

APOLLO

Geometry and trigonometry

HL

Mathematics: analysis and approaches

First assessment 2021

97 questions | questions

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 19]

Consider the line L_1 defined by the Cartesian equation $\frac{x+1}{2} = y = 3 - z$.

(a) (i) Show that the point $(-1, 0, 3)$ lies on L_1 .

(ii) Find a vector equation of L_1 .

[4]

Consider a second line L_2 defined by the vector equation $\mathbf{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + t \begin{pmatrix} a \\ 1 \\ -1 \end{pmatrix}$,

where $t \in \mathbb{R}$ and $a \in \mathbb{R}$.

(b) Find the possible values of a when the acute angle between L_1 and L_2 is 45° .

[8]

It is given that the lines L_1 and L_2 have a unique point of intersection, A, when $a \neq k$.

(c) Find the value of k , and find the coordinates of the point A in terms of a .

[7]

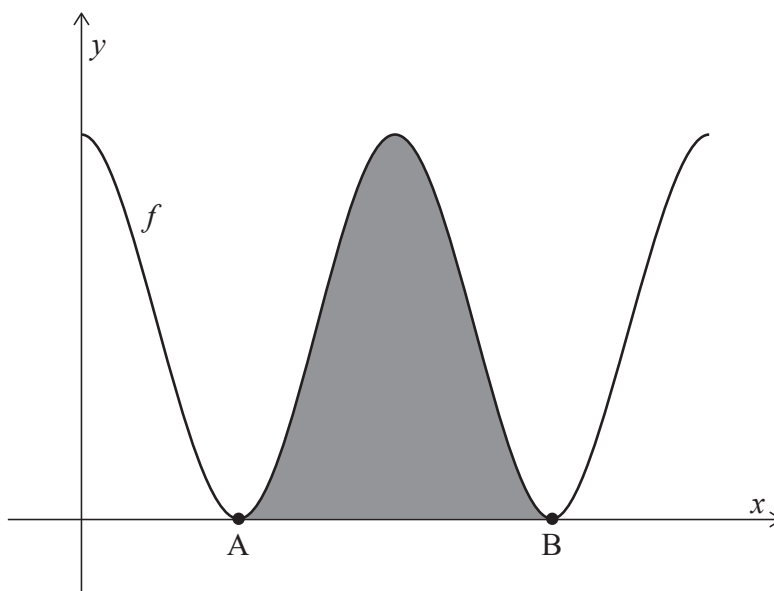
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

Consider the function f defined by $f(x) = 6 + 6 \cos x$, for $0 \leq x \leq 4\pi$.

The following diagram shows the graph of $y = f(x)$.



The graph of f touches the x -axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x -axis, between the points A and B.

- (a) Find the x -coordinates of A and B. [3]
- (b) Show that the area of the shaded region is 12π . [5]

(This question continues on the following page)



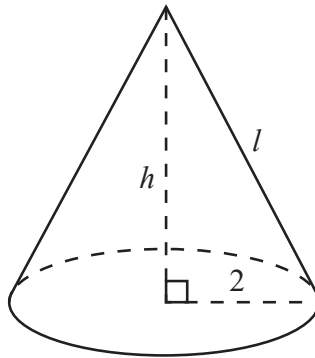
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(Question 10 continued)

The right cone in the following diagram has a total surface area of 12π , equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h , and slant height l .

diagram not to scale



- (c) Find the value of l . [3]
- (d) Hence, find the volume of the cone. [4]



Turn over

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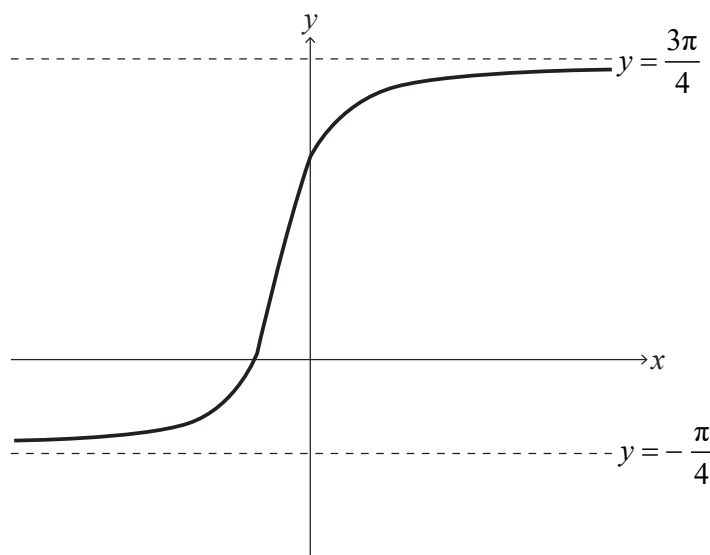
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 19]

The following diagram shows the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$, with asymptotes at $y = -\frac{\pi}{4}$ and $y = \frac{3\pi}{4}$.



- (a) Describe a sequence of transformations that transforms the graph of $y = \arctan x$ to the graph of $y = \arctan(2x+1) + \frac{\pi}{4}$ for $x \in \mathbb{R}$. [3]
- (b) Show that $\arctan p + \arctan q \equiv \arctan\left(\frac{p+q}{1-pq}\right)$ where $p, q > 0$ and $pq < 1$. [4]
- (c) Verify that $\arctan(2x+1) = \arctan\left(\frac{x}{x+1}\right) + \frac{\pi}{4}$ for $x \in \mathbb{R}, x > 0$. [3]
- (d) Using mathematical induction and the result from part (b), prove that $\sum_{r=1}^n \arctan\left(\frac{1}{2r^2}\right) = \arctan\left(\frac{n}{n+1}\right)$ for $n \in \mathbb{Z}^+$. [9]

References:

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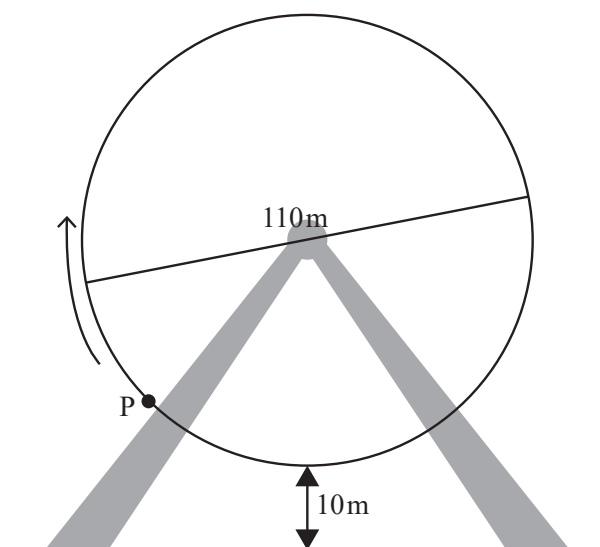
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 5]

A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes.

diagram not to scale



The height, h metres, of P above the ground after t minutes is given by $h(t) = a \cos(bt) + c$, where $a, b, c \in \mathbb{R}$.

Find the values of a , b and c .

[illegible]

Turn over

2. [Maximum mark: 28]

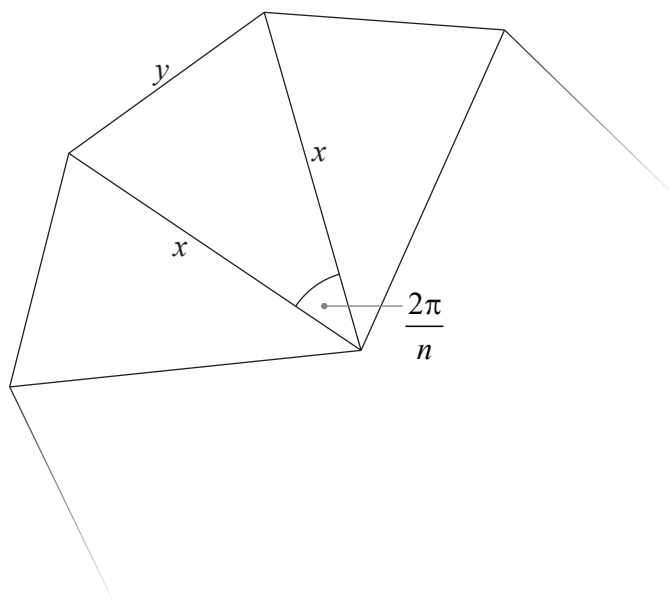
This question asks you to examine various polygons for which the numerical value of the area is the same as the numerical value of the perimeter. For example, a 3 by 6 rectangle has an area of 18 and a perimeter of 18.

For each polygon in this question, let the numerical value of its area be A and let the numerical value of its perimeter be P .

- (a) Find the side length, s , where $s > 0$, of a square such that $A = P$. [3]

An n -sided regular polygon can be divided into n congruent isosceles triangles. Let x be the length of each of the two equal sides of one such isosceles triangle and let y be the length of the third side. The included angle between the two equal sides has magnitude $\frac{2\pi}{n}$.

Part of such an n -sided regular polygon is shown in the following diagram.



- (b) Write down, in terms of x and n , an expression for the area, A_T , of one of these isosceles triangles. [1]

- (c) Show that $y = 2x \sin \frac{\pi}{n}$. [2]

Consider a n -sided regular polygon such that $A = P$.

- (d) Use the results from parts (b) and (c) to show that $A = P = 4n \tan \frac{\pi}{n}$. [7]

(This question continues on the following page)

(Question 2 continued)

The Maclaurin series for $\tan x$ is $x + \frac{x^3}{3} + \frac{2x^5}{15} + \dots$

(e) (i) Use the Maclaurin series for $\tan x$ to find $\lim_{n \rightarrow \infty} \left(4n \tan \frac{\pi}{n} \right)$. [3]

(ii) Interpret your answer to part (e)(i) geometrically. [1]

Consider a right-angled triangle with side lengths a , b and $\sqrt{a^2 + b^2}$, where $a \geq b$, such that $A = P$.

(f) Show that $a = \frac{8}{b-4} + 4$. [7]

(g) (i) By using the result of part (f) or otherwise, determine the three side lengths of the only two right-angled triangles for which a , b , A , $P \in \mathbb{Z}$. [3]

(ii) Determine the area and perimeter of these two right-angled triangles. [1]

References:

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4. [Maximum mark: 6]

A diagram of a semicircle with center O and diameter PQ . A point R is on the arc. The triangle POR is shaded gray. The angle ROQ is labeled θ . The region bounded by the arc RQ and the line segment RQ is shaded gray.

- (a) Given that the areas of the two shaded regions are equal, show that $\theta = 2 \sin \theta$. [5]
- (b) Hence determine the value of θ . [1]

[illegible]

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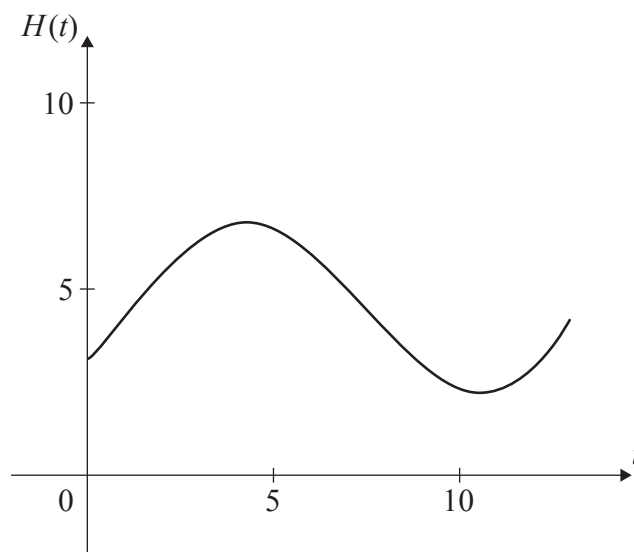
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a \sin(b(t - c)) + d$, where t is the number of hours after midnight, and a , b , c and d are constants, where $a > 0$, $b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at 04:30 and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between 2.2 m and 6.8 m.

