

Mathematics: analysis and approaches
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

--	--	--	--	--	--	--	--	--	--

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the points $A(-2, 20)$, $B(4, 6)$ and $C(-14, 12)$. The line L passes through the point A and is perpendicular to $[BC]$.

(a) Find the equation of L . [3]

The line L passes through the point $(k, 2)$.

(b) Find the value of k . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



2. [Maximum mark: 5]

(a) The expression $\frac{3\sqrt{x}-5}{\sqrt{x}}$ can be written as $3 - 5x^p$. Write down the value of p . [1]

(b) Hence, find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}} \right) dx$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

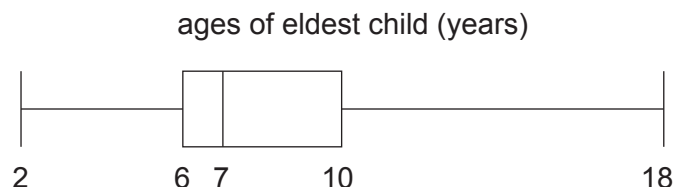


3. [Maximum mark: 7]

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.

diagram not to scale



- (a) Find the largest value of c that would not be considered an outlier. [3]

The regression line of a on c is $a = \frac{7}{4}c + 20$. The regression line of c on a is $c = \frac{1}{2}a - 9$.

- (b) (i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child.
- (ii) Find the mean age of all the adults surveyed.

[illegible]

4. [Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3} \sin x + \cos x$ where $0 \leq x \leq \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

(a) Find $(f \circ g)(x)$. [2]

(b) Solve the equation $(f \circ g)(x) = 2 \cos 2x$ where $0 \leq x \leq \pi$. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



5. [Maximum mark: 5]

Consider the curve with equation $y = (2x - 1)e^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$.

The tangent to the curve at the point where $x = 1$ is parallel to the line $y = 5e^kx$.

Find the value of k .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



6. [Maximum mark: 7]

Consider $f(x) = 4 \sin x + 2.5$ and $g(x) = 4 \sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f .

(a) Describe these two transformations.

[2]

The y -intercept of the graph of g is at $(0, r)$.

(b) Given that $g(x) \geq 7$, find the smallest value of r .

[5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....



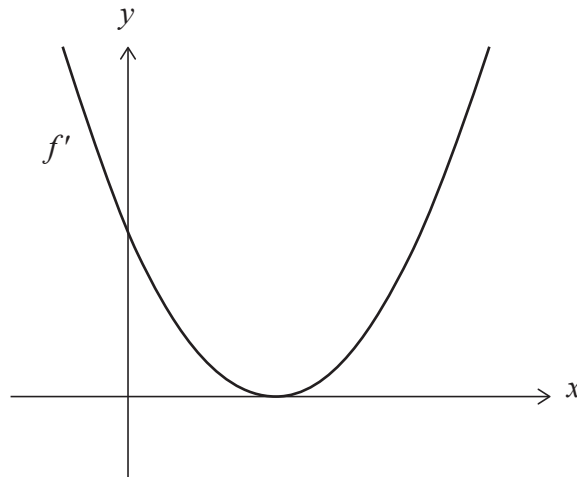
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A function, f , has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f' .



The graph of f' has an axis of symmetry $x = q$.

(a) Find the value of q . [2]

The vertex of the graph of f' lies on the x -axis.

(b) (i) Write down the value of the discriminant of f' .
(ii) Hence or otherwise, find the value of p . [4]

(c) Find the value of the gradient of the graph of f' at $x = 0$. [3]

(d) Sketch the graph of f'' , the second derivative of f . Indicate clearly the x -intercept and the y -intercept. [2]

The graph of f has a point of inflexion at $x = a$.

(e) (i) Write down the value of a .
(ii) Find the values of x for which the graph of f is concave-down. Justify your answer. [3]



Do **not** write solutions on this page.

8. [Maximum mark: 15]

Consider the series $\ln x + p \ln x + \frac{1}{3} \ln x + \dots$, where $x \in \mathbb{R}$, $x > 1$ and $p \in \mathbb{R}$, $p \neq 0$.

(a) Consider the case where the series is geometric.

(i) Show that $p = \pm \frac{1}{\sqrt{3}}$.

(ii) Given that $p > 0$ and $S_{\infty} = 3 + \sqrt{3}$, find the value of x . [5]

(b) Now consider the case where the series is arithmetic with common difference d .

(i) Show that $p = \frac{2}{3}$.

(ii) Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$.

(iii) The sum of the first n terms of the series is $-3 \ln x$.

Find the value of n . [10]

9. [Maximum mark: 15]

(a) (i) Expand and simplify $(1 - a)^3$ in ascending powers of a .

(ii) By using a suitable substitution for a , show that $1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x = 8 \sin^6 x$. [6]

Consider $f(x) = 4 \cos x (1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x)$.

(b) (i) Show that $\int_0^m f(x) dx = \frac{32}{7} \sin^7 m$, where m is a positive real constant.

(ii) It is given that $\int_m^{\frac{\pi}{2}} f(x) dx = \frac{127}{28}$, where $0 \leq m \leq \frac{\pi}{2}$. Find the value of m . [9]

References:

© International Baccalaureate Organization 2022



Please **do not** write on this page.

Answers written on this page
will not be marked.



12EP10

Please **do not** write on this page.

Answers written on this page
will not be marked.



12EP11

Please **do not** write on this page.

Answers written on this page
will not be marked.



12EP12