

APOLLO

Number and algebra

HL

Mathematics: analysis and approaches

First assessment 2021

102 questions | mark schemes

2 METHOD 1 (finding u_1 first, from S_8)

$$4(u_1 + 8) = 8 \quad (A1)$$

$$u_1 = -6 \quad A1$$

$$u_1 + 7d = 8 \text{ OR } 4(2u_1 + 7d) = 8 \text{ (may be seen with their value of } u_1) \quad (A1)$$

$$\text{attempt to substitute their } u_1 \quad (M1)$$

$$d = 2 \quad A1$$

METHOD 2 (solving simultaneously)

$$u_1 + 7d = 8 \quad (A1)$$

$$4(u_1 + 8) = 8 \text{ OR } 4(2u_1 + 7d) = 8 \text{ OR } u_1 = -3d \quad (A1)$$

$$\text{attempt to solve linear or simultaneous equations} \quad (M1)$$

$$u_1 = -6, d = 2 \quad A1A1$$

[5 marks]

7. METHOD 1

other two roots are $a - bi$ and $b - ai$ **A1**

sum of roots = -4 and product of roots = 400 **A1**

attempt to set sum of four roots equal to -4 or 4 OR

attempt to set product of four roots equal to 400 **M1**

$$a + bi + a - bi + b + ai + b - ai = -4$$

$$2a + 2b = -4 (\Rightarrow a + b = -2) \quad \textbf{A1}$$

$$(a + bi)(a - bi)(b + ai)(b - ai) = 400$$

$$(a^2 + b^2)^2 = 400 \quad \textbf{A1}$$

$$a^2 + b^2 = 20$$

attempt to solve simultaneous equations **(M1)**

$$a = 2 \text{ or } a = -4 \quad \textbf{A1A1}$$

[8 marks]

METHOD 2

other two roots are $a - bi$ and $b - ai$ **A1**

$$(z - (a + bi))(z - (a - bi))(z - (b + ai))(z - (b - ai)) = 0 \quad \textbf{A1}$$

$$((z - a)^2 + b^2)((z - b)^2 + a^2) = 0$$

$$(z^2 - 2az + a^2 + b^2)(z^2 - 2bz + b^2 + a^2) = 0 \quad \textbf{A1}$$

Attempt to equate coefficient of z^3 and constant with the given quartic equation **M1**

$$-2a - 2b = 4 \text{ and } (a^2 + b^2)^2 = 400 \quad \textbf{A1}$$

attempt to solve simultaneous equations **(M1)**

$$a = 2 \text{ or } a = -4 \quad \textbf{A1A1}$$

[8 marks]

9. (a) **METHOD 1**
- B has one less pen to select (M1)
- EITHER**
- A and B can be placed in 6×5 ways (A1)
- C, D, E have 6 choices each (A1)
- OR**
- A (or B), C, D, E have 6 choices each (A1)
- B (or A) has only 5 choices (A1)
- THEN**
- $5 \times 6^4 (= 6480)$ A1

METHOD 2

total number of ways $= 6^5$ (A1)

number of ways with Amber and Brownie together $= 6^4$ (A1)

attempt to subtract (may be seen in words) (M1)

$6^5 - 6^4$

$= 5 \times 6^4 (= 6480)$ A1

[4 marks]

- (b) **METHOD 1**
- total number of ways $= 6! (= 720)$ (A1)
- number of ways with Amber and Brownie sharing a boundary
- $= 2 \times 7 \times 4! (= 336)$ (A1)
- attempt to subtract (may be seen in words) (M1)
- $720 - 336 = 384$ A1

METHOD 2

case 1: number of ways of placing A in corner pen

$3 \times 4 \times 3 \times 2 \times 1$

Four corners total no of ways is $4 \times (3 \times 4 \times 3 \times 2 \times 1) = 12 \times 4! (= 288)$ (A1)

case 2: number of ways of placing A in the middle pen

$2 \times 4 \times 3 \times 2 \times 1$

two middle pens so $2 \times (2 \times 4 \times 3 \times 2 \times 1) = 4 \times 4! (= 96)$ (A1)

attempt to add (may be seen in words) (M1)

total no of ways $= 288 + 96$

$= 16 \times 4! (= 384)$ A1

[4 marks]

Total [8 marks]

Section B

1. attempt to subtract squares of integers

(M1)

$$(n+1)^2 - n^2$$

EITHER

correct order of subtraction and correct expansion of $(n+1)^2$, seen anywhere **A1A1**

$$= n^2 + 2n + 1 - n^2 (= 2n + 1)$$

OR

correct order of subtraction and correct factorization of difference of squares **A1A1**

$$= (n+1-n)(n+1+n) (= 2n+1)$$

THEN

$$= n + n + 1 = \text{RHS}$$

A1

Note: Do not award final **A1** unless all previous working is correct.

which is the sum of n and $n+1$

AG

Note: If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n+1$ and then show that the difference of the squares (subtracted in the correct order) is $2n+1$.

[4 marks]

3. EITHER

attempt to use the binomial expansion of $(x+k)^7$ **(M1)**

$${}^7C_0x^7k^0 + {}^7C_1x^6k^1 + {}^7C_2x^5k^2 + \dots \text{ (or } {}^7C_0k^7x^0 + {}^7C_1k^5x^1 + {}^7C_2k^5x^2 + \dots)$$

identifying the correct term ${}^7C_2x^5k^2$ (or ${}^7C_5k^2x^5$) **(A1)**

OR

attempt to use the general term ${}^7C_r x^r k^{7-r}$ (or ${}^7C_r k^r x^{7-r}$) **(M1)**

$$r = 2 \text{ (or } r = 5) \quad \text{ **(A1)**}$$

THEN

$${}^7C_2 = 21 \text{ (or } {}^7C_5 = 21) \text{ (seen anywhere)} \quad \text{ **(A1)**}$$

$$21x^5k^2 = 63x^5 \text{ (} 21k^2 = 63, k^2 = 3) \quad \text{ **A1**}$$

$$k = \pm\sqrt{3} \quad \text{ **A1**}$$

Note: If working shown, award **M1A1A1A1A0** for $k = \sqrt{3}$.

[5 marks]

5. METHOD 1

product of a binomial coefficient, a power of 3 (and a power of x^2) seen
evidence of correct term chosen

(M1)
(A1)

$${}^{n+1}C_2 \times 3^{n+1-2} \times (x^2)^2 \left(= \frac{n(n+1)}{2} \times 3^{n-1} \times x^4 \right) \text{ OR } n-r=1$$

equating their coefficient to 20412 or their term to $20412x^4$

(M1)

EITHER

$${}^{n+1}C_2 \times 3^{n-1} = 20412$$

(A1)

OR

$${}^{r+2}C_r \times 3^r = 20412 \Rightarrow r = 6$$

(A1)

THEN

$$n = 7$$

A1

METHOD 2

$$3^{n+1} \left(1 + \frac{x^2}{3} \right)^{n+1}$$

product of a binomial coefficient and a power of $\frac{x^2}{3}$ OR $\frac{1}{3}$ seen

(M1)

evidence of correct term chosen

(A1)

$$3^{n+1} \times {}^{n+1}C_2 \times \left(\frac{x^2}{3} \right)^2 \left(= 3^{n-1} \frac{n(n+1)}{2} x^4 \right)$$

equating their coefficient to 20412 or their term to $20412x^4$

(M1)

$$3^{n-1} \times \frac{n(n+1)}{2} = 20412$$

(A1)

$$n = 7$$

A1

Total [5 marks]

8. (a) $(|zw| =) 16$

A1**[1 mark]**

(b) attempt to find $\arg(z) + \arg(w)$

(M1)

$$\arg(zw) = \arg(z) + \arg(w)$$

$$= \frac{\pi}{5} - \frac{2k\pi}{5} \left(= \frac{(1-2k)\pi}{5} \right)$$

A1**[2 marks]***continued...*

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Question 8 continued

(c) (i) $zw \in Z \Rightarrow \arg(zw)$ is a multiple of π

(M1)

$$\Rightarrow 1 - 2k \text{ is a multiple of } 5$$

(M1)

$$k = 3$$

A1

(ii) $zw = 16(\cos(-\pi) + i \sin(-\pi))$

$$-16$$

A1**[4 marks]****Total [7 marks]**

2. (a) attempt to use $u_1 + (n-1)d = 0$ (M1)

$$60 - 2.5(k-1) = 0$$

$$k = 25$$

A1

[2 marks]

- (b) **METHOD 1**

attempting to express S_n in terms of n (M1)

use of a graph or a table to attempt to find the maximum sum (M1)

$$= 750$$

A1

METHOD 2

EITHER

recognizing maximum occurs at $n = 25$ (M1)

$$S_{25} = \frac{25}{2}(60 + 0), S_{25} = \frac{25}{2}(2 \times 60 + 24 \times -2.5) \quad (A1)$$

OR

attempting to calculate S_{24} (M1)

$$S_{24} = \frac{24}{2}(2 \times 60 + 23 \times -2.5) \quad (A1)$$

THEN

$$= 750$$

A1

[3 marks]

Total [5 marks]

7. (a) Jack and Andrea finish in that order (as a unit) so we are considering the arrangement of 7 objects (M1)

$$7! (= 5040) \text{ ways} \quad \text{A1}$$

[2 marks]

- (b) **METHOD 1**

the number of ways that Andrea finishes in front of Jack is equal to the number of ways that Jack finishes in front of Andrea (M1)

total number of ways is $8!$ (A1)

$$\frac{8!}{2} (= 20160) \text{ ways} \quad \text{A1}$$

[3 marks]

METHOD 2

the other six runners can finish in $6!$ ($= 720$) ways (A1)

when Andrea finishes first, Jack can finish in 7 different positions

when Andrea finishes second, Jack can finish in 6 different positions etc

$$7 + 6 + 5 + 4 + 3 + 2 + 1 (= 28) \text{ ways} \quad \text{(A1)}$$

hence there are $(7 + 6 + 5 + 4 + 3 + 2 + 1) \times 6!$ ways

$$28 \times 6! (= 20160) \text{ ways} \quad \text{A1}$$

[3 marks]

Total [5 marks]

8. $\frac{1+z}{1-z} = \frac{1+\cos\theta + i\sin\theta}{1-\cos\theta - i\sin\theta}$

attempt to use the complex conjugate of their denominator

M1

$$= \frac{(1+\cos\theta + i\sin\theta)(1-\cos\theta + i\sin\theta)}{(1-\cos\theta - i\sin\theta)(1-\cos\theta + i\sin\theta)}$$

A1

$$\operatorname{Re}\left(\frac{1+z}{1-z}\right) = \frac{1-\cos^2\theta - \sin^2\theta}{(1-\cos\theta)^2 + \sin^2\theta} \left(= \frac{1-\cos^2\theta - \sin^2\theta}{2-2\cos\theta} \right)$$

M1A1

Note: Award **M1** for expanding the numerator and **A1** for a correct numerator. Condone either an incorrect denominator or the absence of a denominator.

using $\cos^2\theta + \sin^2\theta = 1$ to simplify the numerator

(M1)

$$\operatorname{Re}\left(\frac{1+z}{1-z}\right) = 0$$

AG

[5 marks]

2. (a) (i) METHOD 1attempts to expand $(\omega - 1)(\omega^2 + \omega + 1)$ **(M1)**

$$= \omega^3 + \omega^2 + \omega - \omega^2 - \omega - 1$$
 A1

$$= \omega^3 - 1$$
 AG

[2 marks]**METHOD 2**attempts polynomial division on $\frac{\omega^3 - 1}{\omega - 1}$ **M1**

$$= \omega^2 + \omega + 1$$
 A1

$$\text{so } (\omega - 1)(\omega^2 + \omega + 1) = \omega^3 - 1$$
 AG

[2 marks]

$$(ii) \quad (\text{since } \omega \text{ is a root of } z^3 = 1) \Rightarrow \omega^3 - 1 = 0$$
 R1

$$\text{and } \omega \neq 1$$
 R1

$$\Rightarrow \omega^2 + \omega + 1 = 0$$
 AG

Note: In part (a), award marks as appropriate where ω has been converted into Cartesian, modulus-argument (polar) or Euler form.**[2 marks]**

(b) METHOD 1attempts to find either P_0P_1 or P_0P_2 **(M1)**

accept any valid method

$$\text{e.g. } 2\sin\frac{\pi}{3}, 1^2 + 1^2 - 2\cos\frac{2\pi}{3}, \frac{1}{\sin\frac{\pi}{6}} = \frac{P_0P_1}{\sin\frac{2\pi}{3}} \text{ from either } \triangle OP_0P_1 \text{ or } \triangle OP_0P_2$$

e.g. use of Pythagoras' theorem

$$\text{e.g. } \left|1 - e^{i\frac{2\pi}{3}}\right|, \left|1 - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right| \text{ by calculating the distance between 2 points}$$

$$P_0P_1 = \sqrt{3}$$

A1

$$P_0P_2 = \sqrt{3}$$

A1

Note: Award a maximum of **M1A1A0** for any decimal approximation seen in the calculation of either P_0P_1 or P_0P_2 or both.

$$\text{so } P_0P_1 \times P_0P_2 = 3$$

AG**METHOD 2**

$$\text{attempts to find } P_0P_1 \times P_0P_2 = |1 - \omega| |1 - \omega^2|$$

(M1)

$$P_0P_1 \times P_0P_2 = |\omega^3 - \omega^2 - \omega + 1|$$

A1

$$= |1 - (\omega^2 + \omega + 1) + 2| \text{ and since } \omega^2 + \omega + 1 = 0$$

R1

$$\text{so } P_0P_1 \times P_0P_2 = 3$$

AG**[3 marks]**

(c) METHOD 1

$$z^4 - 1 = (z - 1)(z^3 + z^2 + z + 1) \quad \text{A1}$$

$$(\omega \text{ is a root hence}) \omega^4 - 1 = 0 \text{ and } \omega \neq 1 \quad \text{R1}$$

$$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0 \quad \text{AG}$$

Note: Condone the use of ω throughout.

[2 marks]

METHOD 2

$$\text{considers the sum of roots of } z^4 - 1 = 0 \quad \text{(M1)}$$

$$\text{the sum of roots is zero (there is no } z^3 \text{ term)} \quad \text{A1}$$

$$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0 \quad \text{AG}$$

[2 marks]

METHOD 3

$$\text{substitutes for } \omega \quad \text{(M1)}$$

$$\text{e.g. LHS} = e^{i\frac{3\pi}{2}} + e^{i\pi} + e^{i\frac{\pi}{2}} + 1$$

$$= -i - 1 + i + 1 \quad \text{A1}$$

Note: This can be demonstrated geometrically or by using vectors.
Accept Cartesian or modulus-argument (polar) form.

$$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0 \quad \text{AG}$$

[2 marks]

METHOD 4

$$\omega^3 + \omega^2 + \omega + 1 = \frac{\omega^4 - 1}{\omega - 1} \quad \text{A1}$$

$$= \frac{0}{\omega - 1} = 0 \text{ as } \omega \neq 1 \quad \text{R1}$$

$$\Rightarrow \omega^3 + \omega^2 + \omega + 1 = 0 \quad \text{AG}$$

[2 marks]

(d) METHOD 1

$$P_0P_2 = 2$$

A1attempts to find either P_0P_1 or P_0P_3 **(M1)**

Note: For example, $P_0P_1 = |1-i|$ and $P_0P_3 = |1+i|$.

Various geometric and trigonometric approaches can be used by candidates.

$$P_0P_1 = \sqrt{2}, P_0P_3 = \sqrt{2}$$

A1A1

Note: Award a maximum of **A1M1A1A0** if labels such as P_0P_1 are not clearly shown.

Award full marks if the lengths are shown on a clearly labelled diagram.

Award a maximum of **A1M1A1A0** for any decimal approximation seen in the calculation of either P_0P_1 or P_0P_3 or both.

$$P_0P_1 \times P_0P_2 \times P_0P_3 = 4$$

AG**[4 marks]****METHOD 2**

$$\text{attempts to find } P_0P_1 \times P_0P_2 \times P_0P_3 = |1-\omega||1-\omega^2||1-\omega^3|$$

M1

$$P_0P_1 \times P_0P_2 \times P_0P_3 = |-\omega^6 + \omega^5 + \omega^4 - \omega^2 - \omega + 1|$$

A1

$$= | -(-1) + \omega^5 + 1 - (-1) - \omega + 1 | \text{ since } \omega^6 = \omega^2 = -1 \text{ and } \omega^4 = 1$$

A1

$$= | \omega^5 - \omega + 4 | \text{ and since } \omega^5 = \omega$$

R1

$$\text{so } P_0P_1 \times P_0P_2 \times P_0P_3 = 4$$

AG**[4 marks]**

METHOD 3

$$P_0P_2 = 2$$

A1

$$\text{attempts to find } P_0P_1 \times P_0P_3 = |1 - \omega| |1 - \omega^3|$$

M1

$$P_0P_1 \times P_0P_3 = |\omega^4 - \omega^3 - \omega + 1|$$

A1

$$= |2 - (-\omega) - \omega| \text{ since } \omega^4 = 1 \text{ and } \omega^3 = -\omega$$

R1

$$\text{so } P_0P_1 \times P_0P_2 \times P_0P_3 = 4$$

AG**[4 marks]**

(e) $(P_0P_1 \times P_0P_2 \times \dots \times P_0P_{n-1}) = n$

A1**[1 mark]**

(f) (i) $P_0P_2 = |1 - \omega^2|, P_0P_3 = |1 - \omega^3|$

A1A1**[2 marks]**

(ii) $P_0P_{n-1} = |1 - \omega^{n-1}|$

A1

Note: Accept $ 1 - \omega $ from symmetry.

[1 mark]

(g) (i) $z^n - 1 = (z - 1)(z^{n-1} + z^{n-2} + \dots + z + 1)$

considers the equation $z^{n-1} + z^{n-2} + \dots + z + 1 = 0$ **(M1)**

the roots are $\omega, \omega^2, \dots, \omega^{n-1}$ **(A1)**

so $(z - \omega)(z - \omega^2) \dots (z - \omega^{n-1})$ **A1**

[3 marks]

(ii) **METHOD 1**

substitutes $z = 1$ into $(z - \omega)(z - \omega^2) \dots (z - \omega^{n-1}) \equiv z^{n-1} + z^{n-2} + \dots + z + 1$

M1

$$(1 - \omega)(1 - \omega^2) \dots (1 - \omega^{n-1}) = n$$
(A1)

takes modulus of both sides **M1**

$$|(1 - \omega)(1 - \omega^2) \dots (1 - \omega^{n-1})| = |n|$$

$$|1 - \omega||1 - \omega^2| \dots |1 - \omega^{n-1}| = n$$
A1

$$\text{so } P_0 P_1 \times P_0 P_2 \times \dots \times P_0 P_{n-1} = n$$
AG

Note: Award a maximum of **M1A1FTM1A0** from part (e).

[4 marks]

METHOD 2

$(1-\omega), (1-\omega^2), \dots, (1-\omega^{n-1})$ are the roots of $(1-v)^{n-1} + (1-v)^{n-2} + \dots + (1-v) + 1 = 0$

M1

coefficient of v^{n-1} is $(-1)^{n-1}$ and the coefficient of 1 is n

A1

product of the roots is $\frac{(-1)^{n-1} n}{(-1)^{n-1}} = n$

A1

$$|1-\omega||1-\omega^2|\dots|1-\omega^{n-1}| = n$$

A1

$$\text{so } P_0 P_1 \times P_0 P_2 \times \dots \times P_0 P_{n-1} = n$$

AG**[4 marks]****Total[24 marks]**

9. (a) attempt to expand binomial with negative fractional power (M1)

$$\frac{1}{\sqrt{1+ax}} = (1+ax)^{-\frac{1}{2}} = 1 - \frac{ax}{2} + \frac{3a^2x^2}{8} + \dots$$
A1

$$\sqrt{1-x} = (1-x)^{\frac{1}{2}} = 1 - \frac{x}{2} - \frac{x^2}{8} + \dots$$
A1

$$\frac{1}{\sqrt{1+ax}} - \sqrt{1-x} = \frac{(1-a)}{2}x + \left(\frac{3a^2+1}{8}\right)x^2 + \dots$$

attempt to equate coefficients of x or x^2 (M1)

$$x : \frac{1-a}{2} = 4b; \quad x^2 : \frac{3a^2+1}{8} = b$$

attempt to solve simultaneously (M1)

$$a = -\frac{1}{3}, b = \frac{1}{6}$$
A1

[6 marks]

- (b) $|x| < 1$ A1

[1 mark]

Total [7 marks]

Section B

$$12. \quad (a) \quad (i) \quad \left(1 + e^{i\frac{\pi}{6}} - 1\right)$$

$$= \left(e^{i\frac{\pi}{6}}\right)^3$$

A1

$$= e^{i\frac{\pi}{2}}$$

A1

$$= \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$$

$$= i$$

AG

Note: Candidates who solve the equation correctly can be awarded the above two marks. The working for part (i) may be seen in part (ii).

$$(ii) \quad (z-1)^3 = e^{i\left(\frac{\pi}{2} + 2\pi k\right)}$$

(M1)

$$z-1 = e^{i\left(\frac{\pi}{6} + \frac{4\pi k}{6}\right)}$$

(M1)

$$(k=1) \Rightarrow \omega_2 = 1 + e^{i\frac{5\pi}{6}}$$

A1

$$(k=2) \Rightarrow \omega_3 = 1 + e^{i\frac{9\pi}{6}}$$

A1**[6 marks]***continue...*

Question 12 continued.

(b) **EITHER**

attempt to express $e^{i\frac{\pi}{6}}$, $e^{i\frac{5\pi}{6}}$, $e^{i\frac{9\pi}{6}}$ in Cartesian form and translate 1 unit in the positive direction of the real axis

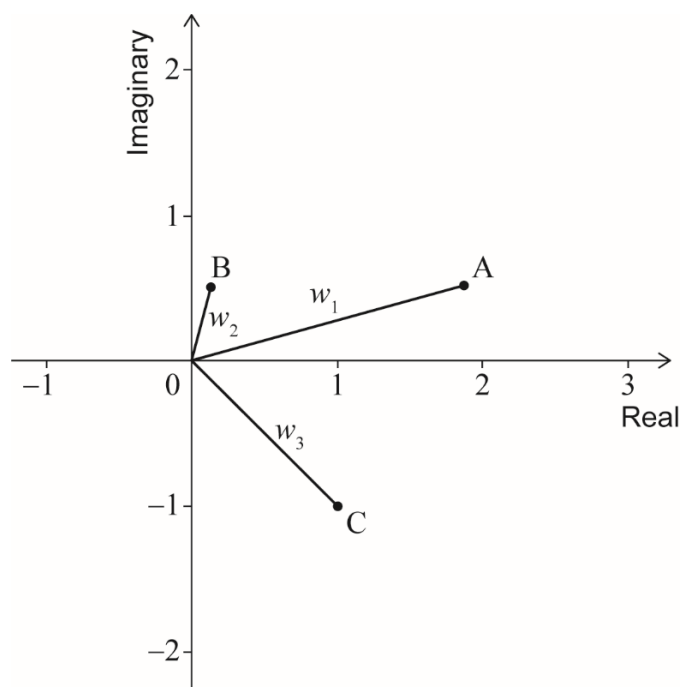
(M1)

OR

attempt to express w_1, w_2 and w_3 in Cartesian form

(M1)

THEN



Note: To award **A** marks, it is not necessary to see A, B or C, the w_i , or the solid lines

A1A1A1

[4 marks]

continue...

Question 12 continued.

(c) valid attempt to find $\omega_1 - \omega_3$ (or $\omega_3 - \omega_1$)

M1

$$\omega_1 - \omega_3 = \left(1 + \frac{\sqrt{3}}{2} + \frac{1}{2}i\right) - (1 - i) = \frac{\sqrt{3}}{2} + \frac{3}{2}i \quad \text{OR} \quad \cos\frac{\pi}{6} + i\sin\frac{\pi}{6} + i\sin\frac{\pi}{2}$$

valid attempt to find $\left|\frac{\sqrt{3}}{2} + \frac{3}{2}i\right|$

M1

$$= \sqrt{\frac{3}{4} + \frac{9}{4}}$$

$$AC = \sqrt{3}$$

A1

[3 marks]

continue...

Question 12 continued.

(d) **METHOD 1**

$$(z-1)^3 = iz^3 \Rightarrow \left(\frac{z-1}{z}\right)^3 = i \quad \textbf{M1}$$

$$\left(\frac{z-1}{z}\right)^3 = e^{i\frac{\pi}{2}} \quad \textbf{A1}$$

$$\frac{\alpha-1}{\alpha} = e^{i\frac{\pi}{6}} \quad \textbf{A1}$$

Note: This step to change from z to α may occur at any point in MS.

$$\alpha-1 = \alpha e^{i\frac{\pi}{6}}$$

$$\alpha - \alpha e^{i\frac{\pi}{6}} = 1$$

$$\alpha \left(1 - e^{i\frac{\pi}{6}}\right) = 1$$

$$\alpha = \frac{1}{1 - e^{i\frac{\pi}{6}}} \quad \textbf{AG}$$

continue...

Question 12 continued.

METHOD 2

$$(z-1)^3 = iz^3 \Rightarrow \left(\frac{z-1}{z}\right)^3 = i \quad \textbf{M1}$$

$$\left(1 - \frac{1}{z}\right)^3 = e^{i\frac{\pi}{2}} \quad \textbf{A1}$$

$$1 - \frac{1}{z} = e^{i\frac{\pi}{6}} \quad \textbf{A1}$$

Note: This step to change from z to α may occur at any point in MS.

$$1 - e^{i\frac{\pi}{6}} = \frac{1}{\alpha}$$

$$\alpha = \frac{1}{1 - e^{i\frac{\pi}{6}}} \quad \textbf{AG}$$

continue...

Question 12 continued.

METHOD 3

$$\begin{aligned}\text{LHS} &= (z-1)^3 = \left(\frac{1}{1 - e^{i\frac{\pi}{6}}} - 1 \right)^3 \\ &= \left(\frac{e^{i\frac{\pi}{6}}}{1 - e^{i\frac{\pi}{6}}} \right)^3 \\ &= \frac{i}{\left(1 - e^{i\frac{\pi}{6}}\right)^3} = \frac{i}{\left(\frac{5}{2} - \frac{3\sqrt{3}}{2} + i\left(\frac{3\sqrt{3}}{2} - \frac{5}{2}\right)\right)}\end{aligned}$$

M1A1

Note: Award **M1** for applying de Moivre's theorem (may be seen in modulus- argument form.)

$$\begin{aligned}\text{RHS} &= iz^3 = i \left(\frac{1}{1 - e^{i\frac{\pi}{6}}} \right)^3 \\ &= \frac{i}{\left(1 - e^{i\frac{\pi}{6}}\right)^3} \\ (z-1)^3 &= iz^3\end{aligned}$$

A1

AG

continue...

Question 12 continued.

METHOD 4

$$(z-1)^3 = iz^3$$

$$z^3 - 3z^2 + 3z - 1 = iz^3$$

$$(1-i)z^3 - 3z^2 + 3z - 1 = 0 \quad \textbf{(M1)}$$

$$\begin{aligned} (1-i) \left(\frac{1}{1-e^{i\frac{\pi}{6}}} \right)^3 - 3 \left(\frac{1}{1-e^{i\frac{\pi}{6}}} \right)^2 + 3 \left(\frac{1}{1-e^{i\frac{\pi}{6}}} \right) - 1 \\ = (1-i) - 3 \left(1-e^{i\frac{\pi}{6}} \right) + 3 \left(1-e^{i\frac{\pi}{6}} \right)^2 - \left(1-e^{i\frac{\pi}{6}} \right)^3 \end{aligned} \quad \textbf{(A1)}$$

$$= (1-i) - 3 \left(1-e^{i\frac{\pi}{6}} \right) + 3 \left(1-2e^{i\frac{\pi}{6}} + e^{i\frac{\pi}{3}} \right) - \left(1-3e^{i\frac{\pi}{6}} + 3e^{i\frac{\pi}{3}} - e^{i\frac{\pi}{2}} \right) \quad \textbf{A1}$$

$$= 0 \quad \textbf{AG}$$

Note: If the candidate does not interpret their conclusion, award
(M1)(A1)A0 as appropriate.

[3 marks]
continue...

Question 12 continued.

(e) **METHOD 1**

$$\frac{1}{1 - e^{i\frac{\pi}{6}}} = \frac{1}{1 - \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)} \quad \text{M1}$$

$$= \frac{2}{2 - \sqrt{3} - i} \quad \text{A1}$$

attempt to use conjugate to rationalise M1

$$= \frac{4 - 2\sqrt{3} + 2i}{(2 - \sqrt{3})^2 + 1} \quad \text{A1}$$

$$= \frac{4 - 2\sqrt{3} + 2i}{8 - 4\sqrt{3}} \quad \text{A1}$$

$$= \frac{1}{2} + \frac{1}{4 - 2\sqrt{3}}i$$

$$\Rightarrow \operatorname{Re}(\alpha) = \frac{1}{2} \quad \text{A1}$$

Note: Their final imaginary part does not have to be correct in order for the final three **A** marks to be awarded

[6 marks]
continue...

Question 12 continued.

METHOD 2

$$\frac{1}{1 - e^{i\frac{\pi}{6}}} = \frac{1}{1 - \left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6} \right)} \quad \text{M1}$$

attempt to use conjugate to rationalise M1

$$= \frac{1}{\left(1 - \cos \frac{\pi}{6} \right) - i \sin \frac{\pi}{6}} \times \frac{\left(1 - \cos \frac{\pi}{6} \right) + i \sin \frac{\pi}{6}}{\left(1 - \cos \frac{\pi}{6} \right) + i \sin \frac{\pi}{6}} \quad \text{A1}$$

$$= \frac{\left(1 - \cos \frac{\pi}{6} \right) + i \sin \frac{\pi}{6}}{\left(1 - \cos \frac{\pi}{6} \right)^2 + \sin^2 \frac{\pi}{6}} \quad \text{A1}$$

$$= \frac{\left(1 - \cos \frac{\pi}{6} \right) + i \sin \frac{\pi}{6}}{1 - 2 \cos \frac{\pi}{6} + \cos^2 \frac{\pi}{6} + \sin^2 \frac{\pi}{6}}$$

$$= \frac{\left(1 - \cos \frac{\pi}{6} \right) + i \sin \frac{\pi}{6}}{2 - 2 \cos \frac{\pi}{6}} \quad \text{A1}$$

$$= \frac{1}{2} + \frac{i \sin \frac{\pi}{6}}{2 - 2 \cos \frac{\pi}{6}}$$

$$\Rightarrow \operatorname{Re}(\alpha) = \frac{1}{2} \quad \text{A1}$$

Note: Their final imaginary part does not have to be correct in order for the final three **A** marks to be awarded

[6 marks]
continue...

Question 12 continued.

METHOD 3

attempt to multiply through by $-\frac{e^{-i\frac{\pi}{12}}}{e^{-i\frac{\pi}{12}}}$ **M1**

$$\frac{1}{1-e^{i\frac{\pi}{6}}} = -\frac{e^{-i\frac{\pi}{12}}}{e^{i\frac{\pi}{12}}-e^{-i\frac{\pi}{12}}} \quad \text{A1}$$

attempting to re-write in r-cis form **M1**

$$= -\frac{\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)}{\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}-\left(\cos\left(-\frac{\pi}{12}\right)+i\sin\left(-\frac{\pi}{12}\right)\right)} \quad \text{A1}$$

$$= -\frac{\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}}{2i\sin\frac{\pi}{12}} \quad \text{A1}$$

$$= \frac{1}{2} - \frac{1}{2i} \cot\frac{\pi}{12} \left(= \frac{1}{2} + \frac{1}{2}i \cot\frac{\pi}{12} \right)$$

$$\Rightarrow \operatorname{Re}(\alpha) = \frac{1}{2} \quad \text{A1}$$

[6 marks]

continue...

Question 12 continued.

METHOD 4

attempt to multiply through by $\frac{1 - e^{-i\frac{\pi}{6}}}{1 - e^{-i\frac{\pi}{6}}}$ **M1**

$$\frac{1}{1 - e^{i\frac{\pi}{6}}} = \frac{1 - e^{-i\frac{\pi}{6}}}{1 - e^{-i\frac{\pi}{6}} - e^{i\frac{\pi}{6}} + 1}$$
A1

attempting to re-write in r-cis form **M1**

$$= \frac{1 - \cos\frac{\pi}{6} - i\sin\frac{\pi}{6}}{2 - 2\cos\frac{\pi}{6}}$$
A1

attempt to re-write in Cartesian form **M1**

$$= \frac{1 - \frac{\sqrt{3}}{2} - \frac{1}{2}i}{2 - \sqrt{3}} \left(= \frac{2 - \sqrt{3}}{2 - \sqrt{3}} + i \frac{1}{2 - \sqrt{3}} \right)$$

$$\Rightarrow \operatorname{Re}(\alpha) = \frac{1}{2}$$
A1

Note: Their final imaginary part does not have to be correct in order for the final **A** mark to be awarded

[6 marks]

Total [22 marks]

5. (a) $u_1 = S_1 = \frac{2}{3} \times \frac{7}{8}$ (M1)

$$= \frac{14}{24} \left(= \frac{7}{12} = 0.583333... \right) \quad \text{A1}$$

[2 marks]

(b) $r = \frac{7}{8} (= 0.875)$ (A1)

substituting their values for u_1 and r into $S_\infty = \frac{u_1}{1-r}$ (M1)

$$= \frac{14}{3} (= 4.66666...) \quad \text{A1}$$

[3 marks]

(c) attempt to substitute their values into the inequality or formula for S_n (M1)

$$\frac{14}{3} - \sum_{r=1}^n \frac{2}{3} \left(\frac{7}{8} \right)^r < 0.001 \quad \text{OR} \quad S_n = \frac{\frac{7}{12} \left(1 - \left(\frac{7}{8} \right)^n \right)}{\left(1 - \frac{7}{8} \right)}$$

attempt to solve their inequality using a table, graph or logarithms
(must be exponential)

(M1)

Note: Award (M0) if the candidate attempts to solve $S_\infty - u_n < 0.001$.

correct critical value or at least one correct crossover value (A1)

$$63.2675... \text{ OR } S_\infty - S_{63} = 0.001036... \text{ OR } S_\infty - S_{64} = 0.000906...$$

$$\text{OR } S_\infty - S_{63} - 0.001 = 0.0000363683... \text{ OR } S_\infty - S_{64} - 0.001 = -0.0000931777...$$

least value is $n = 64$

A1

[4 marks]

Total [9 marks]

6. (a) METHOD 1

$$(p+q)^3 - 3pq(p+q) \equiv p^3 + q^3$$

attempts to expand $(p+q)^3$

M1

$$p^3 + 3p^2q + 3pq^2 + q^3$$

$$(p+q)^3 - 3pq(p+q) \equiv p^3 + 3p^2q + 3pq^2 + q^3 - 3pq(p+q)$$

$$\equiv p^3 + 3p^2q + 3pq^2 + q^3 - 3p^2q - 3pq^2$$

A1

$$\equiv p^3 + q^3$$

AG

Note: Condone the use of equals signs throughout.