All heights are given correct to one decimal place.

- | | |
|---|-----|
| (a) Show that $b = \frac{\pi}{6}$. | [1] |
| (b) Find the value of a . | [2] |
| (c) Find the value of d . | [2] |
| (d) Find the smallest possible value of c . | [3] |
| (e) Find the height of the water at 12:00. | [2] |
| (f) Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. | [3] |

(This question continues on the following page)



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(Question 9 continued)

A fisherman notes that the water height at nearby Folkestone harbour follows the same sinusoidal pattern as that of Dungeness harbour, with the exception that high tides (and low tides) occur 50 minutes earlier than at Dungeness.

- (g) Find a suitable equation that may be used to model the tidal height of water at Folkestone harbour.

[2]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 21]

Three points $A(3, 0, 0)$, $B(0, -2, 0)$ and $C(1, 1, -7)$ lie on the plane Π_1 .

- (a) (i) Find the vector $\vec{AB}$ and the vector $\vec{AC}$.
- (ii) Hence find the equation of Π_1 , expressing your answer in the form $ax + by + cz = d$, where $a, b, c, d \in \mathbb{Z}$. [7]

Plane Π_2 has equation $3x - y + 2z = 2$.

- (b) The line L is the intersection of Π_1 and Π_2 . Verify that the vector equation of L can be written as $\mathbf{r} = \begin{pmatrix} 0 \\ -2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. [2]

- (c) The plane Π_3 is given by $2x - 2z = 3$. The line L and the plane Π_3 intersect at the point P .

- (i) Show that at the point P , $\lambda = \frac{3}{4}$.
- (ii) Hence find the coordinates of P . [3]

- (d) The point $B(0, -2, 0)$ lies on L .

- (i) Find the reflection of the point B in the plane Π_3 .
- (ii) Hence find the vector equation of the line formed when L is reflected in the plane Π_3 . [9]

References:

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1. [Maximum mark: 6]

A diagram of a circular sector AOB with center O. The radius OA is labeled 4 cm. The chord AB is labeled 5 cm. The angle AOB is labeled θ . The sector is shaded grey.

(a) Find the value of θ , giving your answer in radians.

[3]

(b) Find the area of the shaded region.

[3]

[illegible]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 19]

(a) Show that $\cot 2\theta = \frac{1 - \tan^2 \theta}{2 \tan \theta}$. [1]

(b) Verify that $x = \tan \theta$ and $x = -\cot \theta$ satisfy the equation $x^2 + (2 \cot 2\theta)x - 1 = 0$. [7]

(c) Hence, or otherwise, show that the exact value of $\tan \frac{\pi}{12} = 2 - \sqrt{3}$. [5]

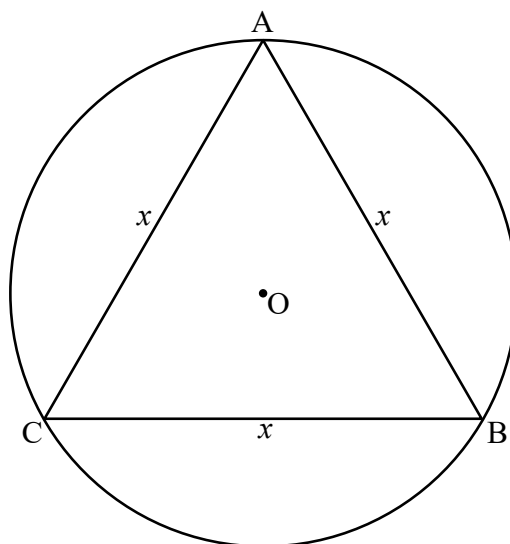
(d) Using the results from parts (b) and (c) find the exact value of $\tan \frac{\pi}{24} - \cot \frac{\pi}{24}$.
Give your answer in the form $a + b\sqrt{3}$ where $a, b \in \mathbb{Z}$. [6]



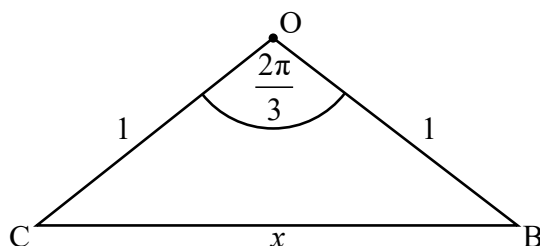
1. [Maximum mark: 30]

This question asks you to investigate regular n -sided polygons inscribed and circumscribed in a circle, and the perimeter of these as n tends to infinity, to make an approximation for π .

- (a) Consider an equilateral triangle ABC of side length, x units, inscribed in a circle of radius 1 unit and centre O as shown in the following diagram.



The equilateral triangle ABC can be divided into three smaller isosceles triangles, each subtending an angle of $\frac{2\pi}{3}$ at O, as shown in the following diagram.



Using right-angled trigonometry or otherwise, show that the perimeter of the equilateral triangle ABC is equal to $3\sqrt{3}$ units. [3]

- (b) Consider a square of side length, x units, inscribed in a circle of radius 1 unit. By dividing the inscribed square into four isosceles triangles, find the exact perimeter of the inscribed square. [3]

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(Question 1 continued)

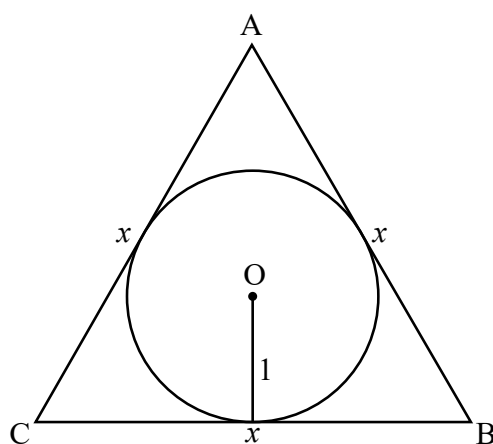
- (c) Find the perimeter of a regular hexagon, of side length, x units, inscribed in a circle of radius 1 unit. [2]

Let $P_i(n)$ represent the perimeter of any n -sided regular polygon inscribed in a circle of radius 1 unit.

- (d) Show that $P_i(n) = 2n \sin\left(\frac{\pi}{n}\right)$. [3]

- (e) Use an appropriate Maclaurin series expansion to find $\lim_{n \rightarrow \infty} P_i(n)$ and interpret this result geometrically. [5]

Consider an equilateral triangle ABC of side length, x units, circumscribed about a circle of radius 1 unit and centre O as shown in the following diagram.



Let $P_c(n)$ represent the perimeter of any n -sided regular polygon circumscribed about a circle of radius 1 unit.

- (f) Show that $P_c(n) = 2n \tan\left(\frac{\pi}{n}\right)$. [4]

- (g) By writing $P_c(n)$ in the form $\frac{2 \tan\left(\frac{\pi}{n}\right)}{\frac{1}{n}}$, find $\lim_{n \rightarrow \infty} P_c(n)$. [5]

- (h) Use the results from part (d) and part (f) to determine an inequality for the value of π in terms of n . [2]

The inequality found in part (h) can be used to determine lower and upper bound approximations for the value of π .

- (i) Determine the least value for n such that the lower bound and upper bound approximations are both within 0.005 of π . [3]

Turn over

3. [Maximum mark: 7]

(a) Find $(f \circ g)(x)$. [2]

(b) Solve the equation $(f \circ g)(x) = 2 \cos 2x$ where $0 \leq x \leq \pi$. [5]

[illegible]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 15]

Consider the three planes

$$\Pi_1 : 2x - y + z = 4$$

$$\Pi_2 : x - 2y + 3z = 5$$

$$\Pi_3 : -9x + 3y - 2z = 32$$

- (a) Show that the three planes do not intersect. [4]
- (b) (i) Verify that the point $P(1, -2, 0)$ lies on both Π_1 and Π_2 .
- (ii) Find a vector equation of L , the line of intersection of Π_1 and Π_2 . [5]
- (c) Find the distance between L and Π_3 . [6]



— 12 —

Turn over

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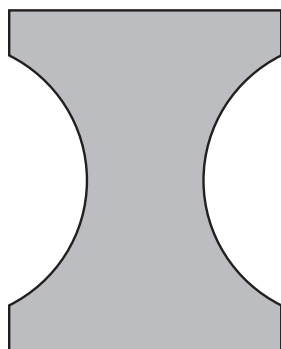
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

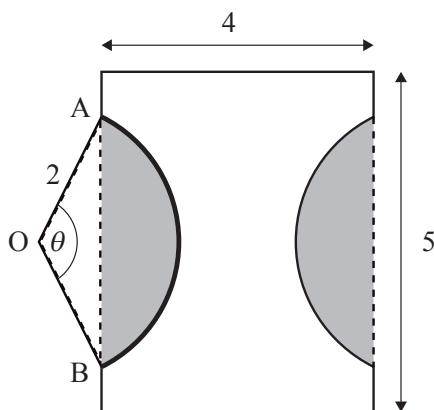
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures 5 cm by 4 cm. The points A and B lie on a circle, with centre O and radius 2 cm, such that $\angle AOB = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



- (a) Find the area of one of the shaded segments in terms of θ . [3]
- (b) Given that the area of the logo is 13.4 cm^2 , find the value of θ . [3]

(This question continues on the following page)



(Question 2 continued)

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Turn over

7. [Maximum mark: 9]

(a) Find the possible range of values for $|a + b|$. [2]

(b) Given that $|a + b|$ is a minimum, find p . [2]

(c) Find $\mathbf{q}$ such that $|\mathbf{q}| = |\mathbf{b}|$ and $\mathbf{q}$ is perpendicular to $\mathbf{a}$. [5]

[illegible]

Section A

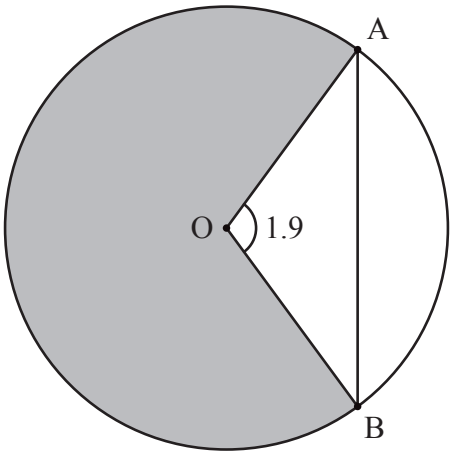
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $\widehat{AOB} = 1.9$ radians.

diagram not to scale



- (a) Find the length of the chord $[AB]$.

[3]
- (b) Find the area of the shaded sector.

[3]

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Turn over

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 20]

Two airplanes, A and B , have position vectors with respect to an origin O given respectively by

$$\mathbf{r}_A = \begin{pmatrix} 19 \\ -1 \\ 1 \end{pmatrix} + t \begin{pmatrix} -6 \\ 2 \\ 4 \end{pmatrix}$$

$$\mathbf{r}_B = \begin{pmatrix} 1 \\ 0 \\ 12 \end{pmatrix} + t \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}$$

where t represents the time in minutes and $0 \leq t \leq 2.5$.

Entries in each column vector give the displacement east of O , the displacement north of O and the distance above sea level, all measured in kilometres.

- (a) Find the three-figure bearing on which airplane B is travelling. [2]
- (b) Show that airplane A travels at a greater speed than airplane B . [2]
- (c) Find the acute angle between the two airplanes' lines of flight. Give your answer in degrees. [4]

The two airplanes' lines of flight cross at point P .

- (d) (i) Find the coordinates of P .
- (ii) Determine the length of time between the first airplane arriving at P and the second airplane arriving at P . [7]

Let $D(t)$ represent the distance between airplane A and airplane B for $0 \leq t \leq 2.5$.

