APOLLO

Mathematics: applications and interpretation
HL Answers

First assessment 2021
Whole questions arranged by syllabus topic.
Original question and scheme crops preserve source diagrams.

APOLLO

Number and algebra

54 questions

3. (a) (i) attempt to find u_{20} using an arithmetic sequence ***(M1)***

e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700, \ldots$

(Charlie ran) $2400\ \text{m}$ ***A1***

(ii) $(r =)\ 1.02$ ***(A1)***

attempt to find u_{20} using a geometric sequence ***(M1)***

e.g. identifying $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** $500,\ 510,\ 520.2, \ldots$

(Daniella ran) $728\ \text{m}\ (728.405...)$ ***A1***

[5 marks]

(b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***

$n > 184.215...$

$n = 185$ ***A1***

[3 marks]

Total [8 marks]

9. (a) (i) $\mathrm{i}^2 = -1$ ***(M1)***

$w = -2 + 1 = -1$ ***A1***

(ii) $w = -1 + \mathrm{i} + 1 = \mathrm{i}$ ***A1***

[3 marks]

(b) **EITHER**

rotation of 90° (anticlockwise, centre at the origin) ***A1A1***

Note: Award ***A1*** for "rotation" and ***A1*** for "90°".

followed by a translation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ***A1***

OR

translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ ***A1***

followed by rotation of 90° (anticlockwise, centre at the origin) ***A1A1***

Note: Award ***A1*** for "rotation" and ***A1*** for "90°".

[3 marks]

(c) **EITHER**

move 1 to left to $1 - \mathrm{i}$ ***(M1)***

then rotate by -90° to

$-1 - \mathrm{i}$ ***A1***

OR

$\mathrm{i}z + 1 = 2 - \mathrm{i}$

$\mathrm{i}z = 1 - \mathrm{i}$

$z = \dfrac{1 - \mathrm{i}}{\mathrm{i}}$ ***(M1)***

$-1 - \mathrm{i}$ ***A1***

[2 marks]

Total [8 marks]

5. (a) $£495 \times 0.9^5 = £292 \ (£292.292...)$ ***(M1)A1***

[2 marks]

(b) $495 \times 0.9^k = 2200 \times 0.85^k$ ***(M1)***

$k = 26.1 \ (26.0968...)$ ***A1***

Note: Award ***M1A0*** for $k - 1$ in place of k.

[2 marks]

(c) depreciation rates unlikely to be constant (especially over a long time period) ***R1***

Note: Accept reasonable answers based on the magnitude of k or the fact that "value" depends on factors other than time.

[1 mark]

Total [5 marks]

7. (a) recognition of geometric sequence *eg* $r = 0.82$ ***(M1)***

$$S_{10} = \frac{450\left(1-0.82^{10}\right)}{1-0.82}$$ ***(A1)***

$= 2160 \text{ m } (2156.37...)$ ***A1***

[3 marks]

(b) $S_{\infty} = \frac{450}{1-0.82}$ ***(M1)***

$= 2500 < 2520$ so the balloon will not reach the required height. ***A1***

[2 marks]

(c) horizontal motion not taken into account,
rate of cooling will not likely be linear,
balloon is considered a point mass / size of balloon not considered,
effects of wind/weather unlikely to be consistent,
a discrete model has been used, whereas a continuous one may offer greater accuracy ***R1***

Note: Accept any other sensible answer.

[1 mark]

Total [6 marks]

12. (a) (i) $z_1^{\,3} = 27e^{\frac{i\pi}{4}} (= 27e^{0.785398\ldots i})$ ***A1A1***

Note: Award ***A1*** for 27 and ***A1*** for the angle in the correct form.

(ii) $\left(\frac{z_1}{z_2}\right)^4 = \left(\frac{81}{16}\right)e^{\frac{i\pi}{2}} (= 5.0625e^{1.57079\ldots i})$ ***A1A2***

Note: Award ***A1*** for $\frac{81}{16}$, ***A2*** for the angle in the correct form and ***A1*** for the angle in incorrect form e.g. $\text{cis}\frac{\pi}{2}$ and/or $\frac{5\pi}{2}$.

Award ***A1*** if i is given in place of $\text{cis}\frac{\pi}{2}$.

[5 marks]

(b) $z_1 z_2 = 6\,\text{cis}\left(\frac{3\pi}{4} + \frac{n\pi}{16}\right)$ ***(M1)***

$= 6\,\text{cis}\left(\frac{12\pi + n\pi}{16}\right)$

$12\pi + n\pi = 32\pi$ ***(M1)***

$n = 20$ ***A1***

[3 marks]

Total [8 marks]

1. (a) **EITHER**

$N = 2$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$N = 8$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 4$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$$FV = 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4\times 2}$$ ***(M1)(A1)***

Note: Award ***M1*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

$= 42\,010 \text{ AUD}$ ***A1***

Note: Award ***(M1)(A1)A0*** for unsupported 42009.87.

[3 marks]

(b) **EITHER**

$PV = -37\,000$
$FV = 50\,000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. The final mark can still be awarded for the correct number of months (multiple of 3).

continued…

Question 1 continued

OR

$PV = -37\,000$

$FV = 50\,000$

$I\% = 6.4$

$P/Y = 4$

$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$50000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4 \times n}$ **OR** $50\,000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{n}$ ***(M1)(A1)***

Note: Award ***M1*** for the correct inequality, $50\,000$ and substituted compound interest formula. Allow an equation. Award ***A1*** for correct substitution.

THEN

$N = 4.74$ (years) $(4.74230...)$ **OR** $N = 18.9692...$ (quarters) ***(A1)***

$m = 57$ months ***A1***

Note: Award ***A1*** for rounding their m to the correct number of months. The final answer must be a multiple of 3. Follow through within this part.

[4 marks]

(c) $150\,000$ AUD ***A1***

[1 mark]

continued…

Question 1 continued

(d) (i) $120 \times 1700 - 150000$ ***(M1)***

$= 54000$ AUD ***A1***

(ii) $N = 120$
$PV = -150000$
$PMT = 1700$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula or $FV = 0$ seen. If a compound interest formula is equated to zero, award ***M1***, otherwise award ***M0*** for a substituted compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in annuity formula, but award ***A0*** for a substituted compound interest formula.
Follow through marks in part (d)(ii) are contingent on working seen.

$r = 6.46$ (%) $(6.45779...)$ ***A1***

[5 marks]

(e) $N = 60$
$I = 6.46$ $(6.45779...)$
$PV = -150000$
$PMT = 1700$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula. Award ***(M0)*** for a substituted compound interest formula. Award ***A1*** for all entries correct. Follow through marks in part (e) are contingent on working seen.

$FV = 86973$ AUD ***A1***

[3 marks]

continued…

Question 1 continued

(f) $204\,000-(60\times1700+86\,973)$ **OR** $204\,000-188\,973$ ***(M1)(M1)***

Note: Award ***M1*** for 60×1700. Award ***M1*** for subtracting their $(60\times1700+86\,973)$ from their $(204\,000)$. Award at most ***M1M0*** for their $204\,000-(60\times1700)$ or ***M0M0*** for their $204\,000-(86\,973)$. Follow through from parts (d)(i) and (e). Follow through marks in part (f) are contingent on working seen.

$15\,027$ AUD ***A1***

[3 marks]

Total [19 marks]

5. (a) $N = 360$
$I\% = 3.8$
$PV = (\pm)170\,000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct. Accept a positive or negative value for PV.

$(PMT =)\ 792.13\ \text{AUD}$ ***A1***

Note: Accept an answer of -792.13. Do not award final ***A1*** if answer is not given correct to 2 dp

[3 marks]

(b) (i) $N = 120$
$I\% = 3.8$
$PV = (\pm)170\,000$
$PMT = (\mp)792.13$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with a least two entries seen, award ***A1*** for all entries correct. PV and PMT must have opposite signs.

$(FV =)\ 133019.94\ \text{AUD}$ ***A1***

Note: Do not award final ***A1*** if answer is not given correct to 2 dp, unless already penalized in part (a). Accept 133020.30 from use of exact value for PMT.

(ii) amount of money paid: $120 \times 792.13\ (= 95055.60)$ ***(M1)***
loan paid off: $170\,000 - 133019.94\ \ (= 36980.06)$ ***(M1)***
interest paid: $(95\,055.60 - 36\,980.06 =)\ 58\,075.54\ \text{AUD}$ ***A1***

Note: Allow 58075.60 or 58075.90 from use of some exact values from parts (a) and (b)(i). If their answer to part (b)(i) is greater than $170\ 000$ then award at most ***(M1)(M1)(A0)*** for follow through in part (b)(ii).

[6 marks]
Total: [9 marks]

6. (a) $10 = \frac{2}{1-r}$ ***(M1)***

$r = 0.8$ ***A1***

[2 marks]

(b) $2\times(0.8)^{n-1} < 0.5$ **OR** $2\times(0.8)^{n-1} = 0.5$ ***(M1)***

$(n >)\ 7.212.....$ ***(A1)***

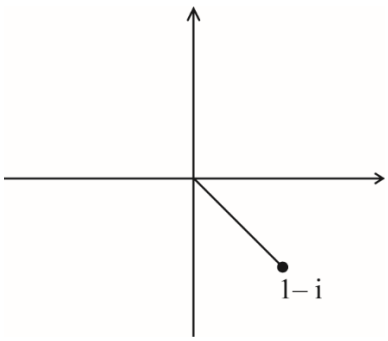
$n = 8$ ***A1***

Note: If $n = 7$ is seen, with or without seeing the value $7.212\ldots$ then award ***M1A1A0***.

[3 marks]

Total: [5 marks]

5. (a) (i)



A1

(ii) $z = \sqrt{2}\mathrm{e}^{-\frac{\mathrm{i}\pi}{4}}$ ***A1A1***

Note: Accept an argument of $\frac{7\pi}{4}$. Do **NOT** accept answers that are not exact.

[3 marks]

(b) (i) $w_1 + w_2 = \mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{\mathrm{i}\left(x-\frac{\pi}{2}\right)}$

$= \mathrm{e}^{\mathrm{i}x}\left(1 + \mathrm{e}^{-\frac{\mathrm{i}\pi}{2}}\right)$ ***(M1)***

$= \mathrm{e}^{\mathrm{i}x}(1 - \mathrm{i})$ ***A1***

(ii) $w_1 + w_2 = \mathrm{e}^{\mathrm{i}x} \times \sqrt{2}\mathrm{e}^{-\frac{\mathrm{i}\pi}{4}}$ ***M1***

$= \sqrt{2}\mathrm{e}^{\mathrm{i}\left(x-\frac{\pi}{4}\right)}$ ***(A1)***

attempt extract real part using cis form ***(M1)***

$\mathrm{Re}(w_1 + w_2) = \sqrt{2}\cos\left(x - \frac{\pi}{4}\right)$ **OR** $1.4142...\cos(x - 0.785398...)$ ***A1***

[6 marks]

continued…

Question 5 continued

(c) (i) $I_t = 12\cos(bt) + 12\cos\left(bt - \frac{\pi}{2}\right)$ **(M1)**

$I_t = 12\,\mathrm{Re}\left(\mathrm{e}^{\mathrm{i}bt} + \mathrm{e}^{\mathrm{i}\left(bt - \frac{\pi}{2}\right)}\right)$ ***(M1)***

$I_t = 12\sqrt{2}\cos\left(bt - \frac{\pi}{4}\right)$

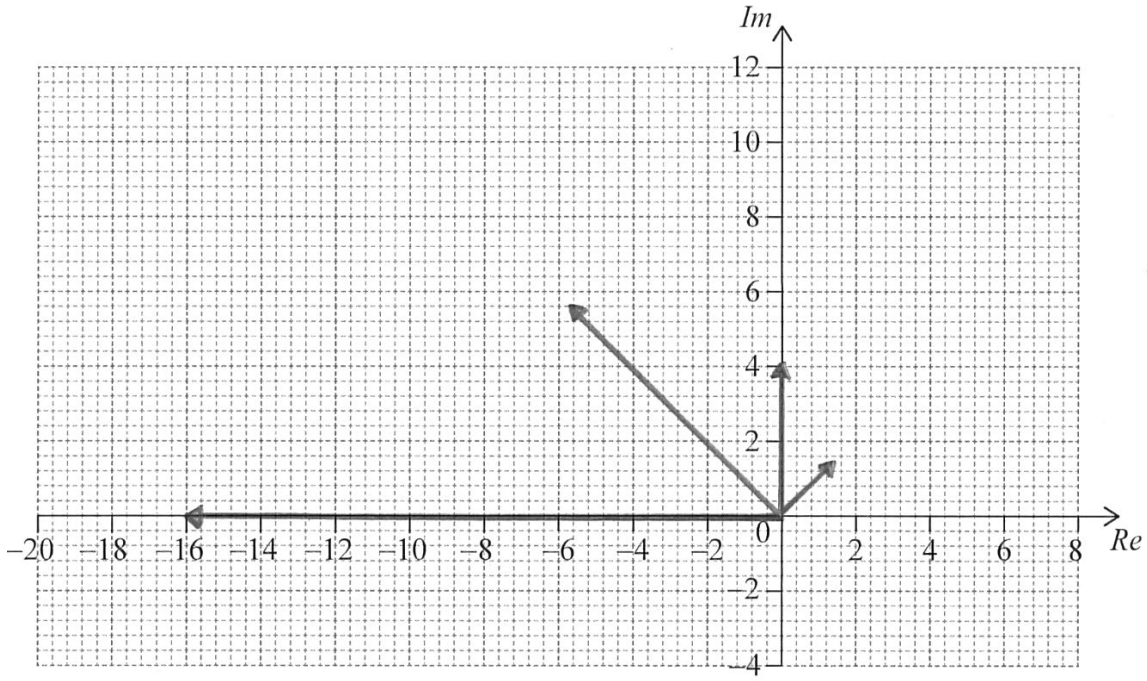
$\text{max} = 12\sqrt{2}\ \ (= 17.0)$ ***A1***

(ii) $\text{phase shift} = \frac{\pi}{4}(= 0.785)$ ***A1***

[4 marks]
Total: [13 marks]

15. (a) (i) $4e^{\frac{\pi}{2}i}, 8e^{\frac{3\pi}{4}i}, 16e^{\pi i}$ $\left(= 4i, -4\sqrt{2} + 4\sqrt{2}i, -16\right)$ ***(M1)A1***

(ii)



A3

Note: Award ***A1*** for correct arguments, award ***A1*** for $4i$ and -16 clearly indicated, award ***A1*** for $|w| < 4$ and $4 < |w^3| < 16$.

[5 marks]

(b) $2^2 + 1^2 = a^2$ ***M1***

$a = \sqrt{5}$ $(= 2.24)$ ***A1***

[2 marks]

Total [7 marks]

18. $\log A = x\log B + y\log C + \log k$ ***(M1)***

$\log 5.74 = x\log 2.1 + y\log 3.4 + \log k$
$\log 2.88 = x\log 1.5 + y\log 2.4 + \log k$
$\log 0.980 = x\log 0.8 + y\log 1.9 + \log k$ ***M1A1***

Note: Allow any consistent base, allow numerical equivalents.

attempting to solve their system of equations ***(M1)***

$x = 1.53,\ y = 0.505$ ***A1***

$k = 0.997$ ***A1***

Total [6 marks]

3. (a) (i) $N = 24$

$I\% = 14$

$PV = -14000$

$FV = 0$

$P/Y = 4$

$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $PV = 14000$.

$(€)871.82$ ***A1***

(ii) $4 \times 6 \times 871.82$ ***(M1)***

$(€)20923.68$ ***A1***

(iii) $20923.68 - 14000$ ***(M1)***

$(€)6923.68$ ***A1***

[7 marks]

(b) (i) 0.9×14000 $(= 14000 - 0.10 \times 14000)$ ***M1***

$(€)12600.00$ ***A1***

(ii) $N = 72$

$PV = 12600$

$PMT = -250$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $PV = -12600$ provided $PMT = 250$.

$12.56(\%)$ ***A1***

[5 marks]

continued…

Question 3 continued

(c) ***EITHER***

Bryan should choose Option A ***A1***

no deposit is required ***R1***

Note: Award ***R1*** for stating that no deposit is required. Award ***A1*** for the correct choice from that fact. Do not award ***R0A1.***

OR

Bryan should choose Option B ***A1***

cost of Option A $(6923.69) >$ cost of Option B $(72 \times 250 - 12\,600 = 5400)$ ***R1***

Note: Award ***R1*** for a correct comparison of costs. Award ***A1*** for the correct choice from that comparison. Do not award ***R0A1***.

[2 marks]

(d) real interest rate is $0.4 - 0.1 = 0.3\%$ ***(M1)***

value of other payments $250 + 250 \times 1.003 + \ldots + 250 \times 1.003^{71}$
use of sum of geometric sequence formula or financial app on a GDC ***(M1)***

$= 20\,058.43$

value of deposit at the end of 6 years
$1400 \times (1.003)^{72} = 1736.98$ ***(A1)***

Total value is $(€)\,21\,795.41$ ***A1***

Note: Both ***M*** marks can awarded for a correct use of the GDC's financial app:

$N = 72\ (6 \times 12)$
$I\% = 3.6\ (0.3 \times 12)$
$PV = 0$
$PMT = -250$
$FV =$
$P/Y = 12$
$C/Y = 12$

OR

$N = 72\ (6 \times 12)$
$I\% = 0.3$
$PV = 0$
$PMT = -250$
$FV =$
$P/Y = 1$
$C/Y = 1$

[4 marks]

Total [18 marks]

$\begin{pmatrix} 30 \end{pmatrix}$ $\begin{pmatrix} -150 \end{pmatrix}$

2. (a) $x + y + z = 600$ ***A1***

$15x + 10y + 12z = 7816$ ***A1***

$x = 2y$ ***A1***

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student). Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

(b) $x = 308,\ y = 154,\ z = 138$ ***A1A1***

Note: Award ***A1*** for all three correct values seen, ***A1*** for correctly labelled as x, y or z. Accept answers written in words: e.g. 308 adult tickets.

[2 marks]

[Total 5 marks]

5. (a) use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68 \text{ m}$

$= 68 \text{ cm}$ ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68 \text{ cm}$ ***AG***

[2 marks]

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If 1.8 m (or 180 cm) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If 1.53 m (or 153 cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114 \text{ m}$ and $(0.85)^{18}(1.8) = 0.0966 \text{ m}$ ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

continued…

Question 5 continued

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \frac{(1.8 \times 0.85)(1 - 0.85^3)}{1 - 0.85}$ $(= 3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(3.935925)$

$= 9.67 \text{ m}$ $(9.67185... \text{ m})$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \frac{(1.8)(1 - 0.85^4)}{1 - 0.85}$ $(= 5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67 \text{ m}$ $(9.67185... \text{ m})$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8)$ $(= 3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$

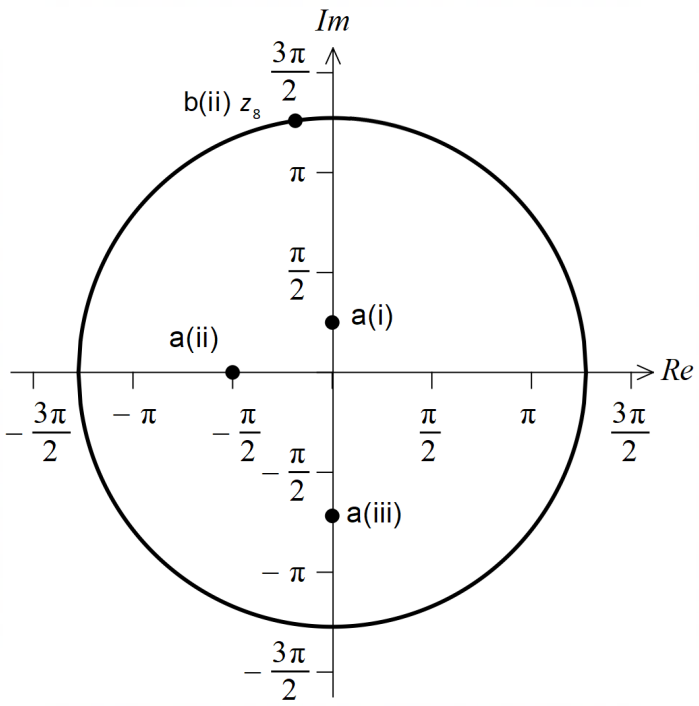
$= 9.67 \text{ m}$ $(9.67185... \text{ m})$ ***A1***

Note: Answers may be given in cm.

[3 marks]
[Total 7 marks]

(-3.2)

10. (a)



A1A1A1

Note:	Award ***A1*** for correct modulus and ***A1*** for correct argument for part (a)(i), and ***A1*** for other two points correct. The points may not be labelled, and they may be shown by line segments.

[3 marks]

(b) (i) $\frac{1}{2}\theta = 4$ ***(M1)***

$\Rightarrow \theta = 8$ ***A1***

(ii) z_8 is shown in the diagram above ***A1A1***

Note:	Award ***A1*** for a point plotted on the circle and ***A1*** for a point plotted in the second quadrant.

[4 marks]
[Total 7 marks]

11. (a) $\lambda = 1$

$\begin{pmatrix} -0.8 & 0.7 \\ 0.8 & -0.7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$0.8x = 0.7y$ ***(A1)***

an eigenvector is $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ (or equivalent with integer values) ***A1***

[3 marks]

(b) **EITHER**
(the long-term probability matrix is given by the eigenvector corresponding to the eigenvalue equal to 1, scaled so that the sum of the entries is 1)
$8 + 7 = 15$ ***(M1)***

OR
$\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}$ ***(M1)***

OR

considering high powers of the matrix *e.g.* $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}^{50}$ ***(M1)***

$\begin{pmatrix} \frac{7}{15} & \frac{7}{15} \\ \frac{8}{15} & \frac{8}{15} \end{pmatrix}$

THEN

probability of being in state A is $\frac{7}{15}$ ***A1***

[2 marks]
[Total 5 marks]

12. (a) $\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

(b) **EITHER**

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000 \ \ (= 10^5)$ ***A1***

[2 marks]

(c) $N = \frac{10^5}{10^{7.2}} = 0.00631 \ \ (0.0063095...)$ ***A1***

Note: Do not accept an answer of $10^{-2.2}$.

[1 mark]

(d) **METHOD 1**

$Y > 100 \Rightarrow$ no earthquakes in the first 100 years ***(M1)***

EITHER

let X be the number of earthquakes of at least magnitude 7.2 in a year

$X \sim \text{Po}(0.0063095\ldots)$

$\left(\text{P}(X = 0)\right)^{100}$ ***(M1)***

OR

let X be the number of earthquakes in 100 years

$X \sim \text{Po}(0.0063095\ldots \times 100)$ ***(M1)***

$\text{P}(X = 0)$

THEN

$0.532 \ (0.532082\ldots)$ ***A1***

METHOD 2

$Y > 100 \Rightarrow$ no earthquakes in the first 100 years ***(M1)***

let X be the number of earthquakes in 100 years

since n is large and p is small

$X \sim \text{B}(100, \ 0.0063095\ldots)$ ***(M1)***

$\text{P}(X = 0)$

$0.531 \ \ (0.531019\ldots)$ ***A1***

[3 marks]

[Total 8 marks]

7. METHOD 1

$\frac{u_1}{1-r} = 9$ ***A1***

therefore $u_1 = 9 - 9r$

$u_1 = 4 + u_1 r$ ***A1***

substitute or solve graphically: ***M1***

$9 - 9r = 4 + (9 - 9r)r$ **OR** $\frac{4}{(1-r)^2} = 9$

$9r^2 - 18r + 5 = 0$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

then $u_1 = 9 - \left(9 \times \frac{1}{3}\right) = 6$

$u_3 = 6 \times \frac{1}{3}^2 = \frac{2}{3}$ ***A1***

METHOD 2

$\frac{u_1}{1-r} = 9$ ***A1***

$r = \frac{u_1 - 4}{u_1}$ ***A1***

attempt to solve ***M1***

$\frac{u_1}{1 - \left(\frac{u_1 - 4}{u_1}\right)} = 9$

$\frac{u_1}{\left(\frac{4}{u_1}\right)} = 9$

$(u_1)^2 = 36$

$u_1 = \pm 6$

attempting to solve both possible sequences

$6, 2, \ldots$ or $-6, -10 \ldots$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

$u_3 = 6 \times \left(\frac{1}{3}\right)^2 = \frac{2}{3}$ ***A1***

Total [5 marks]

11. (a) **METHOD 1** – *(With FV=4000)*

EITHER

N= 10
I=1.5
FV= 4000
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for $(3.5-2=)\ 1.5$ seen and ***M1*** for all other entries correct.

OR

$4000 = A(1+0.015)^{10}$ ***(A1)(M1)***

Note: Award ***A1*** for 1.5 or 0.015 seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN

(PV =) $3447 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 2 – *(With FV including inflation)*

calculate FV with inflation

4000×1.02^{10} ***(A1)***

(=4875.977…)

EITHER

$4000\times1.02^{10} = \text{PV}\times1.035^{10}$ ***(A1)***

OR

N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for *their* FV and all other entries correct.

THEN

(PV =) $3457 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

continued…

Question 11 continued

METHOD 3 – *(Using formula to calculate real rate of return)*
(real rate of return =) $1.47058\ldots(\%)$ ***(A1)***

EITHER
$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***
OR
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for all entries correct.

THEN
(PV **=**) \$3457 ***A1***

[3 marks]

(b) **METHOD 1** – *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) \$4861.87 **OR** \$4875.97 ***(A1)***
so payment required (from TVM) will be \$294 **OR** \$295 ***A1***

Note: Award ***A0*** if a negative sign given, unless already penalized in part (a).

METHOD 2 – *(Using FV)*
N= 10
I=3.5
PV= −1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for I=3.5 **and** FV= $\pm$4875.977…, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) \$295 (295.393) ***A1***

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

[3 marks]
Total [6 marks]

13. (a) **METHOD 1**

recognizing that the real part is distributive ***(M1)***

$V_T = \text{Re}\left(2\text{e}^{3ti} + 5\text{e}^{3ti+4\text{i}}\right)$

$= \text{Re}\left(\text{e}^{3ti}\left(2 + 5\text{e}^{4\text{i}}\right)\right)$ ***(A1)***

(from the GDC) $2 + 5\text{e}^{4\text{i}} = 3.99088\ldots\text{e}^{-1.89418\ldots\text{i}}$ ***(A1)***

Note: Accept arguments differing by 2π e.g. $4.38900\ldots$).

therefore $V_T = 3.99\cos(3t - 1.89)$ $\left(3.99088\ldots\cos(3t - 1.89418\ldots)\right)$ ***A1***

Note: Award the last ***A1*** for the correct values of A, B and C seen either in the required form or not. If method used is unclear and answer is partially incorrect, assume Method 2 and award appropriate marks eg. ***(M1)A1A0A0*** if only A value is correct.

continued…

Question 13 continued

METHOD 2

converting given expressions to cos form ***(M1)***

$V_T = 2\cos 3t + 5\cos(3t+4)$

(from graph) $A = 3.99$ $(3.99088\ldots)$ ***A1***

$V_T = 3.99\cos(Bt+C)$

either by considering transformations or inserting points

$B = 3$ ***A1***

$C = -1.89$ $(-1.89418\ldots)$ ***A1***

Note: Accept arguments differing by 2π e.g. $4.38900\ldots$.

(so, $V_T = 3.99\cos(3t-1.89)$ $\left(3.99088\ldots\cos(3t-1.89418\ldots)\right)$)

Note: It is possible to have $A = 3.99$, $B = -3$ with $C = 1.89$ **OR** $A = -3.99$, $B = 3$ with $C = 1.25$ **OR** $A = -3.99$, $B = -3$ with $C = -1.25$ due to properties of the cosine curve.

[4 marks]

(b) maximum voltage is 3.99 $(3.99088\ldots)$ (units) ***A1***

[1 mark]

Total [5 marks]

1. (a) (i) **EITHER**

$115.5 = u_1 + (3-1)\times d \quad (115.5 = u_1 + 2d)$

$108 = u_1 + (8-1)\times d \quad (108 = u_1 + 7d)$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the arithmetic sequence term formula, ***A1*** for both equations correct. Working for ***M1*** and ***A1*** can be found in parts (i) or (ii).

$(d = -1.5)$

1.5 (cups/day) ***A1***

Note: Answer must be written as a positive value to award ***A1***.

OR

$(d =) \ \frac{115.5-108}{5}$ ***M1A1***

Note: Award ***M1*** for attempting a calculation using the difference between term 3 and term 8; ***A1*** for a correct substitution.

$(d =) \ 1.5$ (cups/day) ***A1***

(ii) $(u_1 =) \ 118.5$ (cups) ***A1***

[4 marks]

(b) attempting to substitute their values into the term formula for arithmetic sequence equated to zero ***(M1)***

$0 = 118.5 + (n-1)\times(-1.5)$

$(n =) \ 80$ days ***A1***

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

(c) $(t_5 =) \ 625\times 1.064^{(5-1)}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the geometric sequence term formula; ***A1*** for a correct substitution.

$\$ 801$ ***A1***

Note: The answer must be rounded to a whole number to award the final ***A1***.

[3 marks]

continued…

Question 1 continued

(d) (i) $(S_{10} =)$ (\$) 8390 (8394.39...) ***A1***

(ii) **EITHER**

the total cost (of dog food) ***R1***
for 10 years beginning in 2021 **OR** 10 years before 2031 ***R1***

OR

the total cost (of dog food) ***R1***
from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 ***R1***

[3 marks]

(e) **EITHER**
According to the model, the cost of dog food per year will eventually be too high to keep a dog.
OR
The model does not necessarily consider changes in inflation rate.
OR
The model is appropriate as long as inflation increases at a similar rate.
OR
The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.
OR
The model is appropriate since dog food bags can only be bought in discrete quantities. ***R1***

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either "model" is mentioned specifically, or other mathematical terms such as "increasing" or "discrete quantities" are seen. Do not accept a contextual argument in isolation, e.g. "The dog will eventually die".

[1 mark]
Total [13 marks]

7. (a) $I\% = 7.5$
$PV = \mp 800$
$PMT = \mp 500$
$FV = \pm 10\,000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award ***A1*** for $\text{PMT} = -500$ or $\text{PMT} = 500$, with same sign to PV and opposite sign to FV.

$17.3070\ldots$ ***(A1)***
$(k =)\ 18$ ***A1***

Note: Award ***(M0)(A0)(A0)A0*** for a final answer of 17 with no working. The final answer must be an integer.

[4 marks]

(b) $10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$ ***(A1)(M1)***

Note: Award ***(A1)*** for 10389 $(10389.38\ldots)$ seen. Award ***(M1)*** for subtraction of their $(18 \times 500 + 800)$ from FV. ***FT*** from their value of k. Award ***A0M1A0*** for $10\,000 - (18 \times 500 + 800)$. Do not award the final ***A1FT*** if their answer is negative.

589 EUR ***A1***

Note: Final answer must be to the nearest euro.

[3 marks]
Total [7 marks]

1. (a) $N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).
Approaches that use the compound interest formula receive no marks.
Award ***A1*** for correct values of PV and PMT (signs must be the same) **and** a correct value of N.

$FV = (\$)3577.43$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to 2 dp.

[3 marks]

(b) $N = 36.5 \ \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

Note: Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that PV and FV have opposite signs and the resulting value of N is positive.

[2 marks]

[Total: 5 marks]

12. METHOD 1 Analytical approach

attempt to express V_1 or V_2 in exponential form ***(M1)***

e.g. $V_1 = \text{Im}(6\text{e}^{\text{i}\left(at+\frac{\pi}{6}\right)}),\ V_2 = \text{Im}(6\text{e}^{\text{i}\left(at+\frac{\pi}{2}\right)})$

Note: Accept angles in radians or degrees.

$(V_1 + V_2 =)\ 6\text{e}^{\text{i}\times\frac{\pi}{6}} + 6\text{e}^{\text{i}\times\frac{\pi}{2}}$ ***(A1)***

Note: This mark can be awarded even if seen as part of a correct larger expression.

$= 10.4\text{e}^{1.05\text{i}}\ \left(6\sqrt{3}\,\text{e}^{\frac{\text{i}\pi}{3}}\right)$ ***(A1)***

so V is 10.4 $(10.3923..., 6\sqrt{3})$ and θ is 60 (degrees) ***A1A1***

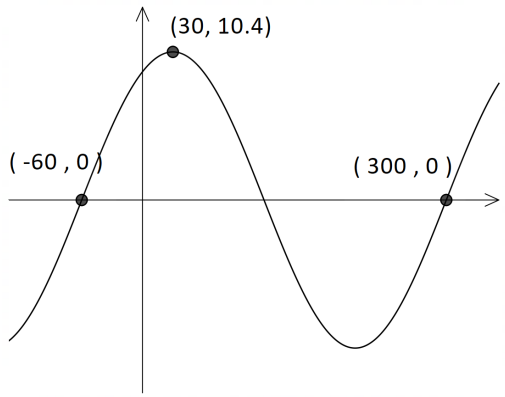
Note: Accept any value for θ that rounds to a 2sf answer of 60.
Do **not** accept a final answer for an angle in radians.
Do **not** award ***A1*** for answer of 60° resulting from incorrect working.

continued…

Question 12 continued

METHOD 2 Graphical approach

let $at = x$ and plot $V_1 + V_2$ curves on GDC ***(M1)***



attempt to find maximum ***(M1)***

$V = 10.4$ ***A1***

attempt to find any x-axis intercept (either -60 or 300) ***(M1)***

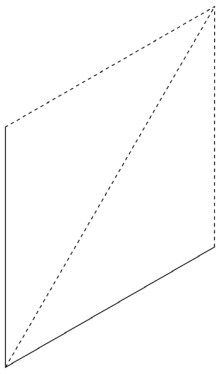
$\theta = 60$ (degrees) $\left(\theta = -300 \text{ (degrees)}\right)$ ***A1***

continued…

Question 12 continued

METHOD 3 Geometric approach

considering the rhombus



(M1)

$V = \sqrt{6^2 + 6^2 - 2 \times 6 \times 6\cos 120^\circ}$ ***(M1)***

$\left(= \sqrt{108} = 6\sqrt{3}\right) = 10.4 \ \ (10.3923\ldots)$ ***A1***

$\theta = 60$ (degrees) ***A2***

Note: An answer of $\theta = -300$. is most likely to be seen in METHOD 2, but should be condoned in METHODS 1 and 3 if seen there.

[Total: 5 marks]

4. (a) **METHOD 1 (use of financial app in GDC)**

$N = 5$	**OR**	$N = 20$
$I\% = 1.2$		$I\% = 1.2$
$PV = \pm 520$		$PV = \pm 520$
$P/Y = 1$		$P/Y = 4$
$C/Y = 4$		$C/Y = 4$

(M1)(A1)

Note: Award ***M1*** for evidence of using the financial app on the calculator, ***A1*** for all correct entries.

$(\$)\ 552.11$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

METHOD 2 (use of formula)

attempt to substitute into compound interest formula ***(M1)***

$$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4}$$ ***(A1)***

$(\$)\ 552.11$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

[3 marks]

(b) **EITHER**

$N = 5$

$I\% = 43.5\ \ (43.4772...(\%))$

$PV = \pm 520$

$FV = \mp 30$ ***(M1)(A1)A1***

Note: Award ***M1*** for evidence of using the finance app on the calculator, ***A1*** for all correct entries, ***A1*** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

OR

$$30 = 520\left(1 - \frac{r}{100}\right)^{5}$$ (or equivalent) ***(M1)(A1)***

$(r =)\ \ 43.5\%\ \ (43.477...\%)$ ***A1***

Note: Award ***M1*** for using the compound interest formula, ***A1*** for correct substitutions and for equating to 30, ***A1*** for correct final answer. Accept $(r =)\ -43.5\%$.
Award ***M1A1A0*** for a final answer of 56.5%.

[3 marks]
Total [6 marks]

11. (a) $30\sin(t+60^\circ)+60\sin(t+10^\circ)$ ***(M1)***

finding maximum graphically

82.5 (V) $(82.5471...)$ ***A1***

Note: Award ***M1A0*** for 83.

[2 marks]

(b) recognizing that a is still 1 ***A1***

$V_0 = 82.5$ ***A1***

attempt to find an x-intercept of combined voltage ***(M1)***

$b = 26.2^\circ$ $(26.1643...^\circ)$ **OR** any other correct x-intercept ***A1***

Note: May be seen in the final answer. Award ***M1A0*** for $b = 26$ with no working.

$\left(V_{\text{TOT}} = 82.5\sin(t+26.2^\circ)\ \left(82.5471...\sin(t+26.1643...^\circ)\right)\right)$

Note: Award at most ***(M1)A1(A1)A0*** if phase shift of $-153.835\ldots$ is seen in the final answer. In part (b), candidates may use $\arg\left(30\text{e}^{60\text{i}}+60\text{e}^{10\text{i}}\right)$ to determine the new phase shift, and hence could be awarded ***M1*** for this valid method.

[4 marks]

Total [6 marks]

1. (a) attempt to use the term formula for a geometric sequence ***(M1)***

$4\times(0.8)^2$

$=2.56\ (\text{mm})\ \left(\frac{64}{25}\right)$ ***A1***

[2 marks]

(b) attempt to use the sum formula for a geometric sequence ***(M1)***

$\frac{4\times(1-0.8^{12})}{1-0.8}$

$=18.6\ (\text{mm})\ \ (18.6256...)$ ***A1***

[2 marks]

(c) attempt to use infinite geometric sum ***(M1)***

e.g. $\frac{4}{1-0.8}$ **OR** 20

adding 25 to their expression or value ***(M1)***

(maximum width $=25+20$)

$45\ (\text{mm})$ ***A1***

[3 marks]

[Total 7 marks]

8. (a) $z = 2\mathrm{e}^{-0.524\mathrm{i}} \left(= 2\mathrm{e}^{-\frac{\pi}{6}\mathrm{i}} \right)$ ***A1A1***

Note: Award ***A1*** for the correct modulus and ***A1*** for the correct argument.

[2 marks]

(b) **METHOD 1**

$z_1 + z_2 = \mathrm{e}^{2t\mathrm{i}} \left(1 + 2\mathrm{e}^{-\frac{\pi}{6}\mathrm{i}} \right)$ ***(M1)***

$= \mathrm{e}^{2t\mathrm{i}} \times 2.90931... \mathrm{e}^{-0.350879...\,\mathrm{i}}$ ***(A1)***

$\mathrm{Im}\,(z_1 + z_2) = 2.91 \sin (2t - 0.351)$ ***A1A1***

METHOD 2

attempt to find the modulus of $\left(1 + 2\mathrm{e}^{-\frac{\pi}{6}\mathrm{i}} \right)$ ***(M1)***

$p = \left| 1 + 2\mathrm{e}^{-\frac{\pi}{6}\mathrm{i}} \right|$

attempt to find the argument of $\left(1 + 2\mathrm{e}^{-\frac{\pi}{6}\mathrm{i}} \right)$ ***(M1)***

$q = \arg\left(1 + 2\mathrm{e}^{-\frac{\pi}{6}\mathrm{i}} \right)$

$\mathrm{Im}(z_1 + z_2) = 2.91 \sin(2t - 0.351)$ ***A1A1***

METHOD 3

sketching $\mathrm{Im}\,(z_1 + z_2)$ ***(M1)***

max is $(0.961, 2.91)$ ***(A1)***

first root is 0.1754 ***(A1)***

$\mathrm{Im}(z_1 + z_2) = 2.91 \sin(2t - 0.351)$ ***A1***

[4 marks]

[Total 6 marks]

3. (a) (i) attempt to find 15% or 85% of 285000 ***(M1)***

285000×0.85

$242\,250$ (USD) ***A1***

Note: Do not award ***A1*** if answer is not given exact.

(ii) $N = 360$

$I\% = 4$

$PV = (\pm)\,242\,250$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct.

$(PMT =)\ 1156.54$ (USD) ***A1***

Note: Do not award final ***A1*** if answer is not given to 2 dp.

[5 marks]

(b) 1156.54×360 ***(M1)***

416354 (USD) ***A1***

Note: Do not award ***A1*** if answer is not given to the nearest dollar, unless already penalized in part (a)(ii).

[2 marks]

(c) $I\% = 4$
$PV = (\pm)242250$
$PMT = (\mp)1300$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(A1)***

Note: Award ***A1*** for $PMT = (\mp)1300$ seen.

$(N =)\ 292$ ***A1***

[2 marks]

(d) **METHOD 1**

$N = 291$
$I\% = 4$
$PV = (\pm)242250$
$PMT = (\mp)1300$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

Note: Award ***A1*** for $N = 291$ seen.

$(FV =)\ 871.91\ (871.908\ldots)$ ***A1***

valid attempt to find interest in final month (e.g. $N = 1$ **OR** $PV = 871.91$) ***(M1)***

$N = 1$
$I\% = 4$
$PV = 871.91\ (871.908\ldots)$
$FV = 0$
$P/Y = 12$
$F/Y = 12$

$(PMT =)\ 874.82$ (USD) ***A1***

Note: Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

METHOD 2

$N = 292$
$I\% = 4$
$PV = (\pm)\,242\,250$
$PMT = (\mp)1300$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

Note: Award ***A1*** for $N = 292$ seen.

$(FV =)\ 425.185\ldots$ ***A1***

$1300 - 425.185...$ ***(A1)***
$(PMT =)\ 874.82$ (USD) ***A1***

Note: Accept 874.81. Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

[4 marks]

(e) $291 \times 1300 + 874.82$ ***(M1)***

379174.82

attempt to find difference between their value and their part (b) ***(M1)***
$(416354 - 379174.82)$

37179 (USD) ***A1***

Note: Accept 37180 (USD) from using the 2 dp. answer from part (b). Do not penalize for not rounding to nearest dollar if this has already been penalized in part (b).

[3 marks]
[Total 16 marks]

1. (a) **EITHER**

$N = 96$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

OR

$N = 8$
$PV = \mp 100000$
$FV = \pm 150000$
$P/Y = 1$
$C/Y = 12$ ***(M1)(A1)***

OR

$150000 = 100000\left(1 + \frac{I}{100 \times 12}\right)^{12 \times 8}$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct) or an attempt to use a compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in compound interest formula.

THEN

$I = 5.08\ (5.07903\ldots)$ ***A1***

[3 marks]

(b) $N = 120$
$I\% = 6.1$
$PV = \mp 150000$
$PMT = \pm 1000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app (at least 3 entries, not necessarily correct). Award ***A1*** for all entries correct in financial app (condone missing -/+ sign if the correct final answer is seen).

$FV = (\$)\ 110867$ ***A1***

Note: Answer must be correct to nearest dollar to award the final ***A1***. Award ***(M1)(A1)A0*** for an unsupported final answer to a greater degree of accuracy eg. (\$) $110866.70\ldots$
Award ***M1A1A0*** for a truncated answer of 110866 if no working is shown.

[3 marks]
[Total: 6 marks]

6. (a) attempt to substitute into geometric sequence formula for twelfth term **OR** at least three correct terms of the sequence ***(M1)***

$u_{12} = 40 \times 1.1^{12-1}$ **OR** $40,\ 44,\ 48.4,\ \ldots$

$114 \quad (114.124\ldots)$ ***A1***

[2 marks]

(b) (i) attempt to substitute into geometric series formula **OR** a sum of at least the first three terms ***(M1)***

$S_{12} = \dfrac{40\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $\sum_{1}^{12}\left(40 \times 1.1^{n-1}\right)$ **OR** $40+44+48.4+\ldots$

Note: Award ***M1*** for $u_1 = 40$ and $r = 1.1$ seen as part of a geometric series formula, or ***M1*** for sigma notation and their u_n formula (condone missing limits), or ***M1*** for the sum of at least the **correct** first three terms of the sequence.

$S_{12} = 855 \quad (855.371\ldots)$ ***A1***

(ii) finding $S_{24} = 3539.89...$ or attempt to find the sum between u_{13} and u_{24} ***(M1)***

Note: Award ***M1*** for $S_{24} = 3539.89\ldots$ **or** sigma notation that includes correct limits and their u_n formula **or** a substituted geometric series formula that includes $125.537\ldots$ and $n = 12$ **or** a list of terms that includes at least the 13th term and the 24th term.

$3539.89\ldots - 855.371\ldots$ **OR** $\sum_{13}^{24}\left(40 \times 1.1^{n-1}\right)$ **OR**

$\left(S_{13\text{ to }24} =\right)\dfrac{125.537\ldots\left(1.1^{12}-1\right)}{1.1-1}$ **OR** $125.537 + \ldots + 358.172\ldots$ ***(A1)***

Note: Accept a calculation using $u_{13} = 125$ or 126.

$2680 \quad \left(2684.52\ldots,\ 2684,\ 2685\right)$ ***A1***

Note: For $u_{13} = 125$, the sum is $2673.03\ldots$ and for $u_{13} = 126$, the sum is $2694.41\ldots$

[5 marks]

[Total: 7 marks]

13. (a) (i) $\left(|z_1| = \sqrt{4^2 + 5^2} =\right)\ 6.40\ \ (6.40312..., \sqrt{41})$ ***A1***

(ii) $\left(\arg(z_1) =\right)\ 0.896\ \left(0.896055..., 51.3401...^\circ, \arctan\left(\frac{5}{4}\right)\right)$ ***A1***

[2 marks]

(b) angle in triangle is $2 - 0.896055\ldots$ **OR** $114.591^\circ - 51.3401...^\circ$ ***(A1)***
use of area of triangle formula ***(M1)***

$\frac{1}{2} \times 6.40312... \times 3 \times \sin(2 - 0.896055...)$ ***(A1)***

$8.58\ (8.57688\ldots)$ ***A1***

Note: Accept methods that use Cartesian form or vector product.

[4 marks]
[Total 6 marks]

1. (a) **EITHER**

$N = 72$ $\quad$ $N = 6$
$I = 1.25$ $\quad$ $I = 1.25$
$PV = -3000$ **OR** $PV = -3000$

$P/Y = 12$ $\quad$ $P/Y = 1$
$C/Y = 12$ $\quad$ $C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for attempt to use finance app on GDC, ***A1*** for all entries correct.

OR

$$3000\left(1+\frac{1.25}{1200}\right)^{72}$$ ***(M1)(A1)***

Note: Award ***M1*** for using CI formula, ***A1*** for correct entries.

THEN

$FV = (\$)\ 3233.53$ ***A1***

Note: Answer must be correct to 2 dp for the final ***A1*** to be awarded.

[3 marks]

(b) **EITHER**

$I = 1.25$ $\quad$ $I = 1.25$
$PV = -3000$ $\quad$ $PV = -3000$
$FV = 3550$ **OR** $FV = 3550$

$P/Y = 12$ $\quad$ $P/Y = 1$
$C/Y = 12$ $\quad$ $C/Y = 12$ ***(A1)***

Note: Award ***A1*** for all entries correct and opposite signs for PV and FV values.

OR

$$3550 = 3000\left(1+\frac{1.25}{1200}\right)^{12N}$$ ***(A1)***

Note: Award ***A1*** for all entries correct.

THEN

$N = 162\ (161.686\ldots)$ (months) ***A1***

[2 marks]

(c) 18450 ***A1***

[1 mark]

(d) **EITHER**

$N = 96$		$N = 96$
$I = 12.6$		$I = 1.05$
$PV = -18450$	**OR**	$PV = -18450$
$FV = 0$		$FV = 0$
$P/Y = 12$		$P/Y = 1$
$C/Y = 12$		$C/Y = 1$

(M1)(A1)

Note: Award ***M1*** for attempt to use finance app on GDC, ***A1*** for $N = 96$, $PV = -18450$ and $FV = 0$,

THEN

$PMT = (\$)\,306$ ***A1***

Note: The answer must be correct to the nearest dollar for the final ***A1*** to be awarded.

[3 marks]

[Total: 9 marks]

5. (a) recognizing that the growth in one year is the difference in the two heights ***(M1)***

$5 \times 0.8 = 4 \text{ (m)}$ ***A1***

[2 marks]

(b) recognizing geometric sequence, $r = 0.8$ ***(M1)***

attempt to find total height by adding initial height to a term in series ***(M1)***

EITHER

$42 + \frac{4(1-(0.8)^6)}{1-(0.8)}$ ***(A1)***

OR

$37 + \frac{5(1-(0.8)^7)}{1-(0.8)}$ ***(A1)***

THEN

$= 56.7571\ldots$

$= 56.76 \text{ (m)}$ **OR** 5676 (cm) ***A1***

Note: Accept an answer in cm or in m to two decimal places.

[4 marks]

(c) attempt to use infinite geometric series to find the total growth of the tree ***(M1)***

e.g. $\frac{5}{1-0.8}$ **OR** $\frac{4}{1-0.8}$

$\left(S_\infty = 37 + \frac{5}{1-0.8}, S_\infty = 42 + \frac{4}{1-0.8} \right)$

$k = 62$ ***A1***

[2 marks]

[Total: 8 marks]

15. (a) $\sqrt{2}\,e^{\frac{3\pi i}{4}}$ ***A1A1***

[2 marks]

(b) (i) enlargement SF $\sqrt{2}$, $(1.41,\ 1.41421\ldots)$, (centre $(0,\ 0)$) ***A1***

rotation of $\frac{3\pi}{4}$, $(2.36,\ 2.35619\ldots)$, (centre $(0,\ 0)$) ***A1***

(ii) $\begin{pmatrix}\sqrt{2} & 0\\ 0 & \sqrt{2}\end{pmatrix}\begin{pmatrix}\cos\left(\frac{3\pi}{4}\right) & -\sin\left(\frac{3\pi}{4}\right)\\ \sin\left(\frac{3\pi}{4}\right) & \cos\left(\frac{3\pi}{4}\right)\end{pmatrix}$ ***(A1)(A1)***

Note: The order of matrix multiplication does not matter in this case.

$=\begin{pmatrix}-1 & -1\\ 1 & -1\end{pmatrix}$ ***A1***

Note: Do not ***FT*** if their matrices in part (b)(i) are not representative of a dilation AND a rotation.

[5 marks]

(c) 8 ***A1***

Note: Since $\frac{3\pi n}{4}$ must be a multiple of 2π.

[1 mark]
[Total: 8 marks]

4. (a) recognition that % increase gives the common difference ***(M1)***

$25000 \times 0.05 = 1250$ ***A1***

[2 marks]

(b) $25000 + 1250n > 44000$ **OR** $25000 + 1250(n-1) > 44000$ (accept equality) ***(A1)***

attempt to solve their inequality **OR** equality (sketch **OR** cross-over values **OR** algebraic manipulation) ***(M1)***

$k = 16$ ***A1***

Note: Award ***(A1)(M1)A0*** for 15.2, 16.2 or 17 seen.

[3 marks]

[Total: 5 marks]

7. (a) $N = 60,\ I = 4.5,\ PV = +400\,000,\ PMT = -3600,\ P/Y = 12,\ C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for use of Finance solver with any 2 entries correct, Award ***A1*** for all correct.

$FV = \$259\,000 (\$258\,994)$ ***A1***

[3 marks]

(b) $I = 4.5,\ PV = +400\,000,\ PMT = -3600,\ FV = 0,\ P/Y = 12,\ C/Y = 12$ ***(A1)(A1)***

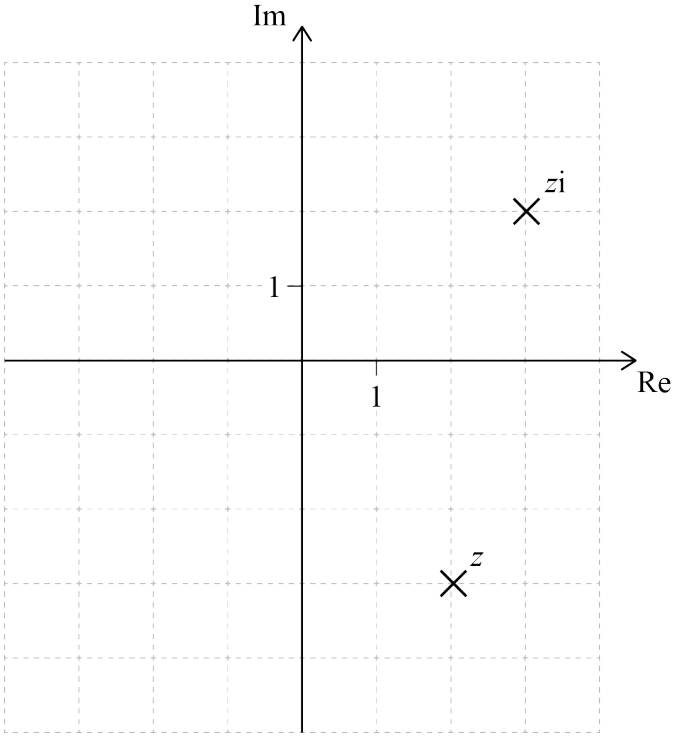
Note: Award ***A1*** for $FV = 0$, award ***A1*** for all correct.

$N = 144$ ***A1***

[3 marks]

[Total 6 marks]

9. (a)



A1
[1 mark]

(b) (i) $3.61\ (3.60555\ldots,\ \sqrt{13}\,)$ ***A1***

(ii) $-0.983\ \ (-0.982793\ldots)$

A1
[2 marks]

(c) $3+2\mathrm{i}$ ***A1***
[1 mark]

(d) **EITHER**

rotation, (anticlockwise) $\frac{\pi}{2}$ (or 90°) (about the origin) ***A1***

OR

translation $\begin{pmatrix}1\\5\end{pmatrix}$ ***A1***

[1 mark]
[Total 5 marks]

11. (a) attempt to find $|\boldsymbol{M}-\lambda\boldsymbol{I}|$ ***(M1)***

$(-4-\lambda)(3-\lambda)+6\ (=0)$ ***(A1)***

$2, -3$ ***A1***

[3 marks]

(b) (i) $\begin{pmatrix}2 & 0\\ 0 & -3\end{pmatrix}$ **OR** $\begin{pmatrix}-3 & 0\\ 0 & 2\end{pmatrix}$ ***A1***

(ii) **EITHER**

attempt to solve $\boldsymbol{M}\begin{pmatrix}x\\ y\end{pmatrix}=2\begin{pmatrix}x\\ y\end{pmatrix}$ **OR** $\boldsymbol{M}\begin{pmatrix}x\\ y\end{pmatrix}=-3\begin{pmatrix}x\\ y\end{pmatrix}$ ***(M1)***

OR

attempt to use $(\boldsymbol{M}-\lambda\boldsymbol{I})\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ ***(M1)***

THEN

$\begin{pmatrix}1\\ 3\end{pmatrix}$ and $\begin{pmatrix}2\\ 1\end{pmatrix}$ (or any multiple) ***(A1)(A1)***

$\boldsymbol{P}=\begin{pmatrix}1 & 2\\ 3 & 1\end{pmatrix}$ **OR** $\boldsymbol{P}=\begin{pmatrix}2 & 1\\ 1 & 3\end{pmatrix}$ ***A1***

Note: Award ***A1*** for correct eigenvectors in a matrix in the correct position for their $\boldsymbol{D}$.

[5 marks]
[Total 8 marks]

3. (a) recognition of a geometric sequence ***(M1)***

$3\left(\frac{2}{3}\right)^4$ **OR** 3, 2, 4/3…

$= 0.593\ (0.592592\ldots,\ 16/27)\ (\text{cm})$ ***A1***

[2 marks]

(b) attempt to sum a geometric sequence ***(M1)***

$\frac{3\left(1-\left(\frac{2}{3}\right)^5\right)}{1-\frac{2}{3}}$ **OR** $3+2+\frac{4}{3}\ldots$

$= 7.81\ (7.81481\ldots,\ 211/27)\ (\text{cm})$ ***A1***

[2 marks]

(c) recognition of need to find sum to infinity ***(M1)***

$\frac{3}{1-\frac{2}{3}}$ ***(A1)***

$= 9\ (\text{cm})$ ***A1***

[3 marks]

(d) Comparing the sum of the widths greater than (or equal to) 8.5 ***(M1)***

e.g. $\frac{3\left(1-\left(\frac{2}{3}\right)^n\right)}{1-\frac{2}{3}} \geq 8.5$ **OR** sketch **OR** list of values with cross-over values

$7.13\ \ (7.1285338740543\ldots)$ seen ***(A1)***

$n \geq 7.13$

$n = 8$ ***A1***

[3 marks]

(e) attempt to divide two adjacent areas **OR** $\left(\frac{2}{3}\right)^2$ ***(M1)***

$\frac{4}{9}\ (0.444,\ 0.444444\ldots)$ ***A1***

[2 marks]

continue…

Question 3 continued.

(f) $u_1 = 13.5$ (may be seen in part (e)) ***(A1)***

attempt to find the sum of their n terms, with their u_1 and their r ***M1***

$$S_8 = \frac{13.5\left(1-\left(\frac{4}{9}\right)^8\right)}{1-\frac{4}{9}}$$

Note: Do not award ***M1*** if 5 or infinity used for n or if $\frac{2}{3}$ used for r.

$= 24.3\ (24.2630\ldots)$ (cm^2) ***A1***

Note: Accept $24.2\ (24.2439\ldots)$ from using 0.444.

[3 marks]
[Total: 15 marks]

3. (a) (i) $N = 72$
$I\% = 7.15$
$PV = \pm 42695$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempt to use finance app in GDC and ***(A1)*** for the first five entries correct.

$(PMT =)\ 730.99$ (USD) ***A1***

Note: Do not award final ***A1*** if answer is not given to 2 d.p. or if given as a negative value.

(ii) attempt to find 80% of $42\,695$ ***(M1)***

$N = 72$
$I\% = 6.40$
$PV = \pm 34156$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(A1)***

$(PMT =)\ 572.54$ (USD) ***A1***

Note: Do not award final ***A1*** if answer is not given to 2 d.p. or if given as a negative value.

[6 marks]

(b) (i) does not have (enough) money for initial payment / does not want to make initial payment ***R1***

(ii) lower monthly payments / will pay less over the life of the loan / less interest will be paid over life of loan ***R1***

[2 marks]

[Total 8 marks]

10. (a) correct use of one log law ***(M1)***

x^{-2} **OR** x^2 **OR** $(2-x)(-2-x)$

$(g(x)=)\dfrac{x^2-4}{x^2}$ **OR** $\dfrac{(2-x)(-2-x)}{x^2}$ **OR** $1-4x^{-2}$ **OR** equivalent ***A1***

[2 marks]

(b) $f^{-1}(x)<-2$ **OR** $(-\infty,-2)$ ***A2***

Note: Award ***A1*** for strictness with -2 using either $<$ or parenthesis and ***A2*** for completely correct inequality. Award ***A2*** for $y<-2$.

[2 marks]

(c) attempt to exchange variables (seen anywhere) ***(M1)***

$x=\ln\left(\dfrac{y^2-4}{y^2}\right)$ **OR** $x=\ln\left(1-4y^{-2}\right)$

$e^x=\dfrac{y^2-4}{y^2}$ **OR** $e^x=1-4y^{-2}$ ***(A1)***

$y^2\left(e^x-1\right)=-4$ ***(A1)***

$(y=)\pm\dfrac{2}{\sqrt{1-e^x}}$ ***A1***

(since domain of $f(x)$ is $x<-2$) $a=-2$ ***A1***

[5 marks]

[Total 9 marks]

11. (a) (i) $(u_2 =)\ 6+6\text{i}$ ***A1***

(ii) $(u_3 =)\ 8\text{i}$ ***A1***

[2 marks]

(b) attempt to find $|r|$ ***(M1)***

$|r| = \sqrt{\left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^2}$ ***A1***

$|r| = \frac{2\sqrt{2}}{3}$ ***A1***

statement that multiplying a number by a complex number changes the modulus of the number by a factor of $|r|$ (may be seen at any stage) ***R1***

Note: Award at most ***(M1)A1A0R1*** if common ratio is given as $0.942809\ldots..$

therefore it is a geometric sequence ***AG***

[4 marks]

(c) substitution of their r and 9 into the sum formula for infinite sequence ***(A1)***

$S_\infty = \frac{9}{1-\frac{2\sqrt{2}}{3}}$

$(S_\infty =)\ 157\ \ (157.367\ldots,\ 81+54\sqrt{2})$ ***A1***

Note: No ***FT*** if their $|r| \geq 1$.

[2 marks]

[Total 8 marks]

14. (a) $L_1(x) = \text{Im}\left(14.8\text{e}^{0.35x\text{i}}\text{e}^{1.27\text{i}}\right)$ ***A1***

$L_2(x) = \text{Im}\left(18.5\text{e}^{0.35x\text{i}}\text{e}^{-2.36\text{i}}\right)$ ***A1***

[2 marks]

(b) $L_3 = \text{Im}\left(14.8\text{e}^{0.35x\text{i}}\text{e}^{1.27\text{i}} + 18.5\text{e}^{0.35x\text{i}}\text{e}^{-2.36\text{i}}\right)$

$L_3 = \text{Im}\left(\text{e}^{0.35x\text{i}}\left(14.8\text{e}^{1.27\text{i}} + 18.5\text{e}^{-2.36\text{i}}\right)\right)$ ***(A1)***

$L_3 = \text{Im}\left(\text{e}^{0.35x\text{i}}\left(8.81559\ldots\text{e}^{3.01604\ldots\text{i}}\right)\right)$ ***(A1)***

$L_3 = 8.81559\ldots\sin(0.35x + 3.01604\ldots)$

$q = 8.82\ (8.81559\ldots)$, $r = 3.02\ (3.01604\ldots)$ ***A1A1***

Note: Award both marks if last line missing, but values are seen in equation for L_3.

[4 marks]

[Total 6 marks]

1. (a) after 10 years

N: 120
I%: 3.6
PV: 0
PMT: (-)1500
P/Y: 12
C/Y: 12 ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct except PV.

FV: 216 278.58 (euros) ***A1***

Note: Award ***A0*** for a negative answer or answer not given to 2 decimal places.

[3 marks]

(b) **METHOD 1 (C.I. formula)**

invests: 116 278.58 ***(M1)***

after 15 years: $116278.58\times(1+\frac{4.5}{400})^{60}$ ***(M1)(A1)***

FV: 227 515.92 (euros) ***A1***

Note: Award the first ***M1*** for subtracting *their* FV from part (a) even if the value of the investment is negative.
Award the second ***M1*** for use of the C.I. formula.
Award first ***A1*** for all entries correct except PV.
Allow full follow through from part (a).
Award final ***A0*** for FV not given to 2 decimal places unless this has already been penalized in (a).
Accept 227 515.93 from use of unrounded value.

METHOD 2 (financial app)

invests: 116 278.58 ***(M1)***

THEN

EITHER		**OR**		**OR**	
N:	60	*N:*	15	*N:*	180
I%:	4.5	*I%:*	4.5	*I%:*	4.5
PV:	116 278.58	*PV:*	116 278.58	*PV:*	116 278.58
PMT:	0	*PMT:*	0	*PMT:*	0
P/Y:	4	*P/Y:*	1	*P/Y:*	12
C/Y:	4	*C/Y:*	4	*C/Y:*	4

(M1)(A1)

continued…

Question 1 continued

Note: Award the first ***M1*** for subtracting *their* FV from part (a) even if the value of the investment is negative.
Award ***(M1)*** for an attempt to use a financial app in their technology with a least two entries seen, award ***A1*** for all entries correct except PV.

THEN

FV: 227 515.92 (euros) ***A1***

Note: Award ***A0*** for a negative answer or answer not given to 2 decimal places unless already penalized in part (a).
Accept 227 515.93 from use of unrounded value.

[4 marks]
Total [7 marks]

5. use of $\text{speed} = \frac{\text{distance}}{\text{time}}$ **OR** $\text{speed} \times \text{time} = \text{distance}$ **OR** $\text{time} = \frac{\text{distance}}{\text{speed}}$

seen anywhere ***(M1)***

attempt to find a bounding value of t (**OR** use appropriate values of t that support and contradict the statement that the car is speeding) ***(M1)***

$(t =) 1.25 (\text{s})$ OR $(t =) 1.15 (\text{s})$ ***(A1)***

THEN

EITHER

use of their upper bound for t to find minimum speed (**OR** of any value of t where $1.20481... < t \leq 1.25$ to find speed)

(speed =) $\frac{10}{1.25}$ (or $\frac{10}{\text{their time}}$) ***A1***

(speed =) $8 (\text{m s}^{-1})$ (or a value where $8 \leq \text{speed} < 8.3$) seen

hence speed $< 8.3 (\text{ms}^{-1})$ ***A1***

OR

use of their upper bound for t to find maximum distance (**OR** of any value of t where $1.20481... < t \leq 1.25$ to find distance)

(distance=) 8.3×1.25 (or $8.3 \times \text{their time}$) ***A1***

(distance =) 10.375 m (or a value where $10 < \text{distance} \leq 10.375$) seen

hence distance $> 10 (\text{m})$ ***A1***

OR

time taken at speed limit $(\text{time} =) \frac{10}{8.3}$ ***A1***

$(\text{time} =) 1.20481... (\text{s})$

hence time of $1.2 (\text{s})$ to the nearest $0.1 (\text{s})$ could be greater than or less than this ***A1***

THEN

conclusion: it is not certain that the car was exceeding the speed limit. ***A1***

Note: The conclusion must have been justified by further showing that for some appropriate value of t the car is speeding

Award at most ***M1M1A0A1A0A1*** for the use of a value of t where $t > 1.25$

Award at most ***M1M0A0A0A0A1*** for $t = 1.2$ used with a corresponding justified conclusion.

Total [6 marks]

16. (a) an attempt to solve the equation for $t = 1$ showing at least one of the solutions $z = -1 + i\sqrt{7}$ or $z = -1 - i\sqrt{7}$ ***M1***

required solution $= -1 + 2.65i \quad (= -1 + 2.64575...i) \quad (= -1 + i\sqrt{7})$ ***A1***

[2 marks]

(b) using the quadratic formula ***M1***

$z = -t \pm \sqrt{t^2 - 8t}$

$= -t \pm i\sqrt{8t - t^2}$ ***A1***

attempt to find $|z|^2$ ***M1***

$|z|^2 = t^2 + 8t - t^2 = 8t$ ***A1***

this is maximised when $t = 2$ ***A1***

[5 marks]

Total [7 marks]

Q2.

(a) attempt to substitute the given values in the function ***(M1)***

$\left(L=\right)10\log_{10}\left(\frac{10^{-5}}{10^{-12}}\right)$

$=70\left(\text{dB}\right)$ ***A1***

[2 marks]

(b) $185=10\log_{10}\left(\frac{I}{10^{-12}}\right)$ ***(A1)***

attempt to solve their logarithmic equation using graph, solver or analytically ***(M1)***

$I=3160000\left(\text{Wm}^{-2}\right)\left(3162277.66\right)$

$I=3.16\times10^{6}\ \ \left(3.16227...\times10^{6}\right)$ ***A1***

Note: Award at most ***A1M1A0*** or ***A0M1A1FT*** for an incorrect answer written correctly in the required form.

[3 marks]

Total [5 marks]

Q5.

(a) evidence of correct method ***(M1)***

$N = 300, I\% = 4.2, PV = \mp 50000, FV = 0, P/Y = 12, C/Y = 12$ OR 269.4711 seen

Note: Award ***M1*** for using the financial app on their GDC with at least 2 correct entries.

269.47(euros) ***A1***

Note: Award at most ***(M1)A0*** for -269.47 . Accept 269.48.

[2 marks]

(b) (i) evidence of correct method $N = 96$ ***(M1)***

$N = 96,\ I\% = 4.2,\ PV = \mp 50000,\ PMT = \pm 269.47,\ P/Y = 12,\ C/Y = 12$

OR

$N = 96,\ I\% = 0.35,\ PV = \mp 50000,\ PMT = \pm 269.47,\ P/Y = 1,\ C/Y = 1$

Note: Award ***M1*** for using the financial app on their GDC with at least 2 correct entries.

$39,\ 243.42$ ***A1***

Note: Accept $39{,}243.29$ which results from the full calculator value for the payment in part (a). Accept $39{,}243.28$ which results from the use of 269.48.

Note: If two decimal place accuracy was penalised in part (a) do not penalise again.

(ii) **METHOD 1**

full payment $12 \times 25 \times 269.47 = 80{,}841.00$ ***(A1)***
actual payment $12 \times 8 \times 269.47 + 39243.42$ ***(M1)***

= $65,\ 112.54$ ***(A1)***

money saved = $15{,}700$ $(15{,}728.46)$ (euros) ***A1***

Note: Accept $15{,}728.59$ from using 269.47 from part (a) and $39{,}243.29$ from part (b)(i).
Accept $15{,}730.64$ from using 269.48 from part (a) and $39{,}243.28$ from part b(i).

continued

Question 5 continued

METHOD 2

if Sophie did not pay back the loan early her subsequent payments would be $269.47 \times 12 \times 17$ ***(M1)***

Note: Award ***M1*** for attempting to find the remaining amount to pay if loan not paid back.

$= 54971.88$ (euros) ***(A1)***

money saved is therefore
$54971.88 - 39243.42$ ***(M1)***

$= 15,700 (15,728.46)$ (euros) ***A1***

[6 marks]
Total [8 marks]

1. (a) (i) 7 ***A1***

(ii) 9 ***A1***

[2 marks]

(b) (i) 2 ***A1***

(ii) $1+2(n-1)\left(=2n-1\right)$ ***(M1)A1***

Note: ***(M1)*** for substituting their (b)(i) into the arithmetic sequence formula.

[3 marks]

(c) (i) 17 ***A1***

(ii) $\frac{9}{2}(1+17)$ OR $\frac{9}{2}(2\times1+(9-1)\times2)$ OR $1+3+\ldots+15+17$ ***(M1)***

$=81$ ***A1***

[3 marks]

(d) $\frac{n}{2}(1+(2n-1))$ OR $\frac{n}{2}(2\times1+(n-1)\times2)$ ***(M1)***

$=n^2$ ***A1***

[2 marks]

(e) 10 ***A1***

[1 mark]

(f) $1+2+3+...+n$ ***(M1)***

$=\frac{n}{2}(1+n)$ ***A1***

Note: Award ***(M1)*** for attempt for use of sum to n terms of an arithmetic sequence.

[2 marks]

(g) $\frac{n(1+n)}{2}+\frac{n(n-1)}{2}$ ***M1***

Note: Award ***M1*** for addition of answers to parts (f) and given expression.

$=\frac{n}{2}\left[\left(1+n\right)+\left(n-1\right)\right]$ OR $\frac{n+n^2+n^2-n}{2}$ ***M1***

Note: Award ***M1*** for valid non-trivial attempt at further simplification of their sum.

$=\frac{n}{2}(2n)$ OR $\frac{2n^2}{2}$

$=n^2$ ***A1***

Note: Award a maximum of ***M1M1A0*** for **ft** from an incorrect part (f).

[3 marks]

Total [16 marks]

Q7. (a) argument is $1.31 \quad (\frac{5\pi}{12},\ 1.30899\ldots)$ ***(A1)***

modulus is 5 ***(A1)***

$5e^{1.31i}$ $(5e^{\frac{5\pi}{12}i})$ ***A1***

Note: Award ***A1A1A0*** for correct answer $1.29409... + 4.82962...i$ in incorrect form

[3 marks]

(b) (i) length of branches in next iteration $= 0.7 \times 5$ ***(M1)***

$2 \times 5 + 4(0.7 \times 5)$ ***A1***

$= 24$ ***AG***

Note: Award ***M1A0*** for $2 \times 5i + 4(0.7 \times 5i)$.

(ii) **EITHER**

recognition of a geometric sequence / series: for example a value given for r, or consideration of the lengths of branches added. ***(M1)***

$r = 2 \times 0.7\ (= 1.4)$ ***(A1)***

use of formula for geometric series for 10 terms ***(M1)***

$\frac{10((0.7 \times 2)^{10} - 1)}{0.7 \times 2 - 1}$ ***(A1)***

OR

attempt to find the formula for n^{th} term, for example correct first three terms found

$2 \times 5 + 2^2 \times 5 \times 0.7 + 2^3 \times 5 \times 0.7^2 + ...$ ***(M1)***

n^{th} term $= 5 \times 0.7^{n-1} \times 2^n$ ***(A1)***

attempt to find a summation for 10 terms clearly shown ***(M1)***

$\sum_{n=1}^{10} 5 \times 0.7^{n-1} \times 2^n$ ***(A1)***

THEN

$= 698(698.136\ldots)$ ***A1***

continued.

Question 7 continued.

(iii) **EITHER**

as common ratio of the geometric series $= 1.4\ (>1)$ ***R1***

OR

as the **total** length added at each stage is increasing ***R1***

THEN

there is no limit ***A1***

Note: Do not award ***R0A1.***

[9 marks]

(c) (i) correct moduli $5,\ 5\times 0.7,\ 5\times 0.7^2\ \ (5,\ 3.5,\ 2.45)$ ***A1***

attempt to calculate the argument by adding or subtracting $\frac{\pi}{12}(0.261799\ldots)$ ***(M1)***

correct arguments $1.31,\ 1.05,\ 1.31\ \left(\frac{5\pi}{12}, \frac{4\pi}{12}, \frac{5\pi}{12}\right)$ ***A1***

$b = 5\mathrm{e}^{1.31\mathrm{i}} + 3.5\times \mathrm{e}^{1.05\mathrm{i}} + 2.45\times \mathrm{e}^{1.31\mathrm{i}}\left(= 5\mathrm{e}^{\frac{5\pi}{12}\mathrm{i}} + 3.5\times \mathrm{e}^{\frac{\pi}{3}\mathrm{i}} + 2.45\times \mathrm{e}^{\frac{5\pi}{12}\mathrm{i}}\right)$ ***A1***

(ii) $(b =) 3.68 + 10.2\mathrm{i}\left(3.67820\ldots + 10.2272\ldots\mathrm{i}\right)$ ***A1***

Note: Accept $3.66 + 10.2\mathrm{i}$ (from using their 3 sf values).

[5 marks]

Total [17 marks]

2. (a) -6 ***A1***
[1 mark]

(b) 394 ***A1***
[1 mark]

(c) attempt to solve $394 - 6(n-1) \geq 0$ **OR** $400 \div 6$ ***(M1)***

Note: Accept equality or any inequality with zero seen, graphical approach to find zero, use of table/list to find zero.

$(n <) \ 66.6666 \ldots$ ***(A1)***
$(n =) \ 66$ ***A1***
[3 marks]

Note: If 400 is given as the answer in (b) then $n = 67$ in part (c).

Total [5 marks]

5. (a) attempt to expand the brackets ***(M1)***

$z^2 = 4 + 4a\mathrm{i} - a^2$

$\mathrm{Re}(z^2) = 4 - a^2$ ***A1***

[2 marks]

(b) attempt to equate Re or Im parts ***(M1)***

$a = -0.5 \left(= -\frac{1}{2}\right)$ ***A1***

$b = -0.375 \left(= -\frac{3}{8}\right)$ ***A1***

Note: Do not follow through if imaginary part is missing in (a) as the resulting equation has no solution.

[3 marks]

Total [5 marks]

7. (a) $\begin{pmatrix}8\\5\end{pmatrix}, \begin{pmatrix}13\\8\end{pmatrix}$ ***A1A1***

[2 marks]

(b) attempt to write LHS as a product of matrices or expand the RHS ***M1***

$\begin{pmatrix}f_n + f_{n-1}\\f_n\end{pmatrix} = \begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}f_n\\f_{n-1}\end{pmatrix}$ ***A1***

$\begin{pmatrix}f_{n+1}\\f_n\end{pmatrix} = \begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}f_n\\f_{n-1}\end{pmatrix}$ ***AG***

[2 marks]

(c) attempt to find the determinant of $\begin{pmatrix}1 & 1\\1 & 0\end{pmatrix} - \lambda\boldsymbol{I}$ ***(M1)***

$(1-\lambda)(-\lambda) - 1 = 0$ ***A1***

$\lambda^2 - \lambda - 1 = 0$ ***AG***

Note: The ***AG*** line must be seen in order to award the final ***A1***. Accept finding the trace and determinant of $\begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}$ and substituting into $\boldsymbol{\lambda^2 - (\mathbf{Trace})\lambda + \mathbf{det} = 0}$.

[2 marks]

(d) $\begin{pmatrix}1 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}\lambda_1\\1\end{pmatrix} = \begin{pmatrix}\lambda_1 + 1\\\lambda_1\end{pmatrix}$ ***A1***

$= \begin{pmatrix}{\lambda_1}^2\\\lambda_1\end{pmatrix}$ (since $\lambda^2 - \lambda - 1 = 0$) ***A1***

$= \lambda_1\begin{pmatrix}\lambda_1\\1\end{pmatrix}$ ***AG***

[2 marks]

(e) (i) $\begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix}$ ***A1***

(ii) $\begin{pmatrix}\lambda_1 & \lambda_2\\1 & 1\end{pmatrix}$ ***A1***

(iii) $\frac{1}{\lambda_1 - \lambda_2}\begin{pmatrix}1 & -\lambda_2\\-1 & \lambda_1\end{pmatrix}$ ***A1***

Note: Accept $\begin{pmatrix}\lambda_2 & 0\\0 & \lambda_1\end{pmatrix}$ in part (i) and FT for parts (ii) and (iii) and part (f).

[3 marks]

(f) attempt to expand $\begin{pmatrix}\lambda_1 & \lambda_2\\ 1 & 1\end{pmatrix}\begin{pmatrix}{\lambda_1}^{n-1} & 0\\ 0 & {\lambda_2}^{n-1}\end{pmatrix}\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}1 & -\lambda_2\\ -1 & \lambda_1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$ ***(M1)***

EITHER (right to left)

$\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}\lambda_1 & \lambda_2\\ 1 & 1\end{pmatrix}\begin{pmatrix}{\lambda_1}^{n-1} & 0\\ 0 & {\lambda_2}^{n-1}\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix}$ ***A1***

$\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}\lambda_1 & \lambda_2\\ 1 & 1\end{pmatrix}\begin{pmatrix}{\lambda_1}^{n-1}\\ -{\lambda_2}^{n-1}\end{pmatrix}$ ***A1***

OR (Left to right)

$\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}{\lambda_1}^{n} & {\lambda_2}^{n}\\ {\lambda_1}^{n-1} & {\lambda_2}^{n-1}\end{pmatrix}\begin{pmatrix}1 & -\lambda_2\\ -1 & \lambda_1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$ ***A1***

$\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}{\lambda_1}^{n}-{\lambda_2}^{n} & \lambda_1{\lambda_2}^{n}-\lambda_2{\lambda_1}^{n}\\ {\lambda_1}^{n-1}-{\lambda_2}^{n-1} & \lambda_1{\lambda_2}^{n-1}-\lambda_2{\lambda_1}^{n-1}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$ ***A1***

THEN

$\begin{pmatrix}f_{n+1}\\ f_n\end{pmatrix}=\frac{1}{\lambda_1-\lambda_2}\begin{pmatrix}{\lambda_1}^{n}-{\lambda_2}^{n}\\ {\lambda_1}^{n-1}-{\lambda_2}^{n-1}\end{pmatrix}$ ***A1***

Note: To award the final ***A1*** the RHS must be correct and must be equated to $\begin{pmatrix}f_{n+1}\\ f_n\end{pmatrix}$ at some point during the proof.
Award marks as appropriate if candidates do a combination of expanding from the left and the right.

so that $f_n=\frac{1}{\lambda_1-\lambda_2}((\lambda_1)^{n-1}-(\lambda_2)^{n-1})$ ***AG***

[4 marks]

(g) (i) attempt to use quadratic formula or complete the square ***(M1)***

$\frac{1\pm\sqrt{5}}{2}$ ***A1***

(ii) $f_n=\frac{1}{\frac{1+\sqrt{5}}{2}-\frac{1-\sqrt{5}}{2}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n-1}-\left(\frac{1-\sqrt{5}}{2}\right)^{n-1}\right)\left(=\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^{n-1}-\left(\frac{1-\sqrt{5}}{2}\right)^{n-1}\right)\right)$ ***A1***

Note: Exact values required.

[3 marks]
Total [18 marks]

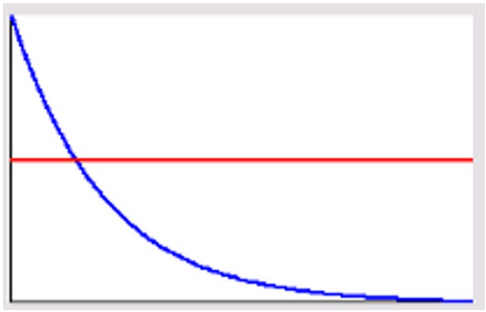
APOLLO

Functions

51 questions

4. (a) $50 = 100\,e^{-1\times p}$ **OR** $0.5 = e^{-1\times p}$ ***(M1)***

OR



(M1)

0.693 $(0.693147..., \ln 2)$ ***A1***

[2 marks]

(b) $R(1.5) = 100\,e^{-0.693147...\times 1.5}$ ***(M1)***

$35.4\,(\%)$ $(35.3553...)$ ***A1***

[2 marks]

(c) $R(t) > 0$ **OR** $R(t)$ has a horizontal asymptote ***R1***

[1 mark]

(d) *Award **A1** for **one** reasonable limitation of the domain:* ***A1***

small values of t produce unrealistic results
$R(0) = 100\%$
large values of t are not possible
people do not live forever
model is not valid at small or large values of t

The reason should focus on the domain $t \geq 0$. Do not accept answers such as:

recollection varies for different people
memories are discrete not continuous
the nature of the information will change how easily it is recalled
emotional/physical stress can affect recollection/concentration

Note: Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

[1 mark]

Total [6 marks]

$\begin{pmatrix} -3 \end{pmatrix}$

17. new function is $f(x-a)+b\left(=\ln(x-a)+b\right)$ ***(M1)***

$f(0)=\ln(-a)+b=1$ ***A1***

$f(\mathrm{e}^3)=\ln(\mathrm{e}^3-a)+b=1+\ln 2$ ***A1***

$\ln(-a)=\ln(\mathrm{e}^3-a)-\ln 2$ ***(M1)***

$$\ln(-a)=\ln\left(\frac{\mathrm{e}^3-a}{2}\right)$$

$$-a=\frac{\mathrm{e}^3-a}{2}$$

$-2a=\mathrm{e}^3-a$

$a=-\mathrm{e}^3\ \ (=-20.0855...)$ ***A1***

$b=1-\ln\mathrm{e}^3=1-3=-2$ ***(M1)A1***

Total [7 marks]

---

2. (a) $(f(-7) =)\ 8$ and $(f(7) =)\ 1$ ***(A1)***

range is $f(x) \leq 1,\ f(x) \geq 8$ ***A1A1***

Note: Award at most ***A1A1A0*** if strict inequalities are used.

[3 marks]

(b) interchanging x, y at any stage ***(A1)***

$$y = 2 - \frac{12}{x+5}$$

$$\frac{12}{x+5} = 2 - y$$

$$\frac{12}{2-y} = x + 5 \quad \textbf{(A1)}$$

$$\frac{12}{2-y} - 5 = x$$

$$\left(f^{-1}(x) =\right)\ \frac{12}{2-x} - 5\ \left(= \frac{2+5x}{2-x}\right) \quad \textbf{A1}$$

[3 marks]

(c) range is $-7 \leq f^{-1}(x) \leq 7,\ f^{-1}(x) \neq -5$ ***A1***

[1 mark]

Total [7 marks]

6. (a) 3 ***A1***

Note: Accept $(3, 0)$ seen.

[1 mark]

(b) **METHOD 1**

$0 = 4a - 2b + c$, $0 = 9a + 3b + c$, $-\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$ ***(M1)(A1)***

(i) 2 ***A1***

(ii) -2 ***A1***

(iii) -12 ***A1***

Note: Award the ***(M1)(A1)*** if at least one correct value is seen.
Do not apply ***FT*** form part (a) if workings are not shown.

METHOD 2

$-12.5 = a(0.5 + 2)(0.5 - 3)$ ***(M1)***

(i) $a = 2$ ***A1***

$0 = 2x(3)^2 + 3b + c$
$0 = 2x(-2)^2 + (-2)b + c$ ***(M1)***

(ii) $b = -2$ ***A1***

(iii) $c = -12$ ***A1***

[5 marks]

(c) $x = 0.5$ ***A1***

Note: Do not ***FT*** from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

4. (a) $y = \frac{5}{8}x + \frac{7}{2}$ $(y = 0.625x + 3.5)$ ***A1A1***

Note: Award ***A1*** for $0.625x$, ***A1*** for 3.5.
Award a maximum of ***A0A1*** if not part of an equation.

[2 marks]

(b) (i) $y = -0.975x^2 + 9.56x - 16.7$ ***(M1)A1***
$(y = -0.974630x^2 + 9.55919x - 16.6569...)$

(ii) gradient of curve is positive at $x = 4$ ***R1***

Note: Accept a sensible rationale that refers to the gradient.

[3 marks]

(c) **METHOD 1**

let $y = ax^2 + bx + c$

differentiating or using $x = \frac{-b}{2a}$ ***(M1)***

$8a + b = 0$

substituting in the coordinates

$7.5^2 a + 7.5b + c = 0$ ***(A1)***

$4^2 a + 4b + c = 6$ ***(A1)***

solve to get

$y = -\frac{24}{49}x^2 + \frac{192}{49}x - \frac{90}{49}$ **OR** $y = -0.490x^2 + 3.92x - 1.84$ ***A1***

Note: Use of quadratic regression with points using the symmetry of the graph is a valid method.

METHOD 2

$y = a(x-4)^2 + 6$ ***(M1)***

$0 = a(7.5-4)^2 + 6$ ***(M1)***

$a = -\frac{24}{49}$ ***(A1)***

$y = -\frac{24}{49}(x-4)^2 + 6$ **OR** $y = -0.490(x-4)^2 + 6$ ***A1***

[4 marks]

continued…

Question 4 continued

(d) (i) $\pi\int_0^4\left(\frac{5}{8}x+3.5\right)^2\mathrm{d}x+\pi\int_4^{7.5}\left(-\frac{24}{49}(x-4)^2+6\right)^2\mathrm{d}x$ ***(M1)(M1)(M1)A1***

Note: Award ***(M1)(M1)(M1)A0*** if π is omitted but response is otherwise correct. Award ***(M1)*** for an integral that indicates volume, ***(M1)*** for their part (a) within their volume integral, ***(M1)*** for their part (b)(i) within their volume integral, ***A1*** for their correct two integrals with all correct limits.

(ii) $501\ \mathrm{cm}^3$ (501.189...) ***A1***

[5 marks]

Total [14 marks]

1. (a) recognition that period $= 669$ ***(M1)***

$b = \frac{2\pi}{669}$ **OR** $b = 0.00939190\ldots$ ***A1***

Note: Award ***A1*** for a correct expression leading to the given value or for a correct value of b to 4 sf or greater accuracy.

$b \approx 0.00939$ ***AG***

[2 marks]

(b) length of day = $24\frac{2}{3}$ hours ***(A1)***

Note: Award ***A1*** for $\frac{2}{3}$, $0.666...$, $0.\overline{6}$ or 0.667.

$\frac{2\pi}{24\frac{2}{3}}$ ***(M1)***

Note: Accept $\left(\frac{360}{24\frac{2}{3}}\right)$.

$= 0.255$ radians $\left(= 0.254723..., \frac{3\pi}{37}, 14.5945...^{\circ}\right)$ ***A1***

[3 marks]

(c) (i) substitution of either value of δ into equation ***(M1)***
correct use of arccos to find a value for ω ***(M1)***

Note: Both ***(M1)*** lines may be seen in either part (c)(i) or part (c)(ii).

$\cos\omega = 0.839\tan(-0.440)$ ***A1***
$\omega = 1.97684...$

≈ 1.98 ***AG***

Note: For substitution of 1.98 award ***M0A0***.

[3 marks]

(ii) $\delta = 0.440$
$\omega = 1.16$ $(1.16474...)$ ***A1***

[1 mark]

continued…

Question 1 continued

(d) (i) $R_{\max} = \dfrac{1.97684...}{0.25472...}$ ***(M1)***

$= 7.76$ hours $(7.76075...)$ ***A1***

Note: Accept 7.70 from use of 1.98.

[2 marks]

(ii) $R_{\min} = \dfrac{1.16474...}{0.25472...}$

$= 4.57$ hours $(4.57258...)$ ***A1***

[1 mark]

Note: Accept 4.55 and 4.56 from use of rounded values.

(e) $a = \dfrac{7.76075... - 4.57258...}{2}$ ***M1***

$\approx 1.59408...$ ***A1***

Note: Award ***M1*** for substituting their values into a correct expression.
Award ***A1*** for a correct value of a from their expression which has at least 3 significant figures and rounds correctly to 1.6.

≈ 1.6 (correct to 2 sf) ***AG***

[2 marks]

(f) **EITHER**

$c = \dfrac{7.76075... + 4.57258...}{2} \left(= \dfrac{12.333...}{2} \right)$ ***(M1)***

OR

$c = 4.57258... + 1.59408...$ or $c = 7.76075... - 1.59408...$

THEN

$= 6.17$ $(6.16666...)$ ***A1***

Note: Accept 6.16 from use of rounded values.
Follow through on their answers to part (d) and 1.6.

[2 marks]

(g) $d = 18.65 - 6.16666...$ ***(M1)***

$= 12.5$ $(12.4833...)$ ***A1***

Note: Follow through for 18.65 minus their answer to part (f).

[2 marks]

continued…

Question 1 continued

(h) (i) at least one expression in the form $re^{g(t)i}$ ***(M1)***

$z_1 = 1.5e^{(0.00939t+2.83)i}$, $z_2 = 1.6e^{(0.00939t)i}$ ***A1A1***

[3 marks]

(ii) **EITHER**

$z_1 - z_2 = 1.5e^{(0.00939t+2.83)i} - 1.6e^{(0.00939t)i}$

$= e^{0.00939ti}\left(1.5e^{2.83i} - 1.6\right)$ ***(M1)***

$= e^{0.00939ti}\left(3.06249...e^{2.99086...i}\right)$ ***(A1)(A1)***

OR

graph of L or f

$p = 3.06249...$ ***(A1)***

$r = -0.150729...$ **OR** $r = 2.99086...$ ***(M1)(A1)***

Note: The p and r variables (or equivalent) must be seen.

THEN

$L(t) = 3.06\sin(0.00939t + 2.99) + 12.5$ ***A1***

$\left(L(t) = 3.06248...\sin(0.00939t + 2.99086...) + 12.4833...\right)$

Note: Accept equivalent forms, e.g. $L(t) = 3.06\sin(0.00939t - 0.151) + 12.5$.
Follow through on their answer to part (g) replacing 12.5.

[4 marks]

(iii) shortest time between sunrise and sunset

$12.4833... - 3.06249...$ ***(M1)***

$= 9.42$ hours $(9.420843...)$ ***A1***

Note: Accept 9.44 from use of 3 sf values.

[2 marks]

[Total 27 marks]

1. (a) $m = \frac{6-0}{4-2} = 3$ ***(M1)A1***

[2 marks]

(b) $(m =)\ -\frac{1}{3}\ (-0.333, -0.333333...)$ ***A1***

[1 mark]

(c) an equation of line with a correct intercept and either of their gradients from (a) or (b) ***(M1)***

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

Note: Award ***(M1)*** for substituting either of their gradients from parts (a) or (b) and point B or $(3, 3)$ into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple ***A1***

[2 marks]

(d) $(x =)\ 12$ ***A1***

[1mark]

Total: [6 marks]

3. (a) $h(4) = \frac{640}{4^2} + 0.5$ **OR** $h(14) = \frac{640}{14^2} + 0.5$ ***(M1)***

Note: Award ***(M1)*** for substituting 4 or 14 into h. This can be implicit from seeing $3.77\ (3.76530\ldots)$ or 40.5.

$3.77 \leq h(x) \leq 40.5\ \ (3.76530... \leq h(x) \leq 40.5)$ ***A1A1***

Note: Award ***A1*** for both correct endpoints seen, ***A1*** for the endpoints in a correct interval.

[3 marks]

(b) (i) $h(x) = 10$ **OR** $h^{-1}(x) = \sqrt{\frac{640}{x - 0.5}}$ **OR** $h^{-1}(10) = \sqrt{\frac{640}{10 - 0.5}}$ ***(M1)***

$(x =)\ 8.21\ \text{cm}\ (8.20782...)$ ***A1***

(ii) a tin that is $10\ \text{cm}$ high will have a diameter of $8.21\ \text{cm}\ (8.20782...)$ ***A1***

Note: Condone a correct answer expressed as the converse.

(iii) $4 \leq h^{-1} \leq 14$ ***A1***

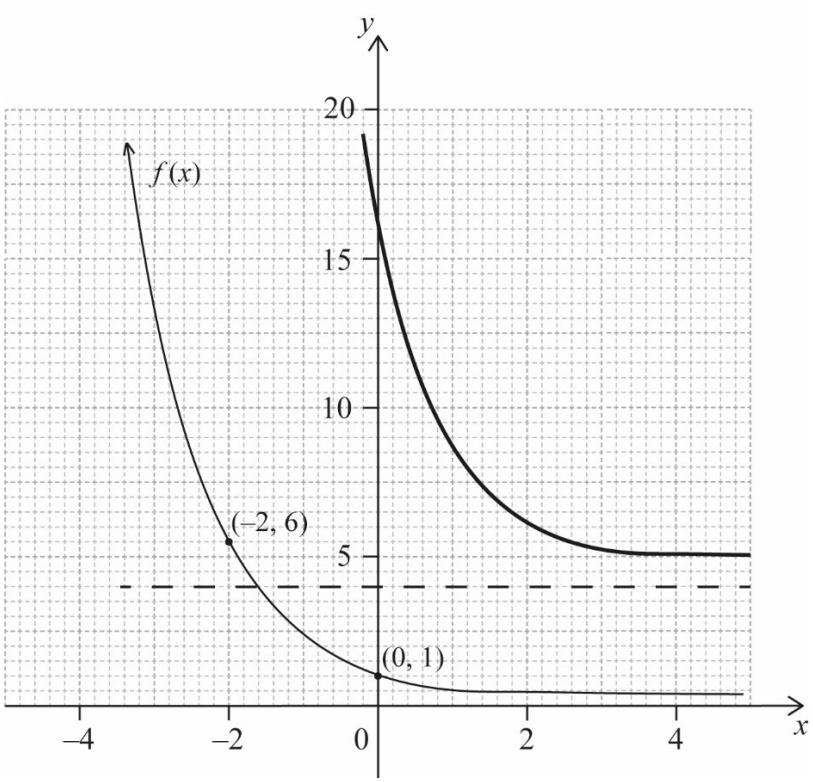
Note: Accept $4 \leq y \leq 14$. Do not ***FT*** in this part.

[4 marks]
Total: [7 marks]

10. (a) $g(0) = 16$ ***M1A1***

[2 marks]

(b)



y-asymptote $(y = 4)$ ***A1***

concave up decreasing curve and passing through $(0, 16)$ ***A1***

[2 marks]

Total: [4 marks]

2. (a) (i) maximum $h = 130$ metres ***A1***

(ii) minimum $h = 50$ metres ***A1***

[2 marks]

(b) (i) $(60 \div 12 =)\ 5$ seconds ***A1***

(ii) $360 \div 5$ ***(M1)***

Note: Award ***(M1)*** for 360 divided by their time for one revolution.

$= 72^\circ$ ***A1***

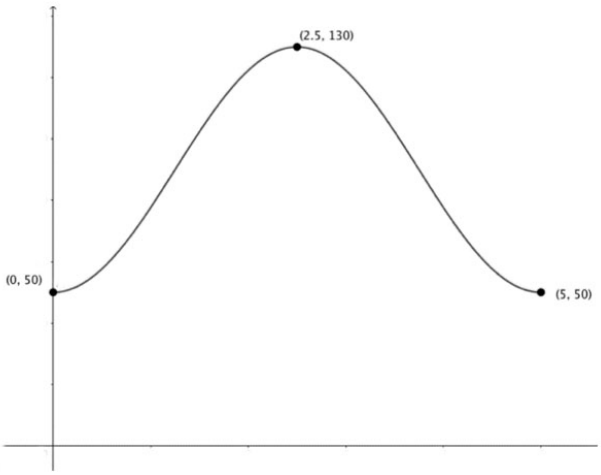
[3 marks]

(c) (i) (amplitude $=)\ 40$ ***A1***

(ii) (period $= \frac{360}{72} =\ )\ 5$ ***A1***

[2 marks]

(d)



Maximum point labelled with correct coordinates. ***A1***

At least one minimum point labelled. Coordinates seen for any minimum points must be correct. ***A1***

Correct shape with an attempt at symmetry and “concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain. ***A1***

[3 marks]

continued…

Question 2 continued

(e) (i) $h = 90 - 40\cos(144^\circ)$ ***(M1)***

$(h =)\ 122\ (\text{m})\ (122.3606.....)$ ***A1***

(ii) evidence of $h = 100$ on graph **OR** $100 = 90 - 40\cos(72t)$ ***(M1)***

t coordinates $3.55\ (3.54892...)$ **OR** $1.45\ (1.45107...)$ or equivalent ***(A1)***

Note: Award ***A1*** for either t-coordinate seen.

$= 2.10$ seconds $(2.09784\ldots)$ ***A1***

[5 marks]

(f) **METHOD 1**

$90 - 40\cos(at^\circ) = 110$ ***(M1)***

$\cos(at^\circ) = -0.5$

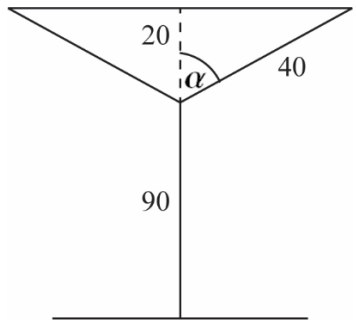
$at^\circ = 120\,,\ 240$ ***(A1)***

$1 = \frac{240}{a} - \frac{120}{a}$ ***(M1)***

$a = 120$ ***(A1)***

period $= \frac{360}{120} = 3$ seconds ***A1***

METHOD 2



attempt at diagram ***(M1)***

$\cos\alpha = \frac{20}{40}$ (or recognizing special triangle) ***(M1)***

angle made by C, $2\alpha = 120^\circ$ ***(A1)***

one third of a revolution in 1 second ***(M1)***

hence one revolution = 3 seconds ***A1***

continued…

Question 2 continued

METHOD 3

considering $h(t) = 110$ on original function ***(M1)***

$t = \frac{5}{3}$ or $\frac{10}{3}$ ***(A1)***

$\frac{10}{3} - \frac{5}{3} = \frac{5}{3}$ ***(A1)***

Note: Accept $t = 1.67$ or equivalent.

so period is $\frac{3}{5}$ of original period ***(R1)***

so new period is 3 seconds ***A1***

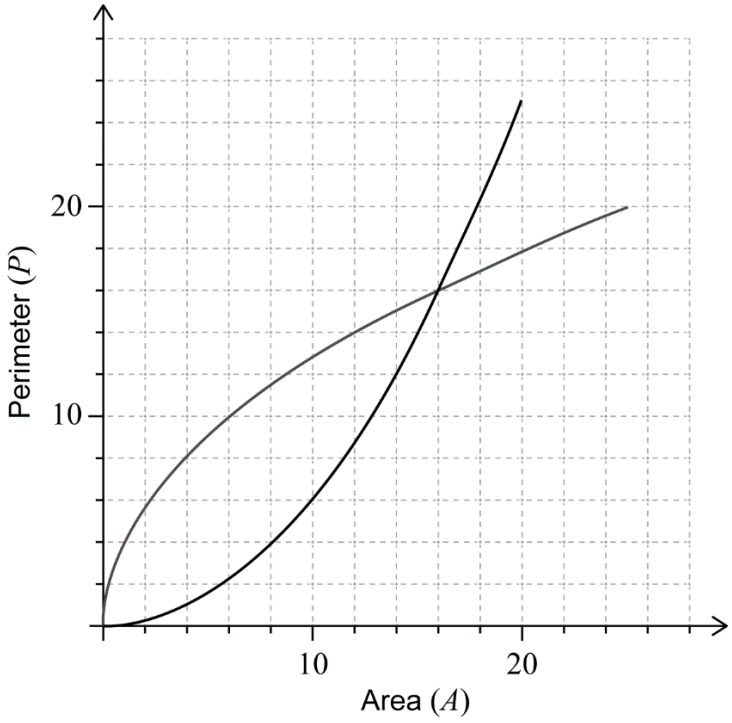
[5 marks]

Total: [20 marks]

2. (a) 20 ***A1***

[1 mark]

(b)



(M1)A1A1

Note: Award ***(M1)*** for reflection in the line $P = A$, award ***A1*** for endpoint at $(20, 25)$, award ***A1*** for passing through $(16, 16)$.