METHOD 2

$$(p+q)^3 - 3pq(p+q) \equiv p^3 + q^3$$

attempts to factorise $(p+q)^3 - 3pq(p+q)$

M1

$$\equiv (p+q)((p+q)^2 - 3pq) \quad (\equiv (p+q)(p^2 - pq + q^2))$$

$$\equiv p^3 - p^2q + pq^2 + p^2q - pq^2 + q^3$$

A1

$$\equiv p^3 + q^3$$

AG

Note: Condone the use of equals signs throughout.

METHOD 3

$$p^3 + q^3 \equiv (p+q)^3 - 3pq(p+q)$$

attempts to factorise $p^3 + q^3$

M1

$$\equiv (p+q)(p^2 - pq + q^2)$$

$$\equiv (p+q)((p+q)^2 - 3pq)$$

A1

$$\equiv (p+q)^3 - 3pq(p+q)$$

AG

Note: Condone the use of the equals sign throughout.

[2 marks]

(b)

Note: Award a maximum of **A1M0A0A1M0A0** for $m = -95$ and $n = 8$ found

by using $\alpha, \beta = \frac{5 \pm \sqrt{17}}{4}$ ($\alpha, \beta = 0.219..., 2.28...$).

Condone, as appropriate, solutions that state but clearly do not use the values of α and β .

Special case: Award a maximum of **A1M1A0A1M0A0** for $m = -95$ and $n = 8$ obtained by solving simultaneously for α and β from product of roots and sum of roots equations.

product of roots of $x^2 - \frac{5}{2}x + \frac{1}{2} = 0$

$$\alpha\beta = \frac{1}{2} \text{ (seen anywhere)}$$

A1

considers $\left(\frac{1}{\alpha^3}\right)\left(\frac{1}{\beta^3}\right)$ by stating $\frac{1}{(\alpha\beta)^3} (= n)$

M1

Note: Award **M1** for attempting to substitute their value of $\alpha\beta$ into $\frac{1}{(\alpha\beta)^3}$.

$$\frac{1}{(\alpha\beta)^3} = \frac{1}{\left(\frac{1}{2}\right)^3}$$

$$n = 8$$

A1

sum of roots of $x^2 - \frac{5}{2}x + \frac{1}{2} = 0$

$$\alpha + \beta = \frac{5}{2} \text{ (seen anywhere)}$$

A1

considers $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ by stating $\frac{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)}{(\alpha\beta)^3} \left(\left(\frac{\alpha + \beta}{\alpha\beta} \right)^3 - \frac{3(\alpha + \beta)}{(\alpha\beta)^2} \right) (= -m)$

M1

Note: Award **M1** for attempting to substitute their values of $\alpha + \beta$ and $\alpha\beta$ into their expression. Award **M0** for use of $(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$ only.

$$= \frac{\left(\frac{5}{2}\right)^3 - \left(\frac{3}{2}\right)\left(\frac{5}{2}\right)}{\frac{1}{8}} (= 125 - 30 = 95)$$

$$m = -95$$

A1

$$(x^2 - 95x + 8 = 0)$$

[6 marks]**Total [8 marks]**

...

1. (a) $f'(t) = \frac{e^t - e^{-t}}{2}$ **A1**

$f''(t) = \frac{e^t + e^{-t}}{2}$ **A1**

$= f(t)$ **AG**

[2 marks]

(b) **METHOD 1**

$$(f(t))^2 + (g(t))^2$$

substituting f and g

M1

$$= \frac{(e^t + e^{-t})^2 + (e^t - e^{-t})^2}{4}$$

$$= \frac{(e^t)^2 + 2 + (e^{-t})^2 + (e^t)^2 - 2 + (e^{-t})^2}{4}$$

(M1)

$$= \frac{(e^t)^2 + (e^{-t})^2}{2} \left(= \frac{e^{2t} + e^{-2t}}{2} \right)$$

A1

$$= f(2t)$$

AG

METHOD 2

$$f(2t) = \frac{e^{2t} + e^{-2t}}{2}$$

$$= \frac{(e^t)^2 + (e^{-t})^2}{2}$$

M1

$$= \frac{(e^t + e^{-t})^2 + (e^t - e^{-t})^2}{4}$$

M1A1

$$= (f(t))^2 + (g(t))^2$$

AG

[3 marks]

Note: Accept combinations of METHODS 1 & 2 that meet at equivalent expressions.

- (c) (i) substituting $e^{iu} = \cos u + i \sin u$ into the expression for f (M1)
 obtaining $e^{-iu} = \cos u - i \sin u$ (A1)

$$f(iu) = \frac{\cos u + i \sin u + \cos u - i \sin u}{2}$$

Note: The **M1** can be awarded for the use of sine and cosine being odd and even respectively.

$$= \frac{2 \cos u}{2}$$

$$= \cos u$$

A1
[3 marks]

(ii) $g(iu) = \frac{\cos u + i \sin u - \cos u + i \sin u}{2}$

substituting and attempt to simplify

(M1)

$$= \frac{2i \sin u}{2}$$

$$= i \sin u$$

A1
[2 marks]

- (d) **METHOD 1**

$$(f(iu))^2 + (g(iu))^2$$

substituting expressions found in part (c)

(M1)

$$= \cos^2 u - \sin^2 u (= \cos 2u)$$

A1

METHOD 2

$$f(2iu) = \frac{e^{2iu} + e^{-2iu}}{2}$$

$$= \frac{\cos 2u + i \sin 2u + \cos 2u - i \sin 2u}{2}$$

M1

$$= \cos 2u$$

A1

Note: Accept equivalent final answers that have been simplified removing all imaginary parts eg $2 \cos^2 u - 1$ etc

[2 marks]

(e) $(f(t))^2 - (g(t))^2 = \frac{(e^t + e^{-t})^2 - (e^t - e^{-t})^2}{4}$ **M1**

$$= \frac{(e^{2t} + e^{-2t} + 2) - (e^{2t} + e^{-2t} - 2)}{4}$$

A1

$$= \frac{4}{4} = 1$$

A1

Note: Award **A1** for a value of 1 obtained from either LHS or RHS of given expression.

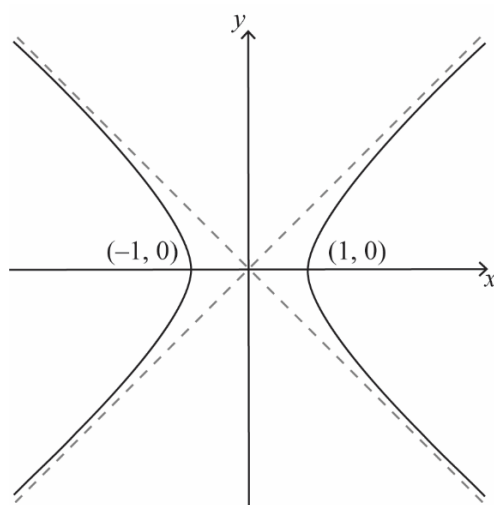
$(f(iu))^2 - (g(iu))^2 = \cos^2 u + \sin^2 u$ **M1**

$= 1$ (hence $(f(t))^2 - (g(t))^2 = (f(iu))^2 - (g(iu))^2$) **AG**

Note: Award full marks for showing that $(f(z))^2 - (g(z))^2 = 1, \forall z \in \mathbb{C}$.

[4 marks]

(f)

**A1A1A1A1**

Note: Award **A1** for correct curves in the upper quadrants, **A1** for correct curves in the lower quadrants, **A1** for correct x-intercepts of $(-1, 0)$ and $(1, 0)$ (condone $x = -1$ and 1), **A1** for $y = x$ and $y = -x$.

[4 marks]

(g) attempt to rotate by 45° in either direction

(M1)

Note: Evidence of an attempt to relate to a sketch of $xy = k$ would be sufficient for this **(M1)**.

attempting to rotate a particular point, eg (1, 0)

(M1)

(1, 0) rotates to $\left(\frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}\right)$ (or similar)

(A1)

hence $k = \pm \frac{1}{2}$

A1A1

[5 marks]

Total [25 marks]

2. (a) attempting to expand the LHS (M1)

$$\text{LHS} = (4n^2 - 4n + 1) + (4n^2 + 4n + 1)$$

$$= 8n^2 + 2 (= \text{RHS})$$
A1
AG
[2 marks]
- (b) **METHOD 1**
- recognition that $2n-1$ and $2n+1$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) R1

$$8n^2 + 2 = 2(4n^2 + 1)$$
A1
valid reason eg divisible by 2 (2 is a factor) R1
so the sum of the squares of any two consecutive odd integers is even AG
[3 marks]
- METHOD 2**
- recognition, eg that n and $n+2$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) R1

$$n^2 + (n+2)^2 = 2(n^2 + 2n + 2)$$
A1
valid reason eg divisible by 2 (2 is a factor) R1
so the sum of the squares of any two consecutive odd integers is even AG
[3 marks]
- Total [5 marks]**

11. (a) attempt to find modulus **(M1)**
 $r = 2\sqrt{3} (= \sqrt{12})$ **A1**
 attempt to find argument in the correct quadrant **(M1)**
 $\theta = \pi + \arctan\left(-\frac{\sqrt{3}}{3}\right)$ **A1**
 $= \frac{5\pi}{6}$ **A1**
 $-3 + \sqrt{3}i = \sqrt{12}e^{\frac{5\pi i}{6}} \left(= 2\sqrt{3}e^{\frac{5\pi i}{6}} \right)$

[5 marks]

- (b) attempt to find a root using de Moivre's theorem **M1**
 $12^{\frac{1}{6}}e^{\frac{5\pi i}{18}}$ **A1**
 attempt to find further two roots by adding and subtracting $\frac{2\pi}{3}$ to
 the argument **M1**
 $12^{\frac{1}{6}}e^{-\frac{7\pi i}{18}}$ **A1**
 $12^{\frac{1}{6}}e^{\frac{17\pi i}{18}}$ **A1**

Note: Ignore labels for u , v and w at this stage.
[5 marks]*continued...*

Question 11 continued

(c) **METHOD 1**

attempting to find the total area of (congruent) triangles UOV, VOW and UOW

M1

$$\text{Area} = 3 \left(\frac{1}{2} \right) \left(12^{\frac{1}{6}} \right) \left(12^{\frac{1}{6}} \right) \sin \frac{2\pi}{3}$$

A1A1

Note: Award **A1** for $\left(12^{\frac{1}{6}} \right) \left(12^{\frac{1}{6}} \right)$ and **A1** for $\sin \frac{2\pi}{3}$.

$$= \frac{3\sqrt{3}}{4} \left(12^{\frac{1}{3}} \right) \text{ (or equivalent)}$$

A1**[4 marks]****METHOD 2**

$$UV^2 = \left(12^{\frac{1}{6}} \right)^2 + \left(12^{\frac{1}{6}} \right)^2 - 2 \left(12^{\frac{1}{6}} \right) \left(12^{\frac{1}{6}} \right) \cos \frac{2\pi}{3} \text{ (or equivalent)}$$

A1

$$UV = \sqrt{3} \left(12^{\frac{1}{6}} \right) \text{ (or equivalent)}$$

A1

attempting to find the area of UVW using $\text{Area} = \frac{1}{2} \times UV \times VW \times \sin \alpha$

for example

M1

$$\text{Area} = \frac{1}{2} \left(\sqrt{3} \times 12^{\frac{1}{6}} \right) \left(\sqrt{3} \times 12^{\frac{1}{6}} \right) \sin \frac{\pi}{3}$$

$$= \frac{3\sqrt{3}}{4} \left(12^{\frac{1}{3}} \right) \text{ (or equivalent)}$$

A1**[4 marks]**(d) $u + v + w = 0$ **R1**

$$12^{\frac{1}{6}} \left(\cos \left(-\frac{7\pi}{18} \right) + i \sin \left(-\frac{7\pi}{18} \right) + \cos \frac{5\pi}{18} + i \sin \frac{5\pi}{18} + \cos \frac{17\pi}{18} + i \sin \frac{17\pi}{18} \right) = 0$$

A1

consideration of real parts

M1

$$12^{\frac{1}{6}} \left(\cos \left(-\frac{7\pi}{18} \right) + \cos \frac{5\pi}{18} + \cos \frac{17\pi}{18} \right) = 0$$

$$\cos \left(-\frac{7\pi}{18} \right) = \cos \frac{7\pi}{18} \text{ explicitly stated}$$

A1

$$\cos \frac{5\pi}{18} + \cos \frac{7\pi}{18} + \cos \frac{17\pi}{18} = 0$$

AG**[4 marks]****Total [18 marks]**

2. (a) $\left(1 + \frac{5.5}{4 \times 100}\right)^4$
1.056

(M1)(A1)

A1

[3 marks]

continued...

Question 2 continued(b) **EITHER**

$$2P = P \times \left(1 + \frac{5.5}{100 \times 4}\right)^{4n} \quad \text{OR} \quad 2P = P \times (\text{their } (a))^m \quad (M1)(A1)$$

Note: Award **(M1)** for substitution into loan payment formula. Award **(A1)** for correct substitution.

OR

$$PV = \pm 1$$

$$FV = \mp 2$$

$$I\% = 5.5$$

$$P/Y = 4$$

$$C/Y = 4$$

$$n = 50.756\dots$$

(M1)(A1)**OR**

$$PV = \pm 1$$

$$FV = \mp 2$$

$$I\% = 100(\text{their } (a) - 1)$$

$$P/Y = 1$$

$$C/Y = 1$$

(M1)(A1)**THEN**

$$\Rightarrow 12.7 \text{ years}$$

Laurie will have double the amount she invested during 2032

A1**[3 marks]****Total [6 marks]**

8. substituting $w = 2iz$ into $z^* - 3w = 5 + 5i$ **M1**
 $z^* - 6iz = 5 + 5i$ **A1**
 let $z = x + yi$
 comparing real and imaginary parts of $(x - yi) - 6i(x + yi) = 5 + 5i$ **M1**
 to obtain $x + 6y = 5$ and $-6x - y = 5$ **A1**
 attempting to solve for x and y **M1**
 $x = -1$ and $y = 1$ and so $z = -1 + i$ **A1**
 hence $w = -2 - 2i$ **A1**

6. EITHER

attempt to obtain the general term of the expansion

$$T_{r+1} = {}^nC_r (8x^3)^{n-r} \left(-\frac{1}{2x}\right)^r \text{ OR } T_{r+1} = {}^nC_{n-r} (8x^3)^r \left(-\frac{1}{2x}\right)^{n-r} \quad (\text{M1})$$

OR

recognize power of x starts at $3n$ and goes down by 4 each time (M1)

THEN

recognizing the constant term when the power of x is zero (or equivalent) (M1)

$$r = \frac{3n}{4} \text{ or } n = \frac{4}{3}r \text{ or } 3n - 4r = 0 \text{ OR } 3r - (n - r) = 0 \text{ (or equivalent)} \quad \text{A1}$$

r is a multiple of 3 ($r = 3, 6, 9, \dots$) or one correct value of n (seen anywhere) (A1)

$$n = 4k, k \in \mathbb{Z}^+ \quad \text{A1}$$

Note: Accept n is a (positive) multiple of 4 or $n = 4, 8, 12, \dots$

Do not accept $n = 4, 8, 12$

Note: Award full marks for a correct answer using trial and error approach showing $n = 4, 8, 12, \dots$ and for recognizing that this pattern continues.

[5 marks]

,

8. Assume that a and b are both odd.

M1

Note: Award **M0** for statements such as “let a and b be both odd”.

Note: Subsequent marks after this **M1** are independent of this mark and can be awarded.

Then $a = 2m + 1$ and $b = 2n + 1$

A1

$$a^2 + b^2 \equiv (2m + 1)^2 + (2n + 1)^2$$

$$= 4m^2 + 4m + 1 + 4n^2 + 4n + 1$$

A1

$$= 4(m^2 + m + n^2 + n) + 2$$

(A1)

$(4(m^2 + m + n^2 + n))$ is always divisible by 4) but 2 is not divisible by 4. (or equivalent)

R1

$\Rightarrow a^2 + b^2$ is not divisible by 4, a contradiction. (or equivalent)

R1

hence a and b cannot both be odd.

AG

Note: Award a maximum of **M1A0A0(A0)R1R1** for considering identical or two consecutive odd numbers for a and b .

[6 marks]

9. (a) $z_1 z_2 = (1+bi)((1-b^2)-(2b)i)$

$$= (1-b^2-2i^2b^2) + i(-2b+b-b^3)$$

M1

$$= (1+b^2) + i(-b-b^3)$$

A1A1

Note: Award **A1** for $1+b^2$ and **A1** for $-bi-b^3i$.

[3 marks]

(b) $\arg(z_1 z_2) = \arctan\left(\frac{-b-b^3}{1+b^2}\right) = \frac{\pi}{4}$

(M1)

EITHER

$$\arctan(-b) = \frac{\pi}{4} \text{ (since } 1+b^2 \neq 0, \text{ for } b \in \mathbb{R})$$

A1

OR

$$-b-b^3 = 1+b^2 \text{ (or equivalent)}$$

A1

THEN

$$b = -1$$

A1

[3 marks]

Total [6 marks]

Section B

10. (a) (i) EITHER

attempt to use a ratio from consecutive terms

M1

$$\frac{p \ln x}{\ln x} = \frac{\frac{1}{3} \ln x}{p \ln x} \quad \text{OR} \quad \frac{1}{3} \ln x = (\ln x) r^2 \quad \text{OR} \quad p \ln x = \ln x \left(\frac{1}{3p} \right)$$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \dots$ and consider the powers of x in geometric sequence.

Award **M1** for $\frac{p}{1} = \frac{\frac{1}{3}}{p}$.

OR

$$r = p \quad \text{and} \quad r^2 = \frac{1}{3}$$

M1**THEN**

$$p^2 = \frac{1}{3} \quad \text{OR} \quad r = \pm \frac{1}{\sqrt{3}}$$

A1

$$p = \pm \frac{1}{\sqrt{3}}$$

AG

Note: Award **M0A0** for $r^2 = \frac{1}{3}$ or $p^2 = \frac{1}{3}$ with no other working seen.

continued...

Question 10 continued(ii) **EITHER**

$$\text{since, } |p| = \frac{1}{\sqrt{3}} \text{ and } \frac{1}{\sqrt{3}} < 1$$

R1**OR**

$$\text{since, } |p| = \frac{1}{\sqrt{3}} \text{ and } -1 < p < 1$$

R1**THEN** $\Rightarrow$ the geometric series converges.**AG****Note:** Accept r instead of p .Award **R0** if both values of p not considered.

$$(iii) \quad \frac{\ln x}{1 - \frac{1}{\sqrt{3}}} \quad (= 3 + \sqrt{3})$$

(A1)

$$\ln x = 3 - \frac{3}{\sqrt{3}} + \sqrt{3} - \frac{\sqrt{3}}{\sqrt{3}} \quad \text{OR} \quad \ln x = 3 - \sqrt{3} + \sqrt{3} - 1 \quad (\Rightarrow \ln x = 2)$$

A1

$$x = e^2$$

A1**[6 marks]***continued...*

Question 10 continued

(b) (i) **METHOD 1**attempt to find a difference from consecutive terms or from u_2 **M1**

correct equation

A1

$$p \ln x - \ln x = \frac{1}{3} \ln x - p \ln x \quad \text{OR} \quad \frac{1}{3} \ln x = \ln x + 2(p \ln x - \ln x)$$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \dots$ and consider the powers of x in arithmetic sequence.

Award **M1A1** for $p - 1 = \frac{1}{3} - p$.

$$2p \ln x = \frac{4}{3} \ln x \quad \left(\Rightarrow 2p = \frac{4}{3} \right)$$

A1

$$p = \frac{2}{3}$$

AG**METHOD 2**attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ **M1**

$$p \ln x = \frac{\ln x + \frac{1}{3} \ln x}{2}$$

A1

$$2p \ln x = \frac{4}{3} \ln x \quad \left(\Rightarrow 2p = \frac{4}{3} \right)$$

A1

$$p = \frac{2}{3}$$

AG

continued...

*Question 10 continued***METHOD 3**attempt to find difference using u_3 **M1**

$$\frac{1}{3}\ln x = \ln x + 2d \quad \left(\Rightarrow d = -\frac{1}{3}\ln x \right)$$

$$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right) \quad \text{OR} \quad p\ln x - \ln x = -\frac{1}{3}\ln x$$

A1

$$p\ln x = \frac{2}{3}\ln x$$

A1

$$p = \frac{2}{3}$$

AG

$$(ii) \quad d = -\frac{1}{3}\ln x$$

A1*continued...*

Question 10 continued

(iii) **METHOD 1**

$$S_n = \frac{n}{2} \left[2 \ln x + (n-1) \times \left(-\frac{1}{3} \ln x \right) \right]$$

attempt to substitute into S_n and equate to $\ln\left(\frac{1}{x^3}\right)$ **(M1)**

$$\frac{n}{2} \left[2 \ln x + (n-1) \times \left(-\frac{1}{3} \ln x \right) \right] = \ln\left(\frac{1}{x^3}\right)$$

$$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 (= \ln x^{-3})$$
 (A1)

$$= -3 \ln x$$
 (A1)

correct working with S_n (seen anywhere) **(A1)**

$$\frac{n}{2} \left[2 \ln x - \frac{n}{3} \ln x + \frac{1}{3} \ln x \right] \quad \text{OR} \quad n \ln x - \frac{n(n-1)}{6} \ln x \quad \text{OR} \quad \frac{n}{2} \left(\ln x + \left(\frac{4-n}{3} \right) \ln x \right)$$

correct equation without $\ln x$ **A1**

$$\frac{n}{2} \left(\frac{7}{3} - \frac{n}{3} \right) = -3 \quad \text{OR} \quad n - \frac{n(n-1)}{6} = -3 \quad (\text{or equivalent})$$

Note: Award as above if the series $1 + p + \frac{1}{3} + \dots$ is considered leading to

$$\frac{n}{2} \left(\frac{7}{3} - \frac{n}{3} \right) = -3.$$

attempt to form a quadratic $= 0$ **(M1)**

$$n^2 - 7n - 18 = 0$$

attempt to solve their quadratic **(M1)**

$$(n-9)(n+2) = 0$$

$$n = 9$$
 A1

continued...

*Question 10 continued***METHOD 2**

$$\ln\left(\frac{1}{x^3}\right) = -\ln x^3 (= \ln x^{-3}) \quad (\text{A1})$$

$$= -3\ln x \quad (\text{A1})$$

listing the first 7 terms of the sequence (A1)

$$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \dots$$

recognizing first 7 terms sum to 0 M1

$$8^{\text{th}} \text{ term is } -\frac{4}{3}\ln x \quad (\text{A1})$$

$$9^{\text{th}} \text{ term is } -\frac{5}{3}\ln x \quad (\text{A1})$$

$$\text{sum of } 8^{\text{th}} \text{ and } 9^{\text{th}} \text{ terms} = -3\ln x \quad (\text{A1})$$

$$n = 9 \quad \text{A1}$$

[12 marks]

Total [18 marks]

1. (a) $u_1 = 12$

A1

[1 mark]

(b) $15 - 3n = -33$

(A1)

$n = 16$

A1

[2 marks]

(c) valid approach to find d

(M1)

$u_2 - u_1 = 9 - 12$ OR recognize gradient is -3 OR attempts to solve
 $-33 = 12 + 15d$

$d = -3$

A1

[2 marks]

Total [5 marks]

2. (a) $(n-1) + n + (n+1)$ **(A1)**
 $= 3n$ **A1**
 which is always divisible by 3 **AG**
[2 marks]

- (b) $(n-1)^2 + n^2 + (n+1)^2$ ($= n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$) **A1**
 attempts to expand either $(n-1)^2$ or $(n+1)^2$ (do not accept $n^2 - 1$ or $n^2 + 1$) **(M1)**
 $= 3n^2 + 2$ **A1**
 demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct
 expression divided by 3 **R1**
 $3n^2$ is divisible by 3 and so $3n^2 + 2$ is never divisible by 3
 OR the first term is divisible by 3, the second is not
 OR $3\left(n^2 + \frac{2}{3}\right)$ OR $\frac{3n^2 + 2}{3} = n^2 + \frac{2}{3}$
 hence the sum of the squares is never divisible by 3 **AG**

[4 marks]

Total [6 marks]

5. (a) EITHER

recognises the required term (or coefficient) in the expansion

(M1)

$$bx^5 = {}^7C_2 x^5 1^2 \quad \text{OR} \quad b = {}^7C_2 \quad \text{OR} \quad {}^7C_5$$

$$b = \frac{7!}{2!5!} \left(= \frac{7!}{2!(7-2)!} \right)$$

correct working

A1

$$\frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1} \quad \text{OR} \quad \frac{7 \times 6}{2!} \quad \text{OR} \quad \frac{42}{2}$$

OR

lists terms from row 7 of Pascal's triangle

(M1)

1, 7, 21, ...

A1**THEN**

$$b = 21$$

AG**[2 marks]****(b) $a = 7$** **(A1)**

correct equation

A1

$$21x^5 = \frac{ax^6 + 35x^4}{2} \quad \text{OR} \quad 21x^5 = \frac{7x^6 + 35x^4}{2}$$

correct quadratic equation

A1

$$7x^2 - 42x + 35 = 0 \quad \text{OR} \quad x^2 - 6x + 5 = 0 \quad (\text{or equivalent})$$

valid attempt to solve **their** quadratic**(M1)**

$$(x-1)(x-5) = 0 \quad \text{OR} \quad x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$$x = 1, x = 5$$

A1

Note: Award final **A0** for obtaining $x = 0, x = 1, x = 5$.

[5 marks]**Total [7 marks]**

9. METHOD 1 (rearranging the equation)

assume there exists some $\alpha \in \mathbb{Z}$ such that $2\alpha^3 + 6\alpha + 1 = 0$

M1

Note: Award **M1** for equivalent statements such as ‘assume that α is an integer root of $2\alpha^3 + 6\alpha + 1 = 0$ ’. Condone the use of x throughout the proof.

Award **M1** for an assumption involving $\alpha^3 + 3\alpha + \frac{1}{2} = 0$.

Note: Award **M0** for statements such as “let’s consider the equation has integer roots...”, “let $\alpha \in \mathbb{Z}$ be a root of $2\alpha^3 + 6\alpha + 1 = 0 \dots$ ”

Note: Subsequent marks after this **M1** are independent of this **M1** and can be awarded.

attempts to rearrange their equation into a suitable form

M1**EITHER**

$$2\alpha^3 + 6\alpha = -1$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow 2\alpha^3 + 6\alpha \text{ is even}$$

R1

$$2\alpha^3 + 6\alpha = -1 \text{ which is not even and so } \alpha \text{ cannot be an integer}$$

R1

Note: Accept ‘ $2\alpha^3 + 6\alpha = -1$ which gives a contradiction’.

OR

$$1 = 2(-\alpha^3 - 3\alpha)$$

A1

$$\alpha \in \mathbb{Z} \Rightarrow (-\alpha^3 - 3\alpha) \in \mathbb{Z}$$

R1

$$\Rightarrow 1 \text{ is even which is not true and so } \alpha \text{ cannot be an integer}$$

R1

Note: Accept ‘ $\Rightarrow 1$ is even which gives a contradiction’.

continued...

*Question 9 continued***OR**

$$\frac{1}{2} = -\alpha^3 - 3\alpha \quad \text{A1}$$

$$\alpha \in \mathbb{Z} \Rightarrow (-\alpha^3 - 3\alpha) \in \mathbb{Z} \quad \text{R1}$$

$$-\alpha^3 - 3\alpha \text{ is not an integer } \left(= \frac{1}{2} \right) \text{ and so } \alpha \text{ cannot be an integer} \quad \text{R1}$$

Note: Accept ‘ $-\alpha^3 - 3\alpha$ is not an integer $\left(= \frac{1}{2} \right)$ which gives a contradiction’.

OR

$$\alpha = -\frac{1}{2(\alpha^2 + 3)} \quad \text{A1}$$

$$\alpha \in \mathbb{Z} \Rightarrow -\frac{1}{2(\alpha^2 + 3)} \in \mathbb{Z} \quad \text{R1}$$

$$-\frac{1}{2(\alpha^2 + 3)} \text{ is not an integer and so } \alpha \text{ cannot be an integer} \quad \text{R1}$$

Note: Accept $-\frac{1}{2(\alpha^2 + 3)}$ is not an integer which gives a contradiction’.

THEN

so the equation $2x^3 + 6x + 1 = 0$ has no integer roots

AG**[5 marks]***continued...*

Question 9 continued

METHOD 2

assume there exists some $\alpha \in \mathbb{Z}$ such that $2\alpha^3 + 6\alpha + 1 = 0$

M1

Note: Award **M1** for statements such as ‘assume that α is an integer root of $2\alpha^3 + 6\alpha + 1 = 0$ ’. Condone the use of x throughout the proof. Award **M1** for an assumption involving $\alpha^3 + 3\alpha + \frac{1}{2} = 0$ and award subsequent marks based on this.

Note: Award **M0** for statements such as “let’s consider the equation has integer roots...”, “let $\alpha \in \mathbb{Z}$ be a root of $2\alpha^3 + 6\alpha + 1 = 0 \dots$ ”

Note: Subsequent marks after this **M1** are independent of this **M1** and can be awarded.

let $f(x) = 2x^3 + 6x + 1$ (and $f(\alpha) = 0$)

$f'(x) = 6x^2 + 6 > 0$ for all $x \in \mathbb{R} \Rightarrow f$ is a (strictly) increasing function

M1A1

$f(0) = 1$ and $f(-1) = -7$

R1

thus $f(x) = 0$ has only one real root between -1 and 0 , which gives a contradiction

(or therefore, contradicting the assumption that $f(\alpha) = 0$ for some $\alpha \in \mathbb{Z}$),

R1

so the equation $2x^3 + 6x + 1 = 0$ has no integer roots

AG**[5 marks]**

Section B

12. (a) $z_2^* = r_2 e^{-i\theta}$ (A1)
- $z_1 z_2^* = r_1 e^{i\alpha} r_2 e^{-i\theta}$ A1
- $z_1 z_2^* = r_1 r_2 e^{i(\alpha-\theta)}$ AG

Note: Accept working in modulus-argument form

[2 marks]

- (b) $\operatorname{Re}(z_1 z_2^*) = r_1 r_2 \cos(\alpha - \theta) (= 0)$ A1
- $\alpha - \theta = \arccos 0 \quad (r_1, r_2 > 0)$
- $\alpha - \theta = \frac{\pi}{2} \text{ (as } 0 < \alpha - \theta < \pi \text{)}$ A1
- so $Z_1 O Z_2$ is a right-angled triangle AG

[2 marks]

- (c) (i) **EITHER**
- $\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\alpha-\theta)} = e^{i\frac{\pi}{3}} \text{ (since } r_1 = r_2 \text{)}$ (M1)
- OR**
- $z_1 = r_2 e^{i\left(\theta + \frac{\pi}{3}\right)} = r_2 e^{i\theta} e^{i\frac{\pi}{3}}$ (M1)
- THEN**
- $z_1 = z_2 e^{i\frac{\pi}{3}}$ A1

Note: Accept working in either modulus-argument form to obtain

$$z_1 = z_2 \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right) \text{ or in Cartesian form to obtain } z_1 = z_2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i \right).$$

continued...

Question 12 continued

(ii) substitutes $z_1 = z_2 e^{i\frac{\pi}{3}}$ into $z_1^2 + z_2^2$ **M1**

$$z_1^2 + z_2^2 = z_2^2 e^{i\frac{2\pi}{3}} + z_2^2 \left(= z_2^2 \left(e^{i\frac{2\pi}{3}} + 1 \right) \right)$$
A1

EITHER

$$e^{i\frac{2\pi}{3}} + 1 = e^{i\frac{\pi}{3}}$$
A1

OR

$$\begin{aligned} z_2^2 \left(e^{i\frac{2\pi}{3}} + 1 \right) &= z_2^2 \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i + 1 \right) \\ &= z_2^2 \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i \right) \end{aligned}$$
A1

THEN

$$\begin{aligned} z_1^2 + z_2^2 &= z_2^2 e^{i\frac{\pi}{3}} \\ &= z_2 \left(z_2 e^{i\frac{\pi}{3}} \right) \text{ and } z_2 e^{i\frac{\pi}{3}} = z_1 \end{aligned}$$
A1

$$\text{so } z_1^2 + z_2^2 = z_1 z_2$$
AG

Note: For candidates who work on the LHS and RHS separately to show equality, award **M1A1** for $z_1^2 + z_2^2 = z_2^2 e^{i\frac{2\pi}{3}} + z_2^2 \left(= z_2^2 \left(e^{i\frac{2\pi}{3}} + 1 \right) \right)$, **A1** for $z_1 z_2 = z_2^2 e^{i\frac{\pi}{3}}$ and **A1** for $e^{i\frac{2\pi}{3}} + 1 = e^{i\frac{\pi}{3}}$. Accept working in either modulus-argument form or in Cartesian form.

