Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Monday 1 November 2021 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

24 pages

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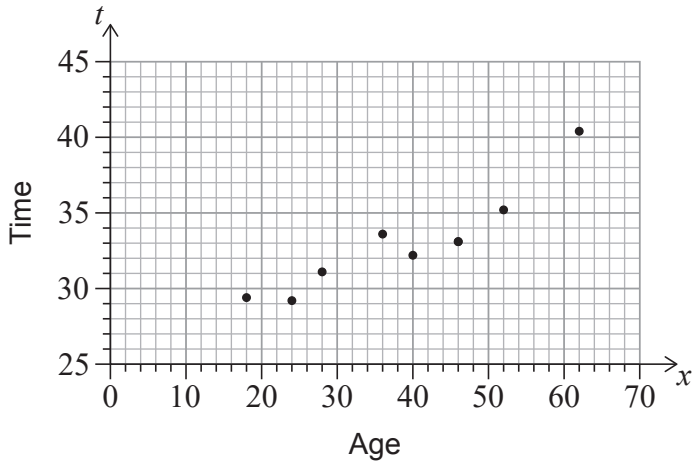
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Eduardo believes that there is a linear relationship between the age of a male runner and the time it takes them to run 5000 metres.

To test this, he recorded the age, x years, and the time, t minutes, for eight males in a single $5000\,\text{m}$ race. His results are presented in the following table and scatter diagram.

***x*, years**	18	24	28	36	40	46	52	62
***t*, minutes**	29.4	29.2	31.1	33.6	32.2	33.1	35.2	40.4



(a) For this data, find the value of the Pearson's product-moment correlation coefficient, r. [2]

Eduardo looked in a sports science text book. He found that the following information about r was appropriate for athletic performance.

Value of $\lvert r \rvert$	**Description of the correlation**
$0 \le \lvert r \rvert < 0.4$	weak
$0.4 \le \lvert r \rvert < 0.8$	moderate
$0.8 \le \lvert r \rvert \le 1$	strong

(b) Comment on your answer to part (a), using the information that Eduardo found. [1]

(c) Write down the equation of the regression line of t on x, in the form $t = ax + b$. [1]

(This question continues on the following page)

(Question 1 continued)

A 57-year-old male also ran in the 5000 m race.

(d) Use the equation of the regression line to estimate the time he took to complete the 5000 m race. [2]

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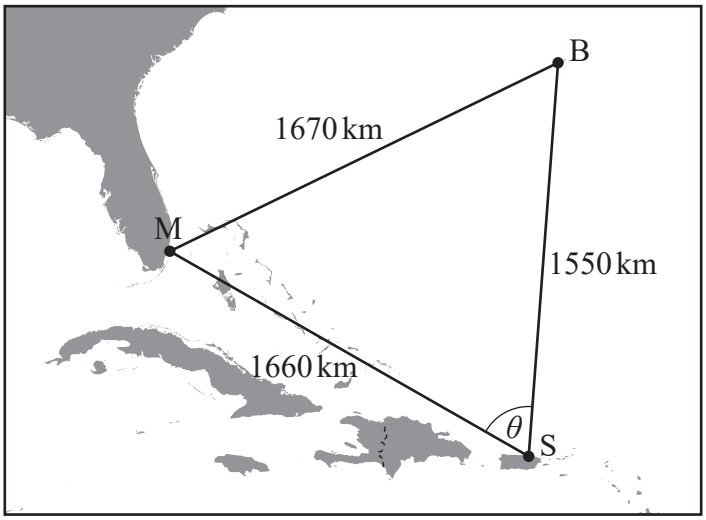
2. [Maximum mark: 5]

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	1670 km
Distance between Bermuda and San Juan	1550 km
Distance between San Juan and Miami	1660 km

(a) Calculate the value of θ, the measure of angle $\mathrm{M\hat{S}B}$. [3]

(b) Find the area of the Bermuda Triangle. [2]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 4]

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = A\mathrm{e}^{kt},$$

where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is $112\,\mathrm{mm}^2$ at the start of the experiment and $360\,\mathrm{mm}^2$ after 5 days.