[3 marks]

(c) when the perimeter is 8, the area is 4 ***A1***

[1 mark]

Total [5 marks]

3. (a) (i) 1750 ***A1***

(ii) $1350 + 400\,(1.25)^{-5}$ ***(M1)***

$= 1480$ ***A1***

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400\,(1.25)^{-t}$ ***(M1)***

9.32 (days ($9.31885\ldots$) (days)) ***A1***

[2 marks]

(c) 1350 ***A1***

Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.

[1 mark]

Total [6 marks]

1. (a) 1.2 metres ***A1***

[1 mark]

(b) $-4.8t^2 + 21t + 1.2 = 0$ ***(M1)***

$(t =)\ 4.43$ s $(4.431415...\ s)$ ***A1***

Note: If both values for t are seen do not award the ***A1*** mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \le t \le 4.43$ **OR** $[0,\ 4.43]$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for expressing answer with correct notation. Award at most ***A1A0*** for use of x instead of t.

[2 marks]

[Total 5 marks]

17. substitute coordinates of A

$f(0) = p\,\mathrm{e}^{q\cos(0)} = 6.5$

$6.5 = p\,\mathrm{e}^{q}$ ***(A1)***

substitute coordinates of B

$f(5.2) = p\mathrm{e}^{q\cos(5.2r)} = 0.2$

EITHER

$f'(t) = -pqr\sin(rt)\,\mathrm{e}^{q\cos(rt)}$ ***(M1)***

minimum occurs when $-pqr\sin(5.2r)\,\mathrm{e}^{q\cos(5.2r)} = 0$

$\sin(rt) = 0$

$r \times 5.2 = \pi$ ***(A1)***

OR

minimum value occurs when $\cos(rt) = -1$ ***(M1)***

$r \times 5.2 = \pi$ ***(A1)***

OR

period = $2 \times 5.2 = 10.4$ ***(A1)***

$r = \dfrac{2\pi}{10.4}$ ***(M1)***

THEN

$r = \dfrac{\pi}{5.2} = 0.604152...\ (0.604)$ ***A1***

$0.2 = p\,\mathrm{e}^{-q}$ ***(A1)***

eliminate p or q ***(M1)***

$\mathrm{e}^{2q} = \dfrac{6.5}{0.2}$ **OR** $0.2 = \dfrac{p^2}{6.5}$

$q = 1.74\ (1.74062...)$ ***A1***

$p = 1.14017...\ (1.14)$ ***A1***

[Total 8 marks]

5. (a) *Accept any one of the following (or equivalent):*
one minimum and one maximum point
three x-intercepts or three roots (or zeroes)
one point of inflexion ***R1***

Note: Do not accept "S shape" as a justification.

[1 mark]

(b) (i) $(d =) -5$ ***A1***

(ii) $8 = a + b + c$
$4 = 8a + 4b + 2c$
$0 = 27a + 9b + 3c$ ***A2***

Note: Award ***A2*** if all three equations are correct.
Award ***A1*** if at least one is correct. Award ***A1*** for three correct equations that include the letter "d".

(iii) $a = 2,\ b = -12,\ c = 18$ ***A1***

[4 marks]

(c) equating found expression to zero ***(M1)***
$0 = 2t^3 - 12t^2 + 18t - 5$
$t = 0.358216\ldots,\ 1.83174\ldots,\ 3.81003\ldots$ ***(A1)***

(so total time in debt is $3.81003\ldots - 1.83174\ldots + 0.358216 \approx$)
$2.34\ \ (2.33650\ldots)$ years ***A1***

[3 marks]
Total [8 marks]

10. (a) $y = \ln\left(\frac{1}{x-2}\right)$

an attempt to isolate x (or y if switched) ***(M1)***

$e^y = \frac{1}{x-2}$

$x - 2 = e^{-y}$

$x = e^{-y} + 2$

switching x and y (seen anywhere) ***M1***

$f^{-1}(x) = e^{-x} + 2$ ***A1***

[3 marks]

(b) sketch of $f(x)$ and $f^{-1}(x)$ ***(M1)***

$x = 2.12 \ \ (2.12002\ldots)$ ***A1***

[2 marks]

Total [5 marks]

4. (a) attempt to use chain rule, including the differentiation of $\frac{1}{T}$ ***(M1)***

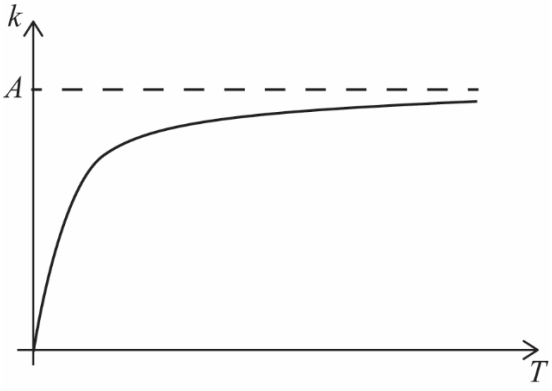
$\frac{\mathrm{d}k}{\mathrm{d}T} = A \times \frac{c}{T^2} \times \mathrm{e}^{-\frac{c}{T}}$ ***A1***

this is the product of positive quantities so must be positive ***R1***

Note: The ***R1*** may be awarded for correct argument from **their** derivative. ***R1*** is not possible if their derivative is not always positive.

[3 marks]

(b)



A1A1A1

Note: Award ***A1*** for an increasing graph, entirely in first quadrant, becoming concave down for larger values of T, ***A1*** for tending towards the origin and ***A1*** for asymptote labelled at $k = A$.

[3 marks]

(c) taking ln of both sides **OR** substituting $y = \ln x$ and $x = \frac{1}{T}$ ***(M1)***

$\ln k = \ln A - \frac{c}{T}$ **OR** $y = -cx + \ln A$ ***(A1)***

(i) so gradient is $-c$ ***A1***

(ii) y-intercept is $\ln A$ ***A1***

Note: The implied ***(M1)*** and ***(A1)*** can only be awarded if **both** correct answers are seen. Award zero if only one value is correct **and** no working is seen.

[4 marks]

continued…

Question 4 continued

(d) an attempt to convert data to $\frac{1}{T}$ and $\ln k$ ***(M1)***

e.g. at least one correct row in the following table

$\frac{1}{T}$	$\ln k$
$1.69491\ldots\times10^{-3}$	$-7.60090\ldots$
$1.66666\ldots\times10^{-3}$	$-7.41858\ldots$
$1.63934\ldots\times10^{-3}$	$-6.90775\ldots$
$1.61290\ldots\times10^{-3}$	$-6.57128\ldots$
$1.58730\ldots\times10^{-3}$	$-6.21460\ldots$
1.5625×10^{-3}	$-5.84304\ldots$
$1.53846\ldots\times10^{-3}$	$-5.62682\ldots$

line is $\ln k = -13400\times\frac{1}{T}+15.0 \quad \left(=-13383.1\ldots\times\frac{1}{T}+15.0107\ldots\right)$ ***A1***

[2 marks]

(e) (i) $c = 13400 \quad (13383.1\ldots)$ ***A1***

(ii) attempt to rearrange or solve graphically $\ln A = 15.0107\ldots$ ***(M1)***

$A = 3300000 \quad (3304258\ldots)$ ***A1***

Note: Accept an A value of $3269017\ldots$ from use of 3sf value.

[3 marks]

Total [15 marks]

2. (a) $71e^{-0.0514(16)} + 23$ ***(M1)***

$54.2\ ^{\circ}\text{C}\ (54.1956...)$ ***A1***

[2 marks]

(b) $23\ ^{\circ}\text{C}$ ***A1***

[1 mark]

(c) $50 = 71e^{-0.0514(k)} + 23$ ***(M1)***

$k = 18.8\ \left(\frac{-5000}{257}\ln\left(\frac{27}{71}\right), 18.8101...\right)$ ***A1***

Note: Award ***M1*** for a sketch showing a point of intersection between the exponential function and $y = 50$.

[2 marks]
Total [5 marks]

6. (a) 78 ***A1***

[1 mark]

(b) (i) 65 ***A1***

(ii) **EITHER**

(period $=$) 16 (could be seen on sketch) ***(M1)***

$b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

OR

$143 = 65\sin(4b) + 78$ ***(M1)***

$(\sin(4b) = 1)$

$(4b = \frac{\pi}{2}$ **OR** $4b = 90^\circ)$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

[3 marks]

(c) 13 ***A1***

Note: Apply follow through marking only if their final answer is positive.

[1 mark]

(d) $(b =)\ 0.196\ \left(0.196349..., \frac{\pi}{16}\right)$ **OR** $(b =) 11.3^\circ\ (11.25^\circ)$ ***A1***

[1 mark]

Total [6 marks]

10. (a) $m = 1 - 2.5\log_{10}(0.0525)$ ***(M1)***

$= 4.20 \ \ (4.19960\ldots)$ ***A1***

[2 marks]

(b) attempt to solve $7 = 1 - 2.5\log_{10}(b)$ ***(M1)***

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5\log_{10}(b)$.

$b = 0.00398 \ \ (0.00398107...)$ ***A1***

[2 marks]

(c) $-3.2 = \left(1 - 2.5\log_{10}(b_n)\right) - \left(1 - 2.5\log_{10}(b_p)\right)$ ***M1***

$-3.2 = -2.5\log_{10}\left(\frac{b_n}{b_p}\right)$ ***A1***

$\frac{b_n}{b_p} = 19.1 \ \ (19.0546...)$ ***A1***

[3 marks]

Total [7 marks]

11. (a) $y = -0.00855x^3 - 0.234x^2 - 0.225x + 3.20$ ***A2***

$(y = -0.00854819...x^3 - 0.234002...x^2 - 0.224884...x + 3.20056...)$

Note: Award ***A0A1*** for at least two terms correct.

[2 marks]

(b) $y(2x)$ (for horizontal stretch) ***(A1)***

attempt to stretch vertically by factor $\frac{1}{2}$ ***(M1)***

$y = 0.0332x^3 - 0.15x^2 - 0.58x \ \ (+1.1)$ ***A1***

Note: Award ***A0M1A0*** for a vertical stretch, factor 2. Although a d value of 1.1 is preferred, technically this value can be wrong/omitted and the question is still answered (hence it is presented in brackets).

[3 marks]
Total [5 marks]

13. (a) Gradient $= \frac{14.9+1.3}{6}$ $(= 2.7)$ ***(M1)***

$\log_{10} Q = 2.7P - 1.3$ ***(A1)***

$Q = 10^{2.7P-1.3}$ **OR** $Q = 0.0501 \times 10^{2.7P}$ $(= 0.0501187... \times 10^{2.7P})$ ***A1***

[3 marks]

(b) $\ln R$ on one axis and Q on the other axis ***A1***

[1 mark]

(c) $\log_{10}(4.3\ln R + 12.1) = 2.7P - 1.3$ **OR** $10^{2.7P-1.3} = 4.3\ln R + 12.1$ ***(M1)***

$P = \frac{\log_{10}(4.3\ln R + 12.1) + 1.3}{2.7}$ ***A1***

[2 marks]

Total [6 marks]

17. (a) $D = S - R$

METHOD 1

$= \text{Re}(1.15\,\text{e}^{(0.0165t-2.97)\text{i}}) - \text{Re}(1.08\,\text{e}^{(0.0165t+0.413)\text{i}})\ \ (+18.9 - 4.94)$ ***(M1)(A1)***

$= \text{Re}\left(\text{e}^{0.0165t\text{i}}(1.15\,\text{e}^{-2.97\text{i}} - 1.08\,\text{e}^{0.413\text{i}})\right)\ \ (+13.96)$ ***(M1)***

$= \text{Re}\left(\text{e}^{0.0165t\text{i}}(2.21379...\text{e}^{-2.85310...\text{i}})\right)\ \ (+13.96)$ ***(A1)***

$= 2.21\cos(0.0165t - 2.85) + 13.96\ \ \left(2.21379...\cos(0.0165t - 2.85310...) + 13.96\right)$ ***A1A1***

$(a = 2.21,\ b = -2.85,\ c = 13.96)$

Note: Award ***A1*** for $2.21\cos(0.0165t - 2.85)$ and ***A1*** for "$+13.96$". The ***A1*** for 13.96 is independent of the previous marks.

METHOD 2

$c = 13.9 - 4.94\ldots = 13.96$ ***A1***

using a graph of D ***M1***

maximum $(172.915\ldots, 16.1738\ldots)$ ***(A1)***

minimum $(-17.4842\ldots, 11.7462\ldots)$ ***(A1)***

EITHER

amplitude $16.1738\ldots - 11.7462\ldots = 4.4276\ldots$

$a = 2.21\ \ (2.2138\ldots)$ ***A1***

OR

$a = 16.1738\ldots - 13.96 = 2.21\ \ (2.2138\ldots)$ ***A1***

THEN

EITHER

when $t = 0$, $D = 11.8377\ldots$

$11.8377\ldots = 2.2138\cos(b) + 13.96$

$b = -2.85\ \ \ (2.85309\ldots)$ ***A1***

OR

$b = -0.0165 \times 172.915\ldots = -2.85\ \ \ (2.85309\ldots)$ ***A1***

[6 marks]

(b) $16.2\ (16.1737\ldots)$ hours on day 173 ***A1A1***

Note: Accept an answer of "day 172" for the second ***A1***.

[2 marks]

Total [8 marks]

6. (a) (i) **METHOD 1**

attempt to find change in height of the ball using gradient ***(M1)***

$\frac{a}{0.43} = (-)0.045$

$a = (-)0.045 \times 0.43$

$a = (-)0.0194\,(\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

METHOD 2

attempt to find height at back of home plate ***(M1)***

horizontal distance to the front of the home plate $= 16.6666\ldots\ (\text{m})$

height at the back of the home plate $= -0.045(16.6666\ldots + 0.43) + 2$

$\left(= 1.23065\ (\text{m})\right)$

Note: The ***M1*** can be awarded for $16.6666\ldots + 0.43$ seen at some point.

$(a = 1.25 - 1.23065\ldots)$

$(a =)\ (-)0.0194\ (\text{m})\ \ \left(0.01935\ (\text{m})\right)$ ***A1***

(ii) $1.25 - 0.01935 = 1.23065$ (may be seen in part (a)(i)) ***A1***

$0.53 <\ 1.23065\ < 1.24$ ***R1***

therefore a strike ***AG***

Note: Do not award ***A0R1***.

[4 marks]

continued…

Question 6 continued

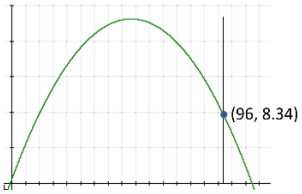
(b) **METHOD 1**

indication of $d = 96$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$\left(h(96) =\right) -0.01(96)^2 + 1.04(96) + 0.66$

OR



THEN

$\left(h(96) =\right)\ 8.34\ (\text{m})$ ***A1***

$8.34 > 5$ so the ball will go over the wall. ***A1***

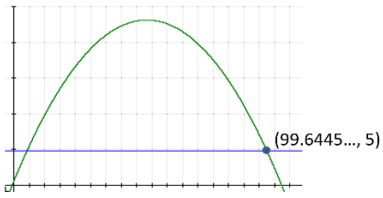
METHOD 2

indication of $h = 5$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$5 = -0.01d^2 + 1.04d + 0.66$

OR



THEN

$d = 99.6\ (\text{m})\ \left(99.6445\ldots\ (\text{m})\right)\ \left(d = 4.35548\ldots\ (\text{m})\right.$ may also be seen$\left.\right)$ ***A1***

$96 < 99.6445\ldots$ so the ball will go over the wall. ***A1***

[3 marks]

[Total: 7 marks]

14. METHOD 1 Analytical approach

recognizing that the linear equation must be expressed in log form ***(M1)***

$\log y = m\log x + \log c$ (or $\log y = m\log x + C$)

EITHER

use of slope formula (must involve logs) ***(M1)***

$m = \dfrac{\log(34.822) - \log(13.1951)}{\log(4) - \log(2)} = 1.4$ ***A1***

attempt to substitute a value ***(M1)***

$\log c = \log(13.1951) - 1.4\log 2 \,(= 0.69897...)$

$\Rightarrow c = 5$ ***A1***

OR

$y = c.x^m$ ***(A1)***

attempt to set up two equations involving power functions ***(M1)***

$13.1951 = c \times 2^m$ and $34.822 = c \times 4^m$

$2^m = \dfrac{34.822}{13.1951} = 2.639... \Rightarrow m = \log_2 2.639... = 1.4$ ***A1***

$c = \dfrac{13.1951}{2.639...} = 5$ ***A1***

THEN

(so the equation is) $y = 5 \times x^{1.4}$ ***A1***

METHOD 2 Regression analysis

recognizing that a log-log graph results in a power function model ***(M1)***

$y = a \times x^b$

attempt to find a power regression model using the given two points ***(M1)***

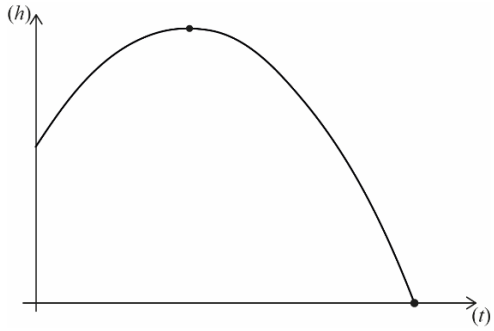
$a = 5$ and $b = 1.4$ ***(A1)(A1)***

(so the equation is) $y = 5 \times x^{1.4}$ ***A2***

[Total: 6 marks]

1. (a) **METHOD 1**

correct sketch with some indication of maximum point ***(M1)***



0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 2

correct substitution into equation for line of symmetry ***(M1)***

$(t =) \ -\frac{8.75}{2 \times -4.75}$

0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 3

equating the correct derivative to 0 ***(M1)***

$-9.5t + 8.75 = 0$

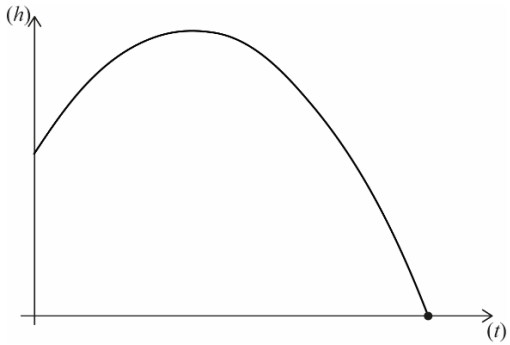
0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

Note: Award ***M1A0*** for a final answer of 0.92 seen with no working.

[2 marks]

(b) **METHOD 1**

correct sketch with some indication of x-intercept



(M1)

Note: May be seen in part (a).

2 (seconds) ***A1***

continued…

Question 1 continued

METHOD 2
setting the equation to zero ***(M1)***

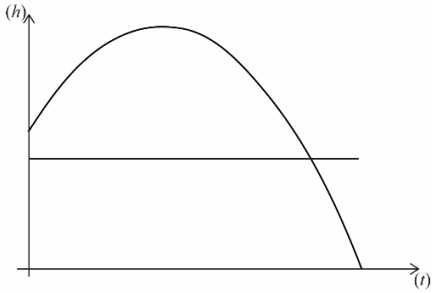
$-4.75t^2 + 8.75t + 1.5 = 0$

2 (seconds) ***A1***

Note: If both roots are given, with or without working, award ***(M1)A0***.

[2 marks]

(c) **METHOD 1**
correct sketch of quadratic function and a straight line in approximate correct position ***(M1)***



1.88 (seconds) $(1.87577\ldots$(seconds)) ***A1***

METHOD 2
setting the equation equal to 1.2 ***(M1)***
$-4.75t^2 + 8.75t + 1.5 = 1.2$
1.88 (seconds) $(1.87577\ldots$(seconds)) ***A1***

Note: Award ***(M1)A0*** if $-0.0336702\ldots$ seen as (part of) a final answer.
Award ***M1A0*** for answer of 1.9 seen without working.

[2 marks]

(d) *Award **R1** for a sensible reason in the context of the question:* ***R1***
e.g.
The model ignores air resistance (or wind)
The model treats the ball as a point
The model assumes gravity is constant
The model assumes that the ball continues to follow the trajectory even after hitting the ground
This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[1 mark]

Total [7 marks]

2. (a) $\frac{18-4}{2}$ ***(M1)***

$(a=)\ 7$ ***A1***

[2 marks]

(b) $\frac{18+4}{2}$ **OR** $18-7$ **OR** $4+7$ ***(M1)***

$(d=)\ 11$ ***A1***

[2 marks]

(c) (time between high and low tide is) $6\text{h}15\text{m}$ **OR** 375 minutes ***(A1)***

multiplying by 2 ***(M1)***

750 minutes ***A1***

[3 marks]

(d) **EITHER**

$\frac{360^\circ}{b}=750$ ***(A1)***

OR

$7\cos(b\times 375)+11=4$ ***(A1)***

THEN

$(b=)\ 0.48$ ***A1***

Note: Award ***A1A0*** for an answer of $\frac{2\pi}{750}\left(=\frac{\pi}{375}=0.00837758...\right)$.

[2 marks]

(e) equating their cos function to 6 **OR** graphing their cos function and 6 ***(M1)***

$7\cos(0.48t)+11=6$

$\Rightarrow t=282.468...$ (minutes) ***(A1)***

$=4.70780...$ (hr) **OR** 4hr 42 mins (4hr $42.4681\ldots$ mins) ***(A1)***

so the time is $10:42$ ***A1***

[4 marks]

continued…

Question 2 continued

(f) next solution is $t = 467.531...$ ***(A1)***

$467.531... - 282.468...$

$185\ (\text{mins})\ (185.063...)$ ***A1***

[2 marks]

[Total: 15 marks]

2. (a) (i) 1200 ***A1***

(ii) the initial population of the bacteria ***A1***

[2 marks]

(b) $1200 \times k^3 = 18750$ ***(A1)***

$(k =)\ 2.5$ ***A1***

[2 marks]

(c) $1200 \times 2.5^{1.5}$ ***(A1)***

$4740\ (4743.41\ldots)$ ***A1***

Note: Do not penalize if final answer is not given as an integer. Award ***(A1)A0*** for an answer of $3950\ (3949.14\ldots)$ from use of 1.3 in the exponent, but only if working is shown.

[2 marks]

(d) equating $P(t)$ and $S(t)$ **OR** equating each function to a common variable ***(M1)***

$1200 \times 2.5^t = 5000 \times 1.65^t$; $1200 \times 2.5^t = x$ and $5000 \times 1.65^t = x$

$t = 3.43$ (hours) $(3.43456...)$ ***A1***

[2 marks]

continued…

Question 2 continued

(e) **METHOD 1**

$5000 \times 1.65^t = 19000$ ***(M1)***

$(t =)$ 2.66586... **OR** $(t - 2 =)$ 0.66586... (seen) ***(A1)***

multiplying by 60 seen to convert to minutes ***(M1)***
$(m = 39.9521\ldots)$

$(m =)$ 40 (minutes) **OR** 2 hours and 40 minutes ***A1***

METHOD 2

equating an expression for $S(t)$ to 19000 ***(M1)***

expressing t as $2 + \frac{m}{60}$ ***(A1)***

$5000 \times 1.65^{2+\frac{m}{60}} = 19000$

$2 + \frac{m}{60} = 2.66586\ldots$ ***A1***

$(m =)$ 40 (minutes) **OR** 2 hours and 40 minutes ***A1***

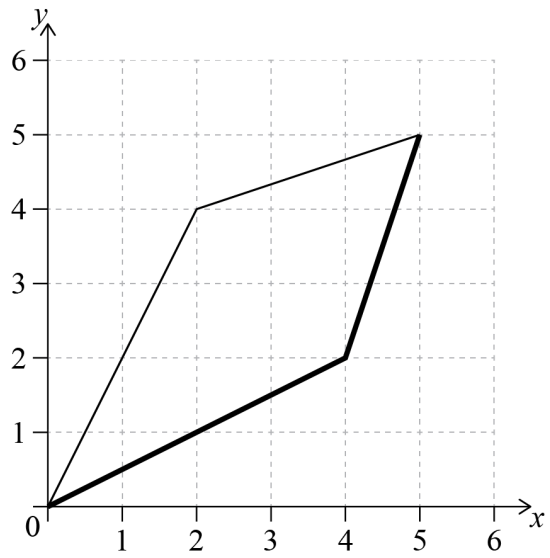
Note: Award ***(M1)(A1)(M1)A0*** for an answer of $39.9521...$ or 39 with or without working.

[4 marks]
Total [12 marks]

2. (a) 4 ***A1***

[1 mark]

(b)



A1A1

Note: Award ***A1*** for passing through $(0, 0)$ and $(4, 2)$, ***A1*** for passing through $(4, 2)$ and $(5, 5)$.

[2 marks]

(c) attempt to solve $y = 3x - 1$ for x **OR** changing variables ***(M1)***

$\left(g^{-1}(x)\right) = \frac{x+1}{3}$ ***A1***

[2 marks]

(d) sketch of $g(x)$ or $g^{-1}(x)$, algebraic approach ***(M1)***

$\frac{1}{2}x = \frac{x+1}{3}$

$(x =)\ 2$ ***A1***

[2 marks]

[Total 7 marks]

5. (a) $2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$ ***A1***

[1 mark]

(b) finding other equations to solve simultaneously ***(M1)***

$5 = a(10)^2 + b(10) + c$ **AND** $7.16 = a(17)^2 + b(17) + c$
OR $5 = 100a + 10b + c$ **AND** $7.16 = 289a + 17b + c$

any one coefficient in equation correct ***(A1)***
$f(x) = -0.00490x^2 + 0.441x + 1.08$ ***A1***

$\left(f(x) = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots\right)$
$\left(f(x) = -\frac{6}{1225}x^2 + \frac{108}{245}x + \frac{53}{49}\right)$

Note: Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 80 into their equation ***(M1)***

$\left(f(80) =\right) 5$ ***A1***

$5 > 4$ **OR** therefore the ball will go over the fence ***R1***

Note: Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero, graph ***(M1)***

$0 = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots$ **OR** $f(x) = 0$

92.4 $(92.3902\ldots)$ (m) ***A1***

[2 marks]
[Total 9 marks]

7. (a) (i) $m = ah^3$ ***(M1)***

$64 = a \times 0.8^3 \ \ (\Rightarrow a = 125)$ ***(A1)***

$m = 125h^3$ ***A1***

Note: The final answer must be written as m in terms of h to award the final ***A1***.

(ii) $m = (125(0.75)^3 =)\ 52.7\ (\text{kg})\ \ (52.7343...)$ ***A1***

[4 marks]

(b) **EITHER (finding the height of the lion)**

$(220 = 125h^3)$

$h = \sqrt[3]{1.76}\ \ (1.20736...)$ ***(A1)***

$E = k_1 h^2$ (seen anywhere) ***(A1)***

$k = \dfrac{\left(\sqrt[3]{1.76}\right)^2}{0.8^2}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

OR (finding a formula for E in terms of m)

$\left(m = 125h^3\right)$

$E = k_1 h^2$ ***(A1)***

$E = k_2 m^{\frac{2}{3}}$ ***(A1)***

$k = \dfrac{220^{\frac{2}{3}}}{64^{\frac{2}{3}}}$ ***(M1)***

$= 2.28\ \ (2.27769\ldots)$ ***A1***

[4 marks]

[Total 8 marks]

14. (a) $x_B = 4\sqrt{2(t-3)} \quad (t \geq 3)$ ***A1A1***

Note: Award ***A1*** for multiplying by 2 and ***A1*** for $t-3$. Award ***A1A0*** for $4\sqrt{2t-3}$.

[2 marks]

(b) equating their x_B to $4\sqrt{t}$ ***(M1)***

$4\sqrt{2(t-3)} = 4\sqrt{t}$

$t = 6$ (seconds) ***A1***

Note: Do not ***FT*** from part (a) to part (b).

[2 marks]
[Total 4 marks]

3. (a) correct substitution of 0.0003 into the formula ***(A1)***

$\text{pH} = -\log_{10}(0.0003)$

$= 3.52 \quad (3.52287\ldots)$ ***A1***

[2 marks]

(b) **EITHER**

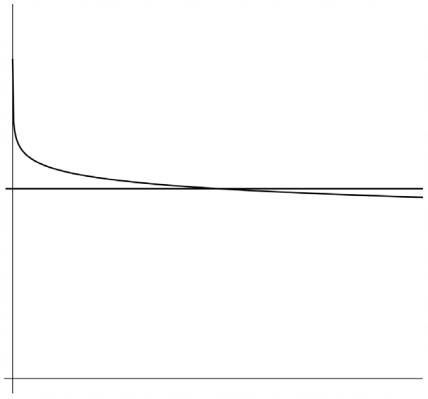
attempt to change to exponential form ***(M1)***

$\left[\text{H}^+\right] = 10^{-6.6}$

OR

attempt to solve $6.6 = -\log_{10}\left[\text{H}^+\right]$

(graphically or using numerical solver)



THEN

$\left[\text{H}^+\right] =$ 0.000000251 (moles per litre) $\quad (0.000000251188..., 2.51\times10^{-7})$ ***A1***

Note: Award ***M1A0*** for an answer of 2.51 $(2.51188...)$ seen.
Award ***M0A0*** if a substitution of 6.6 into the formula is seen without an answer or some indication of using numerical solver.

[2 marks]

(c) $2 = -\log_{10}\left[\text{H}^+\right],\ 4.5 = -\log_{10}\left[\text{H}^+\right]$

10^{-2} (0.01) **OR** $10^{-4.5}$ $(0.0000316227...)$ ***(A1)***

substitution of their values into correct ratio ***(M1)***

$\dfrac{10^{-2}}{10^{-4.5}}$ **OR** $\dfrac{0.01}{0.0000316227...}$

$= 316.227... = 316$ ***A1***

Note: Some candidates may subtract logs and hence look to solve $\log_{10}[\text{H}^+] = 2.5$.

[3 marks]
[Total: 7 marks]

2. (a) 120 (ng) ***A1***

Note: Only accept an exact answer of 120.

[1 mark]

(b) $\left(m(20\,000) = \right) 120\,\mathrm{e}^{-0.000121 \times 20\,000}$ ***(A1)***

10.7 $(10.6705\ldots..)$ (ng) ***A1***

[2 marks]

(c) attempt to write or solve an inequality (or equality) with a value of 60 seen ***(M1)***

$\left(m(t) = \right) 120\,\mathrm{e}^{-0.000121t} \leq 60$ **OR** sketch **OR** table of values

$(t >)$ $5728.489\ldots.$ ***(A1)***

5729 (years) ***A1***

Note: Do not accept 5728 or 5730 for the final ***A1***.

[3 marks]

[Total: 6 marks]

6. (a) (i) $a = 30$ ***A1***

(ii) $p = 11$ ***A1***

[2 marks]

(b) **EITHER**

substituting $n = 11$ and equating to 230 (or equivalent) ***(M1)***

$30(11) + b = 230$

OR

using an arithmetic sequence ***(M1)***

$200 + 30(n - 10)$

THEN

$b = -100$ ***A1***

[2 marks]

[Total 4 marks]

8. (a) $d = \frac{16.63 + 7.83}{2}$ ***(A1)***

$(d =)\ 12.2\ \ (12.23)$ ***A1***

[2 marks]

(b) $(a =)\ 4.4$ ***A1***

[1 mark]

(c) period $= 2(356 - 173)\ \ (= 366)$ ***(A1)***

$(b =)\ \frac{2\pi}{366} = \frac{\pi}{183}\ (= 0.0171671...)$ (Accept $b = \frac{360}{366}\ (= 0.983606\ldots)$) ***A1***

[2 marks]

(d) **EITHER**

attempt to find midpoint of x values of max and min ***(M1)***

$c = \frac{-10 + 173}{2}$

OR

substitute values and solve ***(M1)***

$b(173 - c) = \frac{\pi}{2}$ **OR** $16.63 = 4.4\ \sin\left(b(173 - c)\right) + 12.23$

THEN

$(c =)\ 81.5$ ***A1***

Note: Accept other possible positive values of $c = 81.5 + 366n$.
FT from their "c" consistent with their "a".

[2 marks]
[Total 7 marks]

10. (a) vertical stretch scale factor 4 ***A1***

Note: The vertical stretch can occur in any order.

EITHER

translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ followed by horizontal stretch scale factor $\frac{1}{2}$ ***A1A1***

OR

horizontal stretch scale factor $\frac{1}{2}$ followed by translation $\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$ ***A1A1***

Note: Award ***A1A0*** in either approach, for correct transformations, but incorrect order.

[3 marks]

(b) $g(2) = 5$ ***(A1)***

$f(5) = 7.78 \quad (= 7.78364\ldots,\ 4\ln 7)$ ***A1***

[2 marks]

(c) $4\ln(2g(x) - 3) = 2\ln 9$ ***(M1)***

$g(x) = 3$ ***(A1)***

$x = 1$ ***A1***

[3 marks]

[Total 8 marks]

12. (a)

Bounce number, n	Maximum height, h, according to the model
1	0.613
2	0.514
3	**0.431 (0.430988…)**
4	**0.361 (0.361383…)**

A1A1
[2 marks]

(b) $(0.430988... - 0.439)^2 + (0.361383... - 0.377)^2$ seen ***(A1)***

3.08×10^{-4} $(3.08055\ldots \times 10^{-4})$ ***A1***

[2 marks]

(c) (i) $h = 0.716 \times 0.851^n$ $(0.715766\ldots, 0.850774\ldots)$ ***A2***

Note: Award ***A2*** for $h = 0.716\,\mathrm{e}^{-0.162n}$ $(h = 0.715766\ldots \mathrm{e}^{-0.161608\ldots n})$
Award at most ***A1A0*** for correct coefficients, with any error in notation (e.g. x instead of n, missing $h=$)
Award at most ***A1A0*** for $h = 0.731 \times 0.838^n$ from using the values from the model.

(ii) $(h=)$ 0.716 (metres) ***A1***

[3 marks]
[Total 7 marks]

13. attempt to substitute $t = 9$ and $P = 433$ (**OR** $t = 12$ and $P = 954$) ***(M1)***

$433 = \frac{L}{1 + Ce^{-0.28(9)}}$ and $954 = \frac{L}{1 + Ce^{-0.28(12)}}$ ***(A1)(A1)***

attempt to rearrange into linear form or substitute for L

$433\left(1 + Ce^{-0.28(9)}\right) = L$ and $954\left(1 + Ce^{-0.28(12)}\right) = L$ **OR** $954 = \frac{433\left(1 + Ce^{-0.28(9)}\right)}{1 + Ce^{-0.28(12)}}$ ***(M1)***

attempt to solve simultaneously (e.g. graph or eliminate L) ***(M1)***

$C = 306.187\ldots$

$L = 11100$ $(11100.2\ldots)$ ***A1***

Note: Accept a correct decimal value.

[6 marks]

2. (a) $d = 5.75 \left(d = \frac{23}{4} \right)$ ***A1A1***

Note: Award ***A1*** for 5.75 seen, ***A1*** for completely correct equation.

[2 marks]

(b) attempt to substitute into the axis of symmetry formula using *their* 5.75 **OR**
attempt to substitute the coordinates $(3, 5.1)$ or $(8.5, 5.1)$ into the function ***(M1)***

$5.75 = \frac{-b}{2\times(-0.2)}$ **OR** $5.1 = -0.2(3)^2 + 3b$ **OR** $5.1 = -0.2(8.5)^2 + 8.5b$

$(b =)\ 2.3$ ***A1***

[2 marks]

(c) indication of finding zero or attempt to use symmetry ***(M1)***
$h(d) = 0$ **OR** $0 = -0.2d^2 + 2.3d$ **OR** 2×5.75 **OR** $3 - 0 = d - 8.5$ **OR**
resketch of graph with zero indicated

$(d =)\ 11.5\ (\text{m})$ ***A1***

Note: Award ***A0*** for a FT answer that is less than 8.5 (contradicts the given information).

[2 marks]

(d) **EITHER**
the dolphin is below the surface of the water / the dolphin is underwater / the dolphin is in the water ***R1***

OR

the value of h cannot be negative since the definition of the model only applies to the jump ***R1***

[1 mark]

[Total: 7 marks]

4. (a) $15p$ ***A1***

[1 mark]

(b) $(p-5)$ or $(n-15)$ seen ***A1***

adding a cost expression (multiplying a p and an n) to their $15p$ ***(M1)***

$C(n)=15p+(p-5)(n-15)$ **OR** $C(n)=np-5n+75$ (or equivalent) ***A1***

[3 marks]

(c) (i) $(25.73\times100=)\ 2573$ (EUR) ***A1***

Note: Answer must be exact.

(ii) correctly substituting *their* total cost and $n=100$ into *their* cost function ***(A1)***
$2573=15p+(p-5)(100-15)$

$(p=)\ 29.98$ (EUR) ***A1***

Note: Answer must be written to 2 dp.

[3 marks]

[Total: 7 marks]

6. (a) (i) $f(x) = 0.7x$ **OR** $f(x) = x - 0.3x$ ***A1***

(ii) $g(x) = x - 150$ ***A1***

[2 marks]

(b) attempt to discuss the order in which the reduction/discount are applied ***(M1)***

$(f \circ g)(x)$ describes when the 150 CAD reduction is applied before the 30% discount. ***A1***

[2 marks]

(c) **EITHER**

evidence of solving $(f \circ g)(x)$ and $(g \circ f)(x)$ for a price at 450 CAD ***R1***

OR

$(f \circ g)(x) = 0.7x - 105$ and $(g \circ f)(x) = 0.7x - 150$ seen ***R1***

OR

stating that it is better for 30% to be deducted from a higher price ***R1***

OR

the numbers 165, 210 are seen ***R1***

THEN

$(g \circ f)(x)$ is better for the customer ***A1***

Note: Do not award ***R0A1***.

[2 marks]

[Total 6 marks]

10. (a) 3

A1

[1 mark]

(b) 1

A1

[1 mark]

(c)

x	0	1	2	3	4
$f^{-1}(x)$	2	1	4	0	3

award ***A1*** for 1 correct, ***A2*** for 3 correct, ***A3*** for all 5 correct ***A1A1A1***

[3 marks]
Total [5 marks]

11. (a) $0,\ 1,\ 3$ ***A2***

Note: Award ***A1*** if either 1 or 3 is seen. Award ***A1*** for 0, 1000, 3000

[2 marks]

(b) sketching a graph or using calculus (***M1***)

attempting to find the y coordinate at the local maximum (***M1***)

local max occurs at $(0.451416..., 0.631130...)$ (***A1***)

$w > 0.631$ ***A1***

[4 marks]

Total [6 marks]

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Q10. (a) (i) $F = \frac{k}{d^2}$ (***A1)***

substitution of d and F into their equation (***M1)***

$40000 = \frac{k}{7000^2}$

$k = 1.96 \times 10^{12}$

$F = \frac{1.96 \times 10^{12}}{d^2}$ ***A1***

(ii) $F = \frac{1.96 \times 10^{12}}{3500^2}$

160000 (N) ***A1***

[4 marks]

(b) (i) C ***A1***

D ***A1***

Note: Award at most ***A1A0*** if an additional incorrect answer is given.

(ii) for C the gradient is $k = 1.96 \times 10^{12}$ ***A1***

for D the gradient is -2 ***A1***

[4 marks]

Total [8 marks]

Q2.

(a) $\frac{360}{12}$ ***M1***

$= 30$ ***AG***

Note: Award ***M0*** for use of given value, 30.

[1 mark]

(b) substitution of $t = 5$ into model $\left(-2.5\cos(30 \times 5) + 4.5\right)$ ***(M1)***

$h(5) = 6.67\left(6.66506...\right)$ ***A1***

Note: Award ***M1A0*** for $2.75187...$ (radian answer).

[2 marks]

(c) (i) 2.5 (m) ***A1***

Note: Award ***A0*** for -2.5 (m)

(ii) $h(t) = 4.5$ ***A1A1***

Note: Award ***A1*** for an equation of a horizontal line $h =$ constant (accept $y =$ constant), ***A1*** for that constant being 4.5. Second ***A1*** is dependent on first ***A1***. Do not award ***A0A1***.

[3 marks]

(d) attempt at finding point of intersection between $h(t) = \text{constant}$ and the curve ***(M1)***
using $h(t) = 2.55$ or $h(t) + 0.1 = 2.65$ to find intersection ***(A1)***

13.3 $(13.2913\ldots)$ ***(A1)***
$13{:}17$ (1:17 p.m.) ***A1***

Note: The final ***A1*** can be awarded for correctly converting **their** value of t into a time.

[4 marks]

continued…

Question 2 continued.

(e) using $h(t) = 2.75$ or $h(t) - 0.1 = 2.65$ to find intersection ***(A1)***

$22.4809\ldots - 13.2913... - 0.5$ ***(A1)(M1)(M1)***

Note: Award ***A1*** for $22.4809\ldots$ seen, ***M1*** for subtracting their two times for harbour, ***M1*** for subtracting 0.5 from their answer at any stage of the work.

8.69 (hours) ($8.68958\ldots$ (hours)) ***A1***

Note: Accept minutes used in working, but the final answer must be in hours.

[5 marks]
Total [15 marks]

2. (a) 310 million is the maximum no of EVs ***A1***

OR

the carrying capacity is 310 million ***A1***

OR

the long term behaviour of the system approaches 310 million ***A1***

Note: 'Million' needs to be seen to award the ***A1.***

[1 mark]

(b) (i) correct substitution $0.2 = \frac{310}{1+C\mathrm{e}^{-k}}$ ***A1***

correct rearrangement which leads to correct final answer ***A1***

eg $1+C\mathrm{e}^{-k} = 1550$ OR $C\mathrm{e}^{-k} = 1549$ OR $0.2C\mathrm{e}^{-k} = 309.8$

$C = 1549\mathrm{e}^{k}$ ***AG***

[2 marks]

(ii) correct substitution $3.1 = \frac{310}{1+C\mathrm{e}^{-7k}}$ ***A1***

correct first stage of rearrangement (***A1)***

eg $1+C\mathrm{e}^{-7k} = 100$ OR $3.1+3.1C\mathrm{e}^{-7k} = 310$

$C = 99\mathrm{e}^{7k}$ ***A1***

Note: Allow any correct expression eg $C = \frac{99}{\mathrm{e}^{-7k}}$ or $C = \frac{306.9}{3.1\mathrm{e}^{-7k}}$.

[3 marks]

continued…

Question 2 continued

(c) valid method for finding either variable ***(M1)***

eg graphically or elimination of C, $e^{6k} = \frac{1549}{99}$

(i) $k = 0.458\ (0.458374...)$ ***A1***

Note: Award at most ***M1A0*** if values other than at $t = 1$ or $t = 7$ are used.

[2 marks]

(ii) $C = 2450\ (2449.74...)$ ***A1***

Note: Do not award ***A1FT*** on a wrong value for k if the method used is not clearly shown.

Note: Accept $C = 2440\ (2443.33...)$ from substitution of 3 sf value for k in $C = 99e^{7k}$

Note: Award ***A0*** if values other than at $t = 1$ or $t = 7$ are used.

[1 mark]

$\left(N = \frac{310}{1 + 2449.74...e^{-0.458374...t}} \right)$

(d) substitution of $t = 4$ into their logistic function

$N = \frac{310}{1 + 2449.74...e^{-0.458374... \times 4}}$ ***(M1)***

$a = 0.789606...$

Note: If using 3 sf values for k and C: $a = 0.788346...$. If using $C = 2440$ then $a = 0.792$.

$a = 0.8$ ***A1***

Note: The answer must be given correct to 1 decimal place

[2 marks]

(e) calculation of a squared residual ***(M1)***

$0^2 + 0.1^2 + 0.2^2 + 0.1^2 + 0.1^2$

sum of square residuals $= 0.07$ ***A1***

[2 marks]

continued…

Question 2 continued

(f) substitution of their values into the error function ***M1***

Note: For the ***M1*** allow any integer value of n being substituted, with their sum of square residuals.

$\left(E = \sqrt{\frac{0.07}{5}}\right) = 0.118\ (0.118321...)$ ***A1***

$0.118 < 0.25$ (so can use the model) ***R1***

[3 marks]

(g) number in 2035 is given by value of N when $t = 20$ ***(A1)***

247 million $(246.873\ldots)$ ***A1***

Note: Accept 246 million from using 3 sf values for k and C.

Award ***(A1)A0*** for 247 or 246 without 'million'.

If using $t = 19$ award ***(A0)A1*** for 220 or 221 million. If using $t = 21$ award ***(A0)A1*** for 267 million.

[2 marks]

(h) multiplication of their **model** by one of 0.2 or 0.1 ***M1***

Note: For the ***M1*** ignore any extra incorrect multipliers.

number of public charging points needed in year ***t*** is

$$\frac{310}{1+2449.74...e^{-0.458374...t}} \times 0.2 \times 0.1 \left(= \frac{6.2}{1+2449.74...e^{-0.458374...t}}\right) \text{(million) OR}$$

$$\frac{310,000,000}{1+2449.74...e^{-0.458374...t}} \times 0.2 \times 0.1 \left(= \frac{6,200,000}{1+2449.74...e^{-0.458374...t}}\right)$$ ***A1***

[2 marks]

(i) $\left(\frac{0.54-0.22}{2} =\right) 0.16\ \text{(million)}\ (160,000)$ ***A1***

[1 mark]

continued…

Question 2 continued

(j) attempt to express number of charging points at t as a linear function ***(M1)***

linear function passes through $(5, 0.22)$ or $(7, 0.54)$ ***(A1)***

$0.22 + 0.16(t-5)$ OR $0.54 + 0.16(t-7)$ OR $0.16t - 0.58$ OR $160,000t - 580,000$ ***A1***

Note: Award at most ***(M1)(A1)A1*** for an expression that has the variable in years, for example $0.22 + 0.16(x - 2020)$.

attempt to solve their $\frac{6.2}{1 + 2449.74...e^{-0.458374...t}} > 0.22 + 0.16(t-5)$ ***M1***

Note: Accept $=$ in place of $>$.

$t = 15.1\ (15.1492...)$ ***(A1)***

Note: Accept $15.1665\ldots$ from use of 3 sf values for k and C.

Year is 2031 ***A1***

[6 marks]

Total [27 marks]

6. (a) recognizing the form of inverse variation ***(M1)***

$V = \frac{k}{t}$ **OR** $V \propto \frac{1}{t}$

$300 = \frac{k}{5}$ **OR** $k = 5 \times 300$ (the k must be shown) ***A1***

$V = \frac{1500}{t}$ ***AG***

Note: Award ***M0A0*** for only verifying that 300 and 5 satisfy given equation, e.g $300 = \frac{1500}{5}$ seen.

[2 marks]

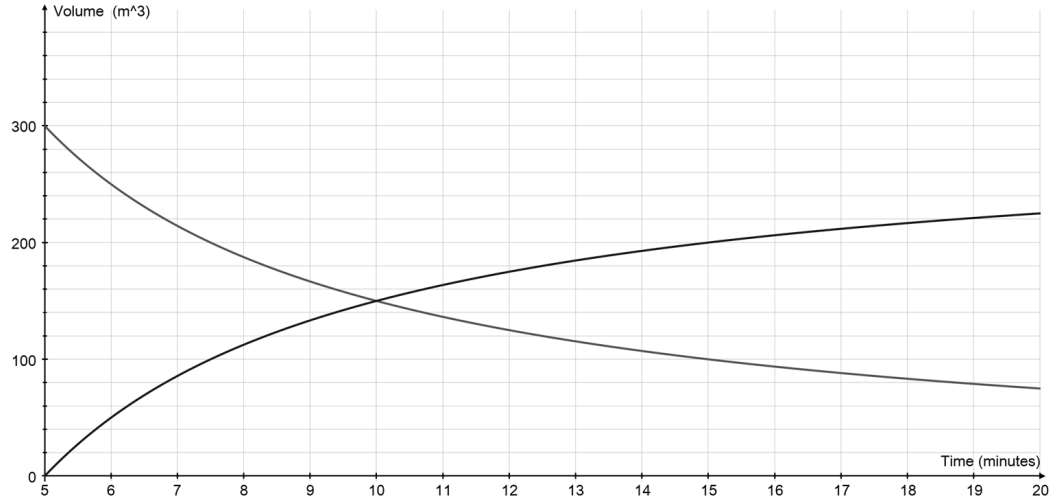
(b) 150 seen ***(A1)***

$150 = \frac{1500}{t}$

$t = 10$ (minutes) ***A1***

[2 marks]

(c)



A1A1

Note: Award ***A1*** for an **increasing curve** from point $(5, 0)$. Award ***A1*** for a smooth curve, with approximately correct shape, eg concave down, approximately passing through $(10,150)$. Accept any endpoint in the range $[150, 300]$.

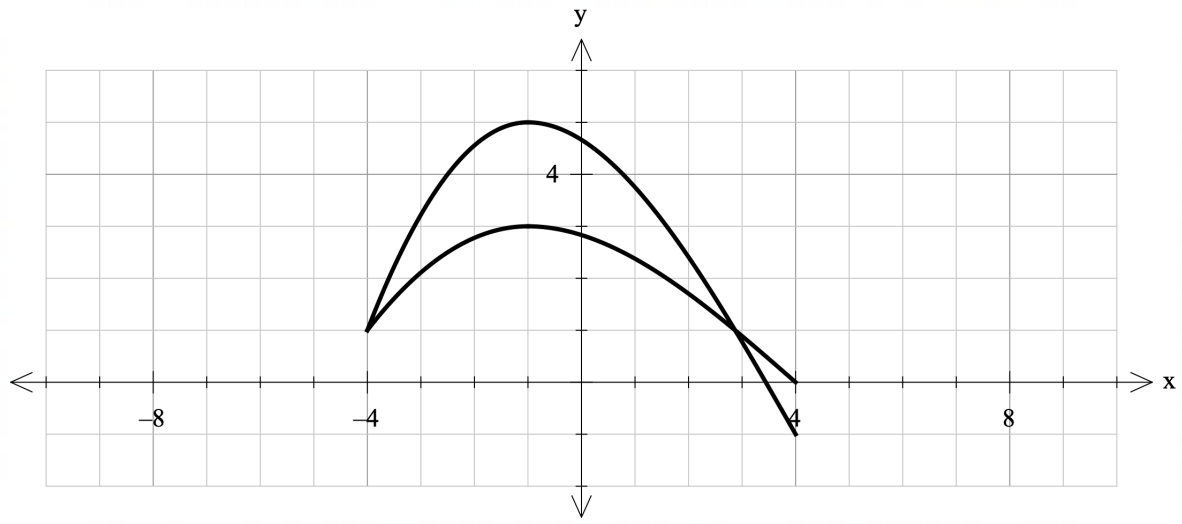
[2 marks]

(d) $(P(t) =) 300 - \frac{1500}{t}$ **OR** $300 - V(t)$ ***A1***

[1 mark]

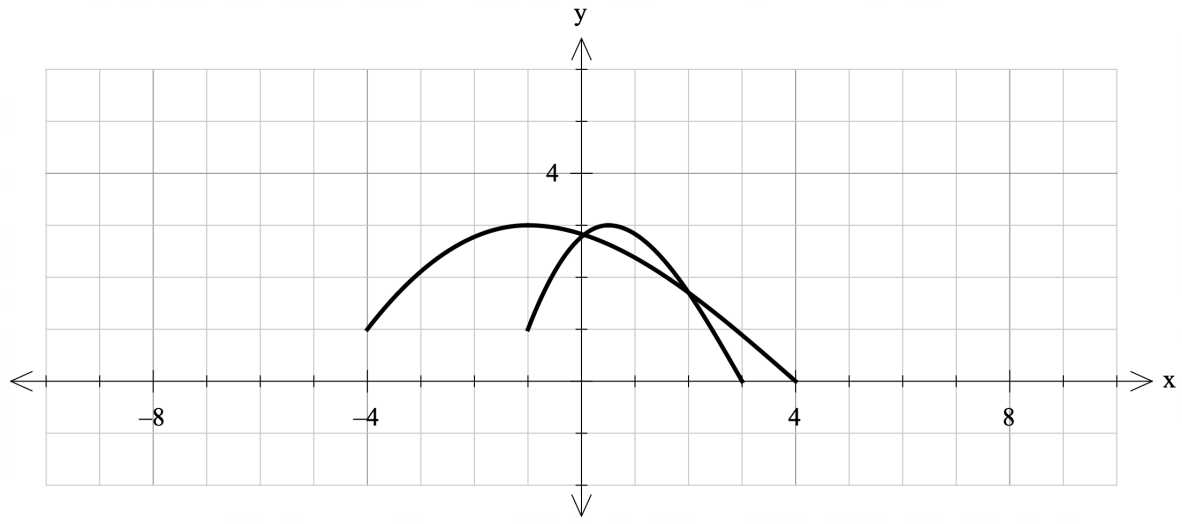
Total [7 marks]

8. (a)



correct maximum of the graph at $(-1, 5)$ ***A1***
correct endpoints at $(-4,\ 1)$ and $(4,\ -1)$ ***A1***
[2 marks]

(b)



maximum at $(\frac{1}{2}, 3)$ ***A1***

Note: Allow ***A1*** if x coordinate of maximum is in the interval $(0, 1)$.

correct endpoints at $(-1,1)$ and $(3,0)$ ***A1***
[2 marks]
Total [4 marks]

3. (a) 4 ***A1***
[1 mark]

(b) (i) **EITHER**
Award ***M1*** for attempting to substitute in $(5,3.9)$ or $(10,4)$ or $(8,3.936)$
in $y = ax^2 + bx + c$ ***(M1)***

$3.9 = a \times 5^2 + b \times 5 + 4$
$-0.1 = 25a + 5b \quad (-0.02 = 5a + b)$ ***A1***

Note: Award at most ***M1A0*** for an equation that is not simplified (e.g. $3.9 = 25a + 5b + 4$).

OR
M1 for attempt at axis of symmetry at $x = 5$ ***(M1)***

$5 = \frac{-b}{2a}$ **OR** $10a + b = 0$ **OR** $b = -10a$ ***A1***

OR
M1 for an attempt to find derivative equated to 0. ***(M1)***
$2a \times 5 + b = 0$
$10a + b = 0$ **OR** $b = -10a$ ***A1***

(ii) $a = 0.004 \left(\frac{1}{250}\right),\ b = -0.04 \left(\frac{-1}{25}\right)$ ***A1A1***

[4 marks]

(c) (i) $\left(\frac{dy}{dx} =\right) 0.008x - 0.04 \quad \left(\frac{1}{125}x - \frac{1}{25}\right)$ ***A1A1***

Note: Award ***A1*** for each correct term. Award at most ***A1A0*** if additional terms are seen.

(ii) substituting 0 or 10 into their derivative ***(M1)***
(steepest gradient=) 0.04 ***A1***
[4 marks]

(d) (i) $r = 0.994 \ (0.994142...)$ ***A2***

(ii) (very) strong ***A1***
[3 marks]

(e) (0.01889 represents) the (increase in) deflection (in m) for each person (on the bridge). ***A1***
[1 mark]

(f) 0.1954 seen ***A1***

0.1 seen **OR** $\mathrm{R}(5,\ 3.8046)$ seen ***A1***

Jenny's model for the height of the walkway has the smaller estimate ***A1***

Note: The final ***A1*** is contingent on both preceding ***A1*** marks being awarded. Accept an equivalent statement that identifies $n = 0$ for the regression line, with comparison of deflections or bridge heights.

[3 marks]
Total [16 marks]

4. (a) attempt to solve $6 = 3\ln(5t + 1)$ ***(M1)***

$(t =)$ 1.28 (hours) $(1.27781 \ldots, \frac{e^2-1}{5})$ ***A1***

[2 marks]

(b) 9.02 $(9.02376 \ldots)$ ***A1***

[1 mark]

(c) attempt to eliminate d ***M1***

$p = 20\left(1 - e^{-0.1\times 3\ln(5t+1)}\right)$ ***A1***

use of log law(s) to simplify ***(M1)***

$p = 20\left(1 - e^{\ln(5t+1)^{-0.3}}\right) = 20(1 - (5t + 1)^{-0.3})$

$p = 20\left(1 - \frac{1}{(5t+1)^{0.3}}\right)$ $(n = 0.3)$ ***A1***

Note: Award ***M0A0M0A1*** for working backwards from the given equation and substituting values of p and t to find $n = 0.3$.

[4 marks]

(d) 10.8 (points) $(10.8392\ldots)$ ***A1***

[1 mark]

(e) considers $t = 12$ ***(M1)***

$p = 14.2$ $(14.1731\ldots)$ ***A1***

Note: Award ***M1A0*** for $p = 15.2554 \ldots$ from using $t = 24$.
Award ***M1A0*** for $p = 20$ from considering the horizontal asymptote.

[2 marks]

(f) recognizes $120 = 7p + b$ ***(M1)***

recognizes $D = 7d$ ***(M1)***

$120 = 140\left(1 - \frac{1}{(5T+1)^{0.3}}\right) + 50\left(1 - \frac{1}{21\ln(5T+1)+1}\right)$ ***A1***

$T = 1.91$ (hours) $(1.91387\ldots)$ ***A1***

[4 marks]

Total [14 marks]

APOLLO

Geometry and trigonometry

101 questions

2. (a) every point in the shaded region is closer to tower T4 ***R1***

Note: Specific reference must be made to the closeness of tower T4.

[1 mark]

(b) $(-9, 1)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate. Accept $x = -9$ and $y = 1$.
Award at most ***A0A1*** if parentheses are missing.

[2 marks]

(c) correct use of gradient formula ***(M1)***

e.g. $(m =) \frac{5-3}{-9--13} \left(= \frac{1}{2}\right)$

taking negative reciprocal of **their** m (at any point) ***(M1)***

edge gradient $= -2$ ***A1***

[3 marks]

Total [6 marks]

5. (a) (i) $\overrightarrow{CA} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$ ***A1***

(ii) $\overrightarrow{CB} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

[2 marks]

(b) $\overrightarrow{CA} \times \overrightarrow{CB} = \begin{pmatrix} 0 \\ -6 \\ 24 \end{pmatrix}$ ***(M1)A1***

Note: Do not award ***(M1)*** if less than 2 entries are correct.

[2 marks]

(c) area is $\frac{1}{2}\sqrt{6^2 + 24^2} = 12.4 \text{ m}^2 \quad (12.3693..., 3\sqrt{17})$ ***(M1)A1***

[2 marks]

Total [6 marks]

6. attempt to find any relevant maximum value ***(M1)***
largest sides are 56.5 and 82.5 ***(A1)***
smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***
$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$

$= 2280 \text{ m}^2 \quad (2275.37...)$ ***A1***

Total [5 marks]

10. (a) (i) use of Prim's algorithm ***M1***

BC 46 ***A1***

BD 58 ***A1***

DE 23

EF 47

Total 174 ***A1***

Note: Award ***M0A0A0A1*** for 174 without correct working e.g. use of Kruskal's, or with no working.
Award ***M1A0A0A1*** for 174 by using Prim's from an incorrect starting point.

(ii) $AB + AC = 55 + 63 = 118$ ***(M1)***

$174 + 118 = 292$ minutes ***A1***

[6 marks]

(b) delete a different vertex ***A1***

[1 mark]

Total [7 marks]

13. (a) $\boldsymbol{r} = \begin{pmatrix} 1.3 \\ -0.3 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for each correct vector. Award ***A0A1*** if their "$\boldsymbol{r} =$" is omitted.

[2 marks]

(b) (i) $-0.3 + \lambda = 0$ ***(M1)***

$\Rightarrow \lambda = 0.3$

$\boldsymbol{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + 0.3 \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.4 \\ 0 \end{pmatrix}$ ***(M1)***

P has coordinates $(0.2, 0.4, 0)$ ***A1***

Note: Accept the coordinates of P in vector form.

(ii) $\sqrt{0.2^2 + 0.4^2}$ ***(M1)***

$= 0.447 \text{ km}$ $(=447 \text{ m})$ ***A1***

[5 marks]

Total [7 marks]

16. (a) $\boldsymbol{M} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$ ***A1A1A1***

Note: Award ***A1*** for each two correct rows.

[3 marks]

(b) calculating $\boldsymbol{M}^6$ ***(M1)***

143 ***A1***

[2 marks]

Total [6 marks]

1. (a) gradient $\mathrm{AB} = \frac{4}{12}\left(\frac{1}{3}\right)$ ***(A1)***

midpoint $\mathrm{AB}: (8, 22)$ ***(A1)***

gradient of bisector $= -\frac{1}{\text{gradient AB}} = -3$ ***(M1)***

perpendicular bisector: $22 = -3 \times 8 + b$ **OR** $(y - 22) = -3(x - 8)$ ***(M1)***

perpendicular bisector: $y = -3x + 46$ ***A1***

[5 marks]

(b) attempt to solve simultaneous equations ***(M1)***

$x + 4 = -3x + 46$

$(10.5, 14.5)$ ***A1***

[2 marks]

Total [7 marks]

4. (a) $\text{AC} = \frac{380}{\tan 25^\circ}$ **OR** $\text{AC} = \sqrt{\left(\frac{380}{\sin 25^\circ}\right)^2 - 380^2}$ **OR** $\frac{380}{\sin 25^\circ} = \frac{\text{AC}}{\sin 65^\circ}$ ***(M1)***

$\text{AC} = 815 \text{ m } (814.912\ldots)$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to find AB ***(M1)***

$\text{AB} = \frac{380}{\tan 40^\circ}$

$= 453 \text{ m } (452.866\ldots)$ ***(A1)***

$\text{BC} = 814.912\ldots - 452.866\ldots$

$= 362 \text{ m } (362.046\ldots)$ ***A1***

METHOD 2

attempt to find HB ***(M1)***

$\text{HB} = \frac{380}{\sin 40^\circ}$

$591 \text{ m } (= 591.175...)$ ***(A1)***

$\text{BC} = \frac{591.175... \times \sin 15^\circ}{\sin 25^\circ}$

$= 362 \text{ m } (362.046...)$ ***A1***

[3 marks]

(c) $362.046\ldots \times 4$

$= 1450 \text{ m h}^{-1} \ (1448.18\ldots)$ ***A1***

[1 mark]

Total [6 marks]

8. (a) setting a dot product of the direction vectors equal to zero ***(M1)***

$$\begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix} . \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix} = 0$$

$p(p+4)+8p-28=0$ ***(A1)***

$p^2+12p-28=0$

$(p+14)(p-2)=0$

$p=-14,\ p=2$ ***A1***

[3 marks]

(b) $p=-14 \Rightarrow$

$$L_1 : r = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -14 \\ -28 \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 4 \\ -7 \end{pmatrix}$$

a common point would satisfy the equations

$2-14\lambda = 14-10\mu$

$-5-28\lambda = 7+4\mu$ ***(M1)***

$-3+4\lambda = -2-7\mu$

METHOD 1

solving the first two equations simultaneously

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}$ ***A1***

substitute into the third equation: ***M1***

$$-3+4\left(-\frac{1}{2}\right) \neq -2+\frac{1}{2}(-7)$$

so lines do not intersect. ***R1***

Note: Accept equivalent methods based on the order in which the equations are considered.

METHOD 2

attempting to solve the equations using a GDC ***M1***

GDC indicates no solution ***A1***

so lines do not intersect ***R1***

[4 marks]

Total [7 marks]

11. (a) Odd vertices are A, B, D, H ***A1***

Consider pairings: ***M1***

Note: Award ***(M1)*** if there are four vertices not necessarily all correct.

AB DH has shortest route AD, DE, EB and DE, EH,
so repeated edges $(19+16+19)+(16+27)=97$

Note: Condone AB in place of AD, DE, EB giving $56+(16+27)=99$.

AD BH has shortest route AD and BE, EH,
so repeated edges $19+(19+27)=65$

AH BD has shortest route AD, DE, EH and BE, ED,
so repeated edges $(19+16+27)+(19+16)=97$ ***A2***

Note: Award ***A1*** if only one or two pairings are correctly considered.

so best pairing is AD, BH
weight of route is therefore $582+65=647$ ***A1***

[5 marks]

(b) least value of the pairings is 19 therefore repeat AD ***R1***

B and H ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [7 marks]

14. (a) $\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 6 \\ -2 \end{pmatrix}+\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$=\begin{pmatrix} 57 \\ 22 \end{pmatrix}$ **OR** $(57,\ 22)$ ***A1***

[2 marks]

(b) $\begin{pmatrix} 2p \\ 2q \end{pmatrix}=\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}+\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$7p-10q-5=2p$

$2p-3q+4=2q$ ***(A1)***

solve simultaneously:

$p=13,\ q=6$ ***A1***

Note: Award ***A0*** if 13 and 6 are not labelled or are labelled the other way around.

[3 marks]

(c) $\det\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}=-1$ $\left(\textbf{OR}\ \left|\det\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\right|=1\right)$ ***A1***

scale factor of image area is therefore $\left(\left|-1\right|=\right)1$ (and the translation does not affect the area)

A1

[2 marks]

Total [7 marks]

1. (a) use of cosine rule ***(M1)***

$$\hat{ACB} = \cos^{-1}\left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225}\right)$$ ***(A1)***

$= 32.0^\circ$ $(31.9980...)$ **OR** 0.558 $(0.558471...)$ ***A1***

[3 marks]

(b) use of sine rule ***(M1)***

$$\frac{\text{DE}}{\sin 31.9980...^\circ} = \frac{210}{\sin 100^\circ}$$ ***(A1)***

$(\text{DE} =)$ 113 m $(112.9937...)$ ***A1***

[3 marks]

(c) **METHOD 1**

$180^\circ - (100^\circ +$ their part $(a))$ ***(M1)***

$= 48.0019\ldots^\circ$ **OR** $0.837791...$ ***(A1)***

substituted area of triangle formula ***(M1)***

$$\frac{1}{2} \times 112.9937... \times 210 \times \sin 48.002^\circ$$ ***(A1)***

$8820\ \text{m}^2$ $(8817.18\ldots)$ ***A1***

METHOD 2

$$\frac{\text{CE}}{\sin(180 - 100 - \text{their part } (a))} = \frac{210}{\sin 100}$$ ***(M1)***

$(\text{CE} =)$ $158.472...$ ***(A1)***

substituted area of triangle formula ***(M1)***

EITHER

$$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$$ ***(A1)***

OR

$$\frac{1}{2} \times 210... \times 158.472... \times \sin(\text{their part } (a))$$ ***(A1)***

THEN

$8820\ \text{m}^2$ $(8817.18\ldots)$ ***A1***

continued…

Question 1 continued

METHOD 3

$\text{CE}^2 = 210^2 + 112.993...^2 - \left(2 \times 210 \times 112.993... \times \cos(180 - 100 - \text{their part } (a))\right)$ ***(M1)***

$(\text{CE} =)\ 158.472\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$ ***(A1)***

$8820\ \text{m}^2\ \ (8817.18\ldots)$ ***A1***

[5 marks]

(d) $1005 - 210$ **OR** 795 ***(A1)***

equating answer to part (c) to area of a triangle formula ***(M1)***

$8817.18\ldots = \frac{1}{2} \times \text{DF} \times (1005 - 210) \times \sin 48.002...^\circ$ ***(A1)***

$(\text{DF} =)\ 29.8\ \text{m}\ \ (29.8473...)$ ***A1***

[4 marks]

Total [15 marks]

4. (a) (the best placement is either point P or point Q)
attempt at using the distance formula ***(M1)***

$\text{AP} = \sqrt{(10-6)^2+(6-2)^2}$ **OR**

$\text{BP} = \sqrt{(10-14)^2+(6-2)^2}$ **OR**

$\text{DP} = \sqrt{(10-10.8)^2+(6-11.6)^2}$ **OR**

$\text{BQ} = \sqrt{(13-14)^2+(7-2)^2}$ **OR**

$\text{CQ} = \sqrt{(13-18)^2+(7-6)^2}$ **OR**

$\text{DQ} = \sqrt{(13-10.8)^2+(7-11.6)^2}$

(AP or BP or DP $=$) $\sqrt{32} = 5.66$ $(5.65685...)$ **AND**
(BQ or CQ or DQ $=$) $\sqrt{26} = 5.10$ $(5.09901...)$ ***A1***

$\sqrt{32} > \sqrt{26}$ **OR** AP (or BP or DP) is greater than BQ (or CQ or DQ) ***A1***

point P is the furthest away ***AG***

Note: Follow through from their values provided their AP (or BP or DP) is greater than their BQ (or CQ or DQ).

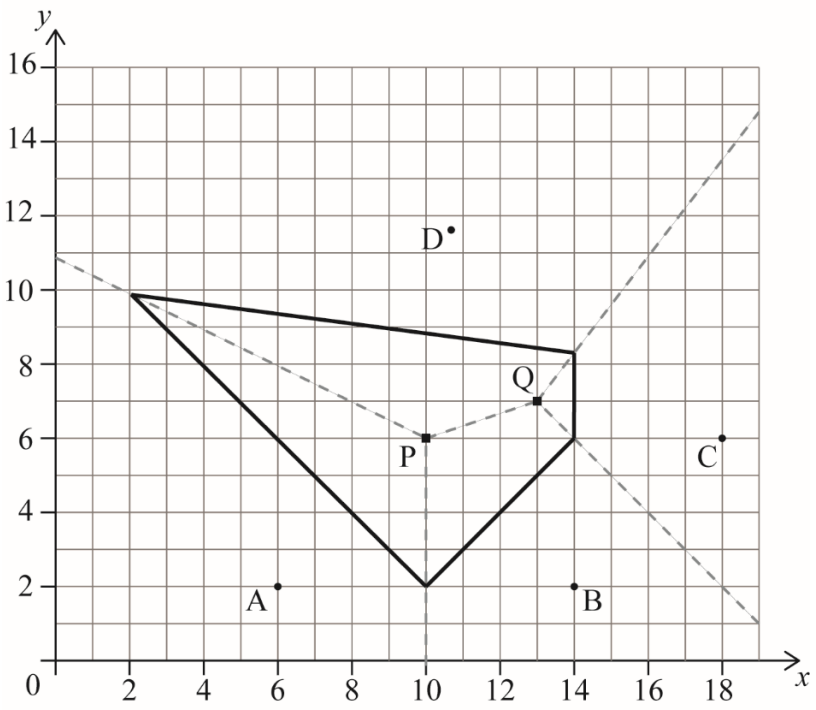
[3 marks]

continued…

Question 4 continued

(b) (i) $x = 14$ ***A1***

(ii)



A1A1

Note: Award ***A1*** for each correct straight line. Do not ***FT*** from their part b(i).

[3 marks]
Total: [6 marks]

11. METHOD 1

attempt to find AC using cosine rule ***M1***

$7^2 = 10^2 + \mathrm{AC}^2 - 2\times 10\times \mathrm{AC}\times \cos 40^\circ$ ***(A1)***

attempt to solve a quadratic equation ***(M1)***

$\mathrm{AC} = 4.888...$ **AND** $10.432...$ ***(A1)***

Note: At least $\mathrm{AC} = 4.888...$ must be seen, or implied by subsequent working.

minimum area = $\frac{1}{2}\times 10\times 4.888...\times \sin(40^\circ)$ ***M1***

Note: Do not award ***M1*** if incorrect value for minimizing the area has been chosen.

$= 15.7\ \mathrm{m}^2$ ***A1***

METHOD 2

attempt to find $\mathrm{A\hat{C}B}$ using the sine Rule ***M1***

$\frac{\sin C}{10} = \frac{\sin 40}{7}$ ***(A1)***

$C = 66.674...^\circ$ **OR** $113.325....^\circ$ ***(A1)***

EITHER

$B = 180 - 40 - 113.325...$

$B = 26.675...^\circ$ ***(A1)***

area $= \frac{1}{2}\times 10\times 7\times \sin(26.675...^\circ)$ ***M1***

OR

sine rule or cosine rule to find $\mathrm{AC} = 4.888...$ ***(A1)***

minimum area = $\frac{1}{2}\times 10\times 4.888...\times \sin(40^\circ)$ ***M1***

THEN

$= 15.7\ \mathrm{m}^2$ ***A1***

Note: Award ***A0M1A0*** if the wrong length AC or the wrong angle B selected but used correctly finding a value of $33.5\ \mathrm{m}^2$ for the area.

Total: [6 marks]

16. (a) $\vec{OS} = \begin{pmatrix} 300 \\ 100 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

[1 mark]

(b) attempt to find the vector from L to S ***(M1)***

$\vec{LS} = \begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

EITHER

$\left|\vec{LS}\right| = \sqrt{(171-12t)^2 + (15t-183)^2}$ ***(M1)(A1)***

minimize to find t on GDC ***(M1)***

OR

S closest when $\vec{LS} \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ ***(M1)***

$\left(\begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}\right) \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$

$-2052 + 144t - 2745 + 225t = 0$ ***(M1)(A1)***

OR

S closest when $\vec{LS} \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ ***(M1)***

$\vec{LS} = \begin{pmatrix} 5k \\ 4k \end{pmatrix}$

$\vec{OS} = \begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix} = \begin{pmatrix} 300 - 12t \\ 100 + 15t \end{pmatrix}$

Solving simultaneously ***(M1)***

THEN

$t = 13$ ***A1***

[6 marks]

(c) the alarm will sound ***A1***

$|\vec{LS}| = 19.2.... < 20$ ***R1***

Note: Do not award ***A1R0***.

[2 marks]

Total: [9 marks]

1. (a) $\tan(\theta) = \frac{6}{10}$ ***(M1)***

$(\theta =)\ 31.0^\circ\ (30.9637...^\circ)$ **OR** $0.540\ (0.540419\ldots)$ ***A1***

[2 marks]

(b) (i) $(\text{CV} =)\ 40\tan(\theta)$ **OR** $(\text{CV} =)\ 4 \times 6$ ***(M1)***

Note: Award ***(M1)*** for an attempt at trigonometry or similar triangles (e.g. ratios).

$(\text{CV} =)\ 24\ \text{m}$ ***A1***

(ii) $(V =)\ \frac{1}{3}80^2 \times 24 - \frac{1}{3}60^2 \times 18$ ***M1A1A1***

Note: Award ***M1*** for finding the difference between the volumes of two pyramids, ***A1*** for each correct volume expression. The final ***A1*** is contingent on correct working leading to the given answer.
If the correct final answer is not seen, award at most ***M1A1A0***. Award ***M0A0A0*** for any height derived from $V = 29\,600$, including 18.875 or 13.875.

$(V =)\ 29\,600\ \text{m}^3$ ***AG***

[5 marks]

(c) **METHOD 1**

$\left(\frac{29\,600}{80} =\right)\ 370$ (days) ***A1***

$(370 > 366)$ Joshua is correct ***A1***

Note: Award ***A0A0*** for unsupported answer of "Joshua is correct". Accept $1.01... > 1$ for the first ***A1*** mark.

METHOD 2

$80 \times 366 = 29280\ \text{m}^3$ **OR** $80 \times 365 = 29200\ \text{m}^3$ ***A1***

$(29280 < 29600)$ Joshua is correct ***A1***

Note: The second ***A1*** can be awarded for an answer consistent with their result.

[2 marks]

continued…

Question 1 continued

(d) height of trapezium is $\sqrt{10^2+6^2}$ $(=11.6619...)$ ***(M1)***

area of trapezium is $\frac{80+60}{2}\times\sqrt{10^2+6^2}$ $(=816.333...)$ ***(M1)(A1)***

$(SA=)\ 4\times\left(\frac{80+60}{2}\times\sqrt{10^2+6^2}\right)+60^2$ ***(M1)***

Note: Award ***M1*** for adding 4 times their (MNOP) trapezium area to the area of the (60×60) base.

$(SA=)\ 6870\ \text{m}^2\ \ (6865.33\ \text{m}^2)$ ***A1***

Note: No marks are awarded if the correct shape is not identified.

[5 marks]
Total: [14 marks]

4.

Note: For clarity, exact answers are used throughout this markscheme. However it is perfectly acceptable for candidates to write decimal values $\left(\text{e.g. } \frac{\sqrt{3}}{2} = 0.866\right)$.

(a) (i) rotation anticlockwise $\frac{\pi}{6}$ is $\begin{pmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ ***(M1)A1***

reflection in $y = \frac{x}{\sqrt{3}}$

$\tan\theta = \frac{1}{\sqrt{3}}$ ***(M1)***

$\Rightarrow 2\theta = \frac{\pi}{3}$ ***(A1)***

matrix is $\begin{pmatrix} 0.5 & 0.866 \\ 0.866 & -0.5 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ ***A1***

rotation clockwise $\frac{\pi}{3}$ is $\begin{pmatrix} 0.5 & 0.866 \\ -0.866 & 0.5 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ ***A1***

(ii) an attempt to multiply three matrices ***(M1)***

$\boldsymbol{P} = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ ***(A1)***

$\boldsymbol{P} = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$ **OR** $\begin{pmatrix} 0.866 & -0.5 \\ -0.5 & -0.866 \end{pmatrix}$ ***A1***

continued…

Question 4 continued

(iii) $\left(\boldsymbol{P}^2 = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} = \right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ***A1***

Note: Do not award ***A1*** if final answer not resolved into the identity matrix I.

(iv) if the overall movement of the drone is repeated ***A1***
the drone would return to its original position ***A1***

[12 marks]

(b) **METHOD 1**

$\left|\det \boldsymbol{P}\right| = \left|\left(-\frac{3}{4}\right) - \left(\frac{1}{4}\right)\right| = 1$ ***A1***

area of triangle ABC = area of triangle $\mathrm{A'B'C'} \times \left|\det \boldsymbol{P}\right|$ ***R1***

area of triangle ABC = area of triangle $\mathrm{A'B'C'}$ ***AG***

Note: Award at most ***A1R0*** for responses that omit modulus sign.

METHOD 2

statement of fact that rotation leaves area unchanged ***R1***
statement of fact that reflection leaves area unchanged ***R1***
area of triangle ABC = area of triangle $\mathrm{A'B'C'}$ ***AG***

[2 marks]

(c) attempt to find angles associated with values of elements in matrix $\boldsymbol{P}$ ***(M1)***

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \cos\left(-\frac{\pi}{6}\right) & \sin\left(-\frac{\pi}{6}\right) \\ \sin\left(-\frac{\pi}{6}\right) & -\cos\left(-\frac{\pi}{6}\right) \end{pmatrix}$$

reflection (in $y = (\tan\theta)x$) ***(M1)***

where $2\theta = -\frac{\pi}{6}$ ***A1***

reflection in $y = \tan\left(-\frac{\pi}{12}\right)x \ \left(= -0.268x\right)$ ***A1***

[4 marks]
Total: [18 marks]

1. (a) e.g. ABCDEGHFA ***A1***

Note: Accept any other correct answers starting at any vertex.

(b) (i) 7 vertices, so 6 edges required for MST ***(M1)***

Note: To award ***(M1)***, their 6 edges should not form a cycle.

Order selected	Edge
1	BC
2	CD
3	DE
4	BH
5	GH
6	FG

M1
A1
A1

Note: Award ***M1*** for the first three edges in correct order, ***A1*** for BH in correct order and ***A1*** for all of the edges correct.

weight of MST = 33 ***A1***

Note: The final ***A1*** can be awarded independently of previous marks.

(ii) lower bound = $33+3+x$ ***(M1)***
$=36+x$ ***A1***

(c) (i) $p=13$ ***A1***

(ii) $q=17$ ***A1***

(iii) $r=14$ ***A1***

(d) (i) attempt to use nearest neighbour algorithm ***(M1)***

any two correct cycles from
ABCDEGHFA, AFGHBCDE(F)A, AB(A)FGHCDE(F)A ***A1A1***

Note: Bracketed vertices may be omitted in candidate's answer.
Award ***M1A0A1*** for candidates who list two correct sequences of vertices, but omit the final vertex A.

(ii) use ABCDEGHFA **OR** their shortest cycle from (d)(i) ***(M1)***
upper bound = 43 ***A1***

continued…

Question 1 continued

(e) (i) cycle starts: ABCDEGHF
return to A has two options, $FA = 18$ or x ***(M1)***
hence least value of $x = 19$ ***A1***

(ii) upper bound = 58 ***A2***

(f) recognition that edges will be repeated / there are odd vertices ***(M1)***
$BH + DG = 21,\ BD + GH = 15, BG + DH = 21$ **OR** $18 + x$ ***A1***
recognizing BD and GH is lowest weight and is repeated ***(M1)***
solution to CPP $= 107 + x$ ***A1***
$x = 13$ ***A1***

Note: Award ***M1A0M0A1A1*** if only pairing BD and GH is considered, leading to a correct answer.

4. (a) $\frac{3-1}{7-3}$ ***(M1)***

$= 0.5$ ***A1***

[2 marks]

(b) $y-2=-2(x-5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

5. (a) $\frac{50 \times \pi}{180} = 0.873\ (0.872664\ldots)$ ***A1***

[1 mark]

(b) volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

Note: Award ***M1*** $240 \times$ area , award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of the angles or their areas.

$= 45800\ (= 45811.96071)$ ***A1***

[4 marks]

Total [5 marks]

8. (a) $\frac{\sin \text{CAB}}{6} = \frac{\sin 15^\circ}{4.5}$ ***(M1)(A1)***

$\text{C}\hat{\text{A}}\text{B} = 20.2^\circ \ (20.187415\ldots)$ ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) $\text{C}\hat{\text{B}}\text{D} = 20.2 + 15 = 35.2^\circ$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots^\circ = \frac{1.8}{\text{BX}}$ ***(M1)***

Note: Award ***A1*** for their correct angle $\text{C}\hat{\text{B}}\text{D}$. Award ***M1*** for correctly substituted trigonometric formula.

$\text{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\text{m})\ (2.44714\ldots)$ ***A1***

[5 marks]

Total [8 marks]

11. (a) $15 \times 0 + 2d + 4 = 0$ ***(M1)***

$d = -2$ ***A1***

[2 marks]

(b) $a\begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ ***(M1)***

$= a\begin{pmatrix} 10 \\ 15 \\ 30 \end{pmatrix} \left(= 5a \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \right)$ ***A1***

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$ ***M1***

$a = \frac{14}{35} \ (= 0.4)$ ***A1***

[4 marks]

Total [6 marks]

4. (a) $\boldsymbol{r} = \begin{pmatrix} 10 \\ 5 \end{pmatrix} + t\begin{pmatrix} -50 \\ -20 \end{pmatrix}$ ***A1A1***

[2 marks]

(b) (i) when $x = 0,\ t = \frac{30}{150} = 0.2$ ***M1***

EITHER

when $y = 0,\ t = \frac{10}{150} = 0.2$ ***A1***

since the two values of t are equal the aircraft passes directly over the airport

OR

$t = 0.2,\ y = 0$ ***A1***

(ii) height $= 5 - 0.2 \times 20 = 1\text{km}$ ***A1***

(iii) time $13{:}12$ ***A1***

[4 marks]

(c) (i) $5 - 20t = 4 \Rightarrow t = \frac{1}{20}$ (3 minutes) ***(M1)***

time $13{:}03$ ***A1***

(ii) displacement is $\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ ***A1***

distance is $\sqrt{22.5^2 + 7.5^2 + 4^2}$ ***(M1)***

$= 24.1\text{km}$ ***A1***

[5 marks]

continued…

Question 4 continued

(d) ***METHOD 1***

time until landing is $12 - 3 = 9$ minutes ***M1***

height to descend $= 4\,\text{km}$

$a = \dfrac{-4}{\frac{9}{60}}$ ***M1***

$= -26.7$ ***A1***

METHOD 2

$\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} = s\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ ***M1***

$-150 = 22.5s \Rightarrow s = -\dfrac{20}{3}$ ***M1***

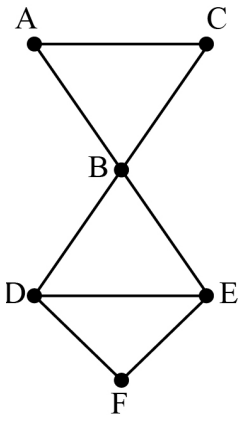
$a = -\dfrac{20}{3} \times 4$

$= -26.7$ ***A1***

[3 marks]

Total [14 marks]

5. (a)



A2

[2 marks]

(b) attempt to form an adjacency matrix ***M1***

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$ ***A1***

[2 marks]

(c) raising the matrix to the power six ***(M1)***

50 ***A1***

[2 marks]

(d) not possible ***A1***

because you must pass through B twice ***R1***

Note: Do not award ***A1R0.***

[2 marks]

(e) $a = 230,\ b = 340$ ***A1A1***

[2 marks]

(f) $\mathrm{A} \to \mathrm{B} \to \mathrm{D} \to \mathrm{E} \to \mathrm{F} \to \mathrm{C} \to \mathrm{A}$ ***(M1)***

$90 + 70 + 100 + 210 + 330 + 150$ ***(A1)***

(US$) 950 ***A1***

[3 marks]

continued…

Question 5 continued

(g) finding weight of minimum spanning tree ***M1***

$70 + 80 + 100 + 180 = (\text{US\$})\ 430$ ***A1***

adding in two edges of minimum weight ***M1***

$430 + 90 + 150 = (\text{US\$})\ 670$ ***A1***

[4 marks]

Total [17 marks]

6. (a) $\begin{pmatrix} -4.5 \\ 6.1 \end{pmatrix}$ ***A1***

[1 mark]

(b) $\sqrt{(-3.2)^2 + (-4.5)^2 + 6.1^2}$ ***(M1)***

$8.22800\ldots \approx 8.23 \text{ m}$ ***A1***

[2 marks]

(c) **EITHER**

$\vec{AO} = \begin{pmatrix} -3.2 \\ -4.5 \\ 0.3 \end{pmatrix}$ ***A1***

$\cos\theta = \dfrac{\vec{AO} \bullet \vec{AF}}{\left|\vec{AO}\right|\left|\vec{AF}\right|}$

$\vec{AO} \bullet \vec{AF} = (-3.2)^2 + (-4.5)^2 + (0.3 \times 6.1) \ (= 32.32)$ ***(A1)***

$\cos\theta = \dfrac{32.32}{\sqrt{3.2^2 + 4.5^2 + 0.3^2} \times 8.22800...}$ ***(M1)***

$= 0.710326\ldots$ ***(A1)***

Note: If $\vec{OA}$ is used in place of $\vec{AO}$ then $\cos\theta$ will be negative.
Award ***A1(A1)(M1)(A1)*** as above. In order to award the final ***A1***, some justification for changing the resulting obtuse angle to its supplementary angle **must** be seen.

OR

$AO = \sqrt{3.2^2 + 4.5^2 + 0.3^2} \ (= 5.52991...)$ ***(A1)***

$\cos\theta = \dfrac{8.22800...^2 + 5.52991...^2 - 5.8^2}{2 \times 8.22800... \times 5.52991...}$ ***(M1)(A1)***

$= 0.710326\ldots$ ***(A1)***

THEN

$\theta = 0.780833... \approx 0.781$ **OR** $44.7384\ldots^\circ \approx 44.7^\circ$ ***A1***

[5 marks]
[Total 8 marks]

13. (a) $(\boldsymbol{r} =) \begin{pmatrix} 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1.2 \\ -0.6 \end{pmatrix}$ ***A1***

Note: Do not condone the use of λ or any other variable apart from t.

[1 mark]

(b) when the bearing from the port is 045°, the distance east from the port is equal to the distance north from the port ***(M1)***

$1 + 1.2t = 4 - 0.6t$ ***(A1)***

$1.8t = 3$

$t = \frac{5}{3}$ $\left(1.6666..., 1 \text{ hour } 40 \text{ minutes}\right)$ ***(A1)***

time is $2{:}40$ pm $(14{:}40)$ ***A1***

[4 marks]

[Total 5 marks]

2. (a) $\sin\theta = \frac{2.1}{2.8}$ **OR** $\tan\theta = \frac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)\ 48.6\ °\quad (48.5903\ldots°)$ ***A1***

[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\frac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

Note: Award ***M1*** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

$1.85\ \text{m}\quad (1.85202\ldots)$ ***(A1)***

Note: Award the ***M1A1*** if 1.85 is seen in part (a).

$(6.4 - 1.85202\ldots)$
$4.55\ \text{m}\quad (4.54797\ldots)$ ***(A1)***

Note: Award ***A1*** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$
$5.01\ \text{m}\quad (5.00939\ldots\text{m})$ ***A1***

METHOD 2
attempt to use cosine rule ***(M1)***
$(c^2 =)\ 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

Note: Award ***A1*** for $48.5903...°$ substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01\ \text{m}\quad (5.00939\ldots\text{m})$ ***A1***

[4 marks]

(c) camera 1 is closer to the cash register than camera 2 (and both cameras are at the same height on the wall) ***R1***
the larger angle of depression is from camera 1 ***A1***

Note: Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept "$1.85<4.55$" or "$2.8<5.01$" as evidence for the ***R1***.

[2 marks]
Total [8 marks]

6. (a)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$ ***A2***

Note: Award ***A2*** for the transposed matrix. Presentation in markscheme assumes columns/rows ordered A-E; accept a matrix with rows and/or columns in a different order only if appropriately communicated. Do not ***FT*** from part (a) into part (b).

[2 marks]

(b) raising their matrix to a power of 5 ***(M1)***

$$\boldsymbol{M}^5 = \begin{pmatrix} 17 & 9 & 2 & 3 & 5 \\ 17 & 10 & 3 & 4 & 4 \\ 13 & 6 & 2 & 2 & 4 \\ 8 & 5 & 1 & 2 & 2 \\ 18 & 11 & 2 & 4 & 5 \end{pmatrix}$$ ***(A1)***

Note: The numbers along the diagonal are sufficient to award ***M1A1***.

(the required number is $17+10+2+2+5=)\ 36$ ***A1***

[3 marks]
Total [5 marks]

8. (a) $\pi \times 2^2 \times \frac{30}{360}$ ***(M1)***

$= 1.047 \text{ cm}^2$ ***A1***

Note: Do not award the final mark if the answer is not correct to 4 sf.

[2 marks]

(b) attempt to substitute any two values from $1.5, 2.5, 25$ or 35 into area of sector formula ***(M1)***

$\left(\text{upper bound} = \pi \times 2.5^2 \times \frac{35}{360} = \right) \ 1.91 \ \text{cm}^2 \ \ (1.90895\ldots)$ ***A1***

$\left(\text{lower bound} = \pi \times 1.5^2 \times \frac{25}{360} = \right) \ 0.491 \ \text{cm}^2 \ \ (0.490873\ldots)$ ***A1***

Note: Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers.

[3 marks]

(c) $\left(\left|\frac{1.047 - 1.90895\ldots}{1.90895\ldots}\right| \times 100 = \right) \ 45.2 \ (\%) \ \ (45.1532\ldots)$ ***A1***

$\left(\left|\frac{1.047 - 0.490873\ldots}{0.490873\ldots}\right| \times 100 = \right) \ 113 \ (\%) \ \ (113.293\ldots)$ ***A1***

so the largest percentage error is $113\ \%$ ***A1***

Note: Accept $45.1\ (\%)\ (45.1428)$, from use of full accuracy answers. Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers. Award ***A0A1A0*** if 113% is the only value found.

[3 marks]

Total [8 marks]

15. (a) (one vector to the line is $\begin{pmatrix}0\\c\end{pmatrix}$ therefore) $\boldsymbol{a}=\begin{pmatrix}0\\c\end{pmatrix}$ ***A1***

the line goes m up for every 1 across

(so the direction vector is) $\boldsymbol{b}=\begin{pmatrix}1\\m\end{pmatrix}$ ***A1***

Note: Although these are the most likely answers, many others are possible.

[2 marks]

(b) (from GDC **OR** $6\times2-4\times3$) $|M|=0$ ***A1***

[1 mark]

(c) **METHOD 1**

$\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}6x+3mx+3c\\4x+2mx+2c\end{pmatrix}$ ***M1A1***

$=\begin{pmatrix}3(2x+mx+c)\\2(2x+mx+c)\end{pmatrix}$ ***A1***

therefore the new line has equation $3Y=2X$ ***A1***

which is independent of m or c ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

METHOD 2

take two points on the line, e.g $(0,\ c)$ and $(1,\ m+c)$ ***M1***

these map to $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}0\\c\end{pmatrix}=\begin{pmatrix}3c\\2c\end{pmatrix}$

and $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}1\\m+c\end{pmatrix}=\begin{pmatrix}6+3m+3c\\4+2m+2c\end{pmatrix}$ ***A1***

therefore a direction vector is $\begin{pmatrix}6+3m\\4+2m\end{pmatrix}=(2+m)\begin{pmatrix}3\\2\end{pmatrix}$

(since $m\neq-2$) a direction vector is $\begin{pmatrix}3\\2\end{pmatrix}$

the line passes through $\begin{pmatrix}3c\\2c\end{pmatrix}-c\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ therefore it always has the origin as a jump-on vector ***A1***

the vector equation is therefore $\boldsymbol{r}=\mu\begin{pmatrix}3\\2\end{pmatrix}$ ***A1***

which is independent of m or c ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

continued…

Question 15 continued

METHOD 3

$\boldsymbol{r} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix}\right) = \begin{pmatrix} 3c \\ 2c \end{pmatrix} + \lambda \begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix}$ ***M1A1***

$= c\begin{pmatrix} 3 \\ 2 \end{pmatrix} + (2+m)\lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ***A1***

$= \mu \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

where $\mu = c + (2+m)\lambda$ is an arbitrary parameter. ***A1***

which is independent of m or c (as μ can take any value) ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

[4 marks]
Total [7 marks]

16. (a) attempt at chain rule ***(M1)***

$$\left(\boldsymbol{v} = \frac{\mathrm{d}OP}{\mathrm{d}t} = \right) \begin{pmatrix} 2t\cos t^2 \\ -2t\sin t^2 \end{pmatrix}$$ ***A1***

[2 marks]

(b) attempt at product rule ***(M1)***

$$\boldsymbol{a} = \begin{pmatrix} 2\cos t^2 - 4t^2\sin t^2 \\ -2\sin t^2 - 4t^2\cos t^2 \end{pmatrix}$$ ***A1***

METHOD 1

let $S = \sin t^2$ and $C = \cos t^2$

finding $\cos\theta$ using

$\boldsymbol{a} \bullet \vec{\mathrm{OP}} = 2SC - 4t^2S^2 - 2SC - 4t^2C^2 = -4t^2$ ***M1***

$|\vec{\mathrm{OP}}| = 1$

$|\boldsymbol{a}| = \sqrt{\left(2C - 4t^2S\right)^2 + \left(-2S - 4t^2C\right)^2}$

$= \sqrt{4 + 16t^4} > 4t^2$

if θ is the angle between them, then

$\cos\theta = -\dfrac{4t^2}{\sqrt{4 + 16t^4}}$ ***A1***

so $-1 < \cos\theta < 0$ therefore the vectors are never parallel ***R1***

continued…

Question 16 continued

METHOD 2

solve

$$\begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \end{pmatrix} = k \begin{pmatrix} \sin t^2 \\ \cos t^2 \end{pmatrix} \quad \textbf{M1}$$

then

$$(k =)\ \frac{2\cos t^2 - 4t^2 \sin t^2}{\sin t^2} = \frac{-2\sin t^2 - 4t^2 \cos t^2}{\cos t^2}$$

Note: Condone candidates not excluding the division by zero case here. Some might go straight to the next line.

$$2\cos^2 t^2 - 4t^2 \cos t^2 \sin t^2 = -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2$$

$$2\cos^2 t^2 + 2\sin^2 t^2 = 0$$

$$2 = 0 \quad \textbf{A1}$$

this is never true so the two vectors are never parallel **R1**

METHOD 3

embedding vectors in a 3d space and taking the cross product: **M1**

$$\begin{pmatrix} \sin t^2 \\ \cos t^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2 - 2\cos^2 t^2 + 4t^2 \cos t^2 \sin t^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix} \quad \textbf{A1}$$

since the cross product is never zero, the two vectors are never parallel **R1**

[5 marks]

Total [7 marks]

2. (a) (i) $\left(\frac{1}{2}\text{A}\hat{\text{O}}\text{B} =\right) \arccos\left(\frac{4}{4.5}\right) = 27.266...$ ***(M1)(A1)***

$\text{A}\hat{\text{O}}\text{B} = 54.532... \approx 54.5^\circ \ (0.951764... \approx 0.952 \text{ radians})$ ***A1***

Note: Other methods may be seen; award ***(M1)(A1)*** for use of a correct trigonometric method to find an appropriate angle and then ***A1*** for the correct answer.

(ii) a finding area of triangle

EITHER

area of triangle = $\frac{1}{2} \times 4.5^2 \times \sin(54.532...)$ ***(M1)***

Note: Award ***M1*** for correct substitution into formula.

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

OR

$\text{AB} = 2 \times \sqrt{4.5^2 - 4^2} = 4.1231\ldots$ ***(M1)***

area triangle $= \frac{4.1231\ldots \times 4}{2}$

$= 8.24621... \approx 8.25 \ (\text{m}^2)$ ***(A1)***

finding area of sector

EITHER

area of sector = $\frac{54.532...}{360} \times \pi \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

OR

area of sector = $\frac{1}{2} \times 0.9517641.. \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

THEN

area of segment = $9.63661... - 8.24621...$

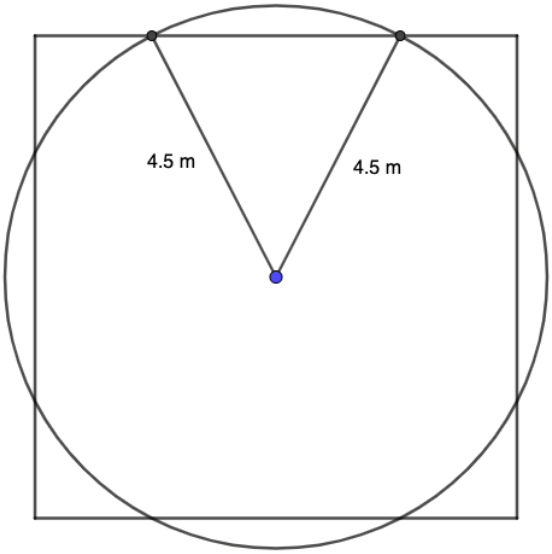
$= 1.39 \text{ m}^2 \ (1.39040...)$ ***A1***

[8 marks]

continued…

Question 2 continued

(b) **METHOD 1**



$\pi \times 4.5^2$ $(63.6172\ldots)$ ***(A1)***

$4 \times 1.39040...$ (5.56160) ***(A1)***

subtraction of four segments from area of circle ***(M1)***

$= 58.1 \text{ m}^2$ $(58.055...)$ ***A1***

METHOD 2

angle of sector = $90 - 54.532...$ $\left(\frac{\pi}{2} - 0.951764...\right)$ ***(A1)***

area of sector = $\frac{90 - 54.532...}{360} \times \pi \times 4.5^2$ $(= 6.26771...)$ ***(A1)***

area is made up of four triangles and four sectors ***(M1)***

total area = $(4 \times 8.2462...) + (4 \times 6.26771...)$

$= 58.1 \text{ m}^2$ $\left(58.055...\right)$ ***A1***

[4 marks]

(c) sketch of $\frac{\mathrm{d}V}{\mathrm{d}t}$ **OR** $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.110363...$ **OR** attempt to find where $\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = 0$ ***(M1)***

$t = 1$ hour ***A1***

[2 marks]

(d) recognizing $V = \int \frac{\mathrm{d}V}{\mathrm{d}t}\,\mathrm{d}t$ ***(M1)***

$\int_0^8 0.3te^{-t}\mathrm{d}t$ ***(A1)***

volume eaten is $0.299... \text{ m}^3$ $(0.299094...)$ ***A1***

[3 marks]

[Total 17 marks]

7. (a) (i) $\boldsymbol{P}\begin{pmatrix}0\\0\end{pmatrix}+\boldsymbol{q}=\begin{pmatrix}0\\1\end{pmatrix}$ ***(M1)***

$\boldsymbol{q}=\begin{pmatrix}0\\1\end{pmatrix}$ ***A1***

(ii) **EITHER**

$\boldsymbol{P}\begin{pmatrix}1\\0\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3}}{4}\\\frac{3}{4}\end{pmatrix}$ ***M1***

hence $\boldsymbol{P}\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}\frac{\sqrt{3}}{4}\\-\frac{1}{4}\end{pmatrix}$ ***A1***

$\boldsymbol{P}\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\frac{1}{4}\\1+\frac{\sqrt{3}}{4}\end{pmatrix}$ ***M1***

hence $\boldsymbol{P}\begin{pmatrix}0\\1\end{pmatrix}=\begin{pmatrix}\frac{1}{4}\\\frac{\sqrt{3}}{4}\end{pmatrix}$ ***A1***

continued…

Question 7 continued

OR

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}+\begin{pmatrix} 0 \\ 1 \end{pmatrix}=\begin{pmatrix} \frac{\sqrt{3}}{4} \\ \frac{3}{4} \end{pmatrix}$$ ***M1***

hence $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}=\begin{pmatrix} \frac{\sqrt{3}}{4} \\ -\frac{1}{4} \end{pmatrix}$ ***A1***

$$\begin{pmatrix} a \\ c \end{pmatrix}=\begin{pmatrix} \frac{\sqrt{3}}{4} \\ -\frac{1}{4} \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}+\begin{pmatrix} 0 \\ 1 \end{pmatrix}=\begin{pmatrix} \frac{1}{4} \\ 1+\frac{\sqrt{3}}{4} \end{pmatrix}$$ ***M1***

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}=\begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{3}}{4} \end{pmatrix}$$ ***A1***

$$\begin{pmatrix} b \\ d \end{pmatrix}=\begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{3}}{4} \end{pmatrix}$$

THEN

$$\Rightarrow \boldsymbol{P}=\begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix}$$ ***AG***

[6 marks]

(b) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ ***A1***

[1 mark]

continued…

Question 7 continued

(c) (i) **EITHER**

$\boldsymbol{S}^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ ***(A1)***

$\boldsymbol{R} = \boldsymbol{P}\boldsymbol{S}^{-1}$ ***(M1)***

Note: The ***M1*** is for an attempt at rearranging the matrix equation. Award even if the order of the product is reversed.