- (e) Find the minimum value of $D(t)$. [5]



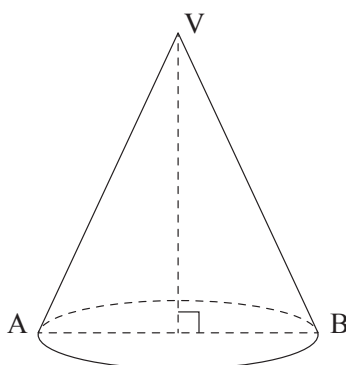
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2. [Maximum mark: 7]

(a) Find

- [4]

diagram not to scale



- [3]

[illegible]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

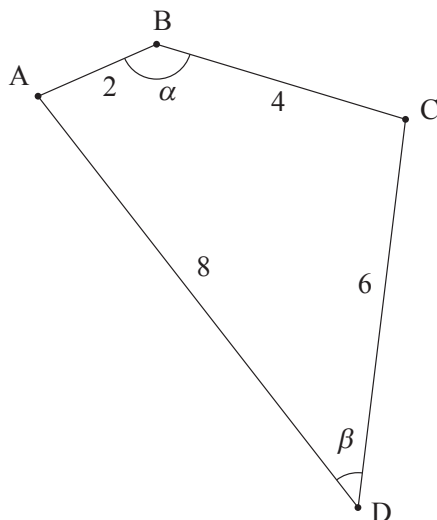
10. [Maximum mark: 20]

The function f is defined by $f(x) = \cos^2 x - 3 \sin^2 x$, $0 \leq x \leq \pi$.

- (a) Find the roots of the equation $f(x) = 0$. [5]
- (b) (i) Find $f'(x)$.
- (ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]
- (c) Sketch the graph of $y = |f(x)|$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [4]
- (d) Hence or otherwise, solve the inequality $|f(x)| > 1$. [4]

9. [Maximum mark: 8]

diagram not to scale



- (a) (i) Find AC in terms of α .
- (ii) Find AC in terms of β .
- (iii) Hence or otherwise, find an expression for α in terms of β . [4]
- (b) Find the maximum area of the quadrilateral ABCD. [4]

[illegible]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 22]

Consider the points $A(1, 2, 3)$, $B(k, -2, 1)$ and $C(5, 0, 2)$, where $k \in \mathbb{R}$.

(a) Write down $\vec{AB}$ and $\vec{AC}$. [2]

(b) Given that the points A , B and C lie on a straight line, show that $k = 9$. [1]

(c) For $k = 9$, let L_1 be the line passing through A , B and C .

(i) Find a vector equation of the line L_1 .

(ii) Line L_2 has the equation $\frac{x-1}{2} = \frac{y}{3} = 1-z$. Show that the lines L_1 and L_2 are skew. [10]

(d) For $k \neq 9$, let Π be the plane containing A , B and C .

(i) Find the Cartesian equation of the plane Π .

(ii) Find the coordinates of the point on the plane Π which is closest to the origin $(0, 0, 0)$. [9]

References:

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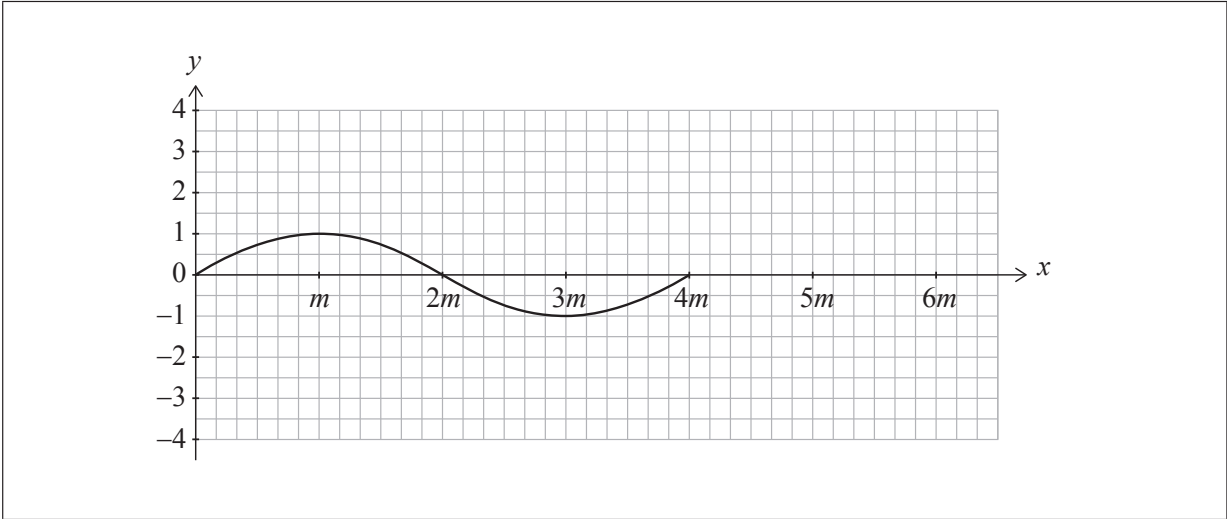


Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

5. [Maximum mark: 6]

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \leq x \leq 4m$, where x is in radians. There are x -intercepts at $x = 0, 2m$ and $4m$.



(a) Find an expression for m in terms of q . [2]

The function g is defined by $g(x) = 3\sin \frac{2qx}{3}$, for $0 \leq x \leq 6m$.

(b) On the axes above, sketch the graph of g . [4]

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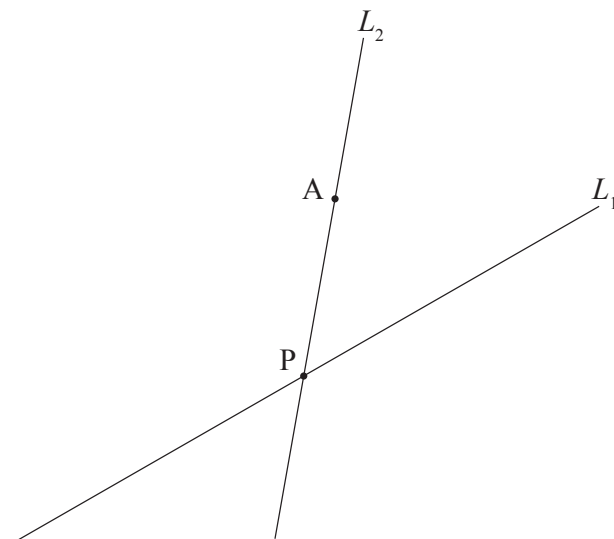
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 21]

Two lines, L_1 and L_2 , intersect at point P. Point A($2t$, 8, 3), where $t > 0$, lies on L_2 . This is shown in the following diagram.

diagram not to scale



The acute angle between the two lines is $\frac{\pi}{3}$.

The direction vector of L_1 is $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\vec{PA} = \begin{pmatrix} 2t \\ 0 \\ 3+t \end{pmatrix}$.

(a) Show that $4t = \sqrt{10t^2 + 12t + 18}$. [4]

(b) Find the value of t . [4]

(c) Hence or otherwise, find the shortest distance from A to L_1 . [4]

A plane, Π , contains L_1 and L_2 .

(d) Find a normal vector to Π . [2]

The base of a right cone lies in Π , centred at A such that L_1 is a tangent to its base. The volume of the cone is $90\pi\sqrt{3}$ cubic units.

(e) Find the two possible positions of the vertex of the cone. [7]

References:

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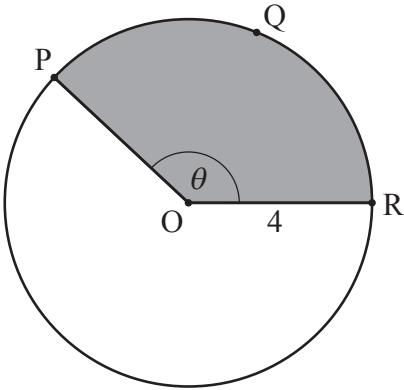
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 4 cm .

diagram not to scale



The points P , Q and R lie on the circumference of the circle and $\widehat{POR} = \theta$, where θ is measured in radians.

The length of arc PQR is 10 cm .

- (a) Find the perimeter of the shaded sector. [2]
- (b) Find θ . [2]
- (c) Find the area of the shaded sector. [2]

(This question continues on the following page)



[illegible]

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

8. [Maximum mark: 7]

The functions f and g are defined by

$$f(x) = \cos x, \quad 0 \leq x \leq \frac{\pi}{2}$$
$$g(x) = \tan x, \quad 0 \leq x < \frac{\pi}{2}.$$

The curves $y = f(x)$ and $y = g(x)$ intersect at a point P whose x -coordinate is k , where $0 < k < \frac{\pi}{2}$.

- (a) Show that $\cos^2 k = \sin k$. [1]
- (b) Hence, show that the tangent to the curve $y = f(x)$ at P and the tangent to the curve $y = g(x)$ at P intersect at right angles. [3]
- (c) Find the value of $\sin k$. Give your answer in the form $\frac{a + \sqrt{b}}{c}$, where $a, c \in \mathbb{Z}$ and $b \in \mathbb{Z}^+$. [3]



Turn over

9. [Maximum mark: 9]

The diagram shows a parallelogram with vertices O, A, B, and C. O is the origin. A is on the horizontal axis. B is at the top right. C is at the bottom right. M is a point on the segment AB. A line segment OM is drawn from O to M. A line segment MC is drawn from M to C. A right angle symbol is shown at M, indicating that OM is perpendicular to MC. The angle AOM is labeled θ .

(c) Find the range of values for θ such that there are two possible positions for M. [4]

[illegible]

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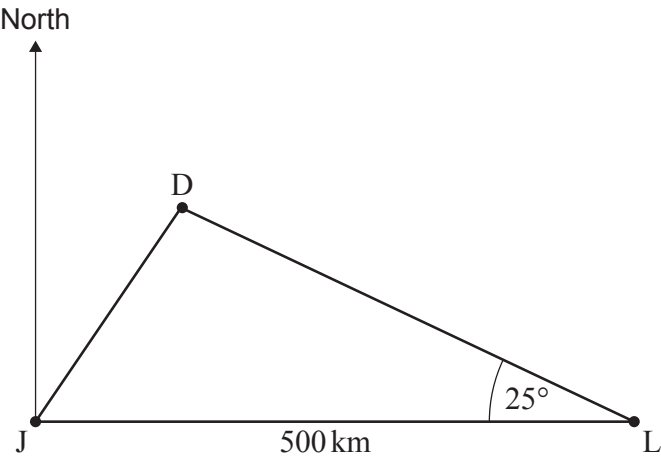
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The cities Lucknow (L), Jaipur (J) and Delhi (D) are represented in the following diagram. Lucknow lies 500 km directly east of Jaipur, and $\angle JDL = 25^\circ$.

diagram not to scale



The bearing of D from J is 034° .

- (a) Find $\angle JDL$.

[2]
- (b) Find the distance between Lucknow and Delhi.

[3]

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Turn over

[illegible]

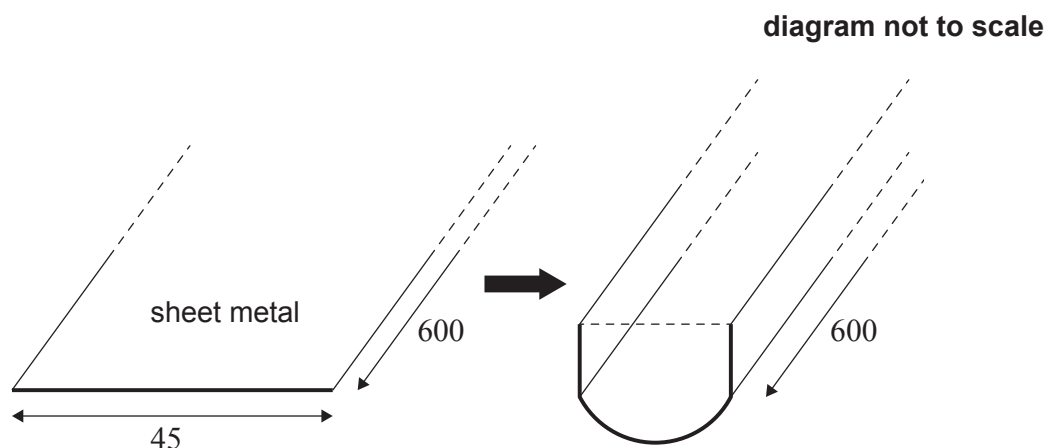
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

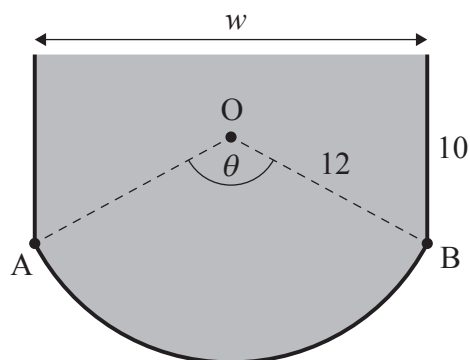
10. [Maximum mark: 15]

An engineer is designing a gutter to catch rainwater from the roof of a house.

