

APOLLO

Calculus

SL

Mathematics: applications and interpretation

First assessment 2021

54 questions | mark schemes

12. (a) (i) $A = \frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times p + 48$ **OR** $A = \frac{1}{2}(p+6)(q+8)$ **OR**
 $A = 3q + 4p + 48$ **A1**

(ii) valid attempt to link p and q , using tangents, similar triangles or other method **(M1)**

eg. $\tan \theta = \frac{8}{p}$ and $\tan \theta = \frac{q}{6}$ **OR** $\tan \theta = \frac{p}{8}$ and $\tan \theta = \frac{6}{q}$ **OR** $\frac{8}{p} = \frac{q}{6}$

correct equation linking p and q **A1**

eg. $pq = 48$ **OR** $p = \frac{48}{q}$ **OR** $q = \frac{48}{p}$

substitute $p = \frac{48}{q}$ into a correct area expression **M1**

eg. $(A =) \frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times \frac{48}{q} + 48$ **OR** $(A =) \frac{1}{2} \left(\frac{48}{q} + 6 \right) (q + 8)$

$$A = 3q + \frac{192}{q} + 48$$

AG

Note: The **AG** line must be seen with no incorrect, intermediate working, for the final **M1** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2} + 3$ **A1A1**

Note: Award **A1** for $\frac{-192}{q^2}$, **A1** for 3. Award **A1A0** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2} + 3 = 0$ **A1**

(ii) $q = 8$ cm **A1**

[2 marks]

Total [8 marks]

13. (a) $l'(50) = -0.2 \times 50 + 9$ (M1)

$= -1$ A1

the curve is decreasing at $\theta = 50^\circ$. A1

Note: For the final **A1**, follow through within this question part for their $l'(50)$ value.
Award **A0** for an answer of "decreasing" with no work shown.

[3 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol or integrating at least one term correctly) (M1)

$l(\theta) = -0.1\theta^2 + 9\theta (+c)$ A1A1

$205.5 = -0.1 \times (40)^2 + 9 \times (40) + c$ (M1)

Note: Award **M1** for correct substitution of $\theta = 40^\circ$ and $l = 205.5$. A constant of integration must be seen (can be implied by a correct answer).

$c = 5.5$

$(l(\theta) =) -0.1\theta^2 + 9\theta + 5.5$ A1

Note: Accept any variable in the working, but for the final **A1**, the variable θ must be used in the expression.

[5 marks]

Total [8 marks]

13. (a) recognition of need to integrate (eg reverse power rule or integral symbol) **(M1)**
 $P(x) = -0.8x^2 + 48x (+c)$ **A1A1**

$$260 = -0.8 \times (15)^2 + 48 \times (15) + c \quad \textbf{(M1)}$$

Note: Award **M1** for correct substitution of $x = 15$ and $P = 260$. A constant of integration must be seen (can be implied by a correct answer).

$$c = -280$$

$$P(x) = -0.8x^2 + 48x - 280 \quad \textbf{A1}$$

[5 marks]

- (b) profit will decrease (with each new car produced) **A1**

EITHER

because the profit function is decreasing / the gradient is negative / the rate of change of P is negative **R1**

OR

$$\int_{30}^{50} -1.6x + 48 \, (dx) = -320 \quad \textbf{R1}$$

OR

evidence of finding $P(30) = 440$ and $P(50) = 120$ **R1**

Note: Award at most **R1A0** if $P(30)$ or $P(50)$ or both have incorrect values.

[2 marks]

Total [7 marks]

5. (a) (i) evidence of power rule (at least one correct term seen) **(M1)**

$$\frac{dy}{dx} = -0.3x^2 + 1.6x \quad \mathbf{A1}$$

(ii) $-0.3x^2 + 1.6x = 0$ **M1**

$$x = 5.33 \left(5.33333..., \frac{16}{3} \right) \quad \mathbf{A1}$$

$$y = -0.1 \times 5.33333...^3 + 0.8 \times 5.33333...^2 \quad \mathbf{(M1)}$$

Note: Award **M1** for substituting their zero for $\frac{dy}{dx}$ (5.333...) into y .

$$7.59 \text{ m } (7.58519...) \quad \mathbf{A1}$$

Note: Award **M0A0M0A0** for an unsupported 7.59.
Award at most **M0A0M1A0** if only the last two lines in the solution are seen.
Award at most **M1A0M1A1** if their $x = 5.33$ is not seen.

[6 marks]

(b) $A = \frac{1}{2} \times 2 \left((2.4 + 0) + 2(6.4 + 7.2) \right)$ **(A1)(M1)**

Note: Award **A1** for $h = 2$ seen. Award **M1** for correct substitution into the trapezoidal rule (the zero can be omitted in working).

$$= 29.6 \text{ m}^2 \quad \mathbf{A1}$$

[3 marks]

(c) (i) $A = \int_2^8 -0.1x^3 + 0.8x^2 \, dx$ **OR** $A = \int_2^8 y \, dx$ **A1A1**

Note: Award **A1** for a correct integral, **A1** for correct limits in the correct location.
Award at most **A0A1** if dx is omitted.

(ii) $A = 32.4 \text{ m}^2$ **A2**

Note: As per the marking instructions, **FT** from their integral in part (c)(i).
Award at most **A1FTA0** if their area is >48 , this is outside the constraints of the question (a 6×8 rectangle).

[4 marks]

Total [13 marks]

5. (a) evidence of splitting diagram into equilateral triangles

M1

$$\text{area} = 6 \left(\frac{1}{2} x^2 \sin 60^\circ \right)$$

A1

$$= \frac{3\sqrt{3}x^2}{2}$$

AG

Note: The **AG** line must be seen for the final **A1** to be awarded.

[2 marks]

- (b) total surface area of prism $1200 = 2 \left(3x^2 \frac{\sqrt{3}}{2} \right) + 6xh$

M1A1

Note: Award **M1** for expressing total surface areas as a sum of areas of rectangles and hexagons, and **A1** for a correctly substituted formula, equated to 1200.

$$h = \frac{400 - \sqrt{3}x^2}{2x}$$

A1

$$\text{volume of prism} = \frac{3\sqrt{3}}{2} x^2 \times h$$

(M1)

$$= \frac{3\sqrt{3}}{2} x^2 \left(\frac{400 - \sqrt{3}x^2}{2x} \right)$$

A1

$$= 300\sqrt{3}x - \frac{9}{4}x^3$$

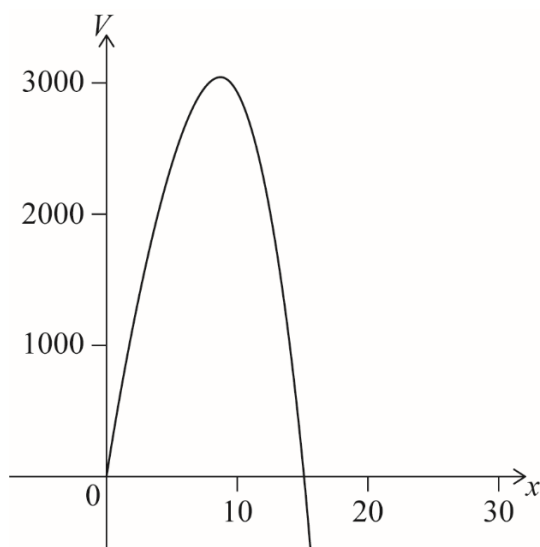
AG

Note: The **AG** line must be seen for the final **A1** to be awarded.

[5 marks]*continued...*

Question 5 continued

(c)

**A1A1**

Note: Award **A1** for correct shape, **A1** for roots in correct place with some indication of scale (indicated by a labelled point).

[2 marks]

(d) $\frac{dV}{dx} = 300\sqrt{3} - \frac{27}{4}x^2$

A1A1

Note: Award **A1** for a correct term.

[2 marks]

(e) from the graph of V or $\frac{dV}{dx}$ **OR** solving $\frac{dV}{dx} = 0$
 $x = 8.77$ (8.77382...)

(M1)**A1****[2 marks]***continued...*

Question 5 continued

- (f) from the graph of V **OR** substituting their value for x into V **(M1)**
 $V_{\max} = 3040 \text{ cm}^3$ (3039.34...) **A1**
[2 marks]
- (g) **EITHER**
wasted space / spheres do not pack densely (tessellate) **A1**
OR
the model uses exterior values / assumes infinite thinness of materials and
hence the modelled volume is not the true volume **A1**
[1 mark]
- Total [16 marks]**
-

6. (a) attempt at using trapezoidal rule formula **(M1)**

$$\frac{1}{2} \left(\frac{2-0}{5} \right) (30 + 50 + 2(50 + 60 + 40 + 20))$$
(A1)

(total carbon =) 84 tonnes

A1

[3 marks]

(b) $\left| \frac{84 - 72}{72} \right| \times 100\%$ **(M1)**

Note: Award **(M1)** for correct substitution of final answer in part (a) into percentage error formula.

$$= 16.7\% \text{ (16.6666...%)}$$

A1

[2 marks]

Total: [5 marks]

12. (a) $(S(x) =) x^2 + 128x^{-1}$ **(M1)**

Note: Award **(M1)** for expressing second term with a negative power. This may be implied by $\frac{1}{x^2}$ seen as part of their answer.

$2x - \frac{128}{x^2}$ **OR** $2x - 128x^{-2}$ **A1A1**

Note: Award **A1** for $2x$ and **A1** for $-\frac{128}{x^2}$. The first **A1** is for x^2 differentiated correctly and is independent of the **(M1)**.

[3 marks]

(b) (i) **EITHER**

any correct manipulation of $2x - \frac{128}{x^2} = 0$ e.g. $2x^3 - 128 = 0$ **(M1)**

OR

sketch of graph of $S'(x)$ with root indicated **(M1)**

OR

sketch of graph of $S(x)$ with minimum indicated **(M1)**

THEN

$x = 4$ **A1**

Note: Value must be positive. Follow through from their part (a) irrespective of working.

(ii) the value of x that will minimize surface area of the box **A1**

Note: Accept 'optimize' in place of minimize.

[3 marks]

Total: [6 marks]

10. (a) $A = \int_0^2 (6-3x)(4+x)dx$

A1A1

Note: Award **A1** for the limits $x = 0$, $x = 2$. Award **A1** for an integral of $f(x)$.

[2 marks]

(b) 28

A1**[1 mark]**

(c) $28 = 0.5 \times a \times 10$

M1

$$5.6 \left(\frac{28}{5} \right)$$

A1**[2 marks]****Total [5 marks]**

4. (a) $2(8 \times 4 + 3 \times 4 + 3 \times 8)$ **M1**
 $= 136 \text{ (cm}^2\text{)}$ **A1**
[2 marks]

(b) $\sqrt{8^2 + 4^2 + 3^2}$ **M1**
 (AG =) 9.43 (cm) (9.4339..., $\sqrt{89}$) **A1**
[2 marks]

(c) $-2x + 220 = 0$ **M1**
 $x = 110$ **A1**
 110 000 (boxes) **A1**
[3 marks]

(d) $P(x) = \int -2x + 220 \, dx$ **M1**

Note: Award **M1** for evidence of integration.