$\boldsymbol{R} = \begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$ ***(A1)***

OR

$\begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix} = \boldsymbol{R} \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$

let $\boldsymbol{R} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

attempt to solve a system of equations ***M1***

$\frac{\sqrt{3}}{4} = 0.5a, \quad \frac{1}{4} = 0.5b$

$-\frac{1}{4} = 0.5c, \quad \frac{\sqrt{3}}{4} = 0.5d$ ***A2***

Note: Award ***A1*** for two correct equations, ***A2*** for all four equations correct.

THEN

$\boldsymbol{R} = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ **OR** $\begin{pmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{pmatrix}$ **OR** $\left(\begin{pmatrix} 0.866025... & 0.5 \\ -0.5 & 0.866025... \end{pmatrix} \right)$ ***A1***

Note: The correct answer can be obtained from reversing the matrices, so do not award if incorrect product seen. If the given answer is obtained from the product $\boldsymbol{R} = \boldsymbol{S}^{-1}\boldsymbol{P}$, award ***(A1)(M1)(A0)A0***.

continued…

Question 7 continued

(ii) clockwise ***A1***
arccosine or arcsine of value in matrix seen ***(M1)***
30° ***A1***

Note: Both ***A1*** marks are dependent on the answer to part (c)(i) and should only be awarded for a valid rotation matrix.

[7 marks]

(d) **METHOD 1**

(i) $\begin{pmatrix} a \\ b \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} a \\ b \end{pmatrix} + \boldsymbol{q}$ ***A1***

(ii) solving $\begin{pmatrix} a \\ b \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} a \\ b \end{pmatrix} + \boldsymbol{q}$ using simultaneous equations or $\boldsymbol{a} = (\boldsymbol{I} - \boldsymbol{P})^{-1}\boldsymbol{q}$

(M1)

$a = 0.651\ (0.651084...),\ b = 1.48\ \ (1.47662...)$ ***A1A1***

$$\left(a = \frac{5 + 2\sqrt{3}}{13},\ b = \frac{14 + 3\sqrt{3}}{13} \right)$$

METHOD 2

(i) $\begin{pmatrix} x' \\ y' \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} x - a \\ y - b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ ***A1***

Note: Accept substitution of x and y (and x' and y') with particular points given in the question.

(ii) $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \boldsymbol{P}\begin{pmatrix} 0 - a \\ 0 - b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ ***(M1)***

Note: This line, with any of the points substituted, may be seen in part (d)(i) and if so the ***M1*** can be awarded there.

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (\boldsymbol{I} - \boldsymbol{P})\begin{pmatrix} a \\ b \end{pmatrix}$$

$a = 0.651084...,\ b = 1.47662...$ ***A1A1***

$$\left(a = \frac{5 + 2\sqrt{3}}{13},\ b = \frac{14 + 3\sqrt{3}}{13} \right)$$

[4 marks]
[Total 18 marks]

3. (a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ ***(M1)***

$(4, 1)$ ***A1***

Note: Award ***A0*** if parentheses are omitted in the final answer.

[2 marks]

(b) attempt to substitute values into gradient formula ***(M1)***

$\left(\frac{0-2}{6-2} = \right) -\frac{1}{2}$ ***(A1)***

therefore the gradient of perpendicular bisector is 2 ***(M1)***

so $y-1=2(x-4)$ $(y=2x-7)$ ***A1***

[4 marks]

(c) identifying the correct equations to use: ***(M1)***

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or finding points of intersection graphically ***(M1)***

$(3, -1)$ ***A1***

Note: Accept an answer expressed as "$x=3$, $y=-1$".

[3 marks]

(d) attempt to use distance formula ***(M1)***

$\text{YZ} = \sqrt{(7-(-1))^2 + (7-3)^2}$

$= \sqrt{80}$ $\left(4\sqrt{5}\right)$ ***A1***

[2 marks]

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule ***(M1)***

$\cos \text{X}\hat{\text{Y}}\text{Z} = \frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}$ $(=0.316227\ldots)$ ***(A1)***

Note: Award ***A1*** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\text{X}\hat{\text{Y}}\text{Z} = \right) 71.6°$ $(71.5650\ldots°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

continued…

Question 3 continued

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ ***(M1)(A1)***

Note: Award ***A1*** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} =\right) 71.6°$ $(71.5650°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\ldots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ ***(M1)***

$= 24\ \mathrm{km}^2$ ***A1***

[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc. ***R1***

[1 mark]

Total [18 marks]

6. (a) (i) $\sqrt{10^2+8^2}$ ***(M1)***

$=12.8\ \ (12.8062\ldots, \sqrt{164})\ (\text{m s}^{-1})$ ***A1***

(ii) $\tan^{-1}\left(\frac{10}{8}\right)$ ***(M1)***

$=0.896$ **OR** $51.3\ \ (0.896055\ldots$ **OR** $51.3401\ldots°)$ ***A1***

Note: Accept 0.897 or 51.4 from use of $\arcsin\left(\frac{10}{12.8}\right)$.

[4 marks]

(b) $y=t(10-5t)$ ***(M1)***

Note: The ***M1*** might be implied by a correct graph or use of the correct equation.

METHOD 1 – graphical Method

sketch graph ***(M1)***

Note: The ***M1*** might be implied by correct graph or correct maximum (eg $t=1$).

max occurs when $y=5\ \text{m}$ ***A1***

METHOD 2 – calculus

differentiating and equating to zero ***(M1)***

$\frac{\text{d}y}{\text{d}t}=10-10t=0$

$t=1$

$y\ (=1(10-5))=5\ \text{m}$ ***A1***

METHOD 3 – symmetry

line of symmetry is $t=1$ ***(M1)***

$y\ (=1(10-5))=5\ \text{m}$ ***A1***

[3 marks]

(c) attempt to solve $t(10-5t)=0$ ***(M1)***

$t=2$ (or $t=0$) ***(A1)***

$x\ (=5+8\times 2)=21\ \text{m}$ ***A1***

Note: Do not award the final ***A1*** if $x=5$ is also seen.

[3 marks]

continued…

Question 6 continued

(d) **METHOD 1**

$t = \frac{x-5}{8}$ ***M1A1***

$y = \left(\frac{x-5}{8}\right)\left(10 - 5 \times \frac{x-5}{8}\right)$ ***A1***

METHOD 2

$y = k(x-5)(x-21)$ ***A1***

when $x = 13,\ y = 5$ so $k = \frac{5}{(13-5)(13-21)} = -\frac{5}{64}$ ***M1A1***

$\left(y = -\frac{5}{64}(x-5)(x-21)\right)$

METHOD 3

if $y = ax^2 + bx + c$

$0 = 25a + 5b + c$

$5 = 169a + 13b + c$

$0 = 441a + 21b + c$ ***M1A1***

solving simultaneously, $a = -\frac{5}{64},\ b = \frac{130}{64},\ c = -\frac{525}{64}$ ***A1***

$(y = -\frac{5}{64}x^2 + \frac{130}{64}x - \frac{525}{64})$

METHOD 4

use quadratic regression on $(5,\ 0),\ (13,\ 5),\ (21,\ 0)$ ***M1A1***

$y = -\frac{5}{64}x^2 + \frac{130}{64}x - \frac{525}{64}$ ***A1***

Note: Question asks for expression; condone omission of "$y =$".

[3 marks]

(e) trajectory of arrow is $y = x\tan 10 + 2$ ***(A1)***

intersecting $y = x\tan 10 + 2$ and their answer to (d) ***(M1)***

$(8.66,\ 3.53)\ \left((8.65705...,\ 3.52647\ldots)\right)$ ***A1***

$(15.1,\ 4.66)\qquad \left((15.0859\ldots,\ 4.66006\ldots)\right)$ ***A1***

[4 marks]

continued…

Question 6 continued

(f) when $x_{\text{target}} = 8.65705\ldots$, $t_{\text{target}} = \frac{8.65705\ldots - 5}{8} = 0.457132\ldots$ s ***(A1)***

attempt to find the distance from point of release to intersection ***(M1)***

$\sqrt{8.65705\ldots^2 + (3.52647\ldots - 2)^2}$ $(= 8.79060\ldots \text{ m})$

time for arrow to get there is $\frac{8.79060\ldots}{60} = 0.146510\ldots$ s ***(A1)***

so the arrow should be released when

$t = 0.311$ (s) $(0.310622\ldots$ (s)) ***A1***

[4 marks]

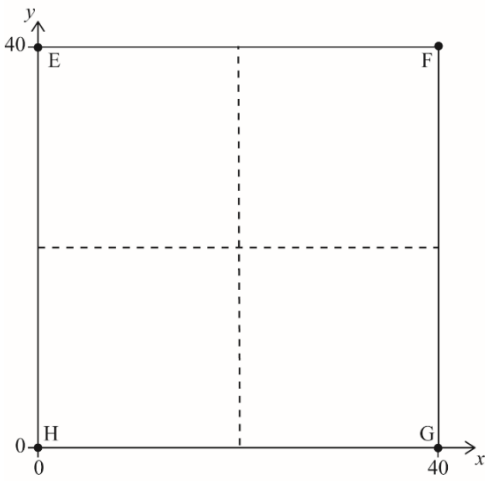
Total [21 marks]

2. (a) the size of each town is small (in comparison with the distance between the towns)
OR
if towns have an identifiable centre
OR
the centre of the town is at that point ***R1***

Note: Accept a geographical landmark in place of "centre", e.g. "town hall" or "capitol".

[1 mark]

(b)



A1

Note: There is no need for a scale / coordinates here. Condone boundaries extending beyond the metropolitan area.

[1 mark]

(c) (i) the gradient of IF is $\frac{40-20}{40-30}=2$ ***(A1)***

negative reciprocal of any gradient ***(M1)***

gradient of perpendicular bisector $=-\frac{1}{2}$

Note: Seeing $-\frac{2}{3}$ (for example) used clearly as a gradient anywhere is evidence of the "negative reciprocal" method despite being applied to an inappropriate gradient.

midpoint is $\left(\frac{40+30}{2}, \frac{40+20}{2}\right)=(35,\ 30)$ ***(A1)***

equation of perpendicular bisector is $y-30=-\frac{1}{2}(x-35)$ ***A1***

Note: Accept equivalent forms *e.g.* $y=-\frac{1}{2}x+\frac{95}{2}$ or $2y+x-95=0$.
Allow ***FT*** for the final ***A1*** from their midpoint and gradient of perpendicular bisector, as long as the ***M1*** has been awarded.

[4 marks]

(ii) the perpendicular bisector of EH is $y = 20$ ***(A1)***

Note: Award this ***A1*** if seen in the y-coordinate of any final answer or if 20 is used as the y-value in the equation of any other perpendicular bisector.

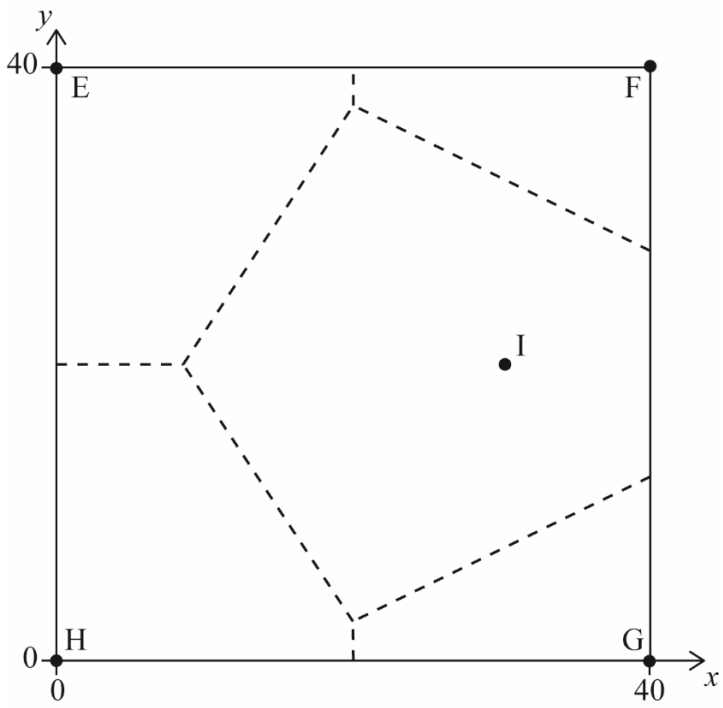
attempt to use symmetry **OR** intersecting two perpendicular bisectors ***(M1)***

$\left(\frac{25}{3}, 20\right)$ ***A1***

$(20, 2.5)$ ***A1***

[4 marks]

(iii)



M1A1

Note: Award ***M1*** for exactly four perpendicular bisectors around I (IE, IF, IG and IH) seen, even if not in exactly the right place.
Award ***A1*** for a completely correct diagram. Scale / coordinates are NOT necessary. Vertices should be in approximately the correct positions but only penalized if clearly wrong (condone northern and southern vertices appearing to be very close to the boundary).

Condone the Voronoi diagram extending outside of the square.
Do not award follow-though marks in this part.

[2 marks]

(d) 30% of 40 is 12 ***(A1)***

recognizing line intersects bisectors at $y = c$ (or equivalent) but different x-values ***(M1)***

$c = \frac{3}{2}x_1 + \frac{15}{2}$ and $c = -\frac{1}{2}x_2 + \frac{95}{2}$

finding an expression for the distance in Isaacopolis in terms of one variable ***(M1)***

$x_2 - x_1 = (95 - 2c) - \frac{2c - 15}{3} = 100 - \frac{8c}{3}$

equating their expression to 12

$100 - \frac{8c}{3} = 0.3 \times 40 = 12$

$c = 33$

distance $= 33$ (km) ***A1***

[4 marks]

(e) (i) must be a vertex (award if vertex given as a final answer) ***(R1)***
attempt to calculate the distance of at least one town from a vertex ***(M1)***

Note: This must be seen as a calculation or a value.

correct calculation of distances ***A1***

$\frac{65}{3}$ **OR** 21.7 **AND** $\sqrt{406.25}$ **OR** 20.2

$\left(\frac{25}{3}, 20\right)$ ***A1***

Note: Award ***R1M0A0A0*** for a vertex written with no other supporting calculations.
Award ***R1M0A0A1*** for correct vertex with no other supporting calculations.
The final ***A1*** is not dependent on the previous ***A1***. There is no follow-through for the final ***A1***.

Do not accept an answer based on "uniqueness" in the question.

[4 marks]

(ii) *For example, any one of the following:*
decision does not take into account the different population densities
closer to a city will reduce travel time/help employees
it is closer to some cites than others ***R1***

Note: Accept any correct reason that engages with the scenario.
Do not accept any answer to do with ethical issues about whether toxic waste should ever be dumped, or dumped in a metropolitan area.

[1 mark]

(f) (i) **METHOD 1**

attempting $\boldsymbol{M}^3$ ***M1***

attempting $\boldsymbol{M}^4$ ***M1***

e.g.

last row/column of $\boldsymbol{M}^3 = \begin{pmatrix} 3 & 5 & 1 & 6 & 0 & 7 \end{pmatrix}$

last row/column of $\boldsymbol{M}^4 = \begin{pmatrix} 10 & 12 & 4 & 16 & 1 & 18 \end{pmatrix}$

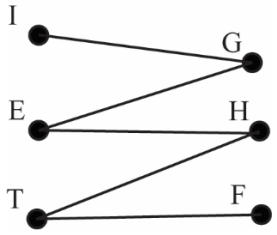
hence Isaacopolis is the last city to be polluted ***A1***

Note: Do not award the ***A1*** unless both $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ are considered.
Award ***M1M0A0*** for a claim that the shortest distance is from T to I and that it is 4, without any support.

METHOD 2

attempting to translate $\boldsymbol{M}$ to a graph or a list of cities polluted on each day ***(M1)***

correct graph or list ***A1***



hence Isaacopolis is the last city to be polluted ***A1***

Note: Award ***M1A1A1*** for a clear description of the graph in words leading to the correct answer.

[3 marks]

(ii) it takes 4 days ***A1***

[1 mark]

(iii) **EITHER**
the orders of the different vertices are:

E	2
F	1
G	2
H	2
I	1
T	2

(A1)

Note: Accept a list where each order is 2 greater than listed above.

OR
a correct diagram/graph showing the connections between the locations ***(A1)***

Note: Accept a diagram with loops at each vertex.
This mark should be awarded if candidate is clearly using their correct diagram from the previous part.

THEN
"Start at F and end at I" **OR** *"Start at I and end at F"* ***A1***

Note: Award ***A1A0*** for *"it could start at either F or I"*.
Award ***A1A1*** for *"IGEHTF"* **OR** *"FTHEGI"*.
Award ***A1A1*** for "*F and I*" **OR** "*I and F*".

[2 marks]

[Total 27 marks]

2. (a) $AF^2 = 89.2^2 + 104.9^2 - 2(89.2)(104.9)\cos 83$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the cosine rule and ***(A1)*** for correct substitution.

$AF = 129 \text{ m} \quad (129.150...)$ ***A1***

[3 marks]

(b) $21310 \div 129.150\ldots$ ***(M1)***

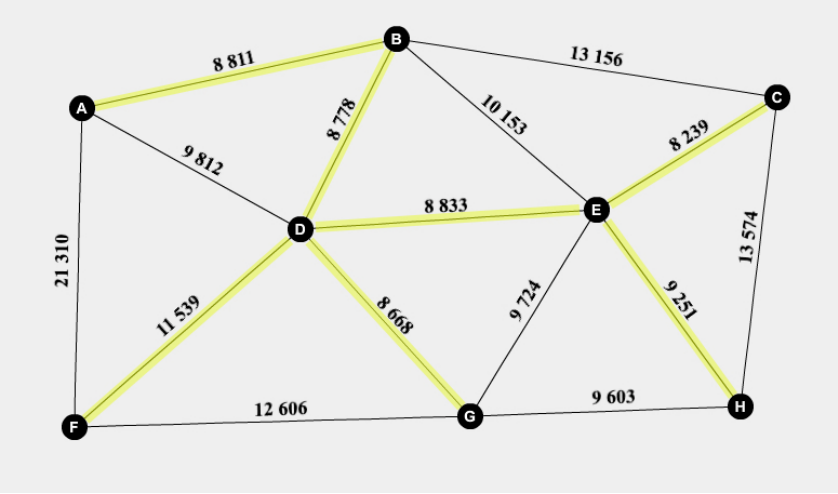
$\$ 165$ ***A1***

[2 marks]

(c) any reasonable statement referring to the lake ***R1***
(eg. there is a lake between A and F, the cables would need to be installed under/over/around the lake, special waterproof cables are needed for lake, etc.)

[1 mark]

(d) (i) edges (or weights) are chosen in the order
CE (8239)
DG (8668)
BD (8778)
AB (8811)
DE (8833)
EH (9251)
DF (11 539) ***A1A1A1***



Note: Award ***A1*** for the first two edges chosen in the correct order. Award ***A1A1*** for the first six edges chosen in the correct order. Award ***A1A1A1*** for all seven edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated.

[3 marks]

(ii) Finding the sum of the weights of their edges ***(M1)***
$8239 + 8668 + 8778 + 8811 + 8833 + 9251 + 11539$

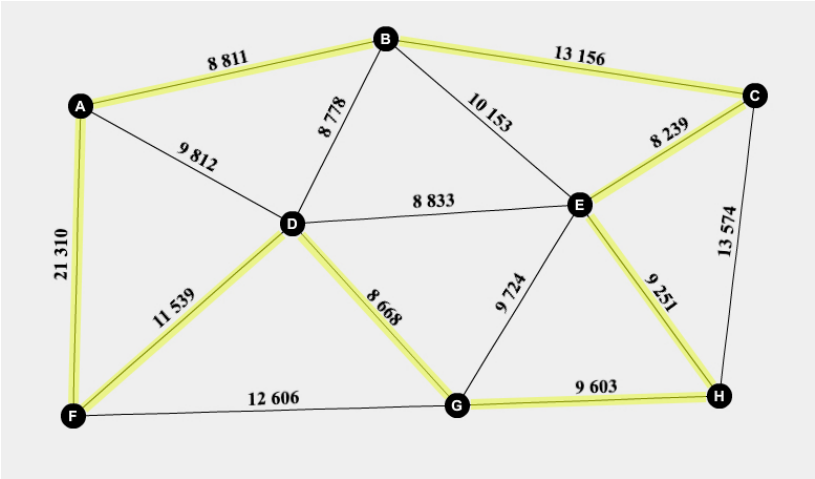
total cost = $\$64\,119$ ***A1***

[2 marks]

(e) a Hamiltonian cycle is not always an Eulerian circuit as it does not have to include all edges of the graph (only all vertices) ***R1***

[1 mark]

(f) edges (or weights) are chosen in the order
DG (8668)
GH (9603)
HE (9251)
EC (8239)
CB (13 156)
BA (8811)
AF (21 310)
FD (11 539) ***A1A1A1***



Note: Award ***A1*** for the first two edges chosen in the correct order. Award ***A1A1*** for the first five edges chosen in the correct order. Award ***A1A1A1*** for all eight edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated.

finding the sum of the weights of their edges ***(M1)***
$8668+9603+9251+8239+13156+8811+21310+11539$

upper bound = $\$90\,577$ ***A1***

[5 marks]

(g) attempt to find MST after deleting vertex D ***(M1)***

these edges (or weights) (in any order)

CE (8239)

AB (8811)

EH (9251)

GH (9603)

BE (10 153)

FG (12 606) ***A1***

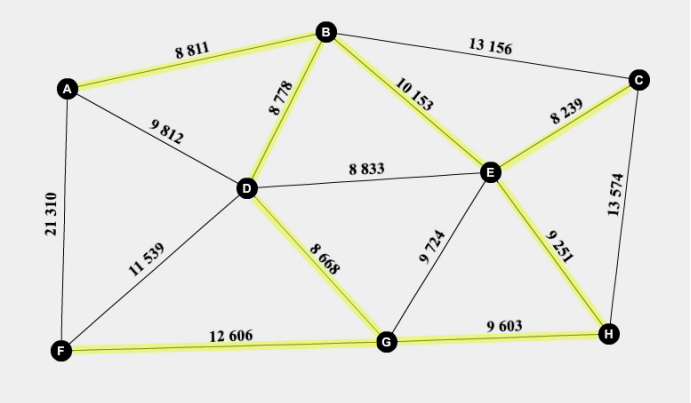
Note: Prim's or Kruskal's algorithm could be used at this stage.

reconnect D to MST with two different edges ***(M1)***

DG (8668)

BD (8778) ***A1***

Note: This ***A1*** is independent of the first ***A*** mark and can be awarded if both DG and BD are chosen to reconnect D to the MST, even if the MST is incorrect.



finding the sum of the weights of their edges ***(M1)***

$8239+8811+9251+9603+10153+12606+8668+8778$

Note: For candidates with an incorrect MST or no MST, the weights of at least seven of the edges being summed (two of which must connect to D) must be shown to award this ***(M1)***.

lower bound = \$76 109 ***A1***

[6 marks]

(h) **METHOD 1**

recognition of a binomial distribution ***(M1)***

$X \sim \mathrm{B}(2,\ 0.014)$

finding the probability that a cable fails (at least one of its connections fails)

$\mathrm{P}(X > 0) = 0.027804$ **OR** $1 - \mathrm{P}(X = 0) = 0.027804$ ***A1***

recognition that **two** cables must fail for the network to go offline ***M1***

recognition of binomial distribution for network, $Y \sim \mathrm{B}(8,\ 0.027804)$ ***(M1)***

$\mathrm{P}(Y \geq 2) = 0.0194\ \ (0.0193602...)$ **OR** $1 - \mathrm{P}(Y < 2) = 0.0194\ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

Note: Evidence of binomial distribution may be seen as combinations.

METHOD 2

recognition of a binomial distribution ***(M1)***

$X \sim \mathrm{B}(16,\ 0.014)$

finding the probability that at least **two** connections fail

$\mathrm{P}(X \geq 2) = 0.0206473\ldots$ **OR** $1 - \mathrm{P}(X < 2) = 0.0206473\ldots$ ***A1***

recognition that the previous answer is an overestimate ***M1***

finding probability of two ends of the same cable failing, $F \sim \mathrm{B}(2,\ 0.014)$,

and the ends of the other 14 cables not failing, $S \sim \mathrm{B}(14, 0.014)$

$\mathrm{P}(F = 2) \times \mathrm{P}(S = 0) = 0.0000160891\ldots$ ***(A1)***

$0.0000160891\ldots \times 8 = 0.00128713\ldots$

$0.0206473\ldots - 0.00128713\ldots = 0.0194\ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

METHOD 3

recognition of a binomial distribution ***M1***

$X \sim \mathrm{B}(16,\ 0.014)$

finding the probability that the network remains secure if 0 or 1 connections fail or if 2 connections fail provided that the second failed connection occurs at the other end of the cable with the first failure ***(M1)***

$\mathrm{P}(\text{remains secure}) = \mathrm{P}(X \leq 1) + \frac{1}{15} \times \mathrm{P}(X = 2)$ ***A1***

$= 0.9806397625$ ***A1***

$\mathrm{P}(\text{network fails}) = 1 - 0.9806397625 = 0.0194\ \ (0.0193602...)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

METHOD 4

$\mathrm{P}(\text{network failing})$

$= 1 - \mathrm{P}(0 \text{ connections failing}) - \mathrm{P}(1 \text{ connection failing})$

$\quad - \mathrm{P}(2 \text{ connections on the same cable failing})$ ***M1***

$= 1 - 0.986^{16} - {}^{16}C_1 \times 0.014 \times 0.986^{15} - {}^{8}C_1 \times 0.014^2 \times 0.986^{14}$ ***A1A1A1***

Note: Award ***A1*** for each of 2nd, 3rd and last terms.

$= 0.0194 \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

[5 marks]

[Total 28 marks]

3. (a) $\sin(21^\circ) = \frac{17}{\text{BF}}$ ***(M1)***

$\text{BF} = 47.4 \text{ m} \ (47.4372...)$ ***A1***

[2 marks]

(b) **EITHER**

$\text{BE} = \sqrt{47.4372...^2 + 44^2} = 64.7015...$ ***(A1)***

$\sin^{-1}\left(\frac{17}{\text{BE}}\right)$ ***(M1)***

$= 15.2^\circ \ (15.2329...^\circ) \quad \left(\text{or } 0.266 \text{ radians } (0.265866...)\right)$ ***A1***

OR

$\text{AD} = \sqrt{47.4372...^2 - 17^2} = 44.2865...$

$\text{DB} = \sqrt{64.7015...^2 + 44^2} = 62.4832...$ ***(A1)***

$\tan^{-1}\left(\frac{17}{62.4832...}\right)$ ***(M1)***

$= 15.2^\circ \ (15.2329...^\circ) \quad \left(\text{or } 0.266 \text{ radians } (0.265866...)\right)$ ***A1***

[3 marks]

Total [5 marks]

4. (a)

$$\begin{array}{c} \\ \mathrm{S} \\ \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \\ \mathrm{D} \\ \mathrm{T} \end{array} \begin{array}{c} \begin{array}{cccccc} \mathrm{S} & \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} & \mathrm{T} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & \boxed{2} & 0 \\ 1 & 0 & 1 & 1 & \boxed{0} & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ \boxed{2} & \boxed{0} & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

$\mathrm{SD} = \mathrm{DS} = 2$ ***A1***
$\mathrm{AD} = \mathrm{DA} = 0$ ***A1***

[2 marks]

(b) attempt to calculate at least one of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***
attempt to calculate all of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***
finding at least one of the top right entries, $4, 10, 64$ ***(A1)***
78 walks ***A1***

Note: If $\mathrm{SD} = \mathrm{DS} = 1$ is their answer in part (a), their ***FT*** answer is $(3 + 8 + 41 =)\ 52$ walks.

[4 marks]

(c) because some of the walks will pass through T, before returning to T ***R1***

[1 mark]

Total [7 marks]

8. (a) setting up at least two simultaneous equations ***(M1)***

$p = -0.8$ **OR** $q = 3.6$ ***(A1)***

M has coordinates $(1.6, -2.6, 8.2)$ ***A1***

[3 marks]

(b) using vectors $\begin{pmatrix}3\\2\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ ***(M1)***

$\begin{pmatrix}3\\2\\1\end{pmatrix} \bullet \begin{pmatrix}1\\-1\\2\end{pmatrix} = 3$ ***(A1)***

$\cos\theta = \dfrac{\begin{pmatrix}3\\2\\1\end{pmatrix} \bullet \begin{pmatrix}1\\-1\\2\end{pmatrix}}{\sqrt{3^2+2^2+1^2}\sqrt{1^2+(-1)^2+2^2}}$ $\left(\cos\theta = \dfrac{3}{\sqrt{14}\sqrt{6}}\right)$ ***(M1)***

Note: Accept correct use of vector product.

$(\theta =)$ 1.24 radians $(1.23732\ldots)$ $(70.9^\circ$ $(70.8933...))$ ***A1***

[4 marks]

Total [7 marks]

9. (a) attempt to find $\det(\boldsymbol{M})$ ***(M1)***

$= 14$

$(12 \times 14) = 168 \text{ cm}^2$ ***A1***

[2 marks]

(b) let X have coordinates (x, y)

METHOD 1

$\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{M}^{-1}\begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$ ***(A1)***

$\boldsymbol{M}^{-1} = \frac{1}{14}\begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix}$ ***A1***

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 2t-3+24-20t \\ -6t+9+12-10t \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 21-18t \\ 21-16t \end{pmatrix}$ **OR** $\left(\frac{21-18t}{14}, \frac{21-16t}{14}\right)$ ***A1A1***

METHOD 2

writing two simultaneous equations ***(M1)***

$2x - 4y = 2t - 3$ ***(A1)***

$3x + y = 6 - 5t$ ***(A1)***

attempting to solve the equations ***(M1)***

$(x, y) = \left(\frac{3}{2} - \frac{9t}{7}, \frac{3}{2} - \frac{8t}{7}\right)$ ***A1A1***

[6 marks]

Total [8 marks]

2. (a) (i) B ***A1***

(ii) F ***A1***

[2 marks]

(b) correct substitution into the midpoint formula ***(M1)***

$\frac{8+5}{2}$

$y = 6.5$ ***A1***

Note: Answer must be an equation for the ***A1*** to be awarded.

[2 marks]

(c) midpoint $= (5, 7)$ ***(A1)***

correct use of gradient formula ***(M1)***

$\frac{8-6}{7-3}$

gradient of $\text{BC} = 0.5$ ***(A1)***

negative reciprocal of gradient ***(M1)***

perpendicular gradient $= -2$

$y - 7 = -2(x - 5)$ (or $y = -2x + 17$) ***A1***

Note: Do not follow through within the part for the final ***A1***.

[5 marks]

(d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) ***(M1)***

Note: This may be seen graphically or algebraically.

$6.5 - 7 = -2(x - 5)$ **OR** $6.5 = -2x + 17$

Note: Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x - 5)$ **OR** $y = 4x - 14.5$

$x = 5.25, y = 6.5$ **OR** $(5.25, 6.5)$ ***A1***

Note: The x-coordinate must be exact or expressed to at least 3 sf.

(ii) their correct substitution into distance formula ***(M1)***

$\sqrt{(5.25-7)^2 + (6.5-5)^2}$

$= 2.30 \text{ km} \left(2.30488..., \frac{\sqrt{85}}{4} \right)$ ***A1***

[4 marks]

Total [13 marks]

4. (a) any correct Hamiltonian cycle e.g. ABCDEFA ***A1***

[1 mark]

(b) no, since not all vertices have an even degree (or equivalent) ***R1***

[1 mark]

(c) (i) 49 ***A1***

(ii) 34 ***A1***

(iii) 50 ***A1***

[3 marks]

(d) cycle is EBCDFAE

$UB = 12 + 25 + 17 + 34 + 18 + 35$ ***(M1)(A1)***

Note: Award ***M1*** for $12 + 25 + 17 + ...$ **OR** EBCD.

$= 141$ ***A1***

[3 marks]

(e) attempt to find MST for vertices A, B, C, D and E ***M1***

$12 + 14 + 17 + 27 \ (= 70)$ ***A1***

$LB = 70 + 18 + 22$ ***(M1)***

$= 110$ ***A1***

[4 marks]

(f) **EITHER**

deleting a different vertex ***A1***

might give a higher value (and hence a better lower bound). ***R1***

OR

the edges selected in part (e) do not form a cycle. ***A1***

so a higher value is possible ***R1***

[2 marks]

[14 marks]

7. (a) (i) use of chain rule ***(M1)***

$\boldsymbol{v} = -9\sin(3t)\boldsymbol{i} + 12\cos(3t)\boldsymbol{j}$ ***A1***

Note: Award ***(M1)*** for at least one correct term seen but condone omission of $\boldsymbol{i}$ or $\boldsymbol{j}$.

(ii) $|\boldsymbol{v}| = \sqrt{(-9\sin(9))^2 + (12\cos(9))^2}$ ***(M1)***

$= 11.5 \text{ m s}^{-1}$ $(11.5455...)$ ***A1***

[4 marks]

(b) (i) $\boldsymbol{a} = -27\cos(3t)\boldsymbol{i} - 36\sin(3t)\boldsymbol{j}$ ***A1***

(ii) $\boldsymbol{a} = -9(3\cos(3t)\boldsymbol{i} - 4\sin(3t)\boldsymbol{j})$ ***M1***

$\boldsymbol{a} = -9\boldsymbol{r}$ (where $\boldsymbol{r}$ is a position vector from the origin) ***A1***

$\boldsymbol{a}$ is in opposite direction to the position vector ***R1***

hence $\boldsymbol{a}$ is always directed towards the origin ***AG***

[4 marks]

(c) relative position $\boldsymbol{d} = \boldsymbol{r}_2 - \boldsymbol{r}_1$ ***(M1)***

distance between particles $= |\boldsymbol{d}|$ $(= |\boldsymbol{r}_2 - \boldsymbol{r}_1|)$ ***(M1)***

$|\boldsymbol{d}| = \sqrt{(-4\sin(4t) - 3\cos(3t))^2 + (3\cos(4t) - 4\sin(3t))^2}$ ***(A1)***

minimum value of $|\boldsymbol{d}|$ when $t = 4.71(\text{s})$ $\left(4.71238..., \frac{3\pi}{2}\right)$ ***(M1)A1***

[5 marks]

(d) (i) for 2nd particle, $\boldsymbol{v} = -16\cos(4t)\boldsymbol{i} - 12\sin(4t)\boldsymbol{j}$ ***(A1)***

EITHER

consider the gradient of either $\boldsymbol{v}$ ***(M1)***

$m_1 = -\frac{12\cos(3t)}{9\sin(3t)}$ and $m_2 = \frac{12\sin(4t)}{16\cos(4t)}$ ***(A1)***

attempt to solve $m_1 = m_2$ ***(M1)***

OR

vectors are parallel therefore one is a multiple of the other, $\boldsymbol{v}_2 = l\,\boldsymbol{v}_1$ ***(M1)***

$(l =)$ $\frac{16\cos(4t)}{9\sin(3t)} = -\frac{\sin(4t)}{\cos(3t)}$ ***(A1)***

attempt to solve ***(M1)***

THEN

$t = 1.30$ s $(1.30135...)$ ***A1***

continued…

Question 7 continued

(ii) **EITHER**

at $t = 1.30$, $\boldsymbol{v}_1 = 6.22\boldsymbol{i} - 8.68\boldsymbol{j}$ and $\boldsymbol{v}_2 = -7.57\boldsymbol{i} + 10.6\boldsymbol{j}$ ***A1***

OR

$l = -1.22$ (following second method in part (d)(i)) ***A1***

THEN

$\boldsymbol{v}_2$ is a negative multiple of $\boldsymbol{v}_1$ $(\boldsymbol{v}_2 = -1.22\boldsymbol{v}_1)$ ***R1***

the two particles are moving in the opposite direction ***AG***

[7 marks]

Total [20 marks]

3. (a) attempt to use distance formula for points D and A ***(M1)***

$\mathrm{DA} = \sqrt{11^2 + 7^2}$

$= 13.0$ (miles) $(13.0384\ldots, \sqrt{170})$ ***A1***

Note: Accept 13 miles. Award ***M0A0*** for finding the equation of the line DA.
DA may be seen in part (b) but this should not be accepted as answer for part (a).

[2 marks]

(b) $\left(\mathrm{DB} = \sqrt{13^2 + 5^2} =\right)\ 13.9\ \left(13.9283..., \sqrt{194}\right)$ **AND**

$\left(\mathrm{DC} = \sqrt{4^2 + 12^2} =\right)\ 12.6\ \left(12.6491..., \sqrt{160}\right)$ ***A1***

recognizing closest town is best estimate ***(M1)***

(town C is closest)

$30\,^{\circ}\mathrm{C}$ ***A1***

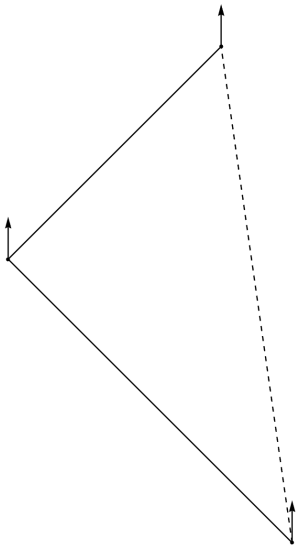
Note: If their DA from part (a) is the shortest length, then allow ***FT*** in (b).

[3 marks]

[Total: 5 marks]

5. METHOD 1

diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

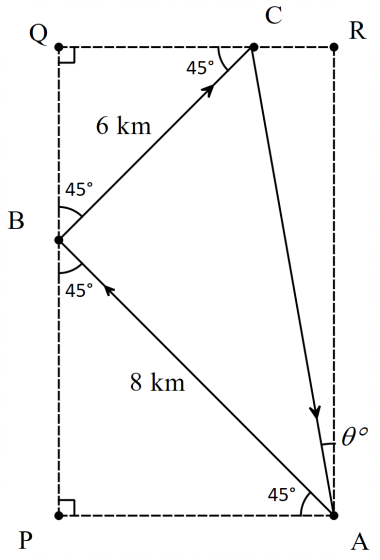
$=172^\circ \ (171.869...^\circ)$ ***A1***

continued…

Question 5 continued

METHOD 2

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$ (these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked $\mathrm{AP}, \mathrm{BP}, \mathrm{CQ}$ and BQ in the diagram ***(M1)***

$$\mathrm{AP} = \mathrm{BP} = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$\mathrm{CQ} = \mathrm{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426...$$

Note: This may be done using a vector approach.

using $\tan\theta° = \dfrac{\mathrm{AP} - \mathrm{CQ}}{\mathrm{PB} + \mathrm{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$= 172° \quad (171.869...°)$ ***A1***

[Total: 5 marks]

7. (a) attempt to find the vector product (e.g. one term correct) ***(M1)***

$$\begin{pmatrix}0\\6\\1\end{pmatrix}\times\begin{pmatrix}7\\3\\0\end{pmatrix}=\begin{pmatrix}-3\\7\\-42\end{pmatrix}$$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to use the vector product formula for the area of triangle

(condone incorrect signs and missing $\frac{1}{2}$) ***(M1)***

$\text{area}=\frac{1}{2}\sqrt{3^2+7^2+42^2}$

$=21.3\ (\text{m}^2)\ \ (21.3424...,\ \frac{1}{2}\sqrt{1822})$ ***A1***

METHOD 2

find θ using $\vec{\text{AB}}\times\vec{\text{AC}}=\left|\vec{\text{AB}}\right|\left|\vec{\text{AC}}\right|\sin\theta$ ***(M1)***

$\theta=67.1^\circ\ \ (67.1350^\circ\ldots,\ 1.171728\ldots.\ \text{radians})$

then $\text{area}=\frac{1}{2}\left|\vec{\text{AB}}\right|\left|\vec{\text{AC}}\right|\sin\theta$

$=21.3\ (\text{m}^2)\ \left(21.3424\ldots,\ \frac{1}{2}\sqrt{1822}\right)$ ***A1***

[2 marks]

continued…

Question 7 continued

(c) $\text{AC} = 7.61577... \left(\sqrt{58}\right)$ ***(A1)***

setting the area formula $\frac{1}{2} \times \text{base} \times \text{height}$ equal to their part (b) ***(M1)***

$$\text{BX} = \frac{2 \times 21.3424...}{\sqrt{58}}$$

$= 5.60 \ \ (5.60480...)$ ***A1***

Note: Award ***A1*** for 5.6.

Award ***A1*** for 5.59 $(5.5936\ldots)$ from the use of 21.3 to 3 sf.

[3 marks]

(d) attempting to set up a trig ratio ***(M1)***

angle is $\arcsin\left(\frac{1}{\text{BX}}\right)$

10.3° $(10.2776...^\circ, 0.179378\ldots$ radians) ***A1***

[2 marks]

[Total: 9 marks]

6. (a) **EITHER**

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Multiplying their volume by $\frac{4}{5}$ ***(M1)***

OR

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Subtracting $\frac{1}{5}$ of their volume ***(M1)***

$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5}\times\frac{4}{3}\pi(3.4)^3\right)$

Note: The ***M1*** can be awarded for a final answer of $32.9272\ldots$ seen without working.

THEN

$132\ \text{cm}^3\ \ (131.708\ldots\ \text{cm}^3)$ ***A1***

[3 marks]

(b) $\pi\times 3\times 11$ ***(A1)***

$103.672\ldots\ (\text{cm}^2)$ **OR** $33\pi(\text{cm}^2)$

$104\ (\text{cm}^2)$ ***A1***

[2 marks]

Total [5 marks]

8. (a) attempt to create a 5x5 adjacency matrix ***(M1)***

$$\boldsymbol{M} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ ***A2***

Note: Allow the transposed matrix. Award ***A2*** for all entries correct, ***A1*** if one or two entries are incorrect, ***A0*** otherwise.
Answer presented in markscheme assumes ABCDE ordering of rows and columns; accept other orders provided they are clearly communicated.
Award ***A1*** if the zeroes are replaced by blank cells.

[3 marks]

(b) (i) recognizing need to find $\boldsymbol{M}^7$ ***(M1)***

$$\boldsymbol{M}^7 = \begin{pmatrix} 8 & 8 & 17 & 8 & 13 \\ 8 & 10 & 19 & 17 & 14 \\ 6 & 11 & 16 & 10 & 17 \\ 11 & 8 & 19 & 14 & 10 \\ 2 & 6 & 8 & 11 & 8 \end{pmatrix}$$

2 (routes) ***A1***

(ii) vertices visited in order are
EITHER
$E \to D \to C \to B \to C \to B \to D \to A$ ***A2***

OR
$E \to D \to C \to B \to C \to E \to D \to A$ ***A2***

[4 marks]
Total [7 marks]

13. (a) **METHOD 1 (find product of matrices first)**

$T \to T'$ is represented by $\boldsymbol{QP} = \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}$ ***(A1)***

recognizing need to find their $(\boldsymbol{QP})^{-1}$ ***(M1)***

$(\boldsymbol{QP})^{-1} = \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}^{-1}$

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

METHOD 2 (find inverses of both matrices first)

recognizing need to find inverse of both $\boldsymbol{P}$ and $\boldsymbol{Q}$ ***(M1)***

$\boldsymbol{P}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix}$ **AND** $\boldsymbol{Q}^{-1} = \begin{pmatrix} -\frac{3}{13} & \frac{1}{13} \\ \frac{1}{13} & \frac{4}{13} \end{pmatrix}$ ***(A1)***

$T' \to T$ is represented by $\boldsymbol{P}^{-1}\boldsymbol{Q}^{-1} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}^{-1}$ ***(M1)***

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

Note: In METHOD 1, award ***M1A0M1A0*** if they multiply the matrices in the wrong order.
In METHOD 2, award ***M1A1M1A0*** if they multiply the matrices in the wrong order.

[4 marks]

(b) $\left(\det\left[-\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}\right] =\right) -\frac{1}{78}$ **OR** $\left(\det\begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix} =\right) -78$ ***(A1)***

area of $T' = \left|\det \boldsymbol{QP}\right| \times$ area of T **OR** area of $T = \left|\det(\boldsymbol{QP})^{-1}\right| \times$ area of T' ***(M1)***

$\Rightarrow$ area of $T = 273 \times \frac{1}{78}$

$= 3.5$ (cm^2) ***A1***

Note: Award ***(A1)(M0)A0*** for an answer of -3.5 (cm^2) with or without working. Accept an answer of 4.04 (cm^2) from use of 3sf values in their answer to part (a).

[3 marks]
Total [7 marks]

14. (a) $\boldsymbol{v}_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ***(A1)***

attempt to find any relevant angle ***(M1)***

$\tan^{-1}\left(\frac{3}{2}\right) \quad \left(= 56.3099...^\circ\right)$

$(90^\circ + 56.3099...^\circ =) \ 146^\circ \ \ (146.3099...^\circ)$ ***A1***

[3 marks]

(b) setting $1 + 2t = -2 + 4t$ ***(M1)***

$t = 1.5$ (hrs.) ***A1***

[2 marks]

(c) $\boldsymbol{r}_B - \boldsymbol{r}_A = (-3 + 2t)\boldsymbol{i} + (-7 + 4t)\boldsymbol{j}$ ***(M1)***

$-3 + 2t = -(-7 + 4t)$ ***(M1)***

$t = 1.67 \ (\text{hrs.}) \ \left(1.66666..., \frac{5}{3}\right)$ ***A1***

[3 marks]

Total [8 marks]

4. (a) there are more than two vertices with odd degree ***R1***

so it is not possible to travel along each road exactly once ***A1***

Note: Do not award ***R0A1***.

Award ***R1*** for "There are 4 vertices with odd degree".

[2 marks]

(b) $a = 11,\ b = 18,\ c = 17,\ d = 15$ ***A2***

Note: Award ***A1*** for any one correct, ***A2*** for all four correct.

[2 marks]

(c) attempt to use nearest neighbour algorithm ***(M1)***

Note: Award ***M1*** for first 3 vertices correct or $11, 4, 3$ seen.

$\mathrm{G}-\mathrm{E}-\mathrm{F}-\mathrm{B}-\mathrm{D}-\mathrm{A}-\mathrm{C}(-\mathrm{E})-\mathrm{G}$ **OR** $11+4+3+5+5+8+\text{their } b$ ***(A1)***

upper bound $= 54\ (\mathrm{km})$ ***A1***

[3 marks]

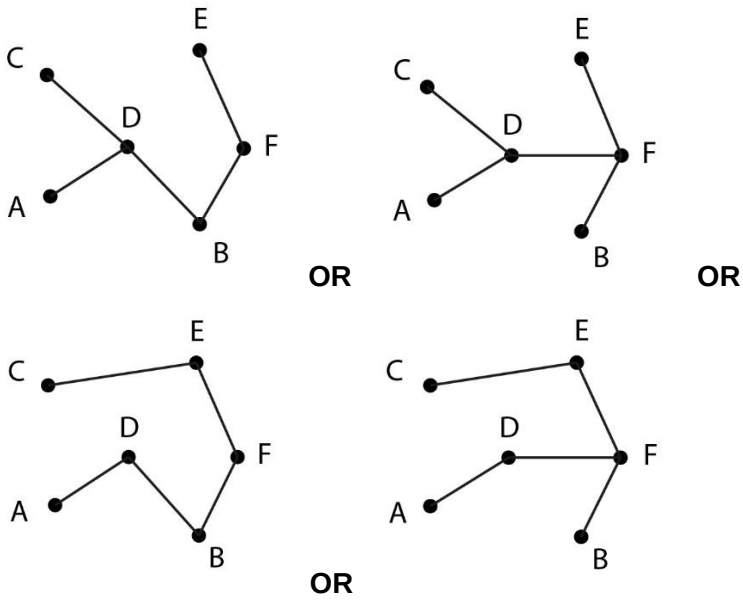
continued…

Question 4 continued

(d) (i) a diagram of **any** spanning tree of the subgraph ABCDEF ***(A1)***

attempt at Kruskal's algorithm or Prim's algorithm ***(M1)***

e.g. edges BF (3), EF (4) and an edge of length 5 listed or seen in any spanning tree



A1

(ii) $24\ (\text{km})$ ***A1***

Note: ***FT*** from their sketch, only if it is a spanning tree. It is not required to see the edge lengths on the sketch, since they are given in the question.

[4 marks]

continued…

Question 4 continued

(e) adding vertex G's two shortest edges to their part (d)(ii) ***(M1)***

$24+11+13$

$=48$ ***A1***

[2 marks]

(f) try removing a different vertex ***A1***

[1 mark]

(g) recognize 7 edges in optimum route ***(M1)***

Note: Award ***M1*** for a total length of 52 seen.

subtracting $0.5\times$ edges from 52 ***(M1)***

$52-7\times 0.5$

$=48.5\ (\text{km})$ ***A1***

[3 marks]

[Total: 17 marks]

7. (a) using area of trapezoid formula ***M1***

$\sin(15^\circ)\times\frac{1+2}{2}$ ***A1***

$=\frac{3}{2}\sin(15^\circ)$ ***AG***

[2 marks]

(b) (i) $\boldsymbol{M}_6=\begin{pmatrix}\frac{1}{2}\cos 90^\circ & -\frac{1}{2}\sin 90^\circ\\ \frac{1}{2}\sin 90^\circ & \frac{1}{2}\cos 90^\circ\end{pmatrix}$ ***(M1)***

$=\begin{pmatrix}0 & -\frac{1}{2}\\ \frac{1}{2} & 0\end{pmatrix}$ ***A1***

(ii) multiplying their part (b)(i) and point $(0,\ -1)$ (in any order) ***M1***

$\begin{pmatrix}0 & -\frac{1}{2}\\ \frac{1}{2} & 0\end{pmatrix}\times\begin{pmatrix}0\\ -1\end{pmatrix}$

$\left(\frac{1}{2},\ 0\right)$ ***A1***

[4 marks]

continued…

Question 7 continued

(c) (i) $\begin{pmatrix} \cos\left(k\times 15^\circ\right) & -\sin\left(k\times 15^\circ\right) \\ \sin\left(k\times 15^\circ\right) & \cos\left(k\times 15^\circ\right) \end{pmatrix}$ ***A1***

(ii) $\begin{pmatrix} 1-\frac{k}{12} & 0 \\ 0 & 1-\frac{k}{12} \end{pmatrix}$ ***A1***

(iii) $k\times 15^\circ$ ***A1***

(iv) $1-\frac{k}{12}$ ***A1***

[4 marks]

(d) **METHOD 1 (using part (c)(iv))**

$\left(1-\frac{k}{12}\right)^2$ ***A2***

METHOD 2 (using full matrix M_k)

$$\left|\begin{pmatrix} \left(1-\frac{k}{12}\right)\cos\left(k\times 15^\circ\right) & -\left(1-\frac{k}{12}\right)\sin\left(k\times 15^\circ\right) \\ \left(1-\frac{k}{12}\right)\sin\left(k\times 15^\circ\right) & \left(1-\frac{k}{12}\right)\cos\left(k\times 15^\circ\right) \end{pmatrix}\right|$$

$$=\left(1-\frac{k}{12}\right)^2\cos^2\left(k\times 15^\circ\right)+\left(1-\frac{k}{12}\right)^2\sin^2\left(k\times 15^\circ\right)$$ ***(M1)***

$$=\left(1-\frac{k}{12}\right)^2\left(\cos^2\left(k\times 15^\circ\right)+\sin^2\left(k\times 15^\circ\right)\right)$$

$$=\left(1-\frac{k}{12}\right)^2$$ ***A1***

[2 marks]

continued…

Question 7 continued

(e) recognizing to multiply by 2 and by original area ***(M1)***

attempt to sum their answer to part (d), $k = 0, 1, ..., 11$ ***(M1)***

a correct expression ***(A1)***

e.g. $0.776457...\left(1^2 + \left(\frac{11}{12}\right)^2 + ... + \left(\frac{1}{12}\right)^2\right)$ **OR** $2\sum_{k=0}^{11}\left(1-\frac{k}{12}\right)^2 \times \frac{3}{2}\sin 15^\circ$

OR $\sum_{k=0}^{11}\left(1-\frac{k}{12}\right)^2 \times 0.776457...$ **OR** $2\sum_{k=1}^{12}\left(\frac{k}{12}\right)^2 \times \frac{3}{2}\sin\left(15^\circ\right)$

$3.50\ (3.50484...)$ (square units)

Note: Award at most ***M0(M1)(A1)A0*** for an unsupported final answer of "1.75242..."

(A1)

[4 marks]

(f) $\boldsymbol{N}_k = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \times \boldsymbol{M}_k$ ***A1A1***

Note: Award ***A1A0*** if correct matrices are written in the wrong order.

[2 marks]

[Total: 18 marks]

1. (a) attempt to use area of triangle formula ***(M1)***

$\frac{1}{2} \times 25.9 \times 6.36 \times \sin(125^\circ)$ ***(A1)***

67.5 m^2 $(67.4700... \text{ m}^2)$ ***A1***

Note: Units are required. The final ***A1*** is only awarded if the correct units are seen in their answer; hence award ***(M1)(A1)A0*** for an unsupported answer of 67.5.

[3 marks]

(b) attempt to use cosine rule ***(M1)***

$(\text{BK} =)\sqrt{12^2 + 6.36^2 - 2 \times 12 \times 6.36 \times \cos 45^\circ}$ ***(A1)***

8.75 (m) $(8.74738\ldots\text{(m)})$ ***A1***

Note: Award ***(M1)(A1)(A0)*** for radian answer of 10.2 (m) $(10.2109\ldots\text{(m)})$ with or without working shown.

[3 marks]

(c) **METHOD 1**

attempt to use sine rule with measurements from triangle OKX ***(M1)***

$\frac{\text{OX}}{\sin 51.1^\circ} = \frac{22.2}{\sin 53.8^\circ}$ ***(A1)***

$(\text{OX} =)\ 21.4 \text{ (m)}$ $(21.4099\ldots)\text{(m)}$ ***A1***

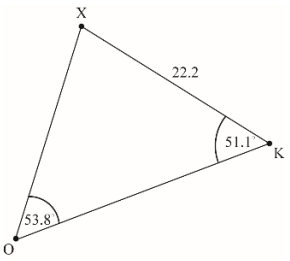
$(21.4 \text{ (m)} < 22.2 \text{ (m)})$

Odette is closer to the football / Khemil is further from the football ***A1***

Note: For the final ***A1*** to be awarded 21.4 $(21.4099\ldots)$ must be seen. Follow through within question part for final ***A1*** for a consistent comparison with their OX.

METHOD 2

sketch of triangle OXK with vertices, angles and lengths ***(A1)***



51.1° is smallest angle in triangle OXK ***R1***

opposite side (OX) is smallest length ***R1***

therefore Odette is closest ***A1***

[4 marks]

continued…

Question 1 continued

(d) attempt to use length of arc formula ***(M1)***

$\frac{135}{360} \times 2\pi \times 12$ ***(A1)***

$28.3\,(\text{m})$ $(9\pi,\ 28.2743...)$ (m) ***A1***

[3 marks]

Total [13 marks]

4. (a) any city can be travelled to or from any other city (so is connected) ***R1***

EITHER
but there is no direct flight between Los Angeles and Dallas (for example) ***R1***

OR
but not every vertex has degree 4 ***R1***

Note: Accept equivalent statements for the cities being connected and the graph not being complete.

[2 marks]

continued…

Question 4 continued

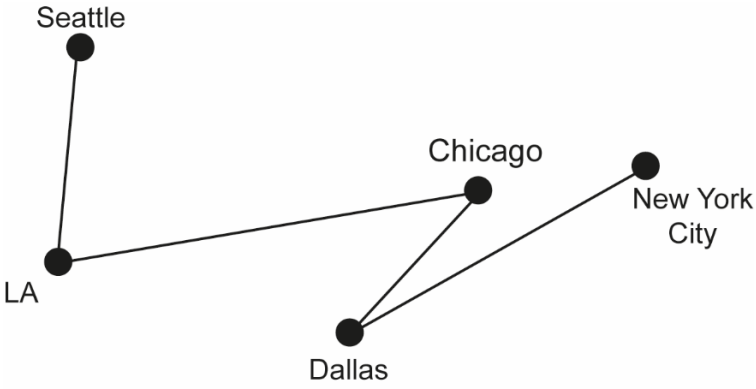
(b) edge CD selected first ***M1***

DN,

CL,

LS ***A1***

Note: Award marks if the answers are written as sums in the correct order.
M1 if 30 is seen first, ***A1*** for $30 + 39 + 41 + 58$.



A1

Note: The final ***A1*** can be awarded independently. Award ***M0A0A1*** for a correct MST graph with no other working. Award ***M1A0A1*** if Prim's algorithm is seen to be used correctly with CD first.

[3 marks]

(c) $2 \times \mathrm{MST}$ weight ***(M1)***

$= \$336$ ***A1***

Note: Allow any integer multiple (>1) of MST weight for ***M1***, and if correctly calculated, award ***M1A1***.

[2 marks]

(d) attempt at nearest neighbour algorithm ***M1***

order is $\mathrm{LA} \to \mathrm{D} \to \mathrm{C} \to \mathrm{NYC} \to \mathrm{S} \to \mathrm{LA}$ ***A1***

Note: Award ***M1*** for a route that begins with LA and then D, this includes seeing 26 as the first value in a sum.
Award ***A1*** if $26{+}30{+}68{+}66{+}58$ seen in order.

Note: Award ***M1A0*** for an incorrect first nearest neighbour proceeding 'correctly' to the next vertex. For example, LA to C and then C to D.

upper bound is $(26 + 30 + 68 + 66 + 58 =)\ \$248$ ***A1***

Note: Award ***M1A0*** for correct nearest neighbour algorithm starting from a vertex other than LA. Condone the correct tour written backwards i.e. $58 + 66 + 68 + 30 + 26 = 248$

[3 marks]

continued…

Question 4 continued

(e) (i) attempt to find MST of L, N, D and S ***(M1)***
by deleting C, Kruskal gives MST for the remainder as LD, DN, LS
weight 123 ***(A1)***
(lower bound is therefore $123 + (30 + 41) =)$ \$194 ***A1***

Note: Award ***(M1)*** for a graph or list of edges that does not include C.
Award ***(A1)*** if $26 + 39 + 58$ seen in any order.

(ii) by deleting S, Kruskal gives MST for the remainder as LD, DC, DN
weight 95 ***(A1)***
(lower bound is therefore $95 + (58 + 66) =)$ \$219 ***A1***

Note: Award ***(A1)*** if $26 + 30 + 39$ seen in any order.

[5 marks]

(f) $219 \le C \le 248$ ***A1A1***

Note: Award ***A1*** for $219 \le C$ and ***A1*** for $C \le 248$. Award at most ***A1A0*** for $219 < C < 248$.
FT for their values from part (e) if higher value from (e)(i) and (e)(ii) used for the lower bound, and part (d) for the upper.

[2 marks]

(g) any valid tour, within their interval from part (f), from any starting point **OR**
any valid tour that starts and finishes at N ***(M1)***
valid tour starting point N **AND** within their interval ***A1***
e.g NDCLSN (weight 234)

Note: If part (f) not correct, **only** award ***A1FT*** if their valid tour begins and ends at N **AND** lies within **BOTH** their interval (including if one-sided) in part (f) **AND** $219 \le C \le 248$.

If no response in the form of an interval seen in part (f) then award ***M1A0*** for a valid tour beginning and ending at N **AND** within $219 \le C \le 248$.

[2 marks]

Total [19 marks]

2. (a) $\vec{PQ} = \begin{pmatrix} 115 \\ 0 \end{pmatrix}$ **OR** $\sqrt{(215-100)^2+(-197+82)^2+0^2}$ ***(M1)***

Note: Accept working using "A and B" in place of "P and Q".

$\sqrt{115^2+115^2+0^2}$

$= 163 \text{ (km)} \ (162.634..., \sqrt{26450})$ ***A1***

[2 marks]

(b) speed of each aircraft = $\sqrt{640^2+640^2+0^2}$ ***(M1)***

$= 905 \ (\text{km h}^{-1}) \ \ (905.096\ldots, \sqrt{819200})$ **OR** $251 \ \text{m s}^{-1} \ (251.388...)$ ***A1***

[2 marks]

(c) time to travel $162.634\ldots \text{ km}$ is $\frac{162.634...}{905.096...}$ $(0.179687...)$ ***(M1)***

Note: Accept $\frac{115}{640}$ from a consideration of the component directions.
Accept use of the 3 significant figure answer from parts (a) and (b):
$\frac{163}{905} = 0.180110\ldots$

EITHER

(multiply by 60 to get) $10.8 \ \ (10.7812...)$ (minutes) or 10 minutes 48 seconds ***A1***

$(10.8 > 10)$ hence not in conflict ***R1***

Note: Award ***R1*** for a correct comparison of their time, in minutes, with 10 minutes.

OR

(convert 10 minutes into hours) $0.167 \ \ (0.166666...)$ (hours) ***A1***

$(0.180 > 0.167)$ hence not in conflict ***R1***

Note: Award ***R1*** for a correct comparison of their time, in hours, with 0.167 hours.

[3 marks]

(d) $\left(\boldsymbol{r}_A = \right) \begin{pmatrix} 100 \\ -82 \\ 10.7 \end{pmatrix} + t \begin{pmatrix} -640 \\ 640 \\ 0 \end{pmatrix}$ ***A1***

[1 mark]

(e) (i) $\boldsymbol{r}_A - \boldsymbol{r}_C = \begin{pmatrix} (100-640t)-(-400-140t) \\ (-82+640t)-(-41+604t) \\ 10.7-(9.1+2t) \end{pmatrix}$ ***(M1)(A1)***

$= \begin{pmatrix} 500-500t \\ -41+36t \\ 1.6-2t \end{pmatrix}$

Note: Award ***M1*** for an attempt to subtract their $\boldsymbol{r}_A$ and $\boldsymbol{r}_C$ in either order, ***(A1)*** for a correct expression, which does not need to be simplfiied and which may be seen in the line below.

$|\boldsymbol{r}_A - \boldsymbol{r}_C| = \sqrt{(500-500t)^2 + (-41+36t)^2 + (1.6-2t)^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct attempt to find the modulus of their $\boldsymbol{r}_A - \boldsymbol{r}_C$.

this is equal to $10\ \text{km}$ when $t = 0.983\ \ (0.983441...)$ and $1.02\ \ (1.01799...)$

A1A1

[5 marks]

(ii) **METHOD 1**

consideration of the vertical component of their $\boldsymbol{r}_A - \boldsymbol{r}_C$ from part (e)(i) ***M1***

$1.6-2t$

$-0.3 < 1.6-2t < 0.3$ ***A1***

Note: Award ***A1*** for relating their expression to -0.3 **and** 0.3; accept an equality.

$(0.65 <)\ t < 0.95$ ***A1***

Note: Award ***A1*** for 0.95 seen.

interval is outside of interval from part (e)(i) (the two conditions are never broken at the same time) ***R1***

Note: The ***R1*** can only be awarded if there is a clear consideration of intervals.

hence regulations are not broken ***A1***

Note: Do not award ***R0A1***.

METHOD 2

consideration of the vertical component of their $\boldsymbol{r}_A - \boldsymbol{r}_C$ from part (e)(i) ***M1***

$1.6 - 2t$

when $t = 0.983441...$ the difference in height is $1.6 - 2 \times 0.983441...$

$= (-)\ 0.366882\ldots \text{ km}$ ***A1***

EITHER

as time increases the vertical displacement between the aircraft also increases ***R2***

OR

when $t = 1.01799...$ the difference in height is $1.6 - 2 \times 1.01799...$

$= (-)\ 0.435970\ldots (\text{km})$ ***A1***

because it is a linear function, the difference in height is also greater than 0.3 km between these values of t. ***R1***

Note: Accept an argument from a graph.

THEN

so regulations are not broken ***A1***

Note: Do not award ***R0A1***.

[5 marks]

(f) circle ***A1***

radius of $6.4\ (\text{km})$ ***A1***

centred on $\text{R} / (0, 0)$ ***A1***

Note: A description that includes only one or two of the points above can be awarded a further ***A1*** for a further correct statement, for example: "the speed is $243.2\ \text{km h}^{-1}$" **OR** "it travels anticlockwise when viewed from above" **OR** "it travels clockwise when viewed from below".

[3 marks]

(g) (i) attempt at scalar product for the correct two vectors ***(M1)***

Note: Award ***(M1)*** for a product and sum of components leading to a single scalar expression.

$\vec{\text{RE}} \bullet \boldsymbol{b} = -1(20-\lambda)+(10+\lambda)$

$= -10+2\lambda$ ***A1***

[2 marks]

(ii) $-10+2\lambda = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their scalar product equal to 0, but only if their scalar product is a single expression.

$\lambda = 5$ ***A1***

[2 marks]

(iii) **METHOD 1**

$\vec{\text{RE}} = \begin{pmatrix}20\\10\end{pmatrix}+5\begin{pmatrix}-1\\1\end{pmatrix} \left(=\begin{pmatrix}15\\15\end{pmatrix}\right)$ ***(M1)***

$\left|\vec{\text{RE}}\right|_{\min} = \sqrt{15^2+15^2} = 21.2 \ \ (21.2132..., 15\sqrt{2}, \sqrt{450})$ (km) ***A1***

METHOD 2

$\left|\vec{\text{RE}}\right| = \sqrt{(20-\lambda)^2+(10+\lambda)^2}$ ***(M1)***

$\left|\vec{\text{RE}}\right|_{\min} = 21.2 \ \ (21.2132..., 15\sqrt{2}, \sqrt{450})$ km ***A1***

[2 marks]

(iv) consideration of at least two distances from (their) 21.2, 6.4 and 10 ***M1***

EITHER

$6.4+10 = 16.4 < 21.2$ ***R1***

OR

$21.2-10 = 11.2 > 6.4$ ***R1***

OR

$21.2-6.4 = 14.8 > 10$ ***R1***

Note: Accept equivalent reasoning in words.

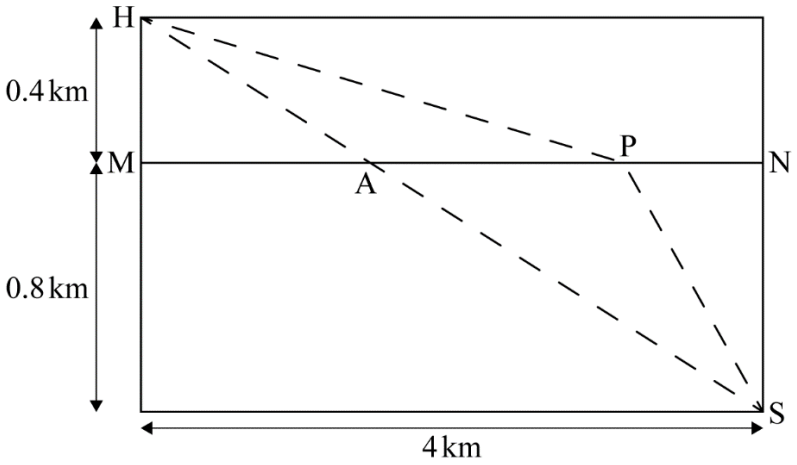
THEN

hence they do not break regulations ***AG***

[2 marks]

[Total 29 marks]

1. (a)



METHOD 1

$\hat{MHS} = \left(\tan^{-1} \frac{4}{1.2} = \right) 73.3007...^{\circ}$ OR $1.27933...$ ***(A1)***

use of trigonometry to find HA or AS ***(M1)***

$HA = \frac{0.4}{\cos \hat{MHS}}$ **AND** $AS = \frac{0.8}{\cos \hat{MHS}}$ ***(A1)***

$(HA = 1.39204...$ and $AS = 2.78408...)$

use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ for either of their distances ***(M1)***

time taken $= \left(\frac{AH}{15} + \frac{AS}{5} \right)$

$0.649618...$ (hours) ***(A1)***

(38.97712… minutes)

therefore 39 (mins) ***A1FT***

Note: Allow ***FT***, within the question part, from their time in hours for the final ***A1***.

METHOD 2

EITHER

use of similar triangles to identify either length MA or AN $\left(\frac{4}{3} \text{ or } \frac{8}{3}\right)$ ***(M1)***

attempt to use Pythagoras for either triangle AMH or ANS ***(M1)***

$\mathrm{AH}^2 = 0.4^2 + \left(\frac{4}{3}\right)^2$ **AND** $\mathrm{AS}^2 = 0.8^2 + \left(\frac{8}{3}\right)^2$ ***(A1)***

OR

attempt to use Pythagoras for larger triangle ***(M1)***

$\mathrm{SH}^2 = 4^2 + 1.2^2$

$\mathrm{AH} = \frac{1}{3}\sqrt{4^2 + 1.2^2}$ **AND** $\mathrm{AS} = \frac{2}{3}\sqrt{4^2 + 1.2^2}$ ***(M1)(A1)***

THEN

$(\mathrm{HA} = 1.39204...$ and $\mathrm{AS} = 2.78408...)$

use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ for either of THEIR distances ***(M1)***

time taken $= \left(\frac{\mathrm{AH}}{15} + \frac{\mathrm{AS}}{5}\right)$

$0.649618...$ (hours) ***(A1)***

$(38.97712\ldots$ minutes)

therefore 39 (mins) ***A1FT***

Note: Allow ***FT***, within the question part, from their time in hours for the final ***A1***.

[6 marks]

(b) (i) $\text{PH}^2 = 0.4^2 + x^2$ **AND** $\text{PS}^2 = 0.8^2 + (4-x)^2$ ***A1***

Note: This ***A1*** can be implied by a clear expression for the time in each region coming from distance / speed below.

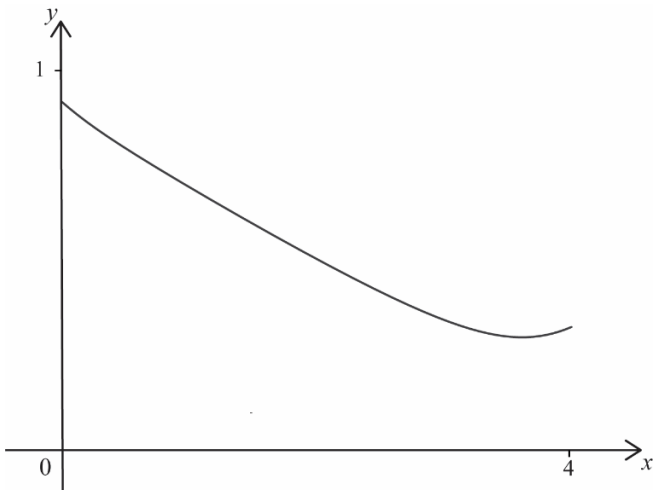
$$T(x) = \frac{\text{PH}}{15} + \frac{\text{PS}}{5}$$ ***(M1)***

$$T(x) = \frac{\sqrt{0.4^2 + x^2}}{15} + \frac{\sqrt{0.8^2 + (4-x)^2}}{5}$$ ***A1***

$$T(x) = \frac{\sqrt{0.4^2 + x^2} + 3\sqrt{0.8^2 + (4-x)^2}}{15}$$ ***AG***

[3 marks]

(ii)



correct shape with minimum point nearer $x = 4$ than $x = 0$ ***A1***
correct (approximate) y-intercept, $0.843\ldots$ (must be clearly below 1) ***A1***

[2 marks]

(iii) using the GDC, at the minimum $x = 3.72$ $(3.71898...)$ ***A1***

Note: Do not accept coordinates of the minimum point.

[1 mark]

(iv) finding their $T(x)$ for their value of x ***M1***
$T(x) = 0.418946\ldots$
so time saved $(= 38.97712\ldots - 25.1367\ldots \text{mins})$ = 14 (mins) ***A1***

[2 marks]

(c) (i) attempt at chain rule ***M1***

$$T'(x) = \frac{1}{15}\left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}}\right)$$ ***A1A1***

Note: Award ***A1*** for each correct term. Accept any equivalent form i.e. condone fractions not simplified.

[3 marks]

(ii) setting their $T'(x) = 0$ ***M1***

Note: This requires more than just a statement that the derivative equals zero – they must use their attempt at $T'(x)$.

$$\frac{1}{15}\left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}}\right) = 0$$

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$ ***AG***

[1 mark]

(iii) **METHOD 1**

$\cos \hat{\text{HPM}} = \frac{x}{\sqrt{0.16 + x^2}}$ **AND** $\cos \hat{\text{SPN}} = \frac{4-x}{\sqrt{0.64 + (4-x)^2}}$ ***A1***

substituting in the above equation and rearranging ***M1***

$\cos \hat{\text{HPM}} = 3\cos \hat{\text{SPN}}$ leading to $\frac{\cos \hat{\text{HPM}}}{\cos \hat{\text{SPN}}} = 3 = \left(\frac{15}{5}\right)$

verifying the result ***AG***

METHOD 2

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$

attempt to rearrange into a quotient ***M1***

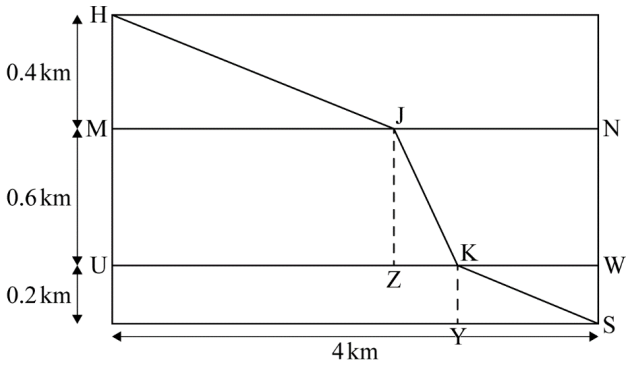
$$\left(\frac{15}{5} = 3 =\right) \frac{\frac{x}{\sqrt{0.16 + x^2}}}{\frac{4-x}{\sqrt{0.64 + (4-x)^2}}}$$

$$= \frac{\cos \hat{\text{HPM}}}{\cos \hat{\text{SPN}}}$$ ***A1***

verifying the result ***AG***

[2 marks]

(d) **METHOD 1**



let $\mathrm{MJ} = y$ km and W and Z be the points on the new boundary directly below N and J

attempt to find ZK in terms of MJ ***M1***

$(\mathrm{KW} = 0.5y)$

$\mathrm{ZK} = (4 - 1.5y)$ km. ***A1***

attempt to use the result from (c)(iii) at J ***M1***

$$\frac{\cos \mathrm{H\hat{J}M}}{\cos \mathrm{Z\hat{K}J}} = \frac{y}{\sqrt{y^2 + 0.4^2}} \div \frac{(4-1.5y)}{\sqrt{(4-1.5y)^2 + 0.6^2}} = \frac{15}{5}$$ ***A1***

Note: Accept $\cos \mathrm{N\hat{J}K}$ in place of $\cos \mathrm{Z\hat{K}J}$.

$$\left(\text{leading to } \frac{y}{\sqrt{y^2 + 0.16}} = \frac{3(4-1.5y)}{\sqrt{(4-1.5y)^2 + 0.36}} \right)$$

valid method for solving this equation, eg drawing graphs of both sides of the equation, using SOLVER, etc. ***(M1)***

solution is $y = 2.53$ ***A1***

METHOD 2

combining the field into one region with height 0.6 km ***M1***

$$\cos \mathrm{H\hat{P}M} = \frac{x}{\sqrt{0.36 + x^2}}$$

$$\cos \mathrm{S\hat{P}N} = \frac{4-x}{\sqrt{0.36 + (4-x)^2}}$$ ***A1***

Note: Both expressions, or their ratio, are required for the ***A1*** to be awarded.

therefore

$$\frac{x\sqrt{0.36 + (4-x)^2}}{(4-x)\sqrt{0.36 + x^2}} = 3$$ ***A1***

valid method for solving ***(M1)***

attempting to find MJ in terms of x e.g. $\mathrm{MJ} = \frac{2}{3}x$ ***M1***

so $\mathrm{MJ} = 2.53$ ***A1***

[6 marks]

Total [26 marks]

3. (a) (upper bound =) $0.525\ (\text{m})$ ***A1***
(lower bound =) $0.515\ (\text{m})$ ***A1***

Note: Accept an answer in interval notation or written as an inequality.

[2 marks]

(b) **METHOD 1 Convert REC to linear metres**
attempt to convert REC to metres using their lower bound ***(M1)***
$440\times0.515\ (=226.6)$ **OR** $280\times0.515\ (=144.2)$ seen

attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440\times0.515)^2(280\times0.515)$ ***(A1)***

$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

METHOD 2 Convert REC to cubic metres
attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440)^2(280)\ \ (=18\,069\,333.33...)$
attempt to convert 1 cubic REC to cubic metres using their lower bound ***(M1)***
(1 cubic REC =) 0.515^3

$(V=)\ \frac{1}{3}(440)^2(280)\times(0.515)^3$ ***(A1)***
$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

[4 marks]
[Total 6 marks]

4. (a) recognizing supplementary angles or acute angles in right-triangles ***(M1)***

$(\text{A}\hat{\text{B}}\text{C} =)\ 41^\circ + (180^\circ - 112^\circ)$, $41^\circ + (90^\circ - 22^\circ)$

Note: Values may be seen on diagram.

$\text{A}\hat{\text{B}}\text{C} = 109^\circ$ ***A1***

[2 marks]

(b) $\text{A}\hat{\text{C}}\text{B} = 49^\circ$ (may be seen in part (a)) ***(A1)***

attempt to substitute into the sine rule (or equivalent) ***(M1)***

$\dfrac{\text{AC}}{\sin 109^\circ} = \dfrac{100}{\sin 49^\circ}$ ***(A1)***

$\text{AC} = 125\ (\text{km})\ \ (= 125.282...)$ ***A1***

[4 marks]

[Total 6 marks]

6. (a) $(4, 8)$ ***A1***

[1 mark]

(b) attempt to find the gradient of AC ***(M1)***

$\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$

attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line ***(M1)***

$y-8=-\frac{4}{5}(x-4)$ **OR** $8=-\frac{4}{5}(4)+c$ **OR** $c=11.2$

$y-8=-\frac{4}{5}(x-4) \quad (y=-0.8x+11.2,\ 4x+5y-56=0)$ ***A1***

[3 marks]

(c) (i) attempt to find one distance from a farm to any closest vertex ***M1***

finding a correct distance from at least two distinct vertices ***A1***

$7.58968\ldots, 4.472135\ldots \left(\sqrt{20}\right), 5.830951\ldots \left(\sqrt{34}\right)$

$\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) ***A1***

(ii) $7.59 \text{ (km)} \ (= 7.58968...)$ ***A1***

[4 marks]

[Total 8 marks]

14. (a) vector from Q to any point in L or vice versa

$$= \begin{pmatrix} 1+\lambda \\ 3+\lambda \\ 2\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -10+\lambda \\ 4+\lambda \\ 2\lambda - 3 \end{pmatrix}$$ ***(M1)***

EITHER (scalar product)
attempt to use scalar product ***(M1)***

$$\begin{pmatrix} -10+\lambda \\ 4+\lambda \\ 2\lambda - 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0$$

$$-10+\lambda+4+\lambda+4\lambda-6=0$$

OR (distance formula)
attempt to use distance formula ***(M1)***
minimizing $(-10+\lambda)^2+(4+\lambda)^2+(-3+2\lambda)^2$

THEN
$\lambda = 2$ ***(A1)***

point $\mathrm{P}(3,\ 5,\ 4)$ ***A1***

Note: Do not award final ***A1*** for P given as a vector.

[4 marks]

(b) $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix}$ ***(A1)***

attempt to use vector product ***(M1)***

(perpendicular vector =) $\begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} -11 \\ -17 \\ 14 \end{pmatrix}$ ***A1***

Note: Award final ***A1*** for any multiple (positive or negative) of the answer given here.

[3 marks]
[Total 7 marks]

1. (a) 25 (m) ***A1***

[1 mark]

(b) (i) recognition of need to use Pythagoras theorem ***(M1)***

$BF^2 = 20^2 + 25^2$

$(BF =)\ 32.0\ \ (32.0156\ldots,\ \sqrt{1025},\ 5\sqrt{41})$ (m) ***A1***

(ii) correct use of trig ratio for $B\hat{F}M$ ***(M1)***

$(B\hat{F}M =)\ \tan^{-1}\left(\frac{25}{20}\right)$ or equivalent

$(B\hat{F}M =)\ 51.3\ (51.3401...)$ ***A1***

Note: Accept radian answer of $0.896\ \ (0.896055\ldots)$ Accept an answer of 51.4 from use of 3sf answer to part (b)(i) and then either cosine rule or inverse sine.

[4 marks]

(c) attempt to use arc length formula ***(M1)***

(arc length =) $\frac{2\times 51.3401...}{360}\times 2\pi(32.0156\ldots)$ ***(A1)***

(arc length =) $57.4\ \ (57.3755\ldots)$ (m) ***A1***

Note: Accept 57.3 from use of 3 sf. values of their answers from parts (b)(i) and (b)(ii).

[3 marks]

(d) $34.0156\ldots$ (seen anywhere) ***(A1)***

use of area of sector formula ***(M1)***

recognition of subtracting areas of two sectors ***(M1)***

(area =) $\frac{102.680\ldots}{360}\times\pi\left((34.0156\ldots)^2-(32.0156\ldots)^2\right)$

(area =) $118\ (m^2)\ \ (118.335\ldots)$ ***A1***

[4 marks]

(e) multiplying their area from part (d) by 0.12 or 12 ***(M1)***

0.12 (m) seen **OR** $1183350\ (cm^2)$ seen ***(A1)***

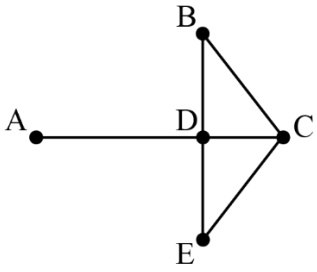
$118.335\ldots\times 0.12$ **OR** 1183350×12

$14.2\ \ (14.2002\ldots)\ m^3$ **OR** $14200000\ (14200236)\ cm^3$ ***A1***

[3 marks]

[Total 15 marks]

5. (a)



A1

[1 mark]

(b) (i) $\boldsymbol{P} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$ ***(M1)***

$\boldsymbol{P}^3 = \begin{pmatrix} 0 & 1 & 2 & 4 & 1 \\ 1 & 2 & 5 & 6 & 2 \\ 2 & 5 & 4 & 6 & 5 \\ 4 & 6 & 6 & 4 & 6 \\ 1 & 2 & 5 & 6 & 2 \end{pmatrix}$

$a = 6$ ***A1***

(ii) 5 (routes) ***A1***

[3 marks]

(c) A and C identified as start/finish points (in either order) ***(A1)***
for example : A – D – E – C – D – B – C ***A1***

[2 marks]

(d) cost of their Eulerian trail A to C (=180) ***(A1)***
consider edges to get from C to A ***(M1)***

235 (USD) ***A1***

[3 marks]

(e) (i) A to C (or C to A) ***A1***

(ii) best is CBDA
55 (USD) ***A1***
[2 marks]

(f) (i) A – D – C – B – E – A **OR** 50, 45, 30, 120, 60 ***(A1)***
summing their 5 edges ***(M1)***
$50 + 45 + 30 + 120 + 60$
(upper bound =) 305 (km) ***A1***

(ii) attempt to find MST without vertex A ***(M1)***
(MST =) 130 ***(A1)***
$130 + 50 + 60$ ***(M1)***
(lower bound =) 240 (km) ***A1***
[7 marks]
[Total 18 marks]

6. (a) $x = -1 + 2\lambda,\ y = 1 - \lambda$ ***A1***

[1 mark]

(b) $\begin{pmatrix} 1 & 7 \\ 7 & -1 \end{pmatrix}\begin{pmatrix} -1+2\lambda \\ 1-\lambda \end{pmatrix} = \begin{pmatrix} 6-5\lambda \\ -8+15\lambda \end{pmatrix}$ ***(M1)(A1)***

$\boldsymbol{r} = \begin{pmatrix} 6 \\ -8 \end{pmatrix} + \lambda\begin{pmatrix} -5 \\ 15 \end{pmatrix}$ (or equivalent) ***(M1)A1***

Note: Award ***(M1)*** for the correct format of a vector equation of a line, ***A1*** for the line being completely correct.

[4 marks]

(c) (i) $\begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix}$ **OR** $\begin{pmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ ***A1***

(ii) $\begin{pmatrix} 5\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{pmatrix}$ ***A1***

[2 marks]

(d) $(\boldsymbol{R} =) \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$ ***A1***

[1 mark]

(e) (i) attempt to multiply matrices from part (c) (in any order) ***(M1)***

e.g. $\boldsymbol{X} = \begin{pmatrix} 5\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{pmatrix} \begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix}$

$\boldsymbol{X} = \begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix}$ ***A1***

(ii) substituting $\boldsymbol{T}$, $\boldsymbol{R}$ and $\boldsymbol{X}$ ***(M1)***

$\begin{pmatrix} 1 & 7 \\ 7 & -1 \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix}$

multiplying by inverse (in any order) ***(M1)***

$\begin{pmatrix} 1 & 7 \\ 7 & -1 \end{pmatrix} \begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$

$\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$ ***A1***

$\cos 2\alpha = -\frac{3}{5}$ **AND** $\sin 2\alpha = \frac{4}{5}$

$\alpha = 1.11 \ (= 1.107148...)$ **OR** $63.4^\circ \ (63.4349\ldots^\circ)$ ***A1***

[6 marks]

[Total 14 marks]

5. (a) $y = 0.5x - 1$ ***A1A1***

Note: Award ***A1*** for $0.5x$ and ***A1*** for -1 (or equivalent equation). Award at most ***A1A0*** if answer is not presented as an equation.

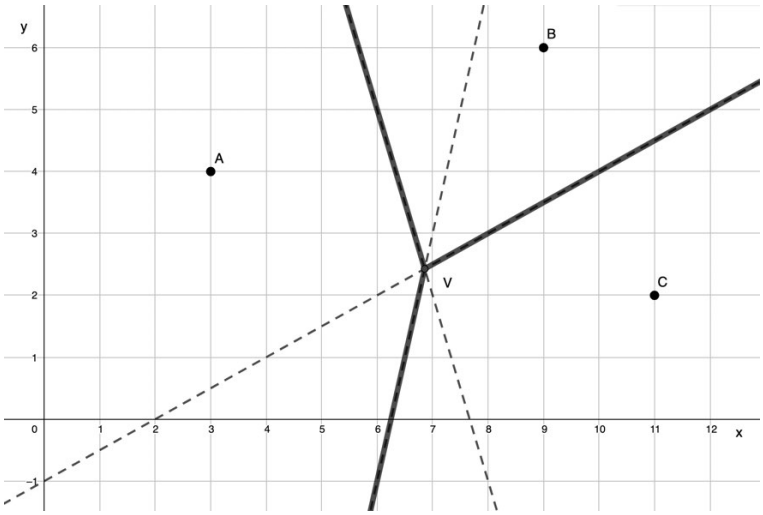
[2 marks]

(b) $(6.857,\ 2.429)$ ***A1A1***

Note: If both answers are not correct to 4 sig figs, award at most ***A1A0***.
Accept $x = 6.857$, $y = 2.429$.
Award ***A1A0*** for $\left(\frac{48}{7}, \frac{17}{7}\right)$. Award ***A0A1*** for $(2.429, 6.857)$.

[2 marks]

(c)



A2

Note: Award marks as shown in the table below. Condone edges that do not extend to the sides of the graph or beyond the x-axis.

Correct edges	Incorrect edges	Marks
3	0	***A2***
3	1	***A1A0***
3	2 or more	***A0A0***
2	0	***A1A0***
2	1	***A1A0***
2	2 or more	***A0A0***
1	0	***A1A0***
1	1 or more	***A0A0***

[2 marks]
[Total 6 marks]

8. (a) B and C ***A1***

[1 mark]

(b) correct intervals seen ($x \leq 5$ (or $x < 5$) **AND** $x \geq 5$ (or $x > 5$)) ***A1***

Note: The case of $x = 5$ must be included for this ***A1*** to be awarded.

attempt to add edges to $33 + x$ ***(M1)***

(If $x < 5$ (or $x \leq 5$) then repeat BC and) length is $33 + 2x$ ***A1***

(If $x > 5$ (or $x \geq 5$) then repeat AB and AC and) length is $(33 + x + 5 =) 38 + x$

A1

Note: If the intervals are not explicit, award at most ***A0(M1)A1A1***.

[4 marks]

[Total 5 marks]

10. (recognition that OB is a radius)

(radius =) $\sqrt{5^2+8^2}$ $(=\sqrt{89})$ ***(A1)***

EITHER (finding angle BOQ)

correct calculation for finding $\mathrm{B\hat{O}A}$ ***(A1)***

$\mathrm{B\hat{O}A} = \arctan\left(\frac{8}{5}\right)$ **OR** $\tan \mathrm{B\hat{O}A} = \frac{8}{5}$

expressing $\mathrm{B\hat{O}Q}$ as $90 + \mathrm{B\hat{O}A}$ ***(M1)***

$\mathrm{B\hat{O}Q} = 90 + \arctan\left(\frac{8}{5}\right)$ **OR** $\mathrm{B\hat{O}Q} = \frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)$

$\left(\mathrm{B\hat{O}Q} =\right) 147.994°\ldots$ **OR** $2.58299\ldots$

substituting *their* radius and angle BOQ correctly into arc length formula ***(M1)***

(arc BQ =) $\frac{90 + \arctan\left(\frac{8}{5}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$ **OR** $\left(\frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)\right) \times \left(\sqrt{5^2+8^2}\right)$

24.4 (m) (24.3679…) ***A1***

OR (finding angle BOP)

correct calculation for finding angle $\mathrm{B\hat{O}P}$ ***(A1)***

$\mathrm{B\hat{O}P} = \arctan\left(\frac{5}{8}\right)$ **OR** $\tan \mathrm{B\hat{O}P} = \frac{5}{8}$

substituting *their* radius and $\mathrm{B\hat{O}P}$ correctly into arc length formula ***(M1)***

(arc BP =) $\frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

subtracting *their* arc BP from arc PQ ***(M1)***

(arc BQ =) $\pi\sqrt{5^2+8^2} - \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi\left(\sqrt{5^2+8^2}\right)$

24.4 (m) (24.3679…) ***A1***

[Total: 5 marks]

11. (a) $\begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for selecting the correct matrix, ***A1*** for substituting 2α into a rotation matrix
Award ***A1A0*** for clockwise rotation. These marks can be awarded independently.
Condone the use of a different symbol.

[2 marks]

(b) $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$

$= \begin{pmatrix} \cos^2\alpha - \sin^2\alpha & -2\cos\alpha\sin\alpha \\ 2\cos\alpha\sin\alpha & \cos^2\alpha - \sin^2\alpha \end{pmatrix}$ ***M1A1***

Note: Award ***M1*** for an attempt to multiply matrices, e.g. at least one entry correct.

[2 marks]

(c) (i) (Because matrix multiplication represents the composition of transformations)
two rotations of α are equivalent to a rotation of 2α ***R1***
so the two matrices are equal (so each of the entries are also equal) ***R1***
$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$ ***AG***

(ii) $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$ ***A1***

replacing $\cos^2\alpha$ with $1 - \sin^2\alpha$ ***M1***
$= 1 - \sin^2\alpha - \sin^2\alpha$

$= 1 - 2\sin^2\alpha$ ***AG***

[4 marks]
[Total 8 marks]

6. attempt to substitute into area of triangle formula ***(M1)***

(sheep's field area =) $0.5\times15\times21\times\sin(78°)$

$=154.058\ldots\ (\text{m}^2)$ ***A1***

EITHER

(goat's field area =) $\frac{282}{360}\times\pi\times8^2$ ***(A1)(A1)***

Note: Award ***A1*** for 282, ***A1*** for correct entries in formula (including their 282).

OR

$\pi\times8^2-\frac{78}{360}\times\pi\times8^2$ ***(M1)(A1)***

Note: Award ***A1*** for minor sector area, ***M1*** for subtracting their sector area from circle area.

THEN

$=157.498\ldots\left(\frac{752\pi}{15}\right)\ (\text{m}^2)$ ***A1***

the goat has most area by $3.44\ (\text{m}^2)\ (3.44026\ldots)$ ***A1***

Note: Accept 154 and 157 for the intermediate ***A1*** marks, but do NOT follow through within the question; a final answer of $3\ \text{m}^2$ is awarded ***A0***.

[Total: 6 marks]

8. (a) $\begin{pmatrix}1\\2\end{pmatrix}$ ***A1***

Note: Accept any valid vector notation. Do not accept $(1, 1, 2)$.

[1 mark]

(b) (i) **EITHER**

use of scalar product formula to find angle $\hat{BAD}$ ***(M1)***

$$\cos \hat{BAD} = \frac{\begin{pmatrix}3\\4\\0\end{pmatrix} \bullet \begin{pmatrix}1\\1\\2\end{pmatrix}}{5\sqrt{6}}$$ ***(A1)***

OR

use of cosine rule to find $\hat{BAD}$ ***(M1)***

$$\cos \hat{BAD} = \frac{6+25-17}{2\times\sqrt{6}\times 5}$$ ***(A1)***

THEN

$$\hat{BAD} = 55.1^\circ \ (55.1417...^\circ, 0.962405...)$$ ***A1***

Note: If the direction of one of the vectors is reversed, leading to an obtuse angle $(124.858...^\circ)$ between the vectors, then award ***M1A1A0***.

(ii) **EITHER**

an attempt at using vector product ***(M1)***

$$\frac{1}{2}\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\times\begin{pmatrix}3\\4\\0\end{pmatrix}\right| \text{ or equivalent}$$

attempt to apply formula (e.g. one correct component) ***(M1)***

$$= \frac{1}{2}\left((1\times 0-4\times 2)\boldsymbol{i}-(1\times 0-2\times 3)\boldsymbol{j}-(1\times 4-1\times 3)\boldsymbol{k}\right)$$

$$\frac{1}{2}\left|\begin{pmatrix}-8\\6\\1\end{pmatrix}\right|$$ ***(A1)***

$$= 5.02 \left(5.02493, \frac{\sqrt{101}}{2}\right)$$ ***A1***

OR

use of formula for the area of a triangle ***(M1)***

area = $\frac{1}{2}\left|\begin{pmatrix}3\\4\\0\end{pmatrix}\right|\left|\begin{pmatrix}1\\1\\2\end{pmatrix}\right|\sin 55.1417\ldots$ ***(A1)(A1)***

$= 5.02$ ***A1***

Note: Award ***A1*** for the lengths AB and AD and ***A1*** for the angle.

[7 marks]

(c) **EITHER**

$\vec{\mathrm{AD}} = \vec{\mathrm{BC}} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ **OR** $\vec{\mathrm{AB}} = \vec{\mathrm{DC}} = \begin{pmatrix}3\\4\\0\end{pmatrix}$ ***A1***

One pair of opposite sides have equal length AND are parallel ***R1***

OR

$\vec{\mathrm{AD}} = \begin{pmatrix}1\\1\\2\end{pmatrix}, \vec{\mathrm{BC}} = \begin{pmatrix}1\\1\\2\end{pmatrix}, \vec{\mathrm{AB}} = \begin{pmatrix}3\\4\\0\end{pmatrix}, \vec{\mathrm{DC}} = \begin{pmatrix}3\\4\\0\end{pmatrix}$ ***A1***

$\vec{\mathrm{AD}} = \vec{\mathrm{BC}} = \begin{pmatrix}1\\1\\2\end{pmatrix}$ and $\vec{\mathrm{AB}} = \vec{\mathrm{DC}} = \begin{pmatrix}3\\4\\0\end{pmatrix}$

both pairs of opposite sides are parallel (or have equal length) ***R1***

Note: Both pairs of opposite angles are equal is also valid.

THEN

hence ABCD is a parallelogram ***AG***

[2 marks]

[Total: 8 marks]

12. (a) $\sqrt{6^2+6^2+3^2}$ ***(A1)***

$=9\ (\text{m s}^{-1})$ ***A1***

[2 marks]

(b) $\boldsymbol{h}=\begin{pmatrix}-38\\134\\315\end{pmatrix}+t\begin{pmatrix}15\\-20\\-60\end{pmatrix}$ ***A1***

[1 mark]

(c) equating one component from each ***(M1)***

e.g. $7+6t=-38+15t$

$t=5$ ***(A1)***

substituting their t-value into either equation ***(M1)***

$\begin{pmatrix}-38\\134\\315\end{pmatrix}+5\begin{pmatrix}15\\-20\\-60\end{pmatrix}$

$=\begin{pmatrix}37\\34\\15\end{pmatrix}$ ***A1***

[4 marks]

[Total: 7 marks]

1. (a) $\mathrm{BC} = 20\ (\mathrm{m})$ ***A1***

[1 mark]

(b) use of Pythagoras ***(M1)***

$\mathrm{AB} = \sqrt{12^2 + 4^2}$

$= 12.6\ (\mathrm{m})\quad (12.6491\ldots, \sqrt{160})$ ***A1***

[2 marks]

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $\mathrm{A\hat{B}C}$ (or finding the bearing of B from A) ***(A1)***

e.g. $\tan\left(\mathrm{A\hat{B}C}\right) = \frac{12}{4}$, $\cos \mathrm{A\hat{B}C} = \frac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649}$, $\cos \mathrm{A\hat{B}C} = \frac{6.3245}{20}$

$\mathrm{A\hat{B}C} = 71.6\ (71.5650\ldots)$ ***(A1)***

Note: Angle $\mathrm{A\hat{B}C}$ can be 71.5 or 72.2 depending on their working out. Bearings should be given in degrees.

$180 + 71.5650\ldots = 252°\ (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 180 plus their 71.6. If radians used, award ***A1A1*** for $1.24904\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

METHOD 2 – finding angle that AB makes with the horizontal (angle *H*)

correct use of a trig ratio to find H, the angle AB makes with horizontal ***(A1)***

e.g. $\tan \hat{H} = \frac{4}{12}$, $\cos \hat{H} = \frac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$

$\hat{H} = 18.4\ (18.4349...)$ ***(A1)***

Note: Accept $18.5\ (18.5078\ldots)$ from use of 3sf answer from part (b). Bearings should be given in degrees.

