

APOLLO

Geometry and trigonometry

HL

Mathematics: applications and interpretation

First assessment 2021

101 questions | mark schemes

2. (a) every point in the shaded region is closer to tower T4

R1

Note: Specific reference must be made to the closeness of tower T4.

[1 mark]

- (b) $(-9, 1)$

A1A1

Note: Award **A1** for each correct coordinate. Accept $x = -9$ and $y = 1$.
Award at most **A0A1** if parentheses are missing.

[2 marks]

- (c) correct use of gradient formula

(M1)

e.g. $(m =) \frac{5-3}{-9-13} \left(= \frac{1}{2} \right)$

taking negative reciprocal of **their** m (at any point)

(M1)

edge gradient $= -2$

A1

[3 marks]

Total [6 marks]

5. (a) (i) $\vec{CA} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$ **A1**

(ii) $\vec{CB} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ **A1**

[2 marks]

(b) $\vec{CA} \times \vec{CB} = \begin{pmatrix} 0 \\ -6 \\ 24 \end{pmatrix}$ **(M1)A1**

Note: Do not award **(M1)** if less than 2 entries are correct.

[2 marks]

(c) area is $\frac{1}{2}\sqrt{6^2 + 24^2} = 12.4 \text{ m}^2$ (12.3693..., $3\sqrt{17}$) **(M1)A1**

[2 marks]

Total [6 marks]

6. attempt to find any relevant maximum value **(M1)**
largest sides are 56.5 and 82.5 **(A1)**
smallest possible angle is 102.5 **(A1)**

attempt to substitute into area of a triangle formula **(M1)**

$$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$$

$$= 2280 \text{ m}^2 \text{ (2275.37...)}$$

A1
Total [5 marks]

10. (a) (i) use of Prim's algorithm **M1**
 BC 46 **A1**
 BD 58 **A1**
 DE 23
 EF 47
 Total 174 **A1**

Note: Award **M0A0A0A1** for 174 without correct working e.g. use of Kruskal's, or with no working.
 Award **M1A0A0A1** for 174 by using Prim's from an incorrect starting point.

- (ii) $AB + AC = 55 + 63 = 118$ **(M1)**
 $174 + 118 = 292$ minutes **A1**

- (b) delete a different vertex **A1** **[6 marks]**
[1 mark]

Total [7 marks]

13. (a) $\mathbf{r} = \begin{pmatrix} 1.3 \\ -0.3 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ 1 \end{pmatrix}$ **A1A1**

Note: Award **A1** for each correct vector. Award **A0A1** if their “ $\mathbf{r} =$ ” is omitted.

[2 marks]

(b) (i) $-0.3 + \lambda = 0$ **(M1)**
 $\Rightarrow \lambda = 0.3$

$$\mathbf{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + 0.3 \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.4 \\ 0 \end{pmatrix}$$
(M1)

P has coordinates (0.2, 0.4, 0) **A1**

Note: Accept the coordinates of P in vector form.

(ii) $\sqrt{0.2^2 + 0.4^2}$ **(M1)**
 $= 0.447 \text{ km } (=447 \text{ m})$ **A1**

[5 marks]

Total [7 marks]

16. (a) $M = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}$

A1A1A1

Note: Award **A1** for each two correct rows.

[3 marks]

(b) calculating M^6
143

(M1)
A1

[2 marks]

Total [6 marks]

1. (a) gradient $AB = \frac{4}{12} \left(\frac{1}{3} \right)$ (A1)
 midpoint AB: (8, 22) (A1)

gradient of bisector $= -\frac{1}{\text{gradient AB}} = -3$ (M1)

perpendicular bisector: $22 = -3 \times 8 + b$ OR $(y - 22) = -3(x - 8)$ (M1)

perpendicular bisector: $y = -3x + 46$ A1

[5 marks]

(b) attempt to solve simultaneous equations (M1)

$x + 4 = -3x + 46$

(10.5, 14.5) A1

[2 marks]

Total [7 marks]

4. (a) $AC = \frac{380}{\tan 25^\circ}$ **OR** $AC = \sqrt{\left(\frac{380}{\sin 25^\circ}\right)^2 - 380^2}$ **OR** $\frac{380}{\sin 25^\circ} = \frac{AC}{\sin 65^\circ}$ **(M1)**

$$AC = 815 \text{ m (814.912...)}$$

A1**[2 marks]**(b) **METHOD 1**

attempt to find AB

(M1)

$$AB = \frac{380}{\tan 40^\circ}$$

$$= 453 \text{ m (452.866...)}$$

(A1)

$$BC = 814.912... - 452.866...$$

$$= 362 \text{ m (362.046...)}$$

A1**METHOD 2**

attempt to find HB

(M1)

$$HB = \frac{380}{\sin 40^\circ}$$

$$591 \text{ m (= 591.175...)}$$

(A1)

$$BC = \frac{591.175... \times \sin 15^\circ}{\sin 25^\circ}$$

$$= 362 \text{ m (362.046...)}$$

A1**[3 marks]**(c) $362.046... \times 4$

$$= 1450 \text{ m h}^{-1} \text{ (1448.18...)}$$

A1**[1 mark]****Total [6 marks]**

8. (a) setting a dot product of the direction vectors equal to zero

(M1)

$$\begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix} \cdot \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix} = 0$$

$$p(p+4) + 8p - 28 = 0$$

(A1)

$$p^2 + 12p - 28 = 0$$

$$(p+14)(p-2) = 0$$

$$p = -14, p = 2$$

A1**[3 marks]**

- (b) $p = -14 \Rightarrow$

$$L_1 : r = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -14 \\ -28 \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 4 \\ -7 \end{pmatrix}$$

a common point would satisfy the equations

$$2 - 14\lambda = 14 - 10\mu$$

$$-5 - 28\lambda = 7 + 4\mu$$

$$-3 + 4\lambda = -2 - 7\mu$$

(M1)**METHOD 1**

solving the first two equations simultaneously

$$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}$$

A1

substitute into the third equation:

M1

$$-3 + 4\left(-\frac{1}{2}\right) \neq -2 + \frac{1}{2}(-7)$$

so lines do not intersect.

R1

Note: Accept equivalent methods based on the order in which the equations are considered.

METHOD 2

attempting to solve the equations using a GDC

M1

GDC indicates no solution

A1

so lines do not intersect

R1**[4 marks]****Total [7 marks]**

11. (a) Odd vertices are A, B, D, H
Consider pairings:

A1
M1

Note: Award (**M1**) if there are four vertices not necessarily all correct.

AB DH has shortest route AD, DE, EB and DE, EH,
so repeated edges $(19 + 16 + 19) + (16 + 27) = 97$

Note: Condone AB in place of AD, DE, EB giving $56 + (16 + 27) = 99$.

AD BH has shortest route AD and BE, EH,
so repeated edges $19 + (19 + 27) = 65$

AH BD has shortest route AD, DE, EH and BE, ED,
so repeated edges $(19 + 16 + 27) + (19 + 16) = 97$

A2

Note: Award **A1** if only one or two pairings are correctly considered.

so best pairing is AD, BH
weight of route is therefore $582 + 65 = 647$

A1
[5 marks]

- (b) least value of the pairings is 19 therefore repeat AD

R1

B and H

A1

Note: Do not award **R0A1**.

[2 marks]

Total [7 marks]

14. (a) $\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 6 \\ -2 \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ **(M1)**

$= \begin{pmatrix} 57 \\ 22 \end{pmatrix}$ **OR** $(57, 22)$ **A1**

[2 marks]

(b) $\begin{pmatrix} 2p \\ 2q \end{pmatrix} = \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ **(M1)**

$7p - 10q - 5 = 2p$

$2p - 3q + 4 = 2q$ **(A1)**

solve simultaneously:

$p = 13, q = 6$ **A1**

Note: Award **A0** if 13 and 6 are not labelled or are labelled the other way around.

[3 marks]

(c) $\det \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} = -1$ **OR** $\left| \det \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \right| = 1$ **A1**

scale factor of image area is therefore $(|-1| =) 1$ (and the translation does not affect the area)

A1

[2 marks]

Total [7 marks]

1. (a) use of cosine rule (M1)

$$\hat{A}CB = \cos^{-1} \left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225} \right) \quad (A1)$$

$$= 32.0^\circ \text{ (31.9980...)} \text{ OR } 0.558 \text{ (0.558471...)} \quad A1$$

[3 marks]

- (b) use of sine rule (M1)

$$\frac{DE}{\sin 31.9980...^\circ} = \frac{210}{\sin 100^\circ} \quad (A1)$$

$$(DE =) 113 \text{ m (112.9937...)} \quad A1$$

[3 marks]

- (c) **METHOD 1**

$$180^\circ - (100^\circ + \text{their part (a)}) \quad (M1)$$

$$= 48.0019...^\circ \text{ OR } 0.837791... \quad (A1)$$

$$\text{substituted area of triangle formula} \quad (M1)$$

$$\frac{1}{2} \times 112.9937... \times 210 \times \sin 48.002^\circ \quad (A1)$$

$$8820 \text{ m}^2 \text{ (8817.18...)} \quad A1$$

METHOD 2

$$\frac{CE}{\sin(180 - 100 - \text{their part (a)})} = \frac{210}{\sin 100} \quad (M1)$$

$$(CE =) 158.472... \quad (A1)$$

$$\text{substituted area of triangle formula} \quad (M1)$$

EITHER

$$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100 \quad (A1)$$

OR

$$\frac{1}{2} \times 210... \times 158.472... \times \sin(\text{their part (a)}) \quad (A1)$$

THEN

$$8820 \text{ m}^2 \text{ (8817.18...)} \quad A1$$

continued...

*Question 1 continued***METHOD 3**

$$CE^2 = 210^2 + 112.993...^2 - (2 \times 210 \times 112.993... \times \cos(180 - 100 - \text{their part } (a))) \quad (M1)$$

$$(\text{CE} =) 158.472... \quad (A1)$$

$$\text{substituted area of triangle formula} \quad (M1)$$

$$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100 \quad (A1)$$

$$8820 \text{ m}^2 \quad (8817.18...) \quad A1$$

[5 marks]

$$(d) \quad 1005 - 210 \quad \text{OR} \quad 795 \quad (A1)$$

$$\text{equating answer to part (c) to area of a triangle formula} \quad (M1)$$

$$8817.18... = \frac{1}{2} \times DF \times (1005 - 210) \times \sin 48.002...^\circ \quad (A1)$$

$$(\text{DF} =) 29.8 \text{ m} \quad (29.8473...) \quad A1$$

[4 marks]**Total [15 marks]**

4. (a) (the best placement is either point P or point Q)
attempt at using the distance formula

(M1)

$$AP = \sqrt{(10-6)^2 + (6-2)^2} \quad \text{OR}$$

$$BP = \sqrt{(10-14)^2 + (6-2)^2} \quad \text{OR}$$

$$DP = \sqrt{(10-10.8)^2 + (6-11.6)^2} \quad \text{OR}$$

$$BQ = \sqrt{(13-14)^2 + (7-2)^2} \quad \text{OR}$$

$$CQ = \sqrt{(13-18)^2 + (7-6)^2} \quad \text{OR}$$

$$DQ = \sqrt{(13-10.8)^2 + (7-11.6)^2}$$

$$(AP \text{ or } BP \text{ or } DP \Rightarrow) \sqrt{32} = 5.66 \text{ (5.65685...)} \quad \text{AND}$$

$$(BQ \text{ or } CQ \text{ or } DQ \Rightarrow) \sqrt{26} = 5.10 \text{ (5.09901...)} \quad \text{A1}$$

$$\sqrt{32} > \sqrt{26} \quad \text{OR} \quad AP \text{ (or } BP \text{ or } DP) \text{ is greater than } BQ \text{ (or } CQ \text{ or } DQ) \quad \text{A1}$$

point P is the furthest away

AG

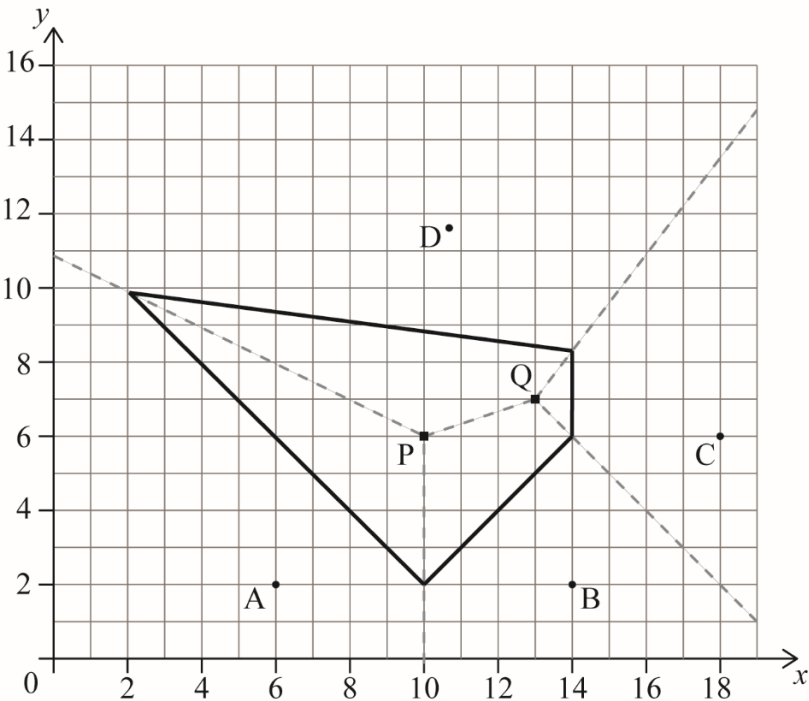
Note: Follow through from their values provided their AP (or BP or DP) is greater than their BQ (or CQ or DQ).

[3 marks]*continued...*

Question 4 continued

- (b) (i) $x = 14$
(ii)

A1



A1A1

Note: Award **A1** for each correct straight line. Do not **FT** from their part b(i).

[3 marks]
Total: [6 marks]

11. METHOD 1

attempt to find AC using cosine rule

M1

$$7^2 = 10^2 + AC^2 - 2 \times 10 \times AC \times \cos 40^\circ$$

(A1)

attempt to solve a quadratic equation

(M1)

$$AC = 4.888... \text{ AND } 10.432...$$

(A1)

Note: At least $AC = 4.888...$ must be seen, or implied by subsequent working.

$$\text{minimum area} = \frac{1}{2} \times 10 \times 4.888... \times \sin(40^\circ)$$

M1

Note: Do not award **M1** if incorrect value for minimizing the area has been chosen.

$$= 15.7 \text{ m}^2$$

A1**METHOD 2**attempt to find $\hat{ACB}$ using the sine Rule**M1**

$$\frac{\sin C}{10} = \frac{\sin 40}{7}$$

(A1)

$$C = 66.674...^\circ \text{ OR } 113.325...^\circ$$

(A1)**EITHER**

$$B = 180 - 40 - 113.325...$$

$$B = 26.675...^\circ$$

(A1)

$$\text{area} = \frac{1}{2} \times 10 \times 7 \times \sin(26.675...^\circ)$$

M1**OR**sine rule or cosine rule to find $AC = 4.888...$ **(A1)**

$$\text{minimum area} = \frac{1}{2} \times 10 \times 4.888... \times \sin(40^\circ)$$

M1**THEN**

$$= 15.7 \text{ m}^2$$

A1

Note: Award **A0M1A0** if the wrong length AC or the wrong angle B selected but used correctly finding a value of 33.5 m^2 for the area.

Total: [6 marks]

16. (a) $\vec{OS} = \begin{pmatrix} 300 \\ 100 \end{pmatrix} + t \begin{pmatrix} -12 \\ 15 \end{pmatrix}$ **A1**

[1 mark]

(b) attempt to find the vector from L to S **(M1)**

$$\vec{LS} = \begin{pmatrix} 171 \\ -183 \end{pmatrix} + t \begin{pmatrix} -12 \\ 15 \end{pmatrix}$$
A1

EITHER

$$|\vec{LS}| = \sqrt{(171 - 12t)^2 + (15t - 183)^2}$$
(M1)(A1)

minimize to find t on GDC **(M1)**

OR

S closest when $\vec{LS} \cdot \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ **(M1)**

$$\left(\begin{pmatrix} 171 \\ -183 \end{pmatrix} + t \begin{pmatrix} -12 \\ 15 \end{pmatrix} \right) \cdot \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$$

$$-2052 + 144t - 2745 + 225t = 0$$
(M1)(A1)

OR

S closest when $\vec{LS} \cdot \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ **(M1)**

$$\vec{LS} = \begin{pmatrix} 5k \\ 4k \end{pmatrix}$$

$$\vec{OS} = \begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix}$$
(A1)

$$\begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix} = \begin{pmatrix} 300 - 12t \\ 100 + 15t \end{pmatrix}$$

Solving simultaneously **(M1)**

THEN

$$t = 13$$
A1

[6 marks]

(c) the alarm will sound **A1**

$$|\vec{LS}| = 19.2\dots < 20$$
R1

Note: Do not award A1R0 .

[2 marks]

Total: [9 marks]

1. (a) $\tan(\theta) = \frac{6}{10}$ (M1)
 $(\theta =) 31.0^\circ (30.9637\dots^\circ)$ OR $0.540 (0.540419\dots)$ A1

[2 marks]

(b) (i) $(CV =) 40 \tan(\theta)$ OR $(CV =) 4 \times 6$ (M1)

Note: Award (M1) for an attempt at trigonometry or similar triangles (e.g. ratios).

$(CV =) 24 \text{ m}$ A1

(ii) $(V =) \frac{1}{3} 80^2 \times 24 - \frac{1}{3} 60^2 \times 18$ M1A1A1

Note: Award M1 for finding the difference between the volumes of two pyramids, A1 for each correct volume expression. The final A1 is contingent on correct working leading to the given answer.
 If the correct final answer is not seen, award at most M1A1A0. Award M0A0A0 for any height derived from $V = 29600$, including 18.875 or 13.875.

$(V =) 29600 \text{ m}^3$ AG

[5 marks]

(c) **METHOD 1**

$\left(\frac{29600}{80} =\right) 370 \text{ (days)}$ A1

$(370 > 366)$ Joshua is correct A1

Note: Award A0A0 for unsupported answer of "Joshua is correct". Accept $1.01\dots > 1$ for the first A1 mark.

METHOD 2

$80 \times 366 = 29280 \text{ m}^3$ OR $80 \times 365 = 29200 \text{ m}^3$ A1

$(29280 < 29600)$ Joshua is correct A1

Note: The second A1 can be awarded for an answer consistent with their result.

[2 marks]

continued...

Question 1 continued

(d) height of trapezium is $\sqrt{10^2 + 6^2}$ (= 11.6619...) **(M1)**

area of trapezium is $\frac{80+60}{2} \times \sqrt{10^2 + 6^2}$ (= 816.333...) **(M1)(A1)**

(SA =) $4 \times \left(\frac{80+60}{2} \times \sqrt{10^2 + 6^2} \right) + 60^2$ **(M1)**

Note: Award **M1** for adding 4 times their (MNOP) trapezium area to the area of the (60 × 60) base.

(SA =) 6870 m² (6865.33 m²) **A1**

Note: No marks are awarded if the correct shape is not identified.

[5 marks]
Total: [14 marks]

4.

Note: For clarity, exact answers are used throughout this markscheme. However it is perfectly acceptable for candidates to write decimal values (e.g. $\frac{\sqrt{3}}{2} = 0.866$).

(a) (i) rotation anticlockwise $\frac{\pi}{6}$ is $\begin{pmatrix} 0.866 & -0.5 \\ 0.5 & 0.866 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ **(M1)A1**

reflection in $y = \frac{x}{\sqrt{3}}$

$\tan \theta = \frac{1}{\sqrt{3}}$ **(M1)**

$\Rightarrow 2\theta = \frac{\pi}{3}$ **(A1)**

matrix is $\begin{pmatrix} 0.5 & 0.866 \\ 0.866 & -0.5 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$ **A1**

rotation clockwise $\frac{\pi}{3}$ is $\begin{pmatrix} 0.5 & 0.866 \\ -0.866 & 0.5 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ **A1**

(ii) an attempt to multiply three matrices **(M1)**

$P = \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$ **(A1)**

$P = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$ **OR** $\begin{pmatrix} 0.866 & -0.5 \\ -0.5 & -0.866 \end{pmatrix}$ **A1**

continued...

Question 4 continued

$$(iii) \quad \left(P^2 = \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \quad A1$$

Note: Do not award **A1** if final answer not resolved into the identity matrix I .

- (iv) if the overall movement of the drone is repeated the drone would return to its original position A1
A1
[12 marks]

(b) **METHOD 1**

$$|\det P| = \left| \left(-\frac{3}{4} \right) - \left(\frac{1}{4} \right) \right| = 1 \quad A1$$

$$\text{area of triangle ABC} = \text{area of triangle A'B'C'} \times |\det P| \quad R1$$

$$\text{area of triangle ABC} = \text{area of triangle A'B'C'} \quad AG$$

Note: Award at most **A1R0** for responses that omit modulus sign.

METHOD 2

statement of fact that rotation leaves area unchanged R1

statement of fact that reflection leaves area unchanged R1

$$\text{area of triangle ABC} = \text{area of triangle A'B'C'} \quad AG$$

[2 marks]

- (c) attempt to find angles associated with values of elements in matrix P (M1)

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \cos\left(-\frac{\pi}{6}\right) & \sin\left(-\frac{\pi}{6}\right) \\ \sin\left(-\frac{\pi}{6}\right) & -\cos\left(-\frac{\pi}{6}\right) \end{pmatrix}$$

reflection (in $y = (\tan \theta)x$) (M1)

$$\text{where } 2\theta = -\frac{\pi}{6} \quad A1$$

$$\text{reflection in } y = \tan\left(-\frac{\pi}{12}\right)x \quad (= -0.268x) \quad A1$$

[4 marks]

Total: [18 marks]

1. (a) e.g. ABCDEGHFA **A1**

Note: Accept any other correct answers starting at any vertex.

- (b) (i) 7 vertices, so 6 edges required for MST **(M1)**

Note: To award **(M1)**, their 6 edges should not form a cycle.

Order selected	Edge
1	BC
2	CD
3	DE
4	BH
5	GH
6	FG

M1**A1****A1**

Note: Award **M1** for the first three edges in correct order, **A1** for BH in correct order and **A1** for all of the edges correct.

weight of MST = 33

A1

Note: The final **A1** can be awarded independently of previous marks.

- (ii) lower bound = $33 + 3 + x$ **(M1)**
 $= 36 + x$ **A1**

- (c) (i) $p = 13$ **A1**

- (ii) $q = 17$ **A1**

- (iii) $r = 14$ **A1**

- (d) (i) attempt to use nearest neighbour algorithm **(M1)**

any two correct cycles from

ABCDEGHFA, AFGHBCDE(F)A, AB(A)FGHCDE(F)A **A1A1**

Note: Bracketed vertices may be omitted in candidate's answer.
Award **M1A0A1** for candidates who list two correct sequences of vertices, but omit the final vertex A.

- (ii) use ABCDEGHFA **OR** their shortest cycle from (d)(i) **(M1)**
upper bound = 43 **A1**

continued...

Question 1 continued

- (e) (i) cycle starts: ABCDEGHF
 return to A has two options, $FA = 18$ or x **(M1)**
 hence least value of $x = 19$ **A1**
- (ii) upper bound = 58 **A2**
- (f) recognition that edges will be repeated / there are odd vertices **(M1)**
 $BH + DG = 21$, $BD + GH = 15$, $BG + DH = 21$ **OR** $18 + x$ **A1**
 recognizing BD and GH is lowest weight and is repeated **(M1)**
 solution to CPP = $107 + x$ **A1**
 $x = 13$ **A1**

Note: Award **M1A0M0A1A1** if only pairing BD and GH is considered, leading to a correct answer.

4. (a) $\frac{3-1}{7-3}$ (M1)

$= 0.5$ A1
[2 marks]

(b) $y-2 = -2(x-5)$ (A1)(M1)

Note: Award (A1) for their -2 seen, award (M1) for the correct substitution of (5, 2) and their normal gradient in equation of a line.

$2x + y - 12 = 0$ A1
[3 marks]

(c) every point in the cell is closer to E than any other snow shelter A1
[1 mark]

Total [6 marks]

5. (a) $\frac{50 \times \pi}{180} = 0.873 \text{ (0.872664...)}$

A1**[1 mark]**

(b) $\text{volume} = 240 \left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664... \right)$

M1M1M1

Note: Award **M1** $240 \times \text{area}$, award **M1** for correctly substituting area sector formula, award **M1** for subtraction of the angles or their areas.

$= 45800 \text{ (= 45811.96071)}$

A1**[4 marks]****Total [5 marks]**

8. (a) $\frac{\sin \hat{CAB}}{6} = \frac{\sin 15^\circ}{4.5}$ **(M1)(A1)**

$\hat{CAB} = 20.2^\circ$ (20.187415...) **A1**

Note: Award **(M1)** for substituted sine rule formula and award **(A1)** for correct substitutions.

[3 marks]

(b) $\hat{CBD} = 20.2 + 15 = 35.2^\circ$ **A1**
(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\dots^\circ = \frac{1.8}{BX}$ **(M1)**

Note: Award **A1** for their correct angle $\hat{CBD}$. Award **M1** for correctly substituted trigonometric formula.

$BX = 2.55285\dots$ **A1**

$5 - 2.55285\dots$ **(M1)**

$= 2.45 \text{ (m)}$ (2.44714...) **A1**

[5 marks]**Total [8 marks]**

11. (a) $15 \times 0 + 2d + 4 = 0$

$$d = -2$$

(M1)

A1

[2 marks]

(b) $a \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$

(M1)

$$= a \begin{pmatrix} 10 \\ 15 \\ 30 \end{pmatrix} \left(= 5a \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \right)$$

A1

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$

M1

$$a = \frac{14}{35} \quad (= 0.4)$$

A1

[4 marks]

Total [6 marks]

4. (a) $r = \begin{pmatrix} 10 \\ 5 \end{pmatrix} + t \begin{pmatrix} -50 \\ -20 \end{pmatrix}$ **A1A1**

[2 marks]

(b) (i) when $x = 0$, $t = \frac{30}{150} = 0.2$ **M1**

EITHER

when $y = 0$, $t = \frac{10}{150} = 0.2$ **A1**

since the two values of t are equal the aircraft passes directly over the airport

OR

$t = 0.2$, $y = 0$ **A1**

(ii) height $= 5 - 0.2 \times 20 = 1 \text{ km}$ **A1**

(iii) time 13:12 **A1**

[4 marks]

(c) (i) $5 - 20t = 4 \Rightarrow t = \frac{1}{20}$ (3 minutes) **(M1)**

time 13:03 **A1**

(ii) displacement is $\begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$ **A1**

distance is $\sqrt{22.5^2 + 7.5^2 + 4^2}$ **(M1)**

$= 24.1 \text{ km}$ **A1**

[5 marks]

continued...