$P(x) = -x^2 + 220x + c$ **A1A1**

Note: Award **A1** for either $-x^2$ or $220x$ award **A1** for both correct terms and constant of integration.

$1700 = -(20)^2 + 220(20) + c$ **M1**
 $c = -2300$
 $P(x) = -x^2 + 220x - 2300$ **A1**
[5 marks]

(e) $-x^2 + 220x - 2300 = 0$ **M1**
 $x = 11.005$ **A1**
 11 006 (boxes) **A1**

Note: Award **M1** for their $P(x) = 0$, award **A1** for their correct solution to x . Award the final **A1** for expressing their solution to the minimum number of boxes. Do not accept 11 005, the nearest integer, nor 11 000, the answer expressed to 3 significant figures, as these will not satisfy the demand of the question.

[3 marks]

Total [15 marks]

6. (a) $\frac{1}{2}(0.6 + 0 + 2(1.2 + 1.2))$ (A1)(M1)

Note: Award **A1** for evidence of $h = 1$, **M1** for a correct substitution into trapezoidal rule (allow for an incorrect h only). The zero can be omitted in the working.

2.7 m^2 **A1**
[3 marks]

(b) $\int_{-1}^2 \frac{-x^3 - 3x^2 + 4x + 12}{10} \text{ dx OR } \int_{-1}^2 f(x) \text{ dx}$ (M1)

Note: Award **M1** for using definite integration with correct limits.

2.925 m^2 **A1**

Note: Question requires exact answer, do not award final **A1** for 2.93.

[2 marks]

(c) $9 - 2.925$ (M1)

Note: Award **M1** for 9 seen as part of a subtraction.

$= 6.08 \text{ m}^2 \text{ (6.075)}$ **A1**
[2 marks]
[Total 7 marks]

9. (a) $f'(x) = -2x^{-2} + 6x$ OR $f'(x) = -\frac{2}{x^2} + 6x$ **A1(M1)A1**

Note: Award **A1** for $6x$ seen, and **(M1)** for expressing $\frac{1}{x}$ as x^{-1} (this can be implied from either x^{-2} or $\frac{2}{x^2}$ seen in their final answer), **A1** for $-\frac{2}{x^2}$. Award at most **A1(M1)A0** if any additional terms are seen.

[3 marks]

(b) finding gradient at $x = 1$

$$\left. \frac{dy}{dx} \right|_{x=1} = 4$$

A1

finding the perpendicular gradient

M1

$$m_{\perp} = -\frac{1}{4}$$

$$2 = -\frac{1}{4}(1) + c \quad \text{OR} \quad y - 2 = -\frac{1}{4}(x - 1)$$

M1

Note: Award **M1** for correctly substituting $x = 1$ and $y = 2$ and their $m_{\perp}$.

$$x + 4y - 9 = 0$$

A1

Note: Do not award the final **A1** if the answer is not in the required form. Accept integer multiples of the equation.

[4 marks]

[Total 7 marks]

12. (a) METHOD 1(when $t = 2$)

$$\frac{dP}{dt} = -4 \quad \text{OR} \quad \frac{dP}{dt} < 0 \text{ (equivalent in words)} \quad \text{OR} \quad 3(2)^2 - 8(2) = -4$$

M1therefore P is decreasing**A1****METHOD 2**sketch with $t = 2$ indicated in 4th quadrant **OR** t -intercepts identified**M1**therefore P is decreasing**A1****[2 marks]**

(b) $(P(t) = t^3 - 4t^2 + c)$

A1A1

$$4 = 1^3 - 4(1)^2 + c$$

(M1)**Note:** Award **M1** for substituting (1, 4) into their equation with $+c$ seen.

$$c = 7$$

$$P(t) = t^3 - 4t^2 + 7$$

A1**[4 marks]****[Total 6 marks]**

6. (a) **EITHER**
 attempt to substitute 3, 4 and 7 into area of a trapezoid formula (M1)
 $(A =) \frac{1}{2}(7+4)(3)$

OR
 given line expressed as an integral (M1)
 $(A =) \int_{-1}^2 (6-x) \, dx$

OR
 attempt to sum area of rectangle and area of triangle (M1)
 $(A =) 4 \times 3 + \frac{1}{2}(3)(3)$

THEN
 16.5 (square units) A1
 [2 marks]

- (b) (i) $(A =) \int_{-1}^2 1.5x^2 - 2.5x + 3 \, dx$ A1A1

Note: Award **A1** for the limits $x = -1$, $x = 2$ in correct location. Award **A1** for an integral of the quadratic function, dx must be included. Do not accept “y” in place of the function, given that two equations are in the question.

(ii) 9.75 (square units) A1
 [3 marks]

- (c) 16.5 – 9.75 (M1)
 6.75 (square units) A1
 [2 marks]
Total [7 marks]

11. (a) $(f'(x) =) 2x + \frac{3}{x^2}$

A1A1

Note: Award **A1** for $2x$, **A1** for $+\frac{3}{x^2}$ **OR** $+3x^{-2}$.

[2 marks]

(b) attempt to substitute 1 into their part (a)

(M1)

$$(f'(1) =) 2(1) + \frac{3}{1^2}$$

5

A1**[2 marks]**

(c) **EITHER**

$$5 = 2x + \frac{3}{x^2}$$

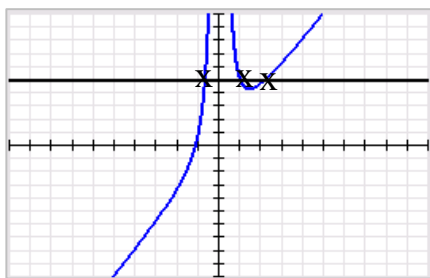
M1

$$x = -0.686, 1, 2.19 \quad (-0.686140\dots, 1, 2.18614\dots)$$

A1

OR

sketch of $y = f'(x)$ with line $y = 5$

M1

three points of intersection marked on this graph
(and it can be assumed no further intersections occur outside of this window)

A1

THEN

there are two other tangent lines to $f(x)$ that are parallel to L

A1

Note: The final **A1** can be awarded provided two solutions other than $x = 1$ are shown **OR** three points of intersection are marked on the graph.

Award **M1A1A1** for an answer of “3 lines” where L is considered to be parallel with itself (given guide definition of parallel lines), but only if working is shown.

[3 marks]**Total [7 marks]**

5. (a) attempt to expand given expression

(M1)

$$C = \frac{xk^2}{10} - \frac{3x^3}{1000}$$

$$\frac{dC}{dx} = \frac{k^2}{10} - \frac{9x^2}{1000}$$

M1A1

Note: Award **M1** for power rule correctly applied to at least one term and **A1** for correct answer.

[3 marks]

- (b) equating their $\frac{dC}{dx}$ to zero

(M1)

$$\frac{k^2}{10} - \frac{9x^2}{1000} = 0$$

$$x^2 = \frac{100k^2}{9}$$

$$x = \frac{10k}{3}$$

(A1)

substituting their x back into given expression

(M1)

$$C_{\max} = \frac{10k}{30} \left(k^2 - \frac{300k^2}{900} \right)$$

$$C_{\max} = \frac{2k^3}{9} (0.222\dots k^3)$$

A1

[4 marks]

- (c) (i) substituting 20 into given expression and equating to 426

M1

$$426 = \frac{20}{10} \left(k^2 - \frac{3}{100} (20)^2 \right)$$

$$k = 15$$

A1

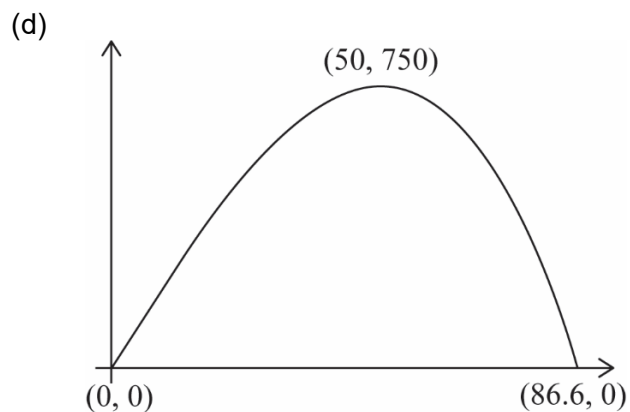
- (ii) 50

A1

[3 marks]

continued...

Question 5 continued

**A1A1A1**

Note: Award **A1** for graph drawn for positive x indicating an increasing and then decreasing function, **A1** for maximum labelled and **A1** for graph passing through the origin and 86.6, marked on the x -axis or whose coordinates are given.

[3 marks]

- (e) setting their expression for C to zero **OR** choosing correct x -intercept on their graph of C

(M1)

$$x_{\max} = 86.6 \text{ (86.6025...)} \text{ litres}$$

A1**[2 marks]****Total [15 marks]**

13. (a) $0 = 20 - \frac{980}{t^2}$ OR $\frac{dP}{dt} = 0$ (M1)

Note: Accept equivalent information presented in a labelled sketch.

$(h =) 7$ hours A1

Note: Award **M1A0** for an answer of (7, 280).

[2 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol) (M1)

$P(t) = 20t + \frac{980}{t} (+c)$ A1A1

$328 = 20 \times 5 + \frac{980}{5} + c$ (M1)

Note: Award (M1) for substitution of $P = 328$ and $t = 5$ into their $P(t)$. A constant of integration must be seen (can be implied by a correct answer).

$c = 32$ A1

$P(7) = 20 \times 7 + \frac{980}{7} + 32$ M1

Note: Award **M1** for substituting 7 and their 32 into their $P(t)$.
Do not award the final **M** mark if their substituted values do not lead to 312.

312 NOK AG

[6 marks]

Total [8 marks]

5. (a) (i) $f'(x) = \frac{-2x}{50} + 2 \left(= \frac{-x}{25} + 2, -0.04x + 2 \right)$ **A1A1**

Note: Award **A1** for each correct term. Award at most **A0A1** if extra terms are seen.

(ii) $0 = \frac{-x}{25} + 2$ **OR** sketch of $f'(x)$ with x -intercept indicated **M1**
 $x = 50$ **A1**
 $y = 80$ **A1**
 $(50, 80)$

Note: Award **M0A0A1** for the coordinate $(50, 80)$ seen either with no working or found from a graph of $f(x)$.

[5 marks]

(b) (i) $\int_0^{70} \frac{-x^2}{50} + 2x + 30 \, dx$ **A1A1**

Note: Award **A1** for a correct integral, **A1** for correct limits in the correct location. Award at most **A0A1** if dx is omitted.

(ii) (Area =) $4710 \, \text{m}^2 \left(4713.33..., \frac{14140}{3} \right)$ **A2**

[4 marks]

(c) (i) $\frac{11.4}{4713.33...} \times 100\%$ **OR** $\left| \frac{4701.93... - 4713.33...}{4713.33...} \right| \times 100\%$ **(M1)**

Note: Award **(M1)** for their correct substitution into the percentage error formula.

$0.242\% \, (0.241867...\%)$ **A1**

Note: Percentage sign is required. Accept $0.242038...\%$ if 4710 is used.