The gutter will be open at the top and is made by folding a piece of sheet metal 45 cm wide and 600 cm long.



The cross-section of the gutter is shaded in the following diagram.



The height of both vertical sides is 10 cm. The width of the gutter is w cm.

Arc AB lies on the circumference of a circle with centre O and radius 12 cm.

(This question continues on the following page)



Do **not** write solutions on this page.

(Question 10 continued)

Let $\widehat{AOB} = \theta$ radians, where $0 < \theta < \pi$.

- (a) Show that $\theta = 2.08$, correct to three significant figures. [3]
- (b) Find the area of the cross-section of the gutter. [7]

In a storm, the total volume, in cm^3 , of rainwater that enters the gutter can be modelled by a function $R(t)$, where t is the time, in seconds, since the start of the storm.

It was determined that the **rate** at which rainwater entered the gutter could be modelled by

$$R'(t) = 50 \cos\left(\frac{2\pi t}{5}\right) + 3000, \quad t \geq 0.$$

During any 60-second period, if the volume of rainwater entering the gutter is greater than the volume of the gutter, it will overflow.

- (c) Determine whether the gutter overflowed in this storm. Justify your answer. [5]



Turn over

— 17 —

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Section A

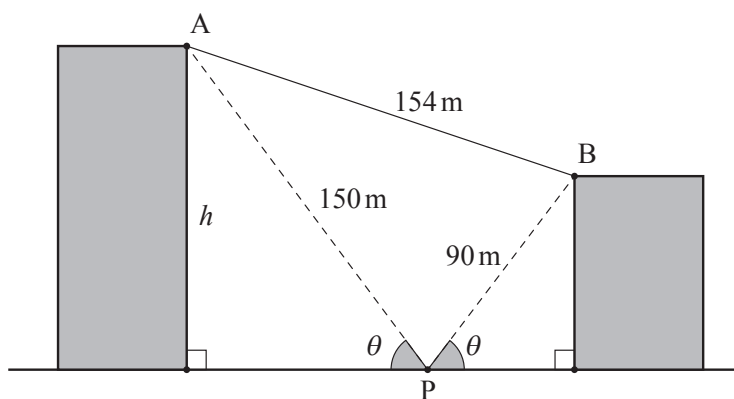
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

2. [Maximum mark: 6]

The following diagram shows two buildings situated on level ground.

From point P on the ground directly between the two buildings, the angle of elevation to the top of each building is θ .

diagram not to scale



The distance from point P to point A at the top of the taller building is 150 metres.

The distance from point P to point B at the top of the shorter building is 90 metres.

The distance between A and B is 154 metres.

Find the height, h , of the taller building.

[illegible]

Turn over

6. [Maximum mark: 8]

$$\Pi_1 : 2x - y + 2z = 6$$

$$P_7: 4x + 3y - z = 2$$

(a) Verify that a vector equation of L is $\mathbf{r} = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$, where $\lambda \in \mathbb{R}$. [3]

(b) Find the coordinates of the point P on L that is nearest to the origin. [5]

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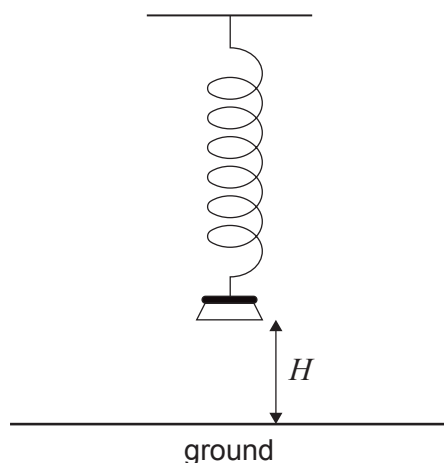
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 13]

A weight suspended on a spring is pulled down and released, so that it moves up and down vertically.

The height, H metres, of the base of the weight above the ground can be modelled by the function $H(t) = a \cos(7.8t) + b$, for $a, b \in \mathbb{R}$ and $0 \leq t \leq 10$, where t is the time in seconds after the weight is released.



(a) Find the period of the function.

[2]

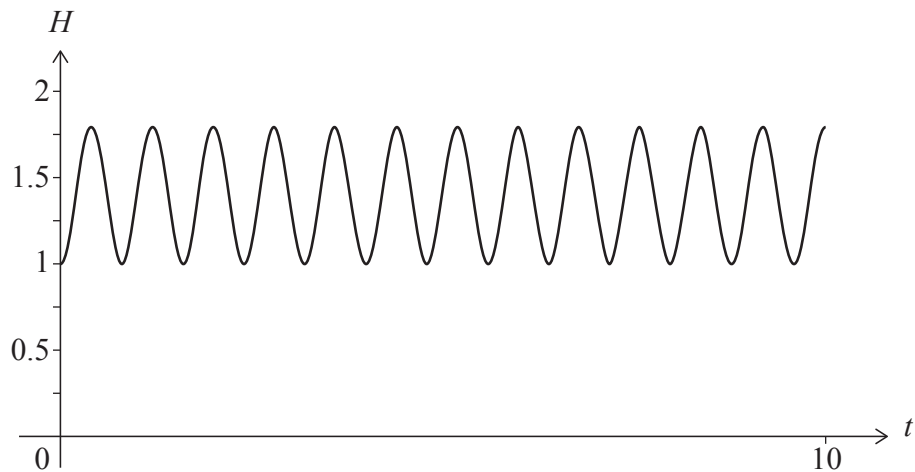
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(Question 10 continued)

The weight is released when its base is at a minimum height of 1 metre above the ground, and it reaches a maximum height of 1.8 metres above the ground. The graph of H is shown in the following diagram.



- (b) Find the value of

(i) a ;

(ii) b .

[3]
- (c) Find the number of times that the weight reaches its maximum height in the first five seconds of its motion.

[2]
- (d) Find the first time that the base of the weight reaches a height of 1.5 metres.

[2]
- A camera is set to take a picture of the weight at a random time during the first five seconds of its motion.

(e) Find the probability that the height of the base of the weight is greater than 1.5 metres at the time the picture is taken.

[4]



— 17 —

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 17]

(a) Find the binomial expansion of $(\cos \theta + i \sin \theta)^5$. Give your answer in the form $a + bi$ where a and b are expressed in terms of $\sin \theta$ and $\cos \theta$. [4]

(b) By using De Moivre's theorem and your answer to part (a), show that $\sin 5\theta \equiv 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$. [6]

(c) (i) Hence, show that $\theta = \frac{\pi}{5}$ and $\theta = \frac{3\pi}{5}$ are solutions of the equation $16 \sin^4 \theta - 20 \sin^2 \theta + 5 = 0$.

(ii) Hence, show that $\sin \frac{\pi}{5} \sin \frac{3\pi}{5} = \frac{\sqrt{5}}{4}$. [7]



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will not be marked.



Section B

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Section A

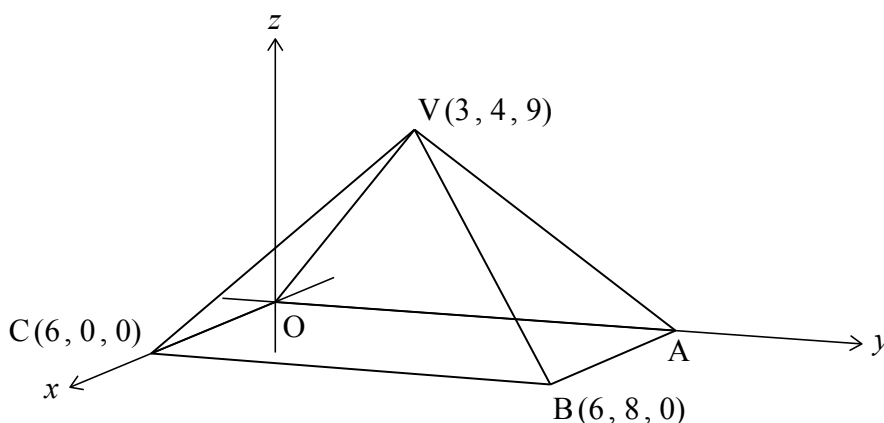
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a pyramid with vertex V and rectangular base $OABC$.

Point B has coordinates $(6, 8, 0)$, point C has coordinates $(6, 0, 0)$ and point V has coordinates $(3, 4, 9)$.

diagram not to scale



- (a) Find BV . [2]
- (b) Find the size of $B\hat{V}C$. [4]

[illegible]

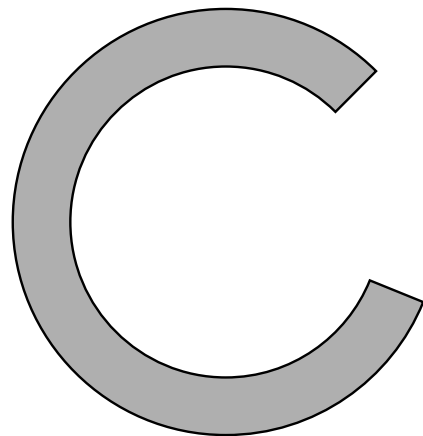
Turn over

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 7]

A company is designing a new logo in the shape of a letter “C”.



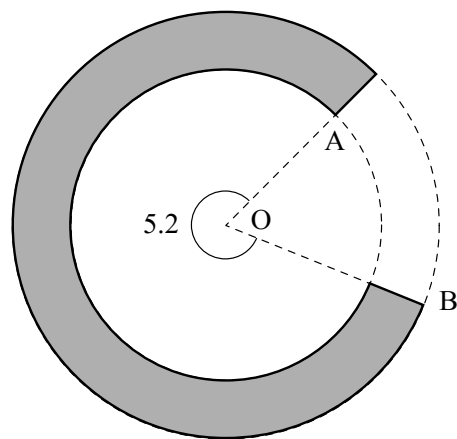
The letter “C” is formed between two circles with centre O .

The point A lies on the circumference of the inner circle with radius $r\text{ cm}$, where $r < 10$.

The point B lies on the circumference of the outer circle with radius 10 cm .

The reflex angle $\widehat{AOB}$ is 5.2 radians. The letter “C” is shown by the shaded area in the following diagram.

diagram not to scale



(This question continues on the following page)



8. [Maximum mark: 9]

(a) Show that $\vec{AB} \times \vec{AC} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$. [4]

(b) Hence, find the smallest possible value of $\left| \vec{AB} \times \vec{AC} \right|^2$. [3]

(c) Hence, find the smallest possible area of triangle ABC . [2]

[illegible]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 18]

Line L is given by the vector equation $\mathbf{r}_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ where $s \in \mathbb{R}$.

Line M is given by the vector equation $\mathbf{r}_2 = \begin{pmatrix} 9 \\ 9 \\ 11 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ where $t \in \mathbb{R}$.

