

APOLLO

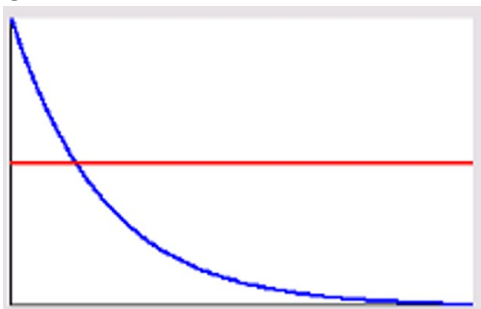
Functions

HL

Mathematics: applications and interpretation
First assessment 2021
51 questions | mark schemes

4. (a) $50 = 100e^{-1 \times p}$ **OR** $0.5 = e^{-1 \times p}$ (M1)

OR



(M1)

0.693 (0.693147..., $\ln 2$)

A1

[2 marks]

(b) $R(1.5) = 100e^{-0.693147... \times 1.5}$ (M1)

35.4(%) (35.3553...)

A1

[2 marks]

(c) $R(t) > 0$ **OR** $R(t)$ has a horizontal asymptote **R1**

[1 mark]

(d) Award **A1** for **one** reasonable limitation of the domain: **A1**

small values of t produce unrealistic results

$R(0) = 100\%$

large values of t are not possible

people do not live forever

model is not valid at small or large values of t

The reason should focus on the domain $t \geq 0$. Do not accept answers such as:

recollection varies for different people

memories are discrete not continuous

the nature of the information will change how easily it is recalled

emotional/physical stress can affect recollection/concentration

Note: Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

[1 mark]

Total [6 marks]

(-3)

17. new function is $f(x-a)+b (= \ln(x-a)+b)$

(M1)

$$f(0) = \ln(-a) + b = 1$$

A1

$$f(e^3) = \ln(e^3 - a) + b = 1 + \ln 2$$

A1

$$\ln(-a) = \ln(e^3 - a) - \ln 2$$

(M1)

$$\ln(-a) = \ln\left(\frac{e^3 - a}{2}\right)$$

$$-a = \frac{e^3 - a}{2}$$

$$-2a = e^3 - a$$

$$a = -e^3 \quad (= -20.0855\dots)$$

A1

$$b = 1 - \ln e^3 = 1 - 3 = -2$$

(M1)A1

Total [7 marks]

2. (a) $(f(-7) =) 8$ and $(f(7) =) 1$
range is $f(x) \leq 1$, $f(x) \geq 8$

(A1)**A1A1**

Note: Award at most **A1A1A0** if strict inequalities are used.

- (b) interchanging x, y at any stage

[3 marks]**(A1)**

$$y = 2 - \frac{12}{x+5}$$

$$\frac{12}{x+5} = 2 - y$$

$$\frac{12}{2-y} = x+5$$

(A1)

$$\frac{12}{2-y} - 5 = x$$

$$(f^{-1}(x) =) \frac{12}{2-x} - 5 \left(= \frac{2+5x}{2-x} \right)$$

A1**[3 marks]**

- (c) range is $-7 \leq f^{-1}(x) \leq 7$, $f^{-1}(x) \neq -5$

A1**[1 mark]****Total [7 marks]**

6. (a) 3

A1

Note: Accept (3, 0) seen.

[1 mark]

(b) **METHOD 1**

$$0 = 4a - 2b + c, \quad 0 = 9a + 3b + c, \quad -\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$$

(M1)(A1)

(i) 2

A1

(ii) -2

A1

(iii) -12

A1

Note: Award the (M1)(A1) if at least one correct value is seen.
 Do not apply **FT** from part (a) if workings are not shown.
METHOD 2

$$-12.5 = a(0.5 + 2)(0.5 - 3)$$

(M1)

(i) $a = 2$

A1

$$0 = 2x(3)^2 + 3b + c$$

$$0 = 2x(-2)^2 + (-2)b + c$$

(M1)

(ii) $b = -2$

A1

(iii) $c = -12$

A1

[5 marks]

(c) $x = 0.5$

A1

Note: Do not **FT** from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

4. (a) $y = \frac{5}{8}x + \frac{7}{2}$ ($y = 0.625x + 3.5$)

A1A1

Note: Award **A1** for $0.625x$, **A1** for 3.5 .
Award a maximum of **A0A1** if not part of an equation.

[2 marks]

(b) (i) $y = -0.975x^2 + 9.56x - 16.7$
($y = -0.974630x^2 + 9.55919x - 16.6569...$)

(M1)A1

(ii) gradient of curve is positive at $x = 4$

R1

Note: Accept a sensible rationale that refers to the gradient.

[3 marks]

(c) **METHOD 1**

let $y = ax^2 + bx + c$

differentiating or using $x = \frac{-b}{2a}$

(M1)

$8a + b = 0$

substituting in the coordinates

$7.5^2a + 7.5b + c = 0$

(A1)

$4^2a + 4b + c = 6$

(A1)

solve to get

$y = -\frac{24}{49}x^2 + \frac{192}{49}x - \frac{90}{49}$ **OR** $y = -0.490x^2 + 3.92x - 1.84$

A1

Note: Use of quadratic regression with points using the symmetry of the graph is a valid method.

METHOD 2

$y = a(x - 4)^2 + 6$

(M1)

$0 = a(7.5 - 4)^2 + 6$

(M1)

$a = -\frac{24}{49}$

(A1)

$y = -\frac{24}{49}(x - 4)^2 + 6$ **OR** $y = -0.490(x - 4)^2 + 6$

A1**[4 marks]***continued...*

Question 4 continued

(d) (i) $\pi \int_0^4 \left(\frac{5}{8}x + 3.5 \right)^2 dx + \pi \int_4^{7.5} \left(-\frac{24}{49}(x-4)^2 + 6 \right)^2 dx$ **(M1)(M1)(M1)A1**

Note: Award **(M1)(M1)(M1)A0** if π is omitted but response is otherwise correct.
Award **(M1)** for an integral that indicates volume, **(M1)** for their part (a) within their volume integral, **(M1)** for their part (b)(i) within their volume integral, **A1** for their correct two integrals with all correct limits.

(ii) 501 cm^3 (501.189...)

A1**[5 marks]****Total [14 marks]**

1. (a) recognition that period = 669 (M1)

$$b = \frac{2\pi}{669} \quad \text{OR} \quad b = 0.00939190\dots$$

A1

Note: Award **A1** for a correct expression leading to the given value or for a correct value of b to 4 sf or greater accuracy.

$$b \approx 0.00939$$

AG

[2 marks]

- (b) length of day = $24\frac{2}{3}$ hours (A1)

Note: Award **A1** for $\frac{2}{3}$, 0.666..., $0.\overline{6}$ or 0.667.

$$\frac{2\pi}{24\frac{2}{3}}$$

(M1)

Note: Accept $\left(\frac{360}{24\frac{2}{3}}\right)$.

$$= 0.255 \text{ radians} \left(= 0.254723\dots, \frac{3\pi}{37}, 14.5945\dots^\circ \right)$$

A1

[3 marks]

- (c) (i) substitution of either value of δ into equation (M1)
correct use of arccos to find a value for ω (M1)

Note: Both (M1) lines may be seen in either part (c)(i) or part (c)(ii).

$$\cos \omega = 0.839 \tan(-0.440)$$

A1

$$\omega = 1.97684\dots$$

$$\approx 1.98$$

AG

Note: For substitution of 1.98 award **M0A0**.

[3 marks]

- (ii) $\delta = 0.440$
 $\omega = 1.16$ (1.16474...)

A1

[1 mark]

continued...

Question 1 continued

$$(d) \quad (i) \quad R_{\max} = \frac{1.97684...}{0.25472...} \quad (M1)$$

$$= 7.76 \text{ hours } (7.76075...) \quad A1$$

Note: Accept 7.70 from use of 1.98.

[2 marks]

$$(ii) \quad R_{\min} = \frac{1.16474...}{0.25472...} \quad A1$$

$$= 4.57 \text{ hours } (4.57258...) \quad [1 \text{ mark}]$$

Note: Accept 4.55 and 4.56 from use of rounded values.

$$(e) \quad a = \frac{7.76075... - 4.57258...}{2} \quad M1$$

$$\approx 1.59408... \quad A1$$

Note: Award **M1** for substituting their values into a correct expression.
Award **A1** for a correct value of a from their expression which has at least 3 significant figures and rounds correctly to 1.6.

$$\approx 1.6 \text{ (correct to 2 sf)} \quad AG$$

[2 marks]

(f) **EITHER**

$$c = \frac{7.76075... + 4.57258...}{2} \left(= \frac{12.333...}{2} \right) \quad (M1)$$

OR

$$c = 4.57258... + 1.59408... \text{ or } c = 7.76075... - 1.59408...$$

THEN

$$= 6.17 \text{ (6.16666...)} \quad A1$$

Note: Accept 6.16 from use of rounded values.
Follow through on their answers to part (d) and 1.6.

[2 marks]

$$(g) \quad d = 18.65 - 6.16666... \quad (M1)$$

$$= 12.5 \text{ (12.4833...)} \quad A1$$

Note: Follow through for 18.65 minus their answer to part (f).

[2 marks]

continued...

Question 1 continued

- (h) (i) at least one expression in the form
- $re^{g(t)i}$

(M1)

$$z_1 = 1.5e^{(0.00939t+2.83)i}, z_2 = 1.6e^{(0.00939t)i}$$

A1A1

[3 marks]

- (ii) EITHER

$$z_1 - z_2 = 1.5e^{(0.00939t+2.83)i} - 1.6e^{(0.00939t)i}$$

$$= e^{0.00939ti} (1.5e^{2.83i} - 1.6)$$

(M1)

$$= e^{0.00939ti} (3.06249...e^{2.99086...i})$$

(A1)(A1)

OR

graph of L or f

$$p = 3.06249...$$

(A1)

$$r = -0.150729... \quad \text{OR} \quad r = 2.99086...$$

(M1)(A1)

Note: The p and r variables (or equivalent) must be seen.