(ii) **EITHER**
 reduce the width of the intervals (trapezoids) **A1**
OR
 increase the number of intervals (trapezoids) **A1**

Note: Accept equivalent statements. Award **A0** for the ambiguous answer "increase the intervals".

[3 marks]

continued...

Question 5 continued

- (d) (i) width of the square is $70 - x$ **OR** the length of the square is $\frac{-x^2}{50} + 2x + 30$

(M1)

Note: Award **(M1)** for $70 - x$ seen anywhere. Accept $\frac{-x^2}{50} + 2x + 30$ but only if this expression is explicitly identified as a dimension of the square.

in term of x , equating the length to the width ED

(M1)

$$\frac{-x^2}{50} + 2x + 30 = 70 - x$$

$$(x = 14.7920... \text{ or } 135.21)$$

$$(x =) 14.8 \text{ m } (14.7920...)$$

A1

Note: Award **M0M0A0** for an unsupported answer of 15. Award at most **M1M0A0** for an approach which leads to $A'(x) = 0$. This will lead to a square base which extends beyond the east boundary of the property. Similar for any solution where F is not on the northern boundary, or GH is not on the east boundary.

- (ii) **EITHER**

$$(70 - 14.7920...)^2$$

(M1)**OR**

$$(55.2079...)^2$$

(M1)**OR**

$$\left(\frac{-(14.7920...)^2}{50} + 2(14.7920...) + 30 \right)^2$$

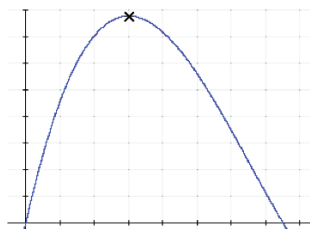
(M1)**THEN**

$$(\text{Area} =) 3050 \text{ m}^2 \text{ (3047.92...)}$$

A1

Note: Follow through from part (d)(i), provided x is between 0 and 70. Award at most **M1A0** if their answer is outside the range of their $[0, 4713.33...]$ from part (b).

[5 marks]**Total [17 marks]**

7. (a) ($k =$) 15**A1****[1 mark]**(b) **EITHER**attempt to sketch the function $V(x)$ with indication of maximum**(M1)****OR**

recognition of setting the derivative to 0

(M1)e.g. $V'(x) = 0$ **THEN**($x =$) 6 (cm)**A1**

Note: Award **(M1)A0** for the maximum given as a coordinate pair.

[2 marks]

(c) 44 or 26 seen

(A1)

attempting to adjust the constant(s) in the given volume formula

(M1)volume of second box = $(44 - 2x)(26 - 2x)(x)$ (New maximum volume $\Rightarrow$) 2730 cm^3 ($2726.13... \text{ cm}^3$)**A1**

Note: Units must be seen to award the final **A1**. Award **(A1)(M1)A0** for the maximum given as a coordinate pair.

[3 marks]**[Total: 6 marks]**

9. (a) attempt to substitute $h = 10$ and at least two different values of y into the trapezoidal rule (M1)

$$\frac{10}{2}((0+0) + 2(3+8+9))$$

$$= 200 \text{ (cm}^2\text{)}$$

A1**[2 marks]**

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \, dx$ OR $\int_0^{40} y \, dx$

A1A1

Note: Award **A1** for a correct integral (including dx), **A1** for correct limits in the correct location.

(ii) $213.33 \text{ (cm}^2\text{)}$

A2

Note: Answer must be given to 2 decimal places to award **A2**. Award **A1A0** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213 \text{ (cm}^2\text{)}$.

[4 marks]

- (c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula (M1)

$$\left| \frac{213.333... - 200}{213.333...} \right| \times 100$$

$$= 6.25(\%) \text{ (6.24999...(\%))}$$

A1

Note: Award **(M1)A0** for a final answer of $-6.25(\%)$ or 0.0625 .

[2 marks]**[Total: 8 marks]**

13. (a) attempt at using the trapezoidal rule (M1)

$$\text{area} = \frac{1}{2}(3 + 2(8 + 19) + 42)$$

$$= 49.5 \text{ (m}^2\text{)}$$

A1

[2 marks]

- (b) recognition of need to integrate (e.g. reverse power rule or integral symbol) (M1)

$$\int 3x^2 + 4 \, dx = x^3 + 4x + c$$

(A1)(A1)

Note: Award **A1** for each correct term.

$$f(x) = x^3 + 4x + 3$$

A1

Note: Award **A1** for simplified correct answer including the value of c . Accept a value of c of 3.005 or 3.025 or 2.975 for using the non-integer x -values and their corresponding y -values.

[4 marks]

- (c) **METHOD 1**

forming expression for sum of integral and deconstructing the trapezoid into a rectangle and triangle

(M1)

$$\int_0^3 x^3 + 4x + 3 \, dx (= 47.25) + 42 \times 1 + \frac{1}{2} \times 2 \times 42 (= 84)$$

(A1)

$$= 131 \text{ (m}^2\text{)} \text{ (131.25)}$$

A1

METHOD 2

forming expression for sum of integral and trapezoid

(M1)

$$\int_0^3 x^3 + 4x + 3 \, dx (= 47.25) + \frac{1}{2} \times 4 \times 42 (= 84)$$

(A1)

$$= 131 \text{ (m}^2\text{)} \text{ (131.25)}$$

A1

Note: Award **(A1)** for their integral with the correct limits added to 84 or their 47.25 added to 84.

[3 marks]

Total [9 marks]

5. (a) equating a volume of a half cylinder (or cylinder) to 0.8 **(M1)**

$$0.8 = \frac{1}{2} \pi r^2 l$$

$$l = \frac{1.6}{\pi r^2} \quad \textbf{A1}$$

Note: Do not accept decimal approximation of π for the **A1** given the demand of question.

Condone the use of h for l for the **M1**

[2 marks]

- (b) calculating area in terms of r and l **M1**

$$C = 2lr + \pi r^2 + \pi r l$$

area with l replaced by $\frac{1.6}{\pi r^2}$ **M1**

apply costs to correct part of each surface **M1**

a correct substitution into an expression for C , leading to given answer

A1

$$\text{e.g. } (C =) 4.40 \times \pi r \left(\frac{1.6}{\pi r^2} \right) + 4.40 \times 2r \left(\frac{1.6}{\pi r^2} \right) + p \times \pi r^2$$

$$(C =) 7.04r^{-1} + \frac{14.08}{\pi} r^{-1} + p\pi r^2 \quad \textbf{AG}$$

Note: The **AG** line must be seen to award the final **A1**.

No incorrect working should be seen after the correct substitution

[4 marks]

continued...

Question 5 continued

(c) EITHER

$$\left(\frac{dC}{dr} =\right) -7.04r^{-2} - \frac{14.08}{\pi}r^{-2} + 2p\pi r$$

A1A1A1

OR

$$-7.04r^{-2} - 4.48r^{-2} + 6.28pr \quad \left(-7.04r^{-2} - (4.48180...)r^{-2} + 6.28318...pr\right)$$

A1A1A1

OR

$$-11.5r^{-2} + 6.28pr \quad \left((-11.5218...)r^{-2} + 6.28318...pr\right)$$

A2A1**Note:** Award **A1** for each correct term.Award at most **A1A1A0** if extra terms are seen.**[3 marks]**

continued...

Question 5 continued

(d) recognition of setting $\frac{dC}{dr}$ to zero (M1)

attempt to substitute 0.7 in for r in their derivative (M1)

$$0 = -7.04(0.7)^{-2} - \frac{14.08}{\pi} \times (0.7)^{-2} + 2p\pi \times 0.7$$

($p =$) (\$5.35 (per square metre) ((\$ 5.34621...)) A1

Note: Accept \$5.34, as this will also lead to a radius of 0.7 (to 3sf).

[3 marks]

(e) attempt to calculate the cost of one container (M1)

$$(C =) 7.04(0.7)^{-1} + \frac{14.08}{\pi}(0.7)^{-1} + 5.34621...\pi \times 0.7^2 \quad (A1)$$

Note: May be shown within a calculation of the cost of all containers.

$$(C =) 24.6895...$$

$$24.6895... \times 350$$

= (\$ 8641 A1

Note: Answer must be rounded to the nearest dollar to award the final A1.

Accept answers between 8641 and 8645 (inclusive), due to rounding the value of p and/or the cost of one container to the nearest cent.

Award (M1)(A1)A0 for an answer rounded to 3sf (e.g. (\$8640) or to 2dp (e.g., (\$8641.35).

Accept an answer of (\$8638 from use of \$5.34 in their cost calculation.

[3 marks]

continued...

Question 5 continued

(f) attempt to apply a discount of 8% to their part (e) **(M1)**

Note: the discount percentage will depend on their answer to part (e)
e.g. $8641.35... \times 0.92$ **OR** $8641.35... \times 0.08$

(\$)7950 ((\$)7950.04...) **A1**

[2 marks]

[Total: 17 marks]

5. (a) (i) $x - 3$ **A1**

(ii) attempt to use 1200 to find width of park in terms of only x **(M1)**

$$\frac{1200}{x} \text{ (seen) OR } 1200 = x \times \text{park width OR } 1200 = x \times (\text{garden width} + 4)$$

$$\frac{1200}{x} - 4$$
 A1

(iii) $A = (x - 3) \times \left(\frac{1200}{x} - 4 \right)$ **A1**

$$= 1200 - 4x - \frac{3600}{x} + 12$$
 A1

Note: Award first **A1FT** for multiplying *their* garden length and width and second **A1** for a simplified (parentheses removed) expression for A that leads to the given answer. The given answer must be shown for the second **A1** mark to be awarded

$$= 1212 - 4x - \frac{3600}{x}$$
 AG

[5 marks]

(b) setting $1212 - 4x - \frac{3600}{x} = 800$ (accept a sketch) **(M1)**

$$x = 9.64 \text{ (9.64011...)} \text{ (m) OR } x = 93.4 \text{ (93.3598...)} \text{ (m)}$$
 A1

$$\text{(width =) } 124 \text{ (124.479...)} \text{ (m)}$$
 A1

$$\text{(width =) } 12.9 \text{ (12.8534...)} \text{ (m)}$$
 A1

Note: To award the final **A1** both values of x **and** both values of the width must be seen. Accept 12.8 for second value of width from candidate dividing 1200 by 3 sf value of 93.4.

[4 marks]

(c) $\left(\frac{dA}{dx} = \right) -4 + \frac{3600}{x^2}$ **OR** $-4 + 3600x^{-2}$ **A1A1A1**

Note: Award **A1** for -4 , **A1** for $+3600$, and **A1** for x^{-2} or x^2 in denominator.

[3 marks]

(d) setting *their* $\frac{dA}{dx}$ equal to 0 **OR** sketch of *their* $\frac{dA}{dx}$ with x -intercept highlighted **M1**

$$(x =) 30 \text{ (m)}$$
 A1

Note: To award **A1FT** the candidate's value of x must be within the domain given in the problem ($3 < x < 300$).

[2 marks]

continued...