[6 marks]

continued...

Question 12 continued

(d) **METHOD 1**

$$z_1 + z_2 = -a \text{ and } z_1 z_2 = b \quad (\text{A1})$$

$$a^2 = z_1^2 + z_2^2 + 2z_1 z_2 \quad \text{A1}$$

$$a^2 = 2z_1 z_2 + z_1 z_2 (= 3z_1 z_2) \quad \text{A1}$$

substitutes $b = z_1 z_2$ into their expression M1

$$a^2 = 2b + b \text{ OR } a^2 = 3b \quad \text{A1}$$

Note: If $z_1 + z_2 = -a$ is not clearly recognized, award maximum (A0)A1A1M1A0.

$$\text{so } a^2 - 3b = 0 \quad \text{AG}$$

METHOD 2

$$z_1 + z_2 = -a \text{ and } z_1 z_2 = b \quad (\text{A1})$$

$$(z_1 + z_2)^2 = z_1^2 + z_2^2 + 2z_1 z_2 \quad \text{A1}$$

$$(z_1 + z_2)^2 = 2z_1 z_2 + z_1 z_2 (= 3z_1 z_2) \quad \text{A1}$$

substitutes $b = z_1 z_2$ and $z_1 + z_2 = -a$ into their expression M1

$$a^2 = 2b + b \text{ OR } a^2 = 3b \quad \text{A1}$$

Note: If $z_1 + z_2 = -a$ is not clearly recognized, award maximum (A0)A1A1M1A0.

$$\text{so } a^2 - 3b = 0 \quad \text{AG}$$

[5 marks]

(e) $a^2 - 3 \times 12 = 0$

$$a = \pm 6 \quad (\Rightarrow z^2 \pm 6z + 12 = 0) \quad \text{A1}$$

for $a = -6$:

$$z_1 = 3 + \sqrt{3}i, z_2 = 3 - \sqrt{3}i \text{ and } \alpha - \theta = -\frac{5\pi}{3} \text{ which does not satisfy } 0 < \alpha - \theta < \pi \quad \text{R1}$$

for $a = 6$:

$$z_1 = -3 - \sqrt{3}i, z_2 = -3 + \sqrt{3}i \text{ and } \alpha - \theta = \frac{\pi}{3} \quad \text{A1}$$

so (for $0 < \alpha - \theta < \pi$), only one equilateral triangle can be formed from point O and the two roots of this equation AG

[3 marks]

Total [18 marks]

9. (a) $6 \times 5!$ (A1)(A1)
 $= 720$ (accept 6!) A1
 [3 marks]

(b) **METHOD 1**

(Peter apart from girls, in an end seat) ${}^8P_4 (= 1680)$ OR

(Peter apart from girls, not in end seat) ${}^7P_4 (= 840)$ (A1)

case 1: Peter at either end

$2 \times {}^8P_4 (= 3360)$ OR $2 \times {}^8C_4 \times 4! (= 3360)$ (A1)

case 2: Peter not at the end

$8 \times {}^7P_4 (= 6720)$ OR $8 \times {}^7C_4 \times 4! (= 6720)$ (A1)

Total number of ways = $3360 + 6720$

$= 10080$ A1

METHOD 2

(Peter next to girl, in an end seat) $4 \times {}^8P_3 (= 1344)$ OR

(Peter next to one girl, not in end seat) $2 \times 4 \times {}^7P_3 (= 1680)$ OR

(Peter next to two girls, not in end seat) $4 \times 3 \times {}^7P_2 (= 504)$ (A1)

case 1: Peter at either end

$2 \times 4 \times {}^8P_3 (= 2688)$ (A1)

case 2: Peter not at the end

$8(2 \times 4 \times {}^7P_3 + 4 \times 3 \times {}^7P_2) (= 17472)$ (A1)

Total number of ways = ${}^{10}P_5 - (2688 + 17472)$

$= 10080$ A1

[4 marks]

Total [7 marks]

Section B

9. (a) $9 \times 9 \times 8 \times 7 \times 6 \times 5 \left(= 9 \times {}^9P_5 \right)$ (M1)

$$= 136080 \left(= 9 \times \frac{9!}{4!} \right) \quad \text{A1}$$

Note: Award **M1A0** for $10 \times 9 \times 8 \times 7 \times 6 \times 5 \left(= {}^{10}P_6 = 151200 = \frac{10!}{4!} \right)$.

Note: Award **M1A0** for ${}^9P_6 = 60480$

[2 marks]

(b) **METHOD 1****EITHER**

every unordered subset of 6 digits from the set of 9 non-zero digits can be arranged in exactly one way into a 6-digit number with the digits in increasing order. A1

OR

$${}^9C_6 (\times 1) \quad \text{A1}$$

THEN

$$= 84 \quad \text{A1}$$

METHOD 2**EITHER**

removes 3 digits from the set of 9 non-zero digits and these 6 remaining digits can be arranged in exactly one way into a 6-digit number with the digits in increasing order. A1

OR

$${}^9C_3 (\times 1) \quad \text{A1}$$

THEN

$$= 84 \quad \text{A1}$$

[2 marks]

Total [4 marks]

Section B

1. (a) (i) $P_3(n) = \frac{(3-2)n - (3-4)n}{2}$ OR $P_3(n) = \frac{n - (-n)}{2}$ **A1**

$$P_3(n) = \frac{n^2 + n}{2} \quad \text{A1}$$

Note: Award **A0A1** if $P_3(n) = \frac{n^2 + n}{2}$ only is seen.

Do not award any marks for numerical verification.

so for triangular numbers, $P_3(n) = \frac{n(n+1)}{2}$ **AG**

[2 marks]

(ii) **METHOD 1**

uses a table of values to find a positive integer that satisfies $P_3(n) = 351$ **(M1)**

for example, a list showing at least 3 consecutive terms (... 325, 351, 378...)

Note: Award **(M1)** for use of a GDC's numerical solve or graph feature.

$n = 26$ (26th triangular number) **A1**

Note: Award **A0** for $n = -27, 26$. Award **A0** if additional solutions besides $n = 26$ are given.

METHOD 2

attempts to solve $\frac{n(n+1)}{2} = 351$ ($n^2 + n - 702 = 0$) for n **(M1)**

$$n = \frac{-1 \pm \sqrt{1^2 - 4(1)(-702)}}{2} \text{ OR } (n-26)(n+27) = 0$$

$n = 26$ (26th triangular number) **A1**

Note: Award **A0** for $n = -27, 26$. Award **A0** if additional solutions besides $n = 26$ are given.

[2 marks]

continued...

Question 1 continued

- (b) (i) attempts to form an expression for $P_3(n) + P_3(n+1)$ in terms of n **M1**

EITHER

$$P_3(n) + P_3(n+1) \equiv \frac{n(n+1)}{2} + \frac{(n+1)(n+2)}{2}$$

$$\equiv \frac{(n+1)(2n+2)}{2} \left(\equiv \frac{2(n+1)(n+1)}{2} \right)$$

A1**OR**

$$P_3(n) + P_3(n+1) \equiv \left(\frac{n^2}{2} + \frac{n}{2} \right) + \left(\frac{(n+1)^2}{2} + \frac{n+1}{2} \right)$$

$$\equiv \left(\frac{n^2 + n}{2} \right) + \left(\frac{n^2 + 2n + 1 + n + 1}{2} \right) \left(\equiv n^2 + 2n + 1 \right)$$

A1**THEN**

$$\equiv (n+1)^2$$

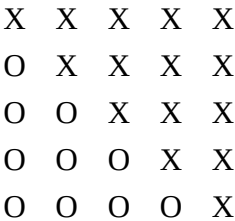
AG**[2 marks]**

- (ii) the sum of the n th and $(n+1)$ th triangular numbers
is the $(n+1)$ th square number

A1**[1 mark]***continued...*

Question 1 continued

(iii)



A1

Note: Accept equivalent single diagrams, such as the one above, where the 4th and 5th triangular numbers and the 5th square number are clearly shown. Award **A1** for a diagram that show $P_3(4)$ (a triangle with 10 dots) and $P_3(5)$ (a triangle with 15 dots) and $P_4(5)$ (a square with 25 dots).

[1 mark]
continued...

Question 1 continued(c) **METHOD 1**

$$8P_3(n)+1=8\left(\frac{n(n+1)}{2}\right)+1 (=4n(n+1)+1) \quad \mathbf{A1}$$

attempts to expand their expression for $8P_3(n)+1$ **(M1)**

$$=4n^2+4n+1$$

$$=(2n+1)^2 \quad \mathbf{A1}$$

and $2n+1$ is odd **AG**

METHOD 2

$$8P_3(n)+1=8\left((n+1)^2-P_3(n+1)\right)+1\left(=8\left((n+1)^2-\frac{(n+1)(n+2)}{2}\right)+1\right) \quad \mathbf{A1}$$

attempts to expand their expression for $8P_3(n)+1$ **(M1)**

$$8(n^2+2n+1)-4(n^2+3n+2)+1 (=4n^2+4n+1)$$

$$=(2n+1)^2 \quad \mathbf{A1}$$

and $2n+1$ is odd **AG**

continued...

*Question 1 continued***METHOD 3**

$$8P_3(n)+1=8\left(\frac{n(n+1)}{2}\right)+1 \left(=(An+B)^2\right) \text{ (where } A,B \in \mathbb{Z}^+)$$

A1attempts to expand their expression for $8P_3(n)+1$ **(M1)**

$$4n^2 + 4n + 1 \left(= A^2n^2 + 2ABn + B^2\right)$$

now equates coefficients and obtains $B=1$ and $A=2$

$$=(2n+1)^2$$

A1and $2n+1$ is odd**AG****[3 marks]***continued...*

Question 1 continued

(d) **EITHER**

$$u_1 = 1 \text{ and } d = 3 \quad (\text{A1})$$

$$\text{substitutes their } u_1 \text{ and their } d \text{ into } P_5(n) = \frac{n}{2}(2u_1 + (n-1)d) \quad (\text{M1})$$

$$P_5(n) = \frac{n}{2}(2 + 3(n-1)) \left(= \frac{n}{2}(2 + 3n - 3) \right) \quad (\text{A1})$$

OR

$$u_1 = 1 \text{ and } u_n = 3n - 2 \quad (\text{A1})$$

$$\text{substitutes their } u_1 \text{ and their } u_n \text{ into } P_5(n) = \frac{n}{2}(u_1 + u_n) \quad (\text{M1})$$

$$P_5(n) = \frac{n}{2}(1 + 3n - 2) \quad (\text{A1})$$

OR

$$P_5(n) = (3(1) - 2) + (3(2) - 2) + (3(3) - 2) + \dots + 3n - 2$$

$$P_5(n) = (3(1) + 3(2) + 3(3) + \dots + 3n) - 2n \quad (= 3(1 + 2 + 3 + \dots + n) - 2n) \quad (\text{A1})$$

$$\text{substitutes } \frac{n(n+1)}{2} \text{ into their expression for } P_5(n) \quad (\text{M1})$$

$$P_5(n) = 3\left(\frac{n(n+1)}{2}\right) - 2n$$

$$P_5(n) = \frac{n}{2}(3(n+1) - 4) \quad (\text{A1})$$

OR

$$\text{attempts to find the arithmetic mean of } n \text{ terms} \quad (\text{M1})$$

$$= \frac{1 + (3n - 2)}{2} \quad (\text{A1})$$

multiplies the above expression by the number of terms n

$$P_5(n) = \frac{n}{2}(1 + 3n - 2) \quad (\text{A1})$$

THEN

$$\text{so } P_5(n) = \frac{n(3n-1)}{2} \quad (\text{AG})$$

[3 marks]

continued...

Question 1 continued(e) **METHOD 1**

forms a table of $P_3(n)$ values that includes some values for $n > 5$ **(M1)**

forms a table of $P_5(m)$ values that includes some values for $m > 5$ **(M1)**

Note: Award **(M1)** if at least one $P_3(n)$ value is correct. Award **(M1)** if at least one $P_5(m)$ value is correct. Accept as above for $(n^2 + n)$ values and $(3m^2 - m)$ values.

$n = 20$ for triangular numbers **(A1)**

$m = 12$ for pentagonal numbers **(A1)**

Note: Award **(A1)** if $n = 20$ is seen in or out of a table. Award **(A1)** if $m = 12$ is seen in or out of a table. Condone the use of the same parameter for triangular numbers and pentagonal numbers, for example, $n = 20$ for triangular numbers and $n = 12$ for pentagonal numbers.

210 (is a triangular number and a pentagonal number) **A1**

Note: Award all five marks for 210 seen anywhere with or without working shown.

continued...

*Question 1 continued***METHOD 2****EITHER**

attempts to express $P_3(n) = P_5(m)$ as a quadratic in n **(M1)**

$$n^2 + n + (m - 3m^2) = 0 \text{ (or equivalent)}$$

attempts to solve their quadratic in n **(M1)**

$$n = \frac{-1 \pm \sqrt{12m^2 - 4m + 1}}{2} \left(= \frac{-1 \pm \sqrt{1^2 - 4(m - 3m^2)}}{2} \right)$$

OR

attempts to express $P_3(n) = P_5(m)$ as a quadratic in m **(M1)**

$$3m^2 - m - (n^2 + n) = 0 \text{ (or equivalent)}$$

attempts to solve their quadratic in m **(M1)**

$$m = \frac{1 \pm \sqrt{12n^2 + 12n + 1}}{6} \left(= \frac{1 \pm \sqrt{(-1)^2 + 12(n^2 + n)}}{6} \right)$$

THEN

$n = 20$ for triangular numbers **(A1)**

$m = 12$ for pentagonal numbers **(A1)**

210 (is a triangular number and a pentagonal number) **A1**

continued...

*Question 1 continued***METHOD 3**

$$\frac{n(n+1)}{2} = \frac{m(3m-1)}{2}$$

let $n = m + k$ ($n > m$) and so $3m^2 - m = (m+k)(m+k+1)$ **M1**

$$2m^2 - 2(k+1)m - (k^2 + k) = 0$$
 A1

attempts to find the discriminant of their quadratic
and recognises that this must be a perfect square **M1**

$$\Delta = 4(k+1)^2 + 8(k^2 + k)$$

$$N^2 = 4(k+1)^2 + 8(k^2 + k) \quad (= 4(k+1)(3k+1))$$

determines that $k = 8$ leading to $2m^2 - 18m - 72 = 0 \Rightarrow m = -3, 12$ and so $m = 12$ **A1**

210 (is a triangular number and a pentagonal number) **A1**

METHOD 4

$$\frac{n(n+1)}{2} = \frac{m(3m-1)}{2}$$

let $m = n - k$ ($m < n$) and so $n^2 + n = (n-k)(3(n-k)-1)$ **M1**

$$2n^2 - 2(3k+1)n + (3k^2 + k) = 0$$
 A1

attempts to find the discriminant of their quadratic
and recognises that this must be a perfect square **M1**

$$\Delta = 4(3k+1)^2 - 8(3k^2 + k)$$

$$N^2 = 4(3k+1)^2 - 8(3k^2 + k) \quad (= 4(k+1)(3k+1))$$

determines that $k = 8$ leading to $2n^2 - 50n + 200 = 0 \Rightarrow n = 5, 20$ and so $n = 20$ **A1**

210 (is a triangular number and a pentagonal number) **A1**

[5 marks]

continued...

Question 1 continued

(f)

Note: Award a maximum of **R1M0M0A1M1A1A1R0** for a ‘correct’ proof using n and $n+1$.

$$\text{consider } n=1: P_r(1) = 1 + (1-1)(r-2) = 1 \text{ and } P_r(1) = \frac{(r-2)(1^2) - (r-4)(1)}{2} = 1$$

so true for $n=1$

R1

Note: Accept $P_r(1) = 1$ and $P_r(1) = \frac{(r-2)(1^2) - (r-4)(1)}{2} = 1$.

Do not accept one-sided considerations such as ‘ $P_r(1) = 1$ and so true for $n=1$ ’.

Subsequent marks after this **R1** are independent of this mark can be awarded.

$$\text{Assume true for } n=k, \text{ ie. } P_r(k) = \frac{(r-2)k^2 - (r-4)k}{2}$$

M1

Note: Award **M0** for statements such as “let $n=k$ ”. The assumption of truth must be clear.
Subsequent marks after this **M1** are independent of this mark and can be awarded.

Consider $n = k + 1$:

$(P_r(k+1))$ can be represented by the sum

$$\sum_{m=1}^{k+1} (1 + (m-1)(r-2)) = \sum_{m=1}^k (1 + (m-1)(r-2)) + (1 + k(r-2)) \text{ and so}$$

$$P_r(k+1) = \frac{(r-2)k^2 - (r-4)k}{2} + (1 + k(r-2)) \quad (P_r(k+1) = P_r(k) + (1 + k(r-2))) \quad \mathbf{M1}$$

$$= \frac{(r-2)k^2 - (r-4)k + 2 + 2k(r-2)}{2} \quad \mathbf{A1}$$

$$= \frac{(r-2)(k^2 + 2k) - (r-4)k + 2}{2}$$

$$= \frac{(r-2)(k^2 + 2k + 1) - (r-2) - (r-4)k + 2}{2} \quad \mathbf{M1}$$

$$= \frac{(r-2)(k+1)^2 - (r-4)k - (r-4)}{2} \quad \mathbf{(A1)}$$

$$= \frac{(r-2)(k+1)^2 - (r-4)(k+1)}{2} \quad \mathbf{A1}$$

hence true for $n=1$ and $n=k$ true $\Rightarrow n=k+1$ true **R1**

therefore true for all $n \in \mathbb{Z}^+$

Note: Only award the final **R1** if the first five marks have been awarded. Award marks as appropriate for solutions that expand both the LHS and (given) RHS of the equation.

[8 marks]

Total [27 marks]

3. (a)

Note: Award a maximum of **M1A0A0** if the candidate manipulates both sides of the equation (such as moving terms from one side to the other).

METHOD 1 (working with LHS)

attempting to expand $(a^2 - 1)^2$ (do not accept $a^4 + 1$ or $a^4 - 1$) **(M1)**

$$\text{LHS} = a^2 + \frac{a^4 - 2a^2 + 1}{4} \text{ or } \frac{4a^2 + a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= \frac{a^4 + 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= \left(\frac{a^2 + 1}{2} \right)^2 (= \text{RHS}) \quad \textbf{AG}$$

Note: Do not award the final A1 if further working contradicts the AG.

METHOD 2 (working with RHS)

attempting to expand $(a^2 + 1)^2$ **(M1)**

$$\text{RHS} = \frac{a^4 + 2a^2 + 1}{4}$$

$$= \frac{4a^2 + a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= a^2 + \frac{a^4 - 2a^2 + 1}{4} \quad \textbf{A1}$$

$$= a^2 + \left(\frac{a^2 - 1}{2} \right)^2 (= \text{LHS}) \quad \textbf{AG}$$

Note: Do not award the final A1 if further working contradicts the AG.

[3 marks]*continued...*

Question 3 continued

- (b) recognise base and height as a and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) **(M1)**

correct substitution into triangle area formula **A1**

$$\text{Area} = \frac{a}{2} \left(\frac{a^2-1}{2} \right) \text{ (or equivalent) } \left(= \frac{a(a^2-1)}{4} = \frac{a^3-a}{4} \right)$$

[2 marks]

Total [5 marks]

5. (a) product of roots = 80 (A1)

$3 - i$ is a root (A1)

attempt to set up an equation involving the product of their four roots and ± 80 (M1)

$$(3+i)(3-i)\alpha^3 = 80 \Rightarrow 10\alpha^3 = 80$$

$$\alpha = 2 \quad \text{A1}$$

[4 marks]

(b) **METHOD 1**

sum of roots = $-p$ (A1)

$$-p = 3 + i + 3 - i + 2 + 4 \quad \text{(M1)}$$

Note: Accept $p = 3 + i + 3 - i + 2 + 4$ for (M1)

$$p = -12 \quad \text{A1}$$

METHOD 2

$$(z - (3+i))(z - (3-i))(z - 2)(z - 4) \quad \text{(M1)}$$

$$((z-3)-i)((z-3)+i)(z-2)(z-4) \quad \text{(A1)}$$

$$(z^2 - 6z + 10)(z^2 - 6z + 8) = z^4 - 12z^3 + \dots$$

$$p = -12 \quad \text{A1}$$

[3 marks]

Total [7 marks]

12. (a) (i) $z_0 = 1 + i$ **(A1)**

$$\arg(z_0) = \arctan(1) = \frac{\pi}{4} = 45^\circ$$
 A1

Note: Accept any of these three forms, including an answer marked on an Argand diagram.

(ii) $\arg(z_n) = \arctan\left(\frac{1}{n^2 + n + 1}\right)$ **A1**

[3 marks]

(b) (i) attempt to use the compound angle formula for tan **M1**

$$\tan(\arctan(a) + \arctan(b)) = \frac{\tan(\arctan(a)) + \tan(\arctan(b))}{1 - \tan(\arctan(a))\tan(\arctan(b))}$$

$$= \frac{a + b}{1 - ab}$$
 A1

$$\Rightarrow \arctan(a) + \arctan(b) = \arctan\left(\frac{a + b}{1 - ab}\right)$$
 AG

(ii) **METHOD 1**

$$\arg(w_1) = \arg(z_0 z_1) = \arg(z_0) + \arg(z_1)$$
 M1

$$= \arctan(1) + \arctan\left(\frac{1}{3}\right)$$
 (A1)

$$= \arctan\left(\frac{1 + \frac{1}{3}}{1 - \frac{1}{3}}\right)$$
 A1

$$= \arctan(2)$$
 AG

continued...

*Question 12 continued***METHOD 2**

$$w_1 = z_0 z_1 = (1 + i)(3 + i) \quad (M1)$$

$$= 2 + 4i \quad A1$$

$$\arg(w_1) = \arctan\left(\frac{4}{2}\right) \text{ or labelled Argand diagram} \quad A1$$

$$= \arctan(2) \quad AG$$

[5 marks]*continued...*

Question 12 continued

(c) let $n = 0$

$$\text{LHS} = \arg(w_0) = \arg(z_0) = \arctan(1) \left(= \frac{\pi}{4} \right)$$

$$\text{RHS} = \arctan(1) \left(= \frac{\pi}{4} \right) \text{ so LHS} = \text{RHS}$$

R1

Note: Award **R0** for not starting at $n = 0$, for example by referring to the result in (b) (ii) for $n = 1$. Award subsequent marks.

assume true for $n = k$, (so $\arg(w_k) = \arctan(k+1)$)

M1

Note: Do not award **M1** for statements such as “let $n = k$ ” or “ $n = k$ is true”. Subsequent marks can still be awarded.

$$\arg(w_{k+1})$$

$$= \arg(w_k z_{k+1}) (= \arg(w_k) + \arg(z_{k+1}))$$

(M1)

$$= \arctan(k+1) + \arctan\left(\frac{1}{(k+1)^2 + (k+1) + 1}\right)$$

A1

$$= \arctan\left(\frac{(k+1) + \left(\frac{1}{(k+1)^2 + (k+1) + 1}\right)}{1 - (k+1)\left(\frac{1}{(k+1)^2 + (k+1) + 1}\right)}\right)$$

M1

$$= \arctan\left(\frac{(k+1) + \left(\frac{1}{k^2 + 3k + 3}\right)}{1 - (k+1)\left(\frac{1}{k^2 + 3k + 3}\right)}\right)$$

(A1)

$$= \arctan\left(\frac{(k+1)(k^2 + 3k + 3) + 1}{(k^2 + 3k + 3) - (k+1)}\right)$$

continued...

Question 12 continued

$$= \arctan\left(\frac{k^3 + 4k^2 + 6k + 4}{k^2 + 2k + 2}\right) \quad \mathbf{A1}$$

$$= \arctan\left(\frac{(k+2)(k^2 + 2k + 2)}{k^2 + 2k + 2}\right) \quad \mathbf{A1}$$

$$= \arctan(k+2) (= \arctan((k+1)+1)) \quad \mathbf{A1}$$

since true for $n = 0$, and true for $n = k + 1$ if true for $n = k$, the statement is true for all $n \in \mathbb{N}$ by mathematical induction

R1

Note: To obtain the final **R1**, four of the previous marks must have been awarded.

[10 marks]**Total [18 marks]**

3. $86.4 = 50r^3$ (A1)

$r = 1.2 \left(= \sqrt[3]{\frac{86.4}{50}} \right)$ seen anywhere (A1)

$\frac{50(1.2^n - 1)}{0.2} > 33500$ OR $250(1.2^n - 1) = 33500$ (A1)

attempt to solve their geometric S_n inequality or equation (M1)

sketch OR $n > 26.9045$, $n = 26.9$ OR $S_{26} = 28368.8$ OR $S_{27} = 34092.6$ OR algebraic

manipulation involving logarithms

$n = 27$ (accept $n \geq 27$) A1

Total [5 marks]

5.

Note: Do not award any marks if there is clear evidence of adding instead of multiplying, for example ${}^9C_r + (ax)^{9-r} + (1)^r$.

valid approach for expansion (must be the product of a binomial coefficient with $n = 9$ and a power of ax)

(M1)

$${}^9C_r(ax)^{9-r}(1)^r \text{ OR } {}^9C_{9-r}(ax)^r(1)^{9-r} \text{ OR } {}^9C_0(ax)^0(1)^9 + {}^9C_1(ax)^1(1)^8 + \dots$$

recognizing that the term in x^6 is needed

(M1)

$$\frac{\text{Term in } x^6}{21x^2} = kx^4 \text{ OR } r = 6 \text{ OR } r = 3 \text{ OR } 9 - r = 6$$

correct term or coefficient in binomial expansion (seen anywhere)

(A1)

$${}^9C_6(ax)^6(1)^3 \text{ OR } {}^9C_3a^6x^6 \text{ OR } 84(a^6x^6)(1) \text{ OR } 84a^6$$

EITHER

correct term in x^4 or coefficient (may be seen in equation)

(A1)

$$\frac{{}^9C_6}{21}a^6x^4 \text{ OR } 4a^6x^4 \text{ OR } 4a^6$$

Set their term in x^4 or coefficient of x^4 equal to $\frac{8}{7}a^5x^4$ or $\frac{8}{7}a^5$ (do not accept other powers of x)

(M1)

$$\frac{{}^9C_3}{21}a^6x^4 = \frac{8}{7}a^5x^4 \text{ OR } 4a^6 = \frac{8}{7}a^5$$

continued...

*Question 5 continued***OR**correct term in x^6 or coefficient of x^6 (may be seen in equation) **(A1)**

$$84a^6x^6 \text{ OR } 84a^6$$

Set their term in x^6 or coefficient of x^6 equal to $24a^5x^6$ or $24a^5$
(do not accept other powers of x) **(M1)**

$$84a^6x^6 = 24a^5x^6 \text{ OR } 84a = 24$$

THEN

$$a = \frac{2}{7} \approx 0.286(0.285714\dots) \quad \mathbf{A1}$$

Note: Award **A0** for the final mark for $a = \frac{2}{7}$ and $a = 0$.**Total [6 marks]**

7. METHOD 1 $3i$ (is a root)**A1**(other complex root is) $-3i$ **A1**

Note: Award **A1A1** for $P(3i) = 0$ and $P(-3i) = 0$ seen in their working.
Award **A1** for each correct root seen in sum or product of their roots.

EITHERattempt to find $P(3i) = 0$ or $P(-3i) = 0$ **(M1)**

$$4m - 3mi + \frac{36}{m}(3i)^2 - (3i)^3 = 0$$

$$4m - 3mi - \frac{36}{m}(-9) + 27i = 0$$

attempt to equate the real or imaginary parts

(M1)

$$27 - 3m = 0 \quad \text{OR} \quad 9 \times \frac{36}{m} = 4m$$

ORattempt to equate sum of three roots to $\frac{36}{m}$ **(M1)**

Note: Accept sum of three roots set to $-\frac{36}{m}$.
Award **M0** for stating sum of roots is $\pm \frac{36}{m}$.

$$3i - 3i + r = \frac{36}{m} \left(\Rightarrow r = \frac{36}{m} \right)$$

substitute their r into product of roots**(M1)**

$$(3i)(-3i)\left(\frac{36}{m}\right) = 4m \quad \text{OR} \quad (z^2 + 9)\left(\frac{36}{m} - z\right)$$

$$9 \times \frac{36}{m} = 4m \quad \text{OR} \quad \frac{4m}{9} = \frac{36}{m}$$

continued...

Question 7 continued

OR

attempt to equate product of three roots to $4m$

(M1)

Note: Accept product of three roots set to $-4m$.
Award **MO** for stating product of roots is $\pm 4m$.

$$(3i)(-3i) \times r = 4m \left(\Rightarrow r = \frac{4m}{9} \right)$$

substitute their r into sum of roots

(M1)

$$3i - 3i + \frac{4m}{9} = \frac{36}{m} \quad \text{OR} \quad (z^2 + 9) \left(\frac{4m}{9} - z \right)$$

$$\frac{4m}{9} = \frac{36}{m}$$

THEN

$$m = 9$$

(A1)

third root is 4

A1

[6 marks]

Question 7 continued**METHOD 2** $3i$ (is a root)**A1**(other complex root is) $-3i$ **A1**recognition that the other factor is $(z + 3i)$ and attempt to write $P(z)$ as product of three linear factors or as product of a quadratic and a linear factor**(M1)**

$$P(z) = (z - 3i)(z + 3i)(r - z) \text{ OR } (z - 3i)(z + 3i) = z^2 + 9 \Rightarrow P(z) = (z^2 + 9)\left(\frac{4m}{9} - z\right)$$

Note: Accept any attempt at long division of $P(z)$ by $z^2 + 9$.Award **M0** for stating other factor is $(z + 3i)$ or obtaining $z^2 + 9$ with no further working.

attempt to compare their coefficients

(M1)

$$-9 = -m \text{ OR } \frac{4m}{9} = \frac{36}{m}$$

$$m = 9$$

(A1)

third root is 4

A1**Note:** Award a maximum of **A0A0(M1)(M1)(A1)A1** for a final answer $P(z) = (z - 3i)(z + 3i)(4 - z)$ seen or stating all three correct factors with no evidence of roots throughout their working.**[6 marks]**

10. (a) (i) recognition that $n = 5$ (M1)
 $S_5 = 45$ A1

(ii) **METHOD 1**
 recognition that $S_5 + u_6 = S_6$ (M1)
 $u_6 = 15$ A1

METHOD 2

recognition that $60 = \frac{6}{2}(S_1 + u_6)$ (M1)
 $60 = 3(5 + u_6)$
 $u_6 = 15$ A1

METHOD 3

substituting their u_1 and d values into $u_1 + (n-1)d$ (M1)
 $u_6 = 15$ A1

[4 marks]

(b) recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their u_6 into S_6 (M1)

OR equations for S_5 and S_6 in terms of u_1 and d

$$1 + 4 \quad \text{OR} \quad 60 = \frac{6}{2}(u_1 + 15)$$

$u_1 = 5$ A1

[2 marks]

continued...

Question 10 continued

(c) **EITHER**valid attempt to find d (may be seen in (a) or (b)) (M1)

$$d = 2 \quad (A1)$$

ORvalid attempt to find $S_n - S_{n-1}$ (M1)

$$n^2 + 4n - (n^2 - 2n + 1 + 4n - 4) \quad (A1)$$

OR

equating $n^2 + 4n = \frac{n}{2}(5 + u_n)$ (M1)

$$2n + 8 = 5 + u_n \text{ (or equivalent)} \quad (A1)$$

THEN

$$u_n = 5 + 2(n - 1) \text{ OR } u_n = 2n + 3 \quad A1$$

[3 marks](d) recognition that $v_2 r^2 = v_4$ OR $(v_3)^2 = v_2 \times v_4$ (M1)

$$r^2 = 3 \text{ OR } v_3 = (\pm)5\sqrt{3} \quad (A1)$$

$$r = \pm\sqrt{3} \quad A1$$

Note: If no working shown, award **M1A1A0** for $\sqrt{3}$.**[3 marks]**(e) recognition that r is negative (M1)

$$v_5 = -15\sqrt{3} \quad \left(= -\frac{45}{\sqrt{3}} \right) \quad A1$$

[2 marks]**Total [14 marks]**

7. let $P(n)$ be the proposition that $\sum_{r=1}^n \frac{r}{(r+1)!} = 1 - \frac{1}{(n+1)!}$ for all integers, $n \geq 1$

considering $P(1)$:

$$\text{LHS} = \frac{1}{2} \text{ and } \text{RHS} = \frac{1}{2} \text{ and so } P(1) \text{ is true} \quad \mathbf{R1}$$

$$\text{assume } P(k) \text{ is true ie, } \sum_{r=1}^k \frac{r}{(r+1)!} = 1 - \frac{1}{(k+1)!} \quad \mathbf{M1}$$

Note: Do not award **M1** for statements such as “let $n = k$ ” or “ $n = k$ is true”.
Subsequent marks after this **M1** are independent of this mark and can be awarded.

considering $P(k+1)$:

$$\sum_{r=1}^{k+1} \frac{r}{(r+1)!} = \sum_{r=1}^k \frac{r}{(r+1)!} + \frac{k+1}{((k+1)+1)!} \quad \mathbf{(M1)}$$

$$= 1 - \frac{1}{(k+1)!} + \frac{k+1}{(k+2)!} \quad \mathbf{A1}$$

$$= 1 - \frac{(k+2) - (k+1)}{(k+2)!} \quad \mathbf{A1}$$

$$= 1 - \frac{1}{(k+2)!} \left(= 1 - \frac{1}{((k+1)+1)!} \right) \quad \mathbf{A1}$$

$P(k+1)$ is true whenever $P(k)$ is true and $P(1)$ is true, so $P(n)$ is true
(for all integers, $n \geq 1$)

R1

Note: To obtain the final **R1**, any four of the previous marks must have been awarded.