(a) Write down the value of A. [1]

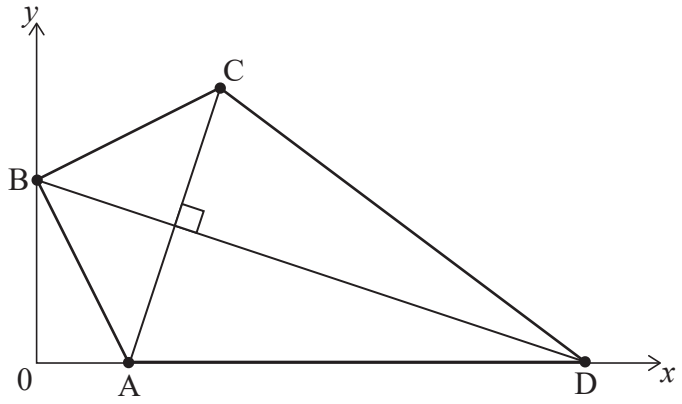
(b) Find the value of k. [3]

4. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents $10\,\text{cm}$.

The coordinates of A, B and C are $(2,\ 0)$, $(0,\ 4)$ and $(4,\ 6)$ respectively. Point D lies on the x-axis. $[\text{AC}]$ is perpendicular to $[\text{BD}]$. This information is shown in the following diagram.

diagram not to scale



(a) Find the gradient of the line through A and C. [2]

(b) Write down the gradient of the line through B and D. [1]

(c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

(d) Write down the x-coordinate of point D. [1]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter $x\,\text{cm}$.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \le x \le 14.$$

(a) Find the range of h. [3]

The function h^{-1} is the inverse function of h.

(b) (i) Find $h^{-1}(10)$.

(ii) In the context of the question, interpret your answer to part (b)(i).

(iii) Write down the range of h^{-1}. [4]

6. [Maximum mark: 5]

Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t, the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t-axis.

The rate, R, is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

(a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours. [3]

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

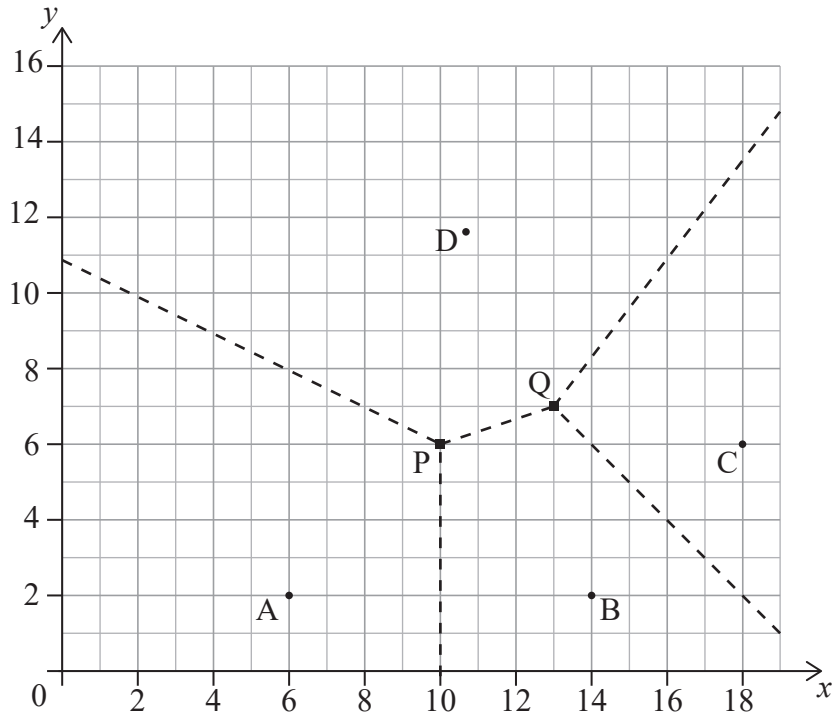
(b) Find the percentage error of the estimate found in part (a). [2]

7. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral $ABCD$. The new station will be located so that it is as far as possible from the nearest existing station.