Question 4 continued(d) **METHOD 1**time until landing is $12 - 3 = 9$ minutes**M1**

height to descend = 4 km

$$a = \frac{-4}{\frac{9}{60}}$$

M1

$$= -26.7$$

A1**METHOD 2**

$$\begin{pmatrix} -150 \\ -50 \\ a \end{pmatrix} = s \begin{pmatrix} 22.5 \\ 7.5 \\ 4 \end{pmatrix}$$

M1

$$-150 = 22.5s \Rightarrow s = -\frac{20}{3}$$

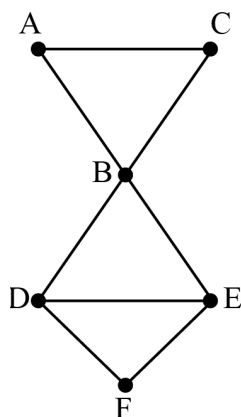
M1

$$a = -\frac{20}{3} \times 4$$

$$= -26.7$$

A1**[3 marks]****Total [14 marks]**

5. (a)

A2
[2 marks]

(b) attempt to form an adjacency matrix

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

M1

A1

[2 marks]

(c) raising the matrix to the power six
50

(M1)

A1

[2 marks]

(d) not possible
because you must pass through B twice

A1

R1

Note: Do not award **A1R0**.

[2 marks]

(e) $a = 230, b = 340$

A1A1

[2 marks]

(f) $A \rightarrow B \rightarrow D \rightarrow E \rightarrow F \rightarrow C \rightarrow A$
 $90 + 70 + 100 + 210 + 330 + 150$
(US\$) 950

(M1)

(A1)

A1

[3 marks]

continued...

Question 5 continued

- (g) finding weight of minimum spanning tree

M1

$$70 + 80 + 100 + 180 = (\text{US\$}) 430$$

A1

adding in two edges of minimum weight

M1

$$430 + 90 + 150 = (\text{US\$}) 670$$

A1**[4 marks]****Total [17 marks]**

(c) (i)

6. (a) $\begin{pmatrix} -4.5 \\ 6.1 \end{pmatrix}$ A1

[1 mark]

(b) $\sqrt{(-3.2)^2 + (-4.5)^2 + 6.1^2}$ (M1)
 $8.22800... \approx 8.23 \text{ m}$ A1

[2 marks]

(c) EITHER

$$\vec{AO} = \begin{pmatrix} -3.2 \\ -4.5 \\ 0.3 \end{pmatrix}$$
 A1

$$\cos \theta = \frac{\vec{AO} \cdot \vec{AF}}{|\vec{AO}| |\vec{AF}|}$$

$$\vec{AO} \cdot \vec{AF} = (-3.2)^2 + (-4.5)^2 + (0.3 \times 6.1) \quad (= 32.32)$$
 (A1)

$$\cos \theta = \frac{32.32}{\sqrt{3.2^2 + 4.5^2 + 0.3^2} \times 8.22800...}$$
 (M1)

$$= 0.710326...$$
 (A1)

Note: If $\vec{OA}$ is used in place of $\vec{AO}$ then $\cos \theta$ will be negative.
 Award **A1(A1)(M1)(A1)** as above. In order to award the final **A1**, some justification for changing the resulting obtuse angle to its supplementary angle **must** be seen.

OR

$$AO = \sqrt{3.2^2 + 4.5^2 + 0.3^2} \quad (= 5.52991...) \quad \text{(A1)}$$

$$\cos \theta = \frac{8.22800...^2 + 5.52991...^2 - 5.8^2}{2 \times 8.22800... \times 5.52991...}$$
 (M1)(A1)

$$= 0.710326... \quad \text{(A1)}$$

THEN

$$\theta = 0.780833... \approx 0.781 \quad \text{OR} \quad 44.7384...^\circ \approx 44.7^\circ$$
 A1

[5 marks]

[Total 8 marks]

13. (a) $(r =) \begin{pmatrix} 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1.2 \\ -0.6 \end{pmatrix}$

A1

Note: Do not condone the use of λ or any other variable apart from t .

[1 mark]

- (b) when the bearing from the port is 045° , the distance east from the port is equal to the distance north from the port

(M1)

$$1 + 1.2t = 4 - 0.6t$$

(A1)

$$1.8t = 3$$

$$t = \frac{5}{3} \quad (1.6666\dots, 1 \text{ hour } 40 \text{ minutes})$$

(A1)

time is 2:40 pm (14:40)

A1**[4 marks]****[Total 5 marks]**

2. (a) $\sin \theta = \frac{2.1}{2.8}$ **OR** $\tan \theta = \frac{2.1}{1.85202\dots}$ (M1)

$(\theta =) 48.6^\circ$ (48.5903...°) **A1**
[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8 \cos(48.5903\dots)$ **OR** $\frac{2.1}{\tan(48.5903\dots)}$ (M1)

Note: Award **M1** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

1.85 m (1.85202...) (A1)

Note: Award the **M1A1** if 1.85 is seen in part (a).

$(6.4 - 1.85202\dots)$
4.55 m (4.54797...) (A1)

Note: Award **A1** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\dots)^2 + 2.1^2}$
5.01 m (5.00939...m) **A1**

METHOD 2

attempt to use cosine rule (M1)
 $(c^2 =) 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\dots)$ (A1)(A1)

Note: Award **A1** for 48.5903...° substituted into cosine rule formula, **A1** for correct substitution.

$(c =) 5.01$ m (5.00939...m) **A1**
[4 marks]

- (c) camera 1 is closer to the cash register than camera 2 (and both cameras are at the same height on the wall) **R1**
the larger angle of depression is from camera 1 **A1**

Note: Do not award **R0A1**. Award **R0A0** if additional calculations are completed and used in their justification, as per the question. Accept "1.85 < 4.55" or "2.8 < 5.01" as evidence for the **R1**.

[2 marks]
Total [8 marks]

6. (a)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$

A2

Note: Award **A2** for the transposed matrix. Presentation in markscheme assumes columns/rows ordered A-E; accept a matrix with rows and/or columns in a different order only if appropriately communicated. Do not **FT** from part (a) into part (b).

[2 marks]

(b) raising their matrix to a power of 5

(M1)

$$M^5 = \begin{pmatrix} 17 & 9 & 2 & 3 & 5 \\ 17 & 10 & 3 & 4 & 4 \\ 13 & 6 & 2 & 2 & 4 \\ 8 & 5 & 1 & 2 & 2 \\ 18 & 11 & 2 & 4 & 5 \end{pmatrix}$$

(A1)

Note: The numbers along the diagonal are sufficient to award **M1A1**.

(the required number is $17 + 10 + 2 + 2 + 5 =$) 36**A1****[3 marks]****Total [5 marks]**

8. (a) $\pi \times 2^2 \times \frac{30}{360}$ (M1)

$= 1.047 \text{ cm}^2$ A1

Note: Do not award the final mark if the answer is not correct to 4 sf.

[2 marks]

(b) attempt to substitute any two values from 1.5, 2.5, 25 or 35 into area of sector formula (M1)

$\left(\text{upper bound} = \pi \times 2.5^2 \times \frac{35}{360} = \right) 1.91 \text{ cm}^2 \text{ (1.90895...)} \quad \text{A1}$

$\left(\text{lower bound} = \pi \times 1.5^2 \times \frac{25}{360} = \right) 0.491 \text{ cm}^2 \text{ (0.490873...)} \quad \text{A1}$

Note: Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers.

[3 marks]

(c) $\left(\left| \frac{1.047 - 1.90895...}{1.90895...} \right| \times 100 = \right) 45.2 \text{ (%) (45.1532...)} \quad \text{A1}$

$\left(\left| \frac{1.047 - 0.490873...}{0.490873...} \right| \times 100 = \right) 113 \text{ (%) (113.293...)} \quad \text{A1}$

so the largest percentage error is 113 % A1

Note: Accept 45.1 (%) (45.1428), from use of full accuracy answers. Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers. Award **A0A1A0** if 113% is the only value found.

[3 marks]

Total [8 marks]

15. (a) (one vector to the line is $\begin{pmatrix} 0 \\ c \end{pmatrix}$ therefore) $\mathbf{a} = \begin{pmatrix} 0 \\ c \end{pmatrix}$ **A1**
 the line goes m up for every 1 across
 (so the direction vector is) $\mathbf{b} = \begin{pmatrix} 1 \\ m \end{pmatrix}$ **A1**

Note: Although these are the most likely answers, many others are possible.

[2 marks]

- (b) (from GDC **OR** $6 \times 2 - 4 \times 3$) $|M| = 0$ **A1**

[1 mark]

- (c) **METHOD 1**
 $\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} 6x + 3mx + 3c \\ 4x + 2mx + 2c \end{pmatrix}$ **M1A1**
 $= \begin{pmatrix} 3(2x + mx + c) \\ 2(2x + mx + c) \end{pmatrix}$ **A1**
 therefore the new line has equation $3Y = 2X$ **A1**
 which is independent of m or c **AG**

Note: The **AG** line (or equivalent) must be seen for the final **A1** line to be awarded.

METHOD 2

take two points on the line, e.g. $(0, c)$ and $(1, m + c)$ **M1**

these map to $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ c \end{pmatrix} = \begin{pmatrix} 3c \\ 2c \end{pmatrix}$

and $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ m + c \end{pmatrix} = \begin{pmatrix} 6 + 3m + 3c \\ 4 + 2m + 2c \end{pmatrix}$ **A1**

therefore a direction vector is $\begin{pmatrix} 6 + 3m \\ 4 + 2m \end{pmatrix} = (2 + m) \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(since $m \neq -2$) a direction vector is $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

the line passes through $\begin{pmatrix} 3c \\ 2c \end{pmatrix} - c \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ therefore it always has the

origin as a jump-on vector **A1**

the vector equation is therefore $\mathbf{r} = \mu \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ **A1**

which is independent of m or c **AG**

Note: The **AG** line (or equivalent) must be seen for the final **A1** line to be awarded.

continued...

*Question 15 continued***METHOD 3**

$$\mathbf{r} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix} \left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix} \right) = \begin{pmatrix} 3c \\ 2c \end{pmatrix} + \lambda \begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix}$$

M1A1

$$= c \begin{pmatrix} 3 \\ 2 \end{pmatrix} + (2+m)\lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

A1

$$= \mu \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

where $\mu = c + (2+m)\lambda$ is an arbitrary parameter.**A1**which is independent of m or c (as μ can take any value)**AG**

Note: The **AG** line (or equivalent) must be seen for the final **A1** line to be awarded.

[4 marks]**Total [7 marks]**

16. (a) attempt at chain rule

(M1)

$$\left(\mathbf{v} = \frac{d\mathbf{OP}}{dt} = \begin{pmatrix} 2t \cos t^2 \\ -2t \sin t^2 \end{pmatrix} \right)$$

A1**[2 marks]**

- (b) attempt at product rule

(M1)

$$\mathbf{a} = \begin{pmatrix} 2 \cos t^2 - 4t^2 \sin t^2 \\ -2 \sin t^2 - 4t^2 \cos t^2 \end{pmatrix}$$

A1**METHOD 1**let $S = \sin t^2$ and $C = \cos t^2$ finding $\cos \theta$ using

$$\mathbf{a} \cdot \vec{\mathbf{OP}} = 2SC - 4t^2 S^2 - 2SC - 4t^2 C^2 = -4t^2$$

M1

$$|\vec{\mathbf{OP}}| = 1$$

$$|\mathbf{a}| = \sqrt{(2C - 4t^2 S)^2 + (-2S - 4t^2 C)^2}$$

$$= \sqrt{4 + 16t^4} > 4t^2$$

if θ is the angle between them, then

$$\cos \theta = -\frac{4t^2}{\sqrt{4 + 16t^4}}$$

A1so $-1 < \cos \theta < 0$ therefore the vectors are never parallel**R1***continued...*

Question 16 continued

METHOD 2

solve

$$\begin{pmatrix} 2 \cos t^2 - 4t^2 \sin t^2 \\ -2 \sin t^2 - 4t^2 \cos t^2 \end{pmatrix} = k \begin{pmatrix} \sin t^2 \\ \cos t^2 \end{pmatrix}$$

M1

then

$$(k =) \frac{2 \cos t^2 - 4t^2 \sin t^2}{\sin t^2} = \frac{-2 \sin t^2 - 4t^2 \cos t^2}{\cos t^2}$$

Note: Condone candidates not excluding the division by zero case here. Some might go straight to the next line.

$$2 \cos^2 t^2 - 4t^2 \cos t^2 \sin t^2 = -2 \sin^2 t^2 - 4t^2 \cos t^2 \sin t^2$$

$$2 \cos^2 t^2 + 2 \sin^2 t^2 = 0$$

$$2 = 0$$

A1

this is never true so the two vectors are never parallel

R1**METHOD 3**

embedding vectors in a 3d space and taking the cross product:

M1

$$\begin{pmatrix} \sin t^2 \\ \cos t^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2 \cos t^2 - 4t^2 \sin t^2 \\ -2 \sin t^2 - 4t^2 \cos t^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \sin^2 t^2 - 4t^2 \cos t^2 \sin t^2 - 2 \cos^2 t^2 + 4t^2 \cos t^2 \sin t^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

A1

since the cross product is never zero, the two vectors are never parallel

R1**[5 marks]****Total [7 marks]**

2. (a) (i) $\left(\frac{1}{2}A\hat{O}B = \right) \arccos\left(\frac{4}{4.5}\right) = 27.266\dots$ **(M1)(A1)**

$A\hat{O}B = 54.532\dots \approx 54.5^\circ$ ($0.951764\dots \approx 0.952$ radians) **A1**

Note: Other methods may be seen; award **(M1)(A1)** for use of a correct trigonometric method to find an appropriate angle and then **A1** for the correct answer.

(ii) a finding area of triangle

EITHER

area of triangle $= \frac{1}{2} \times 4.5^2 \times \sin(54.532\dots)$ **(M1)**

Note: Award **M1** for correct substitution into formula.

$= 8.24621\dots \approx 8.25 \text{ m}^2$ **(A1)**

OR

$AB = 2 \times \sqrt{4.5^2 - 4^2} = 4.1231\dots$ **(M1)**

area triangle $= \frac{4.1231\dots \times 4}{2}$
 $= 8.24621\dots \approx 8.25 \text{ (m}^2\text{)}$ **(A1)**

finding area of sector

EITHER

area of sector $= \frac{54.532\dots}{360} \times \pi \times 4.5^2$ **(M1)**

$= 9.63661\dots \approx 9.64 \text{ m}^2$ **(A1)**

OR

area of sector $= \frac{1}{2} \times 0.951764\dots \times 4.5^2$ **(M1)**

$= 9.63661\dots \approx 9.64 \text{ m}^2$ **(A1)**

THEN

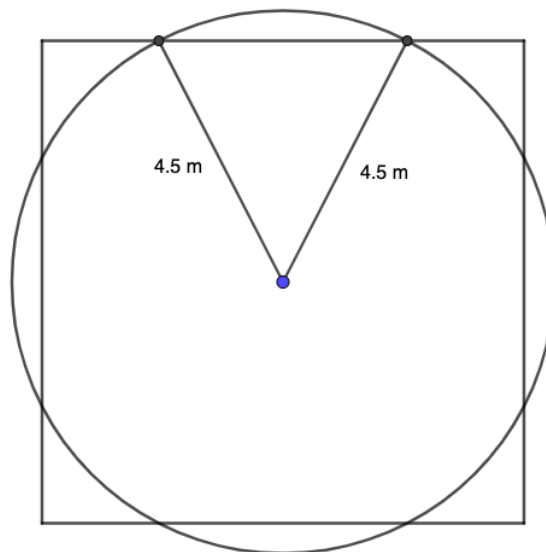
area of segment $= 9.63661\dots - 8.24621\dots$

$= 1.39 \text{ m}^2$ (1.39040...) **A1**

[8 marks]

continued...

Question 2 continued

(b) **METHOD 1**

$$\pi \times 4.5^2 \text{ (63.6172...)}$$

(A1)

$$4 \times 1.39040... \text{ (5.56160)}$$

(A1)

subtraction of four segments from area of circle

(M1)

$$= 58.1 \text{ m}^2 \text{ (58.055...)}$$

A1**METHOD 2**

$$\text{angle of sector} = 90 - 54.532... \left(\frac{\pi}{2} - 0.951764... \right)$$

(A1)

$$\text{area of sector} = \frac{90 - 54.532...}{360} \times \pi \times 4.5^2 \text{ (= 6.26771...)}$$

(A1)

area is made up of four triangles and four sectors

(M1)

$$\text{total area} = (4 \times 8.2462...) + (4 \times 6.26771...)$$

$$= 58.1 \text{ m}^2 \text{ (58.055...)}$$

A1**[4 marks]**

- (c) sketch of $\frac{dV}{dt}$ **OR** $\frac{dV}{dt} = 0.110363...$ **OR** attempt to find where $\frac{d^2V}{dt^2} = 0$
 $t = 1$ hour

(M1)**A1****[2 marks]**

- (d) recognizing $V = \int \frac{dV}{dt} dt$

(M1)

$$\int_0^8 0.3te^{-t} dt$$

(A1)

$$\text{volume eaten is } 0.299... \text{ m}^3 \text{ (0.299094...)}$$

A1**[3 marks]****[Total 17 marks]**

$$7. \quad (a) \quad (i) \quad \mathbf{P} \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \mathbf{q} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (M1)$$

$$\mathbf{q} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad A1$$

(ii) **EITHER**

$$\mathbf{P} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{4} \\ \frac{3}{4} \end{pmatrix} \quad M1$$

$$\text{hence } \mathbf{P} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{4} \\ -\frac{1}{4} \end{pmatrix} \quad A1$$

$$\mathbf{P} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ 1 + \frac{\sqrt{3}}{4} \end{pmatrix} \quad M1$$

$$\text{hence } \mathbf{P} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{3}}{4} \end{pmatrix} \quad A1$$

continued...

*Question 7 continued***OR**

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{4} \\ \frac{3}{4} \end{pmatrix} \quad \textbf{M1}$$

$$\text{hence } \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{4} \\ -\frac{1}{4} \end{pmatrix} \quad \textbf{A1}$$

$$\begin{pmatrix} a \\ c \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{4} \\ -\frac{1}{4} \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ 1 + \frac{\sqrt{3}}{4} \end{pmatrix} \quad \textbf{M1}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{3}}{4} \end{pmatrix} \quad \textbf{A1}$$

$$\begin{pmatrix} b \\ d \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ \frac{\sqrt{3}}{4} \end{pmatrix}$$

THEN

$$\Rightarrow \mathbf{P} = \begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix} \quad \textbf{AG}$$

[6 marks]

$$\text{(b)} \quad \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \quad \textbf{A1}$$

[1 mark]*continued...*

Question 7 continued

(c) (i) **EITHER**

$$S^{-1} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

(A1)

$$R = PS^{-1}$$

(M1)

Note: The **M1** is for an attempt at rearranging the matrix equation. Award even if the order of the product is reversed.

$$R = \begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

(A1)**OR**

$$\begin{pmatrix} \frac{\sqrt{3}}{4} & \frac{1}{4} \\ -\frac{1}{4} & \frac{\sqrt{3}}{4} \end{pmatrix} = R \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}$$

$$\text{let } R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

attempt to solve a system of equations

M1

$$\frac{\sqrt{3}}{4} = 0.5a, \quad \frac{1}{4} = 0.5b$$

$$-\frac{1}{4} = 0.5c, \quad \frac{\sqrt{3}}{4} = 0.5d$$

A2

Note: Award **A1** for two correct equations, **A2** for all four equations correct.

THEN

$$R = \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} 0.866 & 0.5 \\ -0.5 & 0.866 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} 0.866025... & 0.5 \\ -0.5 & 0.866025... \end{pmatrix} \quad \text{A1}$$

Note: The correct answer can be obtained from reversing the matrices, so do not award if incorrect product seen. If the given answer is obtained from the product $R = S^{-1}P$, award **(A1)(M1)(A0)A0**.

continued...

Question 7 continued

- (ii) clockwise **A1**
 arccosine or arcsine of value in matrix seen **(M1)**
 30° **A1**

Note: Both **A1** marks are dependent on the answer to part (c)(i) and should only be awarded for a valid rotation matrix.

[7 marks](d) **METHOD 1**

(i) $\begin{pmatrix} a \\ b \end{pmatrix} = P \begin{pmatrix} a \\ b \end{pmatrix} + q$ **A1**

(ii) solving $\begin{pmatrix} a \\ b \end{pmatrix} = P \begin{pmatrix} a \\ b \end{pmatrix} + q$ using simultaneous equations or $a = (I - P)^{-1} q$ **(M1)**

$a = 0.651 \text{ (0.651084...)}, b = 1.48 \text{ (1.47662...)}$ **A1A1**

$$\left(a = \frac{5 + 2\sqrt{3}}{13}, b = \frac{14 + 3\sqrt{3}}{13} \right)$$

METHOD 2

(i) $\begin{pmatrix} x' \\ y' \end{pmatrix} = P \begin{pmatrix} x - a \\ y - b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ **A1**

Note: Accept substitution of x and y (and x' and y') with particular points given in the question.

(ii) $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = P \begin{pmatrix} 0 - a \\ 0 - b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$ **(M1)**

Note: This line, with any of the points substituted, may be seen in part (d)(i) and if so the **M1** can be awarded there.

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (I - P) \begin{pmatrix} a \\ b \end{pmatrix}$$

$a = 0.651084..., b = 1.47662...$ **A1A1**

$$\left(a = \frac{5 + 2\sqrt{3}}{13}, b = \frac{14 + 3\sqrt{3}}{13} \right)$$

[4 marks]
[Total 18 marks]

3. (a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ (M1)
 (4, 1) A1

Note: Award **A0** if parentheses are omitted in the final answer.

[2 marks]

- (b) attempt to substitute values into gradient formula (M1)
 $\left(\frac{0-2}{6-2}\right) = -\frac{1}{2}$ (A1)
 therefore the gradient of perpendicular bisector is 2 (M1)
 so $y-1 = 2(x-4)$ ($y = 2x-7$) A1

[4 marks]

- (c) identifying the correct equations to use: (M1)
 $y = 2-x$ and $y = 2x-7$
 evidence of solving their correct equations or finding points of intersection graphically (M1)
 (3, -1) A1

Note: Accept an answer expressed as " $x = 3, y = -1$ ".

[3 marks]

- (d) attempt to use distance formula (M1)
 $YZ = \sqrt{(7-(-1))^2 + (7-3)^2}$
 $= \sqrt{80} \quad (4\sqrt{5})$ A1

[2 marks]

- (e) **METHOD 1 (cosine rule)**
 length of XZ is $\sqrt{80} \quad (4\sqrt{5}, 8.94427...)$ (A1)

Note: Accept 8.94 and 8.9.

- attempt to substitute into cosine rule (M1)
 $\cos \hat{XYZ} = \frac{80+32-80}{2 \times \sqrt{80} \sqrt{32}} \quad (= 0.316227...)$ (A1)

Note: Award **A1** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

- $(\hat{XYZ} =) 71.6^\circ \quad (71.5650...^\circ)$ A1

Note: Last **A1** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

continued...

Question 3 continued

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ ($4\sqrt{5}$, 8.94427...) **(A1)**

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ **(M1)(A1)**

Note: Award **A1** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule. Exact values do not need to be used in the substitution.

$(\hat{X}\hat{Y}\hat{Z}) = 71.6^\circ$ (71.5650°) **A1**

Note: Last **A1** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\dots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ **(M1)**

= 24 km² **A1**

[2 marks]

(g) *Any sensible answer such as:*

There might be factors other than proximity which influence shopping choices.

A larger area does not necessarily result in an increase in population.

The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.

Transport links might not be represented by Euclidean distances.

etc.

R1

[1 mark]

Total [18 marks]

6. (a) (i) $\sqrt{10^2 + 8^2}$ (M1)
 $= 12.8$ (12.8062..., $\sqrt{164}$) (ms^{-1}) A1

(ii) $\tan^{-1}\left(\frac{10}{8}\right)$ (M1)
 $= 0.896$ OR 51.3 ($0.896055...$ OR $51.3401...$) A1

Note: Accept 0.897 or 51.4 from use of $\arcsin\left(\frac{10}{12.8}\right)$.

[4 marks]

(b) $y = t(10 - 5t)$ (M1)

Note: The **M1** might be implied by a correct graph or use of the correct equation.

METHOD 1 – graphical Method
 sketch graph

(M1)

Note: The **M1** might be implied by correct graph or correct maximum (eg $t = 1$).

max occurs when $y = 5$ m A1

METHOD 2 – calculus

differentiating and equating to zero

(M1)

$$\frac{dy}{dt} = 10 - 10t = 0$$

$$t = 1$$

$y (= 1(10 - 5)) = 5$ m A1

METHOD 3 – symmetry

line of symmetry is $t = 1$

(M1)

$y (= 1(10 - 5)) = 5$ m A1

[3 marks]

(c) attempt to solve $t(10 - 5t) = 0$ (M1)

$t = 2$ (or $t = 0$) (A1)

$x (= 5 + 8 \times 2) = 21$ m A1

Note: Do not award the final **A1** if $x = 5$ is also seen.

[3 marks]

continued...

Question 6 continued

(d) **METHOD 1**

$$t = \frac{x-5}{8}$$

M1A1

$$y = \left(\frac{x-5}{8} \right) \left(10 - 5 \times \frac{x-5}{8} \right)$$

A1**METHOD 2**

$$y = k(x-5)(x-21)$$

A1

$$\text{when } x=13, y=5 \text{ so } k = \frac{5}{(13-5)(13-21)} = -\frac{5}{64}$$

M1A1

$$\left(y = -\frac{5}{64}(x-5)(x-21) \right)$$

METHOD 3

$$\text{if } y = ax^2 + bx + c$$

$$0 = 25a + 5b + c$$

$$5 = 169a + 13b + c$$

$$0 = 441a + 21b + c$$

M1A1

$$\text{solving simultaneously, } a = -\frac{5}{64}, b = \frac{130}{64}, c = -\frac{525}{64}$$

A1

$$\left(y = -\frac{5}{64}x^2 + \frac{130}{64}x - \frac{525}{64} \right)$$

METHOD 4

use quadratic regression on (5, 0), (13, 5), (21, 0)

M1A1

$$y = -\frac{5}{64}x^2 + \frac{130}{64}x - \frac{525}{64}$$

A1

Note: Question asks for expression; condone omission of “y = ”.

[3 marks](e) trajectory of arrow is $y = x \tan 10 + 2$ **(A1)**intersecting $y = x \tan 10 + 2$ and their answer to (d)**(M1)**

$$(8.66, 3.53) \left((8.65705\dots, 3.52647\dots) \right)$$

A1

$$(15.1, 4.66) \left((15.0859\dots, 4.66006\dots) \right)$$

A1**[4 marks]**

continued...

Question 6 continued

(f) when $x_{\text{target}} = 8.65705\dots$, $t_{\text{target}} = \frac{8.65705\dots - 5}{8} = 0.457132\dots$ s **(A1)**

attempt to find the distance from point of release to intersection **(M1)**

$$\sqrt{8.65705\dots^2 + (3.52647\dots - 2)^2} \quad (= 8.79060\dots \text{ m})$$

time for arrow to get there is $\frac{8.79060\dots}{60} = 0.146510\dots$ s **(A1)**

so the arrow should be released when

$t = 0.311$ (s) (0.310622... (s)) **A1**

[4 marks]

Total [21 marks]

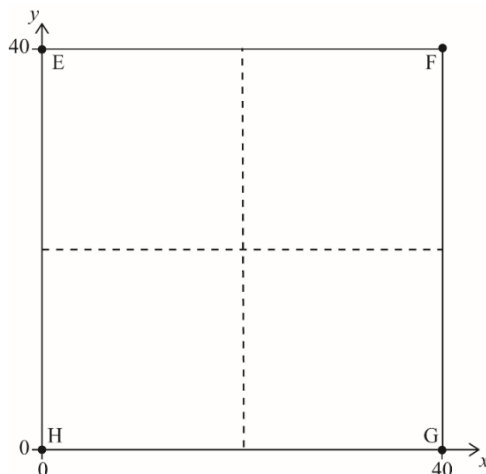
2. (a) the size of each town is small (in comparison with the distance between the towns)
OR
 if towns have an identifiable centre
OR
 the centre of the town is at that point

R1

Note: Accept a geographical landmark in place of “centre”, e.g. “town hall” or “capitol”.