$270 - 18.4348... = 252°\ (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 270 minus their 18.4. If radians used, award ***A1A1*** for $0.321750\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

[3 marks]

(d) (i) $-\frac{4}{3}$ $\left(-\frac{16}{12}\right)$ ***A1***

(ii) $(6, 8)$ ***A1A1***

Note: Award ***A1A0*** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ ***(M1)***

equation of perpendicular bisector of AC ***(A1)***

e.g. $(y-8)=\frac{3}{4}(x-6)$ **OR** $y=\frac{3}{4}x+3.5$

EITHER

equation of perpendicular bisector of BC is $y=10$ ***(A1)***

OR

equation of perpendicular bisector of AB is $y=-3x+36$ ***(A1)***

Note: The ***A1*** is for either equation of perpendicular bisector of BC or AB.

point of intersection $\left(8\frac{2}{3}, 10\right)$ **OR** $(8.67, 10)$ $\left[(8.666\ldots, 10)\right]$ ***(M1)A1***

Note: Award ***M1*** for an attempt to equate their perpendicular bisectors
Award the final ***A1*** for the correct coordinate pair – parentheses omitted or not.

[8 marks]

[Total: 14 marks]

3. (a) (i) correct approach to find missing length ***(A1)***

$\sqrt{4^2-1^2}\quad(=\sqrt{15})$

attempt to find cross-section ***(M1)***

e.g. use of area of trapezoid formula or rectangle+triangle or rectangle – triangle

use of volume of prism formula ***(M1)***

(their cross-section multiplied by 3)

$3\left[\frac{1}{2}(10+11)\left(\sqrt{4^2-1^2}\right)\right]$

$=122\,(\text{m}^3)\ \ (121.998\ldots)$ ***A1***

(ii) correct approach to find missing height ***(A1)***

$\sqrt{4^2-3.2^2}\quad(=2.4)$

attempt to find volume ***(M1)***

(multiplication by 3.2 and 3 seen)

$3\left[\frac{1}{2}\left(10+10+\sqrt{4^2-3.2^2}\right)(3.2)\right]$

$=108\,(\text{m}^3)\ \ (107.52...)$ ***A1***

(iii) correct approach to find missing lengths ***(A1)***

$\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$ **OR** $\sin\left(\frac{\pi}{3}\right)$ and Pythagoras etc seen in work

$3\left[\frac{1}{2}(10+10+4\cos\left(\frac{\pi}{3}\right))4\sin\left(\frac{\pi}{3}\right)\right]$

$=114\,(\text{m}^3)\ \ (114.315\ldots)$ ***A1***

[9 marks]

(b) $V=3\left[\frac{1}{2}(10+10+4\cos(\theta))4\sin(\theta)\right]$ ***A1***

all correct intermediate working leading to given answer ***A1***

e.g. $V=6\sin(\theta)(20+4\cos(\theta))$

$V=24\sin(\theta)(5+\cos(\theta))$ ***AG***

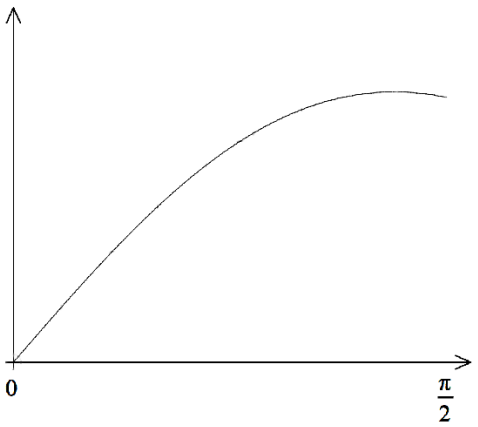
Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(c) *accept any reasoning along the lines:* "skip would have zero volume" or "if the angle is zero, then the contents would fall out" ***R1***

[1 mark]

(d) (i)



A1A1

Note: Award ***A1*** for the correct shape and ***A1*** for the graph on the correct, labelled, domain. Condone omission of θ / V labels (or x/y).

(ii) $\theta = 1.38 \ (1.38356...) \ (79.3^\circ \ (79.2723...^\circ))$ ***A1***

$V_{\max} = 122 \ (122.292...)$ ***A1***

Note: Award ***A0A1*** if values are reversed and ***A0A0*** for a coordinate pair.

[4 marks]

(e) recognizing that derivative is equal to zero (seen at any stage) ***M1***

$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ (accept $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$)

(from graph, turning point is a global maximum)

use of product rule ***M1***

$\left(\frac{\mathrm{d}V}{\mathrm{d}\theta} =\right) 24\cos(\theta)\left(5 + \cos(\theta)\right) + 24\sin(\theta)\left(-\sin(\theta)\right)$ ***A1***

$= 120\cos(\theta) + 24\cos^2(\theta) - 24\sin^2(\theta) \ (= 0)$ (or equivalent) ***A1***

substituting $1 - \cos^2(\theta)$ for $\sin^2(\theta)$ ***M1***

e.g $120\cos(\theta) + 24\cos^2(\theta) - 24(1 - \cos^2(\theta)) \ (= 0)$

correct intermediate steps leading to given answer ***A1***

$2\cos^2(\theta) + 5\cos(\theta) - 1 = 0$ ***AG***

[6 marks]

[Total: 22 marks]

4. (a) AEDCFBA ***A1A1***

Note: Award ***A1*** for AE at start, ***A1*** for correct completed route.

attempt to find the length of their route
length $22+21+19+24+25+31$ ***(M1)***
$=142$ (km) ***A1***

Note: Award ***A1A0M1A0*** for omitted final edge and their sum.

[4 marks]

(b) attempt to form MST without vertex A ***(M1)***

Note: Exactly 4 edges that form a spanning tree are required.

BD DC DE DF **OR** 20, 19, 21, 22 seen in that order ***A1***

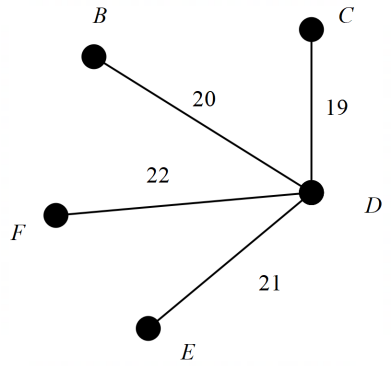
Note: Award ***M1A0*** for diagram of MST.

attempt to reconnect vertex A (one edge is sufficient) ***(M1)***
reconnecting A: AE (22) and AF (23) ***(A1)***
lower bound: $20+19+21+22+22+23$
$=127$ ***A1***

Note: If 127 seen, unsupported or without the explicit evidence of Prim's algorithm, award ***M1A0M1A1A1***.

[5 marks]

(c)



(A1)

Note: Condone the omission of the weights from their diagram. The diagram may include A with its two edges.

correct reasoning based on lack of cycle (once A is reattached) ***R1***
e.g. edges BD and CD would be repeated
this lower bound is not achievable (in this way) ***A1***

Note: Do not award ***R0A1***.

[3 marks]
[Total: 12 marks]

2 (a) (i) $(34, 4.1)$ ***A1A1***

[2 marks]

(ii) recognizing the sequence is arithmetic, with a common difference of 0.5 and 1.0 ***(M1)***

$(30+(n-1),\ 2.1+(n-1)0.5)$ ***A1A1***

$(=(29+n,\ 1.6+0.5n))$

[3 marks]

(b) (i) evidence of integration of the acceleration vector **OR** use of $\boldsymbol{v} = \boldsymbol{u} + \boldsymbol{a}t$ ***(M1)***

$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} c_1 \\ -9.8t + c_2 \end{pmatrix}$

$\left(\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 29.4\cos\theta \\ 29.4\sin\theta - 9.8t \end{pmatrix}\right)$ ***A1A1***

Note: The first ***A1*** is for $\dot{x}$ and the second is for $\dot{y}$.

[3 marks]

(ii) $\theta = 90^\circ$ ***(A1)***

$29.4 - 9.8t = 0$ ***M1A1***

Note: Award ***M1*** for setting their $\dot{y}$ to zero (may still include θ), ***A1*** for correct equation, leading to given result.
If they substitute $t = 3$ award at most ***(A1)M0A0.***

maximum point when $t = \dfrac{29.4}{9.8}$

$= 3$ (seconds) ***AG***

[3 marks]

(c) correct substitution OR use of correct graph ***(M1)***

maximum height is $29.4 \times 3 - 4.9 \times 3^2$

$= 44.1$ (m) ***A1***

[2 marks]

continued…

Question 2 continued.

(d) (i) $29.4\sin\theta t - 4.9t^2 = 0$ ***M1***

$t = 6\sin\theta$ (or $t = 0$) ***A1***

$x = 29.4\cos\theta \times 6\sin\theta$ ***A1***

$= 176.4\cos\theta\sin\theta$ ***AG***

[3 marks]

(ii) valid method to find maximum (e.g. sketch graph, find derivative) ***(M1)***

maximum value of x is 88.2 (m) ***A1***

Note: Award ***(M1)A0*** for an unsupported answer of "45" or $\frac{\pi}{4}$ $(0.785398...)$.

[2 marks]

(e) (i) $(c =)\ 44.1$ ***A1***

Note: follow through from 2c

[1 mark]

(ii) **EITHER**

$\frac{dy}{dx} = 0 \Rightarrow 2a \times 0 + b = 0$ ***(M1)***

OR

vertex is at $x = -\frac{b}{2a} = 0$ ***(M1)***

THEN

$\Rightarrow b = 0$ ***A1***

[2 marks]

(iii) point $(88.2, 0)$ used ***(A1)***

$0 = a \times 88.2^2 + 44.1$ ***(M1)***

$\Rightarrow a = -\frac{44.1}{88.2^2}$

$= -\frac{5}{882} = -0.0056689...$ ***A1***

[3 marks]

Note: The method for finding the parameters might appear in a different part, for example use of $(88.2, 0)$ might appear in part (ii), or the parts might not be clearly numbered. Accept any correct working wherever it is seen, and ascribe the marks to the correct part.

continued…

Question 2 continued.

(f) use of their arithmetic sequence from (a)(ii) with $n = 40$ ***(M1)***
coordinates of seat $A_{40} = (69, 21.6)$ ***(A1)***

EITHER

substitution of their 69 into their $y = -\frac{5}{882}x^2 + 44.1$ from part (e) ***(M1)***

$y = -\frac{5}{882} \times 69^2 + 44.1$
$= 17.1\ (17.1102...)$ ***A1***
$21.6 > 17.1$ ***R1***

OR

substitution of their $y=21.6$ into their $y = -\frac{5}{882}x^2 + 44.1$ from part (e) ***(M1)***

$21.6 = -\frac{5}{882}x^2 + 44.1$

$x = 63$ ***A1***

$69 > 63$ ***R1***

Note: Accept equivalent justification in words, provided both values are seen.
Award ***R1*** for correct reasoning only if ***M1*** has been awarded and their seat coordinates lie outside their equation for the curve.

so the T-shirt cannot reach seat A_{40} ***AG***

[5 marks]

[Total 29 marks]

1. (a) attempt to use the cosine rule to find $\mathrm{C\hat{A}B}$ ***(M1)***

$(\mathrm{C\hat{A}B} =)\ \cos^{-1}\left(\dfrac{42^2+35^2-36^2}{2\times 42\times 35}\right)$ ***(A1)***

$(\mathrm{C\hat{A}B} =)\ \ 54.8^\circ\ \ (54.8407\ldots^\circ)$ **OR** $0.957\ \ (0.957152\ldots)$ ***A1***

[3 marks]

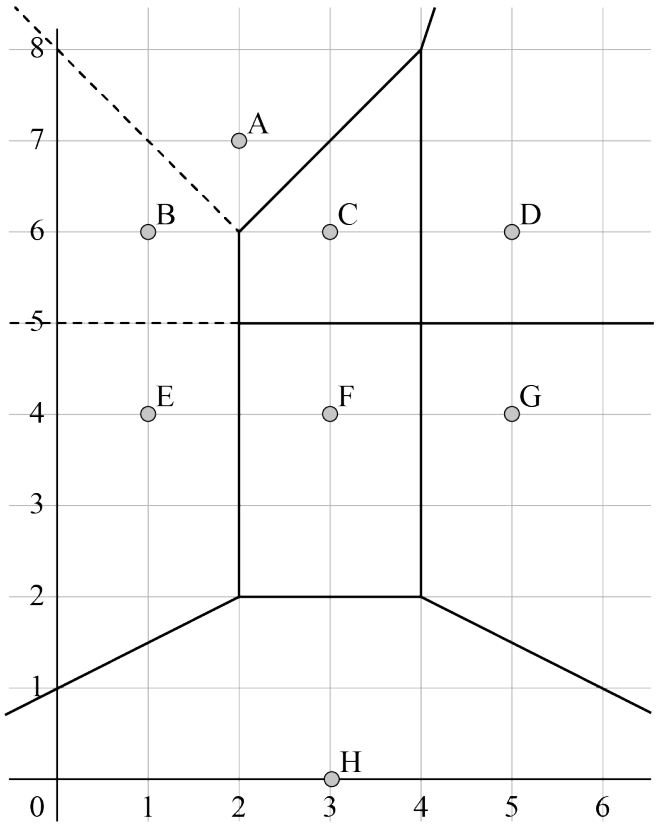
(b) $(\text{Area} =)\ \dfrac{1}{2}\times 42\times 35\times \sin 54.8407\ldots^\circ$ ***(A1)***

$(\text{Area} =)\ 601\ \ (600.903)\ \text{cm}^2$ ***A1***

Note: Correct units must be seen for the final ***A1*** to be awarded.

[2 marks]
[Total 5 marks]

5. (a)



A1A1

Note: Award at most ***A1A0*** if lines are not straight OR not accurate through the midpoint of AB and $(0, 8)$ and through the midpoint of BE and $(0, 5)$.

[2 marks]

(b) (i) area of $F = 6$ **OR** area of $C = 4$ ***(A1)***

$(6 \times 4 =)\ 24\ (\text{km}^2)$ ***A1***

(ii) $(4 \times 4 =)\ 16\ (\text{km}^2)$ ***A1***

[3 marks]

(c) The population in each cell might be equal ***R1***

[1 mark]

[Total 6 marks]

4. (a) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{3.4^2 - 2^2}$

$= 2.75\ (2.74954\ldots)$ (m) ***A1***

[2 marks]

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$60°$ ***(A1)***

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$ ***(M1)***

$\frac{1}{2} \times 2 \times 2 \times \sin(60°)$

$\left(6 \times \frac{1}{2} \times 2 \times 2 \times \sin(60^\circ)\right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 2 (Use of altitude)

(altitude is) $\sqrt{3}$ ***(A1)***

attempt to find the area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude. ***(M1)***

$\frac{1}{2} \times 2 \times \sqrt{3}$

$\left(6 \times \frac{1}{2} \times 2 \times \sqrt{3}\right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 3 (Finding the area of a trapezoid)

(altitude of one trapezoid is) $\sqrt{3}$ ***(A1)***

attempt to find area of one trapezoid using $\frac{1}{2} \times (a + b)\, h$ ***(M1)***

$\frac{1}{2} \times (2 + 4)\sqrt{3} \quad \left(3\sqrt{3}\right)$

$\left(2 \times \frac{1}{2} \times (2 + 4)\sqrt{3}\ \right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

continue…

Question 4 continued.

(ii) $\frac{1}{3} \times 10.3923... \times 2.74954...$ ***(A1)***

$= 9.52 \text{ m}^3 \ (9.52470\ldots)$ ***A1***

Note: Units must be seen.

[5 marks]

(c) $\cos(\hat{MAT}) = \frac{2}{3.4}$ or correct equivalent ***(A1)***

$(MAT =)\ 54.0^\circ$ (53.9681…, 0.941921… radians) ***A1***

[2 marks]

(d) Angle $YAX = 180 - 53.9681... = 126.031\ldots^\circ$ ***(A1)***

Angle $YXA = 180 - 35 - 126.031... = 18.9681\ldots^\circ$ ***(A1)***

Note: These angles may be seen in the sine rule.

Attempt to substitute into sine rule ***(M1)***

$$\frac{AY}{\sin(18.9681...)} = \frac{2.6}{\sin(126.031)}$$

$AY = 1.05 \ \ (1.04503\ldots)$ (m) ***A1***

[4 marks]

(e) **METHOD 1 COSINE RULE**

attempt to substitute into cosine rule to form a quadratic for YZ ***(M1)***

$0.9^2 = YZ^2 + 1.04503...^2 - 2 \times 1.04503... \times YZ \times \cos(35)$ ***(A1)***

$YZ = 0.185\ (0.184692\ldots)$ (m) , $1.53\ \ (1.52739\ldots)$ (m) ***A1A1***

Note: Accept $0.191\ (0.191313\ldots)$ from use of 3 s.f. values.

METHOD 2 SINE RULE

attempt to substitute into sine rule to find $Y\hat{Z}A$

$$\frac{\sin(Y\hat{Z}A)}{1.04503\ldots} = \frac{\sin(35^\circ)}{0.9}$$ ***(M1)***

$Y\hat{Z}A = 41.7597\ldots$ or $Y\hat{Z}A = 138.240\ldots$

$Z\hat{A}Y = 103.240\ldots$ or $Z\hat{A}Y = 6.75972\ldots$ ***(A1)***

Note: Award ***A1*** for any of these angles seen.

$$\frac{YZ}{\sin(Z\hat{A}Y)} = \frac{0.9}{\sin(35^\circ)}$$

$YZ = 0.185\ (0.184692\ldots)$ (m) , $1.53\ \ (1.52739\ldots)$ (m) ***A1A1***

[4 marks]

[Total: 17 marks]

(50)

5. (a) recognition that speed is the magnitude of $\begin{pmatrix} -33 \\ 0 \end{pmatrix}$ ***(M1)***

$\sqrt{50^2 + (-33)^2}$ **OR** $\left\| \begin{pmatrix} 50 \\ -33 \\ 0 \end{pmatrix} \right\|$

$= 59.9 \text{ (km h}^{-1}\text{)}. (59.9082\ldots)$ ***A1***

[2 marks]

(b) $\begin{pmatrix} 50 \\ -33 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix} = -90$ ***(A1)***

$\cos\theta = \dfrac{-90}{59.9082... \times 25}$ ***(M1)***

$\theta = 93.4^\circ \left(93.4450\ldots^\circ,\ 1.63092\ldots\right)$ ***A1***

[3 marks]

(c) $\overrightarrow{\text{OD}} = \begin{pmatrix} 200 \\ -100 \\ 0.02 \end{pmatrix} + t \begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix}$ ***A1***

[1 mark]

(d) (i) $\begin{pmatrix} 200 \\ -100 \\ 0.02 \end{pmatrix} + t \begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix} = \begin{pmatrix} 152 \\ p \\ 0.02 \end{pmatrix}$ **OR** $200 = -15t + 152$ ***(M1)***

$t_1 = 3.2 \left(\dfrac{16}{5}\right)$ ***A1***

(ii) $p = -164$ ***A1***

[3 marks]

continue…

Question 5 continued.

(e) (i) attempt to find difference between the two position vectors ***(M1)***

$\begin{pmatrix} 190-65t \\ -95+13t \\ 0.02 \end{pmatrix}$ ***A1***

Note: Award ***A1M1A0*** for $\begin{pmatrix} -190+65t \\ 95-13t \\ -0.02 \end{pmatrix}$.

(ii) attempt to find $\left|\begin{pmatrix} 190-65t \\ -95+13t \\ 0.02 \end{pmatrix}\right|$ ***(M1)***

$\sqrt{(190-65t)^2+(13t-95)^2+0.02^2}$ ***(A1)***

attempt to find minimum. (e.g. $t = 3.09$ hours) ***(M1)***

closest distance = $55.9 \quad (55.8931\ldots) \quad (\text{km})$ ***A1***

[6 marks]

[Total: 15 marks]

6. (a) anticlockwise rotation of $15°$ about the origin ***A1***
[1 mark]

(b) recognizing that *I* is equivalent to one rotation of $360°$ ***(M1)***

e.g. $\frac{360}{15}$

$= 24$ ***A1***
[2 marks]

(c) (i) $(\boldsymbol{B} =) \begin{pmatrix} 1.05 & 0 \\ 0 & 1.05 \end{pmatrix}$ ***A1***

(ii) $(\boldsymbol{B}^{24} =) \begin{pmatrix} 3.23 & 0 \\ 0 & 3.23 \end{pmatrix}$ ***(A1)***

Note: Award ***A1*** for 3.23 $(3.22509\ldots)$ **OR** 1.05^{24}.

enlargement, with a scale factor of 3.23 (1.05^{24}), (centre $(0, 0)$) ***A1***
[3 marks]

(d) $\boldsymbol{C} = \begin{pmatrix} 1.05\cos(15^\circ) & -1.05\sin(15^\circ) \\ 1.05\sin(15^\circ) & 1.05\cos(15^\circ) \end{pmatrix}$

$\left(= \begin{pmatrix} 1.01\ (1.01422...) & -0.272\ (-0.271759...) \\ 0.272\ (0.271759...) & 1.01\ (1.01422...) \end{pmatrix} = \frac{21}{80}\begin{pmatrix} \sqrt{6}+\sqrt{2} & -\sqrt{6}+\sqrt{2} \\ \sqrt{6}-\sqrt{2} & \sqrt{6}+\sqrt{2} \end{pmatrix}\right)$ ***A2***

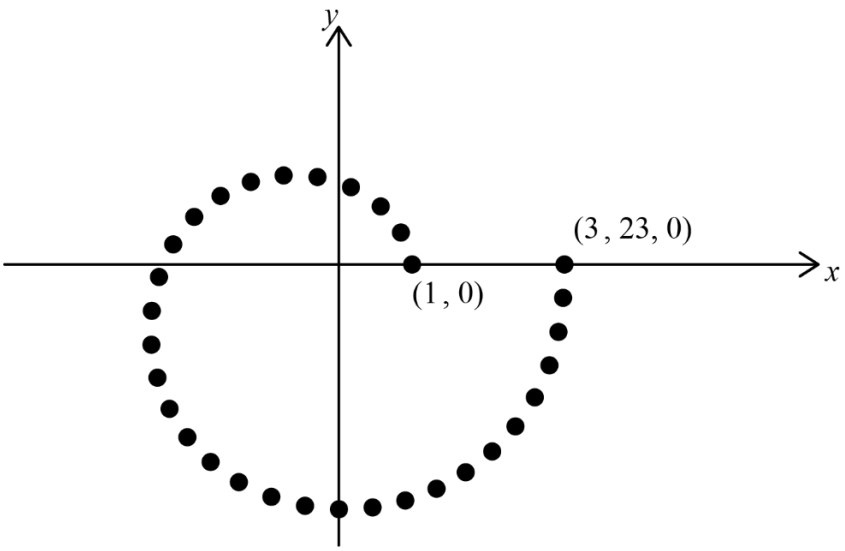
Note: Award ***A1*** for at least two correct elements in the matrix.

[2 marks]

continue…

Question 6 continued.

(e)



anticlockwise spiral ***A1***

starting at $(1, 0)$ and ending at $(3.23, 0)$ ***A1***

Note: Accept a continuous curve instead of discrete points. The two ***A1*** marks can be awarded independently

[2 marks]

(f) $T\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$ ***(M1)***

$0.9p + 2 = p$ **OR** $0.8q + 1 = q$ ***(A1)***

$p = 20, q = 5$ **OR** $\begin{pmatrix} 20 \\ 5 \end{pmatrix}$ ***A1***

[3 marks]

[Total: 13 marks]

1. (a) attempt to calculate area of a circle using 4 or 2.8 ***M1***

recognition of multiplication by $\frac{1}{5}$ or $\frac{72}{360}$ ***M1***

$A = \frac{1}{5}\left(\pi(4)^2 - \pi(2.8)^2\right)$

$5.13\ (\text{m}^2)$ $(5.12707\ldots\ (\text{m}^2),\ \frac{204\pi}{125}\ (\text{m}^2),\ 1.632\pi\ (\text{m}^2))$ ***A1***

$5.12707\ldots < 6$ **OR** Maureen has enough ***AG***

Note: Do not award the final ***A1*** for the area unless a comparison to 6 is shown **OR** a statement of having enough is seen.

[3 marks]

(b) $(C =)\ 2\pi(4)\quad (25.1327\ldots)$ ***(A1)***

recognition to multiply by $\frac{3}{5}$ or $\frac{216}{360}$ ***(M1)***

$\frac{3}{5} \times 2\pi(4) + 2 \times 4$

$23.1\ (\text{m})\quad (23.0796\ldots(\text{m}),\ \frac{24\pi}{5} + 8)$ ***A1***

[3 marks]

[Total: 6 marks]

5. (a) (i) $170°$ ***(A1)***

attempt to substitute into the cosine rule formula ***(M1)***

$(DC^2) = 2^2 + 13^2 - 2(2)(13)\cos(170°)$ ***(A1)***

$(DC =)\ 15.0\ (14.9736\ldots)\ (\text{km})$ ***A1***

Note: Do not accept a 2sf answer of 15. Award ***M0A0A0*** for a 2sf answer of 15 with no working shown OR $2+13=15$.

(ii) correct substitution into sine rule or cosine rule ***(A1)***

$\frac{\sin B\hat{D}C}{13} = \frac{\sin 170°}{14.9736\ldots}$ **OR** $\cos(B\hat{D}C) = \frac{14.9736...^2 + 2^2 - 13^2}{2 \times 14.9736... \times 2}$

$(B\hat{D}C =)\ 8.67°\ (8.67097\ldots°)$ ***(A1)***

(bearing $=$) $098.7°\ (098.6709\ldots°)$ ***A1***

[7 marks]

(b) (distance between points D and E $=$) 13 (km) ***A1***

points B, C, D, and E make a parallelogram **OR** DB and EC are parallel **and** equal in length ***R1***

Note: It is possible to award ***A1R0*** for this question part.

[2 marks]
[Total 9 marks]

13. (a) an anticlockwise/counter-clockwise rotation of $\frac{\pi}{3}$ (about the origin) ***A1A1***

Note: Award ***A1*** for rotation and ***A1*** for correct angle/direction detail.

[2 marks]

(b) attempt to multiply $\boldsymbol{A}$ and $\boldsymbol{B}$ in correct order ***(M1)***

$$\boldsymbol{BA} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix} \left(\begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.5 & -0.866025\ldots \\ 0.866025\ldots & 0.5 \end{bmatrix} \right)$$

$$\begin{bmatrix} 0.119615\ldots & 0.992820\ldots \\ 0.992820\ldots & -0.119615\ldots \end{bmatrix}$$ ***(A1)***

attempt to create an expression with either $\tan 2\theta$ or $\sin 2\theta$ or $\cos 2\theta$ ***(M1)***

$$\tan 2\theta = \frac{0.992820\ldots}{0.119615\ldots}$$

$\theta = 0.725446\ldots$ ***(A1)***

$y = \tan(0.725446\ldots)x$

$a = 0.887 \ \ (0.886751\ldots)$ ***A1***

Note: Award at most ***(M1)(A1)(M1)(A1)A0*** if the extraneous value of a is given alongside a correct answer.

[5 marks]

[Total 7 marks]

2. (a) use of Pythagoras in 3 dimensions ***(M1)***

$\text{AB}^2 = 2^2 + 3^2 + 1^2$

$\text{AB} = 3.74 \quad (= 3.74165... = \sqrt{14})$ ***A1***

[2 marks]

(b) **METHOD 1 (using Pythagoras in 3D)**

$1^2 + 6^2 + (k-1)^2 = 14 + 1^2 + 3^2 + (k-2)^2$ ***(M1)A1***

Note: Award ***M1*** for use of Pythagoras in three dimensions, ***A1*** for correct use.

$38 - 2k = 28 - 4k$ ***(M1)***

Note: Award ***M1*** for reducing equation to linear form.

$k = -5$ ***A1***

METHOD 2 (using vectors)

find the vectors $\vec{\text{AB}}$ and $\vec{\text{CB}}$ ***(M1)***

$\vec{\text{AB}} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad \vec{\text{CB}} = \begin{pmatrix} -1 \\ 3 \\ k-2 \end{pmatrix}$

$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ k-2 \end{pmatrix} = 0$ ***A1***

calculate scalar product ***(M1)***

$-2 + 9 + k - 2 = 0$

$k = -5$ ***A1***

[4 marks]

(c) **EITHER**

$\cos(\text{B}\hat{\text{A}}\text{C}) = \frac{\sqrt{14}}{\sqrt{73}}$ ***(M1)(A1)***

OR

$\cos\left(\text{B}\hat{\text{A}}\text{C}\right) = \frac{\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}}{\sqrt{14}\sqrt{73}}$ ***(M1)(A1)***

OR

$\sin(\text{B}\hat{\text{A}}\text{C}) = \frac{\sqrt{59}}{\sqrt{73}}$ ***(M1)(A1)***

continued…

Question 2 continued

OR

$\tan(\hat{BAC}) = \frac{\sqrt{59}}{\sqrt{14}}$ ***(M1)(A1)***

Note: Award ***M1*** for use of trig ratio, ***A1*** for correct use.

THEN

$\hat{BAC} = 64.0° \quad (64.0282...^{\circ}) \quad (1.12\,(1.11750...))$ ***A1***

Note: Accept 64 for the final ***A1***.

[3 marks]
Total [9 marks]

4. (a) (i) $(8,\ 4)$ ***A1***

Note: Do not condone the omission of parenthesis.

(ii) 5 ***A1***

[2 marks]

(b) (i) 8 ***A1***

(ii) **EITHER**

attempt to use Pythagoras theorem ***(M1)***

$4+(b-5)^2=9+(9-b)^2$ ***(A1)***

$(b=)\frac{61}{8}=7.625\ \ \left(\frac{61}{8}, 7\frac{5}{8}\right)$ ***A2***

Note: Award **A*1A0*** for 7.63

OR

$m(BD)=-\frac{4}{5}$ ***(A1)***

midpoint $[BD]=(7.5,\ 7)$ ***(A1)***

Note: Gradient and midpoint may be implied by further working. Condone as interim coordinate $7.5, 7$ instead of $(7.5, 7)$.

$(y-7)=\frac{5}{4}(x-7.5)$

substitute $x=8$ in their perpendicular bisector ***(M1)***

$b-7=\frac{5}{4}(8-7.5)$

$(b=)7.625\ \ \left(\frac{61}{8}, 7\frac{5}{8}\right)$ ***A1***

Note: Award **A*1A1M1A0*** final answer of 7.63

OR

$m(\mathrm{DF})=-4$ ***(A1)***

midpoint $[\mathrm{DF}]=(5.5\ , 7)$ ***(A1)***

Note: Gradient and midpoint may be implied by further working. Condone as interim coordinate $5.5, 7$ instead of $(5.5, 7)$

continued…

Question 4 continued

$(y-7)=\frac{1}{4}(x-5.5)$

substitute $x=8$ in their perpendicular bisector ***(M1)***

$(8-7)=\frac{1}{4}(b-5.5)$

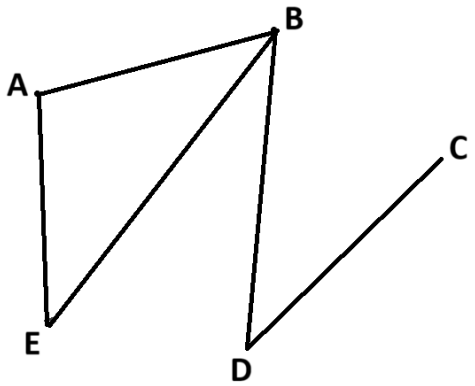
$(b=)\,7.625\ \left(\frac{61}{8}, 7\frac{5}{8}\right)$ ***A1***

Note: Award **A*1A1M1A0*** final answer of 7.63

Note: Both a and b can be found by locating the intersection of two perpendicular bisectors. This is an inefficient method, but correct answers should be awarded full marks.

[5 marks]
Total [7 marks]

8. (a)



A2

[2 marks]

Note: For graphs with edges that cross each other but are otherwise correct, award ***A1***. Award ***A2*** for a directed graph where each edge is bi-directional.

(b) recognition that A^4 is required **(*M1*)**

$$A^4 = \begin{pmatrix} 7 & 6 & 1 & 5 & 6 \\ 6 & 12 & 4 & 2 & 6 \\ 1 & 4 & 2 & 0 & 1 \\ 5 & 2 & 0 & 6 & 5 \\ 6 & 6 & 1 & 5 & 7 \end{pmatrix}$$ **(*A1*)**

recognition that diagonal entries are ones which return to the same town. **(*M1*)**
total of 34 walks ***A1***

Note: For incomplete attempts to list all possible walks of length four award ***M0A0M0A0***.

[4 marks]
Total [6 marks]

Q1.

(a) attempt to use the cosine rule formula ***(M1)***

$(\text{NV} =)\sqrt{20^2 + 25^2 - 2\times 20\times 25\cos 55.5^\circ}$ ***(A1)***

$= 21.4\,(\text{m})\,(21.4148...)$ ***A1***

[3 marks]

(b) **EITHER**

attempt to use the sine rule formula ***(M1)***

$\frac{\sin \hat{\text{PNV}}}{25} = \frac{\sin 55.5^\circ}{21.4148...}$ ***(A1)***

OR

attempt to use the cosine rule formula ***(M1)***

$\cos \hat{\text{PNV}} = \frac{20^2 + 21.4148...^2 - 25^2}{2\times 20\times 21.4148...}$ ***(A1)***

THEN

$\hat{\text{PNV}} = 74.2^\circ\,(74.1749...^\circ)$ ***A1***

[3 marks]

(c) recognition that shortest distance is perpendicular to NV ***(M1)***

(may be seen on diagram)

use of right-angled triangle or other valid method ***(M1)***

$\sin 74.1749...\times 20$ OR $\frac{1}{2}\times 20\times 25\times \sin 55.5^\circ \times \frac{2}{21.4148...}$

$= 19.2\,(\text{m})\,(19.2419...)$ ***A1***

Note: Accept 19.3 (m) from using their 3 sf value in $\frac{1}{2}\times 20\times 25\times \sin 55.5^\circ \times \frac{2}{21.4148...}$

[3 marks]

Total [9 marks]

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Q8. **EITHER**

attempt to find matrix for compound transformation ***(M1)***

$\begin{pmatrix}3 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\left(=\begin{pmatrix}1 & 3\\1 & 0\end{pmatrix}\right)$ ***(A1)***

$\begin{pmatrix}1 & 3\\1 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}4\\7\end{pmatrix}$ ***(A1)***

multiply by the inverse(s) or form two equations ***(M1)***

OR

$\begin{pmatrix}3 & 1\\0 & 1\end{pmatrix}\begin{pmatrix}b_1\\b_2\end{pmatrix}=\begin{pmatrix}4\\7\end{pmatrix}$ ***(A1)***

multiply by the inverse or form two equations ***(M1)***

coordinates of B are $(-1, 7)$ ***(A1)***

attempt to find coordinates for A by setting up matrix ***(M1)***

$\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}a_1\\a_2\end{pmatrix}=\begin{pmatrix}-1\\7\end{pmatrix}$

THEN

$(7,-1)$ ***A1***

Total [5 marks]

Q14. (a) $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} -20 \\ -9.8t \end{pmatrix}$ ***A1A1***

[2 marks]

(b) $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -20t + d \\ -4.9t^2 + 70 \end{pmatrix}$ ***(A1)(A1)***

recognition that package hits the ground when $-4.9t^2 + 70 = 0$ ***(M1)***

$t^2 = \frac{70}{4.9} \Rightarrow t = 3.77964...$ seconds ***(A1)***

attempt to solve $-20t + d = 0$ ***(M1)***

$d = 20 \times 3.7796...$

distance $d = 75.6\ (75.5928...)$ (m) ***A1***

[6 marks]
Total [8 marks]

2. (a) (i) attempt to substitute any given coordinates into midpoint formula ***(M1)***

$(12, 12.5)$ or $\left(12, \frac{25}{2}\right)$ ***A1***

Note: Award ***A0*** if parentheses are omitted.

(ii) attempt to find gradient of BC using coordinates of B and C ***(M1)***

(gradient of $\mathrm{BC}=$) $\frac{12-13}{16-8}\left(=-\frac{1}{8}=-0.125\right)$ ***(A1)***

(gradient of $\perp=$) 8 ***A1***

Note: Follow through within question part for final ***A1*** from their gradient of BC only if the gradient is seen.

(iii) **EITHER**

$8(x-12)=y-12.5 \quad \left(8\left(x-12\right)=y-12\frac{1}{2}\right)$ ***A1***

OR

$y=8x-83.5 \quad \left(y=8x-83\frac{1}{2}\right)$ ***A1***

[6 marks]

(b) (i) **EITHER**

B is expected to respond since $(14, 10)$ lies in the same cell (of the Voronoi diagram) as fire station B ***R1***

OR

B is expected to respond since $(14, 10)$ is closer to B than to any other fire station ***R1***

OR

B is expected to respond since it's the closest (nearest responder) to $(14, 10)$ ***R1***

OR

Fire station B will respond as the fire is within B's boundaries/area/region. ***R1***

Note: Do not award ***R1*** unless the statement refers to both the distance (closer) to B (or located in the B cell) **and** the responder (point B or fire station B) For example, award ***R0*** for "Point B is closer to $(14, 10)$" as there is no reference to a responder.

(ii) *Award **R1** for any viable reason such as:*

- fire station B might have already responded to a different fire
- a different fire station might have more manpower or equipment available to fight the fire
- a different fire station might respond to assist fire station B if the fire is large
- the Voronoi diagram only considers straight-line distances, so a different fire station might have a shorter driving route to the fire ***R1***

[2 marks]

(c) (i) attempt to calculate the length DB ***(M1)***

a correct expression for DB ***A1***

$(\mathrm{DB}=)\ \sqrt{\left(11\frac{1}{3}-16\right)^2+\left(7\frac{1}{6}-12\right)^2}$

$\mathrm{DB}=6.71855$ ***AG***

(ii) **METHOD 1**

$(\mathrm{DC}=)\ \sqrt{\left(11\frac{1}{3}-8\right)^2+\left(7\frac{1}{6}-13\right)^2}$ ***A1***

$\mathrm{DC}=6.71855$ therefore, the fire is equidistant from all 3 fire stations ***R1***

so, any of the fire stations would be expected to respond ***AG***

Note: Do not award ***A0R1***. Do not award ***R1*** if ***AG*** statement is not seen.

METHOD 2

The intersection of bisector AB and bisector BC:

$y=8x-83.5$ and $y=0.5x+1.5$ have an intersection at $\mathrm{D}\left(11\frac{1}{3},\ 7\frac{1}{6}\right)$ ***A1***

Note: Calculator solve is acceptable but D must be clearly identified from a calculation of intersecting lines.

Since D is the intersection of the bisector lines, the fire at D is equidistant from all 3 fire stations ***R1***

so, any of the fire stations would be expected to respond ***AG***

Note: Do not award ***A0R1.*** Do not award ***R1*** if ***AG*** statement is not seen.

[4 marks]

(d) (i) **METHOD 1: Dividing into two triangles**

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

$(20-7.75=)\ 12.25$ **OR** $(20-11.3333\ldots=)\ 8.66666\ldots$ seen ***(A1)***

substitution into triangle area formula for either triangle ***(M1)***

$(A=)\ 0.5\times(20-7.75)\times 7.16666\ldots+0.5\times 11.5\times(20-11.3333\ldots)$ ***(A1)***

$93.7291\ldots\ \left(\frac{4499}{98}\right)$ (sq. units)

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final A1.
Award ***(A1)(A1)(M1)(A1)A0*** for an answer of 93.7 without working.

METHOD 2: Dividing vertically into triangle and trapezoid

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

attempt to substitute into area of trapezoid formula ***(M1)***
$(A=)\ 0.5\times(7.16666\ldots+11.5)\times(20-11.3333\ldots)$

Note: Lengths of parallel sides must be seen to award the ***M1***.

correct calculation of right triangle area ***(A1)***
$(A=)\ 0.5\times(11.3333\ldots-7.75)\times 7.16666\ldots$

$(A=)\ 0.5\times(11.3333\ldots-7.75)\times 7.16666\ldots+0.5\times(7.16666\ldots+11.5)\times(20-11.3333\ldots)$ ***(A1)***

$93.7291\ldots\ \left(\frac{4499}{98}\right)$ (sq. units)

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(M1)(A1)(A1)A0*** for an answer of 93.7 without working.

METHOD 3: Dividing horizontally into triangle and trapezoid

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

attempt to substitute into area of trapezoid formula ***(M1)***

$(A=)\ 0.5\times\left[(20-7.75)+(20-11.3333\ldots)\right]\times 7.16666\ldots$

Note: Lengths of parallel sides must be seen to award the ***M1***.

correct calculation of right triangle area ***(A1)***

$(A=)\ 0.5\times(20-11.3333\ldots)\times(11.5-7.16666\ldots)$

$(A=)\ 0.5\times(20-11.3333\ldots)\times(11.5-7.16666\ldots)+0.5\times[(20-7.75)+(20-11.3333\ldots)]\times 7.16666\ldots$ ***(A1)***

$93.7291\ldots\ \left(\frac{4499}{98}\right)$ (sq. units)

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(M1)(A1)(A1)A0*** for an answer of 93.7 without working.

METHOD 4: Dividing into two triangles and rectangle

$(0.5\times 20+1.5=)11.5$ seen in an area calculation ***(A1)***

$(20-11.3333...)$ **OR** $(11.3333...-7.75)$ **OR** $(11.5-7.16666...)$ seen ***(A1)***

substitution into triangle area formula. ***(M1)***

$0.5\times(11.3333...-7.75)\times 7.16666...+(20-11.3333...)\times 7.16666...+$
$+0.5\times(11.5-7.16666...)\times(20-11.3333...)$

(A1)

$93.7291\ldots\ \left(\frac{4499}{98}\right)$ (sq. units))

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(A1)(M1)(A1)A0*** for an answer of 93.7 without working.

METHOD 5: Calculus

A and C mid point $(13, 10.5)$

(gradient of AC=) $\frac{13-8}{8-18} = -\frac{5}{10}$

(gradient of $\perp$=) 2

AC bisector $2(x-13) = y-10.5$ **OR** $y = 2x-15.5$ ***A1***

attempt to integrate their equation and given equation ***(M1)***

$\text{area} = \int_{7.75}^{11.3} 2x-15.5\,\mathrm{d}x + \int_{11.3}^{20} 0.5x+1.5\,\mathrm{d}x$ ***(A1)(A1)***

Note: Award ***A1*** for each correct expression with correct limits.

$93.7291\ldots \left(\frac{4499}{98}\right)$(sq. units))

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***A1(M1)(A1)(A1)A0*** for an answer of 93.7 without working.

(ii) (1 sq unit = $2.5\text{ km} \times 2.5\text{ km}$ =) $6.25\ (\text{km}^2)$ ***(A1)***

Note: Award ***A1*** for 6.25 seen.

$(94 \times 6.25\,\text{km}^2)$

588 (587.5) (km^2) ***A1***

Note: Accept: 586 (585.807…) (km^2) from $(93.7291\ldots \times 6.25\ \text{km}^2)$.

[7 marks]
[Total: 19 marks]

4. (a) $\sqrt{(-0.40)^2+(-0.35)^2+(0.1)^2}$ ***(A1)***

$0.541 \quad \left(0.540832..., \frac{3\sqrt{13}}{20}\right)$ (kilometres per minute) ***A1***

[2 marks]

(b) any two of the following: ***A1A1***

- flying in a straight line
- flying in an overall NW direction
- flying towards the west at 1.4 km min^{-1} (units required)
- flying towards the north at 1.65 km min^{-1} (units required)
- flying at a constant altitude / at an altitude of 7 kilometres / parallel to the ground
- passes through a point that is 5 kilometres east, 2 kilometres south, and 7 kilometres above the ground

Note: Statement must be about the path and not the speed/velocity. Do not accept any statement referring to moving towards the east at -1.4 km min^{-1}.

[2 marks]

(c) (i) $6+0.1t_1 = 7(+0t_1)$ ***(A1)***

$t_1 = 10$ (min) (accept a final answer of "$10{:}10 \text{ (am)}$") ***A1***

(ii) substitution of $t_1 = 10$ into either vector equation ***(M1)***

airplane (will have a position vector of) $\begin{pmatrix}5\\-2\\7\end{pmatrix}+(10)\begin{pmatrix}-1.4\\1.65\\0\end{pmatrix}=\begin{pmatrix}-9\\14.5\\7\end{pmatrix}$ ***A1***

birds (will have a position vector of) $\begin{pmatrix}-11\\30\\6\end{pmatrix}+(10)\begin{pmatrix}-0.40\\-0.35\\0.05\end{pmatrix}=\begin{pmatrix}-15\\26.5\\7\end{pmatrix}$ ***A1***

Note: All three of the above marks can be awarded to candidates who work with only the x-coordinates or only the y-coordinates. The complete position vectors do not need to be found.

x- **or** y- coordinates are different at the (only) time their z-coordinate is equal ***R1***

Note: The response must include a reference to the x- or y- coordinates being different to award the R1. Simply referring to the two position vectors is not sufficient. A statement saying that the 'north' and/or 'east' positions are different is sufficient. Allow FT within question part for final R mark for a correction statement related to their position vectors

therefore airplane and birds will not collide ***AG***

[6 marks]

(d) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (t_2 + 20)\begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (20)\begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -23 \\ 31 \\ 7 \end{pmatrix} + t_2\begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix}$ ***A1***

[2 marks]

(e) attempt to find difference between any coordinate at time t_2 ***(M1)***

$x = -95 + 1.3t_2 - (-23 - 1.4t_2)$ **and**
$y = 32 + 1.45t_2 - (31 + 1.65t_2)$ **and**
$z = 1.5 + 0.2t_2 - 7$ ***(A1)***

attempt to substitute their expressions into distance formula ***(M1)***
$d = \sqrt{(2.7t_2 - 72)^2 + (-0.2t_2 + 1)^2 + (0.2t_2 - 5.5)^2}$

(using GDC, minimum distance $=) 4.33 (4.32565\ldots)$ (kilometres) ***A1***

therefore the jet and the airplane will break the rule (since $4.33 < 5$) ***R1***

Note: Do not award ***R1*** unless value for minimum distance is seen. If no adjustment is made to the original passenger airplane vector equation, award at most ***(M1)(A0)(M1)(A0)R1***.

Allow FT within question part for final R mark as long as $0 < d < 10$ and their value of d comes from an attempt to use the distance formula.

[5 marks]
[Total 17 marks]

5. (a) correct substitution into distance formula ***(A1)***

$\sqrt{(13-13)^2+(25-1)^2+(7-0)^2}$ **OR** $\sqrt{0^2+24^2+7^2}$

Note: Accept omission of $(13-13)^2$ or 0^2.

$(\text{AC}=)\ 25\ (\text{cm})$ ***A1***

[2 marks]

(b) **METHOD 1**

$\text{AB} \parallel y$-axis **AND** $\text{BC} \parallel z$-axis $(\text{AB}=24\boldsymbol{k}$ **AND** $\text{BC}=7\boldsymbol{j})$ ***A1***

AB perpendicular to BC since y-axis is perpendicular to z-axis **OR** $\text{AB}\bullet\text{BC}=0$ ***R1***

therefore ΔABC must be a right-angled triangle. ***AG***

METHOD 2

24 and 7 seen ***A1***

$24^2+7^2=25^2$ **OR** $625=625$ **OR** (angle $\text{B}=$) $\cos^{-1}\left(\frac{7^2+24^2-25^2}{2(7)(24)}\right)=90^\circ$ ***R1***

Note: Accept $\text{AB}^2+\text{BC}^2=625$ as long as the length of AB and of BC are defined.

therefore ΔABC must be a right-angled triangle ***AG***

[2 marks]

(c) width of 9 or correct cross-section area (84) seen ***(A1)***

correct substitution into volume formula ***(A1)***

$V=\frac{1}{2}(24)(7)(9)$

$(\text{Volume}=)756\ \text{cm}^3$ ***A1***

Note: Do not award final ***A1*** if correct units are not included.

[3 marks]

(d) (i) correct use any trig ratio ***(M1)***

$\tan\theta = \frac{7}{24}$ **OR** $\sin\theta = \frac{7}{25}$ **OR** $\cos\theta = \frac{24}{25}$

$(\theta =)\ 16.3°\ (16.2602\ldots°)$ **OR** $0.284\left(0.283794\ldots\right)$ (radians) ***A1***

(ii) **METHOD 1**

recognition that the width of the door stop is still 9 ***(M1)***

$625 = \frac{1}{2}bh(9)$

$bh = \frac{1250}{9}$

$\tan(16.2602\ldots°) = \frac{h}{b}$ **OR** $\tan(0.283794\ldots) = \frac{h}{b}$ ***(M1)***

$b^2 = \frac{1250}{9\tan(16.2602\ldots°)}$ **OR** $b^2 = \frac{1250}{9\tan(0.283794\ldots)}$ ***(A1)***

$(b = \text{AX} =)\ 21.8\ (21.8217\ldots)$ (cm) ***A1***

METHOD 2 (area similarity)

recognition that the new side face will be similar to ABC ***(M1)***

proportionality of area $k^2 = \frac{625}{756}$ ***(A1)***

proportionality of length $k = \sqrt{\frac{625}{756}} = 0.909241$

new base length $\text{AX} = 24k$ ***(M1)***

$21.8\ \left(21.8217\ldots\right)$ (cm) ***A1***

[6 marks]

(e) multiplying 625 by 1.1 or 1.2 or 0.025 ***(M1)***

$625\times1.1\times0.025\times1.2$ ***(A1)***

(price $=$) $20.63\ (20.625)$ (USD) ***A1***

[3 marks]

[Total 16 marks]

4. (a) (since there are 5 vertices, 4 edges are required)

start with A

add E (AE = 0.86) ***M1***

add C (EC = 0.62)

add B (CB = 0.74)

add D (CD = 0.78) ***A2***

Note: The ***A2*** is awarded for a correct Prim's algorithm. It cannot be split.

the length of cable is $3(.00)$ (km). ***A1***

[4 marks]

(b) attempt to combine MST with AF ***(M1)***

LB = (length of minimum spanning tree) + (FA + FE)

$= 3 + 0.96 + 1.14$

$= 5.1(0)$ (km) ***A1***

[2 marks]

(c) attempt to use the nearest neighbour algorithm ***(M1)***

Note: Award ***M1*** for first 3 edges correct or 0.96, 0.86, 0.62 seen in order

FA , AE , EC, CB, BD, DF OR 0.96, 0.86, 0.62, 0.74, 0.94, 2.24 ***A1***

UB = 6.36 (km) ***A1***

[3 marks]

(d) use A or C for LB and A, B or F for UB ***A1A1***

any attempt to convert distances to time ***(M1)***

$\text{LB} = (5.02 \div 28) + (5 \times 0.2) = 1.18 \text{ hrs} \quad (1.17928...)$

$\text{UB} = (6.36 \div 28) + (5 \times 0.2) = 1.23 \text{ hrs} \quad (1.22714...)$

$1.18 \leq T \leq 1.23$ ***A1***

[4 marks]

Total [13 marks]

Q4. (a) between A and E ***A1***

[1 mark]

(b) (i) attempt to find $\boldsymbol{M}^3$ ***M1***

$$\boldsymbol{M}^3 = \begin{pmatrix} 1 & 3 & 1 & 1 & 3 \\ 3 & 1 & 3 & 1 & 4 \\ 0 & 3 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 2 \\ 0 & 4 & 0 & 2 & 1 \end{pmatrix}$$

contains 0 so some routes are not possible ***R1***

Note: Identification of any route which is not possible, award ***R1,*** provided ***M1*** has been awarded.

(ii) **METHOD 1**

attempt to find $\boldsymbol{M} + \boldsymbol{M}^2 + \boldsymbol{M}^3$ OR $\boldsymbol{M}^2 + \boldsymbol{M}^3$ ***M1***

$$\boldsymbol{M} + \boldsymbol{M}^2 + \boldsymbol{M}^3 = \begin{pmatrix} 2 & 5 & 2 & 2 & 5 \\ 4 & 4 & 4 & 2 & 6 \\ 1 & 4 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 5 & 1 & 3 & 3 \end{pmatrix}, \boldsymbol{M}^2 + \boldsymbol{M}^3 = \begin{pmatrix} 2 & 4 & 2 & 2 & 4 \\ 3 & 4 & 3 & 2 & 5 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 4 & 1 & 2 & 3 \end{pmatrix}$$

all ports can be reached in 3 trips or fewer, ***A1***

because there are no 0 entries ***R1***

Note: The complete matrix does not need to be seen for ***M1*** to be awarded. ***A1*** and ***R1*** are dependent on the ***M1.***

continued…

Question 4 continued.

METHOD 2

attempt to find $\boldsymbol{M}^2$ ***M1***

$$\boldsymbol{M}^2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 \end{pmatrix}$$

all ports can be reached in 3 trips or fewer, ***A1***

because there are no overlapping 0 entries in $\boldsymbol{M}^2$ and $\boldsymbol{M}^3$ ***R1***

Note: The complete matrix does not need to be seen for ***M1*** to be awarded. ***A1*** and ***R1*** are dependent on the ***M1.***

[5 marks]

(c) attempt to find the cost of a route of length 2 ***M1***

route CAD has cost $7+8=15$

route CBD has cost $4+9=13$

route CED has cost $10+3=13$ ***A1A1***

Note: Award ***A1*** for at least one correct answer, ***A1*** for all three correct answers.
Accept CBED has cost $4+6+3=13$ in place of CBD and CED

hence least cost is 13 euros ***AG***

[3 marks]

continued…

Question 4 continued.

(d) evidence of correct algorithm used, $\mathrm{ABC}\ldots$ seen, or in a diagram ***(M1)***

ABCEDA OR $3+4+10+3+8$ ***A1***

cost 28 euros ***A1***

[3 marks]

(e) evidence of selecting three edges not including vertex A ***(M1)***

length of spanning tree $=4+6+3(=13)$ ***(A1)***

reconnect vertex A using two distinct edges ***(M1)***

lower bound $(13+3+5=)21$ ***A1***

[4 marks]

[Total 16 marks]

Q5. (a) $(2,\ 1,\ 0)$ ***A1***

Note: Condone coordinates written as a vector.

[1 mark]

(b) (i) $5\begin{pmatrix}-4\\4\\7\end{pmatrix}$ or $\begin{pmatrix}-18\\21\\35\end{pmatrix}-\begin{pmatrix}2\\1\\0\end{pmatrix}$ ***(M1)***

Note: Condone one error in the position vector for B for ***M1***

$\begin{pmatrix}-20\\20\\35\end{pmatrix}$ ***A1***□

(ii) use of formula for magnitude with their $\vec{AB}$ ***(M1)***

$\sqrt{(-20)^2+20^2+(35)^2}$

$45\ (\text{m})$ ***A1***

Note: Do not accept $\sqrt{2025}$.

[4 marks]

(c) **METHOD 1**

use of two direction vectors ***(M1)***

use of formula for scalar product ***(M1)***

$\begin{pmatrix}-4\\4\\7\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\0\end{pmatrix}=12$ (OR equivalent multiple of 12) ***(A1)***

$\cos\left(C\hat{A}B\right)=\frac{12}{\sqrt{81}\sqrt{5}}$ ***(A1)***

$C\hat{A}B=53.4^{\circ}\left(53.3957\ldots^{\circ},0.932,\ 0.931931...\right)$ ***A1***

continued…

Question 5 continued.

METHOD 2

attempt to find a point C using $\begin{pmatrix}2\\1\\0\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\0\end{pmatrix}$ or some value of λ ***(M1)***

eg $C=(1,3,0)$

$\left|\vec{AC}\right|=\sqrt{5}$, $\left|\vec{BC}\right|=43.7\left(=\sqrt{1910}\right)$ OR equivalent ***(A1)***

attempt to use the cosine rule ***(M1)***

$\cos\left(C\hat{A}B\right)=\dfrac{5+45^2-1910}{2\times\sqrt{5}\times45}$ OR equivalent ***(A1)***

$C\hat{A}B=53.4^\circ\left(53.3957\ldots^\circ, 0.932, 0.931931...\right)$ ***A1***

[5 marks]

(d) let P_2 be at the point D when $t=5$

EITHER

use of distance = speed $\times$ time ***(M1)***

distance travelled by $P_2=12\times5$

$=60$ ***(A1)***

use of cosine rule ***(M1)***

$\left(\left|\vec{BD}\right|^2=\right)45^2+60^2-2\times45\times60\cos(53.3957...)\left(=2405.06...\right)$ ***(A1)***

continued…

Question 5 continued.

OR

position of $P_1 \begin{pmatrix} -18 \\ 21 \\ 35 \end{pmatrix}$ ***(A1)***

position of P_2

attempt to calculate $r = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ for any value of λ ***(M1)***

$\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + 5 \times \frac{12}{\sqrt{5}} \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} \left(= \begin{pmatrix} -24.8328... \\ 54.6656... \\ 0 \end{pmatrix} \right)$ ***(A1)***

subtracting their position vectors for P_1 and P_2 ***(M1)***

$\vec{\mathrm{BD}} = \begin{pmatrix} -24.8328... \\ 54.6656... \\ 0 \end{pmatrix} - \begin{pmatrix} -18 \\ 21 \\ 35 \end{pmatrix}$

THEN

$\left| \vec{\mathrm{BD}} \right| = 49.0 \left(49.0414 \ldots \right)$ (m) ***A1***

[5 marks]

Total [15 marks]

2. (a) (i) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{12500^2 + 6000^2}$

$= 13900$ (km) ***A1***

Note: Answers to more than 3sf are not appropriate and should be penalised. ***A1*** will be given only for 13900km (3sf) or 14000km (nearest 500km).

[2 marks]

(ii) attempt to use an appropriate inverse trig ratio ***(M1)***

e.g. $\arctan\left(\frac{12500}{6000}\right)$ or $\arctan\left(\frac{6000}{12500}\right)$

identifying the correct angle ***(M1)***

$= (0)64.4^\circ$ OR $064^\circ (64.3589...)$ ***A1***

Note: The two method marks can be awarded independently. The first can come from any geometry leading to an inverse trigonometric ratio. The second requires recognizing which angle is the bearing, possibly on a diagram. 64.4 appearing in intermediate working is not sufficient.

[3 marks]

(b) (i) $\boldsymbol{p.n} = 0$ ***M1***

so the angle is 90° OR $\frac{\pi}{2}$ rads ***A1***

Note: Award ***M0A0*** for an answer of 90 degrees without any mention that the scalar product is zero.

[2 marks]

(ii) attempt to use arc length formula $s = r\theta$ OR $\frac{90^\circ}{360}$ of circumference ***(M1)***

$= 6000\left(\frac{\pi}{2}\right)$ OR $\frac{1}{4} \times 2\pi \times 6000$ ***A1***

$= 3\pi$ thousand km OR 3000π km ***AG***

[2 marks]

continued…

Question 2 continued

(c) (i) attempt to find cross product (e.g. correct except for sign error) ***(M1)***

$$\boldsymbol{a}\times\boldsymbol{p}=\begin{pmatrix}0\\-36\\0\end{pmatrix}$$ ***A1***

[2 marks]

(ii) **METHOD 1**

$$\boldsymbol{a}\times\boldsymbol{n}=\begin{pmatrix}0\\0\\36\end{pmatrix}$$ ***A1***

choosing to use scalar product for their $\boldsymbol{a}\times\boldsymbol{p}$ and $\boldsymbol{a}\times\boldsymbol{n}$ ***M1***

$(\boldsymbol{a}\times\boldsymbol{p}).(\boldsymbol{a}\times\boldsymbol{n})=\mathbf{0}$ ***A1***

Note: The final ***A1*** can be awarded for seeing the dot product equals zero without incorrect working. Follow through from (i) is permitted and if they explicitly state that they are using $\boldsymbol{a}\times\boldsymbol{n}$ then allow internal follow through.

so angle is 90° ***AG***

[3 marks]

METHOD 2

$$\boldsymbol{a}\times\boldsymbol{n}=\begin{pmatrix}0\\0\\36\end{pmatrix}$$ ***A1***

attempt to use cross product for their $\boldsymbol{a}\times\boldsymbol{p}$ and $\boldsymbol{a}\times\boldsymbol{n}$. This must include both an attempt to find the cross product and knowledge that $\sin\theta=\frac{|x\times y|}{|x||y|}$ ***M1***

$$(\boldsymbol{a}\times\boldsymbol{n})\times(\boldsymbol{a}\times\boldsymbol{p})=\begin{pmatrix}1296\\0\\0\end{pmatrix} \text{ OR } (\boldsymbol{a}\times\boldsymbol{p})\times(\boldsymbol{a}\times\boldsymbol{n})=\begin{pmatrix}-1296\\0\\0\end{pmatrix}$$

$\sin\theta=\frac{1296}{36\times36}=1$ ***A1***

so angle is 90° ***AG***

continued…

Question 2 continued

(d) attempt to use $\frac{\theta}{360}\times 2\pi r = 6$ OR $s = r\theta$ ***(M1)***

$\frac{\theta}{360}\times 2\pi\times 6 = 6$ OR $6 = 6\theta \Rightarrow \theta = 1$ radian

$\theta = 57.2957...^\circ$ ***A1***

Note: The fourth significant figure must be seen somewhere to award the ***A1***.

$\theta = 57.3^\circ$ to 3 significant figures ***AG***

[2 marks]

(e) **METHOD 1**

EITHER (Scalar product)

attempt to use scalar product to find $\hat{\text{BOM}} = \arccos\left(\frac{\boldsymbol{b}.\boldsymbol{m}}{|\boldsymbol{b}||\boldsymbol{m}|}\right)$ ***(M1)***

Note: This may be written as $\cos(\text{BOM}) = \frac{\boldsymbol{b}.\boldsymbol{m}}{|\boldsymbol{b}||\boldsymbol{m}|}$.

$= \arccos\left(\frac{(6\cos 120^\circ)(6\cos 57.3^\circ)}{6^2}\right)$ ***(A1)***

Note: This ***A1*** can be awarded for $\cos\theta = \frac{6\cos 120^\circ \times 6\cos 57.3^\circ}{6^2}$.

$= 106^\circ\left(= 105.673\ldots^\circ\right)$ OR 1.84 radians $(1.84434\ldots)$ ***(A1)***

OR (Cosine Rule)

$|\text{BM}| = \sqrt{(6\sin 120 - 0)^2 + (6\cos 120 - 6\sin 57.3)^2 + (0 - 6\sin 57.3)^2}$

$= 9.56(9.56299...)$ OR 9560km ***(A1)***

attempt to use the cosine rule to find angle BOM ***(M1)***

$|\text{BM}|^2 = 6^2 + 6^2 - 2\times 6\times 6\times\cos\text{BOM}$

$\text{BOM} = 106^\circ\left(= 105.673\ldots^\circ\right)$ OR 1.84 radians $(1.84434\ldots)$ ***(A1)***

continued…

Question 2 continued

THEN

attempt to use $\frac{\theta}{360}\times 2\pi r$ OR $s = r\theta$ ***(M1)***

$= \frac{105.671^{\circ}}{360}\times 2\pi \times 6$

$= 11.1\ (11.0660\ldots \text{thousand km})$ ***A1***

Note: Accept an answer of 11100 OR $11066.0\ldots$ without units (i.e. "km" is implicit).

[5 marks]

METHOD 2 (Cross product)

attempt to use cross product to find $\text{B}\hat{\text{O}}\text{M} = \arcsin\frac{|\boldsymbol{b}\times\boldsymbol{m}|}{|\boldsymbol{b}||\boldsymbol{m}|}$ ***(M1)***

Note: This may be written as $\sin\text{B}\hat{\text{O}}\text{M} = \frac{|\boldsymbol{b}\times\boldsymbol{m}|}{|\boldsymbol{b}||\boldsymbol{m}|}$.

$= \frac{\left|\begin{pmatrix} 36\cos 120\sin\theta \\ -36\sin 120\sin\theta \\ 36\sin 120\cos\theta \end{pmatrix}\right|}{6\times 6}$

$= \frac{\sqrt{\sin^2\theta + 3}}{2} = 0.962818\ldots$ ***(A1)***

$\text{B}\hat{\text{O}}\text{M} = 180 - \arcsin 0.962818\ldots$

$= 106^{\circ}(105.673\ldots, 1.844346\,\text{rads})$ ***(A1)***

attempt to use $\frac{\theta}{360}\times 2\pi r$ OR $s = r\theta$ ***(M1)***

$= \frac{105.671}{360}\times 2\pi \times 6$

$= 11.1\ (11.0660\ \text{thousand km})$ ***A1***

Note: Award at most ***(M1)(A1)(A0)(M1)A0*** for an answer of 7.78 thousand km from finding the principal root of the arcsin.

[5 marks]

continued…

Question 2 continued

(f) attempt to calculate $\boldsymbol{b}\times\boldsymbol{m}$ ***(M1)***

$$\boldsymbol{b}\times\boldsymbol{m}=\begin{pmatrix}36\cos 120^\circ \sin 57.3^\circ\\ -36\sin 120^\circ \sin 57.3^\circ\\ 36\sin 120^\circ \cos 57.3^\circ\end{pmatrix}\left(=\begin{pmatrix}-15.1464\ldots\\ -26.2344\ldots\\ 16.8449\ldots\end{pmatrix}\right) \quad \textit{(A1)}$$

attempt to use scalar product to find angle between vectors ***(M1)***

$$\left(\cos\theta=\right)\left(\frac{(\boldsymbol{b}\times\boldsymbol{m}).(\boldsymbol{b}\times\boldsymbol{p})}{|\boldsymbol{b}\times\boldsymbol{m}||\boldsymbol{b}\times\boldsymbol{p}|}\right)$$

$$\left(\cos\theta=\right)\left(\frac{1090.54...}{34.6614...\times 36}\right) \quad \textit{(A1)(A1)}$$

Note: Award ***(A1)*** for numerator, ***(A1)*** for denominator.

bearing is $(0)29.1^\circ\left(29.0770...^\circ\right)$ ***A1***

Note: Do not penalise absence of degree symbol.

Award full marks for answer in radians $=0.507$.

Award full marks for 029°.

Special Case: If a candidate calculates $\boldsymbol{m}\times\boldsymbol{b}$ instead of $\boldsymbol{b}\times\boldsymbol{m}$ then you can award at most ***M1A0M1A1A1A0***.

[6 marks]

Total [27 marks]

1. (a) (i) $y = 5$ ***A1A1***

Note: Award ***A1*** for $y =$ a constant, ***A1*** for 5.

[2 marks]

(ii) equating the correct lines ***(M1)***

$5 = 15 - 2x$ OR sketch graph

$x = 5$ $y = 5$ OR $(5,5)$ ***A1***

[2 marks]

(b) station B ***A1***

[1 mark]

(c) nearest neighbour (interpolation) ***A1***

[1 mark]

(d) **EITHER**

because the school is quite close to the other two stations also and their readings are much higher. ***R1***

OR

the reading of 49 is only an average, and hence likely that readings are frequently over 50 at the school. ***R1***

Note: Ignore any extra invalid reasons.

[1 mark]

(e) (i) attempt to find a midpoint ***(M1)***

$\left(\frac{5+8}{2}, \frac{6+9}{2}\right)$

$(6.5, 7.5)$ ***A1***

[2 marks]

continued…

Question 1 continued

(ii) gradient is $\frac{9-6}{8-5}=1$ ***A1***

gradient of perpendicular bisector is -1 ***A1***

substituting their gradient and their mid-point into the equation of a line ***M1***

$y-7.5=-1(x-6.5)$ OR $7.5=-1\times 6.5+c$

$y=14-x$ ***AG***

Note: : If ***A1A1M1*** have been awarded but there is then incorrect work after the final ***M1*** such that the ***AG*** does not follow, do not award the final ***M1***.

[3 marks]

(iii) attempt to find where this line meets the other two lines ***(M1)***

$(9,5)$ ***A1***

$14-x=15-2x$

$(1,13)$ ***A1***

Note: These two marks are un-implied due to the "hence" in the question. At least one needs to be seen for the ***(M1)*** to be awarded.

Area $=\frac{1}{2}\times 4\times 8$ ***(M1)***

$=16\left(\text{km}^2\right)$ ***A1***

Note: Award ***(M1)*** for any valid method which is completed (for example cosine rule, followed by area of triangle formula or using one of the other sides as base and correctly finding the perpendicular height) which leads to an answer, even if the answer is not correct due to a calculation error.

(the side lengths of the triangle are 4, 11.3, 8.94 and the angles 45°, 18.4°, 116.6°)

Note: Award ***(M0)A0*** for an unsupported answer of 16.

[5 marks]

continued…

Question 1 continued

(f) attempt to substitute the values into given formula ***M1***

$W = \dfrac{132 \times 13.7 + 49 \times 16 + 125 \times 6.9}{16 + 13.7 + 6.9}$ ***A1***

$W = 94.4(94.3961\ldots)$ ***AG***

[2 marks]

(g) $H_0 : \mu = 94.4,\ H_1 : \mu > 94.4$ ***A1A1***

Note: Allow the hypotheses in words, but award ***A0A0*** if 'mean' is not seen and, at most ***A1A0*** if it is not clear that the **population** mean is being tested

Accept $\mu \leq 94.4$ for the null hypothesis.

use of t-test ***(M1)***

p-value $= 0.0655\ (0.0655348\ldots)$ ***A1***

$0.0655 < 0.10$ ***R1***

(hence reject the null hypothesis or accept the alternative hypothesis)

(there is significant evidence that) the (mean) AQI value (at the school) is greater than 94.4 ***A1***

Note: Do not award ***R0A1***.

Follow through from an incorrect p-value to award the last two marks,

Award ***A0*** for the conclusion if it is not in context or if the null hypothesis is incorrect, though condone the errors in language detailed in the first note above.

[6 marks]

(h) $\mathrm{P}(X < 50) = 0.00273(0.00272574\ldots)$ ***(A1)***

$\mathrm{P}(X < 50) \times 365$ ***(M1)***

$= 0.994895..$

$= 1$ ***A1***

[3 marks]

Total [28 marks]

4. (a) attempt to use cosine rule ***(M1)***

$\cos\theta = \frac{15^2+10^2-8^2}{2\times15\times10}$ ***(A1)***

$29.5(°)$ **OR** 0.516 $(29.5413\ldots, 0.515594\ldots)$ ***A1***

[3 marks]

(b) attempt to find the area of triangle ABC (substitution into the formula must be attempted) ***(M1)***

$\frac{1}{2}\times 15\times 10\times \sin(29.5413\ldots°)$ ***(A1)***

$(= 36.9788\ldots)$

attempt to find the area of sector ABD (substitution into the formula must be attempted) ***(M1)***

$\frac{29.5413\ldots}{360}\times \pi \times 10^2$ **OR** $\frac{1}{2}\times 10^2 \times 0.515594\ldots$ ***(A1)***

$(= 25.7797\ldots)$

$(36.9788\ldots - 25.7797\ldots)$

$11.2\ \text{cm}^2$ $(11.1991\ldots)$ ***A1***

Note: Correct units are required to award the final ***A1***.

[5 marks]

Total [8 marks]

12. (a) (i) no, since there is more than one edge between A and B. ***R1***

(ii) no, since there is no path from A to F (for example) ***R1***

[2 marks]

(b)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & 2 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$ ***A1A1***

Note: Award ***A1*** for the "1"s and award ***A1*** for the "2"s.

[2 marks]

(c) attempt to calculate at least one of $\boldsymbol{M}^2$ or $\boldsymbol{M}^3$ ***(M1)***

attempt to calculate both $\boldsymbol{M}^2$ and $\boldsymbol{M}^3$ ***(M1)***

finding at least one of the entries for A to B, 1 or 14 ***(A1)***

17 ***A1***

Note: Award ***(M1)(M1)(A1)A0*** for an (unsupported) answer of 32 from including $\boldsymbol{M}^4$.

Accept the use of the matrix for the subgraph with vertices ABCD, which is $\begin{pmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$.

[4 marks]

Total [8 marks]

$\left|\begin{pmatrix} 7 \end{pmatrix}\right|$

13. (a) attempt to find $\left\| \begin{pmatrix} -24 \\ 0 \end{pmatrix} \right\|$ $(=25)$ ***(M1)***

attempt to find scale factor ***(M1)***

$\frac{800}{25}\ (=32)$

$(r_2 =) \begin{pmatrix} -13 \\ 50 \\ 7.2 \end{pmatrix} + 32t \begin{pmatrix} 7 \\ -24 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} -13 \\ 50 \\ 7.2 \end{pmatrix} + t \begin{pmatrix} 224 \\ -768 \\ 0 \end{pmatrix}$ ***A1***

[3 marks]

(b) attempt to find expression for relative position $(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ ***(M1)***

$\begin{pmatrix} 42 \\ -82 \\ -1.4 \end{pmatrix} + t \begin{pmatrix} -544 \\ 1368 \\ 0.5 \end{pmatrix}$ ***(A1)***

attempt to find magnitude (or square of magnitude) for their relative position ***(M1)***

$\sqrt{(42-544t)^2 + (-82+1368t)^2 + (-1.4+0.5t)^2}$

attempt to find minimum value ***(M1)***

e.g. sketch with minimum indicated **OR** attempt to solve derivative = 0

OR $t = 0.0622991\ldots$

8.83 (kilometres) $(8.83380\ldots)$ ***A1***

Note: If $r_2 = \begin{pmatrix} -13 \\ 50 \\ 7.2 \end{pmatrix} + t \begin{pmatrix} 7 \\ -24 \\ 0 \end{pmatrix}$, $t = 0.131$ and least distance $= 1.56$.

[5 marks]

Total [8 marks]

1. (a) ABCDEFA ***(A1)***

Note: Accept any starting point and either direction.

Accept a sum of weights rather than the letters as evidence of finding a Hamiltonian cycle $8+7+9+6+12+5$

Condone ABCDEF seen for ***(A1).***

47 (minutes) ***A1***

[2 marks]

(b) (i) $(a=)12$ ***A1***

[1 mark]

(ii) $(b=)17$ ***A1***

[1 mark]

(c) ADEFBCA (***M1)(A1)***

Note: Award ***(M1)*** for ADEF seen (or $4, 6, 11$).

weight = $(4+6+11+13+7+12=)53$ (minutes) ***A1***

[3 marks]

(d) (i) attempt to use Prim's to find minimum spanning tree ***(M1)***

Note: Award ***M1*** if first two edges seen.

BC, CD, DE, DF ***A1***

weight of MST $(=\ 7+9+6+9)=31$ ***A1***

Note: If using the original graph instead of the table then edges are BC, CD, DE, EF. Award ***(M1)A1A0*** for total weight 34 seen.

[3 marks]

Note: If vertices BCDEF or weights $7, 9, 6, 9$ are given, rather than edges, award ***(M1)A0A1*** if a total of 31 is given, and ***(M0)A0A0*** if a different total (eg 33) is given.

(ii) reconnect A with the two shortest edges $4+5=9$ ***(M1)***

40 (minutes) ***A1***

Note: Follow through from errors in part (b).

[2 marks]

continued…

Question 1 continued

(e) (i) 40 ***A1***

Note: Follow through from (d) (ii). Correct ***FT*** answer is their (d)(ii) if greater than 40, 39 if less than 40.

[1 mark]

(ii) 47 ***A1***

Note: Follow through: the smallest of their answers to (a) and (c).

[1 mark]

(f) (i) $(6+6+9+7+13+5=)46$ (minutes) ***A1***

[1 mark]

(ii) AF ***A1***

[1 mark]

(g) (i) let Q be the total time in queues

$Q \sim \mathrm{N}(80, 47)$ ***A1A1A1***

Note: Award ***A1*** for 'N', ***A1*** for 80 and ***A1*** for 47.

[3 marks]

(ii) attempt to find the total time available for queuing ***(M1)***

time on rides = 10 (minutes), (walking 46 minutes) ***(A1)***

EITHER

(so time in queues needs be less than) $(150-46-10\ =)$

94 (minutes) ***(A1)***

$(\mathrm{P}(Q<94)=)0.979(0.979429...)$ ***A1***

OR

total time has distribution $T \sim \mathrm{N}(136, 47)$ ***(A1)***

$(\mathrm{P}(T<150)=)0.979(0.979429...)$ ***A1***

[4 marks]

continued…

Question 1 continued

(h) total time walking = 46 (minutes), time on rides is $(10+2=)12$ (minutes), expected times waiting for the rides is $(80+10=)90$ (minutes) ***(M1)***

Note: Award ***(M1)*** for any two of the three correct values seen.

total time = 148 minutes ***(A1)***

so $11{:}28$ am ***A1***

Note: Condone use of $\mathrm{E}(2E_t)$ if correct answer seen.

[3 marks]
Total [26 marks]

APOLLO

Statistics and probability

117 questions

1. $X \sim \text{Po}(8.8)$ ***(M1)***

Note: Award ***(M1)*** for calculating the mean, 8.8, of the distribution

$\text{P}(X > 9) = \text{P}(X \geq 10)$ **OR** $\text{P}(X > 9) = 1 - \text{P}(X \leq 9)$ ***(M1)***

$\text{P}(X > 9) = 0.386 \ (0.386260...)$ ***(M1)A1***

Note: Award ***(M1)(M0)(M1)A0*** for finding $\text{P}(X \geq 9) = 0.518 \ (0.517719...)$
OR $\text{P}(X \leq 9) = 0.614 \ (0.613740...)$.

Total [4 marks]

8. (a)

t	1	2	3	4	5	6
$\mathrm{P}(T=t)$	$\frac{1}{36}$ $(0.027777\ldots)$	$\frac{3}{36}$ $(0.083333\ldots)$	$\frac{5}{36}$ $(0.138888\ldots)$	$\frac{7}{36}$ $(0.194444\ldots)$	$\frac{9}{36}$ (0.25)	$\frac{11}{36}$ $(0.305555\ldots)$

A2

Note: Award ***A1*** if three to five probabilities are correct.

[2 marks]

(b) (i) $\frac{32}{36}$ $\left(\frac{8}{9}, 0.888888..., 88.9\%\right)$ ***(A1)***

(ii) use of conditional probability ***(M1)***
e.g. denominator of 32 **OR** denominator of $0.888888\ldots$, etc.
$\frac{11}{32}$ $(0.34375, 34.4\%)$ ***A1***

[3 marks]

(c) $\frac{1\times1+3\times2+5\times3+...+11\times6}{36}$ ***(M1)***

$=\frac{161}{36}$ $\left(4\frac{17}{36}, 4.47, 4.47222...\right)$ ***A1***

[2 marks]

Total [7 marks]

11. (a) Convenience ***A1***

[1 mark]

(b) H_0: 1% of the toys produced are faulty ***A1***

H_1: More than 1% are faulty ***A1***

[2 marks]

(c) $X \sim B(200, 0.01)$ ***(M1)***

$P(X \geq 4) = 0.142$ ***A1***

Note: Any attempt using Normal approximation to find p-value is awarded ***M0A0***.

[2 marks]

(d) $14\% > 10\%$ ***R1***

so there is insufficient evidence to reject H_0. ***A1***

Note: Do not award ***R0A1***. Accept "fail to reject H_0" or "accept H_0".

[2 marks]

Total [7 marks]

14. (a) $158 \times 6 = 948\,(\text{g})$ ***(M1)A1***

[2 marks]

(b) variance 6×13^2 ***(M1)***

$\text{SD} = 31.8\,(\text{g})$ $(13\sqrt{6},\ 31.8433\ldots)$ ***A1***

[2 marks]

(c) $X \sim \text{N}(948,\ 31.8433...^2)$

$\text{P}(X > 1000) = 0.0512$ $(0.0512350\ldots)$ ***(M1)A1***

Note: Accept 0.0510 $(0.0510014\ldots)$ if 3 sf value 31.8 is used.

Award ***(M1)A1FT*** if the answer is correct for their SD, even if no working is shown. e.g. If the SD is 78 then accept 0.252.

[2 marks]

Total [6 marks]

3. (a) (let $\mu_c =$ population mean for chinchilla rabbits, $\mu_s =$ population mean for sable rabbits)

$H_0 : \mu_c = \mu_s$ ***A1***

$H_1 : \mu_c > \mu_s$ ***A1***

Note: Accept an equivalent statement in words, must include mean and reference to "**population** mean" / "mean for **all** chinchilla rabbits" for the first ***A1*** to be awarded. The terms "*on average"* and "*generally"* are also acceptable to indicate populations.
Do not accept an imprecise *"the means are equal".*

[2 marks]

(b) p-value $= 0.0408$ $(0.0408065...)$ ***A2***

Note: Award ***A1*** for an answer of $0.041565\ldots$, from "unpooled" settings on GDC.

[2 marks]

(c) $0.0408 < 0.05$. ***R1***

(there is sufficient evidence to) reject (or not accept) H_0 ***A1***

(there is sufficient evidence to suggest that chinchilla rabbits are (generally) heavier than sable rabbits)

Note: Do not award ***R0A1***. Accept 'accept H_1'.

[2 marks]

Total [6 marks]

9. (a) $\left(\frac{74+97+91+86+112}{5}\right)=92$ ***A1***

[1 mark]

(b) (i) H_0 : The data satisfies the model ***A1***

H_1 : The data does not satisfy the model ***A1***

Note: Do not accept "H_0 : The same number of copies will be sold each day" but accept a similar statement if the word 'expect' or 'expected' is included. Similarly for H_1.

(ii) 4 ***A1***

(iii) $\chi^2_{\text{calc}}=8.54\ \ (8.54347...)$ **OR** $p\text{-value}=0.0736\ \ (0.0735802...)$ ***A2***

$8.54<9.49$ **OR** $0.0736>0.05$ ***R1***

therefore there is insufficient evidence to reject H_0 ***A1***

(i.e. the data satisfies the model)

Note: Do not award ***R0A1***. Accept "accept" or "do not reject" in place of "insufficient evidence to reject".
Award the ***R1*** for comparing their p-value with 0.05 or their χ^2 value with 9.49 and then ***FT*** their final conclusion.

[7 marks]

Total [8 marks]

10. (a) $\overline{x} = \frac{\sum x}{n} = \frac{2506}{30} = 83.5 \ \ (83.5333\ldots)$ ***A1***

[1 mark]

(b) $\left(s_{n-1}^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \right) \ \frac{209738 - \frac{2506^2}{30}}{29}$ ***(M1)***

$= 13.9 \ \ (13.9126...)$ ***A1***

[2 marks]

(c) $(82.1,\ 84.9) \ \ (82.1405...,\ 84.9261...)$ ***A2***

[2 marks]

(d) 85 is outside the confidence interval and therefore Talha would suggest that the manufacturer's claim is incorrect ***R1***

Note: The conclusion must refer back to the original claim.

Allow use of a two sided t-test giving a p-value rounding to $0.04 < 0.05$ and therefore Talha would suggest that the manufacturer's claims in incorrect.

[1 mark]

Total [6 marks]

13.

(a) transition matrix is $\begin{matrix} & A & B & C & D & E & F \\ A & 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 & 0 \\ B & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ C & 0 & \frac{2}{3} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{5} \\ D & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{2}{5} \\ E & \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ F & \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{matrix}$ ***M1A1A1***

Note: Allow the transposed matrix.

Award ***M1*** for a 6x6 matrix with all values between 0 and 1, and all columns (or rows if transposed) adding up to 1, award ***A1*** for one correct row (or column if transposed) and ***A1*** for all rows (or columns if transposed) correct.

[3 marks]

(b) attempting to raise the transition matrix to a large power ***(M1)***

steady state vector is $\begin{pmatrix} (0.157) \\ (0.0868) \\ (0.256) \\ (0.241) \\ (0.0868) \\ 0.173 \end{pmatrix}$ ***(A1)***

so percentage of time spent at vertex F is 17.3% ***A1***

Note: Accept 17.2%.

[3 marks]

(c) the model assumes instantaneous travel from junction to junction, ***R1***
and hence the answer obtained would be an overestimate ***R1***
OR
the mouse may eat the sugar over time ***R1***
and hence the probabilities would change ***R1***

Note: Accept any other sensible answer.

[2 marks]

Total [7 marks]

15. (a) $H_0 : m = 110,\ H_1 : m > 110$ ***A1***

Note: Accept other appropriate variables for the mean.
Accept 22 in place of 110.

[1 mark]

(b) $P(X \geq 128) = 0.05024$ ***(M1)(A1)***

$P(X \geq 129) = 0.04153$ ***(M1)***

(probability of making a type I error is) 0.0415 ***A1***

Note: If other probabilities are seen, the final ***A1*** cannot be awarded unless 0.0415 is clearly identified as the final answer.

[4 marks]

(c) $X \sim \text{Po}(110)$

$P(X \geq 126) = 0.072 > 0.05$ **OR** recognizing $126 < 129$ or ≤ 128 ***R1***

so there is insufficient evidence to reject H_0 ***A1***

(*ie* there is insufficient evidence to suggest that the number of coffees being sold has increased)

Note: Accept 'Accept H_0'.
Do not award ***R0A1***.

[2 marks]

Total [7 marks]

3. (a) (i) evidence of correct probability ***(M1)***
e.g. sketch **OR** correct probability statement $\mathrm{P}(X < 6.5)$

0.0151 ***A1***

(ii) 0.0228 ***A1***

Note: Answers should be given to 4 decimal place.

[3 marks]

(b) (i) multiplying **their** probability by 1000 ***(M1)***
451.7 ***A1***

(ii) 510.5 ***A1***

[3 marks]

Note: Answers should be given to 4 sf.

(c) H_0 : stopping distances can be modelled by $\mathrm{N}(6.76, 0.12^2)$
H_1 : stopping distances cannot be modelled by $\mathrm{N}(6.76, 0.12^2)$ ***A1A1***

Note: Award ***A1*** for correct H_0, including reference to the mean and standard deviation.
Award ***A1*** for the negation of their H_0.

[2 marks]

(d) 15.1 or 22.8 seen ***(M1)***

0.0727 $(0.0726542\ldots, 7.27\%)$ ***A2***

[3 marks]

(e) $0.05 < 0.0727$ ***R1***
there is insufficient evidence to reject H_0 (or "accept H_0") ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [13 marks]

5. (a) finding $\boldsymbol{T}^3$ **OR** use of tree diagram ***(M1)***

$$\boldsymbol{T}^3 = \begin{pmatrix} 0.65 & 0.525 \\ 0.35 & 0.475 \end{pmatrix}$$

the probability of sunny in three days' time is 0.65 ***A1***

[2 marks]

(b) attempt to find eigenvalues ***(M1)***

Note: Any indication that $\det(\boldsymbol{T} - \lambda \boldsymbol{I}) = 0$ has been used is sufficient for the ***(M1)***.

$$\begin{vmatrix} 0.8-\lambda & 0.3 \\ 0.2 & 0.7-\lambda \end{vmatrix} = (0.8-\lambda)(0.7-\lambda) - 0.06 = 0$$

$$(\lambda^2 - 1.5\lambda + 0.5 = 0)$$

$\lambda = 1,\ \lambda = 0.5$ ***A1***

attempt to find either eigenvector ***(M1)***

$0.8x + 0.3y = x \Rightarrow -0.2x + 0.3y = 0$ so an eigenvector is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ***A1***

$0.8x + 0.3y = 0.5x \Rightarrow 0.3x + 0.3y = 0$ so an eigenvector is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1***

Note: Accept multiples of the stated eigenvectors.

[5 marks]

(c) (i) $\boldsymbol{P} = \begin{pmatrix} 3 & 1 \\ 2 & -1 \end{pmatrix}$ **OR** $\boldsymbol{P} = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$ ***A1***

Note: Examiners should be aware that different, correct, matrices $\boldsymbol{P}$ may be seen.

(ii) $\boldsymbol{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0.5 \end{pmatrix}$ **OR** $\boldsymbol{D} = \begin{pmatrix} 0.5 & 0 \\ 0 & 1 \end{pmatrix}$ ***A1***

Note: $\boldsymbol{P}$ and $\boldsymbol{D}$ must be consistent with each other.

[2 marks]

(d) $0.5^n \to 0$ ***(M1)***

$\boldsymbol{D}^n = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ **OR** $\boldsymbol{D}^n = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ ***(A1)***

Note: Award ***A1*** only if their $\boldsymbol{D}^n$ corresponds to their $\boldsymbol{P}$.

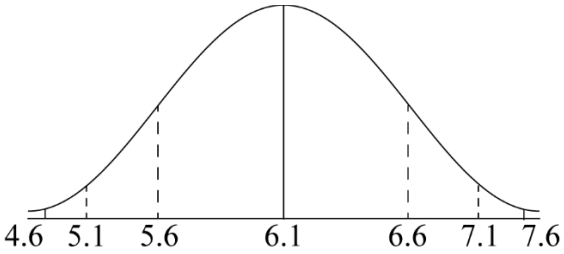
$\boldsymbol{P}\boldsymbol{D}^n\boldsymbol{P}^{-1} = \begin{pmatrix} 0.6 & 0.6 \\ 0.4 & 0.4 \end{pmatrix}$ ***(M1)***

$60\ \%$ ***A1***

[4 marks]

Total [13 marks]

2. (a)



A1A1

Note: Award ***A1*** for a normal curve with mean labelled 6.1 or μ, ***A1*** for indication of SD (0.5): marks on horizontal axis at 5.6 and/or 6.6 **OR** $\mu - 0.5$ and/or $\mu + 0.5$ on the correct side and approximately correct position.

[2 marks]

(b) $X \sim \mathrm{N}(6.1,\ 0.5^2)$

$\mathrm{P}(5.5 < X < 6.5)$ **OR** labelled sketch of region ***(M1)***

$= 0.673\ \ (0.673074...)$ ***A1***

[2 marks]

(c) $(\mathrm{P}(X < 5.3) =)\ \ 0.0547992...$ ***(A1)***

$0.0547992... \times 80$ ***(M1)***

$= 4.38\ \ (4.38393...)$ ***A1***

[3 marks]

(d) (i) $Y \sim \mathrm{N}(4.5,\ 0.45^2)$,

$(\mathrm{P}(Y > 4.62) =)\ \ 0.394862...$ ***(A1)***

use of binomial seen or implied ***(M1)***

using $\mathrm{B}(10,\ 0.394862...)$ ***(M1)***

$0.0430\ \ (0.0429664...)$ ***A1***

(ii) $np(1-p) = 2.39\ \ (2.38946...)$ ***A1***

[5 marks]

(e) $\mathrm{P}\left(F \cap (W > 4.7)\right) = 0.5 \times 0.3284\ \ \left(= 0.1642\right)$ ***(A1)***

attempt use of tree diagram **OR** use of $\mathrm{P}(F \mid W > 4.7) = \dfrac{\mathrm{P}\left(F \cap (W > 4.7)\right)}{\mathrm{P}(W > 4.7)}$ ***(M1)***

$\dfrac{0.5 \times 0.3284}{0.5 \times 0.9974 + 0.5 \times 0.3284}$ ***(A1)***

$= 0.248\ \ (0.247669...)$ ***A1***

[4 marks]

Total [16 marks]

4. (a) $\boldsymbol{T} = \begin{pmatrix} 0.965 & 0.05 \\ 0.035 & 0.95 \end{pmatrix}$ ***M1A1***

Note: Award ***M1A1*** for $\boldsymbol{T} = \begin{pmatrix} 0.95 & 0.035 \\ 0.05 & 0.965 \end{pmatrix}$.

Award the ***A1*** for a transposed $\boldsymbol{T}$ if used correctly in part (b) i.e. preceded by 1×2 matrix $(2100 \quad 3500)$ rather than followed by a 2×1 matrix.

[2 marks]

(b) $\begin{pmatrix} 0.965 & 0.05 \\ 0.035 & 0.95 \end{pmatrix}^2 \begin{pmatrix} 2100 \\ 3500 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 2294 \\ 3306 \end{pmatrix}$

so ratio is $2294 : 3306$ $(= 1147 : 1653,\ 0.693889...)$ ***A1***

[2 marks]

(c) to solve $\boldsymbol{A}x = \lambda x$:

$\begin{vmatrix} 0.965-\lambda & 0.05 \\ 0.035 & 0.95-\lambda \end{vmatrix} = 0$ ***(M1)***

$(0.965-\lambda)(0.95-\lambda) - 0.05\times0.035 = 0$

$\lambda = 0.915$ **OR** $\lambda = 1$ ***(A1)***

attempt to find eigenvectors for at least one eigenvalue ***(M1)***

when $\lambda = 0.915$, $x = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (or any real multiple) ***(A1)***

when $\lambda = 1$, $x = \begin{pmatrix} 10 \\ 7 \end{pmatrix}$ (or any real multiple) ***(A1)***

therefore $\boldsymbol{P} = \begin{pmatrix} 1 & 10 \\ -1 & 7 \end{pmatrix}$ (accept integer valued multiples of their eigenvectors and columns in either order) ***A1***

[6 marks]

continued…

Question 4 continued

(d) $\boldsymbol{P}^{-1} = \begin{pmatrix} 1 & 10 \\ -1 & 7 \end{pmatrix}^{-1} = \frac{1}{17}\begin{pmatrix} 7 & -10 \\ 1 & 1 \end{pmatrix}$ ***(A1)***

Note: This mark is independent, and may be seen anywhere in part (d).

$\boldsymbol{D} = \begin{pmatrix} 0.915 & 0 \\ 0 & 1 \end{pmatrix}$ ***(A1)***

$\boldsymbol{T}^n = \boldsymbol{P}\boldsymbol{D}^n\boldsymbol{P}^{-1} = \begin{pmatrix} 1 & 10 \\ -1 & 7 \end{pmatrix}\begin{pmatrix} 0.915^n & 0 \\ 0 & 1^n \end{pmatrix}\frac{1}{17}\begin{pmatrix} 7 & -10 \\ 1 & 1 \end{pmatrix}$ ***(M1)A1***

Note: Award ***(M1)A0*** for finding $\boldsymbol{P}^{-1}\boldsymbol{D}^n\boldsymbol{P}$ correctly.

as $n \to \infty$, $\boldsymbol{D}^n = \begin{pmatrix} 0.915^n & 0 \\ 0 & 1^n \end{pmatrix} \to \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ ***R1***

so $\boldsymbol{T}^n \to \frac{1}{17}\begin{pmatrix} 1 & 10 \\ -1 & 7 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 7 & -10 \\ 1 & 1 \end{pmatrix}$ ***A1***

$= \begin{pmatrix} \frac{10}{17} & \frac{10}{17} \\ \frac{7}{17} & \frac{7}{17} \end{pmatrix}$ ***AG***

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[6 marks]

(e) **METHOD ONE**

$\begin{pmatrix} \frac{10}{17} & \frac{10}{17} \\ \frac{7}{17} & \frac{7}{17} \end{pmatrix}\begin{pmatrix} 2100 \\ 3500 \end{pmatrix} = \begin{pmatrix} 3294 \\ 2306 \end{pmatrix}$ ***(M1)***

so ratio is $3294 : 2306$ $(1647 : 1153,\ 1.42844...,\ 0.700060...)$

A1

METHOD TWO

long term ratio is the eigenvector associated with the largest eigenvalue ***(M1)***

$10 : 7$ ***A1***

[2 marks]

Total [18 marks]

5. (a) $X_1 \sim \text{Po}(3.1)$

$\text{P}(X_1 = 4) = 0.173 \ \ (0.173349...)$ ***A1***

[1 mark]

(b) (i) $X_2 \sim \text{Po}(3 \times 3.1) = \text{Po}(9.3)$ ***(M1)***

$\text{P}(X_2 = 12) = 0.0799 \ \ (0.0798950...)$ ***A1***

(ii) $\left(\text{P}(X_1 > 0)\right)^2 \times \text{P}(X_1 = 0)$ ***(M1)***

$0.95495^2 \times 0.04505$ ***(A1)***

$= 0.0411 \ \ (0.0410817...)$ ***A1***

[5 marks]

(c) $\text{P}(X_1 = 0) = 0.04505$ ***(A1)***

$X_1 \sim \text{B}(12, 0.04505)$ ***(M1)(A1)***

Note: Award ***M1*** for recognizing binomial probability, and ***A1*** for correct parameters.

$= 0.0133 \ \ (0.013283....)$ ***A1***

[4 marks]

(d) **METHOD ONE**

n	λ	$\text{P}(X \geq 30)$
...	…	…
10	24.1	0.136705
11	26.2	0.253384

(M1)(A1)(A1)

Note: Award ***M1*** for evidence of a cumulative Poisson with $\lambda = 3.1 + 2.1n$, ***A1*** for 0.136705 and ***A1*** for 0.253384.

so require 12 magpies (including Bill) ***A1***

METHOD TWO

evidence of a cumulative Poisson with $\lambda = 3.1 + 2.1n$ ***(M1)***

sketch of curve and $y = 0.2$ ***(A1)***

(intersect at) $10.5810…$ ***(A1)***

rounding up gives $n = 11$

so require 12 magpies (including Bill) ***A1***

[4 marks]

Total [14 marks]

2. (a) Use of χ^2 test for independence ***(M1)***

H_0 : Staying (or leaving) the firm and interview rating are independent.
H_1 : Staying (or leaving) the firm and interview rating are not independent ***A1***

Note: For H_1 accept '…are dependent' in place of '…not independent'.

p-value $= 0.487$ $(0.487221...)$ ***A2***

Note: Award ***A1*** for $\chi^2 = 1.438...$ if p-value is omitted or incorrect.

$0.487 > 0.05$ ***R1***

(the result is not significant at the 5% level)
insufficient evidence to reject the H_0 (or "accept H_0") ***A1***

Note: Do not award ***R0A1***.
The final ***R1A1*** can follow through from their incorrect p-value

[6 marks]

(b) $\frac{55}{91} \times 18 = 10.9$ $(10.8791...)$ ***M1A1***

Note: Award ***A1*** for anything that rounds to 10.9.

≈ 11 ***AG***

[2 marks]

continued…

Question 2 continued

(c) (i) there seems to be a difference between the two departments ***(A1)***

the international department manager seems to be less generous than the national department manager ***R1***

Note: The ***A1*** is for commenting there is a difference between the two departments and the ***R1*** is for correctly commenting on the direction of the difference

[2 marks]

(ii)

	L	M	N	O	P	Q	R
Written assessment rank	1	2	3	4	5	6	7
Manager score rank	1	2.5	4.5	2.5	4.5	6	7

(M1)(A1)

Note: Award ***(M1)*** for an attempt to rank the data, and ***(A1)*** for correct ranks for both variables. Accept either set of rankings in reverse.

$r_s = 0.909 \ \ (0.909241.....)$ ***(M1)A1***

Note: The ***(M1)*** is for calculating the PMCC for their ranks.

Note: If a final answer of 0.9107 is seen, from use of $1-\frac{6\sum d^2}{n(n^2-1)}$, award ***(M1)(A1)A1***.

Accept -0.909 if one set of ranks has been ordered in reverse.

[4 marks]

(iii) **EITHER**

there is a (strong) association between the written assessment mark and the manager scores. ***A1***

OR

there is a (strong) agreement in the rank order of the written assessment marks and the rank order of the manager scores. ***A1***

OR

there is a (strong linear) correlation between the rank order of the written assessment marks and the rank order of the manager scores. ***A1***

Note: Follow through on a value for their value of r_s in c(ii).

THEN

the written assessment is likely to be a valid measure (of the level of employee performance) ***R1***

[2 marks]

continued…

Question 2 continued

(d) (i) test-retest ***A1***

[1 mark]

(ii) p-value $= 0.00209 \ (0.0020939...)$ ***A2***

$0.00209 < 0.05$ ***R1***

(the result is significant at the 5% level)

(there is sufficient evidence to) reject H_0 ***A1***

Note: Do not award ***R0A1***. Accept "accept H_1".
The final ***R1A1*** can follow through from their incorrect p-value.

[4 marks]

(iii) the test seems reliable ***A1***

Note: Follow through from their answer in part (d)(ii). Do not award if there is no conclusion in d(ii).

[1 mark]

(e) (i) 25 ***A1***

[1 mark]

(ii) probability of significant result given no correlation is 0.05 ***(M1)***

probability of at least one significant result in 25 tests is

$1 - 0.95^{25}$ ***(M1)(A1)***

Note: Award ***(M1)*** for use of $1 - \mathrm{P}(0)$ or the binomial distribution with any value of p.

[1 mark]

$= 0.723 \ (0.722610...)$ ***A1***

[4 marks]

(iii) (though the result is significant) it is very likely that one significant result would be achieved by chance, so it should be disregarded or further evidence sought

R1

[1 mark]

[Total 28 marks]

1. (a) (i) Any one from: ***R1***
increase sample size / increase response rate / repeat process
check whether sample is representative
test-retest participants or do a parallel test
use a stratified sample
use a random sample

Note: Do not condone:
Ask different types of doctor
Ask for proof of income
Ask for proof of being a doctor
Remove anonymity
Remove response K.

