrm{P}(X = 8)$ ***(M1)***

**Note:** Award ***(M1)*** for evidence of recognizing binomial probability.
***eg***, $\mathrm{P}(X = 8)$, $X \sim \mathrm{B}\left(20, \frac{6}{15}\right)$.

$0.180\ (0.179705\ldots)$ ***A1***

***[2 marks]***

(c) $\mathrm{P}(\text{male}) = \frac{9}{15}\ (0.6)$ ***A1***

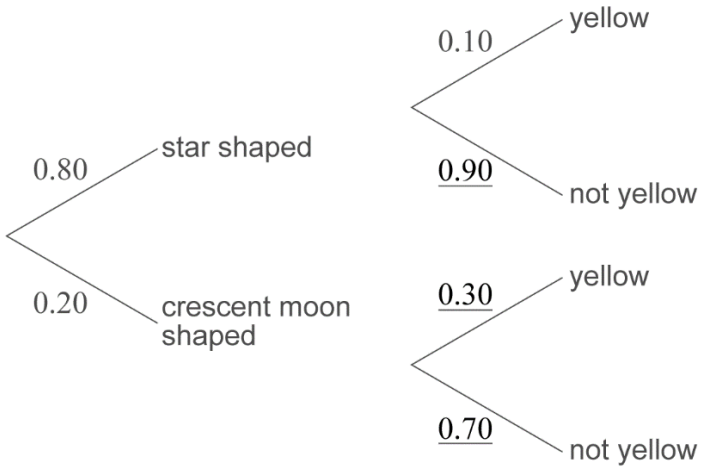
$\mathrm{P}(X \leq 9) = 0.128\ (0.127521\ldots)$ ***(M1)A1***

**Note:** Award ***(M1)*** for evidence of correct approach ***eg***, $\mathrm{P}(X \leq 9)$.

***[3 marks]***

***Total [6 marks]***

**2.** (a)




***A1A1***

**Note:** Award ***A1*** for each correct pair of branches. Accept decimal or percentage responses as equivalent forms on branches.

***[2 marks]***

(b) (i) $\mathrm{P}(Y) = 0.8 \times 0.1 + 0.2 \times 0.3$ ***M1***

$= 0.14$ ***A1***

(ii) $\mathrm{P}(\text{Star} \mid Y) = \dfrac{0.8 \times 0.1}{0.14}$ ***M1***

$= 0.571 \left( \dfrac{4}{7}, 0.571428\ldots \right)$ ***A1***

***[4 marks]***

(c) the colours of the sweets are distributed according to manufacturer specifications ***A1***

***[1 mark]***

(d)

| **Colour** | Brown | Red | Green | Orange | Yellow | Purple |
|---|---|---|---|---|---|---|
| **Expected Frequency** | 12 | 20 | 16 | 16 | 8 | 8 |

***A2***

**Note:** Award ***A2*** for all 6 correct expected values, ***A1*** for 4 or 5 correct values, ***A0*** otherwise.

***[2 marks]***

(e) 5 ***A1***

***[1 mark]***

(f) 0.469 (0.4688117…) ***A2***

***[2 marks]***

*continued…*

*Question 2 continued*

(g) since $0.469 > 0.05$ ***R1***

fail to reject the null hypothesis. There is insufficient evidence to reject the manufacturer's specifications ***A1***

**Note:** Award ***R1*** for a correct comparison of their correct $p$-value to the test level, award ***A1*** for the correct result from that comparison. Do not award ***R0A1***.

***[2 marks]***

***Total [14 marks]***

**3.** (a) (i) $0.909\ (0.909181\ldots)$ ***A2***

(ii) (very) strong and positive ***A1A1***

**Note:** Award ***A1*** for (very) strong ***A1*** for positive.

***[4 marks]***

(b) $y = 1.14x + 0.578\ (y = 1.14033\ldots\ x + 0.578183\ldots)$ ***A1A1***

**Note:** Award ***A1*** for $1.14x$, ***A1*** for $0.578$. Award a maximum of ***A1A0*** if the answer is not an equation in the form $y = mx + c$.

***[2 marks]***

(c) (i) $1.14 \times 10 + 0.578$ ***M1***

$12.0\ (11.9814\ldots)$ ***A1***

(ii) no the estimate is not reliable ***A1***

outside the known data range ***R1***

***OR***

a score greater than $10$ is not possible ***R1***

**Note:** Do not award ***A1R0***.

***[4 marks]***

(d)

| **Competitors** | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| **Stan's rank** | 7 | 8 | 6 | 4 | 2 | 4 | 1 | 4 |
| **Minsun's rank** | 7 | 8 | 6 | 4.5 | 3 | 2 | 1 | 4.5 |

***A1A1***

**Note:** Award ***A1*** for correct ranks for Stan. Award ***A1*** for correct ranks for Minsun.

***[2 marks]***

(e) (i) $0.933\ (0.932673\ldots)$ ***A2***

(ii) Stan and Minsun strongly agree on the ranking of competitors. ***A1A1***

**Note:** Award ***A1*** for "strongly agree", ***A1*** for reference to a rank order.

***[4 marks]***

(f) decreasing the score to $9.1$, does not change the rank of competitor G ***A1***

***[1 mark]***

***Total [17 marks]***

**7.** (a) $H_0$ : The die is fair **OR** $P(\text{any number}) = \frac{1}{6}$ **OR** probabilities are equal

$H_1$ : The die is not fair **OR** $P(\text{any number}) \neq \frac{1}{6}$ **OR** probabilities are not equal ***A1***

***[1 mark]***

(b) $5$ ***A1***

***[1 mark]***

(c) $10$ ***A1***

***[1 mark]***

(d) ($p$-value =) $0.287$ $(0.28724163\ldots.)$ ***A2***

***[2 marks]***

(e) $0.287 > 0.05$ ***R1***

**EITHER**
Insufficient evidence to reject the null hypothesis ***A1***

**OR**
Insufficient evidence to reject that the die is fair ***A1***

**Note:** Do not award ***R0A1***. Condone "*accept* the null hypothesis" or "the die is fair". Their conclusion must be consistent with their $p$-value and their hypothesis.

***[2 marks]***
***[Total 7 marks]***

**8.** (a) $50\%$ ***A1***

**Note:** Do not accept $0.5$ or $\frac{1}{2}$.

***[1 mark]***

(b) $0.0478$ $(0.0477903\ldots, 4.78\%)$ ***A2***

***[2 marks]***

(c) $\mathrm{P}(X < k) = 0.98$ **OR** $\mathrm{P}(X > k) = 0.02$ ***(M1)***

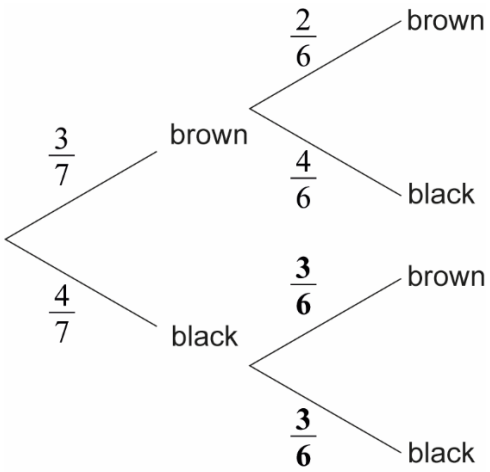
**Note:** Award ***(M1)*** for a sketch with correct region identified.

$506$ g $(506.161\ldots)$ ***A2***

***[3 marks]***

***[Total 6 marks]***

**10.** (a)




***A1***

**Note:** Award ***A1*** for both missing probabilities correct.

***[1 mark]***

(b) multiplying along branches and then adding outcomes ***(M1)***

$$\frac{3}{7}\times\frac{2}{6}+\frac{4}{7}\times\frac{3}{6}$$

$$=\frac{18}{42}\left(=\frac{3}{7}\approx 0.429\ (42.9\%)\right)$$ ***A1***

***[2 marks]***

(c) use of conditional probability formula ***M1***

$$\frac{\left(\frac{3}{7}\times\frac{2}{6}\right)}{\left(\frac{3}{7}\right)}$$ ***A1***

$$=\frac{6}{18}\left(=\frac{1}{3}\right)\ \left(\frac{252}{756},\ 0.333,\ 33.3\%\right)$$ ***A1***

***[3 marks]***
***[Total 6 marks]***

**2.** (a) $\left(\frac{17+25}{130}=\right)\frac{42}{130}$ $\left(\frac{21}{65}, 0.323076\ldots\right)$ ***A1***

***[1 mark]***

(b) $\left(\frac{17}{17+25}=\right)\frac{17}{42}$ $(0.404761\ldots)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator and ***A1*** for correct denominator. Award ***A1A0*** for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

***[2 marks]***

(c) $\frac{41}{130}\times\frac{40}{129}$ ***A1M1***

**Note:** Award ***A1*** for two correct fractions seen, ***M1*** for multiplying their fractions.

$=\frac{1640}{16770}\approx 0.0978$ $\left(0.0977936\ldots, \frac{164}{1677}\right)$ ***A1***

***[3 marks]***
***Total [6 marks]***

**5.** (a) $(\mathrm{E}(X) =) 10 \times 0.8$ ***(M1)***

$8$ (people) ***A1***

***[2 marks]***

(b) recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

***[2 marks]***

(c) $0.8$ and $6$ seen **OR** $0.2$ and $3$ seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

***[3 marks]***

***Total [7 marks]***

**7.** (a) $(88 - 62) \times 1.5$ **OR** $26 \times 1.5$ seen anywhere **OR** $39$ seen anywhere ***(M1)***

$62 - 39$
$23$ ***A1***

$25 > 23$ ***R1***
so is not an outlier ***AG***

***[3 marks]***

(b) The median score for the evening class is higher than the median score for the morning class. ***A1***

**THEN**
but the scores are more spread out in the evening class than in the morning class ***A1***

**OR**
the scores are more inconsistent in the evening class ***A1***
**OR**
the lowest scores are in the evening class ***A1***
**OR**
the interquartile range is lower in the morning class ***A1***
**OR**
the lower quartile is lower in the evening class ***A1***

**Note:** If an incorrect comparison is also made, award at most ***A1A0***.

Award ***A0*** for a comparison that references "the mean score" unless working is shown for the estimated means of the data sets, calculated from the mid-points of the 4 intervals. The estimated mean for the morning class is $71.375$ and the estimated mean for the evening class is $70.5$.

***[2 marks]***
***Total [5 marks]***

**8.** (a) $(H_1:)\ \mu_1 - \mu_2 \neq 0 \quad (\mu_1 \neq \mu_2)$ ***A1***

**Note:** Accept an equivalent statement in words, however reference to "**population** mean" must be explicit for ***A1*** to be awarded.

***[1 mark]***

(b) $0.0778 \quad (0.0778465\ldots)$ ***A2***

**Note:** Award ***A1*** for an answer of $0.0815486\ldots$ from not using a pooled estimate of the variance.

***[2 marks]***

(c) (i) $0.0778 < 0.1$ ***R1***
reject the null hypothesis ***A1***

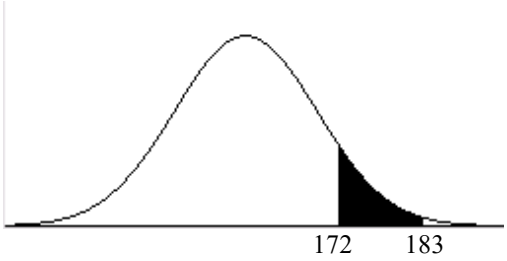
**Note:** Do not award ***R0A1***.

(ii) there is (significant evidence of) a difference between the (population) **mean** reaction times ***A1***

**Note:** Their conclusion in (c)(ii) must match their conclusion in (c)(i) to earn ***A1***. Award ***A0*** if their conclusion refers to mean reaction times in the sample.

***[3 marks]***
***Total [6 marks]***

**10.** (a) sketch of normal curve with shaded region to the right of the mean and correct values ***(M1)***




$0.0921 \quad (0.0920950\ldots)$ ***A1***

***[2 marks]***

(b) **EITHER**

$\left(\mathrm{P}(x < 172)\right)$

$0.906200\ldots$ ***(A1)***

$(0.906200\ldots - 0.68)$

$0.226200\ldots$ ***(A1)***

**OR**

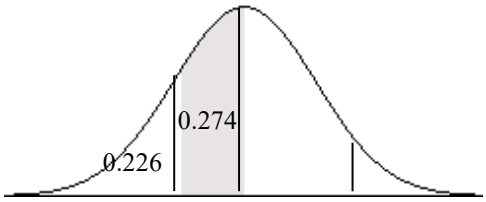
$\left(\mathrm{P}(163 < x < 172)\right)$

$0.406200\ldots$ ***(A1)***

$0.5 - (0.68 - 0.406200\ldots)$ **OR** $0.5 + (0.68 - 0.406200\ldots)$

$0.226200\ldots$ **OR** $0.773799\ldots$ ***(A1)***

**OR**




***(A1)(A1)***

| | |
|---|---|
| **Note:** | Award ***A1*** for a normal distribution curve with a vertical line on each side of the mean and a correct probability of either $0.406$ or $0.274$ or $0.906$ shown, ***A1*** for a probability of $0.226$ seen. |

**THEN**

$(k =)\ 158\ \text{g}\ (157.867\ldots\text{g})$ ***A1***

***[3 marks]***

***Total [5 marks]***

**3.** (a) (i) $\frac{370+472}{2}$ ***(M1)***

**Note:** This ***(M1)*** can also be awarded for either a correct $Q_3$ or a correct $Q_1$ in part (a)(ii).

$Q_3 = 421$ ***A1***

(ii) their part (a)(i) – their $Q_1$ (clearly stated) ***(M1)***

$\text{IQR} = (421 - 318 =)\ 103$ ***A1***

***[4 marks]***

(b) $(Q_3 + 1.5(\text{IQR}) =)\ 421 + (1.5 \times 103)$ ***(M1)***

$= 575.5$

since $498 < 575.5$ ***R1***

Netherlands is not an outlier ***A1***

**Note:** The ***R1*** is dependent on the ***(M1)***. Do not award ***R0A1***.

***[3 marks]***

(c) not appropriate ("no" is sufficient) ***A1***

as $r$ is too close to zero / too weak a correlation ***R1***

***[2 marks]***

(d) (i) $6$ ***A1***

(ii) $4.5$ ***A1***

(iii) $4.5$ ***A1***

***[3 marks]***

(e) (i) $r_s = 0.683\ \ (0.682646...)$ ***A2***

(ii) **EITHER**

there is a (positive) association between the population size and the score ***A1***

**OR**

there is a (positive) linear correlation between the ranks of the population size and the ranks of the scores (when compared with the PMCC of $0.249$). ***A1***

***[3 marks]***

(f) lowering the top score by $20$ does not change its rank so $r_s$ is unchanged ***R1***

**Note:** Accept "this would not alter the rank" or "Netherlands still top rank" or similar. Condone any statement that clearly implies the ranks have not changed, for example: "The Netherlands still has the highest score."

***[1 mark]***

***[Total 16 marks]***

**5.** (a) (let $T$ be the number of passengers who arrive)

$(\mathrm{P}(T > 72) =)\ \mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74, 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379\ (0.00379124...)$ ***A1***

**Note:** Using the distribution $\mathrm{B}(74, 0.1)$, to work with the $10\%$ that do not arrive for the flight, here and throughout this question, is a valid approach.

***[3 marks]***

(b) (i) $72 \times 0.9$ ***(M1)***

$64.8$ ***A1***

(ii) $n \times 0.9 = 72$ ***(M1)***

$80$ ***A1***

***[4 marks]***

(c) **METHOD 1**

**EITHER**

when selling $74$ tickets

| | $T \leq 72$ | $T = 73$ | $T = 74$ |
|---|---|---|---|
| Income minus compensation ($I$) | 11100 | 10800 | 10500 |
| Probability | 0.9962… | 0.003380… | 0.0004110… |

top row ***A1A1***

bottom row ***A1A1***

**Note:** Award ***A1A1*** for each row correct. Award ***A1*** for one correct entry and ***A1*** for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962... + 10800 \times 0.00338... + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

**OR**

income is $74 \times 150 = 11100$ ***(A1)***

expected compensation is

$0.003380... \times 300 + 0.0004110... \times 600\ (= 1.26070...)$ ***(M1)A1A1***

expected income when selling $74$ tickets is $11100 - 1.26070\ldots$ ***(M1)***

$= 11098.73..\ (= \$11099)$ ***A1***

**THEN**

income for $72$ tickets $= 72 \times 150 = 10800$ ***(A1)***

so expected gain $\approx 11099 - 10800 = \$299$ ***A1***

*continued…*

*Question 5 continued*

**METHOD 2**

for 74 tickets sold, let C be the compensation paid out

$\mathrm{P}(T = 73) = 0.00338014\ldots,\ \mathrm{P}(T = 74) = 0.000411098\ldots$ ***A1A1***

$\mathrm{E}(C) = 0.003380...\times 300 + 0.0004110...\times 600\ \ (= 1.26070...)$ ***(M1)A1A1***

extra expected revenue $= 300 - 1.01404... - 0.246658\ldots\ \ (300 - 1.26070\ldots)$ ***(A1)(M1)***

**Note:** Award ***A1*** for the 300 and ***M1*** for the subtraction.

$= \$299$ (to the nearest dollar) ***A1***

**METHOD 3**

let $D$ be the change in income when selling 74 tickets.

| | $T \leq 72$ | $T = 73$ | $T = 74$ |
|---|---|---|---|
| Change in income | 300 | 0 | −300 |

***(A1)(A1)***

**Note:** Award ***A1*** for one error, however award ***A1A1*** if there is no explicit mention that $T = 73$ would result in $D = 0$ and the other two are correct.

$\mathrm{P}(T \leq 73) = 0.9962...,\ \mathrm{P}(T = 74) = 0.000411098\ldots$ ***A1A1***

$\mathrm{E}(D) = 300\times 0.9962... + 0\times 0.003380... - 300\times 0.0004110$ ***(M1)A1A1***

$= \$299$ ***A1***

***[8 marks]***

***[Total 15 marks]***

**1.** (a) (i) $0.58$ (s) ***A1***

(ii) $0.7 - 0.42$ ***(A1)(M1)***

**Note:** Award ***A1*** for correct quartiles seen, ***M1*** for subtraction of their quartiles.

$0.28$ s ***A1***

***[4 marks]***

(b) $9$ (people have reaction time $\leq 0.4$) ***(A1)***

$31$ (people have reaction time $> 0.4$) ***A1***

***[2 marks]***

(c) $(90\% \times 40 =)\ 36$ **OR** $4$ ***(A1)***

$0.8$ s ***A1***

***[2 marks]***

(d) (i) $(a =)\ 6$ ***A1***

(ii) $(b =)\ 4$ ***A1***

***[2 marks]***

(e) $0.6 < t \leq 0.8$ ***A1***

***[1 mark]***

(f) $0.55$ s ***A2***

***[2 marks]***

(g) the mean will increase ***A1***

because the incorrect reaction times are moving from a lower interval to a higher interval which will increase the numerator of the mean calculation ***R1***

the median will stay the same ***A1***

because the median or middle of the data is greater than both intervals being changed ***R1***

**Note:** Do not award ***A1R0.***

***[4 marks]***

***Total [17 marks]***

**4.** (a) The favourite breakfast/berry (of adults) is independent of (their) income (level). ***A1***

***[1 mark]***

(b) $\chi^2 = 2.27 \ (2.26821...)$ ***A2***

***[2 marks]***

(c) **EITHER**

$2.27 < 7.78$ **OR** $2.27 <$ critical value ***R1***

**OR**

$0.687 > 0.1$ (using $p$-value)

**THEN**

(Do not reject $\mathrm{H}_0$)

Insufficient evidence (at the $10\%$ significance level) that the favourite berry depends on income level. ***A1***

**Note:** Do not award ***R0A1***. Accept "$\chi^2$" in place of their "$2.27$", provided an answer was seen in part (b). Their conclusion must be consistent with their $\chi^2$ (or a correct $p$-value) and their hypothesis.

***[2 marks]***

***Total [5 marks]***

**6**. (a) $(H_1 :)\ \mu_1 \neq \mu_2$ ***A1***

**Note:** Accept an equivalent statement in words referring to $\mu_1$ and $\mu_2$ as defined in the question.

***[1 mark]***

(b) $0.97652\ (0.976516...)$ ***A2***

***[2 marks]***

(c) $0.97652 > 0.05\ (0.977 > 0.05)$ ***R1***

Annabelle's conclusion is correct. ***A1***

**Note:** Do not award ***R0A1***. Answer must reference Annabelle's conclusion; do not accept an answer, without context, of "fail to reject $H_0$" for the ***A1*** mark.

***[2 marks]***

***Total [5 marks]***

**8.** (a) $\mathrm{P}(T < 55)$ ***(M1)***

$0.0912\ (0.0912112...)$ ***A1***

**Note:** Award ***M1*** for a correct calculator notation such as $\text{normal cdf}(0, 55, 59, 3)$ or $\text{normal cdf}(-1^{99}, 55, 59, 3)$.

***[2 marks]***

(b) correct use of expected value

$8.6 = 20 \times p$ **OR** $(p =)\ 0.43$ seen ***(M1)***

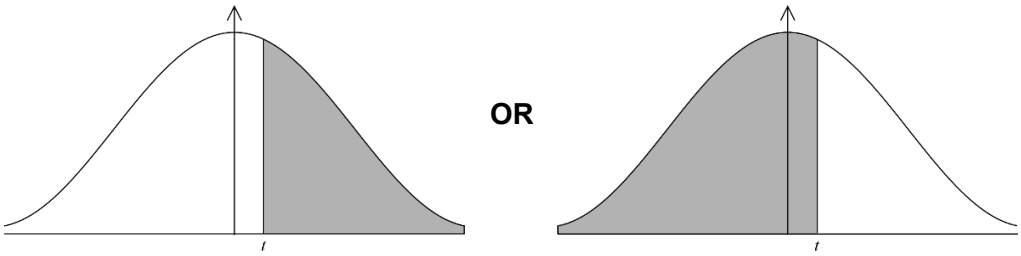
**EITHER**

correct probability statement ***(M1)***

$\mathrm{P}(T > t) = 0.43$ **OR** $\mathrm{P}(T < t) = 0.57$

**OR**

$t$ indicated on sketch to communicate correct area ***(M1)***




**THEN**

$(t =)\ 59.5$ (seconds) (59.5291...) ***A1***

***[3 marks]***

***Total [5 marks]***

**9.** (a) $0.5\times0.1+0.4\times0.4+0.1\times0.5$ ***(M1)(M1)(M1)***

**Note:** Award ***M1*** for $0.5\times0.1$ or $0.1\times0.5$, ***M1*** for $0.4\times0.4$, ***M1*** for adding three correct products.

$0.26$ ***A1***

***[4 marks]***

(b) $0=-8\times0.5+4\times0.4+0.1k$ ***(M1)(M1)***

**Note:** Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

$(k=)$ 24 (points) ***A1***

***[3 marks]***

***Total [7 marks]***

**1.** (a) continuous ***A1***

***[1 mark]***

(b) $160-50-62-14-8$ ***(M1)***

$(k=)\ 26$ ***A1***

***[2 marks]***

(c) (i) $20 \leq T < 40$ ***A1***

(ii) $30$ ***A1***

***[2 marks]***

(d) $33.5$ minutes ***A2***

**Note:** ***FT*** from their value of $k$ and their mid-interval value. Follow through from part (c)(ii) but only if mid-interval value lies in their interval.

***[2 marks]***

(e) $112$ ***A1***

***[1 mark]***

(f) $\frac{22}{160}\ \left[0.138,\ 0.1375,\ 13.75\%,\ \frac{11}{80}\right]$ ***A1A1***

**Note:** Award ***A1*** for correct numerator, ***A1*** for correct denominator.

***[2 marks]***

(g) $26$ minutes ***A1***

***[1 mark]***

(h) $50-16$ ***(M1)***

**Note:** Award ***M1*** for both correct quartiles seen.

$34$ minutes ***A1***

***[2 marks]***

*continued…*

*Question 1 continued*

(i) correct substitution into outlier formula ***(M1)***

$50+1.5\times34$

$=101$ ***A1***

$92<101$ **OR** highest value on diagram $<101$ ***R1***

not an outlier ***AG***

**Note:** Award ***R1*** for their correct comparison. Follow through from their part (h).
Award ***R0*** if their conclusion is "it is an outlier", this contradicts Elsie's belief.

***[3 marks]***

(j) **EITHER**

the diagram is not symmetric or equivalent

e.g the median is not in the center of the box or

the lengths of the whiskers are (very) different or (positive or right) skew

**OR**

the mean and median are (very) different; ***A1***

***[1 mark]***

***Total [17 marks]***

**4.** (a) (i) $(m =)\ 54(\%)$ ***A1***

(ii) $(n =)\ 14(\%)$ ***A1***

(iii) $(p =)\ 22(\%)$ ***A1***

(iv) $(q =)\ 10(\%)$ ***A1***

**Note:** Based on their $n$, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is $100\%$.

***[4 marks]***

(b) $90\ (\%)$ ***A1***

**Note:** Award ***A0*** for a decimal answer.

***[1 mark]***

(c) (i) $0.54\ \left(\frac{54}{100}, \frac{27}{50}, 54\%\right)$ ***A1***

(ii) $\frac{54}{64}\ \left(0.844, \frac{27}{32}, 84.4\%, 0.84375\right)$ ***A1A1***

**Note:** Award ***A1*** for a correct denominator ($0.64$ or $64$ seen), ***A1*** for the correct final answer.

***[3 marks]***

(d) (i) recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10, 0.68)$

$\left(\mathrm{P}(X = 5) =\right)\ 0.123\ \ (0.122940..., 12.3\%)$ ***A1***

(ii) $1 - \mathrm{P}(X \leq 3)$ **OR** $\mathrm{P}(X \geq 4)$ **OR** $\mathrm{P}(4 \leq X \leq 10)$ ***(M1)***

$0.984\ (0.984497..., 98.4\%)$ ***A1***

(iii) $(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2 \times \left((0.68)^9 \times 0.32\right)$

$0.0199\ \ (0.0198957..., 1.99\%)$ ***A1***

***[7 marks]***

(e) **EITHER**

the probability is not constant ***A1***

**OR**

the events are not independent ***A1***

**OR**

the events should be modelled by the hypergeometric distribution instead ***A1***

***[1 mark]***

***Total [16 marks]***

**1.** (a)

| | Event | | Rank | |
|---|---|---|---|---|
| **Country** | **Long Jump (m)** | **High Jump (m)** | **Long Jump Rank** | **High Jump Rank** |
| Germany | 7.64 | 2.11 | 1 | **1** |
| France | 7.52 | 2.08 | 2 | **2** |
| Estonia | 7.49 | 1.84 | 3 | **10** |
| Canada | 7.44 | 2.02 | 4 | **4.5** |
| Netherlands | 7.33 | 2.05 | 5 | **3** |
| Ukraine | 7.28 | 2.02 | 6 | **4.5** |
| Algeria | 7.22 | 1.90 | 7 | **8** |
| Austria | 7.11 | 1.87 | 8 | **9** |
| Grenada | 6.98 | 1.99 | 9 | **6** |
| Japan | 6.64 | 1.96 | 10 | **7** |

***A1A1***

**Note:** Award ***A1*** for ranking of tied heights, ***A1*** for correct ranking of non-tied heights.

***[2 marks]***

(b) $(r_s =) 0.541 \quad (0.541035\ldots)$ ***A2***

**Note:** Award ***A2*** for an answer of $0.539\ (0.539393\ldots)$ from use of the formula for Spearman's rank correlation coefficient when data has tied ranks.

***[2 marks]***

(c) moderate (correlation) ***A1***

as long jump ranking increases, high jump ranking will (likely) increase ***A1***

***[2 marks]***

***[Total: 6 marks]***

**4.** (a) $H_0 : \mu_b = \mu_m$ ***A1***

$H_1 : \mu_b > \mu_m$ ***A1***

**Note:** Accept equivalent statements in words such as "the **mean** score of bilingual people equals the **mean** score of monolingual people".

***[2 marks]***

(b) $0.119 \ (0.119395\ldots)$ ***A2***

***[2 marks]***

(c) $0.119395\ldots > 0.05 \ \ (11.9395\ldots\% > 5\%)$ ***R1***

(fail to reject $H_0$) there is insufficient evidence to suggest that bilingual people have better memory retention than monolingual people ***A1***

**Note:** Do not award ***R0A1***.
The answer to part (c) MUST be consistent with **their** hypotheses and **their** $p$-value.

