

APOLLO

Number and algebra SL

Mathematics: analysis and approaches

First assessment 2021

47 questions | questions

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving \$20 000. They each have \$9000 to invest.

- (a) Amelia invests her \$9000 in an account that offers an interest rate of 7% per annum compounded **annually**.
 - (i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars.
 - (ii) Determine the number of years required for Amelia's investment to reach the target. [5]
- (b) Bill invests his \$9000 in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

- (c) A third friend Chris also wants to reach the \$20 000 target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.
 - (i) Show that Chris will never reach the target if his initial deposit is \$9000.
 - (ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [8]



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(b) Find the maximum value of S_n . [3]

[illegible]

Section B

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8. [Maximum mark: 15]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0, a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

- (a) Show that $a = 8$. [2]
- (b) Write down an expression for $f^{-1}(x)$. [1]
- (c) Find the value of $f^{-1}(\sqrt{32})$. [3]
- (d) Consider the arithmetic sequence $\log_8 27, \log_8 p, \log_8 q, \log_8 125$, where $p > 1$ and $q > 1$.
 - (i) Show that $27, p, q$ and 125 are four consecutive terms in a geometric sequence.
 - (ii) Find the value of p and the value of q . [9]



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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Consider the series $\ln x + p \ln x + \frac{1}{3} \ln x + \dots$, where $x \in \mathbb{R}, x > 1$ and $p \in \mathbb{R}, p \neq 0$.

(a) Consider the case where the series is geometric.

(i) Show that $p = \pm \frac{1}{\sqrt{3}}$.

(ii) Given that $p > 0$ and $S_{\infty} = 3 + \sqrt{3}$, find the value of x . [5]

(b) Now consider the case where the series is arithmetic with common difference d .

(i) Show that $p = \frac{2}{3}$.

(ii) Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$.

(iii) The sum of the first n terms of the series is $-3 \ln x$.

Find the value of n . [10]

2. [Maximum mark: 5]

(a) State the value of the first term, u_1 . [1]

(b) Given that the n^{th} term of this sequence is -33 , find the value of n . [2]

(c) Find the common difference, d . [2]

[illegible]

(b) Prove that the sum of the squares of these three integers is never divisible by 3. [4]

[illegible]

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4. [Maximum mark: 5]

(a) Show that $a^2 + \left(\frac{a^2 - 1}{2}\right)^2 = \left(\frac{a^2 + 1}{2}\right)^2$. [3]

(b) Find an expression for the area of the triangle in terms of a . [2]

[illegible]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

Consider the arithmetic sequence $u_1, u_2, u_3, \dots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a) (i) Find the sum of the first five terms.

(ii) Given that $S_6 = 60$, find u_6 . [4]

(b) Find u_1 . [2]

(c) Hence or otherwise, write an expression for u_n in terms of n . [3]

Consider a geometric sequence, v_n , where $v_2 = u_1$ and $v_4 = u_6$.

(d) Find the possible values of the common ratio, r . [3]

(e) Given that $v_{99} < 0$, find v_5 . [2]



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2. [Maximum mark: 6]

(a) Write down the annual rate of depreciation of the car. [1]

(b) Find the value of the car on 1 January 2028. [2]

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra \$ M .

(c) Find the value of M . [3]

[illegible]

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[illegible]



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5. [Maximum mark: 7]

(a) Find an expression, in terms of h , for

(i) a ;

(ii) b ;

(iii) d .

[4]

(b) Given that a , b , and d are the first three terms of a geometric sequence, find the value of h .

[3]

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Section B

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8. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is \$4200.

Option B The first payment is \$1500. In each month which follows, the payment is 4% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(i) Option A;

(ii) Option B.

[5]

Sorin received an inheritance of \$160 000. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 5% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years.

[1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c) (i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month.

(ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least \$257 000.

[6]

At the end of the three years, Daniela invested \$30 000 for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to \$41 000 after six years.

[3]



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Section B

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8. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is \$5500.