(a) Show that the new station should be built at P. [3]

The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of $[AP]$ and $[BP]$ have been added to the following diagram.

(b) (i) Write down the equation of the perpendicular bisector of $[PC]$.

(ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

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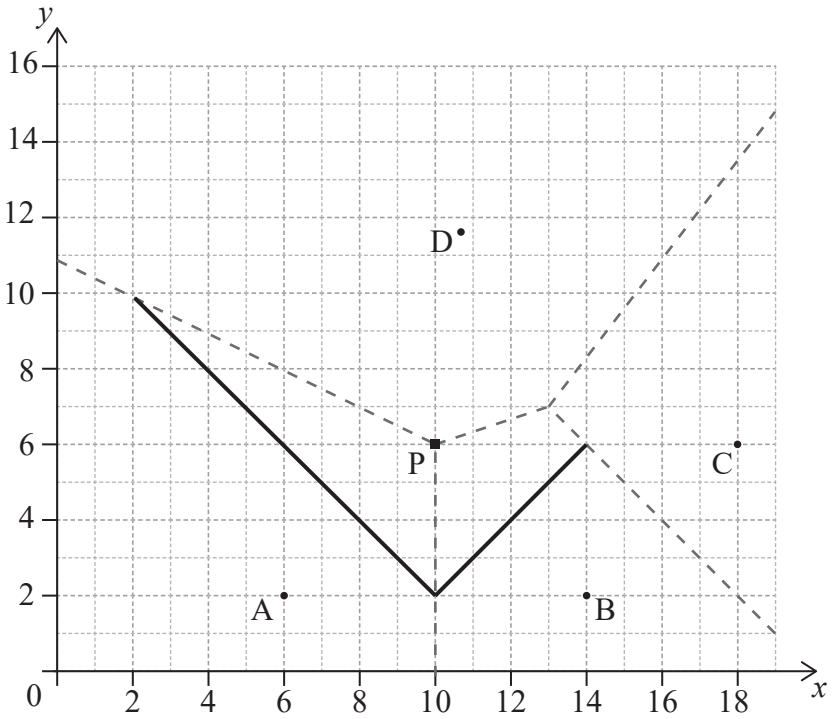
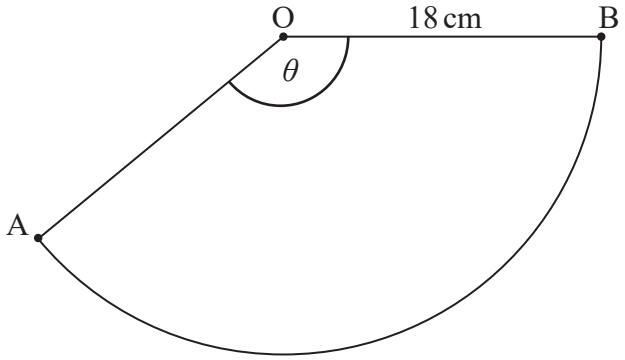
(Question 7 continued)



8. [Maximum mark: 5]

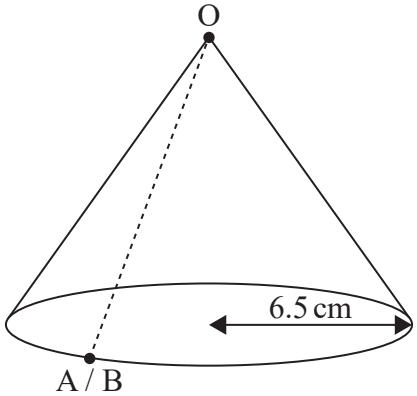
Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB, of a circular piece of paper with a radius of $18\,\mathrm{cm}$ and $\mathrm{A\hat{O}B} = \theta$ as shown in the diagram.

diagram not to scale



To make the hat, sides $[\mathrm{OA}]$ and $[\mathrm{OB}]$ are joined together. The hat has a base radius of $6.5\,\mathrm{cm}$.

diagram not to scale



(a) (i) Write down the perimeter of the base of the hat in terms of π.