THEN

$$L(t) = 3.06 \sin(0.00939t + 2.99) + 12.5$$

A1

$$(L(t) = 3.06248... \sin(0.00939t + 2.99086...) + 12.4833...)$$

Note: Accept equivalent forms, e.g. $L(t) = 3.06 \sin(0.00939t - 0.151) + 12.5$.
Follow through on their answer to part (g) replacing 12.5.

[4 marks]

- (iii) shortest time between sunrise and sunset

$$12.4833... - 3.06249...$$

(M1)

$$= 9.42 \text{ hours } (9.420843...)$$

A1

Note: Accept 9.44 from use of 3 sf values.

[2 marks]

[Total 27 marks]

1. (a) $m = \frac{6-0}{4-2} = 3$

(M1)A1**[2 marks]**

(b) $(m =) -\frac{1}{3} (-0.333, -0.333333...)$

A1**[1 mark]**

- (c) an equation of line with a correct intercept and either of their gradients from (a) or (b)

(M1)

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

Note: Award **(M1)** for substituting either of their gradients from parts (a) or (b) and point B or (3, 3) into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple

A1**[2 marks]**

(d) $(x =) 12$

A1**[1mark]****Total: [6 marks]**

3. (a) $h(4) = \frac{640}{4^2} + 0.5$ **OR** $h(14) = \frac{640}{14^2} + 0.5$ **(M1)**

Note: Award **(M1)** for substituting 4 or 14 into h . This can be implicit from seeing 3.77 (3.76530...) or 40.5.

$$3.77 \leq h(x) \leq 40.5 \quad (3.76530... \leq h(x) \leq 40.5) \quad \textbf{A1A1}$$

Note: Award **A1** for both correct endpoints seen, **A1** for the endpoints in a correct interval.

[3 marks]

(b) (i) $h(x) = 10$ **OR** $h^{-1}(x) = \sqrt{\frac{640}{x-0.5}}$ **OR** $h^{-1}(10) = \sqrt{\frac{640}{10-0.5}}$ **(M1)**
 $(x =) 8.21 \text{ cm } (8.20782...)$ **A1**

(ii) a tin that is 10 cm high will have a diameter of 8.21 cm (8.20782...) **A1**

Note: Condone a correct answer expressed as the converse.

(iii) $4 \leq h^{-1} \leq 14$ **A1**

Note: Accept $4 \leq y \leq 14$. Do not **FT** in this part.

[4 marks]

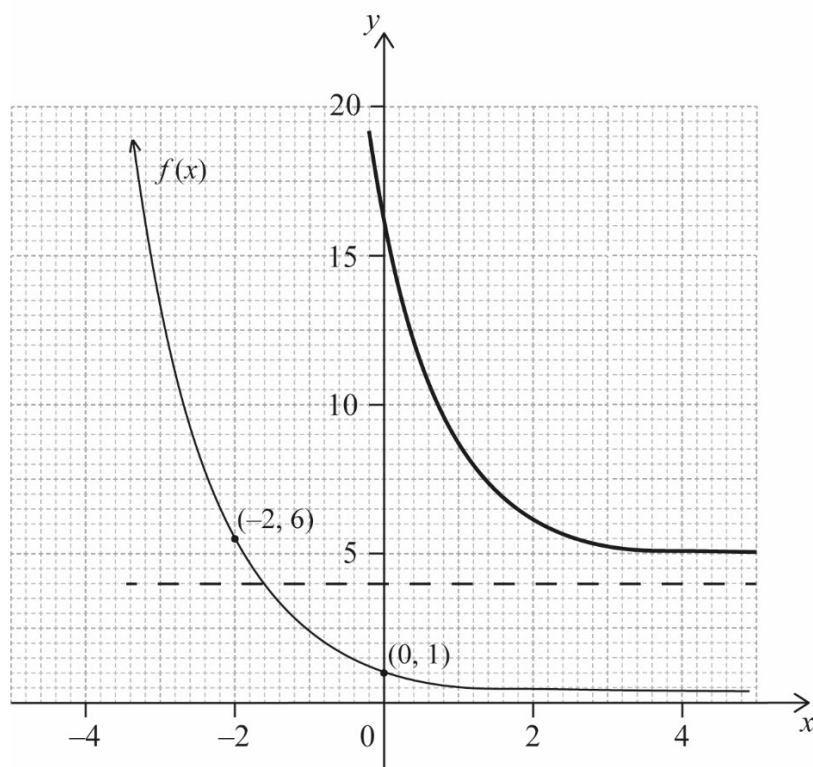
Total: [7 marks]

10. (a) $g(0) = 16$

M1A1

[2 marks]

(b)



y -asymptote ($y = 4$)

A1

concave up decreasing curve and passing through $(0, 16)$

A1

[2 marks]

Total: [4 marks]

2. (a) (i) maximum $h = 130$ metres

A1

(ii) minimum $h = 50$ metres

A1

[2 marks]

(b) (i) $(60 \div 12 =) 5$ seconds

A1

(ii) $360 \div 5$

(M1)

Note: Award **(M1)** for 360 divided by their time for one revolution.

$$= 72^\circ$$

A1

[3 marks]

(c) (i) (amplitude $=$) 40

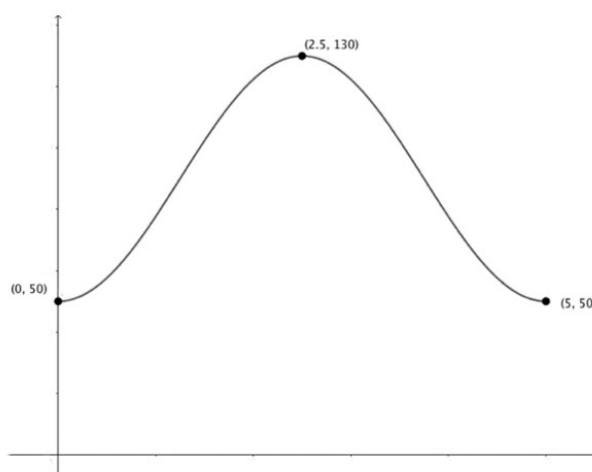
A1

(ii) (period $= \frac{360}{72} =$) 5

A1

[2 marks]

(d)



Maximum point labelled with correct coordinates.

A1

At least one minimum point labelled. Coordinates seen for any minimum points must be correct.

A1

Correct shape with an attempt at symmetry and "concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain.

A1

[3 marks]

continued...

Question 2 continued

(e) (i) $h = 90 - 40 \cos(144^\circ)$ (M1)
 $(h =) 122 \text{ (m) } (122.3606\dots)$ (A1)

(ii) evidence of $h = 100$ on graph **OR** $100 = 90 - 40 \cos(72t)$ (M1)
 t coordinates 3.55 (3.54892...) **OR** 1.45 (1.45107...) or equivalent (A1)

Note: Award **A1** for either t -coordinate seen.

$= 2.10 \text{ seconds } (2.09784\dots)$

(A1)

[5 marks]

(f) **METHOD 1**

$90 - 40 \cos(at^\circ) = 110$ (M1)

$\cos(at^\circ) = -0.5$

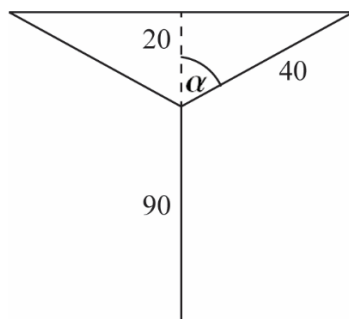
$at^\circ = 120, 240$ (A1)

$1 = \frac{240}{a} - \frac{120}{a}$ (M1)

$a = 120$ (A1)

period $= \frac{360}{120} = 3 \text{ seconds}$ (A1)

METHOD 2



attempt at diagram (M1)

$\cos \alpha = \frac{20}{40}$ (or recognizing special triangle) (M1)

angle made by C, $2\alpha = 120^\circ$ (A1)

one third of a revolution in 1 second (M1)

hence one revolution = 3 seconds (A1)

continued...

*Question 2 continued***METHOD 3**considering $h(t) = 110$ on original function**(M1)**

$$t = \frac{5}{3} \text{ or } \frac{10}{3}$$

(A1)

$$\frac{10}{3} - \frac{5}{3} = \frac{5}{3}$$

(A1)

Note: Accept $t = 1.67$ or equivalent.

so period is $\frac{3}{5}$ of original period**(R1)**

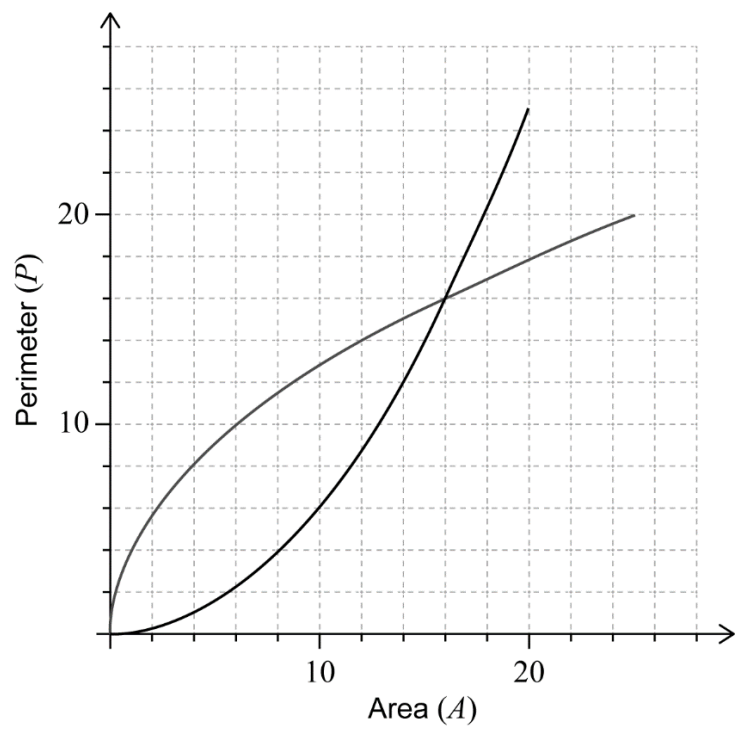
so new period is 3 seconds

A1**[5 marks]****Total: [20 marks]**

2. (a) 20

A1
[1 mark]

(b)



(M1)A1A1

Note: Award **(M1)** for reflection in the line $P = A$, award **A1** for endpoint at $(20, 25)$, award **A1** for passing through $(16, 16)$.