Question 5 continued(e) **EITHER**evidence of using GDC to find maximum of graph of $A = 1212 - 4x - \frac{3600}{x}$ **(M1)****OR**substitution of *their* x into A **(M1)****OR**dividing 1200 by *their* x to find width of park **and** subtracting 3 from *their* x and 4 from the width to find park dimensions **(M1)****Note:** For the last two methods, only follow through if $3 < \text{their } x < 300$.**THEN** $(A =) 972 \text{ (m}^2\text{)}$ **A1****[2 marks]****Total [16 marks]**

3. (a) $x = 0$ **A1**

Note: Answer must be an equation; an answer of “0” or “the y-axis” is awarded **A0**.

[1 mark]

- (b) $(f'(x) =) -16x^{-2} + \frac{x}{4}$ **A1A1A1**

Note: Award **A1** for -16 seen, **A1** for x^{-2} (or $\frac{1}{x^2}$) and **A1** for second term being $\frac{x}{4}$.
Award at most **A1A1A0** if additional terms are seen.

[3 marks]

- (c) $x > 4$ **OR** $(4, \infty)$ **OR** $4 < x < \infty$ **A1A1**

Note: Award **A1** for 4 seen and award **A1** for **correct** inequality.

[2 marks]

[Total 6 marks]

8. (a) $(r'(-2) =) -14$ **A2**
[2 marks]
- (b) $\frac{1}{14}$ seen **(A1)**
 $y - 16 = \frac{1}{14}(x + 2)$ **OR** $y = 0.0714x + 16.1$ ($y = 0.0714285\dots x + 16.1428\dots$) **OR**
 $y = \frac{x}{14} + \frac{113}{7}$ **OR** $x - 14y + 226 = 0$ **A1**
[2 marks]
- (c) attempt to find coordinates of point G **(M1)**
e.g. $(-7.53997\dots, 15.6042\dots)$ **OR** $(0.539978\dots, 13.1814\dots)$ seen
attempt to substitute into distance formula **(M1)**

$$\left(d = \sqrt{(-2 + 7.53997\dots)^2 + (16 - 15.6042\dots)^2} \right)$$

 $(d =) 5.55(\text{km})$ ($5.55409\dots$) **A1**
[3 marks]
[Total 7 marks]

3. (a) $x = 0$ **A1**

Note: Answer must be an equation; an answer of “0” or “the y -axis” is awarded **A0**.

[1 mark]

- (b) $(g'(x) =) -8x^{-2} + x$ **A1A1A1**

Note: Award **A1** for -8 seen, **A1** for x^{-2} (or $\frac{1}{x^2}$) and **A1** for second term being x .
Award at most **A1A1A0** if additional terms are seen.

- (c) $x > 2$ **OR** $(2, \infty)$ **OR** $2 < x < \infty$ **A1A1**

Note: Award **A1** for 2 seen and award **A1** for **correct** inequality.

[2 marks]

[Total 6 marks]

8. (a) $(r'(-1) =) -8$

A2

[2 marks]

(b) $\frac{1}{8}$ seen

(A1)

$y - 6 = \frac{1}{8}(x + 1)$ **OR** $y = 0.125x + 6.13$ (6.125) **OR**

$y = \frac{x}{8} + \frac{49}{8}$ **OR** $x - 8y + 49 = 0$

A1

[2 marks]

(c) attempt to find coordinates of point B
e.g. $(-5.02, 5.50)$ **OR** $(1.02, 6.25)$ seen

(M1)

attempt to substitute into distance formula

(M1)

$\left(d = \sqrt{(-1 + 5.02076\dots)^2 + (6 - 5.49740\dots)^2} \right)$

$(d =) 4.05 \text{ (km)} \quad (4.05205\dots)$

A1

[3 marks]

[Total 7 marks]

1. (a) $(a =) 9$ **A1**
 $(b =) 7.63$ (7.62741...) **A1**
 $(c =) 4.71$ (4.71281...) **A1**
[3 marks]

- (b) attempt to use the trapezoidal rule **(M1)**
 $(\text{area} =) \frac{1}{2}(16)((1+1) + 2(9 + 7.62741... + 4.71281...))$ **(A1)**
 $(\text{area} =) 357(\text{mm}^2)$ (357.443...) **A1**
[3 marks]

- (c) (i) $\int_0^{64} \left(4\sqrt{x} - \frac{x}{2} + 1 \right) dx$ **A1A1**

Note: Award **A1** for correct function seen within the integral and **A1** for correct limits in the correct location and the inclusion of the dx .

- (ii) $405.3(\text{mm}^2)$ **A2**
[4 marks]

- (d) attempt to substitute **their** area values into the percentage error formula **(M1)**
 $\left| \frac{357.443... - 405.3}{405.3} \right| \times 100$
 $11.8(\%)$ (11.8076...) **A1**

Note: Accept an answer of 11.9 from use of 357 from part (b).

[2 marks]
[Total 12 marks]

5. (a) attempt to substitute 16 into the given formula **(M1)**

$$n = 20\,000 - 1\,000(16)$$

$$n = 4\,000$$

A1**[2 marks]**

- (b) multiplying their answer to part (a) by 16
(average monthly income =) $16 \times 4\,000$
64 000 (EUR)

(M1)**A1****[2 marks]**

- (c) $R(x) = x(20\,000 - 1\,000x)$ **OR** $R(x) = 20\,000x - 1\,000x^2$

A1**[1 mark]**

- (d) **EITHER**

attempt to find total costs (both fixed and variable) **AND** subtract from their 64 000

(M1)

$$64\,000 - (10\,000 + 10 \times 4\,000)$$

(A1)**OR**

attempt to find total profit from cases **AND** subtract fixed costs

(M1)

$$(16 - 10) \times 4\,000 - 10\,000$$

(A1)**THEN**

$$= 14\,000 \text{ (EUR)}$$

A1**[3 marks]**

– 14 –

8823 – 7210M

(e) **METHOD 1**attempt to subtract total costs in terms of x from their $R(x)$ **(M1)**

$$(P(x) =)(20000x - 1000x^2) - (10000 + 10(20000 - 1000x))$$

correct intermediate step leading to given answer
(e.g. correct expansion of $10(20000 - 1000x)$)

A1

$$P(x) = -1000x^2 + 30000x - 210000$$

AG

Note: Do not award the **A1** mark if the **AG** line is not stated.

METHOD 2

attempt to express profit per case, and then subtract fixed monthly costs

(M1)

$$(P(x) =)(x - 10)(20000 - 1000x) - 10000$$

correct expansion leading to the given answer

A1

$$P(x) = -1000x^2 + 30000x - 210000$$

AG

Note: Do not award the **A1** mark if the **AG** line is not stated.

[2 marks]

(f) (i) $(P'(x) =) -2000x + 30000$

A1A1

Note: Award at most **A1A0** if additional terms are seen.

(ii) **METHOD 1**

$$P'(x) = 0 \quad \text{OR} \quad \text{sketch of } P(x) \quad \text{OR} \quad \text{use of } x = -\frac{b}{2a}$$

 $x = 15$ is the maximum, not 16**A1R1**

hence salesman's price is not the optimum

AG

Note: Award **A1** for $x = 15$, and **R1** for comparing it to 16 **OR** making a statement that is some version of the **AG** line. It is possible to award **A1R0**.

METHOD 2

$$P'(16) = -2000 \neq 0$$

A1R1

hence salesman's price is not the optimum

AG

Note: Award **A1** for finding an appropriate value, and **R1** for comparing it to zero. It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.

METHOD 3finding $P(x)$ for any value from $14 < x < 16$ **A1**

comparing this value to their part (d)

R1

hence salesman's price is not the optimum

AG

Note: It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.

[4 marks]

– 15 –

8823 – 7210M

(g) substituting the expression for n into cost function, $C(n)$. **(M1)**

$$(\text{cost} =) 10 - 0.0001(20\,000 - 1000x) = 8 + 0.1x$$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ **(M1)**

$$(\text{New } P(x) =) (20\,000x - 1000x^2) - (10\,000 + (8 + 0.1x)(20\,000 - 1000x)) \quad \mathbf{A1}$$

$$((\text{New } P(x) =) -900x^2 + 26\,000x - 170\,000)$$

[3 marks]

(h) 14.4 (EUR) $(14.4444\dots, \frac{130}{9})$ **A2**

[2 marks]**[Total 19 marks]**

1. (a) $(a =) 6$ **A1**
 $(b =) 5.14$ (5.14213...) **A1**
 $(c =) 3.32$ (3.32050...) **A1**
[3 marks]

- (b) attempt to use the trapezoidal rule **(M1)**
 $(\text{area} =) \frac{1}{2}(25)((1+1) + 2(6 + 5.14213... + 3.32050...))$ **(A1)**
 $(\text{area} =) 387(\text{cm}^2)$ (386.566...) **A1**
[3 marks]

- (c) (i) $\int_0^{100} \left(2\sqrt{x} - \frac{x}{5} + 1 \right) dx$ **A1A1**

Note: Award **A1** for correct function seen within the integral and **A1** for correct limits in the correct location and the inclusion of the dx .

- (ii) $433.3(\text{cm}^2)$ **A2**
[4 marks]

- (d) attempt to substitute **their** area values into the percentage error formula **(M1)**
 $\left| \frac{386.566... - 433.3}{433.3} \right| \times 100$
 $10.8(\%)$ (10.7855...) **A1**

Note: Accept an answer of 10.7 from use of 387 from part (b).

[2 marks]
[Total 12 marks]

5. (a) attempt to substitute 16 into the given formula (M1)

$$n = 20000 - 1000(16)$$

$$n = 4000$$

A1

[2 marks]

- (b) multiplying their answer to part (a) by 16 (M1)

(average monthly income $\Rightarrow$) 16×4000

64000 (EUR)

A1

[2 marks]

- (c) $R(x) = x(20000 - 1000x)$ OR $R(x) = 20000x - 1000x^2$ A1

[1 mark]

- (d) EITHER

attempt to find total costs (both fixed and variable) AND subtract from their 64000

(M1)

$$64000 - (10000 + 10 \times 4000)$$

(A1)

OR

attempt to find total profit from mugs AND subtract fixed costs

(M1)

$$(16 - 10) \times 4000 - 10000$$

(A1)

THEN

$$= 14000 \text{ (EUR)}$$

A1

[3 marks]

- (e) METHOD 1

attempt to subtract total costs in terms of x from their $R(x)$ (M1)

$$(P(x) =) (20000x - 1000x^2) - (10000 + 10(20000 - 1000x))$$

correct intermediate step leading to given answer

A1

(e.g. correct expansion of $10(20000 - 1000x)$)

$$P(x) = -1000x^2 + 30000x - 210000$$

AG

Note: Do not award the A1 mark if the AG line is not stated.

METHOD 2

attempt to express profit per mug, and then subtract fixed monthly costs (M1)

$$(P(x) =) (x - 10)(20000 - 1000x) - 10000$$

correct expansion leading to the given answer

A1

$$P(x) = -1000x^2 + 30000x - 210000$$

AG

Note: Do not award the A1 mark if the AG line is not stated.