[1 mark]

(b)

**A1**

Note: There is no need for a scale / coordinates here. Condone boundaries extending beyond the metropolitan area.

[1 mark]

- (c) (i) the gradient of IF is $\frac{40-20}{40-30} = 2$
 negative reciprocal of any gradient
 gradient of perpendicular bisector $= -\frac{1}{2}$

(A1)**(M1)**

Note: Seeing $-\frac{2}{3}$ (for example) used clearly as a gradient anywhere is evidence of the “negative reciprocal” method despite being applied to an inappropriate gradient.

$$\text{midpoint is } \left(\frac{40+30}{2}, \frac{40+20}{2} \right) = (35, 30)$$

(A1)

$$\text{equation of perpendicular bisector is } y - 30 = -\frac{1}{2}(x - 35)$$

A1

Note: Accept equivalent forms e.g. $y = -\frac{1}{2}x + \frac{95}{2}$ or $2y + x - 95 = 0$.

Allow **FT** for the final **A1** from their midpoint and gradient of perpendicular bisector, as long as the **M1** has been awarded.

[4 marks]

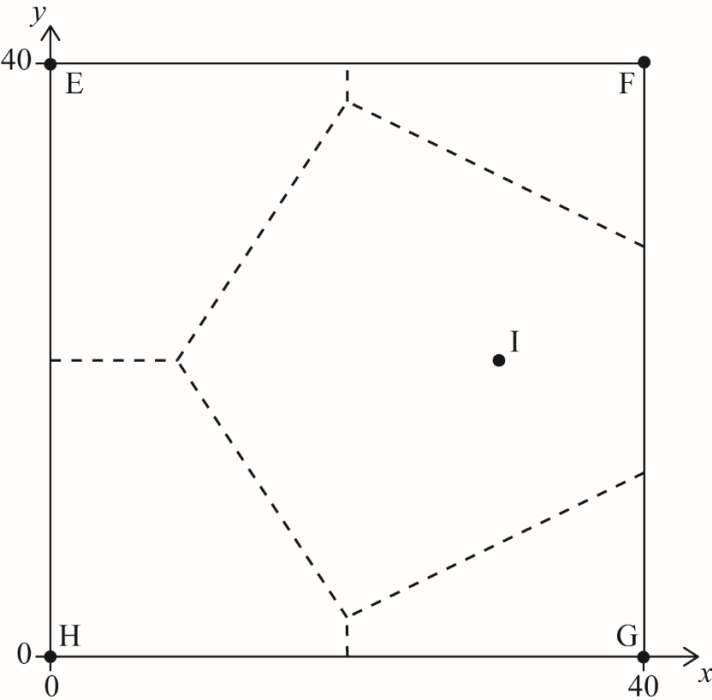
(ii) the perpendicular bisector of EH is $y = 20$ (A1)

Note: Award this **A1** if seen in the y -coordinate of any final answer or if 20 is used as the y -value in the equation of any other perpendicular bisector.

attempt to use symmetry **OR** intersecting two perpendicular bisectors (M1)
 $\left(\frac{25}{3}, 20\right)$ A1
(20, 2.5) A1

[4 marks]

(iii)



M1A1

Note: Award **M1** for exactly four perpendicular bisectors around I (IE, IF, IG and IH) seen, even if not in exactly the right place.
Award **A1** for a completely correct diagram. Scale / coordinates are NOT necessary. Vertices should be in approximately the correct positions but only penalized if clearly wrong (condone northern and southern vertices appearing to be very close to the boundary).

Condone the Voronoi diagram extending outside of the square.
Do not award follow-through marks in this part.

[2 marks]

- (d) 30% of 40 is 12 (A1)

recognizing line intersects bisectors at $y = c$ (or equivalent) but different x -values (M1)

$$c = \frac{3}{2}x_1 + \frac{15}{2} \quad \text{and} \quad c = -\frac{1}{2}x_2 + \frac{95}{2}$$

finding an expression for the distance in Isaacopolis in terms of one variable (M1)

$$x_2 - x_1 = (95 - 2c) - \frac{2c - 15}{3} = 100 - \frac{8c}{3}$$

equating their expression to 12

$$100 - \frac{8c}{3} = 0.3 \times 40 = 12$$

$$c = 33$$

distance = 33 (km)

A1
[4 marks]

- (e) (i) must be a vertex (award if vertex given as a final answer) (R1)
attempt to calculate the distance of at least one town from a vertex (M1)

Note: This must be seen as a calculation or a value.

correct calculation of distances

A1

$$\frac{65}{3} \quad \text{OR} \quad 21.7 \quad \text{AND} \quad \sqrt{406.25} \quad \text{OR} \quad 20.2$$

$$\left(\frac{25}{3}, 20 \right)$$

A1

Note: Award **R1M0A0A0** for a vertex written with no other supporting calculations. Award **R1M0A0A1** for correct vertex with no other supporting calculations. The final **A1** is not dependent on the previous **A1**. There is no follow-through for the final **A1**.

Do not accept an answer based on “uniqueness” in the question.

[4 marks]

- (ii) *For example, any one of the following:*
decision does not take into account the different population densities
closer to a city will reduce travel time/help employees
it is closer to some cities than others

R1

Note: Accept any correct reason that engages with the scenario. Do not accept any answer to do with ethical issues about whether toxic waste should ever be dumped, or dumped in a metropolitan area.

[1 mark]

(f) (i) **METHOD 1**attempting M^3 **M1**attempting M^4 **M1**

e.g.

last row/column of $M^3 = (3 \ 5 \ 1 \ 6 \ 0 \ 7)$ last row/column of $M^4 = (10 \ 12 \ 4 \ 16 \ 1 \ 18)$

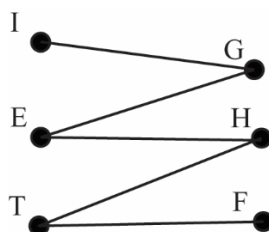
hence Isaacopolis is the last city to be polluted

A1

Note: Do not award the **A1** unless both M^3 and M^4 are considered.
Award **M1M0A0** for a claim that the shortest distance is from T to I and that it is 4, without any support.

METHOD 2attempting to translate M to a graph or a list of cities polluted on each day (**M1**)

correct graph or list

A1

hence Isaacopolis is the last city to be polluted

A1

Note: Award **M1A1A1** for a clear description of the graph in words leading to the correct answer.

[3 marks]

(ii) it takes 4 days

A1**[1 mark]**

- (iii) **EITHER**
the orders of the different vertices are:
E 2
F 1
G 2
H 2
I 1
T 2

(A1)

Note: Accept a list where each order is 2 greater than listed above.

- OR**
a correct diagram/graph showing the connections between
the locations

(A1)

Note: Accept a diagram with loops at each vertex.
This mark should be awarded if candidate is clearly using their
correct diagram from the previous part.

- THEN**
“Start at *F* and end at *I*” **OR** “Start at *I* and end at *F*”

A1

Note: Award **A1A0** for “it could start at either *F* or *I*”.
Award **A1A1** for “*IGEHTF*” **OR** “*FTHEGI*”.
Award **A1A1** for “*F* and *I*” **OR** “*I* and *F*”.

[2 marks]

[Total 27 marks]

2. (a) $AF^2 = 89.2^2 + 104.9^2 - 2(89.2)(104.9)\cos 83$ (M1)(A1)

Note: Award **(M1)** for substitution into the cosine rule and **(A1)** for correct substitution.

$$AF = 129 \text{ m } (129.150\dots)$$

A1

[3 marks]

(b) $21310 \div 129.150\dots$

(M1)

$$\$ 165$$

A1

[2 marks]

- (c) any reasonable statement referring to the lake
(eg. there is a lake between A and F, the cables would need to be installed under/over/around the lake, special waterproof cables are needed for lake, etc.)

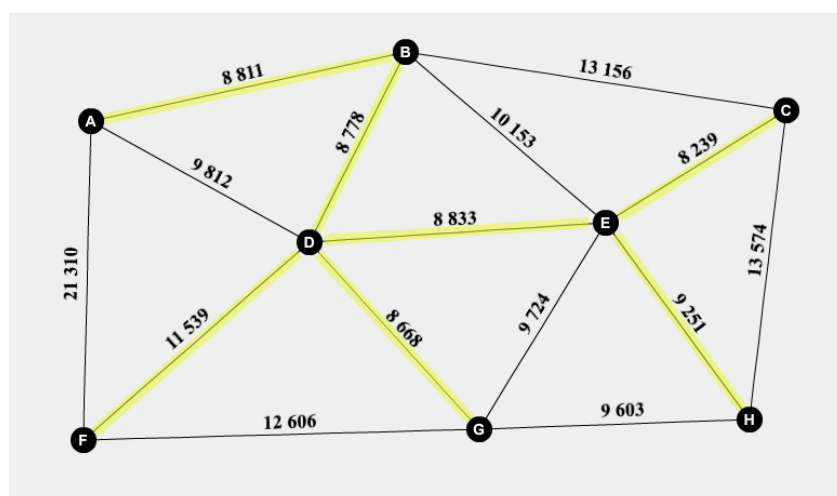
R1

[1 mark]

- (d) (i) edges (or weights) are chosen in the order

CE (8239)
 DG (8668)
 BD (8778)
 AB (8811)
 DE (8833)
 EH (9251)
 DF (11539)

A1A1A1



Note: Award **A1** for the first two edges chosen in the correct order. Award **A1A1** for the first six edges chosen in the correct order. Award **A1A1A1** for all seven edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated.

[3 marks]

- (ii) Finding the sum of the weights of their edges
 $8239 + 8668 + 8778 + 8811 + 8833 + 9251 + 11539$

(M1)

$$\text{total cost} = \$64119$$

A1

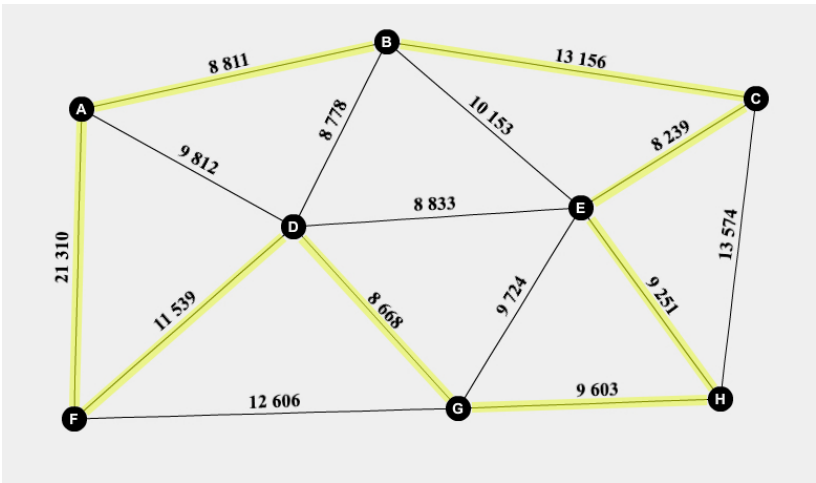
[2 marks]

- (e) a Hamiltonian cycle is not always an Eulerian circuit as it does not have to include all edges of the graph (only all vertices)

R1
[1 mark]

- (f) edges (or weights) are chosen in the order
- DG (8668)
 - GH (9603)
 - HE (9251)
 - EC (8239)
 - CB (13156)
 - BA (8811)
 - AF (21310)
 - FD (11539)

A1A1A1



Note: Award **A1** for the first two edges chosen in the correct order. Award **A1A1** for the first five edges chosen in the correct order. Award **A1A1A1** for all eight edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated.

finding the sum of the weights of their edges
 $8668 + 9603 + 9251 + 8239 + 13156 + 8811 + 21310 + 11539$

(M1)

upper bound = \$90577

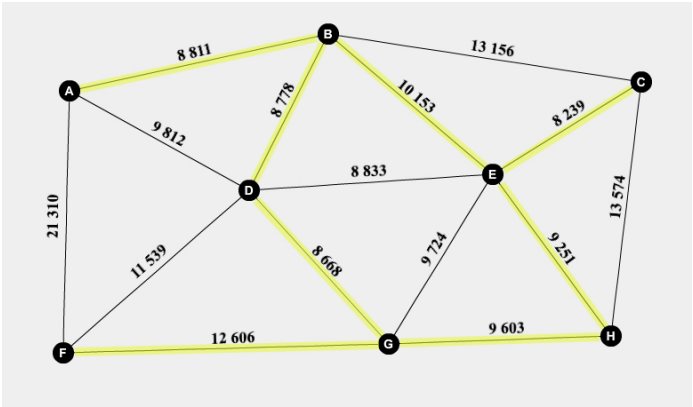
A1
[5 marks]

- (g) attempt to find MST after deleting vertex D (M1)
- these edges (or weights) (in any order)
- CE (8239)
- AB (8811)
- EH (9251)
- GH (9603)
- BE (10153)
- FG (12606)
- A1

Note: Prim's or Kruskal's algorithm could be used at this stage.

- reconnect D to MST with two different edges (M1)
- DG (8668)
- BD (8778)
- A1

Note: This **A1** is independent of the first **A** mark and can be awarded if both DG and BD are chosen to reconnect D to the MST, even if the MST is incorrect.



- finding the sum of the weights of their edges (M1)
- $8239 + 8811 + 9251 + 9603 + 10153 + 12606 + 8668 + 8778$

Note: For candidates with an incorrect MST or no MST, the weights of at least seven of the edges being summed (two of which must connect to D) must be shown to award this (M1).

- lower bound = \$76109
- A1
- [6 marks]

(h) **METHOD 1**

recognition of a binomial distribution

(M1)

$$X \sim B(2, 0.014)$$

finding the probability that a cable fails (at least one of its connections fails)

$$P(X > 0) = 0.027804 \quad \text{OR} \quad 1 - P(X = 0) = 0.027804$$

A1recognition that **two** cables must fail for the network to go offline**M1**recognition of binomial distribution for network, $Y \sim B(8, 0.027804)$ **(M1)**

$$P(Y \geq 2) = 0.0194 \quad (0.0193602...) \quad \text{OR} \quad 1 - P(Y < 2) = 0.0194 \quad (0.0193602...) \quad \text{A1}$$

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ **AG****Note:** Evidence of binomial distribution may be seen as combinations.**METHOD 2**

recognition of a binomial distribution

(M1)

$$X \sim B(16, 0.014)$$

finding the probability that at least **two** connections fail

$$P(X \geq 2) = 0.0206473... \quad \text{OR} \quad 1 - P(X < 2) = 0.0206473...$$

A1

recognition that the previous answer is an overestimate

M1finding probability of two ends of the same cable failing, $F \sim B(2, 0.014)$,and the ends of the other 14 cables not failing, $S \sim B(14, 0.014)$

$$P(F = 2) \times P(S = 0) = 0.0000160891...$$

(A1)

$$0.0000160891... \times 8 = 0.00128713...$$

$$0.0206473... - 0.00128713... = 0.0194 \quad (0.0193602...)$$

A1therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ **AG****METHOD 3**

recognition of a binomial distribution

M1

$$X \sim B(16, 0.014)$$

finding the probability that the network remains secure if 0 or 1 connections fail or if 2 connections fail provided that the second failed connection occurs at the other end of the cable with the first failure

(M1)

$$P(\text{remains secure}) = P(X \leq 1) + \frac{1}{15} \times P(X = 2)$$

A1

$$= 0.9806397625$$

A1

$$P(\text{network fails}) = 1 - 0.9806397625 = 0.0194 \quad (0.0193602...)$$

A1therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ **AG**

METHOD 4

P(network failing)

$$= 1 - P(0 \text{ connections failing}) - P(1 \text{ connection failing}) \\ - P(2 \text{ connections on the same cable failing})$$

M1

$$= 1 - 0.986^{16} - {}^{16}C_1 \times 0.014 \times 0.986^{15} - {}^8C_1 \times 0.014^2 \times 0.986^{14}$$

A1A1A1

Note: Award **A1** for each of 2nd, 3rd and last terms.

$$= 0.0194 \text{ (0.0193602...)}$$

A1therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ **AG****[5 marks]****[Total 28 marks]**

$$3. \quad (a) \quad \sin(21^\circ) = \frac{17}{BF} \quad (M1)$$

$$BF = 47.4 \text{ m } (47.4372...) \quad A1$$

[2 marks]

(b) **EITHER**

$$BE = \sqrt{47.4372...^2 + 44^2} = 64.7015... \quad (A1)$$

$$\sin^{-1}\left(\frac{17}{BE}\right) \quad (M1)$$

$$= 15.2^\circ \text{ (15.2329...}^\circ\text{)} \quad (\text{or } 0.266 \text{ radians (0.265866...)}) \quad A1$$

OR

$$AD = \sqrt{47.4372...^2 - 17^2} = 44.2865...$$

$$DB = \sqrt{64.7015...^2 + 44^2} = 62.4832... \quad (A1)$$

$$\tan^{-1}\left(\frac{17}{62.4832...}\right) \quad (M1)$$

$$= 15.2^\circ \text{ (15.2329...}^\circ\text{)} \quad (\text{or } 0.266 \text{ radians (0.265866...)}) \quad A1$$

[3 marks]

Total [5 marks]

4. (a)

$$\begin{array}{c}
 \begin{array}{cccccc}
 & S & A & B & C & D & T \\
 \begin{array}{c} S \\ A \\ B \\ C \\ D \\ T \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & \boxed{2} & 0 \\ 1 & 0 & 1 & 1 & \boxed{0} & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ \boxed{2} & \boxed{0} & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix}
 \end{array}
 \end{array}$$

$$SD = DS = 2$$

$$AD = DA = 0$$

A1**A1****[2 marks]**

- (b) attempt to calculate at least one of M^2 , M^3 and M^4
 attempt to calculate all of M^2 , M^3 and M^4
 finding at least one of the top right entries, 4, 10, 64
 78 walks

(M1)**(M1)****(A1)****A1**

Note: If $SD = DS = 1$ is their answer in part (a), their **FT** answer is
 $(3 + 8 + 41 =) 52$ walks.

[4 marks]

- (c) because some of the walks will pass through T, before returning to T

R1**[1 mark]****Total [7 marks]**

8. (a) setting up at least two simultaneous equations
 $p = -0.8$ **OR** $q = 3.6$
 M has coordinates $(1.6, -2.6, 8.2)$

(M1)**(A1)****A1****[3 marks]**

- (b) using vectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$

(M1)

$$\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = 3$$

(A1)

$$\cos \theta = \frac{\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}}{\sqrt{3^2 + 2^2 + 1^2} \sqrt{1^2 + (-1)^2 + 2^2}} \quad \left(\cos \theta = \frac{3}{\sqrt{14}\sqrt{6}} \right)$$

(M1)

Note: Accept correct use of vector product.

$$(\theta =) 1.24 \text{ radians } (1.23732\dots) \quad (70.9^\circ \quad (70.8933\dots))$$

A1**[4 marks]****Total [7 marks]**

9. (a) attempt to find $\det(M)$ (M1)
 $= 14$
 $(12 \times 14) = 168 \text{ cm}^2$ A1

[2 marks]

- (b) let X have coordinates (x, y)

METHOD 1

$$M \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix} \quad (M1)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = M^{-1} \begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix} \quad (A1)$$

$$M^{-1} = \frac{1}{14} \begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix} \quad A1$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 2t-3+24-20t \\ -6t+9+12-10t \end{pmatrix} \quad (M1)$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 21-18t \\ 21-16t \end{pmatrix} \quad \text{OR} \quad \left(\frac{21-18t}{14}, \frac{21-16t}{14} \right) \quad A1A1$$

METHOD 2

writing two simultaneous equations (M1)

$$2x - 4y = 2t - 3 \quad (A1)$$

$$3x + y = 6 - 5t \quad (A1)$$

attempting to solve the equations (M1)

$$(x, y) = \left(\frac{3}{2} - \frac{9t}{7}, \frac{3}{2} - \frac{8t}{7} \right) \quad A1A1$$

[6 marks]

Total [8 marks]

2. (a) (i) B A1
- (ii) F A1
- [2 marks]
- (b) correct substitution into the midpoint formula (M1)
- $$\frac{8+5}{2}$$
- $$y = 6.5$$
- A1
- Note:** Answer must be an equation for the **A1** to be awarded.
- [2 marks]
- (c) midpoint = (5, 7) (A1)
- correct use of gradient formula (M1)
- $$\frac{8-6}{7-3}$$
- gradient of BC = 0.5 (A1)
- negative reciprocal of gradient (M1)
- perpendicular gradient = -2
- $$y - 7 = -2(x - 5) \text{ (or } y = -2x + 17 \text{)}$$
- A1
- Note:** Do not follow through within the part for the final **A1**.
- [5 marks]
- (d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) (M1)
- Note:** This may be seen graphically or algebraically.
- $$6.5 - 7 = -2(x - 5) \text{ OR } 6.5 = -2x + 17$$
- Note:** Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x - 5)$ OR $y = 4x - 14.5$
- $$x = 5.25, y = 6.5 \text{ OR } (5.25, 6.5)$$
- A1
- Note:** The x -coordinate must be exact or expressed to at least 3 sf.
- (ii) their correct substitution into distance formula (M1)
- $$\sqrt{(5.25 - 7)^2 + (6.5 - 5)^2}$$
- $$= 2.30 \text{ km} \left(2.30488..., \frac{\sqrt{85}}{4} \right)$$
- A1
- [4 marks]
- Total [13 marks]**

4. (a) any correct Hamiltonian cycle e.g. ABCDEFA **A1**
[1 mark]
- (b) no, since not all vertices have an even degree (or equivalent) **R1**
[1 mark]
- (c) (i) 49 **A1**
- (ii) 34 **A1**
- (iii) 50 **A1**
[3 marks]
- (d) cycle is EBCDFAE
 $UB = 12 + 25 + 17 + 34 + 18 + 35$ **(M1)(A1)**
- Note:** Award **M1** for $12 + 25 + 17 + \dots$ **OR** EBCD.
- $= 141$ **A1**
[3 marks]
- (e) attempt to find MST for vertices A, B, C, D and E **M1**
 $12 + 14 + 17 + 27 (= 70)$ **A1**
 $LB = 70 + 18 + 22$ **(M1)**
 $= 110$ **A1**
[4 marks]
- (f) **EITHER** **A1**
 deleting a different vertex **R1**
 might give a higher value (and hence a better lower bound).
OR
 the edges selected in part (e) do not form a cycle. **A1**
 so a higher value is possible **R1**
[2 marks]
[14 marks]

7. (a) (i) use of chain rule (M1)
 $\mathbf{v} = -9 \sin(3t)\mathbf{i} + 12 \cos(3t)\mathbf{j}$ A1

Note: Award (M1) for at least one correct term seen but condone omission of $\mathbf{i}$ or $\mathbf{j}$.

(ii) $|\mathbf{v}| = \sqrt{(-9 \sin(9))^2 + (12 \cos(9))^2}$ (M1)
 $= 11.5 \text{ m s}^{-1} \text{ (11.5455...)}$ A1

[4 marks]

(b) (i) $\mathbf{a} = -27 \cos(3t)\mathbf{i} - 36 \sin(3t)\mathbf{j}$ A1

(ii) $\mathbf{a} = -9(3 \cos(3t)\mathbf{i} - 4 \sin(3t)\mathbf{j})$ M1
 $\mathbf{a} = -9\mathbf{r}$ (where $\mathbf{r}$ is a position vector from the origin) A1
 $\mathbf{a}$ is in opposite direction to the position vector R1
hence $\mathbf{a}$ is always directed towards the origin AG

[4 marks]

(c) relative position $\mathbf{d} = \mathbf{r}_2 - \mathbf{r}_1$ (M1)

distance between particles $= |\mathbf{d}| \text{ (} = |\mathbf{r}_2 - \mathbf{r}_1| \text{)}$ (M1)

$|\mathbf{d}| = \sqrt{(-4 \sin(4t) - 3 \cos(3t))^2 + (3 \cos(4t) - 4 \sin(3t))^2}$ (A1)

minimum value of $|\mathbf{d}|$ when $t = 4.71 \text{ (s)} \left(4.71238..., \frac{3\pi}{2} \right)$ (M1)A1

[5 marks]

(d) (i) for 2nd particle, $\mathbf{v} = -16 \cos(4t)\mathbf{i} - 12 \sin(4t)\mathbf{j}$ (A1)

EITHERconsider the gradient of either $\mathbf{v}$ (M1)

$m_1 = -\frac{12 \cos(3t)}{9 \sin(3t)}$ and $m_2 = \frac{12 \sin(4t)}{16 \cos(4t)}$ (A1)

attempt to solve $m_1 = m_2$ (M1)

ORvectors are parallel therefore one is a multiple of the other, $\mathbf{v}_2 = l \mathbf{v}_1$ (M1)

$(l =) \frac{16 \cos(4t)}{9 \sin(3t)} = -\frac{\sin(4t)}{\cos(3t)}$ (A1)

attempt to solve (M1)

THEN

$t = 1.30 \text{ s (1.30135...)}$ A1

continued...

Question 7 continued

- (ii) **EITHER**
 at $t = 1.30$, $\mathbf{v}_1 = 6.22\mathbf{i} - 8.68\mathbf{j}$ and $\mathbf{v}_2 = -7.57\mathbf{i} + 10.6\mathbf{j}$ **A1**
- OR**
 $l = -1.22$ (following second method in part (d)(i)) **A1**
- THEN**
 $\mathbf{v}_2$ is a negative multiple of $\mathbf{v}_1$ ($\mathbf{v}_2 = -1.22\mathbf{v}_1$) **R1**
 the two particles are moving in the opposite direction **AG**
- [7 marks]**
Total [20 marks]
-

3. (a) attempt to use distance formula for points D and A (M1)

$$DA = \sqrt{11^2 + 7^2}$$

$$= 13.0 \text{ (miles)} \text{ (13.0384..., } \sqrt{170} \text{)}$$

A1

Note: Accept 13 miles. Award **MOA0** for finding the equation of the line DA.

DA may be seen in part (b) but this should not be accepted as answer for part (a).

[2 marks]

(b) $\left(DB = \sqrt{13^2 + 5^2} = \right) 13.9 \text{ (13.9283..., } \sqrt{194} \text{)}$ **AND**

$$\left(DC = \sqrt{4^2 + 12^2} = \right) 12.6 \text{ (12.6491..., } \sqrt{160} \text{)}$$

A1

recognizing closest town is best estimate

(M1)

(town C is closest)

30 °C

A1

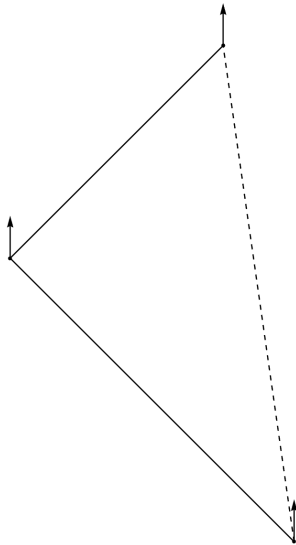
Note: If their DA from part (a) is the shortest length, then allow **FT** in (b).