[3 marks]

(c) when the perimeter is 8, the area is 4

A1
[1 mark]

Total [5 marks]

3. (a) (i) 1750

A1

(ii) $1350 + 400 (1.25)^{-5}$

(M1)

$= 1480$

A1

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400 (1.25)^{-t}$

(M1)

$9.32 \text{ (days (9.31885...)) (days)}$

A1

[2 marks]

(c) 1350

A1

Note: Accept 1351 as a valid interpretation of the model as $P=1350$ is an asymptote.

[1 mark]

Total [6 marks]

1. (a) 1.2 metres **A1**

[1 mark]

(b) $-4.8t^2 + 21t + 1.2 = 0$ **(M1)**
 $(t =) 4.43 \text{ s } (4.431415... \text{ s})$ **A1**

Note: If both values for t are seen do not award the **A1** mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \leq t \leq 4.43$ **OR** $[0, 4.43]$ **A1A1**

Note: Award **A1** for correct endpoints and **A1** for expressing answer with correct notation. Award at most **A1A0** for use of x instead of t .

[2 marks]

[Total 5 marks]

17. substitute coordinates of A

$$f(0) = p e^{q \cos(0)} = 6.5$$

$$6.5 = p e^q \quad (A1)$$

substitute coordinates of B

$$f(5.2) = p e^{q \cos(5.2r)} = 0.2$$

EITHER

$$f'(t) = -pqr \sin(rt) e^{q \cos(rt)} \quad (M1)$$

minimum occurs when $-pqr \sin(5.2r) e^{q \cos(5.2r)} = 0$

$$\sin(rt) = 0$$

$$r \times 5.2 = \pi \quad (A1)$$

OR

minimum value occurs when $\cos(rt) = -1$ (M1)

$$r \times 5.2 = \pi \quad (A1)$$

OR

$$\text{period} = 2 \times 5.2 = 10.4 \quad (A1)$$

$$r = \frac{2\pi}{10.4} \quad (M1)$$

THEN

$$r = \frac{\pi}{5.2} = 0.604152... \quad (0.604) \quad A1$$

$$0.2 = p e^{-q} \quad (A1)$$

eliminate p or q (M1)

$$e^{2q} = \frac{6.5}{0.2} \quad \text{OR} \quad 0.2 = \frac{p^2}{6.5}$$

$$q = 1.74 \quad (1.74062...) \quad A1$$

$$p = 1.14017... \quad (1.14) \quad A1$$

[Total 8 marks]

5. (a) *Accept any one of the following (or equivalent):*
 one minimum and one maximum point
 three x -intercepts or three roots (or zeroes)
 one point of inflexion

R1

Note: Do not accept “S shape” as a justification.

[1 mark]

(b) (i) $(d =) -5$

A1

(ii) $8 = a + b + c$
 $4 = 8a + 4b + 2c$
 $0 = 27a + 9b + 3c$

A2

Note: Award **A2** if all three equations are correct.
 Award **A1** if at least one is correct. Award **A1** for three correct equations that include the letter “ d ”.

(iii) $a = 2, b = -12, c = 18$

A1**[4 marks]**

- (c) equating found expression to zero

(M1)

$$0 = 2t^3 - 12t^2 + 18t - 5$$

$$t = 0.358216..., 1.83174..., 3.81003...$$

(A1)

(so total time in debt is $3.81003... - 1.83174... + 0.358216 \approx$)

2.34 (2.33650...) years

A1**[3 marks]****Total [8 marks]**

10. (a) $y = \ln\left(\frac{1}{x-2}\right)$

an attempt to isolate x (or y if switched)

(M1)

$$e^y = \frac{1}{x-2}$$

$$x-2 = e^{-y}$$

$$x = e^{-y} + 2$$

switching x and y (seen anywhere)

M1

$$f^{-1}(x) = e^{-x} + 2$$

A1

[3 marks]

(b) sketch of $f(x)$ and $f^{-1}(x)$

(M1)

$$x = 2.12 \text{ (2.12002...)}$$

A1

[2 marks]

Total [5 marks]

4. (a) attempt to use chain rule, including the differentiation of $\frac{1}{T}$ (M1)

$$\frac{dk}{dT} = A \times \frac{c}{T^2} \times e^{-\frac{c}{T}}$$

A1

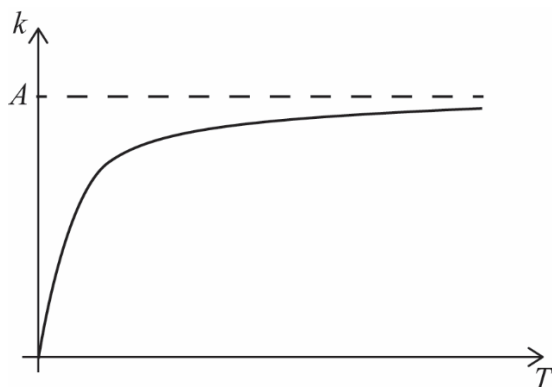
this is the product of positive quantities so must be positive

R1

Note: The **R1** may be awarded for correct argument from **their** derivative.
R1 is not possible if their derivative is not always positive.

[3 marks]

(b)



A1A1A1

Note: Award **A1** for an increasing graph, entirely in first quadrant, becoming concave down for larger values of T , **A1** for tending towards the origin and **A1** for asymptote labelled at $k = A$.

[3 marks]

- (c) taking $\ln$ of both sides **OR** substituting $y = \ln x$ and $x = \frac{1}{T}$ (M1)

$$\ln k = \ln A - \frac{c}{T} \quad \text{OR} \quad y = -cx + \ln A \quad (\text{A1})$$

- (i) so gradient is $-c$ A1

- (ii) y -intercept is $\ln A$ A1

Note: The implied **(M1)** and **(A1)** can only be awarded if **both** correct answers are seen. Award zero if only one value is correct **and** no working is seen.

[4 marks]

continued...

Question 4 continued

- (d) an attempt to convert data to $\frac{1}{T}$ and $\ln k$
e.g. at least one correct row in the following table

(M1)

$\frac{1}{T}$	$\ln k$
$1.69491... \times 10^{-3}$	$-7.60090...$
$1.66666... \times 10^{-3}$	$-7.41858...$
$1.63934... \times 10^{-3}$	$-6.90775...$
$1.61290... \times 10^{-3}$	$-6.57128...$
$1.58730... \times 10^{-3}$	$-6.21460...$
1.5625×10^{-3}	$-5.84304...$
$1.53846... \times 10^{-3}$	$-5.62682...$

$$\text{line is } \ln k = -13400 \times \frac{1}{T} + 15.0 \quad \left(= -13383.1... \times \frac{1}{T} + 15.0107... \right)$$

A1**[2 marks]**

- (e) (i) $c = 13400$ (13383.1...)
(ii) attempt to rearrange or solve graphically $\ln A = 15.0107...$
 $A = 3300000$ (3304258...)

A1**(M1)****A1**

Note: Accept an A value of 3269017... from use of 3sf value.

[3 marks]**Total [15 marks]**

2. (a) $71e^{-0.0514(16)} + 23$ (M1)

54.2°C (54.1956...) A1

[2 marks]

(b) 23°C A1

[1 mark]

(c) $50 = 71e^{-0.0514(k)} + 23$ (M1)

$k = 18.8 \left(\frac{-5000}{257} \ln \left(\frac{27}{71} \right), 18.8101... \right)$ A1

Note: Award **M1** for a sketch showing a point of intersection between the exponential function and $y = 50$.

[2 marks]

Total [5 marks]

6. (a) 78 **A1**
[1 mark]
- (b) (i) 65 **A1**
- (ii) **EITHER**
 (period =) 16 (could be seen on sketch) **(M1)**
 $b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$
 $(b =) 0.393 \left(0.392699..., \frac{\pi}{8} \right)$ **OR** $(b =) 22.5^\circ$ **A1**
- OR**
 $143 = 65 \sin(4b) + 78$ **(M1)**
 $(\sin(4b) = 1)$
 $(4b = \frac{\pi}{2} \text{ **OR** } 4b = 90^\circ)$
 $(b =) 0.393 \left(0.392699..., \frac{\pi}{8} \right)$ **OR** $(b =) 22.5^\circ$ **A1**
- [3 marks]**
- (c) 13 **A1**
- Note:** Apply follow through marking only if their final answer is positive.
- [1 mark]**
- (d) $(b =) 0.196 \left(0.196349..., \frac{\pi}{16} \right)$ **OR** $(b =) 11.3^\circ (11.25^\circ)$ **A1**
- [1 mark]**
Total [6 marks]

10. (a) $m = 1 - 2.5 \log_{10}(0.0525)$ (M1)
 $= 4.20 \text{ (4.19960...)}$ A1

[2 marks]

(b) attempt to solve $7 = 1 - 2.5 \log_{10}(b)$ (M1)

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5 \log_{10}(b)$.