***[2 marks]***

***[Total: 6 marks]***

**12.** (a) attempt to find the difference between $75.7$ and $67.3$ ***(M1)***

$\frac{75.7-67.3}{2}$

$4.2\ (\text{km h}^{-1})$ ***A1***

***[2 marks]***

(b) recognition of normal distribution that includes $72$ ***(M1)***

e.g., sketch of normal distribution curve with $72$ labelled to the right of the mean **OR**

Normal CDF calculation using $72$

$0.132\quad (0.131559\ldots, 13.2\%, 13.1559\ldots\%)$ ***A1***

***[2 marks]***

(c) **METHOD 1 (Comparing areas above and below the mean)**

$\text{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF$(67.3, 74, 67.3, 4.2)$ **OR** sketch of normal distribution with $67.3$ and $74$ labelled and shaded between ***(M1)***

area of region between mean and $q$ is at least $0.445\ \ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ \ (0.375329\ldots)$ between mean and $p$ ***R1***

The region between $p$ and $q$ is not symmetrical ***AG***

**METHOD 2 (Comparing areas in the tails)**

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ ***(M1)***

$\text{P}(\text{speed} < 74) = 0.944670\ldots$

$\text{P}(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ 0.124670\ldots$

$\text{P}(\text{speed} > q) = (1 - 0.944670\ldots =)\ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $\text{P}(\text{speed} > q) \leq 0.0553295$ and $\text{P}(\text{speed} < p) \geq 0.124670$ so

$\text{P}(\text{speed} > q)$ will never equal $\text{P}(\text{speed} < p)$ ***R1***

the region between $p$ and $q$ is not symmetrical ***AG***

*continued…*

*Question 12 continued*

**METHOD 3 (Assumption of symmetry comparing speeds)**

attempt to calculate area below $q$ assuming distribution is symmetrical ***(M1)***

e.g. $\mathrm{P}(\text{speed} < q) = 0.82 + ½ \times 0.18 \quad (0.91)$

**EITHER**

$(q =)\ 72.9 \quad (72.9311\ldots)$ ***A1***

$72.9 < 74$ so 74 would not be in the region ***R1***

the region between $p$ and $q$ is not symmetrical ***AG***

**OR**

$\mathrm{P}(\text{speed} < 74) = 0.945 \quad (0.944670\ldots)$ ***A1***

$0.945 > 0.91$ so 74 would not be in the region ***R1***

the region between $p$ and $q$ is not symmetrical ***AG***

**METHOD 4 (Assumption of symmetry comparing areas)**

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6 < \text{speed} < 74)$ **OR** Normal CDF(60.6, 74, 67.3, 4.2) **OR**

$\mathrm{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2)

**EITHER**

$0.889 \quad (0.889340\ldots)$ ***A1***

$0.889 > 0.82$ so 74 would not be in the region ***R1***

the region between $p$ and $q$ is not symmetrical ***AG***

**OR**

$0.445 \quad (0.444670\ldots)$ ***A1***

$0.445 > 0.82 \div 2$ so 74 would not be in the region ***R1***

the region between $p$ and $q$ is not symmetrical ***AG***

***[3 marks]***

***[Total: 7 marks]***

**3.** (a) (i) $38$ (s) ***A1***

(ii) $32$ (s) ***A1***

(iii) $42$ (s) ***A1***

(iv) $10$ (s) ***A1***

**Note:** Accept a tolerance of $\pm 0.5$ for parts (a)(i)-(iii).

***[4 marks]***

(b) $1.5$ x IQR ***(M1)***

$(32 - 1.5 \times 10 =)\ 17$ (s) ***A1***

$14 < 17$, therefore it is an outlier ***R1***

**Note:** Do not award the ***R1*** unless an explicit comparison of $14$ and their $17$ is seen.
e.g. $14 < 17$
$14$ is outside the interval $[17, 57]$.

***[3 marks]***
***Total [7 marks]***

**4.** (a)

| Athlete | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Age rank | 7 | 6 | 3 | 5 | 4 | 2 | 8 | 1 |
| Time rank | 3.5 | 2 | 3.5 | 6 | 7 | 8 | 1 | 5 |

***A1A1***

**Note:** Award ***A1*** for each correct row.

***[2 marks]***

(b) $r_s = -0.671 \ (-0.670670...)$ ***A2***

**Note:** Only follow through from an incorrect table provided the ranks are all between $1$ and $8$.
Award ***A1*** for $-0.67$ **OR** for the omission of the negative sign, e.g. $0.671$ $(0.670670...)$ or $0.67$

***[2 marks]***

(c) (A value of $r_S = -0.671$) indicates a negative correlation between a person's age and the best time they take to run 100m. ***R1***

**Note:** Condone any comment that includes "weak" or "strong" etc. Accept an interpretation in words, but only if there is a general link described and not a rule: "The older a person gets, the faster they *tend to* run".
Answer must be in context.

***[1 mark]***

(d) *Award **R1** for any sensible reason:* ***R1***
The correlation, such that it is, is unlikely to be linear for this type of data.
Spearman's CC is less sensitive to outliers
Sung-Jin is not sure the data is drawn from a bivariate normal distribution
There are outliers/extreme data
Same time for two athletes with significantly different ages

***[1 mark]***
***Total [6 marks]***

**5.** (a) $34 + p$ ***A1***

***[1 mark]***

(b) attempt to substitute into the mean formula, equating to $4.5$ ***(M1)***

$$\frac{1\times1+2\times4...5\times p+6\times9+7\times4}{34+p}=4.5$$ ***A1***

$(p=)\ 10$ ***A1***

**Note:** Do not award the final ***A1*** if final answer is not an integer.
Award ***(M1)A0A1*** for an unsupported answer of $(p=)\ 10$.

***[3 marks]***
***Total [4 marks]***

**6.** (a) $0.565 \quad \left(0.564655..., \frac{131}{232}, 56.4655...\%\right)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator, ***A1*** for correct denominator.

***[2 marks]***

(b) $11.0 \ (11.0212\ldots)$ ***A2***

**Note:** Award ***A1*** for a final answer of $11$ if no unrounded answer is seen.

***[2 marks]***

(c) **EITHER**
$11.0 > 9.488 \ (11.0212\ldots > 9.488\ )$ ***R1***

**OR**
$0.0263 < 0.05 \ (0.0263264\ldots < 0.05\ )$ ***R1***

**THEN**

**EITHER**
(there is significant evidence to) reject $H_0$ ***A1***

**OR**
(there is significant evidence that) the (food) quality and the type of meal are not independent ***A1***

**Note:** Do not award ***R0A1.***
Award ***R1*** for $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$, provided the calculated value is explicitly seen in part (b).

Accept "$p$-value < significance level" provided their $p$-value is seen and their $p$-value is between $0$ and $1$.

***[2 marks]***
***Total [6 marks]***

**9.** (a) $X \sim \mathrm{N}(4, 0.25^2)$

**EITHER**

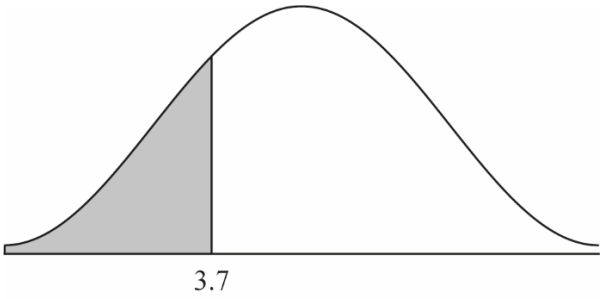
correct probability expression ***(M1)***

$\mathrm{P}(X < 3.7)$

**Note:** Accept a weak or strict inequality, and any label instead of $X$, e.g. $\mathrm{length}$ or $L$.

**OR**

normal curve with vertical line, left of mean, labelled $3.7$, and shaded region ***(M1)***




**THEN**

$0.115$ $(0.115069\ldots, 11.5\%)$ ***A1***

**Note:** Award ***M1A0*** for $0.12$ if no previous working.

***[2 marks]***

(b) **EITHER**

Correct probability expression ***(M1)***

$(\mathrm{P}(X < k) = 0.7$ **OR** $\mathrm{P}(X > k) = 0.3$

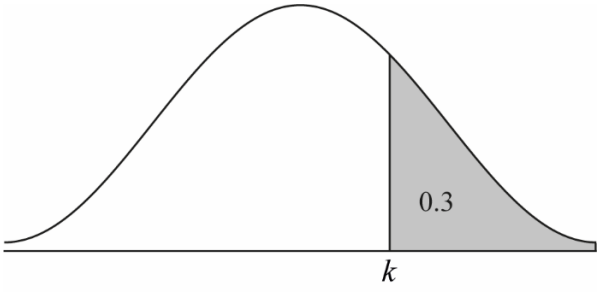
**Note:** Accept a weak or strict inequality, and any label instead of $X$ *e.g.*, $\mathrm{length}$ or $L$.

*continued…*

*Question 9 continued*

**OR**

normal curve with vertical line to the right of the mean and shaded region, correctly labelled either $0.3$ or $0.7$ ***(M1)***




**THEN**

$(k =)\ 4.13\ (4.13110\ldots)$ ***A1***

**Note:** Award ***M1A0*** for $4.1$ if no previous working.

***[2 marks]***

(c) **EITHER**

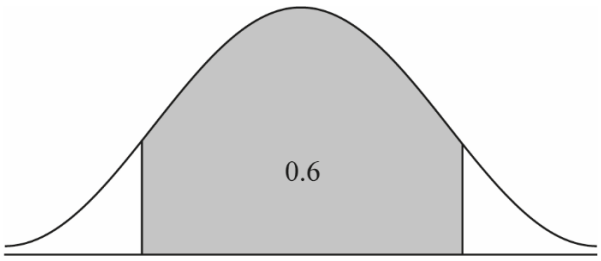
correct probability equation ***(M1)***

$\mathrm{P}(\text{length} < 4 + m) = 0.8$ **OR** $\mathrm{P}(\text{length} < 4 - m) = 0.2$

**Note:** Accept any letter instead of "length" *e.g.*, $X$ or $L$.

**OR**

normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of $0.6$ or $0.2$ or $0.8$ ***(M1)***




**THEN**

$0.210\ (0.210405\ldots)$ ***A1***

**Note:** Award ***(M1)A0*** for an answer of $3.7895$ or $4.2105$ seen without working.

***[2 marks]***

***Total [6 marks]***

**12.** (a) $0.15+0.2+k+0.16+2k+0.25=1$ ***(M1)***

$k=0.08$ ***A1***

***[2 marks]***

(b) $(-4\times0.15)+(-3\times0.2)+(-1\times0.08)+(0\times0.16)+(1\times0.16)+(4\times0.25)$ ***(M1)***

$=-0.12$ ***A1***

$\mathrm{E}(X)\neq0$ therefore the game is not fair ***R1***

**Note:** Do not award ***A0R1*** without an explicit value for $\mathrm{E}(X)$ seen. The ***R1*** can be awarded for comparing **their** $\mathrm{E}(X)$ to zero provided working is shown.

***[3 marks]***

***Total [5 marks]***

**1.** (a) $\frac{9.45-8.73}{1958-1708}$ ***(M1)***

$= 0.00288 \left(\frac{9}{3125}\right)$ ***A1***

***[2 marks]***

(b) (i) the (mean) yearly change in (mean annual) temperature ***A1***

**Note:** Accept equivalent statements, e.g. “rate of change of temperature”.

(ii) $^{\circ}\text{C / year}$ **OR** degrees C per year ***A1***

**Note:** Do not follow through from part (b)(i) into (b)(ii).

***[2 marks]***

(c) attempt to substitute point and gradient into appropriate formula ***(M1)***

$8.73 = 0.00288 \times 1708 + c \Rightarrow c = 3.81096...$

**or**

$9.45 = 0.00288 \times 1958 + c \Rightarrow c = 3.81096.$

equation is $y = 0.00288x + 3.81$ ***A1***

***[2 marks]***

(d) attempt to substitute $2000$ into their part (c) ***(M1)***

$0.00288 \times 2000 + 3.81096...$

$= 9.57\ (^{\circ}\text{C})\ \ (9.57096...)$ ***A1***

***[2 marks]***

*continued…*

*Question 1 continued*

(e) (i) $y = 0.00256x + 4.46$ $(0.00255714...x + 4.46454...)$ ***(M1)A1***

**Note:** Award ***(M1)A0*** for answers that show the correct method, but are presented incorrectly (e.g. no "$y =$" or truncated values etc.). Accept 4.465 as the correct answer to 4 sf.

(ii) $0.861$ $(0.861333...)$ ***A1***

***[3 marks]***

(f) attempt to substitute $2000$ into their part (e)(i) ***(M1)***

$0.00255714... \times 2000 + 4.46454...$

$= 9.58(^{\circ}\text{C})$ $(9.57882...(^{\circ}\text{C}))$ ***A1***

**Note:** Award ***A1*** for $9.57$ from $0.00255714 \times 2000 + 4.46$.

***[2 marks]***

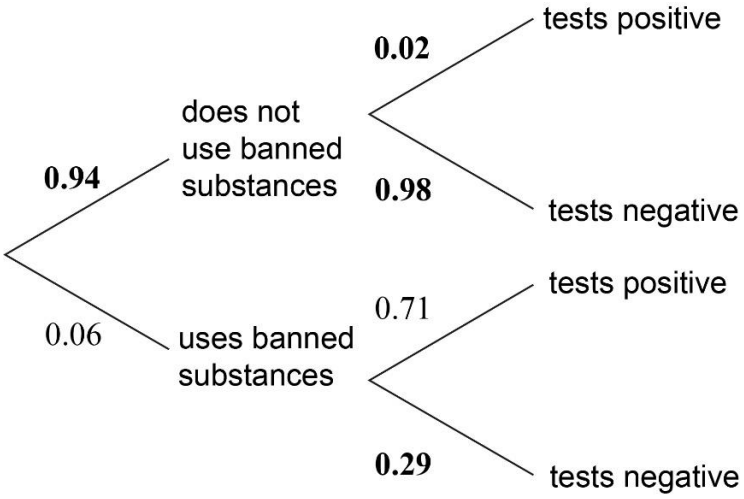
(g) cannot (always reliably) make a prediction of $x$ from a value of $y$, when using $a$ $y$ on $x$ line / regression line is not $x$ on $y$ ***A1***

extrapolation ***A1***

***[2 marks]***

***[Total: 15 marks]***

**4.** (a)




***A1A1***

| **Note:** | Award ***A1*** for any one value correct, ***A1*** for other three values correct. Accept percentage responses as equivalent forms on **all** branches. |
|---|---|

***[2 marks]***

(b) (i) multiplication of two probabilities along the tree diagram ***(M1)***

$0.94 \times 0.98$

$= 0.921 \ (0.9212, 92.1\%, 92.12\%)$ ***A1***

| **Note:** | Do not accept the 2sf value for the final ***A1***. |
|---|---|

(ii) $(0.9212)^2$ ***(A1)***

$= 0.849 \ (0.848609\ldots, 84.9\%, 84.8609...\%)$ ***A1***

| **Note:** | Accept an answer of $0.848$ $(0.848241)$ from use of $3$ sf answer from part (b)(i). |
|---|---|

***[4 marks]***

*continued…*

*Question 4 continued*

(c) (i) $0.94\times0.02+0.06\times0.29$ ***(A1)(M1)***

**Note:** Award ***A1*** for two correct products from their tree diagram seen, ***M1*** for the addition of their two products.

$0.0362$ $(3.62\%)$ ***A1***

(ii) multiplying their part(c)(i) by $1300$

$0.0362\times1300$ ***(M1)***

$47.1$ $(47.06)$ ***A1***

**Note:** accept the 2 sf value of 47 for the final ***A1***

***[5 marks]***

(d) $p=0.02$ **OR** $p=0.98$ ***(A1)***

recognition of binomial probability with $n=20$ ***(M1)***

$\mathrm{P}(X=0)$ **OR** $\mathrm{P}(X=20)$ ***(M1)***

$0.668$ $(0.667607\ldots)$ ***A1***

**Note:** Award ***(A1)(M1)(M1)A0*** for an answer of $0.667$.

$0.98^{20}=0.668$ $(0.667607\ldots)$ is awarded full marks.

***[4 marks]***

(e) $\mathrm{P}(X\geq3)$ **OR** $\mathrm{P}(X\leq17)$ ***(M1)***

$0.00707$ $(0.00706869\ldots)$ ***A1***

**Note:** Award ***(M1)A0*** for an answer of $0.00706$.

***FT*** from their value of $p$ in part (d)

***[2 marks]***

***[Total: 17 marks]***

**4.** (a) recognition of binomial distribution ***(M1)***
e.g. $X \sim \mathrm{B}(115,\ 0.82)$ **OR** binompdf$(115,\ 0.82,\ 90)$ etc.

$(\mathrm{P}(X = 90) =)\ 0.0535\ (0.0535325...)$ ***A2***

**Note:** Award ***(M1)A1A0*** for an answer of $0.054$ with or without working shown.

***[3 marks]***

(b) selecting correct region of distribution ***(M1)***
e.g. $\mathrm{P}(X \geq 95)$ **OR** $1 - \mathrm{P}(X \leq 94)$ **OR** $1 -$ binomcdf$(115,\ 0.82,\ 94)$

$0.491\ (0.491036\ldots)$ ***A1***

***[2 marks]***

(c) substitution in the variance formula for binomial distribution ***(M1)***
$115 \times 0.82 \times 0.18$

$17.0\ \ (16.974)$ ***A1***

**Note:** Allow $17$ for the final answer.

***[2 marks]***

(d) **METHOD 1**
attempt to write an expression containing $n$ inside the brackets of $P(\ )$ **AND** including $0.3$ or $0.7$ ***(M1)***

$\mathrm{P}(X \geq n) < 0.3$ **OR** $\mathrm{P}(X \leq n-1) > 0.7$ ***(A1)***
$n = 98$ ***A1***

**METHOD 2**
using binomcdf in GDC for at least two different values of $n$ greater than $90$ ***(M1)***

**EITHER**
$(\mathrm{P}(X < 97) =)\ 0.696683\ldots$ **AND** $(\mathrm{P}(X < 98) =)\ 0.778249\ldots$ (seen) ***(A1)***

**OR**
$(\mathrm{P}(X > 97) =) 0.303316\ldots$ **AND** $(\mathrm{P}(X > 98) =) 0.221750\ldots$ (seen) ***(A1)***

**THEN**
$n = 98$ ***A1***

***[3 marks]***

*continued…*

*Question 4 continued*

(e) ($\mu_1$ : population mean recovery time for new remedy)

($\mu_2$ : population mean recovery time for old remedy)

$H_0 : \mu_1 = \mu_2 \quad (H_0 : \mu_1 - \mu_2 = 0)$ ***A1***

$H_1 : \mu_1 < \mu_2 \quad (H_1 : \mu_1 - \mu_2 < 0)$ ***A1***

**Note:** Accept an equivalent statement in words, must include mean and reference to "**population** mean", e.g. "mean for **all** patients on old remedy", for the first ***A1*** to be awarded.

Do not accept an imprecise *"the means are equal".*

Award ***A0A1*** for reversed hypotheses ($H_0 : \mu_1 < \mu_2$, $H_1 : \mu_1 = \mu_2$).

***[2 marks]***

(f) $0.0620\ (0.0620061\ldots)$ ***A2***

**Note:** Allow $0.062$ as final answer. Award ***A1*** for an answer of $0.06$. Award ***A1*** for an answer of $0.0527756\ldots$ from use of unpooled setting.

Follow through from an incorrect alternative hypothesis as long as their $p$-value matches their alternative hypothesis.

***[2 marks]***

(g) $0.0620 < 0.1$ ***R1***

(sufficient evidence to) reject $H_0$ ***A1***

**Note:** Do not award ***R0A1***. Accept "$p$-value is less than $0.1$" provided an answer was seen in part (f).

***[2 marks]***

(h) the probability of obtaining results (at least as extreme) as those observed given that the null hypothesis is true ***A1***

***[1 mark]***

***Total [17 marks]***

**1.** (a) (i) $T = 0.552G + 6.36 \quad (= 0.552139...G + 6.35703...)$ ***A1A1***

**Note:** Award ***A1*** for **correct** values of $a$ and $b$, ***A1*** for an equation using these **correct** values.

(ii) $(r =)\ 0.994\ (= 0.993910...)$ ***A1***

there is a (very) strong positive linear correlation ***R1***

**Note:** If $r$ is missing award ***A0R0***.

***[4 marks]***

(b) attempt to substitute $13$ into their regression equation ***(M1)***

$T = 0.552139...(13) + 6.35703...$

$13.5\ (\text{mins})\ (= 13.5348...)$ ***A1***

***[2 marks]***

(c) **EITHER**

using the $T$ on $G$ regression line cannot (always) reliably make a prediction for $G$ ***R1***

**OR**

equation is for Time on Gradient not Gradient on Time. ***R1***

**OR**

this estimate is an extrapolation ***R1***

**OR**

there is no reason to assume this new hill has constant gradient ***R1***

***[1 mark]***

***[Total 7 marks]***

**9.** (a)

| Statement | True (✓) |
|---|---|
| The data for Class A is normally distributed. | |
| A higher percentage of students in Class A received a grade less than $70$ on the exam, than in Class B. | ✓ |
| More students in Class B received a grade greater than 90 on the exam than in Class A. | |
| The interquartile range for Class B is less than the interquartile range for Class A. | ✓ |

***A1A1***

**Note:** Award ***A0A0*** if three or four statements are selected.

***[2 marks]***

(b) **EITHER**

$H_0 : \mu_1 = \mu_2$ ***A1***

$H_1 : \mu_1 \neq \mu_2$ ***A1***

**OR**

$H_0 : \mu_A = \mu_B$ ***A1***

$H_1 : \mu_A \neq \mu_B$ ***A1***

**Note:** Accept an equivalent statement in words, but must include reference to "population mean" / "mean for class A and class B" for the ***A1*** to be awarded.

Do not accept an imprecise "the means are equal".

***[2 marks]***

(c) $p$-value $= 0.0952 \ (0.0952085\ldots)$ ***A2***

***[2 marks]***

(d) $0.0952 > 0.05$ ***R1***

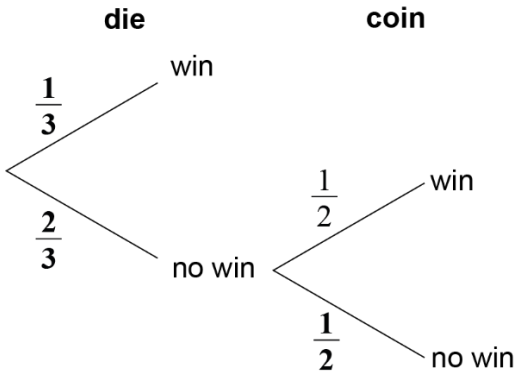
there is insufficient evidence to reject $H_0$ ***A1***

**Note:** Do not award ***R0A1***. The answer to part (d) MUST follow through from their hypotheses seen in part (b) and their $p$-value seen in part (c); if hypotheses are incorrect/reversed, etc., the answer to part (d) must reflect this in order for the ***A1*** to be credited.

***[2 marks]***

***[Total 8 marks]***

**10.** (a)




***A1A1***

**Note:** Award ***A1*** for completing first set of branches, ***A1*** for completing second set of branches.

***[2 marks]***

(b) attempt to multiply along the branches ***(M1)***

$\frac{2}{3}\times\frac{1}{2}$

$=\frac{1}{3}\ \ (=0.333...)$ ***A1***

***[2 marks]***

(c) **EITHER**

$$\frac{\frac{1}{3}}{\frac{1}{3}+\left(\frac{2}{3}\times\frac{1}{2}\right)}$$ ***M1A1***

**Note:** Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

**OR**

$$\frac{\frac{1}{3}}{1-\frac{1}{3}}$$ ***M1A1***

**Note:** Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

**THEN**

$=\frac{1}{2}$ ***A1***

***[3 marks]***
***[Total 7 marks]***

**11.** (a) evidence of using binomial distribution ***(M1)***

**Note:** Evidence is $X \sim \mathrm{B}(5,\ 0.72)$ or binomial with $n = 5,\ p = 0.72$.

$0.293 \quad (0.292626\ldots)$ ***A1***

***[2 marks]***

(b) attempt to find the probability of taking a bus, (or *not* taking a bus); ***(M1)***
$\mathrm{P}(\text{take bus}) = 0.28 \times 0.42,\ \mathrm{P}(\text{not take bus}) = 0.72 + 0.28 \times 0.58$

$0.1176$ or $0.8824$ seen ***(A1)***

**EITHER**
correct use of binomial distribution with their probability
$X \sim \mathrm{B}(5,\ 0.1176),\ X = 0$ **OR** $X \sim \mathrm{B}(5,\ 0.8824),\ X = 5$ ***(A1)***
**OR**
$(1 - 0.1176)^5$ **OR** $(0.8824)^5$ seen ***(A1)***

**THEN**
$0.535 \quad (0.534967\ldots)$ ***A1***

***[4 marks]***
***[Total 6 marks]***

**12.** (a) ***A1***

| | Artist 1 | Artist 2 | Artist 3 | Artist 4 | Artist 5 | Artist 6 | Artist 7 | Artist 8 |
|---|---|---|---|---|---|---|---|---|
| Rank – social media followers | 4 | 3 | 6 | 5 | 7 | 2 | 8 | 1 |
| Rank – albums sold in first week | **2** | **4** | **8** | **3** | **5** | **6** | **7** | 1 |

***[1 mark]***

(b) $(r_s =)\ 0.595\ \ (0.595238\ldots)$ ***A2***

***[2 marks]***

(c) $(\mathrm{H}_1:)$ In the population, there is a **positive** monotonic relationship between the number of social media followers and the number of albums sold in the first week. ***A1***

***[1 mark]***

(d) $0.595 < 0.643$ ***R1***

there is insufficient evidence to reject $\mathrm{H}_0$ ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***
***[Total 6 marks]***

**1.** (a) (i) $T = 0.799G + 2.14 \quad (= 0.798803...G + 2.13972...)$ ***A1A1***

**Note:** Award ***A1*** for **correct** values of $a$ and $b$, ***A1*** for an equation using these **correct** values.

(ii) $(r =)\ 0.996\ (= 0.996247...)$ ***A1***

(there is a very) strong positive linear correlation ***R1***

**Note:** If $r$ is missing award ***A0R0***.

***[4 marks]***

(b) attempt to substitute $17$ into their regression equation ***(M1)***

$0.798803...(17) + 2.13972...$

$15.7\ (\text{mins})\ (= 15.7193...)$ ***A1***

***[2 marks]***

(c) **EITHER**

using the $T$ on $G$ regression line cannot (always) reliably make a prediction for $G$ ***R1***

**OR**

equation is for Time on Gradient, not Gradient on Time ***R1***

**OR**

this estimate is an extrapolation ***R1***

**OR**

there is no reason to assume this new hill has constant gradient ***R1***

***[1 mark]***

***[Total 7 marks]***

**9.** (a)

| Statement | True (✓) |
|---|---|
| A higher percentage of students in Class B received a grade less than $70$ on the exam than Class A. | ✓ |
| The data for Class B is normally distributed. | |
| More students in Class A received a grade greater than 90 on the exam than Class B. | |
| The interquartile range for Class A is less than the interquartile range for Class B. | ✓ |

***A1A1***

**Note:** Award ***A0A0*** if three or four statements are selected.

***[2 marks]***

(b) **EITHER**

$H_0 : \mu_1 = \mu_2$ ***A1***

$H_1 : \mu_1 \neq \mu_2$ ***A1***

**OR**

$H_0 : \mu_A = \mu_B$ ***A1***

$H_1 : \mu_A \neq \mu_B$ ***A1***

**Note:** Accept an equivalent statement in words, but must include reference to "population mean" / "mean for class A and class B" for the ***A1*** to be awarded.

Do not accept an imprecise "the means are equal".

***[2 marks]***

(c) $p$-value $= 0.111 \ (0.110700\ldots)$ ***A2***

***[2 marks]***

(d) $0.111 > 0.05$ ***R1***

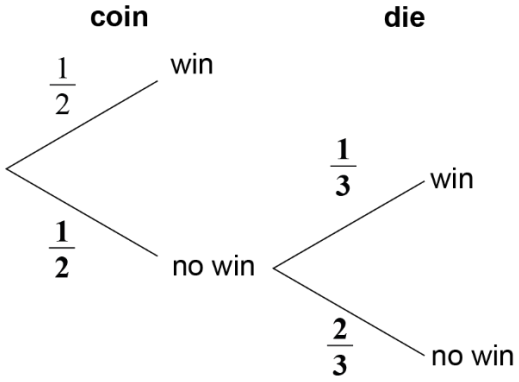
there is insufficient evidence to reject $H_0$ ***A1***

**Note:** Do not award ***R0A1***. The answer to part (d) MUST follow through if hypotheses are incorrect/reversed etc., the answer to part (d) must reflect this in order for the ***A1*** to be credited.