Option B The first payment is \$2000. In each month which follows, the payment is 6% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(i) Option A;

(ii) Option B.

[5]

Sorin received an inheritance of \$120 000. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 4% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years.

[1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c) (i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month.

(ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least \$250 000.

[6]

At the end of the three years, Daniela invested \$40 000 for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to \$53 000 after six years.

[3]



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6. [Maximum mark: 5]

S_n is the sum of the first n terms of the sequence.

- (b) Hence, show that $S_1 + S_2 + S_3 + \dots + S_n = \frac{10(10^n - 1) - 9n}{81}$. [4]

[illegible]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 16]

Consider the arithmetic sequence $a, p, q, \dots$, where $a, p, q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence $a, s, t, \dots$, where $a, s, t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a .

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(i) arithmetic sequence;

(ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n .

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

(ii) Show that $\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]



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2. [Maximum mark: 7]

(a) Find the value of the car at the end of the first year. [2]

(b) Find the value of Darren's car 10 years after he buys it, giving your answer to the nearest dollar. [2]

(c) Find the least value of n . [3]

[illegible]

Find the value of k such that $u_k = 0$.

[illegible]

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3. [Maximum mark: 7]

- Write down the value of k . [1]
- Expand and simplify $(1 + x)^4$. [2]
- Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

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6. [Maximum mark: 8]

Picture frame F_1 has width 4 cm and height 5 cm.

(a) (i) Show that the area of picture frame F_n is $20\left(\frac{9}{4}\right)^{n-1}$ cm².

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the form $p\left(\left(\frac{9}{4}\right)^a - 1\right)\text{cm}^2$, where $p \in \mathbb{Q}^+$, $a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in the form $q\left(\frac{9}{4}\right)^4 \text{ cm}^2$, where $q \in \mathbb{Q}^+$. [3]

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Section B

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8. [Maximum mark: 16]

Consider the sequence $\{u_n\}$, with n th term given by u_n . The first three terms are

$$u_1 = k - 5, u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

(i) Find the value of k .

(ii) Hence, or otherwise, find u_3 . [5]

(b) Consider the case where $k = 12$.

(i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

(ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k .

(i) Show that $k^2 - 10k - 24 = 0$.

(ii) Find the first three terms of $\{u_n\}$ for this second value of k .

(iii) Hence, write down the value of S_{2m} , the sum of the first $2m$ terms, for this second value of k . [7]



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Section B

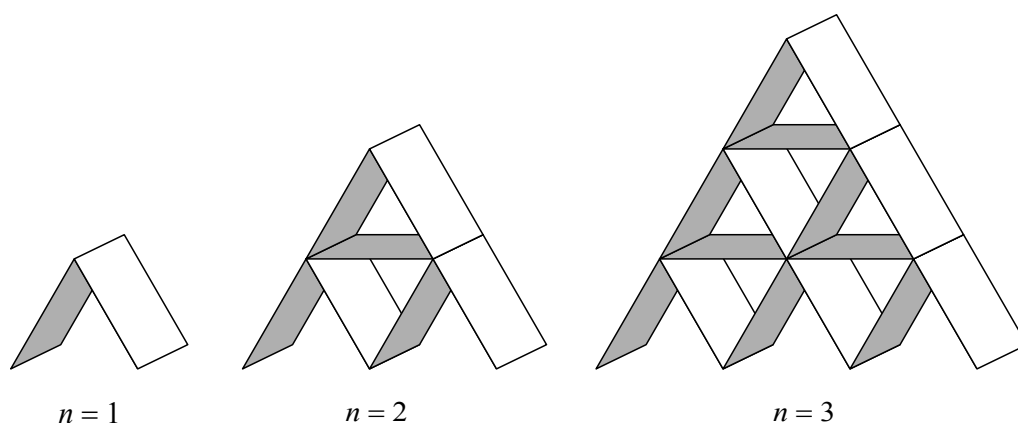
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8. [Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of 60° to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



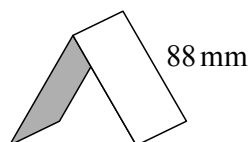
Let t_n represent the number of cards used to create a pyramid stack with n rows.