[1 mark]

(ii) Any one from: ***R1***
non-random sampling means a subset of population might be responding
self-reported happiness is not the same as happiness
happiness is not a constant / cannot be quantified / is difficult to measure
income might include external sources
Juliet is only sampling doctors in her city
correlation does not imply causation
sample might be biased

Note: Do not condone the following common but vague responses unless they make a clear link to validity:
Sample size is too small
Result is not generalizable
There may be other variables Juliet is ignoring
Sample might not be representative

[1 mark]

(b) because the income is very different / implausible / clearly contrived ***R1***

Note: Answers must explicitly reference "income" to get credit.

[1 mark]

continued…

Question 1 continued

(c) (i) $(\$)\ 90\ 200$ ***(M1)A1***

[2 marks]

(ii) $r = 0.558\ \ (0.557723...)$ ***A2***

[2 marks]

(d) (i) **EITHER**

only looking for change in one direction ***R1***

OR

only looking for greater happiness with greater income ***R1***

OR

only looking for evidence of positive correlation ***R1***

[1 mark]

(ii) $\mathrm{H}_0 : \rho = 0;\ \mathrm{H}_1 : \rho > 0$ ***A1A1***

Note: Award ***A1*** for ρ seen (do not accept r), ***A1*** for both correct hypotheses, using **their** ρ or r. Accept an equivalent statement in words, however reference to "correlation for the population" or "association for the population" must be explicit for the first ***A1*** to be awarded.

Watch out for a null hypothesis in words similar to "Annual income is not associated with greater happiness". This is effectively saying $\rho \leq 0$ and should not be condoned.

[2 marks]

continued…

Question 1 continued

(iii) **METHOD 1 – using critical value of *r***

$0.558 > 0.549 \ \ (0.557723... > 0.549)$ ***R1***

(therefore significant evidence of) a positive correlation ***A1***

Note: Do not award ***R0A1***.

METHOD 2 – using *p*-value

$0.0469 < 0.05 \ \ (0.0469463... < 0.05)$ ***A1***

Note: Follow through from their r-value from part (c)(ii).

(therefore significant evidence of) a positive correlation ***A1***

Note: Do not award ***A0A1***.

[2 marks]

(e) (i) $a = 0.000126 \ \ (0.000125842...), \ b = 41.1 \ \ (41.1490...)$ ***A1***

[1 mark]

(ii) **EITHER**

the amount the happiness score increases for every \$1 increase in (annual) income ***A1***

OR

rate of change of happiness with respect to (annual) income ***A1***

Note: Accept equivalent responses e.g. an increase of 1.26 in happiness for every $\$10000$ increase in salary.

[1 mark]

(iii) $c = -2.06 \times 10^{-9} \ \ (-2.06191... \times 10^{-9}),$

$d = 7.05 \times 10^{-4} \ \ (7.05272... \times 10^{-4}),$

$e = 12.6 \ \ (12.5878...)$ ***A1***

[1 mark]

continued…

Question 1 continued

(iv) for quadratic model: $R^2 = 0.659$ $(0.659145...)$ ***A1***

for linear model: $R^2 = 0.311$ $(0.311056...)$ ***A1***

Note: Follow through from their r value from part (c)(ii).

[2 marks]

(v) **EITHER**

quadratic model is a better fit to the data / more accurate ***A1***

OR

quadratic model explains a higher proportion of the variance ***A1***

[1 mark]

(vi) **EITHER**

not valid, R^2 not a useful measure to compare models with different numbers of parameters ***A1***

OR

not valid, quadratic model will always have a better fit than a linear model ***A1***

Note: Accept any other sensible critique of the validity of the method. Do not accept any answers which focus on the conclusion rather than the method of model selection.

[1 mark]

continued…

Question 1 continued

(f) (i) (single sample) t-test ***A1***

[1 mark]

(ii) **EITHER**

$\mathrm{H}_0 : \mu = 80\,000;\ \mathrm{H}_1 : \mu \neq 80\,000$ ***A1***

OR

H_0 : (sample is drawn from a population where) the population mean is $\$80\,000$

H_1 : the population mean is not $\$80\,000$ ***A1***

Note: Do not allow ***FT*** from an incorrect test in part (f)(i) other than a z-test.

[1 mark]

(iii) $p = 0.610\ \ (0.610322...)$ ***A1***

Note: For a z-test follow through from part (f)(i), either 0.578 (from biased estimate of variance) or 0.598 (from unbiased estimate of variance).

$0.610 > 0.05$ ***R1***

EITHER

no (significant) evidence that mean differs from $\$80\,000$ ***A1***

OR

the sample could plausibly have been drawn from the quoted population ***A1***

Note: Allow ***R1FTA1FT*** from an incorrect p-value, but the final ***A1*** must still be in the context of the original research question.

[3 marks]

[Total 24 marks]

7. (a) 75 ***A1***

[1 mark]

(b) recognition that all entries add up to 120 ***(M1)***

$a = 120 - 6 - 13 - 26 - b$ **OR** $a = 75 - b$ ***A1***

[2 marks]

(c) (i) $\dfrac{6\times1+13\times2+26\times3+(75-b)\times4+b\times5}{120} = 3.65$ ***(M1)(A1)***

Note:	Award ***(M1)*** for attempt to substitute into mean formula, LHS expression is sufficient for the ***M*** mark. Award ***(A1)*** for correct substitutions in one variable **OR** in two variables, followed by evidence of solving simultaneously with $a + b = 75$.

$(b =)\ 28$ ***A1***

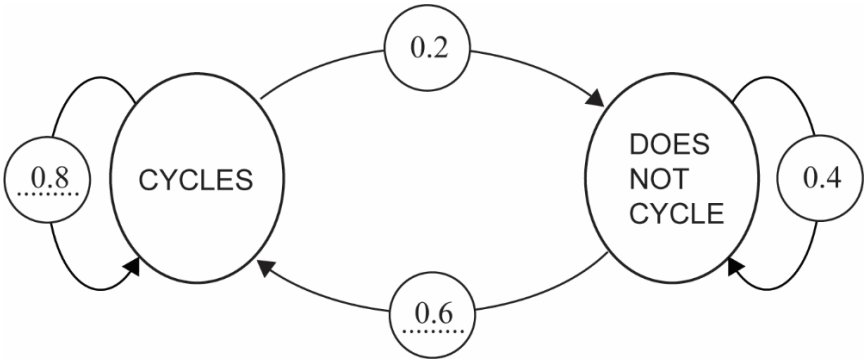
(ii) $120 -$ their part (c)(i) seen (e.g. 92 indicated on graph) ***(M1)***

84 ***A1***

[5 marks]

Total: [8 marks]

9. (a)



A1A1

[2 marks]

(b) $A = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$ ***(A1)***

$A^{180} = \begin{pmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{pmatrix}$ ***(M1)***

0.75 ***A1***

[3 marks]

Total: [5 marks]

12. (a) $m = -0.695 \ (-0.695383...)$; $b = 4.63 \ (4.62974...)$ ***A1A1***

[2 marks]

(b) $\ln x = -0.695(\ln 25) + 4.63$ ***M1***

$\ln x = 2.39288...$ ***(A1)***

$x = 10.9\%$ ***A1***

[3 marks]

Total: [5 marks]

14. (a) let X be the random variable "the weight of a sack of potatoes"

$\mathrm{P}(X < 50)$ ***(M1)***

$= 0.588 \text{ kg} \quad (0.587929...)$ ***A1***

[2 marks]

(b) $\mathrm{P}(X < l) = 0.25$ ***(M1)***

$49.2 \text{ kg} \ (49.1929\ldots)$ ***A1***

[2 marks]

(c) attempt to sum 10 independent random variables ***(M1)***

$Y = \sum_{i=1}^{10} X_i \sim \mathrm{N}\left(498, 10 \times 0.9^2\right)$ ***(A1)***

$\mathrm{P}(Y > 500) = 0.241$ ***A1***

[3 marks]

Total: [7 marks]

3. (a) (i) Let X be the random variable "distance from O".

$X \sim \mathrm{N}(10, 3^2)$

$\mathrm{P}(X < 13) = 0.841 \ (0.841344...)$ ***(M1)A1***

(ii) $\left(\mathrm{P}(X > 15) =\right) \ 0.0478 \ (0.0477903)$ ***A1***

[3 marks]

(b) $\mathrm{P}(X > 15) \times \mathrm{P}(X > 15)$ ***(M1)***

$= 0.00228 \ (0.00228391...)$ ***A1***

[2 marks]

(c) $1 - (0.8143)^3$ ***(M1)***

$= 0.460 \ (0.460050...)$ ***A1***

[2 marks]

(d) (i) let Y be the random variable "number of points scored"

evidence of use of binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10, 0.539949...)$ ***(A1)***

$\left(\mathrm{E}(Y) =\right) 10 \times 0.539949...$ ***(M1)***

$= 5.40$ ***A1***

(ii) $\left(\mathrm{P}(Y \geq 5) =\right) \ 0.717 \ (0.716650...)$ ***A1***

(iii) $\mathrm{P}(5 \leq Y < 8)$ ***(M1)***

$= 0.628 \ (0.627788...)$ ***A1***

Note: Award ***M1*** for a correct probability statement or indication of correct lower and upper bounds, 5 and 7.

(iv) $\dfrac{\mathrm{P}(5 \leq Y < 8)}{\mathrm{P}(Y \geq 5)} \left(= \dfrac{0.627788...}{0.716650...} \right)$ ***(M1)***

$= 0.876 \ (0.876003...)$ ***A1***

[9 marks]

Total: [16 marks]

7. (a) (i) let X be the random variable "number of patients arriving in a minute", such that $X \sim \text{Po}(m)$.

$\text{H}_0 : m = 1.5$ ***A1***

$\text{H}_1 : m > 1.5$ ***A1***

Note: Allow a value of 270 for m. Award at most ***A0A1*** if it is not clear that it is the population mean being referred to e.g

H_0 : *The number of patients is equal to 1.5 every minute*

H_1 : *The number of patients exceeds 1.5 every minute.*

Referring to the "expected" number of patients or the use of μ or λ is sufficient for ***A1A1***.

(ii) under H_0 let Y be the number of patients in 3 hours

$Y \sim \text{Po}(270)$ ***(A1)***

$\text{P}(Y \geq 320) \left(= 1 - \text{P}(Y \leq 319)\right) = 0.00166 \ \ (0.00165874)$ ***(M1)A1***

since $0.00166 < 0.05$ ***R1***

(reject H_0)

Loreto should employ more staff ***A1***

[7 marks]

(b) (i) H_0 : The probability of a patient waiting less than 20 minutes is 0.95 ***A1***

H_1 : The probability of a patient waiting less than 20 minutes is less than 0.95 ***A1***

(ii) under H_0 let W be the number of patients waiting more than 20 minutes

$W \sim \text{B}(150, 0.05)$ ***(A1)***

$\text{P}(W \geq 11) = 0.132 \ \ (0.132215...)$ ***(M1)A1***

since $0.132 > 0.1$ ***R1***

(fail to reject H_0)

insufficient evidence to suggest they are not meeting their target ***A1***

Note: Do not accept "they are meeting target" for the ***A1***.
Accept use of $\text{B}(150, 0.95)$ and $\text{P}(W \leq 139)$ and any consistent use of a random variable, appropriate p-value and significance level.

[7 marks]

Total: [14 marks]

1. (a) discrete ***A1***

[1 mark]

(b) $\frac{24 + 60 + 3k + 40 + 15 + 6}{88 + k} = 2$ ***M1A1***

Note: Award ***M1*** for substitution into the formula for the mean, award ***A1*** for a correct equation.

attempt to solve their equation ***(M1)***

$k = 31$ ***A1***

[4 marks]

(c) systematic ***A1***

[1 mark]

Total [6 marks]

6. (a) $\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

(b) $-3\times\frac{1}{18}+(-1)\times\frac{4}{18}+0\times\frac{3}{18}+\ldots+5\times\frac{7}{18}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the formula for expected value.

$=1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

(c) $2\times\frac{1}{18}\times\frac{3}{18}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{1}{18}\times\frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$=\frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

Total [6 marks]

9. (a) $s_{n-1} = \sqrt{\frac{10}{9}} \times 0.0196 = 0.02066\ldots$ ***(M1)A1***

[2 marks]

(b) $(1.463, 1.493)$ ***(M1)A1***

Note: If s_n used answer is $(1.464, 1.492)$, award ***M1A0.***

[2 marks]

(c) 95% of the time these results would be produced by a population with mean of less than $1.5\,\text{kg}$, so it is likely the mean weight is less than $1.5\,\text{kg}$ ***R1***

[1 mark]

Total [5 marks]

10. let T be the time to serve both customers and T_i the time to serve the ith customer

assuming independence of T_1 and T_2 ***R1***

T is normally distributed and $T = T_1 + T_2$ ***(M1)***

$E(T) = 1.5 + 1.5 = 3$ ***A1***

$\text{Var}(T) = 0.4^2 + 0.4^2 = 0.32$ ***M1A1***

$P(T < 4) = 0.961$ ***A1***

Total [6 marks]

12. *(Model A)*

$R = 3pe^{-0.5p}$ ***M1***

predicted values

p	R
1	1.8196
2	2.2073
3	2.0082

(A1)

$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$ ***(M1)***

$= 0.5263\ldots$ ***A1***

(Model B)

$R = 2.5pe^{-0.6p}$

predicted values

p	R
1	1.372
2	1.506
3	1.2397

(A1)

$SS_{res} = 0.170576\ldots$ ***A1***

chose model B ***A1***

Note: Method marks can be awarded if seen for either model A or model B. Award final ***A1*** if it is a correct deduction from their calculated values for A and B.

Total [7 marks]

16. (a) $H_0 : m = 3,\ H_1 : m < 3$ ***A1***

Note: Accept equivalent statements in words.

[1 mark]

(b) *(let X be the number of fish caught)*

$P(X \leq 1 \mid m = 3) = 0.199$ ***M1A1***

[2 marks]

(c) $P(X \geq 2 \mid m = 2.5)\ \ (= 1 - P(X \leq 1 \mid m = 2.5))$ ***M1A1***

Note: Award ***M1*** for using $m = 2.5$ to evaluate a probability, award ***A1*** for also having $X \geq 2$.

$= 0.713$ ***A1***

[3 marks]

Total [6 marks]

17. (a) $\mathrm{P}(X = 8)$ ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability.
eg, $\mathrm{P}(X = 8)$, $X \sim \mathrm{B}\left(20, \frac{6}{15}\right)$.

$= 0.180\ (0.179705\ldots)$ ***A1***

[2 marks]

(b) let x be the number of male students

recognize that probability of selecting a male is equal to $\frac{x}{80}$ ***(A1)***

$\left(\text{set up equation } {}^{20}\mathrm{C}_8\left(\frac{x}{80}\right)^8\left(\frac{80-x}{80}\right)^{12} = \right) 0.153357$ ***(M1)***

number of male students $= 37$ ***(M1)A1***

Note: Award ***(M1)A0*** for 27.

[4 marks]

Total [6 marks]

2. (a) (i) $\mathrm{P}(Y) = 0.8 \times 0.1 + 0.2 \times 0.3$ ***M1***

$= 0.14$ ***A1***

(ii) $\mathrm{P}(\text{Star} \mid Y) = \dfrac{0.8 \times 0.1}{0.14}$ ***M1***

$= 0.571 \left(\dfrac{4}{7}, 0.571428\ldots \right)$ ***A1***

[4 marks]

(b) the colours of the sweets are distributed according to manufacturer specifications ***A1***

[1 mark]

(c)

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency	12	20	16	16	8	8

A2

Note: Award ***A2*** for all 6 correct expected values, ***A1*** for 4 or 5 correct values, ***A0*** otherwise.

[2 marks]

(d) 5 ***A1***

[1 mark]

(e) $0.469\ (0.4688117\ldots)$ ***A2***

[2 marks]

(f) since $0.469 > 0.05$ ***R1***

fail to reject the null hypothesis. There is insufficient evidence to reject the manufacturer's specifications ***A1***

Note: Award ***R1*** for a correct comparison of their correct p-value to the test level, award ***A1*** for the correct result from that comparison. Do not award ***R0A1***.

[2 marks]

Total [12 marks]

6. (a) $\begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}$ ***M1A1***

[2 marks]

(b) $\begin{vmatrix} 0.8-\lambda & 0.1 \\ 0.2 & 0.9-\lambda \end{vmatrix} = 0$ ***M1***

$\lambda = 1$ and 0.7 ***A1***

eigenvectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***(M1)A1***

Note: Accept any scalar multiple of the eigenvectors.

[4 marks]

(c) ***EITHER***

$\boldsymbol{P} = \begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$ $\boldsymbol{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0.7 \end{pmatrix}$ ***A1A1***

OR

$\boldsymbol{P} = \begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$ $\boldsymbol{D} = \begin{pmatrix} 0.7 & 0 \\ 0 & 1 \end{pmatrix}$ ***A1A1***

[2 marks]

(d) $\boldsymbol{P}^{-1} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}$ ***A1***

$\frac{1}{3}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.7^n \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 1200 \\ 1200 \end{pmatrix}$ ***M1A1***

attempt to multiply matrices ***M1***

so in company A, after n years, $400\left(2+0.7^n\right)$ ***A1***

[5 marks]

(e) $400 \times 2 = 800$ ***A1***

[1 mark]

Total [14 marks]

1. (a) χ^2 (goodness of fit) ***A1***

[1 mark]

(b) ***EITHER***

because aim is to measure improvement

OR

because the students may be of different ability in the two schools ***R1***

[1 mark]

(c) (i) 0.1875 (accept 0.188, 0.19) ***A1***

(ii) 2.46 ***(M1)A1***

Note: Award ***(M1)A0*** for 2.63.

[3 marks]

(d) H_0 : there has been no improvement

H_1: there has been an improvement ***A1***

attempt at a one-tailed paired t-test ***(M1)***

p-value $= 0.423$ ***A1***

there is no significant evidence that the students have improved ***R1***

Note: If the hypotheses are not stated award a maximum of ***A0M1A1R0***.

[4 marks]

(e) (i) H_0 : there is no difference between the schools

H_1: school B did better than school A ***A1***

one-tailed 2 sample t-test ***(M1)***

p-value $= 0.0984$ ***A1***

$0.0984 > 0.05$ (not significant at the 5% level) so do not reject the null hypothesis ***R1A1***

Note: The final ***A1*** cannot be awarded following an incorrect reason.
The final ***R1A1*** can follow through from their incorrect p-value.
Award a maximum of ***A1(M1)A0R1A1*** for p-value $= 0.0993$.

(ii) sample too small for the central limit theorem to apply (and t-tests assume normal distribution) ***R1***

[6 marks]

continued…

Question 1 continued

(f) (i) $H_0: \rho = 0$

$H_1: \rho > 0$ ***A1***

Note: Allow hypotheses to be expressed in words.

p-value $= 0.00157$ ***A1***

$(0.00157 < 0.01)$ there is a significant evidence of a (linear) correlation between effort and improvement (so it is reasonable to assume a linear relationship) ***R1***

(ii) (gradient of line of regression =) 6.6 ***A1***

[4 marks]

(g) H_0: improvement and gender are independent

H_1: improvement and gender are not independent ***A1***

choice of χ^2 test for independence ***(M1)***

groups first two columns as expected values in first column less than 5 ***M1***

new observed table

	$(f-p)<0$	$0 \leq (f-p) < 2$	$(f-p) \geq 2$
Male	14	10	9
Female	11	14	8

(A1)

p-value $= 0.581$ ***A1***

no significant evidence that gender and improvement are dependent ***R1***

[6 marks]

(h) *For example:*
larger samples / include data from whole school
take equal numbers of boys and girls in each sample
have a similar range of abilities in each sample
(if possible) have similar ranges of effort ***R1R1***

Note: Award ***R1*** for each reasonable suggestion to improve the validity of the test.

[2 marks]

Total [27 marks]

3. (a) H_0: The die is fair **OR** $P(\text{any number}) = \frac{1}{6}$ **OR** probabilities are equal

H_1: The die is not fair **OR** $P(\text{any number}) \neq \frac{1}{6}$ **OR** probabilities are not equal ***A1***

[1 mark]

(b) 5 ***A1***

[1 mark]

(c) 10 ***A1***

[1 mark]

(d) (p-value =) 0.287 $(0.28724163\ldots.)$ ***A2***

[2 marks]

(e) $0.287 > 0.05$ ***R1***

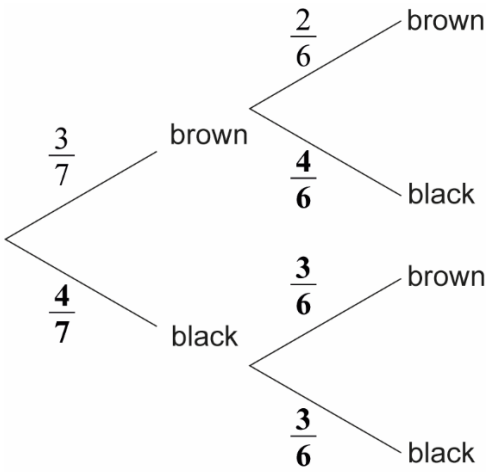
EITHER
Insufficient evidence to reject the null hypothesis ***A1***

OR
Insufficient evidence to reject that the die is fair ***A1***

Note: Do not award ***R0A1***. Condone "*accept* the null hypothesis" or "the die is fair". Their conclusion must be consistent with their p-value and their hypothesis.

[2 marks]
[Total 7 marks]

4. (a)



A1A1

Note: Award ***A1*** for $\frac{4}{7}$ and $\frac{4}{6}$ correctly placed, ***A1*** for final two probabilities correct.

[2 marks]

(b) multiplying along branches and then adding outcomes ***(M1)***

$\frac{3}{7}\times\frac{2}{6}+\frac{4}{7}\times\frac{3}{6}$

$=\frac{18}{42}\left(=\frac{3}{7}\approx 0.429\ (42.9\%)\right)$ ***A1***

[2 marks]

(c) use of conditional probability formula ***M1***

$\frac{\left(\frac{3}{7}\times\frac{2}{6}\right)}{\left(\frac{3}{7}\right)}$ ***A1***

$=\frac{6}{18}\left(=\frac{1}{3}\right)\ \left(\frac{252}{756}, 0.333, 33.3\%\right)$ ***A1***

[3 marks]
[Total 7 marks]

9. (a) let X be the weight of sugar in the bag

$\mathrm{P}(X < 950) = 0.308537... \approx 0.309$ ***(M1)A1***

[2 marks]

(b) **METHOD 1**

let $\bar{X}$ be the mean weight of 5 bags of sugar

$\mathrm{E}(\bar{X}) = 1000$ ***(A1)***

use of $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$ ***(M1)***

$\mathrm{Var}(\bar{X}) = \frac{100^2}{5} \ (= 2000)$ ***(A1)***

$\bar{X} \sim \mathrm{N}(1000,\ 2000)$

$\mathrm{P}(\bar{X} > 950) = 0.868223... \approx 0.868 \ (86.8\%)$ ***A1***

METHOD 2

let T be the total weight of 5 bags of sugar

$\mathrm{E}(T) = 5000$ ***(A1)***

use of $\mathrm{Var}(X_1 + X_2) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2)$ for independent random variables ***(M1)***

$\mathrm{Var}(T) = 5 \times 100^2 \ (= 50\,000)$ ***(A1)***

$T \sim \mathrm{N}(5000,\ 50\,000)$

$\mathrm{P}(T > 4750) = 0.868223... \approx 0.868 \ (86.8\%)$ ***A1***

[4 marks]

[Total 6 marks]

15. (a) $X \sim \text{Po}(324)$ ***A1***

Note: Both distribution and mean must be seen for ***A1*** to be awarded.

[1 mark]

(b) $\text{P}(X \leq 300)$ ***(M1)***

$= 0.0946831... \approx 0.0947$ ***A1***

[2 marks]

(c) (mean number of cars =) $4.5 \times 60 = 270$ ***(A1)***

$\text{P}(X > 300 \mid \lambda = 270)$ ***(M1)***

Note: Award ***M1*** for using $\lambda = 270$ to evaluate a probability.

$\text{P}(X \geq 301)$ **OR** $1 - \text{P}(X \leq 300)$ ***(M1)***

$= 0.0334207... \approx 0.0334$ ***A1***

[4 marks]

[Total 7 marks]

1. (a) $\left(\frac{17+25}{130}=\right)\frac{42}{130}$ $\left(\frac{21}{65}, 0.323076\ldots\right)$ ***A1***

[1 mark]

(b) $\left(\frac{17}{17+25}=\right)\frac{17}{42}$ $(0.404761\ldots)$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator. Award ***A1A0*** for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

[2 marks]

(c) $\frac{41}{130}\times\frac{40}{129}$ ***A1M1***

Note: Award ***A1*** for two correct fractions seen, ***M1*** for multiplying their fractions.

$=\frac{1640}{16770}\approx 0.0978$ $\left(0.0977936\ldots, \frac{164}{1677}\right)$ ***A1***

[3 marks]
Total [6 marks]

3. (a) $(\mathrm{E}(X) =) 10 \times 0.8$ ***(M1)***

8 (people) ***A1***

[2 marks]

(b) recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

[2 marks]

(c) 0.8 and 6 seen **OR** 0.2 and 3 seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

[3 marks]

Total [7 marks]

9. (a) $\overline{x} = 4.63 \ (4.62686\ldots)$ ***A1***

[1 mark]

(b) $s_{n-1} = 1.098702$ ***(A1)***

$s_{n-1}^2 = 1.21 \ (1.207146\ldots)$ ***A1***

Note: Award ***A0A0*** for an answer of 1.19 from biased estimate.

[2 marks]

(c) (i) $H_1 : \mu > 4.4$ ***A1***

(ii) **METHOD 1**

using a z-test ***(M1)***

$p = 0.0454992\ldots$ ***A1***

$p < 0.05$ ***R1***

reject null hypothesis ***A1***

(therefore there is significant evidence that the IB HL math students know more digits of π than the population in general)

Note: Do not award ***R0A1***. Allow ***R1A1*** for consistent conclusion following on from their p-value.

METHOD 2

using a t-test ***(M1)***

$p = 0.0478584\ldots$ ***A1***

$p < 0.05$ ***R1***

reject null hypothesis ***A1***

(therefore there is significant evidence that the IB HL math students know more digits of π than the population in general)

Note: Do not award ***R0A1***. Allow ***R1A1*** for consistent conclusion following on from their p-value.

[5 marks]

Total [8 marks]

12. (a) $\mathrm{P}(\text{Type I error}) = \mathrm{P}(\text{stating female when male})$

$= \mathrm{P}(W_{Male} > 11.5)$ ***(M1)***

$= 0.00135 \quad (0.00134996\ldots)$ ***A1***

[2 marks]

(b) $\mathrm{P}(\text{Type II error}) = \mathrm{P}(\text{stating male when female})$

$= \mathrm{P}(W_{Female} < 11.5)$ ***(M1)***

$= 0.309 \quad (0.308537\ldots)$ ***A1***

[2 marks]

(c) attempt to use the total probability ***(M1)***

$\mathrm{P}(\text{error}) = 0.9 \times 0.00134996\ldots + 0.1 \times 0.308537\ldots$

$= 0.0321 \quad (0.0320687\ldots)$ ***A1***

[2 marks]

Total [6 marks]

1. (a) (i) $\frac{370+472}{2}$ ***(M1)***

Note: This ***(M1)*** can also be awarded for either a correct Q_3 or a correct Q_1 in part (a)(ii).

$Q_3 = 421$ ***A1***

(ii) their part (a)(i) – their Q_1 (clearly stated) ***(M1)***

$\text{IQR} = (421 - 318 =)\ 103$ ***A1***

[4 marks]

(b) $(Q_3 + 1.5(\text{IQR}) =)\ 421 + (1.5 \times 103)$ ***(M1)***

$= 575.5$

since $498 < 575.5$ ***R1***

Netherlands is not an outlier ***A1***

Note: The ***R1*** is dependent on the ***(M1)***. Do not award ***R0A1***.

[3 marks]

(c) not appropriate ("no" is sufficient) ***A1***

as r is too close to zero / too weak a correlation ***R1***

[2 marks]

(d) (i) 6 ***A1***

(ii) 4.5 ***A1***

(iii) 4.5 ***A1***

[3 marks]

(e) (i) $r_s = 0.683\ \ (0.682646\ldots)$ ***A2***

(ii) **EITHER**

there is a (positive) association between the population size and the score ***A1***

OR

there is a (positive) linear correlation between the ranks of the population size and the ranks of the scores (when compared with the PMCC of 0.249) ***A1***

[3 marks]

(f) lowering the top score by 20 does not change its rank so r_s is unchanged ***R1***

Note: Accept "this would not alter the rank" or "Netherlands still top rank" or similar. Condone any statement that clearly implies the ranks have not changed, for example: "The Netherlands still has the highest score."

[1 mark]

[Total 16 marks]

3. (a) quota ***A1***

[1 mark]

(b) (i) $27.125 \approx 27.1$ ***(M1)A1***

(ii) $8.29815\ldots \approx 8.30$ ***A1***

[3 marks]

(c) (let μ be the national mean)

H_0: $\mu = 25.2$
H_1: $\mu > 25.2$ ***A1***

Note: Accept hypotheses in words if they are clearly expressed and '**population** mean' or 'school mean' is referred to. Do not accept H_0: $\mu = \mu_0$ unless μ_0 is explicitly defined as "national standard mark" or given as 25.2.

recognizing t-test ***(M1)***
p-value $= 0.279391\ldots$ ***A1***

$0.279391\ldots > 0.05$ ***R1***

Note: The ***R1*** mark is for the comparison of their p-value with 0.05.

insufficient evidence to reject the null hypothesis (that the mean for the school is 25.2) ***A1***

Note: Award the final ***A1*** only if the **null** hypothesis is also correct (e.g. $\mu_0 = 25.2$ or (population) mean $= 25.2$) and the conclusion is consistent with both the direction of the inequality and the alternative hypothesis.

[5 marks]

(d) **EITHER**
the sampling process is not random ***R1***

For example:
the school asked for volunteers
the students were selected from a single class

OR
the quota might not be representative of the student population ***R1***

For example:
the school may have only 4 boys and 400 girls.

Note: Do not accept 'the sample is too small'.

[1 mark]

(e) (i) $(28.1 \times 2 + 20 =)\ 76.2$ ***A1***

(ii) 8.4×2 ***(A1)***
$= 16.8$ ***A1***

[3 marks]
[Total 13 marks]

5. (a) (let T be the number of passengers who arrive)

$\left(\mathrm{P}(T > 72) =\right)\ \mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74,\ 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379\ \ (0.00379124...)$ ***A1***

Note: Using the distribution $\mathrm{B}(74,\ 0.1)$, to work with the 10% that do not arrive for the flight, here and throughout this question, is a valid approach.

[3 marks]

(b) (i) 72×0.9 ***(M1)***

64.8 ***A1***

(ii) $n \times 0.9 = 72$ ***(M1)***

80 ***A1***

[4 marks]

(c) **METHOD 1**

EITHER

when selling 74 tickets

	$T \leq 72$	$T = 73$	$T = 74$
Income minus compensation (I)	11100	10800	10500
Probability	0.9962…	0.003380…	0.0004110…

top row ***A1A1***

bottom row ***A1A1***

Note: Award ***A1A1*** for **each** row correct. Award ***A1*** for one correct entry and ***A1*** for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962... + 10800 \times 0.00338... + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

continued…

Question 5 continued

OR

income is $74\times150=11100$ ***(A1)***

expected compensation is

$0.003380...\times300+0.0004110...\times600\ \ (=1.26070...)$ ***(M1)A1A1***

Note: The ***(M1)*** is for an attempt to work out expected compensation by multiplying a probability for tickets sold by either 300 or 600.

expected income when selling 74 tickets is $11100-1.26070\ldots$ ***(M1)***

Note: Award ***(M1)*** for subtracting their expected compensation from 11100.

$=11098.73..\ \ (=\$11099)$ ***A1***

THEN

income for 72 tickets $=72\times150=10800$ ***(A1)***

so expected gain $\approx11099-10800=\$299$ ***A1***

METHOD 2

for 74 tickets sold, let C be the compensation paid out

$\mathrm{P}(T=73)=0.00338014\ldots,\ \mathrm{P}(T=74)=0.000411098\ldots$ ***A1A1***

$\mathrm{E}(C)=0.003380...\times300+0.0004110...\times600\ \ (=1.26070...)$ ***(M1)A1A1***

extra expected revenue $=300-1.01404...-0.246658\ldots\ \ (300-1.26070\ldots)$

(A1)(M1)

Note: Award ***A1*** for the 300 and ***M1*** for the subtraction.

$=\$299$ (to the nearest dollar) ***A1***

METHOD 3

let D be the change in income when selling 74 tickets.

	$T\le72$	$T=73$	$T=74$
Change in income	300	0	−300

(A1)(A1)

Note: Award ***A1*** for one error, however award ***A1A1*** if there is no explicit mention that $T=73$ would result in $D=0$ and the other two are correct.

$\mathrm{P}(T\le73)=0.9962...,\ \mathrm{P}(T=74)=0.000411098\ldots$ ***A1A1***

$\mathrm{E}(D)=300\times0.9962...+0\times0.003380...-300\times0.0004110$ ***(M1)A1A1***

$=\$299$ ***A1***

[8 marks]

[Total 15 marks]

5. (a) (i) 0.02 ***A1***

(ii) the probability of mutating from 'not normal state' to 'normal state' ***A1***

Note: The ***A1*** can only be awarded if it is clear that transformation is **from** the mutated state.

[2 marks]

(b) $\det\begin{pmatrix} 0.94-\lambda & 0.02 \\ 0.06 & 0.98-\lambda \end{pmatrix}=0$ ***(M1)***

Note: Award ***M1*** for an attempt to find eigenvalues. Any indication that $\det(\boldsymbol{M}-\lambda\boldsymbol{I})=0$ has been used is sufficient for the ***(M1)***.

$(0.94-\lambda)(0.98-\lambda)-0.0012=0$ **OR** $\lambda^2-1.92\lambda+0.92=0$ ***(A1)***

$\lambda=1,\ 0.92\ \left(\frac{23}{25}\right)$ ***A1***

[3 marks]

(c) $\begin{pmatrix} 0.94 & 0.02 \\ 0.06 & 0.98 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$ **OR** $\begin{pmatrix} 0.94 & 0.02 \\ 0.06 & 0.98 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=0.92\begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

Note: This ***M1*** can be awarded for attempting to find either eigenvector.

$0.02y-0.06x=0$ **OR** $0.02y+0.02x=0$

$\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1A1***

Note: Accept any multiple of the given eigenvectors.

[3 marks]

(d) (i) $\begin{pmatrix} 0.94 & 0.02 \\ 0.06 & 0.98 \end{pmatrix}^5\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.744 & 0.0852 \\ 0.256 & 0.915 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ***(M1)***

Note: Condone omission of the initial state vector for the ***M1***.

$0.744\quad(0.744311\ldots)$ ***A1***

(ii) $\begin{pmatrix} 0.25 \\ 0.75 \end{pmatrix}$ ***(A1)***

Note: Award ***A1*** for $\begin{pmatrix} 0.25 \\ 0.75 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.25 & 0.25 \\ 0.75 & 0.75 \end{pmatrix}$ seen.

0.25 ***A1***

[4 marks]

Total [12 marks]

1. (a) (i) mean $= 4.23\ (4.23333\ldots)$ ***A1***

variance $= 4.27\ (4.26777...)$ ***A1***

[2 marks]

(ii) mean is close to the variance ***A1***

[1 mark]

(b) *One of the following:*
the number of bags sold each day is independent of any other day
the sale of one bag is independent of any other bag sold
the sales of bags of rice (each day) occur at a *constant mean* rate ***A1***

Note: Award ***A1*** for a correct answer in context. Any statement referring to independence must refer to either the independence of each bag sold or the independence of the number of bags sold each day. If the third option is seen, the statement must refer to a "constant mean" or "constant average". Do not accept "the number of bags sold each day is constant".

[1 mark]

(c) attempt to find Poisson probabilities and multiply by 90 ***(M1)***
$a = 7.018$ ***A1***
$b = 17.498$ ***A1***

EITHER
$90 \times \mathrm{P}(X \geq 8) = 90 \times \left(1 - \mathrm{P}(X \leq 7)\right)$ ***(M1)***
$c = 5.755$ ***A1***

OR
$90 - 7.018 - 11.903 - 16.665 - 17.498 - 14.698 - 10.289 - 6.173$ ***(M1)***
$c = 5.756$ ***A1***

Note: Do not penalize the omission of clear a, b and c labelling as this will be penalized later if correct values are interchanged.

[5 marks]

(d) (i) 7 ***A1***

[1 mark]

(ii) H_0 : The number of bags of rice sold each day follows a Poisson distribution with mean 4.2. ***A1***

H_1 : The number of bags of rice sold each day does not follow a Poisson distribution with mean 4.2. ***A1***

Note: Award ***A1A1*** for **both** hypotheses correctly stated and in correct order. Award ***A1A0*** if reference to the data and/or "mean 4.2" is not included in the hypotheses, but otherwise correct.

evidence of attempting to group data to obtain the observed frequencies for ≤ 1 and ≥ 8 ***(M1)***

p-value $= 0.728$ $(0.728100...)$ ***A2***

0.728 $(0.728100...) > 0.05$ ***R1***

the result is not significant so there is no reason to reject H_0
(the number of bags sold each day follows a Poisson distribution) ***A1***

Note: Do not award ***R0A1***. The conclusion MUST follow through from their hypotheses. If no hypotheses are stated, the final ***A1*** can still be awarded for a correct conclusion as long as it is in context (e.g. therefore the data follows a Poisson distribution).

[7 marks]

(e) (i) **METHOD 1**

evidence of multiplying 4.2×60 (seen anywhere) ***M1***

$H_0 : \mu = 252$

$H_1 : \mu > 252$ ***A1***

Note: Accept $H_0 : \mu = 4.2$ and $H_1 : \mu > 4.2$ for the ***A1***.

evidence of finding probabilities around critical region ***(M1)***

Note: Award ***(M1)*** for any of these values seen:
$P(X \geq 277) = 0.0630518\ldots$ **OR** $P(X \leq 276) = 0.936948\ldots$
$P(X \geq 278) = 0.0558415\ldots$ **OR** $P(X \leq 277) = 0.944158\ldots$
$P(X \geq 279) = 0.0493055\ldots$ **OR** $P(X \leq 278) = 0.950694\ldots$.

critical value $= 279$ ***A1***

$282 \geq 279$, ***R1***

the null hypothesis is rejected ***A1***

(the advertising increased the number of bags sold during the 60 days)

Note: Do not award ***R0A1***. Accept statements referring to the advertising being effective for ***A1*** as long as the ***R*** mark is satisfied. For the ***R1A1***, follow through within the part from their critical value.

METHOD 2

evidence of dividing 282 by 60 (or 4.7 seen anywhere) ***M1***

$H_0 : \mu = 4.2$

$H_1 : \mu > 4.2$ ***A1***

attempt to find critical value using central limit theorem ***(M1)***

(e.g. sample standard deviation $= \sqrt{\frac{4.2}{60}}$, $\bar{X} \sim N\left(4.2, \sqrt{\frac{4.2}{60}}\right)$, etc.)

Note: Award ***(M1)*** for a p-value of $0.0293907\ldots$ seen.

critical value $= 4.63518\ldots$ ***A1***

$4.7 > 4.63518\ldots$ ***R1***

the null hypothesis is rejected ***A1***

(the advertising increased the number of bags sold during the 60 days)

Note: Do not award ***R0A1***. Accept statements referring to the advertising being effective for ***A1*** as long as the ***R*** mark is satisfied. For the ***R1A1***, follow through within the part from their critical value.

[6 marks]

(ii) $\left(P(X \geq 279 \mid \mu = 252) =\right)$ 0.0493 $(0.0493055\ldots)$ ***A1***

Note: If a candidate uses **METHOD 2** in part (e)(i), allow an ***FT*** answer of 0.05 for this part but only if the candidate has attempted to find a p-value.

[1 mark]

(f) attempt to compare profit *difference* with cost of advertising ***(M1)***

Note: Award ***(M1)*** for evidence of candidate mathematically comparing a profit difference with the cost of the advertising.

EITHER
(comparing profit from 30 extra bags of rice with cost of advertising)
$14850 < 18000$ ***A1***

OR
(comparing total profit with and without advertising)
$121590 < 124740$ ***A1***

OR
(comparing increase of average daily profit with daily advertising cost)
$247.50 < 300$ ***A1***

THEN

EITHER
Even though the number of bags of rice increased, the advertising is not worth it as the overall profit did not increase. ***R1***

OR
The advertising is worth it even though the cost is less than the increased profit, since the number of customers increased (possibly buying other products and/or returning in the future after advertising stops) ***R1***

Note: Follow through within the part for correct reasoning consistent with their comparison.

[3 marks]

[Total 27 marks]

1. (a) The favourite breakfast/berry (of adults) is independent of (their) income (level). ***A1***

[1 mark]

(b) $\chi^2 = 2.27 \ (2.26821...)$ ***A2***

[2 marks]

(c) **EITHER**

$2.27 < 7.78$ **OR** $2.27 <$ critical value ***R1***

OR

$0.687 > 0.1$ (using p-value)

THEN

(Do not reject H_0)

Insufficient evidence (at the 10% significance level) that the favourite berry depends on income level. ***A1***

Note: Do not award ***R0A1***. Accept "χ^2" in place of their "2.27", provided an answer was seen in part (b). Their conclusion must be consistent with their χ^2 (or a correct p-value) and their hypothesis.

[2 marks]

Total [5 marks]

5. (a) $0.5\times0.1+0.4\times0.4+0.1\times0.5$ ***(M1)(M1)(M1)***

Note: Award ***M1*** for 0.5×0.1 or 0.1×0.5, ***M1*** for 0.4×0.4, ***M1*** for adding three correct products.

0.26 ***A1***

[4 marks]

(b) $0=-8\times0.5+4\times0.4+0.1k$ ***(M1)(M1)***

Note: Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

$(k=)\ 24$ (points) ***A1***

[3 marks]

Total [7 marks]

16. (a) (let p be the probability of a student choosing healthy options)

$\mathrm{H}_0 : p = 0.3$ ***A1***

$\mathrm{H}_1 : p > 0.3$ ***A1***

Note: Award ***A0A1*** for correct hypotheses with μ in place of p. Accept equivalent hypotheses in words.

[2 marks]

(b) a type I error is rejecting H_0 when H_0 is true ***(M1)***

(let N = number of students choosing a healthy option)

$N \sim \mathrm{B}(80, 0.3)$

$\mathrm{P}(31 \leq N \leq 80)$ **OR** $\mathrm{P}(N \geq 31)$ **OR** $1 - \mathrm{P}(N \leq 30)$ ***(M1)***

Note: Do not accept the use of the Normal approximation.

$0.0587 \ (0.0587481\ldots)$ ***A1***

[3 marks]

(c) a type II error is accepting H_0 when H_0 is not true ***(M1)***

$N \sim \mathrm{B}(80, 0.4)$

$\mathrm{P}(0 \leq N \leq 30)$ **OR** $\mathrm{P}(N \leq 30)$ ***(M1)***

$0.369 \ (0.368726\ldots)$ ***A1***

[3 marks]

Total [8 marks]

1. (a) (i) $(m =)\ 54(\%)$ ***A1***

(ii) $(n =)\ 14(\%)$ ***A1***

(iii) $(p =)\ 22(\%)$ ***A1***

(iv) $(q =)\ 10(\%)$ ***A1***

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[4 marks]

(b) (i) $0.54\ \left(\frac{54}{100}, \frac{27}{50}, 54\%\right)$ ***A1***

(ii) $\frac{54}{64}\ \left(0.844, \frac{27}{32}, 84.4\%, 0.84375\right)$ ***A1A1***

Note: Award ***A1*** for a correct denominator (0.64 or 64 seen), ***A1*** for the correct final answer.

[3 marks]

(c) (i) recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10, 0.68)$

$\left(\mathrm{P}(X = 5) =\right)\ 0.123\ \ (0.122940..., 12.3\%)$ ***A1***

(ii) $1 - \mathrm{P}(X \leq 3)$ **OR** $\mathrm{P}(X \geq 4)$ **OR** $\mathrm{P}(4 \leq X \leq 10)$ ***(M1)***

$0.984\ (0.984497..., 98.4\%)$ ***A1***

(iii) $(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2 \times \left((0.68)^9 \times 0.32\right)$

$0.0199\ \ (0.0198957..., 1.99\%)$ ***A1***

[7 marks]

(d) **EITHER**

the probability is not constant ***A1***

OR

the events are not independent ***A1***

OR

the events should be modelled by the hypergeometric distribution instead ***A1***

[1 mark]

Total [15 marks]

6. (a) wood layer, $W \sim \mathrm{N}(7,\ 0.3^2)$; plastic, $P \sim \mathrm{N}(3,\ 0.16^2)$

door: $X = W + P$

$\mathrm{E}(X) = 10$ (mm) ***(A1)***

$\mathrm{Var}(X) = \mathrm{Var}(W) + \mathrm{Var}(P) = 0.1156$ (mm^2) ***(M1)(A1)***

recognizing the distribution is Normal, with their mean and variance ***(M1)***

$X \sim \mathrm{N}(10,\ 0.34^2)$

$\mathrm{P}(X < 9.5) = 0.0707$ $(0.07070125...)$ ***A1***

[5 marks]

(b) $\mathrm{E}(T) = 80$ ***(A1)***

$\mathrm{Var}(T)\left(= 0.1156 \times 8\right) = 0.9248$ ***(M1)(A1)***

$T \sim \mathrm{N}(80,\ 0.9248)$

$\mathrm{P}(T > 82) = 0.0188$ $(0.0187753\ldots)$ ***A1***

[4 marks]

(c) (i) 6.93 mm (6.93428…) ***A1***

(ii) $(s_{n-1} =)$ 0.404 ***(A1)***

$(s_{n-1}^2 =)$ 0.163 mm^2 $(0.162928\ldots)$ ***A1***

[3 marks]

(d) $\mathrm{H}_0 : \mu_A = \mu_B$ and $\mathrm{H}_1 : \mu_A > \mu_B$ ***A1A1***

Note: Award ***A1*** for use of μ or in words "population mean", and ***A1*** for both correct equality in null hypothesis and correct inequality in alternative hypothesis. Accept an equivalent statement in words, must include mean and reference to "**population** mean" / "mean for **all** Machine B layers" for the first ***A1*** to be awarded.

use a two-sample *t*-test ***(M1)***

p-value $= 0.406975...$ ***A1***

since $0.406975... > 0.05$ **OR** p-value > 0.05 ***R1***

Do not reject H_0 (Insufficient evidence to support the employee's claim) ***A1***

Note: Accept a *p*-value of $0.415861\ldots$ from use of 3sf values from part (c). Follow through within the question for the final ***R1*** and ***A1*** for their *p*-value provided $0 \le p \le 1$. Do not award ***R0A1***.

[6 marks]

Total [18 marks]

1. (a) $\frac{12}{5}\ \left(\frac{144}{60}, 2.4\right)$ ***(M1)A1***

[2 marks]

(b) $\frac{3}{5}\ \left(\frac{144}{240}, 0.6\right)$ ***A1***

[1 mark]

(c) (i) wins $\sim \mathrm{B}\left(4, \frac{3}{5}\right)$

$\mathrm{P}(\text{wins} = 0) = 0.0256$ ***A1***

Note: Allow ***FT*** from use of their probability in part (b) but only when used with $n = 4$.

(ii) expected frequency $= 60 \times 0.0256$ ***(M1)***

$= 1.536$ ***A1***

[3 marks]

(d) (i) H_0: data follows a Binomial distribution with $n = 4$ ***A1***

(ii) $(\mathrm{df} = 4 - 1 - 1 =)\ 2$ ***A1***

(iii) p-value $= 0.954\ \ (0.953872...)$ ***(M1)A1***

(iv) $0.954 > 0.05$ ***R1***

insufficient evidence to reject H_0 ***A1***

Note: Condone "accept H_0". Follow through from their p-value in part (d)(iii) if the reasoning is correct and correct conclusions are made. Do NOT award ***R0A1***.

[6 marks]

(e) wins $\sim \mathrm{B}\left(4, \frac{3}{5}\right)$ **OR** $1 - 0.0256$ ***(M1)***

$\mathrm{P}(\text{wins} \geq 1) = 0.974\ \ (0.9744)$ ***A1***

[2 marks]

(f) (i) there are 145 transitions that start with Argentina, ***(M1)***

(of which 85 lead to Argentina winning,) so the probability is $\frac{85}{145}$ ***A1***

$= \frac{17}{29}$ ***AG***

(ii) $\begin{pmatrix} \frac{17}{29} & \frac{31}{47} \\ \frac{12}{29} & \frac{16}{47} \end{pmatrix} \left(= \begin{pmatrix} 0.586 & 0.660 \\ 0.414 & 0.340 \end{pmatrix} \right)$ ***A1A1***

Note: Accept the transposed matrix as correct.
Award ***A1*** for $\frac{17}{29}$ placed in a leading diagonal.
Award ***A1*** for all other values correct and in correct position in the matrix.

[4 marks]

(g) (i) write their matrix with λ subtracted from the leading diagonal ***(M1)***
equate determinant to zero ***(M1)***

$$\det \begin{pmatrix} \frac{17}{29} - \lambda & \frac{31}{47} \\ \frac{12}{29} & \frac{16}{47} - \lambda \end{pmatrix} = 0$$

$\left(\frac{17}{29} - \lambda\right)\left(\frac{16}{47} - \lambda\right) - \frac{12}{29} \times \frac{31}{47} = 0$ ***A1***

correct intermediate step
$1363\lambda^2 - 1263\lambda - 100 = 0$ ***AG***

Note: Do not award ***A1*** if there is no intermediate step leading from determinant to given answer.
Solving $\boldsymbol{T}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$ for x and y may be seen and is a valid alternative method.
Accept working in the form $\det(\lambda I - T) = 0$.

(ii) $\lambda = 1, -\frac{100}{1363}$ $(-0.0733675...)$ ***A1***

(iii) attempt to solve $\boldsymbol{T}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$y = \frac{564}{899}x$

eigenvector for $\lambda = 1$ is $\begin{pmatrix} 1 \\ \frac{564}{899} \end{pmatrix} \left(= \begin{pmatrix} 1 \\ 0.627 \end{pmatrix} \right)$ ***A1***

eigenvector for $\lambda = -\frac{100}{1363}$ is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1***

Note: Allow correct multiples of the eigenvectors.
If eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is stated without the second eigenvector, or any other working, then award ***M0A0A1***.

[7 marks]

(h) **EITHER**

solution found using $\begin{pmatrix} 1 \\ \frac{564}{899} \end{pmatrix} \left(= \begin{pmatrix} 1 \\ 0.627 \end{pmatrix} \right)$

$x + \frac{564}{899}x = 1$ ***(M1)***

$x = 0.614 \left(0.614490..., \frac{899}{1463} \right)$ ***(A1)***

OR

solution can be found from high power of transition matrix

$\begin{pmatrix} \frac{17}{29} & \frac{31}{47} \\ \frac{12}{29} & \frac{16}{47} \end{pmatrix}^{50} = \begin{pmatrix} 0.614 & 0.614 \\ 0.386 & 0.386 \end{pmatrix}$ ***(M1)***

Note: Accept the transposed matrix if consistent with their answer to part (f)(ii).

probability $= 0.614 \left(0.614490..., \frac{899}{1463} \right)$ ***(A1)***

THEN

$\mathrm{P}(3 \text{ wins}) = 0.614 \times 0.586^2 \left(= \frac{899}{1463} \times \left(\frac{17}{29} \right)^2 \right)$ ***(M1)***

$= 0.211 \left(0.211162..., \frac{8959}{42427} \right)$ ***A1***

[4 marks]
[Total: 29 marks]

2. (a) $H_0 : \mu_b = \mu_m$ ***A1***

$H_1 : \mu_b > \mu_m$ ***A1***

Note: Accept equivalent statements in words such as "the **mean** score of bilingual people equals the **mean** score of monolingual people".

[2 marks]

(b) $0.119 \ (0.119395\ldots)$ ***A2***

[2 marks]

(c) $0.119395\ldots > 0.05 \ \ (11.9395\ldots\% > 5\%)$ ***R1***

(fail to reject H_0) there is insufficient evidence to suggest that bilingual people have better memory retention than monolingual people ***A1***

Note: Do not award ***R0A1***.
The answer to part (c) MUST be consistent with **their** hypotheses and **their** p-value.

[2 marks]

[Total: 6 marks]

8. (a) H_0: X and Y are not (linearly) correlated **OR** $\rho = 0$ ***A1***

H_1: X and Y are (linearly) correlated **OR** $\rho \neq 0$ ***A1***

Note: Accept "independent" or "not associated" in place of "not correlated".
If H_0 and H_1 are reversed, then award ***A0A1***.

[2 marks]

(b) (i) $r = 0.849 \ (0.848886...)$ ***A1***

(ii) p-value $= 0.0325 \ (0.0325277...)$ ***A2***

Note: Award ***A1*** for p-value $= 0.033$ or p-value $= 0.03$.
Award ***FT*** for $\rho > 0$ or $\rho < 0$ in part (a), p-value $= 0.0163 \ (0.0162638\ldots)$
or p-value $= 0.984 \ (0.983736\ldots)$
Award the full marks for seeing the values of r and p-value from the markscheme when H_0 and H_1 are reversed in part (a).

[3 marks]

(c) $0.0325 < 0.05$ ***R1***

(so we reject H_0 in favour of H_1)

(there is sufficient evidence to suggest) X and Y are (linearly) correlated ***A1***

Note: Their conclusion must be consistent with their p-value and their hypotheses and it must be in context.

[2 marks]

[Total: 7 marks]

9. (a) attempt to find the difference between 75.7 and 67.3 ***(M1)***

$$\frac{75.7-67.3}{2}$$

$4.2\ (\text{km h}^{-1})$ ***A1***

[2 marks]

(b) **METHOD 1 (Comparing areas above and below the mean)**

$\text{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF$(67.3, 74, 67.3, 4.2)$ **OR** sketch of normal distribution with 67.3 and 74 labelled and shaded between ***(M1)***

area of region between mean and q is at least $0.445\ \ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ \ (0.375329\ldots)$ between mean and p ***R1***

The region between p and q is not symmetrical ***AG***

METHOD 2 (Comparing areas in the tails)

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ ***(M1)***

$\text{P}(\text{speed} < 74) = 0.944670\ldots$

$\text{P}(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ \ 0.124670\ldots$

$\text{P}(\text{speed} > q) = (1 - 0.944670\ldots =)\ \ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $\text{P}(\text{speed} > q) \leq 0.0553295$ and $\text{P}(\text{speed} < p) \geq 0.124670$ so

$\text{P}(\text{speed} > q)$ will never equal $\text{P}(\text{speed} < p)$ ***R1***

the region between p and q is not symmetrical ***AG***

continued…

Question 9 continued

METHOD 3 (Assumption of symmetry comparing speeds)

attempt to calculate area below q assuming distribution is symmetrical ***(M1)***

e.g. $\mathrm{P}(\text{speed} < q) = 0.82 + ½ \times 0.18 \quad (0.91)$

EITHER

$(q =)\ 72.9 \quad (72.9311\ldots)$ ***A1***

$72.9 < 74$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$\mathrm{P}(\text{speed} < 74) = 0.945 \quad (0.944670\ldots)$ ***A1***

$0.945 > 0.91$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 4 (Assumption of symmetry comparing areas)

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6 < \text{speed} < 74)$ **OR** Normal CDF(60.6, 74, 67.3, 4.2) **OR**

$\mathrm{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2)

EITHER

$0.889 \quad (0.889340\ldots)$ ***A1***

$0.889 > 0.82$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$0.445 \quad (0.444670\ldots)$ ***A1***

$0.445 > 0.82 \div 2$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

[3 marks]

[Total: 5 marks]

11. Let $D = O - L - L$ ***(A1)***

$(\text{mean} =)\ 205 - 105 - 105\ (= -5)$ ***(A1)***

manipulating variances (not standard deviations) ***(M1)***

$(\text{variance} =)\ 25 + 9 + 9\ (= 43)$ **OR** $(\text{SD} =)\ 6.55743...$ ***(A1)***

$D \sim \text{N}(205 - 105 - 105,\ 25 + 9 + 9)$

attempt to find the probability that $D > 0$ ***(M1)***

$\text{P}(D > 0)$

$= 0.223\ (0.222882...)$ ***A1***

Note: If $D = O - 2L$ is seen or implied, award at most ***(A0)A1(M0)(A0)M1A0***.

[Total: 6 marks]

16. (a) attempt to find what the model predicts in terms of k ***(M1)***

$k, \frac{k}{4}, \frac{k}{25}$

correct expression for sum of square residuals ***(A1)***

$(k-42)^2+\left(\frac{k}{4}-11\right)^2+\left(\frac{k}{25}-1.5\right)^2$

valid attempt to find c by expanding or recognizing the constant terms ***(M1)***

$c = 42^2+11^2+1.5^2$

$= 1887.25$ ***A1***

[4 marks]

(b) valid method to find the k value at the minimum ***(M1)***

$k = \frac{89.62}{2\times 1.0641}$ $(= 42.1107...)$, graph, completing the square

(so least squares regression is) $I = \frac{42.1}{d^2}$ ***A1***

[2 marks]

[Total: 6 marks]

2. (a) $11.0\ (11.0212\ldots)$ ***A2***

Note: Award ***A1*** for a final answer of 11 if no unrounded answer is seen.

[2 marks]

(b) **EITHER**

$11.0 > 9.488\ (11.0212\ldots > 9.488\ )$ ***R1***

OR

$0.0263 < 0.05\ (0.0263264\ldots < 0.05\ )$ ***R1***

THEN

EITHER

(there is significant evidence to) reject H_0 ***A1***

OR

(there is significant evidence that) the (food) quality and the type of meal are not independent ***A1***

Note: Do not award ***R0A1.***

Award ***R1*** for $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$, provided the calculated value is explicitly seen in part (b).

Accept "p-value < significance level" provided their p-value is seen and their p-value is between 0 and 1.

[2 marks]

Total [4 marks]

3. (a) attempting to use $\mathrm{P}(R \cap S) = \mathrm{P}(R)\mathrm{P}(S)$ ***(M1)***

$0.2 = 0.8(0.2 + x)$ ***(A1)***

$x = 0.05$ ***A1***

[3 marks]

(b) $x + 0.2 + 0.6 + y = 1$ ***(M1)***

$y = 0.15$ ***A1***

[2 marks]

(c) **METHOD 1**

attempting to apply $\mathrm{P}(R' \mid S') = \dfrac{\mathrm{P}(R' \cap S')}{\mathrm{P}(S')}$ ***(M1)***

$\dfrac{0.15}{0.2}$

$= \dfrac{3}{4}$ ***A1***

METHOD 2

$\mathrm{P}(R' \mid S') = \mathrm{P}(R')$ (because R, S are independent) ***(M1)***

$= 1 - 0.25 = 0.75$ ***A1***

Note: ***FT*** from their values of x or y.

[2 marks]

Total [7 marks]

5. (a) $X \sim \mathrm{N}(4, 0.25^2)$

EITHER

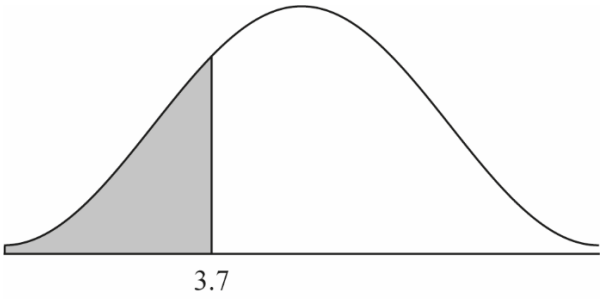
correct probability expression ***(M1)***

$\mathrm{P}(X < 3.7)$

Note: Accept a weak or strict inequality, and any label instead of X, e.g. length or L.

OR

normal curve with vertical line, left of mean, labelled 3.7, and shaded region ***(M1)***



THEN

0.115 $(0.115069\ldots, 11.5\%)$ ***A1***

Note: Award ***M1A0*** for 0.12 if no previous working.

[2 marks]

(b) **EITHER**

Correct probability expression ***(M1)***

$(\mathrm{P}(X < k) = 0.7$ **OR** $\mathrm{P}(X > k) = 0.3$

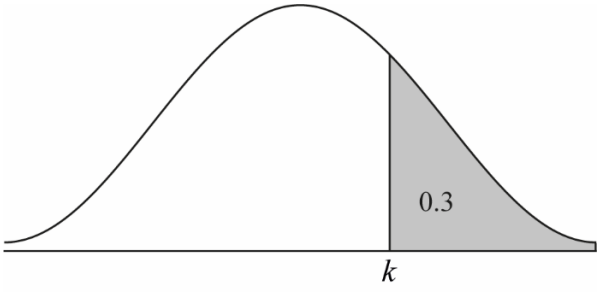
Note: Accept a weak or strict inequality, and any label instead of X *e.g.*, length or L.

continued…

Question 5 continued

OR

normal curve with vertical line to the right of the mean and shaded region, correctly labelled either 0.3 or 0.7 ***(M1)***



THEN

$(k =)\ 4.13\ (4.13110\ldots)$ ***A1***

Note: Award ***M1A0*** for 4.1 if no previous working.

[2 marks]

(c) **EITHER**

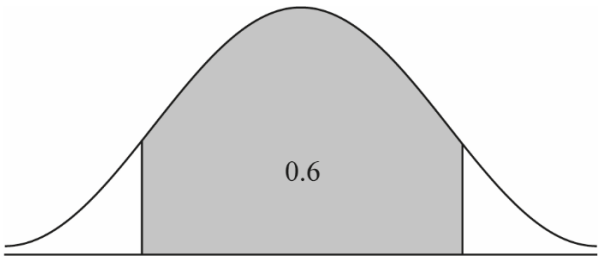
correct probability equation ***(M1)***

$\mathrm{P}(\text{length} < 4 + m) = 0.8$ **OR** $\mathrm{P}(\text{length} < 4 - m) = 0.2$

Note: Accept any letter instead of "length" *e.g.*, X or L.

OR

normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of 0.6 or 0.2 or 0.8 ***(M1)***



THEN

$0.210\ (0.210405\ldots)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 3.7895 or 4.2105 seen without working. Condone 0.21 seen and award ***(M1)A1***.

[2 marks]

Total [6 marks]

7. (a) $(56 \times 0.86) = 48.2 \quad (48.16)$ ***A1***

Note: Accept 48.

[1 mark]

(b) recognizing binomial distribution (may be seen in (a)) ***(M1)***
e.g. $X \sim \mathrm{B}(56,\ 0.86)$
$\left(\mathrm{P}(X \geq 50) =\right)\ 0.316$ ***A2***

[3 marks]

(c) $\mathrm{P}(X \leq n) \geq 0.25$
$n = 46$ ***A2***

[2 marks]
Total [6 marks]

9. (a)

Athlete	A	B	C	D	E	F	G	H
Age rank	7	6	3	5	4	2	8	1
Time rank	3.5	2	3.5	6	7	8	1	5

A1A1

Note: Award ***A1*** for each correct row.

[2 marks]

(b) $r_s = -0.671\ (-0.670670...)$ ***A2***

Note: Only follow through from an incorrect table provided the ranks are all between 1 and 8.
Award ***A1*** for -0.67 **OR** for the omission of the negative sign, e.g. 0.671 $(0.670670...)$ or 0.67

[2 marks]

(c) (A value of $r_S = -0.671$) indicates a negative correlation between a person's age and the best time they take to run 100m. ***R1***

Note: Condone any comment that includes "weak" or "strong" etc. Accept an interpretation in words, but only if there is a general link described and not a rule: "The older a person gets, the faster they *tend to* run".
Answer must be in context.

[1 mark]

(d) *Award **R1** for any sensible reason:* ***R1***
The correlation, such that it is, is unlikely to be linear for this type of data.
Spearman's CC is less sensitive to outliers
Sung-Jin is not sure the data is drawn from a bivariate normal distribution
There are outliers/extreme data
Same time for two athletes with significantly different ages

[1 mark]

(e) (i) $0.264\ (0.263762...)$ ***A2***

Note: Award ***A1*** for 0.26 with no working. Given that the exact model is not specific in the question, accept correct r^2 values from other regression models: 0.631, 0.650, 0.759 and 0.256.

(ii) approximately 26% of the variability in the times taken can be explained by the runner's age. ***R1***

[3 marks]
Total [9 marks]

10. (a) H_0 : there is no particular preference for any of the flavours ***A1***

H_1 : there is a particular preference for some of the flavours ***A1***

Note: Accept equivalent statements such as "H_0 : the population ratio of flavour preferences is $1:1:1:1$" or "H_0 : the population proportions are equal for each flavour" or "H_0 :the data is drawn from a uniform distribution".

[2 marks]

(b) **EITHER**

$p\text{-value} = 0.0629 \ \ (0.0629034...)$ ***A2***

$0.0629 > 0.05$ ***R1***

OR

$\chi^2_{\text{calc}} = 7.30$ ***A2***

$7.30 < 7.82$ ***R1***

Note: Award ***A2*** for either $p\text{-value} = 0.063$ or $\chi^2_{\text{calc}} = 7.3$ seen. Award ***R1*** for a correct comparison involve their $p\text{-value}$ or χ^2_{calc}, and follow through for their conclusion.

THEN

so there is insufficient evidence to reject H_0, i.e.

there is no particular preference for any of the flavours. ***A1***

Note: Do not award ***R0A1***.

[4 marks]

Total [6 marks]

15. (a) (i) $224\text{ g}\ (224.25\text{ g})$ ***A1***

(ii) $[222.1, 226.4]$ ***A1A1***

Note: Award ***A1*** for each correct end of the interval. Accept open or closed (weak or strict) interval notation. Inequalities involving μ would also be accepted, but not involving $\bar{x}$.
Award ***A1A0*** for correct answers not given correct to 4 sf.

[3 marks]

(b) **EITHER**
the (population) weight of granules of Apollo coffee is normally distributed. ***R1***
OR
the readings are independent ***R1***

[1 mark]

(c) 226g lies within the confidence interval, ***R1***
so there is no evidence to dispute the claim on the label. ***A1***

Note: Do not award ***R0A1***.

[2 marks]
Total [6 marks]

1. (a) $\frac{9.45-8.73}{1958-1708}$ ***(M1)***

$= 0.00288 \left(\frac{9}{3125}\right)$ ***A1***

[2 marks]

(b) (i) the (mean) yearly change in (mean annual) temperature ***A1***

Note: Accept equivalent statements, e.g. “rate of change of temperature”.

(ii) $^{\circ}\text{C / year}$ **OR** degrees C per year ***A1***

Note: Do not follow through from part (b)(i) into (b)(ii).

[2 marks]

(c) attempt to substitute point and gradient into appropriate formula ***(M1)***

$8.73 = 0.00288 \times 1708 + c \Rightarrow c = 3.81096...$

or

$9.45 = 0.00288 \times 1958 + c \Rightarrow c = 3.81096.$

equation is $y = 0.00288x + 3.81$ ***A1***

[2 marks]

(d) attempt to substitute 2000 into their part (c) ***(M1)***

$0.00288 \times 2000 + 3.81096...$

$= 9.57\ (^{\circ}\text{C})\ \ (9.57096...)$ ***A1***

[2 marks]

continued…

Question 1 continued

(e) (i) $y = 0.00256x + 4.46$ $(0.00255714...x + 4.46454...)$ ***(M1)A1***

Note: Award ***(M1)A0*** for answers that show the correct method, but are presented incorrectly (e.g. no "$y =$" or truncated values etc.). Accept 4.465 as the correct answer to 4 sf.

(ii) 0.861 $(0.861333...)$ ***A1***

[3 marks]

(f) attempt to substitute 2000 into their part (e)(i) ***(M1)***

$0.00255714... \times 2000 + 4.46454...$

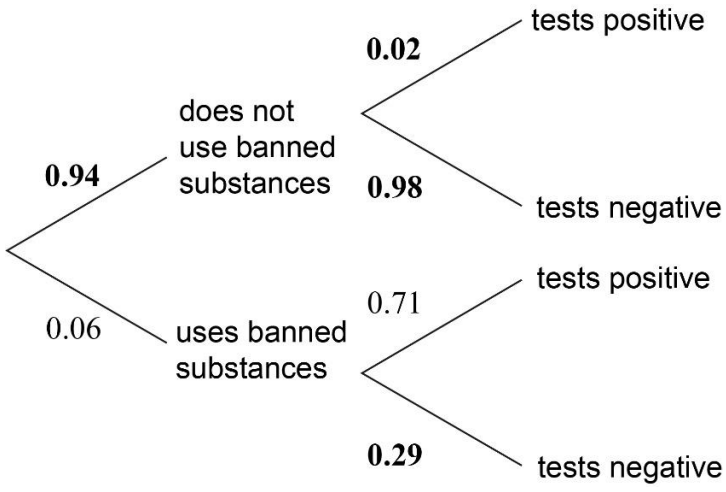
$= 9.58(^{\circ}\text{C})$ $(9.57882...(^{\circ}\text{C}))$ ***A1***

Note: Award ***A1*** for 9.57 from $0.00255714 \times 2000 + 4.46$.

[2 marks]

[Total: 13 marks]

3. (a)



A1A1

Note:	Award ***A1*** for any one value correct, ***A1*** for other three values correct. Accept percentage responses as equivalent forms on **all** branches.

[2 marks]

(b) (i) multiplication of two probabilities along the tree diagram ***(M1)***

0.94×0.98

$= 0.921 \ (0.9212, 92.1\%, 92.12\%)$ ***A1***

(ii) $(0.9212)^2$ ***(A1)***

$= 0.849 \ (0.848609\ldots, 84.9\%, 84.8609...\%)$ ***A1***

[4 marks]

continued…

Question 3 continued

(c) (i) $0.94\times0.02+0.06\times0.29$ ***(A1)(M1)***

Note: Award ***A1*** for two correct products from their tree diagram seen, ***M1*** for the addition of their two products.

0.0362 (3.62%) ***A1***

(ii) multiplying their part (c)(i) by 1300

0.0362×1300 ***(M1)***

$47.1\left(47.06\right)$ ***A1***

[5 marks]

(d) $p=0.02$ **OR** $p=0.98$ ***(A1)***

recognition of binomial probability with $n=20$ ***(M1)***

$\mathrm{P}(X=0)$ **OR** $\mathrm{P}(X=20)$ ***(M1)***

0.668 $(0.667607...)$ ***A1***

Note: Award ***(A1)(M1)(M1)A0*** for an answer of 0.667.

$0.98^{20}=0.668$ $(0.667607\ldots)$ is awarded full marks.

[4 marks]

(e) $\mathrm{P}(X\geq3)$ **OR** $\mathrm{P}(X\leq17)$ ***(M1)***

0.00707 $(0.00706869...)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 0.00706. Award ***(M1)A0*** for an answer of 0.0599 $(0.0598989\ldots)$, obtained from the use of $\mathrm{P}(X\geq2)$.

FT from their value of p in part (d)

[2 marks]

[Total: 17 marks]

5. (a) $(s_{n-1} =) 1.30243...$ ***(M1)(A1)***

$1.70\ (1.69632)$ ***A1***

Note: Award ***(M1)A0A0*** for a value of $(s_n =)\ 1.28934...$ or $(s_n{}^2 =)\ 1.6624$ seen.

[3 marks]

(b) the variance and the mean are similar ***R1***

Note: Do not accept a general statement "the variance and the mean are equal" unless their answer in part (a) is 1.76.

[1 mark]

(c) (i) attempt to find $\mathrm{P}(X = 4)$ under the null hypothesis $(= 0.0687830...)$ ***(M1)***

multiplying by 50 ***(M1)***

$j = 3.44\ (3.43915...)$ ***A1***

(ii) **EITHER**

attempt to find $\mathrm{P}(X \geq 5)$ under the null hypothesis and multiply by 50 ***(M1)***

OR

$50 - (8.60 + 15.14 + 13.32 + 7.82 + 3.44)\quad (= 5.12 - 3.44)$ ***(M1)***

THEN

$k = 1.68\ (1.67925...)$ ***A1***

[5 marks]

(d) there are expected frequencies less than 5 ***A1***

[1 mark]

continued…

Question 5 continued

(e) 3 ***A1***

[1 mark]

(f) $0.991\ (0.991187)$ ***(M1)A1***

Note: Award ***M1*** for a table of observed and expected frequencies with columns for 4 and 5 or more combined.

[2 marks]

(g) $99\% > 5\%$ ***R1***

EITHER

so there is insufficient evidence to reject H_0. ***A1***

OR

we accept that the number of sightings follows a Poisson distribution ***A1***

Note: Do not award ***R0A1***.

A p-value must be seen in part (f) to award ***FT***.

[2 marks]

[Total: 15 marks]

5.

(a) $$(\boldsymbol{T} =) \begin{matrix} \\ \text{(B)} \\ \text{(G)} \\ \text{(N)} \end{matrix} \begin{matrix} \begin{matrix} \text{(B)} & \text{(G)} & \text{(N)} \end{matrix} \\ \begin{pmatrix} 0.945 & 0.015 & 0.02 \\ 0.05 & 0.965 & 0.03 \\ 0.005 & 0.02 & 0.95 \end{pmatrix} \end{matrix}$$ ***M1A1A1***

Note: Accept the columns in any order. Accept the transpose of this matrix.

Award ***M1*** for a 3x3 matrix with all values between (but not including) 0 and 1, and all columns (or rows if transposed) adding up to 1, award ***A1*** for one correct row (or column if transposed) and ***A1*** for all rows (or columns if transposed) correct.

[3 marks]

(b) $$(\boldsymbol{T}^6 =) \begin{pmatrix} 0.72 & 0.077 & 0.098 \\ 0.24 & 0.83 & 0.16 \\ 0.035 & 0.098 & 0.74 \end{pmatrix}$$ ***(M1)***

Note: Accept a transposed matrix.

multiplying their $\boldsymbol{T}^6$ by a correct matrix of the initial populations ***(M1)***

$$\begin{pmatrix} 0.72 & 0.077 & 0.098 \\ 0.24 & 0.83 & 0.16 \\ 0.035 & 0.098 & 0.74 \end{pmatrix} \begin{pmatrix} 26000 \\ 240000 \\ 50000 \end{pmatrix}$$

Note: Award this ***M1*** for a transposed $\boldsymbol{T}$ if used correctly in part (b) i.e. preceded by 1×3 matrix rather than followed by a 3×1 matrix.

$$= \begin{pmatrix} 42133 \\ 212205 \\ 61661 \end{pmatrix}$$ ***(A1)***

so the expected population of the German side would be $212000\ (212205)$ ***A1***

Note: Award ***M0M1A0A1*** for an answer of $174000\ \ (=174031)$. This is the case when $\boldsymbol{T}^{30}$ has been used.

[4 marks]

continued…

Question 5 continued

(c) (i) $\begin{pmatrix} 0.945 & 0.015 & 0.02 \\ 0.05 & 0.965 & 0.03 \\ 0.005 & 0.02 & 0.95 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$

at least two of these three:

$0.945u_1 + 0.015u_2 + 0.02u_3 = u_1$

$0.05u_1 + 0.965u_2 + 0.03u_3 = u_2$ ***A1***

$0.005u_1 + 0.02u_2 + 0.95u_3 = u_3$

and

$u_1 + u_2 + u_3 = 1$ (may be seen in part (c)(ii)) ***A1***

(ii) $(\boldsymbol{u} =) \begin{pmatrix} 0.231 \\ 0.533 \\ 0.236 \end{pmatrix} \left(\boldsymbol{u} = \begin{pmatrix} 0.231155... \\ 0.532663... \\ 0.236180... \end{pmatrix} \right)$ ***A1***

Note: The ***A1*** in part (c)(ii) can be awarded independently of the working in part (c)(i).

[3 marks]

(d) $0.532663... \times (26000 + 240000 + 50000)$ ***(M1)***

$= 168000 \ (168321...)$ ***A1***

Note: Award ***(M1)A1*** for answers using $\boldsymbol{T}^n$ with n large that lead to a correct answer.
Award ***(M0)A0*** for answers that use $\boldsymbol{T}^n$ that lead to an incorrect answer.

[2 marks]

(e) *Award **R1** for each appropriate reason. For example:*

Movement unlikely to be constant

Total population for entire region likely to grow over time

Each power of the transition matrix takes five years; a relatively long time in terms of population movement.

There may be other/new external factors such as wars in other adjoining countries, leading to an influx of economic migrants.

R1R1

Note: Do not award ***R1*** for any response that shows a lack of understanding of the assumption that the total population remains constant.

[2 marks]
Total [14 marks]

6. (a) slugs appear discretely / independently / randomly / at a constant (average) rate / mean is (approximately) equal to variance ***R1R1***

[2 marks]

(b) new $(m=)\ 0.2\times 12\ (=2.4)$ (so $X \sim \text{Po}(2.4)$) ***(A1)***

attempt to use a pdf (e.g $\text{P}(X=4)$) ***(M1)***

$0.125\ (0.125408...)$ ***A1***

[3 marks]

(c) $\text{P}(X<3)$ **OR** $\text{P}(X\leq 2)$ ***(A1)***

$0.570\ (0.569708...)$ ***A1***

[2 marks]

(d) $\text{P}(X\geq 1)=0.909282...$ ***(A1)***

raising a probability to a power of 3 ***(M1)***

$0.909282...^3$

$=0.752\ (0.751788...)$ ***A1***

Note: Award at most ***(A1)(M1)(A0)*** for a final answer of 0.751. Working may not be seen.

[3 marks]

(e) $\text{H}_0: m=2.4,$ ***A1***

$\text{H}_1: m>2.4$ ***A1***

Note: The hypotheses may be written in words but must include reference to the mean (e.g. "number of snails" is not sufficient to award ***A1***), and state clearly for H_1 that the mean increases.

[2 marks]

(f) **EITHER**

finding either $\text{P}(X\geq 7)$ or $\text{P}(X\geq 8)$ ***(M1)***

$\left(\text{P}(X\geq 7)=\right)\ 0.01160...$ **AND** $\left(\text{P}(X\geq 8)=\right)\ 0.00334...$ ***A1***

OR

finding either $\text{P}(X\leq 7)$ or $\text{P}(X\leq 6)$ ***(M1)***

$\left(\text{P}(X\leq 7)=\right)\ 0.996661...$ **AND** $\left(\text{P}(X\leq 6)=\right)\ 0.988405...$ ***A1***

THEN

so critical region is $X\geq 8$ **OR** $X>7$ ***A1***

Note: ***(M1)A0A1*** can be awarded for a correct answer that is unsupported.

[3 marks]

(g) $(0.75\times 12=)\ 9$ ***(A1)***

$\text{P}\left(X\leq 7 \mid m=9\right)$ ***(M1)***

$=0.324$ ***A1***

[3 marks]

Total [18 marks]

2. (a) (i) because the (population) standard deviation(s) are unknown ***A1***

Note: Ignore any references to sample size.

[1 mark]

(ii) **EITHER**
he has no idea beforehand which way the difference would be if there is a difference ***A1***

OR
he is only interested that there is a difference (not the direction) ***A1***

[1 mark]

(b) (i) **EITHER**
$\mathrm{H}_0 : \mu_\mathrm{F} = \mu_\mathrm{G};\ \mathrm{H}_1 : \mu_\mathrm{F} \neq \mu_\mathrm{G}$ ***A1***

OR
$\mathrm{H}_0 : \mu_\mathrm{D} = 0;\ \mathrm{H}_1 : \mu_\mathrm{D} \neq 0$ ***A1***

Note: Accept an equivalent statement in words, must include mean and reference to "population mean" / "mean for all those taking the French exam" etc. for the first ***A1*** to be awarded. The terms "on average" and "generally" are also acceptable to indicate populations. Do not accept an imprecise "the means are equal".

Do not accept "There is (no) (significant) evidence of a difference between μ_F and μ_G" for either hypothesis or "There is (no) significant difference between marks in French and German".

[1 mark]

(ii) Generate a third column giving French mark – German mark or German mark – French mark. ***(M1)***
e.g.

French mark	German mark	Difference
42	39	3
65	66	−1
82	71	11
…	…	…

p-value $= 0.153$. ***A1***

[2 marks]

(iii) The p-value gives the probability of seeing the observed difference in means (or a larger difference) assuming H_0 to be true. ***A1***

Note: Do not accept "the probability that the data occurs by chance" or similar.

[1 mark]

(iv) because $0.153 > 0.05$ ***R1***

EITHER

there is not (significant evidence of) a difference between the (population) means ***A1***

OR

fail to reject H_0 (accept "accept H_0") ***A1***

Note: Do not award ***R0A1***.
Remember to ***FT*** from part (b)(ii).
Do not award the final ***A1*** if the null hypothesis in part (b)(i) is logically wrong (i.e. if the null and the alternative have been reversed or are nonsense) but this can be awarded if part (b)(i) is just poorly communicated.

[2 marks]

(c) (i) $\mathrm{H}_0 : \rho = 0 ;\ \mathrm{H}_1 : \rho > 0$ ***A1***

Note: Condone $\mathrm{H}_0 : \rho \leq 0$.

[1 mark]

(ii) p-value $= 0.00286$ ***A2***

$0.00286 < 0.05$ ***R1***

he should conclude that the two sets of marks are (generally) positively correlated ***A1***

Note: Allow ***FT*** from any test for correlation.
Do not award ***R0A1***.
The final ***R1A1*** should follow through from their p-value.
Do not award the final ***A1*** if the null hypothesis in part (c)(i) is wrong (i.e. if the null and the alternative have been reversed or are nonsense), but this can be awarded if part (c)(i) is just poorly communicated.
The final conclusion must be in context.

[4 marks]

(d) (i) the regression line of German on French is

$\text{German} = 10.2393... + 0.737495... \text{French}$ ***(A1)***

EITHER

substituting $\text{French} = 58$ into their regression line ***(M1)***

OR

sketch showing regression line and $x = 58$ ***(M1)***

THEN

Paul's German mark $= 53$ ***A1***

Note: Accept an answer of 53.0 $(53.0140...)$ or 52.9 as integer results are not explicitly stated in the question.
Regression lines may be written in terms of y and x.

[3 marks]

(ii) recognizing need to use line French on German
$\text{French} = 4.04116... + 1.01122...\text{German}$ ***(A1)***
putting $\text{French} = 71$, Sue's German mark $= 66$ ***A1***

Note: Accept an answer of 66.2 $(66.2158...)$ or 66.3 as integer results are not explicitly stated in the question.
Although not required in the markscheme as presented, candidates may have considered $\text{French} = 70.5$ and $\text{French} = 71.5$; this is valid and will lead to the correct answer.
If the line German on French is used in part (d)(ii) the answer is 63; award ***A0A0***.

[2 marks]

(e) (i) **EITHER**
the maximum value of τ occurs when all pairs are concordant so max $= +1$
the minimum value of τ occurs when all pairs are discordant so min $= -1$
A1

OR

when all concordant $C - D = \frac{n(n-1)}{2}$, and when all discordant $C - D = -\frac{n(n-1)}{2}$
A1

OR

when all concordant $C = \frac{n(n-1)}{2}$, $D = 0$ and when all discordant $C = 0$, $D = \frac{n(n-1)}{2}$
A1

THEN
hence the range is $[-1, +1]$ ***AG***

Note: Accept an answer which is just based on $n = 6$.

[1 mark]

(ii) $(53 - 76)(41 - 70) > 0$ ***A1***
Hence concordant ***AG***

[1 mark]

(iii) Evidence of a valid method, eg ***M1***
P_1: C, D, C, C, C
P_2 : C, C, C, D
P_3 : D, C, D
P_4 : D, C
P_5 : D
P_6 :

Note: At least one pair beyond (P_1, P_2) needs to be compared to award ***M1***.

any evidence (a statement or a list) that 15 pairs need to be considered ***A1***

$C = 9$, $D = 6$ ***A1***

using their stated C and D values in given formula with $n = 6$ ***M1***
$\frac{2(9-6)}{6(6-1)}$ **OR** $\frac{9-6}{15}$
$\tau = 0.2$ ***AG***

[4 marks]

(f) (i) H_0: There is no (underlying) association (or correlation) between the two sets of marks
H_1: There is an (underlying) association (or correlation) between the two sets of marks ***A1***

Note: Do not accept independence in the hypotheses.

[1 mark]

(ii) τ does not lie in the critical region **OR** $0.2<0.733$ ***R1***
EITHER
there is insufficient evidence to indicate that there is an association between the two sets of marks ***A1***

OR
fail to reject H_0 (accept "accept H_0") ***A1***

Note: Do not award ***R0A1***.
In this question the final ***A1*** mark can be awarded for "fail to reject H_0" or "accept H_0" even if the hypotheses in (f)(i) are the wrong way round as the critical region is given.

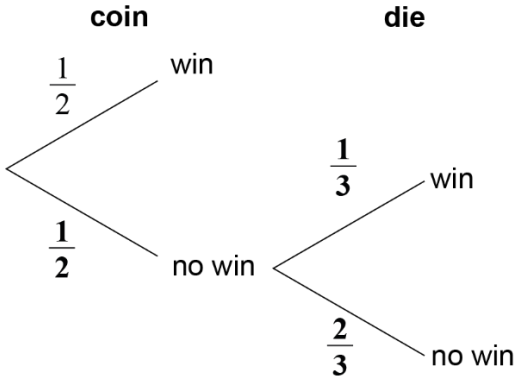
[2 marks]

(g) no ***A1***
because scaling the marks will not affect the concordances/ discordances ***R1***

Note: Do not award ***A1R0***.

[2 marks]
Total [29 marks]

7. (a)



A1A1

Note: Award ***A1*** for completing first set of branches, ***A1*** for completing second set of branches.

[2 marks]

(b) attempt to multiply along the branches ***(M1)***

$\frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3} \ (= 0.333...)$ ***A1***

[2 marks]

(c) **EITHER**

$$\frac{\frac{1}{2}}{\frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{3}\right)}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

OR

$$\frac{\frac{1}{2}}{1 - \frac{1}{3}}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

THEN

$= \frac{3}{4}$ ***A1***

[3 marks]
[Total 7 marks]

10. (a) $M = 1000 \times t^{-0.6}$

$a = 1000 \left(= 999.972...\right) \quad b = -0.600 \ \left(-0.599991...\right)$ ***A1A1***

[2 marks]

(b) $y = -0.600x + 6.908$

$c = -0.600 \quad d = 6.908$ ***A1A1***

Note: Long answer for c is $0.599991\ldots$ and for d is $6.90772\ldots$ If both answers are correct but not given to 3 decimal places award ***A1A0***.

[2 marks]

(c) **METHOD 1 (starting with the result in part (b))**

attempt to apply addition (or subtraction) log laws ***M1***

attempt to apply inverse log ***M1***

Note: These ***M1*** marks can be applied in either order depending on the approach.

e.g. $\ln M = \ln t^{-0.600} + \ln \mathrm{e}^{6.908}$ then $\ln M = \ln\left(\mathrm{e}^{6.908} \times t^{-0.600}\right)$

OR $\ln \dfrac{M}{t^{-0.600}} = 6.908$ then $\dfrac{M}{t^{-0.600}} = \mathrm{e}^{6.908}$

$M = \mathrm{e}^{6.908} t^{-0.600}$ ***A1***

$(M = 1000.24t^{-0.6})$

$M = 1000t^{-0.6}$ and hence (close enough to be) equivalent ***AG***

Note: The ***AG*** line (or something which approximates it) must be seen for the final ***A1*** to be awarded. If 3 sf answers are used from part (b), the coefficient is 1002; this can be condoned in the working, as it equals 1000 when rounded to 3 sf.

METHOD 2 (starting with the result in part (a))

attempt to apply log ***M1***

$\ln M = \ln\left(1000 \times t^{-0.6}\right)$

attempt to apply addition (or subtraction) log laws ***M1***

$\ln M = \ln 1000 + \ln t^{-0.6}$

$\ln M = 6.90775... - 0.6 \ln t$ ***A1***

$(\ln M = y,\ \ln t = x)$

$y = -0.600x + 6.908$ and hence (close enough to be) equivalent ***AG***

Note: The ***AG*** line (or something which approximates it) must be seen for the final ***A1*** to be awarded. Condone $b = -0.6$.

[3 marks]

[Total 7 marks]

15. (a) probability of non veg remaining non veg ***A1***

[1 mark]

(b) attempt to use $\det(\boldsymbol{A}-\lambda\boldsymbol{I})=0$ ***(M1)***

$$\begin{vmatrix} 0.8-\lambda & 0.1 \\ 0.2 & 0.9-\lambda \end{vmatrix} = 0$$

$(0.8-\lambda)(0.9-\lambda)-0.1\times 0.2=0$ ***(A1)***

$\lambda=1;\ \lambda=0.7$ ***A1***

[3 marks]

(c) $-2a+b=0$ ***M1***

$\boldsymbol{v}_1=\begin{pmatrix}1\\2\end{pmatrix}$ (accept any multiples of this answer) ***A1***

$\boldsymbol{v}_1$ means that in the long term the ratio of veg to non-veg is $1:2$
(in the long term one-third of students will be veg and two-thirds will not) ***A1***

[3 marks]

[Total 7 marks]

16. (a) Let X be the random variable number of shots taken in a 12 minute period

$X \sim \text{Po}(5)$ ***(A1)***

$\text{P}(X \leq 6) = 0.762 \ \ (= 0.762183...)$ ***A1***

[2 marks]

(b) $\text{P}(\text{less than 4 shots} \cap \text{success at least once})$

METHOD 1

$= \text{P}(\text{less than 4 shots}) - \text{P}(\text{less than 4 shots} \cap \text{zero success})$ ***(M1)***

Note: Might be communicated in Venn diagram.

attempt to multiply by different powers of 0.6 ***(M1)***

$= \text{P}(X \leq 3) - (\text{P}(X = 0) \times (0.6)^0 + \text{P}(X = 1) \times (0.6)^1 + \text{P}(X = 2) \times (0.6)^2 + \text{P}(X = 3) \times (0.6)^3)$ ***(A1)***

$= 0.414 \ \ (= 0.413845...)$ ***A1***

METHOD 2

attempt to multiply by different powers of 0.4 ***(M1)***

$= \text{P}(X = 1) \times (0.4)^1 + \text{P}(X = 2) \times \left((0.4)^2 + 2 \times 0.4 \times 0.6\right) + \text{P}(X = 3) \times \left((0.4)^3 + 3 \times 0.4^2 \times 0.6 + 3 \times 0.4 \times 0.6^2\right)$ ***(M1)(A1)***

Note: Award ***M1*** for recognizing the six different cases, e.g. $2 \times 0.4 \times 0.6$ (etc.) or equivalent seen, ***A1*** for completely correct expression.

$= 0.414 \ \ (= 0.413845...)$ ***A1***

[4 marks]

[Total 6 marks]

---

2. (a) (i) $150\ (\text{cm})$ ***A1***

(ii) attempt to substitute values in the mean formula with at least one mid-interval value multiplied by a corresponding frequency ***(M1)***

(mean $=$) $176\ (176.3)\ (\text{cm})$ ***A1***

[3 marks]

(b) 183 **OR** 168 seen ***(A1)***

Note: These values may be seen in the working for part (c).

$(\text{IQR} = 183 - 168 =)\ 15\ (\text{cm})$ ***A1***

[2 marks]

(c) (upper bound **=**) $183 + 1.5 \times 15$ **OR** 205.5 seen ***A1***

$205.5 > 204$ **OR** $204 - 183 < 22.5$ **OR** $204 - 22.5 < 183$) ***R1***

Laszlo's height is not an outlier ***A1***

Note: Do not award ***R0A1***.

[3 marks]

(d) H_0: The heights of the students can be modelled by $\text{N}(176, 13.5^2)$

H_1: The heights of the students cannot be modelled by $\text{N}(176, 13.5^2)$ ***A1A1***

Note: Award ***A1*** for each correct hypothesis that includes a reference to normal distribution with a mean of 176 and a standard deviation of 13.5 (or variance of 13.5^2). "Correlation", "independence", "association", and "relationship" are incorrect.

Award at most ***A0A1*** for correctly worded hypotheses that include a reference to a normal distribution but omit the distribution's parameters in one or both hypotheses. Award ***A0A1*** for correct hypotheses that are reversed.

[2 marks]

(e) (i) $h \sim \mathrm{N}(176, 13.5^2)$

attempt to find normal probability in either correct range $\mathrm{P}(170 \leq h < 180)$ **OR** $\mathrm{P}(h \geq 190)$ ***(M1)***

recognition of multiplying either of their probabilities by 200 $0.288137... \times 200$ **OR** $0.149859... \times 200$ ***(M1)***

$a = 57.6 \ (57.6274...)$, $b = 30.0 \ (29.9718...)$ ***A1A1***

(ii) $\mathrm{df} = 4$ ***(A1)***

$(p =) \ 0.0166 \ (= 0.0166282\ldots)$ ***A1***

comparing their p-value to 0.05 $0.0166 < 0.05$ ***R1***

Note: Accept p value of $0.0165 \ (= 0.0164693...)$ from using a and b to 3 sf.

(Reject H_0 There is sufficient evidence to say that) the data has not been drawn from the ($\mathrm{N}(176, 13.5^2)$) distribution. ***A1***

Note: Do not award ***R0A1***.

The conclusion to part (e)(ii) **MUST** follow through from their hypotheses seen in part (d); if hypotheses are incorrect/reversed etc., the answer to part (e)(ii) must reflect this in order for the ***A1*** to be credited.

[8 marks]

[Total 18 marks]

7. (a) (let X be the random variable the weight of an individual in the city of Melba)
$X \sim \mathrm{N}(72, 10^2)$

recognizing need to find $\mathrm{P}(X > 85)$ (condone "86" for the ***M1***) ***(M1)***

e.g. correct sketch of normal curve **OR** $0.0968 \quad (= 0.0968005...)$ seen

let Y be the random variable the number of people more than 85 kg

attempt to use a binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10, 0.0968005...)$ ***(A1)***

Note: This ***(A1)*** can be implied by the value $0.988580\ldots$

$\left(\mathrm{P}(Y \geq 4) =\right) \ 0.0114 \quad (= 0.0114196...)$ ***A1***

[4 marks]

(b) let W be the random variable the total weight of a sample of eight people

$W \sim \mathrm{N}(576, 8\times10^2)$ ***A1A1A1***

Note: Award ***A1*** for normal distribution; ***A1*** correct mean; ***A1*** correct variance or SD (SD = 28.2842…).

[3 marks]

(c) attempt to use inverse normal (or equivalent) ***(M1)***

$\mathrm{P}(W > w) = 0.01$

$(w =)\ 642\ (\mathrm{kg})\ \ (641.799\ldots)$ ***A1***

[2 marks]

(d) (i) *Any two correct assumptions identified,* ***A1A1***

e.g.

That Laetitia's clients are a random sample of the city's population

That people take only one holiday a year

That the choice of individual holidays is independent

That Laetitia is her clients' only agent

Note: Accept "assumes the proportion that takes a holiday abroad is 42%".

(ii) $\mathrm{H}_0 : p = 0.42$ ***A1***

$\mathrm{H}_1 : p < 0.42$ ***A1***

(iii) let Q be the random variable the number who go holiday abroad

$Q \sim \mathrm{B}(200,\ 0.42)$ ***(A1)***

$\left(\mathrm{P}(Q \le 67) =\right)\ 0.00850\ \ (= 0.00849906...)$ ***A1***

$0.00850 < 0.05$ ***R1***

EITHER

there is evidence that Laetitia's claim is reasonable ***A1***

OR

there is insufficient evidence to accept the newspaper's claim ***A1***

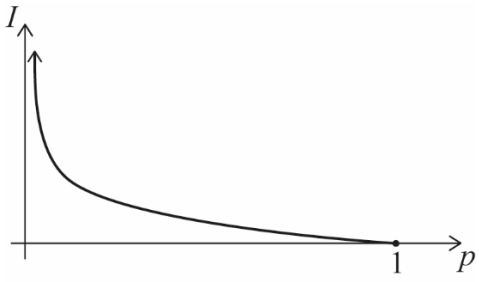
Note: Follow through within this part, for correctly comparing and concluding with their **probability**, e.g. it is possible to award ***A0A0R1A1***.

The conclusion to part (e)(iii) MUST follow through from their hypotheses seen in part (e)(ii); if hypotheses are incorrect/reversed etc., the answer to part (e)(iii) must reflect this in order for the ***A1*** to be credited.

[8 marks]

[Total 17 marks]

2. (a) (i)



approximately correct shape (decreasing and convex) ***A1***
asymptotic behaviour to the I-axis labelled (e.g. arrow) /
equation of asymptote $(p=0)$ seen ***A1***
p-intercept labelled at $p=1$ and graph does not extend beyond ***A1***

Note: Condone I and p being labelled as y and x.
Do not award second ***A1*** if y-intercept label is seen.

[3 marks]

(ii) $\frac{\mathrm{d}I}{\mathrm{d}p}=-\frac{1}{p}$ ***A1A1***

Note: Award ***A1*** for $\frac{\mathrm{d}I}{\mathrm{d}p}$ or equivalent (e.g. I') and ***A1*** for $-\frac{1}{p}$. Do not accept $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the first ***A1*** unless followed by $-\frac{1}{x}$, which can earn ***A1A0.***

(for $0<p\leq 1$) we therefore have $\frac{\mathrm{d}I}{\mathrm{d}p}<0$ ***R1***

hence the function is decreasing ***AG***

[3 marks]

(iii) Any plausible interpretation IN CONTEXT. ***R1***
For example:
More information is gained from a rarer event.
Less information is gained from a more common event.
Information (gained) decreases as probability increases.

[1 mark]

(b) (i) $\frac{5}{10}\quad(=\frac{1}{2})$ ***A1***

[1 mark]

(ii) attempt to substitute $p=$ their (b)(i) into $I=-\ln p$ ***(M1)***

$=0.693\quad(0.693147...,-\ln\left(\frac{1}{2}\right))$ ***A1***

[2 marks]

(iii) $\frac{9}{10}$ ***(A1)***

$= 0.105 \quad (0.105360..., -\ln\left(\frac{9}{10}\right))$ ***A1***

[2 marks]

(c) (i) attempt to substitute into the formula for $\mathrm{E}(I)$ and recognise that $n = 2$ (or two terms are needed) ***M1***

$\mathrm{E}(I) = -\frac{1}{10}\ln\left(\frac{1}{10}\right) - \frac{9}{10}\ln\left(\frac{9}{10}\right)$ ***A1***

$0.325 \quad (0.325082...)$ ***AG***

[2 marks]

(ii) $\mathrm{E}(I) = -\frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right)$ ***A1***

$0.693 \quad (0.693147..., \ln(2))$ ***A1***

$0.693 > 0.325$ ***AG***

[2 marks]

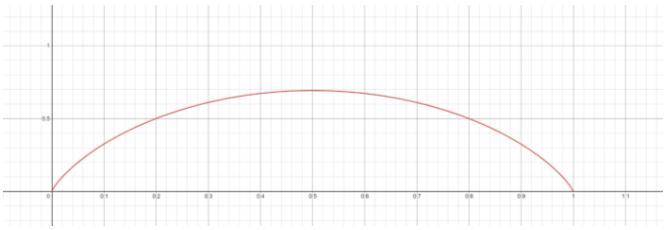
(d) (i) $(I =) -\ln(1-p)$ ***A1***

[1 mark]

(ii) $(\mathrm{E}(I) =) -p\ln p - (1-p)\ln(1-p)$ ***A1***

[1 mark]

(iii) attempt to use graphical method or calculus to maximize $\mathrm{E}(I)$ ***M1***



maximum occurs when $p = \frac{1}{2}$ ***A1***

[2 marks]

(e) (i) $(x=)\frac{6}{12}\quad\left(=\frac{1}{2}\right)$ ***A1***

EITHER
for the scales to balance, the odd ball must be in the six balls not chosen ***R1***

OR
for the scales to balance, all the balls chosen must be of equal weight and hence
$\frac{11}{12}\times\frac{10}{11}\times\frac{9}{10}\times\ldots\times\frac{6}{7}$ ***R1***

[2 marks]

(ii) **EITHER**
recognition that the sum of the probabilities on the third row of the table equals 1 ***(M1)***
e.g. $x+2y=1$

OR

for one side to be heavier, the odd ball must be one of six balls chosen $\left(\frac{6}{12}\right)$ and half the time this will result in left-side being heavier,

therefore $y=\frac{6}{12}\times\frac{1}{2}$ ***(M1)***

$y=\frac{1}{4}$ ***A1***

[2 marks]

(iii) $z=-\frac{1}{6}\ln\frac{1}{6}-\frac{1}{6}\ln\frac{1}{6}-\frac{2}{3}\ln\frac{2}{3}$ ***(M1)***

$=0.868\quad(0.867563...)$ ***A1***

[2 marks]

(iv) 4 balls on each side because that configuration has the largest $\mathrm{E}(I)$ ***R1***

Note: Award ***R1*** for giving a correct reason **AND** stating "4 balls on each side"

[1 mark]

[Total: 27 marks]

2. (a) (i) $r = 0.995 \quad (0.994705\ldots)$ ***A2***

Note: Award ***A1*** for 0.99.