$b = 0.00398 \text{ (0.00398107...)}$ A1

[2 marks]

(c) $-3.2 = (1 - 2.5 \log_{10}(b_n)) - (1 - 2.5 \log_{10}(b_p))$ M1

$-3.2 = -2.5 \log_{10}\left(\frac{b_n}{b_p}\right)$ A1

$\frac{b_n}{b_p} = 19.1 \text{ (19.0546...)}$ A1

[3 marks]

Total [7 marks]

11. (a) $y = -0.00855x^3 - 0.234x^2 - 0.225x + 3.20$
 $(y = -0.00854819...x^3 - 0.234002...x^2 - 0.224884...x + 3.20056...)$

A2

Note: Award **A0A1** for at least two terms correct.

[2 marks]

(b) $y(2x)$ (for horizontal stretch)

(A1)

attempt to stretch vertically by factor $\frac{1}{2}$

(M1)

$y = 0.0332x^3 - 0.15x^2 - 0.58x$ (+1.1)

A1

Note: Award **A0M1A0** for a vertical stretch, factor 2. Although a d value of 1.1 is preferred, technically this value can be wrong/omitted and the question is still answered (hence it is presented in brackets).

[3 marks]**Total [5 marks]**

13. (a) Gradient = $\frac{14.9+1.3}{6}$ (= 2.7) (M1)

$\log_{10} Q = 2.7P - 1.3$ (A1)

$Q = 10^{2.7P-1.3}$ OR $Q = 0.0501 \times 10^{2.7P}$ (= 0.0501187... $\times 10^{2.7P}$) A1

[3 marks]

(b) $\ln R$ on one axis and Q on the other axis A1

[1 mark]

(c) $\log_{10} (4.3 \ln R + 12.1) = 2.7P - 1.3$ OR $10^{2.7P-1.3} = 4.3 \ln R + 12.1$ (M1)

$P = \frac{\log_{10} (4.3 \ln R + 12.1) + 1.3}{2.7}$ A1

[2 marks]

Total [6 marks]

17. (a) $D = S - R$

METHOD 1

$$= \operatorname{Re}(1.15e^{(0.0165t-2.97)i}) - \operatorname{Re}(1.08e^{(0.0165t+0.413)i}) \quad (+18.9 - 4.94) \quad (M1)(A1)$$

$$= \operatorname{Re}(e^{0.0165ti}(1.15e^{-2.97i} - 1.08e^{0.413i})) \quad (+13.96) \quad (M1)$$

$$= \operatorname{Re}(e^{0.0165ti}(2.21379...e^{-2.85310...i})) \quad (+13.96) \quad (A1)$$

$$= 2.21\cos(0.0165t - 2.85) + 13.96 \quad (2.21379...\cos(0.0165t - 2.85310...) + 13.96)$$

A1A1

$$(a = 2.21, b = -2.85, c = 13.96)$$

Note: Award **A1** for $2.21\cos(0.0165t - 2.85)$ and **A1** for “+13.96”. The **A1** for 13.96 is independent of the previous marks.

METHOD 2

$$c = 13.9 - 4.94... = 13.96 \quad A1$$

using a graph of D **M1**

$$\text{maximum } (172.915..., 16.1738...) \quad (A1)$$

$$\text{minimum } (-17.4842..., 11.7462...) \quad (A1)$$

EITHER

$$\text{amplitude } 16.1738... - 11.7462... = 4.4276...$$

$$a = 2.21 \quad (2.2138...) \quad A1$$

OR

$$a = 16.1738... - 13.96 = 2.21 \quad (2.2138...) \quad A1$$

THEN**EITHER**

$$\text{when } t = 0, D = 11.8377...$$

$$11.8377... = 2.2138\cos(b) + 13.96$$

$$b = -2.85 \quad (2.85309...) \quad A1$$

OR

$$b = -0.0165 \times 172.915... = -2.85 \quad (2.85309...) \quad A1$$

[6 marks]

(b) 16.2 (16.1737...) hours on day 173

A1A1

Note: Accept an answer of “day 172” for the second **A1**.

[2 marks]**Total [8 marks]**

6. (a) (i) METHOD 1

attempt to find change in height of the ball using gradient

(M1)

$$\frac{a}{0.43} = (-)0.045$$

$$a = (-)0.045 \times 0.43$$

$$a = (-)0.0194(\text{m}) \quad (0.01935 \text{ (m)})$$

A1**METHOD 2**

attempt to find height at back of home plate

(M1)

horizontal distance to the front of the home plate = 16.6666... (m)

height at the back of the home plate = $-0.045(16.6666... + 0.43) + 2$

$$(= 1.23065 \text{ (m)})$$

Note: The **M1** can be awarded for $16.6666... + 0.43$ seen at some point.

$$(a = 1.25 - 1.23065...)$$

$$(a =) (-)0.0194 \text{ (m)} \quad (0.01935 \text{ (m)})$$

A1

$$(ii) \quad 1.25 - 0.01935 = 1.23065 \quad (\text{may be seen in part (a)(i)})$$

A1

$$0.53 < 1.23065 < 1.24$$

R1

therefore a strike

AG

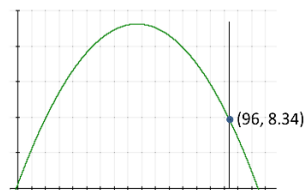
Note: Do not award **A0R1**.

[4 marks]*continued...*

Question 6 continued

(b) **METHOD 1**indication of $d = 96$ in the function $h(d)$ or its graph**(M1)****EITHER**

$$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$$

OR**THEN**

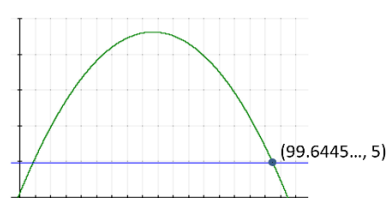
$$(h(96) =) 8.34 \text{ (m)}$$

A1

8.34 > 5 so the ball will go over the wall.

A1**METHOD 2**indication of $h = 5$ in the function $h(d)$ or its graph**(M1)****EITHER**

$$5 = -0.01d^2 + 1.04d + 0.66$$

OR**THEN**

$$d = 99.6 \text{ (m)} \quad (99.6445... \text{ (m)}) \quad (d = 4.35548... \text{ (m)} \text{ may also be seen})$$

A1

96 < 99.6445... so the ball will go over the wall.

A1
[3 marks]**[Total: 7 marks]**

14. METHOD 1 Analytical approach

recognizing that the linear equation must be expressed in log form (M1)

$$\log y = m \log x + \log c \quad (\text{or } \log y = m \log x + C)$$

EITHER

use of slope formula (must involve logs) (M1)

$$m = \frac{\log(34.822) - \log(13.1951)}{\log(4) - \log(2)} = 1.4 \quad \text{A1}$$

attempt to substitute a value (M1)

$$\log c = \log(13.1951) - 1.4 \log 2 (= 0.69897\dots)$$

$$\Rightarrow c = 5 \quad \text{A1}$$

OR

$$y = c \cdot x^m \quad \text{A1}$$

attempt to set up two equations involving power functions (M1)

$$13.1951 = c \times 2^m \quad \text{and} \quad 34.822 = c \times 4^m$$

$$2^m = \frac{34.822}{13.1951} = 2.639\dots \Rightarrow m = \log_2 2.639\dots = 1.4 \quad \text{A1}$$

$$c = \frac{13.1951}{2.639\dots} = 5 \quad \text{A1}$$

THEN

(so the equation is) $y = 5 \times x^{1.4}$ A1

METHOD 2 Regression analysis

recognizing that a log-log graph results in a power function model (M1)

$$y = a \times x^b$$

attempt to find a power regression model using the given two points (M1)

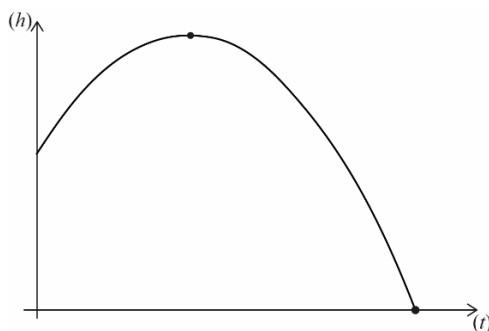
$$a = 5 \quad \text{and} \quad b = 1.4 \quad \text{(A1)(A1)}$$

(so the equation is) $y = 5 \times x^{1.4}$ A2

[Total: 6 marks]

1. (a) METHOD 1

correct sketch with some indication of maximum point

(M1)

$$0.921 \text{ (seconds)} \left(0.921052..., \frac{35}{38} \right)$$

A1**METHOD 2**

correct substitution into equation for line of symmetry

(M1)

$$(t =) -\frac{8.75}{2 \times -4.75}$$

$$0.921 \text{ (seconds)} \left(0.921052..., \frac{35}{38} \right)$$

A1**METHOD 3**

equating the correct derivative to 0

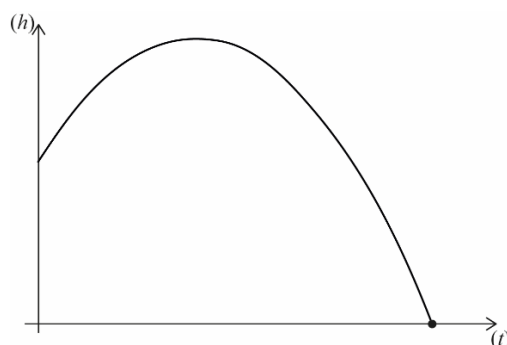
(M1)

$$-9.5t + 8.75 = 0$$

$$0.921 \text{ (seconds)} \left(0.921052..., \frac{35}{38} \right)$$

A1

Note: Award **M1A0** for a final answer of 0.92 seen with no working.