(ii) Find the value of θ. [3]

(b) Find the surface area of the outside of the hat. [2]

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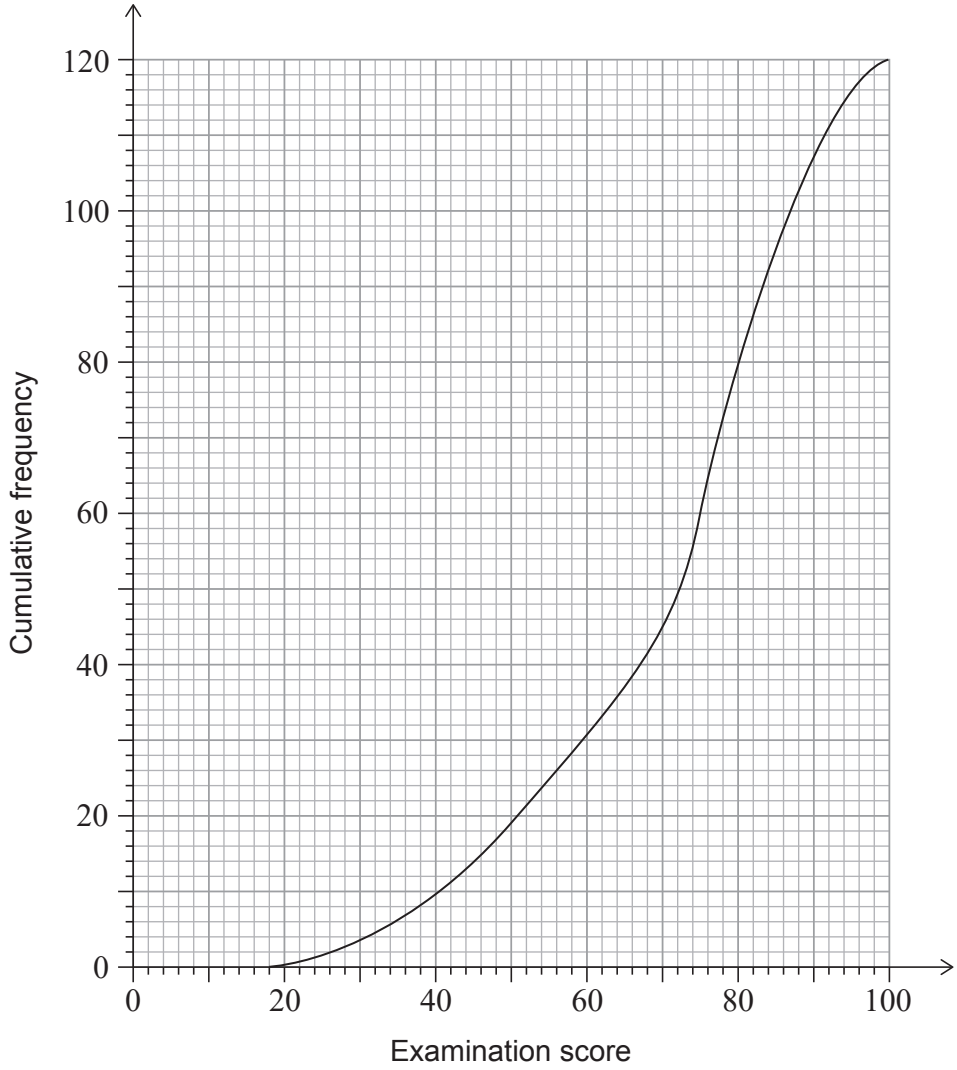
(Question 8 continued)

9. [Maximum mark: 8]

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained. [1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

Grade	1	2	3	4	5
Number of students	6	13	26	a	b

(b) Find an expression for a in terms of b. [2]

(This question continues on the following page)

(Question 9 continued)

(c) The mean grade for these students is 3.65.

(i) Find the number of students who obtained a grade 5.

(ii) Find the minimum score needed to obtain a grade 5. [5]

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10. [Maximum mark: 9]

In this question, give all answers correct to 2 decimal places.