[3 marks]

[Total: 5 marks]

5. METHOD 1

diagram showing (approximately) correct directions (and order) for the 315° and 045° **(A1)**



recognizing right angle triangle **(M1)**

correct expression to find second angle in triangle **(A1)**

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing **(A1)**

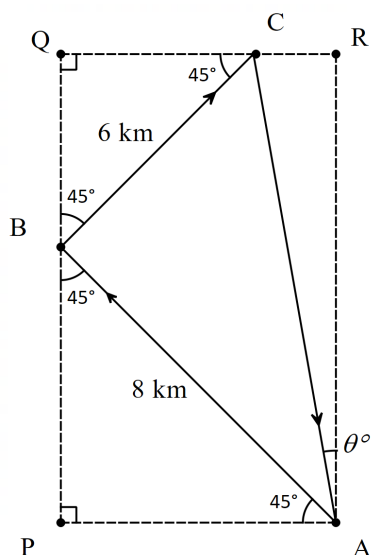
e.g. $\arctan\left(\frac{6}{8}\right) + 135^\circ$ **OR** $360^\circ - \left(\arctan\left(\frac{8}{6}\right) + 135^\circ\right)$

$= 172^\circ$ (171.869...°) **A1**

continued...

Question 5 continued**METHOD 2**

diagram showing (approximately) correct directions (and order) for the 315° and 045° (these may be shown in reverse as the return journey)

(A1)

finding the lengths marked AP, BP, CQ and BQ in the diagram

(M1)

$$AP = BP = 8 \frac{\sqrt{2}}{2} = 5.6568\dots$$

$$CQ = BQ = 6 \frac{\sqrt{2}}{2} = 4.2426\dots$$

Note: This may be done using a vector approach.

using $\tan \theta^\circ = \frac{AP - CQ}{PB + BQ}$ or equivalent to find the direction of AC

(A1)

correct expression to find bearing

(A1)

$$180^\circ - \arctan \left(\frac{8 \frac{\sqrt{2}}{2} + 6 \frac{\sqrt{2}}{2}}{8 \frac{\sqrt{2}}{2} - 6 \frac{\sqrt{2}}{2}} \right)$$

$$= 172^\circ \quad (171.869\dots^\circ)$$

A1**[Total: 5 marks]**

7. (a) attempt to find the vector product (e.g. one term correct) (M1)

$$\begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix} \times \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \\ -42 \end{pmatrix}$$

A1

[2 marks]

- (b) **METHOD 1**

attempt to use the vector product formula for the area of triangle

(condone incorrect signs and missing $\frac{1}{2}$) (M1)

$$\text{area} = \frac{1}{2} \sqrt{3^2 + 7^2 + 42^2}$$

$$= 21.3 \text{ (m}^2\text{)} \quad \left(21.3424\dots, \frac{1}{2} \sqrt{1822} \right) \quad \text{A1}$$

METHOD 2

find θ using $\vec{AB} \times \vec{AC} = \left| \vec{AB} \right| \left| \vec{AC} \right| \sin \theta$ (M1)

$$\theta = 67.1^\circ \quad (67.1350^\circ \dots, 1.171728\dots \text{ radians})$$

$$\text{then area} = \frac{1}{2} \left| \vec{AB} \right| \left| \vec{AC} \right| \sin \theta$$

$$= 21.3 \text{ (m}^2\text{)} \quad \left(21.3424\dots, \frac{1}{2} \sqrt{1822} \right) \quad \text{A1}$$

[2 marks]

continued...

Question 7 continued

(c) $AC = 7.61577... \left(\sqrt{58} \right)$ **(A1)**

setting the area formula $\frac{1}{2} \times \text{base} \times \text{height}$ equal to their part (b) **(M1)**

$$BX = \frac{2 \times 21.3424...}{\sqrt{58}}$$

$$= 5.60 \text{ (5.60480...)} \quad \text{A1}$$

Note: Award **A1** for 5.6.

Award **A1** for 5.59 (5.5936...) from the use of 21.3 to 3 sf.

[3 marks]

(d) attempting to set up a trig ratio **(M1)**

$$\text{angle is } \arcsin\left(\frac{1}{BX}\right)$$

$$10.3^\circ \text{ (10.2776...}^\circ, 0.179378... \text{ radians)} \quad \text{A1}$$

[2 marks]

[Total: 9 marks]

6. (a) **EITHER**

$$\frac{4}{3}\pi(3.4)^3 \quad (\text{A1})$$

Multiplying their volume by $\frac{4}{5}$ (M1)

OR

$$\frac{4}{3}\pi(3.4)^3 \quad (\text{A1})$$

Subtracting $\frac{1}{5}$ of their volume (M1)

$$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5} \times \frac{4}{3}\pi(3.4)^3 \right)$$

Note: The **M1** can be awarded for a final answer of 32.9272... seen without working.

THEN

$$132 \text{ cm}^3 \text{ (131.708... cm}^3\text{)} \quad \text{A1} \quad [3 \text{ marks}]$$

(b) $\pi \times 3 \times 11$ (A1)

$$103.672... (\text{cm}^2) \quad \text{OR} \quad 33\pi(\text{cm}^2)$$

$$104 (\text{cm}^2) \quad \text{A1} \quad [2 \text{ marks}]$$

Total [5 marks]

8. (a) attempt to create a 5x5 adjacency matrix (M1)

$$M = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad \text{A2}$$

Note: Allow the transposed matrix. Award **A2** for all entries correct, **A1** if one or two entries are incorrect, **A0** otherwise.

Answer presented in markscheme assumes ABCDE ordering of rows and columns; accept other orders provided they are clearly communicated.

Award **A1** if the zeroes are replaced by blank cells.

[3 marks]

- (b) (i) recognizing need to find M^7 (M1)

$$M^7 = \begin{pmatrix} 8 & 8 & 17 & 8 & 13 \\ 8 & 10 & 19 & 17 & 14 \\ 6 & 11 & 16 & 10 & 17 \\ 11 & 8 & 19 & 14 & 10 \\ 2 & 6 & 8 & 11 & 8 \end{pmatrix}$$

2 (routes) A1

- (ii) vertices visited in order are

EITHER

$E \rightarrow D \rightarrow C \rightarrow B \rightarrow C \rightarrow B \rightarrow D \rightarrow A$

A2

OR

$E \rightarrow D \rightarrow C \rightarrow B \rightarrow C \rightarrow E \rightarrow D \rightarrow A$

A2

[4 marks]

Total [7 marks]

13. (a) METHOD 1 (find product of matrices first)

$$T \rightarrow T' \text{ is represented by } \mathbf{QP} = \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix} \quad (\mathbf{M1})$$

$$= \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix} \quad (\mathbf{A1})$$

recognizing need to find their $(\mathbf{QP})^{-1}$ (\mathbf{M1})

$$(\mathbf{QP})^{-1} = \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}^{-1}$$

$$= -\frac{1}{78} \begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix} \text{ OR } = \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix} \quad \mathbf{A1}$$

METHOD 2 (find inverses of both matrices first)

recognizing need to find inverse of both $\mathbf{P}$ and $\mathbf{Q}$ (\mathbf{M1})

$$\mathbf{P}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix} \text{ AND } \mathbf{Q}^{-1} = \begin{pmatrix} -\frac{3}{13} & \frac{1}{13} \\ \frac{1}{13} & \frac{4}{13} \end{pmatrix} \quad (\mathbf{A1})$$

$$T' \rightarrow T \text{ is represented by } \mathbf{P}^{-1}\mathbf{Q}^{-1} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}^{-1} \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}^{-1} \quad (\mathbf{M1})$$

$$= -\frac{1}{78} \begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix} \text{ OR } = \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix} \quad \mathbf{A1}$$

Note: In METHOD 1, award **M1A0M1A0** if they multiply the matrices in the wrong order.
In METHOD 2, award **M1A1M1A0** if they multiply the matrices in the wrong order.

[4 marks]

$$(b) \left(\det \left[-\frac{1}{78} \begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix} \right] \right) = -\frac{1}{78} \text{ OR } \left(\det \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix} \right) = -78 \quad (\mathbf{A1})$$

$$\text{area of } T' = |\det \mathbf{QP}| \times \text{area of } T \text{ OR area of } T = |\det (\mathbf{QP})^{-1}| \times \text{area of } T' \quad (\mathbf{M1})$$

$$\Rightarrow \text{area of } T = 273 \times \frac{1}{78}$$

$$= 3.5 \text{ (cm}^2\text{)} \quad \mathbf{A1}$$

Note: Award **(A1)(M0)A0** for an answer of $-3.5 \text{ (cm}^2\text{)}$ with or without working. Accept an answer of $4.04 \text{ (cm}^2\text{)}$ from use of 3sf values in their answer to part (a).

[3 marks]

Total [7 marks]

14. (a) $\mathbf{v}_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ (A1)

attempt to find any relevant angle (M1)

$$\tan^{-1}\left(\frac{3}{2}\right) (= 56.3099...^\circ)$$

$$(90^\circ + 56.3099...^\circ =) 146^\circ \quad (146.3099...^\circ)$$

A1

[3 marks]

(b) setting $1 + 2t = -2 + 4t$ (M1)

$$t = 1.5 \text{ (hrs.)}$$

A1

[2 marks]

(c) $\mathbf{r}_B - \mathbf{r}_A = (-3 + 2t)\mathbf{i} + (-7 + 4t)\mathbf{j}$ (M1)

$$-3 + 2t = -(-7 + 4t)$$

(M1)

$$t = 1.67 \text{ (hrs.)} \left(1.66666..., \frac{5}{3} \right)$$

A1

[3 marks]

Total [8 marks]

4. (a) there are more than two vertices with odd degree **R1**
 so it is not possible to travel along each road exactly once **A1**

Note: Do not award **R0A1**.

Award **R1** for “There are 4 vertices with odd degree”.

[2 marks]

- (b) $a = 11$, $b = 18$, $c = 17$, $d = 15$ **A2**

Note: Award **A1** for any one correct, **A2** for all four correct.

[2 marks]

- (c) attempt to use nearest neighbour algorithm **(M1)**

Note: Award **M1** for first 3 vertices correct or 11, 4, 3 seen.

$G - E - F - B - D - A - C(-E) - G$ **OR** $11 + 4 + 3 + 5 + 5 + 8 +$ their b **(A1)**

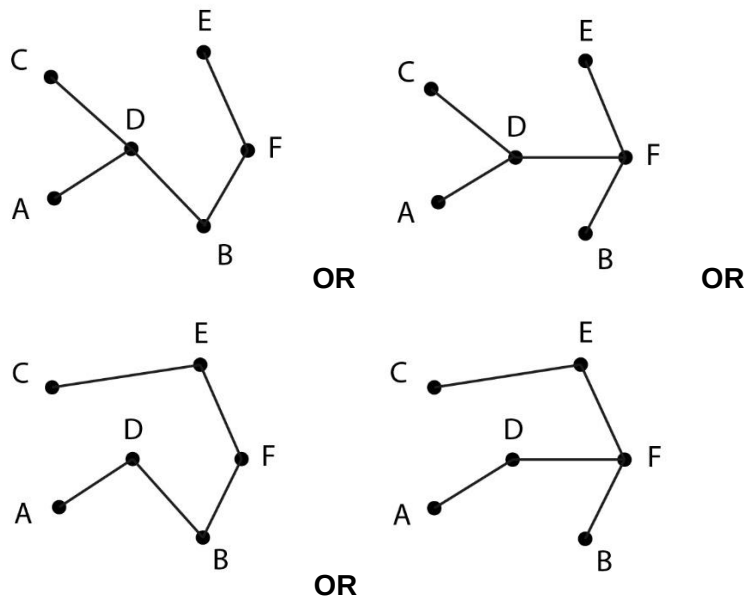
upper bound = 54 (km) **A1**

[3 marks]

continued...

Question 4 continued

- (d) (i) a diagram of **any** spanning tree of the subgraph ABCDEF (A1)
attempt at Kruskal's algorithm or Prim's algorithm (M1)
e.g. edges BF (3), EF (4) and an edge of length 5 listed or seen in any
spanning tree



- (ii) 24 (km) A1

Note: *FT* from their sketch, only if it is a spanning tree. It is not required to see the edge lengths on the sketch, since they are given in the question.

[4 marks]
continued...

Question 4 continued

- (e) adding vertex G's two shortest edges to their part (d)(ii) **(M1)**

$$24 + 11 + 13$$

$$= 48$$

A1

[2 marks]

- (f) try removing a different vertex

A1

[1 mark]

- (g) recognize 7 edges in optimum route

(M1)

Note: Award **M1** for a total length of 52 seen.

subtracting $0.5 \times$ edges from 52

(M1)

$$52 - 7 \times 0.5$$

$$= 48.5 \text{ (km)}$$

A1

[3 marks]

[Total: 17 marks]

7. (a) using area of trapezoid formula

M1

$$\sin(15^\circ) \times \frac{1+2}{2}$$

A1

$$= \frac{3}{2} \sin(15^\circ)$$

AG**[2 marks]**

(b) (i) $M_6 = \begin{pmatrix} \frac{1}{2} \cos 90^\circ & -\frac{1}{2} \sin 90^\circ \\ \frac{1}{2} \sin 90^\circ & \frac{1}{2} \cos 90^\circ \end{pmatrix}$ **(M1)**

$$= \begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$$
 A1

- (ii) multiplying their part (b)(i) and point (0, -1) (in any order)

M1

$$\begin{pmatrix} 0 & -\frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\left(\frac{1}{2}, 0 \right)$$
 A1

[4 marks]*continued...*

Question 7 continued

$$(c) \quad (i) \quad \begin{pmatrix} \cos(k \times 15^\circ) & -\sin(k \times 15^\circ) \\ \sin(k \times 15^\circ) & \cos(k \times 15^\circ) \end{pmatrix} \quad \mathbf{A1}$$

$$(ii) \quad \begin{pmatrix} 1 - \frac{k}{12} & 0 \\ 0 & 1 - \frac{k}{12} \end{pmatrix} \quad \mathbf{A1}$$

$$(iii) \quad k \times 15^\circ \quad \mathbf{A1}$$

$$(iv) \quad 1 - \frac{k}{12} \quad \mathbf{A1}$$

[4 marks]

(d) **METHOD 1 (using part (c)(iv))**

$$\left(1 - \frac{k}{12}\right)^2 \quad \mathbf{A2}$$

METHOD 2 (using full matrix M_k)

$$\begin{aligned} & \left| \begin{pmatrix} \left(1 - \frac{k}{12}\right) \cos(k \times 15^\circ) & -\left(1 - \frac{k}{12}\right) \sin(k \times 15^\circ) \\ \left(1 - \frac{k}{12}\right) \sin(k \times 15^\circ) & \left(1 - \frac{k}{12}\right) \cos(k \times 15^\circ) \end{pmatrix} \right| \\ &= \left(1 - \frac{k}{12}\right)^2 \cos^2(k \times 15^\circ) + \left(1 - \frac{k}{12}\right)^2 \sin^2(k \times 15^\circ) \quad \mathbf{(M1)} \end{aligned}$$

$$= \left(1 - \frac{k}{12}\right)^2 (\cos^2(k \times 15^\circ) + \sin^2(k \times 15^\circ))$$

$$= \left(1 - \frac{k}{12}\right)^2 \quad \mathbf{A1}$$

[2 marks]

continued...

Question 7 continued

(e) recognizing to multiply by 2 and by original area **(M1)**

attempt to sum their answer to part (d), $k = 0, 1, \dots, 11$ **(M1)**

a correct expression **(A1)**

e.g. $0.776457... \left(1^2 + \left(\frac{11}{12} \right)^2 + \dots + \left(\frac{1}{12} \right)^2 \right)$ **OR** $2 \sum_{k=0}^{11} \left(1 - \frac{k}{12} \right)^2 \times \frac{3}{2} \sin 15^\circ$

OR $\sum_{k=0}^{11} \left(1 - \frac{k}{12} \right)^2 \times 0.776457... \quad \text{OR} \quad 2 \sum_{k=1}^{12} \left(\frac{k}{12} \right)^2 \times \frac{3}{2} \sin(15^\circ)$

3.50 (3.50484...) (square units)

Note: Award at most **M0(M1)(A1)A0** for an unsupported final answer of "1.75242..."

(A1)

[4 marks]

(f) $N_k = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \times M_k$ **A1A1**

Note: Award **A1A0** if correct matrices are written in the wrong order.

[2 marks]

[Total: 18 marks]

1. (a) attempt to use area of triangle formula (M1)
 $\frac{1}{2} \times 25.9 \times 6.36 \times \sin(125^\circ)$ (A1)

67.5 m^2 ($67.4700... \text{ m}^2$) A1

Note: Units are required. The final **A1** is only awarded if the correct units are seen in their answer; hence award **(M1)(A1)A0** for an unsupported answer of 67.5.

[3 marks]

- (b) attempt to use cosine rule (M1)
 $(BK =) \sqrt{12^2 + 6.36^2 - 2 \times 12 \times 6.36 \times \cos 45^\circ}$ (A1)
 8.75 (m) ($8.74738... \text{ (m)}$) A1

Note: Award **(M1)(A1)(A0)** for radian answer of 10.2 (m) ($10.2109... \text{ (m)}$) with or without working shown.

[3 marks]

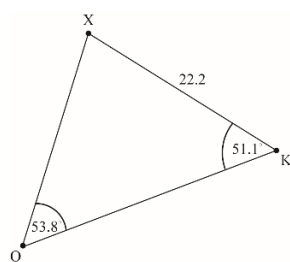
- (c) **METHOD 1**
 attempt to use sine rule with measurements from triangle OKX (M1)
 $\frac{OX}{\sin 51.1^\circ} = \frac{22.2}{\sin 53.8^\circ}$ (A1)
 $(OX =) 21.4 \text{ (m)}$ ($21.4099... \text{ (m)}$) A1
 $(21.4 \text{ (m)} < 22.2 \text{ (m)})$

Odette is closer to the football / Khemil is further from the football A1

Note: For the final **A1** to be awarded 21.4 ($21.4099...$) must be seen. Follow through within question part for final **A1** for a consistent comparison with their OX.

METHOD 2

sketch of triangle OXK with vertices, angles and lengths (A1)



51.1° is smallest angle in triangle OXK R1
 opposite side (OX) is smallest length R1
 therefore Odette is closest A1

[4 marks]

continued...

Question 1 continued

- (d) attempt to use length of arc formula

(M1)

$$\frac{135}{360} \times 2\pi \times 12$$

(A1)

$$28.3(\text{m}) \quad (9\pi, 28.2743\dots) \quad (\text{m})$$

A1**[3 marks]****Total [13 marks]**

4. (a) any city can be travelled to or from any other city (so is connected) **R1**

EITHER

but there is no direct flight between Los Angeles and Dallas (for example) **R1**

OR

but not every vertex has degree 4 **R1**

Note: Accept equivalent statements for the cities being connected and the graph not being complete.
--

[2 marks]

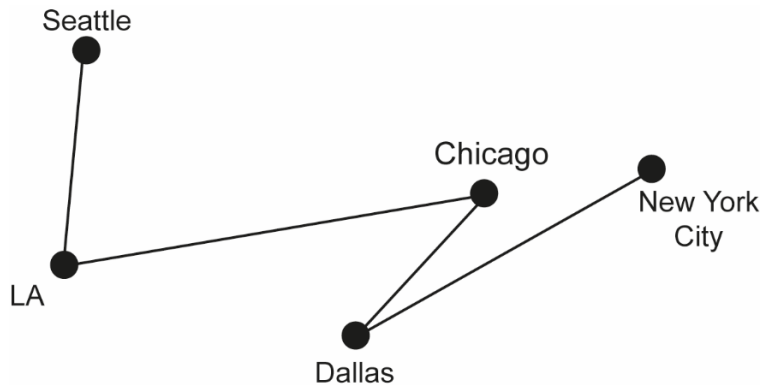
continued...

Question 4 continued

- (b) edge CD selected first
DN,
CL,
LS

M1**A1**

Note: Award marks if the answers are written as sums in the correct order.
M1 if 30 is seen first, **A1** for $30 + 39 + 41 + 58$.

**A1**

Note: The final **A1** can be awarded independently. Award **M0A0A1** for a correct MST graph with no other working. Award **M1A0A1** if Prim's algorithm is seen to be used correctly with CD first.

[3 marks]

- (c) $2 \times \text{MST weight}$
= \$336

(M1)**A1**

Note: Allow any integer multiple (>1) of MST weight for **M1**, and if correctly calculated, award **M1A1**.

[2 marks]

- (d) attempt at nearest neighbour algorithm
order is $LA \rightarrow D \rightarrow C \rightarrow NYC \rightarrow S \rightarrow LA$

M1**A1**

Note: Award **M1** for a route that begins with LA and then D, this includes seeing 26 as the first value in a sum.
Award **A1** if $26+30+68+66+58$ seen in order.

Note: Award **M1A0** for an incorrect first nearest neighbour proceeding 'correctly' to the next vertex. For example, LA to C and then C to D.

upper bound is $(26 + 30 + 68 + 66 + 58 =)$ \$248

A1

Note: Award **M1A0** for correct nearest neighbour algorithm starting from a vertex other than LA. Condone the correct tour written backwards i.e. $58 + 66 + 68 + 30 + 26 = 248$

[3 marks]

continued...

Question 4 continued

- (e) (i) attempt to find MST of L, N, D and S (M1)
 by deleting C, Kruskal gives MST for the remainder as LD, DN, LS (A1)
 weight 123 A1
 (lower bound is therefore $123 + (30 + 41) =$ \$194

Note: Award **(M1)** for a graph or list of edges that does not include C.
 Award **(A1)** if $26 + 39 + 58$ seen in any order.

- (ii) by deleting S, Kruskal gives MST for the remainder as LD, DC, DN (A1)
 weight 95 A1
 (lower bound is therefore $95 + (58 + 66) =$ \$219

Note: Award **(A1)** if $26 + 30 + 39$ seen in any order.

[5 marks]

- (f) $219 \leq C \leq 248$ A1A1

Note: Award **A1** for $219 \leq C$ and **A1** for $C \leq 248$. Award at most **A1A0** for $219 < C < 248$.
FT for their values from part (e) if higher value from (e)(i) and (e)(ii) used for the lower bound, and part (d) for the upper.

[2 marks]

- (g) any valid tour, within their interval from part (f), from any starting point **OR**
 any valid tour that starts and finishes at N (M1)
 valid tour starting point N **AND** within their interval A1
 e.g. NDCLSN (weight 234)

Note: If part (f) not correct, **only** award **A1FT** if their valid tour begins and ends at N **AND** lies within **BOTH** their interval (including if one-sided) in part (f) **AND** $219 \leq C \leq 248$.
 If no response in the form of an interval seen in part (f) then award **M1A0** for a valid tour beginning and ending at N **AND** within $219 \leq C \leq 248$.

[2 marks]

Total [19 marks]

2. (a) $\vec{PQ} = \begin{pmatrix} 115 \\ 0 \end{pmatrix}$ **OR** $\sqrt{(215-100)^2 + (-197+82)^2 + 0^2}$ **(M1)**

Note: Accept working using “A and B” in place of “P and Q”.

$$\begin{aligned} & \sqrt{115^2 + 115^2 + 0^2} \\ & = 163 \text{ (km)} \text{ (162.634..., } \sqrt{26450}) \end{aligned}$$

A1
[2 marks]

(b) speed of each aircraft = $\sqrt{640^2 + 640^2 + 0^2}$ **(M1)**

$$= 905 \text{ (km h}^{-1}\text{)} \text{ (905.096..., } \sqrt{819200}) \text{ **OR** } 251 \text{ ms}^{-1} \text{ (251.388...)} \quad \textbf{A1}$$

[2 marks]

(c) time to travel 162.634... km is $\frac{162.634...}{905.096...}$ (0.179687...) **(M1)**

Note: Accept $\frac{115}{640}$ from a consideration of the component directions.
Accept use of the 3 significant figure answer from parts (a) and (b):
 $\frac{163}{905} = 0.180110...$

EITHER

(multiply by 60 to get) 10.8 (10.7812...) (minutes) or 10 minutes 48 seconds **A1**
(10.8 > 10) hence not in conflict **R1**

Note: Award **R1** for a correct comparison of their time, in minutes, with 10 minutes.

OR

(convert 10 minutes into hours) 0.167 (0.166666...) (hours) **A1**
(0.180 > 0.167) hence not in conflict **R1**

Note: Award **R1** for a correct comparison of their time, in hours, with 0.167 hours.

[3 marks]

(d) $(\mathbf{r}_A =) \begin{pmatrix} 100 \\ -82 \\ 10.7 \end{pmatrix} + t \begin{pmatrix} -640 \\ 640 \\ 0 \end{pmatrix}$ **A1**

[1 mark]

$$\begin{aligned}
 \text{(e) (i)} \quad \mathbf{r}_A - \mathbf{r}_C &= \begin{pmatrix} (100 - 640t) - (-400 - 140t) \\ (-82 + 640t) - (-41 + 604t) \\ 10.7 - (9.1 + 2t) \end{pmatrix} & \mathbf{(M1)(A1)} \\
 &= \begin{pmatrix} 500 - 500t \\ -41 + 36t \\ 1.6 - 2t \end{pmatrix}
 \end{aligned}$$

Note: Award **M1** for an attempt to subtract their $\mathbf{r}_A$ and $\mathbf{r}_C$ in either order, **(A1)** for a correct expression, which does not need to be simplified and which may be seen in the line below.

$$|\mathbf{r}_A - \mathbf{r}_C| = \sqrt{(500 - 500t)^2 + (-41 + 36t)^2 + (1.6 - 2t)^2} \quad \mathbf{(M1)}$$

Note: Award **(M1)** for a correct attempt to find the modulus of their $\mathbf{r}_A - \mathbf{r}_C$.

this is equal to 10 km when $t = 0.983$ (0.983441...) and 1.02 (1.01799...)

A1A1

[5 marks]

(ii) **METHOD 1**

consideration of the vertical component of their $\mathbf{r}_A - \mathbf{r}_C$ from part (e)(i) **M1**

$$1.6 - 2t$$

$$-0.3 < 1.6 - 2t < 0.3$$

A1

Note: Award **A1** for relating their expression to -0.3 and 0.3 ; accept an equality.

$$(0.65 <) t < 0.95$$

A1

Note: Award **A1** for 0.95 seen.

interval is outside of interval from part (e)(i) (the two conditions are never broken at the same time)

R1

Note: The **R1** can only be awarded if there is a clear consideration of intervals.

hence regulations are not broken

A1

Note: Do not award **R0A1**.

METHOD 2

consideration of the vertical component of their $r_A - r_C$ from part (e)(i) **M1**

$$1.6 - 2t$$

when $t = 0.983441\dots$ the difference in height is $1.6 - 2 \times 0.983441\dots$

$$= (-) 0.366882\dots \text{ km} \quad \textbf{A1}$$

EITHER

as time increases the vertical displacement between the aircraft also increases **R2**

OR

when $t = 1.01799\dots$ the difference in height is $1.6 - 2 \times 1.01799\dots$

$$= (-) 0.435970\dots(\text{km}) \quad \textbf{A1}$$

because it is a linear function, the difference in height is also greater than 0.3 km between these values of t . **R1**

Note: Accept an argument from a graph.