[7 marks]

11. (a) METHOD 1

$$|u| = \sqrt{(-1)^2 + (\sqrt{3})^2} (= \sqrt{1+3})$$

A1

$$= 2$$

AG

$$\text{reference angle} = \frac{\pi}{3} \text{ OR } \arg u = \pi - \tan^{-1}(\sqrt{3}) \text{ OR } \arg u = \pi + \tan^{-1}(-\sqrt{3})$$

M1

$$= \pi - \frac{\pi}{3}$$

A1

Note: Award the above **M1A1** for a labelled diagram that convincingly shows that $\arg u = \frac{2\pi}{3}$.

$$= \frac{2\pi}{3} \text{ and so } u = 2e^{i\frac{2\pi}{3}}$$

AG**[3 marks]****METHOD 2**

$$\text{reference angle} = \frac{\pi}{3} \text{ OR } \arg u = \pi - \tan^{-1}(\sqrt{3}) \text{ OR } \arg u = \pi + \tan^{-1}(-\sqrt{3})$$

M1

$$= \pi - \frac{\pi}{3}$$

A1

Note: Award the above **M1A1** for a labelled diagram that convincingly shows that $\arg u = \frac{2\pi}{3}$.

$$= \frac{2\pi}{3}$$

AG

$$r \left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} \right) = -1 + \sqrt{3}i$$

$$r = \frac{-1}{\cos \frac{2\pi}{3}} = \frac{-1}{-\frac{1}{2}} \text{ OR } r = \frac{\sqrt{3}}{\sin \frac{2\pi}{3}} = \frac{\sqrt{3}}{\frac{\sqrt{3}}{2}}$$

A1

$$= 2 \text{ and so } u = 2e^{i\frac{2\pi}{3}}$$

AG**[3 marks]***continued...*

Question 11 continued

(b) (i) $u^n \in \mathbb{R} \Rightarrow \frac{2n\pi}{3} = k\pi \quad (k \in \mathbb{Z})$ **(M1)(A1)**

Note: Award **M1** for noting that $\sin \frac{2n\pi}{3} = 0$ from $u^n = 2^n \left(\cos \frac{2n\pi}{3} + i \sin \frac{2n\pi}{3} \right)$.
Award **(A1)** for a multiple of 3 considered.

$n = 3$ **A1**

(ii) substitutes their value (must be a multiple of 3) for n into u^n **(M1)**

$$u^3 = 2^3 \cos 2\pi$$

$= 8$ **A1**

[5 marks]

continued...

Question 11 continued

- (c) (i)
- $-1 - \sqrt{3}i$
- is a root (by the conjugate root theorem)

A1**Note:** Accept $2e^{-i\frac{2\pi}{3}}$.let $z = c$ be the real root**EITHER**uses sum of roots (equated to ± 5)**(M1)**

$$\left((-1 + \sqrt{3}i) + (-1 - \sqrt{3}i) + c\right) = -5$$

(A1)

$$-2 + c = -5$$

(A1)**OR**uses product of roots (equated to ± 12)**(M1)**

$$(-1 + \sqrt{3}i)(-1 - \sqrt{3}i)c = -12$$

(A1)

$$4c = -12$$

(A1)**OR**

$$\left(z - (-1 + \sqrt{3}i)\right)\left(z - (-1 - \sqrt{3}i)\right) = z^2 + 2z + 4$$

(A1)

compares coefficients eg

(M1)

$$(z - c)(z^2 + 2z + 4) = z^3 + 5z^2 + 10z + 12$$

$$-4c = 12$$

(A1)**THEN** $c = -3$ (and so $z = -3$ is a root)**A1***continued...*

Question 11 continued

(ii) **METHOD 1**compares $z^3 + 5z^2 + 10z + 12 = 0$ and $1 + 5w + 10w^2 + 12w^3 = 0$

$$z = \frac{1}{w} \Rightarrow w = \frac{1}{z} \quad \mathbf{A2}$$

$$w = -\frac{1}{3}, \frac{1}{-1 \pm \sqrt{3}i} \left(= \frac{-1 \pm \sqrt{3}i}{4} \right) \quad \mathbf{A1A1}$$

METHOD 2attempts to factorize into a product of a linear factor and a quadratic factor **(M1)**

$$1 + 5w + 10w^2 + 12w^3 = (3w + 1)(4w^2 + 2w + 1) \quad \mathbf{A1}$$

$$w = -\frac{1}{3}, \frac{1}{-1 \pm \sqrt{3}i} \left(= \frac{-1 \pm \sqrt{3}i}{4} \right) \quad \mathbf{A1A1}$$

[9 marks]

$$(d) \quad (a + bi)^2 = 2(a - bi) \quad \mathbf{A1}$$

attempts to expand and equate real and imaginary parts: **M1**

$$a^2 - b^2 + 2abi = 2a - 2bi$$

$$a^2 - b^2 = 2a \text{ and } 2ab = -2b$$

attempts to find the value of a or b **M1**

$$2b(a + 1) = 0$$

$$b = 0 \Rightarrow a^2 = 2a \Rightarrow a = 2 \text{ (real root)} \quad \mathbf{A1}$$

$$a = -1 \Rightarrow 1 - b^2 = -2 \Rightarrow b = \pm\sqrt{3} \text{ (complex roots } -1 \pm \sqrt{3}i) \quad \mathbf{A1}$$

[5 marks]**Total [22 marks]**

5. product of a binomial coefficient, a power of ax^3 and a power of b seen **(M1)**
evidence of correct term chosen

for $n=8$: $r=2$ (or $r=6$) OR for $n=10$: $r=2$ (or $r=8$) **(A1)**

correct equations (may include powers of x) **A1A1**

$${}^8C_2 a^2 b^6 = 448 \left(28a^2 b^6 = 448 \Rightarrow a^2 b^6 = 16 \right), {}^{10}C_2 a^2 b^8 = 2880 \left(45a^2 b^8 = 2880 \Rightarrow a^2 b^8 = 64 \right)$$

attempt to solve their system in a and b algebraically or graphically **(M1)**

$$b = 2; a = \frac{1}{2} \quad \mathbf{A1A1}$$

Note: Award a maximum of **(M1)(A1)A1A1(M1)A1A0** for $b = \pm 2$ and/or $a = \pm \frac{1}{2}$.

[7 marks]

6. (a) attempt to use De Moivre's theorem (M1)

$$\left(\cos \frac{11\pi}{18} + i \sin \frac{11\pi}{18} \right)^n = \cos \frac{11\pi n}{18} + i \sin \frac{11\pi n}{18} \left(= e^{\frac{11\pi n i}{18}} \right) \text{ OR } \cos(110^\circ n) + i \sin(110^\circ n)$$

EITHER

attempt to consider imaginary part (M1)

$$\sin \frac{11\pi n}{18} = -1 \text{ OR } \sin(110^\circ n) = -1$$

OR

attempt to consider argument of $-i$ (M1)

$$e^{\frac{11\pi n i}{18}} = e^{\frac{3\pi i}{2}}$$

THEN

$$\frac{11\pi n}{18} = \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \dots \left(= \frac{3\pi}{2} + 2\pi k, k \in \mathbb{Z} \right) \text{ OR}$$

$$110^\circ n = 270^\circ, 630^\circ, 990^\circ, \dots \left(= 270^\circ + 360^\circ k, k \in \mathbb{Z} \right) \quad \text{(A1)}$$

$$11n = 27, 63, 99, \dots$$

$$n = 9$$

A1

[4 marks]

continued...

Question 6 continued

(b) **EITHER**

$$z^{10} = e^{10\left(\frac{11\pi}{18}i\right)} \left(= e^{\frac{55\pi}{9}i} = e^{\frac{\pi}{9}i} \right) \text{ OR } \arg(z^{10}) = \frac{\pi}{9} \text{ OR } \arg(z^{10}) = 20^\circ \quad (\mathbf{A1})$$

Note: Accept equivalent arguments given in any interval, in degrees or radians.

recognising that the difference between $\arg(z^{10})$ and $\arg(z)$ is needed (M1)

$$\arg(z^{10}) - \arg(z) = \frac{\pi}{9} - \frac{11\pi}{18} = -\frac{\pi}{2}$$

OR

recognising that $z^{10} = z^9 \times z$ (M1)

$$z^9 = e^{9\left(\frac{11\pi}{18}i\right)} \left(= e^{\frac{11\pi}{2}i} = e^{\frac{3\pi}{2}i} \right) \text{ OR } \arg(z^9) = \frac{3\pi}{2} \text{ or } -\frac{\pi}{2} \text{ OR } \arg(z^9) = 270^\circ \text{ or } -90^\circ \quad (\mathbf{A1})$$

Note: Accept equivalent arguments given in any interval, in degrees or radians.

THEN

a rotation $\frac{3\pi}{2}$ OR $-\frac{\pi}{2}$ OR equivalent angle about the origin. A1

Note: Accept correct answer given in degrees.

Accept $\frac{\pi}{2}$ clockwise or $\frac{11\pi}{2}$ or $\frac{(4k-1)\pi}{2}$ for $k \in \mathbb{Z}$.

The centre must be stated to gain the final **A1**.

[3 marks]

Total [7 marks]

9. Assume $p^2 - 8q - 11 = 0$, ($p, q \in \mathbb{Z}$) **M1**

Note: This **M1** is dependent on the assumption of truth (implied by “assume” or “suppose that ... is true”.)
Subsequent marks should be awarded independently.

EITHER

$$p^2 = 8q + 11 (= 2(4q + 5) + 1) \text{ so } p^2 \text{ odd} \Rightarrow p \text{ odd} \quad \textbf{R1}$$

OR

$$p \text{ even} \Rightarrow p^2 - 8q = 11 \text{ even which is a contradiction so } p \text{ is odd} \quad \textbf{R1}$$

Note: This **R1** should be awarded for any valid reason to conclude that p must be odd.

THEN

$$p = 2k + 1, (k \in \mathbb{Z}) \quad \textbf{M1}$$

$$(2k + 1)^2 = 8q + 11$$

$$4k^2 + 4k + 1 = 8q + 11 \quad \textbf{(A1)}$$

$$4k^2 + 4k = 8q + 10$$

$$2k^2 + 2k = 4q + 5 \text{ or equivalent with one side odd and one side even} \quad \textbf{A1}$$

a contradiction as LHS is even and RHS is odd **R1**

Note: This **R1** is dependent on all previous marks.

Accept correct variations such as work based on $p = 2k - 1$.

therefore, if $p, q \in \mathbb{Z}$ then $p^2 - 8q - 11 \neq 0$ **AG**

Total [6 marks]

Section B

4. attempt to use the binomial expansion of $(x+h)^8$ (M1)

$${}^8C_0x^8h^0 + {}^8C_1x^7h^1 + {}^8C_2x^6h^2 + \dots$$

$$a = 8h \text{ (accept } {}^8C_1 h \text{)} \quad \text{A1}$$

$$b = 28h^2 \text{ (accept } {}^8C_2 h^2 \text{)} \quad \text{A1}$$

$$d = 70h^4 \text{ (accept } {}^8C_4 h^4 \text{)} \quad \text{A1}$$

recognition that there is a common ratio between their terms (M1)

$$8h \times r = 28h^2 \text{ OR } 28h^2 \times r = 70h^4 \text{ OR } 8h \times r^2 = 70h^4$$

correct equation in terms of h (A1)

$$\frac{28h^2}{8h} = \frac{70h^4}{28h^2} \text{ (or equivalent)}$$

$$h = 1.4 \quad \text{A1}$$

[7 marks]

9. METHOD 1

10 numbers of the form $3n$, 10 numbers of the form $(3n-1)$ and 10 numbers of the form $(3n-2)$ (may be seen anywhere) **(M1)**

considers one of the following two cases of forming a sum divisible by 3 **(M1)**

case 1:

chooses 3 numbers of the form $3n$ or chooses 3 numbers of the form $(3n-1)$ or chooses 3 numbers of the form $(3n-2)$

$${}^{10}C_3 + {}^{10}C_3 + {}^{10}C_3 \left(= 3 \times {}^{10}C_3 = 3 \times 120 = 360 \right) \text{ ways}$$

A1

case 2:

chooses 1 number of the form $3n$ and chooses 1 number of the form $(3n-1)$ and chooses 1 number of the form $(3n-2)$

$${}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \left(= ({}^{10}C_1)^3 = 10^3 = 1000 \right) \text{ ways OR } \frac{{}^{30}C_1 \times {}^{20}C_1 \times {}^{10}C_1}{3!} (= 1000) \text{ ways}$$

A1

$$\text{total number of ways is } 3 \times {}^{10}C_3 + {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \left(= 360 + 1000 \right)$$

$$= 1360$$

A1**METHOD 2**

total number of ways of choosing 3 numbers (without restriction) is ${}^{30}C_3 = 4060$

A1

attempts to find the total number of ways of choosing 3 numbers whose sum is not divisible by 3 **(M1)**

chooses 2 numbers from one group and chooses 1 number from another group

eg chooses 2 numbers of the form $3n$ and chooses 1 number of the form $3n-1$

$$3! \times {}^{10}C_2 \times {}^{10}C_1 = 2700$$

(M1)A1

Note: Award **(M1)** for any integer multiple of ${}^{10}C_2 \times {}^{10}C_1$.

$$\text{total number of ways is } 4060 - 2700$$

$$= 1360$$

A1**[5 marks]**

Section B

2. (a) $m = 2$, $c = -1$

$$r = \frac{1}{2} \quad \text{(A1)}$$

$$2, \frac{1}{2}, -1$$

EITHER

$$d \left(= \frac{1}{2} - 2 = -1 - \frac{1}{2} \right) = -\frac{3}{2} \quad \text{A1}$$

OR

this sequence has a common difference of $-\frac{3}{2}$ A1

OR

the (arithmetic) mean of 2 and -1 is $\frac{1}{2}$ A1

THEN

hence $L(x) = 2x - 1$ is an AS-linear function AG
[2 marks]

(b) (i) $(L(r) = 0 \Rightarrow) mr + c = 0$ A1

$$r = -\frac{c}{m} \quad \text{AG}$$

Note: Award **A0** for numerical verification from $L(x) = 2x - 1$ in part (a).

[1 mark]

continued...

Question 2 continued

(ii) **METHOD 1****EITHER**attempts to use $(d =) r - m = c - r$ **(M1)**

Note: Award **(M1)** for attempting to use $(d =) m - r = r - c$.

$$(d =) -\frac{c}{m} - m = c - \left(-\frac{c}{m}\right) \quad \mathbf{A1}$$

Note: Award **A1** for $(d =) -\frac{c}{m} - m = \frac{c - m}{2}$.

removes the denominator m from their expression involving m and c **(M1)**
 $m^2 + cm + 2c = 0$ (or equivalent)

OR

attempts to use $\frac{m + c}{2} = r$ **(M1)**

$$m + c = -\frac{2c}{m} \quad \mathbf{A1}$$

removes the denominator m from their expression involving m and c **(M1)**
 $m^2 + cm + 2c = 0$ (or equivalent)

OR

attempts to use $c = m + 2d$ **(M1)**

$$c = m + 2\left(-\frac{c}{m} - m\right) \quad \mathbf{A1}$$

Note: Award **A1** for $c = m + 2\left(c - \left(-\frac{c}{m}\right)\right)$.

removes the denominator m from their expression involving m and c **(M1)**
 $m^2 + cm + 2c = 0$ (or equivalent)

continued...

Question 2 continued

ORattempts to use $r = m + d$ and $c = m + 2d$ ($c = m + 2(r - m)$) **(M1)** $m^2 + dm + m + 2d = 0$ (or equivalent) **A1**substitutes $d = \frac{c - m}{2}$ into their expression involving m and d **(M1)** $m^2 + cm + 2c = 0$ (or equivalent)**THEN** $c(m + 2) = -m^2 \Rightarrow c = -\frac{m^2}{m + 2}$ **A1****Note:** Award **A1** for a convincing demonstration that $c = -\frac{m^2}{m + 2}$.so $L(x) = mx - \frac{m^2}{m + 2}$ **AG****Note:** Do not accept working backwards from the **AG**.**METHOD 2**considers $L(x) = mx - mr$ attempts to use $(d =)r - m = c - r$ **(M1)****Note:** Award **(M1)** for attempting to use $(d =)m - r = r - c$. $(d =)r - m = -mr - r$ **A1**attempts to express r in terms of m **(M1)** $2r + mr = m \Rightarrow r = \frac{m}{m + 2}$ **A1**so $L(x) = mx - \frac{m^2}{m + 2}$ **AG****Note:** Do not accept working backwards from the **AG**.**[4 marks]**

continued...

Question 2 continued

(iii) $m \neq -2$ ($m \neq 0$)

A1**[1 mark]**

(c)

attempts to find an integer value of m **(M1)**e.g. uses the result that $m+2$ exactly divides 2 OR uses a table OR uses a graph and slider OR uses systematic trial and error

Note: Award **(M1)** for solving $m^2 = k(m+2)$ for m or solving $mr - \frac{m^2}{m+2} = 0$ for m or solving $m^2 + cm + 2c = 0$ for m .

$m = -4$ OR $m = -3$

(A1)

$-4, 2, 8$ OR $-3, 3, 9$

$L(x) = -4x + 8$, $L(x) = -3x + 9$

A1

Note: Award **(M1)(A1)A0** for $-4x + 8$ and $-3x + 9$.

[3 marks]

(d) (i) $-\frac{b}{a}$

A1**[1 mark]**

(ii) $\frac{c}{a}$

A1**[1 mark]**

continued...

Question 2 continued(e) (i) $b - a$ **A1**

Note: Award marks as appropriate in parts (e) (ii) and (iii) for use of $r_1 - r_2 = a - b$.

[1 mark](ii) attempts to eliminate r_2 **M1**

$$2r_1 = -\frac{b}{a} - (b - a) \Rightarrow 2r_1 = \frac{a^2 - ab - b}{a} \text{ (or equivalent)}$$

A1

Note: Award **A1** for a correct alternative form of $\pm r_1$ or $\pm 2r_1$.

$$\text{so } r_1 = \frac{a^2 - ab - b}{2a}$$

AG

Note: Do not accept working backwards from the **AG**.

[2 marks]*continued...*

Question 2 continued

(iii) **METHOD 1****EITHER**

$$(r_1 =) \frac{a+b}{2} \quad (\text{A1})$$

attempts to equate two expressions for either r_1 or $2r_1$ in terms of a and b **M1**

$$\frac{a+b}{2} = \frac{a^2 - ab - b}{2a} \text{ OR } a+b = \frac{a^2 - ab - b}{a}$$

OR

$$b - r_1 = r_1 - a \quad (\text{A1})$$

attempts to use $b - r_1 = r_1 - a$ with $r_1 = \frac{a^2 - ab - b}{2a}$ **M1**

$$b - \left(\frac{a^2 - ab - b}{2a} \right) = \frac{a^2 - ab - b}{2a} - a$$

OR

$$(r_1 =) a + d \quad (\text{A1})$$

attempts to use $r_1 = a + d$ with $r_1 = \frac{a^2 - ab - b}{2a}$ and $d = \frac{b-a}{2}$ **M1**

$$\frac{a^2 - ab - b}{2a} = a + \frac{b-a}{2}$$

THEN

$$2a^2 + 2ab = 2a^2 - 2ab - 2b \text{ OR } a^2 + ab = a^2 - ab - b$$

$$4ab + 2b = 0 \text{ OR } 2ab + b = 0$$

$$2b(2a+1) = 0 \text{ OR } b(2a+1) = 0 \quad \text{A1}$$

Note: Award **(A1)M1** for any valid approach that correctly leads to $2ab + b = 0$ (or equivalent).

Do not accept numerical verification from the **AG**.

$$\text{so } b = 0 \text{ or } a = -\frac{1}{2} \quad \text{AG}$$

continued...

*Question 2 continued***METHOD 2**

$$(b =) a + 2d \text{ OR } (r_1 =) a + d \quad \textbf{(A1)}$$

attempts to equate two expressions for either r_1 or $2r_1$ in terms of a and d **M1**

$$a + d = \frac{a^2 - a(a + 2d) - (a + 2d)}{2a} \textbf{ OR } 2(a + d) = \frac{a^2 - a(a + 2d) - (a + 2d)}{a}$$

$$2a^2 + 4ad + a + 2d = 0$$

$$(2a + 1)(a + 2d) = 0 \quad \textbf{A1}$$

Note: Do not accept numerical verification from the **AG**.

$$\text{so } b = 0 \text{ or } a = -\frac{1}{2} \quad \textbf{AG}$$

[3 marks]

continued...

Question 2 continued

(f) METHOD 1

$$r_1 = \frac{a}{2} \text{ OR } r_2 = -\frac{a}{2} \text{ OR } d = -\frac{a}{2} \quad (\text{A1})$$

$$c = -a \quad (\text{A1})$$

attempts to find the values of a (M1)

EITHER

the roots of $ax^2 - a = 0$ are ± 1 and $\frac{a}{2} = \pm 1$

OR

substitutes $x = \pm \frac{a}{2}$ into $ax^2 - a = 0$ giving $\frac{a^3}{4} - a = 0$

OR

$$(r_1 r_2 =) \frac{c}{a} = -\frac{a^2}{4} \Rightarrow c = -\frac{a^3}{4} \text{ and so } -a = -\frac{a^3}{4} \Rightarrow \frac{a^3}{4} - a = 0$$

Note: Award **(M1)** for attempting to find the values of a from their arithmetic sequence expressed in terms of a .

OR

$$c - r_2 = r_2 - b \Rightarrow -\frac{a^3}{4} - \left(-\frac{a}{2}\right) = -\frac{a}{2} \Rightarrow \frac{a^3}{4} - a = 0$$

THEN

$$a = \pm 2 \quad (\text{A1})$$

$$(r_1 = \pm 1, b = 0, r_2 = \mp 1, c = \mp 2)$$

$$\text{so } Q(x) = 2x^2 - 2, Q(x) = -2x^2 + 2 \quad \text{A1}$$

Note: Award **A0** for $2x^2 - 2$ and $-2x^2 + 2$.
Award **(A1)(A1)(M1)(A0)A0** for obtaining either $a = 2$ or $a = -2$.

continued...

*Question 2 continued***METHOD 2**

$$r_1 = -d \text{ OR } r_2 = d \text{ OR } a = -2d \quad (\text{A1})$$

$$c = 2d \quad (\text{A1})$$

attempts to find the values of d (M1)

EITHER

the roots of $-2dx^2 + 2d = 0$ are ± 1

OR

substitutes $x = \pm d$ into $-2dx^2 + 2d = 0$ giving $-2d^3 + 2d = 0$

OR

attempts to use $r_1 r_2 = \frac{c}{a}$ to form $-d^2 = \frac{2d}{-2d}$

Note: Award **(M1)** for attempting to find the values of d from their arithmetic sequence expressed in terms of d .

THEN

$$d = \pm 1 \quad (\text{A1})$$

$$(a = \pm 2, r_1 = \pm 1, b = 0, r_2 = \mp 1, c = \mp 2)$$

$$\text{so } Q(x) = 2x^2 - 2, Q(x) = -2x^2 + 2 \quad \text{A1}$$

Note: Award **A0** for $2x^2 - 2$ and $-2x^2 + 2$.
Award **(A1)(A1)(M1)(A0)A0** for obtaining either $d = 1$ or $d = -1$.

continued...

*Question 2 continued***METHOD 3**

$$a = 2r_1 \text{ OR } r_2 = -r_1 \text{ OR } d = -r_1 \quad (\text{A1})$$

$$c = -2r_1 \quad (\text{A1})$$

attempts to find the values of r_1 (M1)

EITHER

the roots of $2r_1x^2 - 2r_1 = 0$ are ± 1

OR

substitutes $x = \pm r_1$ into $2r_1x^2 - 2r_1 = 0$ giving $2r_1^3 - 2r_1 = 0$

OR

attempts to use $r_1r_2 = \frac{c}{a}$ to form $-r_1^2 = \frac{-2r_1}{2r_1}$

Note: Award **(M1)** for attempting to find the values of r_1 from their arithmetic sequence expressed in terms of r_1 .

THEN

$$r_1 = \pm 1 \quad (\text{A1})$$

$$(a = \pm 2, r_1 = \pm 1, b = 0, r_2 = \mp 1, c = \mp 2)$$

$$\text{so } Q(x) = 2x^2 - 2, Q(x) = -2x^2 + 2 \quad \text{A1}$$

Note: Award **A0** for $2x^2 - 2$ and $-2x^2 + 2$.
Award **(A1)(A1)(M1)(A0)A0** for obtaining either $r_1 = 1$ or $r_1 = -1$.

[5 marks]

continued...

Question 2 continued

- (g) (i) attempts to express r_1 in terms of b with $a = -\frac{1}{2}$ **(M1)**

Note: Do not award **(M1)** if $a = \frac{1}{2}$ is used.

EITHER

uses $r_1 = \frac{a+b}{2}$

OR

uses $r_1 = \frac{a^2 - ab - b}{2a}$

OR

uses $r_1 - a = b - r_1$

THEN

$$r_1 = \frac{2b-1}{4} \left(= \frac{b}{2} - \frac{1}{4}, = \frac{b - \frac{1}{2}}{2} \right)$$

A1

[2 marks]

continued...

Question 2 continued

(ii) **METHOD 1****EITHER**substitutes their expression for r_1 with $a = -\frac{1}{2}$ into $Q(x)(=0)$ **(M1)**

$$Q\left(\frac{2b-1}{4}\right)(=0) \Rightarrow -\frac{1}{2}\left(\frac{2b-1}{4}\right)^2 + b\left(\frac{2b-1}{4}\right) + c(=0)$$

OR

$$r_2 = \frac{6b+1}{4} \left(= \frac{3b}{2} + \frac{1}{4} \right)$$

substitutes their expression for r_2 with $a = -\frac{1}{2}$ into $Q(x)(=0)$ **(M1)**

$$Q\left(\frac{6b+1}{4}\right)(=0) \Rightarrow -\frac{1}{2}\left(\frac{6b+1}{4}\right)^2 + b\left(\frac{6b+1}{4}\right) + c(=0)$$

THEN

$$c = \frac{4b+1}{2} \left(= 2b + \frac{1}{2} \right) \text{ (seen anywhere)} \quad \mathbf{A1}$$

$$4b^2 + 20b + 5 = 0$$

attempts to solve their quadratic in b **(M1)**

$$b = \frac{-5 \pm 2\sqrt{5}}{2} \quad \mathbf{A1}$$

$$\text{substitutes into } c = \frac{4b+1}{2}$$

$$c = \frac{-9 \pm 4\sqrt{5}}{2} \quad \mathbf{A1}$$

Note: Award **A0A0** for b and c expressed as decimal values.

Note: Award a maximum of **(M1)A1(M1)A0A0FT** for **FT** from part (g) (i).

continued...

Question 2 continued**METHOD 2**

substitutes their expressions for r_1 and r_2 with $a = -\frac{1}{2}$ into $Q(x)$ **(M1)**

$$-\frac{1}{2}\left(x - \left(\frac{2b-1}{4}\right)\right)\left(x - \left(\frac{6b+1}{4}\right)\right)$$

$$-\frac{1}{2}x^2 + bx - \frac{3}{8}b^2 + \frac{1}{8}b + \frac{1}{32}$$

$$c = \frac{4b+1}{2} \left(= 2b + \frac{1}{2}\right) \text{ (seen anywhere)} \quad \textbf{A1}$$

$$2b + \frac{1}{2} = -\frac{3}{8}b^2 + \frac{1}{8}b + \frac{1}{32}$$

$$4b^2 + 20b + 5 = 0$$

attempts to solve their quadratic in b **(M1)**

$$b = \frac{-5 \pm 2\sqrt{5}}{2} \quad \textbf{A1}$$

substitutes into $c = \frac{4b+1}{2}$

$$c = \frac{-9 \pm 4\sqrt{5}}{2} \quad \textbf{A1}$$

Note: Award **A0A0** for b and c expressed as decimal values.

Note: Award a maximum of **(M1)A1(M1)A0A0FT** for **FT** from part (g) (i).

continued...

Question 2 continued

METHOD 3

$$r_2 = \frac{6b+1}{4} \left(= \frac{3b}{2} + \frac{1}{4} \right)$$

substitutes their expressions for r_1 and r_2 with $a = -\frac{1}{2}$ into $r_1 r_2 = \frac{c}{a}$ **(M1)**

$$\left(\frac{2b-1}{4} \right) \left(\frac{6b+1}{4} \right) = \frac{c}{-\frac{1}{2}} \text{ (or equivalent)}$$

EITHER

$$c = \frac{4b+1}{2} \left(= 2b + \frac{1}{2} \right) \text{ (seen anywhere)} \quad \mathbf{A1}$$

OR

$$-\frac{1}{2} \left(\frac{2b-1}{4} \right) \left(\frac{6b+1}{4} \right) - \left(\frac{6b+1}{4} \right) = \frac{2b-1}{4} - \left(-\frac{1}{2} \right) \quad \mathbf{A1}$$

THEN

$$4b^2 + 20b + 5 = 0$$

attempts to solve their quadratic in b **(M1)**

$$b = \frac{-5 \pm 2\sqrt{5}}{2} \quad \mathbf{A1}$$

substitutes into $c = \frac{4b+1}{2}$

$$c = \frac{-9 \pm 4\sqrt{5}}{2} \quad \mathbf{A1}$$

Note: Award **A0A0** for b and c expressed as decimal values.

Note: Award a maximum of **(M1)A1(M1)A0A0FT** for **FT** from part (g) (i).

continued...

Question 2 continued

METHOD 4

attempts to equate two expressions for r_1 with $a = -\frac{1}{2}$ **(M1)**

$$\frac{-b \pm \sqrt{b^2 + 2c}}{-1} = \frac{2b-1}{4} \left(\pm \sqrt{b^2 + 2c} = \frac{2b+1}{4} \right)$$

$$c = \frac{4b+1}{2} \left(= 2b + \frac{1}{2} \right) \text{ (seen anywhere)} \quad \textbf{A1}$$

$$12b^2 - 4b - 1 + 32 \left(2b + \frac{1}{2} \right) = 0 \quad (4b^2 + 20b + 5 = 0)$$

attempts to solve their quadratic in b **(M1)**

$$b = \frac{-5 \pm 2\sqrt{5}}{2} \quad \textbf{A1}$$

substitutes into $c = \frac{4b+1}{2}$

$$c = \frac{-9 \pm 4\sqrt{5}}{2} \quad \textbf{A1}$$

Note: Award **A0A0** for b and c expressed as decimal values.

Note: Award a maximum of **(M1)A1(M1)A0A0FT** for **FT** from part (g) (i).

continued...

Question 2 continued

METHOD 5**EITHER**

$$r_1 = d - \frac{1}{2}$$

substitutes their expression for r_1 in terms of d with $a = -\frac{1}{2}$ into $Q(x)(=0)$ **(M1)**

$$Q\left(d - \frac{1}{2}\right)(=0) \Rightarrow -\frac{1}{2}\left(d - \frac{1}{2}\right)^2 + b\left(d - \frac{1}{2}\right) + c(=0)$$

OR

$$r_2 = 3d - \frac{1}{2}$$

substitutes their expression for r_2 in terms of d with $a = -\frac{1}{2}$ into $Q(x)(=0)$ **(M1)**

$$Q\left(3d - \frac{1}{2}\right)(=0) \Rightarrow -\frac{1}{2}\left(3d - \frac{1}{2}\right)^2 + b\left(3d - \frac{1}{2}\right) + c(=0)$$

THEN

$$b = 2d - \frac{1}{2} \text{ and } c = 4d - \frac{1}{2} \text{ (seen anywhere)} \quad \mathbf{A1}$$

$$4d^2 + 8d - 1 = 0$$

attempts to solve their quadratic in d **(M1)**

$$d = \frac{-2 \pm \sqrt{5}}{2}$$

$$b = \frac{-5 \pm 2\sqrt{5}}{2} \quad \mathbf{A1}$$

$$\text{substitutes into } c = \frac{4b+1}{2}$$

$$c = \frac{-9 \pm 4\sqrt{5}}{2} \quad \mathbf{A1}$$

Note: Award **A0A0** for b and c expressed as decimal values.

Note: Award a maximum of **(M1)A1(M1)A0A0FT** for **FT** from part (g) (i).

continued...

Question 2 continued

METHOD 6

$$r_1 = d - \frac{1}{2} \text{ and } r_2 = 3d - \frac{1}{2}$$

substitutes their expressions for r_1 and r_2 in terms of d with $a = -\frac{1}{2}$ into $r_1 r_2 = \frac{c}{a}$ **(M1)**

$$\left(d - \frac{1}{2}\right)\left(3d - \frac{1}{2}\right) = \frac{c}{-\frac{1}{2}} \text{ (or equivalent)}$$

$$c = 4d - \frac{1}{2} \text{ (seen anywhere)} \quad \mathbf{A1}$$

$$4d^2 + 8d - 1 = 0$$

attempts to solve their quadratic in d **(M1)**

$$d = \frac{-2 \pm \sqrt{5}}{2}$$

$$b = \frac{-5 \pm 2\sqrt{5}}{2} \quad \mathbf{A1}$$

substitutes into $c = \frac{4b+1}{2}$

$$c = \frac{-9 \pm 4\sqrt{5}}{2} \quad \mathbf{A1}$$

Note: Award **A0A0** for b and c expressed as decimal values.