(a) Show that lines L and M intersect at a point A and find the position vector of A . [5]

(b) Verify that the lines L and M both lie in the plane Π given by $\mathbf{r} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$. [3]

Point B has position vector $\begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix}$. A line through B perpendicular to Π intersects Π at point C .

(c) (i) Find the position vector of C .

(ii) Hence, find $|\vec{BC}|$. [7]

(d) Find the reflection of the point B in the plane Π . [3]



Section A

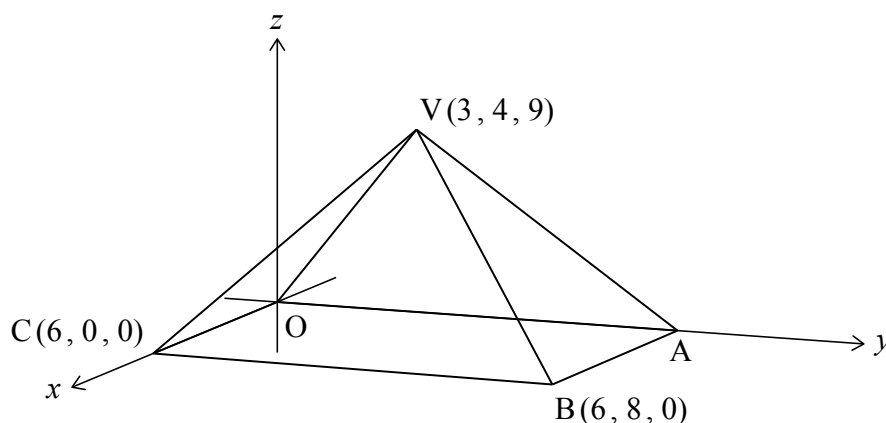
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

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Point B has coordinates $(6, 8, 0)$, point C has coordinates $(6, 0, 0)$ and point V has coordinates $(3, 4, 9)$.

diagram not to scale



- Find BV . [2]
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[illegible]

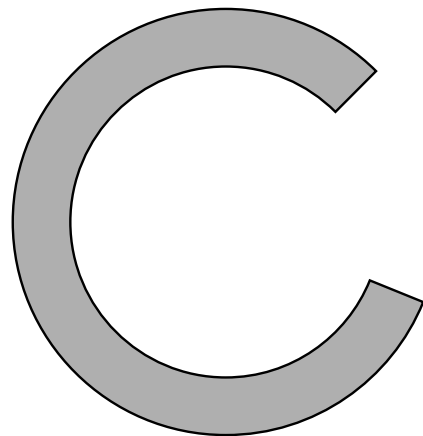
Turn over

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 7]

A company is designing a new logo in the shape of a letter “C”.



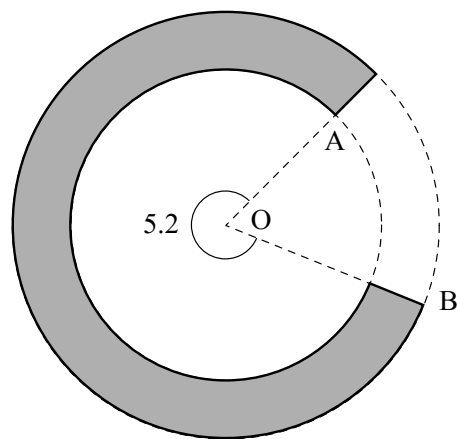
The letter “C” is formed between two circles with centre O .

The point A lies on the circumference of the inner circle with radius $r\text{ cm}$, where $r < 10$.

The point B lies on the circumference of the outer circle with radius 10 cm .

The reflex angle $\widehat{AOB}$ is 5.2 radians. The letter “C” is shown by the shaded area in the following diagram.

diagram not to scale



(This question continues on the following page)



8. [Maximum mark: 9]

(a) Show that $\vec{AB} \times \vec{AC} = \begin{pmatrix} 2-3p \\ -2-p \\ p^2-2p \end{pmatrix}$. [4]

(b) Hence, find the smallest possible value of $\left| \vec{AB} \times \vec{AC} \right|^2$. [3]

(c) Hence, find the smallest possible area of triangle ABC. [2]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 18]

Line L is given by the vector equation $\mathbf{r}_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + s \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ where $s \in \mathbb{R}$.

Line M is given by the vector equation $\mathbf{r}_2 = \begin{pmatrix} 9 \\ 9 \\ 11 \end{pmatrix} + t \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$ where $t \in \mathbb{R}$.

(a) Show that lines L and M intersect at a point A and find the position vector of A . [5]

(b) Verify that the lines L and M both lie in the plane Π given by $\mathbf{r} \cdot \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix} = 7$. [3]

Point B has position vector $\begin{pmatrix} -3 \\ 12 \\ 2 \end{pmatrix}$. A line through B perpendicular to Π intersects Π at point C .

(c) (i) Find the position vector of C .

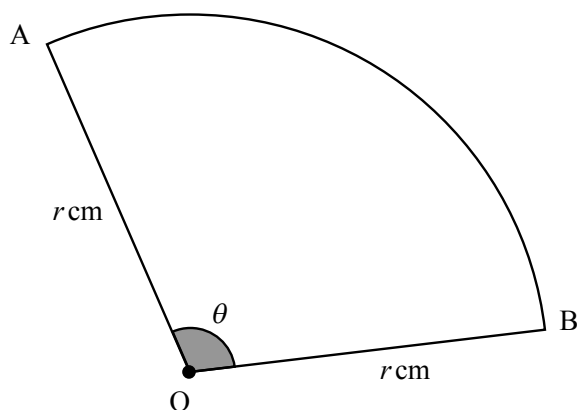
(ii) Hence, find $|\vec{BC}|$. [7]

(d) Find the reflection of the point B in the plane Π . [3]



3. [Maximum mark: 8]

diagram not to scale



(a) Show that $4r^2 - 20r + 25 = 0$. [4]

(b) Hence, or otherwise, find the value of r and the value of θ . [4]

[illegible]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 19]

The plane Π_1 has equation $2x + 6y - 2z = 5$.

- (a) Verify that the point $A\left(2, \frac{1}{2}, 1\right)$ lies on the plane Π_1 . [1]

The plane Π_2 is given by $(k^2 - 6)x + (2k + 3)y + pz = q$, where $p, q, k \in \mathbb{R}$ and $p \neq 0$.

- (b) In the case where $p = -6$, Π_2 is perpendicular to Π_1 and A lies on Π_2 .
Find the value of k and the value of q . [5]

For parts (c), (d) and (e) it is now given that Π_2 is parallel to Π_1 with $k = 3$.

- (c) Determine the value of p . [2]

It is also given that $q = -\frac{51}{2}$.

The line through A that is perpendicular to Π_1 meets Π_2 at the point B.

- (d) (i) Find the coordinates of B.
(ii) Hence, show that the perpendicular distance between Π_1 and Π_2 is $\sqrt{11}$. [7]
- (e) Find the equation of a third parallel plane Π_3 which is also a perpendicular distance of $\sqrt{11}$ from Π_1 . [4]

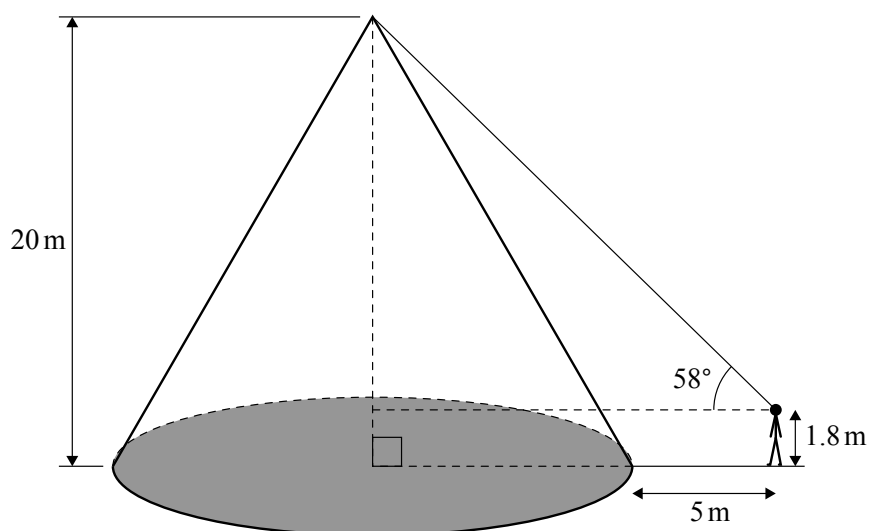


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2. [Maximum mark: 5]

diagram not to scale



- (a) Find the radius of the base of the cone. [3]
- (b) Find the volume of the monument. [2]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

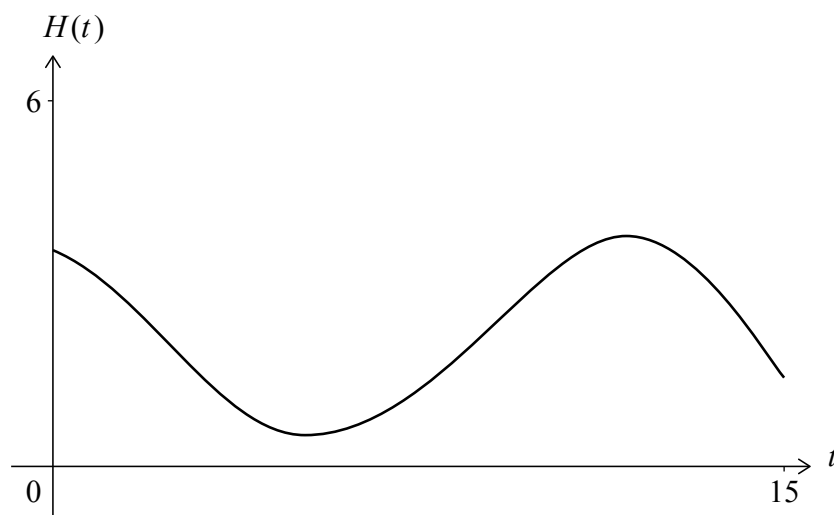
Sule Skerry and Rockall are small islands in the Atlantic Ocean, in the same time zone.

On a given day, the height of water in metres at Sule Skerry is modelled by the function $H(t) = 1.63 \sin(0.513(t - 8.20)) + 2.13$, where t is the number of hours after midnight.

The following graph shows the height of the water for 15 hours, starting at midnight.

At low tide the height of the water is 0.50 m. At high tide the height of the water is 3.76 m.

All heights are given correct to two decimal places.



- (a) The length of time between the first low tide and the first high tide is 6 hours and m minutes. Find the value of m to the nearest integer. [3]
- (b) Between two consecutive high tides, determine the length of time, in hours, for which the height of the water is less than 1 metre. [2]
- (c) Find the rate of change of the height of the water when $t = 13$, giving your answer in metres per hour. [2]

(This question continues on the following page)



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(Question 10 continued)

On the same day, the height of water at the second island, Rockall, is modelled by the function $h(t) = a \sin(b(t - c)) + d$, where t is the number of hours after midnight, and $a, b, c, d > 0$.

The first low tide occurs at 02:41 when the height of the water is 0.40 m.

The first high tide occurs at 09:02 when the height of the water is 2.74 m.

(d) Find the values of a , b , c and d . [7]

When $t = T$, the height of the water at Sule Skerry is the same as the height of the water at Rockall for the first time.

(e) Find the value of T . [2]



Turn over

— 17 —

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 20]

Consider the non-zero vectors $\mathbf{u}$ and $\mathbf{v}$. Let θ be the angle between $\mathbf{u}$ and $\mathbf{v}$.

- (a) Using the definitions of $\mathbf{u} \cdot \mathbf{v}$ and $\mathbf{u} \times \mathbf{v}$ in terms of $|\mathbf{u}|$, $|\mathbf{v}|$ and θ , show that $(\mathbf{u} \cdot \mathbf{v})^2 + |\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2 |\mathbf{v}|^2$. [2]

A triangle ABC has vertices A(0, 1, 2), B(p , q , 3) and C(3, 2, 1), $p, q \in \mathbb{Q}$.