THEN

so regulations are not broken **A1**

Note: Do not award **R0A1**.

[5 marks]

- (f) circle **A1**
 radius of 6.4 (km) **A1**
 centred on R / (0, 0) **A1**

Note: A description that includes only one or two of the points above can be awarded a further **A1** for a further correct statement, for example: “the speed is 243.2 km h^{-1} ” **OR** “it travels anticlockwise when viewed from above” **OR** “it travels clockwise when viewed from below”.

[3 marks]

- (g) (i) attempt at scalar product for the correct two vectors

(M1)

Note: Award **(M1)** for a product and sum of components leading to a single scalar expression.

$$\begin{aligned}\vec{RE} \cdot \mathbf{b} &= -1(20 - \lambda) + (10 + \lambda) \\ &= -10 + 2\lambda\end{aligned}$$

A1**[2 marks]**

- (ii)
- $-10 + 2\lambda = 0$

(M1)

Note: Award **(M1)** for setting their scalar product equal to 0, but only if their scalar product is a single expression.

$$\lambda = 5$$

A1**[2 marks]**

- (iii)
- METHOD 1**

$$\vec{RE} = \begin{pmatrix} 20 \\ 10 \end{pmatrix} + 5 \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad \left(= \begin{pmatrix} 15 \\ 15 \end{pmatrix} \right)$$

(M1)

$$\left| \vec{RE} \right|_{\min} = \sqrt{15^2 + 15^2} = 21.2 \quad (21.2132..., 15\sqrt{2}, \sqrt{450}) \text{ (km)}$$

A1**METHOD 2**

$$\left| \vec{RE} \right| = \sqrt{(20 - \lambda)^2 + (10 + \lambda)^2}$$

(M1)

$$\left| \vec{RE} \right|_{\min} = 21.2 \quad (21.2132..., 15\sqrt{2}, \sqrt{450}) \text{ km}$$

A1**[2 marks]**

- (iv) consideration of at least two distances from (their) 21.2, 6.4 and 10

M1**EITHER**

$$6.4 + 10 = 16.4 < 21.2$$

R1**OR**

$$21.2 - 10 = 11.2 > 6.4$$

R1**OR**

$$21.2 - 6.4 = 14.8 > 10$$

R1

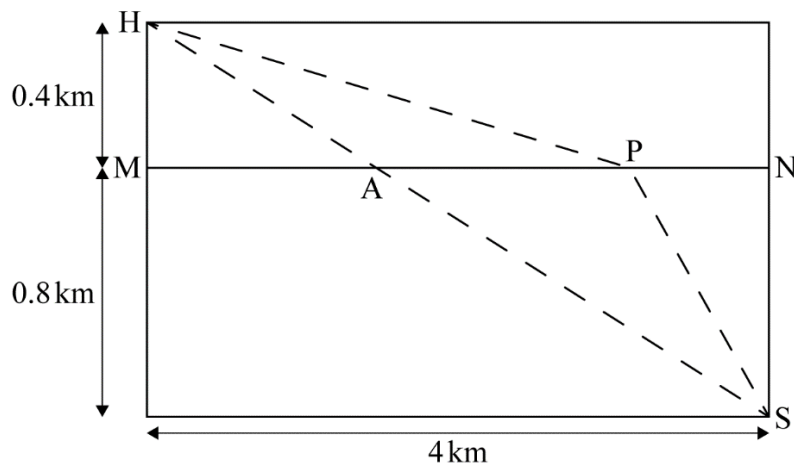
Note: Accept equivalent reasoning in words.

THEN

hence they do not break regulations

AG**[2 marks]****[Total 29 marks]**

1. (a)

**METHOD 1**

$$\hat{MHS} = \left(\tan^{-1} \frac{4}{1.2} \right) 73.3007...^\circ \text{ OR } 1.27933... \quad (\text{A1})$$

use of trigonometry to find HA or AS (M1)

$$HA = \frac{0.4}{\cos \hat{MHS}} \quad \text{AND} \quad AS = \frac{0.8}{\cos \hat{MHS}} \quad (\text{A1})$$

(HA = 1.39204... and AS = 2.78408...)

use of time = $\frac{\text{distance}}{\text{speed}}$ for either of their distances (M1)

$$\text{time taken} = \left(\frac{AH}{15} + \frac{AS}{5} \right)$$

0.649618... (hours) (A1)

(38.97712... minutes)

therefore 39 (mins)

A1FT

Note: Allow **FT**, within the question part, from their time in hours for the final **A1**.

METHOD 2**EITHER**

use of similar triangles to identify either length MA or AN **(M1)**

$$\left(\frac{4}{3} \text{ or } \frac{8}{3}\right)$$

attempt to use Pythagoras for either triangle AMH or ANS **(M1)**

$$AH^2 = 0.4^2 + \left(\frac{4}{3}\right)^2 \quad \textbf{AND} \quad AS^2 = 0.8^2 + \left(\frac{8}{3}\right)^2 \quad \textbf{(A1)}$$

OR

attempt to use Pythagoras for larger triangle **(M1)**

$$SH^2 = 4^2 + 1.2^2$$

$$AH = \frac{1}{3}\sqrt{4^2 + 1.2^2} \quad \textbf{AND} \quad AS = \frac{2}{3}\sqrt{4^2 + 1.2^2} \quad \textbf{(M1)(A1)}$$

THEN

(HA = 1.39204... and AS = 2.78408...)

use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ for either of THEIR distances **(M1)**

$$\text{time taken} = \left(\frac{AH}{15} + \frac{AS}{5}\right)$$

0.649618... (hours) **(A1)**

(38.97712... minutes)

therefore 39 (mins) **A1FT**

Note: Allow **FT**, within the question part, from their time in hours for the final **A1**.

[6 marks]

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M23/5/MATHY/HP3/ENG/TZ2/XX/M

(b) (i) $PH^2 = 0.4^2 + x^2$ **AND** $PS^2 = 0.8^2 + (4-x)^2$

A1

Note: This **A1** can be implied by a clear expression for the time in each region coming from distance / speed below.

$$T(x) = \frac{PH}{15} + \frac{PS}{5}$$

(M1)

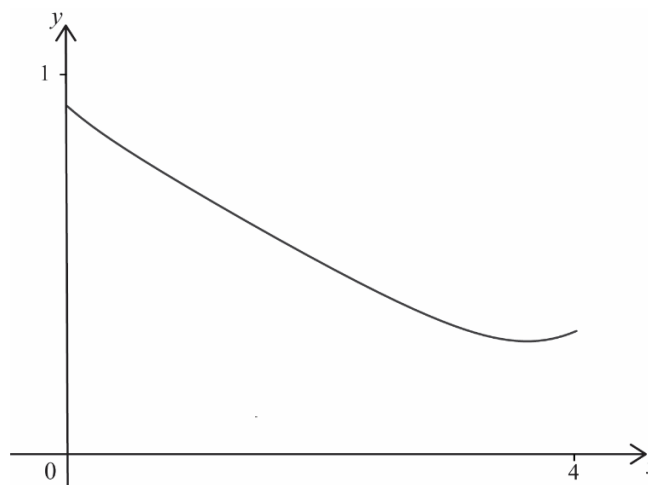
$$T(x) = \frac{\sqrt{0.4^2 + x^2}}{15} + \frac{\sqrt{0.8^2 + (4-x)^2}}{5}$$

A1

$$T(x) = \frac{\sqrt{0.4^2 + x^2} + 3\sqrt{0.8^2 + (4-x)^2}}{15}$$

AG**[3 marks]**

(ii)



correct shape with minimum point nearer $x = 4$ than $x = 0$

A1

correct (approximate) y-intercept, 0.843... (must be clearly below 1)

A1**[2 marks]**

(iii) using the GDC, at the minimum $x = 3.72$ (3.71898...)

A1

Note: Do not accept coordinates of the minimum point.

[1 mark]

(iv) finding their $T(x)$ for their value of x

M1

$$T(x) = 0.418946\dots$$

$$\text{so time saved } (= 38.97712\dots - 25.1367\dots \text{mins}) = 14 \text{ (mins)}$$

A1**[2 marks]**

- (c) (i) attempt at chain rule

M1

$$T'(x) = \frac{1}{15} \left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}} \right)$$

A1A1

Note: Award **A1** for each correct term. Accept any equivalent form i.e. condone fractions not simplified.

[3 marks]

- (ii) setting their
- $T'(x) = 0$

M1

Note: This requires more than just a statement that the derivative equals zero – they must use their attempt at $T'(x)$.

$$\frac{1}{15} \left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}} \right) = 0$$

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$

AG**[1 mark]**

- (iii)
- METHOD 1**

$$\cos \hat{H}PM = \frac{x}{\sqrt{0.16 + x^2}} \text{ AND } \cos \hat{S}PN = \frac{4-x}{\sqrt{0.64 + (4-x)^2}}$$

A1

substituting in the above equation and rearranging

M1

$$\cos \hat{H}PM = 3 \cos \hat{S}PN \text{ leading to } \frac{\cos \hat{H}PM}{\cos \hat{S}PN} = 3 = \left(\frac{15}{5} \right)$$

verifying the result

AG**METHOD 2**

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$

attempt to rearrange into a quotient

M1

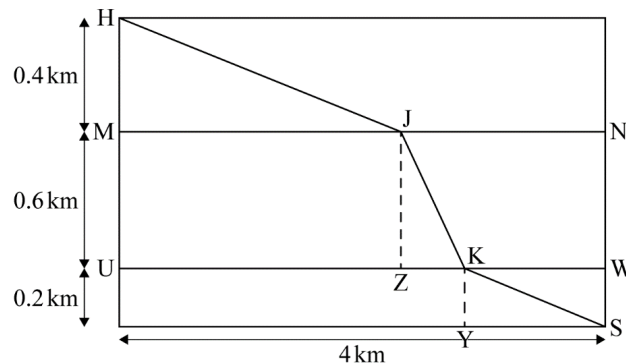
$$\left(\frac{15}{5} = 3 \right) = \frac{\frac{x}{\sqrt{0.16 + x^2}}}{\frac{4-x}{\sqrt{0.64 + (4-x)^2}}}$$

$$= \frac{\cos \hat{H}PM}{\cos \hat{S}PN}$$

A1

verifying the result

AG**[2 marks]**

(d) **METHOD 1**

let $MJ = y$ km and W and Z be the points on the new boundary directly below N and J

attempt to find ZK in terms of MJ

M1

($KW = 0.5y$)

$ZK = (4 - 1.5y)$ km.

A1

attempt to use the result from (c)(iii) at J

M1

$$\frac{\cos \hat{HJM}}{\cos \hat{ZKJ}} = \frac{y}{\sqrt{y^2 + 0.4^2}} \div \frac{(4 - 1.5y)}{\sqrt{(4 - 1.5y)^2 + 0.6^2}} = \frac{15}{5}$$

A1

Note: Accept $\cos \hat{NJK}$ in place of $\cos \hat{ZKJ}$.

$$\left(\text{leading to } \frac{y}{\sqrt{y^2 + 0.16}} = \frac{3(4 - 1.5y)}{\sqrt{(4 - 1.5y)^2 + 0.36}} \right)$$

valid method for solving this equation, eg drawing graphs of both sides of the equation, using SOLVER, etc.

(M1)

solution is $y = 2.53$

A1**METHOD 2**

combining the field into one region with height 0.6 km

M1

$$\cos \hat{HPM} = \frac{x}{\sqrt{0.36 + x^2}}$$

$$\cos \hat{SPN} = \frac{4 - x}{\sqrt{0.36 + (4 - x)^2}}$$

A1

Note: Both expressions, or their ratio, are required for the **A1** to be awarded.

therefore

$$\frac{x\sqrt{0.36 + (4 - x)^2}}{(4 - x)\sqrt{0.36 + x^2}} = 3$$

A1

valid method for solving

(M1)

attempting to find MJ in terms of x e.g. $MJ = \frac{2}{3}x$

M1

so $MJ = 2.53$

A1**[6 marks]****Total [26 marks]**

3. (a) (upper bound =) 0.525 (m) A1
 (lower bound =) 0.515 (m) A1

Note: Accept an answer in interval notation or written as an inequality.

[2 marks]

- (b) **METHOD 1 Convert REC to linear metres**
 attempt to convert REC to metres using their lower bound (M1)
 440×0.515 (= 226.6) **OR** 280×0.515 (= 144.2) seen

attempt to use the formula for the volume of a right pyramid (M1)

$$(V =) \frac{1}{3} (440 \times 0.515)^2 (280 \times 0.515) \quad \text{(A1)}$$

$$2470000 \text{ (m}^3\text{)} \quad (2468106.051\dots, 2.47 \times 10^6) \quad \text{A1}$$

METHOD 2 Convert REC to cubic metres
 attempt to use the formula for the volume of a right pyramid (M1)

$$(V =) \frac{1}{3} (440)^2 (280) \quad (= 18069333.33\dots)$$

attempt to convert 1 cubic REC to cubic metres using their lower bound (M1)
 (1 cubic REC =) 0.515^3

$$(V =) \frac{1}{3} (440)^2 (280) \times (0.515)^3 \quad \text{(A1)}$$

$$2470000 \text{ (m}^3\text{)} \quad (2468106.051\dots, 2.47 \times 10^6) \quad \text{A1}$$

[4 marks]

[Total 6 marks]

4. (a) recognizing supplementary angles or acute angles in right-triangles
 $(\hat{A}BC =) 41^\circ + (180^\circ - 112^\circ), 41^\circ + (90^\circ - 22^\circ)$

(M1)

Note: Values may be seen on diagram.

$$\hat{A}BC = 109^\circ$$

A1**[2 marks]**

- (b) $\hat{A}CB = 49^\circ$ (may be seen in part (a))
 attempt to substitute into the sine rule (or equivalent)

(A1)**(M1)**

$$\frac{AC}{\sin 109^\circ} = \frac{100}{\sin 49^\circ}$$

(A1)

$$AC = 125 \text{ (km)} (=125.282\dots)$$

A1**[4 marks]****[Total 6 marks]**

6. (a) (4, 8) **A1**
[1 mark]
- (b) attempt to find the gradient of AC **(M1)**
 $\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$
 attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line **(M1)**
 $y-8 = -\frac{4}{5}(x-4)$ **OR** $8 = -\frac{4}{5}(4) + c$ **OR** $c = 11.2$
 $y-8 = -\frac{4}{5}(x-4)$ ($y = -0.8x + 11.2, 4x + 5y - 56 = 0$) **A1**
[3 marks]
- (c) (i) attempt to find one distance from a farm to any closest vertex **M1**
 finding a correct distance from at least two distinct vertices **A1**
 $7.58968\dots, 4.472135\dots \left(\sqrt{20}\right), 5.830951\dots \left(\sqrt{34}\right)$
 $\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) **A1**
- (ii) 7.59 (km) (= 7.58968...) **A1**
[4 marks]
[Total 8 marks]

14. (a) vector from Q to any point in L or vice versa

$$= \begin{pmatrix} 1+\lambda \\ 3+\lambda \\ 2\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -10+\lambda \\ 4+\lambda \\ 2\lambda-3 \end{pmatrix} \quad (M1)$$

EITHER (scalar product)

attempt to use scalar product

(M1)

$$\begin{pmatrix} -10+\lambda \\ 4+\lambda \\ 2\lambda-3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0$$

$$-10 + \lambda + 4 + \lambda + 4\lambda - 6 = 0$$

OR (distance formula)

attempt to use distance formula

(M1)

minimizing $(-10 + \lambda)^2 + (4 + \lambda)^2 + (-3 + 2\lambda)^2$

THEN

$$\lambda = 2$$

(A1)

point P(3, 5, 4)

A1

Note: Do not award final **A1** for P given as a vector.

[4 marks]

(b) $\vec{PQ} = \begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix}$ (A1)

attempt to use vector product

(M1)

(perpendicular vector =) $\begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} -11 \\ -17 \\ 14 \end{pmatrix}$$

A1

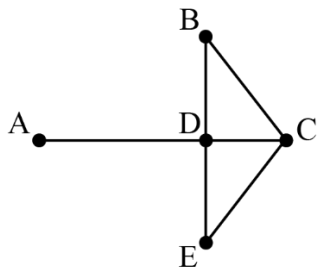
Note: Award final **A1** for any multiple (positive or negative) of the answer given here.

[3 marks]

[Total 7 marks]

1. (a) 25 (m) **A1**
[1 mark]
- (b) (i) recognition of need to use Pythagoras theorem **(M1)**
 $BF^2 = 20^2 + 25^2$
 (BF =) 32.0 (32.0156..., $\sqrt{1025}$, $5\sqrt{41}$) (m) **A1**
- (ii) correct use of trig ratio for $\hat{BFM}$ **(M1)**
 ($\hat{BFM}$ =) $\tan^{-1}\left(\frac{25}{20}\right)$ or equivalent
 ($\hat{BFM}$ =) 51.3 (51.3401...) **A1**
- Note:** Accept radian answer of 0.896 (0.896055...) Accept an answer of 51.4 from use of 3sf answer to part (b)(i) and then either cosine rule or inverse sine.
- [4 marks]**
- (c) attempt to use arc length formula **(M1)**
 (arc length =) $\frac{2 \times 51.3401...}{360} \times 2\pi(32.0156...)$ **(A1)**
 (arc length =) 57.4 (57.3755...) (m) **A1**
- Note:** Accept 57.3 from use of 3 sf. values of their answers from parts (b)(i) and (b)(ii).
- [3 marks]**
- (d) 34.0156... (seen anywhere) **(A1)**
 use of area of sector formula **(M1)**
 recognition of subtracting areas of two sectors **(M1)**
 (area =) $\frac{102.680...}{360} \times \pi((34.0156...)^2 - (32.0156...)^2)$
 (area =) 118 (m²) (118.335...) **A1**
- [4 marks]**
- (e) multiplying their area from part (d) by 0.12 or 12 **(M1)**
 0.12 (m) seen **OR** 1183350 (cm²) seen **(A1)**
 118.335... × 0.12 **OR** 1183350 × 12
 14.2 (14.2002...) m³ **OR** 14200000 (14200236) cm³ **A1**
- [3 marks]**
[Total 15 marks]

5. (a)



A1
[1 mark]

(b) (i) $P = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$ (M1)

$$P^3 = \begin{pmatrix} 0 & 1 & 2 & 4 & 1 \\ 1 & 2 & 5 & 6 & 2 \\ 2 & 5 & 4 & 6 & 5 \\ 4 & 6 & 6 & 4 & 6 \\ 1 & 2 & 5 & 6 & 2 \end{pmatrix}$$

$a = 6$ A1

(ii) 5 (routes) A1
[3 marks]

(c) A and C identified as start/finish points (in either order) (A1)
for example : A – D – E – C – D – B – C A1
[2 marks]

(d) cost of their Eulerian trail A to C (=180) (A1)
consider edges to get from C to A (M1)
235 (USD) A1
[3 marks]

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- (e) (i) A to C (or C to A) **A1**
- (ii) best is CBDA
55 (USD) **A1**
[2 marks]
- (f) (i) A – D – C – B – E – A **OR** 50, 45, 30, 120, 60 **(A1)**
summing their 5 edges **(M1)**
 $50 + 45 + 30 + 120 + 60$
(upper bound =) 305 (km) **A1**
- (ii) attempt to find MST without vertex A **(M1)**
(MST =) 130 **(A1)**
 $130 + 50 + 60$ **(M1)**
(lower bound =) 240 (km) **A1**
[7 marks]
[Total 18 marks]

6. (a) $x = -1 + 2\lambda$, $y = 1 - \lambda$

A1**[1 mark]**

(b) $\begin{pmatrix} 1 & 7 \\ 7 & -1 \end{pmatrix} \begin{pmatrix} -1 + 2\lambda \\ 1 - \lambda \end{pmatrix} = \begin{pmatrix} 6 - 5\lambda \\ -8 + 15\lambda \end{pmatrix}$

(M1)(A1)

$$\mathbf{r} = \begin{pmatrix} 6 \\ -8 \end{pmatrix} + \lambda \begin{pmatrix} -5 \\ 15 \end{pmatrix} \text{ (or equivalent)}$$

(M1)A1

Note: Award **(M1)** for the correct format of a vector equation of a line, **A1** for the line being completely correct.

[4 marks]

(c) (i) $\begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix}$ **OR** $\begin{pmatrix} 0.707 & -0.707 \\ 0.707 & 0.707 \end{pmatrix}$ **OR** $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$

A1

(ii) $\begin{pmatrix} 5\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{pmatrix}$

A1**[2 marks]**

(d) $(\mathbf{R} =) \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$

A1**[1 mark]**

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- (e) (i) attempt to multiply matrices from part (c) (in any order)

(M1)

$$\text{e.g. } X = \begin{pmatrix} 5\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{pmatrix} \begin{pmatrix} \cos\left(\frac{\pi}{4}\right) & -\sin\left(\frac{\pi}{4}\right) \\ \sin\left(\frac{\pi}{4}\right) & \cos\left(\frac{\pi}{4}\right) \end{pmatrix}$$

$$X = \begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix}$$

A1

- (ii) substituting
- T**
- ,
- R**
- and
- X**

(M1)

$$\begin{pmatrix} 1 & 7 \\ 7 & -1 \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} \begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix}$$

multiplying by inverse (in any order)

(M1)

$$\begin{pmatrix} 1 & 7 \\ 7 & -1 \end{pmatrix} \begin{pmatrix} 5 & -5 \\ 5 & 5 \end{pmatrix}^{-1} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix}$$

$$\begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ \sin 2\alpha & -\cos 2\alpha \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

A1

$$\cos 2\alpha = -\frac{3}{5} \quad \text{AND} \quad \sin 2\alpha = \frac{4}{5}$$

$$\alpha = 1.11 \quad (=1.107148...) \quad \text{OR} \quad 63.4^\circ \quad (63.4349...^\circ)$$

A1**[6 marks]****[Total 14 marks]**

5. (a) $y = 0.5x - 1$

A1A1

Note: Award **A1** for $0.5x$ and **A1** for -1 (or equivalent equation). Award at most **A1A0** if answer is not presented as an equation.

[2 marks]

(b) $(6.857, 2.429)$

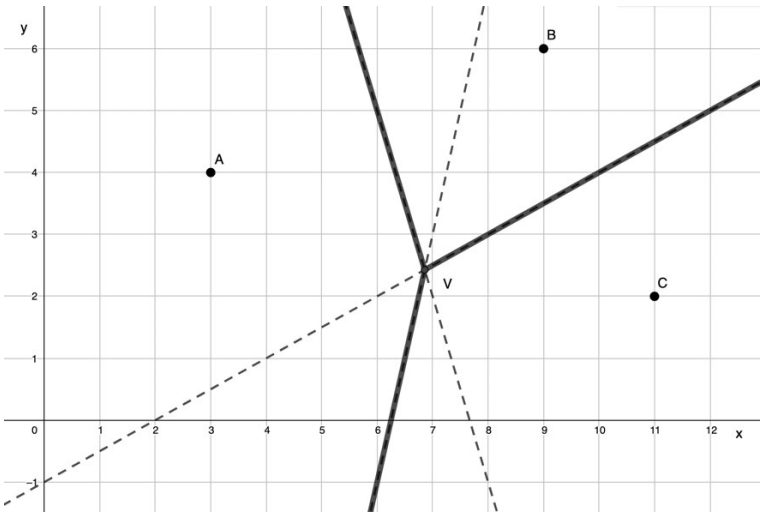
A1A1

Note: If both answers are not correct to 4 sig figs, award at most **A1A0**.
Accept $x = 6.857$, $y = 2.429$.

Award **A1A0** for $\left(\frac{48}{7}, \frac{17}{7}\right)$. Award **A0A1** for $(2.429, 6.857)$.

[2 marks]

(c)



A2

Note: Award marks as shown in the table below. Condone edges that do not extend to the sides of the graph or beyond the x -axis.

Correct edges	Incorrect edges	Marks
3	0	A2
3	1	A1A0
3	2 or more	A0A0
2	0	A1A0
2	1	A1A0
2	2 or more	A0A0
1	0	A1A0
1	1 or more	A0A0

[2 marks]
[Total 6 marks]

8. (a) B and C

A1**[1 mark]**(b) correct intervals seen ($x \leq 5$ (or $x < 5$) **AND** $x \geq 5$ (or $x > 5$))**A1****Note:** The case of $x = 5$ must be included for this **A1** to be awarded.attempt to add edges to $33 + x$ **(M1)**(If $x < 5$ (or $x \leq 5$) then repeat BC and) length is $33 + 2x$ **A1**(If $x > 5$ (or $x \geq 5$) then repeat AB and AC and) length is $(33 + x + 5)38 + x$ **A1****Note:** If the intervals are not explicit, award at most **A0(M1)A1A1**.**[4 marks]****[Total 5 marks]**

10. (recognition that OB is a radius)

$$(\text{radius} =) \sqrt{5^2 + 8^2} (= \sqrt{89}) \quad (\text{A1})$$

EITHER (finding angle BOQ)

correct calculation for finding $\hat{BOA}$ (A1)

$$\hat{BOA} = \arctan\left(\frac{8}{5}\right) \quad \text{OR} \quad \tan \hat{BOA} = \frac{8}{5}$$

expressing $\hat{BOQ}$ as $90 + \hat{BOA}$ (M1)

$$\hat{BOQ} = 90 + \arctan\left(\frac{8}{5}\right) \quad \text{OR} \quad \hat{BOQ} = \frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)$$

$$(\hat{BOQ} =) 147.994^\circ \dots \quad \text{OR} \quad 2.58299 \dots$$

substituting *their* radius and angle BOQ correctly into arc length formula (M1)

$$(\text{arc BQ} =) \frac{90 + \arctan\left(\frac{8}{5}\right)}{360} \times 2\pi(\sqrt{5^2 + 8^2}) \quad \text{OR} \quad \left(\frac{\pi}{2} + \arctan\left(\frac{8}{5}\right)\right) \times (\sqrt{5^2 + 8^2})$$

$$24.4 \text{ (m)} \quad (24.3679 \dots) \quad \text{A1}$$

OR (finding angle BOP)

correct calculation for finding angle $\hat{BOP}$ (A1)

$$\hat{BOP} = \arctan\left(\frac{5}{8}\right) \quad \text{OR} \quad \tan \hat{BOP} = \frac{5}{8}$$

substituting *their* radius and $\hat{BOP}$ correctly into arc length formula (M1)

$$(\text{arc BP} =) \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi(\sqrt{5^2 + 8^2})$$

subtracting *their* arc BP from arc PQ (M1)

$$(\text{arc BQ} =) \pi\sqrt{5^2 + 8^2} - \frac{\arctan\left(\frac{5}{8}\right)}{360} \times 2\pi(\sqrt{5^2 + 8^2})$$

$$24.4 \text{ (m)} \quad (24.3679 \dots) \quad \text{A1}$$

[Total: 5 marks]

11. (a) $\begin{pmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{pmatrix}$ **A1A1**

Note: Award **A1** for selecting the correct matrix, **A1** for substituting 2α into a rotation matrix
Award **A1A0** for clockwise rotation. These marks can be awarded independently.
Condone the use of a different symbol.