[2 marks]

– 15 –

8823 – 7215M

(f) (i) $(P'(x) =) -2000x + 30000$

A1A1**Note:** Award at most **A1A0** if additional terms are seen.(ii) **METHOD 1**

$$P'(x) = 0 \text{ OR sketch of } P(x) \text{ OR use of } x = -\frac{b}{2a}$$

 $x = 15$ is the maximum, not 16**A1R1**

hence salesman's price is not the optimum

AG**Note:** Award **A1** for $x = 15$, and **R1** for either comparing it to 16 **OR** making a statement that is some version of the **AG** line. It is possible to award **A1R0**.**METHOD 2**

$$P'(16) = -2000 \neq 0$$

A1R1

hence salesman's price is not the optimum

AG**Note:** Award **A1** for finding an appropriate value, and **R1** for comparing it to zero. It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.**METHOD 3**finding $P(x)$ for any value from $14 < x < 16$ **A1**

comparing this value to their part (d)

R1

hence salesman's price is not the optimum

AG**Note:** It is possible to award **A1R0**. To award the **R1** a statement that is some version of the **AG** line must also be given.**[4 marks]**(g) substituting the expression for n into cost function, $C(n)$.**(M1)**

$$(\text{cost} =) 10 - 0.0001(20000 - 1000x) = 8 + 0.1x$$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ **(M1)**

$$(\text{New } P(x) =) (20000x - 1000x^2) - (10000 + (8 + 0.1x)(20000 - 1000x))$$

A1

$$((\text{New } P(x) =) -900x^2 + 26000x - 170000)$$

[3 marks]

(h) 14.4 (EUR) $(14.4444\dots, \frac{130}{9})$

A2**[2 marks]****[Total 19 marks]**

9. (a) (i) recognizing that 0.2 is removed 20 times (M1)
 $10 - 0.2 \times 20$
 6 (mm) A1

- (ii) ($V =$) 216 mm^3 A1

Note: Units are required for the **A1** to be awarded.

[3 marks]

- (b) attempt at power rule (M1)
 $(V'(t) =) -60 + 2.4t - 0.024t^2$ A1

[2 marks]

- (c) recognizing need to find $V'(20)$ (M1)
 $(V'(20) =) -60 + 2.4(20) - 0.024(20)^2$
 $= -21.6 \text{ (mm}^3 \text{ s}^{-1}\text{)}$ A1

[2 marks]

- (d) decreasing/change in coffee temperature R1
 decreasing/change in surface area of the cube
 the cube breaks apart into smaller pieces
 dissolution rate related to volume of the cube

Note: Award **R1** for a reasonable explanation of a change in rate.

[1 mark]

[Total: 8 marks]

10. 3 (seen at any stage of their work)

(A1)

$$\frac{dy}{dx} = 2ax + b$$

(M1)(A1)

Note: Award **M1** for an attempt to find $\frac{dy}{dx}$.

substituting $x = 2$ and their gradient into their derivative

(M1)

$$3 = 2a(2) + b$$

substituting (2, 4) into original equation

(M1)

$$4 = a(2)^2 + b(2) - 10$$

solving equations $a = -2$
 $b = 11$

A1A1

Note: Using $m = -\frac{1}{3}$, gives $a = -\frac{11}{3}, b = \frac{43}{3}$, award **A0M1A1M1M1A0A1**.

For the final **A** marks, award at most **A1A0** if correct values are unlabelled or incorrectly labelled.

[Total: 7 marks]

12. (a) $\pi x + 2y (= 20)$

A1**Note:** Award **A0** for an unsimplified answer.**[1 mark]**(b) attempt to combine area formulas to express A in terms of x and y **M1**

$$A = xy + \frac{\pi x^2}{4}$$

$$y = \frac{20 - \pi x}{2} (= 10 - \frac{\pi x}{2})$$

A1correct substitution for y and expansion of brackets**A1**

$$A = x \left(\frac{20 - \pi x}{2} \right) + \frac{\pi x^2}{4} = \frac{20x - \pi x^2}{2} + \frac{\pi x^2}{4}$$

$$= 10x - \frac{\pi x^2}{4}$$

AG**Note:** The **AG** line must be stated for the final **A1** to be awarded.**[3 marks]**

(c) attempt at power rule e.g. one correct term

(M1)

$$10 - \frac{\pi x}{2}$$

A1**[2 marks]**(d) **EITHER**

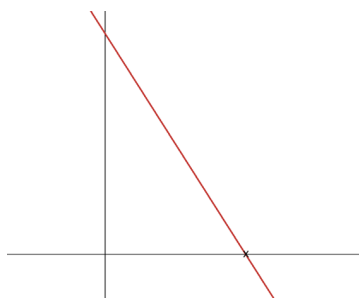
setting their derivative to zero

M1

$$10 - \frac{\pi x}{2} = 0 \quad \text{OR} \quad \frac{dA}{dx} = 0$$

OR

attempt at finding the root of the graph of their derivative

M1**THEN**

$$x = \frac{20}{\pi} \text{ (m)}$$

A1

(given that function is a negative quadratic, only stationary point is a maximum)

Note: Do not award the M mark if their derivative is not used to find the x -value.
Award at most **M1A0** if their answer is not given in exact form.**[2 marks]****[Total: 8 marks]**

3. (a) $\frac{40000}{x^2} = 400$ **(M1)**
 $x = 10$ (pesos) (since x is positive) **A1**
[2 marks]

(b) (i) $\left(\frac{40000}{50^2} = \right) 16$ **A1**

(ii) $(50 \times 16 =) 800$ (pesos) **A1**
[2 marks]

(c) (i) **EITHER**
profit for each smoothie = $x - 20$ **(M1)**
 $P = \frac{40000}{x^2} \times (x - 20)$ **A1**

OR
profit = revenue – costs = $nx - 20n$ **(M1)**
 $P = x \times \frac{40000}{x^2} - 20 \times \frac{40000}{x^2}$ **A1**

Note: Do not award **A1** if $\frac{40000}{x}$ seen as first term unless explained (in part (a) or (b)), as it is given in question.

THEN

$P = \frac{40000}{x} - \frac{800000}{x^2}$ **AG**

(ii) attempt to express P ready for power rule **(M1)**
 $P = 40000x^{-1} - 800000x^{-2}$
 $\frac{dP}{dx} = -\frac{40000}{x^2} + \frac{1600000}{x^3}$ **OR** $\frac{dP}{dx} = -40000x^{-2} + 1600000x^{-3}$ **A1A1**

Note: The **(M1)** can be awarded for either of the correct terms seen.
A1 for each correct term.
At most **M1A1A0** if additional terms seen.

continued...

Question 3 continued(iii) attempt to find x -value (M1)

e.g. sketch of $\frac{dP}{dx}$ with x -intercept indicated **OR** recognition that it occurs at the maximum of P **OR** algebraic approach (requires multiplication by x^3)

$$x = 40$$
A1

Note: $\frac{-40000}{x^2} + \frac{1600000}{x^3} = 0$ is insufficient to award **M1**, this is given in the question. There must be an “attempt to find x -value”.
Award **M1A0** for a coordinate pair $(40, 500)$.

(iv) attempt to substitute their x -value into equation for n (M1)

$$n = \frac{40000}{40^2}$$

$$= 25$$

A1

Note: Given the nature of the function P , the local maximum is also the global maximum. This is often the case in examinations, but should not always be assumed.

[9 marks]
[Total 13 marks]

5. (a) attempt to substitute 2 into the DE (M1)

$$0.6 \times 2^2 - 1.8 \times 2$$

$$-1.2$$

A1
[2 marks]

- (b) recognition of need to integrate (M1)

Note: An increase of powers (expressions with $x^2 + x^3$) is sufficient evidence of the need to integrate.

$$0.2x^3 - 0.9x^2 (+c)$$

(A1)

attempt to substitute given condition to find c (M1)

$$0.2(4)^3 - 0.9(4)^2 + c = 1.4 \quad \text{OR} \quad c = 3$$

$$(h(x) =) 0.2x^3 - 0.9x^2 + 3$$

A1
[4 marks]
continued...

Question 5 continued

(c) (i) $\int_0^4 0.2x^3 - 0.9x^2 + 3dx$ OR $\int_0^4 h dx$ **A1A1**

Note: Award **A1** for a correct integral, **A1** for correct limits in the correct location.
Award at most **A0A1** if dx is omitted.
Award at most **A0A1** for "integral ydx " or integral $f(x)dx$, this is not the correct integrand.

(ii) $5.6 (m^2)$ **A2**
[4 marks]

Note: A negative area for their integrand is unrealistic. Award at most **A1A0** for their **FT** answer expressed as negative area or a negative area converted to a positive answer."

(d) $(5.6 \times 1.2 =) 6.72 m^3$ **A1**

Note: Correct unit must be seen for the **A1** to be awarded.

[1 mark]

- (e) attempt to substitute one of the given conditions, both x and h , into either the function or the derivative **(M1)**

$1 = 16a + 4b + c$ **A1**

attempt to differentiate $h(x)$ **(M1)**

$h'(x) = 2a(x) + b$ **(A1)**

$-1.2 = 4a + b$ **A1**

$0 = 6a + b$ OR $3 = \frac{-b}{2a}$ **A1**

Note: The equations can be found in any order, and hence the associated mark(s) should be awarded independently.

Accept $1 = 4a + 2b + c$ and $1.2 = 8a + b$ as correct equations, from symmetry.

Award at most **M1A1M1A1A1A0** if the equations are not simplified to integer values.

E.g. $1 = 4^2a + 4b + c$, $-1.2 = 2 \times 2a + b$, $0 = 2 \times 3a + b$

[6 marks]

(f) $a = 0.6 \left(\frac{3}{5} \right)$, $b = -3.6 \left(-\frac{18}{5} \right)$, $c = 5.8 \left(\frac{29}{5} \right)$ **A2**

Note: Award **A1** if only two are correct, **A0** otherwise.
Only follow through from three explicit equations given in part (e).

[2 marks]

- (g) gradients are the same OR $h'(2) = -1.2$ for both curves **A1**
heights are the same OR $h(2) = 1$ for both curves **A1**

[2 marks]

[Total 21 marks]

3. (a) heights, 0, 4, 1.75, 3 and 3.75 seen (A2)

Note: Award **A1A0** if **two** of 1.75, 3 or 3.75 are seen.

attempt to use trapezoidal rule formula for their heights (M1)

$$\frac{1}{2} \times 1 \times \{0 + 4 + 2(1.75 + 3 + 3.75)\} \quad (A1)$$

Note: Award **(M1)(A1)** for correctly expressing this as 3 trapezoids and a triangle. The “×1” need not be seen.

$$= 10.5 \text{ (m}^2\text{)} \quad A1$$

[5 marks]

(b) $-\frac{1}{12}x^3 + x^2 + c \quad A1A1A1$

[3 marks]

(c) $\int_0^4 \left(-\frac{1}{4}x^2 + 2x\right) dx + 1 \times 4 + \frac{1}{2} \times 7 \times 4 \quad (A1)(M1)(A1)$

Note: Award **A1** for correct area of rectangle **OR** triangle, **M1** for substituting correct limits into given integral (may be seen in part (b)), and **A1** for entire expression correct.

$$= 10.6666\dots + 4 + 14$$

$$= 28\frac{2}{3} \text{ (m}^2\text{)} \left(\frac{86}{3}\right) \quad A1$$

Note: The answer must be **exact** for the **A1** to be awarded. For an answer of 28.7 or 28.66 award **(A1)(M1)(A1)A0**.