[2 marks]**(b) METHOD 1**correct sketch with some indication of x -intercept**(M1)**

Note: May be seen in part (a).

$$2 \text{ (seconds)}$$

A1*continued...*

Question 1 continued

METHOD 2

setting the equation to zero

(M1)

$$-4.75t^2 + 8.75t + 1.5 = 0$$

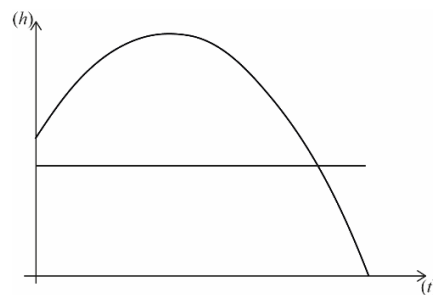
2 (seconds)

A1

Note: If both roots are given, with or without working, award **(M1)A0**.

[2 marks]**(c) METHOD 1**

correct sketch of quadratic function and a straight line in approximate correct position

(M1)

1.88 (seconds) (1.87577...(seconds))

A1**METHOD 2**

setting the equation equal to 1.2

(M1)

$$-4.75t^2 + 8.75t + 1.5 = 1.2$$

1.88 (seconds) (1.87577...(seconds))

A1

Note: Award **(M1)A0** if -0.0336702... seen as (part of) a final answer.
Award **M1A0** for answer of 1.9 seen without working.

[2 marks]**(d) Award R1 for a sensible reason in the context of the question:****R1**

e.g.

The model ignores air resistance (or wind)

The model treats the ball as a point

The model assumes gravity is constant

The model assumes that the ball continues to follow the trajectory even after hitting the ground

This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[1 mark]**Total [7 marks]**

2. (a) $\frac{18-4}{2}$ (M1)
 (a =) 7 A1

[2 marks]

(b) $\frac{18+4}{2}$ OR $18-7$ OR $4+7$ (M1)
 (d =) 11 A1

[2 marks]

(c) (time between high and low tide is) 6h15m OR 375 minutes (A1)
 multiplying by 2 (M1)
 750 minutes A1

[3 marks]

(d) EITHER
 $\frac{360^\circ}{b} = 750$ (A1)
 OR
 $7 \cos(b \times 375) + 11 = 4$ (A1)
 THEN
 (b =) 0.48 A1

Note: Award **A1A0** for an answer of $\frac{2\pi}{750} \left(= \frac{\pi}{375} = 0.00837758... \right)$.

[2 marks]

(e) equating their cos function to 6 OR graphing their cos function and 6 (M1)
 $7 \cos(0.48t) + 11 = 6$
 $\Rightarrow t = 282.468... \text{ (minutes)}$ (A1)
 $= 4.70780... \text{ (hr)}$ OR 4hr 42 mins (4hr 42.4681... mins) (A1)
 so the time is 10:42 A1

[4 marks]

continued...

Question 2 continued

(f) next solution is $t = 467.531\dots$

(A1)

$467.531\dots - 282.468\dots$

185 (mins) (185.063...)

A1

[2 marks]

[Total: 15 marks]

2. (a) (i) 1200 **A1**
- (ii) the initial population of the bacteria **A1**
[2 marks]
- (b) $1200 \times k^3 = 18750$ **(A1)**
- $(k =) 2.5$ **A1**
[2 marks]
- (c) $1200 \times 2.5^{1.5}$ **(A1)**
- 4740 (4743.41...) **A1**
- Note:** Do not penalize if final answer is not given as an integer. Award **(A1)A0** for an answer of 3950 (3949.14...) from use of 1.3 in the exponent, but only if working is shown.
- [2 marks]
- (d) equating $P(t)$ and $S(t)$ **OR** equating each function to a common variable **(M1)**
- $1200 \times 2.5^t = 5000 \times 1.65^t$; $1200 \times 2.5^t = x$ and $5000 \times 1.65^t = x$
- $t = 3.43$ (hours) (3.43456...) **A1**
[2 marks]

continued...

Question 2 continued(e) **METHOD 1**

$$5000 \times 1.65^t = 19000 \quad (M1)$$

$$(t =) 2.66586... \quad \text{OR} \quad (t - 2 =) 0.66586... \text{ (seen)} \quad (A1)$$

multiplying by 60 seen to convert to minutes (M1)
 $(m = 39.9521...)$

$(m =) 40$ (minutes) **OR** 2 hours and 40 minutes A1

METHOD 2

equating an expression for $S(t)$ to 19000 (M1)

expressing t as $2 + \frac{m}{60}$ (A1)

$$5000 \times 1.65^{2 + \frac{m}{60}} = 19000$$

$$2 + \frac{m}{60} = 2.66586... \quad A1$$

$(m =) 40$ (minutes) **OR** 2 hours and 40 minutes A1

Note: Award **(M1)(A1)(M1)A0** for an answer of 39.9521... or 39 with or without working.

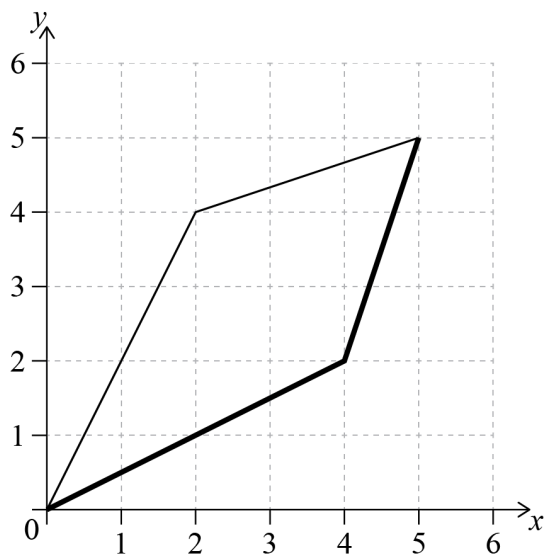
[4 marks]

Total [12 marks]

2. (a) 4

A1**[1 mark]**

(b)

**A1A1**

Note: Award **A1** for passing through (0, 0) and (4, 2),
A1 for passing through (4, 2) and (5, 5).

[2 marks](c) attempt to solve $y = 3x - 1$ for x **OR** changing variables**(M1)**

$$(g^{-1}(x)) = \frac{x+1}{3}$$

A1**[2 marks]**(d) sketch of $g(x)$ or $g^{-1}(x)$, algebraic approach**(M1)**

$$\frac{1}{2}x = \frac{x+1}{3}$$

$$(x =) 2$$

A1**[2 marks]****[Total 7 marks]**

5. (a) $2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$

A1**[1 mark]**

(b) finding other equations to solve simultaneously

(M1)

$$5 = a(10)^2 + b(10) + c \quad \textbf{AND} \quad 7.16 = a(17)^2 + b(17) + c$$

$$\textbf{OR} \quad 5 = 100a + 10b + c \quad \textbf{AND} \quad 7.16 = 289a + 17b + c$$

any one coefficient in equation correct

(A1)

$$f(x) = -0.00490x^2 + 0.441x + 1.08$$

A1

$$\left(f(x) = -0.00489795\dots x^2 + 0.440816\dots x + 1.08163\dots \right)$$

$$\left(f(x) = -\frac{6}{1225}x^2 + \frac{108}{245}x + \frac{53}{49} \right)$$

Note: Award at most **(M1)(A1)A0** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 80 into their equation

(M1)

$$(f(80) =) 5$$

A1

$5 > 4$ **OR** therefore the ball will go over the fence

R1

Note: Do not award **A0R1**; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero, graph

(M1)

$$0 = -0.00489795\dots x^2 + 0.440816\dots x + 1.08163\dots \quad \textbf{OR} \quad f(x) = 0$$

$$92.4 \quad (92.3902\dots) \quad (\text{m})$$

A1**[2 marks]****[Total 9 marks]**

7. (a) (i) $m = ah^3$ (M1)
 $64 = a \times 0.8^3 \Rightarrow a = 125$ (A1)

$m = 125h^3$ A1

Note: The final answer must be written as m in terms of h to award the final A1.

(ii) $m = (125(0.75)^3) = 52.7 \text{ (kg)} \text{ (52.7343...)} \quad \text{A1}$

[4 marks]

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$
 $h = \sqrt[3]{1.76} \text{ (1.20736...)} \quad \text{(A1)}$

$E = k_1 h^2 \text{ (seen anywhere)} \quad \text{(A1)}$

$k = \frac{(\sqrt[3]{1.76})^2}{0.8^2} \quad \text{(M1)}$
 $= 2.28 \text{ (2.27769...)} \quad \text{A1}$

OR (finding a formula for E in terms of m)

$(m = 125h^3)$
 $E = k_1 h^2 \quad \text{(A1)}$

$E = k_2 m^{\frac{2}{3}} \quad \text{(A1)}$

$k = \frac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}} \quad \text{(M1)}$

$= 2.28 \text{ (2.27769...)} \quad \text{A1}$

[4 marks]

[Total 8 marks]

14. (a) $x_B = 4\sqrt{2(t-3)}$ ($t \geq 3$) **A1A1**

Note: Award **A1** for multiplying by 2 and **A1** for $t-3$. Award **A1A0** for $4\sqrt{2t-3}$.

[2 marks]

(b) equating their x_B to $4\sqrt{t}$ **(M1)**

$$4\sqrt{2(t-3)} = 4\sqrt{t}$$

$$t = 6 \text{ (seconds)}$$

A1

Note: Do not **FT** from part (a) to part (b).