[2 marks]

(b) $\begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$
 $= \begin{pmatrix} \cos^2 \alpha - \sin^2 \alpha & -2 \cos \alpha \sin \alpha \\ 2 \cos \alpha \sin \alpha & \cos^2 \alpha - \sin^2 \alpha \end{pmatrix}$ **M1A1**

Note: Award **M1** for an attempt to multiply matrices, e.g. at least one entry correct.

[2 marks]

- (c) (i) (Because matrix multiplication represents the composition of transformations)
 two rotations of α are equivalent to a rotation of 2α **R1**
 so the two matrices are equal (so each of the entries are also equal) **R1**
 $\sin(2\alpha) = 2 \sin(\alpha) \cos(\alpha)$ **AG**

(ii) $\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$ **A1**

replacing $\cos^2 \alpha$ with $1 - \sin^2 \alpha$ **M1**
 $= 1 - \sin^2 \alpha - \sin^2 \alpha$

$= 1 - 2 \sin^2 \alpha$ **AG**

[4 marks]**[Total 8 marks]**

6. attempt to substitute into area of triangle formula (M1)
 (sheep's field area =) $0.5 \times 15 \times 21 \times \sin(78^\circ)$
 $= 154.058... \text{ (m}^2\text{)}$ A1

EITHER

(goat's field area =) $\frac{282}{360} \times \pi \times 8^2$ (A1)(A1)

Note: Award A1 for 282, A1 for correct entries in formula (including their 282).

OR

$\pi \times 8^2 - \frac{78}{360} \times \pi \times 8^2$ (M1)(A1)

Note: Award A1 for minor sector area, M1 for subtracting their sector area from circle area.

THEN

$= 157.498... \left(\frac{752\pi}{15} \right) \text{ (m}^2\text{)}$ A1

the goat has most area by 3.44 (m²) (3.44026...) A1

Note: Accept 154 and 157 for the intermediate A1 marks, but do NOT follow through within the question; a final answer of 3 m² is awarded A0.

[Total: 6 marks]

8. (a) $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ A1

Note: Accept any valid vector notation. Do not accept (1, 1, 2).

[1 mark]

(b) (i) **EITHER**
use of scalar product formula to find angle $\hat{B}AD$ (M1)

$$\cos \hat{B}AD = \frac{\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}}{5\sqrt{6}} \quad \text{(A1)}$$

OR

use of cosine rule to find $\hat{B}AD$ (M1)

$$\cos \hat{B}AD = \frac{6 + 25 - 17}{2 \times \sqrt{6} \times 5} \quad \text{(A1)}$$

THEN

$$\hat{B}AD = 55.1^\circ \text{ (55.1417...}^\circ, 0.962405\text{...)} \quad \text{A1}$$

Note: If the direction of one of the vectors is reversed, leading to an obtuse angle (124.858... $^\circ$) between the vectors, then award **M1A1A0**.

(ii) **EITHER**
an attempt at using vector product (M1)

$$\frac{1}{2} \left| \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} \right| \text{ or equivalent}$$

attempt to apply formula (e.g. one correct component) (M1)

$$= \frac{1}{2} ((1 \times 0 - 4 \times 2)\mathbf{i} - (1 \times 0 - 2 \times 3)\mathbf{j} - (1 \times 4 - 1 \times 3)\mathbf{k})$$

$$\frac{1}{2} \left| \begin{pmatrix} -8 \\ 6 \\ 1 \end{pmatrix} \right| \quad \text{(A1)}$$

$$= 5.02 \left(5.02493, \frac{\sqrt{101}}{2} \right) \quad \text{A1}$$

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OR

use of formula for the area of a triangle

(M1)

$$\text{area} = \frac{1}{2} \begin{vmatrix} 3 \\ 4 \\ 0 \end{vmatrix} \begin{vmatrix} 1 \\ 1 \\ 2 \end{vmatrix} \sin 55.1417\dots$$

$$= 5.02$$

(A1)(A1)**A1**

Note: Award **A1** for the lengths AB and AD and **A1** for the angle.

[7 marks]**(c) EITHER**

$$\vec{AD} = \vec{BC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{OR} \quad \vec{AB} = \vec{DC} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

A1

One pair of opposite sides have equal length AND are parallel

R1**OR**

$$\vec{AD} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \vec{BC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, \vec{AB} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \vec{DC} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

A1

$$\vec{AD} = \vec{BC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{AB} = \vec{DC} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$$

both pairs of opposite sides are parallel (or have equal length)

R1

Note: Both pairs of opposite angles are equal is also valid.

THEN

hence ABCD is a parallelogram

AG**[2 marks]****[Total: 8 marks]**

12. (a) $\sqrt{6^2 + 6^2 + 3^2}$
 $= 9 \text{ (ms}^{-1}\text{)}$

(A1)

A1

[2 marks]

(b) $\mathbf{h} = \begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix} + t \begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$

A1

[1 mark]

(c) equating one component from each

(M1)

e.g. $7 + 6t = -38 + 15t$

$t = 5$

(A1)

substituting their t -value into either equation

(M1)

$$\begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix} + 5 \begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$$

$$= \begin{pmatrix} 37 \\ 34 \\ 15 \end{pmatrix}$$

A1

[4 marks]

[Total: 7 marks]

1. (a) $BC = 20$ (m) A1

[1 mark]

(b) use of Pythagoras (M1)

$$AB = \sqrt{12^2 + 4^2}$$

$$= 12.6 \text{ (m)} \quad (12.6491\dots, \sqrt{160})$$

A1

[2 marks]

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $\hat{ABC}$ (or finding the bearing of B from A)

(A1)

$$\text{e.g. } \tan(\hat{ABC}) = \frac{12}{4}, \cos \hat{ABC} = \frac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649}, \cos \hat{ABC} = \frac{6.3245}{20}$$

$$\hat{ABC} = 71.6 \text{ (71.5650\dots)}$$

(A1)

Note: Angle $\hat{ABC}$ can be 71.5 or 72.2 depending on their working out. Bearings should be given in degrees.

$$180 + 71.5650\dots = 252^\circ \text{ (251.565\dots)}$$

A1

Note: The final **A1** can be awarded for 180 plus their 71.6. If radians used, award **A1A1** for 1.24904... or 4.39063... seen, and then **A0** for the radian answer.

METHOD 2 – finding angle that AB makes with the horizontal (angle H)

correct use of a trig ratio to find H , the angle AB makes with horizontal (A1)

$$\text{e.g. } \tan \hat{H} = \frac{4}{12}, \cos \hat{H} = \frac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$$

$$\hat{H} = 18.4 \text{ (18.4349\dots)}$$

(A1)

Note: Accept 18.5 (18.5078...) from use of 3sf answer from part (b). Bearings should be given in degrees.

$$270 - 18.4348\dots = 252^\circ \text{ (251.565\dots)}$$

A1

Note: The final **A1** can be awarded for 270 minus their 18.4. If radians used, award **A1A1** for 0.321750... or 4.39063... seen, and then **A0** for the radian answer.

[3 marks]

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(d) (i) $-\frac{4}{3} \left(-\frac{16}{12} \right)$ **A1**

(ii) $(6, 8)$ **A1A1**

Note: Award **A1A0** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ **(M1)**

equation of perpendicular bisector of AC **(A1)**

e.g. $(y-8) = \frac{3}{4}(x-6)$ **OR** $y = \frac{3}{4}x + 3.5$

EITHER

equation of perpendicular bisector of BC is $y = 10$ **(A1)**

OR

equation of perpendicular bisector of AB is $y = -3x + 36$ **(A1)**

Note: The **A1** is for either equation of perpendicular bisector of BC or AB.

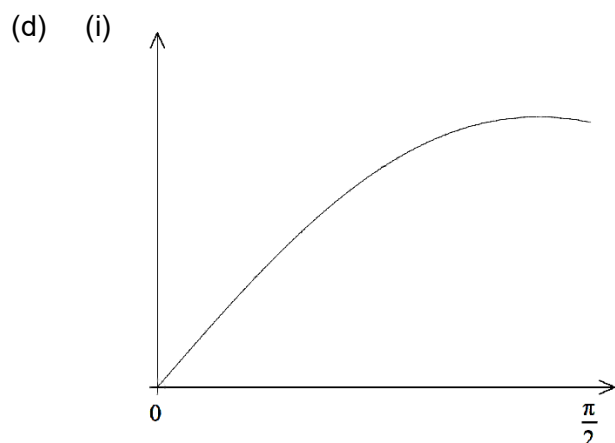
point of intersection $\left(8\frac{2}{3}, 10 \right)$ **OR** $(8.67, 10)$ $[(8.666..., 10)]$ **(M1)A1**

Note: Award **M1** for an attempt to equate their perpendicular bisectors
Award the final **A1** for the correct coordinate pair – parentheses omitted or not.

[8 marks]

[Total: 14 marks]

3. (a) (i) correct approach to find missing length **(A1)**
 $\sqrt{4^2 - 1^2} (= \sqrt{15})$
 attempt to find cross-section **(M1)**
 e.g. use of area of trapezoid formula or rectangle+triangle or rectangle – triangle
 use of volume of prism formula **(M1)**
 (their cross-section multiplied by 3)
 $3 \left[\frac{1}{2} (10 + 11) (\sqrt{4^2 - 1^2}) \right]$
 $= 122 (\text{m}^3) \text{ (121.998...)} \quad \mathbf{A1}$
- (ii) correct approach to find missing height **(A1)**
 $\sqrt{4^2 - 3.2^2} (= 2.4)$
 attempt to find volume **(M1)**
 (multiplication by 3.2 and 3 seen)
 $3 \left[\frac{1}{2} (10 + 10 + \sqrt{4^2 - 3.2^2}) (3.2) \right]$
 $= 108 (\text{m}^3) \text{ (107.52...)} \quad \mathbf{A1}$
- (iii) correct approach to find missing lengths **(A1)**
 $\sin\left(\frac{\pi}{3}\right)$ and $\cos\left(\frac{\pi}{3}\right)$ **OR** $\sin\left(\frac{\pi}{3}\right)$ and Pythagoras etc seen in work
 $3 \left[\frac{1}{2} (10 + 10 + 4 \cos\left(\frac{\pi}{3}\right)) 4 \sin\left(\frac{\pi}{3}\right) \right]$
 $= 114 (\text{m}^3) \text{ (114.315...)} \quad \mathbf{A1}$
- [9 marks]**
- (b) $V = 3 \left[\frac{1}{2} (10 + 10 + 4 \cos(\theta)) 4 \sin(\theta) \right] \quad \mathbf{A1}$
 all correct intermediate working leading to given answer **A1**
 e.g. $V = 6 \sin(\theta)(20 + 4 \cos(\theta))$
 $V = 24 \sin(\theta)(5 + \cos(\theta)) \quad \mathbf{AG}$
- Note:** The **AG** line must be seen for the final **A1** to be awarded.
- [2 marks]**
- (c) *accept any reasoning along the lines:* “skip would have zero volume” or
 “if the angle is zero, then the contents would fall out” **R1**
[1 mark]

**A1A1**

Note: Award **A1** for the correct shape and **A1** for the graph on the correct, labelled, domain. Condone omission of θ/V labels (or x/y).

(ii) $\theta = 1.38$ (1.38356...) (79.3° ($79.2723\dots^\circ$))

A1

$$V_{\max} = 122 \text{ (122.292...)}$$

A1

Note: Award **A0A1** if values are reversed and **A0A0** for a coordinate pair.

[4 marks]

- (e) recognizing that derivative is equal to zero (seen at any stage)

M1

$$\frac{dV}{d\theta} = 0 \text{ (accept } \frac{dy}{dx} = 0 \text{)}$$

(from graph, turning point is a global maximum)

use of product rule

M1

$$\left(\frac{dV}{d\theta} = \right) 24 \cos(\theta)(5 + \cos(\theta)) + 24 \sin(\theta)(-\sin(\theta))$$

A1

$$= 120 \cos(\theta) + 24 \cos^2(\theta) - 24 \sin^2(\theta) (= 0) \text{ (or equivalent)}$$

A1

substituting $1 - \cos^2(\theta)$ for $\sin^2(\theta)$

M1

e.g. $120 \cos(\theta) + 24 \cos^2(\theta) - 24(1 - \cos^2(\theta)) (= 0)$

correct intermediate steps leading to given answer

A1

$$2 \cos^2(\theta) + 5 \cos(\theta) - 1 = 0$$

AG**[6 marks]****[Total: 22 marks]**

4. (a) AEDCFBA A1A1

Note: Award **A1** for AE at start, **A1** for correct completed route.

attempt to find the length of their route
length $22 + 21 + 19 + 24 + 25 + 31$ (M1)
 $= 142$ (km) A1

Note: Award **A1A0M1A0** for omitted final edge and their sum.

[4 marks]

(b) attempt to form MST without vertex A (M1)

Note: Exactly 4 edges that form a spanning tree are required.

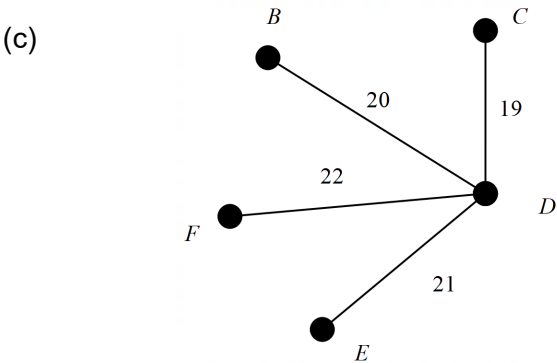
BD DC DE DF **OR** 20, 19, 21, 22 seen in that order A1

Note: Award **M1A0** for diagram of MST.

attempt to reconnect vertex A (one edge is sufficient) (M1)
reconnecting A: AE (22) and AF (23) (A1)
lower bound: $20 + 19 + 21 + 22 + 22 + 23$
 $= 127$ A1

Note: If 127 seen, unsupported or without the explicit evidence of Prim's algorithm, award **M1A0M1A1A1**.

[5 marks]



(A1)

Note: Condone the omission of the weights from their diagram.
The diagram may include A with its two edges.

correct reasoning based on lack of cycle (once A is reattached) R1
e.g. edges BD and CD would be repeated
this lower bound is not achievable (in this way) A1

Note: Do not award **R0A1**.

[3 marks]

[Total: 12 marks]

2 (a) (i) (34, 4.1)

A1A1

[2 marks]

(ii) recognizing the sequence is arithmetic, with a common difference of 0.5 and 1.0

(M1)

$$(30 + (n-1), 2.1 + (n-1)0.5)$$

A1A1

$$= (29 + n, 1.6 + 0.5n)$$

[3 marks]

(b) (i) evidence of integration of the acceleration vector **OR** use of $v = u + at$ (M1)

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} c_1 \\ -9.8t + c_2 \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} 29.4 \cos \theta \\ 29.4 \sin \theta - 9.8t \end{pmatrix}$$

A1A1

Note: The first **A1** is for $\dot{x}$ and the second is for $\dot{y}$.

[3 marks]

(ii) $\theta = 90^\circ$

(A1)

$$29.4 - 9.8t = 0$$

M1A1

Note: Award **M1** for setting their $\dot{y}$ to zero (may still include θ), **A1** for correct equation, leading to given result.
If they substitute $t = 3$ award at most **(A1)M0A0**.

$$\text{maximum point when } t = \frac{29.4}{9.8}$$

$$= 3 \text{ (seconds)}$$

AG

[3 marks]

(c) correct substitution OR use of correct graph

(M1)

$$\text{maximum height is } 29.4 \times 3 - 4.9 \times 3^2$$

$$= 44.1 \text{ (m)}$$

A1

[2 marks]

continued...

Question 2 continued.

(d) (i) $29.4 \sin \theta t - 4.9t^2 = 0$
 $t = 6 \sin \theta$ (or $t = 0$)

M1
A1

$$x = 29.4 \cos \theta \times 6 \sin \theta$$

$$= 176.4 \cos \theta \sin \theta$$

A1
AG

[3 marks]

- (ii) valid method to find maximum (e.g. sketch graph, find derivative)
 maximum value of x is 88.2 (m)

(M1)
A1

Note: Award **(M1)A0** for an unsupported answer of “45” or $\frac{\pi}{4}$ (0.785398...) .

[2 marks]

(e) (i) ($c =$) 44.1

A1

Note: follow through from 2c

[1 mark]

- (ii) **EITHER**

$$\frac{dy}{dx} = 0 \Rightarrow 2a \times 0 + b = 0$$

(M1)**OR**

$$\text{vertex is at } x = -\frac{b}{2a} = 0$$

(M1)**THEN**

$$\Rightarrow b = 0$$

A1

[2 marks]

- (iii) point (88.2, 0) used

(A1)

$$0 = a \times 88.2^2 + 44.1$$

(M1)

$$\Rightarrow a = -\frac{44.1}{88.2^2}$$

$$= -\frac{5}{882} = -0.0056689...$$

A1

[3 marks]

Note: The method for finding the parameters might appear in a different part, for example use of (88.2, 0) might appear in part (ii), or the parts might not be clearly numbered. Accept any correct working wherever it is seen, and ascribe the marks to the correct part.

continued...

Question 2 continued.

- (f) use of their arithmetic sequence from (a)(ii) with $n = 40$
coordinates of seat $A_{40} = (69, 21.6)$

(M1)**(A1)****EITHER**

substitution of their 69 into their $y = -\frac{5}{882}x^2 + 44.1$ from part (e)

(M1)

$$y = -\frac{5}{882} \times 69^2 + 44.1$$

$$= 17.1 \text{ (17.1102...)}$$

A1

$$21.6 > 17.1$$

R1**OR**

substitution of their $y=21.6$ into their $y = -\frac{5}{882}x^2 + 44.1$ from part (e)

(M1)

$$21.6 = -\frac{5}{882}x^2 + 44.1$$

$$x = 63$$

A1

$$69 > 63$$

R1

Note: Accept equivalent justification in words, provided both values are seen.

Award **R1** for correct reasoning only if **M1** has been awarded and their seat coordinates lie outside their equation for the curve.

so the T-shirt cannot reach seat A_{40}

AG**[5 marks]****[Total 29 marks]**

1. (a) attempt to use the cosine rule to find $\hat{CAB}$ **(M1)**

$$(\hat{CAB} =) \cos^{-1} \left(\frac{42^2 + 35^2 - 36^2}{2 \times 42 \times 35} \right) \quad \textbf{(A1)}$$

$$(\hat{CAB} =) 54.8^\circ \text{ (54.8407...}^\circ) \textbf{ OR } 0.957 \text{ (0.957152...)} \quad \textbf{A1}$$

[3 marks]

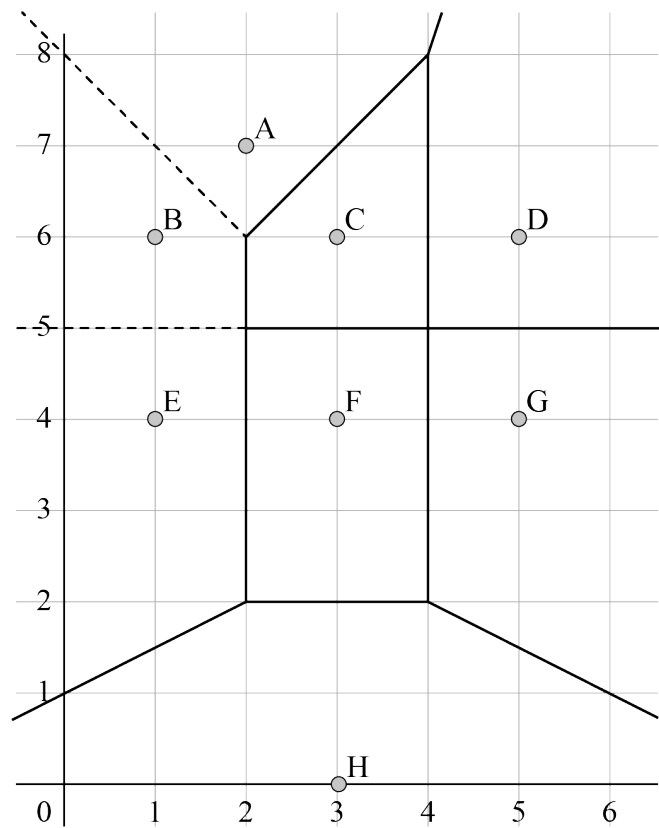
(b) (Area =) $\frac{1}{2} \times 42 \times 35 \times \sin 54.8407...^\circ$ **(A1)**

$$(\text{Area} =) 601 \text{ (600.903) cm}^2 \quad \textbf{A1}$$

Note: Correct units must be seen for the final **A1** to be awarded.

[2 marks]
[Total 5 marks]

5. (a)



A1A1

Note: Award at most **A1A0** if lines are not straight OR not accurate through the midpoint of AB and (0, 8) and through the midpoint of BE and (0, 5).

[2 marks]

(b) (i) area of $F = 6$ OR area of $C = 4$
($6 \times 4 =$) 24 (km²)

(A1)
A1

(ii) ($4 \times 4 =$) 16 (km²)

A1
[3 marks]

(c) The population in each cell might be equal

R1

[1 mark]
[Total 6 marks]

4. (a) attempt to use Pythagoras' theorem (M1)
- $$\sqrt{3.4^2 - 2^2}$$
- $$= 2.75 \text{ (2.74954...) (m)}$$

A1
[2 marks]

- (b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)
- 60° (A1)
- attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$ (M1)
- $$\frac{1}{2} \times 2 \times 2 \times \sin(60^\circ)$$

$$\left(6 \times \frac{1}{2} \times 2 \times 2 \times \sin(60^\circ) \right) = 10.4 \text{ (10.3923..., } 6\sqrt{3}) \text{ (m}^2\text{)} \quad \text{A1}$$

Note: Award **A0M0A0** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

- METHOD 2** (Use of altitude)
- (altitude is) $\sqrt{3}$ (A1)
- attempt to find the area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude. (M1)

$$\frac{1}{2} \times 2 \times \sqrt{3}$$

$$\left(6 \times \frac{1}{2} \times 2 \times \sqrt{3} \right) = 10.4 \text{ (10.3923..., } 6\sqrt{3}) \text{ (m}^2\text{)} \quad \text{A1}$$

Note: Award **A0M0A0** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

- METHOD 3** (Finding the area of a trapezoid)
- (altitude of one trapezoid is) $\sqrt{3}$ (A1)
- attempt to find area of one trapezoid using $\frac{1}{2} \times (a + b) h$ (M1)

$$\frac{1}{2} \times (2 + 4) \sqrt{3} \quad (3\sqrt{3})$$

$$\left(2 \times \frac{1}{2} \times (2 + 4) \sqrt{3} \right) = 10.4 \text{ (10.3923..., } 6\sqrt{3}) \text{ (m}^2\text{)} \quad \text{A1}$$

continue...

Question 4 continued.

$$(ii) \quad \frac{1}{3} \times 10.3923... \times 2.74954... \quad (A1)$$

$$= 9.52 \text{ m}^3 (9.52470...) \quad A1$$

Note: Units must be seen.

[5 marks]

$$(c) \quad \cos(\hat{MAT}) = \frac{2}{3.4} \text{ or correct equivalent} \quad (A1)$$

$$(\hat{MAT} =) 54.0^\circ \quad (53.9681..., 0.941921... \text{ radians}) \quad A1$$

[2 marks]

$$(d) \quad \text{Angle } YAX = 180 - 53.9681... = 126.031...^\circ \quad (A1)$$

$$\text{Angle } YXA = 180 - 35 - 126.031... = 18.9681...^\circ \quad (A1)$$

Note: These angles may be seen in the sine rule.

Attempt to substitute into sine rule

(M1)

$$\frac{AY}{\sin(18.9681...)} = \frac{2.6}{\sin(126.031)}$$

$$AY = 1.05 \quad (1.04503...) \text{ (m)}$$

A1
[4 marks]

(e) **METHOD 1 COSINE RULE**

attempt to substitute into cosine rule to form a quadratic for YZ

(M1)

$$0.9^2 = YZ^2 + 1.04503...^2 - 2 \times 1.04503... \times YZ \times \cos(35)$$

(A1)

$$YZ = 0.185 \quad (0.184692...) \text{ (m)}, 1.53 \quad (1.52739...) \text{ (m)}$$

A1A1

Note: Accept 0.191 (0.191313...) from use of 3 s.f. values.

METHOD 2 SINE RULEattempt to substitute into sine rule to find $\hat{YZA}$

$$\frac{\sin(\hat{YZA})}{1.04503...} = \frac{\sin(35^\circ)}{0.9}$$

(M1)

$$\hat{YZA} = 41.7597... \text{ or } \hat{YZA} = 138.240...$$

$$\hat{ZAY} = 103.240... \text{ or } \hat{ZAY} = 6.75972...$$

(A1)

Note: Award **A1** for any of these angles seen.

$$\frac{YZ}{\sin(\hat{ZAY})} = \frac{0.9}{\sin(35^\circ)}$$

$$YZ = 0.185 \quad (0.184692...) \text{ (m)}, 1.53 \quad (1.52739...) \text{ (m)}$$

A1A1

[4 marks]

[Total: 17 marks]

– 15 –

8824 – 9741M

(50)

5. (a) recognition that speed is the magnitude of $\begin{pmatrix} -33 \\ 0 \end{pmatrix}$ (M1)

$$\sqrt{50^2 + (-33)^2} \quad \text{OR} \quad \left| \begin{pmatrix} 50 \\ -33 \\ 0 \end{pmatrix} \right|$$

$$= 59.9 \text{ (km h}^{-1}\text{)}. (59.9082\dots)$$

A1
[2 marks]

(b) $\begin{pmatrix} 50 \\ -33 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix} = -90$ (A1)

$$\cos \theta = \frac{-90}{59.9082\dots \times 25}$$
(M1)

$$\theta = 93.4^\circ (93.4450\dots^\circ, 1.63092\dots)$$
A1
[3 marks]

(c) $\vec{OD} = \begin{pmatrix} 200 \\ -100 \\ 0.02 \end{pmatrix} + t \begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix}$ A1

[1 mark]

(d) (i) $\begin{pmatrix} 200 \\ -100 \\ 0.02 \end{pmatrix} + t \begin{pmatrix} -15 \\ -20 \\ 0 \end{pmatrix} = \begin{pmatrix} 152 \\ p \\ 0.02 \end{pmatrix}$ **OR** $200 = -15t + 152$ (M1)

$$t_1 = 3.2 \left(\frac{16}{5} \right)$$
A1

(ii) $p = -164$ **A1**
[3 marks]

continue...

Question 5 continued.

- (e) (i) attempt to find difference between the two position vectors

(M1)

$$\begin{pmatrix} 190 - 65t \\ -95 + 13t \\ 0.02 \end{pmatrix}$$

A1

Note: Award **A1M1A0** for $\begin{pmatrix} -190 + 65t \\ 95 - 13t \\ -0.02 \end{pmatrix}$.