- (a) Write down t_3 . [1]
- (b) Find t_4 . [2]
- (c) Show that $t_n = \frac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

- (d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]
- (e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

The long edge of each playing card measures 88 mm as illustrated in the following diagram.



- (f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]



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Given that the coefficient of x^6 is 8.4×10^6 , find the possible values of k .

[illegible]

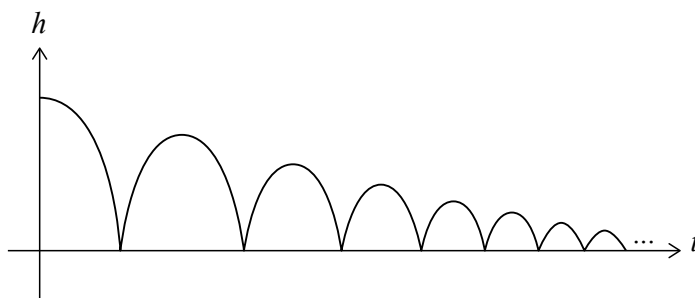
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Section B

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9. [Maximum mark: 14]

A tennis ball is dropped from a height. After each bounce the maximum height reached by the ball is $\frac{2}{3}$ of its previous maximum height. This can be seen in the diagram below where h , in metres, is the height of a ball after t seconds.



A box contains tennis balls. Each ball satisfies the condition of rebounding to $\frac{2}{3}$ of their previous maximum height. The tennis balls are numbered Ball 1, 2, 3, ...

Ball 1 is dropped from a height of 10 metres.

(a) Find the maximum height of Ball 1 after the 5th bounce. [3]

(b) Find the total distance travelled by Ball 1 immediately before the 5th bounce. [3]

Let δ be the total distance travelled by any of these balls.

(c) A ball is dropped from a height of x metres. Show that $\delta = 5x$ metres. [3]

Let δ_1 be the total distance travelled by Ball 1.

(d) Write down the value of δ_1 . [1]

Ball 2 is dropped from a height of 9.56 metres.

Let δ_2 be the total distance travelled by Ball 2, and so on for each ball in the box.

It is given that $\delta_1, \delta_2, \delta_3, \dots$ form an arithmetic sequence.

(e) Determine which tennis ball is the first ball to travel less than 25 metres. [4]



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The 1st and 5th terms of an arithmetic sequence are 36 and 12 respectively.

(a) Find the 13th term of this arithmetic sequence. [4]

The sum of the first n terms of this arithmetic sequence is zero.

(b) Find the value of n . [2]

[illegible]

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Andy and Jess each have \$5000.

Andy invests the money in a new savings plan that will pay interest at the end of each month.

Andy will receive a fixed amount of interest each month. The amount received is 0.315% of the initial investment.

- (a) (i) Determine the amount of interest Andy will receive at the end of each month. Give this answer correct to two decimal places.
- (ii) Hence, determine the amount of interest Andy will receive each year.
- (iii) Write down an expression in the form $5000 + pn$, where $p \in \mathbb{Z}^+$, for the amount of money Andy will have in his savings plan at the end of n years.
- (iv) Hence, show that Andy will have \$5945 in his savings plan at the end of 5 years. [4]

In this part, where appropriate, give all answers to the nearest dollar.

Jess invests her \$5000 in a new account that pays 3% interest compounded annually.

- (b) (i) Determine the amount of money that will be in Jess's account at the end of 5 years.
- (ii) Hence, find the amount of interest Jess will receive in the 5 years. [3]
- (c) (i) Write an expression in the form $5000 \times q^n$, where $q \in \mathbb{R}^+$, for the amount of money that Jess will have in her account at the end of n years.
- (ii) Hence, determine the smallest number of complete years that it will take for Jess to have more money in her account than Andy has in his savings plan. [5]



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