[2 marks]

[Total 4 marks]

3. (a) correct substitution of 0.0003 into the formula (A1)

$$\text{pH} = -\log_{10}(0.0003)$$

$$= 3.52 \quad (3.52287\dots)$$

A1

[2 marks]

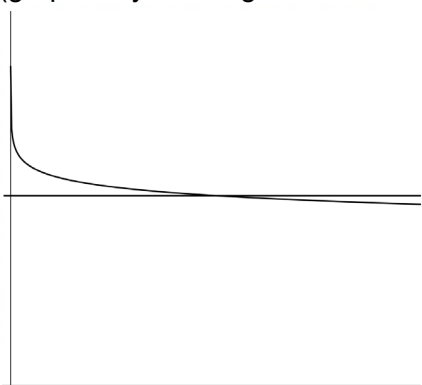
- (b) EITHER attempt to change to exponential form (M1)

$$[\text{H}^+] = 10^{-6.6}$$

OR

$$\text{attempt to solve } 6.6 = -\log_{10}[\text{H}^+]$$

(graphically or using numerical solver)



THEN

$$[\text{H}^+] = 0.000000251 \text{ (moles per litre)} \quad (0.000000251188\dots, 2.51 \times 10^{-7}) \quad \text{A1}$$

Note: Award **M1A0** for an answer of 2.51 (2.51188...) seen.

Award **M0A0** if a substitution of 6.6 into the formula is seen without an answer or some indication of using numerical solver.

[2 marks]

- (c) $2 = -\log_{10}[\text{H}^+]$, $4.5 = -\log_{10}[\text{H}^+]$ (A1)
 10^{-2} (0.01) OR $10^{-4.5}$ (0.0000316227...)

substitution of their values into correct ratio (M1)

$$\frac{10^{-2}}{10^{-4.5}} \quad \text{OR} \quad \frac{0.01}{0.0000316227\dots}$$

$$= 316.227\dots = 316$$

A1

Note: Some candidates may subtract logs and hence look to solve $\log_{10}[\text{H}^+] = 2.5$.

[3 marks]

[Total: 7 marks]

2. (a) 120 (ng)

A1**Note:** Only accept an exact answer of 120.**[1 mark]**

(b) $(m(20000) =) 120e^{-0.000121 \times 20\,000}$
 10.7 (10.6705.....) (ng)

(A1)**A1****[2 marks]**

(c) attempt to write or solve an inequality (or equality) with a value of 60 seen
 $(m(t) =) 120e^{-0.000121t} \leq 60$ **OR** sketch **OR** table of values
 $(t >) 5728.489....$
 5729 (years)

(M1)**(A1)****A1****Note:** Do not accept 5728 or 5730 for the final **A1**.**[3 marks]****[Total: 6 marks]**

6. (a) (i) $a=30$ **A1**
- (ii) $p=11$ **A1**
- [2 marks]**
- (b) **EITHER**
- substituting $n = 11$ and equating to 230 (or equivalent) **(M1)**
- $$30(11) + b = 230$$
- OR**
- using an arithmetic sequence **(M1)**
- $$200 + 30(n - 10)$$
- THEN**
- $$b = -100$$
- A1**
- [2 marks]**
- [Total 4 marks]**

8. (a) $d = \frac{16.63 + 7.83}{2}$ (A1)
 $(d =) 12.2 \text{ (12.23)}$ A1
[2 marks]

(b) $(a =) 4.4$ A1
[1 mark]

(c) period = $2(356 - 173)$ (= 366) (A1)
 $(b =) \frac{2\pi}{366} = \frac{\pi}{183}$ (= 0.0171671...) (Accept $b = \frac{360}{366}$ (= 0.983606...)) A1
[2 marks]

(d) **EITHER**
 attempt to find midpoint of x values of max and min (M1)

$$c = \frac{-10 + 173}{2}$$

OR
 substitute values and solve (M1)

$$b(173 - c) = \frac{\pi}{2} \quad \text{OR} \quad 16.63 = 4.4 \sin(b(173 - c)) + 12.23$$

THEN

$(c =) 81.5$ A1

Note: Accept other possible positive values of $c = 81.5 + 366n$.
 FT from their “c” consistent with their “a”.

[2 marks]
[Total 7 marks]

10. (a) vertical stretch scale factor 4

A1**Note:** The vertical stretch can occur in any order.**EITHER**translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ followed by horizontal stretch scale factor $\frac{1}{2}$ **A1A1****OR**horizontal stretch scale factor $\frac{1}{2}$ followed by translation $\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$ **A1A1****Note:** Award **A1A0** in either approach, for correct transformations, but incorrect order.**[3 marks]**(b) $g(2) = 5$ **(A1)**

$$f(5) = 7.78 \quad (= 7.78364\dots, 4 \ln 7)$$

A1**[2 marks]**(c) $4 \ln(2g(x) - 3) = 2 \ln 9$ **(M1)**

$$g(x) = 3$$

(A1)

$$x = 1$$

A1**[3 marks]****[Total 8 marks]**

12. (a)

Bounce number, n	Maximum height, h , according to the model
1	0.613
2	0.514
3	0.431 (0.430988...)
4	0.361 (0.361383...)

A1A1
[2 marks]

(b) $(0.430988... - 0.439)^2 + (0.361383... - 0.377)^2$ seen (A1)
 3.08×10^{-4} ($3.08055... \times 10^{-4}$) A1

[2 marks]

(c) (i) $h = 0.716 \times 0.851^n$ (0.715766..., 0.850774...) A2

Note: Award **A2** for $h = 0.716e^{-0.162n}$ ($h = 0.715766...e^{-0.161608...n}$)
Award at most **A1A0** for correct coefficients, with any error in notation (e.g. x instead of n , missing $h =$)
Award at most **A1A0** for $h = 0.731 \times 0.838^n$ from using the values from the model.

(ii) ($h =$) 0.716 (metres) A1
[3 marks]
[Total 7 marks]

13. attempt to substitute $t = 9$ and $P = 433$ (OR $t = 12$ and $P = 954$) (M1)

$$433 = \frac{L}{1 + Ce^{-0.28(9)}} \quad \text{and} \quad 954 = \frac{L}{1 + Ce^{-0.28(12)}} \quad (\text{A1})(\text{A1})$$

attempt to rearrange into linear form or substitute for L

$$433(1 + Ce^{-0.28(9)}) = L \quad \text{and} \quad 954(1 + Ce^{-0.28(12)}) = L \quad \text{OR} \quad 954 = \frac{433(1 + Ce^{-0.28(9)})}{1 + Ce^{-0.28(12)}} \quad (\text{M1})$$

attempt to solve simultaneously (e.g. graph or eliminate L)

$$C = 306.187\dots$$

(M1)

$$L = 11100 \quad (11100.2\dots)$$

A1

Note: Accept a correct decimal value.
--

[6 marks]

2. (a) $d = 5.75 \left(d = \frac{23}{4} \right)$ **A1A1**

Note: Award **A1** for 5.75 seen, **A1** for completely correct equation.

[2 marks]

- (b) attempt to substitute into the axis of symmetry formula using *their* 5.75 **OR**
attempt to substitute the coordinates (3, 5.1) or (8.5, 5.1) into the function **(M1)**

$$5.75 = \frac{-b}{2 \times (-0.2)} \quad \text{OR} \quad 5.1 = -0.2(3)^2 + 3b \quad \text{OR} \quad 5.1 = -0.2(8.5)^2 + 8.5b$$

$$(b =) 2.3$$

A1
[2 marks]

- (c) indication of finding zero or attempt to use symmetry **(M1)**
 $h(d) = 0$ **OR** $0 = -0.2d^2 + 2.3d$ **OR** 2×5.75 **OR** $3 - 0 = d - 8.5$ **OR**
resketch of graph with zero indicated

$$(d =) 11.5 \text{ (m)}$$

A1

Note: Award **A0** for a FT answer that is less than 8.5 (contradicts the given information).

[2 marks]

- (d) **EITHER**
the dolphin is below the surface of the water / the dolphin is underwater / the dolphin
is in the water **R1**

OR

the value of h cannot be negative since the definition of the model only applies
to the jump

R1
[1 mark]

[Total: 7 marks]

4. (a) $15p$ A1

[1 mark]

(b) $(p-5)$ or $(n-15)$ seen A1

adding a cost expression (multiplying a p and an n) to their $15p$ (M1)

$C(n) = 15p + (p-5)(n-15)$ **OR** $C(n) = np - 5n + 75$ (or equivalent) A1
[3 marks]

(c) (i) $(25.73 \times 100 =) 2573$ (EUR) A1

Note: Answer must be exact.

(ii) correctly substituting *their* total cost and $n = 100$ into *their* cost function (A1)
 $2573 = 15p + (p-5)(100-15)$

$(p =) 29.98$ (EUR) A1

Note: Answer must be written to 2 dp.

[3 marks]

[Total: 7 marks]

6. (a) (i) $f(x) = 0.7x$ **OR** $f(x) = x - 0.3x$ **A1**
- (ii) $g(x) = x - 150$ **A1**
- [2 marks]**
- (b) attempt to discuss the order in which the reduction/discount are applied **(M1)**
- $(f \circ g)(x)$ describes when the 150 CAD reduction is applied before the 30% discount. **A1**
- [2 marks]**
- (c) **EITHER**
- evidence of solving $(f \circ g)(x)$ and $(g \circ f)(x)$ for a price at 450 CAD **R1**
- OR**
- $(f \circ g)(x) = 0.7x - 105$ and $(g \circ f)(x) = 0.7x - 150$ seen **R1**
- OR**
- stating that it is better for 30% to be deducted from a higher price **R1**
- OR**
- the numbers 165, 210 are seen **R1**
- THEN**
- $(g \circ f)(x)$ is better for the customer **A1**

Note: Do not award **R0A1**.