(ii) attempt to find $\left| \begin{pmatrix} 190 - 65t \\ -95 + 13t \\ 0.02 \end{pmatrix} \right|$

(M1)

$$\sqrt{(190 - 65t)^2 + (13t - 95)^2 + 0.02^2}$$

(A1)attempt to find minimum. (e.g. $t = 3.09$ hours)**(M1)**

closest distance = 55.9 (55.8931...) (km)

A1**[6 marks]****[Total: 15 marks]**

6. (a) anticlockwise rotation of 15° about the origin

A1
[1 mark]

- (b) recognizing that I is equivalent to one rotation of 360°

(M1)

e.g. $\frac{360}{15}$
 $= 24$

A1
[2 marks]

(c) (i) $(B =) \begin{pmatrix} 1.05 & 0 \\ 0 & 1.05 \end{pmatrix}$

A1

(ii) $(B^{24} =) \begin{pmatrix} 3.23 & 0 \\ 0 & 3.23 \end{pmatrix}$

(A1)

Note: Award **A1** for 3.23 (3.22509...) **OR** 1.05^{24} .

enlargement, with a scale factor of 3.23 (1.05^{24}), (centre (0, 0))

A1
[3 marks]

(d) $C = \begin{pmatrix} 1.05 \cos(15^\circ) & -1.05 \sin(15^\circ) \\ 1.05 \sin(15^\circ) & 1.05 \cos(15^\circ) \end{pmatrix}$

$$\left(= \begin{pmatrix} 1.01 \text{ (1.01422...)} & -0.272 \text{ (-0.271759...)} \\ 0.272 \text{ (0.271759...)} & 1.01 \text{ (1.01422...)} \end{pmatrix} = \frac{21}{80} \begin{pmatrix} \sqrt{6} + \sqrt{2} & -\sqrt{6} + \sqrt{2} \\ \sqrt{6} - \sqrt{2} & \sqrt{6} + \sqrt{2} \end{pmatrix} \right)$$

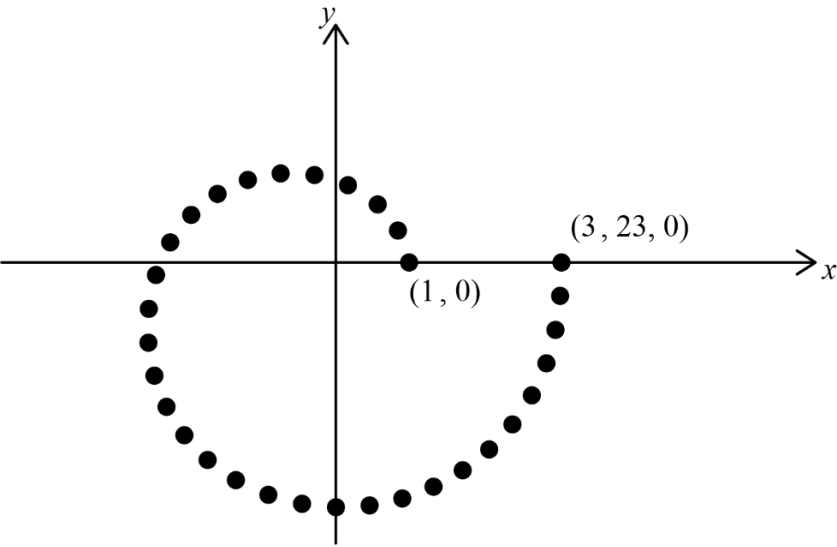
A2

Note: Award **A1** for at least two correct elements in the matrix.

[2 marks]
continue...

Question 6 continued.

(e)



anticlockwise spiral
starting at (1, 0) and ending at (3.23, 0)

A1
A1

Note: Accept a continuous curve instead of discrete points. The two **A1** marks can be awarded independently

[2 marks]

(f) $T\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$

(M1)

$0.9p + 2 = p$ **OR** $0.8q + 1 = q$

(A1)

$p = 20, q = 5$ **OR** $\begin{pmatrix} 20 \\ 5 \end{pmatrix}$

A1

[3 marks]
[Total: 13 marks]

1. (a) attempt to calculate area of a circle using 4 or 2.8

M1

recognition of multiplication by $\frac{1}{5}$ or $\frac{72}{360}$

M1

$$A = \frac{1}{5}(\pi(4)^2 - \pi(2.8)^2)$$

$$5.13 \text{ (m}^2\text{)} \text{ (5.12707... (m}^2\text{), } \frac{204\pi}{125} \text{ (m}^2\text{), } 1.632\pi \text{ (m}^2\text{))}$$

A1

$$5.12707... < 6 \quad \text{OR} \quad \text{Maureen has enough}$$

AG

Note: Do not award the final **A1** for the area unless a comparison to 6 is shown **OR** a statement of having enough is seen.

[3 marks]

(b) $(C =) 2\pi(4) \text{ (25.1327...)}$

(A1)

recognition to multiply by $\frac{3}{5}$ or $\frac{216}{360}$

(M1)

$$\frac{3}{5} \times 2\pi(4) + 2 \times 4$$

$$23.1 \text{ (m)} \text{ (23.0796...(m), } \frac{24\pi}{5} + 8)$$

A1**[3 marks]****[Total: 6 marks]**

✓ ✓

5. (a) (i) 170° (A1)

attempt to substitute into the cosine rule formula (M1)

$$(DC)^2 = 2^2 + 13^2 - 2(2)(13)\cos(170^\circ) \quad (A1)$$

$$(DC \Rightarrow) 15.0 \text{ (14.9736...)} \text{ (km)} \quad A1$$

Note: Do not accept a 2sf answer of 15. Award **M0A0A0** for a 2sf answer of 15 with no working shown OR $2+13=15$.

(ii) correct substitution into sine rule or cosine rule (A1)

$$\frac{\sin \hat{BDC}}{13} = \frac{\sin 170^\circ}{14.9736...} \quad \text{OR} \quad \cos(\hat{BDC}) = \frac{14.9736...^2 + 2^2 - 13^2}{2 \times 14.9736... \times 2}$$

$$(\hat{BDC} \Rightarrow) 8.67^\circ \text{ (8.67097...}^\circ) \quad (A1)$$

$$(\text{bearing} \Rightarrow) 098.7^\circ \text{ (098.6709...}^\circ) \quad A1$$

[7 marks]

(b) (distance between points D and E $\Rightarrow$) 13 (km) A1

points B, C, D, and E make a parallelogram **OR** DB and EC are parallel
and equal in length R1

Note: It is possible to award **A1R0** for this question part.

[2 marks]

[Total 9 marks]

13. (a) an anticlockwise/counter-clockwise rotation of $\frac{\pi}{3}$ (about the origin)

A1A1

Note: Award **A1** for rotation and **A1** for correct angle/direction detail.

[2 marks]

- (b) attempt to multiply **A** and **B** in correct order

(M1)

$$\mathbf{BA} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} \cos \frac{\pi}{3} & -\sin \frac{\pi}{3} \\ \sin \frac{\pi}{3} & \cos \frac{\pi}{3} \end{bmatrix} \left(\begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0.5 & -0.866025... \\ 0.866025... & 0.5 \end{bmatrix} \right)$$

$$\begin{bmatrix} 0.119615... & 0.992820... \\ 0.992820... & -0.119615... \end{bmatrix}$$

(A1)

attempt to create an expression with either $\tan 2\theta$ or $\sin 2\theta$ or $\cos 2\theta$

(M1)

$$\tan 2\theta = \frac{0.992820...}{0.119615...}$$

$$\theta = 0.725446...$$

(A1)

$$y = \tan(0.725446...)x$$

$$a = 0.887 \text{ (0.886751...)}$$

A1

Note: Award at most **(M1)(A1)(M1)(A1)A0** if the extraneous value of a is given alongside a correct answer.

[5 marks]**[Total 7 marks]**

2. (a) use of Pythagoras in 3 dimensions (M1)
 $AB^2 = 2^2 + 3^2 + 1^2$
 $AB = 3.74$ ($= 3.74165... = \sqrt{14}$) A1

[2 marks]

- (b) **METHOD 1 (using Pythagoras in 3D)**

$$1^2 + 6^2 + (k-1)^2 = 14 + 1^2 + 3^2 + (k-2)^2 \quad (M1)A1$$

Note: Award **M1** for use of Pythagoras in three dimensions, **A1** for correct use.

$$38 - 2k = 28 - 4k \quad (M1)$$

Note: Award **M1** for reducing equation to linear form.

$$k = -5 \quad A1$$

METHOD 2 (using vectors)

find the vectors $\vec{AB}$ and $\vec{CB}$ (M1)

$$\vec{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad \vec{CB} = \begin{pmatrix} -1 \\ 3 \\ k-2 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 3 \\ k-2 \end{pmatrix} = 0 \quad A1$$

calculate scalar product (M1)

$$-2 + 9 + k - 2 = 0$$

$$k = -5 \quad A1$$

[4 marks]

- (c) **EITHER**

$$\cos(\hat{BAC}) = \frac{\sqrt{14}}{\sqrt{73}} \quad (M1)(A1)$$

OR

$$\cos(\hat{BAC}) = \frac{\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 6 \\ -6 \end{pmatrix}}{\sqrt{14}\sqrt{73}} \quad (M1)(A1)$$

OR

$$\sin(\hat{BAC}) = \frac{\sqrt{59}}{\sqrt{73}} \quad (M1)(A1)$$

continued...

*Question 2 continued***OR**

$$\tan(\hat{BAC}) = \frac{\sqrt{59}}{\sqrt{14}}$$

(M1)(A1)
Note: Award **M1** for use of trig ratio, **A1** for correct use.
THEN

$$\hat{BAC} = 64.0^\circ \quad (64.0282\dots^\circ) \quad (1.12 (1.11750\dots))$$

A1
Note: Accept 64 for the final **A1**.
[3 marks]**Total [9 marks]**

4. (a) (i) (8, 4) A1

Note: Do not condone the omission of parenthesis.

(ii) 5 A1

[2 marks]

(b) (i) 8 A1

(ii) **EITHER**
attempt to use Pythagoras theorem (M1)

$$4 + (b - 5)^2 = 9 + (9 - b)^2 \quad (A1)$$

$$(b =) \frac{61}{8} = 7.625 \quad \left(\frac{61}{8}, 7\frac{5}{8} \right) \quad A2$$

Note: Award **A1A0** for 7.63

OR

$$m(BD) = -\frac{4}{5} \quad (A1)$$

$$\text{midpoint } [BD] = (7.5, 7) \quad (A1)$$

Note: Gradient and midpoint may be implied by further working. Condone as interim coordinate 7.5, 7 instead of (7.5, 7).

$$(y - 7) = \frac{5}{4}(x - 7.5)$$

substitute $x = 8$ in their perpendicular bisector (M1)

$$b - 7 = \frac{5}{4}(8 - 7.5)$$

$$(b =) 7.625 \quad \left(\frac{61}{8}, 7\frac{5}{8} \right) \quad A1$$

Note: Award **A1A1M1A0** final answer of 7.63

OR

$$m(DF) = -4 \quad (A1)$$

$$\text{midpoint } [DF] = (5.5, 7) \quad (A1)$$

Note: Gradient and midpoint may be implied by further working. Condone as interim coordinate 5.5, 7 instead of (5.5, 7)

continued...

Question 4 continued

$$(y - 7) = \frac{1}{4}(x - 5.5)$$

substitute $x = 8$ in their perpendicular bisector**(M1)**

$$(8 - 7) = \frac{1}{4}(b - 5.5)$$

$$(b =) 7.625 \left(\frac{61}{8}, 7\frac{5}{8} \right)$$

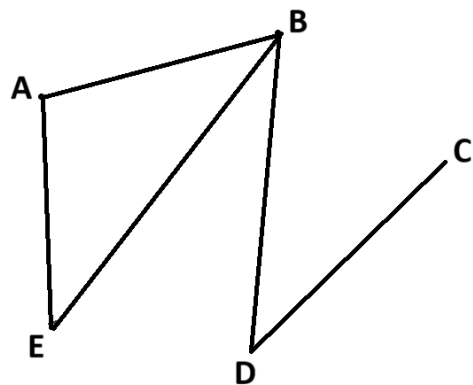
A1

Note: Award **A1A1M1A0** final answer of 7.63

Note: Both a and b can be found by locating the intersection of two perpendicular bisectors.
This is an inefficient method, but correct answers should be awarded full marks.

[5 marks]**Total [7 marks]**

8. (a)



A2

[2 marks]

Note: For graphs with edges that cross each other but are otherwise correct, award **A1**.
Award **A2** for a directed graph where each edge is bi-directional.

(b) recognition that A^4 is required

(M1)

$$A^4 = \begin{pmatrix} 7 & 6 & 1 & 5 & 6 \\ 6 & 12 & 4 & 2 & 6 \\ 1 & 4 & 2 & 0 & 1 \\ 5 & 2 & 0 & 6 & 5 \\ 6 & 6 & 1 & 5 & 7 \end{pmatrix}$$

(A1)

recognition that diagonal entries are ones which return to the same town.
total of 34 walks

(M1)

A1

Note: For incomplete attempts to list all possible walks of length four award **M0A0M0A0**.

[4 marks]

Total [6 marks]

Q1.

- (a) attempt to use the cosine rule formula (M1)

$$(NV =) \sqrt{20^2 + 25^2 - 2 \times 20 \times 25 \cos 55.5^\circ} \quad (A1)$$

$$= 21.4(\text{m})(21.4148...) \quad A1$$

[3 marks]

- (b) **EITHER**

- attempt to use the sine rule formula (M1)

$$\frac{\sin \hat{PNV}}{25} = \frac{\sin 55.5^\circ}{21.4148...} \quad (A1)$$

OR

- attempt to use the cosine rule formula (M1)

$$\cos \hat{PNV} = \frac{20^2 + 21.4148...^2 - 25^2}{2 \times 20 \times 21.4148...} \quad (A1)$$

THEN

$$\hat{PNV} = 74.2^\circ (74.1749...) \quad A1$$

[3 marks]

- (c) recognition that shortest distance is perpendicular to NV (M1)
(may be seen on diagram)

- use of right-angled triangle or other valid method (M1)

$$\sin 74.1749... \times 20 \quad \text{OR} \quad \frac{1}{2} \times 20 \times 25 \times \sin 55.5^\circ \times \frac{2}{21.4148...}$$

$$= 19.2(\text{m})(19.2419...) \quad A1$$

Note: Accept 19.3 (m) from using their 3 sf value in $\frac{1}{2} \times 20 \times 25 \times \sin 55.5^\circ \times \frac{2}{21.4148...}$

[3 marks]**Total [9 marks]**

Q8. EITHER

attempt to find matrix for compound transformation

(M1)

$$\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & 0 \end{pmatrix}$$

(A1)

$$\begin{pmatrix} 1 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

(A1)

multiply by the inverse(s) or form two equations

(M1)**OR**

$$\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$$

(A1)

multiply by the inverse or form two equations

(M1)coordinates of B are $(-1, 7)$ **(A1)**

attempt to find coordinates for A by setting up matrix

(M1)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 7 \end{pmatrix}$$

THEN

$$(7, -1)$$

A1**Total [5 marks]**

A1A1

(A1)(A1)

(M1)

(A1)

(M1)

A1

Total [8 marks]

2. (a) (i) attempt to substitute any given coordinates into midpoint formula (M1)

$$(12, 12.5) \text{ or } \left(12, \frac{25}{2}\right) \quad \text{A1}$$

Note: Award **A0** if parentheses are omitted.

- (ii) attempt to find gradient of BC using coordinates of B and C (M1)

$$(\text{gradient of BC} =) \frac{12-13}{16-8} \left(= -\frac{1}{8} = -0.125 \right) \quad \text{(A1)}$$

$$(\text{gradient of } \perp =) 8 \quad \text{A1}$$

Note: Follow through within question part for final **A1** from their gradient of BC only if the gradient is seen.

- (iii) **EITHER**

$$8(x-12) = y-12.5 \quad \left(8(x-12) = y-12\frac{1}{2} \right) \quad \text{A1}$$

OR

$$y = 8x - 83.5 \quad \left(y = 8x - 83\frac{1}{2} \right) \quad \text{A1}$$

[6 marks]

- (b) (i) **EITHER**

B is expected to respond since (14, 10) lies in the same cell (of the Voronoi diagram) as fire station B **R1**

OR

B is expected to respond since (14, 10) is closer to B than to any other fire station **R1**

OR

B is expected to respond since it's the closest (nearest responder) to (14, 10) **R1**

OR

Fire station B will respond as the fire is within B's boundaries/area/region. **R1**

Note: Do not award **R1** unless the statement refers to both the distance (closer) to B (or located in the B cell) **and** the responder (point B or fire station B) For example, award **R0** for "Point B is closer to (14, 10)" as there is no reference to a responder.

- (ii) Award **R1** for any viable reason such as:

- fire station B might have already responded to a different fire
- a different fire station might have more manpower or equipment available to fight the fire
- a different fire station might respond to assist fire station B if the fire is large
- the Voronoi diagram only considers straight-line distances, so a different fire station might have a shorter driving route to the fire

R1
[2 marks]

(c) (i) attempt to calculate the length DB (M1)

a correct expression for DB A1

$$(DB =) \sqrt{\left(11\frac{1}{3} - 16\right)^2 + \left(7\frac{1}{6} - 12\right)^2}$$

$$DB = 6.71855$$

AG

(ii) **METHOD 1**

$$(DC =) \sqrt{\left(11\frac{1}{3} - 8\right)^2 + \left(7\frac{1}{6} - 13\right)^2}$$

A1

DC = 6.71855 therefore, the fire is equidistant from all 3 fire stations R1

so, any of the fire stations would be expected to respond AG

Note: Do not award **A0R1**. Do not award **R1** if **AG** statement is not seen.

METHOD 2

The intersection of bisector AB and bisector BC :

$$y = 8x - 83.5 \text{ and } y = 0.5x + 1.5 \text{ have an intersection at } D\left(11\frac{1}{3}, 7\frac{1}{6}\right)$$

A1

Note: Calculator solve is acceptable but D must be clearly identified from a calculation of intersecting lines.

Since D is the intersection of the bisector lines, the fire at D is equidistant from all 3 fire stations R1

so, any of the fire stations would be expected to respond AG

Note: Do not award **A0R1**. Do not award **R1** if **AG** statement is not seen.

[4 marks]

- (d) (i) **METHOD 1: Dividing into two triangles**
- $(0.5 \times 20 + 1.5 =) 11.5$ seen in an area calculation (A1)
- $(20 - 7.75 =) 12.25$ **OR** $(20 - 11.3333... =) 8.66666...$ seen (A1)
- substitution into triangle area formula for either triangle (M1)
- $(A =) 0.5 \times (20 - 7.75) \times 7.16666... + 0.5 \times 11.5 \times (20 - 11.3333...)$ (A1)
- $93.7291... \left(\frac{4499}{98} \right)$ (sq. units)
- 94 (sq. units) A1

Note: Answer must be given as a whole number to award final A1.
Award (A1)(A1)(M1)(A1)A0 for an answer of 93.7 without working.

- METHOD 2: Dividing vertically into triangle and trapezoid**
- $(0.5 \times 20 + 1.5 =) 11.5$ seen in an area calculation (A1)
- attempt to substitute into area of trapezoid formula (M1)
- $(A =) 0.5 \times (7.16666... + 11.5) \times (20 - 11.3333...)$

Note: Lengths of parallel sides must be seen to award the M1.

- correct calculation of right triangle area (A1)
- $(A =) 0.5 \times (11.3333... - 7.75) \times 7.16666...$
- $(A =) 0.5 \times (11.3333... - 7.75) \times 7.16666... + 0.5 \times (7.16666... + 11.5) \times (20 - 11.3333...)$ (A1)
- $93.7291... \left(\frac{4499}{98} \right)$ (sq. units)
- 94 (sq. units) A1

Note: Answer must be given as a whole number to award final A1.
Award (A1)(M1)(A1)(A1)A0 for an answer of 93.7 without working.

METHOD 3: Dividing horizontally into triangle and trapezoid $(0.5 \times 20 + 1.5 =) 11.5$ seen in an area calculation**(A1)**

attempt to substitute into area of trapezoid formula

(M1)

$$(A =) 0.5 \times [(20 - 7.75) + (20 - 11.3333...)] \times 7.16666...$$

Note: Lengths of parallel sides must be seen to award the **M1**.

correct calculation of right triangle area

(A1)

$$(A =) 0.5 \times (20 - 11.3333...) \times (11.5 - 7.16666...)$$

$$(A =) 0.5 \times (20 - 11.3333...) \times (11.5 - 7.16666...) + 0.5 \times [(20 - 7.75) + (20 - 11.3333...)] \times 7.16666...$$

(A1)

$$93.7291... \left(\frac{4499}{98} \right) \text{ (sq. units)}$$

94 (sq. units)

A1

Note: Answer must be given as a whole number to award final **A1**.
Award **(A1)(M1)(A1)(A1)A0** for an answer of 93.7 without working.

METHOD 4: Dividing into two triangles and rectangle $(0.5 \times 20 + 1.5 =) 11.5$ seen in an area calculation**(A1)** $(20 - 11.3333...)$ **OR** $(11.3333... - 7.75)$ **OR** $(11.5 - 7.16666...)$ seen**(A1)**

substitution into triangle area formula.

(M1)

$$0.5 \times (11.3333... - 7.75) \times 7.16666... + (20 - 11.3333...) \times 7.16666... + 0.5 \times (11.5 - 7.16666...) \times (20 - 11.3333...)$$

(A1)

$$93.7291... \left(\frac{4499}{98} \right) \text{ (sq. units)}$$

94 (sq. units)

A1

Note: Answer must be given as a whole number to award final **A1**.
Award **(A1)(A1)(M1)(A1)A0** for an answer of 93.7 without working.

METHOD 5: Calculus

A and C mid point (13, 10.5)

$$(\text{gradient of AC} =) \frac{13-8}{8-18} = -\frac{5}{10}$$

(gradient of $\perp$ =) 2

$$\text{AC bisector } 2(x-13) = y-10.5 \quad \text{OR} \quad y = 2x - 15.5$$

A1

attempt to integrate their equation and given equation

(M1)

$$\text{area} = \int_{7.75}^{11.3} 2x - 15.5 \, dx + \int_{11.3}^{20} 0.5x + 1.5 \, dx$$

(A1)(A1)

Note: Award **A1** for each correct expression with correct limits.

$$93.7291... \left(\frac{4499}{98} \right) (\text{sq. units})$$

$$94 \text{ (sq. units)}$$

A1

Note: Answer must be given as a whole number to award final **A1**.

Award **A1(M1)(A1)(A1)A0** for an answer of 93.7 without working.

$$(ii) \quad (1 \text{ sq unit} = 2.5 \text{ km} \times 2.5 \text{ km} =) 6.25 \text{ (km}^2\text{)}$$

(A1)

Note: Award **A1** for 6.25 seen.

$$(94 \times 6.25 \text{ km}^2)$$

$$588 \text{ (587.5) (km}^2\text{)}$$

A1

Note: Accept: 586 (585.807...) (km²) from (93.7291... × 6.25 km²).

[7 marks]**[Total: 19 marks]**

4. (a) $\sqrt{(-0.40)^2 + (-0.35)^2 + (0.1)^2}$ (A1)

0.541 $\left(0.540832\dots, \frac{3\sqrt{13}}{20}\right)$ (kilometres per minute) A1

[2 marks]

- (b) any two of the following: A1A1
- flying in a straight line
 - flying in an overall NW direction
 - flying towards the west at 1.4 km min^{-1} (units required)
 - flying towards the north at 1.65 km min^{-1} (units required)
 - flying at a constant altitude / at an altitude of 7 kilometres / parallel to the ground
 - passes through a point that is 5 kilometres east, 2 kilometres south, and 7 kilometres above the ground

Note: Statement must be about the path and not the speed/velocity. Do not accept any statement referring to moving towards the east at -1.4 km min^{-1} .

[2 marks]

(c) (i) $6 + 0.1t_1 = 7(+0t_1)$ (A1)

$t_1 = 10$ (min) (accept a final answer of "10:10 (am)") A1

- (ii) substitution of $t_1 = 10$ into either vector equation (M1)

airplane (will have a position vector of) $\begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (10) \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} = \begin{pmatrix} -9 \\ 14.5 \\ 7 \end{pmatrix}$ A1

birds (will have a position vector of) $\begin{pmatrix} -11 \\ 30 \\ 6 \end{pmatrix} + (10) \begin{pmatrix} -0.40 \\ -0.35 \\ 0.05 \end{pmatrix} = \begin{pmatrix} -15 \\ 26.5 \\ 7 \end{pmatrix}$ A1

Note: All three of the above marks can be awarded to candidates who work with only the x -coordinates or only the y -coordinates. The complete position vectors do not need to be found.

x - or y - coordinates are different at the (only) time their z -coordinate is equal R1

Note: The response must include a reference to the x - or y - coordinates being different to award the R1. Simply referring to the two position vectors is not sufficient. A statement saying that the 'north' and/or 'east' positions are different is sufficient. Allow FT within question part for final R mark for a correction statement related to their position vectors

therefore airplane and birds will not collide AG

[6 marks]

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$$(d) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (t_2 + 20) \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} \quad \text{OR} \quad \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ -2 \\ 7 \end{pmatrix} + (20) \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} \quad (A1)$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -23 \\ 31 \\ 7 \end{pmatrix} + t_2 \begin{pmatrix} -1.4 \\ 1.65 \\ 0 \end{pmatrix} \quad A1$$

[2 marks]

(e) attempt to find difference between any coordinate at time t_2 (M1)

$$x = -95 + 1.3t_2 - (-23 - 1.4t_2) \quad \text{and}$$

$$y = 32 + 1.45t_2 - (31 + 1.65t_2) \quad \text{and}$$

$$z = 1.5 + 0.2t_2 - 7 \quad (A1)$$

attempt to substitute their expressions into distance formula (M1)

$$d = \sqrt{(2.7t_2 - 72)^2 + (-0.2t_2 + 1)^2 + (0.2t_2 - 5.5)^2}$$

(using GDC, minimum distance $\Rightarrow$) 4.33 (4.32565...) (kilometres) A1

therefore the jet and the airplane will break the rule (since $4.33 < 5$) R1

Note: Do not award **R1** unless value for minimum distance is seen. If no adjustment is made to the original passenger airplane vector equation, award at most **(M1)(A0)(M1)(A0)R1**.

Allow FT within question part for final R mark as long as $0 < d < 10$ and their value of d comes from an attempt to use the distance formula.

[5 marks]**[Total 17 marks]**

5. (a) correct substitution into distance formula (A1)
 $\sqrt{(13-13)^2 + (25-1)^2 + (7-0)^2}$ **OR** $\sqrt{0^2 + 24^2 + 7^2}$

Note: Accept omission of $(13-13)^2$ or 0^2 .

(AC =) 25 (cm)

A1
[2 marks]

- (b) **METHOD 1**

AB $\parallel$ y-axis **AND** BC $\parallel$ z-axis (AB = 24*k* **AND** BC = 7*j*) A1

AB perpendicular to BC since y-axis is perpendicular to z-axis **OR** AB • BC = 0 R1

therefore $\triangle ABC$ must be a right-angled triangle. AG

METHOD 2

24 and 7 seen A1

$24^2 + 7^2 = 25^2$ **OR** $625 = 625$ **OR** (angle B =) $\cos^{-1}\left(\frac{7^2 + 24^2 - 25^2}{2(7)(24)}\right) = 90^\circ$ R1

Note: Accept $AB^2 + BC^2 = 625$ as long as the length of AB and of BC are defined.

therefore $\triangle ABC$ must be a right-angled triangle **AG**
[2 marks]

- (c) width of 9 or correct cross-section area (84) seen (A1)

correct substitution into volume formula (A1)

$$V = \frac{1}{2}(24)(7)(9)$$

(Volume =) 756 cm³ A1

Note: Do not award final **A1** if correct units are not included.