[4 marks]

(d) (Total area using part (a) =) 28.5 (A1)

$$\text{Percentage error} = \left| \frac{28.5 - 28.6666\dots}{28.6666\dots} \right| \times 100 \quad (M1)$$

Note: if their trapezoid value is incorrect but is used correctly in the percentage error formula, award at most **A0M1A0**. If it is clear from the answer that ×100 has been used, then condone the omission and award the **M** mark.

$$= 0.581 \text{ (\%)} \text{ (0.581395\dots)} \quad A1$$

(accept 0.697 from use of 28.7)

[3 marks]

[Total: 15 marks]

12. (a) $\left(\frac{dy}{dx} = \right) 10x + 9x^{-4}$ $\left(\left(\frac{dy}{dx} = \right) 10x + \frac{9}{x^4}\right)$ **A1(M1)A1**

Note: Award **A1** for $10x$, **M1** for expressing $\frac{1}{x^3}$ as x^{-3} , **A1** for $+9x^{-4}$.

[3 marks]

(b) 19 **A1**
[1 mark]

(c) $m_{\perp} = -\frac{1}{19}$ (seen anywhere) **(A1)**

$y = 2$ (seen anywhere) **(A1)**

$y - 2 = -\frac{1}{19}(x - 1)$ **OR** $y = -\frac{1}{19}x + \frac{39}{19}$ ($y = -0.0526x + 2.05$, $y = -0.0526315...x + 2.05263...$)

A1

Note: Follow through within the question for the final **A1**, which can be awarded for correctly substituting **their** gradient (seen) and **their** point (seen) into the equation for a line.

[3 marks]**[Total: 7 marks]**

12. (a) $\left(\frac{dy}{dx} = \right) 12x^2 + 4x^{-3} \quad \left(\left(\frac{dy}{dx} = \right) 12x^2 + \frac{4}{x^3}\right)$ **A1(M1)A1**

Note: Award **A1** for $12x^2$, **M1** for expressing $\frac{1}{x^3}$ as x^{-3} , **A1** for $+4x^{-3}$.

[3 marks]

(b) 16 **A1**
[1 mark]

(c) $m_{\perp} = -\frac{1}{16}$ (seen anywhere) **(A1)**

$y = 2$ (seen anywhere) **(A1)**

$y - 2 = -\frac{1}{16}(x - 1)$ **OR** $y = -\frac{1}{16}x + \frac{33}{16}$ ($y = -0.0625x + 2.0625$) **OR** $y = -0.0625x + 2.06$

A1

Note: Follow through within the question for the final **A1**, which can be awarded for correctly substituting **their** gradient (seen) and **their** point (seen) into the equation for a line.

[3 marks]
[Total: 7 marks]

4. (a) $\frac{1}{2} \times 1 \times ((6 + 4.3) + 2(1.7 + 1.3 + 3 + 4.7))$ **A1A1**

$= 15.9 \text{ (15.85) (cm}^2\text{)}$ **A1**

[3 marks]

(b) (i) $\int_0^5 \left(-\frac{x^3}{3} + 3x^2 - 7x + 6 \right) dx$ **A2**

Note: Award **A2** for an integral with correct integrand and correct limits.

(ii) $15.4 \text{ (15.4166..., } \frac{185}{12}) \text{ (cm}^2\text{)}$ **A2**

Note: If their limits are outside the domain, award at most **A1A0FT**.

[4 marks]

(c) $400 = \pi r^2 h$ **A1**

(d) **METHOD 1** (Substitution of $h = \frac{400}{\pi r^2}$ in $A = 2\pi r^2 + 2\pi r h$)

$A = 2\pi r^2 + 2\pi r h$ **(A1)**
 attempt to isolate h **(M1)**

$$h = \frac{400}{\pi r^2}$$

correct substitution of their h into correct expression **(M1)**

$$A = 2\pi r^2 + 2\pi r \left(\frac{400}{\pi r^2} \right) \left(A = 2\pi r^2 + \frac{800}{r} \right)$$

$k = 800$ **A1**

METHOD 2 (Equating $2\pi r h$ and $\frac{k}{r}$)

$A = 2\pi r^2 + 2\pi r h$ **OR** $2\pi r h = \frac{k}{r}$ **(A1)**

attempt to isolate h **(M1)**

$$h = \frac{400}{\pi r^2}$$

correct substitution of their h into $2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{k}{r}$ **(M1)**

$$2\pi r \left(\frac{400}{\pi r^2} \right) = \frac{k}{r}$$

$k = 800$ **A1**

[4 marks]

continue...

Question 4 continued.

(e) (i) $\frac{dA}{dr} = 4\pi r - 800r^{-2}$

A1(M1)A1

Note: Award **A1** for $4\pi r$ seen, and **(M1)** for expressing $\frac{800}{r}$ as $800r^{-1}$ (can be implied through $\mp \frac{800}{r^2}$ seen), **A1** for $-800r^{-2}$. Award at most **A1(M1)A0** if any additional terms are seen.

(ii) $0 = 4\pi r - 800r^{-2}$

(M1)

$$r = 3.99 \left(3.99294\dots, \sqrt[3]{\frac{200}{\pi}} \right) \text{ (cm)}$$

A1

Note: Award at most **M1A0** if k is not found.

[5 marks]

(f) ($h =$) 7.99 (7.98589...) (cm)

A1

($C =$) $2\pi (3.99294\dots) \approx 25.1 (25.0884\dots)$ (cm)

A1

EITHER

the longest dimension of the label (7 cm) is less than both values and hence the label will fit (in any rotation)

R1

OR

$7 < 7.99$ and $5 < 25.1$

R1

Note: Do not accept an argument based on the comparison of areas.

[3 marks]

[Total 20 marks]

4. (a) $\frac{1}{2} \times 1 \times ((4 + 0.7) + 2(7.3 + 6.7 + 4 + 1.3))$ (A1)(A1)

$= 21.7 \text{ (21.65) (cm}^2\text{)}$ A1
[3 marks]

(b) (i) $\int_0^5 \left(\frac{x^3}{3} - 3x^2 + 6x + 4 \right) dx$ A2

Note: Award **A2** for an integral with correct integrand and correct limits.

(ii) $22.1 \text{ (22.08333..., } \frac{265}{12} \text{) (cm}^2\text{)}$ A2

Note: If their limits are outside the domain, award at most **A1A0FT**.

[4 marks]

(c) $600 = \pi r^2 h$ A1
[1 mark]

(d) **METHOD 1** (Substitution of $h = \frac{600}{\pi r^2}$ in $A = 2\pi r^2 + 2\pi r h$)

$A = 2\pi r^2 + 2\pi r h$ (A1)
attempt to isolate h (M1)

$h = \frac{600}{\pi r^2}$

correct substitution of their h into correct expression (M1)

$A = 2\pi r^2 + 2\pi r \left(\frac{600}{\pi r^2} \right) \quad \left(A = 2\pi r^2 + \frac{1200}{r} \right)$

$k = 1200$ A1

METHOD 2 (Equating $2\pi r h$ and $\frac{k}{r}$)

$A = 2\pi r^2 + 2\pi r h$ OR $2\pi r h = \frac{k}{r}$ (A1)

attempt to isolate h (M1)

$h = \frac{600}{\pi r^2}$

correct substitution of their h into $2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{k}{r}$ (M1)

$2\pi r \left(\frac{600}{\pi r^2} \right) = \frac{k}{r}$

$k = 1200$ A1
[4 marks]

continue...

Question 4 continued.

(e) (i) $\frac{dA}{dr} = 4\pi r - 1200r^{-2}$

A1(M1)A1

Note: Award **A1** for $4\pi r$ seen, and **(M1)** for expressing $\frac{1200}{r}$ as $1200r^{-1}$ (can be implied through $\mp \frac{1200}{r^2}$ seen), **A1** for $-1200r^{-2}$. Award at most **A1(M1)A0** if any additional terms are seen.

(ii) $0 = 4\pi r - 1200r^{-2}$

(M1)

$$r = 4.57 \left(4.570781\dots, \sqrt[3]{\frac{300}{\pi}} \right) \text{ (cm)}$$

A1

Note: Award at most **M1A0** if the final answer is in terms of k .

[5 marks]

(f) ($h =$) 9.14 (9.14156.....) (cm)

A1

($C =$) $2\pi (4.570781\dots) \approx 28.7 (28.7190\dots)$ (cm)

A1

EITHER

the longest dimension of the label (9 cm) is less than both values and hence the label will fit (in any rotation)

R1

OR

$9 < 9.14$ and $5 < 28.7$

R1

Note: Do not accept an argument based on the comparison of areas.

[3 marks]

[Total 20 marks]

8. (a) evidence of finding the derivative of $f(x)$ (M1)
 e.g., $f'(x)$ seen **OR** attempt to use power rule

$$f'(x) = -4x^{-3} - bx^{-2} \quad (A1)(A1)$$

Note: Award **A1** for each correct term. Award at most **(A1)(A0)** if extra terms are seen.

Substitution of 5 for x and 0.208 for $f'(x)$ in *their* derivative (M1)

$$0.208 = -4(5)^{-3} - b(5)^{-2}$$

$$(b =) -6$$

A1
[5 marks]

(b) **METHOD 1**

attempt to find the value of the derivative at $x = 3.5$ **OR** $f'(3.5)$ seen **M1**

$$-4(3.5)^{-3} - b(3.5)^{-2}$$

$0.397 > 0$ ($0.396501... > 0$) **OR** 0.397 ($0.396501...$) is positive **R1**

$f(x)$ is increasing at $x = 3.5$ **AG**

METHOD 2

sketch of derivative with indication of $x = 3.5$ **M1**

$f'(3.5) > 0$ **OR** $f'(3.5)$ is positive **R1**

$f(x)$ is increasing at $x = 3.5$ **AG**

[2 marks]

[Total: 7 marks]

9. (a) $f(x) = 0$ seen **OR** sketch of the graph with all three x -intercepts marked **OR** evidence of using product of roots (M1)

(a =) 3 A1
[2 marks]

- (b) evidence of using an integral to find the area (M1)

$\int f(x), \int_1^3$, etc.

(area =) 4 A1
[2 marks]

[Total: 4 marks]

12. (a) indication of setting the derivative to 0 (M1)
 $\frac{dV}{dw} = 0$ OR $690w - 36w^2 = 0$ OR sketch of the graph of the derivative with the zero marked
 $(w =) 19.2 \text{ (cm)}$ $(19.1666... \text{ (cm)}, \frac{115}{6} \text{ (cm)})$ A1

[2 marks]

- (b) evidence of the need to integrate (M1)
 integral symbol seen OR $\int 690w - 36w^2 dw$ OR one term integrated correctly

$$(V =) 345w^2 - 12w^3 (+c) \quad \text{A1}$$

$$c = 0 \text{ since } V = 0 \text{ when } w = 0 \quad \text{R1}$$

$$V = 345 \times (19.1666...)^2 - 12 \times (19.1666...)^3$$

$$(V =) 42\,246.5... \text{ (cm}^3\text{)} \text{ OR } (V =) 42247 \text{ (cm}^3\text{)} \text{ seen} \quad \text{A1}$$

$$42\,200 \text{ cm}^3 \quad \text{AG}$$

Note: It is possible to award (M1)A1R0A1. No marks are awarded for part (b) if 42 200 is used to work backward.