The vectors $\mathbf{u}$ and $\mathbf{v}$ are defined as $\mathbf{u} = \vec{AB}$ and $\mathbf{v} = \vec{AC}$.

It is given that $\mathbf{u} \cdot \mathbf{v} = 3$ and the area of triangle ABC is $\sqrt{6}$.

- (b) (i) Find the value of $|\mathbf{u} \times \mathbf{v}|$.
 (ii) Hence, or otherwise, find the value of $|\mathbf{u}|$.
 (iii) Hence, or otherwise, find the possible values of p and the corresponding values of q . [13]

Consider a new point D, the vector $\mathbf{w}$ is defined as $\mathbf{w} = \vec{CD}$.

It is given that $\mathbf{u} \cdot \mathbf{w} = \mathbf{v} \cdot \mathbf{w} = 0$ and the area of triangle ACD is 5 square units.

- (c) Assuming that $p = 1$, find the possible vectors for $\mathbf{w}$. [5]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 17]

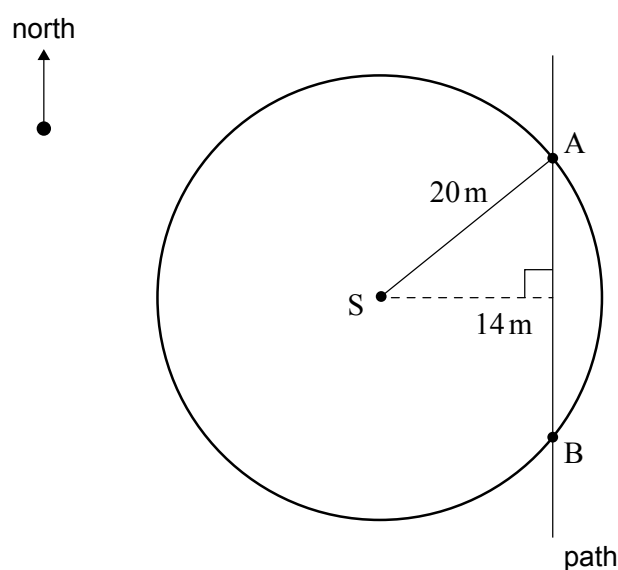
A rotating sprinkler is at a fixed point S .

It waters all points inside and on a circle of radius 20 metres.

Point S is 14 metres from the edge of a path which runs in a north-south direction.

The edge of the path intersects the circle at points A and B .

This information is shown in the following diagram.



- (a) Show that $AB = 28.57$, correct to four significant figures. [3]

The sprinkler rotates at a constant rate of one revolution every 16 seconds.

- (b) Show that the sprinkler rotates through an angle of $\frac{\pi}{8}$ radians in one second. [1]

Let T seconds be the time that $[AB]$ is watered in each revolution.

- (c) Find the value of T . [4]

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(Question 11 continued)

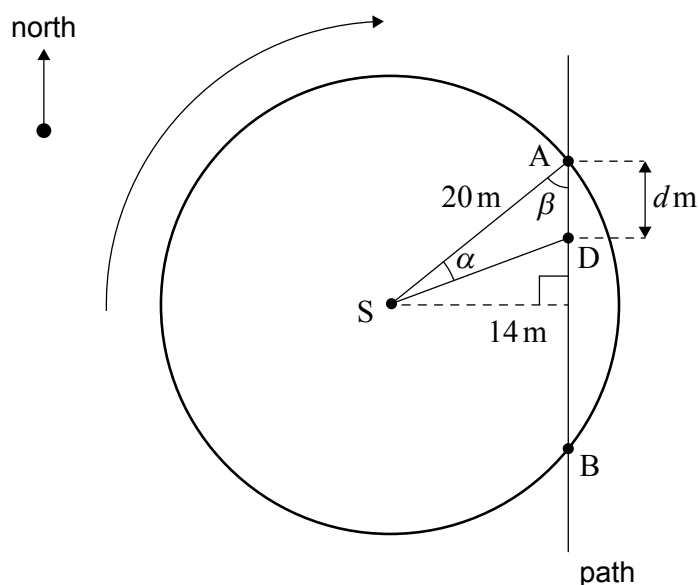
Consider one clockwise revolution of the sprinkler.

At $t = 0$, the water crosses the edge of the path at A.

At time t seconds, the water crosses the edge of the path at a movable point D which is a distance d metres south of point A.

Let $\alpha = \widehat{ASD}$ and $\beta = \widehat{SAB}$, where α, β are measured in radians.

This information is shown in the following diagram.



- (d) Write down an expression for α in terms of t .

[1]

It is known that $\beta = 0.7754$ radians, correct to four significant figures.

- (e) By using the sine rule in $\triangle ASD$, show that the distance, d , at time t , can be modelled by

$$d(t) = \frac{20 \sin\left(\frac{\pi t}{8}\right)}{\sin\left(2.37 - \frac{\pi t}{8}\right)}.$$

[3]

(This question continues on the following page)



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— 17 —

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(Question 11 continued)

A turtle walks south along the edge of the path.

At time t seconds, the turtle's distance, g metres south of A, can be modelled by

$$g(t) = 0.05t^2 + 1.1t + 18, \text{ where } t \geq 0.$$

- (f) At $t = 0$, state how far south the turtle is from A. [1]

Let w represent the distance between the turtle and point D at time t seconds.

- (g) (i) Use the expressions for $g(t)$ and $d(t)$ to write down an expression for w in terms of t .
 (ii) Hence find when and where on the path the water first reaches the turtle. [4]



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Section A

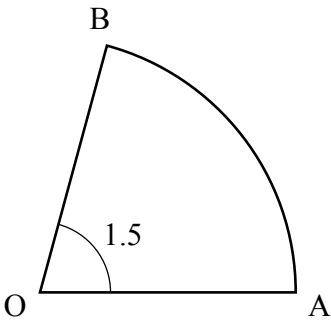
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius r cm, where $\widehat{AOB} = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is 48 cm^2 .

- (a) Find the value of r . [3]
- (b) Hence, find the perimeter of sector OAB . [2]

(This question continues on the following page)



(Question 1 continued)

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Turn over

8. [Maximum mark: 8]

It is given that $|\mathbf{a}| = 1$, $|\mathbf{b}| = 2$ and $\mathbf{a} \cdot \mathbf{b} = \frac{1}{4}$.

(b) In the case that $\vec{OP}$ is perpendicular to $\vec{AB}$, find the value of λ . [7]

[illegible]

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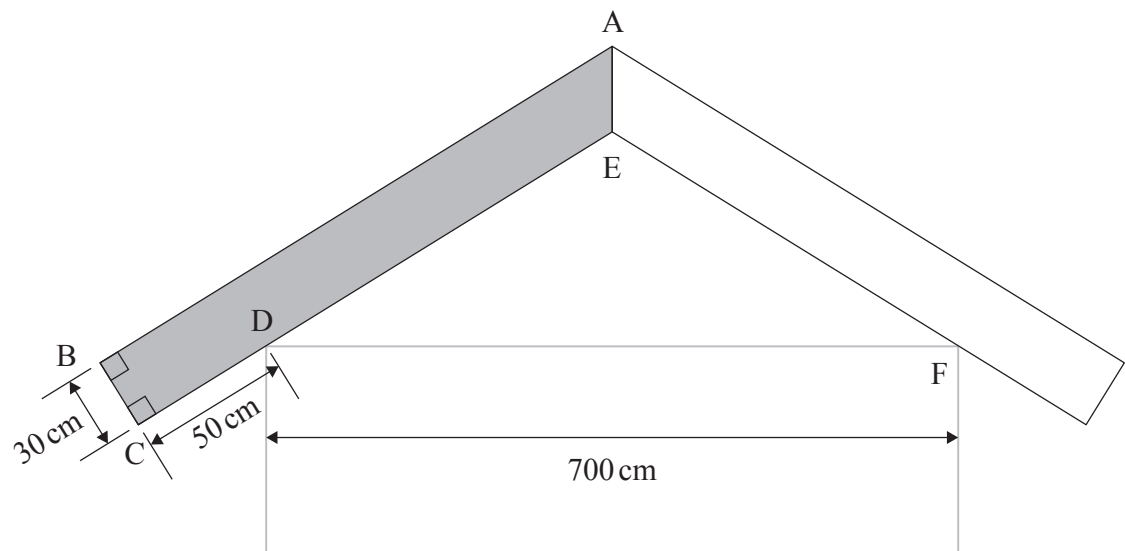
(Question 4 continued)

A builder requires the lengths of the sides [BA] and [CE].

The builder has the following measurements:

$\hat{ABC} = \hat{BCE} = 90^\circ$, $DC = 50\text{ cm}$, $BC = 30\text{ cm}$, and $DF = 700\text{ cm}$.

diagram not to scale



(b) Find

(i) CE;

(ii) BA.

[5]

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Turn over

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 18]

A line L is defined by $L: -\frac{x}{2} + 1 = y + 4 = \frac{z}{3}$.

(a) Find the equation of L , expressing your answer in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$, where $\lambda \in \mathbb{R}$. [3]

(b) Determine the minimum distance from the origin O to the line L . [5]

A plane Π is defined by $\Pi: 6x - 3y + 5z = 24$.

(c) Verify that Π contains L . [3]

A second line M is parallel to Π .

The line M passes through the point $(4, 1, 2)$ and intersects the z -axis.

(d) Find the equation of M , expressing your answer in the form $\mathbf{s} = \mathbf{c} + \mu \mathbf{d}$, where $\mu \in \mathbb{R}$. [7]



Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 17]

The plane Π_1 has equation $x + 2y + z = 0$ and the plane Π_2 has equation $x - y - 2z = 0$.

The acute angle between the planes Π_1 and Π_2 is θ .

(a) Show that $\theta = 60^\circ$. [6]

A third plane Π_3 is perpendicular to both Π_1 and Π_2 .

The unique point of intersection of all three planes is the point $R(5, -5, 5)$.

(b) Find the Cartesian equation of Π_3 . [4]

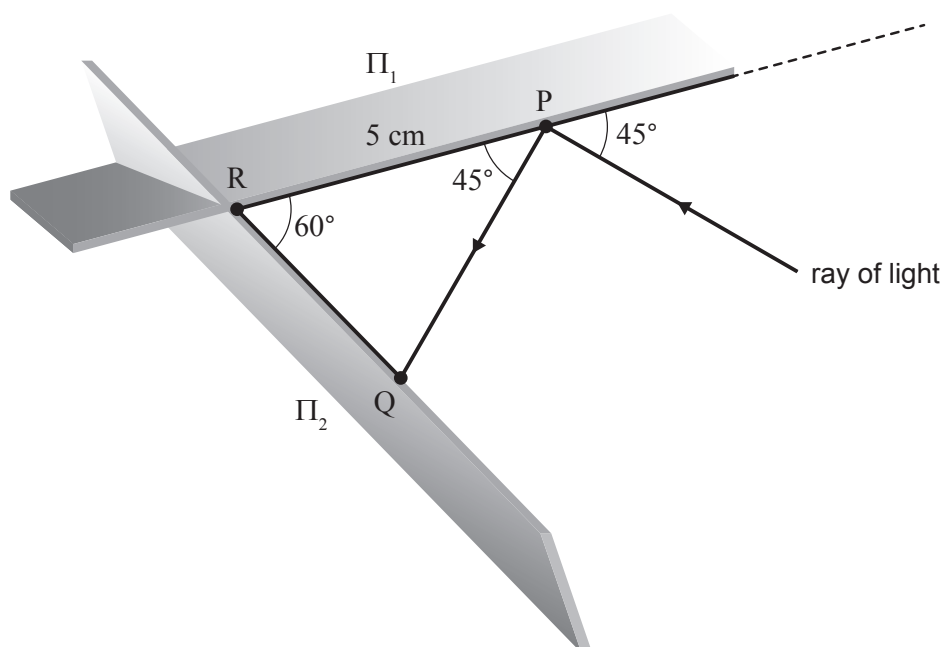
Each of the planes Π_1 and Π_2 contains a mirror.