***[2 marks]***

***[Total 8 marks]***

**10.** (a)




***A1A1***

**Note:** Award ***A1*** for completing first set of branches, ***A1*** for completing second set of branches.

***[2 marks]***

(b) attempt to multiply along the branches ***(M1)***

$\frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3} \ (= 0.333...)$ ***A1***

***[2 marks]***

(c) **EITHER**

$$\frac{\frac{1}{2}}{\frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{3}\right)}$$ ***M1A1***

**Note:** Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

**OR**

$$\frac{\frac{1}{2}}{1 - \frac{1}{3}}$$ ***M1A1***

**Note:** Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

**THEN**

$= \frac{3}{4}$ ***A1***

***[3 marks]***
***[Total 7 marks]***

**11.** (a) evidence of using binomial distribution ***(M1)***

**Note:** Evidence is $X \sim \mathrm{B}(5,\ 0.65)$ or binomial with $n = 5,\ p = 0.65$.

$0.181\ \ (0.181146\ldots)$ ***A1***

***[2 marks]***

(b) attempt to find the probability of taking a taxi, (or *not* taking a taxi); ***(M1)***

$\mathrm{P}(\text{take taxi}) = 0.35 \times 0.45$, $\mathrm{P}(\text{not take taxi}) = 0.65 + 0.35 \times 0.55$

$0.1575$ or $0.8425$ seen ***(A1)***

**EITHER**

correct use of binomial distribution with their probability

$X \sim \mathrm{B}(5,\ 0.1575)$, $X = 0$ **OR** $X \sim \mathrm{B}(5,\ 0.8425)$, $X = 5$ ***(A1)***

**OR**

$(1 - 0.1575)^5$ **OR** $(0.8425)^5$ seen ***(A1)***

**THEN**

$0.424\ \ (0.424472\ldots)$ ***A1***

***[4 marks]***

***[Total 6 marks]***

**12.** (a) ***A1***

| | Artist 1 | Artist 2 | Artist 3 | Artist 4 | Artist 5 | Artist 6 | Artist 7 | Artist 8 |
|---|---|---|---|---|---|---|---|---|
| Rank – social media followers | 4 | 3 | 6 | 5 | 7 | 2 | 8 | 1 |
| Rank – albums sold in first week | **2** | **4** | **8** | **3** | **5** | **6** | **7** | 1 |

***[1 mark]***

(b) $(r_s =)\ 0.595\ \ (0.595238\ldots)$ ***A2***

***[2 marks]***

(c) $(\mathrm{H}_1:)$ In the population, there is a **positive** monotonic relationship between the number of social media followers and the number of albums sold in the first week. ***A1***

***[1 mark]***

(d) $0.595 < 0.643$ ***R1***

there is insufficient evidence to reject $\mathrm{H}_0$ ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***

***[Total 6 marks]***

**3.** (a) (i) $75$ (minutes) ***A1***

(ii) attempt to substitute values in the mean formula with at least one mid-interval value multiplied by a corresponding frequency ***(M1)***

(mean $=$) $88.2$ $(88.15)$ (minutes) ***A1***

***[3 marks]***

(b) $91.5$ **OR** $84$ seen ***(A1)***

**Note:** These values may be seen in the working for part (c).

$(\text{IQR} = 91.5 - 84 =)$ $7.5$ (minutes) ***A1***

***[2 marks]***

(c) (upper bound =) $91.5 + 1.5 \times 7.5$ **OR** $102.75$ seen ***A1***

$102.75 > 100$ **OR** $100 - 91.5 < 11.25$ **OR** $100 - 11.25 < 91.5$ ) ***R1***

Star Feud is not an outlier ***A1***

**Note:** Do not award ***R0A1***.

***[3 marks]***

(d) $\text{H}_0$ : The running times of the movies can be modelled by $\text{N}(88, 6.75^2)$

$\text{H}_1$ : The running times of the movies cannot be modelled by $\text{N}(88, 6.75^2)$ ***A1A1***

**Note:** Award ***A1*** for each correct hypothesis that includes a reference to normal distribution with a mean of $88$ and a standard deviation of $6.75$ (or variance of $6.75^2$). "Correlation", "independence", "association", and "relationship" are incorrect.

Award at most ***A0A1*** for correctly worded hypotheses that include a reference to a normal distribution but omit the distribution's parameters in one or both hypotheses. Award ***A0A1*** for correct hypotheses that are reversed.

***[2 marks]***

(e) (i) $T \sim \mathrm{N}(88, 6.75^2)$

attempt to find normal probability in either correct range ***(M1)***
$\mathrm{P}(85 \leq T < 90)$ **OR** $\mathrm{P}(T \geq 95)$

recognition of multiplying either of their probabilities by $200$ ***(M1)***
$0.288137... \times 200$ **OR** $0.149859... \times 200$

$a = 57.6 \ (57.6274...)$, $b = 30.0 \ (29.9718...)$ ***A1A1***

(ii) $\mathrm{df} = 4$ ***(A1)***

$(p =) \ 0.0166 \ (= 0.0166282\ldots)$ ***A1***

comparing their $p$-value to $0.05$ ***R1***
$0.0166 < 0.05$

**Note:** Accept $p$ value of $0.0165 \ (= 0.0164693...)$ from using $a$ and $b$ to 3 sf.

(Reject $\mathrm{H}_0$, There is sufficient evidence to say that) the data has not been drawn from the ($\mathrm{N}(88, 6.75^2)$) distribution. ***A1***

**Note:** Do not award ***R0A1***.

The conclusion to part (e)(ii) **MUST** follow through from their hypotheses seen in part (d); if hypotheses are incorrect/reversed etc., the answer to part (e)(ii) must reflect this in order for the ***A1*** to be credited.

***[8 marks]***
***[Total 18 marks]***

**3.** (a) (i) $150\ (\text{cm})$ ***A1***

(ii) attempt to substitute values in the mean formula with at least one mid-interval value multiplied by a corresponding frequency ***(M1)***

(mean $=$) $176\ (176.3)\ (\text{cm})$ ***A1***

***[3 marks]***

(b) $183$ **OR** $168$ seen ***(A1)***

**Note:** These values may be seen in the working for part (c).

$(\text{IQR} = 183 - 168 =)\ 15\ (\text{cm})$ ***A1***

***[2 marks]***

(c) (upper bound =) $183 + 1.5 \times 15$ **OR** $205.5$ seen ***A1***

$205.5 > 204$ **OR** $204 - 183 < 22.5$ **OR** $204 - 22.5 < 183$ ***R1***

Laszlo's height is not an outlier ***A1***

**Note:** Do not award ***R0A1***.

***[3 marks]***

(d) $H_0$: The heights of the students can be modelled by $N(176, 13.5^2)$

$H_1$: The heights of the students cannot be modelled by $N(176, 13.5^2)$ ***A1A1***

**Note:** Award ***A1*** for each correct hypothesis that includes a reference to normal distribution with a mean of $176$ and a standard deviation of $13.5$ (or variance of $13.5^2$). "Correlation", "independence", "association", and "relationship" are incorrect.

Award at most ***A0A1*** for correctly worded hypotheses that include a reference to a normal distribution but omit the distribution's parameters in one or both hypotheses. Award ***A0A1*** for correct hypotheses that are reversed.

***[2 marks]***

(e) (i) $h \sim \mathrm{N}(176, 13.5^2)$

attempt to find normal probability in either correct range $\mathrm{P}(170 \leq h < 180)$ **OR** $\mathrm{P}(h \geq 190)$ ***(M1)***

recognition of multiplying either of their probabilities by $200$ $0.288137... \times 200$ **OR** $0.149859... \times 200$ ***(M1)***

$a = 57.6 \ (57.6274...)$, $b = 30.0 \ (29.9718...)$ ***A1A1***

(ii) $\mathrm{df} = 4$ ***(A1)***

$(p =) \ 0.0166 \ (= 0.0166282\ldots)$ ***A1***

comparing their $p$-value to $0.05$ $0.0166 < 0.05$ ***R1***

**Note:** Accept $p$ value of $0.0165 \ (= 0.0164693...)$ from using $a$ and $b$ to 3 sf.

(Reject $\mathrm{H}_0$, There is sufficient evidence to say that) the data has not been drawn from the ($\mathrm{N}(176, 13.5^2)$) distribution. ***A1***

**Note:** Do not award ***R0A1***.

The conclusion to part (e)(ii) **MUST** follow through from their hypotheses seen in part (d); if hypotheses are incorrect/reversed etc., the answer to part (e)(ii) must reflect this in order for the ***A1*** to be credited.

***[8 marks]***

***[Total 18 marks]***

**1.** (a) (i) $1.96$ (m) ***A2***

**Note:** Award ***A1*** for substitution into the formula for the mean e.g. $\frac{1.67+1.60+1.68+...}{6}$.

(ii) $1.94$ (m) $(1.935)$ ***A1***

(iii) $2.31$ (m) ***A1***

(iv) $2.31 - 1.60$ ***(M1)***

**Note:** Award ***M1*** for recognizing $2.31$ **and** $1.60$ as the critical values.

$0.71$ (m) ***A1***

***[6 marks]***

(b) $1.975$ (m) **OR** $197.5$ (cm) ***A1***

***[1 mark]***

***[Total: 7 marks]***

**2.** (a) $33$ ***A1***

***[1 mark]***

(b) $12$ ***A1***

***[1 mark]***

(c) $\frac{13}{33}$ $(0.394, 0.393939\ldots, 39.4\%)$ ***A1***

***[1 mark]***

(d) $\left(\mathrm{P}(T \cup I) =\right)$ $\frac{31}{33}$ $(0.939, 0.939393\ldots, 93.9\%)$ ***A1***

**Note:** For ***A1*(ft)** to be awarded, the numerator must be $31$ and the denominator must be their answer to part (a).

***[1 mark]***

(e) $\mathrm{P}(T \cap I) \neq 0$ **OR** $n(T \cap I) \neq 0$ ***R1***

**Note:** Accept $\mathrm{P}(T) + \mathrm{P}(I) \neq \mathrm{P}(T \cup I)$ provided probabilities are shown. Accept an equivalent statement in words such as "some (13) students went to both cafes" or "students could go to both cafes". Condone $\mathrm{P}(T \text{ and } I) \neq 0$ **OR** $n(T \text{ and } I) \neq 0$

no, they are not mutually exclusive ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***
***[Total: 6 marks]***

**8.** (a) (i) attempt to rearrange to isolate $C$ ***(M1)***
e.g., subtracting $32$ or dividing the equation by $1.8$

$C = \frac{5}{9}(F - 32)$ $\left(C = \frac{F - 32}{1.8},\ C = 0.556F - 17.8\right)$ ***A1***

**Note:** If the answer is not written as an equation, award at most ***M1A0***.

(ii) $C = \left(\frac{77 - 32}{1.8} =\right)\ 25\ (^{\circ}\text{C})$ ***A1***

***[3 marks]***

(b) (i) $(1.8 \times 17 + 32 =)\ 62.6\quad (^{\circ}\text{F})$ ***A1***

(ii) recognizing that the "+32" does not affect the SD ***(M1)***
$(1.8 \times 9 =)\ 16.2\quad (^{\circ}\text{F})$ ***A1***

**Note:** Award ***M0A0*** for $1.8 \times 9 + 32\ \ (= 48.2)$.

***[3 marks]***
***[Total: 6 marks]***

**12.** (a) (i) use of expected value formula. ***(M1)***

$\mathrm{E}(X) = 5\times 0.40 + (-8)\times 0.1 + (-5)\times 0.2 + (-10)\times 0.3$

$(\$)\ -2.8$ ***A1***

(ii) Any one of the following ***A1***

- on average, players will lose $\$2.80$ (per game)
- players are expected to/are more likely to lose $\$2.80$ (per game)
- this is the long-term expected average when playing the game many times
- the expected value/it does not equal $0$, so the game is not fair

Do not accept:

- players will lose $\$2.80$ (per game)
- players will/are expected to win $-\$2.80$
- on average, players will lose money
- players are expected to lose money (per game)
- there is more chance of losing money than winning
- the game is not fair

***[3 marks]***

(b) (i) $\mathrm{E}(X) = 0$ **OR** $\dfrac{2.80}{0.40}$ ***(M1)***

**EITHER**

evidence of increase in winning prize ***(M1)***

$5 + (k-1)\times 1$ **OR** (number of price increases $=$) $7$ **OR**

$\mathrm{E}(X)$ for game $1 = -2.80$, $\mathrm{E}(X)$ for game $2 = -2.40$, etc.

$\big(5 + (k-1)1\big)\times 0.40 + (-8)\times 0.1 + (-5)\times 0.2 + (-10)\times 0.3 = 0$ ***(A1)***

**OR** $(4+k)\times 0.40 + (-8)\times 0.1 + (-5)\times 0.2 + (-10)\times 0.3 = 0$

**OR** $(k=)\ \dfrac{2.80}{0.40} + 1$

$k = 8$ (games) ***A1***

**OR**

(calculation of winnings to make the game fair)

$\big(w\times 0.40 + (-8)\times 0.1 + (-5)\times 0.2 + (-10)\times 0.3 = 0\big)$

$(w=)\ (\$)\ 12$ ***(A1)***

evidence of increase in winnings per game up to $\$12$ ***(M1)***

$\$5, \$6, \$7, \ldots \$12$

$k = 8$ (games) ***A1***

(ii) $\mathrm{E}(X) < 0$ for each (any) of the first $7$ games (or equivalent) ***R1***

***[5 marks]***

***[Total: 8 marks]***

**1.** (a) identifying the largest and smallest values: $(\$)255$, $(\$)49$ ***(M1)***
$(\$)206$ ***A1***

***[2 marks]***

(b) (i) $(\$)137$ $(137.1)$ ***(M1)A1***

(ii) $(\$)74.5$ $(74.4693\ldots)$ ***A1***

**Note:** The ***(M1)*** mark is for correct GDC use and hence can be awarded if either of the values is correct. An answer of $78.4976\ldots$ in (b)(ii) is awarded ***A0*** but is sufficient to credit the ***(M1)***.

***[3 marks]***

(c) (i) (mean =) $(\$)117$ $(117.1)$ ***A1***

(ii) (standard deviation =) $(\$)74.5$ $(74.4693\ldots)$ ***A1***

**Note:** If their answer to part (c)(ii) is incorrect, it should match their answer to part (b)(ii) to be awarded ***A1(FT)***.

***[2 marks]***
***[Total: 7 marks]***

**3.** (a) $H_0$: the size of peppers is independent of colour
$H_1$: the size of peppers is not independent of colour ***A1***

**Note:** Award ***A1*** for both hypotheses correct. Accept "not associated" in place of independent. Do not accept "correlated" or "related" or "affected".

***[1 mark]***

(b) (i) $\chi^2_{\text{calc}} = 22.5 \ (22.5483...)$ ***A2***

(ii) $22.5483... > 9.49$ **OR** $0.000155837\ldots < 0.05$ ***R1***
(there is sufficient evidence to) reject the null hypothesis ***A1***

**Note:** Do not award ***R0A1***.
Accept "accept the alternative hypothesis".
Their conclusion must be consistent with their $\chi^2_{\text{calc}}$ (or $p$-value) and their hypothesis.
Accept $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$ or $p <$ sig level provided their $\chi^2_{\text{calc}}$ value or $p$-value is seen.

***[4 marks]***
***[Total: 5 marks]***

**6.** (a) (i) recognition of binomial distribution (condone incorrect parameter) ***(M1)***

e.g. $M \sim \mathrm{B}(20,\ 0.04)$ **OR** $\mathrm{P}(M = 2) = \mathrm{binpdf}(20,\ 0.04,\ 2)$

$= 0.146\ \ (0.145799...)$ ***A1***

(ii) recognition the cumulative probability required ***(M1)***

e.g. $\mathrm{P}(M \geq 3) = 1 - \mathrm{bincdf}(20,\ 0.04,\ 2)$ **OR** $\mathrm{bincdf}(20,\ 0.04,\ 3,\ 20)$

$= 0.0439\ \ (0.0438627...)$ ***A1***

***[4 marks]***

(b) either one of two terms in expected value formula correct ***(M1)***

$50\big(20(0.96)\big) + 15\big(20(0.04)\big)$

= 972 (pesos) ***A1***

***[2 marks]***

***[Total: 6 marks]***

**8.** (a) recognizing that only way to score $7$ is to achieve a head and a $6$ on die ***(M1)***

e.g. $\frac{1}{6}$ and $\frac{1}{2}$ seen in an attempt to combine probabilities

$\left(\frac{1}{6}\times\frac{1}{2}=\right)\ \frac{1}{12}\ (0.0833333...)$ ***A1***

**Note:** Accept $0.0835$ from the use of $0.167$.

***[2 marks]***

(b) there are two ways to score (e.g.) 5

achieve a head and a 4 on die, or a tail and a 5 on die ***(M1)***

$\left(2\left(\frac{1}{6}\times\frac{1}{2}\right)=\right)\ \frac{2}{12}\quad\left(\frac{1}{6},\ 0.167,\ 0.16666...\right)$ ***A1***

**Note:** Award these marks for equivalent working for the $2,\ 3,\ 4$ or $6$ point scenarios.

| Final Score | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Probability | $\frac{1}{12}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{12}$ |

***A1***

**Note:** Award ***A1*** for a completely correct table. Award at most ***(M1)A1A0*** if their follow-through answer from part (a) leads to a total probability not equal to $1$.

***[3 marks]***

(c) **EITHER**

multiplying at least two columns from their table ***(M1)***

$1\times\frac{1}{12}+2\times\frac{1}{6}+...+6\times\frac{1}{6}+7\times\frac{1}{12}$

**OR**

recognizing the probabilities in the table are symmetric ***(M1)***

**OR** *(for HL markscheme and/or removed from SL at publication)*

Considering the sum of two random variables ***(M1)***

$\mathrm{E}(X+Y)=\mathrm{E}(X)+\mathrm{E}(Y)\ \ (=3.5+0.5)$

**THEN**

(expected value $=$) $4$ ***A1***

**Note:** Accept $4.01\ (4.00640\ldots)$ from use of their 3 sf values from (b).
Award at most ***M1A0*** if their final answer is not in the range 1 – 7

***[2 marks]***
***[Total: 7 marks]***

**1.** (a) $r_s = -1$ ***A1***

***[1 mark]***

(b) (i) $r = -0.979 \quad (-0.979191\ldots)$ ***A2***

(ii) strong **AND** negative ***A1A1***

**Note:** Award at most ***A1A0*** if additional answers are seen.
Due to the demand of the question, do not accept "negative (from the graph)" if their $r$ value is positive.

***[4 marks]***

(c) (i) $a = -0.0992 \qquad (a = -0.0992075\ldots)$ ***A1***

(ii) $b = 3.19 \qquad (b = 3.19150\ldots)$ ***A1***

(iii) $b$ represents the (typical) price of an apartment in the centre (of the city) ***A1***

**Note:** To award the ***A1***, some reference to "centre" or "zero distance from the city" needs to be seen.

***[3 marks]***

(d) (i) attempt to substitute $19.6$ for $x$ ***(M1)***

$y = -0.0992075\ldots \times 19.6 + 3.19150\ldots$

$= 1.25 \qquad (1.24704\ldots)$ ***A1***

price $= 1.25 \text{ million (AUD)} \quad (1.24704\ldots \text{million})$ ***A1***

(ii) interpolation ***R1***

strong correlation. ***R1***

***[5 marks]***

(e) $\mu_A \neq \mu_B$ ***A1***

***[1 mark]***

(f) $p = 0.0224 \quad (0.0223977\ldots)$ ***A2***

**Note:** Award ***A1*** for $0.022$ (2sf)
Award ***A1*** for an answer of $p = 0.0265 \quad (0.0265017\ldots)$, from use of unpooled GDC settings.

***[2 marks]***

(g) $0.0223977\ldots < 0.05$ ***R1***

(there is sufficient evidence to) reject the null hypothesis ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***

(h) (the two populations are) normally distributed ***A1***

**Note:** Do not accept "independent" as that applies to the samples, not the populations.

***[1 mark]***

***[Total 19 marks]***

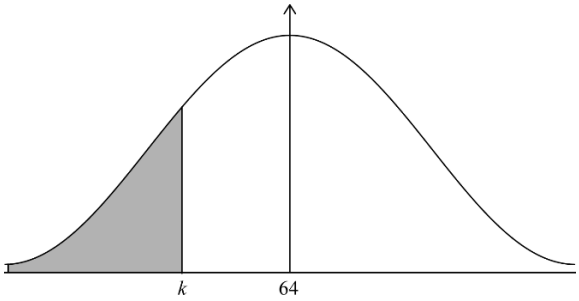
**2.** (a) $0.5$ ***A1***

***[1 mark]***

(b) $0.452 \ (0.452209\ldots)$ ***A2***

***[2 marks]***

(c) (i)




***A1A1***

**Note:** Award ***A1*** for a normal curve (with symmetry and some evidence of change of curvature towards the extreme values).
Award ***A1*** for a shaded region $x < k$, where $k$<mean.

(ii) $\mathrm{P}(T < k) = 0.3$

solving a cumulative distribution function **OR**
use of inverse function on GDC ***(M1)***

$k = 57.7 \ \ (57.7071\ldots)$ ***A1***

***[4 marks]***

(d) recognizing binomial distribution ***(M1)***

$\mathrm{B}(5,\ 0.3) \quad \left(\mathrm{P}(X = 2)\right)$ ***(A1)***

$0.309 \ \ (0.3087)$ ***A1***

***[3 marks]***

(e) $2(x-1)+4.5$ **OR** $2x+2.5$ ***A1A1***

**Note:** Award ***A1*** for a linear expression with a gradient of $2$,
***A1*** for a completely correct expression in $x$.

***[2 marks]***

(f) $(\$)13.10$ (accept $13.1$) ***A1***

***[1 mark]***

(g) attempt to solve $2(x-1)+4.5=7.2$ **OR** $2x+2.5=7.2$ ***(M1)***

$2.35$ (kg) ***A1***

**Note:** Award ***M1A1FT*** for an answer of $1.35$ (kg) from $2x+4.5$ seen in (e).

***[2 marks]***

***[Total 15 marks]***

**4.** (a) (i) $0.9$ (ii) $0.3$ (iii) $0.7$ ***A2***

**Note:** Award ***A1A0*** if one of the values is incorrect, ***A0A0*** otherwise.

***[2 marks]***

(b) $(0.1 \times 0.3 =)\ 0.03$ ***A1***

***[1 mark]***

(c) $\mathrm{P}(\text{no fail}) = 0.63$ ***A1***

multiplying by $200$ ***M1***

$= 126$ ***AG***

**Note:** Award ***A0M0*** for a flawed approach to find $\mathrm{P}(\text{no fail}) = 0.63$, e.g. $\frac{126}{200} = 0.63$, which is reverse engineering.

***[2 marks]***

*continued...*

*Question 4 continued*

(d) **EITHER**

attempt to find probability one switch failing ***(M1)***

$\mathrm{P}(\text{one failing}) = 0.34$ ***(A1)***

**OR**

expected value for two switches failing $= 6$ ***(A1)***

expected value for one switch failing $= 200 - 126 - 6$ ***(M1)***

**THEN**

| No switch fails | One switch fails | Two switches fail |
|---|---|---|
| 126 | 68 | 6 |

***(A1)***

degrees of freedom $= 2$ ***(A1)***

**Note***:* Award ***A1*** for df = $2$ seen anywhere and may be awarded independent of the ***M1*** mark.
The df cannot be implied from chi sq statistic $= 3.40989$

$p$-value $0.182$ $(0.181781\ldots)$ ***A1***

$0.182 > 0.05$ ***R1***

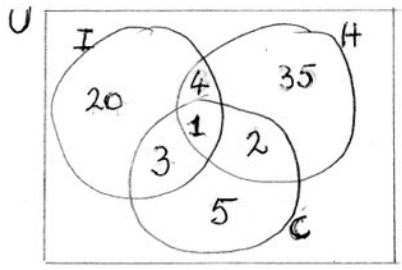
hence insufficient evidence to reject $H_0$ (that the manufacturers claims are correct) ***A1***

**Note:** The ***R1A1*** can be awarded as follow through within part (d) from their (explicitly labelled) incorrect p-value.
An unrealistic p-value ( $p \geq 1$ ) should preclude awarding the final ***R1A1***.
Accept either a conclusion to not reject the null hypothesis or the manufacturers claims are correct.
Do not award ***R0A1***.

***[7 marks]***

***[Total 12 marks]***

**1.** (a)




***A3***

**Note:** Award ***A1*** for $1$ in correct place, ***A1*** for $3$, $2$ and $4$ correct, ***A1*** for $35$, $20$ and $5$ correct. Award at most ***A0A1A1*** if the rectangle is omitted. Condone the omission of the $430$, as explicitly asked for in part (b).

***[3 marks]***

(b) $35+4+20+3+1+2+5$ ***(M1)***

$430$ ***A1***

**Note:** The $430$ may be seen in the Venn diagram.

***[2 marks]***

(c) (i) $\frac{42}{500}$ $(\frac{21}{250}, 0.084, 8.4\%)$ ***A2***

**Note:** Award ***A1*** for numerator, ***A1*** for denominator.

(ii) $\frac{5}{42}$ $(0.119,\ 0.119047\ldots, 11.9\%)$ ***A2***

**Note:** ***A1*** for numerator, ***A1*** for denominator.
The first ***A1*** can be awarded for an attempt to use conditional probability with $0.084$ on the denominator.

***[4 marks]***

(d) substituting $t=0$ into given expression ***(M1)***

$10$ ***A1***

***[2 marks]***

(e) $130=250-240(2)^{10k}$ ***(M1)***

$k=-0.1$ ***A1***

***[2 marks]***

(f) $200=250-240(2)^{-0.1t}$ ***(M1)***

$t=22.6$ $(22.6303\ldots, 23)$ (days) ***A1***

***[2 marks]***

(g) **EITHER**
model does not predict $n$ to go above $250$ / reach $300$ ***A1***

**OR**
$250 - 240 \times 2^{-0.1 \times 365} = 250$, so does not reach 300 ***A1***

**OR**
there is no solution to $n(t) = 300$ ***A1***

**OR**
correct sketch graph, with $250$ and/or $300$ labelled, and a supporting comment ***A1***

**THEN**
hence Diego's model is not appropriate.

| **Note:** Do not credit reasoning based on selecting arbitrary high values of $t$ and finding the associated $n$ value. |
|---|

***[1 mark]***
***[Total: 16 marks]***

**4.** (a) (i) $\mathrm{P}(X < 160)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

**Note:** Accept either zero or a large negative value as the lower bound.

$= 0.0447 \ \ (0.0447149\ldots, 4.47\%)$ ***A1***

(ii) $\mathrm{P}(160 < X < 170)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 0.411 \ \ (0.411049\ldots, 41.1\%)$ ***A1***

**Note:** Award ***A0A2*** for answers of $0.045$ and $0.41$ both given to $2$ sf.

***[4 marks]***

(b) $\mathrm{P}(X > h) = 0.27$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 1.75 \text{ (m)} \ \ (1.74560\ldots)$ ***A1***

**Note:** Accept $175$ (cm).

***[2 marks]***

(c) (i) $82.21$ ***A1***

(ii) $94.86$ ***A1***

**Note:** Follow through from an incorrect part (c)(i) if fourth value is found by subtracting first three values from $200$. Award at most ***A0A1*** if both answers are not given to four significant figures.
Award ***A0A1*** for an answer of $a = 82.2$ and $b = 94.8$.

***[2 marks]***

(d) $3$ ***A1***

***[1 mark]***

(e) $p$-value $= 0.616 \ \ (0.615583\ldots.)$ **OR** $\chi^2 = 1.80 \ \ (1.79702\ldots.)$ ***A2***

**Note:** Award ***A1A0*** if the $p$-value or $\chi^2$-value is given correct to $2$ dp.

$0.615583\ldots > 0.05$ **OR** $1.79702\ldots. < 7.815$ ***R1***

**EITHER**

fail to reject the null hypothesis ***A1***

**OR**

the heights are normally distributed with mean $170.7$ cm and standard deviation $6.3$ cm ***A1***

**Note:** Do not award ***R0A1***. Condone "accept" in place of "fail to reject".

The ***R1A1*** can be awarded as follow through within part (e) from their (explicitly labelled) $p$-value or $\chi^2$-value. Accept comparison in words.