[2 marks]
[Total 6 marks]

10. (a) 3

A1
[1 mark]

(b) 1

A1
[1 mark]

(c)

x	0	1	2	3	4
$f^{-1}(x)$	2	1	4	0	3

award **A1** for 1 correct, **A2** for 3 correct, **A3** for all 5 correct

A1A1A1
[3 marks]
Total [5 marks]

11. (a) 0, 1, 3

A2

Note: Award **A1** if either 1 or 3 is seen. Award **A1** for 0, 1000, 3000

[2 marks]

- (b) sketching a graph or using calculus
- (M1)
- attempting to find the y coordinate at the local maximum
- (M1)
- local max occurs at (0.451416..., 0.631130...)
- (A1)
- $w > 0.631$
- A1

[4 marks]

Total [6 marks]

- Q10.** (a) (i) $F = \frac{k}{d^2}$ (A1)
- substitution of d and F into their equation (M1)
- $$40000 = \frac{k}{7000^2}$$
- $$k = 1.96 \times 10^{12}$$
- $$F = \frac{1.96 \times 10^{12}}{d^2}$$
- A1
- (ii) $F = \frac{1.96 \times 10^{12}}{3500^2}$
- 160000 (N) A1
- [4 marks]
- (b) (i) C A1
- D A1

Note: Award at most A1A0 if an additional incorrect answer is given.
--

- (ii) for C the gradient is $k = 1.96 \times 10^{12}$ A1
- for D the gradient is -2 A1
- [4 marks]
- Total [8 marks]**

Q2.

(a)	$\frac{360}{12}$	M1
	$= 30$	AG

Note: Award **M0** for use of given value, 30.

(b)	substitution of $t = 5$ into model $(-2.5 \cos(30 \times 5) + 4.5)$	[1 mark] (M1)
	$h(5) = 6.67 (6.66506\dots)$	A1

Note: Award **M1A0** for 2.75187... (radian answer).

(c)	(i)	2.5 (m)	[2 marks] A1
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Note: Award **A0** for -2.5 (m)

(ii)	$h(t) = 4.5$	A1A1
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Note: Award **A1** for an equation of a horizontal line $h = \text{constant}$ (accept $y = \text{constant}$), **A1** for that constant being 4.5. Second **A1** is dependent on first **A1**. Do not award **A0A1**.

[3 marks]

(d)	attempt at finding point of intersection between $h(t) = \text{constant}$ and the curve using $h(t) = 2.55$ or $h(t) + 0.1 = 2.65$ to find intersection	(M1) (A1)
	13.3 (13.2913...)	(A1)
	13:17 (1:17 p.m.)	A1

Note: The final **A1** can be awarded for correctly converting **their** value of t into a time.

[4 marks]*continued...*

Question 2 continued.

- (e)

using $h(t) = 2.75$ or $h(t) - 0.1 = 2.65$ to find intersection
 $22.4809... - 13.2913... - 0.5$

(A1)
(A1)(M1)(M1)

Note: Award **A1** for 22.4809... seen, **M1** for subtracting their two times for harbour, **M1** for subtracting 0.5 from their answer at any stage of the work.

8.69 (hours) (8.68958... (hours))

A1

Note: Accept minutes used in working, but the final answer must be in hours.

[5 marks]
Total [15 marks]

2. (a) 310 million is the maximum no of EVs **A1**
OR
 the carrying capacity is 310 million **A1**
OR
 the long term behaviour of the system approaches 310 million **A1**

Note: 'Million' needs to be seen to award the **A1**.

[1 mark]

- (b) (i) correct substitution $0.2 = \frac{310}{1 + Ce^{-k}}$ **A1**

correct rearrangement which leads to correct final answer **A1**

eg $1 + Ce^{-k} = 1550$ OR $Ce^{-k} = 1549$ OR $0.2Ce^{-k} = 309.8$

$C = 1549e^k$ **AG**

[2 marks]

- (ii) correct substitution $3.1 = \frac{310}{1 + Ce^{-7k}}$ **A1**

correct first stage of rearrangement **(A1)**

eg $1 + Ce^{-7k} = 100$ OR $3.1 + 3.1Ce^{-7k} = 310$

$C = 99e^{7k}$ **A1**

Note: Allow any correct expression eg $C = \frac{99}{e^{-7k}}$ or $C = \frac{306.9}{3.1e^{-7k}}$.

[3 marks]

continued...

Question 2 continued

- (c) valid method for finding either variable

(M1)

eg graphically or elimination of C , $e^{6k} = \frac{1549}{99}$

- (i)
- $k = 0.458$
- (0.458374...)

A1

Note: Award at most **M1A0** if values other than at $t = 1$ or $t = 7$ are used.

[2 marks]

- (ii)
- $C = 2450$
- (2449.74...)

A1

Note: Do not award **A1FT** on a wrong value for k if the method used is not clearly shown.

Note: Accept $C = 2440$ (2443.33...) from substitution of 3 sf value for k in $C = 99e^{7k}$

Note: Award **A0** if values other than at $t = 1$ or $t = 7$ are used.

[1 mark]

$$\left(N = \frac{310}{1 + 2449.74...e^{-0.458374...t}} \right)$$

- (d) substitution of
- $t = 4$
- into their logistic function

$$N = \frac{310}{1 + 2449.74...e^{-0.458374... \times 4}}$$

(M1)

$$a = 0.789606...$$

Note: If using 3 sf values for k and C : $a = 0.788346...$. If using $C = 2440$ then $a = 0.792$.

$$a = 0.8$$

A1

Note: The answer must be given correct to 1 decimal place

[2 marks]

- (e) calculation of a squared residual

(M1)

$$0^2 + 0.1^2 + 0.2^2 + 0.1^2 + 0.1^2$$

$$\text{sum of square residuals} = 0.07$$

A1

[2 marks]

continued...

Question 2 continued

- (f) substitution of their values into the error function

M1

Note: For the **M1** allow any integer value of n being substituted, with their sum of square residuals.

$$\left(E = \sqrt{\frac{0.07}{5}}\right) = 0.118 \text{ (0.118321...)}$$

A1

$$0.118 < 0.25 \text{ (so can use the model)}$$

R1**[3 marks]**

- (g) number in 2035 is given by value of
- N
- when
- $t = 20$

(A1)

247 million (246.873...)

A1

Note: Accept 246 million from using 3 sf values for k and C .

Award **(A1)A0** for 247 or 246 without 'million'.

If using $t = 19$ award **(A0)A1** for 220 or 221 million. If using $t = 21$ award **(A0)A1** for 267 million.

[2 marks]

- (h) multiplication of their
- model**
- by one of 0.2 or 0.1

M1

Note: For the **M1** ignore any extra incorrect multipliers.

number of public charging points needed in year t is

$$\frac{310}{1 + 2449.74...e^{-0.458374...t}} \times 0.2 \times 0.1 \left(= \frac{6.2}{1 + 2449.74...e^{-0.458374...t}} \right) \text{ (million) OR}$$

$$\frac{310,000,000}{1 + 2449.74...e^{-0.458374...t}} \times 0.2 \times 0.1 \left(= \frac{6,200,000}{1 + 2449.74...e^{-0.458374...t}} \right)$$

A1**[2 marks]**

$$(i) \left(\frac{0.54 - 0.22}{2} \right) = 0.16 \text{ (million) (160,000)}$$

A1**[1 mark]**

continued...

Question 2 continued

(j) attempt to express number of charging points at t as a linear function **(M1)**

linear function passes through $(5, 0.22)$ or $(7, 0.54)$ **(A1)**

$0.22 + 0.16(t - 5)$ OR $0.54 + 0.16(t - 7)$ OR $0.16t - 0.58$ OR $160,000t - 580,000$ **A1**

Note: Award at most **(M1)(A1)A1** for an expression that has the variable in years, for example $0.22 + 0.16(x - 2020)$.

attempt to solve their $\frac{6.2}{1 + 2449.74...e^{-0.458374...t}} > 0.22 + 0.16(t - 5)$ **M1**

Note: Accept = in place of $>$.

$t = 15.1$ (15.1492...) **(A1)**

Note: Accept 15.1665... from use of 3 sf values for k and C .

Year is 2031 **A1**

[6 marks]

Total [27 marks]

6. (a) recognizing the form of inverse variation (M1)

$$V = \frac{k}{t} \quad \text{OR} \quad V \propto \frac{1}{t}$$

$$300 = \frac{k}{5} \quad \text{OR} \quad k = 5 \times 300 \text{ (the } k \text{ must be shown)}$$

$$V = \frac{1500}{t}$$

A1

AG

Note: Award **MOA0** for only verifying that 300 and 5 satisfy given equation, e.g. $300 = \frac{1500}{5}$ seen.