A ray of light is directed towards the mirror in Π_1 . The ray of light forms an angle of 45° with Π_1 and meets it at the point P .

The ray of light is then reflected towards the mirror in Π_2 , and meets Π_2 at the point Q . The points P and Q are contained in Π_3 .

It is given that $PR = 5$ cm.

This information is shown on the following diagram.



(This question continues on the following page)



Do **not** write solutions on this page.

(Question 11 continued)

(c) (i) Using an appropriate compound angle identity, show that $\sin 75^\circ = \frac{\sqrt{2} + \sqrt{6}}{4}$.

(ii) Find QR, giving your answer in the form $p(\sqrt{q} - 1)$ cm where $p, q \in \mathbb{Z}$. [7]



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2. [Maximum mark: 5]

(a) Find a vector equation of L_1 .

[2]

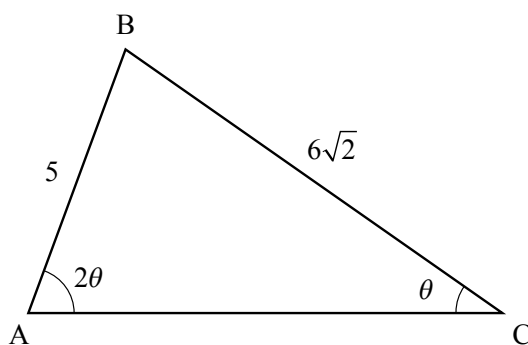
(b) Find the coordinates of the point where L_1 and L_2 intersect.

[3]

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3. [Maximum mark: 7]

diagram not to scale



(b) Hence, find $\sin \theta$. [2]

(c) Find DC. [2]

[illegible]

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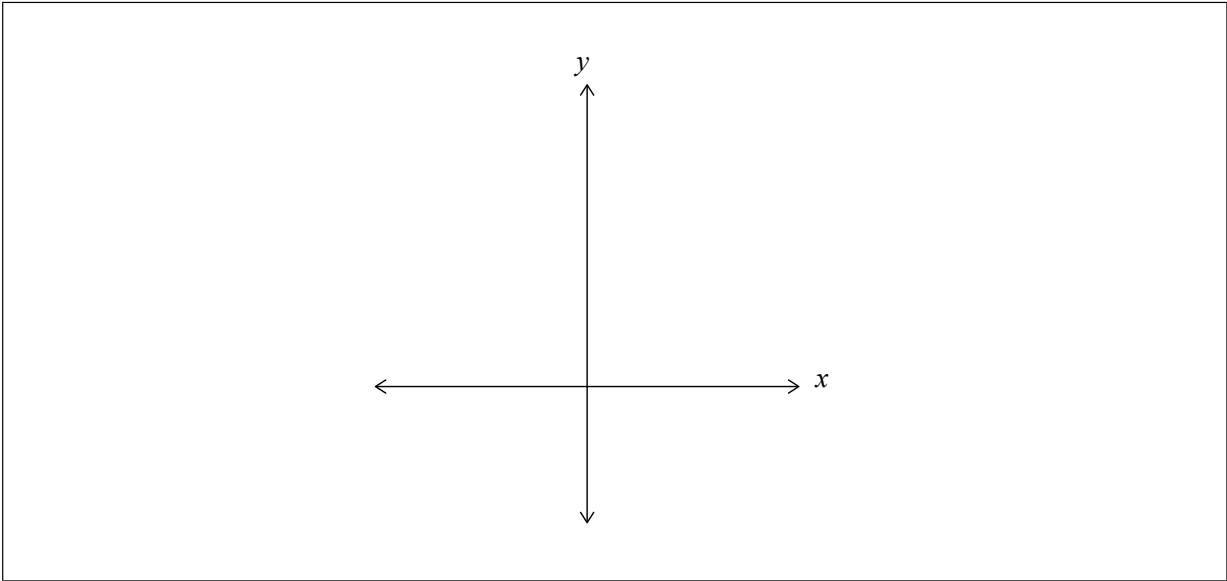
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

8. [Maximum mark: 8]

Consider the function $f(x) = \arccos x$ for $-1 \leq x \leq 1$.

- (a) On the set of axes below sketch the graph of $y = f(x)$.
On your sketch clearly indicate the y -intercept and coordinates of the end points. [2]



- (b) Solve $\arccos(x) + \arccos(x\sqrt{3}) = \frac{3\pi}{2}$, for $-\frac{1}{\sqrt{3}} \leq x \leq \frac{1}{\sqrt{3}}$. [6]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 18]

The points $A(1, -4, 0)$, $B(-3, -6, 2)$, $C(-1, -2, 4)$ and D form a parallelogram, $ABCD$, where D is diagonally opposite B .

(a) Find the coordinates of D . [2]

The diagonals of the parallelogram, $[AC]$ and $[BD]$, intersect at point E .

(b) Find the coordinates of E . [2]

(c) (i) Given that $\vec{AB} \times \vec{AD} = m \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$, where $m \in \mathbb{Z}^+$, find the value of m .

(ii) Hence, find the area of parallelogram $ABCD$. [4]

The plane, Π_1 , contains the parallelogram $ABCD$.

(d) Find the Cartesian equation of Π_1 . [2]

A second plane, Π_2 , has Cartesian equation $5x + y - 7z = 1$.

The acute angle between Π_1 and Π_2 is θ .

(e) Show that $\cos \theta = \frac{1}{5}$. [3]

The line L passes through E and is perpendicular to Π_1 .

The line L intersects the plane Π_2 at point F .

(f) Find the coordinates of F . [5]



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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

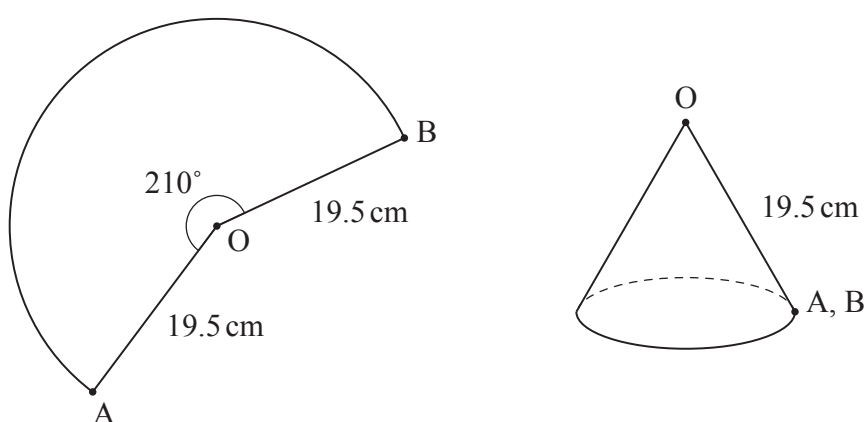
1. [Maximum mark: 6]

The points A and B lie on a circle, with centre O and radius 19.5 cm , such that $\widehat{BOA} = 210^\circ$.

A piece of paper is cut into the shape of the sector BOA .

A hollow cone with no base is constructed from the sector by joining the points A and B . The sector forms the curved surface of the cone.

This is shown in the following diagrams.



Find

- | | |
|------------------------------------|-----|
| (a) the area of the sector BOA ; | [3] |
| (b) the radius of the cone. | [3] |

(This question continues on the following page)



(Question 1 continued)

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2. [Maximum mark: 6]

Point $X(-3, 4, 2)$ is the centre of the base ABCD.

A diagram of a triangular pyramid with apex V and base triangle ABC . The base is a triangle with vertices A , B , and C . A point D is located on the edge AC . A dashed line segment connects V and D . Another point X is located on the segment VD , and a dashed line segment connects V and X .

- (c) Find the size of the angle between the edge $[VC]$ and the base of the pyramid. [2]

[illegible]

9. [Maximum mark: 8]

The plane Π_1 contains the line L_1 and passes through the point $(2, 1, 5)$.

(a) Show that the Cartesian equation of the plane Π_1 is $x + y - z = -2$. [4]

Consider the three planes

$$\Pi_1 : x + y - z = -2$$

$$\Pi_7 : 2x + by - z = 3$$

$$\Pi_3 : x - y + 2z = d$$

where $b, d \in \mathbb{Q}^+$.

The three planes intersect in a line.

(b) Find the value of b and the value of d . [4]

[illegible]

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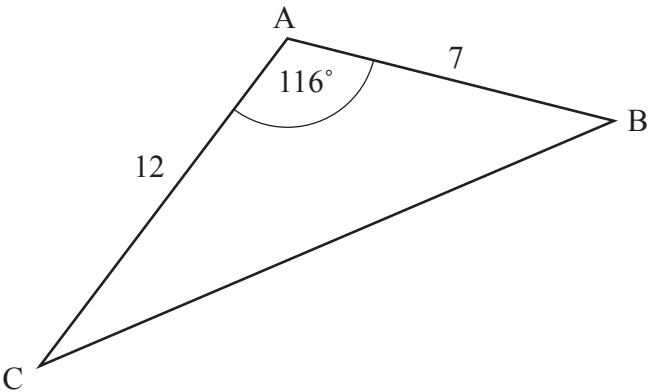
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a triangle ABC , with $AB = 7$, $AC = 12$ and $\hat{BAC} = 116^\circ$.

diagram not to scale



- (a) Find BC .

[3]
- (b) Find $\hat{ACB}$.

[3]

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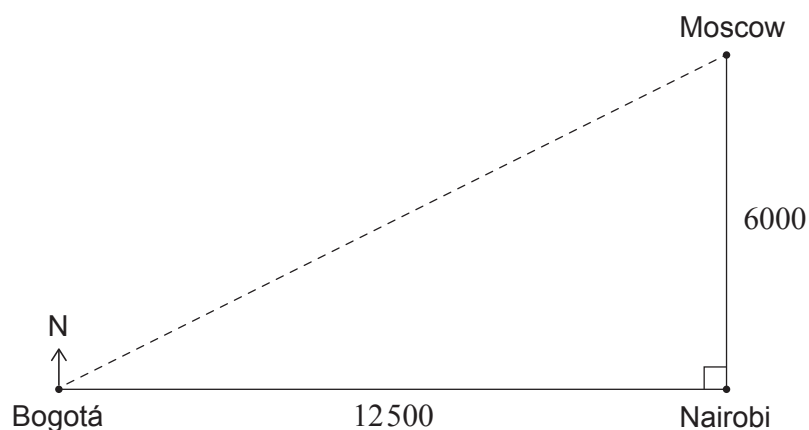


2. [Maximum mark: 27]

The following question compares the distance and direction between cities on a flat surface to the distance and direction between cities on a sphere.

Consider a model where the cities of Bogotá, Moscow, and Nairobi lie on a flat surface. In this model, Nairobi is 6000 km due south of Moscow and Bogotá is 12 500 km due west of Nairobi, as shown in the following diagram.

diagram not to scale



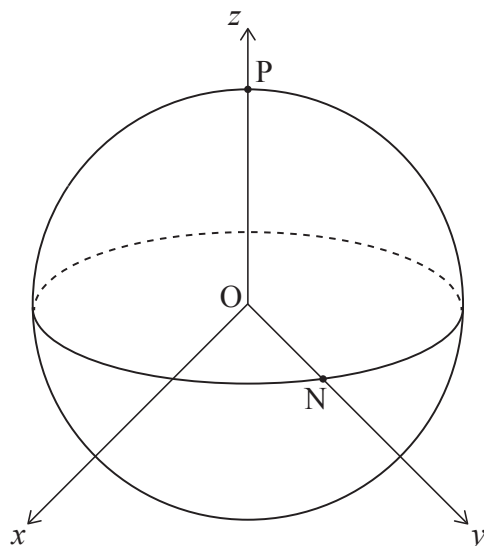
- (a) (i) Find the distance from Bogotá to Moscow. [2]
- (ii) Find the bearing of Moscow from Bogotá. Give your answer in degrees. [3]

(This question continues on the following page)

(Question 2 continued)

In reality, these three cities lie on the curved surface of the Earth which will change the distances and directions found in part (a).