Raul and Rosy want to buy a new house and they need a loan of $170\,000$ Australian dollars (AUD) from a bank. The loan is for 30 years and the annual interest rate for the loan is $3.8\,\%$, compounded monthly. They will pay the loan in fixed monthly instalments at the end of each month.

(a) Find the amount they will pay the bank each month. [3]

(b) (i) Find the amount Raul and Rosy will still owe the bank at the end of the first 10 years.

(ii) Using your answers to parts (a) and (b)(i), calculate how much interest they will have paid in total during the first 10 years. [6]

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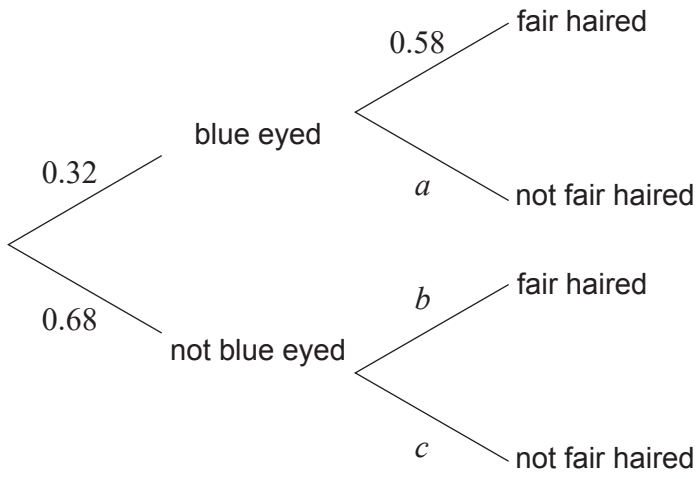
11. [Maximum mark: 5]

In a city, 32% of people have blue eyes. If someone has blue eyes, the probability that they also have fair hair is 58%. This information is represented in the following tree diagram.



(a) Write down the value of a. [1]

(b) Find an expression, in terms of b, for the probability of a person not having blue eyes **and** having fair hair. [1]

It is known that 41% of people in this city have fair hair.

(c) Calculate the value of

(i) b.

(ii) c. [3]

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(Question 11 continued)

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12. [Maximum mark: 6]

The surface area of an open box with a volume of $32\,\text{cm}^3$ and a square base with sides of length $x\,\text{cm}$ is given by $S(x) = x^2 + \frac{128}{x}$ where $x > 0$.

(a) Find $S'(x)$. [3]

(b) (i) Solve $S'(x) = 0$.

(ii) Interpret your answer to (b)(i) in context. [3]

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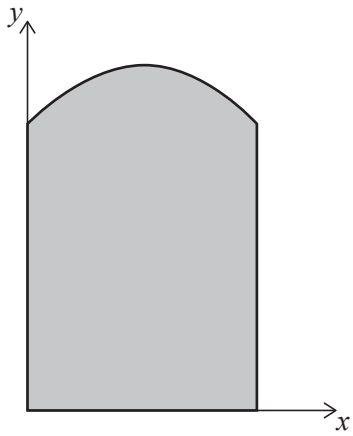
13. [Maximum mark: 8]

Irina uses a set of coordinate axes to draw her design of a window. The base of the window is on the x-axis, the upper part of the window is in the form of a quadratic curve and the sides are vertical lines, as shown on the diagram. The curve has end points $(0, 10)$ and $(8, 10)$ and its vertex is $(4, 12)$. Distances are measured in centimetres.



The quadratic curve can be expressed in the form $y = ax^2 + bx + c$ for $0 \le x \le 8$.

(a) (i) Write down the value of c.

(ii) Hence form two equations in terms of a and b.

(iii) Hence find the equation of the quadratic curve. [5]

(b) Find the area of the shaded region in Irina's design. [3]

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(Question 13 continued)

References:

2. Bermuda Triangle map [online] Available at: https://commons.wikimedia.org/wiki/File:Bermuda_Triangle_map_(de).svg Thomas Römer. This file is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license. (CC BY-SA 3.0) https://creativecommons.org/licenses/by-sa/3.0/deed.en [Accessed 17 December 2020] Source adapted.

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