(ii) $m = 10.6t + 43.9 \quad (10.6032...t + 43.8780...)$ ***A1A1***

Note: Second ***A1*** is for the correct variables.

[4 marks]

(b) **EITHER**

$10.6032... \times 1.5$ ***(M1)***

OR

$\left(10.6032...(t + 1.5) + 43.8780...\right) - \left(10.6032...(t) + 43.8780...\right)$ ***(M1)***

THEN

15.9 (marks) $(15.9048\ldots)$ ***A1***

Note: Accept 16.

[2 marks]

(c) *Accept any valid reason* ***R1***

e.g:

The students in the sample might not be of equal ability / she has not controlled for ability

She might have originally obtained close to full marks so an extra 15.9 would not be possible.

[1 mark]

[Total: 7 marks]

3. (a) (i) attempt to rearrange to isolate C ***(M1)***
e.g., subtracting 32 or dividing the equation by 1.8

$$C = \frac{5}{9}(F - 32) \quad \left(C = \frac{F - 32}{1.8},\ C = 0.556F - 17.8) \right)$$ ***A1***

Note: If the answer is not written as an equation, award at most ***M1A0***.

(ii) $C = \left(\frac{77 - 32}{1.8} = \right) \ 25 \ (^{\circ}\text{C})$ ***A1***

[3 marks]

(b) (i) $(1.8 \times 17 + 32 =) \ 62.6 \ (^{\circ}\text{F})$ ***A1***

(ii) recognizing that the "+32" does not affect the SD ***(M1)***
$(1.8 \times 9 =) \ 16.2 \ (^{\circ}\text{F})$ ***A1***

Note: Award ***M0A0*** for $1.8 \times 9 + 32 \ (= 48.2)$.

[3 marks]
[Total 6 marks]

12. (a) $40 = 100 - 100 \times 2^{-t}$ ***(A1)***

0.737 (hours) $(0.736965\ldots)$ ***A1***

Note: Accept 44.2 minutes.

[2 marks]

(b) values of P are $50, 75, 87.5, 93.75$ ***(A1)***

$SS_{res} = (50-48)^2 + (75-74)^2 + (87.5-86)^2 + (93.75-91)^2$ ***(M1)(A1)***

$= 14.8 \ (14.8125)$ ***A1***

[4 marks]

(c) (i) The sum of the square residuals is smaller so it is a better fit ***R1***

(ii) *Accept a valid argument in favour of model P or against the quadratic model.* ***R1***

e.g. any one of the following:
quadratic has no asymptote
quadratic will begin to go down
quadratic will become negative
quadratic might not go through $(0, 0)$
model P is the manufacturer's model

[2 marks]
[Total 8 marks]

17. (a) $\begin{pmatrix} 0.25 & 0.25 \\ 0.1 & 0.1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***(M1)***

Note: Accept equivalent methods including only using one line of the matrix.

$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ (or any multiple) ***A1***

[2 marks]

(b) $D^n = \begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}$ ***A1***

[1 mark]

(c) $\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1}$ ***(M1)***

EITHER

multiplying by the initial state ***(M1)***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1}\begin{pmatrix} 7000 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2000 \\ 1000 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 2000 \times 0.65^n \\ 1000 \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 2000 \times 0.65^n + 5000 \\ -2000 \times 0.65^n + 2000 \end{pmatrix}$ ***(A1)***

Note: Award ***A0*** if either term in the matrix is incorrect.

OR

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}^{-1} = \frac{1}{7}\begin{pmatrix} 2 & -5 \\ 1 & 1 \end{pmatrix}$ ***A1***

$\begin{pmatrix} 1 & 5 \\ -1 & 2 \end{pmatrix}\begin{pmatrix} 0.65^n & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0.65^n & 5 \\ -0.65^n & 2 \end{pmatrix}$ ***A1***

Note: The preceding ***A1*** marks can be awarded independently.

$\frac{1}{7}\begin{pmatrix} 5+2\times 0.65^n & 5-5\times 0.65^n \\ 2-2\times 0.65^n & 2+5\times 0.65^n \end{pmatrix}$ ***A1***

Note: Award ***A0*** if any term in the matrix is incorrect.

multiplying by the initial state ***(M1)***

$\frac{1}{7}\begin{pmatrix} 5+2\times 0.65^n & 5-5\times 0.65^n \\ 2-2\times 0.65^n & 2+5\times 0.65^n \end{pmatrix}\begin{pmatrix} 7000 \\ 0 \end{pmatrix}$

THEN

$2000 - 2000\times 0.65^n \quad \left(= 2000\left(1-0.65^n\right)\right)$ ***A1***

Note: For the final ***A1***, follow through within the question part from the bottom-left entry of their 2x2 matrix or the bottom entry of their 2x1 matrix but only if "in terms of n".

If any mistake in the matrices is seen that DOES NOT affect the correct final answer, do not award the penultimate ***A1*** mark.

[6 marks]
[Total 9 marks]

2. (a) (i) recognition of binomial distribution (condone incorrect parameter) ***(M1)***

e.g. $M \sim \mathrm{B}(20,\ 0.04)$ **OR** $\mathrm{P}(M = 2) = \mathrm{binpdf}(20,\ 0.04,\ 2)$

$= 0.146\ \ (0.145799...)$ ***A1***

(ii) recognition the cumulative probability required ***(M1)***

e.g. $\mathrm{P}(M \geq 3) = 1 - \mathrm{bincdf}(20,\ 0.04,\ 2)$ **OR** $\mathrm{bincdf}(20,\ 0.04,\ 3,\ 20)$

$= 0.0439\ \ (0.0438627...)$ ***A1***

[4 marks]

(b) either one of two terms in expected value formula correct ***(M1)***

$50\big(20(0.96)\big) + 15\big(20(0.04)\big)$

= 972 (pesos) ***A1***

[2 marks]

[Total: 6 marks]

4. (a) recognizing that only way to score 7 is to achieve a head and a 6 on die ***(M1)***

e.g. $\frac{1}{6}$ and $\frac{1}{2}$ seen in an attempt to combine probabilities

$\left(\frac{1}{6}\times\frac{1}{2}=\right)\ \frac{1}{12}\ (0.0833333...)$ ***A1***

Note: Condone 0.0835 from the use of 0.167.

[2 marks]

(b) there are two ways to score (e.g.) 5

achieve a head and a 4 on die, or a tail and a 5 on die ***(M1)***

$\left(2\left(\frac{1}{6}\times\frac{1}{2}\right)=\right)\ \frac{2}{12}\quad\left(\frac{1}{6}, 0.167, 0.16666...\right)$ ***A1***

Note: Award these marks for equivalent working for the 2, 3, 4 or 6 point scenarios.

Final Score	1	2	3	4	5	6	7
Probability	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{12}$

A1

Note: Award ***A1*** for a completely correct table. Award at most ***(M1)A1A0*** if their follow-through answer from part (a) leads to a total probability not equal to 1.

[3 marks]

(c) **EITHER**

multiplying at least two columns from their table ***(M1)***

$1\times\frac{1}{12}+2\times\frac{1}{6}+...+6\times\frac{1}{6}+7\times\frac{1}{12}$

OR

recognizing the probabilities in the table are symmetric ***(M1)***

OR *(for HL markscheme and/or removed from SL at publication)*

Considering the sum of two random variables ***(M1)***

$\mathrm{E}(X+Y)=\mathrm{E}(X)+\mathrm{E}(Y)\ \ (=3.5+0.5)$

THEN

(expected value $=$) 4 ***A1***

Note: Accept $4.01\ (4.00640\ldots)$ from use of their 3 sf values from (b).
Award at most ***M1A0*** if their final answer is not in the range 1 – 7

[2 marks]
[Total: 7 marks]

9. (a) (i) let X be number of accidents per week $\Rightarrow X \sim \text{Po}(0.76)$

recognition cumulative Poisson distribution ***(M1)***

$\text{P}(X \geq 2) = 1 - \text{P}(X \leq 1)$

$= 0.177 \ \ (0.176907...)$ ***A1***

(ii) let F be number of accidents per 4-week $\Rightarrow F \sim \text{Po}(3.04)$ ***(M1)***

$\text{P}(Y = 3) = 0.224 \ \ (0.223982...)$ ***A1***

[4 marks]

(b) binomial model ***A1***

with $p = 0.177$ **OR** $p = 1 - 0.177 \left(= 0.823\right)$ and $n = 8$ ***A1***

[2 marks]

[Total: 6 marks]

14. (a) room temperature / the temperature below which the hot water will not cool ***A1***

Note: Accept answers similar to "the temperature of the water in the cup after 25 minutes".

[1 mark]

(b) evidence of subtracting 25 from the temperature data ***(M1)***

$a = 244\ (243.920...)$ and $b = -1.03\left(-1.02965...\right)$ ***A1A1***

Note: Award ***M0A1A0*** for both answers $a = 116.764...$ and $b = -0.375218...$ seen, from not subtracting the 25.

[3 marks]

(c) $k = 61.1\ (61.0848...)$ and $c = 0.923\ (0.923029...)$ ***A1***

Note: If not subtracting 25 was penalized in part (b), award ***A1*** for $k = 68.9023...$ and $c = 0.972506...$ seen in part (c).

[1 mark]

(d) *Award **R1** for any appropriate reason that supports Soo Min's model*
e.g.
EITHER
compare r^2 values: $0.76573583\ldots$ and $0.9233117\ldots$ ***R1***

OR
consider value at $t = 0$ / "water cannot reach a temperature more than 100 degrees" ***R1***

Note: If subtracting 25 was penalized in (b) then award ***R1*** for comparing r^2 values $0.904086\ldots$ and $0.994522\ldots$

Award ***R1*** for correct reasoning seen for the use of comparison of the sum of square residuals.

[1 mark]
[Total: 8 marks]

5. (a) $\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}$ ***M1A1***

Note: Award ***M1*** for correct values used, ***A1*** if in correct positions.
Accept alternative consistent matrix (e.g. the transpose or diagonal elements exchanged) and follow through to eigenvectors and initial state vector.

[2 marks]

(b) 5 (seen) ***(A1)***

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}^5 \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.596608 \\ 0.403392 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix}^5 = \begin{pmatrix} 0.596608 & 0.268928 \\ 0.403392 & 0.731072 \end{pmatrix}$ ***(M1)***

$\mathrm{P}(\text{Friday evening}) = 0.403 \ \ (0.403392)$ ***A1***

Note: Award ***A0M1A0*** for use of 4 (and resulting probability 0.354).

[3 marks]

(c) attempt to find $\det(\boldsymbol{A} - \lambda \boldsymbol{I})$ ***(M1)***

$\begin{vmatrix} 0.88-\lambda & 0.08 \\ 0.12 & 0.92-\lambda \end{vmatrix}$ **OR** $(0.88-\lambda)(0.92-\lambda) - (0.12)(0.08)$

$\lambda^2 - 1.8\lambda + 0.8$ ***A1***

[2 marks]

(d) eigenvalues are 0.8 and 1 ***(A1)***

Note: If no attempt is made to find eigenvectors, do not award ***A1*** for finding eigenvalues.

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0.8 \begin{pmatrix} x \\ y \end{pmatrix}$

$0.88x + 0.08y = 0.8x$

eigenvector = eg. $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1***

EITHER

$\begin{pmatrix} 0.88 & 0.08 \\ 0.12 & 0.92 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 1 \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$0.88x + 0.08y = x$

$0.08y = 0.12x$

OR

eigenvalue 1 gives

$\begin{pmatrix} -0.12 & 0.08 \\ 0.12 & -0.08 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***(M1)***

$-0.12x + 0.08y = 0$

$0.08y = 0.12x$

Note: Award ***M1*** for an attempt to find the eigenvector with eigenvalue 1.

THEN

eigenvector = eg. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ***A1***

Note: Award ***A0A1M0A0*** if only $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is seen and no eigenvalues are found.

[4 marks]

(e) $\boldsymbol{D} = \begin{pmatrix} 1 & 0 \\ 0 & 0.8 \end{pmatrix}$, $\boldsymbol{P} = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$ **OR** $\boldsymbol{D} = \begin{pmatrix} 0.8 & 0 \\ 0 & 1 \end{pmatrix}$, $\boldsymbol{P} = \begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for one of $\boldsymbol{P}$ or $\boldsymbol{D}$ correct. Do not award the second ***A1*** unless $\boldsymbol{P}$ and $\boldsymbol{D}$ are consistent.

[2 marks]

(f) **EITHER**

attempt to use $\boldsymbol{T}^n = (\boldsymbol{PDP}^{-1})^n = \boldsymbol{PD}^n\boldsymbol{P}^{-1}$ ***M1***

Note: Award ***M1*** for their $\boldsymbol{D}^n$ seen.

limit of $\boldsymbol{D}^n$ calculated ***A1***

$\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}^{-1}$

Note: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ must be seen to award ***A1***.

OR

attempt to expand their $\boldsymbol{PD}^n\boldsymbol{P}^{-1}$ using explicit $\boldsymbol{P}$, $\boldsymbol{P}^{-1}$ ***M1***

$(\boldsymbol{T}^n =) \ \frac{1}{5}\begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0.8^n \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 3 & -2 \end{pmatrix}$

$(\boldsymbol{T}^n =) \ \frac{1}{5}\begin{pmatrix} 2+3(0.8^n) & 2-2(0.8^n) \\ 3-3(0.8^n) & 3+2(0.8^n) \end{pmatrix}$ ***A1***

Note: Using this method, the limit of 0.8^n may be inferred and ***M1A1*** awarded.

THEN

0.6 ***A1***

Note: Multiplication by initial condition $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ may be seen at any point as part of their method.

For an answer of 0.6 from incomplete methods award a maximum of ***M1A0A0***, or if no working is seen, award ***M0A0A1***.

[3 marks]

[Total: 16 marks]

7. (a) $\mathrm{P}(X < 340)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 0.159 \quad (0.158656\ldots)$ ***A1***

[2 marks]

(b) recognizing endpoint occurs at either 0.975 or 0.025 ***(M1)***

$\mathrm{P}(X < k) = 0.975$ **OR** $\mathrm{P}(X < m) = 0.025$

$330 < X < 370 \quad (330.400... < X < 369.599...)$ ***A1A1***

[3 marks]

(c) (i) recognizing mean of W is sum of individual means within wall ***(M1)***

$W = C_1 + C_2 + L$ may be seen

$\mathrm{E}(W) = 2\mathrm{E}(C) + \mathrm{E}(L)$

$= 800$ ***A1***

recognizing **variance** of W is sum of individual **variances** within wall ***(M1)***

$\mathrm{Var}(W) = 2\mathrm{Var}(C) + \mathrm{Var}(L)$ **OR** 225 seen ***(A1)***

$(\mathrm{SD}(W) =)\ 15$ ***A1***

Note: Award ***M1A0A0*** for an answer of 20.6 from using $\mathrm{Var}(2C)$ in place of $2\mathrm{Var}(C)$.

(ii) recognizing that W is modelled by a normal distribution ***(M1)***

$(\mathrm{P}(780 < W < 810) =)\ 0.656 \quad (0.656296...)$ ***A1***

Note: The answer is $0.521\ (0.520)$ from using $\mathrm{SD} = 20.6 \quad (5\sqrt{17})$. Follow through from part (c)(i) without working seen.

[7 marks]

(d) $810 = 350 + 350 + \mathrm{E}(L)$ (or equivalent) ***(A1)***

$(\mathrm{E}(L) =)\ 110$ ***A1***

$\mathrm{Var}(W) = 2\mathrm{Var}(C) + \mathrm{Var}(L)$ **OR** $256 = 2(100) + \mathrm{Var}(L)$ ***(A1)***

$(\mathrm{SD}(L) =)\ 7.48 \quad (7.48331..., \sqrt{56})$ ***A1***

[4 marks]

(e) $116 \quad (116.298)$ ***A1***

Note: Do not follow through from either a negative variance or a negative SD.

[1 mark]

[Total: 17 marks]

1. (a) (i) $141-88$ ***A1***

$=53$ ***AG***

[1 mark]

(ii) $(120-53-54=)\ 13$ ***A1***

[1 mark]

(b) Restricting the size of the sample space to 150 ($54+96$) ***(M1)***

$=0.36\ \left(\frac{54}{150}, \frac{9}{25}\right)$ ***A1***

[2 marks]

(c) (i) H_0: The development of cracks and the technique used are independent

H_1: The development of cracks and the technique used are not independent ***A1***

Note: Condone equivalent statements such as 'not dependent' but do not accept "uncorrelated" or "not related" in place of "independent".

[1 mark]

(ii) (p-value$=$) $0.0170\ \ (0.0169864\ldots)$ ***(M1)A1***

[2 marks]

(iii) $0.0170<0.05$ ***R1***

hence there is sufficient evidence to reject the null hypothesis that the development of cracks and the technique used are independent. ***A1***

Note: Do not award ***R0A1***.

[2 marks]

(d) $\frac{96+56}{240}\ \left(=\frac{152}{240}\right)$ ***A1***

$\frac{19}{30}$ ***AG***

[1 mark]

continued…

Question 1 continued.

(e) **EITHER**

the probability of each component developing cracks is independent of all the other components in the sample. ***R1***

Note: Do not accept the word "independence" on its own. Appropriate context must be seen.

OR

the development of cracks can be partitioned into two clear groups ***R1***

Note: Do not accept 'the samples are representative of the population'.

[1 mark]

(f) 67 seen ***(A1)***

EITHER

attempt to find a probability ≤ 67 (condone strict inequality for ***(M1)***) ***(M1)***

$(\mathrm{P}(X \leq 67) =)\ 0.0549\ \ (0.0549093\ldots)$ ***A1***

Note: Award ***(A1)(M1)A0*** for an unsupported $p = 0.0372$, from use of strict inequality.

$0.0549 > 0.05$ ***R1***

OR

attempt to find the critical region ***(M1)***

critical region is $X \leq 66$ ***A1***

$66 < 67$ or '67 is not in the critical region' ***R1***

THEN

EITHER
do not reject the null hypothesis (as there is insufficient evidence that the new technique reduces the number of cracks). ***A1***

OR
do not accept the alternative hypothesis (as there is insufficient evidence that the new technique reduces the number of cracks). ***A1***

continued…

Question 1 continued.

Note: Do not award ***R0A1***.
Only follow through within part (f) for final ***R1A1*** if the ***(M1)*** has been awarded.
Do not condone "accept the null hypothesis".

[5 marks]

(g) (i) the test for a proportion is directional and so considers whether the new treatment reduces the number of components developing cracks. ***R1***

[1 mark]

(ii) **EITHER**

there could be variation in the value of p chosen for the null hypothesis / the value of p from the sample might not be a representative of the current technique ***R1***

OR
the test in (f) does not treat minor and major cracks as different attributes / the test in (c) does treat minor and major cracks as different attributes ***R1***

OR
the test in (f) has to make an additional assumption (for example 'independence') ***R1***

[1 mark]

continued…

Question 1 continued.

(h) **EITHER**

let μ_1 be the mean length of time before cracks appear with the new technique and μ_2 be the mean length with the current technique

H_0: $\mu_1 = \mu_2$ ***A1***

H_1: $\mu_1 > \mu_2$ ***A1***

Note: Award ***A1A0*** for correct hypotheses in which the two population means are not clearly defined (e.g. unsupported μ_1 and μ_2).

OR

H_0: the POPULATION mean length of time before cracks appear is the same for both groups ***A1***

H_1:the new technique increases the POPULATION mean length of time before cracks appear. ***A1***

OR

H_0: the mean length of time before cracks appear in ALL components made with the new technique is the same as for ALL components made with the current technique. ***A1***

H_1: the mean length of time before cracks appear in ALL components made with the new technique is greater than the mean for ALL components made with the current technique. ***A1***

Note: Award ***A1A0*** if "population" (or equivalent, such as "all") is omitted from an otherwise correct answer.

THEN

recognition of the need to use of a two-sample test ***(M1)***

p-value $= 0.0162$ $(0.0162328\ldots)$ ***A2***

Note: If not pooled, answer is $0.0164368\ldots$ award ***(M1)A2***.

$0.0162 < 0.05$ ***R1***

reject the null hypothesis (**OR** accept the alternative hypothesis) ***A1***
(there is sufficient evidence to that the new technique increases the mean length of time before the cracks appear)

Note: Do not award ***R0A1***.

Follow through within part (h) for the last ***R1A1***, provided their p-value is between 0 and 1 inclusive.

[7 marks]

continued…

Question 1 continued.

(i) **EITHER**

(though statistically significant) the new technique only seems to increase the time before cracks appear by 1 hour out of 250, so it is not a significant increase
(i.e. the effect size is small) ***R1***

OR

the minimum time (not mean time) before cracks appear should be considered given the context / An appropriate confidence interval should be considered, and not simply the mean. ***R1***

Note: If a not significant p-value was seen in part (h), do not award ***R1*** for an answer of "the result is not significant" in part (i).

[1 mark]

[Total 26 marks]

1. (a) $a = 70.35$ **OR** 70.4 ***A1***

$b = 73.45$ **OR** 73.5 ***A1***

$c = 5.45$ ***A1***

Note: Follow through within the question from their (labelled) value of b, when finding c (applies both if b is incorrect or correct to a different level of accuracy).

[3 marks]

(b) **EITHER**

median/average dart score has increased ***R1***

OR

Darts players are showing (some) improvement ***R1***

OR

Spread in average dart score has decreased ***R1***

OR

Players have become more similar ***R1***

Note: Answers without context "e.g. the median is higher in year 2" score ***R0***.

[1 mark]

(c) **EITHER**

(difference in averages is approximately) normally distributed ***R1***

OR

the individuals' changes in mean are independent ***R1***

Note: Do not award ***R1*** if there is also a reference to variances being pooled / equal variances or to independence <u>between the two years</u> or anything else which shows a misunderstanding of the test used.

[1 mark]

(d) (i) let μ_{d} represent the mean of the differences from table 1 (e.g. column 2-column 1)

$\text{H}_0 : \mu_{\text{d}} = 0$ ***A1***

$\text{H}_1 : \mu_{\text{d}} > 0$ ***A1***

Note: Condone an alternative hypothesis of $\text{H}_1 : \mu_{\text{d}} < 0$, provided it is consistent with their subsequent work (e.g. their "differences" are year 1 – year 2). Accept hypotheses communicated in words, e.g. "H_0 : there has been no change in the mean of the players' averages". The word "mean" must be seen in any word-based response.

Allow $\mu_1 = \mu_2$ and $\mu_2 > \mu_1$.

Allow a compound null hypothesis (i.e. $\mu_d \leq 0$ or equivalent).

Condone $\mu = 0$ or $\mu_0 = 0$ for null hypothesis and equivalent for alternative hypothesis.

[2 marks]

(ii) finding/entering differences in 'means' across both years ***(M1)***

p-value $= 0.134$ $(0.133797\ldots)$ ***A1***

[2 marks]

(iii) $0.134 > 0.1$, ***R1***
(so do not reject H_0)
there is insufficient evidence to suggest that the darts players' averages have increased ***A1***

Note: Do not award ***R0A1***. Award ***A0*** if the conclusion is not in context or if the null hypothesis was logically incorrect (i.e. an error of confusing null and alternative rather than just an error in communication).

[2 marks]

(e) (i) decrease the significance level ***A2***

Note: Award ***A1*** if type I error has been defined (e.g. As rejecting H_0 when H_0 is true) but correct answer not seen.

Increased sample size is incorrect.
If more than one answer is given, mark only the first one.

[2 marks]

(ii) increase the sample size / increase the significance level ***A2***

Note: Condone "Collect more data".
If more than one answer is given, mark only the first one.

[2 marks]

(f)

Year 1 mean	Year 2 mean	Difference	*P*	*N*
68.1	70.1	2.0	**6**	
72.2	72.0	–0.2		1
65.8	65.0	–0.8		2
73.7	74.9	1.2	3	
69.8	68.0	–1.8		**5**
64.5	68.0	3.5	**7**	
69.2	70.6	1.4	**4**	
64.9	92.9	28	8	
		Total	28	**8**

(i) $D = 4$ ***A1***
$B = 5$ ***A1***
$A = 6$, $C = 7$ ***A1***

[3 marks]

(ii) $(\sum N) = 8$ ***A1***

[1 mark]

(g) (i) (H_1 :) the (population's) median (three dart average) increased (from year 1 to year 2).

A1

[1 mark]

(ii) $(T =)\ 8$ ***A1***

[1 mark]

(iii) $(T =)\ 8 > 5$ ***R1***

Note: Condone $8 \geq 5$.

(so do not reject H_0 (allow 'accept H_0'))

there is insufficient evidence to suggest that the median of the players' averages have increased ***A1***

Note: Do not award ***R0A1***. Award ***A0*** if the conclusion is not in context.

Nearly anything better than "Do not reject H_0" will be accepted as being in context.

[2 marks]

(h) **EITHER**

she could repeat the test(s) the following year ***R1***

OR

she could repeat the test(s) with a different sample (for the same years) ***R1***

OR

apply test-retest ***R1***

Note: Increasing sample size is NOT a test for reliability. Any reference to this would score ***R0***.

[1 mark]

Total [24 marks]

3. (a) recognizing need to find $\mathrm{P}(W > 3.5)$ **OR** $1 - \mathrm{P}(W < 3.5)$ ***(M1)***

$(\mathrm{P}(W > 3.5) =)$ 0.994 $(0.993790\ldots)$ ***A1***

Note: Award ***M1A0*** for an answer of 0.99 with no working.

[2 marks]

(b) (i) appropriate use of the GDC to find either a or b seen in either (b)(i) or (b)(ii) ***(M1)***

$a = 3.99$ $(3.98737\ldots)$ ***A1***

(ii) $b = 5.01$ $(5.01262\ldots)$ ***A1***

Note: The working for ***(M1)*** may be seen and awarded in part (b)(ii) if part (b)(i) has not been answered.

[3 marks]

(c) multiplying two identical probabilities ***(M1)***

$\mathrm{P}(W < 3.5) \times \mathrm{P}(W < 3.5)$ **OR** $(1 - \text{their (a)}) \times (1 - \text{their (a)})$

Note: Accept a lower bound of zero for W, given the context.

$(0.00620967)^2$

0.0000386 $(0.0000385601\ldots$ **OR** $3.86 \times 10^{-5})$ ***A1***

Note: Award ***M0A0*** for an answer greater than 1 such as 3.86.
Award ***M1A0*** for 3.86E-5.

[2 marks]
[Total: 7 marks]

15. (a) normal distribution ***A1***

Mean $= 15\ (\text{cm})$ ***A1***

Standard deviation $= 0.4\ (\text{cm})$ **OR** variance $= 0.16\ (\text{cm}^2)$ ***A1***

[3 marks]

(b) $\text{E}(W) = 3$ ***(A1)***

$\text{Var}(W) = 0.4^2 + 0.25^2$ ***(A1)***

$= 0.223\ \ (0.2225)$ ***(A1)***

W is normally distributed ***(M1)***

$(\text{P}(W < 2.5) =)\ \ 0.145\ \ (0.144572\ldots)$ ***A1***

Note: Accept $\text{P}(0 < W < 2.5)$.

[5 marks]

[Total 8 marks]

1. (a) (i) $(Q_3 =)\ 7.2$ ***A2***

Note: Award ***A1A0*** if the lower quartile, 6.5, is given as the answer.
Award ***A1A0*** for a correct ordered list of happiness scores, when the correct Q_3 is not seen.

(ii) $Q_1 = 6.5$ ***(A1)***

$IQR = 7.2 - 6.5$

$= 0.7$ ***A1***

[4 marks]

(b) $Q_3 + 1.5 \times IQR$ ***(A1)***

$\left(7.2 + 1.5 \times 0.7 =\right)\ 8.25$ ***A1***

since $7.5 < 8.25$ ***R1***

Switzerland is not an outlier ***AG***

Note: Do not award ***A0A0R1***.

[3 marks]

(c) (i) $a = 3.5$ ***A1***

(ii) $b = 8$ ***A1***

(iii) $c = 3.5$ ***A1***

[3 marks]

(d) (i) $(r_s =)\ 0.164\ \ (0.164134\ldots)$ ***A2***

(ii) France rank (of sixth) is unchanged (so the r_s is unchanged) ***R1***

[3 marks]

(e) Because r_s is too close to zero and hence Jose's conclusion is not appropriate ***R1***

Note: Award ***R1*** for a comment on the value r_s and "not appropriate" as a conclusion.
Accept "r_s indicates a weak correlation of the ranks".
Do not accept "r_s indicates a weak correlation of happiness score and country rank".

[1 mark]

[Total 14 marks]

1. (a) evidence of using the nearest neighbour algorithm ***(M1)***

$\mathrm{HAED}(\mathrm{CBH})$ **OR** $40+50+60+...$

$40+50+60+90+80+85$ ***(A1)***

upper bound $=405$ ***A1***

[3 marks]

(b) (i) evidence of using Prim's algorithm to find MST ***M1***

AE, DE, CD, … **OR** $50+60+90+...$

AE, DE, CD, BC **OR** $50+60+90+80$ ***A1***

$=280$ ***A1***

Note: Award at most ***M1A0A1*** for using Prim's from an incorrect starting point.
Award ***M0A0A1*** for obtaining 280 using Kruskal's or if no working seen.

[3 marks]

(ii) evidence of reconnecting H with two edges to MST ***(M1)***

$280+40+70$

lower bound $=390$ ***A1***

[2 marks]

(c) (i) 0.4

A1

[1 mark]

(ii) 0.3

A1

[1 mark]

(iii) 0.2

Note: ***FT*** from 0.6 – their p.

A1

[1 mark]

continued…

Question 1 continued.

(d) (i) $\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}$

A1

[1 mark]

(ii) recognition to find $\boldsymbol{T}^4$ or $\boldsymbol{s_4}$ ***(M1)***

$$\boldsymbol{T}^4\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}=\begin{pmatrix}0.1025\\0.1832\\0.2162\\0.2513\\0.2468\end{pmatrix}$$

0.216 (0.2162) ***A1***

[2 marks]

(iii) recognition to use $\boldsymbol{T}^3$ ***(M1)***

recognition of two cases DD or EE ***(M1)***

$$0.7\boldsymbol{T}^3\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}\left(=\begin{pmatrix}0.0749\\0.1274\\0.1526\\0.1736\\0.1715\end{pmatrix}\right) \textbf{ AND } 0.3\boldsymbol{T}^3\begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}\left(=\begin{pmatrix}0.0276\\0.0558\\0.0636\\0.0777\\0.0753\end{pmatrix}\right)$$ ***A1A1***

$(0.1736+0.0753)=0.249$ (0.2489) ***A1***

[5 marks]

continued…

Question 1 continued.

(e) (i) attempt to find steady state matrix $\begin{pmatrix} 0.104072\ldots \\ 0.190045\ldots \\ 0.226244\ldots \\ 0.235294\ldots \\ 0.244343\ldots \end{pmatrix}$ ***(M1)***

Note: May be seen in part (e)(ii).

$0.104 \ (0.104072\ldots)\left(\frac{23}{221}\right)$ ***A1***

[2 marks]

(ii) attempt to multiply distances by probabilities from steady state matrix ***(M1)***

$2\begin{pmatrix} 40 & 85 & 160 & 100 & 70 \end{pmatrix}\begin{pmatrix} 0.104 \\ 0.190 \\ 0.226 \\ 0.235 \\ 0.244 \end{pmatrix}$ ***(A1)***

$194.298...$

$194 \ (\text{km})$ ***A1***

Note: Award ***(M1)(A1)A0*** for 97.1, from omitting the 2.

[3 marks]

Total [24 marks]

2. (a) $2.88 \left(\frac{72}{25}\right)$ ***A2***

Note: Award ***M1A0*** for an attempt to calculate mean by hand, leading to an incorrect answer e.g values and frequencies must be seen substituted into formula.

[2 marks]

(b) using Poisson distribution with their part (a) as m ***(M1)***

$\mathrm{P}(X = 0)$

$= 0.05613476\ldots$ ***(A1)***

$(0.05613476\ldots \times 50)$

$= 2.81\ (2.80673\ldots)$ ***A1***

[3 marks]

(c) to ensure the expected frequency is greater than 5 ***R1***

Note: Accept equivalent reasoning, provided it includes "expected" and "5".

[1mark]

(d) (i) H_0: the data can be modelled by a Poisson distribution ***A1***

Note: Award ***A0*** if they include mean $= 2.88$. However, in this specific case it is possible to award ***A1*** in part (iv).

[1 mark]

(ii) $\mathrm{d}f = 3$ ***A1***

[1 mark]

(iii) Evidence of combining columns for observed frequencies ***(M1)***

e.g. $9, 12, 11, 10, 8$

p-value $= 0.843\ (0.842640\ldots)$ ***A2***

Note: If their $\mathrm{d}f = 4$, then award ***FT*** for a p-value of $0.934587\ldots$

If their $\mathrm{d}f = 2$, then award ***FT*** for a p -value of $0.660837\ldots$

Award ***M1A1A0*** for an answer of 0.84 (2 sf value) or 0.842 (truncated value)

[3 marks]

continued…

Question 2 continued.

(iv) since p-value > 0.05 ***R1***

Insufficient evidence to reject H_0, (data can be modelled by a Poisson) ***A1***

[2 marks]

Note: Award ***R1*** for correctly comparing their p-value to 0.05, and ***A1*** for a consistent conclusion.
Do not award final ***A1*** if the null hypothesis is incorrect.
Do not award ***R0A1***.
Condone "accept the null hypothesis" or "do not accept the alternative hypothesis".

(e) use of t-test ***(M1)***

p-value $= 0.0263$ $(0.0262738\ldots)$ ***A1***

since p-value < 0.05 ***R1***

sufficient evidence to reject H_0 /

there is evidence of a decrease in the population mean ***AG***

Note: Award ***M0A0R0*** for use of the z-test (p-value = 0.0170858…)
Award a maximum of ***M1A0R1*** for a p-value of 0.026 (only 2 sf)

[3 marks]

(f) (i) must be positive, ***A1***

since $\frac{\mathrm{d}L}{\mathrm{d}t} > 0$ (and $L_\infty - L > 0$) ***R1***

Note: Do not award ***A1R0***.
Accept equivalent reasoning in words, e.g. the growth is positive.

[2 marks]

(ii) 0 ***A1***

Since $L \to L_\infty$ ***R1***

Note: Accept equivalent reasoning in words, e.g. the fish will stop growing.

[2 marks]

continued…

Question 2 continued.

(iii) **EITHER**

L_∞ , since the length decreased ***A2***

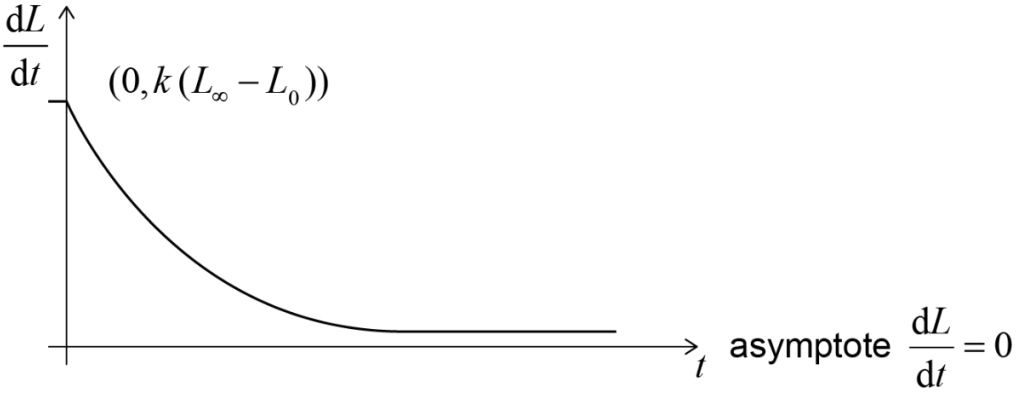
OR

k **,** not growing as quickly, finite lifespan ***A2***

Note: ***Award A1 for "L, since the length decreased"***

[2 marks]

(g)



Award ***A1*** for a concave-up curve which is positive and decreasing

Award ***A1*** for a $\frac{dL}{dt}$-intercept at $k(L_\infty - L_0)$ (condone omission of t coordinate)

Award ***A1*** for a horizontal asymptote $\frac{dL}{dt} = 0$ (accept $y = 0$)

[3 marks]

continued…

Question 2 continued.

(h) attempt to separate variables ***(M1)***

$$\int \frac{1}{L_\infty - L}\,\mathrm{d}L = \int k\,\mathrm{d}t$$

$$-\ln\left|L_\infty - L\right| = kt + c$$ ***A1A1***

Note: Award ***A1*** for correct integration of RHS including "$+c$"***A1*** for correct integration of LHS. Accept absence of absolute value, since $L_\infty - L > 0$.

attempt to remove ln using e from their integrated equation ***(M1)***

$$L_\infty - L = \mathrm{e}^{-kt-c} = A\mathrm{e}^{-kt}$$

substitute initial conditions into their integrated equation, to find constant of integration ***(A1)***

$$L_\infty - L_0 = \mathrm{e}^{-c}\,(= A)$$

Note: The previous two marks may be seen in either order.

$$L = L_\infty - (L_\infty - L_0)\,\mathrm{e}^{-kt}$$ ***A1***

[6 marks]

Total [31 marks]

9. (a) Evidence of use of normal cdf with 45 **OR** sketch of bell curve with area above 45 labelled. ***(M1)***

$0.0338556\ldots$ ***(A1)***

recognition of binomial distribution with probability of their $0.0338556\ldots$ ***(M1)***

$X \sim \mathrm{B}(180,\ 0.0338556\ldots)$

$\mathrm{P}(X \geq 10)$ **OR** $1-\mathrm{P}(X \leq 9)$

$0.0869\ (0.0869379\ldots)$ ***A1***

[4 marks]

(b) defining event as being late and lateness marked correctly

$0.0338556\ldots \times (1-0.35) = 0.0220061\ldots$ ***(A1)***

Y is the number of occasions of being marked late

$Y \sim \mathrm{B}(180,\ 0.0220061)$ **AND** $\mathrm{P}(Y \geq 4)$ **OR** $\mathrm{P}(Y > 3)$ **OR** $1-\mathrm{P}(Y \leq 3)$ ***(M1)***

$0.561\ (0.561005\ldots)$ ***A1***

[3 marks]

[Total 7 marks]

3. (a) 4 ***A1***

[1 mark]

(b)

	A	B	C	D	E	F	G	H
Brett	1	4	2	3	5	6	7	8
Clarence	1	2	4	3	5	7	6	8

(A1)(A1)

Note: Award **(A*1)*** for each line in table

0.881 $(0.880952\ldots)$ ***A2***

Note: The ranks maybe seen within printed table.
Award ***A1*** for 0.88 (2sf) answer.

For a common mistake:
Brett: 1 3 4 2 5 6 7 8 & Clarence correct
Award (***A0)(A1)A2*FT** if 0.952 seen.

Note: The use of $r_s = 1 - \frac{6\sum d^2}{n(n-1)}$ i.e. $(r_s =) 1 - \frac{6\left(0^2+2^2+2^2+0^2+0^2+1^2+1^2+0^2\right)}{8\left(8^2-1\right)}$ may be seen which is not in the syllabus, but is a valid method because there are no ties. As they need the table to get the differences, there is no change to MS for this method.

[4 marks]

(c) (since r_s is close to 1) there is a strong (positive) correlation between the **rankings** of the sheepdogs by Brett and the **rankings** of the sheepdogs by Clarence
(since r_s is close to 1) there is a strong (positive) association between Brett's **rankings** and Clarence's **rankings**.
since r_s is not equal to 1, Brett and Clarence do not always agree on the **rankings** of the sheepdogs. ***A1***

Note: For ***A1*** to be awarded, a numerical value must be seen in part (b), and the conclusion consistent with that numerical value.
"Strong positive" is insufficient because it is not in terms of the **ranks**.
"Similar" in isolation is insufficient, however if they state "the order in which the sheepdogs were **ranked** by the judges is similar" would be awarded ***A1***.
The names of judges do not need to be seen for the ***A*** mark.

[1 mark]
Total [6 marks]

6. (a) attempt to use formula $S_{n-1}^2 = \frac{n}{n-1} S_n^2$ **(*M1*)**

$= \frac{5}{4} \times 3.2^2$

$= 12.8 \left({}^\circ\text{C}^2 \right)$ ***A1***

[2 marks]

(b) indication of use of t-interval **(*M1*)**

$[16.1267,\ 24.0733]$ from use of 3.2

from GDC $[15.7,\ 24.5]$ $([15.6577...,\ 24.5423...])$ ***A1***

Note: Do not penalize use of round brackets, strong inequalities or using terms such as "upper" and "lower" to describe the endpoints of the interval.

[2 marks]

(c) 17 is within the confidence interval, so it is a plausible mean ***R1***

[1 mark]

Total [5 marks]

13. let X be the number of worms in 1 square metre then $X \sim \text{Po}(1.2)$

(a) $\text{P}(X = 1) = 0.361 \quad (0.361433...)$ ***A1***

[1 mark]

(b) $\text{P}(X \geq 1) = 0.699 \quad (0.698805...)$ ***A1***

[1 mark]

(c) if Y is the number of worms in 5 square metres then $Y \sim \text{Po}(6)$ ***(A1)***

$\text{P}(Y = 5) = 0.161 \quad (0.160623...)$ ***A1***

[2 marks]

(d) **EITHER**

attempting to find an expression in terms of $\text{P}(X = 1)$ ***(M1)***

$\text{P}(X = 1)^5 = 0.00617 \quad (0.00616792...)$ ***A1***

OR

let V be the number of square metres containing exactly one worm
attempting to use the binomial with $\text{P}(X = 1)$ ***(M1)***

then $V \sim B(5, \text{P}(X = 1))$

$\text{P}(V = 5) = 0.00617 \quad (0.00616792...)$ ***A1***

Note: Accept 0.00613 from 3sf **FT** from part (a).

[2 marks]

(e) let W be the number of square metres containing at least one worm.
attempting to use binomial ***(M1)***
then $W \sim \text{B}(5,\ \text{P}(X \geq 1))$ ***(A1)***

$\text{P}(W = 3) = 0.310 \quad (0.309572...)$ ***A1***

Note: Accept 0.31 for final ***A1.*** Accept 0.309 from 3sf **FT** from part (b).

[3 marks]

Total [9 marks]

14. use the second line since it has a higher R^2 ***A1R1***

Note: Do not award ***R0A1.*** The ***R1*** can be awarded independently within the question.

$\ln w = 0.302 \ln d + 0.693$ ***A1***

attempt to exponentiate both sides ***M1***

$w = e^{0.693} \times d^{0.302}$ ***A1***

$w = 2.00 d^{0.302} \quad \left(= 1.99970... d^{0.302}\right)$ ***A1***

Note: Accept $w = 2d^{0.302}$ for final ***A1.***
Award ***A0R0A1M1A1A0*** for use of the first line

Total [6 marks]

Q3.

(a) H_1: satisfaction level and the type of service interaction are not independent. ***A1***

[1 mark]

(b) 4 ***A1***

[1 mark]

(c) $3.32\ (3.31906\ldots)$ ***A2***

Note: Award ***A1A0*** for a 2 sf answer of 3.3 as recognition of correct method.

[2 marks]

(d) (i) $3.319106\ldots < 9.488$ OR $0.506\ (\ 0.505917\ldots) > 0.05$ ***R1***

No, he isn't correct (insufficient evidence to reject H_0) ***A1***

Note: Accept a comparison in words (e.g. $\chi^2_{calc} < \chi^2_{crit}$ or '*p*-value < sig level') for the ***R*** mark only if their χ^2_{calc} value or *p*-value is seen. Do not award ***A1R0.***

(ii) insufficient evidence to say ''satisfaction level and the type of service interaction are not independent''. ***A1***

Note: Answer to part d (ii) follows through from their answer to part d(i).

[3 marks]

Total [7 marks]

Q4.

(a) 4 ***A1***

[1 mark]

(b) identification of lower quartile as 24.28 ***(A1)***

$24.70 - 24.28$

0.42 (seconds) ***A1***

[2 marks]

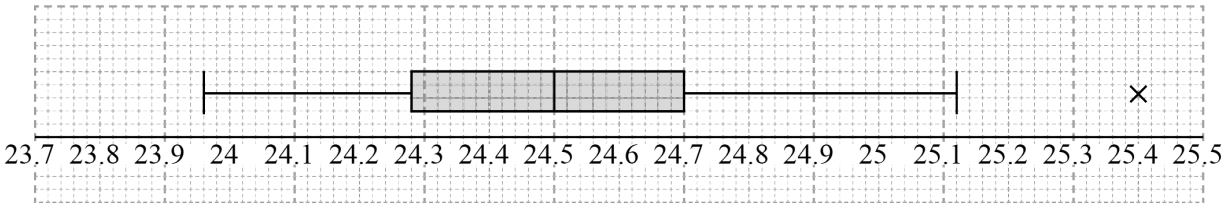
(c) (i) attempt to use correct formula to find boundaries for outliers ***(M1)***

$24.70 + 1.5 \times 0.42$ OR $24.28 - 1.5 \times 0.42$

25.33 and 23.65 ***A1***

only 25.4 is an outlier ***A1***

(ii)



A1

Note: Award the ***A1*** only if both whiskers and the outlier are correctly marked.
Accept symbols other than a cross to represent the outlier.
The whiskers should not be drawn through the box.

[4 marks]
Total [7 marks]

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Q6

(a) use of formula for sum of an arithmetic sequence or first few terms seen ***(M1)***

$\frac{10}{2}(2\times 6+9\times 3)$ OR $6+9+12+15+...$

sum of angles is 195° ***(A1)***

$p=\frac{195}{360}\left(0.542,\ \frac{13}{24},\ 0.541666...\right)$ ***A1***

[3 marks]

(b) recognition of binomial distribution ***(M1)***

$n=4,\ p=\frac{195}{360}$ OR $X\sim \mathrm{B}\left(4,\frac{195}{360}\right)$ ***(A1)***

let X be the number of times the arrow lands in white

$\mathrm{P}(X\geq 3)=0.377\ (0.377450\ldots)$ ***A1***

Note: Accept $0.378\ \ (0.377988...)$ from use of 0.542 from part (a)***.***

[3 marks]
Total [6 marks]

Q7.

(a) stratified (sampling) ***A1***

[1 mark]

(b) (i) $r = 0.854\ (0.853899...)$ ***A2***

Note: Award ***(M1)A0*** for a truncated value $r = 0.853$ or incorrectly rounded value eg. $r = 0.85$.

(ii) it is valid (since $r > 0.8$) ***A1***

Note: Follow through from their value of r, for the final ***A1.***

[3 marks]

(c) $y = 0.266x + 2.40\ (y = 0.265851...x + 2.39573...)$ ***A1***

Note: Accept 2.4.

[1 mark]

(d) substitution of 17.2 into their regression line ***(M1)***

$y = 0.265851... \times 17.2 + 2.39573...$

$= 6.97\left(6.96837...\right)$ ***A1***

Note: Accept $6.98\left(6.9752\right)$ from use of 3sf answers from part (c).

[2 marks]

Total [7 marks]

Q12. (a) (i) rejecting the null hypothesis when it is true ***A1***

OR

rejecting the mean as 12.4 / min ($124 / 10$ mins) when it is true ***A1***

(ii) let X be the number of cars that arrive in 10 minutes

attempt to find $\mathrm{P}(X \geq 131)$ ***(M1)***

use of $\mathrm{Po}(124)$ ***(M1)***

$\mathrm{P}(X \geq 131) = 0.276 \ (0.276397\ldots)$ ***A1***

[4 marks]

(b) any valid reason ***R1***

for example:

the arrival of cars are not independent (there might be a traffic light shortly before the intersection)

the rate of arrival is not uniform (some times of day might be busier than others)

Note: Do not accept ‘’mean and variance are not the same’’.

[1 mark]

Total [5 marks]

Q15. (a) $\begin{pmatrix} p-1 & 1-q \\ 1-p & q-1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} p & 1-q \\ 1-p & q \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$(p-1)x+(1-q)y=0$ (or equivalent) ***(A1)***

$\begin{pmatrix} 1-q \\ 1-p \end{pmatrix}$ (or any multiple) ***A1***

[3 marks]

(b) $\dfrac{1-q}{2-p-q}$ ***A1A1***

Note: Award ***A1*** for numerator, ***A1*** for denominator.
For an answer of $1-q$ only, award ***A0A0***.

[2 marks]

(c) $\dfrac{4}{11}(0.364, 0.363636...)$ ***A1***

[1 mark]
Total [6 marks]

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1. (a) $(k =)\ 5$ ***A1***

[1 mark]

(b) (i) attempt to substitute into the mean formula with a sum of at least two correct products **and** equating to 2.2 (or $2.2\ (p+27)$ if denominator not seen) or equating to "mean". Equation may be seen in part b(ii). ***(M1)***

$$2.2 = \frac{0\times 3+1\times 11+2\times 7+3\times 5+4\times p+5\times 1}{3+11+7+5+p+1} \quad \textbf{OR} \quad 2.2 = \frac{45+4p}{p+27}$$ ***A1***

(ii) $(p =)\ 8$ ***A1***

Note: Follow through from part (b)(i) only if their value of p is a whole number.

[3 marks]

(c) finding the median $(=2)$, quartiles ($Q_1 = 1$ and $Q_3 = 4$), or interquartile range of the data $IQR = 3$ from the 2024 tournament ***(A1)***

Note: Award ***(A1)*** for any correctly identified/labelled value referring to 2024 data as listed in the question. Accept "Range from 0 to 5" without reference to 2024.

Any two of the following statements: ***R1R1***

The median number of goals per game **in both years** is 2.
In both years, (the lower quartile is 1 and the upper quartile is 4) the IQR is 3.
Both distributions are (roughly) skewed to the right

Note: Award ***R1*** for each correct statement. Each statement must reference values from both the box and whisker diagram **and** the data from the 2024 tournament to award ***R1***. (e.g. "the median is 2" earns ***R0***, "the median for both years is 2" earns ***R1***.) If an incorrect statement is made, award at most ***R1R0***.

[3 marks]

(d) B and C ***A1A1***

Note: Award ***A1*** for each correct event, if two events are selected.
Award ***A1A0*** for two correct events and one additional incorrect event selected.
Award ***A0A1*** for an answer of "A, D, and E" from misreading the meaning of F'.
Award ***A0A0*** for any other combination of events selected.

[2 marks]

(e) $\frac{5}{17}\left(\frac{10}{34}\right)$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator. Award at most ***A1A0*** for a decimal answer $(0.294117\ldots)$ if no fraction is shown. Do not award ***A1ft*** for their denominator if their answer to part (a) or part (b)(ii) is not a whole number.

[2 marks]

(f) $\frac{1}{35}\times\frac{3}{34}$ ***(A1)(A1)(M1)***

Note: Award ***A1*** for each correct probability, ***M1*** for multiplying their two probabilities. Do not award any marks if their answer to part (a) or part (b)(ii) is not a whole number.

$0.00252 \quad (0.00252100\ldots, \frac{3}{1190}, 0.252\%)$ ***A1***

[4 marks]
Total [15 marks]

5. (a) 186 ***A1***

[1 mark]

(b) using midpoints within formula for mean ***M1***

$$\frac{11\times103+14\times52+17\times31}{186}$$

$12.8\ (12.8387\ldots)\ (\text{mm})$ ***A1***

[2 marks]

(c) $\frac{42}{86}$ or $\frac{21}{43}$ seen $(0.488372\ldots)$ ***(A1)***

$$\left(\frac{21}{43}\right)^2$$

method mark for squaring their probability ***(M1)***

$= 0.239 \quad (0.238507...) \quad \left(=\frac{441}{1849}\right)$ ***A1***

[3 marks]

(d) choosing a chi-squared test for independence ***(M1)***

p-value = $0.127\ (0.126687\ldots)$ ***A1***

p-value > 0.05 therefore do not reject H_0 ***R1***

Note: Accept p-value > significance level, as this is stated in the question.

there is no significant evidence that length category and sex are not independent ***A1***

Note: Do not award ***R0A1***. Accept length category and sex are independent. Allow internal follow through in this part for a consistent comparison and conclusion from their p-value.

[4 marks]

(e) $H_0 : \phi = 0.45;\quad H_1 : \phi > 0.45$ ***A1***

choosing a binomial test ***(M1)***

Note: can be implied by mention of Binomial, calculator notation, recognisable p-value

under the null hypothesis, if X is the number of males then $X \sim B(186, 0.45)$

p-value $= P(X \geq 100)$ ***(M1)***

$=0.0101\ (0.010103\ldots)$ ***A1***

since p-value < 0.05 we reject H_0

there is significant evidence that more than 45% of cane beetles are male. ***A1***

Note: Do not award the final ***A1*** unless the correct p-value is seen.

[5 marks]

Total [15 marks]

7. (a) (i) 0.4 ***A1***

(ii) 0.6 ***A1***

[2 marks]

(b) (i) $0.4^4 = 0.0256$ ***(M1)A1***

(ii) considering $M^4 = \begin{pmatrix} 0.5422 & 0.5341 \\ 0.4578 & 0.4659 \end{pmatrix}$ ***(M1)(A1)***

required probability is therefore 0.542 ***A1***

[5 marks]

(c) attempting Mv with given eigenvector ***M1***

$$\begin{pmatrix} 0.4 & p \\ 0.6 & 1-p \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

correct matrix multiplication

$$\begin{pmatrix} 0.4-p \\ -0.4+p \end{pmatrix}$$ ***A1***

writing as a multiple of given vector

$$=\begin{pmatrix} 0.4 & -p \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ ***(A1)***

therefore the eigenvalue is $0.4-p$ ***A1***

[4 marks]

(d) consider eigenvector with eigenvalue 1 $\boldsymbol{Mv} = v$ ***(M1)***

$$\begin{pmatrix} 0.4 & p \\ 0.6 & 1-p \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

turn into equations:

$0.4x + py = x$ OR $0.6x + (1-p)y = y$ ***A1***

also use $x + y = 1$ ***A1***

then $x + \frac{0.6x}{p} = 1$

$x = \frac{p}{p+0.6}$ ***A1***

[4 marks]

(e) graphical, inequality or equality approach to solve $\frac{p}{p+0.6} \geq 0.6$ ***(M1)***

$p = 0.9$ ***A1***

[2 marks]

Total [17 marks]

Q6. (a) (i) 0.45 ***A1***

(ii) attempt to find the expected value (at least two correct products) ***(M1)***

$0\times0.21+1\times0.34+2\times0.28+3\times0.17$

1.41 (customers) ***A1***

[3 marks]

(b) $\mathrm{E}(T)=115$ ***(A1)***

1.41×115

$162\ (162.15)$ (seconds) $(2$ minutes 42 seconds$)$ ***A1***

[2 marks]

(c) $\mathrm{P}(T>180)$ ***(M1)***

$0.0101\left(0.0101318\ldots\right)$ ***A1***

Note: Award ***M1A0*** for an answer of 0.467.

[2 marks]

(d) attempt at linear combination of two normal variables ***(M1)***

time to serve two people, $T_2\sim\mathrm{N}(230,2\times28^2)$ ***(A1)(A1)***

$\mathrm{P}\left(T_2>180\right)=0.897\left(0.896649\ldots\right)$ ***A1***

[4 marks]

(e) (i) (if no-one in the queue the waiting time is 0)

$(0.21\times0)+0.34\times0.0101318...+0.28\times0.896649+0.17\times1$ ***(M1)(A1)***

$0.425\ \left(0.424506\ldots\right)$ ***A1***

(ii) $\left(0.425>0.4\right)$ employ more staff ***A1***

Note: Award if consistent with their probability from part (e) (i).

[4 marks]

[Total 15 marks]

1. (a)

$$(\boldsymbol{T} =)\begin{pmatrix} 0.91 & 0.04 & 0.06 \\ 0.05 & 0.95 & 0.02 \\ 0.04 & 0.01 & 0.92 \end{pmatrix}$$ ***A1A1A1***

Note: Award ***A1*** for each correct column.

[3 marks]

(b) recognizing the need to calculate $\boldsymbol{T}^5$ ***(M1)***

$$(\boldsymbol{T}^5 =)\begin{pmatrix} 0.659 & 0.156 & 0.219 \\ 0.196 & 0.792 & 0.101 \\ 0.146 & 0.0514 & 0.680 \end{pmatrix} \left(\begin{pmatrix} 0.658687... & 0.156128... & 0.219028... \\ 0.195559... & 0.792469... & 0.101208... \\ 0.145753... & 0.0514021... & 0.679762... \end{pmatrix}\right)$$ ***(A1)***

selects relevant cell (e.g. 0.219 seen separately) ***(M1)***

21.9% ***A1***

Note: Final answer must be a percentage. Follow through from their $\boldsymbol{T}$, including the transpose.

[4 marks]

(c) attempt to find steady state vector (e.g. raising $\boldsymbol{T}$ to a high power) ***(M1)***

$$\begin{pmatrix} 0.342 \\ 0.432 \\ 0.225 \end{pmatrix} \left(\begin{pmatrix} 0.342342... \\ 0.432432... \\ 0.225225... \end{pmatrix}\right)$$ ***A1***

[2 marks]

Note: Award ***M1A0*** if they express steady state vector as $\begin{pmatrix} 0.342 & 0.342 & 0.342 \\ 0.432 & 0.432 & 0.432 \\ 0.225 & 0.225 & 0.225 \end{pmatrix}$

(d) 34.2% ***A1***

[1 mark]

(e) require new steady state vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ such that $a \geq 0.40$ (seen anywhere) ***(R1)***

any correct manipulation of Clover's entries in $\boldsymbol{T}$ (e.g. complementary increase and decrease of $\boldsymbol{T}_{1,3}$ and $\boldsymbol{T}_{3,3}$) ***(A1)***

Note: This ***A1*** can be awarded independently of the ***R1***.

$$(\boldsymbol{T} =) \begin{pmatrix} 0.91 & 0.04 & 0.08 \\ 0.05 & 0.95 & 0.02 \\ 0.04 & 0.01 & 0.90 \end{pmatrix}$$

$$\begin{pmatrix} 0.403 \\ 0.459 \\ 0.138 \end{pmatrix}$$

when $C \to A$ is 13% ***A1***

Note: If candidates go on to state 13% - 6% = 7%, then ignore this and award ***A1***.If their answer is just '7%' then award ***A0***.

[3 marks]

(f) since $0.459 > 0.432$ ***R1***
therefore Bodega will benefit ***A1***

Note: Do not award ***R0A1***.

[2 marks]

(g) Accept any appropriate reason that shows engagement with the scenario.
e.g.
(Convergence to steady state vector takes a relatively long time and …)

…during which time either of the three stores may notice a drop/influx of customers and change their advertising accordingly, thus changing $\boldsymbol{T}$. ***A1***

…during this time, a fourth (etc.) store could open, impacting the model. ***A1***

…during this time, customers may move in and out of the region in a non-consistent rate. ***A1***

Note: 'Customers may change their minds' or similar, award ***A0***.

[1 mark]

(h) attempt to apply $u_n = u_1 + (n-1)d$ ***(M1)***
$= 600 + 9 \times 30$
$= 870$ ***A1***

[2 marks]

(i) 720 seen anywhere ***A1***

$720+1.12\times 30$ ***A1***

$=753.6$ ***AG***

[2 marks]

(j) attempts to sum geometric series with $r=1.12$ ***(M1)***

$720+30(1.12+1.12^2+...+1.12^5)$ ***(A1)***

$=933$ $(933.455...)$ ***A1***

[3 marks]

(k) **METHOD 1 (AS formula from booklet and adapted general formula from part (j))**

general formula for AS: $600+30(n-1)$ ***(A1)***

attempt to adapt GS formula (from part (j)) to include "$n-5$" ***(M1)***

general formula for GS: $720+30\left[\frac{1.12(1.12^{n-5}-1)}{1.12-1}\right]$ ***(A1)***

EITHER

attempts to solve GS $-$ AS $=500$ ***(M1)***

$n=17.5052...$ ***A1***

Note: Other correct n values, depending on original expressions, are possible but only differing by an integer. ***FT*** within the question.

OR

creates list of both sequences ***(M1)***

$1080, 1530.87\ldots$ **AND** $1110, 1661.77\ldots$ ***A1***

Note: If list method is used, award ***A1*** for the cross-over values clearly stated.

THEN

$\Rightarrow$ week 18 ***A1***

Note: Award at most ***(A1)(M1)(A1)(M0)A0A1*** for a correct answer with no working.

[6 marks]

METHOD 2 (general formula from part (j) and adapted AS formula)

general formula for GS: $720+30\left[\frac{1.12(1.12^n-1)}{1.12-1}\right]$ ***(A1)***

attempt to express AS formula for weeks after fifth week ***(M1)***

general formula for AS: $720+30n$ ***(A1)***

Note: The n values can be offset (e.g $n-1$ or $n-5$ etc.) provided the approach is consistent between GS and AS.

EITHER

attempts to solve GS $-$ AS $=500$ ***(M1)***

$n=12.5052...$ ***A1***

Note: Other correct n values, depending on original expressions, are possible but only differing by an integer. ***FT*** within the question.

OR

creates list of both sequences ***(M1)***

$1080, 1530.87\ldots$ **AND** $1110, 1661.77\ldots$ ***A1***

Note: If list method is used, award ***A1*** for the cross-over values clearly stated.

THEN

$\Rightarrow$ week 18 ***A1***

Note: Award at most ***(A1)(M1)(A1)(M0)A0A1*** for a correct answer with no working.

[6 marks]

[Total: 29 marks]

2. (a) $\left(\mathrm{P}(X > 55) =\right)\ 0.208737...$ ***(A2)***

recognizing Binomial distribution ***(M1)***

$\left(\mathrm{P}(Y \geq 5) =\right)\ 0.621\ (0.621150...)$ ***A1***

Note: Award ***(A2)(M1)A0*** for an answer of 0.622 (from use of 0.209).

[4 marks]

(b) $(\text{variance} =)\ \dfrac{3.7^2}{25}$ **OR** $(\text{SD} =)\ \dfrac{3.7}{5}$ ***(A1)***

$\overline{X} \sim \mathrm{N}\left(52.0, \dfrac{3.7^2}{25}\right)$

$\mathrm{P}(51 < \overline{X} < 53) = 0.823\ (0.823417...)$ ***A2***

[3 marks]

(c) $\mathrm{H}_0 : \mu = 52.0$ ***A1***

$\mathrm{H}_1 : \mu > 52.0$ ***A1***

Note: Accept hypotheses in words if they are clearly expressed and 'population mean' is referred to. Do not accept $\mathrm{H}_0 : \mu = \mu_0$ unless μ_0 is explicitly defined to be 52.0.

p-value $= 0.105055\ldots$ ***A2***

$0.105 > 0.05$ ***R1***

therefore there is insufficient evidence to reject H_0 ***A1***

(i.e. the population mean weight of eggs is 52.0 g)

Note: Follow through within the question for the ***R1A1***, from their p-value seen. Award at most ***A0A0A2R1A0*** if hypotheses are omitted or incorrect. Award the final ***A1*** only if the null hypothesis is also correct and the conclusion is consistent with both the direction of the inequality and the alternative hypothesis.

[6 marks]

(d) **EITHER**

52.66 ± 1.5888 ***A1A1***

Note: Award ***A1*** for each correct value. Award at most ***A1A0*** if the $\pm$ is incorrect or omitted.

OR

$(51.1, 54.2)\ \left((51.0712..., 54.2488...)\right)$ ***A1A1***

Note: Award ***A1*** for each correct end point. Award at most ***A1A0*** if interval notation is incorrect or omitted.
Allow $51.1 \leq \mu \leq 54.2$

[2 marks]

(e) (i) paired *t*-test ***A1***
[1 mark]

(ii) **EITHER**
That the differences in weights are normally distributed. ***A1***
OR
The weights of eggs from any two chickens are independent. ***A1***
[1 mark]

(f) $H_0 : \mu_d = 0$ **OR** $H_0 : \mu_2 - \mu_1 = 0$ ***A1***
$H_1 : \mu_d > 0$ **OR** $H_1 : \mu_2 - \mu_1 > 0$ ***A1***

$p\text{-value} = 0.03377479...$ ***A2***
$0.0338 < 0.05$ ***R1***
(there is sufficient evidence that) the mean weight of the chickens' eggs has increased following the new feed ***A1***

Note: The conclusion must be in context for the final ***A1*** to be awarded. Follow through within the question for the ***R1A1***, from their *p*-value seen. Award at most ***A0A0A2R1A0*** if hypotheses are omitted or incorrect. Do not follow through from an incorrect part (e)(i).

[6 marks]

(g) $2.39...$ $(2.391555\ldots)$ ***A1***
[1 mark]

(h) an attempt to find s_R^2 using the given expression and their part (g) ***(M1)***
$s_R^2 = 2.391555\ldots \times 3.1789$
$= 7.60$ ***A1***
[2 marks]

[Total: 26 marks]

1. (a) (i) any sensible comment, eg:

the sample will be more reliable / (likely to be) more representative / valid ***R1***

the sample mean is likely to be closer to the population mean ***[1 mark]***

Note: Do not accept "The sample will be (more) accurate, precise or exact", or anything referring to outliers or SD only.

(ii) more time consuming / expensive / more open to human error ***R1***

[1 mark]

Note: Do not accept anything referring to outliers or SD only

(b) 7.60 $(7.597537...)$ ***(M1)A1***

Note: M1 for 7.60 seen anywhere, ***A1*** for correct final answer.

[2 marks]

(c) any sensible comment, eg: ***R1***

the population standard deviation may be different from the sample

these s_{n-1} values are only estimates / approximations and the values are too close together.

standard deviation is not the only measure of spread

Note: Award ***R0*** if part (b) is not in range $7\text{-}7.65$

[1 mark]

(d) (i) (the population variances) are the same. ***A1***

Note: Must say 'the same/equal'. 'Similar' or 'close to each other' is not correct.

[1 mark]

(ii) any attempt to compare sample standard deviations or sample variances ***M1***

the sample variances/sample standard deviation/ s_{n-1} are similar, so assumption is plausible. (Therefore Augustin should use a pooled t-test.) ***R1***

Note: Award ***R0*** if part (b) is not in range $7\text{-}8$

[2 marks]

continued…

Question 1 continued.

(e) (i) $H_0 : \mu_A = \mu_B$ ***A1***

$H_1 : \mu_A < \mu_B$ ***A1***

Note: Award ***A1*** for a correct null hypothesis and ***A1*** for choosing the correct tail for *their* alternative hypothesis. Accept correct hypotheses in words, but only if "population" is clear.

Award at most ***A1A0*** if μ_1 or μ_2 used without definition.

[2 marks]

(ii) $0.0445 \quad (0.0444586...)$ ***A2***

Note: Accept anything in the range 0.044-0.045 due to rounding issues.

[2 marks]

(iii) $0.0445 < 0.05$ ***R1***

there is (significant) evidence that the (population) mean of school B is higher than the (population) mean of school A **OR** Augustin's belief is supported ***A1***

Note: Do not award **R0A1**. Do not award the final **A1** if the null hypothesis is incorrect or the conclusion is not in context.

[2 marks]

(f) (i) $H_0 : \rho = 0; H_1 : \rho \neq 0$ ***A1A1***

Note: Hypotheses must involve ρ. Hypotheses in terms of r are not correct. In words 'population correlation' is equally valid.

$r > 0.576$ **OR** $0.876 > 0.576$ ***R1***

Note: Accept $|r| > 0.576$

therefore reject H_0, there is significant evidence of correlation between entry exam and final exam results. ***A1***

Note: Do not award ***R0A1.*** Do not award final ***A1*** for just 'reject H_0', there must be some interpretation.

Condone 'there is evidence of correlation'. Of course, the final two marks can still be given if hypotheses were in terms of r.

[4 marks]

(ii) could use Spearman Rank. ***A1***

[1 mark]

continued…

Question 1 continued.

(g) as entry exam results goes up by one, final exam result goes up by 0.37 ***A1***

Note: Statement should clearly be in context. Anything vague about the gradient of the regression line is not acceptable.

[1 mark]

(h) (i) **METHOD 1**

the exam board model predicts $\hat{y} = 0.37 \times 38.7 + 37.6 = 51.919$ ***M1***

therefore the improvement is $52.5 - 51.919 = 0.581$ ***A1***

so the answer is 0.6 to 1 decimal place. ***AG***

[2 marks]

(ii) $H_0 : \mu_A = \mu_B; H_1 : \mu_A > \mu_B$ ***A1***

so the p value is $0.0213 \quad (0.0212715...)$ ***A1***

Note: First two ***A1*** marks are independent. Hypotheses must clearly be in terms of population means. Expressions involving $\mu_{(y-\bar{y})A}$ or μ_1 are acceptable.

since $p < 0.05$ ***R1***

therefore reject H_0, evidence of higher school value added in School A ***A1***

Note: 'Reject H_0' alone is not enough. Answer must be in context.

[4 marks]

(i) school A outperforms school B is supported by the test in part (g)/better added value ***A1***

school B outperforms school A is supported by the test in part (e)/higher mean ***A1***

[2 marks]

Total [28 marks]

1. (a) **EITHER**
the data points come from normal populations ***A1***
OR
the variances (standard deviations) of the two groups are (approximately) equal. ***A1***
OR
data points of each group are independent. ***A1***
OR
the sample is representative of the population ***A1***
[1 mark]

(b) (i) $\mu_s = \mu_P$ **OR** $\mu_1 = \mu_2$ ***A1***
(no difference in the mean (population) distance travelled using premium/standard fuel)
(ii) $\mu_s < \mu_p$ **OR** $\mu_1 < \mu_2$ ***A1***
(mean (population) distance travelled using premium fuel is greater than using standard fuel)

[2 marks]

(c) (i) $0.0264\ (0.0263584\ldots)$ ***A2***

Note: Their p-value must be consistent with *their* H_1.

[2 marks]

(ii) $0.0263584\ldots < 0.05$ ***R1***

Note: Accept "$p < 0.05$" or "p-value is less than the significance level", provided their p-value is shown in part (c)(i).

EITHER
(there is sufficient evidence that) the car can travel further using premium fuel. ***A1***
OR
(there is sufficient evidence that) the car uses less premium fuel compared to standard fuel. ***A1***

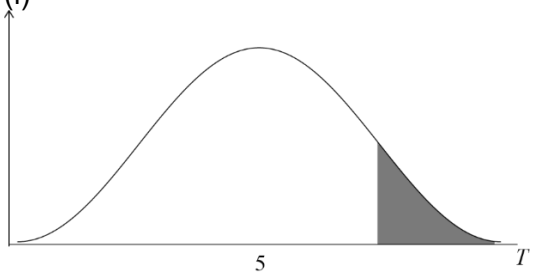
Note: Do not award ***R0A1***. The conclusion must be in context to award ***A1****;* "reject H_0" earns ***A0***. Their conclusion must be consistent with *their* p-value and *their* hypothesis.

[2 marks]
Total [7 marks]

3. (a) 0.159 $(0.158655\ldots)$ ***A2***

[2 marks]

(b) (i)



A1

Note: Accept any shaded region as shown provided it is to the right of 5.

(ii) Recognition to use the inverse normal function on GDC ***(M1)***

Note: Award ***(M1)*** for $\mathrm{P}(X > k) = 0.15$ **OR** $P(X < k) = 0.85$ **OR** inverse normal (invNorm) seen.

6.04 (seconds) $(6.03643\ldots)$ ***A1***

[3 marks]
Total [5 marks]

9. (a) (for a 30 minute period,) mean = 17 ***(A1)***

recognize lower bound is 23 **OR** P(number of calls > 22) ***(M1)***

0.0953 (0.0952720…) ***A1***

[3 marks]

(b) recognition of binomial distribution ***(M1)***

with $n = 10$ and $p = 0.0953\ldots$ OR $p = 0.904\ldots$ ***(A1)***

$0.754 \quad (0.754360...)$ ***A1***

[3 marks]

Total [6 marks]

16. (a) $(q =)\ 0.3$ ***A1***

[1 mark]

(b) 0.7 ***A1***

[1 mark]

(c) 0.2×0.2 **OR** 0.1×0.1 **OR** 0.7×0.4 ***(A1)***

recognize 3 possible ways BB or CC or TT ***(M1)***

$0.2 \times 0.2 + 0.1 \times 0.1 + 0.7 \times 0.4$

0.33 ***A1***

[3 marks]

(d) $\begin{pmatrix} 0.2 & 0.4 & 0.3 \\ 0.1 & 0.1 & 0.3 \\ 0.7 & 0.5 & 0.4 \end{pmatrix}$ ***A1***

[1 mark]

(e) **EITHER**

recognize need to find large power of transition matrix (such that a steady state is found) ***(M1)***

OR

attempt to find eigenvector with eigenvalue 1 ***(M1)***

e.g. $\begin{pmatrix} 0.2 & 0.4 & 0.3 \\ 0.1 & 0.1 & 0.3 \\ 0.7 & 0.5 & 0.4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ **OR** $0.2x + 0.4y + 0.3z = x$

eigenvector $= \begin{pmatrix} \frac{39}{68} \\ \frac{27}{68} \\ 1 \end{pmatrix}$

THEN

0.201 $(0.201492\ldots)$ ***A1***

[2 marks]

Total [8 marks]

1. (a) (i) 4 ***A1***

(ii) 6 ***A1***

(iii) 3 ***A1***
[3 marks]

(b) $(1.5\times 3)+6$
$= 10.5$ ***(A1)***
$(k=)\ 11$ ***A1***
[2 marks]

(c) at least one correct mid-interval value identified ***(A1)***

Note: Mid-interval value may be seen in mean formula.

$4.54 \quad (4.53650..., \ \frac{5716}{1260})$ ***A2***

[3 marks]

(d) the frequency of the number of letters (in each word in his essay) is **not** consistent with the English language. ***A1***
[1 mark]

(e) (i) 6 ***A1***

(ii) $20.3 \quad (20.3131...)$ ***A2***

(iii) $0.00244 \quad (0.00243537... \ \ 2.44\times 10^{-3})$ ***A1***
[4 marks]

(f) **EITHER**
$20.3 > 16.812$ ***R1***
OR
$0.00244 < 0.01$ ***R1***
THEN
Dr. Petrillo should reject the null hypothesis ***A1***

Note: Do not award ***R0A1***.

[2 marks]
Total [15 marks]

5. (a) convenience ***A1***

Note: Accept opportunity.

[1 mark]

(b) (i) $p > 0.6$ ***A1***

(ii) recognition to use binomial distribution (with $n = 80$ and $p = 0.6$) ***(M1)***
recognize need to find $\mathrm{P}(X \geq 54)$ ***(A1)***
p -value $= 0.104\ (0.103731\ldots)$ ***A1***
since $0.104 > 0.05$, ***R1***
insufficient evidence to reject H_0
insufficient evidence that the new recipe is an improvement. ***A1***

Note: ***FT*** from their p-value for the final ***R1A1***. Award ***A1*** only if reference is specifically made to the context. Do not accept "the new recipe is not an improvement". Do not award ***R0A1***.

[6 marks]

(c) (i)

m	$\mathrm{P}(X \geq m)$
55	$0.067464\ldots$
56	$0.041747\ldots$

a region with a lower bound of 56 e.g. $[56, 80]$ **OR** $X \geq 56$ ***A2***

(ii) attempt to find $P(X \leq 55)$ when $p = 0.65$ ***M1***
$0.793\ (0.792833\ldots)$ ***A1***

[4 marks]

(d) H_0: buying snacks is independent of age
H_1: buying snacks is not independent of age ***A1***

Note: Award ***A1*** for both hypotheses correct.

recognize need to combine cells as $fe < 5$ ***(M1)***
new table is ***(A1)***

Age, x	Will buy	Will not buy
$x < 20$	22	8
$20 \leq x < 30$	18	5
$30 \leq x$	14	13

p-value $= 0.0957\ (0.0956988\ldots)$ ***A2***
$0.0957 > 0.05$
insufficient evidence to reject H_0 ***A1***

Note: Award at most ***A1M0A0A2(FT)A1(FT)***, if a candidate does not combine cells, obtaining a p-value of $0.123596\ldots$
Award ***A1*** for a p-value$= 0.096$ (2 s.f.) or 0.0956 (truncated).
FT from their p-value for the final ***A1***, providing H_0 is stated. Award ***A1*** for "Accept H_0".

[6 marks]
Total [17 marks]

APOLLO

Calculus

104 questions

7. (a) (i) $A = \frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times p + 48$ **OR** $A = \frac{1}{2}(p+6)(q+8)$ **OR**
$A = 3q + 4p + 48$ ***A1***

(ii) valid attempt to link p and q, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta = \frac{8}{p}$ and $\tan\theta = \frac{q}{6}$ **OR** $\tan\theta = \frac{p}{8}$ and $\tan\theta = \frac{6}{q}$ **OR** $\frac{8}{p} = \frac{q}{6}$

correct equation linking p and q ***A1***

eg. $pq = 48$ **OR** $p = \frac{48}{q}$ **OR** $q = \frac{48}{p}$

substitute $p = \frac{48}{q}$ into a correct area expression ***M1***

eg. $(A =)\frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times \frac{48}{q} + 48$ **OR** $(A =)\frac{1}{2}\left(\frac{48}{q} + 6\right)(q+8)$

$A = 3q + \frac{192}{q} + 48$ ***AG***

Note: The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2} + 3$ ***A1A1***

Note: Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for 3. Award ***A1A0*** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2} + 3 = 0$ ***A1***

(ii) $q = 8$ cm ***A1***

[2 marks]

Total [8 marks]

12. (a) $\frac{\mathrm{d}V}{\mathrm{d}t} = -kV^{\frac{1}{2}}$

use of separation of variables ***(M1)***

$\Rightarrow \int V^{-\frac{1}{2}}\,\mathrm{d}V = \int -k\,\mathrm{d}t$ ***A1***

$2V^{\frac{1}{2}} = -kt\ (+c)$ ***A1***

considering initial conditions $40 = c$ ***A1***

$2\sqrt{324} = -10k + 40$

$\Rightarrow k = 0.4$ ***A1***

$2\sqrt{V} = -0.4t + 40$

$\Rightarrow \sqrt{V} = 20 - 0.2t$ ***A1***

Note: Award ***A1*** for any correct intermediate step that leads to the ***AG***.

$\Rightarrow V = \left(20 - \frac{t}{5}\right)^2$ ***AG***

Note: Do not award the final ***A1*** if the ***AG*** line is not stated.

[6 marks]

(b) $0 = \left(20 - \frac{t}{5}\right)^2 \Rightarrow t = 100$ minutes ***(M1)A1***

[2 marks]

Total [8 marks]

$\begin{pmatrix} 0.8 \\ \end{pmatrix} \quad \begin{pmatrix} -2 \\ \end{pmatrix}$

15. (a) $\sin(x+y)=0$ ***A1***

$\Rightarrow x+y=0$ ***(M1)***

(the equation of L_1 is) $y=-x$ ***A1***

[3 marks]

(b) $x+y=\pi$ **OR** $y=-x+\pi$ ***(M1)A1***

[2 marks]

Total [5 marks]

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix}$$

16. (a) $x_n = x_{n-1} + h\,f(x_{n-1}, t_{n-1})$

$h = 0.1,\ f(x, t) = x\cos t\,(\mathrm{e}^{-\sin t})$

$x_n = x_{n-1} + 0.1x_{n-1}\cos t_{n-1}(\mathrm{e}^{-\sin t_{n-1}})$ ***(M1)***

Note: Award ***M1*** for a valid start.

n	t_n	x_n
0	0	0.367879
1	0.1	0.404667
2	0.2	0.441106
3	0.3	0.476548

(A1)

Note: Award ***A1*** for a correct x value when $n = 1$.

$x(0.3) \approx 0.477\ \ (0.476548...)$ ***A1***

[3 marks]

(b) **EITHER**

$\int \frac{\mathrm{d}x}{x} = \int \cos t(\mathrm{e}^{-\sin t})\ \mathrm{d}t\ \ (+c)$ ***M1***

$\ln x = -\mathrm{e}^{-\sin t} + c$ ***A1***

$t = 0,\ x = \frac{1}{\mathrm{e}} \Rightarrow c = 0$ ***M1***

$x = \mathrm{e}^{(-\mathrm{e}^{-\sin t})}$

$x(0.3) \approx 0.475140...$ ***A1***

OR

$\int_{1/\mathrm{e}}^{x} \frac{\mathrm{d}u}{u} = \int_{0}^{0.3} \cos t(\mathrm{e}^{-\sin t})\,\mathrm{d}t$ ***M1***

$[\ln u]_{1/\mathrm{e}}^{x} = 0.255855...$ (from GDC) ***A1***

$\ln x + 1 = 0.255855...$

$\ln x = -0.744145...$ ***A1***

$x = \mathrm{e}^{-0.744145} = 0.475140....$ ***A1***

THEN

percentage error $= \left| \frac{0.476548... - 0.475140...}{0.475140...} \right| \times 100 = 0.296\%\ \ (2.96192...)$ ***A1***

Note: If candidates do not attempt to find c, they may score ***M1A0M0A1A1***.

[5 marks]

Total [8 marks]

2. (a) (i) evidence of power rule (at least one correct term seen) ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -0.3x^2 + 1.6x$ ***A1***

(ii) $-0.3x^2 + 1.6x = 0$ ***M1***

$x = 5.33 \left(5.33333..., \frac{16}{3} \right)$ ***A1***

$y = -0.1 \times 5.33333...^3 + 0.8 \times 5.33333...^2$ ***(M1)***

Note: Award ***M1*** for substituting their zero for $\frac{\mathrm{d}y}{\mathrm{d}x}$ $(5.333\ldots)$ into y.

$7.59 \text{ m } (7.58519...)$ ***A1***

Note: Award ***M0A0M0A0*** for an unsupported 7.59.
Award at most ***M0A0M1A0*** if only the last two lines in the solution are seen.
Award at most ***M1A0M1A1*** if their $x = 5.33$ is not seen.

[6 marks]

(b) One correct substitution seen ***(M1)***

(i) 6.4 m ***A1***

(ii) 7.2 m ***A1***

[3 marks]

continued…

Question 2 continued

(c) $A = \frac{1}{2} \times 2\big((2.4+0)+2(6.4+7.2)\big)$ ***(A1)(M1)***

Note: Award ***A1*** for $h = 2$ seen. Award ***M1*** for correct substitution into the trapezoidal rule (the zero can be omitted in working).

$= 29.6 \text{ m}^2$ ***A1***

[3 marks]

(d) (i) $A = \int_2^8 -0.1x^3 + 0.8x^2 \text{ d}x$ **OR** $A = \int_2^8 y \text{ d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location. Award at most ***A0A1*** if $\text{d}x$ is omitted.

(ii) $A = 32.4 \text{ m}^2$ ***A2***

Note: As per the marking instructions, ***FT*** from their integral in part (d)(i). Award at most ***A1FTA0*** if their area is >48, this is outside the constraints of the question (a $6\text{x}8$ rectangle).

[4 marks]

Total [16 marks]

6. (a) use of product rule ***(M1)***

$$\begin{pmatrix}\dot{x}\\ \dot{y}\end{pmatrix}=\begin{pmatrix}ab\mathrm{e}^{bt}\cos t-a\mathrm{e}^{bt}\sin t\\ ab\mathrm{e}^{bt}\sin t+a\mathrm{e}^{bt}\cos t\end{pmatrix}$$ ***A1A1***

[3 marks]

(b) $|\boldsymbol{v}|^2=\dot{x}^2+\dot{y}^2=\left[ab\mathrm{e}^{bt}\cos t-a\mathrm{e}^{bt}\sin t\right]^2+\left[ab\mathrm{e}^{bt}\sin t+a\mathrm{e}^{bt}\cos t\right]^2$ ***M1***

Note: It is more likely that an expression for $|\boldsymbol{v}|$ is seen.
$\sqrt{\dot{x}^2+\dot{y}^2}$ is not sufficient to award the ***M1***, their part (a) must be substituted.

$=\left[a^2\sin^2 t-2a^2b\sin t\cos t+a^2b^2\cos^2 t+a^2\cos^2 t+2a^2b\sin t\cos t+a^2b^2\sin^2 t\right]\mathrm{e}^{2bt}$ ***A1***

use of $\sin^2 t+\cos^2 t=1$ within a factorized expression that leads to the final answer ***M1***

$=a^2\left(b^2+1\right)\mathrm{e}^{2bt}$ ***A1***

magnitude of velocity is $a\mathrm{e}^{bt}\sqrt{\left(1+b^2\right)}$ ***AG***

[4 marks]

(c) when $t=0$, $a\mathrm{e}^{bt}\cos t=5$

$a=5$ ***A1***

$ab\mathrm{e}^{bt}\cos t-a\mathrm{e}^{bt}\sin t=-3.5$ ***(M1)***

$b=-0.7$ ***A1***

Note: Use of $a\mathrm{e}^{bt}\sqrt{\left(1+b^2\right)}$ result from part (b) is an alternative approach.

[3 marks]

(d) $5\mathrm{e}^{-0.7\times2}\sqrt{\left(1+(-0.7)^2\right)}$ ***(M1)***

$1.51\ \ (1.50504\ldots)$ ***A1***

[2 marks]

(e) $\dot{x}=0$ ***(M1)***

$a\mathrm{e}^{bt}(b\cos t-\sin t)=0$

$\tan t=b$

$t=2.53\ \ (2.53086...)$ ***(A1)***

correct substitution of their t to find x or y ***(M1)***

$x=-0.697\ \ (-0.696591...)$ **and** $y=0.488\ \ (0.487614...)$ ***(A1)***

use of Pythagoras / distance formula ***(M1)***

$\mathrm{OP}=0.850\ \mathrm{m}\ \ (0.850297\ldots)$ ***A1***

[6 marks]

Total [18 marks]

7. (a) $\int \frac{1}{x}\mathrm{d}x = \int 2\mathrm{d}t$ ***(M1)***

$\ln x = 2t + c$
$x = A\mathrm{e}^{2t}$ ***(A1)***

$x(0) = 100 \Rightarrow A = 100$ ***(M1)***

$x = 100\mathrm{e}^{2t}$ ***(A1)***

$x(1) = 739$ ***A1***

Note: Accept 738 for the final ***A1***.

[5 marks]

(b) $t_{n+1} = t_n + 0.25$ ***(A1)***

Note: This may be inferred from a correct t column, where this is seen.

$x_{n+1} = x_n + 0.25\,x_n\left(2 - 0.01y_n\right)$ ***(A1)***

$y_{n+1} = y_n + 0.25\,y_n\left(0.0002x_n - 0.8\right)$ ***(A1)***

t	x	y
0	1000	100
0.25	1250	85
0.5	1609	73
0.75	2119	65
1	2836	58

(A1)

Note: Award ***A1*** for whole line correct when $t = 0.5$ or $t = 0.75$. The t column may be omitted and implied by the correct x and y values. The formulas are implied by the correct x and y columns.

(i) 2840 $(2836$ **OR** $2837)$ ***A1***

(ii) 58 **OR** 59 ***A1***

[6 marks]

(c) (i) both populations are increasing ***A1***

(ii) rabbits are decreasing and foxes are increasing ***A1A1***

[3 marks]

(d) setting at least one DE to zero ***(M1)***
$x = 4000,\ y = 200$ ***A1A1***

[3 marks]

Total [17 marks]

3. (a) evidence of splitting diagram into equilateral triangles ***M1***

area $= 6\left(\frac{1}{2}x^2 \sin 60^\circ\right)$ ***A1***

$= \frac{3\sqrt{3}x^2}{2}$ ***AG***

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(b) total surface area of prism $1200 = 2\left(3x^2 \frac{\sqrt{3}}{2}\right) + 6xh$ ***M1A1***

Note: Award ***M1*** for expressing total surface areas as a sum of areas of rectangles and hexagon(s), and ***A1*** for a correctly substituted formula, equated to 1200.

[5 marks]

$h = \frac{400 - \sqrt{3}x^2}{2x}$ ***A1***

volume of prism $= \frac{3\sqrt{3}}{2}x^2 h$ ***(A1)***

$= \frac{3\sqrt{3}}{2}x^2\left(\frac{400 - \sqrt{3}x^2}{2x}\right)$ ***A1***

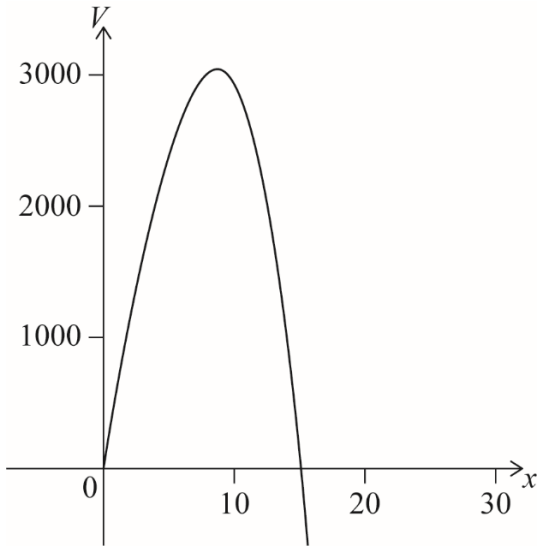
$= 300\sqrt{3}x - \frac{9}{4}x^3$ ***(AG)***

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

continued…

Question 3 continued

(c)



A1A1

Note: Award ***A1*** for correct shape, ***A1*** for roots in correct place with some indication of scale (indicated by a labelled point).

[2 marks]

(d) $\frac{\mathrm{d}V}{\mathrm{d}x} = 300\sqrt{3} - \frac{27}{4}x^2$ ***A1A1***

Note: Award ***A1*** for a correct term.

[2 marks]

(e) from the graph of V or $\frac{\mathrm{d}V}{\mathrm{d}x}$ **OR** solving $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ ***(M1)***

$x = 8.77 \ \ (8.77382...)$ ***A1***

[2 marks]

(f) from the graph of V **OR** substituting their value for x into V ***(M1)***

$V_{\max} = 3040 \text{ cm}^3 \ \ (3039.34...)$ ***A1***

[2 marks]

Total [15 marks]

6. (a) solving $v = 0$ ***M1***

$t = 2,\ t = 6$ ***A1***

[2 marks]

(b) use of power rule ***(M1)***

$\frac{\mathrm{d}v}{\mathrm{d}t} = -4t + 16$ ***(A1)***

$(t = 6)$

$\Rightarrow a = -8$ ***(A1)***

magnitude $= 8\ \mathrm{m\,s^{-2}}$ ***A1***

[4 marks]

(c) using a sketch graph of v ***(M1)***

$24\ \mathrm{m\,s^{-1}}$ ***A1***

[2 marks]

(d) **METHOD ONE**

$x = \int v\ \mathrm{d}t$

attempt at integration of v ***(M1)***

$-\frac{2t^3}{3} + 8t^2 - 24t\ (+c)$ ***A1***

attempt to find c (use of $t = 0,\ x = 0$) ***(M1)***

$c = 0$ ***A1***

$\left(x = -\frac{2t^3}{3} + 8t^2 - 24t\right)$

METHOD TWO

$x = \int_0^t v\ \mathrm{d}t$

attempt at integration of v ***(M1)***

$\left[-\frac{2t^3}{3} + 8t^2 - 24t\right]_0^t$ ***A1***

attempt to substituted limits into their integral ***(M1)***

$x = -\frac{2t^3}{3} + 8t^2 - 24t$ ***A1***

[4 marks]

(e) $\int_0^4 |v|\ \mathrm{d}t$ ***(M1)(A1)***

Note: Award ***M1*** for using the absolute value of v, or separating into two integrals, ***A1*** for the correct expression.

$= 32\ \mathrm{m}$ ***A1***

[3 marks]

Total [15 marks]

7. (a) $\begin{vmatrix} -4-\lambda & 0 \\ 3 & -2-\lambda \end{vmatrix} = 0$ ***(M1)***

$(-4-\lambda)(-2-\lambda) = 0$ ***(A1)***

$\lambda = -4$ **OR** $\lambda = -2$ ***A1***

$\lambda = -4$

$\begin{pmatrix} -4 & 0 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -4x \\ -4y \end{pmatrix}$ ***(M1)***

Note: This ***M1*** can be awarded for attempting to find either eigenvector.

$3x - 2y = -4y$

$3x = -2y$

possible eigenvector is $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$ (or any real multiple) ***A1***

$\lambda = -2$

$\begin{pmatrix} -4 & 0 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2x \\ -2y \end{pmatrix}$

$x = 0,\ y = 1$

possible eigenvector is $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ (or any real multiple) ***A1***

[6 marks]

(b) $\begin{pmatrix} x \\ y \end{pmatrix} = A\mathrm{e}^{-4t}\begin{pmatrix} -2 \\ 3 \end{pmatrix} + B\mathrm{e}^{-2t}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ***(M1)A1***

Note: Award ***M1A1*** for $x = -2A\mathrm{e}^{-4t},\ y = 3A\mathrm{e}^{-4t} + B\mathrm{e}^{-2t}$, ***M1A0*** if LHS is missing or incorrect.

[2 marks]

(c) two (distinct) real negative eigenvalues ***R1***

(or equivalent (*eg* both $\mathrm{e}^{-4t} \to 0, \mathrm{e}^{-2t} \to 0$ as $t \to \infty$))

$\Rightarrow$ stable equilibrium point ***A1***

Note: Do not award ***R0A1***.

[2 marks]

continued…

Question 7 continued

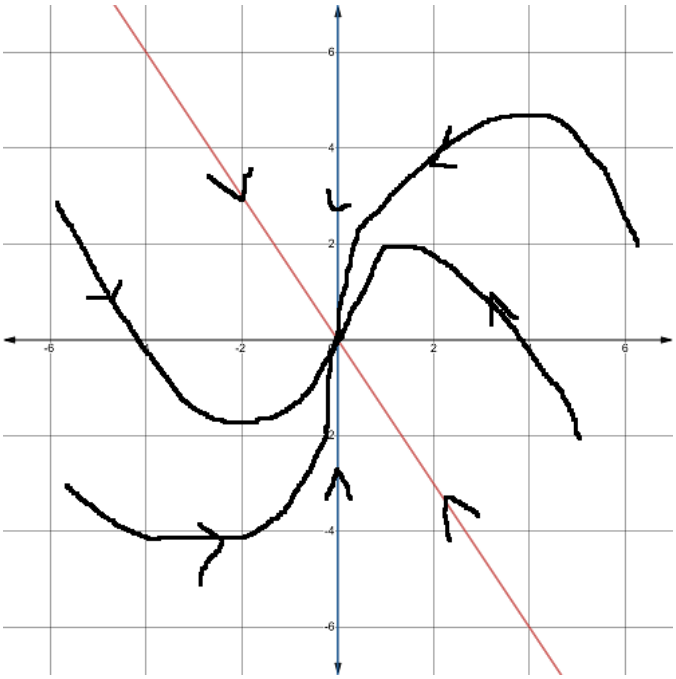
(d) $\frac{dy}{dx} = \frac{3x-2y}{-4x}$ ***(M1)***

(i) $(4, 0) \Rightarrow \frac{dy}{dx} = -\frac{3}{4}$ ***A1***

(ii) $(-4, 0) \Rightarrow \frac{dy}{dx} = -\frac{3}{4}$ ***A1***

[3 marks]

(e)



A1A1A1A1

Note: Award ***A1*** for a phase plane, with correct axes (condone omission of labels) and at least three non-overlapping trajectories. Award ***A1*** for all trajectories leading to a stable node at $(0, 0)$. Award ***A1*** for showing gradient is negative at $x = 4$ and -4. Award ***A1*** for both eigenvectors on diagram.

[4 marks]

Total [17 marks]

2. (a) (i) population growth rate / birth rate of sharks (due to eating mackerel) ***A1***

[1 mark]

(ii) (net) death rate of sharks ***A1***

[1 mark]

(b) (i) $\gamma MS - \delta S = 0$ ***A1***

since $S \neq 0$ ***R1***

Note: Accept $S > 0$.

getting to given answer without further error by either cancelling or factorizing ***A1***

$M = \frac{\delta}{\gamma}$ ***AG***

[3 marks]

(ii) $\frac{\mathrm{d}M}{\mathrm{d}t} = 0$

$\alpha M - \beta MS = 0$ ***(M1)***

(since $M \neq 0$) $S = \frac{\alpha}{\beta}$ ***A1***

[2 marks]

(c) (i) $M_{eq} = \frac{\delta}{\gamma} \Rightarrow \frac{\delta}{\frac{1}{2}\gamma} = 2M_{eq}$ ***M1***

Note: Accept equivalent in words.

Doubles ***A1***

Note: Do not accept "increases".

[2 marks]

(ii) $M_{eq} = \frac{\delta}{\gamma}$ is not dependent on α ***R1***

Note: Award ***R0*** for any contextual argument.

no change ***A1***

Note: Do not award ***R0A1***.

[2 marks]

continued…

Question 2 continued

(d) (i) $\frac{\mathrm{d}M}{\mathrm{d}t} = \alpha M$ ***A1***

[1 mark]

(ii) $\int \frac{1}{M}\mathrm{d}M = \int \alpha \,\mathrm{d}t$ ***M1***

Note: Award ***M1*** is for an attempt to separate variables. This means getting to the point $\int f(M)\,\mathrm{d}M = \int g(t)\,\mathrm{d}t$ where the integral can be seen or implied by further work.

$\ln|M| = \alpha t + c$ ***A1***

Note: Accept $\ln M$. Condone missing constant of integration for this mark.

$M = k\mathrm{e}^{\alpha t}$
when $t = 0$, $M_0 = k$ ***M1***

Note: Award ***M1*** for a clear attempt at using initial conditions to find a constant of integration. Only possible if the constant of integration exists. $t = 0$ or "initially" or similar must be seen. Substitution may appear earlier, following the integration.

initial conditions and all other manipulations correct and clearly communicated to get to the final answer ***A1***
$M = M_0\mathrm{e}^{\alpha t}$ ***AG***

[4 marks]

(iii) $M = 3M_0$ seen anywhere ***(A1)***
substituting $t = 2$, $M = 3M_0$ into equation $M = M_0\mathrm{e}^{\alpha t}$ ***(M1)***
$3M_0 = M_0\mathrm{e}^{2\alpha}$
$\alpha = \frac{1}{2}\ln 3$ **OR** $0.549306...$ ***A1***

Note: The ***A1*** requires either the exact answer or an answer to at least 4 sf.

≈ 0.549 ***AG***

[3 marks]

continued…

Question 2 continued

(e) (i) an attempt to set up one recursive equation ***(M1)***

Note: Must include **two** given parameters **and** M_n and S_n **and** M_{n+1} or S_{n+1} for the ***(M1)*** to be awarded.