***[4 marks]***

(f) **EITHER**

$H_0 : \mu_L = \mu_D$ ***A1***

$H_1 : \mu_L > \mu_D$ ***A1***

**OR**

$H_0$ : The (population) mean height of Latvian women is equal to the (population) mean height of Dutch women ***A1***

$H_1$ : The (population) mean height of Latvian women is greater than the (population) mean height of Dutch women ***A1***

**Note:** Award at most ***A0A1*** if the hypotheses explicitly refer to the "sample" and not the population. For $H_0 : m_1 = m_2$ and $H_1 : m_1 > m_2$ award ***A0A1***.

***[2 marks]***

(g) $p$-value $= 0.673$ $(0.673205\ldots)$ ***A2***

**Note:** In this question the $p$-value is the same 3 sf value for unpooled GDC settings so will be awarded ***A2***.
If using a two-tailed test, the answer is $p$-value $= 0.654$ $(0.653589\ldots)$ ; award ***A1*** if alternative hypothesis was correct or ***A2*** if it follows through correctly from their alternative hypothesis (i.e. two-tailed test was penalized in part (f)).

$0.673205 > 0.05$ ***R1***

fail to reject the null hypothesis (Gundega is not correct) ***A1***

**Note:** Do not award ***R0A1***. Condone "accept" in place of "fail to reject".

The ***R1A1*** can be awarded as follow through within part (g) from their (explicitly labelled) $p$-value. Accept comparison in words.

***[4 marks]***

***[Total: 19 marks]***

**2.** (a) $\overline{x} = 4.44166...$ $\sigma_x = 1.657286...$ ***(A1)(A1)***

$\overline{x} = 4.442$ $\sigma_x = 1.657$ (both correct to 4 sf) ***A1***

***[3 marks]***

(b) $\overline{x} = 44.4$ $(44.4166\ldots)$ $\sigma_x = 16.6$ $(16.5728\ldots)$ ***A1A1***

***[2 marks]***

(c) $\overline{x} = 74.4$ $(74.4166\ldots)$ $\sigma_x = 16.6$ $(16.5728\ldots)$ ***A1A1***

***[2 marks]***

***[Total 7 marks]***

**7.** (a) recognizing need to find $\mathrm{P}(W > 7.2)$ **OR** $1 - \mathrm{P}(W < 7.2)$ ***(M1)***

$(\mathrm{P}(W > 7.2) =)$ $0.922$ $(0.921709\ldots, 92.2\%)$ ***A1***

**Note:** Award ***M1A0*** for an answer of $0.92$ with no working.

***[2 marks]***

(b) (i) appropriate use of the GDC to find either $a$ or $b$ seen in either (b)(i) or (b)(ii) ***(M1)***

$(a =)$ $7.36$ $(7.36213\ldots)$ ***A1***

(ii) $(b =)$ $10.4$ $(10.4378\ldots)$ ***A1***

**Note:** The working for ***(M1)*** may be seen and awarded in part (b)(ii) if part (b)(i) has not been answered.

***[3 marks]***

(c) multiplying two identical probabilities ***(M1)***

$\mathrm{P}(W < 7.2) \times \mathrm{P}(W < 7.2)$ **OR** $(1 - \text{their (a)}) \times (1 - \text{their (a)})$

**Note:** Accept a lower bound of zero for $W$, given the context.

$(0.0782902...)^2$

$0.00613$ $(0.00612935\ldots$ **OR** $6.13 \times 10^{-3})$ ***A1***

**Note:** Award ***M0A0*** for an answer greater than $1$ such as $6.13$.
Award ***M1A0*** for $6.13\text{E-}3$.

***[2 marks]***
***[Total: 7 marks]***

**9.** (a) (i) (mean =) $18$ ***A1***

(ii) (variance =) $7.2$ ***A1***

***[2 marks]***

(b) (i) $\left(\mathrm{P}(X = 21) =\right)$ $0.0823$ $(0.0822752\ldots)$ ***A2***

**Note:** Award ***A1A0*** for an answer of $0.0822$ or $0.082$.

(ii) recognizing cumulative probability with upper bound of $11$ ***(M1)***

$\mathrm{P}(X \leq 11)$ **OR** $\mathrm{P}(X < 12)$

$= 0.00830$ $(0.0083016\ldots, 8.30\times10^{-3}, 0.830\%)$ ***A1***

***[4 marks]***

(c) **EITHER**

the outcomes of the test are independent of each other ***A1***

**OR**

the probability is constant ***A1***

**Note:** Do not accept "$60\%$ will pass". Accept "all students have the same probability of passing the test".

**OR**

there are only two possible outcomes ***A1***

**Note:** The number of trials is given in the question, and is NOT an assumption.

**Do not accept:**

"All students have the same driving ability"
"The driving conditions are the same"
"All test are done on sunny days and there is no rain"
"The instructors are fair" etc.

***[1 mark]***

***[Total 7 marks]***

**11.** (a) recognition that the total probability is $1$ ***(M1)***

$0.35 + 0.4 + p = 1$

$p = 0.25$ ***A1***

***[2 marks]***

(b) (expected losses is) $60 \times$ their $p$ from part (a) ***(A1)***

$60 \times 0.25$

$15$ ***A1***

***[2 marks]***

(c) interpreting that fair game means $\mathrm{E}(X) = 0$ (seen anywhere) ***(M1)***

correct expression to calculate **their** $\mathrm{E}(X)$ ***(M1)***

$0.35w + 0 \times 0.4 - 7 \times$ their $p$

$(w =)\ 5$ ***A1***

***[3 marks]***

***[Total 7 marks]***

**2.** (a) $\bar{x} = 4.454545...$ $\sigma_x = 1.63560...$ ***(A1)(A1)***

$\bar{x} = 4.455$ $\sigma_x = 1.636$ (both correct to 4 sf) ***A1***

***[3 marks]***

(b) $\bar{x} = 44.5$ $(44.5454\ldots)$ $\sigma_x = 16.4$ $(16.3560\ldots)$ ***A1A1***

***[2 marks]***

(c) $\bar{x} = 74.5$ $(74.5454\ldots)$ $\sigma_x = 16.4$ $(16.3560\ldots)$ ***A1A1***

***[2 marks]***

***[Total 7 marks]***

**7.** (a) recognizing need to find $\mathrm{P}(W > 3.5)$ **OR** $1 - \mathrm{P}(W < 3.5)$ ***(M1)***

$(\mathrm{P}(W > 3.5) =)$ $0.994$ $(0.993790\ldots)$ ***A1***

**Note:** Award ***M1A0*** for an answer of $0.99$ with no working.

***[2 marks]***

(b) (i) appropriate use of the GDC to find either $a$ or $b$ seen in either (b)(i) or (b)(ii) ***(M1)***

$(a =)$ $3.99$ $(3.98737\ldots)$ ***A1***

(ii) $(b =)$ $5.01$ $(5.01262\ldots)$ ***A1***

**Note:** The working for ***(M1)*** may be seen and awarded in part (b)(ii) if part (b)(i) has not been answered.

***[3 marks]***

(c) multiplying two identical probabilities ***(M1)***

$\mathrm{P}(W < 3.5) \times \mathrm{P}(W < 3.5)$ **OR** $(1 - \text{their (a)}) \times (1 - \text{their (a)})$

**Note:** Accept a lower bound of zero for $W$, given the context.

$(0.00620967)^2$

$0.0000386$ $(0.0000385601\ldots$ **OR** $3.86 \times 10^{-5})$ ***A1***

**Note:** Award ***M0A0*** for an answer greater than $1$ such as $3.86$.
Award ***M1A0*** for $3.86$E-5.

***[2 marks]***
***[Total: 7 marks]***

**9.** (a) (i) (mean =) 11 ***A1***

(ii) (variance =) 4.95 ***A1***

***[2 marks]***

(b) (i) $(P(X = 14) =)$ 0.0746 (0.0745996…) ***A2***

**Note:** Award ***A1A0*** for an answer of 0.0745 or 0.075.

(ii) recognizing cumulative probability with upper bound of 4 ***(M1)***

$P(X \leq 4)$ **OR** $P(X < 5)$

$= 0.00153$ $(0.0015307442\ldots, 1.53 \times 10^{-3}, 0.153\%)$ ***A1***

***[4 marks]***

(c) **EITHER**

the outcomes of the test are independent of each other ***A1***

**OR**

the probability is constant ***A1***

**Note:** Do not accept "55% will pass". Accept "all students have the same probability of passing the test".

**OR**

there are only two possible outcomes ***A1***

**Note:** The number of trials is given in the question, and is NOT an assumption.

**Do not accept:**

"All students have the same driving ability"
"The driving conditions are the same"
"All test are done on sunny days and there is no rain"
"The instructors are fair" etc.

***[1 mark]***

***[Total 7 marks]***

**11.** (a) recognition that the total probability is $1$ ***(M1)***

$0.25 + 0.4 + p = 1$

$p = 0.35$ ***A1***

***[2 marks]***

(b) (expected losses is) $60 \times$ their $p$ from part (a) ***(A1)***

$60 \times 0.35$

$21$ ***A1***

***[2 marks]***

(c) interpreting that fair game means $\mathrm{E}(X) = 0$ (seen anywhere) ***(M1)***

correct expression to calculate **their** $\mathrm{E}(X)$ ***(M1)***

$0.25w + 0 \times 0.4 - 5 \times$ their $p$

$(w =)\ 7$ ***A1***

***[3 marks]***

***[Total 7 marks]***

**3.** (a) (i) $(Q_3 =)\ 7.2$ ***A2***

**Note:** Award ***A1A0*** for a correct ordered list of happiness scores, when the correct $Q_3$ is not seen.

(ii) $Q_1 = 6.5$ ***(A1)***

$IQR = 7.2 - 6.5$

$= 0.7$ ***A1***

***[4 marks]***

(b) $Q_3 + 1.5 \times IQR$ ***(A1)***

$(7.2 + 1.5 \times 0.7 =)\ 8.25$ ***A1***

since $7.5 < 8.25$ ***R1***

Switzerland is not an outlier ***AG***

**Note:** Do not award ***A0A0R1***.

***[3 marks]***

(c) (i) $a = 3.5$ ***A1***

(ii) $b = 8$ ***A1***

(iii) $c = 3.5$ ***A1***

***[3 marks]***

(d) (i) $(r_s =)\ 0.164\ (0.164134\ldots)$ ***A2***

(ii) France rank (of sixth) is unchanged (so the $r_s$ is unchanged) ***R1***

***[3 marks]***

(e) Because $r_s$ is too close to zero and hence Jose's conclusion is not appropriate ***R1***

**Note:** Award ***R1*** for a comment on the value $r_s$ and "not appropriate" as a conclusion.
Accept "$r_s$ indicates a weak correlation of the ranks".
Do not accept "$r_s$ indicates a weak correlation of happiness score and country rank".

***[1 mark]***

***[Total 14 marks]***

**3.** (a) (i) $(Q_3 =)\ 7.2$ ***A2***

**Note:** Award ***A1A0*** for a correct ordered list of happiness scores, when the correct $Q_3$ is not seen.

(ii) $Q_1 = 6.5$ ***(A1)***

$IQR = 7.2 - 6.5$

$= 0.7$ ***A1***

***[4 marks]***

(b) $Q_3 + 1.5 \times IQR$ ***(A1)***

$\left(7.2 + 1.5 \times 0.7 =\right)\ 8.25$ ***A1***

since $7.5 < 8.25$ ***R1***

Switzerland is not an outlier ***AG***

**Note:** Do not award ***A0A0R1***.

***[3 marks]***

(c) (i) $a = 3.5$ ***A1***

(ii) $b = 8$ ***A1***

(iii) $c = 3.5$ ***A1***

***[3 marks]***

(d) (i) $(r_s =)\ 0.164\ \ (0.164134\ldots)$ ***A2***

(ii) France rank (of sixth) is unchanged (so the $r_s$ is unchanged) ***R1***

***[3 marks]***

(e) Because $r_s$ is too close to zero and hence Jose's conclusion is not appropriate ***R1***

**Note:** Award ***R1*** for a comment on the value $r_s$ and "not appropriate" as a conclusion.
Accept "$r_s$ indicates a weak correlation of the ranks".
Do not accept "$r_s$ indicates a weak correlation of happiness score and country rank".

***[1 mark]***

***[Total 14 marks]***

**1.** (a) $(r =)\ 0.999\ \ (0.998886\ldots)$ ***A2***

***[2 marks]***

(b) (i) $y = 340x + 742\ \left(y = 340.379\ldots x + 742.015\ldots\right)$ ***A1***

**Note:** Answer must be written as an equation. If the correct answer is seen here but no marks were awarded in part (a), then award ***A1A0*** for part (a) for accurate GDC data input.

(ii) the (recommended minimum) volume (of the hot air balloon) increases by (approximately) $340\ \text{m}^3$ for each additional passenger ***A1***

***[2 marks]***

(c) indication of $x = 10$ in regression equation or graph ***(M1)***
e.g. $(y =)\ 340.379\ldots \times 10 + 742.015\ldots$ **OR** graph of regression equation in approximate correct location with $x = 10$ indicated

$4150\ (\text{m}^3)\ \ (4145.81\ldots\ (\text{m}^3))$ ***A1***

***[2 marks]***

***[Total: 6 marks]***

**2.** (a) $(b =)\ 21$ ***A1***

***[1 mark]***

(b) $(\chi^2_{\text{calc}} =)\ 18.3\ (18.3313\ldots)$ ***A2***

***[2 marks]***

(c) **EITHER**

$18.3 > 16.81\ (18.3313\ldots > 16.81)$ ***R1***

**OR**

$(p =)\ 0.00546 < 0.01\ \ (0.00545524\ldots < 0.01)$ ***R1***

**Note:** The $p$-value $0.00546\ (0.00545524\ldots)$ and the significance level $0.01$ or $1\%$ must both be explicitly stated to award ***R1***.

**THEN**

**EITHER**

(reject $\text{H}_0$) there is insufficient evidence to suggest that age and car colour preference are independent / not dependent ***A1***

**OR**

(accept $\text{H}_1$) there is sufficient evidence to suggest that age and car colour preference are not independent ***A1***

**Note:** The answer to part (c) must be consistent with their $\chi^2_{\text{calc}}$ in part (b). Do not award ***R0A1***.

***[2 marks]***

***[Total: 5 marks]***

**7.** (a) Any one of these criteria:

- number of trials/children is fixed (at 5)
- two possible outcomes (for each trial/child with success being brown eyes, failure being not brown eyes)
- the probability of brown eyes/success is constant/the same (for each child/outcome) ***R1***

***[1 mark]***

(b) (i) $X \sim \mathrm{B}(5,\ 0.75)$

$0.264\ (0.263671\ldots)$ ***A2***

(ii) **METHOD 1**

recognizing cumulative probability with a lower bound of 4 ***(M1)***
$\mathrm{P}(X \geq 4)$ **OR** $\mathrm{P}(X > 3)$ **OR** $1 - \mathrm{P}(X \leq 3)$

$0.633\ (0.632812\ldots)$ ***A1***

**METHOD 2**

recognizing a sum of probabilities with a lower bound of 4 ***(M1)***
$\mathrm{P}(X = 4) + \mathrm{P}(X = 5)$ **OR** $0.75^4 \times 0.25 \times 5 + 0.75^5$

$0.633\ (0.632812\ldots)$ ***A1***

***[4 marks]***

***[Total: 5 marks]***

**2.** (a) (i) $\frac{1}{6}\left(=\frac{6}{36}, 0.167, 0.166666...\right)$ ***A1***

(ii) $\frac{1}{18}\left(=\frac{2}{36}, 0.056, 0.055555...\right)$ ***A1***

**Note:** The table value takes precedent if no working is seen before the table.

***[2 marks]***

(b)

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(X=x)$ | | $\frac{10}{36}$ $(0.277777...)$ | $\frac{8}{36}$ $(0.222222...)$ | | $\frac{4}{36}$ $(0.111111...)$ | |

***A1A1***

**Note:** ***A1*** for one of these three correct probabilities, ***A1*** these three probabilities correct.

***[2 marks]***

(c) attempt to find $E(X)$ ***(M1)***

$(\mathrm{E}(X)=)0\times\frac{6}{36}+1\times\frac{10}{36}...+5\times\frac{2}{36}$

$\frac{35}{18}\left(=\frac{70}{36}=1.94...\right)$ ***A1***

***[2 marks]***

(d) $\frac{1}{9}\left(=\frac{2}{18}\right)$ $0.111, 0.1111111111$ ... ... ***A1A1***

**Note:** Award ***A1*** for their 2, award ***A1*** for their 18 (these values may be implied in terms of probabilities.)
Accept $\frac{2}{x}$ for a numerator of $2$ irrespective of incorrect working in previous parts so long as their final answer gives $0<$probability$<1$.

***[2 marks]***
***Total [8 marks]***

**4.** (a) $H_0$: Animal type and flavour preference are independent

$H_1$: Animal type and flavour preference are not independent ***A1***

***[1 mark]***

(b) $2$ ***A1***

***[1 mark]***

(c) $(p=)0.0297 \quad (=0.029739...)$ ***A2***

**Note:** Award ***A1*** for $(p=)0.030$; award ***A0*** for $(p=)0.03$.

Award ***A1*** if unrounded correct answer (or correct 3sf answer) is seen in part (d) and part (c) is unanswered or wrong such as $0.029$

***[2 marks]***

(d) $1\% < p < 5\%$ OR $p > 1\%$ OR $p < 5\%$ ***R1***

(i) (since $p > 1\%$) do not reject $H_0$ (accept $H_0$) OR

there is not significant evidence that animal type and flavour preference are not independent ***A1***

(ii) (since $p < 5\%$) reject $H_0$ OR there is significant evidence at the $5\%$ level that animal type and flavour preference are not independent ***A1***

**Note:** Do not award ***R0A1.*** Award ***A1*** only if conclusions are consistent with their $p$-value.

***[3 marks]***

(e) $5\%$ (level) ***A1***

**Note:** No **FT** from previous parts.

***[1 mark]***

***Total [8 marks]***

**8.** (a) $\frac{80-72}{80}$ ***(M1)***

**Note:** Award ***(M1)*** for 72 or 8 seen.

10% ***A1***
***[2 marks]***

(b) (i) 44 ***A1***
(ii) 60 ***A1***
(iii) 70 ***A1***
***[3 marks]***

(c) attempt to use mean formula using at least one correct midpoint ***(M1)***
fully correct formula for mean ***(A1)***
$\frac{20\ (22+52+65+85)}{80}$
56 (km/hr) ***A1***
***[3 marks]***
***Total [8 marks]***

**10.** (a) 4 ***A1***

***[1 mark]***

(b)

| | A | B | C | D | E | F | G | H |
|---|---|---|---|---|---|---|---|---|
| Brett | 1 | 4 | 2 | 3 | 5 | 6 | 7 | 8 |
| Clarence | 1 | 2 | 4 | 3 | 5 | 7 | 6 | 8 |

***(A1)(A1)***

**Note:** Award **(A*1)*** for each line in table

$0.881$ $(0.880952\ldots)$ ***A2***

**Note:** The ranks maybe seen within printed table.
Award ***A1*** for 0.88 (2sf) answer.

For a common mistake:
Brett: 1 3 4 2 5 6 7 8 & Clarence correct
Award (***A0)(A1)A2*FT** if $0.952$ seen.

**Note:** The use of $r_s = 1 - \frac{6\sum d^2}{n(n-1)}$ i.e. $(r_s =) 1 - \frac{6\left(0^2+2^2+2^2+0^2+0^2+1^2+1^2+0^2\right)}{8\left(8^2-1\right)}$ may be seen which is not in the syllabus, but is a valid method because there are no ties. As they need the table to get the differences, there is no change to MS for this method.

***[4 marks]***

(c) (since $r_s$ is close to $1$) there is a strong (positive) correlation between the **rankings** of the sheepdogs by Brett and the **rankings** of the sheepdogs by Clarence
(since $r_s$ is close to $1$) there is a strong (positive) association between Brett's **rankings** and Clarence's **rankings**.
since $r_s$ is not equal to $1$, Brett and Clarence do not always agree on the **rankings** of the sheepdogs. ***A1***

**Note:** For ***A1*** to be awarded, a numerical value must be seen in part (b), and the conclusion consistent with that numerical value.
"Strong positive" is insufficient because it is not in terms of the **ranks**.
"Similar" in isolation is insufficient, however if they state "the order in which the sheepdogs were **ranked** by the judges is similar" would be awarded ***A1***.
The names of judges do not need to be seen for the ***A*** mark.

***[1 mark]***
***Total [6 marks]***

**Q2.**

(a) (i) attempt to find mid-values $600, 800, 1000...$ or to substitute in the mean formula ***(M1)***

$$\frac{600\times 38+800\times 45+1000\times 25+...1600\times 8}{150}$$

$937$ (kWh) $(937.333\ldots)$ ***A2***

| |
|---|
| **Note**: Award at most ***(M1)M1A0*** for an incorrect answer but evidence of correct use of mean formula.<br>Award ***(M1)A0*** for an unsupported 1100 , use of the mid-values can be inferred. |

(ii) $299$ (kWh) $(299.454\ldots)$ ***A1***

| |
|---|
| **Note**: Accept $299.5$. If no mark has been awarded in part (a)(i), award A2 for a correct answer seen in part (a)(ii). |

***[4 marks]***

(b) the monthly energy consumption in *Eureka* is less spread out than that of *Helvetia* (or equivalent) ***R1***

| |
|---|
| **Note**: Do not accept eg standard deviation in Helvetia is 3 times that in Eureka or interpretations referring to lower average, efficiency, or lower consumption unless clearly related to spread. Some mention of spread is required for the ***R1*** e.g less variation, more consistent, closer together, smaller range, tighter clustering. |

***[1 mark]***

***Total [5 marks]***



Apollo | Original source crop | 435

**Q8.**

(a) $H_1$: satisfaction level and the type of service interaction are not independent. ***A1***

***[1 mark]***

(b) 4 ***A1***

***[1 mark]***

(c) $3.32\ (3.31906\ldots)$ ***A2***

**Note**: Award ***A1A0*** for a 2 sf answer of $3.3$ as recognition of correct method.

***[2 marks]***

(d) (i) $3.319106\ldots < 9.488$ OR $0.506\ (\ 0.505917\ldots) > 0.05$ ***R1***

No, he isn't correct (insufficient evidence to reject $H_0$) ***A1***

**Note**: Accept a comparison in words (e.g. $\chi^2_{calc} < \chi^2_{crit}$ or '*p*-value < sig level') for the ***R*** mark only if their $\chi^2_{calc}$ value or *p*-value is seen. Do not award ***A1R0.***

(ii) insufficient evidence to say ''satisfaction level and the type of service interaction are not independent''. ***A1***

**Note**: Answer to part d (ii) follows through from their answer to part d(i).

***[3 marks]***

***Total [7 marks]***

**Q10.**

(a) $4$ ***A1***

***[1 mark]***

(b) identification of lower quartile as $24.28$ ***(A1)***

$24.70 - 24.28$

$0.42$ (seconds) ***A1***

***[2 marks]***

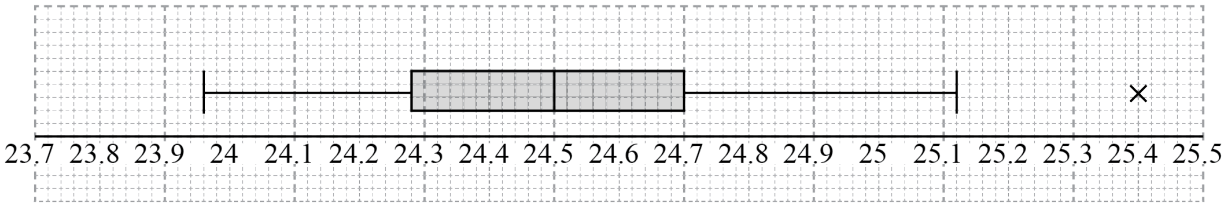
(c) (i) attempt to use correct formula to find boundaries for outliers ***(M1)***

$24.70 + 1.5 \times 0.42$ OR $24.28 - 1.5 \times 0.42$

$25.33$ and $23.65$ ***A1***

only $25.4$ is an outlier ***A1***

(ii)




***A1***

**Note**: Award the ***A1*** only if both whiskers and the outlier are correctly marked.
Accept symbols other than a cross to represent the outlier.
The whiskers should not be drawn through the box.

***[4 marks]***
***Total [7 marks]***

**Q12.**

(a) stratified (sampling) ***A1***

***[1 mark]***

(b) (i) $r = 0.854\ (0.853899...)$ ***A2***

**Note**: Award ***(M1)A0*** for a truncated value $r = 0.853$ or incorrectly rounded value eg. $r = 0.85$.

(ii) it is valid ( since $r > 0.8$ ) ***A1***

**Note**: Follow through from their value of $r$, for the final ***A1.***

***[3 marks]***

(c) $y = 0.266x + 2.40\ (y = 0.265851...x + 2.39573...)$ ***A1***

**Note**: Accept $2.4$.

***[1 mark]***

(d) substitution of $17.2$ into their regression line ***(M1)***

$y = 0.265851... \times 17.2 + 2.39573...$

$= 6.97\left(6.96837...\right)$ ***A1***

**Note**: Accept $6.98\left(6.9752\right)$ from use of 3sf answers from part (c).

***[2 marks]***

***Total [ 7 marks ]***

**1.** (a) $(k =)\ 5$ ***A1***

***[1 mark]***

(b) (i) attempt to substitute into the mean formula with a sum of at least two correct products **and** equating to $2.2$ (or $2.2\ (p+27)$ if denominator not seen) or equating to "mean". Equation may be seen in part b(ii). ***(M1)***

$$2.2 = \frac{0\times 3+1\times 11+2\times 7+3\times 5+4\times p+5\times 1}{3+11+7+5+p+1} \quad \textbf{OR} \quad 2.2 = \frac{45+4p}{p+27}$$ ***A1***

(ii) $(p =)\ 8$ ***A1***

**Note:** Follow through from part (b)(i) only if their value of $p$ is a whole number.

***[3 marks]***

(c) finding the median $(=2)$, quartiles ($Q_1 = 1$ and $Q_3 = 4$), or interquartile range of the data $IQR = 3$ from the 2024 tournament ***(A1)***

**Note:** Award ***(A1)*** for any correctly identified/labelled value referring to $2024$ data as listed in the question. Accept "Range from $0$ to $5$" without reference to $2024$.

Any two of the following statements: ***R1R1***

The median number of goals per game **in both years** is 2.
**In both years**, (the lower quartile is 1 and the upper quartile is 4) the IQR is 3.
**Both** distributions are (roughly) skewed to the right

**Note:** Award ***R1*** for each correct statement. Each statement must reference values from both the box and whisker diagram **and** the data from the 2024 tournament to award ***R1***. (e.g. "the median is 2" earns ***R0***, "the median for both years is 2" earns ***R1***.) If an incorrect statement is made, award at most ***R1R0***.

***[3 marks]***

(d) $B$ and $C$ ***A1A1***

**Note:** Award ***A1*** for each correct event, if one or two events are selected.
Award ***A1A0*** for two correct events and one additional incorrect event selected.
Award ***A0A1*** for an answer of "$A$, $D$, and $E$" from misreading the meaning of $F'$.
Award ***A0A0*** for any other combination of events selected.

***[2 marks]***

(e) $\frac{5}{17}\left(\frac{10}{34}\right)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator, ***A1*** for correct denominator. Award at most ***A1A0*** for a decimal answer $(0.294117\ldots)$ if no fraction is shown. Do not award ***A1ft*** for their denominator if their answer to part (a) or part (b)(ii) is not a whole number.

***[2 marks]***

(f) $\frac{1}{35}\times\frac{3}{34}$ ***(A1)(A1)(M1)***

**Note:** Award ***A1*** for each correct probability, ***M1*** for multiplying their two probabilities. Do not award any marks if their answer to part (a) or part (b)(ii) is not a whole number.