Note: Award a maximum of **(M1)A1(M1)A0A0FT** for **FT** from part (g) (i).

[5 marks]

Total [31 marks]

3. (a) METHOD 1

attempt to form at least one equation, using either S_4 or S_5 (M1)

$$65 = 25p - 5q \quad (13 = 5p - q) \quad \text{and} \quad 40 = 16p - 4q \quad (10 = 4p - q) \quad (A1)$$

valid attempt to solve simultaneous linear equations in p and q by substituting or eliminating one of the variables. (M1)

$$p = 3, \quad q = 2 \quad A1A1$$

Note: If candidate does not explicitly state their values of p and q , but gives $S_n = 3n^2 - 2n$, award final two marks as **A1A0**.

METHOD 2

attempt to form at least one equation, using either S_4 or S_5 (M1)

$$65 = \frac{5}{2}(2u_1 + 4d) \quad (26 = 2u_1 + 4d) \quad \text{and} \quad 40 = 2(2u_1 + 3d) \quad (20 = 2u_1 + 3d) \quad (A1)$$

valid attempt to solve simultaneous linear equations in u_1 and d by substituting or eliminating one of the variables. (M1)

$$u_1 = 1, \quad d = 6 \quad A1$$

$$S_n = \frac{n}{2}(2 + 6(n-1)) = 3n^2 - 2n$$

$$p = 3 \text{ and } q = 2 \quad A1$$

Note: If candidate does not explicitly state their values of p and q , do not award the final mark.

[5 marks]

(b) $u_5 = S_5 - S_4$ OR substituting their values of u_1 and d into $u_5 = u_1 + 4d$

OR substituting their value of u_1 into $65 = \frac{5}{2}(u_1 + u_5)$ (M1)

$$(u_5 =) 65 - 40 \quad \text{OR} \quad (u_5 =) 1 + 4 \times 6 \quad \text{OR} \quad 65 = \frac{5}{2}(1 + u_5)$$

$$= 25 \quad A1$$

[2 marks]**Total [7 marks]**

5. attempt to apply binomial expansion

(M1)

$$(1+kx)^n = 1 + {}^nC_1 kx + {}^nC_2 k^2 x^2 + \dots \quad \text{OR} \quad {}^nC_1 k = 12 \quad \text{OR} \quad {}^nC_2 = 28$$

$$nk = 12$$

(A1)

$$\frac{n(n-1)}{2} = 28 \quad \text{OR} \quad \frac{n!}{(n-2)!2!} = 28$$

(A1)

$$n^2 - n - 56 = 0 \quad \text{OR} \quad n(n-1) = 56$$

valid attempt to solve

(M1)

$$(n-8)(n+7) = 0 \quad \text{OR} \quad 8(8-1) = 56 \quad \text{OR} \quad \text{finding correct value in Pascal's triangle}$$

$$\Rightarrow n = 8$$

A1

$$\Rightarrow k = \frac{3}{2}$$

A1

Note: If candidate finds $n = 8$ with no working shown, award **M1A0A0M1A1A0**.

If candidate finds $n = 8$ and $k = \frac{3}{2}$ with no working shown, award

M1A0A0M1A1A1.

[6 marks]

6. base case $n = 1$: $5^2 - 2^3 = 25 - 8 = 17$ so true for $n = 1$

A1

assume true for $n = k$ ie $5^{2k} - 2^{3k} = 17s$ for $s \in \mathbb{Z}$ OR $5^{2k} - 2^{3k}$ is divisible by 17

M1

Note: The assumption of truth must be clear. Do not award the **M1** for statements such as “let $n = k$ ” or “ $n = k$ is true”. Subsequent marks can still be awarded.

EITHER

consider $n = k + 1$:

M1

$$5^{2(k+1)} - 2^{3(k+1)}$$

$$= (5^2)^{2k} - (2^3)^{2k}$$

A1

$$= (25)^{2k} - (8)^{2k}$$

$$= (17)^{2k} + (8)^{2k} - (8)^{2k} \text{ OR } (25)^{2k} - (25)^{2k} + (17)^{2k}$$

A1

$$= (17)^{2k} + 8(5^{2k} - 2^{3k}) \text{ OR } 25(5^{2k} - 2^{3k}) + (17)^{2k}$$

$$= (17)^{2k} + 8(17s) \text{ OR } 25(17s) + (17)^{2k}$$

$$= 17(5^{2k} + 8s) \text{ OR } 17(25s + 2^{3k}) \text{ which is divisible by 17}$$

A1

OR

$$(5^{2k} - 2^{3k}) \times 5^2 = 5^{2k+2} - 25 \times 2^{3k} = 17s \times 25$$

M1

$$= 5^{2k+2} - 8 \times 2^{3k} - 17 \times 2^{3k} = 17s \times 25$$

A1

$$= 5^{2k+2} - 2^{3k+3} - 17 \times 2^{3k} = 17s \times 25$$

$$= 5^{2(k+1)} - 2^{3(k+1)} - 17 \times 2^{3k} = 17s \times 25$$

A1

$$= 5^{2(k+1)} - 2^{3(k+1)} = 17s \times 25 + 17 \times 2^{3k}$$

hence for $n = k + 1$: $5^{2(k+1)} - 2^{3(k+1)} = 17(25s + 2^{3k})$ is divisible by 17

A1

THEN

since true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$,

therefore true for all $n \in \mathbb{Z}^+$

R1

Note: Only award **R1** if 4 of the previous 6 marks have been awarded

Note: 5^{2k} and 2^{3k} may be replaced by 25^k and 8^k throughout.

[7 marks]

7. METHOD 1

attempt to substitute solution into given equation **(M1)**

$$(5 + qi)^2 + i(5 + qi) = -p + 25i$$

$$25 - q^2 + 10qi - q + 5i + p - 25i = 0 \quad \text{OR} \quad 25 - q^2 + 10qi - q + 5i = -p + 25i \quad \textbf{A1}$$

$$25 - q^2 + p - q + (10q - 20)i = 0$$

attempt to equate real or imaginary parts: **(M1)**

$$10q - 20 = 0 \quad \text{OR} \quad 25 - q^2 + p - q = 0$$

$$q = 2, \quad p = -19 \quad \textbf{A1A1}$$

METHOD 2

$$z^2 + iz + p - 25i = 0$$

sum of roots = $-i$, product of roots = $p - 25i$ **M1**

one root is $(5 + qi)$ so other root is $(-5 - qi - i)$ **A1**

$$\text{product}(5 + qi)(-5 - qi - i) = -25 - 5qi - 5i - 5qi + q^2 + q = p - 25i$$

equating real and imaginary parts for product of roots **(M1)**

$$\text{Im: } -25 = -10q - 5 \quad \text{Re: } p = -25 + q^2 + q$$

$$q = 2, \quad p = -19 \quad \textbf{A1A1}$$

[5 marks]

3. (a) METHOD 1

attempt to form at least one equation, using either S_4 or S_5 (M1)

$$65 = 25p - 5q \quad (13 = 5p - q) \quad \text{and} \quad 40 = 16p - 4q \quad (10 = 4p - q) \quad (A1)$$

valid attempt to solve simultaneous linear equations in p and q by substituting or eliminating one of the variables. (M1)

$$p = 3, \quad q = 2 \quad A1A1$$

Note: If candidate does not explicitly state their values of p and q , but gives $S_n = 3n^2 - 2n$, award final two marks as **A1A0**.

METHOD 2

attempt to form at least one equation, using either S_4 or S_5 (M1)

$$65 = \frac{5}{2}(2u_1 + 4d) \quad (26 = 2u_1 + 4d) \quad \text{and} \quad 40 = 2(2u_1 + 3d) \quad (20 = 2u_1 + 3d) \quad (A1)$$

valid attempt to solve simultaneous linear equations in u_1 and d by substituting or eliminating one of the variables. (M1)

$$u_1 = 1, \quad d = 6 \quad A1$$

$$S_n = \frac{n}{2}(2 + 6(n-1)) = 3n^2 - 2n$$

$$p = 3 \text{ and } q = 2 \quad A1$$

Note: If candidate does not explicitly state their values of p and q , do not award the final mark.

[5 marks]

(b) $u_5 = S_5 - S_4$ OR substituting their values of u_1 and d into $u_5 = u_1 + 4d$

OR substituting their value of u_1 into $65 = \frac{5}{2}(u_1 + u_5)$ (M1)

$$(u_5 =) 65 - 40 \quad \text{OR} \quad (u_5 =) 1 + 4 \times 6 \quad \text{OR} \quad 65 = \frac{5}{2}(1 + u_5)$$

$$= 25 \quad A1$$

[2 marks]

Total [7 marks]

5. attempt to apply binomial expansion

(M1)

$$(1+kx)^n = 1 + {}^nC_1 kx + {}^nC_2 k^2 x^2 + \dots \quad \text{OR} \quad {}^nC_1 k = 12 \quad \text{OR} \quad {}^nC_2 = 28$$

$$nk = 12$$

(A1)

$$\frac{n(n-1)}{2} = 28 \quad \text{OR} \quad \frac{n!}{(n-2)!2!} = 28$$

(A1)

$$n^2 - n - 56 = 0 \quad \text{OR} \quad n(n-1) = 56$$

valid attempt to solve

(M1)

$$(n-8)(n+7) = 0 \quad \text{OR} \quad 8(8-1) = 56 \quad \text{OR} \quad \text{finding correct value in Pascal's triangle}$$

$$\Rightarrow n = 8$$

A1

$$\Rightarrow k = \frac{3}{2}$$

A1

Note: If candidate finds $n = 8$ with no working shown, award **M1A0A0M1A1A0**.

If candidate finds $n = 8$ and $k = \frac{3}{2}$ with no working shown, award

M1A0A0M1A1A1.

[6 marks]

6. base case $n = 1$: $5^2 - 2^3 = 25 - 8 = 17$ so true for $n = 1$

A1

assume true for $n = k$ ie $5^{2k} - 2^{3k} = 17s$ for $s \in \mathbb{Z}$ OR $5^{2k} - 2^{3k}$ is divisible by 17

M1

Note: The assumption of truth must be clear. Do not award the **M1** for statements such as “let $n = k$ ” or “ $n = k$ is true”. Subsequent marks can still be awarded.

EITHER

consider $n = k + 1$:

M1

$$5^{2(k+1)} - 2^{3(k+1)}$$

$$= (5^2)^{2k} - (2^3)^{2k}$$

A1

$$= (25)^{2k} - (8)^{2k}$$

$$= (17)^{2k} + (8)^{2k} - (8)^{2k} \text{ OR } (25)^{2k} - (25)^{2k} + (17)^{2k}$$

A1

$$= (17)^{2k} + 8(5^{2k} - 2^{3k}) \text{ OR } 25(5^{2k} - 2^{3k}) + (17)^{2k}$$

$$= (17)^{2k} + 8(17s) \text{ OR } 25(17s) + (17)^{2k}$$

$$= 17(5^{2k} + 8s) \text{ OR } 17(25s + 2^{3k}) \text{ which is divisible by 17}$$

A1

OR

$$(5^{2k} - 2^{3k}) \times 5^2 = 5^{2k+2} - 25 \times 2^{3k} = 17s \times 25$$

M1

$$= 5^{2k+2} - 8 \times 2^{3k} - 17 \times 2^{3k} = 17s \times 25$$

A1

$$= 5^{2k+2} - 2^{3k+3} - 17 \times 2^{3k} = 17s \times 25$$

$$= 5^{2(k+1)} - 2^{3(k+1)} - 17 \times 2^{3k} = 17s \times 25$$

A1

$$= 5^{2(k+1)} - 2^{3(k+1)} = 17s \times 25 + 17 \times 2^{3k}$$

hence for $n = k + 1$: $5^{2(k+1)} - 2^{3(k+1)} = 17(25s + 2^{3k})$ is divisible by 17

A1

THEN

since true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$,

therefore true for all $n \in \mathbb{Z}^+$

R1

Note: Only award **R1** if 4 of the previous 6 marks have been awarded

Note: 5^{2k} and 2^{3k} may be replaced by 25^k and 8^k throughout.

[7 marks]

7. METHOD 1

attempt to substitute solution into given equation

(M1)

$$(5 + qi)^2 + i(5 + qi) = -p + 25i$$

$$25 - q^2 + 10qi - q + 5i + p - 25i = 0 \quad \text{OR} \quad 25 - q^2 + 10qi - q + 5i = -p + 25i$$

A1

$$25 - q^2 + p - q + (10q - 20)i = 0$$

attempt to equate real or imaginary parts:

(M1)

$$10q - 20 = 0 \quad \text{OR} \quad 25 - q^2 + p - q = 0$$

$$q = 2, \quad p = -19$$

A1A1**METHOD 2**

$$z^2 + iz + p - 25i = 0$$

sum of roots = $-i$, product of roots = $p - 25i$ **M1**one root is $(5 + qi)$ so other root is $(-5 - qi - i)$ **A1**

$$\text{product}(5 + qi)(-5 - qi - i) = -25 - 5qi - 5i - 5qi + q^2 + q = p - 25i$$

equating real and imaginary parts for product of roots

(M1)

$$\text{Im: } -25 = -10q - 5 \quad \text{Re: } p = -25 + q^2 + q$$

$$q = 2, \quad p = -19$$

A1A1**[5 marks]**

7. (a) (i) $(9! =) 362880$

A1

Note: Accept 9! or 363000.

(ii) attempt to consider girls as a single object

(M1)

$(3! \times 7! =) 30240$

A1

Note: Accept 30200.

[3 marks]

(b) **METHOD 1**

recognition of the two different cases for 2 girls and 3 girls

(M1)

exactly 2 girls is ${}^6C_3 \times {}^3C_2 = 60$ and exactly 3 girls $({}^3C_3 \times) {}^6C_2 = 15$

(A1)

total $(= 60 + 15) = 75$

A1

METHOD 2

recognition of the three different cases: total choices, 1 girl and no girls

(M1)

total choices ${}^9C_5 = 126$, one girl case ${}^3C_1 \times {}^6C_4 = 45$, no girl case ${}^6C_5 = 6$

(A1)

total $(= 126 - 45 - 6) = 75$

A1

[3 marks]

Total [6 marks]

$(\quad 1 \quad)$

7. (a) (i) $(9! =) 362880$

A1

Note: Accept 9! or 363000.

(ii) attempt to consider girls as a single object

(M1)

$(3! \times 7! =) 30240$

A1

Note: Accept 30200.

[3 marks]

(b) **METHOD 1**

recognition of the two different cases for 2 girls and 3 girls

(M1)

exactly 2 girls is ${}^6C_3 \times {}^3C_2 = 60$ and exactly 3 girls $({}^3C_3 \times) {}^6C_2 = 15$

(A1)

total $(= 60 + 15) = 75$

A1

METHOD 2

recognition of the three different cases: total choices, 1 girl and no girls

(M1)

total choices ${}^9C_5 = 126$, one girl case ${}^3C_1 \times {}^6C_4 = 45$, no girl case ${}^6C_5 = 6$

(A1)

total $(= 126 - 45 - 6) = 75$

A1

[3 marks]

Total [6 marks]

$$\begin{pmatrix} 1 \\ \end{pmatrix}$$

5. (a) $S_n = \frac{10^n - 1}{9}$ **A1**

$$(a=10, b=9)$$

[1 mark]

(b) **METHOD 1**

$$S_1 + S_2 + S_3 + \dots + S_n$$

$$= \frac{10-1}{9} + \frac{10^2-1}{9} + \dots + \frac{10^n-1}{9} \quad \text{(A1)}$$

$$= \frac{10-1+10^2-1+10^3-1+\dots+10^n-1}{9} \quad \text{OR} \quad \frac{9(10-1+10^2-1+10^3-1+\dots+10^n-1)}{81}$$

attempt to use geometric series formula on powers of 10, and collect -1's together **M1**

$$10+10^2+10^3+\dots+10^n = \frac{10(10^n-1)}{10-1} \quad \text{and} \quad -1-1-1\dots = -n \quad \text{A1}$$

$$= \frac{\frac{10(10^n-1)}{10-1} - n}{9} \quad \text{OR} \quad \frac{9\left(\frac{10(10^n-1)}{10-1}\right) - 9n}{81} \quad \text{A1}$$

Note: Award **A1** for any correct intermediate expression.

$$= \frac{10(10^n-1) - 9n}{81} \quad \text{AG}$$

continued...

*Question 5 continued***METHOD 2**attempt to create sum using sigma notation with S_n **M1**

$$\sum_{i=1}^n \frac{10^i - 1}{9} \quad \left(= \frac{1}{9} \left(\sum_{i=1}^n 10^i - \sum_{i=1}^n 1 \right) \right)$$

$$\sum_{i=1}^n 10^i = \frac{10(10^n - 1)}{9}$$

A1

$$\sum_{i=1}^n 1 = n$$

A1

$$= \frac{1}{9} \left(\frac{10(10^n - 1)}{9} - n \right) \text{ OR } \frac{1}{9} \left(\frac{10(10^n - 1) - 9n}{9} \right)$$

A1

$$= \frac{10(10^n - 1) - 9n}{81}$$

AG*continued...*

Question 5 continued

METHOD 3

let $P(n)$ be the proposition that $S_1 + S_2 + S_3 + \dots + S_n = \frac{10(10^n - 1) - 9n}{81}$

considering $P(1)$:

$$\text{LHS} = S_1 = \frac{10^1 - 1}{9} = 1 \quad \text{and} \quad \text{RHS} = \frac{10(10^1 - 1) - 9(1)}{81} = 1 \quad \text{and so } P(1) \text{ is true} \quad \mathbf{R1}$$

$$\text{assume } P(k) \text{ is true i.e. } S_1 + S_2 + S_3 + \dots + S_k = \frac{10(10^k - 1) - 9k}{81} \quad \mathbf{M1}$$

Note: Do not award **M1** for statements such as “let $n = k$ ” or “ $n = k$ is true”. Subsequent marks after this **M1** are independent of this mark and can be awarded.

considering $P(k + 1)$:

$$\begin{aligned} S_1 + S_2 + S_3 + \dots + S_{k+1} &= \frac{10(10^k - 1) - 9k}{81} + \frac{10^{k+1} - 1}{9} \\ &= \frac{10^{k+1} - 10 - 9k + 9(10^{k+1}) - 9}{81} \quad \mathbf{A1} \\ &= \frac{10(10^{k+1} - 1) - 9(k + 1)}{81} \end{aligned}$$

$P(k + 1)$ is true whenever $P(k)$ is true and $P(1)$ is true, so $P(n)$ is true $\mathbf{R1}$

(for all integers $n \geq 1$)

Note: To obtain the final **R1**, the first **R1** and **A1** must have been awarded.

[4 marks]

Total [5 marks]

7. Base case $n = 1$: $LHS = {}^1C_1 = 1$ and $RHS = {}^2C_2 = 1$, so true for $n = 1$

R1

Note: Award **R0** if the value of ${}^1C_1$ and ${}^2C_2$ are not evaluated.

Subsequent marks can still be awarded.

assume true for $n = k$ ie $\sum_{r=1}^k {}^rC_1 = {}^{k+1}C_2$ for some $k \in \mathbb{Z}^+$

M1

Note: The assumption of truth must be clear.

Award **M0** for statements such as “let $n = k$ ” or “ $n = k$ is true”.

Subsequent marks can still be awarded.

consider $n = k + 1$

$$LHS = \sum_{r=1}^{k+1} {}^rC_1$$

$$= \sum_{r=1}^k {}^rC_1 + {}^{k+1}C_1$$

(M1)

$$= {}^{k+1}C_2 + {}^{k+1}C_1 \text{ OR } \frac{(k+1)!}{2(k-1)!} + \frac{(k+1)!}{k!}$$

A1

EITHER

attempt to cancel factorials and use a common denominator

M1

$$= \frac{(k+1)k + 2(k+1)}{2} \left(= \frac{(k+2)(k+1)}{2} \right)$$

OR

attempt to use a common denominator

M1

$$= \frac{k(k+1)!}{2k!} + \frac{2(k+1)!}{2k!} \left(= \frac{(k+2)(k+1)!}{2k!} \right)$$

THEN

$$= \frac{(k+2)!}{2!k!} \left(= \frac{(k+2)!}{2!(k+2-2)!} \right)$$

A1

$$= {}^{k+2}C_2$$

since true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$,

therefore true for all $n \in \mathbb{Z}^+$

R1

Note: Only award the final **R1** if 4 of the previous 6 marks have been awarded.

Total [7 marks]

Note: Throughout this question, condone presence of any additional terms once the first two correct terms are seen.

$\frac{1}{n^3}$

12. (a) attempt to expand the brackets or attempt to find modulus and argument of ϕ **(M1)**

$$(a + bi)^3 = a^3 + 3a^2bi + 3a(bi)^2 + (bi)^3 \quad \text{OR} \quad \left(\sqrt{a^2 + b^2}\right)^3 \operatorname{cis}\left(3 \arctan\left(\frac{b}{a}\right)\right)$$

(i) real part is $a^3 - 3ab^2$ OR $(a^2 + b^2)^{\frac{3}{2}} \cos\left(3 \arctan\left(\frac{b}{a}\right)\right)$ **A1**

(ii) imaginary part is $3a^2b - b^3$ OR $(a^2 + b^2)^{\frac{3}{2}} \sin\left(3 \arctan\left(\frac{b}{a}\right)\right)$ **A1**

Note: Award **(M1)A1A0** for $(a^3 - 3ab^2) + (3a^2b - b^3)i$ OR

$$(a^2 + b^2)^{\frac{3}{2}} \cos\left(3 \arctan\left(\frac{b}{a}\right)\right) + (a^2 + b^2)^{\frac{3}{2}} i \sin\left(3 \arctan\left(\frac{b}{a}\right)\right)$$

For (ii) condone $(3a^2b - b^3)i$ OR $(a^2 + b^2)^{\frac{3}{2}} i \sin\left(3 \arctan\left(\frac{b}{a}\right)\right)$

[3 marks]

- (b) attempt to substitute $a = 1$ and $b = \sqrt{3}$ into their real or imaginary part found in (a)
OR to expand the brackets OR to use polar form **M1**

$$(1 - 9) + (3\sqrt{3} - 3\sqrt{3})i \quad \text{OR} \quad (2\sqrt{3}i - 2)(1 + \sqrt{3}i) = 2\sqrt{3}i - 2 - 6 - 2\sqrt{3}i \quad \text{OR}$$

$$\left(2e^{\frac{i\pi}{3}}\right)^3 = 8e^{i\pi} \quad \text{OR} \quad (2\operatorname{cis}(60^\circ))^3 = 8\operatorname{cis}(180^\circ)$$
 A1

$$= -8$$
 AG

[2 marks]

- (c) $v = -2, w = 1 - \sqrt{3}i$ **A1A1**

Note: Award **A1A0** for $v = 1 - \sqrt{3}i, w = -2$ or if the labels v and w are not clearly specified or missing. Candidates may be awarded full **FT** marks for subsequent parts.

[2 marks]

continued...

*Question 12 continued***(d) METHOD 1**

triangle UVW has height $h = 3$ and base $b = 2\sqrt{3}$ **(A1)**

attempt to find area of triangle with their height and base **(M1)**

$$\text{area} = \frac{1}{2} \times 2\sqrt{3} \times 3$$

$$= 3\sqrt{3} \text{ (square units)} \quad \textbf{A1}$$

METHOD 2

triangle UVW has sides of length $\left(\sqrt{3^2 + (\sqrt{3})^2}\right)\sqrt{12}$ **(A1)**

attempt to find area of equilateral triangle with their side length **(M1)**

$$\text{area} = \frac{1}{2}(\sqrt{12})^2 \sin \frac{\pi}{3} \text{ OR } \frac{1}{2}\sqrt{12}(3) \text{ OR } (\sqrt{12})^2 \times \frac{\sqrt{3}}{4}$$

$$= 3\sqrt{3} \text{ (square units)} \quad \textbf{A1}$$

METHOD 3

triangle UVO has sides of length $\left(\sqrt{1^2 + (\sqrt{3})^2}\right)2$ **(A1)**

attempt to find area of three isosceles triangles with their side length and angle $\frac{2\pi}{3}$ **(M1)**

$$\text{area} = 3\left(\frac{1}{2}(2)^2 \sin \frac{2\pi}{3}\right)$$

$$= 3\sqrt{3} \text{ (square units)} \quad \textbf{A1}$$

[3 marks]*continued...*

Question 12 continued

- (e) attempt to express u, v or w in the form $re^{i\theta}$ and multiply by $e^{\frac{\pi}{4}}$ (M1)

$$\left(u' = 2e^{\frac{\pi}{3}} e^{\frac{\pi}{4}} = \right) 2e^{\frac{7\pi}{12}} \quad \text{A1}$$

$$\left(v' = 2e^{\pi i} e^{\frac{\pi}{4}} = 2e^{\frac{5\pi}{4}} = \right) 2e^{-\frac{3\pi}{4}} \quad \text{A1}$$

$$\left(w' = 2e^{-\frac{\pi}{3}} e^{\frac{\pi}{4}} = \right) 2e^{-\frac{\pi}{12}} \quad \text{A1}$$

Note: These **A1** marks should be awarded independently and in any order.

[4 marks]

- (f) **EITHER**

attempt to find one of $(u')^3, (v')^3$ or $(w')^3$ (M1)

$$(u')^3 = \left(2e^{\frac{7\pi}{12}} \right)^3 = 8e^{\frac{7\pi}{4}} = 8e^{-\frac{\pi}{4}} \quad \text{OR} \quad (v')^3 = \left(2e^{-\frac{3\pi}{4}} \right)^3 = 8e^{-\frac{9\pi}{4}} = 8e^{-\frac{\pi}{4}} \quad \text{OR}$$

$$(w')^3 = \left(2e^{-\frac{\pi}{12}} \right)^3 = 8e^{-\frac{\pi}{4}} \quad \text{(A1)}$$

OR

attempt to find product of their three roots u', v' and w' (M1)

$$u' \times v' \times w' = (c + di)$$

$$2e^{\frac{7\pi}{12}} \times 2e^{-\frac{3\pi}{4}} \times 2e^{-\frac{\pi}{12}} = 8e^{-\frac{\pi}{4}} \quad \text{OR} \quad 2e^{\frac{7\pi}{12}} \times 2e^{\frac{5\pi}{4}} \times 2e^{-\frac{\pi}{12}} = 8e^{-\frac{\pi}{4}} \quad \text{(or equivalent)} \quad \text{(A1)}$$

OR

attempt to find $\left(ze^{\frac{\pi}{4}} \right)^3$ for any z such that $z^3 = -8$ OR to rotate -8 by $\frac{3\pi}{4}$ (M1)

$$\left(\left(ze^{\frac{\pi}{4}} \right)^3 = z^3 e^{\frac{3\pi}{4}} = \right) -8e^{\frac{3\pi}{4}} \quad \text{OR} \quad 8e^{-\frac{\pi}{4}} \quad \text{OR} \quad 8\text{cis}(-45^\circ) \quad \text{(A1)}$$

continued...

Question 12 continued

THEN

$$8e^{\frac{\pi}{4}i} = 8\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right) = \frac{8}{\sqrt{2}}(1-i)$$

$$= 4\sqrt{2} - 4\sqrt{2}i$$

A1

Note: Accept $c = \frac{8}{\sqrt{2}}, d = -\frac{8}{\sqrt{2}}$.

[3 marks](g) **METHOD 1**

attempt to write the arguments of u, v, w, u', v' and w' over a common denominator OR to write the arguments in degrees

(M1)

$$\frac{-9\pi}{12}, \frac{-4\pi}{12}, \frac{-\pi}{12}, \frac{4\pi}{12}, \frac{7\pi}{12}, \frac{12\pi}{12} \text{ OR } -135^\circ, -60^\circ, -15^\circ, 60^\circ, 105^\circ, 180^\circ$$

THEN

arguments of u, v, w, u', v' and w' differ by $\frac{3\pi}{12}$ and $\frac{5\pi}{12}$ OR 45° and 75°

so arguments of polygon vertices differ by $\frac{\pi}{12}$ or 15° **(A1)**

$n = 24$ **A1**

METHOD 2

Let $z = r \operatorname{cis} \theta \Rightarrow z^n = r^n \operatorname{cis}(n\theta) = r^n \operatorname{cis}(n\theta)$, where θ is the argument of u, v, w, u', v' and w' .

recognition to find $n\theta$ where $n = 6, 12, 18, \dots$ and $\theta = -\frac{3\pi}{4}, -\frac{\pi}{3}, -\frac{\pi}{12}, \frac{\pi}{3}, \frac{7\pi}{12}, \pi$ **(M1)**

$$\text{when } n = 6 \Rightarrow (n\theta) = -\frac{9\pi}{2}, -2\pi, -\frac{\pi}{2}, 2\pi, \frac{7\pi}{2}, 6\pi$$

when $n = 12 \Rightarrow (n\theta) = -9\pi, -4\pi, -\pi, 4\pi, 7\pi, 12\pi$ (which is not a multiple of 2π) **(A1)**

$n = 24$ **A1**

[3 marks]**Total [20 marks]**

7. (a) $\alpha + \beta + \gamma = \frac{7}{2}$

A1**[1 mark]**

(b) $p - 3i$ is also a root (seen anywhere)

A1

recognition of 5 roots and attempt to sum these roots

(M1)

$$p + 3i + p - 3i + \frac{7}{2}$$

$$p + 3i + p - 3i + \frac{7}{2} = \frac{11}{2}$$

A1

$$p = 1$$

AG**[3 marks]**

(c) (i) attempt to find product of 5 roots and equate to ± 10

(M1)

$$(1 + 3i)(1 - 3i)\frac{1}{2}\alpha\beta = 10$$

$$\alpha\beta = 2$$

A1

(ii) $\alpha = 1$ and $\beta = 2$

A1**[3 marks]****Total [7 marks]**

9. (a) ${}^nC_3$ **A1****[1 mark]**(b) **EITHER**

finding the number of ways to assign the students with the two students apart

number of ways to assign two students ${}^2C_1$ (seen anywhere) **(A1)**number of ways to assign others ${}^{n-2}C_2$ to have one group of 3 (seen anywhere) **(A1)**number of ways = ${}^2C_1 \times {}^{n-2}C_2$ attempt to set up an equation involving either half of their answer to part (a) and their number of ways or their answer to part (a) is twice their number of ways **M1**

$$\frac{1}{2} {}^nC_3 = {}^2C_1 \times {}^{n-2}C_2 \quad \text{OR} \quad {}^nC_3 = 2 \times {}^2C_1 \times {}^{n-2}C_2$$

valid attempt to eliminate all factorials from their equation **(M1)**

$$\frac{n(n-1)(n-2)}{3 \times 2} = 2 \times 2 \times \frac{(n-2)(n-3)}{2} \quad \text{or equivalent with no factorials}$$

$$n(n-1) = 12(n-3)$$

continued...

Question 9 continued.

OR

finding the number of ways to assign the students with the two students together

number of ways to assign two students and one other to the first group ${}^{n-2}C_1$
(seen anywhere) **(A1)**

number of ways to assign three other students the first group ${}^{n-2}C_3$ (seen
anywhere) **(A1)**

$$\text{number of ways} = {}^{n-2}C_1 + {}^{n-2}C_3$$

attempt to set up an equation involving either half of their answer to part (a) and
their number of ways or their answer to part (a) is twice their number of ways **M1**

$$\frac{1}{2} {}^nC_3 = {}^{n-2}C_1 + {}^{n-2}C_3 \text{ OR } {}^nC_3 = 2({}^{n-2}C_1 + {}^{n-2}C_3)$$

valid attempt to eliminate all factorials from their equation **(M1)**

$$\frac{n(n-1)(n-2)}{3 \times 2} = 2 \times (n-2) + 2 \times \frac{(n-2)(n-3)(n-4)}{3 \times 2}$$

$$n(n-1) = 12 + 2(n-3)(n-4)$$

THEN

$$n^2 - 13n + 36 = 0 \quad \textbf{A1}$$

$$(n-9)(n-4) = 0$$

$$n = 9 \quad \textbf{A1}$$

Note: Do not award the final **A1** if additional values of n are given.

[6 marks]

Total [7 marks]

SECTION B

10. (a) attempt to find a difference (M1)

$$d = p - a, 2d = q - a, d = q - p \quad \text{OR} \quad p = a + d, q = a + 2d, q = p + d$$

correct equation A1

$$p - a = q - p \quad \text{OR} \quad q - a = 2(p - a) \quad \text{OR} \quad p = \frac{a + q}{2} \quad (\text{or equivalent})$$

$$2p - q = a \quad \text{AG}$$

[2 marks]

- (b) attempt to find a ratio (M1)

$$r = \frac{s}{a}, r^2 = \frac{t}{a}, r = \frac{t}{s} \quad \text{OR} \quad s = ar, t = ar^2, t = sr$$

correct equation A1

$$\left(\frac{s}{a}\right)^2 = \frac{t}{a} \quad \text{OR} \quad \frac{s}{a} = \frac{t}{s} \quad (\text{or equivalent})$$

$$s^2 = at \quad \text{AG}$$

[2 marks]

continued...