$M_{n+1} = M_n + 0.1(0.549M_n - 0.236M_nS_n)$ ***A1***

$S_{n+1} = S_n + 0.1(0.244M_nS_n - 1.39S_n)$ ***A1***

[3 marks]

(ii) **EITHER**

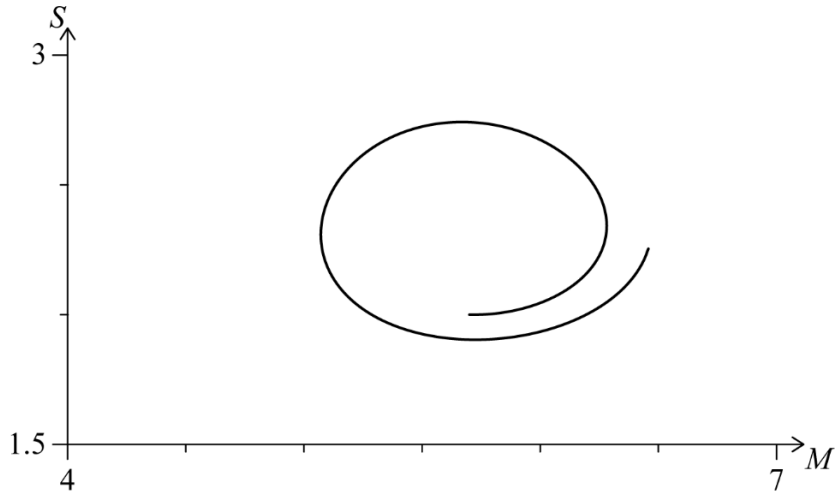
$6.12 \ (6.11609...)$ ***A2***

OR

$6120 \ (6116.09...)$ (mackerel per km^3) ***A2***

[2 marks]

(f) (i)



spiral or closed loop shape ***A1***

approximately 1.25 rotations (can only be awarded if a spiral) ***A1***

correct shape, in approximately correct position (centred at approx. $(5.5, 2.5)$) ***A1***

Note: Award ***A0A0A0*** for any plot of S or M against t.

[3 marks]

continued…

Question 2 continued

(ii) **EITHER**

approximate minimum is $(5.07223\ldots)\ 5.07$ (which is greater than 5) ***A1***

OR

the line $M = 5$ clearly labelled on their phase portrait ***A1***

THEN

(the density will not fall below 5000) hence sufficient for sustainable fishing

A1

Note: Do not award ***A0A1***. Only if the minimum point is labelled on the sketch then a statement here that *"the mackerel population is always above 5000*" would be sufficient. Accept the value 5.07 seen within a table of values.

[2 marks]

(iii) Any two from: ***A1A1***

- Current values / parameters are only an estimate,
- The Euler method is only an approximate method / choosing $h = 0.1$ might be too large.
- There might be random variation / the model has no stochastic component
- Conditions / parameters might change over the nine years,
- A discrete system is being approximated by a continuous system,

Allow any other sensible critique.

If a candidate identifies factors which the model ignores, award ***A1*** per factor identified. These factors could include:

- Other predators
- Seasonality
- Temperature
- The effect of fishing
- Environmental catastrophe
- Migration

Note: Do not allow:
"You cannot have 5.07 mackerel".
It is only a model (as this is too vague).
Some factors have been ignored (without specifically identifying the factors).
Values do not always follow the equation / model. (as this is too vague)

[2 marks]

[Total 31 marks]

2. (a) attempt at using trapezoidal rule formula ***(M1)***

$$\frac{1}{2}\left(\frac{2-0}{5}\right)\left(30+50+2(50+60+40+20)\right)$$ ***A1***

$(\text{total carbon} =)\ 84$ tonnes ***A1***

[3 marks]

(b) $\left|\frac{84-72}{72}\right| \times 100\%$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of final answer in part (a) into percentage error formula.

$= 16.7\%\ \ (16.6666...\%)$ ***A1***

[2 marks]

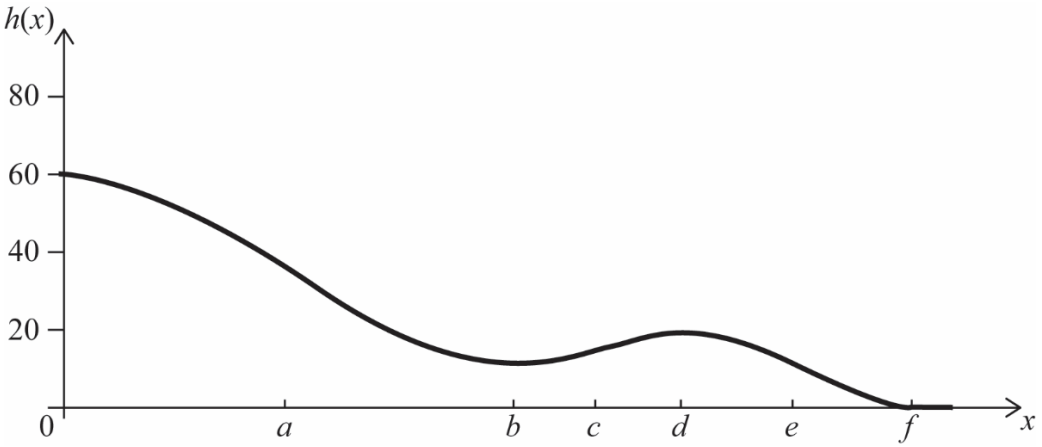
Total: [5 marks]

8. (a) (i) a ***A1***

(ii) the hill is at its steepest / largest slope of hill ***A1***

[2 marks]

(b)



A1A1A1

Note:	Award ***(A1)*** for decreasing function from 0 to b and d to f and increasing from b to d; ***(A1)*** for minimum at b and max at d; ***(A1)*** for starting at height of 60 and finishing at a height of 0 at f. If reasonable curvature not evident on graph (i.e. only straight lines used) award ***A1A0A1***.

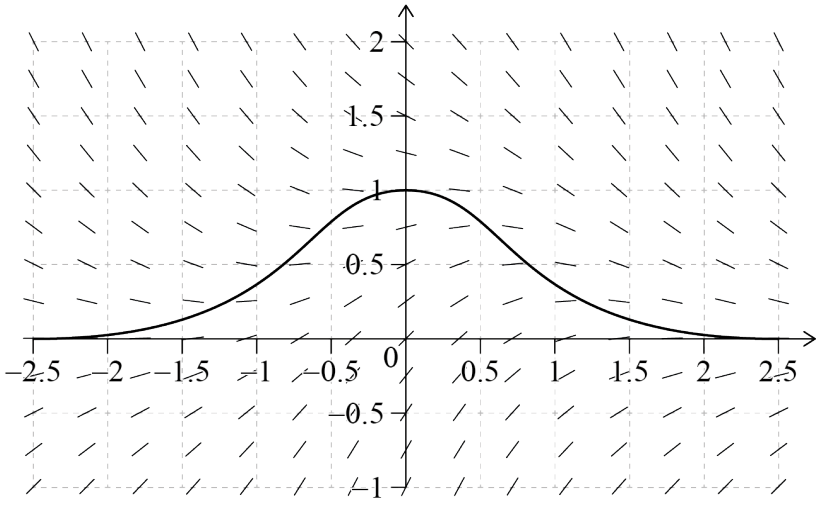
[3 marks]

Total: [5 marks]

13. (a) $\left(\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^0 - 1\right) = 0$ ***A1***

[1 mark]

(b)



gradient $= 0$ at $(0, 1)$ ***A1***
correct shape ***A1***

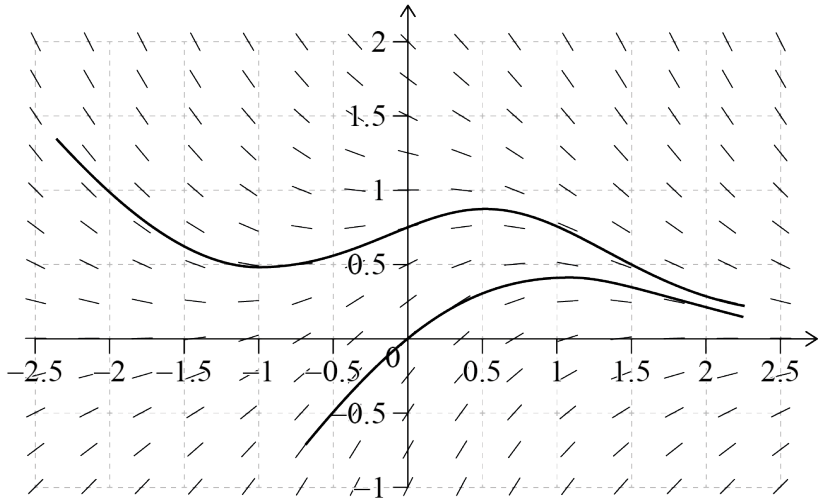
Note:	Award second ***A1*** for horizontal asymptote of $y = 0$, and general symmetry about the y-axis.

[2 marks]

continued…

Question 13 continued

(c)



(i) positive gradient at origin ***A1***
correct shape ***A1***

Note: Award second ***A1*** for a single maximum in 1st quadrant and tending toward an asymptote.

(ii) positive gradient at $(0, 0.75)$ ***A1***
correct shape ***A1***

Note: Award second ***A1*** for a single minimum in 2nd quadrant, single maximum in 1st quadrant and tending toward an asymptote.

[4 marks]
Total: [7 marks]

15. (a) $15 = 3 + 4r + 2r\theta$ ***M1***

$12 = 2r(2 + \theta)$ ***A1***

Note: Award ***A1*** for any reasonable working leading to expected result e,g, factorizing r.

$r = \dfrac{6}{2+\theta}$ ***AG***

[2 marks]

(b) (i) attempt to use sector area to find volume ***(M1)***

volume $= \dfrac{1}{2} r^2 \theta \times 1$

$= \dfrac{1}{2} \times \dfrac{36}{(2+\theta)^2} \times \theta \quad \left(= \dfrac{18\theta}{(2+\theta)^2} \right)$ ***A1***

(ii) $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = \dfrac{(2+\theta)^2 \times 18 - 36\theta(2+\theta)}{(2+\theta)^4}$ ***M1A1A1***

$\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = \dfrac{36 - 18\theta}{(2+\theta)^3}$

(iii) $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = \dfrac{36 - 18\theta}{(2+\theta)^3} = 0$ ***M1***

Note: Award this ***M1*** for simplified version equated to zero. The simplified version may have been seen in part (b)(ii).

$\theta = 2$ ***A1***

[7 marks]

Total: [9 marks]

17. (a) attempt to use $V = \pi\int_a^b x^2\,\mathrm{d}y$ ***(M1)***

$x = \mathrm{e}^{\frac{y}{6}}$ or any reasonable attempt to find x in terms of y ***(M1)***

$V = \pi\int_0^h \mathrm{e}^{\frac{y}{3}}\,\mathrm{d}y$ ***A1***

Note: Correct limits must be seen for the ***A1*** to be awarded.

$= \pi\left[3\mathrm{e}^{\frac{y}{3}}\right]_0^h$ ***(A1)***

Note: Condone the absence of limits for this ***A1*** mark.

$= 3\pi\left[\mathrm{e}^{\frac{h}{3}} - \mathrm{e}^0\right]$ ***A1***

$= 3\pi\left[\mathrm{e}^{\frac{h}{3}} - 1\right]$ ***AG***

Note: If the variable used in the integral is x instead of y (i.e. $V = \pi\int_0^h \mathrm{e}^{\frac{x}{3}}\mathrm{d}x$) and the candidate has not stated that they are interchanging x and y then award at most ***M1M1A0A1A1AG***.

[5 marks]

(b) maximum volume when $h = 9\ \mathrm{cm}$ ***(M1)***

max volume $= 180\ \mathrm{cm}^3$ ***A1***

[2 marks]
Total: [7 marks]

6. (a) $y = \dot{x} \Rightarrow \dot{y} = \ddot{x}$ ***A1***

$\dot{y} + 3(y) + 1.25x = 0$ ***R1***

Note: If no explicit reference is made to $\dot{y} = \ddot{x}$, or equivalent, award ***A0R1*** if second line is seen.
If $\frac{\mathrm{d}y}{\mathrm{d}x}$ used instead of $\frac{\mathrm{d}y}{\mathrm{d}t}$, award ***A0R0***.

$\dot{y} = -3y - 1.25x$ ***AG***

[2 marks]

(b) $\boldsymbol{A} = \begin{pmatrix} 0 & 1 \\ -1.25 & -3 \end{pmatrix}$ ***A1***

[1 mark]

(c) (i) $\begin{vmatrix} -\lambda & 1 \\ -1.25 & -3-\lambda \end{vmatrix} = 0$ ***(M1)***

$\lambda(\lambda + 3) + 1.25 = 0$ ***(A1)***

$\lambda = -2.5 \;\; ; \;\; \lambda = -0.5$ ***A1***

continued…

Question 6 continued

(ii) $\begin{pmatrix} 2.5 & 1 \\ -1.25 & -0.5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***(M1)***

$2.5a + b = 0$

$\boldsymbol{v}_1 = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ ***A1***

$\begin{pmatrix} 0.5 & 1 \\ -1.25 & -2.5 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$0.5a + b = 0$

$\boldsymbol{v}_2 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$ ***A1***

Note: Award ***M1*** for a valid attempt to find either eigenvector. Accept equivalent forms of the eigenvectors.
Do not award ***FT*** for eigenvectors that do not satisy both rows of the matrix.

[6 marks]

(d) $\begin{pmatrix} x \\ y \end{pmatrix} = A\mathrm{e}^{-2.5t}\begin{pmatrix} -2 \\ 5 \end{pmatrix} + B\mathrm{e}^{-0.5t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ ***M1A1***

$t = 0 \Rightarrow x = 8,\ \dot{x} = y = 0$ ***(M1)***

$-2A - 2B = 8$
$5A + B = 0$ ***(M1)***

$A = 1;\ B = -5$ ***A1***

$x = -2\mathrm{e}^{-2.5t} + 10\mathrm{e}^{-0.5t}$ ***A1***

Note: Do not award the final ***A1*** if the answer is given in the form $\begin{pmatrix} x \\ y \end{pmatrix} = A\mathrm{e}^{-2.5t}\begin{pmatrix} -2 \\ 5 \end{pmatrix} + B\mathrm{e}^{-0.5t}\begin{pmatrix} -2 \\ 1 \end{pmatrix}$.

[6 marks]
Total: [15 marks]

2. (a) (i) $h(t) = -0.134t + 3.1$ ***A1A1***

Note: Award ***A1*** for an equation in h and t and ***A1*** for the coefficient -0.134 and constant 3.1.

(ii) **EITHER**
the rate of change of height (of water in metres per minute) ***A1***

Note: Accept "rate of decrease" or "rate of increase" in place of "rate of change".

OR
the (average) amount that the height (of the water) decreases each minute ***A1***

(iii) **EITHER**
unreliable to use h on t equation to estimate t ***A1***

OR
unreliable to extrapolate from original data ***A1***

OR
rate of change (of height) might not remain constant (as the water drains out) ***A1***

(b) (i) $h(t) = 0.002t^2 - 0.174t + 3.2$ ***A1***

(ii) $0.002t^2 - 0.174t + 3.2 = 0$ ***(M1)***
26.4 $(26.4046\ldots)$ ***A1***

(iii) **EITHER**
$(0 \leq)\ t \leq 26.4$ $(t \leq 26.4046...)$ ***A1***

OR
$(0 \leq)\ t \leq 20$ (due to range of original data / interpolation) ***A1***

continued…

Question 2 continued

(c) $V = \pi(1)^2 h$ ***(A1)***

EITHER

$\frac{\mathrm{d}V}{\mathrm{dt}} = \pi\frac{\mathrm{d}h}{\mathrm{dt}}$ ***M1***

OR

attempt to use chain rule ***M1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$

THEN

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{\pi} \times -\pi R^2\sqrt{70560h}$ ***A1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = -R^2\sqrt{70560h}$ ***AG***

(d) attempt to separate variables ***M1***

$\int \frac{1}{\sqrt{70560\,h}}\,\mathrm{d}h = \int -R^2\mathrm{d}t$ ***A1***

$\frac{2\sqrt{h}}{\sqrt{70560}} = -R^2t + c$ ***A1A1***

Note: Award ***A1*** for each correct side of the equation.

$\sqrt{h} = \frac{\sqrt{70560}}{2}\left(c - R^2t\right)$ ***A1***

Note: Award the final ***A1*** for any correct intermediate step that clearly leads to the given equation.

$h = 17640\left(c - R^2t\right)^2$ ***AG***

(e) $t = 0 \Rightarrow 3.2 = 17640c^2$ ***(M1)***

$c = 0.0134687...$ ***(A1)***

substituting $h = 0$ and their non-zero value of c ***(M1)***

$T = \frac{c}{R^2} = \frac{0.0134687...}{0.023^2}$

$= 25.5$ (minutes) $(25.4606\ldots)$ ***A1***

(f) $h = 0 \Rightarrow c = R^2t$

$c = 0.023^2 \times 15 \ (= 0.007935)$ ***(A1)***

$t = 0 \Rightarrow h = 17640(0.023^2 \times 15)^2$ ***(M1)***

$h = 1.11$ (metres) $(1.11068\ldots)$ ***A1***

continued…

Question 2 continued

(g) (i) let h be the height of water in the highest container
from parts (d) and (e) we get

$\frac{\mathrm{d}h}{\mathrm{d}t} = -35280R^2\left(0.0134687... - R^2t\right)$ ***(M1)(A1)***

so $\frac{\mathrm{d}H}{\mathrm{d}t} = 35280R^2\left(0.0135 - R^2t\right) - R^2\sqrt{70560\,H}$ ***M1A1***

$\left(\frac{\mathrm{d}H}{\mathrm{d}t} = 18.6631...\left(0.0134687... - 0.000529t\right) - 0.000529\sqrt{70560\,H}\right)$

$\left(\frac{\mathrm{d}H}{\mathrm{d}t} = 0.251367... - 0.0987279... - 0.140518...\sqrt{H}\right)$

$\frac{\mathrm{d}H}{\mathrm{d}t} \approx 0.2514 - 0.009873t - 0.1405\sqrt{H}$ ***AG***

(ii) evidence of using Euler's method correctly
e.g. $y_1 = 1.05545...$ ***(A1)***

maximum value of $H = 1.45$ (metres) (at 8.5 minutes) ***A2***
($1.44678\ldots$ metres)

7. (a) $E = 5(2\sin t)^2 \left(= 20\sin^2 t\right)$ ***A1***

[1mark]

(b) $\dfrac{\mathrm{d}E}{\mathrm{d}t} = 40\sin t\cos t$ ***(M1)A1***

[2 marks]

(c) $t = 0.126$ ***(M1)A1***

[2 marks]

Total [5 marks]

13. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16-20}{24-20}$ ***M1***

$= -1$ ***A1***

[2 marks]

(b) asymptote of trajectory along $\boldsymbol{r} = k\begin{pmatrix}2\\1\end{pmatrix}$ ***M1A1***

Note: Award ***M1A0*** if asymptote along $\begin{pmatrix}1\\2\end{pmatrix}$.

trajectory begins at $(8, 10)$ with negative gradient ***A1A1***

[4 marks]

Total [6 marks]

14. (a) (i) $a = 33$ ***A1***

(ii) $\frac{1}{\sqrt[3]{0.08}} = 2.32$ ***M1A1***

[3 marks]

(b) volume within outer dome

$\frac{2}{3}\pi \times 16^3 + \pi \times 16^2 \times 17 = 22\,250.85$ ***M1A1***

volume within inner dome

$\pi\int_0^{33}\left(\frac{33-y}{0.08}\right)^{\frac{2}{3}}\mathrm{d}y = 3446.92$ ***M1A1***

volume between $= 22\,250.85 - 3446.92 = 18\,803.93\,\mathrm{m}^3$ ***A1***

[5 marks]

Total [8 marks]

1. (a) $2(8\times4+3\times4+3\times8)$ ***M1***

$=136\ (\text{cm}^2)$ ***A1***

[2 marks]

(b) $\sqrt{8^2+4^2+3^2}$ ***M1***

$(\text{AG}=)\ 9.43\ (\text{cm})\ (9.4339\ldots, \sqrt{89})$ ***A1***

[2 marks]

(c) $-2x+220=0$ ***M1***

$x=110$ ***A1***

$110\,000$ (boxes) ***A1***

[3 marks]

(d) $P(x)=\int -2x+220\,\text{d}x$ ***M1***

Note: Award ***M1*** for evidence of integration.

$P(x)=-x^2+220x+c$ ***A1A1***

Note: Award ***A1*** for either $-x^2$ or $220x$ award ***A1*** for both correct terms and constant of integration.

$1700=-(20)^2+220(20)+c$ ***M1***

$c=-2300$

$P(x)=-x^2+220x-2300$ ***A1***

[5 marks]

(e) $-x^2+220x-2300=0$ ***M1***

$x=11.005$ ***A1***

$11\,006$ (boxes) ***A1***

Note: Award ***M1*** for their $P(x)=0$, award ***A1*** for their correct solution to x. Award the final ***A1*** for expressing their solution to the minimum number of boxes. Do not accept $11\,005$, the nearest integer, nor $11\,000$, the answer expressed to 3 significant figures, as these will not satisfy the demand of the question.

[3 marks]

Total [15 marks]

7. (a) $\frac{\mathrm{d}v}{\mathrm{d}t} = 9.81 - 0.9v$ ***M1***

$\int \frac{1}{9.81 - 0.9v}\,\mathrm{d}v = \int 1\,\mathrm{d}t$ ***M1***

$-\frac{1}{0.9}\ln(9.81 - 0.9v) = t + c$ ***A1***

$9.81 - 0.9v = A\mathrm{e}^{-0.9t}$ ***A1***

$v = \frac{9.81 - A\mathrm{e}^{-0.9t}}{0.9}$ ***A1***

when $t = 0$, $v = 0$ hence $A = 9.81$ ***A1***

$v = \frac{9.81(1 - \mathrm{e}^{-0.9t})}{0.9}$

$v = 10.9(1 - \mathrm{e}^{-0.9t})$ ***A1***

[7 marks]

(b) **either** let t tend to infinity, or $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ ***(M1)***

$v = 10.9$ ***A1***

[2 marks]

(c) $\frac{\mathrm{d}x}{\mathrm{d}t} = y$ ***M1***

$\frac{\mathrm{d}y}{\mathrm{d}t} = 9.81 - 0.9y^2$ ***A1***

[2 marks]

(d) $x_{n+1} = x_n + 0.2y_n$, $y_{n+1} = y_n + 0.2(9.81 - 0.9(y_n)^2)$ ***(M1)(A1)***

$x = 1.04$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 3.31$ ***(M1)A1***

[4 marks]

(e) 3.3015 ***A1***

[1 mark]

(f) $0 = 9.81 - 0.9(v)^2$ ***M1***

$\Rightarrow v = \sqrt{\frac{9.81}{0.9}} = 3.301511\ldots (= 3.30)$ ***A1***

[2 marks]

continued…

Question 7 continued

(g) the model found the terminal velocity very accurately, so good approximation ***R1***

intermediate values had object exceeding terminal velocity so not good approximation ***R1***

[2 marks]

Total [20 marks]

2. (a) (i) 2000 ***(M1)A1***

(ii) because the value of $\frac{\mathrm{d}x}{\mathrm{d}t}$ is positive (for $x > 0$) ***R1***

[3 marks]

(b) (i) substitute $x = 800,\ y = 600$ into both equations ***M1***

both equations equal 0 ***A1***

hence an equilibrium point ***AG***

(ii) $x = 0,\ y = 0$ ***A1***

$x = 2000,\ y = 0,\ x = 0,\ y = 3000$ ***M1A1A1***

Note: Award ***M1*** for an attempt at solving the system provided some values of x and y are found.

[6 marks]

(c) (i) $\int \frac{1}{x}\mathrm{d}x = \int 2\,\mathrm{d}t$ ***M1***

$\ln x = 2t + c$ ***A1A1***

Note: Award ***A1*** for RHS, ***A1*** for LHS.

$x = \mathrm{e}^{c}\mathrm{e}^{2t}$ ***M1***

$x = A\mathrm{e}^{2t}\ \left(\text{where } A = \mathrm{e}^{c}\right)$ ***AG***

(ii) $y = B\mathrm{e}^{3t}$ ***A1***

Note: Allow any letter for the constant term, including A.

(iii) $x = 15,\ y = 18$ ***(M1)A1***

[7 marks]

continued…

Question 2 continued

(d) (i) $x_{n+1} = x_n + 0.2\frac{x_n}{1000}(2000 - x_n - 2y_n)$

$y_{n+1} = y_n + 0.2\frac{y_n}{1000}(3000 - 3x_n - y_n)$ ***M1A1***

Note: Accept equivalent forms.

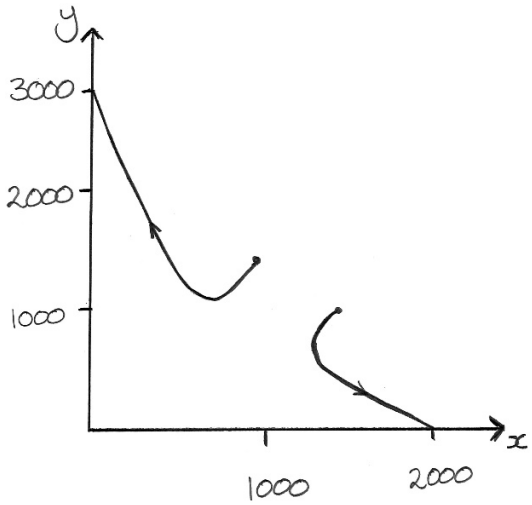
(ii) $x = 319,\ y = 617$ ***(M1)A1A1***

(iii) number of brown squirrels go down to 0,
black squirrels to a population of 3000 ***A1***

(iv) number of brown squirrels go to 2000,
number of black squirrels goes down to 0 ***A1***

[7 marks]

(e) (i) ***AND*** (ii)



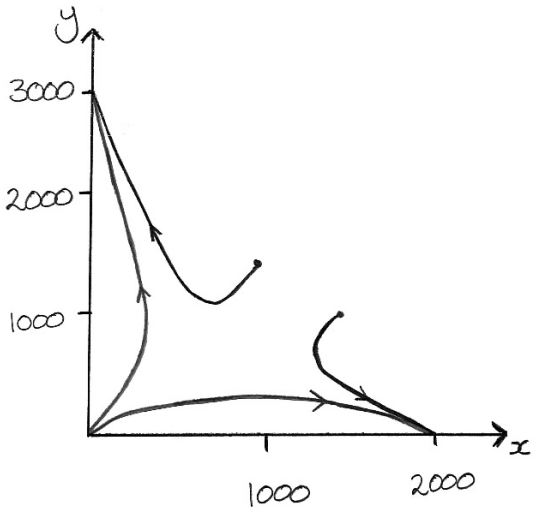
M1A1A1

[3 marks]

continued…

Question 2 continued

(f)



A1A1

Note: Award ***A1*** for a trajectory beginning close to $(0, 0)$ and going to $(0, 3000)$ and ***A1*** for a trajectory beginning close to $(0, 0)$ and going to $(2000, 0)$ in approximately the correct places.

[2 marks]

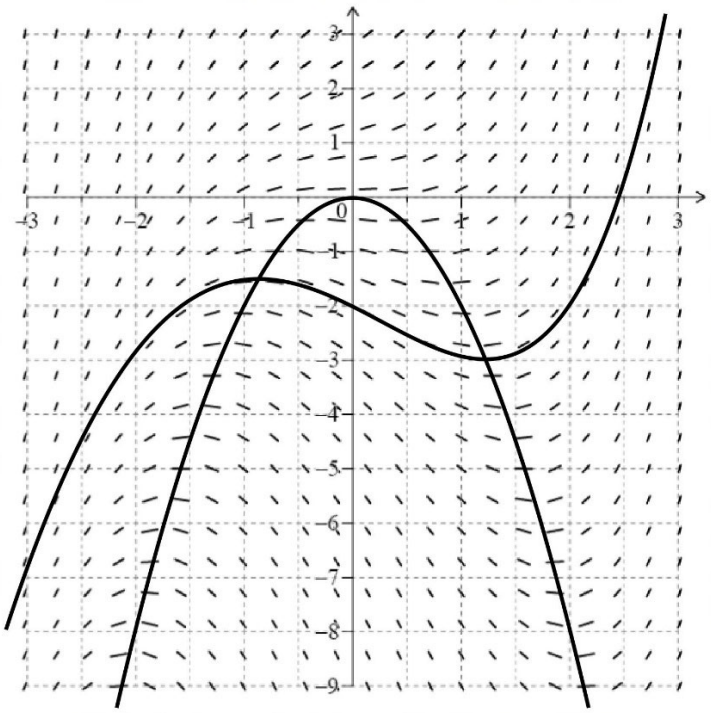
Total [28 marks]

7. (a) (i) $x^2 + \frac{y}{2} = 0 \ (y = -2x^2)$ ***A1***

(ii) $y = -2x^2$ drawn on diagram (correct shape with a maximum at $(0, 0)$) ***A1***

[2 marks]

(b)



correct shape with a local maximum and minimum, passing through $(0, -2)$ ***A1***

local maximum and minimum on the graph of $y = -2x^2$ ***A1***

[2 marks]

[Total 4 marks]

8. (a) (i) use of product rule ***(M1)***

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\left(4-\mathrm{e}^{x}\right)+2x\left(-\mathrm{e}^{x}\right) \quad \textit{A1}$$
$$= 8-2\mathrm{e}^{x}-2x\mathrm{e}^{x}$$

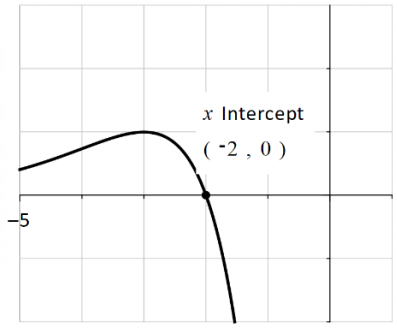
(ii) use of product rule ***(M1)***

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2\mathrm{e}^{x}-2\mathrm{e}^{x}-2x\mathrm{e}^{x} \quad \textit{A1}$$
$$= -4\mathrm{e}^{x}-2x\mathrm{e}^{x}$$
$$= -2(2+x)\mathrm{e}^{x}$$

[4 marks]

(b) $-2(2+a)\mathrm{e}^{a}=0$ **OR** sketch of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ with x-intercept indicated

OR finding the local maximum of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $(-2, 8.27)$ ***(M1)***



$(a=)\ -2$ ***A1***

[2 marks]
[Total 6 marks]

14. (a) (i) $\frac{1}{u^2}+\frac{2}{u}+1$ ***A1***

(ii) $\int\left(\frac{1}{(x+2)}+1\right)^2 \mathrm{d}x$

$=\int\left(\frac{1}{(x+2)^2}+\frac{2}{x+2}+1\right)\mathrm{d}x$ **OR** $\int\left(\frac{1}{u^2}+\frac{2}{u}+1\right)\mathrm{d}u$ ***(M1)***

$=-\frac{1}{(x+2)}+2\ln|x+2|+x(+c)$ ***A1A1***

Note: Award ***A1*** for first expression, ***A1*** for second two expressions. Award ***A1A0*** for a final answer of $=-\frac{1}{u}+2\ln(u)+u(+c)$.

[4 marks]

continued…

Question 14 continued

(b) volume $= \pi\left[-\frac{1}{(x+2)} + 2\ln(x+2) + x\right]_0^2$ ***M1***

$= \pi\left(-\frac{1}{4} + 2\ln(4) + 2 + \frac{1}{2} - 2\ln 2\right)$ ***A1***

$= \pi\left(\frac{9}{4} + 2\ln(4) - 2\ln 2\right)$

use of log laws seen, for example ***M1***

$\pi\left(\frac{9}{4} + 4\ln(2) - 2\ln 2\right)$ **OR** $\pi\left(\frac{9}{4} + 2\ln\left(\frac{4}{2}\right)\right)$

$= \frac{\pi}{4}(9 + 8\ln(2))$ **OR** $a = 9$, $b = 8$ and $c = 2$ ***A1***

Note: Other correct integer solutions are possible and should be accepted for example $a = 9$, $b = c = 4$.

[4 marks]
Total [8 marks]

16. (a) use of power rule ***(M1)***

$\frac{dW}{dv} = -1.1848v^{-0.84}$ **OR** $-1.18v^{-0.84}$ ***A1***

[2 marks]

(b) $\frac{dv}{dt} = 5$ ***(A1)***

$\frac{dW}{dt} = \frac{dv}{dt} \times \frac{dW}{dv}$ ***(M1)***

$\left(\frac{dW}{dt} = -5 \times 1.1848v^{-0.84}\right)$

when $v = 10$

$\frac{dW}{dt} = -5 \times 1.1848 \times 10^{-0.84}$ ***(M1)***

-0.856 $(-0.856278\ldots)^{\circ}\text{C}\,\text{min}^{-1}$ ***A2***

Note: Accept a negative answer communicated in words, "decreasing at a rate of…".
Accept a final answer of $-0.852809\ldots\,^{\circ}\text{C}\,\text{min}^{-1}$ from use of -1.18.
Accept 51.4 (or 51.2) $^{\circ}\text{C}\,\text{hour}^{-1}$.

[5 marks]
[Total 7 marks]

4. (a) **EITHER**

attempt to substitute 3, 4 and 7 into area of a trapezoid formula ***(M1)***

$(A =) \frac{1}{2}(7+4)(3)$

OR

given area expressed as an integral ***(M1)***

$(A =) \int_{-1}^{2}(6-x)\ \mathrm{d}x$

OR

attempt to sum area of rectangle and area of triangle ***(M1)***

$(A =) 4 \times 3 + \frac{1}{2}(3)(3)$

THEN

16.5 (square units) ***A1***

[2 marks]

(b) (i) $(A =) \int_{-1}^{2} 1.5x^2 - 2.5x + 3\ \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x = -1$, $x = 2$ in correct location. Award ***A1*** for an integral of the quadratic function, $\mathrm{d}x$ must be included. Do not accept "y" in place of the function, given that two equations are in the question.

(ii) 9.75 (square units) ***A1***

[3 marks]

(c) $16.5 - 9.75$ ***(M1)***

6.75 (square units) ***A1***

[2 marks]

Total [7 marks]

14. $V = \pi\int_0^{10} y^2\,\mathrm{d}x$ **OR** $\pi\int_0^{10} x^2\,\mathrm{d}y$ ***(M1)***

$h = 2$

$\approx \pi \times \frac{1}{2} \times 2 \times \left(\left(4^2 + 5^2\right) + 2 \times \left(6^2 + 8^2 + 7^2 + 3^2\right)\right)$ ***M1A1***

$= 1120\ \text{cm}^3 \quad (1121.548\ldots)$ ***A1***

Note: Do not award the second ***M1*** If the terms are not squared.

Total [4 marks]

17. (a) use of chain rule ***(M1)***

$$\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=1$ ***(M1)***

$$0.2=0.04\times\frac{\mathrm{d}x}{\mathrm{d}t}$$

$\left(\frac{\mathrm{d}x}{\mathrm{d}t}=\right)\ 5\ \mathrm{m\,h^{-1}}$ ***A1***

[3 marks]

(b) (i) if the position of the snail is (X, Y)

from part (a) $\frac{\mathrm{d}X}{\mathrm{d}t}=\frac{1}{0.04X}\frac{\mathrm{d}Y}{\mathrm{d}t}$

since speed is 1:

finding modulus of velocity vector and equating to 1 ***(M1)***

$$1=\sqrt{\left(\frac{\dot{Y}}{0.04X}\right)^2+\dot{Y}^2}\quad\textbf{OR}\quad 1=\sqrt{\dot{X}^2+0.0016X^2\dot{X}^2}$$

$$1=\dot{Y}^2\left(\frac{1}{0.0016X^2}+1\right)\quad\textbf{OR}\quad 1=\dot{X}^2(1+0.0016X^2)$$

$$\dot{Y}=\sqrt{\frac{1}{\frac{1}{0.08Y}+1}}\quad\textbf{OR}\quad \dot{X}=\sqrt{\frac{1}{1+0.0016X^2}}$$ ***(A1)***

$$\int_{0.02}^{2}\sqrt{\frac{1}{0.08Y}+1}\ \mathrm{d}Y=\int_{0}^{T}\mathrm{d}t\quad\textbf{OR}\quad \int_{1}^{10}\sqrt{1+0.0016X^2}\ \mathrm{d}X=\int_{0}^{T}\mathrm{d}t$$ ***(M1)***

$T=9.26$ hours ***A1***

(ii) **EITHER**

time for water to reach top is $\frac{2}{0.2}=10$ hours (seen anywhere) ***A1***

OR

or at time $t=9.26$, height of water is $0.2\times9.26=1.852$ ***A1***

THEN

so the water will not reach the snail ***AG***

[5 marks]

Total [8 marks]

4. (a) (i) $y = \frac{\mathrm{d}x}{\mathrm{d}t} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}t} + 5\frac{\mathrm{d}x}{\mathrm{d}t} + 6x = 0$ **OR** $\frac{\mathrm{d}y}{\mathrm{d}t} + 5y + 6x = 0$ ***M1***

Note: Award ***M1*** for substituting $\frac{\mathrm{d}y}{\mathrm{d}t}$ for $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$.

$$\begin{pmatrix} \frac{\mathrm{d}x}{\mathrm{d}t} \\ \frac{\mathrm{d}y}{\mathrm{d}t} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -6 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$ ***AG***

(ii) $\det \begin{pmatrix} -\lambda & 1 \\ -6 & -5-\lambda \end{pmatrix} = 0$ ***(M1)***

Note: Award ***M1*** for an attempt to find eigenvalues. Any indication that $\det(\boldsymbol{M} - \lambda \boldsymbol{I}) = 0$ has been used is sufficient for the ***(M1)***.

$-\lambda(-5-\lambda) + 6 = 0$ **OR** $\lambda^2 + 5\lambda + 6 = 0$ ***(A1)***

$\lambda = -2, -3$ ***A1***

(iii) (on a phase portrait the particle approaches $(0, 0)$ as t increases so long term velocity (y) is)

0 ***A1***

Note: Only award ***A1*** for 0 if both eigenvalues in part (a)(ii) are negative. If at least one is positive accept an answer of '*no limit*' or '*infinity*', or in the case of one positive and one negative also accept '*no limit or 0 (depending on initial conditions)*'.

[5 marks]

(b) (i) $y = \frac{\mathrm{d}x}{\mathrm{d}t}$

$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{\mathrm{d}y}{\mathrm{d}t}$ ***(A1)***

$\frac{\mathrm{d}y}{\mathrm{d}t} + 5y + 6x = 3t + 4$ ***A1***

(ii) recognition that $h = 0.1$ in any recurrence formula ***(M1)***

$(t_{n+1} = t_n + 0.1)$

$x_{n+1} = x_n + 0.1y_n$ ***(A1)***

$y_{n+1} = y_n + 0.1(3t_n + 4 - 5y_n - 6x_n)$ ***(A1)***

(when $t = 1$,) $x = 0.64402... \approx 0.644$ m ***A2***

(iii) recognizing that y is the velocity

$0.5\ \mathrm{m\,s^{-1}}$ ***A1***

[8 marks]

[Total 13 marks]

6. (a) (i) $y = x^{\frac{1}{2}}$ ***(M1)***

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$ ***A1***

(ii) gradient at $x = 0.16$ is $\frac{1}{2} \times \frac{1}{\sqrt{0.16}}$ ***M1***

$= 1.25$

EITHER

$y - 0.4 = 1.25(x - 0.16)$ ***M1***

OR

$0.4 = 1.25(0.16) + b$ ***M1***

Note: Do not allow working backwards from the given answer.

THEN

hence $y = 1.25x + 0.2$ ***AG***

[4 marks]

(b) $p = 0.45,\ q = 0.4125$ (or 0.413) (accept "$(0.45,\ 0.4125)$") ***A1A1***

[2 marks]

(c) (i) $(h(x) =)\ \frac{1}{2}\sqrt{2(x - 0.2)}$ ***A2***

Note: Award ***A1*** if only two correct transformations are seen.

(ii) $(a =)\ 0.28$ ***A1***

(iii) **EITHER**

Correct substitution of their part (b) (or $(0.28,\ 0.2)$) into the given expression ***(M1)***

OR

$\frac{1}{2}\left(1.25 \times 2(x - 0.2) + 0.2\right)$ ***(M1)***

Note: Award ***M1*** for transforming the equivalent expression for f correctly.

THEN

$(b =)\ -0.15$ ***A1***

[5 marks]

continued…

Question 6 continued

(d) (i) recognizing need to add two integrals ***(M1)***

$\int_0^{0.16} \sqrt{x}\ \mathrm{d}x + \int_{0.16}^{0.5} (1.25x + 0.2)\ \mathrm{d}x$ ***(A1)***

Note: The second integral could be replaced by the formula for the area of a trapezoid $\frac{1}{2} \times 0.34(0.4 + 0.825)$.

$0.251\ \mathrm{m}^2\ (0.250916\ldots)$ ***A1***

(ii) **EITHER**

area of trapezoid $\frac{1}{2} \times 0.05(0.4125 + 0.825) = 0.0309375$ ***(M1)(A1)***

OR

$\int_{0.45}^{0.5} (8.25x - 3.3)\,\mathrm{d}x = 0.0309375$ ***(M1)(A1)***

Note: If the rounded answer of 0.413 from part (b) is used, the integral is $\int_{0.45}^{0.5} (8.24x - 3.295)\,\mathrm{d}x = 0.03095$ which would be awarded ***(M1)(A1)***.

THEN

shaded area $= 0.250916\ldots - 0.0627292 - 0.0309375$ ***(M1)***

Note: Award ***(M1)*** for the subtraction of both $0.0627292\ldots$ and their area for the trapezoid from their answer to (a)(i).

$= 0.157\ \mathrm{m}^2\ (0.15725)$ ***A1***

[7 marks]

[Total 18 marks]

2. (a) attempt to expand given expression **OR** attempt at product rule ***(M1)***

$$C = \frac{xk^2}{10} - \frac{3x^3}{1000}$$

$$\frac{\mathrm{d}C}{\mathrm{d}x} = \frac{k^2}{10} - \frac{9x^2}{1000}$$ ***M1A1***

Note: Award ***M1*** for power rule correctly applied to at least one term and ***A1*** for correct answer.

[3 marks]

(b) equating their $\frac{\mathrm{d}C}{\mathrm{d}x}$ to zero ***(M1)***

$$\frac{k^2}{10} - \frac{9x^2}{1000} = 0$$

$$x^2 = \frac{100k^2}{9}$$

$$x = \frac{10k}{3}$$ ***(A1)***

substituting their x back into given expression ***(M1)***

$$C_{\max} = \frac{10k}{30}\left(k^2 - \frac{300k^2}{900}\right)$$

$$C_{\max} = \frac{2k^3}{9}\left(0.222\ldots k^3\right)$$ ***A1***

[4 marks]

(c) (i) substituting 20 into given expression and equating to 426 ***M1***

$$426 = \frac{20}{10}\left(k^2 - \frac{3}{100}(20)^2\right)$$

$$k = 15$$ ***A1***

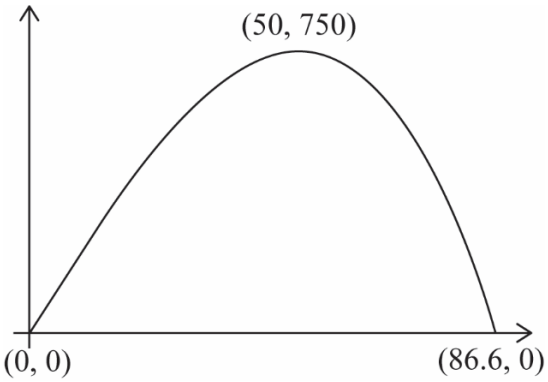
(ii) 50 ***A1***

[3 marks]

continued…

Question 2 continued

(d)



A1A1A1

Note: Award ***A1*** for graph indicating an increasing and then decreasing function (drawn in first quadrant), ***A1*** for maximum labelled and ***A1*** for graph drawn for positive x, passing through the origin and 86.6 which is marked on the x-axis or its coordinates are given.

[3 marks]

(e) setting their expression for C to zero **OR** choosing correct x-intercept on their graph of C ***(M1)***

$x_{\max} = 86.6 \quad (86.6025\ldots)$ litres ***A1***

[2 marks]
Total [15 marks]

7. (a) differentiating first equation. ***M1***

$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = \frac{\mathrm{d}y}{\mathrm{d}t}$

substituting in for $\frac{\mathrm{d}y}{\mathrm{d}t}$ ***M1***

$= -2x - 3y = -2x - 3\frac{\mathrm{d}x}{\mathrm{d}t}$

therefore $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 3\frac{\mathrm{d}x}{\mathrm{d}t} + 2x = 0$ ***AG***

Note: The ***AG*** line must be seen to award the final ***M1*** mark.

[2 marks]

(b) the relevant matrix is $\begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix}$ ***(M1)***

Note: $\begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}$ is also possible.

(this has characteristic equation) $-\lambda(-3-\lambda)+2=0$ ***(A1)***

$\lambda = -1, -2$ ***A1***

[3 marks]

continued…

Question 7 continued

(c) **EITHER**

the general solution is $x = Ae^{-t} + Be^{-2t}$ ***M1***

Note: Must have constants, but condone sign error for the ***M1***.

so $\frac{dx}{dt} = -Ae^{-t} - 2Be^{-2t}$ ***M1A1***

OR

attempt to find eigenvectors ***(M1)***

respective eigenvectors are $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ (or any multiple)

$\begin{pmatrix} x \\ y \end{pmatrix} = Ae^{-t}\begin{pmatrix} 1 \\ -1 \end{pmatrix} + Be^{-2t}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ ***(M1)A1***

THEN

the initial conditions become:

$0 = A + B$

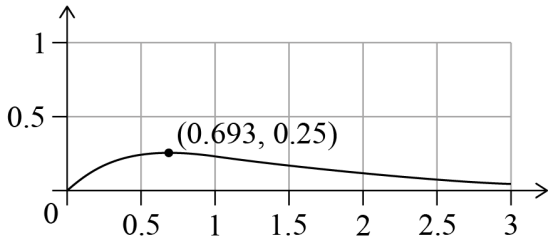
$1 = -A - 2B$ ***M1***

this is solved by $A = 1,\ B = -1$

so the solution is $x = e^{-t} - e^{-2t}$ ***A1***

[5 marks]

(d)



A1A1

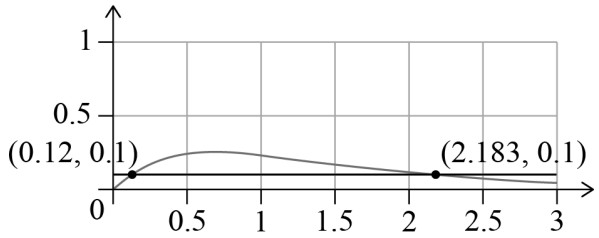
Note: Award ***A1*** for correct shape (needs to go through origin, have asymptote at $y = 0$ and a single maximum; condone $x < 0$). Award ***A1*** for correct coordinates of maximum.

[2 marks]

continued…

Question 7 continued

(e) intersecting graph with $y = 0.1$ ***(M1)***



so the time fishing is stopped between $2.1830\ldots$ and $0.11957\ldots$ ***(A1)***
$= 2.06(343\ldots)$ days ***A1***

[3 marks]

(f) *Any reasonable answer. For example:*
There are greater downsides to allowing fishing when the levels may be dangerous than preventing fishing when the levels are safe.
The concentration of mercury may not be uniform across the river due to natural variation / randomness.
The situation at the power plant might get worse.
Mercury levels are low in water but still may be high in fish. ***R1***

Note: Award ***R1*** for a reasonable answer that refers to this specific context (and not a generic response that could apply to *any* model).

[1 mark]
Total [16 marks]

1. (a) (i) $Q(t) = 3090t - 54000 \quad (3094.27...t - 54042.3...)$ ***A1A1***

Note: Award at most ***A1A0*** if answer is not an equation. Award ***A1A0*** for an answer including either x or y.

[2 marks]

(ii) $0.755 \quad (0.754741\ldots)$ ***A1***

[1 mark]

(iii) t is not a random variable **OR** it is not a (bivariate) normal distribution
OR data is not a sample from a population
OR data appears nonlinear
OR r only measures linear correlation ***R1***

Note: Do not accept "r is not large enough".

[1 mark]

(b) (i) attempt to separate variables ***(M1)***

$$\int \frac{1}{Q}\,\mathrm{d}Q = \int \beta N\,\mathrm{d}t$$

$$\ln|Q| = \beta Nt + c$$ ***A1A1A1***

Note: Award ***A1*** for LHS, ***A1*** for βNt, and ***A1*** for $+c$.
Award full marks for $Q = \mathrm{e}^{\beta Nt + c}$ **OR** $Q = A\mathrm{e}^{\beta Nt}$.
Award ***M1A1A1A0*** for $Q = \mathrm{e}^{\beta Nt}$

[4 marks]

(ii) attempt at exponential regression ***(M1)***

$Q = 1.15\mathrm{e}^{0.292t} \quad (Q = 1.14864...\mathrm{e}^{0.292055...t})$ ***A1***

OR

attempt at exponential regression ***(M1)***

$Q = 1.15 \times 1.34^{t} \quad (1.14864... \times 1.33917...^{t})$ ***A1***

Note: Condone answers involving y or x. Condone absence of "$Q =$"
Award ***M1A0*** for an incorrect answer in correct format.

[2 marks]

(iii) $0.999 \quad (0.999431...)$ ***A1***

[1 mark]

(iv) comparing something to do with R^2 and something to do with r ***M1***

Note: Examples of where the ***M1*** should be awarded:
$R^2 > r$
$R > r$
$0.999 > 0.755$
$0.999 > 0.755^2 \quad (= 0.563)$
The "correlation coefficient" in the exponential model is larger.
Model B has a larger R^2
Examples of where the ***M1*** should **not** be awarded:
The exponential model shows better correlation (since not clear how it is being measured)
Model 2 has a better fit
Model 2 is more correlated

an unambiguous comparison between R^2 and r^2 or R and r leading to the conclusion that the model in part (b) is more suitable / better ***A1***

Note: Condone candidates claiming that R is the "correlation coefficient" for the non-linear model.

[2 marks]

(v) it suggests that there will be more infected computers than the entire population ***R1***

Note: Accept any response that recognizes unlimited growth.

[1 mark]

(c) $1.15\,\mathrm{e}^{0.292t} = 2.3$ **OR** $1.15 \times 1.34^t = 2.3$ **OR** $t = \dfrac{\ln 2}{0.292}$ **OR** using the model to find two specific times with values of $Q(t)$ which double ***M1***

$t = 2.37$ (days) ***A1***

Note: Do not ***FT*** from a model which is not exponential. Award ***M0A0*** for an answer of 2.13 which comes from using $(10, 20)$ from the data or any other answer which finds a doubling time from figures given in the table.

[2 marks]

(d) an attempt to calculate β for city X ***(M1)***

$\beta = \dfrac{0.292055...}{2.6 \times 10^6}$ **OR** $\beta = \dfrac{\ln 1.33917...}{2.6 \times 10^6}$

$= 1.12328... \times 10^{-7}$ ***A1***

this is larger than 9.64×10^{-8} so the virus spreads more easily in city X ***R1***

Note: It is possible to award ***M1A0R1***.
Condone "so the virus spreads faster in city X" for the final ***R1***.

[3 marks]

(e) $a = 38.3,\ b = 3086.1$ ***A1A1***

Note: Award ***A1A0*** if values are correct but not to 1 dp.

[2 marks]

(f) (i) $\frac{Q'}{Q} = 0.42228 - 2.5561\times10^{-6}Q$ ***(A1)(A1)***

Note: Award ***A1*** for each coefficient seen – not necessarily in the equation. Do not penalize seeing in the context of y and x.

identifying that the constant is k **OR** that the gradient is $-\frac{k}{L}$ ***(M1)***

therefore $k = 0.422 \quad (0.422228...)$ ***A1***

$\frac{k}{L} = 2.5561\times10^{-6}$

$L = 165000 \quad (165205)$ ***A1***

Note: Accept a value of L of 164843 from use of 3 sf value of k, or any other value from plausible pre-rounding.
Allow follow-through **within** the question part, from the equation of their line to the final two ***A1*** marks.

[5 marks]

(ii) recognizing that their L is the eventual number of infected ***(M1)***

$\frac{165205...}{2600000} = 6.35\% \quad (6.35403...\%)$ ***A1***

Note: Accept any final answer consistent with their answer to part (f)(i) unless their L is less than 120146 in which case award at most ***M1A0***.

[2 marks]

[Total 28 marks]

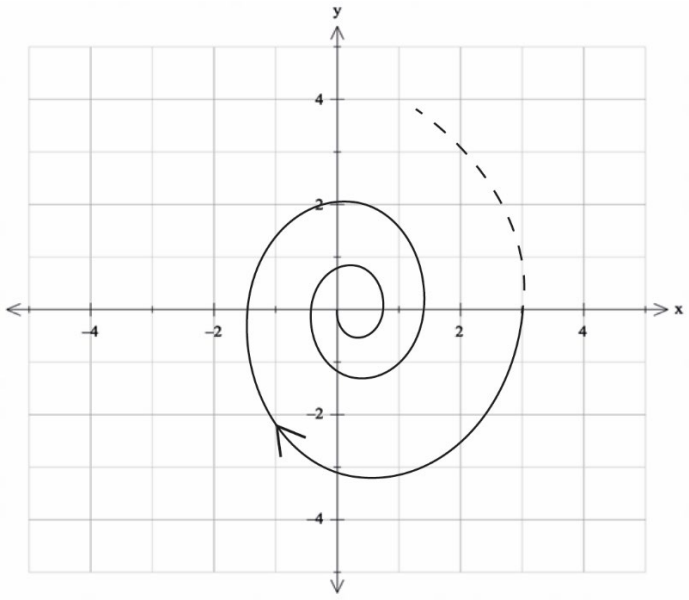
12. (a)

Description	Phase portrait
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	**D**
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = -3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	**C**
$\lambda_1 = -2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	**B**

A1A1A1

[3 marks]

(b)



spiral (crossing x-axis at least twice), centre at origin ***A1***

arrow indicating clockwise, passing through or starting from $(3, 0)$ ***A1***

[2 marks]

Total [5 marks]

14. (a) attempt to use product rule ***(M1)***

$a = 2t^2\cos(t^2) + \sin(t^2)$ ***A1***

[2 marks]

(b) graph of a ***(M1)***

$126\ (\text{m s}^{-2})\ (125.699...)$ ***A1***

[2 marks]

(c) attempt at integration by substitution or inspection ***(M1)***

$s = -\frac{1}{2}\cos(t^2)\ (+c)$ ***A1***

$(s = 0 \text{ when } t = 0) \Rightarrow c = \frac{1}{2}$ ***A1***

$\left(s = -\frac{1}{2}\cos(t^2) + \frac{1}{2}\right)$

[3 marks]

(d) $\cos(t^2) \leq 1$ ***A1***

$-\frac{1}{2}\cos(t^2) \geq -\frac{1}{2}$

so $\frac{1}{2} - \frac{1}{2}\cos(t^2) \geq 0$ ***R1***

hence the particle never has a negative displacement. ***AG***

Note: Do not accept reasoning based on a sketch of the graph.

[2 marks]

Total [9 marks]

15. EITHER

$$q_{n+1} = q_n + 0.1\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_n$$

$$\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_{n+1} = \left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_n + 0.1\left(\frac{\mathrm{d}^2q}{\mathrm{d}t^2}\right)_n \quad \textit{(M1)}$$

OR

let $\frac{\mathrm{d}q}{\mathrm{d}t} = y$

$$q_{n+1} = q_n + 0.1y_n$$

$$y_{n+1} = y_n + 0.1\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)_n \quad \textit{(M1)}$$

THEN
EITHER

$$\frac{\mathrm{d}y}{\mathrm{d}t} = 200 - 5y - 20q \quad \textit{(A1)}$$

OR

$$\frac{\mathrm{d}^2q}{\mathrm{d}t^2} = 200 - 5\frac{\mathrm{d}q}{\mathrm{d}t} - 20q \quad \textit{(A1)}$$

THEN
evidence of using Euler's method (e.g.) *(M1)*

0	1	8	140
0.1	1.8	22	54

maximum charge $= 12.7$ (Coulombs, at $t = 0.7$) *A2*

Note: Award ***A0A1*** for a final answer of 10.8, from reading the value at $t = 1$.

Total [5 marks]

3. (a) (i) $f'(x) = \frac{-2x}{50} + 2 \quad \left(= \frac{-x}{25} + 2, -0.04x + 2 \right)$ ***A1A1***

Note: Award ***A1*** for each correct term. Award at most ***A0A1*** if extra terms are seen.

(ii) $0 = \frac{-x}{25} + 2$ **OR** sketch of $f'(x)$ with x-intercept indicated ***M1***

$x = 50$ ***A1***

$y = 80$ ***A1***

$(50, 80)$

Note: Award ***M0A0A1*** for the coordinate $(50, 80)$ seen either with no working or found from a graph of $f(x)$.

[5 marks]

(b) (i) $\int_0^{70} \frac{-x^2}{50} + 2x + 30 \, \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location. Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.

(ii) $(\text{Area} =) \ 4710 \ \text{m}^2 \quad \left(4713.33\ldots, \frac{14140}{3} \right)$ ***A2***

[4 marks]

(c) (i) $\frac{11.4}{4713.33...} \times 100\%$ **OR** $\left| \frac{4701.93... - 4713.33...}{4713.33...} \right| \times 100\%$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the percentage error formula.

$0.242\% \ \ (0.241867...\%)$ ***A1***

Note: Percentage sign is required. Accept $0.242038...\%$ if 4710 is used.

(ii) **EITHER**

reduce the width of the intervals (trapezoids) ***A1***

OR

increase the number of intervals (trapezoids) ***A1***

Note: Accept equivalent statements. Award ***A0*** for the ambiguous answer "increase the intervals".

[3 marks]

continued…

Question 3 continued

(d) (i) width of the square is $70-x$ **OR** the length of the square is $\frac{-x^2}{50}+2x+30$

(M1)

Note: Award ***(M1)*** for $70-x$ seen anywhere. Accept $\frac{-x^2}{50}+2x+30$ but only if this expression is explicitly identified as a dimension of the square.

in term of x, equating the length to the width ED ***(M1)***

$\frac{-x^2}{50}+2x+30=70-x$

$(x=14.7920...$ or $135.21)$

$(x=)\ 14.8\ \mathrm{m}\ (14.7920...)$ ***A1***

Note: Award ***M0M0A0*** for an unsupported answer of 15. Award at most ***M1M0A0*** for an approach which leads to $A'(x)=0$. This will lead to a square base which extends beyond the east boundary of the property. Similar for any solution where F is not on the northern boundary, or GH is not on the east boundary.

(ii) **EITHER**

$(70-14.7920...)^2$ ***(M1)***

OR

$(55.2079...)^2$ ***(M1)***

OR

$\left(\frac{-(14.7920...)^2}{50}+2(14.7920...)+30\right)^2$ ***(M1)***

THEN

$(\mathrm{Area}=)\ 3050\ \mathrm{m}^2\ (3047.92...)$ ***A1***

Note: Follow through from part (d)(i), provided x is between 0 and 70. Award at most ***M1A0*** if their answer is outside the range of their $[0,\ 4713.33\ldots]$ from part (b).

[5 marks]

Total [17 marks]

5. (a) $\frac{1}{2}x^3+1=(x-1)^4$ ***(M1)***

$(p=)\ 2.91\ \text{cm}\ (2.91082...)$ ***A1***

[2 marks]

(b) attempt to make x (or x^2) the subject of $y=\frac{1}{2}x^3+1$ ***(M1)***

$x=\sqrt[3]{2(y-1)}$ (or $x^2=\left(2(y-1)\right)^{\frac{2}{3}}$) ***(A1)***

(upper limit $=$) $13.3(315\ldots)$ ***(A1)***

$V=\int_1^{13.3315...}\pi\left(2(y-1)\right)^{\frac{2}{3}}\ \mathrm{d}y$ ***(M1)***

Note: Award ***(M1)*** for setting up correct integral squaring their expression for x with both correct lower limit and their upper limit, and π. Condone omission of $\mathrm{d}y$.

$=197\ \text{cm}^3\ (196.946...)$ ***A1***

[5 marks]

(c) $x=y^{\frac{1}{4}}+1$ (or $x^2=\left(y^{\frac{1}{4}}+1\right)^2$) ***(A1)***

$V_2=\int_0^{13.3315...}\pi(y^{\frac{1}{4}}+1)^2\ \mathrm{d}y$ ***(M1)(A1)***

Note: Award ***(M1)*** for setting up correct integral squaring their expression for x with their upper limit, and π. Award ***(A1)*** for lower limit of 0, dependent on ***M1***. Condone omission of $\mathrm{d}y$. If a candidate found an area in part (b), do not award ***FT*** for another area calculation seen in part (c).

$=271.87668\ldots$ ***(A1)***

Note: Accept $271.038\ldots$ from use of 3sf in the upper limit.

subtracting their volumes ***(M1)***

$271.87668\ldots-196.946...$

$=74.9\ \text{cm}^3\ (74.93033\ldots)$ ***A1***

Note: Accept any answer that rounds to $75\ (\text{cm}^3)$. If a candidate found an area in part (b), do not award ***FT*** for another area calculation seen in part (c).

[6 marks]

[13 marks]

2. (a) attempt to separate variable ***M1***

$$\int \frac{1}{C}\,\mathrm{d}C = \int -k\,\mathrm{d}t$$

$\ln|C| = -kt\ (+c)$ ***A1***

$C = A\mathrm{e}^{-kt}$

substituting $t = 0,\ C = d$

$A = d$ ***A1***

Note: To award the ***A1***, $t = 0$ must be seen.

$C = d\mathrm{e}^{-kt}$ ***AG***

[3 marks]

(b) $0.05d = d\mathrm{e}^{-0.2t}$ ***(A1)***

$15.0\ \ (14.9786\ldots)$ ***A1***

[2 marks]

(c) **EITHER**

first dose: when $t = 2T$ then $C_1 = d\mathrm{e}^{-0.4T}$ ***A1***

second dose: when $t = T$ then $C_2 = d\mathrm{e}^{-0.2T}$ ***A1***

third dose: when $t = 0$ then $C_3 = d$ ***A1***

sum the three doses ***M1***

Note: A timing of the dose and the expression must be clearly indicted and correct to award ***A1***. Condone absence of d for ***A1A1A1***.

OR

considering values of C before and after a dose ***M1***

immediately before the second dose, $C = d\,\mathrm{e}^{-0.2T}$

immediately after the second dose, $C = d\,\mathrm{e}^{-0.2T} + d$ ***A1***

immediately before the third dose, $C = (d\,\mathrm{e}^{-0.2T} + d)\,\mathrm{e}^{-0.2T}$ ***(A1)***

immediately after the third dose, $C = (d\,\mathrm{e}^{-0.2T} + d)\,\mathrm{e}^{-0.2T} + d$ ***A1***

THEN

$\Rightarrow C = d\,(1 + \mathrm{e}^{-0.2T} + \mathrm{e}^{-0.4T})$ ***AG***

Note: Do not accept any response using the general formula given before part (d). Award ***M1A1A1A0*** if d not included in the final formula.

[4 marks]

(d) $r = \mathrm{e}^{-0.2T}$ and $u_1 = d$ ***A1***

correct substitution into geometric series formula ***A1***

$$C = \frac{d\left(1-\left(\mathrm{e}^{-0.2T}\right)^n\right)}{1-\mathrm{e}^{-0.2T}}$$

$$= d\left(\frac{1-\mathrm{e}^{-0.2nT}}{1-\mathrm{e}^{-0.2T}}\right)$$ ***AG***

[2 marks]

(e) (i) as $n \to \infty$, $\mathrm{e}^{-0.2nT} \to 0$ (since $T > 0$) ***(M1)***

$$H_\infty = \frac{d}{1-\mathrm{e}^{-0.2T}}$$ ***A1***

(ii) **METHOD 1**

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} H_n \times \mathrm{e}^{-0.2T}$$ ***(M1)(A1)***

$$L_\infty = \left(\frac{d}{1-\mathrm{e}^{-0.2T}}\right)\mathrm{e}^{-0.2T} \quad \textbf{OR} \quad = \frac{d}{\mathrm{e}^{0.2T}-1}$$ ***A1***

METHOD 2

L_n occurs immediately before the $(n+1)^{\text{th}}$ dose ***(M1)***

$$L_n = d\left(\frac{1-\mathrm{e}^{-0.2nT}}{1-\mathrm{e}^{-0.2T}}\right)\mathrm{e}^{-0.2T}$$ ***(A1)***

as $n \to \infty$, $\mathrm{e}^{-0.2nT} \to 0$ (since $T > 0$)

$$L_\infty = \left(\frac{d}{1-\mathrm{e}^{-0.2T}}\right)\mathrm{e}^{-0.2T} \quad \textbf{OR} \quad = \frac{d}{\mathrm{e}^{0.2T}-1}$$ ***A1***

[5 marks]

(f) (i) **METHOD 1**

$H_\infty - L_\infty = \frac{d}{1-\mathrm{e}^{-0.2T}} - \frac{d\mathrm{e}^{-0.2T}}{1-\mathrm{e}^{-0.2T}}$ ***M1***

$= \frac{d\,(1-\mathrm{e}^{-0.2T})}{1-\mathrm{e}^{-0.2T}}$ ***A1***

$= d$ ***AG***

METHOD 2

$H_\infty - L_\infty = \frac{d}{1-\mathrm{e}^{-0.2T}} - \frac{d}{\mathrm{e}^{0.2T}-1}$ ***M1***

$= \frac{d\,\mathrm{e}^{0.2T} - d - d + d\,\mathrm{e}^{-0.2T}}{\mathrm{e}^{0.2T} - 1 - \mathrm{e}^{0} + \mathrm{e}^{-0.2T}}$

$= \frac{d\,(\mathrm{e}^{0.2T} + \mathrm{e}^{-0.2T} - 2)}{\mathrm{e}^{0.2T} + \mathrm{e}^{-0.2T} - 2}$ ***A1***

$= d$ ***AG***

Note: Award ***M1A0*** for the use of their formulae from part (e).
Award ***M0A0*** if at no point $n \to \infty$ in their response.

(ii) **EITHER**

$\ln\left(\frac{H_\infty}{L_\infty}\right) = \ln\left(\frac{\frac{d}{1-\mathrm{e}^{-0.2T}}}{\frac{d\mathrm{e}^{-0.2T}}{1-\mathrm{e}^{-0.2T}}}\right)$ ***M1***

$= \ln(\mathrm{e}^{0.2T})$ ***A1***

$= 0.2T$ ***A1***

OR

$H_\infty \mathrm{e}^{-0.2T} = L_\infty$ ***(M1)***

$\ln\left(\frac{L_\infty}{H_\infty}\right) = -0.2T$ ***A1***

$\ln\left(\frac{L_\infty}{H_\infty}\right)^{-1} = 0.2T$ ***A1***

THEN

$5\ln\left(\frac{H_\infty}{L_\infty}\right) = T$ ***AG***

Note: Award ***M1A0*** for the use of their incorrect formulae from part (e).
Award ***M0A0*** if at no point $n \to \infty$ in their response.

[5 marks]

(g) (i) $d = 0.22$ ***A1***

(ii) $T = 7.70$ ***A1***

Note: Accept $T = 7.7$.

[2 marks]

(h) $0.06 = 0.22\,\mathrm{e}^{-0.2t}$

proportion of time $= \dfrac{6.49641...}{7.702...}$ ***(M1)***

84.3% ***A1***

[2 marks]

(i) **EITHER**

rounding to 8 hours leads to 3 times a day ***R1***

OR

7.7 hours is difficult to schedule ***R1***

OR

a tolerance may be necessary to not be too close to unsafe boundary ***R1***

OR

avoid giving too much of the drug and giving the patient an overdose ***R1***

Note: Accept any reasonable argument that engages with the model in a practical sense.
Do not accept an answer based on part (h) along the lines “it should be given more often”.

[1 mark]

[Total: 26 mark]

4. (a) attempt to substitute $h = 10$ and at least two different values of y into the trapezoidal rule ***(M1)***

$$\frac{10}{2}\big((0+0)+2(3+8+9)\big)$$

$$= 200\ (\text{cm}^2)$$ ***A1***

[2 marks]

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \,\text{d}x$ **OR** $\int_0^{40} y \,\text{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral (including $\text{d}x$), ***A1*** for correct limits in the correct location.

(ii) $213.33\ (\text{cm}^2)$ ***A2***

Note: Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213\ (\text{cm}^2)$.

[4 marks]

(c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$$\left|\frac{213.333... - 200}{213.333...}\right| \times 100$$

$$= 6.25(\%)\ \ (6.24999...(\%))$$ ***A1***

Note: Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or 0.0625.

[2 marks]

[Total: 8 marks]

10. (a) $y = x$ ***A1***

[1 mark]

(b) **METHOD 1**

equation has the form $y = ax^2 + bx + c$

when $x = 0,\ y = 0$ so $c = 0$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b$

attempt to find the value of b by setting *their* derivative equal to 1 when x is 0 ***(M1)***

$2a(0) + b = 1$

$b = 1$ ***(A1)***

when $x = -2,\ y = 0$

$a = \frac{1}{2}$ (and hence $y = \frac{1}{2}x^2 + x$) ***A1***

METHOD 2

equation has the form $y = ax(x+2)$ **OR** $y = ax^2 + 2ax$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + 2a$

attempt to find the value of a by setting *their* derivative equal to 1 when x is 0 ***(M1)***

$a = \frac{1}{2}$ (and hence $y = \frac{1}{2}x^2 + x$) ***A1***

Note: Writing $y = x(x+2)$ is incorrect and gains no marks.

[3 marks]

continued…

Question 10 continued

(c) equation is $y = ax^2 + bx + c$

finding an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ with unknown coefficients ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b$

setting up two equations using two points AND/OR one equation using the gradient function ***(M1)***

three correct equations ***(A1)***

$9a + 3b + c = 3$
$36a + 6b + c = 2$
$6a + b = 1$

$a = -\frac{4}{9}, b = \frac{11}{3}, c = -4$ ($a = -0.444444..., b = 3.66666..., c = -4$) ***A1***

(and hence $y = -\frac{4}{9}x^2 + \frac{11}{3}x - 4$)

[4 marks]

(d) $f(x) = \begin{cases} \frac{1}{2}x^2 + x & , \quad -2 \le x < 0 \\ x & , \quad 0 \le x \le 3 \\ -\frac{4}{9}x^2 + \frac{11}{3}x - 4 & , \quad 3 < x \le 6 \end{cases}$ ***A1***

Note: Condone open or closed endpoints for all intervals.
Condone y in place of $f(x)$.
Allow ***FT*** from parts (a), (b) and (c).

[1 mark]

[Total: 9 marks]

13. $\frac{\mathrm{d}x}{\mathrm{d}t} = y$ ***(A1)***

$\frac{\mathrm{d}y}{\mathrm{d}t} = -10x - 2y$ ***(A1)***

Note: Writing $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -10x - 2\frac{\mathrm{d}x}{\mathrm{d}t}$ is a valid approach and should be awarded ***A1A1***.

attempt to use the Euler equations shown by finding either a correct x_{n+1} or y_{n+1} ***(M1)***

correct equations for both x_{n+1} and y_{n+1} ***(A1)***

$x_{n+1} = x_n + 0.1(y_n)$, $y_{n+1} = y_n + 0.1(-10x_n - 2y_n)$ (accept equivalent notation)

$(t_{n+1} = t_n + 0.1)$

Note: All of the above marks can be implied by a correct second row in a table **OR** by a correct f_1 and f_2 clearly identified for use in Euler's method formula.

T	x	y
0	0.75	0
0.1	**0.75**	**-0.75**
0.2	0.675	-1.35
0.3	0.54	-1.755
0.4	0.3645	-1.944
0.5	0.1701	

so estimate is 0.170 ***A2***

Note: Accept 0.17 rounded to 2 sf.

[Total: 6 marks]

15. METHOD 1 Using the volume formula

volume of a "full" or "half" cylinder (seen anywhere) ***(A1)***

$\pi\int_2^4 4^2\,\mathrm{d}y$, $\pi\times 4^2\times 2$, 32π $(100.53\ldots)$ **OR**

$\pi\int_2^3 4^2\,\mathrm{d}y$, $\pi\times 4^2\times 1$, 16π $(50.265\ldots)$

one correct equation for the diagonal lines (seen anywhere) ***(A1)***

$y=\frac{1}{2}x$ or $y=6-\frac{1}{2}x$

attempt to write one equation x in terms of y ***(M1)***

$x=2y$, $x=12-2y$

EITHER (symmetry plus the volume of the "half" cylinder)

recognition of symmetry between $y=1$ and $y=3$ ***(M1)***

$2\pi\left(\int_1^2 (2y)^2\,\mathrm{d}y+\int_2^3 4^2\,\mathrm{d}y\right)$ ***(A1)***

OR (symmetry plus volume of the "full" cylinder)

recognition of symmetry between $y=1$ and $y=2$ ***(M1)***

$2\pi\left(\int_1^2 (2y)^2\,\mathrm{d}y\right)+\int_2^4 4^2\,\mathrm{d}y$ ***(A1)***

OR (calculation of separate parts) ***(M1)***

$\pi\left(\int_1^2 (2y)^2\,\mathrm{d}y+\int_2^4 4^2\,\mathrm{d}y+\int_4^5 (-2y+12)^2\,\mathrm{d}y\right)$ ***(A1)***

THEN

(volume of the solid=) 159 $\left(159.174\ldots,\frac{152\pi}{3}\right)$ ***A1***

continued…

Question 15 continued

METHOD 2 Geometric approach using cones and cylinders

volume of a cylinder (seen anywhere) ***(A1)***

$\pi \times 4^2 \times 2,\ 32\pi\ (100.53\ldots)$ (a *full* cylinder) **OR**

$\pi \times 4^2 \times 1,\ 16\pi\ (50.265\ldots)$ (a *half* cylinder)

using volume of cone formula to find the volume of the truncated cone ***(M1)***

correct expression to find the volume of the truncated cone (seen anywhere) ***(A1)***

$\frac{1}{3}\left(\pi \times 4^2 \times 2 - \pi \times 2^2 \times 1\right)$

attempt to find an expression for total volume using symmetry or individual parts ***(M1)***

correct expression for total volume ***(A1)***

$2\left(\frac{1}{3}\left(\pi 4^2 \times 2 - \pi 2^2 \times 1\right) + \pi 4^2 \times 1\right)$ **OR** $\frac{1}{3}\left(\pi 4^2 \times 2 - \pi 2^2 \times 1\right) + \pi 4^2 \times 2 + \frac{1}{3}\left(\pi 4^2 \times 2 - \pi 2^2 \times 1\right)$

(volume of the solid=) $159\ \left(159.174..., \frac{152\pi}{3}\right)$ ***A1***

Note: There are other valid approaches possible.

[Total: 6 marks]

17. (a) attempt to use the chain rule to set up a related rate ***(M1)***

correct expression ***A1***

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{\mathrm{d}x}{\mathrm{d}t} \div \frac{\mathrm{d}\theta}{\mathrm{d}t}$ **OR** $\frac{-250}{0.075}$

$= -\frac{10000}{3}$ ***AG***

[2 marks]

(b) $x(\theta) = \frac{3000}{\tan\theta}$ ***A1***

[1 mark]

(c) attempt to use chain rule **OR** quotient rule ***(M1)***

$\frac{-3000}{\tan^2\theta \times \cos^2\theta}$, $\frac{-3000\left(\sin\theta(-\sin\theta) - \cos^2\theta\right)}{\sin^2\theta}$ ***(A1)***

$= -\frac{3000}{\sin^2\theta}$ ***A1***

[3 marks]

(d) setting their equation in part (c) equal to the given expression in part (a) ***(M1)***

$-\frac{3000}{\sin^2\theta} = -\frac{10000}{3}$

$\theta = 1.24904...$ ***(A1)***

$x(1.24904...) = 1000\,\mathrm{m}$ ***A1***

[3 marks]

[Total: 9 marks]

12. (a) equating volume of sphere formula to 288π ***(M1)***

$$\frac{4}{3}\pi r^3 = 288\pi$$

$$\Rightarrow r = 6 \text{ (cm)}$$ ***A1***

[2 marks]

(b) $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ (seen anywhere) ***(A1)***

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t}$$ ***(M1)***

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$15 = 4\pi \times 6^2 \times \frac{\mathrm{d}r}{\mathrm{d}t}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{15}{144\pi}(\text{cm s}^{-1}) \ (0.0332,\ 0.0331572...)$$ ***A1***

[3 marks]

Total [5 marks]

16. (a) $(1.04,\ 0.509)$ $((1.03667...,0.509085...))$ ***A1A1***

[2 marks]

(b) attempt to make x the subject for either function ***(M1)***

$x = 4y^2$, $x = \cos^{-1} y$ ***A1A1***

attempt to use $V = \pi\int x^2 \mathrm{d}y$ ***(M1)***

$$V = \pi \int_0^{0.509085...} \left(4y^2\right)^2 \mathrm{d}y + \pi \int_{0.509085...}^{1} \left(\cos^{-1} y\right)^2 \mathrm{d}y$$ ***A1***

[5 marks]

(c) $= 1.15$ (units^3) ***A2***

Note: Do not ***FT*** from part (b) to part (c).
Award ***A1*** for $1.1.$ with no previous working.

[2 marks]
Total [9 marks]

17. (a) $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undetermined at $(0, 1)$ ***R1***

$\left(\text{so cannot use } y_n = y_{n-1} + h\left(\frac{x}{(x^2+1)(2y-2)}\right)\right)$

Note: Accept "undefined", "indeterminate" or "division by zero" in place of "undetermined".

[1 mark]

(b) $\int (2y-2)\,\mathrm{d}y = \int \frac{x}{x^2+1}\mathrm{d}x$ ***M1***

$y^2 - 2y = \frac{1}{2}\ln(x^2+1) + c$ ***A1***

substituting $x = 0,\ y = 1$ ***M1***

$c = -1$

$y^2 - 2y + 1 = \frac{1}{2}\ln(x^2+1)$

$(y-1)^2 = \frac{1}{2}\ln(x^2+1)$ ***A1***

$y - 1 = \sqrt{\frac{1}{2}\ln(x^2+1)}$ (where positive root required as $y \geq 1$)

$y = 1 + \sqrt{\frac{\ln(x^2+1)}{2}}$ ***AG***

[4 marks]

(c) (when $x = 0.1$) $y = 1.07$ $(1.07053...)$ ***A1***

[1 mark]

Total [6 marks]

6. (a) attempt to solve $\det(A-\lambda I)=0$ ***(M1)***

$(-0.05-\lambda)^2+25=0$ ***(A1)***

$-0.05-\lambda=\pm 5\text{i}$ ***(A1)***

$\lambda=-0.05\pm 5\text{i}$ ***A1***

[4 marks]

(b) (i) spiral ***A1***

(ii) inwards / towards O ***A1***

[2 marks]

(c) (i) attempt to substitute $(20,\ 0)$ into expression for $\frac{\text{d}y}{\text{d}t}$ ***(M1)***

$-5(20)-0.05(0)$

$\frac{\text{d}y}{\text{d}t}=-100\ (\text{m s}^{-1})$ ***A1***

(ii) $\frac{\text{d}x}{\text{d}t}=-1$ ***(A1)***

$\frac{\text{d}y}{\text{d}x}=\frac{\text{d}y}{\text{d}t}\div\frac{\text{d}x}{\text{d}t}$ **OR** $\frac{\text{d}y}{\text{d}x}=\frac{\text{d}y}{\text{d}t}\times\frac{\text{d}t}{\text{d}x}$ ***(M1)***

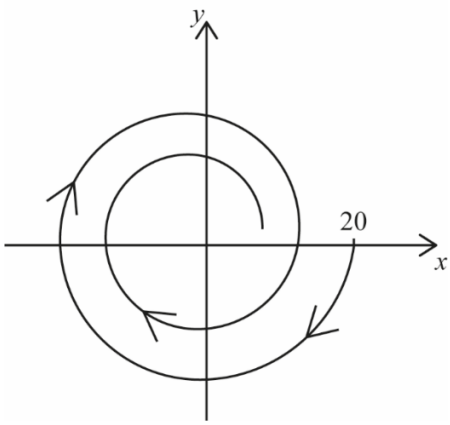
$(=-100\div -1=)\ 100$ ***A1***

[5 marks]

continued…

Question 6 continued

(d)



A4

Note: Award ***A1*** for starting at $(20,\ 0)$, ***A1*** for spiral inwards, ***A1*** for clockwise, ***A1*** for non-negative gradient at $(20,\ 0)$.

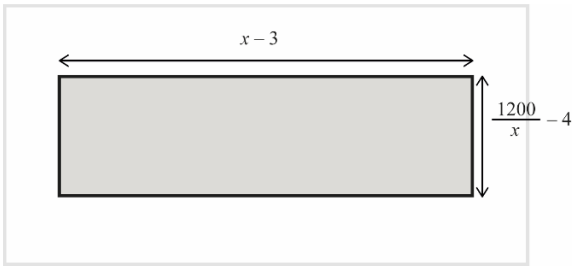
[4 marks]

[Total: 15 marks]

3. (a)

Note: In methods 1 and 2, full marks are available for candidates who work with a dummy variable, e.g. y, that represents the width of the park and hence is equal to $\frac{1200}{x}$.
The substitution to express an answer in only x may come as late as the final line.

METHOD 1 (finding dimensions of garden)



(width of park =) $\frac{1200}{x}$ ***(A1)***

(length of garden =) $x-3$, (width of garden =) $\frac{1200}{x}-4$ ***(A1)(A1)***

$A=(x-3)\times\left(\frac{1200}{x}-4\right)$ ***A1***

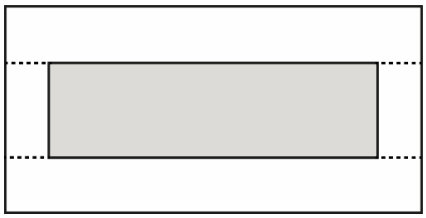
$=1200-4x-\frac{3600}{x}+12$ ***A1***

$=1212-4x-\frac{3600}{x}$ ***AG***

continued…

Question 3 continued

METHOD 2 (subtracting the area of the path)



width of park = $\frac{1200}{x}$ ***(A1)***

attempt to cut path into 4 (or 8) pieces ***(M1)***

four (or eight) areas of the path expressed in terms of x ***(A1)***

$A = 1200 - 2x - 2x - 1.5\left(\frac{1200}{x} - 4\right) - 1.5\left(\frac{1200}{x} - 4\right)$ ***A1***

correct manipulation leading to given result ***A1***

$= 1212 - 4x - \frac{1800}{x} - \frac{1800}{x}$

$= 1212 - 4x - \frac{3600}{x}$ ***AG***

Note: To award ***(M1)(A1)*** without a diagram the division of the park must be clear.

[5 marks]

(b) setting $1212 - 4x - \frac{3600}{x} = 800$ (accept a sketch) ***(M1)***

$x = 9.64\ (9.64011\ldots)$ (m) **OR** $x = 93.4\ (93.3598\ldots)$ (m) ***A1***

(width $=$) $124\ (124.479\ldots)$ (m) ***A1***

(width $=$) $12.9\ (12.8534\ldots)$ (m) ***A1***

Note: To award the final ***A1*** both values of x **and** both values of the width must be seen. Accept 12.8 for second value of width from candidate dividing 1200 by 3 sf value of 93.4.

[4 marks]

(c) $\left(\frac{dA}{dx} =\right) -4 + \frac{3600}{x^2}$ **OR** $-4 + 3600x^{-2}$ ***A1A1A1***

Note: Award ***A1*** for -4, ***A1*** for $+3600$, and ***A1*** for x^{-2} or x^2 in denominator.

[3 marks]

continued…

Question 3 continued

(d) setting *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ equal to 0 **OR** sketch of *their* $\frac{\mathrm{d}A}{\mathrm{d}x}$ with x-intercept highlighted ***M1***

$(x =)\ 30\ (\mathrm{m})$ ***A1***

Note: To award ***A1FT*** the candidate's value of x must be within the domain given in the problem $(3 < x < 300)$.

[2 marks]

(e) **EITHER**

evidence of using GDC to find maximum of graph of $A = 1212 - 4x - \frac{3600}{x}$ ***(M1)***

OR

substitution of *their* x into A ***(M1)***

OR

dividing 1200 by *their* x to find width of park **and** subtracting 3 from *their* x and 4 from the width to find park dimensions ***(M1)***

Note: For the last two methods, only follow through if $3 <$ their $x < 300$.

THEN

$(A =)\ 972\ (\mathrm{m}^2)$ ***A1***

[2 marks]

Total [16 marks]

7. (a) $\begin{vmatrix} -4-\lambda & 6 \\ 9 & -1-\lambda \end{vmatrix} = 0$ ***(M1)***

Note: Do not accept $\det(\boldsymbol{A}-\lambda\boldsymbol{I})=0$ or similar as evidence of a correct method unless A is explicitly defined to be the given matrix.

$(-4-\lambda)(-1-\lambda)-54=0$

$\lambda=-10,\ \lambda=5$ ***A1A1***

For $\lambda=-10$

$\begin{pmatrix} -4 & 6 \\ 9 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} -10x \\ -10y \end{pmatrix}$ ***(M1)***

$-4x+6y=-10x$

$x+y=0$

possible eigenvector is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$ (or equivalent) ***A1***

for $\lambda=5$

$\begin{pmatrix} -4 & 6 \\ 9 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 5x \\ 5y \end{pmatrix}$

$-4x+6y=5x$

$3x=2y$

possible eigenvector is $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ (or equivalent) ***A1***

Note: If both eigenvalues are incorrect then award at most ***M1A0A0M1A0A0***.

[6 marks]

(b) attempt to substitute their eigenvalues and eigenvectors equation ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix}=A\mathrm{e}^{-10t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}+B\mathrm{e}^{5t}\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ ***A1***

Note: Award at most ***(M1)A0*** if $\begin{pmatrix} x \\ y \end{pmatrix}$ not seen.

[2 marks]

continued…

Question 7 continued

(c) At $t = 0$, $x = 500$ and $y = 125$

$x = -A + 2B$ and $y = A + 3B$

Solving simultaneously: ***(M1)***

$A = -250$ and $B = 125$ ***A1***

$$\left(\begin{pmatrix} x \\ y \end{pmatrix} = -250\mathrm{e}^{-10t}\begin{pmatrix} -1 \\ 1 \end{pmatrix} + 125\mathrm{e}^{5t}\begin{pmatrix} 2 \\ 3 \end{pmatrix}\right)$$

Note: Follow through from their eigenvectors.
Accept equivalent values for A and B based on the direction of their eigenvectors and the order of their eigenvalues in the equation.

[2 marks]

(d) $2:3$ ***A1***

[1 mark]

(e) attempt to eliminate $\mathrm{d}t$ from the two differential equations ***M1***

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{9x - y}{-4x + 6y}$$

substituting initial conditions ***(M1)***

$$= \frac{9(500) - 125}{-4(500) + 6(125)}$$

$$= -3.5$$ ***A1***

Note: Award ***M1*** for $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4x + 6y}{9x - y}$.

[3 marks]

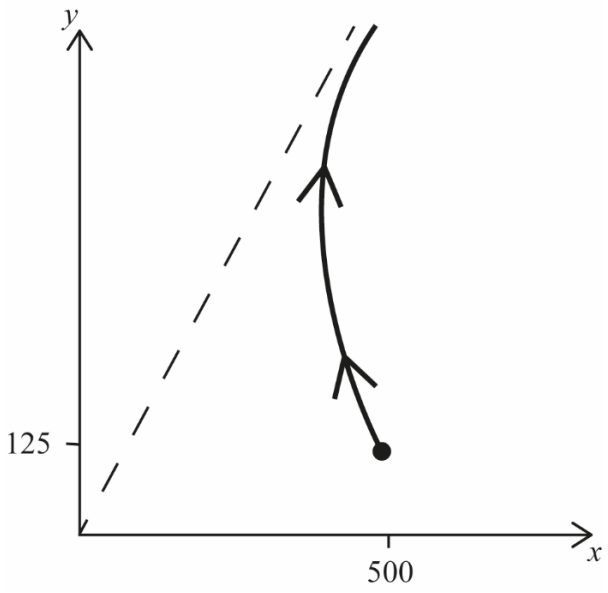
continued…

Question 7 continued

(f) trajectory or trajectories that are consistent with their eigenvalues ***A1***
a trajectory that passes through the point $(500, 125)$ with gradient that is consistent with the response to part (e) ***A1***
the diagram contains at least one of their eigenvectors ***A1***
(e.g. labelled $y = 1.5x$; $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\lambda = 5$ etc.)
the trajectory that passes through $(125, 500)$ tends towards an oblique asymptote that corresponds to their eigenvector and the direction is indicated by at least one arrow on the trajectory ***A1***

Note: For the second ***A1***, the point $(500, 125)$ may not be labelled but there should be a point marked on the trajectory that is consistent with these coordinates.

The final ***A1*** will depend on their eigenvalues. Follow through can be awarded as long as the direction of the trajectory is consistent with the nature of their eigenvalues and eigenvectors.



[4 marks]
Total [18 marks]

1. (a) $C = kd$ ***(M1)***

$0.80 = 0.5k$ **OR** $\frac{C}{d} = \frac{0.8}{0.5}$ ***(A1)***

$k = 1.6$

$C = 1.6d$ **OR** $C = \frac{d}{0.625}$ ***A1***

Note: For the final ***A1*** do not accept $C = \frac{0.8}{0.5}d$ or a correct equation which does not have C as the subject.

[3 marks]

(b) $d = \frac{0.96}{1.6}$ ***M1***

Note: Award ***M1*** for the substitution of $C = 0.96$ into a correct equation, award ***M0*** for substitution of $d = 0.6$.

$= 0.6$ ***AG***

[1 mark]

(c) attempt at using Pythagoras ***M1***

$d^2 = h^2 + (2r)^2$ ***(A1)***

$r^2 = \frac{1}{4}\left(d^2 - h^2\right)$ ***A1***

[3 marks]

(d) $V = \pi r^2(2h)$ ***(A1)***

$V = \frac{\pi}{4}\left(d^2 - h^2\right)(2h)$ ***M1***

Note: The ***M1*** is for the substitution of their expression for r^2 into their formula for V.

$V = \frac{\pi}{2}\left(d^2h - h^3\right)$ ***AG***

[2 marks]

(e) (i) $V = \frac{\pi}{2}\left(d^2h - h^3\right)$

$= \frac{\pi}{2}\left(0.6^2 \times 0.4 - 0.4^3\right)$ ***(M1)***

Note: The ***M1*** is for substituting correct values of both d and h in the formula from part (d).

$= 0.126\ (\text{m}^3)\ \left(0.12566\ldots, \frac{\pi}{25}, 0.04\pi\right)$ ***A1***

[2 marks]

(ii) $V = \frac{\pi}{2}\left(0.36h - h^3\right)$ ***M1***

Note: Award ***M1*** for substitution of $d = 0.6$. This may be seen anywhere.

$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{2}\left(0.36 - 3h^2\right) = 0$ ***A1***

Note: Award ***A1*** for correct differentiation.

$h^2 = \frac{0.36}{3}\ (= 0.12)$ ***A1***

Note: Do not award the final ***A1*** if the working is done with approximate values or if $\sqrt{0.12}$ is substituted into $\frac{\mathrm{d}V}{\mathrm{d}h}$.

$h = \sqrt{0.12}$ ***AG***

[3 marks]

(iii) substituting $h = \sqrt{0.12}$ into equation for V **OR** use of graph ***(M1)***

$V = \frac{\pi}{2}(0.36 - 0.12)\sqrt{0.12}$

$0.131\ (\text{m}^3)\ (0.130593..., 0.0416\pi)$ ***A1***

[2 marks]

(f) x-coordinate of S is 0.4 ***(A1)***

let the y-coordinate be y_S

attempt at Pythagoras ***(M1)***

$(y_S + 0.2)^2 = 0.6^2 - 0.4^2$

$y_S + 0.2 = \sqrt{0.2}$

$y_S = 0.247\ (0.247213...)$ ***(A1)***

Note: The ***(M1)*** mark can be implied by a y-coordinate of 0.447 or 0.647 seen.

any valid method to find equation ***(M1)***

e.g. quadratic regression, vertex form, simultaneous equations

Note: Award only if the student has found three points on the curve.

EITHER

$y = -0.295x^2 + 0.236x + 0.2$ ***A2***

$(y = -0.295081...x^2 + 0.236065...x + 0.2)$

Note: Award ***A1*** if one coefficient ($-0.295081...$ or $0.236065...$) is correct or if "$y =$" is missing, ***A2*** for completely correct equation.
Award ***A1*** for $y = -0.294x^2 + 0.235x + 0.2$ obtained from using the 3 sf value of y_S.

OR

$y = -0.295(x - 0.4)^2 + 0.247$ ***A2***

$(y = -0.295081...(x - 0.4)^2 + 0.247213...)$

Note: Award ***A1*** for -0.295, ***A2*** for completely correct equation.

[6 marks]

(g) volume $= \pi\int_0^{0.8}\left(-0.295081...x^2 + 0.236065...x + 0.2\right)^2 \mathrm{d}x$ ***M1***

Note: Award ***M1*** for the minimum of an integral with the correct limits and their function squared.

$= 0.135\ \ (0.135161\ldots)$ ***A1***

$0.135 > 0.131$ ***R1***

Note: Award ***R1*** independently of the previous marks for a correct comparison of their (clearly stated) volume with their answer to part (e)(iii).

Hence the volume is greater than any cylinder volume ***AG***

[3 marks]

(h) *Award **A1** for at least one reasonable answer, for example:*

the barrel is full of wine when sold
the barrel/stick is constructed with zero thickness
the stick is straight and inflexible
the hole has no diameter
volume of wine is the only important factor in value
aesthetics are not important

Note: Do not accept statements that relate to the barrel having circular cross-section, for example, 'there are no deformities', 'it is perfectly smooth' as these assumptions have already been made with the chosen model.

[1 mark]

[Total 26 marks]

$\begin{pmatrix} -115 \end{pmatrix}$

9. (a) $p = 1.5;\ q = 2$ ***A1A1***

[2 marks]

(b) attempt at using chain rule ***(M1)***

(i) $h'(t) = -\frac{\pi}{4}\sin\left(\frac{\pi}{6}t\right)\ \left(= -0.785\sin\left(\frac{\pi}{6}t\right)\right)$ ***A1***

(ii) $h''(t) = -\frac{\pi^2}{24}\cos\left(\frac{\pi}{6}t\right)\ \left(= -0.411233...\cos\left(\frac{\pi}{6}t\right)\right)$ ***A1***

[3 marks]

(c) (i) attempt to locate points of inflexion or max value of $h'(t)$ ***(M1)***

$h''(t) = -\frac{\pi^2}{24}\cos\left(\frac{\pi}{6}t\right) = 0$ **OR** sketch on graph **OR** $t = 3$ **OR** $\frac{\pi}{6}k = \frac{3\pi}{2}$

$(k =)\ 9$ ***A1***

(ii) $(h(9) =)\ 2\ \ (\text{m})$ ***A1***

[3 marks]
[Total 8 marks]

11. (a) (i) **EITHER**

$(\text{area of } R =) \int_{-1}^{1} \left| x^3 - x \right| \mathrm{d}x$ ***A1***

OR

$(\text{area of } R =) \ 2 \times \int_{-1}^{0} x^3 - x \, \mathrm{d}x$ **OR** $(\text{area of } R =) \ -2 \times \int_{0}^{1} x^3 - x \, \mathrm{d}x$ ***A1***

OR

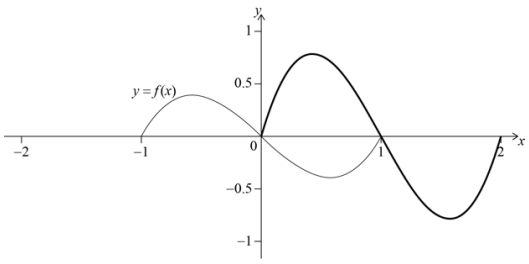
$(\text{area of } R =) \int_{-1}^{0} x^3 - x \, \mathrm{d}x - \int_{0}^{1} x^3 - x \, \mathrm{d}x$ ***A1***

(ii) $(\text{area of } R =) \ 0.5$ ***A1***

Note: Follow through from part (a)(i) only if answer is greater than zero.

[2 marks]

(b)



A1A1

Note: Award ***A1*** for sketch with correct shape on $[0, 2]$, ***A1*** for vertical stretch x2. Condone max/min of g extending to 1 / -1.

[2 marks]

(c) attempt to use $\pi \int y^2 \mathrm{d}x$ ***(M1)***

volume $= \pi \int_{-1}^{1} \left(x^3 - x \right)^2 \mathrm{d}x$ ***(A1)***

volume $= 0.479$ (cubic units) $\left(= 0.478718..., \frac{16\pi}{105} \right)$ ***A1***

[3 marks]
[Total 7 marks]

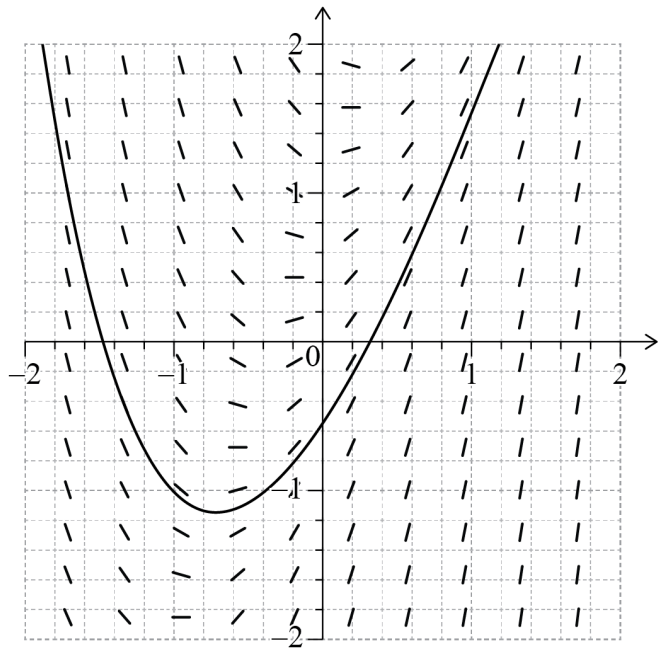
12. (a) gradient $(=-3+1+1)=-1$ ***A1***

$y+1=-1(x+1)$

$x+y+2=0$ ***A1***

[2 marks]

(b)



A1A1

Note:	Award ***A1*** for (approximately) intersecting $(-1, -1)$ and with correct gradient, ***A1*** for generally plausible shape (e.g. not crossing over LOTS of isoclines).

[2 marks]
[Total 4 marks]

13. (a) attempt to solve $u_1(t) = u_2(t)$ **OR** sketch of two graphs ***(M1)***

$(T =)\ 2$ ***A1***

Note: Award ***(M1)A0*** if additional values of T are seen **OR** if $T = -2$ is their final answer.

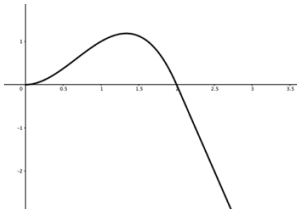
[2 marks]

(b) $u_1'(t) = 4t - 3t^2$ ***A1***

$u_1'(2) = -4 = u_2'(2)$ ***R1***

[2 marks]

(c)



recognition of integrating **AND** equating to zero ***(M1)***

$\int_0^2 2t^2 - t^3 \,\mathrm{d}t + \int_2^k 8 - 4t \,\mathrm{d}t = 0$ ***(A1)***

$\frac{4}{3} + (8k - 2k^2) - 8 = 0$ **OR** $1.18350\ldots \left(\frac{6-\sqrt{6}}{3}\right)$ seen ***(A1)***

Note: Award ***(M1)(A1)A0*** if integration done correctly but limits are not substituted.

$(k =)\ 2.82 \left(= 2.81649..., \frac{6+\sqrt{6}}{3}\right)$ ***A1***

[4 marks]
[Total 8 marks]

4. (a) (i) $h(0) = 0.00623 \text{ (km)} \ (= 0.00622517)$ ***A1***

(ii) this is the height of the nose of the plane (above the runway), when the plane is on the runway ***A1***

[2 marks]

(b) (i) $y = 9.94$ ***A1***

Note: Accept $h = 9.94$.