[4 marks]

- (c) **METHOD 1**
 attempt to calculate height using volume, width, and length numerical values (M1)

$$h = \frac{42\,200}{19.1666... \times (19.1666... \times 3)} \quad \text{(may be seen in part (b))}$$

 $(h =) 38.3 \text{ (cm)} \text{ (38.2911... (cm))} \quad \text{(A1)}$

Note: Accept a substitution of 42 246.5... for the volume ($h = 38.3333...$).

$$19.1666... + 57.5 + 38.2911... \\ (M =) 115 \text{ (114.957... (cm))} \quad \text{A1}$$

- METHOD 2**
 attempt to calculate height using volume, width, and length expressions (M1)

$$h = \frac{345w^2 - 12w^3}{w \times 3w} \\ (h =) 115 - 4w \quad \text{(A1)}$$

$$w + 3w + (115 - 4w)$$

$$(M =) 115 \text{ (cm)} \quad \text{A1}$$

[3 marks]

[Total: 9 marks]

Q4.(a) (i) -2 **A1**(ii) $0.667\left(0.66666\dots, \frac{2}{3}\right)$ **A1****Note:** Award **A1A0** for answers given as coordinates $(-2, 6)$ and $(0.667, -3.48)$.Accept answers which round to the correct 3 sf value (-2.00) , e.g. -1.9999 **[2 marks]**(b) $-2 < x < \frac{2}{3}$ **A1****Note:** Award **A1** for correct inequalities and their endpoints.**[1 mark]**(c) (i) 3 **A1**(ii) $y + 3 = 3(x - 1)$ OR $y = 3x - 6$ **A1****[2 marks]****Total [5 marks]**

Q9.

- (a) evidence of integrating $\frac{dP}{dx}$ (M1)

$$P = \int -10x + 460 \, dx$$

$$= -5x^2 + 460x (+c)$$

A1A1

Note: Award **A1** for each correct term. Omission of the constant c may be ignored at this stage.

- evidence of substituting $x = 10$ and $P = 3300$ in their profit expression (M1)

$$P = -5x^2 + 460x - 800$$

A1**[5 marks]**

- (b) attempt at using definite integral with limits 25 and 50 OR attempt at finding the difference between $P(50)$ and $P(25)$ M1

$$\int_{25}^{50} -10x + 460 \, dx \text{ OR } P(50) - P(25)$$

$$= 2130(\text{MUR})(2125)$$

A1

Note: Do not apply FT in part (b) if their final answer in part (a) is a constant, as it implies profit does not depend on x .

[2 marks]**Total [7 marks]**

2. (a) 0.638 (0.637623...) and 4.10 (4.09811...) **A2A1**

Note: Award **A2** for one correct answer, accept 4.1. Award **A2A0** if coordinate pair is given.

[3 marks]

(b) (i) $\int_{0.638}^{4.1} 1.5^x dx$ OR $\int_{0.638}^{4.1} f(x) dx$

A1A1

Note: **A1** for function and dx , **A1** for limits. Award **A1** for $\int_{0.638}^{4.1} y dx$.

Condone the use of modulus within the integrand.

(ii) 9.81 (9.80788...) (from 3sf)

9.80 (9.79844...) (from full)

A2

Note: Accept 9.8 for **A2**.

(iii) correct expression for area

(**A1**)

$\int_{0.638}^{4.1} g(x) - f(x) dx$ OR $\int_{0.638}^{4.1} 6 - \frac{3}{x} - 1.5^x dx$

5.38 (5.38290...)

A1

Note: Accept 5.39

[6 marks]

(c) $f'(x) = g'(x)$

(**M1**)

correct graphs of $f'(x)$ and $g'(x)$

(**M1**)

Note: Award **M1** if $\frac{3}{x^2}$ seen.

($k =$) 1.86 (1.86406...)

A1

[3 marks]

Total [12 marks]

5. (a) $(A =) 2\pi r^2 + 2\pi r h$ A1A1

Note: Award **A1** for either term correct (even if unsimplified) and **A1** for all correct (including simplification and no extra term).

[2 marks]

(b) $V = \frac{2\pi r^3}{3} + \pi r^2 h$ A1A1

$V = \frac{2\pi r^3 + 3\pi r^2 h}{3}$ AG

Note: Award **A1** for one correct term, **A1** for both and completion to required form.

[2 marks]

- (c) 10000 substituted into (b)
correct rearrangement with at least one line of intermediate working leading to the given value.

M1

M1

$h = \frac{30000 - 2\pi r^3}{3\pi r^2}$ AG

[2 marks]

- (d) h substituted into their A

M1

$(A =) 2\pi r^2 + 2\pi r \left(\frac{30000 - 2\pi r^3}{3\pi r^2} \right)$

further consistent rearrangement that leads to **given** A

A1

[2 marks]

(e) $\frac{4\pi r}{3} - \frac{20000}{r^2}$ A1A1A1

Note: Award **A1** for $\frac{4\pi r}{3}$, **A1** for -20000 seen, and **A1** for a correct index for r in second term.

[3 marks]

(f) $\frac{dA}{dr} = 0$ (M1)

EITHER

rearrangement leading to $r^3 = \frac{15000}{\pi}$ (M1)

OR

graph of $\frac{dA}{dr}$ (M1)

THEN

$r = 16.8$ (16.8389...) OR $\sqrt[3]{\frac{15000}{\pi}}$ A1

Note: Accept correct graph of Area function leading to correct minimum value for r for **M1M1A1**.

$h = 0$ A1
[4 marks]

- (g) hemisphere A1

Note: Do not **FT** from incorrect answers to part (f).

[1 mark]

Total [16 marks]

Q3.

(a) $(d=)3l$ **A1**

[1 mark]

(b) substitution into volume of cuboid formula $V = l^2 \times 3l (= 375)$ **(M1)**

attempting to solve their $3l^3 = 375$ **(M1)**

$$l^3 = 125 \text{ OR } l = \sqrt[3]{125} \text{ OR } l^3 = \frac{375}{3}$$

$l = 5, d = 15$ **A1**

[3 marks]

(c) finding the area of a square and the area of a (non-square) rectangle 5×5 and 5×15 **(M1)**

$(5 \times 5 + 5 \times 15 + 5 \times 15) \times 2$ **(A1)**

$= 350\text{cm}^2$ **A1**

Note: The final answer is 350cm^2 . Units are required.

[3 marks]

(d) $\pi r^2 h = 375$ **(A1)**

$h = \frac{375}{\pi r^2}$ **A1**

Note: Award at most **A0A1** for a final answer $h = \frac{V}{\pi r^2}$.

[2 marks]

(e) attempt to substitute their h into the curved surface area $2\pi r h$ **M1**

$A = 2\pi r^2 + 2\pi r \left(\frac{375}{\pi r^2} \right)$ **A1**

$A = 2\pi r^2 + \frac{750}{r}$ **AG**

[2 marks]

(f) $\left(\frac{dA}{dr} \right) 4\pi r - 750r^{-2}$ **A1A1A1**

Note: Award **A1** for $4\pi r$, **A1** for -750 , **A1** for r^{-2} or equivalent. Award at most **A1A1A0** for additional terms seen.

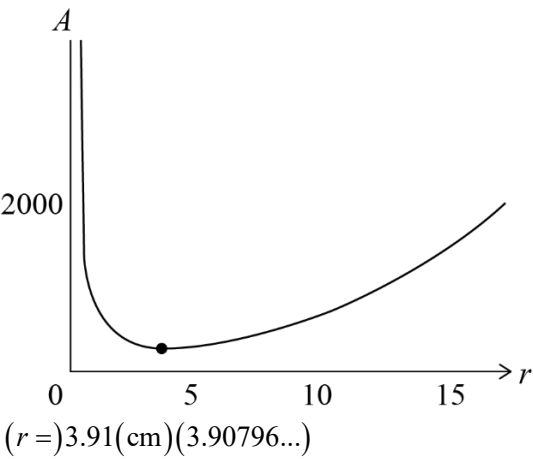
[3 marks]

continued...

Question 3 continued.

(g) (i) **METHOD 1**

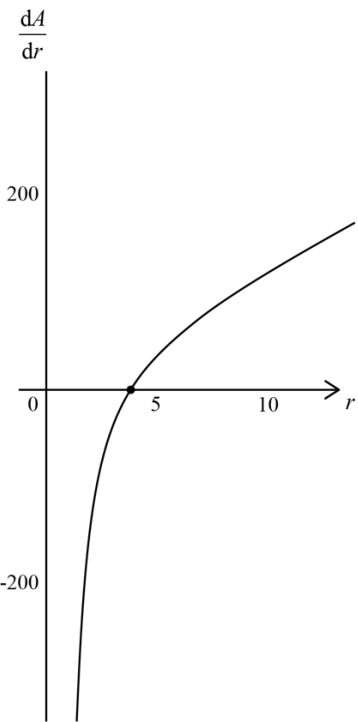
correct sketch with some indication of min point (need not be labelled) **(M1)**



A1

METHOD 2

attempt to solve $4\pi r - \frac{750}{r^2} = 0$, graphically or otherwise **(M1)**



(r =) 3.91 (cm) (3.90796...) **A1**

(ii) $288(\text{cm}^2)$ (287.873...) **A1**
[3 marks]

continued...

Question 3 continued.

- (h)

cuboid: 1.10×350 and cylinder: $1.25 \times 287.873\dots$

cuboid: 385 and cylinder: 359.842...

he should choose the cylinder

(M1)

R1

A1

Note: Do not award **R0A1**.

[3 marks]

Total [20 marks]

Q5.

(a) attempt to substitute into trapezoidal formula (M1)

$$(\text{area of cross-section}) \approx \frac{1.9}{2}(0 + 0 + 2(1.68 + 2.81 + 2.32)) \quad (\text{A1})$$

$$= 12.9 \text{ (m}^2\text{)}(12.939) \quad \text{A1}$$

[3 marks]

(b) attempt to find the volume of water in one second, their (a) $\times 0.3$ (M1)

$$12.939 \times 0.3$$

$$= 3.88(3.8817) \text{ (m}^3\text{)} \quad \text{A1}$$

Note: Accept 3.87 for use of 3sf answer to part (a) 12.9.