Question 10 continued(c) **EITHER**

$$2p - 1 = s^2 \text{ (or equivalent)} \quad \mathbf{A1}$$

$$(s^2 > 0) \Rightarrow 2p - 1 > 0 \text{ OR } s = \sqrt{2p - 1} \Rightarrow 2p - 1 > 0 \text{ OR } p = \frac{s^2 + 1}{2} \text{ (and } s^2 > 0) \quad \mathbf{R1}$$

OR

$$2p - 1 = a \text{ and } s^2 = a \quad \mathbf{A1}$$

$$(s^2 > 0, \text{ so }) a > 0 \Rightarrow 2p - 1 > 0 \text{ OR } p = \frac{a + 1}{2} \text{ and } a > 0 \quad \mathbf{R1}$$

$$\Rightarrow p > \frac{1}{2} \quad \mathbf{AG}$$

Note: Do not award **A0R1**.

[2 marks]*continued...*

Question 10 continued

(d) (i) 9, 5, 1, –3

A1A1**Note:** Award **A1** for each of 2nd term and 4th term(ii) 9, 3, 1, $\frac{1}{3}$ **A1A1****Note:** Award **A1** for each of 2nd term and 4th term**[4 marks]**

(e) (i) attempt to find the difference between two consecutive terms

(M1)

$$d = u_2 - u_1 = 5 + \ln 3 - 9 - \ln 9 \quad \text{OR} \quad d = u_3 - u_2 = 1 + \ln 1 - 5 - \ln 3$$

$$\ln 9 = 2 \ln 3 \quad \text{OR} \quad \ln 1 = 0 \quad \text{OR} \quad \ln 3 - \ln 9 = \ln \frac{1}{3} \left(= \ln 3^{-1} = -\ln 3 \right) \quad (\text{seen anywhere}) \quad \textbf{(A1)}$$

$$d = -4 - \ln 3$$

A1*continued...*

Question 10 continued.

(ii) **METHOD 1**

attempt to substitute first term and their common difference into S_{10} **(M1)**

$$\frac{10}{2}(2(9 + \ln 9) + 9(-4 - \ln 3)) \text{ OR } \frac{10}{2}(2(9 + 2 \ln 3) + 9(-4 - \ln 3)) \text{ (or equivalent) } \quad \mathbf{A1}$$

$$= 5(-18 - 5 \ln 3) \text{ (or equivalent in terms of } \ln 3) \quad \mathbf{A1}$$

$$\sum_{i=1}^{10} u_i = -90 - 25 \ln 3 \quad \mathbf{AG}$$

METHOD 2

$$u_{10} = 9 + \ln 9 + 9(-4 - \ln 3) (= -27 + \ln 9 - 9 \ln 3)$$

attempt to substitute first term and their u_{10} into S_{10} **(M1)**

$$\frac{10}{2}(2(9 + \ln 9) + 9(-4 - \ln 3)) \text{ OR } \frac{10}{2}(9 + \ln 9 - 27 + \ln 9 - 9 \ln 3) \text{ OR}$$

$$\frac{10}{2}(2(9 + 2 \ln 3) + 9(-4 - \ln 3)) \text{ OR } \frac{10}{2}(9 + \ln 9 - 27 - 7 \ln 3) \text{ (or equivalent) } \quad \mathbf{A1}$$

$$= 5(-18 - 5 \ln 3) \text{ (or equivalent in terms of } \ln 3) \quad \mathbf{A1}$$

$$\sum_{i=1}^{10} u_i = -90 - 25 \ln 3 \quad \mathbf{AG}$$

[6 marks]

Total [16 marks]

1. (a) recognition that a 15% loss leaves 85% OR finding 15% and subtracting from original **(M1)**
 0.85×35000 OR $35000 - 0.15 \times 35000$
 $= (\$)29750$ **A1**

Note: Accept $(\$)29800$.

[2 marks]

- (b) **EITHER**

$$29750 \times 0.89^9 \quad \textbf{(A1)}$$

OR

$$N = 9$$

$$I\% = -11$$

$$PV = \mp 29750$$

(A1)

THEN

$$\text{value}(FV) = (\$)10423 \quad \textbf{A1}$$

Note: For this **A1** the answer must be rounded to the nearest dollar.

Accept $(\$)10441$ from using 3 sf answer from part (a).

[2 marks]

continued...

Question 1 continued(c) **METHOD 1**

attempt to solve the inequality (or equation) $29750 \times 0.89^{n-1} < 3500$ OR table of values **(M1)**

19.3643... OR $(n = 19 \Rightarrow) 3651.80...$ OR $(n = 20 \Rightarrow) 3250.10...$ **(A1)**

Note: For candidates using (\$)29800, $n > 19.3787...$, $(n = 19 \Rightarrow) 3657.93...$,
 $(n = 20 \Rightarrow) 3255.56...$

$n = 20$

A1

[3 marks]

METHOD 2

use of the finance app with $I\% = -11$, $PV = \mp 29750$, $FV = \pm 3500$

OR $29750 \times 0.89^N < 3500$ (condone the use of n or x) **(M1)**

$(N =) 18.3643...$ **(A1)**

Note: For candidates using (\$)29800, $N = 18.3787...$

$n = 20$

A1

[3 marks]

Total [7 marks]

9. (a) METHOD 1

the number of ways Alvin and Bobby can be seated is $2 \times 8 (= 16)$ **(A1)**

the number of ways the other children can be seated is $8! (= 40320)$ **(A1)**

Note: These **A1** marks may be awarded independently.

total number of ways is $(16 \times 8! =) 645120$ **A1**

Note: Accept $16 \times 8!$ and 645000.

METHOD 2

the number of ways children can be seated in a row of 10 seats is $2 \times 9! (= 725760)$ **(A1)**

the number of ways the children can be seated with Alvin and Bobby in seats 5 and 6 is $2 \times 8! (= 80640)$ **(A1)**

Note: These **A1** marks may be awarded independently.

total number of ways is $(2 \times 9! - 2 \times 8! =) 645120$ **A1**

Note: Accept $16 \times 8!$ and 645000.

[3 marks]

(b) METHOD 1

attempt to find number of ways that A and B are seated next to each other AND C and D are seated next to each other and subtract from part a) **(M1)**

Case 1: A and B are sat at the end of a row (8 ways)

$6(2) = 12$ ways to seat C and D together

$12 \times 6! (= 8640)$ ways **(A1)**

the total number of ways is $8 \times 12 \times 6! (= 69120)$

continued...

Question 9 continued**Case 2:** A and B are not sat at the end of a row (8 ways)

5(2) = 10 ways to seat C and D together

 $10 \times 6! = 7200$ ways**(A1)**the total number of ways is $8 \times 10 \times 6! (= 57600)$ total number of ways is $645120 - (69120 + 57600)$ $= 518400$ **A1****Note:** Accept 518000 or 518280 (from use of 645000).**METHOD 2**

attempt to split into cases based on position of A and B and adding all possibilities

(M1)**Case 1:** A and B are sat at the end of a row (8 ways)

the number of ways C and D can be seated:

with at least one in the same row as A and B $2(1 + 15) = 32$ with both in a different row to A and B $2(6) = 12$ the number of ways C and D can be seated is $44 \times 6! (= 31680)$ **(A1)**the total number of ways is $8 \times 31680 = 253440$ **Case 2:** A and B are not sat at the end of a row (8 ways)

the number of ways C and D can be seated:

with at least one in the same row as A and B $2(2 + 15) = 34$ with both in a different row to A and B $2(6) = 12$ the number of ways C and D can be seated is $46 \times 6! (= 33120)$ **(A1)**the total number of ways is $8 \times 33120 = 264960$ total number of ways is $253440 + 264960$ $= 518400$ **A1****Note:** Accept 518000.**[4 marks]****Total [7 marks]**

Section B

8. (a) METHOD 1

$$(i) \quad \arg z = \arctan\left(\frac{\sin 2\theta}{1 + \cos 2\theta}\right) \left(\tan(\arg z) = \frac{\sin 2\theta}{1 + \cos 2\theta} \right) \quad \mathbf{A1}$$

uses $2 \sin \theta \cos \theta$ in the numerator and any double angle identity for $\cos 2\theta$ in the denominator

M1

$$\arg z = \arctan\left(\frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta}\right) \left(\tan(\arg z) = \frac{2 \sin \theta \cos \theta}{2 \cos^2 \theta} \right)$$

$$\Rightarrow \arg z = \arctan(\tan \theta) \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right) \quad \mathbf{A1}$$

$$= \theta \quad \mathbf{AG}$$

[3 marks]

$$(ii) \quad \text{attempts to express } |z| \text{ in the form } \sqrt{(\operatorname{Re} z)^2 + (\operatorname{Im} z)^2} \quad \mathbf{(M1)}$$

$$|z| = \sqrt{(1 + \cos 2\theta)^2 + \sin^2 2\theta}$$

attempts to expand $(1 + \cos 2\theta)^2$ and then uses

$$\cos^2 2\theta + \sin^2 2\theta = 1 \text{ in an attempt to simplify} \quad \mathbf{(M1)}$$

$$|z| = \sqrt{2 + 2 \cos 2\theta} \quad \mathbf{A1}$$

$$|z| = \sqrt{4 \cos^2 \theta} (= 2|\cos \theta|) \quad \mathbf{A1}$$

$$= 2 \cos \theta \left(-\frac{\pi}{2} < \theta < \frac{\pi}{2} \right) \quad \mathbf{AG}$$

[4 marks]**METHOD 2 (i) and (ii)**

$$z = 1 + 2 \cos^2 \theta - 1 + 2 \sin \theta \cos \theta i \quad \mathbf{M1A1A1}$$

$$z = 2 \cos^2 \theta + 2 \sin \theta \cos \theta i \quad \mathbf{A1}$$

$$\text{attempt to form } z = r \operatorname{cis} \theta \quad \mathbf{M1}$$

$$z = 2 \cos \theta (\cos \theta + i \sin \theta) \quad \mathbf{A1A1}$$

$$\therefore |z| = 2 \cos \theta \text{ and } \arg z = \theta. \quad \mathbf{AG}$$

continued...

Question 8 continued.

(b) $2\theta = (2\cos\theta)^3$ **(A1)**

attempts to solve for θ **(M1)**

$\theta = 0.913236\dots$

$\theta = 0.913$ **A1**

Note: Award all marks for $\theta = 0.913$ found directly without using part (a).

Note: Award **(A1)(M1)A0** for $\theta = 3.97^\circ$.

[3 marks]

Total [10 marks]

3. METHOD 1

attempt to expand $(3n+2)^2 - (3n-2)^2$ **M1**

Note: Award **M0** for invalid attempts such as $(3n+2)^2 = 9n^2 + 4$.

$= 9n^2 + 12n + 4 - (9n^2 - 12n + 4)$ or equivalent **A1**

$= 24n$ OR $12n + 12n$ **A1**

$= 12(2n)$ OR $\frac{24}{12} = 2$ OR $\frac{24n}{2} = 12n$ OR $12n + 12n = 12(n+n)$ (or equivalent) **R1**

so is a multiple of 12 **AG**

Note: Do not award the **R1** unless both **A** marks have been awarded.

METHOD 2

use of $a^2 - b^2 = (a+b)(a-b)$ where $a = 3n+2$, $b = 3n-2$ **M1**

$= (3n+2+3n-2)(3n+2-3n+2)$

$= 6n \times 4$ **A1**

$= 24n$ **A1**

$= 12(2n)$ OR $\frac{24n}{12} = 2n$ OR $\frac{24n}{2} = 12n$ OR $12n + 12n = 12(n+n)$ (or equivalent)

R1

so is a multiple of 12 **AG**

Note: Do not award the **R1** unless both **A** marks have been awarded.

continued...

Question 3 continued.

METHOD 3

base case $n = 1$: $(3(1)+2)^2 - (3(1)-2)^2 = 25 - 1 = 24$

so true for $n = 1$

A1

assume true for $n = k$ i.e. $(3k+2)^2 - (3k-2)^2$ is a multiple of 12

M1

consider $n = k + 1$:

$$(3(k+1)+2)^2 - (3(k+1)-2)^2$$

$$((3k+2)+3)^2 - ((3k-2)+3)^2$$

$$(3k+2)^2 + 6(3k+2) + 9 - ((3k-2)^2 + 6(3k-2) + 9)$$

$$(3k+2)^2 - (3k-2)^2 + 24$$

using the assumption $(3k+2)^2 - (3k-2)^2 = 12M$

$$12M + 24$$

$$12(M+2)$$

which is a multiple of 12, hence true for $n = k + 1$

A1

since true for $n = 1$, and true for $n = k$ implies true for $n = k + 1$

therefore, true for all $n \in \mathbb{Z}^+$

R1

Note: Do not award the **R1** unless both **A** marks have been awarded.

[4 marks]

5. attempt to use $u_n = u_1 + (n-1)d$ or $S_n = \frac{n}{2}[2u_1 + (n-1)d]$ or $S_n = \frac{n}{2}[u_1 + u_n]$ to

set up at least one equation in u_1 and d **(M1)**

$$16 = u_1 + 9d \text{ and } 100 = \frac{25}{2}[2u_1 + 24d] \quad \text{ **(A1)**}$$

attempt to solve their two linear equations in u_1 and d simultaneously (must eliminate one variable) **(M1)**

$$d = -4 (\Rightarrow u_1 = 52) \quad \text{ **A1**}$$

attempt to solve $u_k = 0$ with their d (or with their d and u_1) **(M1)**

$$\Rightarrow k = 14 \quad \text{ **A1**}$$

[6 marks]

12. (a) $|16i| = 16$ and $\arg(16i) = \frac{\pi}{2}$ (A1)

attempt to use De Moivre's Theorem (M1)

$$z_1 = 2 \left(\cos\left(\frac{\pi}{8}\right) + i \sin\left(\frac{\pi}{8}\right) \right) \quad \text{A1}$$

attempts to find other solutions using $z = 2 \left(\cos\left(\frac{\pi}{8} + \frac{\pi k}{2}\right) + i \sin\left(\frac{\pi}{8} + \frac{\pi k}{2}\right) \right)$ or equivalent (M1)

$$z_2 = 2 \left(\cos\left(\frac{5\pi}{8}\right) + i \sin\left(\frac{5\pi}{8}\right) \right) \text{ (or any other root)} \quad \text{A1}$$

$$z_3 = 2 \left(\cos\left(\frac{9\pi}{8}\right) + i \sin\left(\frac{9\pi}{8}\right) \right) \text{ and } z_4 = 2 \left(\cos\left(\frac{13\pi}{8}\right) + i \sin\left(\frac{13\pi}{8}\right) \right) \quad \text{A1}$$

Note: Award a maximum of (A1)(M1)A1(M1)A1A0 for more than four roots or any roots outside the range.

Note: Allow use of r-cis form throughout.

[6 marks]

(b) attempt to evaluate a ratio with their roots eg $\frac{z_2}{z_1}$ (M1)

$$\frac{z_2}{z_1} = \frac{2 \left(\cos\left(\frac{5\pi}{8}\right) + i \sin\left(\frac{5\pi}{8}\right) \right)}{2 \left(\cos\left(\frac{\pi}{8}\right) + i \sin\left(\frac{\pi}{8}\right) \right)} \quad \text{or equivalent}$$

$$= \left(\cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right) \right) \quad \text{(A1)}$$

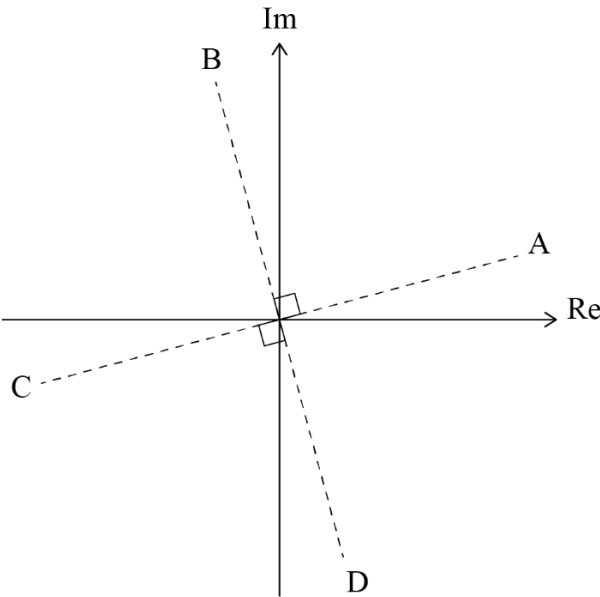
$$= i \quad \text{A1}$$

[3 marks]

continued...

Question 12 continued.

(c)



- point A in approximately correct place in first quadrant
- A1
- points A, B, C and D approximately the same distance from the origin
- A1
- approximate angular separation of $\frac{\pi}{2}$
- A1

Note: Dotted lines not required.

[3 marks]

continued...

Question 12 continued.

(d) **EITHER**

$$(z_i^*)^4 = (z_i^4)^* \quad (\text{A1})$$

$$= (16i)^* \quad (\text{A1})$$

OR

$$z_1^* = 2 \left(\cos \left(-\frac{\pi}{8} \right) + i \sin \left(-\frac{\pi}{8} \right) \right) \quad (\text{A1})$$

$$(z_1^*)^4 = 2^4 \left(\cos \left(-\frac{4\pi}{8} \right) + i \sin \left(-\frac{4\pi}{8} \right) \right) \quad (\text{A1})$$

OR

$$z_1 z_2 z_3 z_4 = -16i \quad (\text{A1})$$

$$(z_1 z_2 z_3 z_4)^* = (-16i)^*$$

$$z_1^* z_2^* z_3^* z_4^* = 16i \quad (\text{A1})$$

THEN

$$(z^4) = -16i \quad \text{A1}$$

$$(a = 0, b = -16)$$

[3 marks]

$$(e) \quad \arg w_1 \left(= \frac{\pi}{8} + \frac{\pi}{4} \right) = \frac{3\pi}{8} \quad \text{A1}$$

$$AB = \sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\Rightarrow AA' = \sqrt{2}$$

$$\Rightarrow OA' = |w_1| = \sqrt{2} \quad \text{A1}$$

$$\therefore w_1 = \sqrt{2} \operatorname{cis} \frac{3\pi}{8}$$

$$\text{considers when } \arg(w_1^p) \in \mathbb{Z}^+ \text{ (multiple of } 2\pi) \quad (\text{M1})$$

$$\Rightarrow p = 16 \quad \text{A1}$$

$$\Rightarrow q = 8 \quad \text{A1}$$

[5 marks]**Total [20 marks]**

2. EITHER

attempt to form a product of binomial coefficient, a power of $2x$ and a power of -5 seen (M1)

${}^9C_3(2x)^6(-5)^3$ OR ${}^9C_6(2x)^6(-5)^3$ OR $84 \times (2x)^6(-5)^3$ (A1)(A1)

Note: Award **A1** for ${}^9C_6$ or ${}^9C_3$ or 84 , **A1** for $(2x)^6(-5)^3$.

OR

attempt to use the general term (M1)

${}^9C_r(2x)^{9-r}(-5)^r$ and $r = 3$ (A1)(A1)

THEN

-672000 (exact) **A1**

Note: Award **A0** for a final answer of $-672000x^6$.

[4 marks]

7. (a) total ways = $3! {}^{12}C_3 (= {}^{12}P_3 = 1320)$ OR total ways together = $3! \times 10 (= 60)$ (A1)
 attempt to consider the total ways of sitting – total ways of sitting together (M1)
 $3! {}^{12}C_3 - 3! \times 10$
 $= 1260$ A1

[3 marks]

(b) **METHOD 1**

attempt to multiply ways of seating AVP by ways of sitting additional people (M1)

AVP can sit in $3! \times 10 (= 60)$ ways (may be seen in part (a))other 3 then have $9 \times 8 \times 7 (= {}^9P_3)$ ways to sit (A1)total ways = $3! \times 10 \times 9 \times 8 \times 7$ $= 30240$ A1

Note: Award (M1)(A0)A0 for $3! \times 10 \times {}^9C_3 = 5040$.

METHOD 2

attempt to consider 'AVP' as one item, so 4 'items' in total (M1)

 ${}^{10}C_4 \times 3! \times 4! (= {}^{10}P_4 \times 3!)$ (A1) $= 30240$ A1

[3 marks]

Total [6 marks]

8. (a) METHOD 1suppose $w = x + iy$

$$ww^* = (x + iy)(x - iy) \quad \text{A1}$$

$$= x^2 + y^2 \quad \text{A1}$$

$$= |w|^2 \quad \text{AG}$$

METHOD 2suppose $w = re^{i\theta}$

$$ww^* = (re^{i\theta})(re^{-i\theta}) \quad \text{A1}$$

$$= r^2 \quad \text{A1}$$

$$= |w|^2 \quad \text{AG}$$

METHOD 3suppose $w = r(\cos \theta + i \sin \theta)$

$$ww^* = (r(\cos \theta + i \sin \theta))(r(\cos \theta - i \sin \theta)) \quad \text{A1}$$

$$= r^2 (\cos^2 \theta + \sin^2 \theta) (= r^2) \quad \text{A1}$$

$$= |w|^2 \quad \text{AG}$$

[2 marks]**(b) EITHER**multiplying first equation by w OR multiplying LHS and RHS of both equations **M1**

$$5w^*w = (1 - 2i)z^2w \quad \text{OR} \quad 5w^*zw = (1 - 2i)z^2(10 + 10i)$$

$$5(2\sqrt{5})^2 = (1 - 2i)(10 + 10i)z (\Rightarrow 100 = (30 - 10i)z) \quad \text{A1}$$

ORattempt to eliminate w and w^* using $ww^* = 20$ **M1**

$$\left(\frac{(1 - 2i)}{5}z^2\right)\left(\frac{10 + 10i}{z}\right) = 20$$

$$\frac{(1 - 2i)(10 + 10i)}{5}z = 20 \quad \text{A1}$$

continued...

Question 8 continued.

THEN

$$z = \frac{10}{(1-2i)(1+i)} \left(= \frac{10}{3-i} \right) \quad \textbf{(A1)}$$

$$z = 3 + i \quad \textbf{A1}$$

$$(a = 3, b = 1)$$

[4 marks]

Total [6 marks]

9. (a) recognition that $|\log_2 c| < 1$ (M1)
- $0.5 < c < 2, (c \neq 1)$ A1A1

Note: Award **A1** for endpoints and **A1** for strict inequalities.

[3 marks]

- (b) attempt to find $S_\infty = \frac{u_1}{1-r}$ (M1)

$$= \frac{5}{1 - \log_2(1.5)} \left(= \frac{5}{1 - 0.58496\dots} = 12.0471\dots \right) \quad (\text{A1})$$

- attempt to solve their $|S_\infty - S_n| < 0.1$ (M1)

$$\left| \frac{5}{1 - \log_2(1.5)} - \sum_{r=0}^{n-1} 5(\log_2(1.5))^r \right| < 0.1 \quad \text{OR} \quad \left| \frac{5}{1 - \log_2(1.5)} - \frac{5(1 - (\log_2(1.5))^n)}{1 - \log_2(1.5)} \right| < 0.1$$

Note: Award (M1) for solving an equality. Condone absence of absolute value signs.

$$n = 8.93574\dots$$

$$n = 9$$

A1

[4 marks]

Total [7 marks]

Section B

1. (a) $T_2 = 1.1(6000) - 500$ OR $T_2 = 6600 - 500$
 $= 6100$

A2**AG****[2 marks]**

(b) (i) $T_3 = 1.1(6100) - 500$ ($T_3 = 1.1(1.1(6000) - 500) - 500$) OR $T_3 = 6710 - 500$
 $= 6210$

A1**AG****[1 mark]**

(ii) $T_4 = 1.1(6210) - 500$ OR $T_4 = 6831 - 500$
 $= 6331$

(A1)**A1****[2 marks]***continued...*

Question 1 continued.

(c) (i) **EITHER**

attempts to eliminate the denominator

(M1)

$$T_n = 6000(1.1)^{n-1} - 5000((1.1)^{n-1} - 1) \quad (= (6000 - 5000)(1.1)^{n-1} + 5000)$$

A1

Note: Accept correct equivalent forms, eg. $T_n = 6000(1.1)^{n-1} - 5000(1.1)^{n-1} + 5000$.

ORattempts to express T_n with a common denominator of 0.1**(M1)**

$$T_n = \frac{600(1.1)^{n-1} - 500((1.1)^{n-1} - 1)}{0.1} \quad \left(= \frac{600(1.1)^{n-1} - 500(1.1)^{n-1} + 500}{0.1} \right)$$

A1

Note: Accept correct equivalent forms.

THEN

$$T_n = 1000(1.1)^{n-1} + 5000$$

AG**[2 marks]**(c) (ii) **EITHER**attempts to find T_6 using the explicit formula for T_n **(M1)**

$$T_6 = 1000(1.1)^{6-1} + 5000 \quad \text{OR} \quad T_6 = 6610.51$$

ORattempts to find T_6 using $T_{n+1} = 1.1T_n - 500$ **(M1)**

$$T_4 = 6331, \quad T_5 = 6464.1, \quad T_6 = 6610.51$$

ORattempts to find T_6 using a finance solver application**(M1)**

$$N = 5, \quad I = 10\%, \quad PV = -6000 \quad \text{and} \quad PMT = 500$$

THEN

$$6611$$

A1

Note: Accept 6610.

[2 marks]

continued...

Question 1 continued.

(d) (i) **METHOD 1**

attempts to use the explicit formula for D_n to find an expression for $D_{n+1} - D_n$ **(M1)**

$$= 1.1D_n - 750 - (-1500(1.1)^{n-1} + 7500)$$

$$= 1.1(-1500(1.1)^{n-1} + 7500) - 750 - (-1500(1.1)^{n-1} + 7500) \quad \mathbf{A1}$$

EITHER

$$= -1650(1.1)^{n-1} + 8250 - 750 + 1500(1.1)^{n-1} - 7500 \quad \mathbf{A1}$$

OR

$$= 1500(1.1)^{n-1} - 1500(1.1)^n$$

$$= 1500(1.1)^{n-1}(1 - 1.1) \quad (= 1500(1.1)^{n-1}(-0.1)) \quad \mathbf{A1}$$

THEN

$$D_{n+1} - D_n = -150(1.1)^{n-1} \quad \mathbf{AG}$$

METHOD 2

attempts to express $D_{n+1} - D_n$ in terms of D_n **(M1)**

$$D_{n+1} - D_n = 1.1D_n - 750 - D_n$$

$$= 0.1D_n - 750 \quad \mathbf{(A1)}$$

$$= 0.1(-1500(1.1)^{n-1} + 7500) - 750 \quad (= -150(1.1)^{n-1} + 750 - 750) \quad \mathbf{A1}$$

$$D_{n+1} - D_n = -150(1.1)^{n-1} \quad \mathbf{AG}$$

[3 marks]

continued...

Question 1 continued.

$$(ii) \quad D_{n+1} - D_n < 0 \text{ (because } (1.1)^{n-1} > 0 \text{ and so } -150(1.1)^{n-1} < 0)$$

R1

Note: Award **R1** for responses such as:

‘The difference is negative’.

$$-150(1.1)^{n-1} < 0$$

$$D_n = D_{n+1} + 150(1.1)^{n-1}$$

Accept responses such as:

$$D_n - D_{n+1} > 0 \text{ or ‘the difference is positive’ as long as } D_n - D_{n+1} \text{ is referred to.}$$

Do not accept arguments based on specific numerical examples.

$$\text{therefore } D_{n+1} < D_n \Rightarrow D_n > D_{n+1}$$

AG

(and hence the predicted number of trout at the start of any year will be greater than the predicted number at the start of the next year)

[1 mark]

continued...

Question 1 continued.

(e) **METHOD 1**

attempts (numerically, graphically, algebraically or by trial & improvement) to find the least value of n such that $D_n \leq 0$ ($D_n = 0$ OR $D_n < 0$)

(M1)

Note: Award **(M1)** for attempting to obtain $-1500(1.1)^{n-1} \leq -7500$ or

$-1500(1.1)^{n-1} = -7500$ (or equivalent) and then attempting to take the log of both sides.

For the use of trial & improvement, award **(M1)**, for considering values of n either side of $D_n = 0$.

EITHER

$$17.8863... \left(= \log_{1.1} 5 + 1, = \frac{\ln 5}{\ln 1.1} + 1 \right) \text{ (critical value of } n \text{ seen anywhere)} \quad \textbf{(A2)}$$

OR

$$D_{17} = 607.540... \text{ (} 607.540... < 750 \text{)} \quad \textbf{(A2)}$$

OR

$$D_{18} = -81.7054... (< 0) \quad \textbf{(A2)}$$

THEN

trout disappears from the lake during (at the end of) the 17th year

A1

Note: Award **A1** for $n = 18$ OR ('during the) 18th year' or 'start of the 18th year'.

Note: Award as above for $D_{n+1} \leq 0$ ($D_{n+1} = 0$ OR $D_{n+1} < 0$).

continued...

Question 1 continued.

METHOD 2

$S_n = \sum (D_{n+1} - D_n)$ is the total decrease in the number of trout by the end of year n or start of year $n + 1$

attempts (numerically, graphically, algebraically or by trial & improvement) to find the least value of n such that $S_n \leq -6000$ ($S_n = -6000$ OR $S_n < -6000$)

(M1)

$$\frac{-150((1.1)^n - 1)}{1.1 - 1} \left(= -1500((1.1)^n - 1) \right) \leq -6000$$

Note: Award **(M1)** for attempting to obtain $-1500((1.1)^n - 1) \leq -6000$ or

$-1500((1.1)^n - 1) = -6000$ (or equivalent) and then attempting to take the log of both sides.

For the use of trial & improvement, award **(M1)**, for considering values of n either side of $S_n = -6000$.

$$n = 16.8863\dots$$

(A2)

trout disappears from the lake during (at the end of) the 17th year

A1

Note: Award **A1** for $n = 18$ OR ('during the) 18th year' or 'start of the 18th year'.

[4 marks]

(f) **METHOD 1**

$$C_n = (6000 - 10d)(1.1)^{n-1} + 10d$$

(A1)

EITHER

$$6000 - 10d = 0 \text{ (recognizes that } C_n \text{ is independent of } n \text{)}$$

(M1)

OR

$$10d = 6000 \text{ (sets } C_n = 6000 \text{ and compares coefficients)}$$

(M1)

THEN

$$d = 600$$

A1

continued...

Question 1 continued.

METHOD 2

EITHER

$$C_{n+1} - C_n (= 0)$$

$$6000(1.1)^n - 10d \left((1.1)^n - 1 \right) - \left(6000(1.1)^{n-1} - 10d \left((1.1)^{n-1} - 1 \right) \right) (= 0) \quad (\mathbf{A1})$$

OR

$$0.1C_n - d (= 0) \text{ (similar to part (d) (i) method 2)}$$

$$0.1 \left(6000(1.1)^{n-1} - 10d \left((1.1)^{n-1} - 1 \right) \right) - d (= 0) \quad (\mathbf{A1})$$

THEN

attempts to expand to eliminate the linear terms in d and then attempts to factorize (\mathbf{M1})

$$600(1.1)^{n-1} - d(1.1)^{n-1} (= 0) \left((600 - d)(1.1)^{n-1} (= 0) \right)$$

$$d = 600 \quad \mathbf{A1}$$

METHOD 3

considers $C_{n+1} = 1.1C_n - d$ and $C_1 = 6000$ (\mathbf{A1})

EITHER

$$6000 = 1.1 \times 6000 - d \quad (C_1 = C_2 = 6000) \quad (\mathbf{A1})$$

OR

systematically varies the value of d (\mathbf{M1})

THEN

$$d = 600 \quad \mathbf{A1}$$

continued...

Question 1 continued.

METHOD 4

EITHER

considers a specific $C_n = C_{n+1}$, for example, $C_2 = C_3$ **(A1)**

$$6000(1.1) - 10d(1.1 - 1) = 6000(1.1)^2 - 10d((1.1)^2 - 1)$$

attempts to solve their $C_n = C_{n+1}$ for d **(M1)**

for example, $1.1d = 660$

OR

considers a specific $C_n = 6000$, for example, $C_2 = 6000$ **(A1)**

$$6000(1.1)^{2-1} - 10d((1.1)^{2-1} - 1) = 6000$$

attempts to solve their $C_n = 6000$ for d **(M1)**

$$6600 - d = 6000$$

THEN

$$d = 600$$

A1

[3 marks]

continued...

Question 1 continued.

$$(g) \quad n = 1: \text{LHS} = u_1 \text{ and } \text{RHS} = \left(u_1 r^{1-1} - \frac{d(r^{1-1} - 1)}{r-1} \right) = u_1 r^0 - \frac{d(r^0 - 1)}{r-1} = u_1 \quad \mathbf{R1}$$

LHS = RHS and so true for $n = 1$

Note: Award **R0** for considering $n = 2$.

Subsequent marks after this **R1** are independent of this mark and can be awarded.

$$\text{assume true for } n = k \left(u_k = u_1 r^{k-1} - \frac{d(r^{k-1} - 1)}{r-1} \right) \quad \mathbf{M1}$$

Note: The assumption of truth must be apparent. Do not award **M1** for statements such as “let $n = k$ ” or “assume that $n = k$ is true”. Subsequent marks after this **M1** are independent of this mark and can be awarded.

attempts to substitute for u_k in $u_{k+1} = ru_k - d$ **M1**

$$= r \left(u_1 r^{k-1} - \frac{d(r^{k-1} - 1)}{r-1} \right) - d$$

$$= u_1 r^k - \frac{dr(r^{k-1} - 1)}{r-1} - \frac{d(r-1)}{r-1} \left(= u_1 r^k - \frac{dr^k - dr}{r-1} - d \right) \quad \mathbf{A1}$$

$$= u_1 r^k - \frac{dr^k + dr - dr - d}{r-1} \left(= u_1 r^k - \frac{dr^k - d}{r-1} \right) \quad \mathbf{A1}$$

$$= u_1 r^k - \frac{d(r^k - 1)}{r-1} \quad \mathbf{A1}$$

since true for $n = 1$ and true for $n = k + 1$ if true for $n = k$, hence true for all $n (\in \mathbb{Z}^+)$ **R1**

Note: To obtain the final **R1**, four of the previous six marks must have been awarded.