$0.00252 \quad (0.00252100\ldots, \frac{3}{1190}, 0.252\%)$ ***A1***

***[4 marks]***
***Total [15 marks]***

**3.** (a) (i) $20$ (min) ***A1***

(ii) $100$ ($\text{min}^2$) (accept $10^2$) ***A1***

***[2 marks]***

(b) $\text{P}(15 < X < 25)$ ***(M1)***

**Note:** Condone calculator syntax.

$0.383$ $(0.382924...)$ $(38.3\%)$ ***A1***

***[2 marks]***

(c) $\text{P}(X > 30) \times 365$ ***(M1)(M1)***

**Note:** Award ***M1*** for attempt to find the probability $\text{P}(X > 30)$ and ***M1*** for their probability multiplied by $365$. Condone calculator syntax

$57.9$ $(57.9091...)$ ***A1***

**Note:** For a final answer of $57$ or $58$ irrespective of $57.9$ seen award ***M1M1A0***.

***[3 marks]***

(d) $\text{P}(X < k) = 0.99$ ***(M1)***

**Note:** Condone calculator syntax.

$43.2634... + 10 = 53.2634...$ ***(M1)(A1)***

time is $6{:}54$ pm $(18{:}54)$ ***A1***

**Note:** Award ***M1M1A1A0*** for a final answer of $6{:}53$ $(18{:}53)$

***[4 marks]***

(e) (i) $\text{P}(X > 0)$ ***(M1)***

**Note:** Condone calculator syntax.

$0.977$ $(0.9772499...)$ ***A1***

(ii) recognition of binomial ***(M1)***

$n = 7$, $p = 0.9772499...$ ***(A1)***

**Note:** Condone calculator syntax.

$5$ successes

$0.00969$ $(0.00968755...)$ ***A1***

**Note:** Accept $0.00989$ from use of $3$sf answers

(iii) three terms ***(M1)***

correct powers of probabilities ***(M1)***

correct coefficients ***(M1)***

$1(0.9772499...)^7 + 2(0.9772499...)^6(0.02275) + 3(0.9772499...)^5(0.02275)^2$

$0.892$ $\left(0.892231...\right)$ ***A1***

**Note:** Accept $0.891$ from use of $3$sf answers only if working is seen.

***[9 marks]***

***Total [20 marks]***

**Q2.**

(a) let $W$ be the "weights of components" $W \sim \mathrm{N}\left(2.5, 0.15^2\right)$

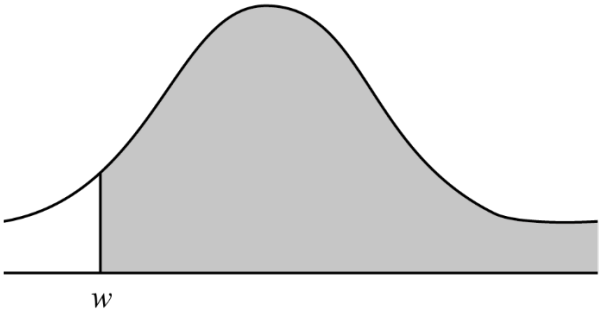
$\mathrm{P}(W > 2.8)$ ***(M1)***

$= 0.0228 (0.0227500...)$ ***A1***

**Note:** Award ***M1*** for correct calculator notation seen eg normcdf(2.8, 9E99, 2.5, 0.15).

***[2 marks]***

(b) (i)




***A1A1***

Award ***A1*** for normal curve with reasonable shape

**Note**: Reasonable shape = approximately symmetric, change of curvature (not parabolic), approximately asymptotic (no clear upward direction in extreme values).

Award ***A1*** for area shaded to the right of $w$, with left hand boundary to the left of centre of the curve.

*continued…*

*Question 2 continued.*

(ii) $\mathrm{P}(X > w) = 0.8$ or $\mathrm{P}(X < w) = 0.2$ ***(M1)***

$= 2.37\ (2.37375\ldots)$ ***A1***

**Note:** Award ***M1*** for correct calculator notation that represents the intended shaded region eg $(m =)$ InvNorm$(0.2, 2.5, 0.15)$
Award ***M1A0*** for $2.63\ (2.62624\ldots)$ seen. If their diagram shows $w$ located right of centre, award ***M1A1FT*** for $w = 2.63$.

***[4 marks]***

(c) (i) $(\mathrm{P}(2.3 < W < 2.7) =) 0.818 (= 0.817577...)$ ***A1***

(ii) $200 \times \mathrm{P}(2.3 < W < 2.7)$ ***(M1)***

$164\ (163.515\ldots)$ ***A1***

**Note**: Accept $163$ and correct non-integer values i.e. $163.515...$

***[3 marks]***

(d) recognition of binomial ***(M1)***

$X$ is the random variable "number of components passing the test" $X \sim \mathrm{B}(200, 0.95)$

$0.128\ (0.128357\ldots)$ ***A1***

***[2 marks]***

(e) $\mathrm{P}(X \geq 188)$ OR $1 - \mathrm{P}(X \leq 187)$ ***(M1)***

$0.796 (0.796484...)$ ***A1***

**Note:** Award ***M1*** for calculator notation communicating correct parameters and range.
eg binomcdf(200, 0.95, 188, 200) OR 1-binomcdf(200, 0.95, 187) OR binomcdf(200, 0.05, 12)

***[2 marks]***

*continued…*

*Question 2 continued.*

(f) multiplication of probabilities for independent events ***(M1)***

let $\mathrm{P}(T_1)$ and $\mathrm{P}(T_2)$ be the probabilities of passing Test 1 and Test 2, respectively,

$\mathrm{P}(T_1 \cap T_2) = 0.817577... \times 0.95$

$= 0.776698...$ ***A1***

recognition of a binomial distribution with $n = 12, p = 0.776698...$ ***(M1)***

let $Y$ be the random variable "number of components which pass both tests,

$Y \sim \mathrm{B}(12, 0.776698...)$

$\mathrm{P}(Y \geq 10)$

$0.477 (0.477419..)$ ***A1***

***[4 marks]***
***Total [17 marks]***

**4.** (a) **EITHER**
the data points come from normal populations ***A1***
**OR**
the variances (standard deviations) of the two groups are (approximately) equal. ***A1***
**OR**
data points of each group are independent. ***A1***
**OR**
the sample is representative of the population ***A1***
***[1 mark]***

(b) (i) $\mu_s = \mu_P$ **OR** $\mu_1 = \mu_2$ ***A1***
(no difference in the mean (population) distance travelled using premium/standard fuel)
(ii) $\mu_s < \mu_p$ **OR** $\mu_1 < \mu_2$ ***A1***
(mean (population) distance travelled using premium fuel is greater than using standard fuel)
***[2 marks]***

(c) (i) $0.0264\ (0.0263584\ldots)$ ***A2***

**Note:** Their $p$-value must be consistent with *their* $H_1$.
***[2 marks]***

(ii) $0.0263584\ldots < 0.05$ ***R1***

**Note:** Accept "$p < 0.05$" or "$p$-value is less than the significance level", provided their $p$-value is shown in part (c)(i).

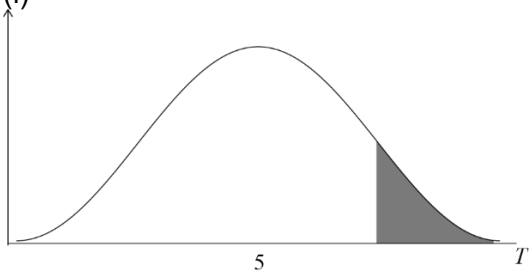
**EITHER**
(there is sufficient evidence that) the car can travel further using premium fuel. ***A1***
**OR**
(there is sufficient evidence that) the car uses less premium fuel compared to standard fuel. ***A1***

**Note:** Do not award ***R0A1***. The conclusion must be in context to award ***A1***; "reject $H_0$" earns ***A0***. Their conclusion must be consistent with *their* $p$-value and *their* hypotheses.
***[2 marks]***
***Total [7 marks]***

**7.** (a) $0.159$ $(0.158655\ldots)$ ***A2***

***[2 marks]***

(b) (i)




***A1***

**Note:** Accept any shaded region as shown provided it is to the right of 5.

(ii) recognition to use the inverse normal function on GDC ***(M1)***

**Note:** Award ***(M1)*** for $P(X > k) = 0.15$ **OR** $P(X < k) = 0.85$ **OR** inverse normal (invNorm) seen.

$6.04$ (seconds) $(6.03643\ldots)$ ***A1***

***[3 marks]***

***Total [5 marks]***

**9.** (a) (i) $X \sim \mathrm{B}(10, 0.25)$ ***(M1)***

**Note:** Award ***M1*** for recognizing binomial probability.

$(\mathrm{P}(X = 5) =)\ 0.0584 \quad (0.0583992\ldots)$ ***A1***

(ii) recognizing cumulative probability with a lower bound of 3 ***(M1)***
$\mathrm{P}(X \geq 3)$ **OR** $\mathrm{P}(X > 2)$ **OR** $1 - \mathrm{P}(X \leq 2)$ **OR** $1 - 0.525592\ldots$

$0.474 (0.474407\ldots)$ ***A1***

***[4 marks]***

(b) an attempt at calculating expected value ($0.3 \times 500\,000$ must be seen) ***M1***
$0.3 \times 500\,000 + 0.7 \times 0$ ***A1***
($) $150\,000$ ***AG***

**Note:** The $0.7 \times 0$, or equivalent, must be seen for the ***A1*** to be awarded.

***[2 marks]***

(c) **EITHER**
Option B, because the expected value of a Head ($\$150\,000$) is greater than the cash prize ($\$50\,000$). ***R1***

**OR**
Option B, because there is a chance to win a lot more money ***R1***

**OR**
Option A, because prize is guaranteed / less of a risk (safer) / Option B is a single trial, therefore the expected value is unachievable ***R1***

**OR**
Option A, because the probability of flipping a tail **which has no prize** is greater than the probability of flipping a head. ***R1***

**Note:** The ***R1*** is awarded for correctly considering the value from part (b), considering the amount that can be won, or reflecting on the (accurate) likelihood of winning money in a one-off game. Do not accept contextual reasons beyond that (e.g. "Option B, as she is very lucky having answered $10$ already" is awarded ***R0***).

***[1 mark]***
***Total [7 marks]***

**1.** (a) 21 ***A1***
***[1 mark]***

(b) 19 ***A1***
***[1 mark]***

(c) 4.96 (4.96487…) ***A1***
***[1 mark]***

(d) 32 and 12 seen ***(A1)***
20 ***A1***
***[2 marks]***
***Total [5 marks]***

**3.** (a) $150$ ***A1***

***[1 mark]***

(b) (choice of) main course is independent of (choice of) ice cream (flavour) ***A1***

***[1 mark]***

(c) $p = 0.0926(0.0926402\ldots)$ ***A2***

**Note:** Award ***A1*** for truncated $0.092$ or 2sf value of $0.093$.

***[2 marks]***

(d) $0.0926 > 0.05$ ***R1***
(insufficient evidence to reject the null hypothesis / fail to reject null hypothesis)

**Note:** Award ***R1*** for $p > \alpha$ so long as the $p$-value was seen in previous part.

insufficient evidence to say (choice of) main course is dependent of (choice of) ice cream (flavour).
**OR** main course choice is independent of ice-cream flavour choice ***A1***

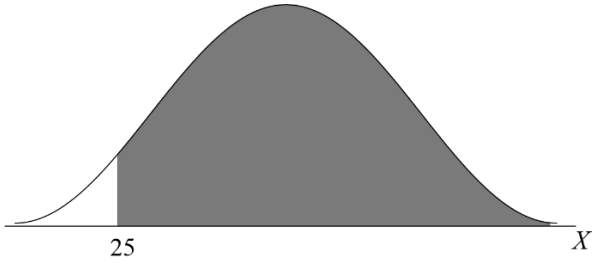
**Note:** Do not award the ***A1*** unless conclusion is in context.
Do not award ***R0A1***. Accept "not independent" in place of "dependent".

***[2 marks]***
***Total [6 marks]***

**4.** (a) $0.5\left(50\%, \frac{1}{2}\right)$ ***A1***

***[1 mark]***

(b) sketch of correct region ***(M1)***

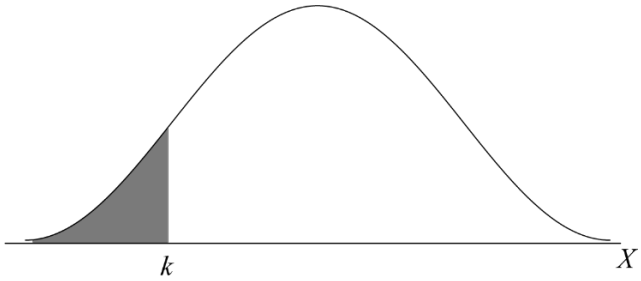



**Note:** Award ***M1*** for a sketch, provided the $25$ is labelled **AND** to the left of the mean.

**OR** normalcdf(25,1E99,35,5.5)
$(\mathrm{P}(X > 25) =)\ \ 0.965(0.965481\ldots, 96.5\%)$ ***A1***

***[2 marks]***

(c) sketch of correct region ***(M1)***




**Note:** Award ***M1*** for a sketch, provided the $k$ is labelled **AND** in approximately correctly position.

$\mathrm{P}(X < k) = 0.08$
**OR** invNorm(0.08,35,5.5)
$(k =) 27.3\ \ (27.2721\ldots)(\mathrm{m})$ ***A1***

**Note:** Accept an answer of $27.2$ from rounding down, to not exceed $8\%$.

***[2 marks]***
***Total [5 marks]***

**7.** (a) $0.2$ $\left(\frac{1}{5}, 20\%\right)$ ***A1***

***[1 mark]***

(b) $0.5 \times 0.3$ ***(A1)***

$15\%$ **OR** $0.15$ **OR** $\frac{3}{20}$ ***A1***

***[2 marks]***

(c) $0.3b$ ***A1***

***[1 mark]***

(d) (i) an attempt to sum probabilities related to ice-skating and equate to $87.5\%$ ***(M1)***

| | |
|---|---|
| **Note:** | Two of the following three correct either $0.5$ or $0.3b$ or their $a \times 0.625$ as part of their sum. **AND** no probabilities greater than 1. |

$0.5 + 0.3b + 0.2 \times 0.625 = 0.875$

$(b =) 0.833$ $\left(0.833333\ldots, \frac{5}{6}, 83.3\%\right)$ ***A1***

(ii) $(c =) 0.167$ $\left(0.166666\ldots, \frac{1}{6}, 16.7\%\right)$ ***A1***

| | |
|---|---|
| **Note:** | Award at most ***(M1)A0A1FT*** if part (a) is incorrect. Follow through within the question part for correctly finding *their* c (following an incorrect $b$). |

***[3 marks]***

***Total [7 marks]***

**10.** (a) $0.2$ seen ***(A1)***

$15 \times 0.2$

$3$ ***A1***

***[2 marks]***

(b) recognition of binomial distribution (may be seen in part (a) or part c) ***(M1)***

**Note:** Award ***(M1)*** for recognition of binomial by statement such as binomial with $n = 15$ and $p = 0.2$ or $\mathrm{B}(15,0.2)$ or binomialpdf(15,0.2,3) or correct value seen.

$(\mathrm{P}(X = 3) =)\ 0.250\ \ (0.250138\ldots)$ ***A2***

**Note:** Award ***(M1)A1A0*** for the correct 2sf answer, $0.25$.

***[3 marks]***

(c) identifying the correct region ***(M1)***

$P(X \geq 7)$ **OR** $1 - \mathrm{P}(X \leq 6)$ or 1-binomialcdf(15,0.2,6)

$0.0181(0.0180588\ldots)$ ***A1***

**Note:** Award ***(M1)A0*** for an answer of $0.018$, with or without working.

***[2 marks]***

***Total [7 marks]***

**1.** (a) (i) $4$ ***A1***

(ii) $6$ ***A1***

(iii) $3$ ***A1***

***[3 marks]***

(b) $(1.5\times 3)+6$

$= 10.5$ ***(A1)***

$(k=)\ 11$ ***A1***

***[2 marks]***

(c) at least one correct mid-interval value identified ***(A1)***

**Note:** Mid-interval value may be seen in mean formula.

$4.54 \quad (4.53650..., \ \frac{5716}{1260})$ ***A2***

***[3 marks]***

(d) the frequency of the number of letters in each word in his essay is **not** consistent with the English language. ***A1***

***[1 mark]***

(e) (i) $6$ ***A1***

(ii) $20.3 \quad (20.3131...)$ ***A2***

(iii) $0.00244 \quad (0.00243537... \ \ 2.44\times 10^{-3})$ ***A1***

**Note:** If ***A0*** awarded for part (e)(ii), ***A2*** can be awarded for a correct part (e)(iii).

***[4 marks]***

(f) **EITHER**

$20.3 > 16.812$ ***R1***

**OR**

$0.00244 < 0.01$ ***R1***

**THEN**

Dr. Petrillo should reject the null hypothesis ***A1***

**Note:** Do not award ***R0A1***.

***[2 marks]***

***Total [15 marks]***

**3.** (a) (i) $\frac{35}{50}\ \left(0.7,\ \frac{7}{10}\right)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator and ***A1*** for correct denominator.

(ii) $\frac{14}{35}\ \left(0.4,\ \frac{2}{5}\right)$ ***A1A1***

**Note:** Award ***A1*** for correct numerator and ***A1*** for correct denominator .
Award ***A1A0*** for $\frac{\left(\frac{14}{50}\right)}{\left(\frac{35}{50}\right)}$ seen, leading to an incorrect final answer.

***[4 marks]***

(b) $n(F \cap M) \neq 0$ **OR** $\mathrm{P}(F \cap M) \neq 0$ **OR** $\mathrm{P}(\mathrm{F} \cup \mathrm{M}) \neq \mathrm{P}(F) + P(M)$ **OR**
$\mathrm{n}(\mathrm{F} \cup \mathrm{M}) \neq \mathrm{n}(F) + n(M)$ ***R1***
no (not mutually exclusive) ***A1***

**Note:** Accept equivalent statements in words. Do not award ***R0A1***.

***[2 marks]***

(c) (i) $\frac{14}{x+14} = 0.7$ ***(M1)(A1)***

**Note:** Award ***M1*** for restricting set to $M$, e.g. $x+14$ seen, ***A1*** for completely correct equation.

$x = 6$ ***A1***

**Note:** Award at most ***(M1)(A1)A0*** for answer of $5.99999$.

(ii) $50 - 21 - 14 - 6$ **OR** $50 = 21 + 14 + 6 + y$ ***(A1)***
$(y =)\ 9$ ***A1***

**Note:** Award at most ***A1A0*** if their $y$ is a negative or non-integer value.

(iii) $36$ ***A1***

***[6 marks]***

(d) $\frac{35}{50} \times \frac{34}{49} \times \frac{33}{48}$ ***(M1)(A1)***

**Note:** Award ***M1*** for multiplying three probabilities, ***A1*** for a completely correct expression.

$= 0.334\ \ (0.333928..., \frac{187}{560}, \frac{39270}{117600})$ ***A1***

***[3 marks]***
***Total [15 marks]***

**2.** (a) $0.648\ (0.64848\ldots)$ ***A2***

strong positive (correlation between the ranks (of the data)) ***A1A1***

**Note:** Award ***A1A0*** if only the strength is correctly commented upon. ***FT*** their value for $r$.

***[4 marks]***

(b) evidence of constructing $\mathrm{N}$ from separate shapes ***(M1)***
rectangle minus $2$ triangles **OR** two rectangles + parallelogram
$13\times14-2\left(\frac{1}{2}\times7\times5\right)$ OR $2(4\times14)+7\times5$ ***(A1)***
$147\ (\mathrm{cm}^2)$ ***A1***

***[3 marks]***

(c) $588\ (\mathrm{cm}^3)$ ***A1***

***[1 mark]***

(d) (i) $\mathrm{H}_0:\mu_A=\mu_B;\ \mathrm{H}_1:\mu_A\neq\mu_B$ ***A1A1***

**Note:** Accept an equivalent statement in words, however reference to "**population** mean" must be explicit for both ***A1*** marks to be awarded.
Award at most ***A1A0*** for an imprecise *"the means are (not) equal"*.
Accept $\mu_1$ in place of $\mu_A$ and $\mu_2$ in place of $\mu_B$.

(ii) $p=0.102\ \ (0.101900\ldots)$ ***A2***

(iii) $p$-value > significance level or 0.05 **OR** $0.102>0.05$ ***R1***
insufficient evidence to reject null hypothesis ***A1***

**Note:** Do not award ***R0A1***.
Their conclusion must be consistent with their hypothesis and their $p$-value.
Do not accept a $p$-value >1

***[6 marks]***
***Total [14 marks]***

APOLLO

# Calculus

54 questions

**12.** (a) (i) $A = \frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times p + 48$ ***OR*** $A = \frac{1}{2}(p+6)(q+8)$ ***OR***
$A = 3q + 4p + 48$ ***A1***

(ii) valid attempt to link $p$ and $q$, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta = \frac{8}{p}$ and $\tan\theta = \frac{q}{6}$ **OR** $\tan\theta = \frac{p}{8}$ and $\tan\theta = \frac{6}{q}$ **OR** $\frac{8}{p} = \frac{q}{6}$

correct equation linking $p$ and $q$ ***A1***

eg. $pq = 48$ **OR** $p = \frac{48}{q}$ **OR** $q = \frac{48}{p}$

substitute $p = \frac{48}{q}$ into a correct area expression ***M1***

eg. $(A =)\frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times \frac{48}{q} + 48$ **OR** $(A =)\frac{1}{2}\left(\frac{48}{q} + 6\right)(q+8)$

$A = 3q + \frac{192}{q} + 48$ ***AG***

**Note:** The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

***[4 marks]***

(b) $\frac{-192}{q^2} + 3$ ***A1A1***

**Note:** Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for $3$. Award ***A1A0*** if extra terms are seen.

***[2 marks]***

(c) (i) $\frac{-192}{q^2} + 3 = 0$ ***A1***

(ii) $q = 8$ cm ***A1***

***[2 marks]***

***Total [8 marks]***

**13.** (a) $l'(50) = -0.2 \times 50 + 9$ ***(M1)***

$= -1$ ***A1***

the curve is decreasing at $\theta = 50^\circ$. ***A1***

**Note:** For the final ***A1***, follow through within this question part for their $l'(50)$ value. Award ***A0*** for an answer of "decreasing" with no work shown.

***[3 marks]***

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol or integrating at least one term correctly) ***(M1)***

$l(\theta) = -0.1\theta^2 + 9\theta\ (+c)$ ***A1A1***

$205.5 = -0.1 \times (40)^2 + 9 \times (40) + c$ ***(M1)***

**Note:** Award ***M1*** for correct substitution of $\theta = 40^\circ$ **and** $l = 205.5$. A constant of integration must be seen (can be implied by a correct answer).

$c = 5.5$

$\left(l(\theta) =\right)\ -0.1\theta^2 + 9\theta + 5.5$ ***A1***

**Note:** Accept any variable in the working, but for the final ***A1***, the variable $\theta$ must be used in the expression.

***[5 marks]***

***Total [8 marks]***

**13.** (a) recognition of need to integrate (*eg* reverse power rule or integral symbol) ***(M1)***

$P(x) = -0.8x^2 + 48x\ (+c)$ ***A1A1***

$260 = -0.8 \times (15)^2 + 48 \times (15) + c$ ***(M1)***

**Note:** Award ***M1*** for correct substitution of $x = 15$ and $P = 260$. A constant of integration must be seen (can be implied by a correct answer).

$c = -280$

$P(x) = -0.8x^2 + 48x - 280$ ***A1***

***[5 marks]***

(b) profit will decrease (with each new car produced) ***A1***

**EITHER**

because the profit function is decreasing / the gradient is negative / the rate of change of $P$ is negative ***R1***

**OR**

$\int_{30}^{50} -1.6x + 48\ (\mathrm{d}x) = -320$ ***R1***

**OR**

evidence of finding $P(30) = 440$ and $P(50) = 120$ ***R1***

**Note:** Award at most ***R1A0*** if $P(30)$ or $P(50)$ or both have incorrect values.

***[2 marks]***

***Total [7 marks]***

**5.** (a) (i) evidence of power rule (at least one correct term seen) ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -0.3x^2 + 1.6x$ ***A1***

(ii) $-0.3x^2 + 1.6x = 0$ ***M1***

$x = 5.33 \left( 5.33333..., \frac{16}{3} \right)$ ***A1***

$y = -0.1 \times 5.33333...^3 + 0.8 \times 5.33333...^2$ ***(M1)***

**Note:** Award ***M1*** for substituting their zero for $\frac{\mathrm{d}y}{\mathrm{d}x}$ $(5.333\ldots)$ into $y$.

$7.59 \text{ m} \ (7.58519...)$ ***A1***

**Note:** Award ***M0A0M0A0*** for an unsupported $7.59$.
Award at most ***M0A0M1A0*** if only the last two lines in the solution are seen.
Award at most ***M1A0M1A1*** if their $x = 5.33$ is not seen.

***[6 marks]***

(b) $A = \frac{1}{2} \times 2\big((2.4 + 0) + 2(6.4 + 7.2)\big)$ ***(A1)(M1)***

**Note:** Award ***A1*** for $h = 2$ seen. Award ***M1*** for correct substitution into the trapezoidal rule (the zero can be omitted in working).

$= 29.6 \text{ m}^2$ ***A1***

***[3 marks]***

(c) (i) $A = \int_2^8 -0.1x^3 + 0.8x^2 \ \mathrm{d}x$ **OR** $A = \int_2^8 y \ \mathrm{d}x$ ***A1A1***

**Note:** Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location.
Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.

(ii) $A = 32.4 \text{ m}^2$ ***A2***

**Note:** As per the marking instructions, ***FT*** from their integral in part (c)(i).
Award at most ***A1FTA0*** if their area is $>48$, this is outside the constraints of the question (a 6x8 rectangle).

***[4 marks]***

***Total [13 marks]***

**5.** (a) evidence of splitting diagram into equilateral triangles ***M1***

area $= 6\left(\frac{1}{2}x^2 \sin 60^\circ\right)$ ***A1***

$= \frac{3\sqrt{3}x^2}{2}$ ***AG***

**Note:** The ***AG*** line must be seen for the final ***A1*** to be awarded.

***[2 marks]***

(b) total surface area of prism $1200 = 2\left(3x^2 \frac{\sqrt{3}}{2}\right) + 6xh$ ***M1A1***

**Note:** Award ***M1*** for expressing total surface areas as a sum of areas of rectangles and hexagons, and ***A1*** for a correctly substituted formula, equated to $1200$.

$h = \frac{400 - \sqrt{3}x^2}{2x}$ ***A1***

volume of prism $= \frac{3\sqrt{3}}{2}x^2 \times h$ ***(M1)***

$= \frac{3\sqrt{3}}{2}x^2\left(\frac{400 - \sqrt{3}x^2}{2x}\right)$ ***A1***

$= 300\sqrt{3}x - \frac{9}{4}x^3$ ***AG***

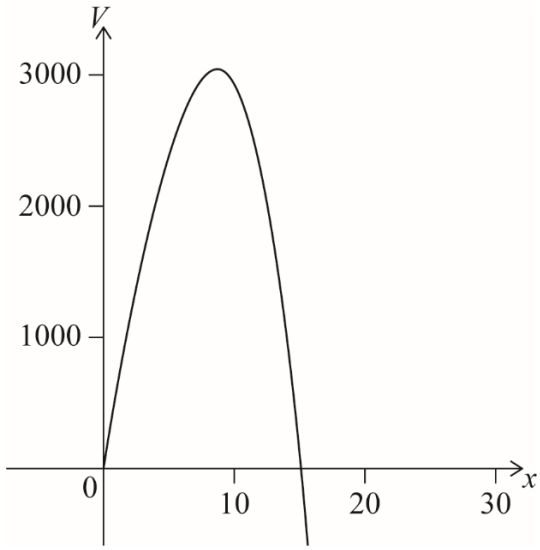
**Note:** The ***AG*** line must be seen for the final ***A1*** to be awarded.

***[5 marks]***

*continued…*

*Question 5 continued*

(c)




***A1A1***

**Note:** Award ***A1*** for correct shape, ***A1*** for roots in correct place with some indication of scale (indicated by a labelled point).