Now consider a curved model using a coordinate system (x, y, z) with its origin, O , at the centre of the Earth. The units of this system are thousands of kilometres and the Earth is modelled as a sphere with radius 6000 km. The North Pole, P , lies on the z -axis, and Nairobi, N , is modelled as being on the equator and lying on the y -axis.



P has position vector $\vec{OP} = \mathbf{p} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$ and N has position vector $\vec{ON} = \mathbf{n} = \begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$.

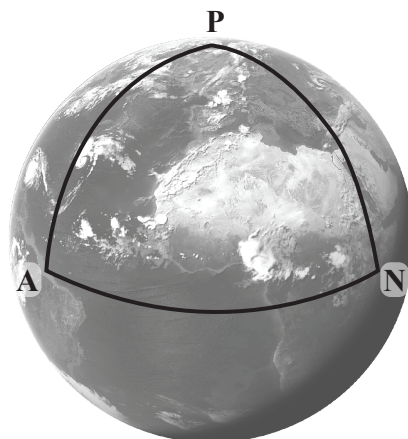
- (b) (i) Use the scalar product to find the angle between $\mathbf{p}$ and $\mathbf{n}$. [2]
- (ii) Show that the distance between P and N along the arc from P to N is 3000π km. [2]

(This question continues on the following page)

Turn over

(Question 2 continued)

Point A, which is also on the equator, has position vector $\mathbf{a} = \begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ as shown in the following diagram.



P, N and A, and the arcs connecting them, form a spherical triangle.

The angle at vertex A is defined as the angle between the vectors $\mathbf{a} \times \mathbf{p}$ and $\mathbf{a} \times \mathbf{n}$.

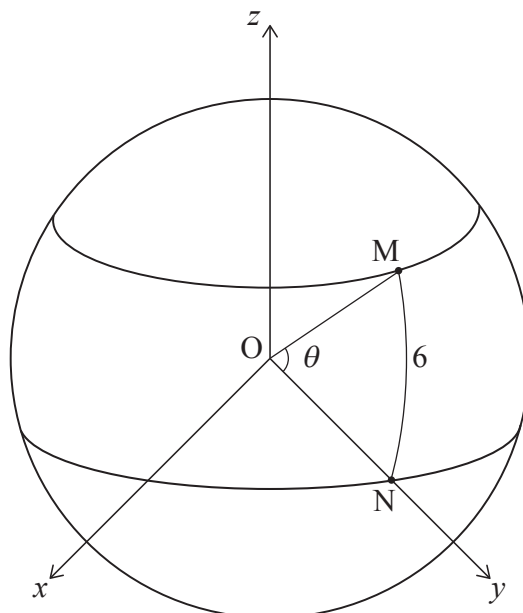
(c) (i) Find the vector $\mathbf{a} \times \mathbf{p}$. [2]

(ii) Show that the angle at vertex A in the spherical triangle is 90° . [3]

(This question continues on the following page)

(Question 2 continued)

Moscow, M, has position vector $\vec{OM} = \mathbf{m} = \begin{pmatrix} 0 \\ 6 \cos \theta \\ 6 \sin \theta \end{pmatrix}$, as shown in the following diagram.



The shortest distance between two points on the sphere lies along an arc of a circle on the sphere with centre O. In this model the shortest distance from Moscow to Nairobi is 6000 km.

- (d) Show that $\theta = 57.3^\circ$, correct to three significant figures. [2]

Bogotá, B, is west of Nairobi and has position vector $\vec{OB} = \mathbf{b} = \begin{pmatrix} 6 \sin 120^\circ \\ 6 \cos 120^\circ \\ 0 \end{pmatrix}$.

- (e) Find the shortest distance from Bogotá to Moscow on the sphere. [5]

The bearing from B to M is defined as the angle at vertex B in the spherical triangle

containing B, M and P. It is given that $\mathbf{b} \times \mathbf{p} = \begin{pmatrix} 36 \cos 120^\circ \\ -36 \sin 120^\circ \\ 0 \end{pmatrix}$.

- (f) Using the method from part (c), find the bearing from Bogotá to Moscow. [6]

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References:

- 2(c)** Google Maps/Google Earth. Data from Data SIO, NOAA, U.S. Navy, NGA, GEBCOLandsat / Copernicus, IBCAO, U.S. Geological Survey, PGC/NASA. Imagery from 14/12/2015. Image available at: https://earth.google.com/web/@12.01529518,-18.56070747,-158.39383184a,23597813.93249989d,30.00008083y,359.99981502h,0t,0r/data=CgRCAggBOgMKATBCAggASg0I_____ARAA.

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8. [Maximum mark: 8]

(a) Show that $\sin 3\theta \equiv 3\sin \theta - 4\sin^3 \theta$. [4]

(b) Hence, describe a sequence of transformations that maps the graph of $y = \sin \theta$ onto the graph of $y = 6\sin\left(\theta + \frac{\pi}{6}\right) - 8\sin^3\left(\theta + \frac{\pi}{6}\right)$. [4]

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0020-7100

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 19]

The point $P(-1, 1, -13)$ lies on the line L_1 . The line L_1 has a direction vector $\begin{pmatrix} 7 \\ 1 \\ 2 \end{pmatrix}$.

(a) Write down a vector equation for L_1 in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b}$. [1]

(b) Find a vector equation for line L_2 in the form $\mathbf{s} = \mathbf{c} + \mu \mathbf{d}$, given that L_2 passes through the points $A(2, -4, 2)$ and $B(7, -6, 1)$. [2]

(c) Show that L_1 and L_2 are skew. [5]

The point N lies on L_2 .

(d) Find $\vec{PN} \cdot \vec{AB}$ in terms of μ . [4]

(e) Given that N is the point on L_2 that lies closest to P , find the coordinates of N . [3]

(f) Point O denotes the origin $(0, 0, 0)$.
Find the equation of the plane containing points O , P and N , giving your answer in the form $\alpha x + \beta y + \gamma z = \delta$, where $\alpha, \beta, \gamma, \delta \in \mathbb{Z}$. [4]



Turn over

— 17 —

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

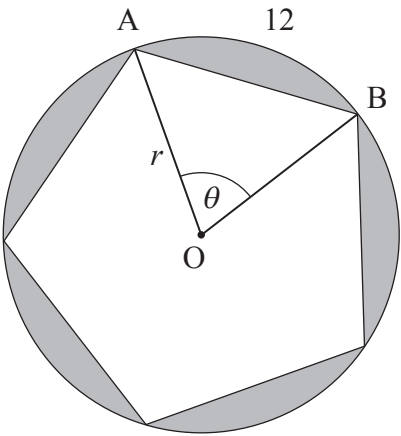
1. [Maximum mark: 6]

The following diagram shows a regular pentagon inscribed in a circle with centre O and radius r cm.

The angle $\widehat{AOB}$ is θ , where θ is measured in radians.

The arc AB is 12 cm.

diagram not to scale



(a) Find

(i) θ ;

(ii) r .

[3]

(b) Find the area of the shaded region.

[3]

(This question continues on the following page)



(Question 1 continued)

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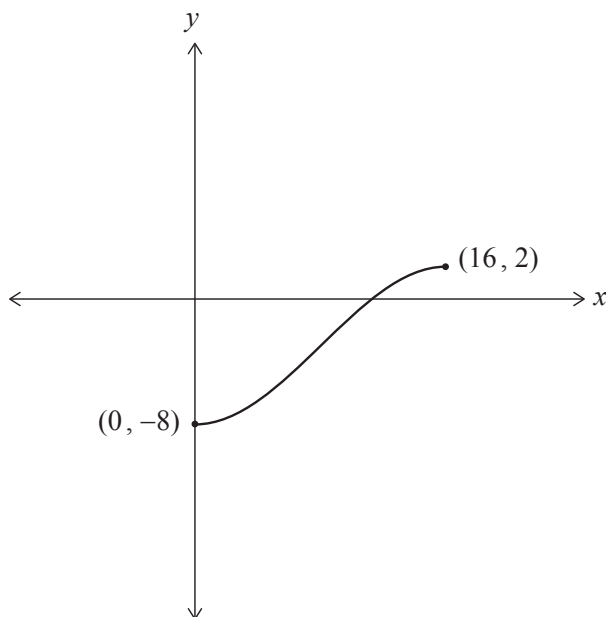
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

4. [Maximum mark: 7]

Consider the function $g(x) = a \cos(bx) - 3$, where $x \in \mathbb{R}$ and $a, b \in \mathbb{R}$.

The following diagram shows part of the graph of g .



The graph of g has a local minimum at $(0, -8)$ and a local maximum at $(16, 2)$.

(a) Find the value of

(i) a ;

(ii) b , where $b > 0$.

[3]

(b) Write down the smallest positive value of the constant k such that $g(x + k) = g(x)$ for all x .

[1]

(c) The function $g(x)$ can be written in the form $f(x) = p \sin b(x - q) - 3$ where $p, q \in \mathbb{Z}^+$.

(i) Find the smallest positive value of q .

(ii) For this value of q , write down the value of p .

[3]

(This question continues on the following page)



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

12. [Maximum mark: 21]

The equations of two lines, L_1 and L_2 , are given by:

$$L_1 : \mathbf{r}_1 = \begin{pmatrix} 5 \\ 4 \\ 2 \end{pmatrix} + s \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \text{ where } s \in \mathbb{R}$$

$$L_2 : \frac{x+1}{2} = y-7 = \frac{z+5}{3}$$

- (a) Show that the position vector of the point of intersection, L_1 and L_2 is $\begin{pmatrix} 1 \\ 8 \\ -2 \end{pmatrix}$. [3]

The plane Π contains the lines L_1 and L_2 .

- (b) (i) Write down the equation of Π , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$ where $\lambda, \mu \in \mathbb{R}$.
- (ii) Given that $\mathbf{b} \times \mathbf{c} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$, show that the Cartesian equation of Π is $4x + y - 3z = 18$. [3]

The plane intersects the coordinate axes at $P(4.5, 0, 0)$, $Q(0, q, 0)$, and $R(0, 0, r)$.

- (c) Write down the value of
- (i) q ;
- (ii) r . [2]
- (d) Use a vector method to find the area of the triangle PQR. [5]

Another line, L_3 , is normal to Π and passes through the point of intersection of L_1 and L_2 .

- (e) Write down an equation for L_3 in the form $\mathbf{r}_3 = \mathbf{m} + \gamma \mathbf{n}$. [2]
- (f) Given that the point $S(-11, 5, 7)$ lies on the line L_3 , find γ . [2]
- (g) Hence, find the volume of pyramid PQRS. [4]



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1. [Maximum mark: 5]

(b) Hence, or otherwise, solve the equation $3\cos 2x + 11\sin x - 6 = 0$ for $0^\circ \leq x \leq 180^\circ$. [3]

[illegible]

Turn over

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

3. [Maximum mark: 6]

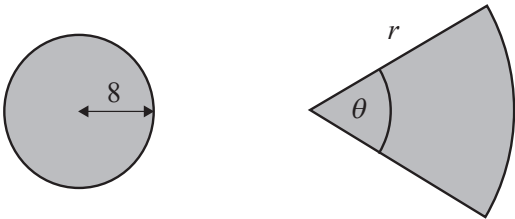
Consider a circle and a sector.

The radius of the circle is 8 mm.

The radius of the sector is r mm and the acute angle at the centre is θ radians.

This is shown in the following diagram.

diagram not to scale



The perimeter of the sector is 1.5 times the circumference of the circle.

(a) Show that $r = \frac{24\pi}{2 + \theta}$. [3]

It is given that the area of the circle is the same as the area of the sector.

(b) Determine the value of θ . [3]

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