[2 marks]

- (b) 150 seen (A1)

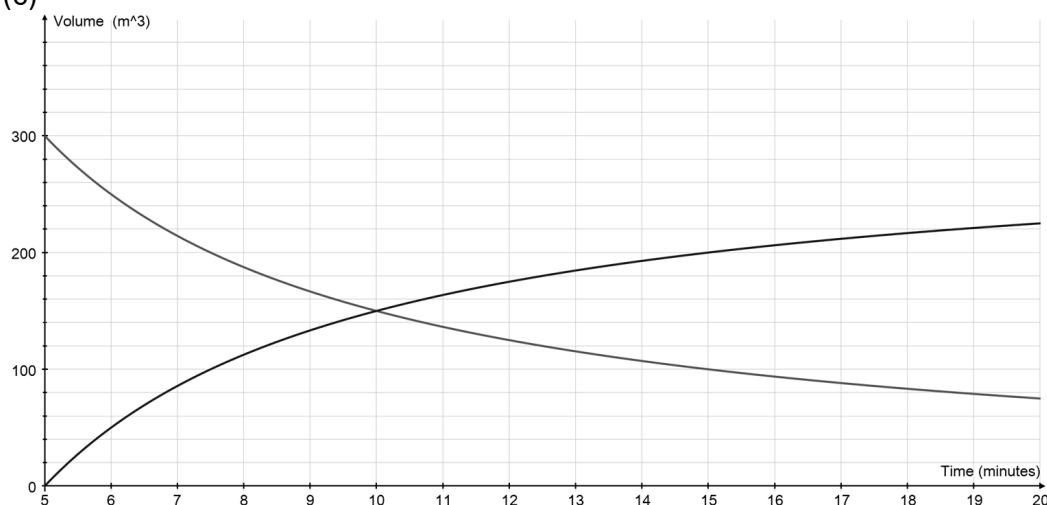
$$150 = \frac{1500}{t}$$

$$t = 10 \text{ (minutes)}$$

A1

[2 marks]

- (c)



A1A1

Note: Award **A1** for an **increasing curve** from point (5, 0). Award **A1** for a smooth curve, with approximately correct shape, eg concave down, approximately passing through (10,150). Accept any endpoint in the range [150, 300].

[2 marks]

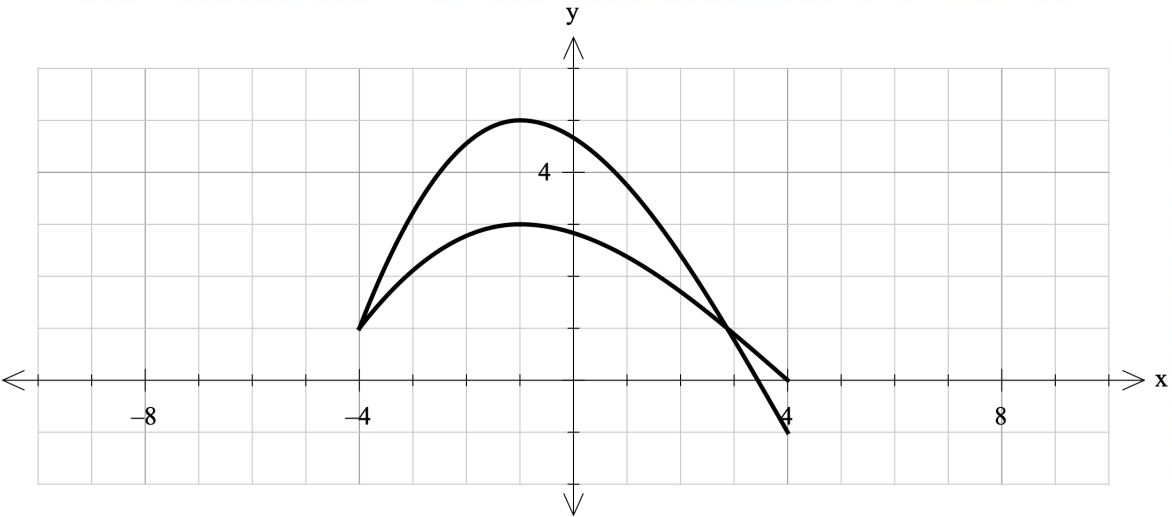
- (d) $(P(t) =) 300 - \frac{1500}{t}$ OR $300 - V(t)$

A1

[1 mark]

Total [7 marks]

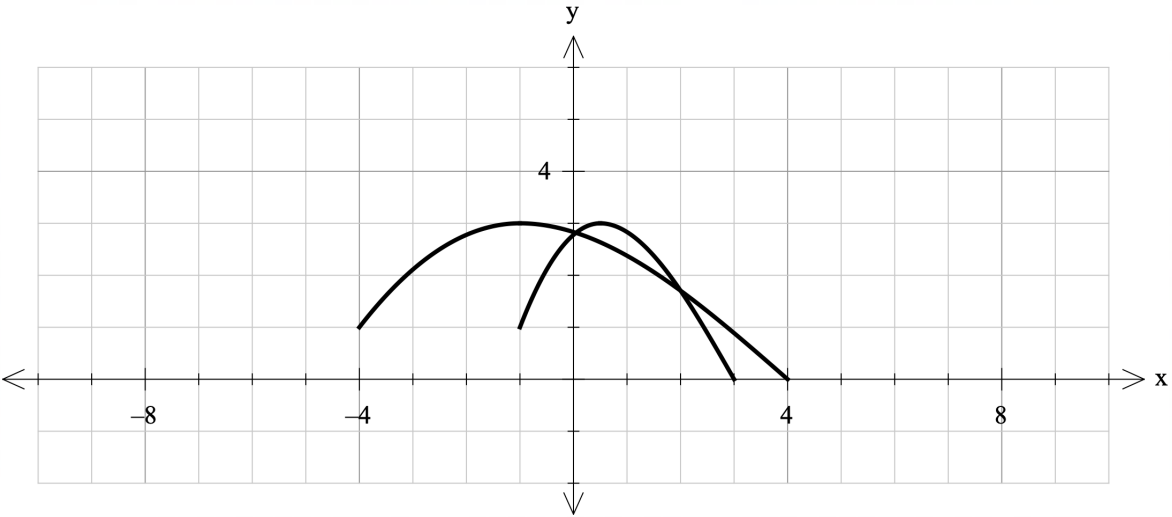
8. (a)



correct maximum of the graph at $(-1, 5)$
correct endpoints at $(-4, 1)$ and $(4, -1)$

A1
A1
[2 marks]

(b)



maximum at $(\frac{1}{2}, 3)$

Note: Allow **A1** if x coordinate of maximum is in the interval $(0, 1)$.

A1

correct endpoints at $(-1, 1)$ and $(3, 0)$

A1
[2 marks]
Total [4 marks]

3. (a) 4

A1
[1 mark](b) (i) **EITHER**Award **M1** for attempting to substitute in (5,3.9) or (10,4) or (8,3.936)

in $y = ax^2 + bx + c$

(M1)

$3.9 = a \times 5^2 + b \times 5 + 4$

$-0.1 = 25a + 5b \quad (-0.02 = 5a + b)$

A1**Note:** Award at most **M1A0** for an equation that is not simplified (e.g. $3.9 = 25a + 5b + 4$).**OR****M1** for attempt at axis of symmetry at $x = 5$ **(M1)**

$5 = \frac{-b}{2a} \quad \text{OR} \quad 10a + b = 0 \quad \text{OR} \quad b = -10a$

A1**OR****M1** for an attempt to find derivative equated to 0.**(M1)**

$2a \times 5 + b = 0$

$10a + b = 0 \quad \text{OR} \quad b = -10a$

A1

(ii) $a = 0.004 \left(\frac{1}{250} \right), b = -0.04 \left(\frac{-1}{25} \right)$

A1A1**[4 marks]**

(c) (i) $\left(\frac{dy}{dx} = \right) 0.008x - 0.04 \quad \left(\frac{1}{125}x - \frac{1}{25} \right)$

A1A1**Note:** Award **A1** for each correct term. Award at most **A1A0** if additional terms are seen.(ii) substituting 0 or 10 into their derivative
(steepest gradient=) 0.04**(M1)****A1****[4 marks]**

(d) (i) $r = 0.994 \quad (0.994142\dots)$

A2

(ii) (very) strong

A1**[3 marks]**

(e) (0.01889 represents) the (increase in) deflection (in m) for each person (on the bridge).

A1**[1 mark]**

- (f)

0.1954 seen

A1
- 0.1 seen **OR** $R(5, 3.8046)$ seen

A1
- Jenny’s model for the height of the walkway has the smaller estimate

A1

Note: The final **A1** is contingent on both preceding **A1** marks being awarded.
Accept an equivalent statement that identifies $n = 0$ for the regression line, with comparison of deflections or bridge heights.

[3 marks]

Total [16 marks]

4. (a) attempt to solve $6 = 3\ln(5t + 1)$ (M1)
 $(t \Rightarrow) 1.28 \text{ (hours)} \left(1.27781 \dots, \frac{e^2 - 1}{5}\right)$ A1
 [2 marks]

- (b) 9.02 (9.02376 ...) A1
 [1 mark]

- (c) attempt to eliminate d M1
 $p = 20(1 - e^{-0.1 \times 3 \ln(5t+1)})$ A1
 use of log law(s) to simplify (M1)
 $p = 20(1 - e^{\ln(5t+1)^{-0.3}}) = 20(1 - (5t + 1)^{-0.3})$
 $p = 20\left(1 - \frac{1}{(5t+1)^{0.3}}\right) \quad (n = 0.3)$ A1

Note: Award **M0A0M0A1** for working backwards from the given equation and substituting values of p and t to find $n = 0.3$.

[4 marks]

- (d) 10.8 (points) (10.8392...) A1
 [1 mark]

- (e) considers $t = 12$ (M1)
 $p = 14.2$ (14.1731...) A1

Note: Award **M1A0** for $p = 15.2554 \dots$ from using $t = 24$.
 Award **M1A0** for $p = 20$ from considering the horizontal asymptote.

[2 marks]

- (f) recognizes $120 = 7p + b$ (M1)
 recognizes $D = 7d$ (M1)
 $120 = 140\left(1 - \frac{1}{(5T+1)^{0.3}}\right) + 50\left(1 - \frac{1}{21\ln(5T+1)+1}\right)$ A1
 $T = 1.91 \text{ (hours)} (1.91387\dots)$ A1

[4 marks]

Total [14 marks]