[2 marks]

(c) $(2.81 - 1.6 =) 1.21 \text{ (m)}$ A1

[1 mark]

(d) attempt to find maximum eg graphically or $\frac{dy}{dx} = 0$ or using height at $x = 3.8$ (M1)

$$3.07 \text{ (m)} \text{ OR } 3.066 \text{ (m)} \quad \text{A1}$$

Note: Award **M1A0** for coordinate pair (3.8, 3.066)

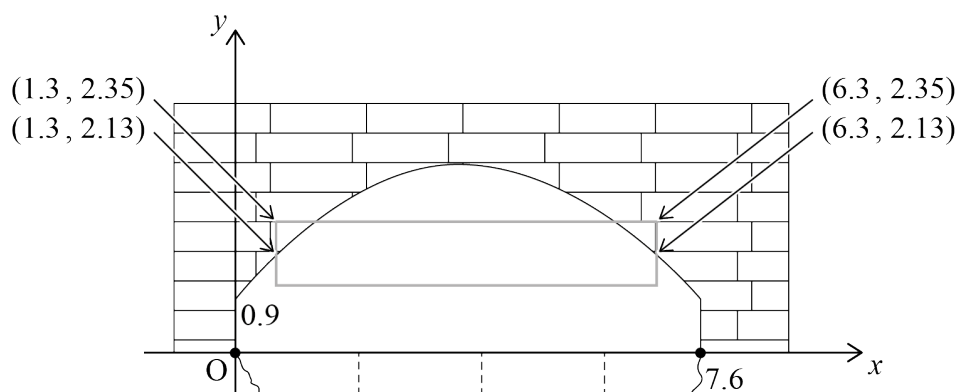
[2 marks]

continued...

Question 5 continued.

(e) **METHOD 1**

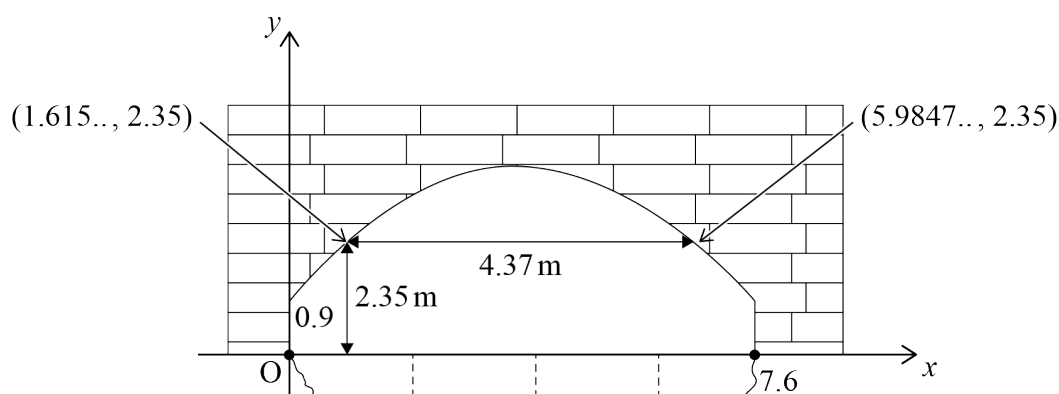
height of bridge < height of boat at
 $x = 1.3$ or $x = 6.3$ (5 m wide boat)

use of $3.8 + 2.5$ or $3.8 - 2.5$ **(M1)**

attempt to find the height of the bridge at either 1.3 or 6.3 m

(M1) $2.13(\text{m})(2.1285)$ **A1** $(2.13 < 2.35)$ the boat cannot pass under the bridge**A1****METHOD 2**

boat is wider (5 m) than the width between
the arch at $h = 2.35$ m (height of boat)



attempt to find the values of x (1.61520... and 5.98479...) when the bridge is 2.35 m
above the water

(M1)

continued...

Question 5 continued.

subtract their two values to find the width of bridge at that point at this height **(M1)**

$5.984795... - 1.615204...$ OR $3.8 - 1.615204...$ OR $5.98479... - 3.8$

$4.37(4.36959...)(\text{m})$ OR $2.18 (2.18479...)(\text{m})$ **A1**

$(4.37 < 5)$ OR $(2.18 < 2.5)$ the boat cannot pass under the bridge **A1**

METHOD 3 (see the diagram for method 2)

use of $3.8 + 2.5 (6.3)$ or $3.8 - 2.5 (1.3)$ **(M1)**

attempt to find the values of x when the bridge is 2.35 m above the water **(M1)**

$5.98479...$ or $1.61520... (\text{m})$ **A1**

$6.3 > 5.98...(5.98479...)$ or $1.3 < 1.62 (1.61520...)$ the boat cannot pass under the

bridge **A1**

Note: In all 3 methods the final **A1** is dependent on the previous **A1**.

[4 marks]

[Total 12 marks]

11. (a) (i) $\left(B'(x) = \frac{-32}{x^2} + 2\right)$ **OR** $\left(B'(x) = -32x^{-2} + 2\right)$ **A1(M1)A1**

Note: Award **A1** for 2 seen, **(M1)** for expressing $\frac{1}{x}$ as x^{-1} (this can be implied from, either $\frac{1}{x^2}$ or x^{-2} seen in final answer), **A1** for $\frac{-32}{x^2}$. Award at most **A1(M1)A0** if additional terms are seen.

(ii) $B'(x) = 0$ **OR** $0 = \frac{-32}{x^2} + 2$ **OR** sketch of $B'(x)$ with root identified **M1**

Note: It is specified that the student must use their derivative from part (i) to answer this question.

$x = 4$ **(A1)**

Note: FT marks can only be awarded if *their* x -value lies within the given domain of $x \geq 1$.

$(B(x) =) 36$ (minutes) **A1**
[6 marks]

(b) $40 > 20 + \frac{32}{k} + 2k$ **OR** 2 and 8 seen **OR** sketch of $T(n)$ and $T(n) = 40$ **(M1)**

$2 < k < 8$ **OR** $3 \leq k \leq 7$ **OR** 3, 4, 5, 6, 7 **A1**
[2 marks]
Total [8 marks]

9. (a) $(f(1) =) \frac{1^3 - 2(1)^2 + 5(1) - 7}{-3}$ **OR** $1 - 2 + 5 - 7$

(A1)
A1
[2 marks]

(b) $f'(x) = 3x^2 - 4x + 5$

M1A1

Note: Award **M1** for an attempt at power rule (e.g. one term correct),
A1 for a completely correct answer.

[2 marks]

(c) $f'(1) = 4$
 $y + 3 = 4(x - 1)$ **OR** $y = 4x - 7$ (or equivalent)

(A1)
A1
[2 marks]
Total [6 marks]

13. $SA = 2(1.5w)w + 2(1.5w)h + 2wh$ **OR** $10000 = (1.5w)wh$ (M1)

Note: Award **M1** for an attempt to substitute $l = 1.5w$ into either surface area or volume.

correctly isolating h from volume expression (A1)

$$h = \frac{20000}{3w^2} \text{ OR } h = \frac{10000}{1.5w^2}$$

substituting their expression for h into their expression for SA (M1)

$$SA = 2(1.5w)w + 2(1.5w)\frac{20000}{3w^2} + 2w\frac{20000}{3w^2}$$

$$(SA = 3w^2 + \frac{20\,000}{w} + \frac{40\,000}{3w} \text{ OR } SA = 3w^2 + \frac{100\,000}{3w})$$

EITHER

sketch of their graph with minimum indicated (M1)

$(w \Rightarrow) 17.7 \text{ (cm)} \text{ (17.7109 ... (cm))}$ A1

OR

attempt to differentiate expression in terms of only w **AND** setting to zero (M1)

$$0 = 6w - \frac{100\,000}{3w^2}$$

$(w \Rightarrow) 17.7 \text{ (cm)} \text{ (17.7109 ... (cm))}$

A1

Total [5 marks]

2. (a) (i) 0.9 A1
- (ii) the rate of change of the height of the plant is 0.9 cm per day
3 days after it is planted A1
- [2 marks]**
- (b) attempt to solve $\frac{dh}{dt} = 0$ (algebraically or graphically) (M1)
($t =$) 12 (days) A1
- [2 marks]**
- (c) recognition of the need to integrate (M1)
 $h = 1.2t - 0.05t^2 (+c)$ A1A1
attempt to use given conditions (2,6.5) to find c (M1)
 $c = 4.3$
($h =$) $1.2t - 0.05t^2 + 4.3$ A1
- [5 marks]**
- (d) **EITHER**
attempt to find maximum graphically (M1)
- OR**
attempt to substitute their part (b) for t (M1)
- THEN**
 $h = 11.5$ (cm) A1
- Note:** If their answer to part (c) is linear or any other function that substantially changes the nature of the question, award at most **M1A0**. Award at most **M1A0** if the x value of their max point (FT from part (c)), contradicts their $x = 12$ found in part (b).
- [2 marks]**
- (e) 2 (cm) A1
- [1 mark]**
- (f) **EITHER**
attempt to write 1.2 as a percentage or percentage increase (M1)
- OR**
attempt to find two values of H and % increase (M1)
- THEN**
($p =$) 20 A1
- [2 marks]**
- (g) attempt to solve $H(t) = h(t)$ (M1)
 $t = 9.43$ (days) (9.43483...) A1
- [2 marks]**
- Total [16 marks]**

5. (a) (i) $\int_0^{60} 0.001x^3 - 0.12x^2 + 4.05x + 3 \, dx$ **OR** $\int_0^{60} f(x) \, dx$ **A1A1**

Note: Award **A1** for $\int_0^{60}$, **A1** for $0.001x^3 - 0.12x^2 + 4.05x + 3$ or " $f(x)$ " and dx .

- (ii) 2070 (m²) **A2**
[4 marks]

- (b) (i) $x = 24.2$ (24.1886...), $y = 44.9$ (44.9056...) or (24.2, 44.9) **A1A1**

Note: Award **A1A0** if parentheses are omitted.

- (ii) attempt to use inverse tangent **(M1)**

$$\tan^{-1}\left(\frac{y}{x}\right)$$

$$61.7^\circ \text{ (61.6907...}^\circ\text{)}$$

A1
[4 marks]

- (c) **METHOD 1**

attempt to use trapezoidal rule, for $h = 20$.

(M1)

$$\frac{1}{2}(20)((3+30) + 2(44+37))$$

(A1)

$$1950 \text{ (m}^2\text{)}$$

A1

METHOD 2

attempt to find area of three trapezoids, each with $h = 20$

(M1)

correct expressions of values seen

(A1)

$$\frac{1}{2}(3+44)20, \frac{1}{2}(44+37)20, \frac{1}{2}(37+30)20 \text{ **OR** 470, 810, 670}$$

$$1950 \text{ (m}^2\text{)}$$

A1
[3 marks]

– 15 –

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(d) recognition of need to integrate (e.g. reverse power rule or integral symbol) **(M1)**

$$\int -0.00292a + 4.71 \, da = -0.00146a^2 + 4.71a (+c) \quad \textbf{(A1)(A1)}$$

Note: Award **A1** for each correct term.

$$1750 = -0.00146(500)^2 + 4.71(500) + c \quad \textbf{(M1)}$$

Note: Award **M1** for correct substitution of 500 **and** 1750 into their expression.
A constant of integration must be seen (can be implied by a correct c value seen).

$$c = -240$$

$$(C(a) =) -0.00146a^2 + 4.71a - 240 \quad \textbf{(A1)}$$

[5 marks]

$$\textbf{(e)} \quad (C(1950) =) -0.00146(1950)^2 + 4.71(1950) - 240 \quad \textbf{(A1)}$$

Note: Award **A1** for correctly substituting their part (c) into their $C(a)$ from part (d).

$$(\$) 3390 \text{ (3392.85)} \quad \textbf{A1}$$

[2 marks]
Total [18 marks]