[7 marks]

Total [27 marks]

2. (a) $k = \frac{4}{400} \left(= \frac{1}{100} = 0.01 \right)$ **A1**

[1 mark]

(b) attempt to find binomial coefficients or multiply out brackets **(M1)**

e.g. Pascal's triangle down to correct row OR $(1 + 2x + x^2)^2$ OR substitute into binomial expansion

$$(1 + x)^4$$

$$= 1 + 4x + 6x^2 + 4x^3 + x^4$$

A1

[2 marks]

(c) **METHOD 1**

recognition that the expansion can be used with x replaced with k **(M1)**

$$\left(1 + \frac{1}{100} \right)^4$$

$$= 1 + \frac{4}{100} + \frac{6}{100^2} + \dots (= 1 + 0.04 + 0.0006 + \dots)$$

(A1)

multiplies by 1000 (seen anywhere) **(M1)**

$$1000 \left(1 + \frac{1}{100} \right)^4$$

$$= 1000 + 40 + 0.6 + \dots (= 1040.6 \dots)$$

$$= 1041 \text{ (dinar)}$$

A1

METHOD 2

attempt to find the value of $(1 + k)^4$ by hand **(M1)**

$$(1.01)^4 = (1.0201)(1.01)^2 = (1.030301)(1.01)$$

$$= 1.0406 \dots$$

(A1)

multiplies by 1000 (seen anywhere) **(M1)**

$$1000(1.01)^4$$

$$= 1040.6 \dots$$

$$= 1041 \text{ (dinar)}$$

A1

[4 marks]

Total [7 marks]

5. (a) (i) for a sequence of areas, uses two consecutive terms to find a common ratio OR for sequences of both widths and heights uses two consecutive terms for both sequences to find both common ratios OR recognises that both widths and heights are geometric sequences with common ratio $\frac{3}{2}$ **M1**

areas form a geometric sequence with first term 20 and common ratio $\frac{45}{20}$ **A1**

OR area of picture frame F_n is $4\left(\frac{3}{2}\right)^{n-1} \times 5\left(\frac{3}{2}\right)^{n-1}$

area of F_n is $20\left(\frac{9}{4}\right)^{n-1}$ **AG**

- (ii) attempt to find the sum of the areas using $S_n = \frac{u_1(r^n - 1)}{r - 1}$ **(M1)**

sum of areas $\frac{20\left(\left(\frac{9}{4}\right)^{10} - 1\right)}{\frac{9}{4} - 1} \left(= 16\left(\left(\frac{9}{4}\right)^{10} - 1\right) \right)$ **(A1)**

mean area $= \frac{1}{10} \left(\frac{20\left(\left(\frac{9}{4}\right)^{10} - 1\right)}{\frac{9}{4} - 1} \right) \left(= \frac{1}{10} \left(16\left(\left(\frac{9}{4}\right)^{10} - 1\right) \right) \right)$

$= \frac{16}{10} \left(\left(\frac{9}{4}\right)^{10} - 1 \right) \left(= \frac{8}{5} \left(\left(\frac{9}{4}\right)^{10} - 1 \right) \right)$ **A1**

$p = \frac{8}{5}, a = 10$

[5 marks]

continued...

Question 5 continued.

- (b) recognition that median is between 5th and 6th picture frame

(M1)

$$\text{median area} = \frac{20\left(\frac{9}{4}\right)^4 + 20\left(\frac{9}{4}\right)^5}{2}$$

(A1)

$$= \frac{20\left(\frac{9}{4}\right)^4 \left(1 + \frac{9}{4}\right)}{2}$$

$$= \frac{65}{2} \left(\frac{9}{4}\right)^4$$

A1

$$q = \frac{65}{2}$$

[3 marks]

Total [8 marks]

((0))

((1))

7. (a) $(3 =) e^{\ln 3}$ OR $a = \ln 3$

A1**[1 mark]**

(b) (i) $z = \frac{1}{3} e^{i \ln 3}$ OR $(\operatorname{Re}(z)) = e^{-\ln 3} \cos(\ln 3)$

(A1)

$$(\operatorname{Re}(z)) = \frac{1}{3} \cos(\ln 3)$$

A1

(ii) $\frac{1}{z} = 3e^{-i \ln 3} (= 3(\cos(-\ln 3) + i \sin(-\ln 3)))$ OR $\left(\operatorname{Re}\left(\frac{1}{z}\right) =\right) e^{\ln 3} \cos(-\ln 3)$ **(A1)**

$$\operatorname{Re}\left(\frac{1}{z}\right) = 3 \cos(-\ln 3)$$

$$= 3 \cos(\ln 3) \text{ OR } 3 \cos\left(\ln \frac{1}{3}\right)$$

A1**[4 marks]****Total [5 marks]**

8. (a) $1 + \log_2 n = \log_2 2 + \log_2 n$

$$= \log_2(2n)$$

A1

$$2n \geq n+1 \text{ OR } \log_2(2n) \geq \log_2(n+1)$$

R1

(for $n \in \mathbb{Z}^+$) (since $\log_2 x$ is an increasing function)

$$1 + \log_2 n \geq \log_2(n+1) \text{ (for } n \in \mathbb{Z}^+)$$

AG

Note: Do not award **A0R1**.

[2 marks]

(b) for $n = 1$

$$\log_2 1 = 0 \text{ and } 1 > 0 \text{ OR LHS} = 1, \text{ RHS} = \log_2 1 = 0 \text{ OR } \log_2 2 > \log_2 1$$

R1

(so true for $n = 1$)

assume true for $n = k$, ie $k > \log_2 k$

M1

Note: Award **M0** for statements such as “let $n = k$ ”, “assume $n = k$ is true”. The assumption of truth must be clear. “Assume P_k true” is accepted.

The following two marks after this **M1** are independent of this mark and can be awarded.

hence

$$1 + k > 1 + \log_2 k \text{ (using assumption)}$$

M1

$$\geq \log_2(k+1) \text{ (using result from part a)}$$

A1

hence if true for $n = k$ then true for $n = k + 1$

R1

and as true for $n = 1$, therefore true for all $n \in \mathbb{Z}^+$.

Note: Only award the final **R1** if the first three marks have been awarded.

[5 marks]**Total [7 marks]**

9. METHOD 1

assume that $\frac{1}{x(1-x)} < 4$ (for $0 < x < 1$)

M1

Note: Award **M0** for statements such as “let $\frac{1}{x(1-x)} < 4$ ”.

Award **M0** for incorrect use of weak inequality signs such as “assume that $\frac{1}{x(1-x)} \leq 4$ ”.

since $0 < x < 1$ ($\Rightarrow x > 0$ and $1-x > 0$, therefore) $x(1-x) > 0$ (seen anywhere)

R1

Note: Subsequent marks after this **M1R1** are independent of these two marks and can only be awarded provided a correct or weak inequality ($<$ or $\leq$) is used at the assumption step.

attempt to form a quadratic inequality in x (using their assumption)

(M1)

$$1 < 4x(1-x) \text{ OR } \frac{1-4x(1-x)}{x(1-x)} < 0$$

$$4x^2 - 4x + 1 < 0 \text{ (or equivalent)}$$

A1**EITHER**

$$(2x-1)^2 < 0 \text{ (or equivalent)}$$

A1

a contradiction, since $(2x-1)^2 \geq 0$ (for $0 < x < 1$)

R1**OR**

a graph of $y = 4x^2 - 4x + 1$ with the vertex touching the x-axis at $x = \frac{1}{2}$

A1

a contradiction, since $4x^2 - 4x + 1 \geq 0$ (for $0 < x < 1$)

R1**THEN**

$$\text{hence } \frac{1}{x(1-x)} \geq 4 \text{ for } x \in \mathbb{R}, 0 < x < 1.$$

AG

Note: Award a maximum of **M0R1(M1)A1A1R0** for incorrect use of weak inequality signs.

continued...

Question 9 continued.

METHOD 2

assume that $\frac{1}{x(1-x)} < 4$ (for $0 < x < 1$)

M1

Note: Award **M0** for statements such as “let $\frac{1}{x(1-x)} < 4$ ”.

Award **M0** for incorrect use of weak inequality signs such as “assume that $\frac{1}{x(1-x)} \leq 4$ ”.

Subsequent marks after this **M1** are independent and can only be awarded provided a correct or weak inequality ($<$ or $\leq$) is used at the assumption step.

$$\frac{1}{x(1-x)} \equiv \frac{A}{x} + \frac{B}{1-x} \Rightarrow 1 \equiv A(1-x) + Bx$$

attempt to equate both coefficients OR substitute two values eg 0 and 1

(M1)

$$A = 1 \text{ and } B = 1$$

A1

Note: Award **A1** for both correct values.

$$\text{Let } f(x) = \frac{1}{x} + \frac{1}{1-x}$$

equating their $f'(x) = 0$ and solve for x OR a concave upward graph for $0 < x < 1$

(M1)

$$f'(x) = -\frac{1}{x^2} + \frac{1}{(1-x)^2} = 0 \Rightarrow x^2 - 2x + 1 = x^2 \Rightarrow x = \frac{1}{2}$$

$$\text{minimum value is } f\left(\frac{1}{2}\right) = 4$$

A1

a contradiction, since $4 < 4$

R1

hence $\frac{1}{x(1-x)} \geq 4$ for $x \in \mathbb{R}, 0 < x < 1$.

AG

Note: Award a maximum of **M0(M1)A1(M1)A1R0** for incorrect use of weak inequality signs.

continued...

Question 9 continued.

METHOD 3

assume that $\frac{1}{x(1-x)} < 4$ (for $0 < x < 1$)

M1

Note: Award **M0** for statements such as “let $\frac{1}{x(1-x)} < 4$ ”.

Award **M0** for incorrect use of weak inequality signs such as “assume that $\frac{1}{x(1-x)} \leq 4$ ”.

Subsequent marks after this **M1** are independent and can only be awarded provided a correct or weak inequality ($<$ or $\leq$) is used at the assumption step.

Let $f(x) = \frac{1}{x(1-x)}$

attempt to use product rule to find $f'(x)$

(M1)

$$f'(x) = -\frac{1}{x^2(1-x)} + \frac{1}{x(1-x)^2} \left(= \frac{2x-1}{x^2(1-x)^2} \right)$$

A1

equating their $f'(x) = 0$ and solve for x

(M1)

$$x = \frac{1}{2}$$

minimum value is $f\left(\frac{1}{2}\right) = 4$

A1

a contradiction, since $4 < 4$

R1

hence $\frac{1}{x(1-x)} \geq 4$ for $x \in \mathbb{R}, 0 < x < 1$.

AG

Note: Award a maximum of **M0(M1)A1(M1)A1R0** for incorrect use of weak inequality signs.

[6 marks]

Section B

11. (a) attempt to use the binomial expansion with $n = \frac{1}{2}$ (M1)

$$= 1 + \left(\frac{1}{2}\right)(5x) + \dots$$

correct substitution into at least three terms (A1)

$$= 1 + \left(\frac{1}{2}\right)(5x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(5x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(5x)^3 + \dots$$

$$= 1 + \frac{5}{2}x - \frac{25}{8}x^2 + \frac{125}{16}x^3 (+\dots)$$

A1A1

Note: Award **A1** for $1 + \frac{5}{2}x - \frac{25}{8}x^2$ and **A1** for $\frac{125}{16}x^3$.

[4 marks]

- (b) at least 3 correct terms in expansion of $(1 + qx)^{-1}$ **A1**

$$= 1 + (-1)(qx) + \frac{(-1)(-2)}{2!}(qx)^2 + \dots (= 1 - qx + q^2x^2 + \dots)$$

attempt to find product (M1)

$$(1 + px)(1 - qx + q^2x^2 + \dots) = 1 - qx + q^2x^2 + \dots + px - pqx^2 + \dots$$

$$= 1 + px - qx + q^2x^2 - pqx^2 + \dots (= 1 + (p - q)x + (q^2 - pq)x^2 + \dots)$$

A1

Note: Award **A0** if the final answer is not in ascending order.

[3 marks]

continued...

Question 11 continued.

(c) equating their coefficients of either x or x^2 (M1)

one correct equation A1

$$p - q = \frac{5}{2} \quad \text{OR} \quad q^2 - pq = -\frac{25}{8}$$

valid attempt to solve for q or p (M1)

$$-\frac{5}{2}q = -\frac{25}{8} \quad \text{OR} \quad q^2 - q\left(q + \frac{5}{2}\right) = -\frac{25}{8} \quad \text{OR} \quad \frac{15}{4} - q = \frac{5}{2} \quad \text{A1}$$

Note: Award full marks for $\frac{15}{4} - q = \frac{5}{2}$ provided a full solution for $p = \frac{15}{4}$ is seen

first. Using $p = \frac{15}{4}$ from part (d)'s stem is not sufficient.

$$q = \frac{5}{4}$$

AG

[4 marks]

(d) (i) attempt to find x (M1)

$$1 + 5x = 1.2$$

$$x = \frac{1}{25} (= 0.04) \quad (\text{or equivalent}) \quad \text{A1}$$

substituting their value of x with p and q (M1)

$$\frac{1 + \frac{15}{4}\left(\frac{1}{25}\right)}{1 + \frac{5}{4}\left(\frac{1}{25}\right)}$$

$$\frac{115}{105} \left(= \frac{23}{21} \right) \quad \text{A1}$$

Note: Award (M1)A1(M1)A1 for an answer of $\frac{219}{200}$ or $\frac{2191}{2000}$ using the expansion from part (b) or (a).

continued...

Question 11 continued.

$$(ii) \quad \frac{\sqrt{5}}{2} = \sqrt{\frac{5}{4}} = \sqrt{1.25}$$

$$x = \frac{1}{20}$$

A1

$$\text{recognising } \frac{1}{20} > \frac{1}{25}$$

R1

therefore, approximation for $\frac{\sqrt{5}}{2}$ is not as accurate as the approximation

for $\sqrt{1.2}$

AG

[6 marks]

Total [17 marks]

.....

12. (a) $z^2 = 2\left(\cos\left(-\frac{2\pi}{3}\right) + i\sin\left(-\frac{2\pi}{3}\right)\right)$ OR $z^2 = 2\left(\cos\left(\frac{4\pi}{3}\right) + i\sin\left(\frac{4\pi}{3}\right)\right)$ **A2**

Note: Award **A1** for correct modulus or argument seen.

$$z^2 = 2\left(\cos\left(-\frac{2\pi}{3} + 2k\pi\right) + i\sin\left(-\frac{2\pi}{3} + 2k\pi\right)\right) \text{ OR } z^2 = 2\left(\cos\left(\frac{4\pi}{3} + 2k\pi\right) + i\sin\left(\frac{4\pi}{3} + 2k\pi\right)\right)$$

attempt to use De Moivre's theorem

M1

$$z = \sqrt{2}\left(\cos\left(-\frac{\pi}{3} + k\pi\right) + i\sin\left(-\frac{\pi}{3} + k\pi\right)\right) \text{ OR } z = \sqrt{2}\left(\cos\left(\frac{2\pi}{3} + k\pi\right) + i\sin\left(\frac{2\pi}{3} + k\pi\right)\right)$$

$$z = \sqrt{2}\left(\cos\left(-\frac{\pi}{3}\right) + i\sin\left(-\frac{\pi}{3}\right)\right); z = \sqrt{2}\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$$

A1

Note: accept $z = \sqrt{2}\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)$ in place of the 1st term above.

accept $r \operatorname{cis} \theta$ form throughout.

[4 marks]

(b) (i) $z_1 = \frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$ and $z_2 = -\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$

A1A1

Note: may be seen in part (a).

Award **A1A0** for z_1 and z_2 interchanged.

Accept factorised form, e.g. $z_1 = \sqrt{2}\left(\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)$.

(ii) $z_3 = \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$ and $z_4 = -\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i$

A1A1

Note: Award **A1A0** for z_3 and z_4 interchanged.

Accept factorised form, e.g. $z_3 = \sqrt{2}\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$.

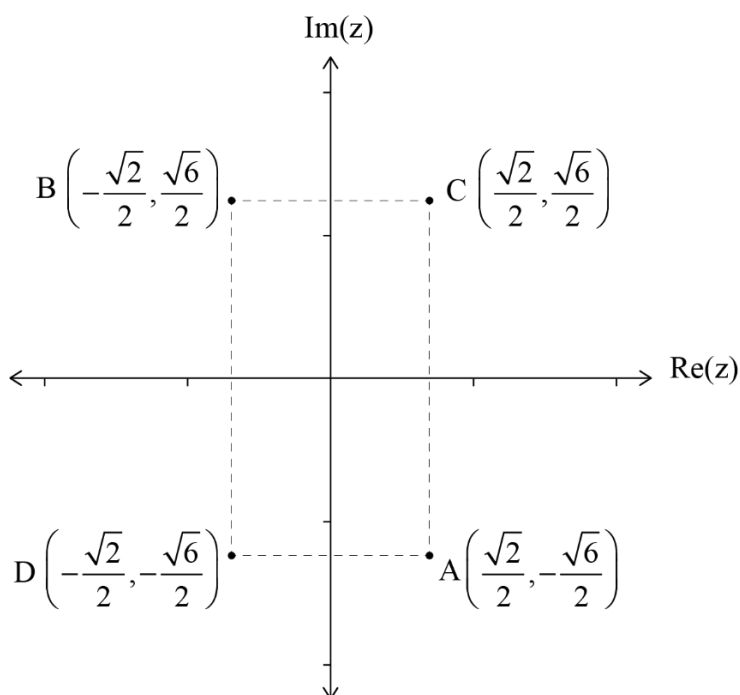
Award **A1FTA1FT** for $z_3 = \operatorname{their}(z_1)^*$ with $\operatorname{Re}(z_3) > 0$ and $z_4 = \operatorname{their}(z_2)^*$.

[4 marks]

continued...

Question 12 continued.

- (c) (i) Argand diagram with all four points plotted in approximately correct positions forming a rectangle.

A2

Note: Award **A1** for any two points plotted correctly on an argand diagram.

Award **A1A0** if the four points form a square.

Condone absence of labels, coordinates if they are approximately in correct positions and same distance from the origin. Dotted lines not required.

- (ii) **METHOD 1**

attempt to find the length OR the width of rectangle

(M1)

$$2 \times \frac{\sqrt{2}}{2} \quad \text{OR} \quad 2 \times \frac{\sqrt{6}}{2}$$

$$\text{length} = \sqrt{6} \quad \text{and} \quad \text{width} = \sqrt{2}$$

$$\text{area of rectangle} = \sqrt{12} \quad (= 2\sqrt{3})$$

A1

continued...

Question 12 continued.

METHOD 2

attempt to find the area of triangle

(M1)

$$\frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times \sin \frac{2\pi}{3} \quad \text{OR} \quad \frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times \sin \frac{\pi}{3}$$

$$4 \times \left(\frac{1}{2} \times \sqrt{2} \times \sqrt{2} \times \frac{\sqrt{3}}{2} \right)$$

$$\text{area of rectangle} = 2\sqrt{3}$$

A1

[4 marks]

(d) recognition that $z = \frac{1}{w}$

(M1)

$$\left(\frac{1}{w} \right)^4 + 2 \left(\frac{1}{w} \right)^2 + 4 = 0 \quad \text{OR} \quad \frac{1 + 2w^2 + 4w^4}{w^4} = 0$$

(A1)

$$4w^4 + 2w^2 + 1 = 0$$

$$p = 4, q = 2, r = 1$$

A1

[3 marks]

(e) (i) **EITHER**

attempt to take conjugate

M1

$$\frac{1}{\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{6}}{2}i \right)} \times \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i \right) \quad \text{OR} \quad \frac{2}{(\sqrt{2} - \sqrt{6}i)} \times \frac{(\sqrt{2} + \sqrt{6}i)}{(\sqrt{2} + \sqrt{6}i)}$$

OR

attempt to use De Moivre's theorem

M1

$$\frac{1}{\sqrt{2}} \left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3} \right)$$

THEN

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}i \quad (\text{or equivalent})$$

A1

continued...

Question 12 continued

- (ii) recognition of dilation/stretch by a scale factor of $\frac{1}{2}$ for both dimensions **(M1)**

$$\text{area of rectangle} = \frac{\sqrt{12}}{4} \left(= \frac{\sqrt{3}}{2} \right) \quad \textbf{A1}$$

Note: Award follow through from their area found in part (c)(ii).

[4 marks]

Total [19 marks]

1. (a) correct application of $\log_a xy = \log_a x + \log_a y$ or $\log_a x^m = m \log_a x$ (M1)

correct expression in terms of $\log_{10} 2$ AND $\log_{10} 3$ (arguments must be 2 and 3) (A1)

$$3 \log_{10} 2 + \log_{10} 3 \quad \text{OR} \quad \log_{10} 2 + \log_{10} 2 + \log_{10} 2 + \log_{10} 3$$

$$3p + q$$

A1

[3 marks]

(b) $(\log_3 8) = \frac{\log_{10} 8}{\log_{10} 3} \left(= \frac{3 \log_{10} 2}{\log_{10} 3} \right)$ (A1)

$$= \frac{3p}{q}$$

A1

[2 marks]

Total [5 marks]

Total [5 marks]
(M1)

8. (a) attempt to find modulus
- OR**
- argument of
- z_1

$$r = \sqrt{12} \left(= 2\sqrt{3} \right) \text{ or } \theta = -\frac{\pi}{3} \quad \textbf{A1}$$

$$z_1 = \sqrt{12}e^{-i\frac{\pi}{3}} \left(= 2\sqrt{3}e^{-i\frac{\pi}{3}} \right) \quad \textbf{A1}$$

[3 marks]

$$(b) \left(\frac{z_2}{z_1} = \frac{2\sqrt{3}e^{i\frac{5\pi}{6}}}{2\sqrt{3}e^{-i\frac{\pi}{3}}} = e^{i\frac{7\pi}{6}} \left(= e^{-i\frac{5\pi}{6}} \right) \right) \quad \textbf{(A1)}$$

recognizing three equivalent arguments **(M1)**

$$\text{eg } \frac{z_2}{z_1} = e^{i\left(-\frac{5\pi}{6} + 2\pi k\right)} \quad \textbf{OR} \quad \frac{z_2}{z_1} = e^{-\frac{5\pi}{6}i}, e^{-\frac{17\pi}{6}i}, e^{\frac{7\pi}{6}i}$$

attempt to find a root using de Moivre's theorem **(M1)**

$$e^{i\left(-\frac{5\pi}{18} + \frac{2\pi k}{3}\right)} \left(= e^{i\left(\frac{(12k-5)\pi}{18}\right)} = e^{i\left(\frac{(12k+7)\pi}{18}\right)} \right) \quad \textbf{(A1)}$$

$$e^{-i\frac{17\pi}{18}}, e^{-i\frac{5\pi}{18}}, e^{i\frac{7\pi}{18}} \quad \textbf{A1}$$

[5 marks]**Total [8 marks]**

10. (a) (i) METHOD 1attempt to equate differences of consecutive terms **(M1)**

$$(3-2k)-(k-5)=(5k+3)-(3-2k) \text{ OR } (k-5)-(3-2k)=(3-2k)-(5k+3) \quad \mathbf{A1}$$

$$8-3k=7k$$

$$(k=) \frac{4}{5} \quad \mathbf{A1}$$

METHOD 2 (system of equations)**TWO** correct equations involving k and d **A1**

$$k-5+d=3-2k \text{ OR } 3-2k+d=5k+3 \text{ OR } k-5+2d=5k+3$$

$$\text{OR } \frac{3}{2}(2(k-5)+2d)=k-5+3-2k+5k+3 \text{ (or equivalent)}$$

valid attempt to solve their system of equations using substitution or elimination **(M1)**

$$(d=5.6)$$

$$(k=) \frac{4}{5} \quad \mathbf{A1}$$

METHOD 3 (in terms of k)

$$\frac{3}{2}(k-5+5k+3)=k-5+3-2k+5k+3 \text{ (or equivalent)} \quad \mathbf{A1}$$

combining like terms **(M1)**

$$9k-3=4k+1 \text{ OR } 5k=4 \text{ (or equivalent)}$$

$$(k=) \frac{4}{5} \quad \mathbf{A1}$$

METHOD 4 (arithmetic mean)attempt to find mean of u_1 and u_3 **(M1)**

$$\frac{(k-5)+(5k+3)}{2}=3-2k \quad \mathbf{A1}$$

$$3k-1=3-2k$$

$$(k=) \frac{4}{5} \quad \mathbf{A1}$$

(ii) substituting their value of k into expression for u_3 **(A1)**

$$(u_3=) 5 \times \frac{4}{5} + 3$$

$$=7 \quad \mathbf{A1}$$

[5 marks]

continued...

Question 10 continued

(b) (i) substituting $k = 12$ into u_1 , u_2 or u_3 (M1)

$(u_1 =) 7$ **AND** $(u_2 =) -21$ **AND** $(u_3 =) 63$ (A1)

$(r =) \frac{-21}{7} = \frac{63}{-21} (= -3)$ **OR** $(-21)^2 = 7 \times 63$ **OR** $r = -3$ R1

u_1 , u_2 and u_3 are in geometric sequence AG

(ii) since $|r| \geq 1$ (accept $|r| > 1$ or $r < -1$) R1

hence not convergent AG

[4 marks]

(c) (i) attempts to find ratios, in terms of k , of consecutive terms and equating (M1)

$\frac{(3-2k)}{(k-5)} = \frac{(5k+3)}{(3-2k)}$ **OR** $(3-2k)^2 = (k-5)(5k+3)$ (or equivalent)

$9-12k+4k^2 = 5k^2-22k-15$ A1

Note: Award **A1** for correct expansion of all brackets leading to given result.

$k^2-10k-24=0$ AG

(ii) recognizing need to factorize, complete the square or substitute into quadratic formula (M1)

$(k+2)(k-12) (= 0)$ **OR** $(k-5)^2-49 (= 0)$ **OR** $k = \frac{10 \pm \sqrt{196}}{2}$

$k = -2$ (accept $k = -2$ **and** $k = 12$) A1

substituting their value of k (other than $k = 12$) to find u_1 , u_2 or u_3 (M1)

$(u_1 =) -7$ **AND** $(u_2 =) 7$ **AND** $(u_3 =) -7$ A1

(iii) $(S_{2m} =) 0$ A1

[7 marks]

Total [16 marks]

$\binom{2}{}$ $\binom{4}{}$

12. (a) substituting into modulus formula with either correct real part or correct imaginary part

(M1)

$$\sqrt{(x-2)^2 + (y-1)^2} (=3)$$

(A1)

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 9$$

A1

$$x^2 + y^2 - 4x - 2y - 4 = 0$$

AG

[3 marks]

- (b) **METHOD 1**

recognizing the need to express $\frac{z+p}{z-1}$ in the form $a+ib$

(M1)

$$\frac{z+p}{z-1} \times \frac{z^*-1}{z^*-1} \quad \text{OR} \quad \frac{z+p}{z-1} \times \frac{(x-yi)-1}{(x-yi)-1} \quad \text{OR} \quad \frac{(x+p)+iy}{(x-1)+iy} \times \frac{(x-1)-iy}{(x-1)-iy}$$

A1

expanding their $\frac{z+p}{z-1} \times \frac{z^*-1}{z^*-1}$

(M1)

$$\frac{zz^* - z + pz^* - p}{zz^* - z - z^* + 1} \quad \text{OR} \quad \frac{x^2 + y^2 - x - yi + px - pyi - p}{x^2 + y^2 - x - yi - x + yi + 1}$$

$$\text{OR} \quad \frac{(x+p)(x-1) + y^2 + ((x-1)y - (x+p)y)i}{(x-1)^2 + y^2}$$

$$\frac{x^2 + y^2 + (p-1)x - p - (p+1)yi}{x^2 + y^2 - 2x + 1} \quad (\text{or equivalent}) \quad (\text{seen anywhere})$$

A1

$$\left(\Rightarrow \arg\left(\frac{z+p}{z-1}\right) = \right) \arctan\left(\frac{-(p+1)y}{x^2 + y^2 + (p-1)x - p}\right)$$

(A1)

equating their $\arg\left(\frac{z+p}{z-1}\right)$ to $\frac{\pi}{4}$ OR their expression for $\frac{z+p}{z-1}$ in x and y to 1

OR equating real and imaginary parts

(M1)

$$\arctan\left(\frac{-(p+1)y}{x^2 + y^2 + (p-1)x - p}\right) = \frac{\pi}{4} \quad \text{OR} \quad \frac{-(p+1)y}{x^2 + y^2 + (p-1)x - p} = 1$$

$$-(p+1)y = x^2 + y^2 + (p-1)x - p$$

A1

$$x^2 + y^2 + (p-1)x + (p+1)y - p = 0$$

AG

continued...

Question 12 continued

METHOD 2

$$\arg\left(\frac{z+p}{z-1}\right) = \arg(z+p) - \arg(z-1) \quad (M1)$$

$$= \arg(x+p+yi) - \arg(x-1+yi) \quad A1$$

$$= \arctan\left(\frac{y}{x+p}\right) - \arctan\left(\frac{y}{x-1}\right) \quad A1$$

attempt to use compound angle formula for tan (M1)

$$\tan\left(\arctan\left(\frac{y}{x+p}\right) - \arctan\left(\frac{y}{x-1}\right)\right) = \frac{\tan\left(\arctan\left(\frac{y}{x+p}\right)\right) - \tan\left(\arctan\left(\frac{y}{x-1}\right)\right)}{1 + \tan\left(\arctan\left(\frac{y}{x+p}\right)\right) \times \tan\left(\arctan\left(\frac{y}{x-1}\right)\right)}$$

$$= \frac{\frac{y}{x+p} - \frac{y}{x-1}}{1 + \frac{y}{x+p} \times \frac{y}{x-1}} \quad A1$$

equating their expression for $\tan\left(\arctan\left(\frac{y}{x+p}\right) - \arctan\left(\frac{y}{x-1}\right)\right)$ to $\tan\frac{\pi}{4}$ (M1)

$$\frac{\frac{y}{x+p} - \frac{y}{x-1}}{1 + \frac{y}{x+p} \times \frac{y}{x-1}} = \tan\frac{\pi}{4} \left(\Rightarrow \frac{(x-1)y - (x+p)y}{(x+p)(x-1) + y^2} = 1 \right)$$

$$(x+p)(x-1) + y^2 = -y - py \quad A1$$

$$x^2 - x + px - p + y^2 = -y - py$$

$$x^2 + y^2 + (p-1)x + (p+1)y - p = 0 \quad AG$$

[7 marks]

continued...

Question 12 continued

- (c) $\left(\arg\left(\frac{z+4}{z-1}\right) = \frac{\pi}{4} \Rightarrow \right) x^2 + y^2 + 3x + 5y - 4 = 0$ (seen anywhere) **A1**
- solving simultaneously their equations in x and y **(M1)**
- $7x + 7y = 0$ ($\Rightarrow x = -y$) (or equivalent) **A1**
- substituting to obtain an equation in either x or y **(M1)**
- $x^2 - x - 2 = 0$ **OR** $y^2 + y - 2 = 0$
- solving for either x or y **(M1)**
- $x = -1$ and $x = 2$ **OR** $y = -2$ and $y = 1$ **A1**
- $(z_1 =) -1 + i$ **AND** $(z_2 =) 2 - 2i$ (or equivalent) **A1**

Note: Accept $x = -1$, $y = 1$ **AND** $x = 2$, $y = -2$.

[7 marks]

- (d) let z_1, z_2, z_3 and z_4 be roots of $z^4 + az^3 + bz^2 + cz + d = 0$
- EITHER**
- $a = -(z_1 + z_2 + z_3 + z_4)$ **(A1)**
- OR**
- expanding $(z - z_1)(z - z_2)(z - z_3)(z - z_4)$ to obtain term in z^3
- $-(z_1 + z_2)z^3 - (z_3 + z_4)z^3$ **(A1)**
- THEN**
- recognizing that z_3 and z_4 are conjugates of z_1 and z_2 (seen anywhere) **(M1)**
- roots are $-1 + i, -1 - i, 2 - 2i, 2 + 2i$ **A1**
- $a = -((-1 + i) + (-1 - i) + (2 - 2i) + (2 + 2i))$
- $= -2$ **A1**

[4 marks]

Total [21 marks]

8. (a) $\text{Im}\left(e^{2ti}\left(e^i + e^{3i}\right)\right)$

$$= \text{Im}\left(e^{(2t+1)i} + e^{(2t+3)i}\right)$$

A1

Note: This **A1** is for clearly showing that the powers are added.