***[2 marks]***

(d) $\frac{\mathrm{d}V}{\mathrm{d}x} = 300\sqrt{3} - \frac{27}{4}x^2$ ***A1A1***

**Note:** Award ***A1*** for a correct term.

***[2 marks]***

(e) from the graph of $V$ or $\frac{\mathrm{d}V}{\mathrm{d}x}$ **OR** solving $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ ***(M1)***

$x = 8.77 \ (8.77382...)$ ***A1***

***[2 marks]***

*continued…*

*Question 5 continued*

(f) from the graph of $V$ **OR** substituting their value for $x$ into $V$ ***(M1)***

$V_{\max} = 3040 \text{ cm}^3$ $(3039.34...)$ ***A1***

***[2 marks]***

(g) **EITHER**

wasted space / spheres do not pack densely (tesselate) ***A1***

**OR**

the model uses exterior values / assumes infinite thinness of materials and hence the modelled volume is not the true volume ***A1***

***[1 mark]***

***Total [16 marks]***

---

**6.** (a) attempt at using trapezoidal rule formula ***(M1)***

$$\frac{1}{2}\left(\frac{2-0}{5}\right)\left(30+50+2(50+60+40+20)\right)$$ ***(A1)***

$(\text{total carbon} =)\ 84$ tonnes ***A1***

***[3 marks]***

(b) $\left|\frac{84-72}{72}\right| \times 100\,\%$ ***(M1)***

**Note:** Award ***(M1)*** for correct substitution of final answer in part (a) into percentage error formula.

$= 16.7\,\%\ \ (16.6666...\%)$ ***A1***

***[2 marks]***
***Total: [5 marks]***

**12.** (a) $(S(x) =)\ x^2 + 128x^{-1}$ ***(M1)***

**Note:** Award ***(M1)*** for expressing second term with a negative power. This may be implied by $\frac{1}{x^2}$ seen as part of their answer.

$2x - \frac{128}{x^2}$ **OR** $2x - 128x^{-2}$ ***A1A1***

**Note:** Award ***A1*** for $2x$ and ***A1*** for $-\frac{128}{x^2}$. The first ***A1*** is for $x^2$ differentiated correctly and is independent of the ***(M1)***.

***[3 marks]***

(b) (i) **EITHER**

any correct manipulation of $2x - \frac{128}{x^2} = 0$ e.g. $2x^3 - 128 = 0$ ***(M1)***

**OR**

sketch of graph of $S'(x)$ with root indicated ***(M1)***

**OR**

sketch of graph of $S(x)$ with minimum indicated ***(M1)***

**THEN**

$x = 4$ ***A1***

**Note:** Value must be positive. Follow through from their part (a) irrespective of working.

(ii) the value of $x$ that will minimize surface area of the box ***A1***

**Note:** Accept 'optimize' in place of minimize.

***[3 marks]***

***Total: [6 marks]***

**10.** (a) $A = \int_0^2 (6-3x)(4+x)\mathrm{d}x$ ***A1A1***

**Note:** Award ***A1*** for the limits $x = 0,\ x = 2$. Award ***A1*** for an integral of $f(x)$.

***[2 marks]***

(b) $28$ ***A1***

***[1 mark]***

(c) $28 = 0.5 \times a \times 10$ ***M1***

$5.6\left(\frac{28}{5}\right)$ ***A1***

***[2 marks]***

***Total [5 marks]***

**4.** (a) $2(8\times4+3\times4+3\times8)$ ***M1***

$=136\ (\text{cm}^2)$ ***A1***

***[2 marks]***

(b) $\sqrt{8^2+4^2+3^2}$ ***M1***

$(\text{AG}=)\ 9.43\ (\text{cm})\ (9.4339\ldots, \sqrt{89})$ ***A1***

***[2 marks]***

(c) $-2x+220=0$ ***M1***

$x=110$ ***A1***

$110\,000$ (boxes) ***A1***

***[3 marks]***

(d) $P(x)=\int -2x+220\,\text{d}x$ ***M1***

**Note:** Award ***M1*** for evidence of integration.

$P(x)=-x^2+220x+c$ ***A1A1***

**Note:** Award ***A1*** for either $-x^2$ or $220x$ award ***A1*** for both correct terms and constant of integration.

$1700=-(20)^2+220(20)+c$ ***M1***

$c=-2300$

$P(x)=-x^2+220x-2300$ ***A1***

***[5 marks]***

(e) $-x^2+220x-2300=0$ ***M1***

$x=11.005$ ***A1***

$11\,006$ (boxes) ***A1***

**Note:** Award ***M1*** for their $P(x)=0$, award ***A1*** for their correct solution to $x$. Award the final ***A1*** for expressing their solution to the minimum number of boxes. Do not accept $11\,005$, the nearest integer, nor $11\,000$, the answer expressed to $3$ significant figures, as these will not satisfy the demand of the question.

***[3 marks]***

***Total [15 marks]***

**6.** (a) $\frac{1}{2}\left(0.6+0+2(1.2+1.2)\right)$ ***(A1)(M1)***

**Note:** Award ***A1*** for evidence of $h=1$, ***M1*** for a correct substitution into trapezoidal rule (allow for an incorrect $h$ only). The zero can be omitted in the working.

$2.7\ \text{m}^2$ ***A1***

***[3 marks]***

(b) $\int_{-1}^{2}\frac{-x^3-3x^2+4x+12}{10}\,\text{d}x$ **OR** $\int_{-1}^{2}f(x)\,\text{d}x$ ***(M1)***

**Note:** Award ***M1*** for using definite integration with correct limits.

$2.925\ \text{m}^2$ ***A1***

**Note:** Question requires exact answer, do not award final ***A1*** for $2.93$.

***[2 marks]***

(c) $9-2.925$ ***(M1)***

**Note:** Award ***M1*** for $9$ seen as part of a subtraction.

$=6.08\ \text{m}^2\ \ (6.075)$ ***A1***

***[2 marks]***
***[Total 7 marks]***

**9.** (a) $f'(x) = -2x^{-2} + 6x$ **OR** $f'(x) = -\frac{2}{x^2} + 6x$ ***A1(M1)A1***

**Note:** Award ***A1*** for $6x$ seen, and ***(M1)*** for expressing $\frac{1}{x}$ as $x^{-1}$ (this can be implied from either $x^{-2}$ or $\frac{2}{x^2}$ seen in their final answer), ***A1*** for $-\frac{2}{x^2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

***[3 marks]***

(b) finding gradient at $x = 1$

$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1} = 4$ ***A1***

finding the perpendicular gradient ***M1***

$m_{\perp} = -\frac{1}{4}$

$2 = -\frac{1}{4}(1) + c$ **OR** $y - 2 = -\frac{1}{4}(x - 1)$ ***M1***

**Note:** Award ***M1*** for correctly substituting $x = 1$ and $y = 2$ and their $m_{\perp}$.

$x + 4y - 9 = 0$ ***A1***

**Note:** Do not award the final ***A1*** if the answer is not in the required form. Accept integer multiples of the equation.

***[4 marks]***
***[Total 7 marks]***

**12.** (a) **METHOD 1**

(when $t = 2$)

$\frac{\mathrm{d}P}{\mathrm{d}t} = -4$ **OR** $\frac{\mathrm{d}P}{\mathrm{d}t} < 0$ (equivalent in words) **OR** $3(2)^2 - 8(2) = -4$ ***M1***

therefore $P$ is decreasing ***A1***

**METHOD 2**

sketch with $t = 2$ indicated in 4th quadrant **OR** $t$-intercepts identified ***M1***

therefore $P$ is decreasing ***A1***

***[2 marks]***

(b) $(P(t) =)\ t^3 - 4t^2\ (+c)$ ***A1A1***

$4 = 1^3 - 4(1)^2 + c$ ***(M1)***

**Note:** Award ***M1*** for substituting $(1, 4)$ into their equation with $+c$ seen.

$c = 7$

$P(t) = t^3 - 4t^2 + 7$ ***A1***

***[4 marks]***

***[Total 6 marks]***

**6.** (a) **EITHER**

attempt to substitute $3$, $4$ and $7$ into area of a trapezoid formula ***(M1)***

$(A =) \frac{1}{2}(7+4)(3)$

**OR**

given line expressed as an integral ***(M1)***

$(A =) \int_{-1}^{2}(6-x)\ \mathrm{d}x$

**OR**

attempt to sum area of rectangle and area of triangle ***(M1)***

$(A =) 4 \times 3 + \frac{1}{2}(3)(3)$

**THEN**

$16.5$ (square units) ***A1***

***[2 marks]***

(b) (i) $(A =) \int_{-1}^{2} 1.5x^2 - 2.5x + 3\ \mathrm{d}x$ ***A1A1***

**Note:** Award ***A1*** for the limits $x = -1$, $x = 2$ in correct location. Award ***A1*** for an integral of the quadratic function, $\mathrm{d}x$ must be included. Do not accept "$y$" in place of the function, given that two equations are in the question.

(ii) $9.75$ (square units) ***A1***

***[3 marks]***

(c) $16.5 - 9.75$ ***(M1)***

$6.75$ (square units) ***A1***

***[2 marks]***

***Total [7 marks]***

**11.** (a) $\left(f'(x)=\right)\ 2x+\frac{3}{x^2}$ ***A1A1***

**Note:** Award ***A1*** for $2x$, ***A1*** for $+\frac{3}{x^2}$ **OR** $+3x^{-2}$.

***[2 marks]***

(b) attempt to substitute $1$ into their part (a) ***(M1)***

$\left(f'(1)=\right)\ 2(1)+\frac{3}{1^2}$

$5$ ***A1***

***[2 marks]***

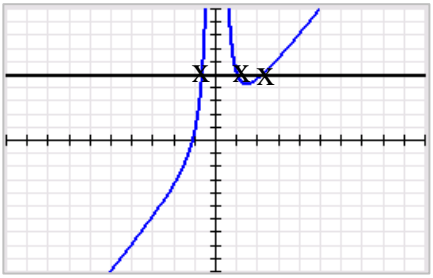
(c) **EITHER**

$5=2x+\frac{3}{x^2}$ ***M1***

$x=-0.686,\ 1,\ 2.19\ \ (-0.686140...,\ 1,\ 2.18614\ldots)$ ***A1***

**OR**

sketch of $y=f'(x)$ with line $y=5$ ***M1***



three points of intersection marked on this graph ***A1***
(and it can be assumed no further intersections occur outside of this window)

**THEN**

there are two other tangent lines to $f(x)$ that are parallel to $L$ ***A1***

**Note:** The final ***A1*** can be awarded provided two solutions other than $x=1$ are shown **OR** three points of intersection are marked on the graph.

Award ***M1A1A1*** for an answer of "3 lines" where $L$ is considered to be parallel with itself (given guide definition of parallel lines), but only if working is shown.

***[3 marks]***
***Total [7 marks]***

**5.** (a) attempt to expand given expression ***(M1)***

$$C = \frac{xk^2}{10} - \frac{3x^3}{1000}$$

$$\frac{\mathrm{d}C}{\mathrm{d}x} = \frac{k^2}{10} - \frac{9x^2}{1000}$$ ***M1A1***

**Note:** Award ***M1*** for power rule correctly applied to at least one term and ***A1*** for correct answer.

***[3 marks]***

(b) equating their $\frac{\mathrm{d}C}{\mathrm{d}x}$ to zero ***(M1)***

$$\frac{k^2}{10} - \frac{9x^2}{1000} = 0$$

$$x^2 = \frac{100k^2}{9}$$

$$x = \frac{10k}{3}$$ ***(A1)***

substituting their $x$ back into given expression ***(M1)***

$$C_{\max} = \frac{10k}{30}\left(k^2 - \frac{300k^2}{900}\right)$$

$$C_{\max} = \frac{2k^3}{9}\left(0.222\ldots k^3\right)$$ ***A1***

***[4 marks]***

(c) (i) substituting $20$ into given expression and equating to $426$ ***M1***

$$426 = \frac{20}{10}\left(k^2 - \frac{3}{100}(20)^2\right)$$

$$k = 15$$ ***A1***

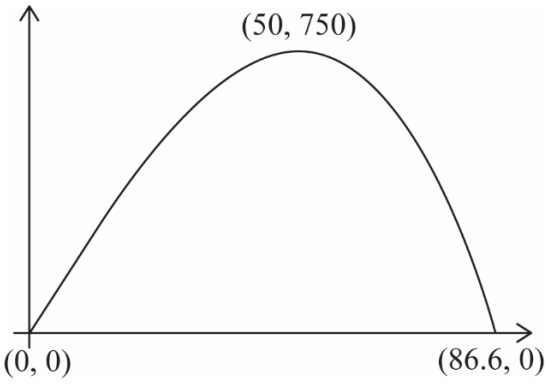
(ii) $50$ ***A1***

***[3 marks]***

*continued…*

*Question 5 continued*

(d)




***A1A1A1***

**Note:** Award ***A1*** for graph drawn for positive $x$ indicating an increasing and then decreasing function, ***A1*** for maximum labelled and ***A1*** for graph passing through the origin and $86.6$, marked on the $x$-axis or whose coordinates are given.

***[3 marks]***

(e) setting their expression for $C$ to zero **OR** choosing correct $x$-intercept on their graph of $C$ ***(M1)***

$x_{\max} = 86.6 \quad (86.6025\ldots)$ litres ***A1***

***[2 marks]***

***Total [15 marks]***

**13.** (a) $0 = 20 - \frac{980}{t^2}$ **OR** $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$ ***(M1)***

**Note:** Accept equivalent information presented in a labelled sketch.

$(h =)\ 7$ hours ***A1***

**Note:** Award ***M1A0*** for an answer of $(7, 280)$.

***[2 marks]***

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol) ***(M1)***

$P(t) = 20t + \frac{980}{t}\ (+c)$ ***A1A1***

$328 = 20 \times 5 + \frac{980}{5} + c$ ***(M1)***

**Note:** Award ***(M1)*** for substitution of $P = 328$ and $t = 5$ into their $P(t)$. A constant of integration must be seen (can be implied by a correct answer).

$c = 32$ ***A1***

$P(7) = 20 \times 7 + \frac{980}{7} + 32$ ***M1***

**Note:** Award ***M1*** for substituting $7$ and their $32$ into their $P(t)$.
Do not award the final ***M*** mark if their substituted values do not lead to $312$.

$312$ NOK ***AG***

***[6 marks]***
***Total [8 marks]***

**5.** (a) (i) $f'(x) = \frac{-2x}{50} + 2 \quad \left( = \frac{-x}{25} + 2, -0.04x + 2 \right)$ ***A1A1***

**Note:** Award ***A1*** for each correct term. Award at most ***A0A1*** if extra terms are seen.

(ii) $0 = \frac{-x}{25} + 2$ **OR** sketch of $f'(x)$ with $x$-intercept indicated ***M1***

$x = 50$ ***A1***

$y = 80$ ***A1***

$(50, 80)$

**Note:** Award ***M0A0A1*** for the coordinate $(50, 80)$ seen either with no working or found from a graph of $f(x)$.

***[5 marks]***

(b) (i) $\int_0^{70} \frac{-x^2}{50} + 2x + 30 \, \mathrm{d}x$ ***A1A1***

**Note:** Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location. Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.

(ii) $(\text{Area} =)\ 4710\ \text{m}^2 \quad \left( 4713.33\ldots, \frac{14140}{3} \right)$ ***A2***

***[4 marks]***

(c) (i) $\frac{11.4}{4713.33...} \times 100\%$ **OR** $\left| \frac{4701.93... - 4713.33...}{4713.33...} \right| \times 100\%$ ***(M1)***

**Note:** Award ***(M1)*** for their correct substitution into the percentage error formula.

$0.242\% \ \left( 0.241867...\% \right)$ ***A1***

**Note:** Percentage sign is required. Accept $0.242038...\%$ if $4710$ is used.

(ii) **EITHER**

reduce the width of the intervals (trapezoids) ***A1***

**OR**

increase the number of intervals (trapezoids) ***A1***

**Note:** Accept equivalent statements. Award ***A0*** for the ambiguous answer "increase the intervals".

***[3 marks]***

*continued…*

*Question 5 continued*

(d) (i) width of the square is $70-x$ **OR** the length of the square is $\frac{-x^2}{50}+2x+30$

***(M1)***

**Note:** Award ***(M1)*** for $70-x$ seen anywhere. Accept $\frac{-x^2}{50}+2x+30$ but only if this expression is explicitly identified as a dimension of the square.

in term of $x$, equating the length to the width $\mathrm{ED}$ ***(M1)***

$\frac{-x^2}{50}+2x+30=70-x$

$(x=14.7920...$ or $135.21)$

$(x=)\ 14.8\ \mathrm{m}\ (14.7920...)$ ***A1***

**Note:** Award ***M0M0A0*** for an unsupported answer of $15$. Award at most ***M1M0A0*** for an approach which leads to $A'(x)=0$. This will lead to a square base which extends beyond the east boundary of the property. Similar for any solution where $\mathrm{F}$ is not on the northern boundary, or $\mathrm{GH}$ is not on the east boundary.

(ii) **EITHER**

$(70-14.7920...)^2$ ***(M1)***

**OR**

$(55.2079...)^2$ ***(M1)***

**OR**

$\left(\frac{-(14.7920...)^2}{50}+2(14.7920...)+30\right)^2$ ***(M1)***

**THEN**

$(\mathrm{Area}=)\ 3050\ \mathrm{m}^2\ \ (3047.92...)$ ***A1***

**Note:** Follow through from part (d)(i), provided $x$ is between $0$ and $70$. Award at most ***M1A0*** if their answer is outside the range of their $[0,\ 4713.33\ldots]$ from part (b).

***[5 marks]***

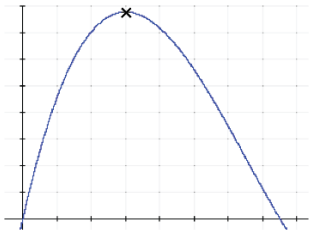
***Total [17 marks]***

**7.** (a) $(k =)\ 15$ ***A1***

***[1 mark]***

(b) **EITHER**

attempt to sketch the function $V(x)$ with indication of maximum ***(M1)***



**OR**

recognition of setting the derivative to $0$ ***(M1)***

e.g. $V'(x) = 0$

**THEN**

$(x =)\ 6\ (\text{cm})$ ***A1***

**Note:** Award ***(M1)A0*** for the maximum given as a coordinate pair.

***[2 marks]***

(c) $44$ or $26$ seen ***(A1)***

attempting to adjust the constant(s) in the given volume formula ***(M1)***

volume of second box $= (44 - 2x)(26 - 2x)(x)$

(New maximum volume $=)\ 2730\ \text{cm}^3\quad (2726.13...\ \text{cm}^3)$ ***A1***

**Note:** Units must be seen to award the final ***A1***. Award ***(A1)(M1)A0*** for the maximum given as a coordinate pair.

***[3 marks]***

***[Total: 6 marks]***

**9.** (a) attempt to substitute $h = 10$ and at least two different values of $y$ into the trapezoidal rule ***(M1)***

$$\frac{10}{2}\big((0+0)+2(3+8+9)\big)$$

$$= 200\ (\text{cm}^2)$$ ***A1***

***[2 marks]***

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \,\text{d}x$ **OR** $\int_0^{40} y \,\text{d}x$ ***A1A1***

**Note:** Award ***A1*** for a correct integral (including $\text{d}x$), ***A1*** for correct limits in the correct location.

(ii) $213.33\ (\text{cm}^2)$ ***A2***

**Note:** Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least $3$ significant figures, e.g. $213\ (\text{cm}^2)$.

***[4 marks]***

(c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$$\left|\frac{213.333... - 200}{213.333...}\right| \times 100$$

$$= 6.25(\%)\ \ (6.24999...(\%))$$ ***A1***

**Note:** Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or $0.0625$.

***[2 marks]***

***[Total: 8 marks]***

**13.** (a) attempt at using the trapezoidal rule ***(M1)***

$$\text{area} = \frac{1}{2}\left(3 + 2(8+19) + 42\right)$$

$$= 49.5 \ (\text{m}^2)$$ ***A1***

***[2 marks]***

(b) recognition of need to integrate (*e.g.* reverse power rule or integral symbol) ***(M1)***

$$\int 3x^2 + 4 \, \text{d}x = x^3 + 4x + c$$ ***(A1)(A1)***

**Note:** Award ***A1*** for each correct term.

$$f(x) = x^3 + 4x + 3$$ ***A1***

**Note:** Award ***A1*** for simplified correct answer including the value of $c$. Accept a value of $c$ of $3.005$ or $3.025$ or $2.975$ for using the non-integer $x$-values and their corresponding $y$-values.

***[4 marks]***

(c) **METHOD 1**

forming expression for sum of integral and deconstructing the trapezoid into a rectangle and triangle ***(M1)***

$$\int_0^3 x^3 + 4x + 3 \ \text{d}x (= 47.25) + 42 \times 1 + \frac{1}{2} \times 2 \times 42 \ (= 84)$$ ***(A1)***

$$= 131 \ (\text{m}^2) \ \ (131.25)$$ ***A1***

**METHOD 2**

forming expression for sum of integral and trapezoid ***(M1)***

$$\int_0^3 x^3 + 4x + 3 \ \text{d}x (= 47.25) \ + \frac{1}{2} \times 4 \times 42 \ (= 84)$$ ***(A1)***

$$= 131 \ (\text{m}^2) \ \ (131.25)$$ ***A1***

**Note:** Award ***(A1)*** for their integral with the correct limits added to $84$ or their $47.25$ added to $84$.

***[3 marks]***

***Total [9 marks]***

**5.** (a) equating a volume of a half cylinder (or cylinder) to $0.8$ ***(M1)***

$$0.8 = \frac{1}{2}\pi r^2 l$$

$$l = \frac{1.6}{\pi r^2}$$ ***A1***

**Note:** Do not accept decimal approximation of $\pi$ for the ***A1*** given the demand of question.
Condone the use of $h$ for $l$ for the ***M1***

***[2 marks]***

(b) calculating area in terms of $r$ and $l$ ***M1***

$$C = 2lr + \pi r^2 + \pi rl$$

area with $l$ replaced by $\frac{1.6}{\pi r^2}$ ***M1***

apply costs to correct part of each surface ***M1***

a correct substitution into an expression for $C$, leading to given answer

***A1***

*e.g.* $(C =)\ 4.40 \times \pi r\left(\frac{1.6}{\pi r^2}\right) + 4.40 \times 2r\left(\frac{1.6}{\pi r^2}\right) + p \times \pi r^2$

$$(C =)\ 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$$ ***AG***

**Note:** The ***AG*** line must be seen to award the final ***A1***.
No incorrect working should be seen after the correct substitution

***[4 marks]***

*continued…*

*Question 5 continued*

(c) **EITHER**

$$\left(\frac{\mathrm{d}C}{\mathrm{d}r} =\right) -7.04r^{-2} - \frac{14.08}{\pi}r^{-2} + 2p\pi r$$ ***A1A1A1***

**OR**

$$-7.04r^{-2} - 4.48r^{-2} + 6.28pr \quad \left(-7.04r^{-2} - (4.48180...)r^{-2} + 6.28318...pr\right)$$

***A1A1A1***

**OR**

$$-11.5r^{-2} + 6.28pr \quad \left((-11.5218...)r^{-2} + 6.28318...pr\right)$$ ***A2A1***

**Note:** Award ***A1*** for each correct term.
Award at most ***A1A1A0*** if extra terms are seen.

***[3 marks]***

*continued…*

*Question 5 continued*

(d) recognition of setting $\frac{\mathrm{d}C}{\mathrm{d}r}$ to zero ***(M1)***

attempt to substitute $0.7$ in for $r$ in their derivative ***(M1)***

$0 = -7.04(0.7)^{-2} - \frac{14.08}{\pi} \times (0.7)^{-2} + 2p\pi \times 0.7$

$(p =)$ ($)5.35 (per square metre) $((\$)\ 5.34621\ldots)$ ***A1***

**Note:** Accept $5.34, as this will also lead to a radius of $0.7$ (to $3$sf).

***[3 marks]***

(e) attempt to calculate the cost of one container ***(M1)***

$(C =)\ 7.04(0.7)^{-1} + \frac{14.08}{\pi}(0.7)^{-1} + 5.34621...\pi \times 0.7^2$ ***(A1)***

**Note:** May be shown within a calculation of the cost of all containers.

$(C =)\ 24.6895\ldots$

$24.6895\ldots \times 350$

= ($) 8641 ***A1***

**Note:** Answer must be rounded to the nearest dollar to award the final ***A1.***
Accept answers between $8641$ and $8645$ (inclusive), due to rounding the value of $p$ and/or the cost of one container to the nearest cent.
Award ***(M1)(A1)A0*** for an answer rounded to 3sf (e.g. ($)8640) or to 2dp (e.g., ($)8641.35).
Accept an answer of ($)8638 from use of $5.34 in their cost calculation.

***[3 marks]***

*continued…*

*Question 5 continued*

(f) attempt to apply a discount of 8% to their part (e) ***(M1)***

| |
|---|
| **Note:** the discount percentage will depend on their answer to part (e)<br>e.g. $8641.35... \times 0.92$ **OR** $8641.35... \times 0.08$ |

($)7950 (($)7950.04…) ***A1***

***[2 marks]***

***[Total: 17 marks]***

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**5.** (a) (i) $x-3$ ***A1***

(ii) attempt to use $1200$ to find width of park in terms of only $x$ ***(M1)***

$\frac{1200}{x}$ (seen) **OR** $1200 = x \times$ park width **OR** $1200 = x \times (\text{garden width} + 4)$

$\frac{1200}{x} - 4$ ***A1***

(iii) $A = (x-3) \times \left( \frac{1200}{x} - 4 \right)$ ***A1***

$= 1200 - 4x - \frac{3600}{x} + 12$ ***A1***

**Note:** Award first ***A1FT*** for multiplying *their* garden length and width and second ***A1*** for a simplified (parentheses removed) expression for $A$ that leads to the given answer. The given answer must be shown for the second ***A1*** mark to be awarded

$= 1212 - 4x - \frac{3600}{x}$ ***AG***

***[5 marks]***

(b) setting $1212 - 4x - \frac{3600}{x} = 800$ (accept a sketch) ***(M1)***

$x = 9.64 \ (9.64011\ldots)$ (m) **OR** $x = 93.4 \ (93.3598\ldots)$ (m) ***A1***

(width $=$) $124 \ (124.479\ldots)$ (m) ***A1***

(width $=$) $12.9 \ (12.8534\ldots)$ (m) ***A1***

**Note:** To award the final ***A1*** both values of $x$ **and** both values of the width must be seen. Accept $12.8$ for second value of width from candidate dividing $1200$ by $3$ sf value of $93.4$.

***[4 marks]***

(c) $\left( \frac{\mathrm{d}A}{\mathrm{d}x} = \right) -4 + \frac{3600}{x^2}$ **OR** $-4 + 3600x^{-2}$ ***A1A1A1***

**Note:** Award ***A1*** for $-4$, ***A1*** for $+3600$, and ***A1*** for $x^{-2}$ or $x^2$ in denominator.

***[3 marks]***

(d) setting *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ equal to $0$ **OR** sketch of *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ with $x$-intercept highlighted ***M1***

$(x =) \ 30$ (m) ***A1***

**Note:** To award ***A1FT*** the candidate's value of $x$ must be within the domain given in the problem $(3 < x < 300)$.

***[2 marks]***

*continued…*

*Question 5 continued*

(e) **EITHER**

evidence of using GDC to find maximum of graph of $A = 1212 - 4x - \frac{3600}{x}$ ***(M1)***

**OR**

substitution of *their* $x$ into $A$ ***(M1)***

**OR**

dividing $1200$ by *their* $x$ to find width of park **and** subtracting $3$ from *their* $x$ and $4$ from the width to find park dimensions ***(M1)***

**Note:** For the last two methods, only follow through if $3 <$ their $x < 300$.

**THEN**

$(A =)\ 972\ (\text{m}^2)$ ***A1***

***[2 marks]***

***Total [16 marks]***

**3.** (a) $x = 0$ ***A1***

**Note:** Answer must be an equation; an answer of "0" or "the $y$-axis" is awarded ***A0***.

***[1 mark]***

(b) $\left(f'(x) =\right) -16x^{-2} + \frac{x}{4}$ ***A1A1A1***

**Note:** Award ***A1*** for $-16$ seen, ***A1*** for $x^{-2}$ $\left(\text{or } \frac{1}{x^2}\right)$ and ***A1*** for second term being $\frac{x}{4}$.
Award at most ***A1A1A0*** if additional terms are seen.