(ii) **EITHER**
this is the height that the (nose of the) plane approaches (but does not reach) ***A1***

OR
this is the maximum possible height of the (nose of the) plane ***A1***

OR
the (nose of the) plane does not exceed this height ***A1***

[2 marks]

(c) **METHOD 1 (chain rule)**

$h(x) = 10\left(1+150\mathrm{e}^{-0.07x}\right)^{-1} - 0.06$ ***(M1)***

find $h'(x) = -10\left(1+150\mathrm{e}^{-0.07x}\right)^{-2} \times 150\mathrm{e}^{-0.07x} \times -0.07$ ***A1M1A1***

$\left(= \dfrac{105\mathrm{e}^{-0.07x}}{(1+150\mathrm{e}^{-0.07x})^2}\right)$

Note: Award ***A1*** for correct first term $\left(-10\left(1+150\mathrm{e}^{-0.07x}\right)^{-2}\right)$, ***M1*** for attempt to use the chain rule, ***A1*** for correct use of chain rule $\left(\times 150\mathrm{e}^{-0.07x} \times -0.07\right)$. Award at most ***A1M1A0*** if additional terms are seen. The answer is not required to be simplified beyond what is shown in the markscheme.

METHOD 2 (quotient rule)

$$\frac{\left(1+150\,\mathrm{e}^{-0.007x}\right)(0) - 10\left(150\mathrm{e}^{-0.007x} \times -0.007\right)}{\left(1+150\,\mathrm{e}^{-0.007x}\right)^2}$$ ***M1A1***

Note: Award ***M1*** for attempt to use quotient rule, ***A1*** for correct use.

$$= \frac{-10\left(150\mathrm{e}^{-0.007x} \times -0.007\right)}{\left(1+150\,\mathrm{e}^{-0.007x}\right)^2} \quad \left(= \frac{105\mathrm{e}^{-0.07x}}{(1+150\mathrm{e}^{-0.07x})^2}\right)$$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator.

[4 marks]

(d) evidence of a graph of $h'(x)$ ***(M1)***

maximum at $x = 71.6 \quad (= 71.58051...)$ ***(A1)***

$h'(71.58051...) = 0.175$ ***A1***

maximum gradient is less than 0.2
and hence the regulation is being followed ***A1***

[4 marks]

[Total 12 marks]

1. (a) (i) $\frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds}$ ***A1***

$(v = \frac{ds}{dt})$

$\frac{dv}{dt} = v\frac{dv}{ds}$ ***AG***

[1 mark]

(ii) $v\frac{dv}{ds} = g$

attempt to separate variables ***M1***

$\int v\,dv = \int g\,ds$

$\frac{v^2}{2} = gs\ (+c)$ ***A1***

using initial conditions (can be done at any point) ***M1***

$50 = c$

so $v = \sqrt{2gs + 100}$ ***A1***

Note: Marks are intentionally unimplied to ensure on-syllabus techniques are used.

[4 marks]

(iii) **EITHER**

attempt to use their part (a)(ii) to find a value of s when $v = 330$ ***(M1)***

$330 = \sqrt{2gs + 100}$

therefore $s = 5551.02...$ ***A1***

$(5551.02 < 40000)$

so (the model does predict) he will reach the speed of sound ***A1***

OR

attempt to use their part (a)(ii) to find a value of v when $s = 40000$ ***(M1)***

$v = \sqrt{2g(40000) + 100}$

$= 885 \quad (885.49...)$ ***A1***

$(885 > 330)$

so (the model does predict) he will reach the speed of sound (before $s = 40000$) ***A1***

Note: For the **OR** method, accept any large s that leads to $v = 330$.
FT from $\sqrt{2gs}$ gives $885\ (885.437\ldots)$ for v and $5560\ (5556.12\ldots)$ for s
FT from their *v* or their *s* for the final A1, provided M1 is awarded

[3 marks]

(b) (i) $v = gt\ (+c)$ OR gradient is a constant ***(M1)***

so the graph should be a straight line ***A1***

[2 marks]

(ii) the graph is not a straight line / only (approx.) straight for small t, so the model does not appear to be valid ***R1***

Note: Award ***R1*** for recognising that the graph is non-linear **AND** stating that the model does not appear to be valid

[1 mark]

(c) (i) $v\dfrac{\mathrm{d}v}{\mathrm{d}s} = g - kv^2$

separating variables ***(M1)***

$\displaystyle\int \frac{v}{g - kv^2}\,\mathrm{d}v = \int \mathrm{d}s$

$-\dfrac{1}{2k}\ln(g - kv^2) = s\ (+c)$ **OR** $-\dfrac{1}{2k}\ln\left|g - kv^2\right| = s\ (+c)$ ***(A1)***

rearranging to make v the subject ***(M1)***

Note: Award ***(M1)*** for making v the subject of their equation and not just an attempt, or an erroneous equation with v also on the RHS.

$g - kv^2 = A\mathrm{e}^{-2ks}$

$v = \sqrt{\dfrac{g - A\mathrm{e}^{-2ks}}{k}}$

applying initial conditions (here or elsewhere) ***(M1)***

$100 = \dfrac{g - A}{k}$

$A = g - 100k$

so

$v = \sqrt{\dfrac{g - (g - 100k)\mathrm{e}^{-2ks}}{k}}$ ***A1***

[5 marks]

(ii) $9.672 = 9.8 - 1600k$ ***A1A1***

Note: Award ***A1*** for correct left-hand side and ***A1*** for correct right-hand side.

$k = \dfrac{9.8 - 9.672}{1600}$

$k = 8 \times 10^{-5}$ ***AG***

Note: Award ***A1A0*** for $k = 8\times10^{-5}$ substituted into the right-hand side of the expression, leading to 9.672.

[2 marks]

(iii) $s \to \infty, \mathrm{e}^{-2ks} \to 0$ **OR** $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ **OR** graph/table ***(M1)***

$(v_{\max} = \sqrt{\frac{g}{k}} =) 350 \quad (ms^{-1})$ ***A1***

[2 marks]

(iv) upper limit occurs when $s = 40000$ ***(M1)***

Note: The ***M1*** can be implied by 40000 substituted into their part (c)(i).

$349.7\ (\mathrm{ms}^{-1})$ ***A1***

Note: Answer must be to 4 sf.

[2 marks]

(d) $s_{n+1} = s_n + 4000$ ***(A1)***

$v_{n+1} = v_n + 4000\times\left(\frac{3.98\times10^{14}}{v_n(6.41\times10^6 - s_n)^2} - (8\times10^{-5})v_n\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempt to use Euler method formula **AND** dividing through by v.

if $v_0 = 10$, then $v_{10} = 361\ (360.658\ldots)$ ***A1***

[4 marks]

(e) (i) Use a smaller step length ***R1***
OR
Use a better method such as Runge-Kutta ***R1***
OR
(Try to) solve the equation exactly ***R1***

[1 mark]

(ii) Any reasonable response: ***R1***
For example:
Ignoring parachute / end point of motion / only valid for certain domain.
Treating Felix as a point object.
Ignoring weather / wind / air currents.
Assuming path is directly downwards.
Assuming perfect measurement of initial speed.

[1 mark]

[Total: 28 marks]

4. attempt to use Euler

$y_{n+1} = y_n + 0.1\log(x_n + y_n)$ ***(A1)***

$y_1 \left(= 1 + 0.1 \times \log_{10}(1)\right) = 1$ ***(A1)***

$y_2 = 1.004139...$ ***(A1)***

THEN

when $x = 2$ $y(2) \approx 1.61\ (1.60536...)$ ***A1***

[Total: 4 marks]

9. (a) attempt to integrate by substitution or inspection ***(M1)***

$4\ln|2x+3|+c$ **OR** $4\ln|x+1.5|+c$ ***A1A1***

Note: Award ***M1*** for $\ln(2x+3)$ or $\ln(x+1.5)$, ***A1*** for the 4 and ***A1*** for c. The ***A*** marks can only be awarded if the ***M*** mark is awarded. Condone absence of modulus signs.

[3 marks]

(b) recognizing that area is $\left[4\ln(2x+3)\right]_0^6$ ***(M1)***

$=4\ln(15)-4\ln(3)$ ***(A1)***

use of log laws for their expression ***(M1)***

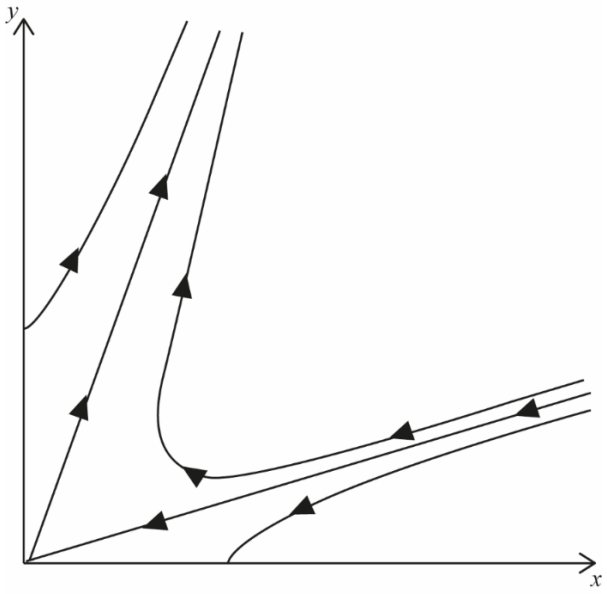
$=4\ln(5)\ \ (=2\ln(25)=1\ln(625))$ ***A1***

Note: Award ***(M1)A0M0A0*** for an unsupported final answer of $6.43775\ldots$
Award at most ***(M1)A1FTM0A0*** if their answer from part (a) does not include $\ln$.

[4 marks]
[Total 7 marks]

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15. (a) (i)&(ii)



A1A1A1

Note: Award ***A1*** for correct directions on eigenvectors, ***A1*** for correct trajectories, ***A1*** for correct arrows on trajectories.

[3 marks]

(b) for Y not to die out $y > \frac{1}{3}x$ ***(R1)***

as $x = 252$, $y > 84$ ***(M1)***

(minimum number of new animals is) 25 ***A1***

Note: Award ***(R1)(M1)A0*** for an unsupported 24.

[3 marks]
[Total 6 marks]

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16. attempt to find gradient ***(M1)***

EITHER

gradient of tangent $= -\tan 75^\circ \ (= -3.73205\ldots, -2-\sqrt{3})$ ***(A1)(A1)***

Note: Award ***A1*** for negative and ***A1*** for $\tan 75^\circ$ (or equivalent).

OR

gradient of tangent $= \tan 105^\circ \ (= -3.73205\ldots)$ ***(A2)***

THEN

$\frac{\mathrm{d}y}{\mathrm{d}x} = -5.5\sin(1.1x)$ ***(A1)***

Note: Award ***(A1)*** for a labelled sketch of the derivative function.

equating derivative to their gradient ***(M1)***

$-5.5\sin(1.1x) = -3.73205...$ **OR** line on graph

$x = 0.677993...$ ***(A1)***

Note: Award ***(A1)(M1)A0*** for an answer of $x = 38.8$, from calculator being in degrees.
Award ***A0M1A0*** if "$\frac{\mathrm{d}}{\mathrm{d}x}(5\cos(1.1x)) = -3.73205...$" seen, but leading to an incorrect x-value.

height = $= 5\cos(1.1\times 0.677993...)$ ***(M1)***

$= 3.67 \text{ (m)} \ (3.67274...)$ ***A1***

[Total 8 marks]

7. attempt at Euler ***(M1)***

$y_{n+1} = (y_n x_n - 1)\,0.1 + y_n$ ***(A1)***

x	1	1.1	1.2	1.3	1.4	1.5
y	2	2.1	2.231	2.39872	2.61055…	2.87603…

(A1)

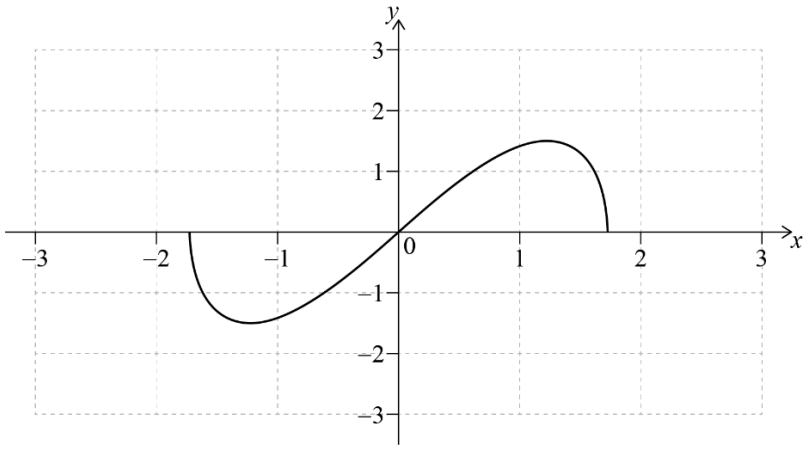
Note: Any correct value either as a separate calculation or as part of a table, (including the result).

$y = 2.88 \;\; (2.87603...)$ ***A1***

[Total: 4 marks]

(1)

10. (a)



A1A1

Note: Award ***A1*** smooth curve, through origin, with maximum in 1st quadrant and minimum in 3rd quadrant (approximately correct position).
Award ***A1*** correct (approximate) position of endpoints and rotational symmetry.
Condone a (small) gap between end of curve and the x-axis.

[2 marks]

(b) (i) $2\pi\int_0^{\sqrt{3}}\left(x\sqrt{3-x^2}\right)^2\,\mathrm{d}x$ **OR** $\pi\int_{-\sqrt{3}}^{\sqrt{3}}\left(x\sqrt{3-x^2}\right)^2\,\mathrm{d}x$ (or equivalent) ***A1A1***

Note: Award ***A1*** for integral with correct limits in the correct places, ***A1*** for completely correct expression;
$\mathrm{d}x$ must be included. Condone a correct answer presented as two integrals, despite the demand of the question: $\pi\left(\int_{-\sqrt{3}}^{0}\left(x\sqrt{3-x^2}\right)^2\,\mathrm{d}x+\int_0^{\sqrt{3}}\left(x\sqrt{3-x^2}\right)^2\,\mathrm{d}x\right)$.

(ii) $13.1\left(13.0593...,\ \frac{12\pi\sqrt{3}}{5}\right)$ ***A2***

[4 marks]

(c) attempt to express $g(x)$ in terms of $f(x)$ ***(M1)***

$g(x)=\frac{1}{2}\left(\frac{x}{2}\sqrt{3-\left(\frac{x}{2}\right)^2}\right)$, $g(x)=\frac{1}{2}f\left(\frac{x}{2}\right)$

correct limits for g seen: $-2\sqrt{3}$ and $2\sqrt{3}$ ***(A1)***

$\pi\int_{-2\sqrt{3}}^{2\sqrt{3}}\left(\frac{1}{2}\left(\frac{x}{2}\sqrt{3-\left(\frac{x}{2}\right)^2}\right)\right)^2 \mathrm{d}x$ **OR** $\pi\int_{-2\sqrt{3}}^{2\sqrt{3}}\left(\frac{1}{2}f\left(\frac{x}{2}\right)\right)^2 \mathrm{d}x$ **OR** $2\times\frac{1}{2}^2\times\left(\frac{12\pi\sqrt{3}}{5}\right)$

$6.53\ \left(6.52967\ldots, \frac{6\pi\sqrt{3}}{5}\right)$ ***A1***

Note: Some candidates may answer question by transforming their part (b)(ii): $2\times\frac{1}{2}^2\times\left(\frac{12\pi\sqrt{3}}{5}\right)$. This is a valid method and leads to the correct answer.

[3 marks]
[Total: 9 marks]

11. (a) $y = kx^2 - x$

$\frac{dy}{dx} = 2kx - 1$ ***(A1)***

$m_{\tan} = 1$ ***A1***

finding negative reciprocal of their gradient ***(M1)***

$m_{\text{normal}} = -1$

normal: $y = -\left(x - \frac{1}{k}\right)$ ***A1***

equating normal line to quadratic curve ***M1***

$kx^2 - x = -\left(x - \frac{1}{k}\right)$

correct simplification leading to given result ***A1***

e.g. quadratic formula or difference of two squares

$kx^2 - \frac{1}{k} = 0$

$k^2x^2 - 1 = 0$

$(kx - 1)(kx + 1) = 0$

$x = -\frac{1}{k}$ ***AG***

[6 marks]

(b) area $= \int_{-\frac{1}{2}}^{\frac{1}{2}} -\left(x - \frac{1}{2}\right) - (2x^2 - x)\ \mathrm{d}x$ ***(M1)(A1)***

Note: Award ***M1*** for evidence of subtracting the functions and ***A1*** for a completely correct integral. (Condone a missing $\mathrm{d}x$.)

$= 0.333\ \left(0.333333..., \frac{1}{3}\right)$ ***A1***

[3 marks]

[Total: 9 marks]

13. (a) $3\sin(a)(1+\cos(a)) = 0$ ***(M1)***

$a = 3.14 \quad (\pi,\ 3.14159\ldots)$ ***A1***

[2 marks]

(b) attempt to find the maximum velocity ***(M1)***

e.g. $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ **OR** sketch of v with maximum marked

$v_{\max} = 3.90\,(\mathrm{m\,s^{-1}})$ $(3.89711...)$ ***A1***

Note: Accept 3.9 rounded to 2 sf.

[2 marks]

(c) attempt to find integral of speed (condone omission of modulus signs) ***(M1)***

$\int_0^{\pi} \left| 3\sin(t)(1+\cos(t)) \right| \mathrm{d}t$ **OR** 6

attempt to substitute into $\text{speed} = \frac{\text{distance}}{\text{time}}$ ***(M1)***

$= \frac{\int_0^{\pi} \left| 3\sin(t)(1+\cos(t)) \right| \mathrm{d}t}{\pi}$ ***(A1)***

$= 1.91\ (\mathrm{ms^{-1}}) \quad (1.90985\ldots,\ \frac{6}{\pi})$ ***A1***

[4 marks]
[Total: 8 marks]

2. (a) heights, 0, 4, 1.75, 3 and 3.75 seen ***(A2)***

Note: Award ***A1A0*** if **two** of 1.75, 3 or 3.75 are seen.

attempt to use trapezoidal rule formula for their heights ***(M1)***

$\frac{1}{2}\times 1\times\{0+4+2(1.75+3+3.75)\}$ ***(A1)***

Note: Award ***(M1)(A1)*** for correctly expressing this as 3 trapezoids and a triangle. The "$\times 1$" need not be seen.

$=10.5\ (\text{m}^2)$ ***A1***

[5 marks]

(b) $-\frac{1}{12}x^3+x^2+c$ ***A1A1A1***

[3 marks]

(c) $\int_0^4\left(-\frac{1}{4}x^2+2x\right)\text{d}x+1\times 4+\frac{1}{2}\times 7\times 4$ ***(A1)(M1)(A1)***

Note: Award ***A1*** for correct area of rectangle **OR** triangle, ***M1*** for substituting correct limits into given integral (may be seen in part (b)), and ***A1*** for entire expression correct.

$=10.6666\ldots. + 4 + 14$

$= 28\frac{2}{3}\ (\text{m}^2)\left(\frac{86}{3}\right)$ ***A1***

Note: The answer must be **exact** for the ***A1*** to be awarded. For an answer of 28.7 or 28.66 award ***(A1)(M1)(A1)A0***.

[4 marks]

(d) (Total area using part (a) =) 28.5 ***(A1)***

Percentage error $=\left|\frac{28.5-28.6666\ldots}{28.6666\ldots}\right|\times 100$ ***(M1)***

Note: if their trapezoid value is incorrect but is used correctly in the percentage error formula, award at most ***A0M1A0***. If it is clear from the answer that ×100 has been used, then condone the omission and award the ***M*** mark.

$=0.581\ (\%)\ (0.581395\ldots)$ ***A1***

(accept 0.697 from use of 28.7)

[3 marks]

[Total: 15 marks]

2. (a) attempt to substitute $x = 5,\ y = 5$ into either of the given equations ***(M1)***

Note: Award ***M1*** for substituting $x = 500,\ y = 500$.

(i) $\frac{\mathrm{d}x}{\mathrm{d}t} = 2(5) - \frac{(5)(5)}{2}$

-2.5 (hundred geese per year) / -250 (geese per year) ***A1***

[2 marks]

(ii) $\frac{\mathrm{d}y}{\mathrm{d}t} = -3(5) + (5)(5)$

10 (hundred foxes per year) / 1000 (foxes per year) ***A1***

[1 mark]

(b) (i) **EITHER**

both $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 0$ at A ***R1***

OR

if $x = 0$ and $y = 0$ then there are no foxes or geese, so their populations cannot change ***R1***

Note: Award ***R0*** for just stating "there are no foxes or geese".

[1 mark]

(ii) attempt to put $\frac{\mathrm{d}x}{\mathrm{d}t}$ $\left(\text{or } \frac{\mathrm{d}y}{\mathrm{d}t}\right)$ equal to zero and solve ***(M1)***

$\frac{\mathrm{d}x}{\mathrm{d}t} = 0 \Rightarrow x\left(2 - \frac{y}{2}\right) = 0,\ \frac{\mathrm{d}y}{\mathrm{d}t} = 0 \Rightarrow y(-3 + x) = 0$

$p = 3$ (accept $x = 3$) ***A1***

$q = 4$ (accept $y = 4$) ***A1***

[3 marks]

(c) (i) attempt to solve by separation of variables ***(M1)***

$\int \frac{\mathrm{d}x}{x} = \int 2\mathrm{d}t$

$\ln x = 2t + c$ ***(A1)(A1)***

Note: Award ***(A1)*** for each side correct.
Award ***(A1)(A0)*** if both sides are integrated correctly but $+c$ is not seen.
Also accept $0.5\ln x = t + c$ **OR** $0.5\ln 2x = t + c$.

$x = A\mathrm{e}^{2t}$ (accept $x = \mathrm{e}^{2t + c_A}$) ***A1***

[4 marks]

(ii) $y = B\mathrm{e}^{-3t}$ (accept $y = \mathrm{e}^{-3t + c_B}$) ***A1***

Note: Accept alternative (distinct) labels for their A and B.

[1 mark]

(d) (i) **METHOD 1**

attempt to find x^3 and y^2 in terms of t
(e.g. any attempt to substitute their solutions to part (c)) ***M1***

$x^3 = A^3 e^{6t}, \quad y^2 = B^2 e^{-6t}$

$x^3 y^2 = A^3 e^{6t} \times B^2 e^{-6t}$ ***A1***

$= A^3 B^2$ ***A1***

$= k$ ***AG***

Note: There will probably be lots of students who assume what is to be proved by introducing a k earlier on. Do not penalize this.

METHOD 2

attempting to isolate t ***M1***

$e^{2t} = \frac{x}{A}$ and $e^{3t} = \frac{B}{y}$

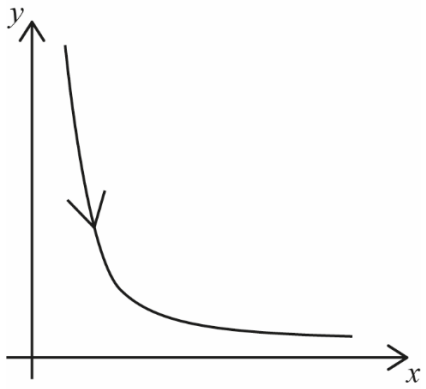
$e^{6t} = \frac{x^3}{A^3}$ **OR** $e^{6t} = \frac{B^2}{y^2}$ ***A1***

$\frac{x^3}{A^3} = \frac{B^2}{y^2}$ ***A1***

$x^3 y^2 = k$ ***AG***

[3 marks]

(ii) $y = \sqrt{\frac{k}{x^3}}$ ***(A1)***



A1A1

Note: Award ***A1*** for correct shape (i.e. decreasing, convex and not touching axes), ***A1*** for correct direction indicated.
Ignore any trajectories not in the first quadrant.
The first ***A1*** can only be implied by a correct graph.

[3 marks]

(e) attempt to substitute $x = 3 + X$, $y = 4 + Y$ **OR** $x = p + X, y = q + Y$ into $\frac{\mathrm{d}x}{\mathrm{d}t} = 2x - \frac{xy}{2}$ ***M1***

$$\dot{X} = 2(3+X) - \frac{1}{2}(3+X)(4+Y)$$

$$= 6 + 2X - \frac{1}{2}(12 + 4X + 3Y + XY)$$

$$= 6 + 2X - 6 - 2X - \frac{3}{2}Y - \frac{XY}{2}$$

$= -\frac{3Y}{2} - \frac{XY}{2}$ ***A1***

The term in XY can be ignored. ***R1***

$= -\frac{3Y}{2} + \cdots \approx -\frac{3Y}{2}$ ***AG***

Note: Follow through can be awarded for the ***M1*** and ***R1*** but not the ***A1***.

[3 marks]

(f) $\boldsymbol{M} = \begin{pmatrix} 0 & -\frac{3}{2} \\ 4 & 0 \end{pmatrix}$ ***A1***

[1 mark]

(g) setting $\begin{vmatrix} -\lambda & -\frac{3}{2} \\ 4 & -\lambda \end{vmatrix} = 0$ ***(M1)***

Note: Setting their determinant to zero can be implied by any correct root seen.

$\lambda^2 + 6 = 0$ ***(A1)***

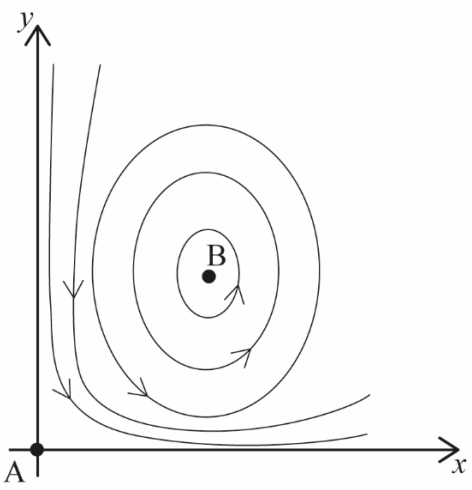
$\lambda = \pm \mathrm{i}\sqrt{6}$ **OR** $\pm 2.45i$ ($\pm 2.44948\ldots i$) **OR** noting 2 purely imaginary eigenvalues ***A1***

$\Rightarrow$ ellipse (or circle) ***R1***

Note: Accept “closed loops” or “oval” as descriptions of the trajectories.
Allow follow through for the final ***R1*** from their eigenvalues as long as they are either both real or complex conjugates. If there are two real eigenvalues the description should be a “node” and if there are two complex eigenvalues the description should be a “focus” or a “spiral”.

[4 marks]

(h)



A1A1A1

Note: Award ***A1***: At least one ellipse, oval or closed loop seen. Allow parts of ellipses shown (as in the diagram) as long as at least one complete loop observed.
Award ***A1***: Arrows showing correct direction seen on at least one loop. Must be anti-clockwise on a closed loop.
Award ***A1***: Ellipses remaining entirely in the 1st quadrant **AND** centre B marked in some way (either directly or via coordinates consistent with their B).

Do not follow through into part 2(h) from part 2(g).

[3 marks]

(i) The park-keeper is incorrect ***A1***
The phase portrait shows that the trajectory (starting at $(5, 5)$) is a closed loop (around the equilibrium point) (despite having a negative initial dx/dt in comparison with large positive $\frac{\mathrm{d}y}{\mathrm{d}t}$) ***R1***

Note: Do not award ***A1R0***.

[2 marks]

Total [31 marks]

14. (a) attempt to solve $f(x) = 0.25$ ***(M1)***

$0.302\ (0.301928\ldots)$ ***A1***

[2 marks]

(b) evidence of using chain rule (multiple of $-4x$ seen with $e^{\langle something \rangle}$) ***(M1)***

$f'(x) = -1.2x\,e^{-2x^2}$ ***A1***

[2 marks]

(c) $f'(0.302) = -0.302\ \ (-0.301928\ldots)$ ***(A1)***

finding the negative reciprocal of their gradient ***(M1)***

gradient of normal $= \dfrac{-1}{-0.302}\ \ (= 3.31)\ \ (3.312104\ldots)$

recognition that gradient equals tan **OR** finding the x-intercept of the flagpole and using right-angled triangle to find $\tan\theta$. ***(M1)***

$\theta = \arctan(3.31)$

$= 1.28\ (1.27757\ldots)$ radians $(=73.2°)\ (73.1994\ldots°)$ ***A1***

[4 marks]

[Total 8 marks]

16. (a) $0.781\ (0.781124\ldots)$ (accept $4-2\ln(5)$) ***A1***

[1 mark]

(b) recognizing need to find $\pi\int x^2\ \mathrm{d}y$ ***(M1)***

$\pi\int_{0.781}^{11.8} x^2\ \mathrm{d}y$ ***(A1)***

Note: Award ***A1*** for correct limits and squaring **their** x, seen anywhere.

attempt to make x the subject. ***(M1)***

$2\ln(5-x)=4-y$

attempt to remove ln by using exponent base e. ***(M1)***

$5-x=\mathrm{e}^{\frac{4-y}{2}}$

$x=5-\mathrm{e}^{\frac{4-y}{2}}$ ***(A1)***

$(V=)\ 631\ (\mathrm{cm}^3)\ \ (631.071\ldots)$ ***A1***

[6 marks]

[Total 7 marks]

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17. (a) substituting h and f_1 into formula for x_{n+1} **OR** h and f_2 into y_{n+1} ***(M1)***

Note: ***M1*** may be awarded for an appropriate table with at least one correct value for F or L, even if there is no evidence of substitution into the Euler equations.

$F_{n+1} = F_n + 4(0.000004F_nL_n - 0.2F_n)$ AND $L_{n+1} = L_n + 4(0.06L_n - 0.00003F_nL_n)$ ***(A1)***

evidence of correct use of Euler's method ***(A1)***

Note: Award ***A1*** for any correct value of F_{n+1} AND L_{n+1} for $t = 4, 8, \ldots$ seen (including this intermediate value presented as a final answer).

t	F	L
0	5000	80000
4	7400	51200
8	7542	18022
12	3683	6037
16	1092	4817
20	303	5342

(i) maximum value of $F = 7542$ (at $t = 8$) ***A1***

(ii) minimum value of $L = 4817$ (at $t = 16$) ***A1***

[5 marks]

(b) (i) they die out (population becomes zero) ***A1***

(ii) Any valid reason ***R1***
E.g. Euler's method is only an approximation
Other factors might change the model over time

[2 marks]
[Total 7 marks]

2. (a) $\frac{1}{2}\times 1\times\big((4+0.7)+2(7.3+6.7+4+1.3)\big)$ ***(A1)(A1)***

$=21.7\ \ (21.65)\ \ (\text{cm}^2)$ ***A1***

[3 marks]

(b) $22.1\ (22.08333\ldots, \frac{265}{12})\ \ (\text{cm}^2)$ ***A2***

[2 marks]

(c) $600=\pi r^2 h$ ***A1***

[1 mark]

(d) **METHOD 1** (Substitution of $h=\frac{600}{\pi r^2}$ in $A=2\pi r^2+2\pi rh$)

$A=2\pi r^2+2\pi rh$ ***(A1)***

attempt to isolate h or πrh ***(M1)***

$h=\frac{600}{\pi r^2}$ **OR** $\pi rh=\frac{600}{r}$

correct substitution of their h into correct expression ***(M1)***

$A=2\pi r^2+2\pi r\left(\frac{600}{\pi r^2}\right)\quad\left(A=2\pi r^2+\frac{1200}{r}\right)$

$k=1200$ ***A1***

METHOD 2 (Equating $2\pi rh$ and $\frac{k}{r}$)

$A=2\pi r^2+2\pi rh$ **OR** $2\pi rh=\frac{k}{r}$ ***(A1)***

attempt to isolate h or $(\pi)rh$ or r or r^2 ***(M1)***

$h=\frac{600}{\pi r^2}$ **OR** $\pi rh=\frac{600}{r}$ **OR** $r=\sqrt{\frac{600}{\pi h}}$ **OR** $r^2=\frac{600}{\pi h}$

correct substitution of their h or πrh or r or r^2 into $2\pi r^2+2\pi rh=2\pi r^2+\frac{k}{r}$ ***(M1)***

$2\pi r\left(\frac{600}{\pi r^2}\right)=\frac{k}{r}$ **OR** $2\left(\frac{600}{r}\right)=\frac{k}{r}$ **OR** $2(600)=k$ **OR** $2\pi\left(\frac{600}{\pi h}\right)h=k$

$k=1200$ ***A1***

[4 marks]

continue…

Question 2 continued.

(e) (i) $\frac{\mathrm{d}A}{\mathrm{d}r} = 4\pi r - 1200r^{-2}$ ***A1(M1)A1***

Note: Award ***A1*** for $4\pi r$ seen, and ***(M1)*** for expressing $\frac{1200}{r}$ as $1200r^{-1}$ (can be implied through $\mp\frac{1200}{r^2}$ seen), ***A1*** for $-1200r^{-2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

(ii) $0 = 4\pi r - 1200r^{-2}$ **OR** $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ ***(M1)***

$r = 4.57\left(4.570781\ldots, \sqrt[3]{\frac{300}{\pi}}\right)\ (\mathrm{cm})$ ***A1***

Note: Award at most ***M1A0*** if the final answer is in terms of k.

[5 marks]

(f) $(h =)\ 9.14\ (9.14156\ldots..)\ (\mathrm{cm})$ ***A1***

$(C =)\ 2\pi\ (4.570781\ldots.) \approx 28.7\ \left(28.7190\ldots\right)(\mathrm{cm})$ ***A1***

EITHER

the longest dimension of the label $(9\ \mathrm{cm})$ is less than both values and hence the label will fit (in any rotation) ***R1***

OR

$9 < 9.14$ and $5 < 28.7$ ***R1***

Note: Do not accept an argument based on the comparison of areas.

[3 marks]

[Total 18 marks]

7. (a) $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}^2x}{\mathrm{d}t^2}$

A1

$\frac{\mathrm{d}y}{\mathrm{d}t} + ay + bx = 0$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}t} = -bx - ay$ ***AG***

[2 marks]

(b) $\begin{pmatrix} 0 & 1 \\ -77 & -18 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -7\begin{pmatrix} x \\ y \end{pmatrix}$ **OR** $\begin{pmatrix} 0 & 1 \\ -77 & -18 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -11\begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$y = -7x$

eigenvector is $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$ (or any multiple) ***A1***

$y = -11x$

eigenvector is $\begin{pmatrix} 1 \\ -11 \end{pmatrix}$ (or any multiple) ***A1***

[3 marks]

(c) (i) $\begin{pmatrix} x \\ y \end{pmatrix} = A\mathrm{e}^{-7t}\begin{pmatrix} 1 \\ -7 \end{pmatrix} + B\mathrm{e}^{-11t}\begin{pmatrix} 1 \\ -11 \end{pmatrix}$ ***(A1)***

substitution of initial values ***(M1)***

two correct equations (not in vector form) ***(A1)***

$5 = A + B,\ 2 = -7A - 11B$

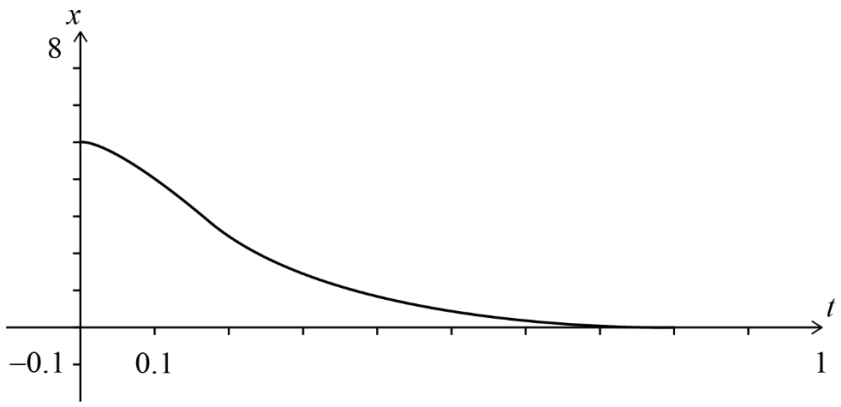
$A = \frac{57}{4},\ B = -\frac{37}{4}$ ***(A1)***

$x = \frac{57}{4}\mathrm{e}^{-7t} - \frac{37}{4}\mathrm{e}^{-11t}$ ($x = 14.25\mathrm{e}^{-7t} - 9.25\mathrm{e}^{-11t}$) ***A1***

continue…

Question 7 continued.

(ii)



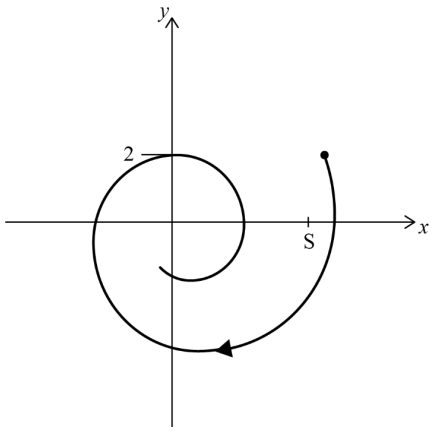
decreasing curve ***A1***

Note: There is a maximum at $t = 0.00496$, but this does not need to be shown.

starting at $x=5$ and asymptote at $x = 0$ ***A1***

[7 marks]

(d) (i)



spiral towards origin ***A1***

starting at $(5, 2)$ ***A1***

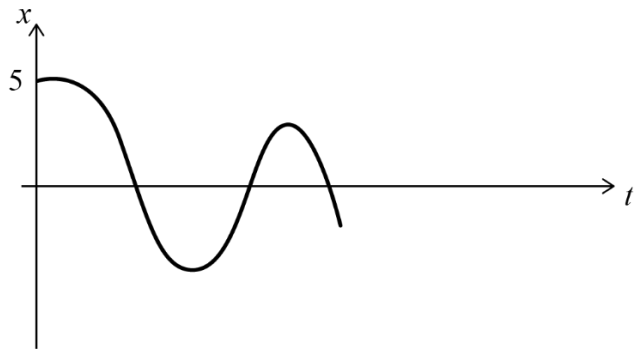
$\frac{dy}{dt} = -85(5) - 18(2) = -461 < 0$

Clockwise spiral ***A1***

continue…

Question 7 continued.

(ii)



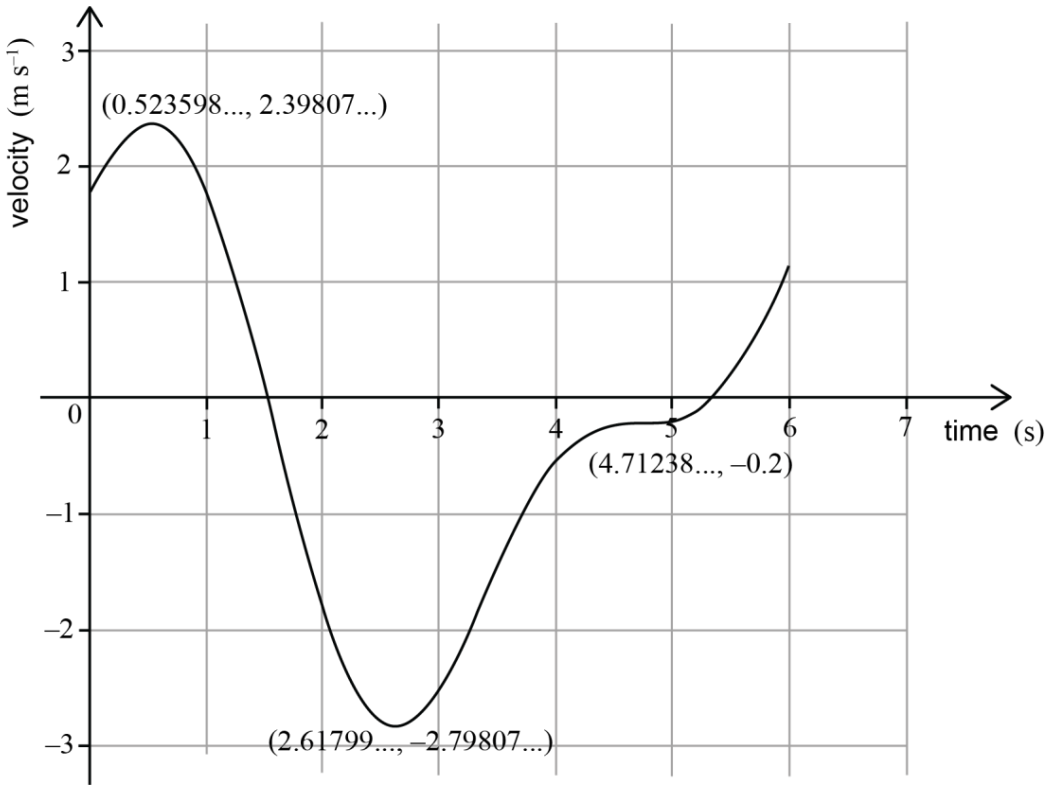
oscillations ***A1***
starting at $x = 5$, decreasing amplitude ***A1***
[5 marks]

(e) spring 1 is better as it gives a smoother ride (less oscillations, less bouncy) ***R1***
[1 mark]
[Total:18 marks]

7. (a) (i) correct general shape ***A1***

curve begins at $(0, 1.8)$ and does not extend beyond $t = 6$ ***A1***

Note: Do not award the second ***A1*** if the first ***A1*** is not awarded.



(ii) $(0.524, 2.40)$ ***A1***

$(2.62, -2.80)$ ***A1***

$(4.71, -0.2)$ ***A1***

Note: Award ***A1*** for each correct coordinate. Award marks even if the labels are not shown on the graph near the points.

[5 marks]

(b) (i) **EITHER**

$\int_0^6 \left| 2\cos t + \sin 2t - 0.2 \right| \mathrm{d}t$ ***A1A1***

Note: Award ***A1*** for correct boundaries and use of $\mathrm{d}t$ and ***A1*** for completely correct expression.

OR

$\left| \int_0^{1.52074..} \left(2\cos t + \sin 2t - 0.2\right)\mathrm{d}t \right| + \left| \int_{1.52074..}^{5.31535\ldots} \left(2\cos t + \sin 2t - 0.2\right)\mathrm{d}t \right| + \left| \int_{5.31535\ldots}^{6} \left(2\cos t + \sin 2t - 0.2\right)\mathrm{d}t \right|$

A1A1

Note: Award ***A1*** for $1.52074...$ and $5.31535\ldots$ as boundaries and ***A1*** for completely correct expression.
Allow + - + instead of modulus signs

(ii) 7.76 $(7.76487\ldots)$ (m) ***A1***

[3 marks]

[Total 8 marks]

8. (a) $f(x) = 0$ seen **OR** sketch of the graph with all three x-intercepts marked **OR** evidence of using sum or product of roots ***(M1)***

$(a =)\ 3$ ***A1***

[2 marks]

(b) evidence of using an integral to find the area ***(M1)***

$\int f(x), \int_1^3$, etc.

(area $=$) 4 ***A1***

[2 marks]

(c) attempt to equate two integrals ***(M1)***

$$\int_1^b f(x)\mathrm{d}x = \int_1^b g(x)\mathrm{d}x$$

evidence of using their GDC to solve the equation **OR** attempt to expand both sides of the equation into expressions in terms of b ***(M1)***

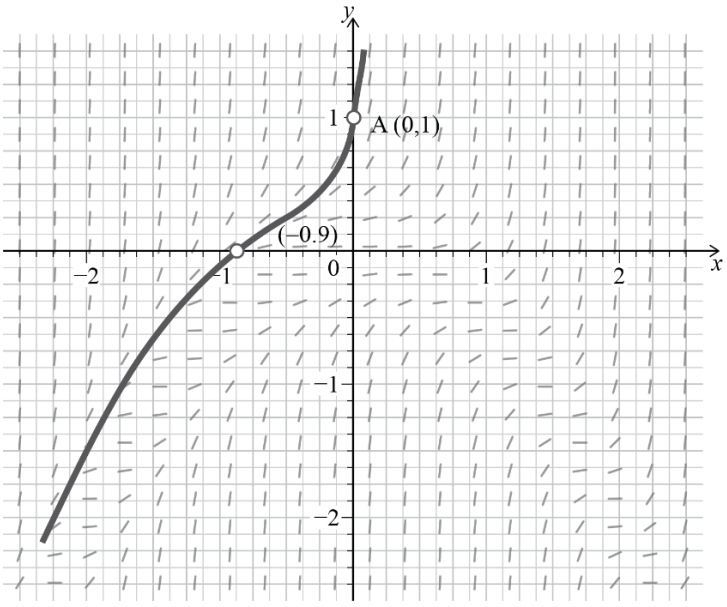
$(b =)\ 2.57\ \ (2.56949\ldots)$ ***A1***

[3 marks]

[Total 7 marks]

12. (a) correct shape that goes through point $(0, 1)$ ***A1***

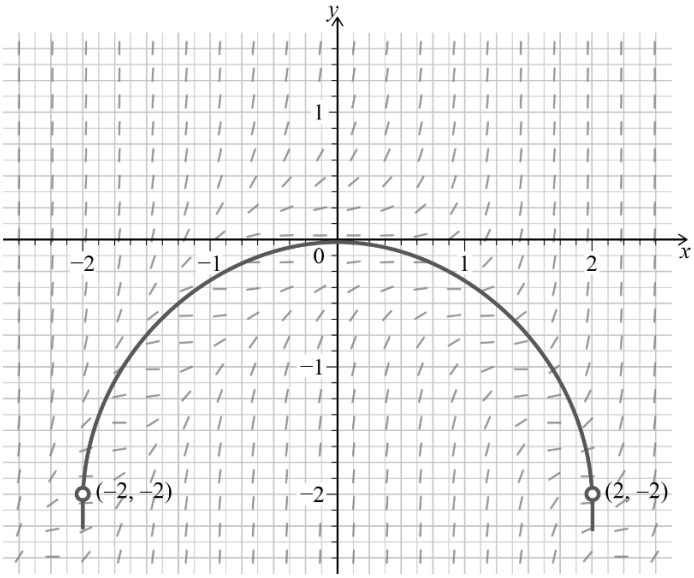
crosses the x-axis is at $x = -0.9$ (± 1 small grid box) ***A1***



[2 marks]

(b) correct shape with maximum at $(0, 0)$ ***A1***

smooth curve that goes approximately through points $(-2, -2)$ and $(2, -2)$ ***A1***



[2 marks]

(c) **EITHER**

substituting $y = ax^2$ into differential equation ***(M1)***

$0 = x^4 + 4x^2(ax^2) + 4(ax^2)^2$

choosing an appropriate method to solve for a ***(M1)***

$0 = x^4(1 + 4a + 4a^2)$

$a = -\frac{1}{2}$ ***A1***

OR

$x^4 + 4x^2y + 4y^2 = (x^2 + 2y)^2$ ***(M1)***

$(x^2 + 2y)^2 = 0$ ***(M1)***

$y = -\frac{1}{2}x^2$ (so $a = -\frac{1}{2}$) ***A1***

[3 marks]

[Total 7 marks]

15. (a) (i) $x_{n+1} = x_n + 0.1\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_n$

$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{n+1} = \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_n + 0.1\left(\frac{\mathrm{d}^2x}{\mathrm{d}t^2}\right)_n$ ***(M1)***

$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 4.9 - 1.2\frac{\mathrm{d}x}{\mathrm{d}t} - 4x$ ***(A1)***

$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_{n+1} = \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)_n + 0.1\left(4.9 - 1.2\frac{\mathrm{d}x}{\mathrm{d}t} - 4x\right)_n$ ***(A1)***

(when $t = 1$ or $n = 10$) $x = 4.36$ $(4.36332\ldots)$ (m) ***A1***

(ii) $(t =)$ 2 (seconds) ***A1***

Note: Award ***A0*** for either $n = 20$ or $t = 20$.

[5 marks]

(b) $x = 3.845$ seen **OR** $x = 3.855$ seen ***(A1)***

substitution into percentage error formula using their value for part (a)(i) and 3.845 or 3.855 or 3.85 ***(M1)***

$$\left|\frac{4.36332\ldots - 3.845}{3.845}\right| \times 100$$

Note: Accept methods using centimetres instead of metres as long as both values use the same unit.

13.5 $(13.4803\ldots)(\%)$ ***A1***

[3 marks]

[Total 8 marks]

7. (a) a valid/coherent attempt to use the substitution method **(*M1*)**

$u = (-)x^2$

$\frac{du}{dx} = (-)2x$ **(*A1*)**

Note: To award ***M1***, both $u =$ and $\frac{du}{dx} =$ must be seen.

integral becomes

$\frac{1}{2}\int e^{-(u)}\,du$ OR $-\frac{1}{2}\int e^{u}\,du$ **(*M1*)**

$-\frac{1}{2}e^{-x^2} + c$ ***A1***

Note: Award ***M1A1M1A0*** for a final answer of $-\frac{1}{2}e^{-x^2}$ with c omitted.

Accept integration by inspection, e.g. using $\frac{d}{dx}\left(e^{-x^2}\right) = -2xe^{-x^2}$

[4 marks]

(b) recognition that 0 is the lower limit ***(A1)***

substituting both 0 and k into previous expression **(*M1*)**

$\int_0^k xe^{-x^2}\,dx = \left[-\frac{1}{2}e^{-x^2}\right]_0^k$

$= \frac{1}{2} - \frac{1}{2}e^{-k^2}$ ***A1***

[3 marks]

Total [7 marks]

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9. (a) **METHOD 1**

$A = xy$ ***A1***

attempt to use product rule ***M1***

$$\frac{\mathrm{d}A}{\mathrm{d}t} = x\frac{\mathrm{d}y}{\mathrm{d}t} + y\frac{\mathrm{d}x}{\mathrm{d}t}$$

$75\times(-5)+40\times10$

$= 25$ (km^2/year) ***A1***

METHOD 2

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 10, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = -5$$

$A = (10t+75)(40-5t)$ ***A1***

attempt to differentiate ***M1***

$\frac{\mathrm{d}A}{\mathrm{d}t} = 10(40-5t) - 5(10t+75)$ OR $\frac{\mathrm{d}A}{\mathrm{d}t} = -100t + 25$

$\left(\text{at } t = 0, \ \frac{\mathrm{d}A}{\mathrm{d}t}\right) = 25$ (km^2/year) ***A1***

[3 marks]

(b) increasing since the sign of $\frac{\mathrm{d}A}{\mathrm{d}t}$ is positive ***R1***

[1 mark]

(c) new x is 85 and new y is 35, therefore new A is 2975 ***A1***

old A was 3000 ***A1***

therefore (a decrease of) $25\ (\mathrm{km}^2)$ ***A1***

Note: Accept an answer of $-25\ (\mathrm{km}^2)$

[3 marks]

Total [7 marks]

12. (a) $\frac{\mathrm{d}v}{\mathrm{d}t} = kv^{1.5}$ ***A1***

[1 mark]

(b) using initial condition to find an equation in k ***M1***

$-3 = k \times 8$

$k = -\frac{3}{8}\ \left(-0.375\right)$ ***A1***

attempt to separate variables ***M1***

$\int v^{-1.5}\,\mathrm{d}v = \int -\frac{3}{8}\mathrm{d}t$

$2v^{-0.5} = \frac{3}{8}t + c \quad \left(2v^{-0.5} = 0.375t + c\right)$ ***A1A1***

Note: Allow negative signs on each side.
Award ***A1*** for each integral

using initial conditions to find constant of integration ***M1***

$c = 1$ ***A1***

Note: Might be a different value of c depending on rearrangement.

$v = \frac{1}{\left(\frac{3}{16}t + \frac{1}{2}\right)^2} \quad \left(= \frac{256}{\left(3t+8\right)^2}\right) \quad \left(= \frac{1}{\left(0.1875\,t + 0.5\right)^2}\right)$ OE ***A1***

[8 marks]
Total [9 marks]

15. (a) attempt to find gradient **(*M1*)**

$\frac{dy}{dx} = 2x$

gradient of tangent is 4

gradient of normal is $-\frac{1}{4}$ **(*A1*)**

equation of normal is $y - 4 = -\frac{1}{4}(x-2)$ $\left(y = -\frac{1}{4}x + \frac{9}{2}\right)$ OE ***A1***

[3 marks]

(b) attempt to find gradient ***M1***

gradient is $\frac{10-4}{0-2} = -3$ ***A1***

[2 marks]

(c) **METHOD 1**

$\theta_1 = \arctan(-3)$ $\left(\approx -1.249, -71.57^\circ\right)$ ***A1***

$\theta_2 = \arctan(-\frac{1}{4})$ $\left(\approx -0.2449, -14.04^\circ\right)$ ***A1***

Note: Award ***A1A1*** for $\arctan(3) - \arctan\left(\frac{1}{4}\right)$ OR $\arctan(-3) - \arctan\left(-\frac{1}{4}\right)$ seen

OR $\arctan\left(\frac{-3-\left(-\frac{1}{4}\right)}{1+(-3)\times\left(-\frac{1}{4}\right)}\right)$ OE.

difference between them is $1.00\ (1.00407\ldots)$ radians (57.5°) $(57.5288...^\circ)$ ***A1***

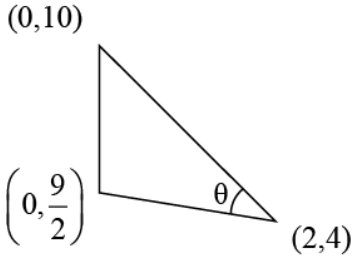
Note: Accept an obtuse angle between the normal and the ray. $2.14\ (2.13752...)$ radians, $122^\circ\ (122.471...^\circ)$.

continued...

Question 15 continued

METHOD 2

a point on the normal is $\left(0, \frac{9}{2}\right)$ ***A1***



$\cos\theta = \frac{2^2 + 6^2 + 2^2 + (0.5)^2 - (5.5)^2}{2\sqrt{4^2 + 1^2}\sqrt{1^2 + 3^2}}$ ***A1***

$\theta = 1.00\ (1.00407\ldots)\text{radians}\ \ (57.5^\circ)\ \ (57.5288...^\circ)$ ***A1***

Note: Other points on the normal such as $(18, 0)$ may be seen

Note: Accept an obtuse angle between the normal and the ray. $2.14\ (2.13752...)$ radians, $122^\circ\ (122.471...^\circ)$.

METHOD 3

writing the ray and normal as vectors

$\begin{pmatrix}2\\4\end{pmatrix} + \lambda\begin{pmatrix}-4\\1\end{pmatrix}$ and $\begin{pmatrix}2\\4\end{pmatrix} + \mu\begin{pmatrix}-1\\3\end{pmatrix}$ ***A1***

the angle between them is given by

$\cos\theta = \frac{\begin{pmatrix}-4\\1\end{pmatrix}.\begin{pmatrix}-1\\3\end{pmatrix}}{\sqrt{(-4)^2 + 1^2}\sqrt{(-1)^2 + 3^2}}$ ***A1***

$\theta = 1.00\ (1.00407\ldots)\text{radians}\ \ (57.5^\circ)\ \ (57.5288...^\circ)$ ***A1***

Note: Accept an obtuse angle between the normal and the ray. $2.14\ (2.13752...)$ radians, $122^\circ\ (122.471...^\circ)$.

[3 marks]
Total [8 marks]

Q9 (a) attempt to use product rule, two terms with one term correct ***(M1)***

attempt to use the chain rule to find the derivative of $\sqrt{3x-2}$ ***(M1)***

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 4(3x-2)^{0.5} + 4x \times \frac{3}{2}(3x-2)^{-0.5}$$ ***A1***

$$\left(= 4(3x-2)^{0.5} + 6x(3x-2)^{-0.5}\right)\left(= \frac{2(9x-4)}{\sqrt{3x-2}}\right)$$

[3 marks]

(b) substitution of $x = 6$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ ***(M1)***

gradient of tangent = $(16+9=)25$ ***A1***

Note: Follow through for this ***A1*** from their $\frac{\mathrm{d}y}{\mathrm{d}x}$.

at $(6, 96)$ tangent is

EITHER

$y - 96 = 25(x-6)$ ***A1***

OR

$96 = 25 \times 6 + c$ ***A1***

Note: Do not allow follow through for this ***A1***.

THEN

$y = 25x - 54$ ***AG***

[3 marks]

Total [6 marks]

Q11. (a) C ***A1***

[1 mark]

(b) attempting to substitute values into the correct formulae ***(M1)***

$x_{n+1} = x_n + 0.1(3x_n - 2y_n)$, $y_{n+1} = y_n + 0.1(4x_n - y_n)$ ***(A1)***

$(t_{n+1} = t_n + 0.1)$

any correct value for x_{n+1} and y_{n+1} for $n = 2, 3, ...$ ***(A1)***

n	t	x	y
0	5	1	0
1	5.1	1.3	0.4
2	5.2	1.61	0.88
3	5.3	1.917	1.436
4	5.4	2.2049	2.0592

$x = 2.45$ $(2.45453...)$ ***A1***

Note: Accept $x = 2.455$ from rounding to 4 sf.

$y = 2.74$ $(2.73524...)$ ***A1***

[5 marks]

Total [6 marks]

Q13. (a) $\frac{\mathrm{d}P}{\mathrm{d}t} = 0.2(1-P)$

attempt to separate variables ***M1***

$\int \frac{1}{1-P}\mathrm{d}P = \int 0.2\mathrm{d}t$

attempt to integrate left hand side by substitution or inspection. ***(M1)***

Note: The ***(M1)*** may be awarded if their LHS is $\ln(1-P)$ without the minus.

$-\ln(1-P) = 0.2t(+c)$ ***A1A1***

Note: Modulus sign not required as $0 \leq P < 1$.

EITHER

substitution of $P = 0$ when $t = 0$ ***M1***

$-\ln(1-0) = 0.2(0) + c \Rightarrow c = 0$ ***A1***

eliminating the natural logarithm ***M1***

$1 - P = \mathrm{e}^{-0.2t}$

OR

eliminating the natural logarithm ***M1***

$1 - P = \mathrm{e}^{-0.2t-c}$ or $1 - P = A\mathrm{e}^{-0.2t}$

substitution of $P = 0$ when $t = 0$ ***M1***

$1 = \mathrm{e}^{-c} \Rightarrow c = 0$ OR $1 = A$ ***A1***

THEN

$P = 1 - \mathrm{e}^{-0.2t}$ ***AG***

[7 marks]

(b) attempt to solve $0.9 = 1 - \mathrm{e}^{-0.2t}$ ***(M1)***

11.5 (days) (11.5129…) ***A1***

[2 marks]

Total [9 marks]

Q16. (a) $g(x) = 4\cos\left(\frac{\pi}{8}(x+0.3)\right) - 0.2$ ***A1A1***

Note: Accept $g(x) = f(x+0.3) - 0.2$

[2 marks]

(b) $3.57\ (3.57262\ldots)$ ***A1***

[1 mark]

(c) (i) use of formula for volume in at least one integral ***M1***

Note: In this formula condone one of:
missing or incorrect limits, absence of π, absence of $\mathrm{d}x$ for the ***M1.***

subtracting two volumes ***M1***

$\frac{\pi}{2}\int_0^4 \left(4\cos\left(\frac{\pi}{8}x\right)\right)^2 \mathrm{d}x - \frac{\pi}{2}\int_0^{3.57262\ldots} \left(4\cos\left(\frac{\pi}{8}(x+0.3)\right) - 0.2\right)^2 \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for each completely correct expression (including the $\mathrm{d}x$).
Award ***A1A0*** for everything correct except not dividing by two.

(ii) $\frac{\pi}{2}(32 - 23.7729\ldots)$

$12.9\ \text{cm}^3\ (12.9229\ldots)$ ***A1***

Note: Do not award ***A1FT*** follow through for this part.

[5 marks]
Total [8 marks]

3. (a) evidence of using the values $0,20,40,60,\ldots$ for their x data ***(M1)***

$f(x) = -0.0989x^2 + 45.9x - 67.8$ ***A2***

$\left(f(x) = -0.0989285\ldots x^2 + 45.925x - 67.7857\ldots\right)$

Note: Accept models using full population, $f(x) = -98.9285\ldots x^2 + 45925x - 67785.7\ldots$

Award ***(M0)A2*** for $f(x) = -0.0989285\ldots x^2 + 421.853\ldots x - 444457$ or $f(x) = -98.9285\ldots x^2 + 421853.5\ldots x - 444457428.6\ldots$ from finding regression without converting years. Award at most ***(M1)A1A0*** for a correct model with one rounding error or misread. Award at most ***(M1)A0A0*** for models with multiple rounding errors. Award at most ***(M1)A1A0*** if the answer is not given in equation form.

[3 marks]

(b) **EITHER**

R^2 is (very) close to 1 **OR** 0.98843 is (very) close to 1 ***R1***

OR

R^2 shows that the model explains 98.843% of the total variation ***R1***

THEN

therefore, it supports Ethan's proposal ***A1***

Note: Do not award ***R0A1***. Accept "Yes" for the A mark as long as reasoning is correct. To award the R1, a comparison to 1 must be seen if not using the percentage method.

[2 marks]

(c) **EITHER**

the negative value of c is unrealistic as you cannot have a negative population ***R1***

OR

the negative value of a is unrealistic because it suggests that the population of Los Angeles will decrease back to 0 at some point in the future / be negative in the future / was negative before 1900 ***R1***

[1 mark]

(d) (i) $\left(f'(110) =\right) 24.2\ (24.1607\ldots)$ ***A1***

Note: Accept $24.1(24.142)$ from use of 3sf values in model. Accept $400(400.089\ldots)$ or $400{,}000\ (400{,}089.28\ldots)$ from use of a model from part (a) where years were not converted.

(recognition that value predicts the) annual growth (in 2010 as) $24{,}200(24{,}160.7\ldots)$ per year **OR** $24.2(24.1607\ldots)$ thousands per year ***A1***

Note: Allow FT within question part for the final A mark for a correct interpretation of their incorrect value for $f'(110)$. Do not follow through for interpreting a value for $f(110)$.

(ii) **EITHER**
population growth is clearly slowing down / only averaged a growth of $10\,700$ per year between 2000 and 2020 ***A1***

validity questioned since the model's predicted growth per year in 2010 is significantly higher ***R1***

OR
total population change between 2000 and 2020 is $214,000$ and the predicted change using $f'(110)$ over that time frame $483,200$ ***A1***

validity supported since prediction is in the correct order of magnitude / low percentage error for the predicted population of 2020. ***R1***

Note: For either of the arguments, the response must include a comparison of their answer for part (d)(i) with specific values from the actual data. FT marks can only be awarded in this question part if value if $15 < f'(110) < 30$. .

Do not award FT marks in this question part for responses that include a model where the years were not converted as their value in (d)(i) would tell them the population rate of change in the year 110.

[4 marks]

(e) the model predicts the slowing growth rate of the population over the long term / the population is trending towards a carrying capacity ***R1***
[1 mark]

(f) (i) attempt to differentiate $g(x)$ using chain/quotient rule ***(M1)***

$$\left(g'(x) = \right)\frac{2800e^{-0.05x}}{\left(1+14e^{-0.05x}\right)^2}$$ ***A1A1***

(ii) recognition to find maximum of $g'(x)$ ***(M1)***

(maximum occurs when $x =) \ 52.7806\ldots$ ***A1***

(therefore, greatest growth rate occurred during the year) 1952 ***A1***

Note: FT within the question part for the final A1 as long as the candidate's given value for x is not a whole number (must be evidence of rounding down to match the context of the problem).

[6 marks]
[Total 17 marks]

6. (a) $\frac{\mathrm{d}h}{\mathrm{d}t} = k \times \frac{h}{v}$ ***(A1)***

attempt to substitute given values into their equation with a constant of proportionality ***M1***

(When $t = 0$) $-5 = k \times \frac{1350}{27}$ **OR** $-5 = k \times \frac{1350}{27000}$

$k = -\frac{1}{10}$ ***A1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h}{10v}$ ***AG***

[3 marks]

(b) $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h}{10(27+0.3t)}$ ***(A1)***

Note: Follow through within question part for all remaining marks for responses that lose the initial A mark here from omitting the 10 $\left(\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{h}{27+0.3t}\right)$

attempt to separate variables to setup integration ***(M1)***

$\int \frac{1}{h}\mathrm{d}h = \int -\frac{1}{10(27+0.3t)}\mathrm{d}t$ ***(A1)***

EITHER

$\ln h = -\frac{1}{3}\ln(27+0.3t) + c$ ***A1***

Note: This ***A1*** can be awarded even if $+c$ is not seen.

$h = A(27+0.3t)^{-\frac{1}{3}}$ ***(A1)***

$A = 4050$ (from using $t = 0$ and $h = 1350$) ***(A1)***

$h = 4050(27+0.3t)^{-\frac{1}{3}}$ ***A1***

Note: The follow through answer here for those that omit 10 in the first step is.

$h = 79716150\left(27+0.3t\right)^{-\frac{10}{3}}$

OR

$\ln h = -\frac{1}{3}\ln(270+3t)+c$ ***A1***

Note: This ***A1*** can be awarded even if $+c$ is not seen.

$h = A(270+3t)^{-\frac{1}{3}}$ ***(A1)***

$A = 4050\sqrt[3]{10}$ $(8725.46\ldots)$ (from using $t=0$ and $h=1350$) ***(A1)***

$h = 8730(270+3t)^{-\frac{1}{3}}$

$\left(h = 8725.46\ldots(270+3t)^{-\frac{1}{3}},\ 4050\sqrt[3]{10}(270+3t)^{-\frac{1}{3}}\right)$ ***A1***

[7 marks]

(c) attempt to setup equation or inequality comparing the number of virus particles to the number of healthy cells ***(M1)***

recognition to multiply expression for v by 1000 **OR** dividing expression for h by 1000 ***(M1)***

$1000(27+0.28t) \geq 100\left(4380(27+0.28t)^{-0.36}\right)$ ***(A1)***

Note: Accept an equation.

$t = 216$ $(216.253\ldots)$ (hours) ***A1***

Note: Accept 217 as a contextual answer. Award at most ***(M1)(M0)A0A1FT*** for an answer of $50100\left(50136.06\ldots\right)$ or 50137 from correct solution of inequality where the candidate has not appreciated the 1000.

[4 marks]
[Total 14 marks]

7. (a) **METHOD 1**

attempt to represent area under curve with integral ***(M1)***

$\int_{0.1}^{1} \frac{2}{3}\cos^{-1}\left(\frac{1}{10x}\right)\mathrm{d}x$ ***(A1)***

$0.780871\ldots$ seen ***(A1)***

$1-0.780871\ldots$

$0.219\ (0.219128\ldots)\ (\mathrm{m}^2)$ ***A1***

METHOD 2
recognition to split area into three areas ***(M1)***

$(1\times(1-0.980419\ldots)=)\ 0.0195807\ldots$ seen **AND**
$(0.1\times 0.980419\ldots=)\ 0.0980419\ldots$ seen ***(A1)***

$\int_{0.1}^{1}\left(0.980419\ldots-\frac{2}{3}\cos^{-1}\left(\frac{1}{10x}\right)\right)\mathrm{d}x$ ***(M1)***

$0.0195807\ldots+0.0980419\ldots+0.101505\ldots$

$0.219\ (0.219128\ldots)\ (\mathrm{m}^2)$ ***A1***

METHOD 3
attempt to rearrange to make x a function of y ***(M1)***

$x=\frac{1}{10\cos\left(\frac{3y}{2}\right)}$

Integral upper bound $\frac{2}{3}\cos^{-1}(0.1)=0.980419$ seen ***(A1)***

attempt to represent area to left of curve with sum of integral **AND** rectangle area ***(M1)***

$\int_{0}^{0.980419\ldots}\frac{1}{10\cos\left(\frac{3y}{2}\right)}\mathrm{d}y+1\times(1-0.980419\ldots)$

Note: Must see attempt at both an integral and a rectangle area to award ***M1A0***.

$0.219\ (0.219128\ldots)\ (\mathrm{m}^2)$ ***A1***

[4 marks]

(b) recognition of rectangles forming cylinders when rotated ***(M1)***

Note: Evidence of recognition includes:
- the word 'cylinder' written in work
- $\pi r^2 h$ seen anywhere
- $\pi(1)^2$ or $\pi(0.3)^2$ multiplied to anything

attempt to set up integral for middle section ***(M1)***

Note: Attempt includes any integral that includes π and squaring some form of the given expression, regardless of whether they have attempted to rearrange to solve for x. It is also not necessary to see the $\mathrm{d}y$ or proper limits here for the M1.

attempt to rearrange expression to solve for x (possibly seen in part(a)) ***(M1)***

Note: If this is seen in part (a), it must be seen within an integral here to award the mark.

value of $0.980419\ldots$ or $0.0195807\ldots$ seen anywhere (possibly seen in part (a)) ***A1***

correct volume expression seen for one of the cylinders ***A1***

fully correct expression ***A1***

$$(V =)\ \pi(1)^2(0.0196) + \int_0^{0.980} \pi \left(\frac{1}{10\cos\left(\frac{3y}{2}\right)} \right)^2 \mathrm{d}y + \pi(0.3)^2(0.075)$$

$$\left(\pi(1)^2(0.0195807\ldots) + \int_0^{0.980419\ldots} \pi \left(\frac{1}{10\cos\left(\frac{3y}{2}\right)} \right)^2 \mathrm{d}y + \pi(0.3)^2(0.075) \right)$$

[6 marks]

(c) $0.291\ (0.291110\ldots)\ (\mathrm{m}^3)$ ***A2***

Note: FT from their expression in part (b) as long as their expression contains three parts and their final value is positive.

[2 marks]
[Total 12 marks]

2. (a) $0.638\ (0.637623...)$ and $4.10\ (4.09811...)$ ***A2A1***

Note: Award ***A2*** for one correct answer, accept 4.1. Award ***A2A0*** if coordinate pair is given.

[3 marks]

(b) (i) $\int_{0.638}^{4.1} 1.5^x \,\mathrm{d}x$ OR $\int_{0.638}^{4.1} f(x)\,\mathrm{d}x$ ***A1A1***

Note: ***A1*** for function and $\mathrm{d}x$, ***A1*** for limits. Award ***A1*** for $\int_{0.638}^{4.1} y\mathrm{d}x$.
Condone the use of modulus within the integrand.

(ii) $9.81\ (9.80788...)$ (from 3sf)

$9.80\ (9.79844...)$ (from full) ***A2***

Note: Accept 9.8 for ***A2***.

(iii) correct expression for area ***(A1)***

$\int_{0.638}^{4.1} g(x) - f(x)\,\mathrm{d}x$ **OR** $\int_{0.638}^{4.1} 6 - \frac{3}{x} - 1.5^x \,\mathrm{d}x$

$5.38\ (5.38290...)$ ***A1***

Note: Accept 5.39

[6 marks]

(c) $f'(x) = g'(x)$ ***(M1)***

correct graphs of $f'(x)$ and $g'(x)$ ***(M1)***

Note: Award ***M1*** if $\frac{3}{x^2}$ seen.

$(k =)1.86\ (1.86406...)$ ***A1***

[3 marks]
Total [12 marks]

3. (a) $(A=)2\pi r^2+2\pi rh$ ***A1A1***

Note: Award ***A1*** for either term correct (even if unsimplified) and ***A1*** for all correct (including simplification and no extra term).

[2 marks]

(b) $V=\frac{2\pi r^3}{3}+\pi r^2h$ ***A1A1***

$V=\frac{2\pi r^3+3\pi r^2h}{3}$ ***AG***

Note: Award ***A1*** for one correct term, ***A1*** for both and completion to required form.

[2 marks]

(c) 10000 substituted into (b) ***M1***

correct rearrangement with at least one line of intermediate working leading to the given value.

M1

$h=\frac{30000-2\pi r^3}{3\pi r^2}$ ***AG***

[2 marks]

(d) h substituted into their A ***M1***

$(A=)2\pi r^2+2\pi r\left(\frac{30000-2\pi r^3}{3\pi r^2}\right)$

further consistent rearrangement that leads to **given** A ***A1***

[2 marks]

(e) $\frac{4\pi r}{3}-\frac{20000}{r^2}$ ***A1A1A1***

Note: Award ***A1*** for $\frac{4\pi r}{3}$, ***A1*** for -20000 seen, and ***A1*** for a correct index for r in second term.

[3 marks]

(f) $\frac{\mathrm{d}A}{\mathrm{d}r}=0$ ***(M1)***

EITHER

rearrangement leading to $r^3=\frac{15000}{\pi}$ ***(M1)***

OR

graph of $\frac{\mathrm{d}A}{\mathrm{d}r}$ ***(M1)***

THEN

$r=16.8\ (16.8389...)$ OR $\sqrt[3]{\frac{15000}{\pi}}$ ***A1***

Note: Accept correct graph of Area function leading to correct minimum value for r for ***M1M1A1***.

$h=0$ ***A1***

[4 marks]

(g) hemisphere ***A1***

Note: Do not ***FT*** from incorrect answers to part (f).

[1 mark]

Total [16 marks]

6. (a) associated matrix is $\begin{pmatrix} 0 & 1 \\ -3 & -4 \end{pmatrix}$ **(A1)**

attempt to find eigenvalues **(M1)**

$-\lambda(-4-\lambda)+3=0$

eigenvalues are -3 and -1 **(A1)(A1)**

therefore the general solution for x is $Ae^{-3t}+Be^{-t}$ **A1**

[5 marks]

(b) (i) using initial condition for x, $A+B=0$ **A1**

attempting to find derivative of general solution **(M1)**

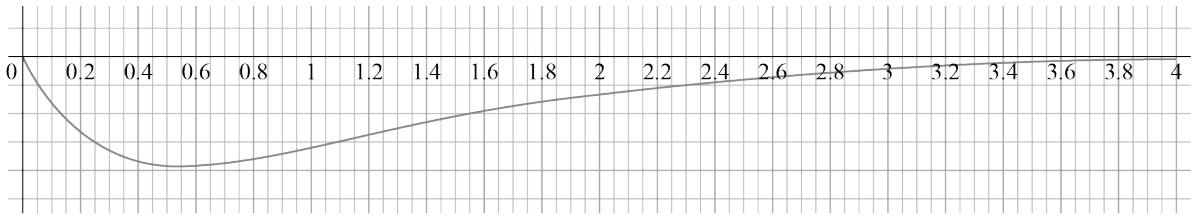
$\frac{\mathrm{d}x}{\mathrm{d}t}=-3Ae^{-3t}-Be^{t}$

$-3A-B=-1$ **A1**

$A=\frac{1}{2},\ B=-\frac{1}{2}$

$x=\frac{1}{2}e^{-3t}-\frac{1}{2}e^{-t}$ **A1**

(ii)



any curve starting at 0 and initially decreasing. **A1**

all else correct (minimum and some indication of POI / asymptote. **A1**

[6 marks]

(c) (i) recognition that minimum is relevant to the question **(M1)**

0.549 (minutes) $(0.549306...)$ **A1**

(ii) any consideration of the y-coordinate of the minimum point **(M1)**

0.192 dollars $(0.192450...)$ **A1**

[4 marks]

continued…

Question 6 continued

(d) attempting to write out the iterative relationship in terms of the variables **(*M1*)**

$x_{n+1} = x_n + 0.1(y_n)$ **(*A1*)**

$y_{n+1} = y_n + 0.1(-4y_n - 3x_n + x_n \sin t_n)$ **(*A1*)**

$t_n = 0.1n$

using $y_0 = x_0' = -1$ **(*A1*)**

attempt to find x_{10} **(*M1*)**

Note: Award the ***M1*** for $x_1 = -0.1$ and $y_1 = -0.6$ seen as the first iterative stage.

$x_{10} = -0.172 \ (-0.171795...)$ ***A1***

Note: a common error may be $0.104 \ (0.104330...)$.

[6 marks]

Total [21 marks]

Q1.

(a) $(d=)3l$ ***A1***

[1 mark]

(b) substitution into volume of cuboid formula $V = l^2 \times 3l\,(=375)$ ***(M1)***

attempting to solve their $3l^3 = 375$ ***(M1)***

$l^3 = 125$ OR $l = \sqrt[3]{125}$ OR $l^3 = \frac{375}{3}$

$l = 5,\ d = 15$ ***A1***

[3 marks]

(c) finding the area of a square and the area of a (non-square) rectangle 5×5 and 5×15 ***(M1)***

$(5\times5 + 5\times15 + 5\times15)\times2$ ***(A1)***

$= 350\text{cm}^2$ ***A1***

Note: The final answer is 350cm^2. Units are required.

[3 marks]

(d) $\pi r^2 h = 375$ ***(A1)***

$h = \frac{375}{\pi r^2}$ ***A1***

Note: Award at most ***A0A1*** for a final answer $h = \frac{V}{\pi r^2}$.

[2 marks]

(e) attempt to substitute their h into the curved surface area $2\pi rh$ ***M1***

$A = 2\pi r^2 + 2\pi r\left(\frac{375}{\pi r^2}\right)$ ***A1***

$A = 2\pi r^2 + \frac{750}{r}$ ***AG***

[2 marks]

(f) $\left(\frac{\mathrm{d}A}{\mathrm{d}r} =\right) 4\pi r - 750r^{-2}$ ***A1A1A1***

Note: Award ***A1*** for $4\pi r$, ***A1*** for -750, ***A1*** for r^{-2} or equivalent. Award at most ***A1A1A0*** for additional terms seen.

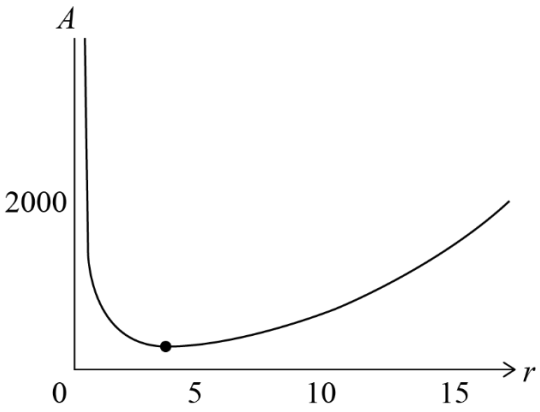
[3 marks]

continued…

Question 1 continued.

(g) (i) **METHOD 1**

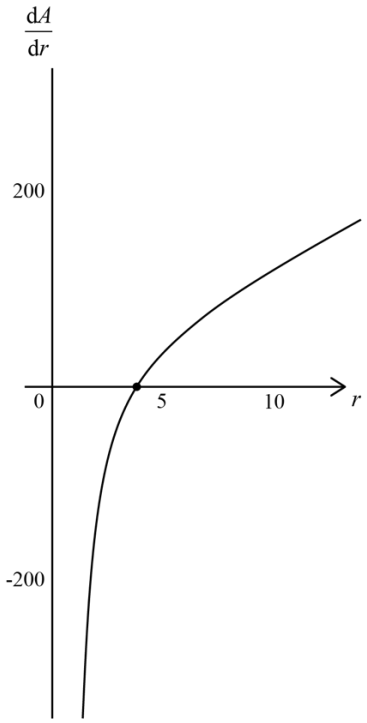
correct sketch with some indication of min point (need not be labelled) ***(M1)***



$(r =) 3.91 (\text{cm}) (3.90796...)$ ***A1***

METHOD 2

attempt to solve $4\pi r - \frac{750}{r^2} = 0$, graphically or otherwise ***(M1)***



$(r =) 3.91 (\text{cm}) (3.90796...)$ ***A1***

(ii) $288 (\text{cm}^2)$ $(287.873...)$ ***A1***

[3 marks]

continued…

Question 1 continued.

(h) (i) $\left(\frac{\mathrm{d}^2A}{\mathrm{d}r^2} = \right) 4\pi + \frac{1500}{r^3}$ ***A1***

(ii) (as $r > 0$), $\frac{\mathrm{d}^2A}{\mathrm{d}r^2} > 0$ ***R1***

hence graph is concave-up. ***A1***

Note: Do not award ***R0A1***.Marks can only be awarded in part (ii) if an expression is seen in part (i).

[3 marks]

Total [20 marks]

Q3.

(a) attempt to substitute into trapezoidal formula ***(M1)***

(area of cross-section) $\approx \frac{1.9}{2}\left(0+0+2\left(1.68+2.81+2.32\right)\right)$ ***(A1)***

$= 12.9\ (\text{m}^2)(12.939)$ ***A1***

[3 marks]

(b) attempt to find the volume of water in one second, their (a) $\times 0.3$ ***(M1)***

12.939×0.3

$= 3.88\left(3.8817\right)(\text{m}^3)$ ***A1***

Note: Accept 3.87 for use of 3sf answer to part (a)12.9.

[2 marks]

(c) $\left(2.81-1.6=\right)1.21$ (m) ***A1***

[1 mark]

(d) attempt to find maximum eg graphically or $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ or using height at $x=3.8$ ***(M1)***

3.07 (m) OR 3.066 (m) ***A1***

Note: Award ***M1A0*** for coordinate pair $(3.8, 3.066)$

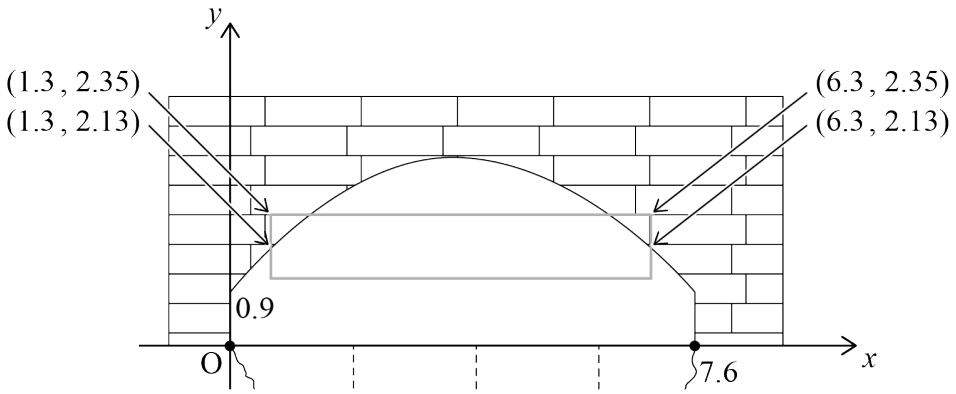
[2 marks]

continued...

Question 3 continued.

(e) **METHOD 1**

height of bridge < height of boat at
$x = 1.3$ or $x = 6.3$ (5 m wide boat)



use of $3.8 + 2.5$ or $3.8 - 2.5$ ***(M1)***

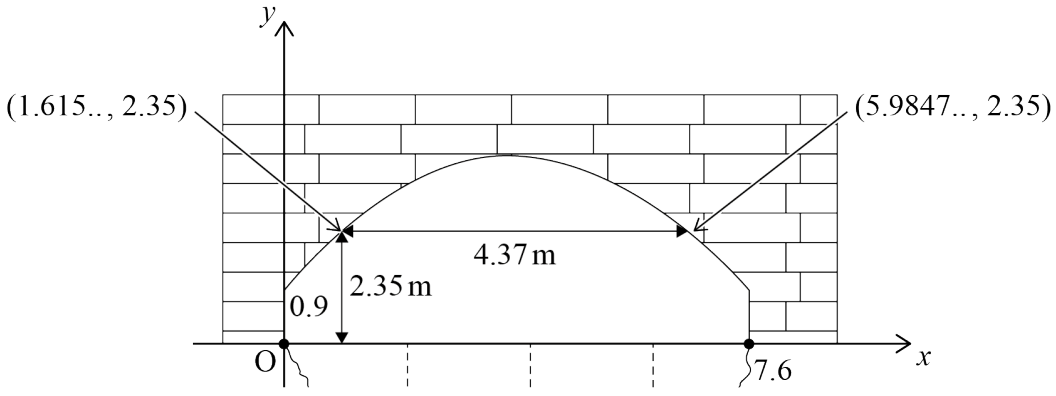
attempt to find the height of the bridge at either 1.3 or 6.3 m ***(M1)***

$2.13(\text{m})(2.1285)$ ***A1***

$(2.13 < 2.35)$ the boat cannot pass under the bridge ***A1***

METHOD 2

boat is wider $(5\,\text{m})$ than the width between
the arch at $h = 2.35$ m (height of boat)



attempt to find the values of x $(1.61520\ldots$ and $5.98479\ldots)$ when the bridge is 2.35 m above the water ***(M1)***

continued.

Question 3 continued.

subtract their two values to find the width of bridge at that point at this height ***(M1)***

$5.984795... - 1.615204...$ OR $3.8 - 1.615204\ldots$ OR $5.98479\ldots - 3.8$

$4.37(4.36959...)(\text{m})$ OR $2.18\ (2.18479\ldots)(\text{m})$ ***A1***

$(4.37 < 5)$ OR $(2.18 < 2.5)$ the boat cannot pass under the bridge ***A1***

METHOD 3 (see the diagram for method 2)

use of $3.8 + 2.5\ \ (6.3)$ or $3.8 - 2.5\ \ (1.3)$ ***(M1)***

attempt to find the values of x when the bridge is 2.35 m above the water ***(M1)***

$5.98479\ldots$ or $1.61520\ldots\ (\text{m})$ ***A1***

$6.3 > 5.98\ldots(5.98479\ldots)$ or $1.3 <\ 1.62\ (1.61520\ldots)$ the boat cannot pass under the bridge ***A1***

Note: In all 3 methods the final ***A1*** is dependent on the previous ***A1.***

[4 marks]

[Total 12 marks]

7. (a) (i) $\left(B'(x) =\right)\ \frac{-32}{x^2} + 2$ **OR** $\left(B'(x) =\right)\ -32x^{-2} + 2$ ***A1(M1)A1***

Note: Award ***A1*** for 2 seen, ***(M1)*** for expressing $\frac{1}{x}$ as x^{-1} (this can be implied from, either $\frac{1}{x^2}$ or x^{-2} seen in final answer), ***A1*** for $\frac{-32}{x^2}$. Award at most ***A1(M1)A0*** if additional terms are seen.

(ii) $B'(x) = 0$ **OR** $0 = \frac{-32}{x^2} + 2$ **OR** sketch of $B'(x)$ with root identified ***M1***

Note: It is specified that the student must use their derivative from part (i) to answer this question.

$x = 4$ ***(A1)***

Note: FT marks can only be awarded if *their* x-value lies within the given domain of $x \geq 1$.

$(B(x) =)36$ (minutes) ***A1***

[6 marks]

(b) $40\ > 20 + \frac{32}{k} + 2k$ **OR** 2 and 8 seen **OR** sketch of T(n) **and** T(n) = 40 ***(M1)***

$2 < k < 8$ **OR** $3 \leq k \leq 7$ **OR** $3, 4, 5, 6, 7$ ***A1***

[2 marks]

Total [8 marks]

10. (a) attempt to use the chain rule or the quotient rule ***(M1)***
e.g. $(4x+5)$ seen as part of a product

$\frac{-6(4x+5)}{\left(2x^2+5x+4\right)^2}$ **OR** $-6(4x+5)\left(2x^2+5x+4\right)^{-2}$ ***A1***

[2 marks]

(b) recognize need to find max/min of graph of f' **OR** zero of graph of f'' ***(M1)***

$-1.63, -0.868$ $(-1.63188\ldots, -0.86811\ldots)$ ***A1A1***

[3 marks]
Total [5 marks]

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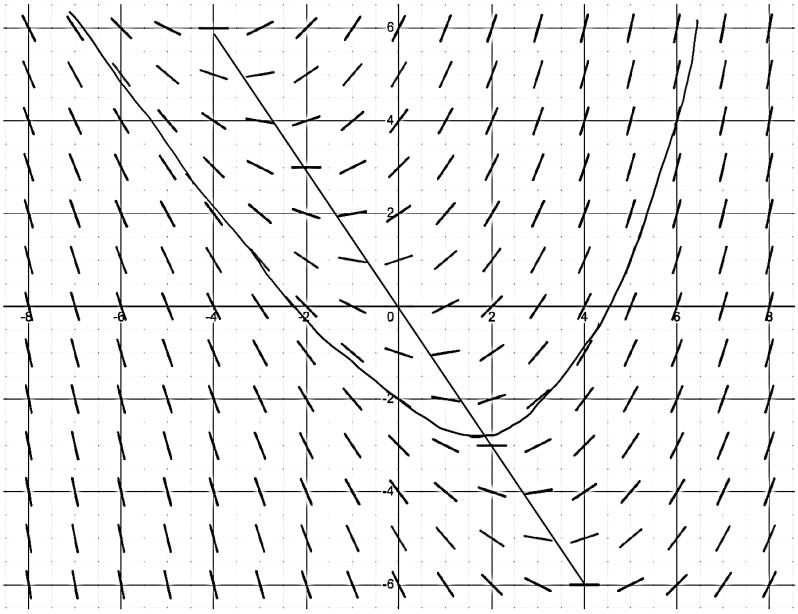
11. (a) (i) $y = -1.5x$ ***A1***

Note: Allow equivalent versions, e.g. $3x + 2y = 0$, $\frac{x}{2} + \frac{y}{3} = 0$.

(ii) straight line from $(-4,6)$ to $(4,-6)$ shown on diagram ***A1***

[2 marks]

(b)



A1A1

Note: Award ***A1*** for approx. correct shape passing through $(6,\ 4)$ and ***A1*** for minimum on line $y = -1.5x$.

[2 marks]
Total [4 marks]

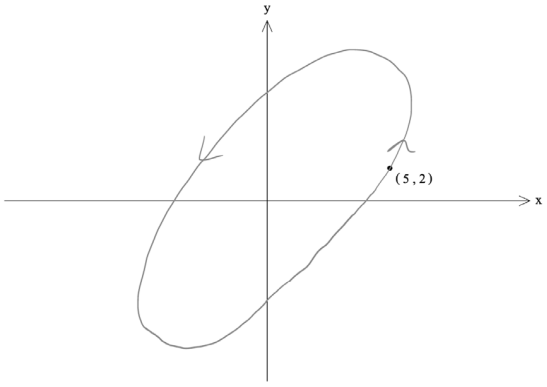
14. (a) **EITHER** $\frac{dx}{dt} = 2.2(5) - 2.6(2)(= 5.8)$ **OR** $\frac{dy}{dt} 2.2(5) - 2.6(2)(= 12.6)$ ***(A1)***

recognize need to use $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ **OR** $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ ***(M1)***

$\left(\frac{dy}{dx} =\right) 2.17 \quad (2.17241...)$ ***A1***

[3 marks]

(b)



ellipse or circle, centre $(0, 0)$ ***A1***

anticlockwise ***A1***

ellipse, with positive gradient at $(5, 2)$ ***A1***

[3 marks]

Total [6 marks]

15. (a) attempt to find an expression for x^2 or x ***(M1)***

$x^2 = \sqrt[3]{65536y}$ **OR** $x = \sqrt[6]{65536y}$ ***(A1)***

attempt to substitute their expression for x^2 into $\pi\int x^2 \mathrm{d}y$ ***(M1)***

$\pi\int_0^4 \sqrt[3]{65536y}\,\mathrm{d}y$ ***(A1)***

$603\ (\mathrm{cm}^3)$ $(603.185\ldots, 192\ \pi)$ ***A1***

Note: Award at most ***(M1)(A1)(M1)(A0)A0*** for an answer of 192 from omission of π.

[5 marks]

(b) recognize need to find the upper limit of an integral ***(M1)***

attempt to solve $\pi\int_0^h \sqrt[3]{65536y}\,\mathrm{d}y = 250$ ***(M1)***

$2.07\ (\mathrm{cm})$ $(2.06622\ldots)$ ***A1***

Note: Award at most ***(M1)(M1)A0*** for an answer of 4.88 from omission of π.

[3 marks]
Total [8 marks]

17. (a) 15 (metres) ***A1***

[1 mark]

(b) (i) recognize need to integrate ***(M1)***

$$\int_0^{4.5} \frac{\pi^2}{24}\left|\sin\left(\frac{\pi}{2}t\right)\right| \mathrm{d}t \quad \textit{(A1)}$$

$1.12 \quad (1.12387\ldots)$ ***A1***

Note: Award at most ***(M1)(A1)(A0)*** for an unsupported answer of $0.114006\ldots$ from use of degrees.
Award ***(M1)(A0)(A0)*** for an unsupported answer of $0.0766792\ldots$ from omission of absolute value.

(ii) $\frac{\mathrm{d}h}{\mathrm{d}\theta} = 15\sin\theta$ ***(A1)***

use of chain rule $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{d}t}$ ***(M1)***

substitute $\theta = 1.12387\ldots$ and $t = 4.5$ ***(M1)***
$\frac{\mathrm{d}h}{\mathrm{d}\theta} = 13.5267\ldots$, $\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0.290786\ldots$

$\frac{\mathrm{d}h}{\mathrm{d}t} = 3.93 \ (3.93338\ldots)$ (metres per minute) ***A1***

[7 marks]
Total [8 marks]

2. (a) (i) 0.9 ***A1***

(ii) the rate of change (of the height of the plant is 0.9 cm per day)
3 days after it is planted (accept $t = 3$) ***A1***
[2 marks]

(b) attempt to solve $\frac{\mathrm{d}h}{\mathrm{d}t} = 0$ (algebraically or graphically) ***(M1)***
$(t =)\ 12$ (days) ***A1***
[2 marks]

(c) recognition of the need to integrate ***(M1)***
$h = 1.2t - 0.05t^2 (+c)$ ***A1A1***

Note: Award ***A1*** for each correct term.

attempt to use given conditions (2,6.5) to find c ***(M1)***
$c = 4.3$
$(h =) 1.2t - 0.05t^2 + 4.3$ ***A1***
[5 marks]

(d) **EITHER**
attempt to find maximum graphically ***(M1)***

OR
attempt to substitute their part (b) for t ***(M1)***

THEN
$h = 11.5$ (cm) ***A1***

NOTE: If their answer to part (c) is linear or any other function that substantially changes the nature of the question, award at most ***M1A0***. Award at most ***M1A0*** if the x value of their max point (FT from part (c)), contradicts their $x = 12$ found in part (b).

[2 marks]

(e) 2 (cm) ***A1***
[1 mark]

(f) **EITHER**
attempt to write 1.2 as a percentage or percentage increase ***(M1)***

OR
attempt to find two values of H and % increase ***(M1)***

THEN
$(p =)\ 20$ ***A1***
[2 marks]

(g) attempt to solve $H(t) = h(t)$ ***(M1)***
$t = 9.43$ (days) $(9.43483\ldots)$ ***A1***
[2 marks]
Total [16 marks]

6. (a) attempt to integrate $12\cos\theta$ with respect to t ***(M1)***

$(x =)\quad (12\cos\theta)t + 2$ ***A1A1***

[3 marks]

(b) attempt to solve $y = 0$ ***M1***

$t(12\sin\theta - 4.9t) = 0$ ***A1***

$t = 0$ or $t = \frac{12\sin\theta}{4.9}$

$t = 0$ is the starting position **OR** it hits the ground when $t > 0$ ***R1***

so the ball hits the ground at $t = \frac{12\sin\theta}{4.9} = \frac{120\sin\theta}{49}$ ***AG***

[3 marks]

(c) attempt to substitute $t = \frac{120\sin\theta}{49}$ into their expression for x. ***(M1)***

$(x_g =)\ 12\cos\theta\frac{120\sin\theta}{49} + 2\ \left(= \frac{1440}{49}\sin\theta\cos\theta + 2 = (29.3877\ldots)\sin\theta\cos\theta + 2\right)$ ***A1***

[2 marks]

(d) sketch of graph showing maximum ***(M1)***

$0.785\ (0.785398\ldots, \frac{\pi}{4},\ 45°)$ ***A1***

Note: The answer must follow from their expression for x_g.

[2 marks]

(e) (i) $(x =)\quad (12\cos\theta - 3)t\ \ (+2)$ ***A1***

$(x_g =)\quad (12\cos\theta - 3)\frac{120\sin\theta}{49} + 2$ ***A1***

(ii) sketch of graph showing maximum ***(M1)***

$0.688\ \ (0.688242\ldots, 39.433°\ldots)$ ***A1***

[4 marks]

Total [14 marks]

2. (a) (i) that the stable/long term population will be the same as before the reduction. ***A1***

Note: Condone 'the maximum value / carrying capacity of the population will be 200'.

[1 mark]

(ii) attempt to substitute into logistic function ***(M1)***

$$40 = \frac{200}{1 + C\mathrm{e}^{0}}$$

$$40 + 40C = 200$$

$$C = 4$$ ***A1***

[2 marks]

(iii) correct substitution into their logistic function ***(A1)***

$$70 = \frac{200}{1 + 4\mathrm{e}^{-5k}}$$

$$k = 0.153\left(0.153451..., -\frac{1}{5}\ln\left(\frac{130}{280}\right)\right)$$ ***A1***

[2 marks]

(iv) correct substitution into their logistic function ***(A1)***

$$x = \frac{200}{1 + 4\mathrm{e}^{-10\times 0.153451...}} \left(= 107.397....\right)$$

$$x = 107$$ ***A1***

[2 marks]

continued…

Question 2 continued

(b) (i) $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ ***A1***

[1 mark]

(ii) the population is steady/stable (at 200) OR this (200) is the equilibrium population ***A1***

Note: Accept phrases such as 'the rate of change of wolf population is zero'. Do not accept 200 is the maximum. Do not follow through from an incorrect (b)(i).

[1 mark]

(iii) **EITHER**

$\frac{1}{\ln 200 - \ln x} \times$ an attempt to differentiate $\ln 200 - \ln x$ ***M1***

$f'(x) = \frac{1}{\ln 200 - \ln x} \times \left(-\frac{1}{x}\right)$ ***A1***

OR

$\frac{1}{\ln\left(\frac{200}{x}\right)} \times \frac{x}{200} \times -\frac{200}{x^2}$ ***M1A1***

Note: Award ***M1*** if there is a product of three expressions and two are correct.

$= \frac{-1}{x \ln\left(\frac{200}{x}\right)}$ ***AG***

[2 marks]

continued…

Question 2 continued

(iv) attempt to separate variables ***M1***

$$\int \frac{-1}{x\ln\left(\frac{200}{x}\right)}\mathrm{d}x = \int -a\mathrm{d}t$$ ***A1***

Note: Award ***A1*** also for $\int \frac{1}{x\left(\ln\left(\frac{200}{x}\right)\right)}\mathrm{d}x = \int a\mathrm{d}t$.

$$\ln\left(\ln 200 - \ln x\right) = -at\left(+c\right)$$ ***A1A1***

Note: Award ***A1*** for LHS and ***A1*** for RHS.

$\ln 200 - \ln x = \mathrm{e}^{-at+c}$ or $\ln 200 - \ln x = A\mathrm{e}^{-at}$, $(A = \mathrm{e}^{c})$ ***A1***

$\ln x = \ln 200 - A\mathrm{e}^{-at}$ ***AG***

[5 marks]

(v) correct substitution $\ln 40 = \ln 200 - A\mathrm{e}^{-0}$ ***(A1)***

$A = \ln\left(5\right)$ ***A1***

$\left(p = 5\right)$

Note: Award ***A1A0*** for an unsupported 1.61 $(1.60943\ldots)$.

[2 marks]

continued…

Question 2 continued

(vi) correct substitution $\ln 70 = \ln 200 - \ln 5 \times e^{-5a}$ ***A1***

$a = 0.0854528...$ ***A1***

Note: Award ***A1*** if their unrounded answer has at least four significant figures and rounds to 0.0855 OR if an exact value equivalent to $0.2\ln\left(\frac{\ln 5}{\ln\left(\frac{200}{70}\right)}\right)$ is seen.

$a = 0.0855$ to 3 significant figures ***AG***

[2 marks]

(vii) correct substitution $\ln x = \ln 200 - \ln 5 \times e^{-10\times 0.0854528...}$ ***(A1)***

$100.839\ldots$ ***(A1)***

Note: Accept $100.871\ldots$ from use of 3sf 0.0855.

101 to the nearest whole number ***A1***

[3 marks]

(c) both are overestimates OR the Gompertz is closer to the true value ***A1***

[1 mark]

Note: Follow through on their answers for 2(a)(iv) and 2 (b)(vii) if both positive values and these values are compared with 85.

continued…

Question 2 continued

(d) (i) attempt to use Euler ***(M1)***

Note: Condone maximum of one error from missing $x_n +$ at start, or incorrect/missing subscripts, absence of 0.5.

$$x_{n+1} = x_n + 0.5\left(0.0855x_n \ln\left(\frac{200}{x_n}\right) - 0.03x_n\right)$$ ***(A1)***

evidence of correct use of Euler's method ***(A1)***

Note: Award ***(A1)*** for any correct value of x_{n+1} for $n = 2,3,...$ seen (including this intermediate value presented as a final answer).

after 10 years the population will be 89 wolves ($89.42803\ldots$) ***A1***

[4 marks]

(ii) correctly compare their result to the actual number of wolves or to the other models or both. Allow even if this is implicit, for example, it is more accurate / realistic than the other models ***A1***

[1 mark]

Total [29 marks]