[3 marks]

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- (d) (i) correct use any trig ratio (M1)

$$\tan \theta = \frac{7}{24} \quad \text{OR} \quad \sin \theta = \frac{7}{25} \quad \text{OR} \quad \cos \theta = \frac{24}{25}$$

$$(\theta =) 16.3^\circ (16.2602\dots^\circ) \quad \text{OR} \quad 0.284 (0.283794\dots) \text{ (radians)} \quad \text{A1}$$

- (ii) **METHOD 1**

recognition that the width of the door stop is still 9 (M1)

$$625 = \frac{1}{2}bh(9)$$

$$bh = \frac{1250}{9}$$

$$\tan(16.2602\dots^\circ) = \frac{h}{b} \quad \text{OR} \quad \tan(0.283794\dots) = \frac{h}{b} \quad \text{(M1)}$$

$$b^2 = \frac{1250}{9 \tan(16.2602\dots^\circ)} \quad \text{OR} \quad b^2 = \frac{1250}{9 \tan(0.283794\dots)} \quad \text{(A1)}$$

$$(b = AX =) 21.8 (21.8217\dots) \text{ (cm)} \quad \text{A1}$$

METHOD 2 (area similarity)

recognition that the new side face will be similar to ABC (M1)

$$\text{proportionality of area } k^2 = \frac{625}{756} \quad \text{(A1)}$$

$$\text{proportionality of length } k = \sqrt{\frac{625}{756}} = 0.909241$$

$$\text{new base length } AX = 24k \quad \text{(M1)}$$

$$21.8 (21.8217\dots) \text{ (cm)} \quad \text{A1}$$

[6 marks]

- (e) multiplying 625 by 1.1 or 1.2 or 0.025 (M1)

$$625 \times 1.1 \times 0.025 \times 1.2 \quad \text{(A1)}$$

$$(\text{price} =) 20.63 (20.625) \text{ (USD)} \quad \text{A1}$$

[3 marks]

[Total 16 marks]

4. (a) (since there are 5 vertices, 4 edges are required)
 start with A
 add E ($AE = 0.86$) **M1**
 add C ($EC = 0.62$)
 add B ($CB = 0.74$)
 add D ($CD = 0.78$) **A2**

Note: The **A2** is awarded for a correct Prim's algorithm. It cannot be split.

the length of cable is $3(.00)$ (km). **A1**

[4 marks]

- (b) attempt to combine MST with AF **(M1)**
 $LB = (\text{length of minimum spanning tree}) + (FA + FE)$
 $= 3 + 0.96 + 1.14$
 $= 5.1(0)$ (km) **A1**

[2 marks]

- (c) attempt to use the nearest neighbour algorithm **(M1)**

Note: Award **M1** for first 3 edges correct or 0.96, 0.86, 0.62 seen in order

FA, AE, EC, CB, BD, DF OR 0.96, 0.86, 0.62, 0.74, 0.94, 2.24 **A1**
 $UB = 6.36$ (km) **A1**

[3 marks]

- (d) use A or C for LB and A, B or F for UB **A1A1**
 any attempt to convert distances to time **(M1)**

$$LB = (5.02 \div 28) + (5 \times 0.2) = 1.18 \text{ hrs } (1.17928\dots)$$

$$UB = (6.36 \div 28) + (5 \times 0.2) = 1.23 \text{ hrs } (1.22714\dots)$$

$$1.18 \leq T \leq 1.23$$

A1

[4 marks]

Total [13 marks]

Q4. (a) between A and E

A1
[1 mark]

(b) (i) attempt to find M^3

M1

$$M^3 = \begin{pmatrix} 1 & 3 & 1 & 1 & 3 \\ 3 & 1 & 3 & 1 & 4 \\ 0 & 3 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 2 \\ 0 & 4 & 0 & 2 & 1 \end{pmatrix}$$

contains 0 so some routes are not possible

R1

Note: Identification of any route which is not possible, award **R1**, provided **M1** has been awarded.

(ii) **METHOD 1**

attempt to find $M + M^2 + M^3$ OR $M^2 + M^3$

M1

$$M + M^2 + M^3 = \begin{pmatrix} 2 & 5 & 2 & 2 & 5 \\ 4 & 4 & 4 & 2 & 6 \\ 1 & 4 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 5 & 1 & 3 & 3 \end{pmatrix}, M^2 + M^3 = \begin{pmatrix} 2 & 4 & 2 & 2 & 4 \\ 3 & 4 & 3 & 2 & 5 \\ 1 & 3 & 1 & 1 & 2 \\ 1 & 1 & 1 & 1 & 2 \\ 1 & 4 & 1 & 2 & 3 \end{pmatrix}$$

all ports can be reached in 3 trips or fewer,

A1

because there are no 0 entries

R1

Note: The complete matrix does not need to be seen for **M1** to be awarded. **A1** and **R1** are dependent on the **M1**.

continued...

Question 4 continued.

METHOD 2

attempt to find M^2

M1

$$M^2 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 2 \end{pmatrix}$$

all ports can be reached in 3 trips or fewer,

A1

because there are no overlapping 0 entries in M^2 and M^3

R1

Note: The complete matrix does not need to be seen for **M1** to be awarded. **A1** and **R1** are dependent on the **M1**.

[5 marks]

(c) attempt to find the cost of a route of length 2

M1

route CAD has cost $7 + 8 = 15$

route CBD has cost $4 + 9 = 13$

route CED has cost $10 + 3 = 13$

A1A1

Note: Award **A1** for at least one correct answer, **A1** for all three correct answers. Accept CBED has cost $4 + 6 + 3 = 13$ in place of CBD and CED

hence least cost is 13 euros

AG
[3 marks]

continued...

Question 4 continued.

- | | | |
|-----|--|-------------------------|
| (d) | evidence of correct algorithm used, ABC... seen, or in a diagram | (M1) |
| | ABCEDA OR $3 + 4 + 10 + 3 + 8$ | A1 |
| | cost 28 euros | A1 |
| | | [3 marks] |
| (e) | evidence of selecting three edges not including vertex A | (M1) |
| | length of spanning tree = $4 + 6 + 3 (= 13)$ | (A1) |
| | reconnect vertex A using two distinct edges | (M1) |
| | lower bound $(13 + 3 + 5 =) 21$ | A1 |
| | | [4 marks] |
| | | [Total 16 marks] |

Q5. (a) (2, 1, 0)

A1

Note: Condone coordinates written as a vector.

[1 mark]

$$(b) \quad (i) \quad 5 \begin{pmatrix} -4 \\ 4 \\ 7 \end{pmatrix} \text{ or } \begin{pmatrix} -18 \\ 21 \\ 35 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

(M1)

Note: Condone one error in the position vector for B for **M1**

$$\begin{pmatrix} -20 \\ 20 \\ 35 \end{pmatrix}$$

A1□

(ii) use of formula for magnitude with their $\vec{AB}$

(M1)

$$\sqrt{(-20)^2 + 20^2 + (35)^2}$$

45 (m)

A1

Note: Do not accept $\sqrt{2025}$.

[4 marks]

(c) **METHOD 1**

use of two direction vectors

(M1)

use of formula for scalar product

(M1)

$$\begin{pmatrix} -4 \\ 4 \\ 7 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = 12 \quad (\text{OR equivalent multiple of 12})$$

(A1)

$$\cos(\hat{CAB}) = \frac{12}{\sqrt{81}\sqrt{5}}$$

(A1)

$$\hat{CAB} = 53.4^\circ (53.3957\dots^\circ, 0.932, 0.931931\dots)$$

A1

continued...

Question 5 continued.

METHOD 2

attempt to find a point C using $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ or some value of λ (M1)

eg $C = (1, 3, 0)$

$|\vec{AC}| = \sqrt{5}$, $|\vec{BC}| = 43.7 (= \sqrt{1910})$ OR equivalent (A1)

attempt to use the cosine rule (M1)

$\cos(\hat{CAB}) = \frac{5 + 45^2 - 1910}{2 \times \sqrt{5} \times 45}$ OR equivalent (A1)

$\hat{CAB} = 53.4^\circ (53.3957...^\circ, 0.932, 0.931931...)$ A1

[5 marks]

(d) let P_2 be at the point D when $t = 5$

EITHER

use of distance = speed $\times$ time (M1)

distance travelled by $P_2 = 12 \times 5$

$= 60$ (A1)

use of cosine rule (M1)

$\left(|\vec{BD}|^2 = \right) 45^2 + 60^2 - 2 \times 45 \times 60 \cos(53.3957...) (= 2405.06...)$ (A1)

continued...

Question 5 continued.

OR

position of P_1 $\begin{pmatrix} -18 \\ 21 \\ 35 \end{pmatrix}$ **(A1)**

position of P_2

attempt to calculate $r = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$ for any value of λ **(M1)**

$r = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + 5 \times \frac{12}{\sqrt{5}} \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} -24.8328... \\ 54.6656... \\ 0 \end{pmatrix}$ **(A1)**

subtracting their position vectors for P_1 and P_2 **(M1)**

$\vec{BD} = \begin{pmatrix} -24.8328... \\ 54.6656... \\ 0 \end{pmatrix} - \begin{pmatrix} -18 \\ 21 \\ 35 \end{pmatrix}$

THEN

$|\vec{BD}| = 49.0(49.0414...) \text{ (m)}$ **A1**

[5 marks]

Total [15 marks]

2. (a) (i) attempt to use Pythagoras' theorem (M1)

$$\sqrt{12500^2 + 6000^2}$$

$$= 13900 \text{ (km)}$$

A1

Note: Answers to more than 3sf are not appropriate and should be penalised. **A1** will be given only for 13900km (3sf) or 14000km (nearest 500km).

[2 marks]

- (ii) attempt to use an appropriate inverse trig ratio (M1)

$$\text{e.g. } \arctan\left(\frac{12500}{6000}\right) \text{ or } \arctan\left(\frac{6000}{12500}\right)$$

identifying the correct angle (M1)

$$= (0)64.4^\circ \text{ OR } 064^\circ (64.3589\dots)$$

A1

Note: The two method marks can be awarded independently. The first can come from any geometry leading to an inverse trigonometric ratio. The second requires recognizing which angle is the bearing, possibly on a diagram. 64.4 appearing in intermediate working is not sufficient.

[3 marks]

- (b) (i) $p \cdot n = 0$ M1

$$\text{so the angle is } 90^\circ \text{ OR } \frac{\pi}{2} \text{ rads}$$

A1

Note: Award **M0A0** for an answer of 90 degrees without any mention that the scalar product is zero.

[2 marks]

- (ii) attempt to use arc length formula $s = r\theta$ OR $\frac{90^\circ}{360}$ of circumference (M1)

$$= 6000\left(\frac{\pi}{2}\right) \text{ OR } \frac{1}{4} \times 2\pi \times 6000$$

A1

$$= 3\pi \text{ thousand km OR } 3000\pi \text{ km}$$

AG

[2 marks]

continued...

Question 2 continued

- (c) (i) attempt to find cross product (e.g. correct except for sign error)

(M1)

$$\mathbf{a} \times \mathbf{p} = \begin{pmatrix} 0 \\ -36 \\ 0 \end{pmatrix}$$

A1

[2 marks]

- (ii)
- METHOD 1**

$$\mathbf{a} \times \mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 36 \end{pmatrix}$$

A1

choosing to use scalar product for their $\mathbf{a} \times \mathbf{p}$ and $\mathbf{a} \times \mathbf{n}$

M1

$$(\mathbf{a} \times \mathbf{p}) \cdot (\mathbf{a} \times \mathbf{n}) = 0$$

A1

Note: The final **A1** can be awarded for seeing the dot product equals zero without incorrect working. Follow through from (i) is permitted and if they explicitly state that they are using $\mathbf{a} \times \mathbf{n}$ then allow internal follow through.

so angle is 90°

AG

[3 marks]

METHOD 2

$$\mathbf{a} \times \mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 36 \end{pmatrix}$$

A1

attempt to use cross product for their $\mathbf{a} \times \mathbf{p}$ and $\mathbf{a} \times \mathbf{n}$. This must include both an attemptto find the cross product and knowledge that $\sin \theta = \frac{|\mathbf{x} \times \mathbf{y}|}{|\mathbf{x}||\mathbf{y}|}$

M1

$$(\mathbf{a} \times \mathbf{n}) \times (\mathbf{a} \times \mathbf{p}) = \begin{pmatrix} 1296 \\ 0 \\ 0 \end{pmatrix} \text{ OR } (\mathbf{a} \times \mathbf{p}) \times (\mathbf{a} \times \mathbf{n}) = \begin{pmatrix} -1296 \\ 0 \\ 0 \end{pmatrix}$$

$$\sin \theta = \frac{1296}{36 \times 36} = 1$$

A1

so angle is 90°

AG

continued...

Question 2 continued

(d) attempt to use $\frac{\theta}{360} \times 2\pi r = 6$ OR $s = r\theta$ (M1)

$$\frac{\theta}{360} \times 2\pi \times 6 = 6 \quad \text{OR} \quad 6 = 6\theta \Rightarrow \theta = 1 \text{ radian}$$

$$\theta = 57.2957\dots^\circ$$

A1

Note: The fourth significant figure must be seen somewhere to award the A1.

$$\theta = 57.3^\circ \text{ to 3 significant figures}$$

AG

[2 marks]

(e) **METHOD 1****EITHER** (Scalar product)

attempt to use scalar product to find $\hat{\text{BOM}} = \arccos\left(\frac{\mathbf{b} \cdot \mathbf{m}}{|\mathbf{b}||\mathbf{m}|}\right)$ (M1)

Note: This may be written as $\cos(\text{BOM}) = \frac{\mathbf{b} \cdot \mathbf{m}}{|\mathbf{b}||\mathbf{m}|}$.

$$= \arccos\left(\frac{(6 \cos 120^\circ)(6 \cos 57.3^\circ)}{6^2}\right) \quad (\text{A1})$$

Note: This A1 can be awarded for $\cos \theta = \frac{6 \cos 120^\circ \times 6 \cos 57.3^\circ}{6^2}$.

$$= 106^\circ (= 105.673\dots^\circ) \text{ OR } 1.84 \text{ radians } (1.84434\dots) \quad (\text{A1})$$

OR (Cosine Rule)

$$|\text{BM}| = \sqrt{(6 \sin 120^\circ - 0)^2 + (6 \cos 120^\circ - 6 \sin 57.3^\circ)^2 + (0 - 6 \sin 57.3^\circ)^2}$$

$$= 9.56 (9.56299\dots) \text{ OR } 9560 \text{ km} \quad (\text{A1})$$

attempt to use the cosine rule to find angle BOM

(M1)

$$|\text{BM}|^2 = 6^2 + 6^2 - 2 \times 6 \times 6 \times \cos \text{BOM}$$

$$\text{BOM} = 106^\circ (= 105.673\dots^\circ) \text{ OR } 1.84 \text{ radians } (1.84434\dots) \quad (\text{A1})$$

continued...

Question 2 continued

THEN

attempt to use $\frac{\theta}{360} \times 2\pi r$ OR $s = r\theta$ (M1)

$$= \frac{105.671^\circ}{360} \times 2\pi \times 6$$

$$= 11.1 \text{ (11.0660...thousand km)} \quad \text{A1}$$

Note: Accept an answer of 11100 OR 11066.0... without units (i.e. “km” is implicit).

[5 marks]**METHOD 2** (Cross product)

attempt to use cross product to find $\hat{BOM} = \arcsin \frac{|b \times m|}{|b||m|}$ (M1)

Note: This may be written as $\sin \hat{BOM} = \frac{|b \times m|}{|b||m|}$.

$$= \frac{\begin{vmatrix} 36 \cos 120 \sin \theta \\ -36 \sin 120 \sin \theta \\ 36 \sin 120 \cos \theta \end{vmatrix}}{6 \times 6}$$

$$= \frac{\sqrt{\sin^2 \theta + 3}}{2} = 0.962818... \quad \text{(A1)}$$

$$\hat{BOM} = 180 - \arcsin 0.962818...$$

$$= 106^\circ (105.673..., 1.844346 \text{rads}) \quad \text{(A1)}$$

attempt to use $\frac{\theta}{360} \times 2\pi r$ OR $s = r\theta$ (M1)

$$= \frac{105.671}{360} \times 2\pi \times 6$$

$$= 11.1 \text{ (11.0660 thousand km)} \quad \text{A1}$$

Note: Award at most (M1)(A1)(A0)(M1)A0 for an answer of 7.78 thousand km from finding the principal root of the arcsin.

[5 marks]

continued...

Question 2 continued

(f) attempt to calculate $\mathbf{b} \times \mathbf{m}$ (M1)

$$\mathbf{b} \times \mathbf{m} = \begin{pmatrix} 36 \cos 120^\circ \sin 57.3^\circ \\ -36 \sin 120^\circ \sin 57.3^\circ \\ 36 \sin 120^\circ \cos 57.3^\circ \end{pmatrix} = \begin{pmatrix} -15.1464... \\ -26.2344... \\ 16.8449... \end{pmatrix} \quad (\text{A1})$$

attempt to use scalar product to find angle between vectors (M1)

$$(\cos \theta) = \frac{(\mathbf{b} \times \mathbf{m}) \cdot (\mathbf{b} \times \mathbf{p})}{|\mathbf{b} \times \mathbf{m}| |\mathbf{b} \times \mathbf{p}|}$$

$$(\cos \theta) = \left(\frac{1090.54...}{34.6614... \times 36} \right) \quad (\text{A1})(\text{A1})$$

Note: Award (A1) for numerator, (A1) for denominator.

bearing is $(0)29.1^\circ (29.0770...^\circ)$ A1

Note: Do not penalise absence of degree symbol.

Award full marks for answer in radians = 0.507.

Award full marks for 029° .

Special Case: If a candidate calculates $\mathbf{m} \times \mathbf{b}$ instead of $\mathbf{b} \times \mathbf{m}$ then you can award at most **M1A0M1A1A1A0**.

[6 marks]

Total [27 marks]

1. (a) (i) $y = 5$ **A1A1****Note:** Award **A1** for $y =$ a constant, **A1** for 5.**[2 marks]**

(ii) equating the correct lines

(M1)

$$5 = 15 - 2x \text{ OR sketch graph}$$

$$x = 5 \ y = 5 \text{ OR } (5, 5)$$

A1**[2 marks]**

(b) station B

A1**[1 mark]**

(c) nearest neighbour (interpolation)

A1**[1 mark]**(d) **EITHER**

because the school is quite close to the other two stations also and their readings are much higher.

R1**OR**

the reading of 49 is only an average, and hence likely that readings are frequently over 50 at the school.

R1**Note:** Ignore any extra invalid reasons.**[1 mark]**

(e) (i) attempt to find a midpoint

(M1)

$$\left(\frac{5+8}{2}, \frac{6+9}{2} \right)$$

$$(6.5, 7.5)$$

A1**[2 marks]***continued...*

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Question 1 continued

(ii) gradient is $\frac{9-6}{8-5} = 1$ **A1**

gradient of perpendicular bisector is -1 **A1**

substituting their gradient and their mid-point into the equation of a line **M1**

$$y - 7.5 = -1(x - 6.5) \text{ OR } 7.5 = -1 \times 6.5 + c$$

$$y = 14 - x$$
 AG

Note: : If **A1A1M1** have been awarded but there is then incorrect work after the final **M1** such that the **AG** does not follow, do not award the final **M1**.

[3 marks]

(iii) attempt to find where this line meets the other two lines **(M1)**

$(9,5)$ **A1**

$$14 - x = 15 - 2x$$

$(1,13)$ **A1**

Note: These two marks are un-implicit due to the “hence” in the question. At least one needs to be seen for the **(M1)** to be awarded.

$$\text{Area} = \frac{1}{2} \times 4 \times 8$$
 (M1)

$$= 16 (\text{km}^2)$$
 A1

Note: Award **(M1)** for any valid method which is completed (for example cosine rule, followed by area of triangle formula or using one of the other sides as base and correctly finding the perpendicular height) which leads to an answer, even if the answer is not correct due to a calculation error.

(the side lengths of the triangle are 4, 11.3, 8.94 and the angles 45° , 18.4° , 116.6°)

Note: Award **(M0)A0** for an unsupported answer of 16.

[5 marks]*continued...*

Question 1 continued

- (f) attempt to substitute the values into given formula

M1

$$W = \frac{132 \times 13.7 + 49 \times 16 + 125 \times 6.9}{16 + 13.7 + 6.9}$$

A1

$$W = 94.4(94.3961\dots)$$

AG**[2 marks]**

- (g)
- $H_0 : \mu = 94.4, H_1 : \mu > 94.4$

A1A1

Note: Allow the hypotheses in words, but award **A0A0** if 'mean' is not seen and, at most **A1A0** if it is not clear that the **population** mean is being tested

Accept $\mu \leq 94.4$ for the null hypothesis.

use of t -test**(M1)**

$$p\text{-value} = 0.0655 \text{ (0.0655348\dots)}$$

A1

$$0.0655 < 0.10$$

R1

(hence reject the null hypothesis or accept the alternative hypothesis)

(there is significant evidence that) the (mean) AQI value (at the school) is greater than 94.4

A1

Note: Do not award **R0A1**.

Follow through from an incorrect p -value to award the last two marks,

Award **A0** for the conclusion if it is not in context or if the null hypothesis is incorrect, though condone the errors in language detailed in the first note above.

[6 marks]

- (h)
- $P(X < 50) = 0.00273(0.00272574\dots)$

(A1)

$$P(X < 50) \times 365$$

(M1)

$$= 0.994895\dots$$

$$= 1$$

A1**[3 marks]****Total [28 marks]**

4. (a) attempt to use cosine rule (M1)
 $\cos\theta = \frac{15^2 + 10^2 - 8^2}{2 \times 15 \times 10}$ (A1)
 29.5(°) **OR** 0.516 (29.5413..., 0.515594...) A1
[3 marks]

- (b) attempt to find the area of triangle ABC (substitution into the formula must be attempted) (M1)
 $\frac{1}{2} \times 15 \times 10 \times \sin(29.5413 \dots^\circ)$ (A1)
 (= 36.9788...)

attempt to find the area of sector ABD (substitution into the formula must be attempted) (M1)
 $\frac{29.5413 \dots}{360} \times \pi \times 10^2$ **OR** $\frac{1}{2} \times 10^2 \times 0.515594 \dots$ (A1)
 (= 25.7797...)

(36.9788... – 25.7797...)
 11.2 cm² (11.1991...)

A1

Note: Correct units are required to award the final A1 .
--

[5 marks]
Total [8 marks]

- R1**
[2 marks]

A1A1

[2 marks]

- A1**

Accept the use of the matrix for the subgraph with vertices ABCD, which is $\begin{pmatrix} 0 & 2 & 0 & 1 \\ 2 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{pmatrix}$.

Total [8 marks]

$\sqrt{7}$

13. (a) attempt to find $\begin{vmatrix} -24 \\ 0 \end{vmatrix}$ (=25) (M1)

attempt to find scale factor (M1)

$$\frac{800}{25} (=32)$$

$$(r_2 =) \begin{pmatrix} -13 \\ 50 \\ 7.2 \end{pmatrix} + 32t \begin{pmatrix} 7 \\ -24 \\ 0 \end{pmatrix} \text{ OR } \begin{pmatrix} -13 \\ 50 \\ 7.2 \end{pmatrix} + t \begin{pmatrix} 224 \\ -768 \\ 0 \end{pmatrix} \quad \text{A1}$$

[3 marks]

(b) attempt to find expression for relative position ($r_1 - r_2$) (M1)

$$\begin{pmatrix} 42 \\ -82 \\ -1.4 \end{pmatrix} + t \begin{pmatrix} -544 \\ 1368 \\ 0.5 \end{pmatrix} \quad \text{(A1)}$$

attempt to find magnitude (or square of magnitude) for their relative position (M1)

$$\sqrt{(42 - 544t)^2 + (-82 + 1368t)^2 + (-1.4 + 0.5t)^2}$$

attempt to find minimum value (M1)

e.g. sketch with minimum indicated OR attempt to solve derivative = 0

OR $t = 0.0622991\dots$

8.83 (kilometres) (8.83380...) A1

Note: If $r_2 = \begin{pmatrix} -13 \\ 50 \\ 7.2 \end{pmatrix} + t \begin{pmatrix} 7 \\ -24 \\ 0 \end{pmatrix}$, $t = 0.131$ and least distance = 1.56.

[5 marks]

Total [8 marks]

1. (a) ABCDEFA (A1)

Note: Accept any starting point and either direction.

Accept a sum of weights rather than the letters as evidence of finding a Hamiltonian cycle $8+7+9+6+12+5$

Condone ABCDEF seen for (A1).

47 (minutes)

A1
[2 marks]

- (b) (i) $(a=)12$

A1

[1 mark]

- (ii) $(b=)17$

A1

[1 mark]

- (c) ADEFBCA

(M1)(A1)

Note: Award (M1) for ADEF seen (or 4, 6, 11).

weight = $(4+6+11+13+7+12=)53$ (minutes)

A1
[3 marks]

- (d) (i) attempt to use Prim's to find minimum spanning tree

(M1)

Note: Award M1 if first two edges seen.

BC, CD, DE, DF

A1

weight of MST $(= 7+9+6+9) = 31$

A1

Note: If using the original graph instead of the table then edges are BC, CD, DE, EF. Award (M1)A1A0 for total weight 34 seen.

[3 marks]

Note: If vertices BCDEF or weights 7, 9, 6, 9 are given, rather than edges, award (M1)A0A1 if a total of 31 is given, and (M0)A0A0 if a different total (eg 33) is given.

- (ii) reconnect A with the two shortest edges $4+5=9$
40 (minutes)

(M1)
A1

Note: Follow through from errors in part (b).

[2 marks]

continued...

Question 1 continued

(e) (i) 40

A1

Note: Follow through from (d) (ii). Correct **FT** answer is their (d)(ii) if greater than 40, 39 if less than 40.

(ii) 47

[1 mark]

A1

Note: Follow through: the smallest of their answers to (a) and (c).

[1 mark]

(f) (i) $(6 + 6 + 9 + 7 + 13 + 5 =) 46$ (minutes)

A1

(ii) AF

[1 mark]

A1

[1 mark]

(g) (i) let Q be the total time in queues
 $Q \sim N(80, 47)$

A1A1A1

Note: Award **A1** for 'N', **A1** for 80 and **A1** for 47.

[3 marks]

(ii) attempt to find the total time available for queuing

(M1)

time on rides = 10 (minutes), (walking 46 minutes)

(A1)

EITHER(so time in queues needs be less than) $(150 - 46 - 10 =)$

94 (minutes)

(A1)

 $(P(Q < 94) =) 0.979(0.979429...)$

A1

ORtotal time has distribution $T \sim N(136, 47)$

(A1)

 $(P(T < 150) =) 0.979(0.979429...)$

A1

[4 marks]

continued...

Question 1 continued

- (h) total time walking = 46 (minutes), time on rides is $(10 + 2 =) 12$ (minutes), expected times waiting for the rides is $(80 + 10 =) 90$ (minutes) **(M1)**

Note: Award **(M1)** for any two of the three correct values seen.

total time = 148 minutes **(A1)**

so 11:28 am **A1**

Note: Condone use of $E(2E_t)$ if correct answer seen.

[3 marks]
Total [26 marks]