***[3 marks]***

(c) $x > 4$ **OR** $(4, \infty)$ **OR** $4 < x < \infty$ ***A1A1***

**Note:** Award ***A1*** for $4$ seen and award ***A1*** for **correct** inequality.

***[2 marks]***
***[Total 6 marks]***



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**8.** (a) $(r'(-2) =)\ -14$ ***A2***

***[2 marks]***

(b) $\frac{1}{14}$ seen ***(A1)***

$y - 16 = \frac{1}{14}(x + 2)$ **OR** $y = 0.0714x + 16.1$ $(y = 0.0714285...x + 16.1428...)$ **OR**

$y = \frac{x}{14} + \frac{113}{7}$ **OR** $x - 14y + 226 = 0$ ***A1***

***[2 marks]***

(c) attempt to find coordinates of point G ***(M1)***

e.g. $(-7.53997\ldots, 15.6042\ldots)$ **OR** $(0.539978..., 13.1814\ldots)$ seen

attempt to substitute into distance formula ***(M1)***

$\left(d = \sqrt{(-2 + 7.53997\ldots)^2 + (16 - 15.6042\ldots)^2}\right)$

$(d =)\ 5.55\,(\text{km})\ (5.55409\ldots)$ ***A1***

***[3 marks]***

***[Total 7 marks]***

**3.** (a) $x = 0$ ***A1***

**Note:** Answer must be an equation; an answer of "0" or "the $y$-axis" is awarded ***A0***.

***[1 mark]***

(b) $(g'(x) =) -8x^{-2} + x$ ***A1A1A1***

**Note:** Award ***A1*** for $-8$ seen, ***A1*** for $x^{-2}$ $\left(\text{or } \frac{1}{x^2}\right)$ and ***A1*** for second term being $x$.
Award at most ***A1A1A0*** if additional terms are seen.

(c) $x > 2$ **OR** $(2, \infty)$ **OR** $2 < x < \infty$ ***A1A1***

**Note:** Award ***A1*** for $2$ seen and award ***A1*** for **correct** inequality.

***[2 marks]***
***[Total 6 marks]***



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**8.** (a) $\left(r'(-1) =\right) -8$ ***A2***

***[2 marks]***

(b) $\frac{1}{8}$ seen ***(A1)***

$y-6=\frac{1}{8}(x+1)$ **OR** $y=0.125x+6.13$ $(6.125)$ **OR**

$y=\frac{x}{8}+\frac{49}{8}$ **OR** $x-8y+49=0$ ***A1***

***[2 marks]***

(c) attempt to find coordinates of point B ***(M1)***
e.g. $(-5.02,\ 5.50)$ **OR** $(1.02,\ 6.25)$ seen

attempt to substitute into distance formula ***(M1)***
$\left(d=\sqrt{(-1+5.02076\ldots)^2+(6-5.49740\ldots)^2}\right)$

$(d =)$ $4.05$ (km) $(4.05205\ldots)$ ***A1***

***[3 marks]***
***[Total 7 marks]***

**1.** (a) $(a =)\ 9$ ***A1***

$(b =)\ 7.63\ (7.62741...)$ ***A1***

$(c =)\ 4.71\ (4.71281...)$ ***A1***

***[3 marks]***

(b) attempt to use the trapezoidal rule ***(M1)***

(area =) $\frac{1}{2}(16)\big((1+1)+2(9+7.62741...+4.71281\ldots)\big)$ ***(A1)***

(area =) $357\,(\text{mm}^2)\ (357.443\ldots)$ ***A1***

***[3 marks]***

(c) (i) $\int_0^{64}\left(4\sqrt{x}-\frac{x}{2}+1\right)\text{d}x$ ***A1A1***

**Note:** Award ***A1*** for correct function seen within the integral and ***A1*** for correct limits in the correct location and the inclusion of the $\text{d}x$.

(ii) $405.3\,(\text{mm}^2)$ ***A2***

***[4 marks]***

(d) attempt to substitute **their** area values into the percentage error formula ***(M1)***

$$\left|\frac{357.443\ldots-405.3}{405.3}\right|\times 100$$

$11.8\,(\%)\ (11.8076...)$ ***A1***

**Note:** Accept an answer of $11.9$ from use of $357$ from part (b).

***[2 marks]***

***[Total 12 marks]***

**5.** (a) attempt to substitute $16$ into the given formula ***(M1)***

$n = 20000 - 1000(16)$
$n = 4000$ ***A1***

***[2 marks]***

(b) multiplying their answer to part (a) by $16$ ***(M1)***
(average monthly income =) $16 \times 4000$
$64000$ (EUR) ***A1***

***[2 marks]***

(c) $R(x) = x(20000 - 1000x)$ **OR** $R(x) = 20000x - 1000x^2$ ***A1***

***[1 mark]***

(d) **EITHER**

attempt to find total costs (both fixed and variable) **AND** subtract from their $64000$ ***(M1)***
$64000 - (10000 + 10 \times 4000)$ ***(A1)***

**OR**
attempt to find total profit from cases **AND** subtract fixed costs ***(M1)***
$(16 - 10) \times 4000 - 10000$ ***(A1)***

**THEN**
$= 14000$ (EUR) ***A1***

***[3 marks]***

(e) **METHOD 1**

attempt to subtract total costs in terms of $x$ from their $R(x)$ ***(M1)***

$(P(x) =)\left(20000x - 1000x^2\right) - \left(10000 + 10(20000 - 1000x)\right)$

correct intermediate step leading to given answer ***A1***
(e.g. correct expansion of $10(20000 - 1000x)$)

$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

**Note:** Do not award the ***A1*** mark if the ***AG*** line is not stated.

**METHOD 2**

attempt to express profit per case, and then subtract fixed monthly costs ***(M1)***

$(P(x) =)(x - 10)(20000 - 1000x) - 10000$

correct expansion leading to the given answer ***A1***

$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

**Note:** Do not award the ***A1*** mark if the ***AG*** line is not stated.

***[2 marks]***

(f) (i) $(P'(x) =) \ -2000x + 30000$ ***A1A1***

**Note:** Award at most ***A1A0*** if additional terms are seen.

(ii) **METHOD 1**

$P'(x) = 0$ **OR** sketch of $P(x)$ **OR** use of $x = -\frac{b}{2a}$

$x = 15$ is the maximum, not $16$ ***A1R1***

hence salesman's price is not the optimum ***AG***

**Note:** Award ***A1*** for $x = 15$, and ***R1*** for comparing it to $16$ **OR** making a statement that is some version of the ***AG*** line. It is possible to award ***A1R0***.

**METHOD 2**

$P'(16) = -2000 \neq 0$ ***A1R1***

hence salesman's price is not the optimum ***AG***

**Note:** Award ***A1*** for finding an appropriate value, and ***R1*** for comparing it to zero. It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

**METHOD 3**

finding $P(x)$ for any value from $14 < x < 16$ ***A1***

comparing this value to their part (d) ***R1***

hence salesman's price is not the optimum ***AG***

**Note:** It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

***[4 marks]***

(g) substituting the expression for $n$ into cost function, $C(n)$. ***(M1)***

(cost =) $10 - 0.0001(20000 - 1000x) = 8 + 0.1x$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ ***(M1)***

(New $P(x)$ =) $\left(20000x - 1000x^2\right) - \left(10000 + (8 + 0.1x)(20000 - 1000x)\right)$ ***A1***

((New $P(x)$ =) $-900x^2 + 26000x - 170000$)

***[3 marks]***

(h) 14.4 (EUR) $(14.4444\ldots, \frac{130}{9})$ ***A2***

***[2 marks]***
***[Total 19 marks]***

---

**1.** (a) $(a =)\ 6$ ***A1***

$(b =)\ 5.14\ (5.14213...)$ ***A1***

$(c =)\ 3.32\ (3.32050\ldots)$ ***A1***

***[3 marks]***

(b) attempt to use the trapezoidal rule ***(M1)***

(area =) $\frac{1}{2}(25)\big((1+1)+2(6+5.14213...+3.32050\ldots)\big)$ ***(A1)***

(area =) $387\,(\text{cm}^2)\ (386.566\ldots)$ ***A1***

***[3 marks]***

(c) (i) $\int_0^{100}\left(2\sqrt{x}-\frac{x}{5}+1\right)\mathrm{d}x$ ***A1A1***

**Note:** Award ***A1*** for correct function seen within the integral and ***A1*** for correct limits in the correct location and the inclusion of the $\mathrm{d}x$.

(ii) $433.3\,(\text{cm}^2)$ ***A2***

***[4 marks]***

(d) attempt to substitute **their** area values into the percentage error formula ***(M1)***

$$\left|\frac{386.566\ldots-433.3}{433.3}\right|\times 100$$

$10.8\ (\%)\ (10.7855...)$ ***A1***

**Note:** Accept an answer of $10.7$ from use of $387$ from part (b).

***[2 marks]***

***[Total 12 marks]***

**5.** (a) attempt to substitute $16$ into the given formula ***(M1)***

$n = 20000 - 1000(16)$
$n = 4000$ ***A1***

***[2 marks]***

(b) multiplying their answer to part (a) by $16$ ***(M1)***
(average monthly income =) $16 \times 4000$
$64000$ (EUR) ***A1***

***[2 marks]***

(c) $R(x) = x(20000 - 1000x)$ **OR** $R(x) = 20000x - 1000x^2$ ***A1***

***[1 mark]***

(d) **EITHER**
attempt to find total costs (both fixed and variable) **AND** subtract from their $64000$ ***(M1)***
$64000 - (10000 + 10 \times 4000)$ ***(A1)***

**OR**
attempt to find total profit from mugs **AND** subtract fixed costs ***(M1)***
$(16 - 10) \times 4000 - 10000$ ***(A1)***

**THEN**
$= 14000$ (EUR) ***A1***

***[3 marks]***

(e) **METHOD 1**
attempt to subtract total costs in terms of $x$ from their $R(x)$ ***(M1)***
$\left(P(x) = \right)\left(20000x - 1000x^2\right) - \left(10000 + 10(20000 - 1000x)\right)$

correct intermediate step leading to given answer ***A1***
(e.g. correct expansion of $10(20000 - 1000x)$ )
$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

**Note:** Do not award the ***A1*** mark if the ***AG*** line is not stated.

**METHOD 2**
attempt to express profit per mug, and then subtract fixed monthly costs ***(M1)***
$\left(P(x) = \right)(x - 10)(20000 - 1000x) - 10000$
correct expansion leading to the given answer ***A1***
$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

**Note:** Do not award the ***A1*** mark if the ***AG*** line is not stated.

***[2 marks]***

(f) (i) $(P'(x) =) \ -2000x + 30000$ ***A1A1***

**Note:** Award at most ***A1A0*** if additional terms are seen.

(ii) **METHOD 1**

$P'(x) = 0$ **OR** sketch of $P(x)$ **OR** use of $x = -\frac{b}{2a}$

$x = 15$ is the maximum, not $16$ ***A1R1***

hence salesman's price is not the optimum ***AG***

**Note:** Award ***A1*** for $x = 15$, and ***R1*** for either comparing it to $16$ **OR** making a statement that is some version of the ***AG*** line. It is possible to award ***A1R0***.

**METHOD 2**

$P'(16) = -2000 \neq 0$ ***A1R1***

hence salesman's price is not the optimum ***AG***

**Note:** Award ***A1*** for finding an appropriate value, and ***R1*** for comparing it to zero. It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

**METHOD 3**

finding $P(x)$ for any value from $14 < x < 16$ ***A1***

comparing this value to their part (d) ***R1***

hence salesman's price is not the optimum ***AG***

**Note:** It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

***[4 marks]***

(g) substituting the expression for $n$ into cost function, $C(n)$. ***(M1)***

(cost =) $10 - 0.0001(20000 - 1000x) = 8 + 0.1x$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ ***(M1)***

(New $P(x)$ =) $\left(20000x - 1000x^2\right) - \left(10000 + (8 + 0.1x)(20000 - 1000x)\right)$ ***A1***

((New $P(x)$ =) $-900x^2 + 26000x - 170000$)

***[3 marks]***

(h) 14.4 (EUR) $(14.4444\ldots, \frac{130}{9})$ ***A2***

***[2 marks]***

***[Total 19 marks]***

**9.** (a) (i) recognizing that $0.2$ is removed $20$ times ***(M1)***

$10 - 0.2 \times 20$

$6$ (mm) ***A1***

(ii) $(V =) \ 216 \text{ mm}^3$ ***A1***

**Note:** Units are required for the ***A1*** to be awarded.

***[3 marks]***

(b) attempt at power rule ***(M1)***

$\left(V'(t) =\right) \ -60 + 2.4t - 0.024t^2$ ***A1***

***[2 marks]***

(c) recognizing need to find $V'(20)$ ***(M1)***

$\left(V'(20) =\right) \ -60 + 2.4(20) - 0.024(20)^2$

$= -21.6 \ (\text{mm}^3 \text{ s}^{-1})$ ***A1***

***[2 marks]***

(d) decreasing/change in coffee temperature ***R1***

decreasing/change in surface area of the cube

the cube breaks apart into smaller pieces

dissolution rate related to volume of the cube

**Note:** Award ***R1*** for a reasonable explanation of a change in rate.

***[1 mark]***

***[Total: 8 marks]***



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**10.** $3$ (seen at any stage of their work) ***(A1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b$ ***(M1)(A1)***

**Note:** Award ***M1*** for an attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

substituting $x = 2$ and their gradient into their derivative ***(M1)***

$3 = 2a(2) + b$

substituting $(2, 4)$ into original equation ***(M1)***

$4 = a(2)^2 + b(2) - 10$

solving equations $a = -2$

$b = 11$ ***A1A1***

**Note:** Using $m = -\frac{1}{3}$, gives $a = -\frac{11}{3}, b = \frac{43}{3}$ , award ***A0M1A1M1M1A0A1***.
For the final ***A*** marks, award at most ***A1A0*** if correct values are unlabelled or incorrectly labelled.

***[Total: 7 marks]***

**12.** (a) $\pi x + 2y \ (= 20)$ ***A1***

**Note:** Award ***A0*** for an unsimplified answer.

***[1 mark]***

(b) attempt to combine area formulas to express $A$ in terms of $x$ and $y$ ***M1***

$A = xy + \frac{\pi x^2}{4}$

$y = \frac{20 - \pi x}{2} \ (= 10 - \frac{\pi x}{2})$ ***A1***

correct substitution for y and expansion of brackets ***A1***

$A = x\left(\frac{20 - \pi x}{2}\right) + \frac{\pi x^2}{4} = \frac{20x - \pi x^2}{2} + \frac{\pi x^2}{4}$

$= 10x - \frac{\pi x^2}{4}$ ***AG***

**Note:** The ***AG*** line must be stated for the final ***A1*** to be awarded.

***[3 marks]***

(c) attempt at power rule e.g. one correct term ***(M1)***

$10 - \frac{\pi x}{2}$ ***A1***

***[2 marks]***

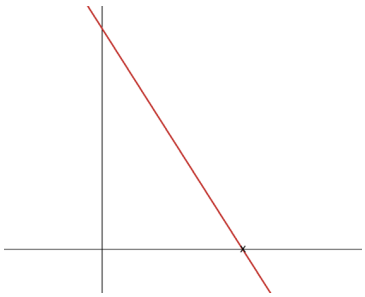
(d) **EITHER**

setting their derivative to zero ***M1***

$10 - \frac{\pi x}{2} = 0$ **OR** $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$

**OR**

attempt at finding the root of the graph of their derivative ***M1***



**THEN**

$x = \frac{20}{\pi}$ (m) ***A1***

(given that function is a negative quadratic, only stationary point is a maximum)

**Note:** Do not award the M mark if their derivative is not used to find the $x$-value. Award at most ***M1A0*** if their answer is not given in exact form.

***[2 marks]***

***[Total: 8 marks]***

**3.** (a) $\frac{40000}{x^2} = 400$ ***(M1)***

$x = 10$ (pesos) (since $x$ is positive) ***A1***

***[2 marks]***

(b) (i) $\left(\frac{40000}{50^2} =\right) 16$ ***A1***

(ii) $(50 \times 16 =)\ 800$ (pesos) ***A1***

***[2 marks]***

(c) (i) **EITHER**

profit for each smoothie = $x - 20$ ***(M1)***

$P = \frac{40000}{x^2} \times (x - 20)$ ***A1***

**OR**

profit = revenue – costs = $nx - 20n$ ***(M1)***

$P = x \times \frac{40000}{x^2} - 20 \times \frac{40000}{x^2}$ ***A1***

**Note:** Do not award ***A1*** if $\frac{40000}{x}$ seen as first term unless explained (in part (a) or (b)), as it is given in question.

**THEN**

$P = \frac{40000}{x} - \frac{800000}{x^2}$ ***AG***

(ii) attempt to express $P$ ready for power rule ***(M1)***

$P = 40000x^{-1} - 800000x^{-2}$

$\frac{dP}{dx} = -\frac{40000}{x^2} + \frac{1600000}{x^3}$ **OR** $\frac{dP}{dx} = -40000x^{-2} + 1600000x^{-3}$ ***A1A1***

**Note:** The ***(M1)*** can be awarded for either of the correct terms seen.
***A1*** for each correct term.
At most ***M1A1A0*** if additional terms seen.

*continued...*

*Question 3 continued*

(iii) attempt to find $x$-value ***(M1)***

e.g. sketch of $\frac{\mathrm{d}P}{\mathrm{d}x}$ with $x$-intercept indicated **OR** recognition that it occurs at the maximum of $\mathrm{P}$ **OR** algebraic approach (requires multiplication by $x^3$)

$x = 40$ ***A1***

**Note:** $\frac{-40000}{x^2} + \frac{1600000}{x^3} = 0$ is insufficient to award ***M1***, this is given in the question. There must be an "attempt to find $x$-value".
Award ***M1A0*** for a coordinate pair $(40, 500)$.

(iv) attempt to substitue their $x$-value into equation for $n$ ***(M1)***

$n = \frac{40000}{40^2}$

$= 25$ ***A1***

**Note:** Given the nature of the function *P*, the local maximum is also the global maximum. This is often the case in examinations, but should not always be assumed.

***[9 marks]***
***[Total 13 marks]***

**5.** (a) attempt to substitute $2$ into the DE ***(M1)***

$0.6\times 2^2 - 1.8\times 2$

$-1.2$ ***A1***

***[2 marks]***

(b) recognition of need to integrate ***(M1)***

**Note:** An increase of powers (expressions with $x^2 + x^3$) is sufficient evidence of the need to integrate.

$0.2x^3 - 0.9x^2 \ \ (+c)$ (***A1)***

attempt to substitute given condition to find $c$ ***(M1)***

$0.2(4)^3 - 0.9(4)^2 + c = 1.4$ OR $c = 3$

$\big(h(x) =\big) \ \ 0.2x^3 - 0.9x^2 + 3$ ***A1***

***[4 marks]***

*continued…*

*Question 5 continued*

(c) (i) $\int_0^4 0.2x^3 - 0.9x^2 + 3\mathrm{d}x$ **OR** $\int_0^4 h\,\mathrm{d}x$ ***A1A1***

**Note:** Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location.
Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.
Award at most ***A0A1*** for "integral $y\mathrm{d}x$ " or integral $f(x)\mathrm{d}x$, this is not the correct integrand.

(ii) $5.6\ (\mathrm{m}^2)$ ***A2***

***[4 marks]***

**Note:** A negative area for their integrand is unrealistic. Award at most ***A1A0*** for their ***FT*** answer expressed as negative area or a negative area converted to a positive answer."

(d) $(5.6\times 1.2 =)\ 6.72\ \mathrm{m}^3$ ***A1***

**Note:** Correct unit must be seen for the ***A1*** to be awarded.

***[1 mark]***

(e) attempt to substitute one of the given conditions, both $x$ and $h$, into either the function or the derivative ***(M1)***

$1 = 16a + 4b + c$ ***A1***

attempt to differentiate $h(x)$ ***(M1)***

$h'(x) = 2a(x) + b$ ***(A1)***

$-1.2 = 4a + b$ ***A1***

$0 = 6a + b$ **OR** $3 = \frac{-b}{2a}$ ***A1***

**Note:** The equations can be found in any order, and hence the associated mark(s) should be awarded independently.
Accept $1 = 4a + 2b + c$ and $1.2 = 8a + b$ as correct equations, from symmetry.
Award at most ***M1A1M1A1A1A0*** if the equations are not simplified to integer values.
E.g . $1 = 4^2 a + 4b + c$, $-1.2 = 2\times 2a + b$, $0 = 2\times 3a + b$

***[6 marks]***

(f) $a = 0.6\left(\frac{3}{5}\right),\ b = -3.6\left(-\frac{18}{5}\right),\ c = 5.8\left(\frac{29}{5}\right)$ ***A2***

**Note:** Award ***A1*** if only two are correct, ***A0*** otherwise.
Only follow through from three explicit equations given in part (e).

***[2 marks]***

(g) gradients are the same **OR** $h'(2) = -1.2$ for both curves ***A1***
heights are the same **OR** $h(2) = 1$ for both curves ***A1***

***[2 marks]***

***[Total 21 marks]***

**3.** (a) heights, $0$, $4$, $1.75$, $3$ and $3.75$ seen ***(A2)***

**Note:** Award ***A1A0*** if **two** of $1.75$, $3$ or $3.75$ are seen.

attempt to use trapezoidal rule formula for their heights ***(M1)***

$\frac{1}{2}\times 1\times\{0+4+2(1.75+3+3.75)\}$ ***(A1)***

**Note:** Award ***(M1)(A1)*** for correctly expressing this as $3$ trapezoids and a triangle. The "$\times 1$" need not be seen.

$=10.5\ (\text{m}^2)$ ***A1***

***[5 marks]***

(b) $-\frac{1}{12}x^3+x^2+c$ ***A1A1A1***

***[3 marks]***

(c) $\int_0^4\left(-\frac{1}{4}x^2+2x\right)\text{d}x+1\times 4+\frac{1}{2}\times 7\times 4$ ***(A1)(M1)(A1)***

**Note:** Award ***A1*** for correct area of rectangle **OR** triangle, ***M1*** for substituting correct limits into given integral (may be seen in part (b)), and ***A1*** for entire expression correct.

$=10.6666\ldots. + 4 + 14$

$= 28\frac{2}{3}\ (\text{m}^2)\left(\frac{86}{3}\right)$ ***A1***

**Note:** The answer must be **exact** for the ***A1*** to be awarded. For an answer of $28.7$ or $28.66$ award ***(A1)(M1)(A1)A0***.

***[4 marks]***

(d) (Total area using part (a) =) $28.5$ ***(A1)***

Percentage error $=\left|\frac{28.5-28.6666\ldots}{28.6666\ldots}\right|\times 100$ ***(M1)***

**Note:** if their trapezoid value is incorrect but is used correctly in the percentage error formula, award at most ***A0M1A0***. If it is clear from the answer that ×100 has been used, then condone the omission and award the ***M*** mark.

$=0.581\ (\%)\ (0.581395\ldots)$ ***A1***

(accept $0.697$ from use of $28.7$)

***[3 marks]***

***[Total: 15 marks]***

**12.** (a) $\left(\frac{dy}{dx} =\right) 10x + 9x^{-4}$ $\left(\left(\frac{dy}{dx} =\right) 10x + \frac{9}{x^4}\right)$ ***A1(M1)A1***

**Note:** Award ***A1*** for $10x$, ***M1*** for expressing $\frac{1}{x^3}$ as $x^{-3}$, ***A1*** for $+9x^{-4}$.

***[3 marks]***

(b) $19$ ***A1***

***[1 mark]***

(c) $m_{\perp} = -\frac{1}{19}$ (seen anywhere) ***(A1)***

$y = 2$ (seen anywhere) ***(A1)***

$y - 2 = -\frac{1}{19}(x - 1)$ **OR** $y = -\frac{1}{19}x + \frac{39}{19}$ $(y = -0.0526x + 2.05,\ y = -0.0526315...x + 2.05263...)$

***A1***

**Note:** Follow through within the question for the final ***A1***, which can be awarded for correctly substituting **their** gradient (seen) and **their** point (seen) into the equation for a line.

***[3 marks]***
***[Total: 7 marks]***



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**12.** (a) $\left(\frac{dy}{dx} =\right)\ 12x^2 + 4x^{-3}$ $\left(\left(\frac{dy}{dx} =\right)\ 12x^2 + \frac{4}{x^3}\right)$ ***A1(M1)A1***

**Note:** Award ***A1*** for $12x^2$, ***M1*** for expressing $\frac{1}{x^2}$ as $x^{-2}$, ***A1*** for $+4x^{-3}$.

***[3 marks]***

(b) $16$ ***A1***

***[1 mark]***

(c) $m_{\perp} = -\frac{1}{16}$ (seen anywhere) ***(A1)***

$y = 2$ (seen anywhere) ***(A1)***

$y - 2 = -\frac{1}{16}(x - 1)$ **OR** $y = -\frac{1}{16}x + \frac{33}{16}$ $(y = -0.0625x + 2.0625)$ **OR** $y = -0.0625x + 2.06$

***A1***

**Note:** Follow through within the question for the final ***A1***, which can be awarded for correctly substituting **their** gradient (seen) and **their** point (seen) into the equation for a line.

***[3 marks]***
***[Total: 7 marks]***



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**4.** (a) $\frac{1}{2}\times 1\times\left((6+4.3)+2(1.7+1.3+3+4.7)\right)$ ***A1A1***

$=15.9\ \ (15.85)\ (\text{cm}^2)$ ***A1***

***[3 marks]***

(b) (i) $\int_0^5\left(-\frac{x^3}{3}+3x^2-7x+6\right)\mathrm{d}x$ ***A2***

**Note:** Award ***A2*** for an integral with correct integrand and correct limits.

(ii) $15.4\ (15.4166\ldots,\frac{185}{12})\ \ (\text{cm}^2)$ ***A2***

**Note**: If their limits are outside the domain, award at most ***A1A0FT***.

***[4 marks]***

(c) $400=\pi r^2 h$ ***A1***

(d) **METHOD 1** (Substitution of $h=\frac{400}{\pi r^2}$ in $A=2\pi r^2+2\pi rh$)

$A=2\pi r^2+2\pi rh$ ***(A1)***

attempt to isolate $h$ ***(M1)***

$h=\frac{400}{\pi r^2}$

correct substitution of their $h$ into correct expression ***(M1)***

$A=2\pi r^2+2\pi r\left(\frac{400}{\pi r^2}\right)\left(A=2\pi r^2+\frac{800}{r}\right)$

$k=800$ ***A1***

**METHOD 2** (Equating $2\pi rh$ and $\frac{k}{r}$)

$A=2\pi r^2+2\pi rh$ **OR** $2\pi rh=\frac{k}{r}$ ***(A1)***

attempt to isolate $h$ ***(M1)***

$h=\frac{400}{\pi r^2}$

correct substitution of their $h$ into $2\pi r^2+2\pi rh=2\pi r^2+\frac{k}{r}$ ***(M1)***

$2\pi r\left(\frac{400}{\pi r^2}\right)=\frac{k}{r}$

$k=800$ ***A1***

***[4 marks]***

*continue…*

*Question 4 continued.*

(e) (i) $\frac{\mathrm{d}A}{\mathrm{d}r} = 4\pi r - 800r^{-2}$ ***A1(M1)A1***

**Note:** Award ***A1*** for $4\pi r$ seen, and ***(M1)*** for expressing $\frac{800}{r}$ as $800r^{-1}$ (can be implied through $\mp\frac{800}{r^2}$ seen), ***A1*** for $-800r^{-2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

(ii) $0 = 4\pi r - 800r^{-2}$ ***(M1)***

$r = 3.99 \left(3.99294\ldots, \sqrt[3]{\frac{200}{\pi}}\right)$ (cm) ***A1***

**Note**: Award at most ***M1A0*** if $k$ is not found.

***[5 marks]***

(f) $(h =)\ 7.99\ (7.98589\ldots)\ (\mathrm{cm})$ ***A1***

$(C =)\ 2\pi\ (3.99294...) \approx 25.1\ (25.0884\ldots)(\mathrm{cm})$ ***A1***

**EITHER**

the longest dimension of the label (7 cm) is less than both values and hence the label will fit (in any rotation) ***R1***

**OR**

$7 < 7.99$ and $5 < 25.1$ ***R1***

**Note:** Do not accept an argument based on the comparison of areas.

***[3 marks]***

***[Total 20 marks]***

**4.** (a) $\frac{1}{2}\times 1\times\big((4+0.7)+2(7.3+6.7+4+1.3)\big)$ ***(A1)(A1)***

$=21.7\ \ (21.65)\ \ (\text{cm}^2)$ ***A1***

***[3 marks]***

(b) (i) $\int_0^5\left(\frac{x^3}{3}-3x^2+6x+4\right)\mathrm{d}x$ ***A2***

**Note:** Award ***A2*** for an integral with correct integrand and correct limits.

(ii) $22.1\ (22.08333\ldots,\frac{265}{12})\ \ (\text{cm}^2)$ ***A2***

**Note:** If their limits are outside the domain, award at most ***A1A0FT***.

***[4 marks]***

(c) $600=\pi r^2 h$ ***A1***

***[1 mark]***

(d) **METHOD 1** (Substitution of $h=\frac{600}{\pi r^2}$ in $A=2\pi r^2+2\pi rh$)

$A=2\pi r^2+2\pi rh$ ***(A1)***

attempt to isolate $h$ ***(M1)***

$h=\frac{600}{\pi r^2}$

correct substitution of their $h$ into correct expression ***(M1)***

$A=2\pi r^2+2\pi r\left(\frac{600}{\pi r^2}\right)\quad\left(A=2\pi r^2+\frac{1200}{r}\right)$

$k=1200$ ***A1***

**METHOD 2** (Equating $2\pi rh$ and $\frac{k}{r}$)

$A=2\pi r^2+2\pi rh$ **OR** $2\pi rh=\frac{k}{r}$ ***(A1)***

attempt to isolate $h$ ***(M1)***

$h=\frac{600}{\pi r^2}$

correct substitution of their $h$ into $2\pi r^2+2\pi rh=2\pi r^2+\frac{k}{r}$ ***(M1)***

$2\pi r\left(\frac{600}{\pi r^2}\right)=\frac{k}{r}$

$k=1200$ ***A1***

***[4 marks]***

*continue…*

*Question 4 continued.*

(e) (i) $\frac{\mathrm{d}A}{\mathrm{d}r} = 4\pi r - 1200r^{-2}$ ***A1(M1)A1***

**Note:** Award ***A1*** for $4\pi r$ seen, and ***(M1)*** for expressing $\frac{1200}{r}$ as $1200r^{-1}$ (can be implied through $\mp\frac{1200}{r^2}$ seen), ***A1*** for $-1200r^{-2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

(ii) $0 = 4\pi r - 1200r^{-2}$ ***(M1)***

$r = 4.57 \left( 4.570781\ldots, \sqrt[3]{\frac{300}{\pi}} \right)$ (cm) ***A1***

**Note:** Award at most ***M1A0*** if the final answer is in terms of $k$.

***[5 marks]***

(f) $(h =)\ 9.14\ (9.14156\ldots..)$ (cm) ***A1***

$(C =)\ 2\pi\ (4.570781\ldots.) \approx 28.7\ \left(28.7190\ldots\right)(\mathrm{cm})$ ***A1***

**EITHER**

the longest dimension of the label (9 cm) is less than both values and hence the label will fit (in any rotation) ***R1***

**OR**

$9 < 9.14$ and $5 < 28.7$ ***R1***

**Note:** Do not accept an argument based on the comparison of areas.

***[3 marks]***

***[Total 20 marks]***

**8.** (a) evidence of finding the derivative of $f(x)$ ***(M1)***
e.g., $f'(x)$ seen **OR** attempt to use power rule

$f'(x) = -4x^{-3} - bx^{-2}$ ***(A1)(A1)***

**Note:** Award ***A1*** for each correct term. Award at most ***(A1)(A0)*** if extra terms are seen.

Substitution of $5$ for $x$ and $0.208$ for $f'(x)$ in *their* derivative ***(M1)***
$0.208 = -4(5)^{-3} - b(5)^{-2}$

$(b =)\ -6$ ***A1***
***[5 marks]***

(b) **METHOD 1**
attempt to find the value of the derivative at $x = 3.5$ **OR** $f'(3.5)$ seen ***M1***
$-4(3.5)^{-3} - b(3.5)^{-2}$

$0.397 > 0\ \ (0.396501\ldots > 0)$ **OR** $0.397\ (0.396501\ldots)$ is positive ***R1***

$f(x)$ is increasing at $x = 3.5$ ***AG***

**METHOD 2**
sketch of derivative with indication of $x = 3.5$ ***M1***

$f'(3.5) > 0$ **OR** $f'(3.5)$ is positive ***R1***

$f(x)$ is increasing at $x = 3.5$ ***AG***
***[2 marks]***

***[Total: 7 marks]***

**9.** (a) $f(x) = 0$ seen **OR** sketch of the graph with all three $x$-intercepts marked **OR** evidence of using product of roots ***(M1)***

$(a =)\ 3$ ***A1***

***[2 marks]***

(b) evidence of using an integral to find the area ***(M1)***

$\int f(x), \int_1^3$, etc.