Accept alternative notation for the step of adding the arguments e.g.

$$(\cos(2t) + i \sin(2t))(\cos(1) + i \sin(1)) = \cos(2t+1) + i \sin(2t+1)$$

$$= \sin(2t+1) + \sin(2t+3)$$

A1

$$= h(t)$$

AG

Note: Accept argument in reverse

[2 marks]

(b) $e^i + e^{3i} = 1.08060\dots e^{2i}$

$$1.08e^{2i}$$

A1A1

Note: Award **A1** for modulus **A1** for argument

$$r = 1.08 (= 2 \cos(1)) \quad , \quad \theta = 2$$

[2 marks]

(c) **METHOD 1**

attempt to use their answers to part (a) and (b) to write $h(t)$ as the imaginary part of a number in the form $re^{i\theta}$

(M1)

$$h(t) = \text{Im}\left(e^{2ti}\left(1.08060\dots e^{2i}\right)\right)$$

$$= \text{Im}\left(1.08060\dots e^{(2t+2)i}\right)$$

(A1)

$$h(t) = 1.08060\dots \sin(2t+2)$$

$$= 1.08 \sin(2t+2) (= 2 \cos(1) \sin(2t+2))$$

A1

METHOD 2

Considering the graph of $h(t)$

amplitude = 1.08060...

$$p = 1.08 (= 2 \cos(1))$$

(A1)

attempt to consider the horizontal shift of the graph of h

(M1)

first negative zero of graph is -1

$$h(t) = 1.08060\dots \sin(2(t+1)) (= 1.08060\dots \sin(2t+2))$$

$$= 1.08 \sin(2t+2) (= 2 \cos(1) \sin(2t+2))$$

A1**[3 marks]****Total [7 marks]**

10. (a) 15

A1**[1 mark]**(b) attempt to add 11 cards onto a stack with 3 rows OR attempt to consider all 4 rows **(M1)**valid diagram with 4 rows OR $t_4 = 15 + 11$ OR $t_4 = 2 + 5 + 8 + 11$ $= 26$ **A1****[2 marks]**(c) **METHOD 1**recognition that t_n is a sum of an arithmetic sequence**(M1)**

$$t_n = 2 + 5 + 8 + 11 + \dots$$

attempt to use formula for the sum of n terms of an arithmetic sequence**M1**

$$t_n = \frac{n}{2}(2(2) + 3(n-1))$$

A1

$$t_n = \frac{n}{2}(3n+1)$$

AG**METHOD 2**attempt to split t_n into the total number of stacked and horizontal cards**(M1)**

$$\text{stacked } 2 + 4 + 6 + \dots = \frac{n}{2}(4 + 2(n-1)) \left(= \frac{n}{2}(2n+2) \right)$$

A1

$$\text{horizontal } 0 + 1 + 2 + \dots = \frac{n}{2}(0 + 1(n-1)) \left(= \frac{n}{2}(n-1) \right)$$

A1

$$t_n = \frac{n}{2}(4 + 2(n-1)) + \frac{n}{2}(0 + 1(n-1)) \left(= \frac{n}{2}(2n+2) + \frac{n}{2}(n-1) \right)$$

$$t_n = \frac{n}{2}(3n+1)$$

AG*continued...*

Question 10 continued.

METHOD 3

recognition that a stack with n rows is made up of complete triangles with the bottom row of horizontal cards removed and that the numbers of complete triangle cards form an arithmetic sequence (M1)

$$t_n = (3 + 6 + 9 + 12 + \dots + 3n) - n \text{ OR } t_n = 3(1 + 2 + 3 + 4 + \dots + n) - n$$

attempt to use formula for the sum of n terms of an arithmetic sequence M1

$$t_n = \frac{n}{2}(2(3) + 3(n-1)) - n \text{ OR } t_n = 3 \times \frac{n}{2}(1+n) - n \quad \text{A1}$$

$$t_n = \frac{n}{2}(3n+1) \quad \text{AG}$$

[3 marks]

(d) attempt to solve $\frac{n(3n+1)}{2} \leq 14(52) (= 728)$ (M1)

Note: Accept an attempt to solve an equation for (M1).

$$21.8642\dots \text{ OR } n = 21, t_n = 672 \text{ and } n = 22, t_n = 737 \quad \text{(A1)}$$

max number of rows is 21 A1

[3 marks]

continued...

Question 10 continued.

(e) **EITHER**

attempt to solve by listing at least six values of t_n **(M1)**

2, 7, 15, 26, 40, 57...

OR

recognition that $\frac{\frac{1}{2}n(3n+1)}{52}$ must be an integer **(M1)**

$\frac{1}{2}n(3n+1) = 52k$ (where k is an integer)

THEN

min number of rows is 13 **A1**

Note: Award **(M1)A0** for an answer of 5 packs.

Award **M0A0** for any answer resulting from solving $\frac{1}{2}n(3n+1) = 52$.

[2 marks]

continued...

Question 10 continued.

(f) **EITHER**

attempt to use Pythagoras's Theorem or trigonometry to find the height of an equilateral triangle with sides 88mm **(M1)**

$$\text{height} = \sqrt{88^2 - 44^2} \text{ OR } 88\sin 60^\circ \text{ OR } 88\cos 30^\circ \text{ OR } 44\tan 60^\circ \text{ OR}$$

$$\frac{44}{\tan 30^\circ} \text{ OR } 44\sqrt{3} (= 76.2102...) \quad \textbf{(A1)}$$

attempt to solve $44n\sqrt{3} > 2000$ OR their perpendicular height $\times n > 2000$ **(M1)**

Note: Accept an attempt to solve an equation for **(M1)**.

OR

attempt to use trigonometry to find the side of an equilateral triangle with height 2000mm **(M1)**

$$\text{side} = \frac{2000}{\sin 60^\circ} \text{ OR } \frac{2000}{\cos 30^\circ} \text{ OR } \frac{4000}{\sqrt{3}} (= 2309.40...) \quad \textbf{(A1)}$$

attempt to solve $88n > 2309.40...$ OR $88n > \text{their side}$ **(M1)**

Note: Accept an attempt to solve an equation for **(M1)**.

THEN

$$n > 26.2431...$$

so min number of rows is 27 **(A1)**

$$t_{27} = 1107 \quad \textbf{A1}$$

[5 marks]

Total [16 marks]

12. (a) attempt to replace x with $-x$ **M1**

$$f_n(-x) = \sum_{r=0}^n \left(-2(-x)^2 \right)^r \text{ OR } -2(-x)^2 = -2x^2 \text{ (seen anywhere)} \quad \textbf{A1}$$

$$f_n(-x) = f_n(x) \quad \textbf{A1}$$

so f_n is even for all values of n **AG**

[3 marks]

(b) (i) $f_3(x) = 1 - 2x^2 + (-2x^2)^2 + (-2x^2)^3$ **A1**

$$= 1 - 2x^2 + 4x^4 - 8x^6 \quad \textbf{AG}$$

(ii) $= 1 - 2x^2 + 4x^4 - 8x^6 + 16x^8$ **A1**

[2 marks]

(c) (i) recognition of geometric series with common ratio $-2x^2$ **(M1)**

$$\text{converges for } |-2x^2| < 1 \left(\Rightarrow x^2 < \frac{1}{2} \right) \quad \textbf{(A1)}$$

$$-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}} \left(\Rightarrow -\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2} \right)$$

$$\text{largest } K = \frac{1}{\sqrt{2}} \left(= \frac{\sqrt{2}}{2} \right) \quad \textbf{A1}$$

(ii) use of formula for S_∞ of a geometric series with first term 1, common ratio $-2x^2$ **(M1)**

$$f(x) = \frac{1}{1 - (-2x^2)}$$

$$(f(x) =) \frac{1}{1 + 2x^2} \quad \textbf{A1}$$

$$a = 1, b = 2$$

[5 marks]

continued...

Question 12 continued.

(d) (i) g is a one-to-one function (A1)

since g is a (strictly) decreasing function OR g has no points of zero gradient (turning points) R1

(ii) attempt to rearrange and swop x and y (at any stage) (M1)

$$x = \frac{1}{1+2y^2} \Rightarrow x + 2xy^2 = 1$$

$$y^2 = \frac{1-x}{2x} \quad (A1)$$

$$y = \pm \sqrt{\frac{1-x}{2x}}$$

$$g^{-1}(x) = \sqrt{\frac{1-x}{2x}} \quad A1$$

Note: Award **A0** if $g^{-1}(x)$ is missing.

Award **FT A1** marks only if f is of the correct form $f(x) = \frac{1}{a+bx^2}$

(domain) $\frac{1}{2} < x \leq 1$ A1

Note: This **A1** can be awarded independently.

[6 marks]

continued...

Question 12 continued.

(e)

Note: Throughout part (e), do not award **A1** marks as **FT** from part (d)

METHOD 1

curves intersect at $x = 0.5897545107\dots$

(A1)

attempt to add the areas to the left and to the right of the point of intersection

(M1)

$$\int_0^{0.589\dots} g(x)dx + \int_{0.589\dots}^1 g^{-1}(x)dx \left(= \int_0^{0.589\dots} \frac{1}{1+2x^2} dx + \int_{0.589\dots}^1 \sqrt{\frac{1-x}{2x}} dx \right)$$

$$= 0.491548\dots + 0.143738\dots$$

(A1)

Note: Award **A1** for one correct value seen, dependent on **(M1)**

$$= 0.635286\dots$$

$$\text{Area of } R = 0.635$$

A1

METHOD 2

curves intersect at $x = 0.5897545107\dots$

(A1)

attempt to find the area between $y = g(x)$ and $y = x$ to the left of the point of intersection

(M1)

$$\int_0^{0.589\dots} g(x)dx - \int_0^{0.589\dots} xdx = \int_0^{0.589\dots} \left(\frac{1}{1+2x^2} - x \right) dx$$

$$= 0.317643\dots$$

(A1)

$$2(0.3176432617\dots) = 0.635286\dots$$

$$\text{Area of } R = 0.635$$

A1

continued...

Question 12 continued.

METHOD 3

curves intersect at $x = 0.5897545107\dots$

A1

attempt to find the area under $y = g^{-1}(x)$ and $y = x$ to the right of the point of intersection and the area of a square of side $x = 0.589\dots$

(M1)

$$0.589\dots^2 + 2 \int_{0.589\dots}^1 g^{-1}(x) dx \left(= 0.589\dots^2 + 2 \int_{0.589\dots}^1 \sqrt{\frac{1-x}{2x}} dx \right)$$

$$= 0.347810\dots + 2(0.143738\dots)$$

A1

Note: Award **A1** for 0.143..seen, dependent on **(M1)**

$$= 0.635286\dots$$

Area of $R = 0.635$

A1

[4 marks]

Total [20 marks]

4. (a) METHOD 1

$$30000\left(1 - \frac{15}{100}\right)^{10} \text{ OR } 30000(0.85)^{10} \quad (M1)$$

5906.232130...

€5906.23

A1**Note:** Award **(M1)** for the correct list of values**METHOD 2**

PV = ±30000

FV = ∓5906.23213...

I% = -15%

P / Y = 1

C / Y = 1

N = 10

€5906.23

(M1)**A1****[2 marks]****(b) METHOD 1**

$$r = c - i (= 1.5 - 0.8)$$

$$r = 0.7(\%) \quad (A1)$$

attempt to substitute their values into inequality **(M1)**

$$50000\left(1 + \frac{0.7}{100}\right)^n > 55000 \text{ (accept equality)}$$

correct critical value OR at least one correct crossover value **(A1)**

$$n = 13.6633... \text{ OR } n = 13, €54746.09 \text{ OR } n = 14, €55129.31$$

14 (months)

A1**[4 marks]****Note:** Award **(A1)(M1)(A0)A0** for 164 (months)*continued...*

Question 4 continued.

METHOD 2

$$\frac{1 + \frac{1.5}{100}}{1 + \frac{0.8}{100}} = 1.00694... \quad (r = 0.694...(\%)) \quad (\text{A1})$$

attempt to substitute their values into inequality (M1)

$$50000(1.00694...)^n > 55000 \quad (\text{accept equality})$$

correct critical value OR at least one correct crossover value (A1)

$$n = 13.7722... \text{ OR } n = 13, \text{€}54706.84 \text{ OR } n = 14, \text{€}55086.75$$

14 (months) A1

Note: Award (A1)(M1)(A0)A0 for 164 (months)

[4 marks]

Total [6 marks]

7. (a) attempt to find common ratio (M1)

$$80r^3 = 0.74088$$

$$r = 0.21 \quad (A1)$$

$$u_2 = 16.8 \quad A1$$

[3 marks]

- (b) **METHOD 1**

attempt to find common difference (M1)

$$u_1 + 10d - u_1 = 16.8 - 80 \quad (10d = -63.2)$$

$$d = -6.32 \quad (A1)$$

EITHER

attempt to find the value of n using table or graph of S_n (M1)

$$n = 13$$

$$S_{13} = \frac{13}{2}(2 \times 80 - 6.32 \times 12)$$

OR

attempt to find greatest value of S_n using table or graph (M1)

THEN

$$= 547.04 \quad A1$$

Note: Award (M1)(A1)(M1)A0 for candidates who give maximum at (13.1582..., 547.119...).

[4 marks]

METHOD 2

attempt to find common difference (M1)

$$u_1 + 10d - u_1 = 16.8 - 80 \quad (10d = -63.2)$$

$$d = -6.32 \quad (A1)$$

attempt to find the value of n using $u_n > 0$ (M1)

$$80 + (n-1)(-6.32) > 0 \quad (\text{accept } =) \Rightarrow n < 13.6582... \text{ OR } n = 13$$

$$S_{13} = \frac{13}{2}(2 \times 80 - 6.32 \times 12)$$

$$= 547.04 \quad A1$$

[4 marks]

Total [7 marks]

$$7. \quad \frac{1}{\sqrt{q-x^2}} = q^{-\frac{1}{2}} \left(1 - \frac{x^2}{q}\right)^{-\frac{1}{2}} \text{ OR } \frac{1}{\sqrt{q-x^2}} = (q-x^2)^{-\frac{1}{2}} \quad (\text{A1})$$

substituting into extension of binomial theorem formula to find term in x^6 (M1)

$$-\frac{1}{2} \times \left(-\frac{1}{2} - 1\right) \times \left(-\frac{1}{2} - 2\right) \times \frac{1}{3!} \times \left(-\frac{x^2}{q}\right)^3 \left(= \left(\left(-\frac{5}{16}\right) \times \left(-\frac{x^2}{q}\right)^3\right) \right) \text{ OR } \binom{-\frac{1}{2}}{r} \times q^{\frac{1}{2}-3} \times (-x^2)^3 \quad (\text{A1})$$

$$q^{-\frac{1}{2}} \times \left(-\frac{5}{16}\right) \times \left(-\frac{1}{q}\right)^3 \left(= \frac{5}{16} q^{-\frac{7}{2}} \right) \text{ OR } \frac{5}{16} q^{-\frac{7}{2}} x^6 \quad (\text{A1})$$

equating their coefficient to 5120 or their term in x^6 to $5120x^6$ (M1)

$$\frac{5}{16} q^{-\frac{7}{2}} = 5120 \text{ OR } \frac{5}{16} q^{-\frac{7}{2}} x^6 = 5120x^6$$

$$q = 0.0625 \left(= \frac{1}{16} \right) \quad \text{A1}$$

[6 marks]

8. METHOD 1**EITHER**

number of games played by $n-1$ players ${}^{n-1}C_2 \times 2$ **OR** $(n-1)(n-2)$ **(A2)**

total number of games is ${}^{n-1}C_2 \times 2 + 7$ **OR** $(n-1)(n-2) + 7$ **(A1)**

OR

total number of games that should have been played is $2 \times {}^nC_2$ **OR** $n(n-1)$ **(A1)**

Stephen should have played $2(n-1)$ so he missed $2(n-1) - 7$ games **(A1)**

total number of games played is $n(n-1) - (2(n-1) - 7)$ **(A1)**

THEN

equating their total number of games to 513 **(M1)**

$n = 24$ **A1**

METHOD 2

there are N students other than Stephen

number of games played by N students is $2((N-1) + (N-2) + \dots + 2 + 1)$ $(= N(N-1))$ **(A1)**

total number of games that are played $N(N-1) + 7$ **(A1)**

equating their total number of games to 513 **(M1)**

$N = 23$ **(A1)**

number of students in class = 24 **A1**

[5 marks]

1. (a) attempt to find the common difference (M1)

$$12 = 36 + 4d \quad \text{OR} \quad \frac{12 - 36}{5 - 1}$$

$$d = -6 \quad \text{(A1)}$$

attempt to use the term formula for an arithmetic sequence (M1)

$$u_{13} = u_1 + 12(-6) \quad \text{OR} \quad u_5 + 8(-6)$$

$$u_{13} = -36 \quad \text{A1}$$

[4 marks]

- (b) **METHOD 1**

attempt to set $S_n = 0$ (M1)

$$\frac{n}{2}(2(36) + (n-1)(-6)) = 0 \quad \text{OR} \quad \frac{n(36 + u_n)}{2} = 0 \Rightarrow u_n = -36$$

$$n = 13 \quad \text{A1}$$

Note: Condone $n = 0, 13$

METHOD 2

attempt to recognize symmetry (M1)

$$36 + 30 + \dots + 0 + \dots - 30 - 36$$

$$n = 13 \quad \text{A1}$$

[2 marks]

Total [6 marks]

6. METHOD 1 Row Reduction

attempt to reduce a matrix or eliminate one variable

(M1)

two correct rows or equivalent equations

A1A1

$$\begin{bmatrix} 1 & 0 & -1 & 4 \\ 2 & 2 & 1 & 14 \\ 1 & 2 & \alpha & \beta \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -1 & 4 \\ 0 & 2 & 3 & 6 \\ 0 & 2 & 1+\alpha & \beta-4 \end{bmatrix}$$

correct third row or equivalent equation

A1

$$\begin{bmatrix} 1 & 0 & -1 & 4 \\ 0 & 2 & 3 & 6 \\ 0 & 0 & \alpha-2 & \beta-10 \end{bmatrix}$$

equating the third row coefficients to zero

(M1)

$$\alpha = 2 \text{ and } \beta = 10$$

A1**METHOD 2 Writing an equation in one variable**attempt to eliminate either x or z using the first two equations (seen anywhere)**(M1)**

$$2(z+4)+2y+z=14 \text{ OR } 2x+2y+(x-4)=14$$

one correct equation

A1

$$2y+3z=6 \text{ OR } 3x+2y=18$$

correct substitution into the third equation to create an equation in either x or z **A2**

$$x+18-3x+\alpha(x-4)=\beta \text{ OR } z+4+6-3z+\alpha z=\beta$$

$$\text{OR } (\alpha-2)x=\beta+4\alpha-18 \text{ OR } z(\alpha-2)=\beta-10$$

attempt to set coefficients equal to zero for infinite solutions

(M1)

$$\alpha = 2 \text{ and } \beta = 10$$

A1*continued...*

Question 6 continued.

METHOD 3 Subtracting and comparing equations

attempt to add or subtract two equations

(M1)

$$x + 2y + 2z = 10 \quad \text{OR} \quad 2x + 2y + (\alpha - 1)z = \beta + 4 \quad \text{OR} \quad x + (1 - \alpha)z = 14 - \beta$$

A2

attempt to compare coefficients with the third equation

(M1)

$$\alpha - 1 = 1, \beta + 4 = 14, 1 - \alpha = -1, 14 - \beta = 4$$

$$\alpha = 2 \quad \text{and} \quad \beta = 10$$

A1A1

METHOD 4 Vector approach

attempt to find direction vector of line of intersection using π_1 and π_2

(M1)

$$\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$$(\vec{d} =) \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$$

A1

$$\text{normal of } \pi_3 \text{ is perpendicular to } \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$$

(M1)

$$\begin{pmatrix} 1 \\ 2 \\ \alpha \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix} = 0$$

$$\alpha = 2$$

A1

correct substitution of a point common to the first two planes into the third plane

A1

$$0 + 2(9) + 2(-4), 4 + 2(3) + 2(0)$$

$$\beta = 10$$

A1

continued...

Question 6 continued.

METHOD 5 Finding a general point on the line of intersection

attempt to find intersection of π_1 and π_2 **(M1)**

$$x = 4 + z, \quad y = 3 - \frac{3}{2}z$$

$x = 4 + 2t, \quad y = 3 - 3t, \quad z = 2t$ **OR** $\mathbf{r} = (4, 3, 0) + t(2, -3, 2)$ **A1A1**

correct substitution of general point into π_3 **A1**

$$(4 + 2t) + 2(3 - 3t) + \alpha(2t), \quad 10 + t(2\alpha - 4)$$

recognizing that the coefficient of t must be zero OR the constant term must be β **(M1)**

$$2\alpha - 4 = 0 \quad \text{OR} \quad \beta = 10$$

$\alpha = 2$ and $\beta = 10$ **A1**

Total [6 marks]

11. (a) attempt to multiply by conjugate

M1

$$\frac{4i}{1+i\sqrt{3}} \times \frac{1-i\sqrt{3}}{1-i\sqrt{3}}$$

$$\frac{4\sqrt{3}+4i}{4}, i(1-i\sqrt{3})$$

A1

$$w = \sqrt{3} + i$$

AG

[2 marks]

- (b) (i) attempt to find either $|w|$ or $\arg w$ (could be seen in (ii))

(M1)

$$|w| = \sqrt{1^2 + 3} \text{ OR } \arctan\left(\frac{1}{\sqrt{3}}\right) \text{ OR labelled diagram}$$

$$|w| = 2$$

A1

[2 marks]

- (ii) $\arg w = \frac{\pi}{6}$ (accept 30°)

A1

[1 mark]

- (c) attempt to find a root using de Moivre's theorem with $n = \frac{1}{2}$

(M1)

$$\sqrt{2}e^{\frac{i\pi}{12}}$$

A1

attempt to find second root by adding or subtracting π from *their* argument

(M1)

$$\sqrt{2}e^{\frac{-11\pi i}{12}}$$

A1

[4 marks]

continued...

Question 11 continued.

(d) (i) **METHOD 1**

attempt to expand and equate to $\sqrt{3} + i$ **(M1)**

$$a^2 - b^2 + 2abi = \sqrt{3} + i$$

at least one correct equation **A1**

$$a^2 - b^2 = \sqrt{3} \quad \text{OR} \quad 2ab = 1$$

substituting $b = \frac{1}{2a}$ **(M1)**

$$a^2 - \left(\frac{1}{2a}\right)^2 = \sqrt{3}$$

$$4a^4 - 4\sqrt{3}a^2 - 1 = 0 \quad \text{OR} \quad a^4 - \sqrt{3}a^2 - \frac{1}{4} = 0 \quad \text{A1}$$

attempt to solve quadratic in a^2 **M1**

$$a^2 = \frac{4\sqrt{3} \pm \sqrt{48 + 16}}{8} \quad \text{OR} \quad \frac{\sqrt{3}}{2} \pm 1$$

since $a^2 > 0$ (accept $a > 0$) **R1**

$$a^2 = \frac{\sqrt{3}}{2} + 1 \quad \text{AG}$$

continued...

Question 11 continued.

METHOD 2

$$\sqrt{w} = a + bi \quad (\text{A1})$$

$$a + bi = \sqrt{2} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12} \right) \quad \text{A1}$$

$$\text{attempt to equate } \operatorname{Re}(\sqrt{w}) \quad \text{M1}$$

$$a = \sqrt{2} \cos \frac{\pi}{12}$$

$$a^2 = 2 \cos^2 \frac{\pi}{12} \quad \text{A1}$$

$$\text{attempt to use double angle formula} \quad (\text{M1})$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$a^2 = 2 \left(\frac{\cos \frac{\pi}{6} + 1}{2} \right) = \left(\cos \frac{\pi}{6} + 1 \right) \quad \text{A1}$$

$$\left(a^2 = \right) \frac{\sqrt{3}}{2} + 1 \quad \text{AG}$$

[6 marks]

$$(ii) \quad \text{attempt to substitute } a^2 \text{ into } b^2 = a^2 - \sqrt{3} \text{ OR } b^2 = \frac{1}{4a^2} \quad (\text{M1})$$

$$\left(\frac{\sqrt{3}}{2} + 1 \right) - \sqrt{3}, \frac{1}{4 \left(\frac{\sqrt{3}}{2} + 1 \right)}$$

$$b^2 = 1 - \frac{\sqrt{3}}{2} \quad \text{A1}$$

Note: Accept the unrationalized answer of $\frac{1}{2\sqrt{3} + 4}$.

[2 marks]

continued...

Question 11 continued.

- (e) attempt to set up an equation using an argument of $\frac{\pi}{12}$ **(M1)**

$$\tan\left(\frac{\pi}{12}\right) = \frac{b}{a} = \sqrt{\frac{b^2}{a^2}} \quad \text{A1}$$

substitution of the given a and their b **M1**

$$\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}} \quad \text{OR} \quad \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}} \quad \text{A1}$$

$$\sqrt{\frac{(2 - \sqrt{3})^2}{2^2 - 3}}$$

$$\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}, \text{ accept } p = 2, q = -1 \quad \text{A1}$$

[5 marks]

Total [22 marks]

1.

METHOD 1attempt to use $u_n = u_1 + (n - 1)d$ (M1)

$$u_1 + 6d = 6$$
 (A1)

$$u_1 + 5d + u_1 + 11d = 24$$
 (A1)

$$2u_1 + 16d = 24 \quad (u_1 + 8d = 12)$$

attempt to solve their equations simultaneously (M1)

$$u_1 = -12 \text{ and } d = 3$$
 A1

METHOD 2attempt to express u_6 and u_{12} in terms of the 7th term and d (M1)

$$u_6 = 6 - d$$
 (A1)

$$u_{12} = 6 + 5d$$
 (A1)

setting their sum of u_6 and u_{12} equal to 24 (M1)

$$12 + 4d = 24$$

$$d = 3 \text{ and } u_1 = -12$$
 A1

[5 marks]

11. (a) (i) $w = 2e^{i\frac{\pi}{3}}$

A1A1

Note: Award **A1** for $r = 2$ and **A1** for $\alpha = \frac{\pi}{3}$.

(ii) attempt to apply De Moivre's theorem:

M1

$$(1 + i\sqrt{3})^8 = 2^8 e^{i\frac{8\pi}{3}} \text{ OR } 2^8 \operatorname{cis}\left(\frac{8\pi}{3}\right) \text{ (or equivalent)}$$

A1

$$(1 - i\sqrt{3})^8 = 2^8 e^{-i\frac{8\pi}{3}} \text{ OR } 2^8 \operatorname{cis}\left(-\frac{8\pi}{3}\right) \text{ (or equivalent)}$$

A1

$$\left((1 + i\sqrt{3})^8 + (1 - i\sqrt{3})^8\right) = 256\left(-\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) + 256\left(-\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) \text{ OR}$$

$$2^8 \left(\cos\left(\frac{2\pi}{3}\right) + \cos\left(-\frac{2\pi}{3}\right) \right)$$

A1

$$= -256$$

AG**[6 marks]**

(b) recognition that $1 - i \tan \theta = 1 + i \tan(-\theta)$

(M1)

$$(1 + i \tan \theta)^4 + (1 - i \tan \theta)^4 \equiv (1 + i \tan \theta)^4 + (1 + i \tan(-\theta))^4$$

$$\equiv \frac{\cos 4\theta + i \sin 4\theta}{\cos^4 \theta} + \frac{\cos(-4\theta) + i \sin(-4\theta)}{\cos^4(-\theta)}$$

(A1)

$$\equiv \frac{\cos 4\theta + i \sin 4\theta}{\cos^4 \theta} + \frac{\cos 4\theta - i \sin 4\theta}{\cos^4 \theta}$$

A1

$$\equiv \frac{2 \cos 4\theta}{\cos^4 \theta}$$

AG**[3 marks]***continued...*

Question 11 continued

(c) considering $\cos 4\theta = 0$ (M1)

$$4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2} \text{ or equivalent} \quad (\text{A1})$$

$$\theta = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8} \text{ or equivalent} \quad \text{A1}$$

$$\text{hence roots are } z = i \tan\left(\frac{k\pi}{8}\right) \text{ where } k = 1, 3, 5, 7 \text{ (or equivalent)} \quad \text{A1}$$

Note: For the first two **A1**'s allow extra correct solutions.
For the final mark award **A0** for more than four solutions.

Note: For candidates who use $4\theta = \frac{\pi}{2}, \frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{3\pi}{2}$, award **M1A1A0A0**.

[4 marks]

(d) attempt to use binomial expansion (M1)

$$(1+z)^4 + (1-z)^4 = (1+4z+6z^2+4z^3+z^4) + (1-4z+6z^2-4z^3+z^4)$$

$$= 2 + 12z^2 + 2z^4 \quad \text{A1}$$

attempt to solve (for z^2) $z^4 + 6z^2 + 1 = 0$ using formula or completing the square (M1)

$$z^2 = -3 \pm 2\sqrt{2} \text{ or equivalent} \quad \text{A1}$$

$$\text{substituting } z = i \tan\left(\frac{\pi}{8}\right) \quad \text{M1}$$

$$-\tan^2\left(\frac{\pi}{8}\right) = -3 \pm 2\sqrt{2} \text{ OR } \tan^2\left(\frac{\pi}{8}\right) = 3 \pm 2\sqrt{2}$$

$$\tan\left(\frac{\pi}{8}\right) = \pm\sqrt{3 \pm 2\sqrt{2}} \quad \text{A1}$$

continued...

*Question 11 continued***EITHER**

$\tan\left(\frac{\pi}{8}\right)$ is the smallest positive root (hence take positive square root and negative sign inside)

R1**OR**

$$0 < \tan \frac{\pi}{8} < \tan \frac{3\pi}{8}$$

R1**THEN**

$$\tan\left(\frac{\pi}{8}\right) = \sqrt{3 - 2\sqrt{2}}$$

AG**[7 marks]****Total [20 marks]**

2. ${}^{10}P_3$ OR $10 \times 9 \times 8$ **OR** ${}^{10}C_3 \times 3!$ **OR** ${}^{10}C_7 \times 3!$ (= 720) (A1)

correct number of ways the remaining four members can be selected

${}^7C_4$ **OR** 35 (A1)

Note: The first two **A1** marks are to be awarded independently.

attempt to use the multiplication principle with their values (M1)

720×35

25 200 A1

Total [4 marks]

6. (a) $u_1 = \frac{53}{1000}$

$$r = \frac{1}{100}$$

$$\frac{\frac{53}{1000}}{1 - \frac{1}{100}}$$

(M1)(A1)

$$S_{\infty} = \frac{53}{990} (= 0.0\overline{53}) (= 0.0\dot{5}\dot{3})$$

A1

Note: Award (M1) for an attempt to use the sum to infinity formula.

[3 marks]

(b) $0.2 + \frac{53}{990}$

M1

$$\frac{251}{990}$$

A1

[2 marks]

Total [5 marks]

7. (a) attempt to solve using gdc or manual row operations or elimination (M1)

$$\begin{pmatrix} 1 & 0 & \frac{1}{2} & \frac{17}{4} \\ 0 & 1 & -\frac{3}{2} & \frac{3}{4} \end{pmatrix} \text{ OR } \begin{pmatrix} 6 & 18 & -24 & 39 \\ 6 & -2 & 6 & 24 \end{pmatrix} \text{ OR } \begin{cases} 6x + 18y - 24z = 39 \\ 6x - 2y + 6z = 24 \end{cases}$$

$$x = -\frac{1}{2}t + \frac{17}{4}, y = \frac{3}{2}t + \frac{3}{4}, z = t \quad (x = -0.5t + 4.25, y = 1.5t + 0.75, z = t) \text{ OR}$$

$$y = t, z = \frac{2}{3}t - \frac{1}{2}, x = \frac{9}{2} - \frac{1}{3}t \quad (y = t, z = 0.667t - 0.5, x = 4.5 - 0.333t) \text{ OR}$$

$$x = t, y = \frac{27}{2} - 3t, z = \frac{17}{2} - 2t \quad x = t, y = 13.5 - 3t, z = 8.5 - 2t \text{ or equivalent}$$

A1

[2 marks]

- (b) **METHOD 1**

attempt to use GDC or manual row operations or elimination to solve the 3 equations (M1)

$$x = 29, y = -73.5 \left(-\frac{147}{2} \right), z = -49.5 \left(-\frac{99}{2} \right)$$

A1

METHOD 2

attempt to substitute their parametric solution from a) into the third equation with $a = 3$ and $k = -3$ and solve to find the value of the parameter

(M1)

$$3 \left(-\frac{1}{2}t + \frac{17}{4} \right) + 12 \left(\frac{3}{2}t + \frac{3}{4} \right) - 16t = -3$$

$$x = 29, y = -73.5 \left(-\frac{147}{2} \right), z = -49.5 \left(-\frac{99}{2} \right)$$

A1

[2 marks]

- (c) (i) $a = 4$
(ii) $k = 26$

A1

A1

[2 marks]

Total [6 marks]

8. (a) $15!$ OR $1.31 \times 10^{12} (1.30767 \dots \times 10^{12})$ **A1**

[1 mark]

(b) recognise that the books need to be arranged in separate groups **(M1)**

$6!(720)$ OR $5!(120)$ OR $4!(24)$ seen

recognise that the three groups can be placed in any order **(M1)**

$3!$ seen

$(6! \times 5! \times 4! \times 3!) = 12441600$ **A1**

Note: Accept 12400000 or $6! \times 5! \times 4! \times 3!$.

[3 marks]

(c) recognition of the combination(s) required **(M1)**

${}^6C_4 (=15)$ OR ${}^5C_4 (=5)$ OR ${}^4C_4 (=1)$ seen

recognition that the sum of 3 different cases is required **(M1)**

${}^6C_4 + {}^5C_4 + {}^4C_4 = 15 + 5 + 1$

21 **A1**

[3 marks]

Total [7 marks]

9. assume $\sqrt{2xy} > \frac{1}{2}(2x + y)$

M1

Note: A statement of assumption of truth is required for example “assume”, “presume”, “suppose” or “let ... be true”.

Award **M0** for use of ‘let’ only or “consider”.

attempt to square both sides (which is valid since both x, y are positive)

M1

Note: This **M1** is independent of the previous method mark.

$$2xy > \frac{1}{4}(2x + y)^2$$

$$8xy > 4x^2 + 4xy + y^2$$

$$0 > 4x^2 - 4xy + y^2 \text{ OR } 0 > x^2 - xy + \frac{y^2}{4}$$

A1

$$0 > (2x - y)^2 \text{ OR } 0 > \left(x - \frac{y}{2}\right)^2$$

A1

a square number can never be negative hence contradiction

R1

Note: Do not award **A0R1**

therefore $\sqrt{2xy} \leq \frac{1}{2}(2x + y)$

AG**[5 marks]**

Section B