(area $=$) 4 ***A1***

***[2 marks]***

***[Total: 4 marks]***

**12.** (a) indication of setting the derivative to $0$ ***(M1)***

$\frac{\mathrm{d}V}{\mathrm{d}w}=0$ **OR** $690w-36w^2=0$ **OR** sketch of the graph of the derivative with the zero marked

$(w=)\ 19.2\ (\text{cm})\ (19.1666\ldots\ (\text{cm}),\ \frac{115}{6}\ (\text{cm}))$ ***A1***

***[2 marks]***

(b) evidence of the need to integrate ***(M1)***

integral symbol seen **OR** $\int 690w-36w^2\ \mathrm{d}w$ **OR** one term integrated correctly

$(V=)\ 345w^2-12w^3\ (+c)$ ***A1***

$c=0$ since $V=0$ when $w=0$ ***R1***

$V=345\times(19.1666\ldots)^2-12\times(19.1666\ldots)^3$

$(V=)\ 42\,246.5\ldots\ (\text{cm}^3)$ **OR** $(V=)\ 42247\ (\text{cm}^3)$ seen ***A1***

$42\,200\ \text{cm}^3$ ***AG***

**Note:** It is possible to award ***(M1)A1R0A1***. No marks are awarded for part (b) if $42\,200$ is used to work backward.

***[4 marks]***

(c) **METHOD 1**

attempt to calculate height using volume, width, and length numerical values ***(M1)***

$h=\frac{42\,200}{19.1666\ldots\times(19.1666\ldots\times3)}$ (may be seen in part (b))

$(h=)38.3\ (\text{cm})\ (38.2911\ldots\ (\text{cm}))$ ***(A1)***

**Note:** Accept a substitution of $42\,246.5\ldots$ for the volume $(h=38.3333\ldots)$.

$19.1666\ldots+57.5+38.2911\ldots$

$(M=)\ 115\ (114.957\ldots)\ (\text{cm})$ ***A1***

**METHOD 2**

attempt to calculate height using volume, width, and length expressions ***(M1)***

$h=\frac{345w^2-12w^3}{w\times3w}$

$(h=)\ 115-4w$ ***(A1)***

$w+3w+(115-4w)$

$(M=)\ 115\ (\text{cm})$ ***A1***

***[3 marks]***

***[Total: 9 marks]***

**Q4.**

(a) (i) $-2$ ***A1***

(ii) $0.667\left(0.66666..., \frac{2}{3}\right)$ ***A1***

**Note**: Award ***A1A0*** for answers given as coordinates $(-2, 6)$ and $(0.667, -3.48)$.

Accept answers which round to the correct 3 sf value $(-2.00)$, e.g. $-1.9999$

***[2 marks]***

(b) $-2 < x < \frac{2}{3}$ ***A1***

**Note**: Award ***A1*** for correct inequalities and their endpoints.

***[1 mark]***

(c) (i) $3$ ***A1***

(ii) $y + 3 = 3(x - 1)$ OR $y = 3x - 6$ ***A1***

***[2 marks]***

***Total [5 marks]***



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**Q9**.

(a) evidence of integrating $\frac{\mathrm{d}P}{\mathrm{d}x}$ (***M1)***

$P = \int -10x + 460\,\mathrm{d}x$

$= -5x^2 + 460x(+c)$ ***A1A1***

**Note**: Award ***A1*** for each correct term. Omission of the constant $c$ may be ignored at this stage.

evidence of substituting $x = 10$ and $P = 3300$ in their profit expression ***(M1)***

$P = -5x^2 + 460x - 800$ ***A1***

***[5 marks]***

(b) attempt at using definite integral with limits $25$ and $50$ OR attempt at finding the difference between $P(50)$ and $P(25)$ ***M1***

$\int_{25}^{50} -10x + 460\,\mathrm{d}x$ OR $P(50) - P(25)$

$= 2130(\mathrm{MUR})(2125)$ ***A1***

**Note**: Do not apply FT in part (b) if their final answer in part (a) is a constant, as it implies profit does not depend on $x$.

***[2 marks]***

***Total [7 marks]***

**2.** (a) $0.638\ (0.637623...)$ and $4.10\ (4.09811...)$ ***A2A1***

**Note:** Award ***A2*** for one correct answer, accept $4.1$. Award ***A2A0*** if coordinate pair is given.

***[3 marks]***

(b) (i) $\int_{0.638}^{4.1} 1.5^x \,\mathrm{d}x$ OR $\int_{0.638}^{4.1} f(x)\,\mathrm{d}x$ ***A1A1***

**Note:** ***A1*** for function and $\mathrm{d}x$, ***A1*** for limits. Award ***A1*** for $\int_{0.638}^{4.1} y\,\mathrm{d}x$.
Condone the use of modulus within the integrand.

(ii) $9.81\ (9.80788...)$ (from 3sf)

$9.80\ (9.79844...)$ (from full) ***A2***

**Note:** Accept $9.8$ for ***A2***.

(iii) correct expression for area ***(A1)***

$\int_{0.638}^{4.1} g(x) - f(x)\,\mathrm{d}x$ **OR** $\int_{0.638}^{4.1} 6 - \frac{3}{x} - 1.5^x \,\mathrm{d}x$

$5.38\ (5.38290...)$ ***A1***

**Note:** Accept $5.39$

***[6 marks]***

(c) $f'(x) = g'(x)$ ***(M1)***

correct graphs of $f'(x)$ and $g'(x)$ ***(M1)***

**Note:** Award ***M1*** if $\frac{3}{x^2}$ seen.

$(k =) 1.86\ (1.86406...)$ ***A1***

***[3 marks]***
***Total [12 marks]***

**5.** (a) $(A=)2\pi r^2+2\pi rh$ ***A1A1***

**Note:** Award ***A1*** for either term correct (even if unsimplified) and ***A1*** for all correct (including simplification and no extra term).

***[2 marks]***

(b) $V=\frac{2\pi r^3}{3}+\pi r^2h$ ***A1A1***

$V=\frac{2\pi r^3+3\pi r^2h}{3}$ ***AG***

**Note:** Award ***A1*** for one correct term, ***A1*** for both and completion to required form.

***[2 marks]***

(c) $10000$ substituted into (b) ***M1***

correct rearrangement with at least one line of intermediate working leading to the given value.

***M1***

$h=\frac{30000-2\pi r^3}{3\pi r^2}$ ***AG***

***[2 marks]***

(d) $h$ substituted into their $A$ ***M1***

$(A=)2\pi r^2+2\pi r\left(\frac{30000-2\pi r^3}{3\pi r^2}\right)$

further consistent rearrangement that leads to **given** $A$ ***A1***

***[2 marks]***

(e) $\frac{4\pi r}{3}-\frac{20000}{r^2}$ ***A1A1A1***

**Note:** Award ***A1*** for $\frac{4\pi r}{3}$, ***A1*** for $-20000$ seen, and ***A1*** for a correct index for $r$ in second term.

***[3 marks]***

(f) $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ ***(M1)***

**EITHER**

rearrangement leading to $r^3=\frac{15000}{\pi}$ ***(M1)***

OR

graph of $\frac{\mathrm{d}A}{\mathrm{d}r}$ ***(M1)***

**THEN**

$r=16.8\ (16.8389...)$ OR $\sqrt[3]{\frac{15000}{\pi}}$ ***A1***

**Note:** Accept correct graph of Area function leading to correct minimum value for $r$ for ***M1M1A1***.

$h=0$ ***A1***

***[4 marks]***

(g) hemisphere ***A1***

**Note:** Do not ***FT*** from incorrect answers to part (f).

***[1 mark]***

***Total [16 marks]***

**Q3.**

(a) $(d=)3l$ ***A1***

***[1 mark]***

(b) substitution into volume of cuboid formula $V=l^2\times 3l\,(=375)$ ***(M1)***

attempting to solve their $3l^3=375$ ***(M1)***

$l^3=125$ OR $l=\sqrt[3]{125}$ OR $l^3=\dfrac{375}{3}$

$l=5,\ d=15$ ***A1***

***[3 marks]***

(c) finding the area of a square and the area of a (non-square) rectangle $5\times 5$ and $5\times 15$ ***(M1)***

$(5\times 5+5\times 15+5\times 15)\times 2$ ***(A1)***

$=350\text{cm}^2$ ***A1***

**Note**: The final answer is $350\text{cm}^2$. Units are required.

***[3 marks]***

(d) $\pi r^2h=375$ ***(A1)***

$h=\dfrac{375}{\pi r^2}$ ***A1***

**Note**: Award at most ***A0A1*** for a final answer $h=\dfrac{V}{\pi r^2}$.

***[2 marks]***

(e) attempt to substitute their $h$ into the curved surface area $2\pi rh$ ***M1***

$A=2\pi r^2+2\pi r\left(\dfrac{375}{\pi r^2}\right)$ ***A1***

$A=2\pi r^2+\dfrac{750}{r}$ ***AG***

***[2 marks]***

(f) $\left(\dfrac{\mathrm{d}A}{\mathrm{d}r}=\right)4\pi r-750r^{-2}$ ***A1A1A1***

**Note**: Award ***A1*** for $4\pi r$, ***A1*** for $-750$, ***A1*** for $r^{-2}$ or equivalent. Award at most ***A1A1A0*** for additional terms seen.

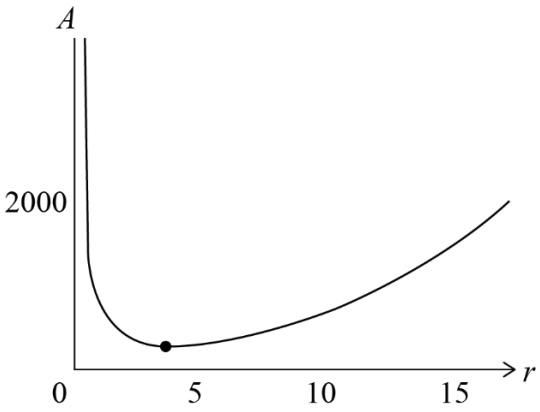
***[3 marks]***

*continued…*

*Question 3 continued.*

(g) (i) **METHOD 1**

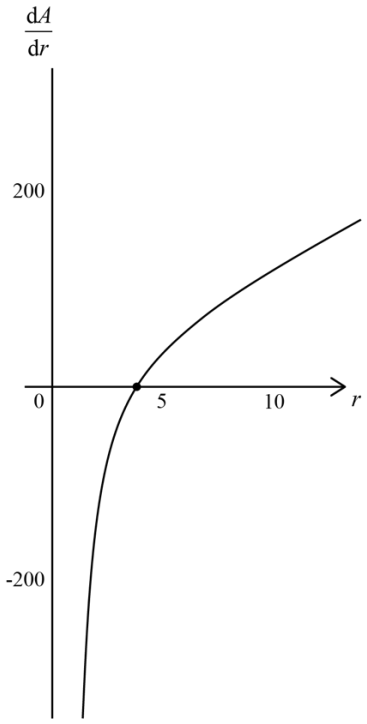
correct sketch with some indication of min point (need not be labelled) ***(M1)***




$(r =) 3.91 (\text{cm}) (3.90796...)$ ***A1***

**METHOD 2**

attempt to solve $4\pi r - \frac{750}{r^2} = 0$ , graphically or otherwise ***(M1)***




$(r =) 3.91 (\text{cm}) (3.90796...)$ ***A1***

(ii) $288 (\text{cm}^2)$ $(287.873...)$ ***A1***

***[3 marks]***

*continued…*

*Question 3 continued.*

(h) cuboid: $1.10 \times 350$ and cylinder: $1.25 \times 287.873...$ ***(M1)***

cuboid: $385$ and cylinder: $359.842...$ ***R1***
he should choose the cylinder ***A1***

**Note**: Do not award ***R0A1.***

***[3 marks]***

***Total [20 marks]***

**Q5.**

(a) attempt to substitute into trapezoidal formula ***(M1)***

(area of cross-section) $\approx \frac{1.9}{2}\left(0+0+2\left(1.68+2.81+2.32\right)\right)$ ***(A1)***

$= 12.9\ (\text{m}^2)(12.939)$ ***A1***

***[3 marks]***

(b) attempt to find the volume of water in one second, their (a) $\times 0.3$ ***(M1)***

$12.939 \times 0.3$

$= 3.88\left(3.8817\right)(\text{m}^3)$ ***A1***

**Note**: Accept $3.87$ for use of 3sf answer to part (a)$12.9$.

***[2 marks]***

(c) $\left(2.81-1.6=\right)1.21$ (m) ***A1***

***[1 mark]***

(d) attempt to find maximum eg graphically or $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ or using height at $x=3.8$ ***(M1)***

$3.07$ (m) OR $3.066$ (m) ***A1***

**Note**: Award ***M1A0*** for coordinate pair $(3.8, 3.066)$

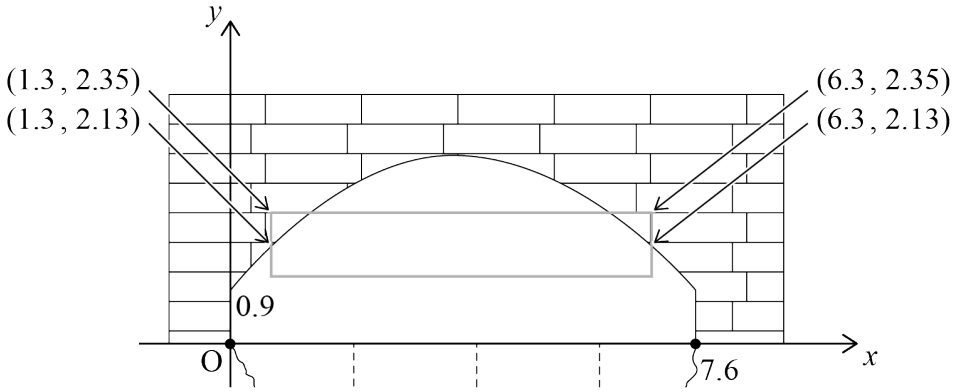
***[2 marks]***

*continued…*

*Question 5 continued.*

(e) **METHOD 1**

height of bridge < height of boat at
$x = 1.3$ or $x = 6.3$ (5 m wide boat)




use of $3.8 + 2.5$ or $3.8 - 2.5$ ***(M1)***

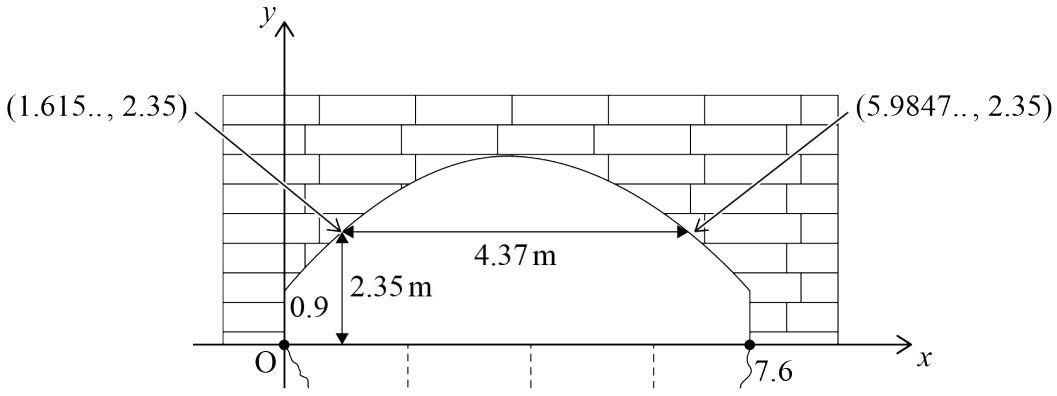
attempt to find the height of the bridge at either $1.3$ or $6.3$ m ***(M1)***

$2.13(\text{m})(2.1285)$ ***A1***

$(2.13 < 2.35)$ the boat cannot pass under the bridge ***A1***

**METHOD 2**

boat is wider $(5\,\text{m})$ than the width between
the arch at $h = 2.35$ m (height of boat)




attempt to find the values of $x$ $(1.61520\ldots$ and $5.98479\ldots)$ when the bridge is $2.35$ m above the water ***(M1)***

*continued…*

*Question 5 continued.*

subtract their two values to find the width of bridge at that point at this height ***(M1)***

$5.984795... - 1.615204...$ OR $3.8 - 1.615204\ldots$ OR $5.98479\ldots - 3.8$

$4.37(4.36959...)(\text{m})$ OR $2.18\ (2.18479\ldots)(\text{m})$ ***A1***

$(4.37 < 5)$ OR $(2.18 < 2.5)$ the boat cannot pass under the bridge ***A1***

**METHOD 3 (see the diagram for method 2)**

use of $3.8 + 2.5\ \ (6.3)$ or $3.8 - 2.5\ \ (1.3)$ ***(M1)***

attempt to find the values of $x$ when the bridge is $2.35$ m above the water ***(M1)***

$5.98479\ldots$ or $1.61520\ldots$ (m) ***A1***

$6.3 > 5.98\ldots(5.98479\ldots)$ or $1.3 <\ 1.62\ (1.61520\ldots)$ the boat cannot pass under the bridge ***A1***

**Note**: In all 3 methods the final ***A1*** is dependent on the previous ***A1.***

***[4 marks]***

***[Total 12 marks]***

**11.** (a) (i) $\left(B'(x) =\right)\ \frac{-32}{x^2} + 2$ **OR** $\left(B'(x) =\right)\ -32x^{-2} + 2$ ***A1(M1)A1***

**Note:** Award ***A1*** for $2$ seen, ***(M1)*** for expressing $\frac{1}{x}$ as $x^{-1}$ (this can be implied from, either $\frac{1}{x^2}$ or $x^{-2}$ seen in final answer), ***A1*** for $\frac{-32}{x^2}$. Award at most ***A1(M1)A0*** if additional terms are seen.

(ii) $B'(x) = 0$ **OR** $0 = \frac{-32}{x^2} + 2$ **OR** sketch of $B'(x)$ with root identified ***M1***

**Note:** It is specified that the student must use their derivative from part (i) to answer this question.

$x = 4$ ***(A1)***

**Note:** FT marks can only be awarded if *their* $x$-value lies within the given domain of $x \geq 1$.

$(B(x) =)36$ (minutes) ***A1***

***[6 marks]***

(b) $40\ > 20 + \frac{32}{k} + 2k$ **OR** $2$ and $8$ seen **OR** sketch of T(n) **and** T(n) = 40 ***(M1)***

$2 < k < 8$ **OR** $3 \leq k \leq 7$ **OR** $3, 4, 5, 6, 7$ ***A1***

***[2 marks]***

***Total [8 marks]***

**9.** (a) $(f(1) =)\quad 1^3 - 2(1)^2 + 5(1) - 7$ **OR** $1 - 2 + 5 - 7$ ***(A1)***
$-3$ ***A1***
***[2 marks]***

(b) $f'(x) = 3x^2 - 4x + 5$ ***M1A1***

**Note:** Award ***M1*** for an attempt at power rule (e.g. one term correct), ***A1*** for a completely correct answer.

***[2 marks]***

(c) $f'(1) = 4$ ***(A1)***
$y + 3 = 4(x - 1)$ **OR** $y = 4x - 7$ (or equivalent) ***A1***
***[2 marks]***
***Total [6 marks]***

**13.** $SA = 2(1.5w)w + 2(1.5w)h + 2wh$ **OR** $10000 = (1.5w)wh$ ***(M1)***

> **Note:** Award ***M1*** for an attempt to substitute $l = 1.5w$ into either surface area or volume.

correctly isolating $h$ from volume expression ***(A1)***

$h = \frac{20000}{3w^2}$ **OR** $h = \frac{10000}{1.5w^2}$

substituting their expression for $h$ into their expression for SA ***(M1)***

$SA = 2(1.5w)w + 2(1.5w)\frac{20000}{3w^2} + 2w\frac{20000}{3w^2}$

$(SA = 3w^2 + \frac{20\ 000}{w} + \frac{40\ 000}{3w}$ **OR** $SA = 3w^2 + \frac{100\ 000}{3w})$

**EITHER**

sketch of their graph with minimum indicated ***(M1)***

$(w =) 17.7\ (\text{cm})\ (17.7109\ldots(\text{cm}))$ ***A1***

**OR**

attempt to differentiate expression in terms of only $w$ **AND** setting to zero ***(M1)***

$0 = 6w - \frac{100\ 000}{3w^2}$

$(w =) 17.7\ (\text{cm})\ (17.7109\ldots(\text{cm}))$ ***A1***

***Total [5 marks]***

**2.** (a) (i) $0.9$ ***A1***

(ii) the rate of change of the height of the plant is $0.9$ cm per day
3 days after it is planted ***A1***

***[2 marks]***

(b) attempt to solve $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$ ( algebraically or graphically) ***(M1)***

$(t =)\ 12$ (days) ***A1***

***[2 marks]***

(c) recognition of the need to integrate ***(M1)***

$h = 1.2t - 0.05t^2 (+c)$ ***A1A1***

attempt to use given conditions $(2, 6.5)$ to find $c$ ***(M1)***

$c = 4.3$

$(h =) 1.2t - 0.05t^2 + 4.3$ ***A1***

***[5 marks]***

(d) **EITHER**

attempt to find maximum graphically ***(M1)***

**OR**

attempt to substitute their part (b) for $t$ ***(M1)***

**THEN**

$h = 11.5$ (cm) ***A1***

**Note:** If their answer to part (c) is linear or any other function that substantially changes the nature of the question, award at most ***M1A0***. Award at most ***M1A0*** if the $x$ value of their max point (FT from part (c)), contradicts their $x = 12$ found in part (b).

***[2 marks]***

(e) $2$ (cm) ***A1***

***[1 mark]***

(f) **EITHER**

attempt to write $1.2$ as a percentage or percentage increase ***(M1)***

**OR**

attempt to find two values of H and % increase ***(M1)***

**THEN**

$(p =)\ 20$ ***A1***

***[2 marks]***

(g) attempt to solve $H(t) = h(t)$ ***(M1)***

$t = 9.43$ (days) $(9.43483\ldots)$ ***A1***

***[2 marks]***

***Total [16 marks]***

**5.** (a) (i) $\int_0^{60} 0.001x^3 - 0.12x^2 + 4.05x + 3 \, \mathrm{d}x$ **OR** $\int_0^{60} f(x) \, \mathrm{d}x$ ***A1A1***

**Note:** Award ***A1*** for $\int_0^{60}$, ***A1*** for $0.001x^3 - 0.12x^2 + 4.05x + 3$ or "$f(x)$" **and** $\mathrm{d}x$.

(ii) $2070 \ (\mathrm{m}^2)$ ***A2***

***[4 marks]***

(b) (i) $x = 24.2 \ (24.1886\ldots)$, $y = 44.9 \ (44.9056\ldots)$ or (24.2, 44.9) ***A1A1***

**Note:** Award ***A1A0*** if parentheses are omitted.

(ii) attempt to use inverse tangent ***(M1)***

$\tan^{-1}\left(\frac{y}{x}\right)$

$61.7° \ (61.6907\ldots°)$ ***A1***

***[4 marks]***

(c) **METHOD 1**

attempt to use trapezoidal rule, for $h = 20$. ***(M1)***

$\frac{1}{2}(20)\big((3+30)+2(44+37)\big)$ ***(A1)***

$1950 \ (\mathrm{m}^2)$ ***A1***

**METHOD 2**

attempt to find area of three trapezoids, each with $h = 20$ ***(M1)***

correct expressions of values seen ***(A1)***

$\frac{1}{2}(3+44)20, \frac{1}{2}(44+37)20, \frac{1}{2}(37+30)20$ **OR** $470, 810, 670$

$1950 \ (\mathrm{m}^2)$ ***A1***

***[3 marks]***

(d) recognition of need to integrate (e.g. reverse power rule or integral symbol) ***(M1)***

$\int -0.00292a + 4.71 \,\mathrm{da} = -0.00146a^2 + 4.71a \,(+c)$ ***(A1)(A1)***

**Note:** Award ***A1*** for each correct term.

$1750 = -0.00146(500)^2 + 4.71(500) + c$ ***(M1)***

**Note:** Award ***M1*** for correct substitution of $500$ **and** $1750$ into their expression.
A constant of integration must be seen (can be implied by a correct $c$ value seen).

$c = -240$

$(C(a) =) -0.00146a^2 + 4.71a - 240$ ***(A1)***

***[5 marks]***

(e) $\left(C(1950) =\right) -0.00146(1950)^2 + 4.71(1950) - 240$ ***(A1)***

**Note:** Award ***A1*** for correctly substituting their part (c) into their $C(a)$ from part (d).

(\$) 3390 (3392.85) ***A1***

***[2 marks]***

***Total [18 marks]***